KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2025 Mathematics (L2FM) 2026

Unit 16 Tasks – Part 1

DO NOT WRITE INSIDE

KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2025 Mathematics (L2FM) 2026

Unit 16 Tasks – Part 2

DO NOT WRITE INSIDE

KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2025 Mathematics (L2FM) 2026

Unit 16 Tasks – Part 3

DO NOT WRITE INSIDE

Contents

1 Recurring Decimals
2 Parallel and Perpendicular Lines
3 Graphical Inequalities
4 Graphical Simultaneous Equations
5 Linear Simultaneous Equations
6 Non-Linear Graphs

1 Recurring Decimals

Intelligent Practice

Write the following out fully:

1) $0.\dot{5}$

2) $0.4\dot{5}$

3) $0.\dot{4}\dot{5}$

4) $0.3\dot{4}\dot{5}$

5) $0.\dot{3}4\dot{5}$

6) $0.2\dot{3}4\dot{5}$

7) $0.\dot{2}34\dot{5}$

8) $1.\dot{2}34\dot{5}$

Write the following using dot notation:

1) $0.666\ldots$

2) $0.7666\ldots$

3) $0.767676\ldots$

4) $0.8767676\ldots$

5) $0.876876876\ldots$

6) $0.9876876876\ldots$

7) $0.987698769876\ldots$

8) $10.987698769876\ldots$

Purposeful Practice

changing a fraction into a decimal

(1) work out what these fractions are as decimals by division (without using a calculator)

(a) $\frac{2}{5}$ (b) $\frac{3}{8}$ (c) $\frac{7}{8}$ (d) $\frac{4}{5}$ (e) $\frac{3}{25}$ (f) $\frac{7}{20}$ (g) $\frac{3}{80}$

why do you think these fractions as decimals terminate?

(2) work out what these fractions are as decimals by division (without using a calculator)

(a) $\frac{1}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{9}$ (d) $\frac{2}{9}$ (e) $\frac{1}{11}$ (f) $\frac{2}{11}$ (g) $\frac{1}{22}$

(3) work out what these fractions are as decimals by division (without using a calculator)

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{4}{7}$ (e) $\frac{5}{7}$ (f) $\frac{6}{7}$

2 × 7 = 14	6 × 7 = 42
3 × 7 = 21	7 × 7 = 49
4 × 7 = 28	8 × 7 = 56
5 × 7 = 35	9 × 7 = 63

what patterns can you find in the recurring decimal values of sevenths?

Purposeful Practice

changing a fraction into a decimal

(1) work out what these fractions are as decimals by division (without using a calculator)

(a) $\frac{1}{25}$ (b) $\frac{1}{40}$ (c) $\frac{1}{80}$ (d) $\frac{1}{16}$ (e) $\frac{1}{32}$

why do you think these fractions as decimals terminate?

(2) work out what these fractions are as decimals by division (without using a calculator)

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$ (e) $\frac{5}{13}$

$2 \times 13 = 26$ $6 \times 13 = 78$
$3 \times 13 = 39$ $7 \times 13 = 91$
$4 \times 13 = 52$ $8 \times 13 = 104$
$5 \times 13 = 65$ $9 \times 13 = 117$

(3) work out which fraction is bigger (with a calculator)

(a) $\frac{5}{9}$ or $\frac{6}{11}$

(b) $\frac{7}{8}$ or $\frac{17}{20}$

(c) $\frac{2}{11}$ or $\frac{3}{17}$

(d) $\frac{4}{17}$ or $\frac{15}{62}$

(4) without a calculator

$\frac{27}{40} = 0.675$

$\frac{29}{40} = 0.725$

what are:

(a) $\frac{31}{40}$ (b) $\frac{37}{40}$

(5) without a calculator

$\frac{29}{32} = 0.90625$

what are:

(a) $\frac{29}{320}$ (b) $\frac{290}{32}$

(c) $\frac{13}{32}$ (d) $\frac{21}{32}$

Fluency Practice

Ordering Recurring Decimals

A

0.678	0	•	6					
$0.\dot{6}7$	0	•	6					

[ ] < [ ]

B

0.252	0	•						
$0.\dot{2}$	0	•						

[ ] < [ ]

C

$0.\dot{5}\dot{6}$	0	•	5	6	5			
0.566	0	•						

[ ] < [ ]

D

$0.\dot{7}\dot{8}$	0	•			8			
$0.7\dot{8}$	0	•						8

[ ] < [ ]

E

$0.\dot{1}0\dot{2}$	0	•	1	0	2	1		
$0.1\dot{2}$	0	•				2		

[ ] < [ ]

F

$0.\dot{5}$	0	•						
$0.\dot{5}\dot{6}$	0	•						
$0.\dot{5}6\dot{5}$	0	•	5	6	5	5		

[ ] < [ ] < [ ]

G

$0.\dot{0}8\dot{1}$		•			0			
$0.0\dot{8}\dot{1}$		•	0	8				
$0.08\dot{1}$		•				1		

[ ] < [ ] < [ ]

H

$0.13\dot{1}$		•						1
$0.\dot{1}3\dot{1}$		•			1			
$0.\dot{1}\dot{3}$		•				1		

[ ] < [ ] < [ ]

I

$9.\dot{7}9\dot{8}$		•						
$9.\dot{7}9\dot{8}$		•						
$9.\dot{7}98\dot{7}$		•						

[ ] < [ ] < [ ]

By writing the denominator as a product of its prime factors, decide if each of these fractions would convert to a terminating or recurring decimal.

(a) $\frac{1}{8}$ (b) $\frac{1}{25}$ (c) $\frac{1}{15}$

(d) $\frac{1}{14}$ (e) $\frac{1}{50}$ (f) $\frac{1}{16}$

(g) $\frac{1}{30}$ (h) $\frac{1}{12}$ (i) $\frac{1}{40}$

Write out the following recurring decimals to show the first 10 decimal places.

(a) $0.\dot{4}$ (b) $0.\dot{7}$

(c) $0.\dot{1}\dot{4}$ (d) $0.\dot{2}\dot{3}$

(e) $0.\dot{1}2\dot{3}$ (f) $0.\dot{4}6\dot{1}$

(g) $0.0\dot{5}$ (h) $0.1\dot{7}\dot{2}$

Use your calculator to convert the following fractions into terminating or recurring decimals.

(a) $\frac{4}{9}$ (b) $\frac{2}{5}$ (c) $\frac{3}{10}$

(d) $\frac{7}{11}$ (e) $\frac{5}{16}$ (f) $\frac{1}{8}$

(g) $\frac{4}{7}$ (h) $\frac{29}{100}$ (i) $\frac{3}{35}$

Use your calculator to convert the following fractions into recurring decimals.

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$ (c) $\frac{3}{9}$

Can you spot a pattern?

Use your calculator to convert the following fractions into recurring decimals.

(a) $\frac{12}{99}$ (b) $\frac{13}{99}$ (c) $\frac{14}{99}$

Can you spot a pattern?

Fluency Practice

For each of the following values of x, find

a) $10x$
b) $100x$
c) $1000x$
d) $2x$
e) $20x$

1) $x = 0.\dot{1}$
2) $x = 0.0\dot{1}$
3) $x = 0.00\dot{1}$
4) $x = 0.\dot{2}$
5) $x = 0.\dot{3}$
6) $x = 0.\dot{2}\dot{5}$
7) $x = 0.\dot{7}\dot{2}$
8) $x = 0.0\dot{2}\dot{5}$
9) $x = 0.1\dot{2}\dot{5}$
10) $x = 1.34\dot{2}\dot{4}$
11) $x = 0.\dot{2}2\dot{4}$
12) $x = 0.0\dot{2}2\dot{4}$
13) $x = 0.2\dot{2}2\dot{4}$
14) $x = 1.01\dot{1}0\dot{1}$

Given that $0.333... = \frac{1}{3}$

Write these as fractions:

a) 1.333…

b) 0.4333…

c) 0.35333…

d) 3.333…

Given that $\frac{5}{17} = 0.\dot{2}941176470588235\dot{}$, **write down** the following fractions as recurring decimals.

a) $\frac{12}{17}$

b) $\frac{1}{34}$

c) $\frac{27}{34}$

Which if the following fractions is equivalent to a recurring decimal?

(a) $\frac{7}{10}$ (b) $\frac{7}{9}$ (c) $\frac{7}{100}$

(d) $\frac{7}{11}$ (e) $\frac{7}{20}$ (f) $\frac{7}{30}$

Using an algebraic method, write the following recurring decimals as a fraction.

(a) $0.\dot{4}$ (b) $0.\dot{8}$

(c) $0.\dot{1}\dot{3}$ (d) $0.\dot{4}\dot{5}$

(e) $0.\dot{5}\dot{7}$ (f) $0.\dot{4}1\dot{2}$

(g) $0.\dot{1}2\dot{7}$ (h) $0.\dot{6}7\dot{5}$

Using an algebraic method, write the following recurring decimals as a fraction.

(a) $0.0\dot{4}$ (b) $0.0\dot{6}$

(c) $0.2\dot{3}$ (d) $0.1\dot{6}$

(e) $0.2\dot{1}\dot{7}$ (f) $0.00\dot{4}\dot{5}$

(g) $0.0\dot{1}5\dot{5}$ (h) $0.3\dot{6}9\dot{5}$

Use an algebraic method to show that:

(a) $0.\dot{1}\dot{5} = \frac{5}{33}$

(b) $0.\dot{1}4\dot{4} = \frac{16}{111}$

(c) $0.7\dot{1} = \frac{32}{45}$

Using an algebraic method, find $0.\dot{9}$ as a fraction.

Fluency Practice

Q	Express as a simplified fraction	Answers
1	0.222 …	
2	0.222	
3	$0.\dot{2}\dot{3}$	
4	0.23	
5	$0.\dot{2}0\dot{3}$	
6	0.203	
7	$0.02\dot{0}\dot{3}$	
8	0.0203	
9	$2.4\dot{6}0\dot{5}$	
10	2.4605	

Purposeful Practice

A1 State the conditions under which a fraction can be written as a **terminating** decimal.	**A2** State the conditions under which a fraction can be written as a **recurring** decimal.	**A3** Which of the following can be written as **terminating** decimals: $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{9}$ $\frac{5}{6}$ $\frac{5}{8}$ $\frac{3}{7}$ $\frac{3}{5}$	**A4** Which of the following can be written as **recurring** decimals: $\frac{5}{12}$ $\frac{7}{25}$ $\frac{3}{14}$ $\frac{5}{16}$ $\frac{5}{32}$ $\frac{5}{11}$
B1 Show that $0.\dot{5}=\frac{5}{9}$	**B2** Show that $0.7\dot{3}=\frac{11}{15}$	**B3** Show that $0.61\dot{6}=\frac{37}{60}$	**B4** Show that $3.5\dot{2}=3\frac{47}{90}$
C1 Show that $0.\dot{2}\dot{7}=\frac{3}{11}$	**C2** Show that $0.2\dot{5}\dot{7}=\frac{17}{66}$	**C3** Show that $0.\dot{4}4\dot{7}=\frac{149}{333}$	**C4** Show that $2.5\dot{1}=2\frac{17}{33}$
D1 Work out $0.2\dot{7}\times 3$, writing your answer as a fraction in its simplest terms.	**D2** Work out $0.5\dot{7}-0.2\dot{6}$, writing your answer as a fraction in its simplest terms.	**D3** x is a whole number such that $1\le x\le 9$ Write the recurring decimal $0.1\dot{x}$ as a fraction in its simplest terms.	**D4** y is a whole number such that $1\le y\le 9$ Show that $0.\dot{3}\dot{y}=\frac{y}{33}$

Problem Solving

These decimals have one digit recurring:

$$0.\dot{2} = 0.22222222 \ldots \qquad 0.07\dot{2} = 0.07222222 \ldots$$

1) Write three different decimals with one digit recurring.

2) Convert you decimals in part one into fractions in their simplest form. Use the algebraic method.

3) Find five fractions with different denominators when in simplest form that have one digit recurring.

4) Find the prime factors of the denominators. What do you notice?

5) Find five fractions with different denominators when in simplest form that have two digits recurring.

6) Find the prime factors of the denominators. What do you notice?

7) Repeat 5 and 6 for 3 digits recurring.

8) Investigate for other decimals with different number of digits recurring.

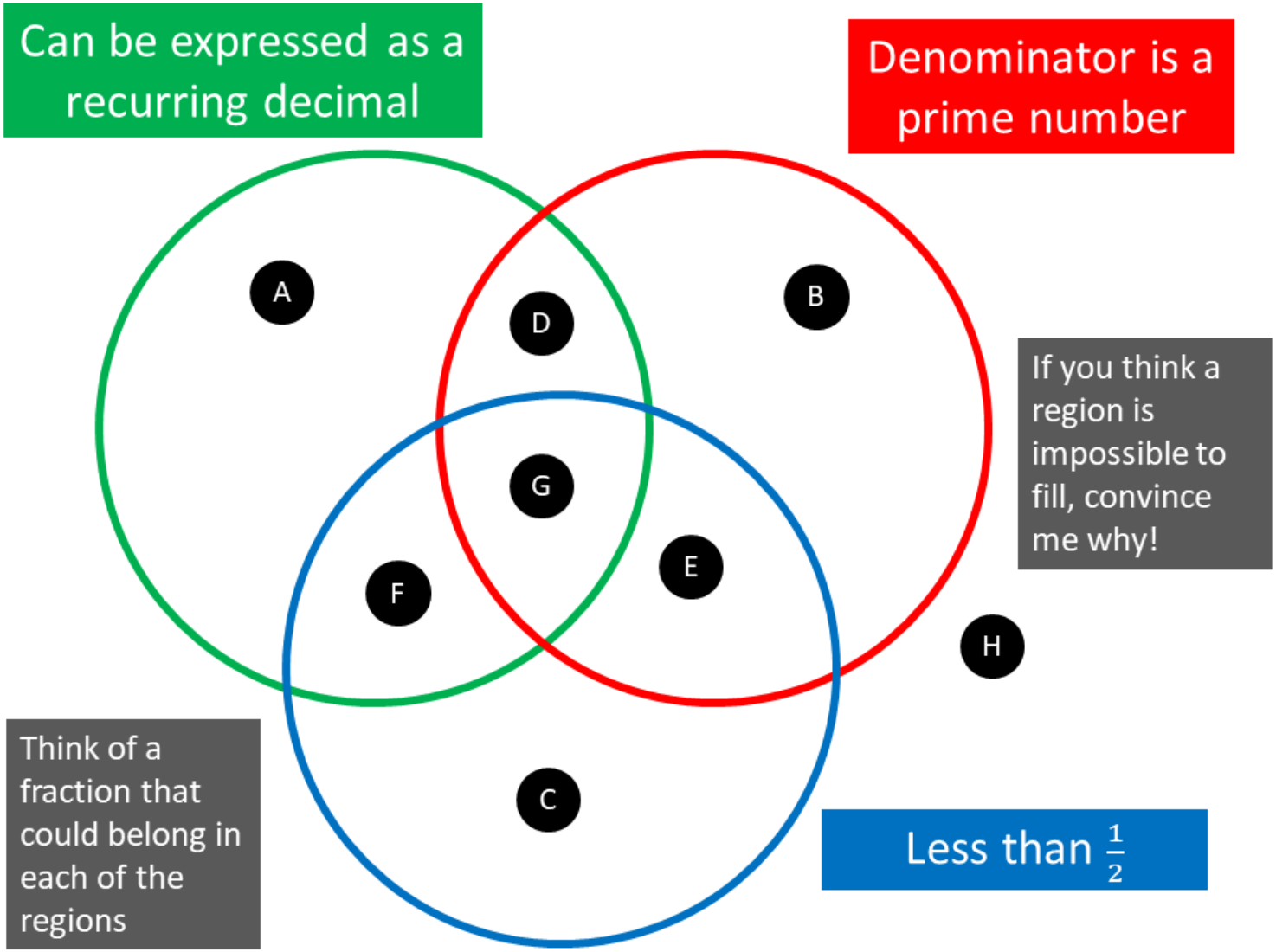
Purposeful Practice

Instructions: Convert the decimal in the middle box to a fraction, giving your answer in its simplest form. Then fill in the remaining boxes, making the minimum change possible from the middle box.

Number of digits which recur when expressed as a decimal

Difference between numerator and denominator when expressed as a fraction in its simplest form		Less	Same	More
	More			
	Same		$0.\dot{3}\dot{6}$	
	Less			

Problem Solving



Prove each of the following sums using recurring decimals.

Prove that $0.\dot{1} \times 0.\dot{2}\dot{7} = \frac{1}{33}$

Prove that $0.\dot{5} \times 0.\dot{8}\dot{1} = \frac{5}{11}$

Prove that $0.\dot{1}\dot{8} \div 0.\dot{4} = \frac{9}{22}$

Prove that $0.\dot{8} \div 0.\dot{7}\dot{2} = 1\frac{2}{9}$

Prove that $0.1\dot{2}\dot{6} \div 0.\dot{2} = \frac{5}{8}$

Prove that $0.19\dot{4} \div 0.\dot{4} = \frac{7}{16}$

(a) Using algebra, show that

$$0.\dot{2} + 0.\dot{2}\dot{3} = \frac{5}{11}$$

(b) Using algebra, show that

$$1.3\dot{8}\dot{1} - 0.\dot{8}\dot{1} = \frac{31}{55}$$

(a) Using algebra, show that

$$0.\dot{5} \times 0.\dot{5}\dot{4} = \frac{10}{33}$$

(b) Using algebra, show that

$$4 \times 0.8\dot{5} \times 0.\dot{1}\dot{5} = \frac{14}{27}$$

(a) Using algebra, show that

$$0.\dot{7} \div 0.2\dot{1} = 3\frac{13}{19}$$

(b) Using algebra, show that

$$0.3\dot{5} \div 1.2\dot{7} = \frac{32}{115}$$

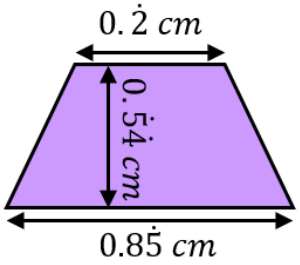
(a) Show that the mean of the three numbers

$$0.\dot{8}, 0.8\dot{1} \text{ and } 0.\dot{8}\dot{1}$$

can be written in its simplest form as a fraction $\frac{a}{b}$, where a and b are integers to be found.

(b) Using algebra, prove that the area of the trapezium shown is

$$\frac{97}{330}\ cm^2$$



Question 1: Use division to convert these fractions to recurring decimals.

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{4}{9}$ (d) $\frac{7}{9}$ (e) $\frac{1}{6}$ (f) $\frac{5}{6}$

(g) $\frac{3}{11}$ (h) $\frac{8}{15}$ (i) $\frac{5}{22}$ (j) $\frac{1}{7}$ (k) $\frac{1}{30}$ (l) $\frac{6}{7}$

Question 2: Convert the following recurring decimals to fractions.
Give each answer in its simplest form.

(a) 0.5555... (b) 0.1111... (c) 0.121212...

(d) 0.363636... (e) 0.919191... (f) 0.727272...

(g) 0.125125... (h) 0.621621... (i) 0.204204...

Question 3: Convert the following recurring decimals to fractions.
Give each answer in its simplest form.

(a) $0.\dot{2}$ (b) $0.\dot{8}$ (c) $0.\dot{1}\dot{8}$

(d) $0.\dot{5}\dot{3}$ (e) $0.\dot{7}\dot{5}$ (f) $0.\dot{6}\dot{3}$

(g) $0.\dot{1}1\dot{2}$ (h) $0.\dot{3}3\dot{9}$ (i) $0.\dot{1}7\dot{1}$

Question 4: Convert the following recurring decimals to fractions.
Give each answer in its simplest form.

(a) 0.53333... (b) 0.26666... (c) 0.08888...

(d) 0.1353535... (e) 0.4505050... (f) 0.9121212...

(g) 0.0152152... (h) 0.123333... (i) 0.354141414...

Question 5: Convert the following recurring decimals to fractions.
Give each answer in its simplest form.

(a) $0.2\dot{8}$ (b) $0.0\dot{3}$ (c) $0.\dot{9}\dot{6}$ (d) $0.5\dot{2}\dot{1}$

(e) $0.3\dot{9}\dot{0}$ (f) $0.1\dot{2}3\dot{5}$ (g) $0.12\dot{6}$ (h) $0.5\dot{0}3\dot{5}$

Question 6: Convert the following recurring decimals to fractions.
Give each answer in its simplest form.

(a) 1.555... (b) 1.45454545... (c) 1.24444...

(d) 2.0717171... (e) 1.3599999... (f) 3.8123123...

Question 7: Convert the following recurring decimals to fractions.
Give each answer in its simplest form.

(a) $1.\dot{2}$ (b) $1.\dot{6}\dot{4}$ (c) $1.9\dot{2}$

(d) $2.0\dot{3}$ (e) $3.6\dot{5}\dot{9}$ (f) $8.6\dot{7}\dot{9}$

Apply

Question 1: Work out the following addition.
Give your answer as a simplified fraction

$$0.\dot{5} + 0.\dot{2}\dot{1}$$

Question 2: Work out the following
Give your answer as a simplified fraction

$$0.\dot{2}\dot{7} + 0.\dot{6}\dot{4} \div 0.5\dot{3}$$

Purposeful Practice

Question 3: Arrange in order from smallest to largest.

$$\frac{61}{330} \qquad 0.1\dot{7}\dot{8} \qquad 3^{-2} \qquad \frac{19}{110}$$

Question 4: Mark is converting $0.2\dot{5}\dot{1}$ into a fraction.

Can you spot any mistakes?

$$\begin{aligned} x &= 0.2\dot{5}\dot{1} \\ x &= 0.2515151\ldots \\ 10x &= 2.515151\ldots \\ 100x &= 251.515151\ldots \\ 90x &= 249 \\ x &= \frac{249}{90} \end{aligned}$$

In each question, the variables x and y represent a single digit from 1 to 9. Give all your answers in their lowest terms.

1. Convert $0.\dot{x}$ to a fraction.
2. Convert $0.3\dot{x}$ to a fraction.
3. Convert $0.x\dot{5}$ to a fraction.
4. Convert $0.\dot{7}\dot{x}$ to a fraction.
5. Convert $0.\dot{x}\dot{y}$ to a fraction.
6. Work out $0.\dot{x}\dot{y} - 0.\dot{y}\dot{x}$, giving your answer as a fraction.
7. $0.\dot{x}\dot{y} - 0.\dot{y}\dot{x} = 0.\dot{5}\dot{4}$. How many possible pairs of values of x and y are there?
8. The fraction $\frac{1}{n}$, where n is a positive integer, is converted to a decimal. What values of n will give a terminating decimal? [Hint – use your calculator to try various values of n]
9. How many fractions of the form $\frac{1}{n}$, where n is an integer such that $1 \leq n \leq 30$, give a terminating decimal?
10. Giving your answer as a fraction, work out the value of

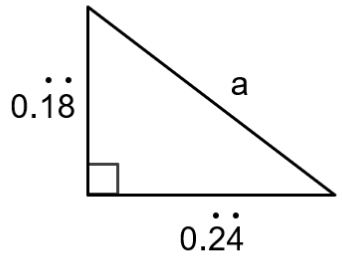
$$\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \cdots$$

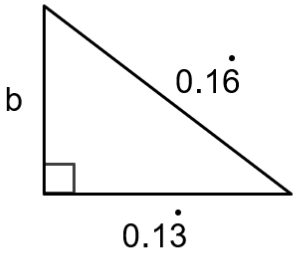
Fill in each box in the table with either "always recurring", "always terminating", or "sometimes recurring, sometimes terminating". Provide an example for each case.

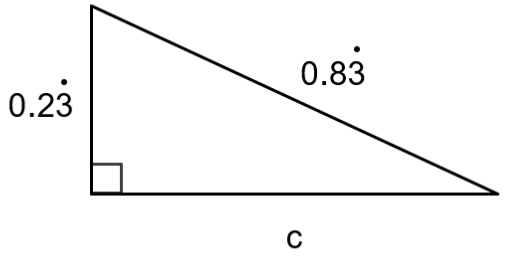
a	b	$a \times b$	$a + b$	$a \div b$
Recurring	Recurring			
Recurring	Terminating			
Terminating	Terminating			

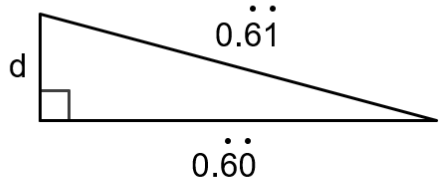
Fluency Practice

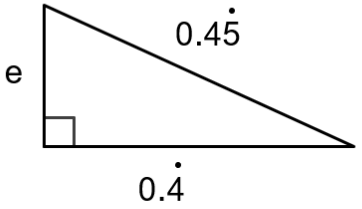
find the length of the missing side as a fraction in its simplest form

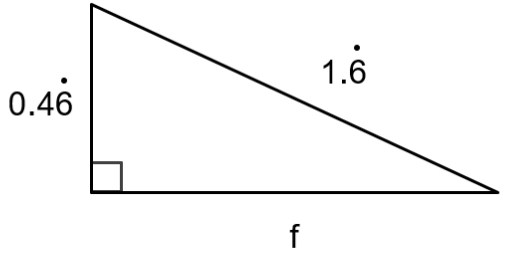


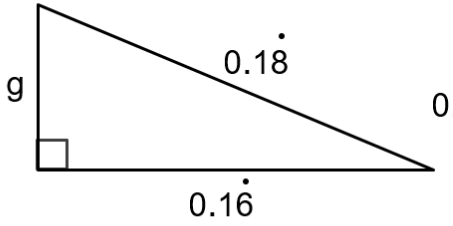


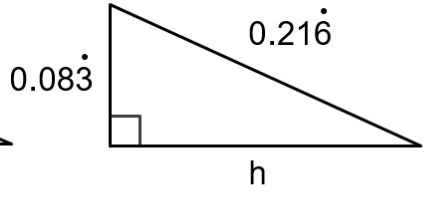


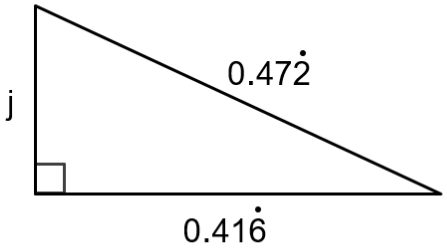






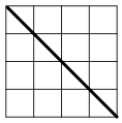
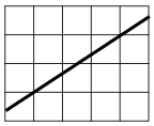






2 Parallel and Perpendicular Lines

Decide whether each pair of lines is parallel or not parallel.

1	The line with equation $y = -2x + 9$	The line with equation $10 - 2x = y$	Parallel / Not Parallel
2	The line with equation $y = 4x + 3$	The line with equation $y = 3 - 4x$	Parallel / Not Parallel
3	The line with equation $x - 1 = y$	The line shown on the grid 	Parallel / Not Parallel
4	The line with equation $y = 2x - 4$	The line with equation $3y = 9 + 6x$	Parallel / Not Parallel
5	The line with equation $\frac{1}{2}x - 4 = y$	The line with equation $2y = 8 + x$	Parallel / Not Parallel
6	The line shown on the grid 	The line with equation $2y = 3x - 11$	Parallel / Not Parallel
7	The line that passes through $(0, 0)$ and $(3, 9)$	The line with equation $y - 3x = 12$	Parallel / Not Parallel
8	The line with equation $5x + 2y = 20$	The line with equation $20 - \frac{5}{2}x = y$	Parallel / Not Parallel
9	The line with equation $2y + x - 8 = 0$	The line that passes through $(-1, 3)$ and $(2, -3)$	Parallel / Not Parallel
10	The line with table x: 2, 4, 6 y: 9, 6, 3	The line with equation $6x - 4y = 12$	Parallel / Not Parallel

Intelligent Practice

Write down the equation:

Parallel to the line	Goes through the point	Answer
$y = 3x + 2$	$(0,7)$	
$y = 3x + 2$	$(3,0)$	
$y = 3x + 2$	$(3,7)$	
$y = 3x + 10$	$(3,7)$	
$y = 2x - 2$	$(3,7)$	
$y = 2x - 2$	$(-3,7)$	
$y = 2x - 2$	$(0,0)$	

Decide whether each of these pairs of straight lines is parallel or not parallel:

(a) $y = 2x + 7$ and $y = 2x - 5$

(b) $y = 3x + 4$ and $y = 5x + 4$

(c) $y = 5x - 3$ and $y = 5x$

(d) $y = -4x + 1$ and $y = 4x + 2$

(e) $y = \frac{1}{2}x - 8$ and $y = 9 + \frac{1}{2}x$

(f) $y = -5 + 2x$ and $y = 5 - 2x$

(a) Write down the equation of the straight line that is parallel to $y = 4x - 1$ and passes through $(0, 5)$

(b) Write down the equation of the straight line that is parallel to $y = -2x + 7$ and passes through $(0, 3)$

(c) Write down the equation of the straight line that is parallel to $y = \frac{3}{4}x - 2$ and passes through $(0, -8)$

(d) Write down the equation of the straight line that is parallel to $y = \frac{7}{2}x + \frac{1}{2}$ and passes through the origin

(a) Write down the equation of the straight line that is parallel to $y = 1 - 3x$ and passes through $(0, -2)$

(b) Write down the equation of the straight line that is parallel to $y - 4x = 1$ and passes through $(0, -\frac{5}{2})$

(c) Write down the equation of the straight line that is parallel to $3x + y - 5 = 0$ and passes through $(0, 1)$

Match the pairs of parallel lines:

$y = -7x + 3$	$y + 3x = 7$
$7 + 3x = y$	$7x + y + 3 = 0$
$7y = 7 - 21x$	$y = 3x$

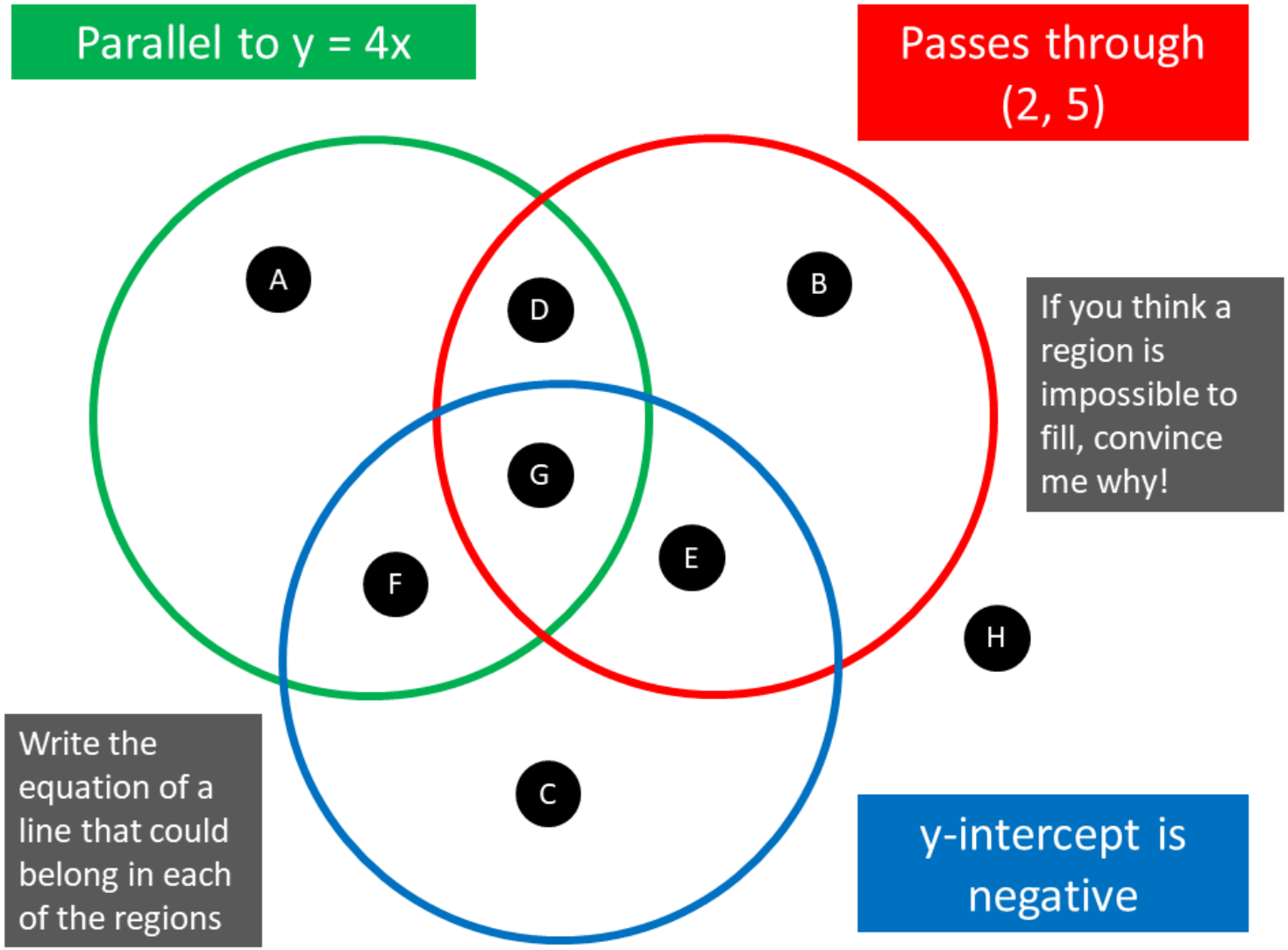
Purposeful Practice

Line A passes through the points (2, 1) and (5, 10)
Find the equation of the line parallel to A that passes through (2,5) [3]

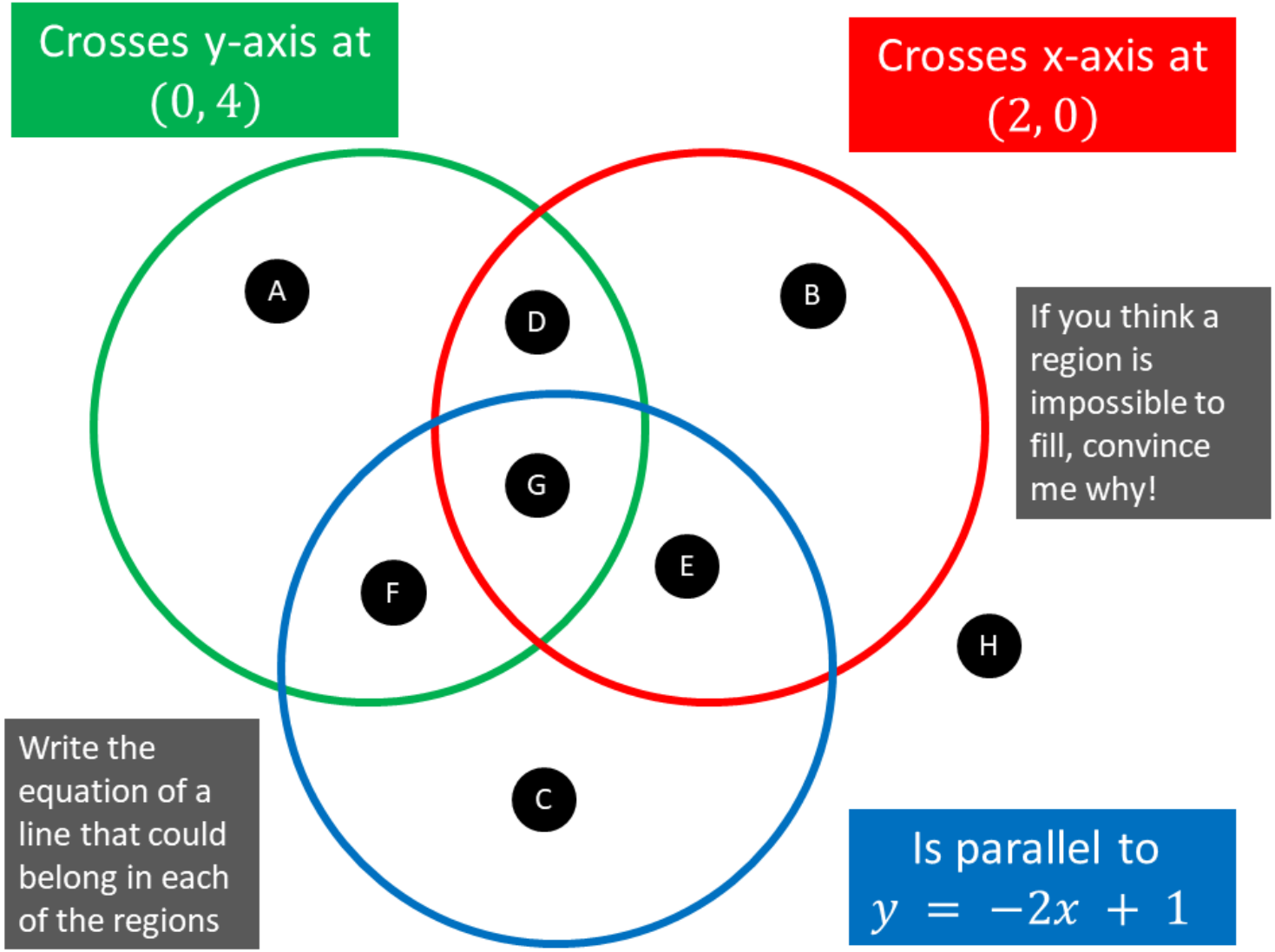
Line A passes through the points (2, 1) and (5, 10)
Line B passes through the points (4, 7) and (2, 1)
Show that Line A and Line B are parallel [4]

Line A passes through the points (3, 6) and (5, -2)
Line B passes through the points (2, 5) and (8, k)
Line A and Line B are parallel. Find the value of k. [4]

Problem Solving



Problem Solving



Fluency Practice

Q	Work out the:	Answers
1	reciprocal of 0.6	
2	negative reciprocal of $\frac{5}{3}$	
3	reciprocal of 1.4	
4	negative reciprocal of $1\frac{4}{5}$	
5	reciprocal of $-\frac{9}{5}$	
6	negative reciprocal of $\frac{5}{9}$	
7	reciprocal of 0.35	
8	negative reciprocal of $-2\frac{6}{7}$	
9	product of the reciprocal and negative reciprocal of x	
10	sum of the reciprocal and negative reciprocal of x	

Intelligent Practice

Write down the equation:

Perpendicular to the line	Goes through the point	Answer
$y = -x + 2$	$(0,7)$	
$y = 2x + 2$	$(0,7)$	
$y = 2x + 2$	$(12,7)$	
$y = 3x + 2$	$(12,7)$	
$y = 3x + 2$	$(10,7)$	
$y = 3x + 2$	$(-12,7)$	
$y = 2x + 2$	$(0,0)$	

Find the equation of the line that has:

(a) Gradient 2 and goes through (0, 5)

(b) Gradient -3 and goes through (0, 7)

(c) Gradient $\frac{2}{3}$ and goes through (0, 4)

(d) Gradient -4 and goes through (0, -1)

(e) Gradient 1 and goes through (0, -6)

Find the equation of the line that is:

(a) Parallel to the line $y = 4x + 7$ and passes through (0, 2)

(b) Parallel to the line $y = -2x + 4$ and passes through (0, 6)

(c) Parallel to the line $y = 3x + 1$ and passes through (0, -4)

(d) Parallel to the line $y = x + 6$ and passes through (0, 5)

(e) Parallel to the line $y = \frac{1}{2}x + 3$ and passes through (0, -1)

Find the equation of the line that is:

(a) Perpendicular to the line $y = 2x + 5$ and passes through (0, 7)

(b) Perpendicular to the line $y = \frac{1}{3}x + 4$ and passes through (0, -5)

(c) Perpendicular to the line $y = -5x + 1$ and passes through (0, 2)

(d) Perpendicular to the line $y = -\frac{1}{4}x + 5$ and passes through (0, -4)

(e) Perpendicular to the line $y = 3x - 1$ and passes through (0, 3)

Match the pairs of perpendicular lines.

$x = 6$ $\quad x + y = 5$ $\quad y = 8x - 9$

$2y = x + 4$ $\quad 2x + y = 9$ $\quad y = -\frac{1}{8}x + 6$

$5y = 2x + 15$ $\quad y = 0.1x + 2$ $\quad y = -2$

$y = 33 - 10x$ $\quad 2y + 5x = 2$ $\quad y = x + 4$

Fluency Practice

Equation of line l_1	Two points that l_1 passes through.		Equation of a line perpendicular to l_1, through point A, in the form $ax + by + c = 0$
	Point A	Point B	
$2x + 3y - 5 = 0$	$(-5, a)$ a	$(b, -5)$ $b =$	
	$(-4,1)$	$(-3, -2)$	
	$(a, 1)$ $a =$	$(b, 3)$ $b =$	$x + 3y - 3 = 0$
$-x + 6y + 17 = 0$		$(b, -2)$ $b =$	$6x + y - 65 = 0$

Purposeful Practice

Write down the equation of a line perpendicular to $y = 3x + 3$ [1]

Write down the equation of the line perpendicular to $y = \frac{1}{2}x + 4$ which passes through (0,7) [2]

Line A passes through the points (-3, -1) and (-1, 9).
Line B passes through the points (-2, 1) and (k, 4).
Line A and Line B are perpendicular.
Find the value of k. [4]

In each question you will need to use the formula $y=mx+c$.

Find the equation of the line perpendicular to the line $y=2x+3$ and passes through the point (4,11)

Where does the line cross the x and y axis?

Find the equation of the line perpendicular to the line $y=3x-5$ and passes through the point (3,4)

Where does the line cross the x and y axis?

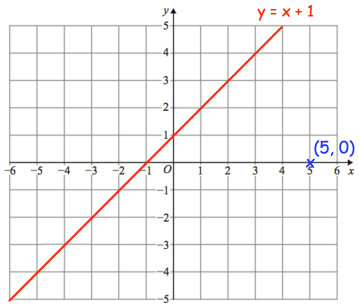
Find the equation of the line perpendicular to the line $y=-2x+8$ and passes through the point (6,-4)

Find the equation of the line perpendicular to the line $y=-3x+5$ and passes through the point (6,-13)

Find the equation of the line perpendicular to the line $y=\frac{1}{2}x+4$ and passes through the point (6,7)

Find the equation of the line perpendicular to the line $y=5x-9$ and passes through the point (10,41)

Question 1: Shown on the grid is the line, $y = x + 1$, and the point (5, 0)



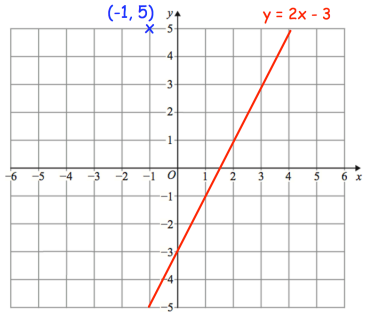
(a) Find the gradient of a line perpendicular to $y = x + 1$

(b) Find the equation of the line perpendicular to $y = x + 1$ that passes through the point (5, 0)

(c) Find the point where your answer to (b) and the line $y = x + 1$ intersect.

(d) Calculate the distance between your answer to (c) and the point (5, 0)

Question 2: Shown on the grid is the line, $y = 2x - 3$, and the point (−1, 5)



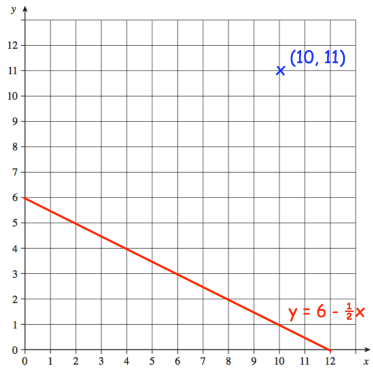
(a) Find the gradient of a line perpendicular to $y = 2x - 3$

(b) Find the equation of the line perpendicular to $y = 2x - 3$ that passes through (−1, 5)

(c) Find the point where your answer to (b) and the line $y = 2x - 3$ intersect.

(d) Calculate the distance between your answer to (c) and the point (−1, 5)

Question 3: Shown on the grid is the line, $y = 6 - \frac{1}{2}x$ and the point (10, 11)



(a) Find the equation of the line perpendicular to $y = 6 - \frac{1}{2}x$ that passes through (10, 11).

(b) Find where your answer to (a) intersects $y = 6 - \frac{1}{2}x$

(c) Calculate the shortest distance between the (10, 11) and the line $y = 6 - \frac{1}{2}x$

Question 4: Calculate the shortest distances between the following lines and points.

(a) The line $y = -x + 2$ and the point (8, 4)

(b) The line $y = 4x + 7$ and the point (−13, 6)

(c) The line $x - 3y + 9 = 0$ and the point (5, 38)

Question 5: Calculate the shortest distances between the following lines and points.

(a) The line $y = 5x + 1$ and the point (3, 3)

(b) The line $y = -2x - 15$ and the point (−1, −4)

(c) The line $2x + y + 5 = 0$ and the point (0, 0)

Apply

Question 1: The line L_1 has equation $y = 2x - 4$
The line L_2 has equation $y = -3x + 11$
The line L_3 has equation $y = x$
L_1 and L_2 meet at the point A.

Work out the shortest distance between the line L_3 and the point A.

Question 2: ABC is a triangle.
A is the point (−5, 2)
B is the point (0, 8)
C is the point (9, 3)

Work out the area of triangle ABC.

PAR. or PERP. ?

For lines 1-10, which line is **PARALLEL** and which is **PERPENDICULAR**?

L_1	$y = 5 - x$	$2y = 4x - 6$	$y + x = 3$	$y = x + 5$
L_2	$y = 3x + 5$	$y = 6 - \frac{x}{3}$	$y = 5x - 3$	$y = 5 + 3x$
L_3	$y = 4 - 2x$	$y + 2x = 7$	$y = 2x - 3$	$2y = x - 3$
L_4	$x + y = 7$	$2y = 5 - x$	$y = \frac{x}{2} - 6$	$x + y = 6$
L_5	$2y = 4x - 5$	$y - 2x = 8$	$y - 3x = 9$	$y = 1 - \frac{1}{2}x$
L_6	$y - 3x = 8$	$3y = 6 - x$	$2y - 6x = 10$	$6x + 3y = 7$
L_7	$2x + y = 4$	$4y - 6x = 9$	$y = \frac{x}{2} - 3$	$4x + 2y = 7$
L_8	$2y = x - 10$	$4y = 2x + 7$	$3y - x = 6$	$2y = 10 - 4x$
L_9	$4y + x = 16$	$3y - 12x = 3$	$8y = 4 - 2x$	$2y + 4x = -3$
L_{10}	$4y - 3x = 12$	$8y = 12 - 6x$	$6y + 8x = 5$	$8 = 3x - 4y$

Fluency Practice

Q	Find the equation of the line that is:	Answers
1	Parallel to $y = 2x + 5$ and passes through $(4, 9)$	
2	Perpendicular to $y = 2x + 5$ and passes through $(4, 9)$	
3	Parallel to $y = 3x - 4$ and passes through $(9, 4)$	
4	Perpendicular to $y = 3x - 4$ and passes through $(9, 4)$	
5	Parallel to $y = \frac{1}{2}x + 5$ and passes through $(4, 9)$	
6	Perpendicular to $y = \frac{1}{2}x + 5$ and passes through $(4, 9)$	
7	Parallel to $y = -\frac{1}{3}x - 4$ and passes through $(-12, 1)$	
8	Perpendicular to $y = -\frac{1}{3}x - 4$ and passes through $(-12,1)$	
9	Parallel to $2x + 3y = 5$ and passes through $(12, -5)$	
10	Perpendicular to $2x + 3y = 5$ and passes through $(12, -5)$	
11	Parallel to $3x - 5y = 2$ and passes through $(-15, 2)$	
12	Perpendicular to $3x - 5y = 2$ and passes through $(-15, 2)$	

Fluency Practice

Q	Find the equation of the line that is:	Answers
1	Parallel to the line connecting $(2, 5)$ and $(7, 15)$ and passes through $(1, -4)$	
2	Perpendicular to the line connecting $(2, 5)$ and $(7, 15)$ and passes through $(4, -1)$	
3	Parallel to the line connecting $(2, 5)$ and $(7, -15)$ and passes through $(1, -4)$	
4	Perpendicular to the line connecting $(2, 5)$ and $(7, -15)$ and passes through $(4, -1)$	
5	Parallel to the line connecting $(-2, 5)$ and $(-7, -15)$ and passes through $(1, -4)$	
6	Perpendicular to the line connecting $(2, -5)$ and $(7, -25)$ and passes through $(4, 1)$	
7	Perpendicular to the line connecting $(2, 5)$ and $(8, 17)$ and passes through its midpoint	
8	Perpendicular to the line connecting $(1, 4)$ and $(7, 22)$ and passes through its midpoint	

Decide whether each of these pairs of lines is perpendicular, parallel or neither:

(a) $y = 2x - 1$ and $y = -\frac{1}{2}x + 5$

(b) $y = \frac{1}{3}x + 2$ and $y = \frac{1}{3}x - 4$

(c) $y = 1 - 4x$ and $y = -\frac{1}{4}x + \frac{3}{4}$

(d) $y = \frac{2}{3}x$ and $y = -\frac{3}{2}x - 6$

(a) Write down the equation of the straight line that is perpendicular to the line $y = -3x + 1$ and passes through $(0, 2)$

(b) Write down the equation of the straight line that is perpendicular to the line $y = \frac{1}{4}x - 5$ and passes through $(0, 7)$

(c) Write down the equation of the straight line that is perpendicular to the line $y = -\frac{1}{2}x$ and passes through $(0, -4)$

(a) Write down the equation of the straight line that is perpendicular to the line $y = 4 - 5x$ and passes through $(0, -8)$

(b) Write down the equation of the straight line that is perpendicular to the line $y + 3x = 9$ and passes through $(0, 0)$

(c) Write down the equation of the straight line that is perpendicular to the line $2y = -5x + 6$ and passes through $(0, 4)$

Match the pairs of perpendicular lines:

$y = \frac{2}{3}x - 1$	$y = \frac{1}{2}x + \frac{3}{2}$
$y - 3x = 2$	$2 - 3x = 2y$
$3 - 2x = y$	$3y + x + 2 = 0$

(a) Find the equation of the line with gradient 7 that passes through (1, 3)
(b) Find the equation of the line with gradient -2 that passes through (4, 3)
(c) Find the equation of the line with gradient 2 that passes through (1, -4)
(d) Find the equation of the line with gradient -3 that passes through (-1, 6)

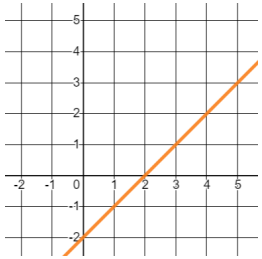
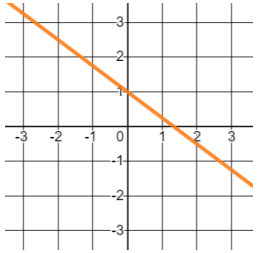
(e) Find the equation of a line which is parallel to $y = 2x + 1$ and passes through (3, 1).
(f) Find the equation of a line which is parallel to $y = 3x + 1$ and passes through (6, 10).
(g) Find the equation of a line which is parallel to $y = 5x - 2$ and passes through (5, 7).
(h) Find the equation of a line which is parallel to $y = 4x - 7$ and passes through (4, -3).

(i) Find the equation of a line which is perpendicular to $y = 2x + 6$ and passes through (6, 4).
(j) Find the equation of a line which is perpendicular to $y = -4x + 7$ and passes through (12, 15).
(k) Find the equation of a line which is perpendicular to $y = \frac{1}{5}x + 6$ and passes through (3, 1).
(l) Find the equation of a line which is perpendicular to $y = -\frac{1}{6}x + 3$ and passes through (2, 10).

Fluency Practice

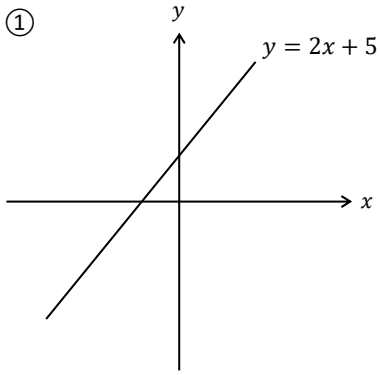
Coordinates and Straight Lines			
(a)	**(b)**	**(c)**	**(d)**
Write down the gradient of the line with equation $y = -3x + 5$	Write down the y-intercept of the line with equation $y = 5x - 1$	Write down the gradient of the line with equation $y = \frac{2}{3}x - 1$	Write down the y-intercept of the line with equation $y = 6 - 5x$
(e)	**(f)**	**(g)**	**(h)**
Write down the equation of the line with gradient 4 and y-intercept $(0, -3)$	Find the midpoint of the line segment joining $(5, 2)$ and $(9, -2)$	Write down the equation of the line with y-intercept $(0, 7)$ and gradient $-\frac{1}{2}$	Find the equation of the line parallel to $y = 3x - 1$ that passes through $(0, 6)$
(i)	**(j)**	**(k)**	**(l)**
Find the midpoint of the line segment joining $(-4, 1)$ and $(-8, 5)$	Find the length of the line joining $(3, 1)$ and $(7, 4)$	Find the equation of the line parallel to $y = -\frac{3}{2}x$ that passes through $(0, 5)$	Find the length of the line joining $(-1, 3)$ and $(4, 12)$
(m)	**(n)**	**(o)**	**(p)**
Find the equation of the line with gradient 2 that passes through $(5, 3)$	Find the equation of the line parallel to $y = -3x$ that passes through $(2, 4)$	Find the equation of the line that passes through $(5, 4)$ and $(3, 10)$	Find the equation of the line that is perpendicular to $y = -2x + 1$ and passes through $(-3, 5)$

Fluency Practice

(a)	(b)	(c)	(d)
Write down the gradient and y-intercept of the straight line with equation $y = 5x - 2$	Write down the gradient and y-intercept of the straight line with equation $y = -\frac{1}{2}x + 7$	Write down the gradient and y-intercept of the straight line with equation $3y = 2x - 9$	Find the gradient of the line joining $(2, 5)$ and $(4, 11)$
(e)	**(f)**	**(g)**	**(h)**
Find the equation of the line. 	Find the equation of the line. 	Write down the equation of the line that is parallel to $y = -4x - 9$ and passes through $(0, 2)$	Write down the equation of the line that is perpendicular to $y = -3x$ and passes through the point $(0, -5)$

(i)	(j)	(k)
Find the equation of the line that has a gradient of 2 and passes through $(4, 3)$	Find the equation of the line that is perpendicular to the line $2y = x - 8$ and passes through $(-1, 9)$	Find the equation of the line that passes through $(2, 9)$ and $(5, 3)$.

Fluency Practice

① 

On the axes, sketch the graphs:

$y = 3x$ $y = 2x - 6$

$y = \frac{x}{2} + 5$

② $y = 3x + 5$

Gradient = _____

y-intercept = _____

$y = 8 - 2x$

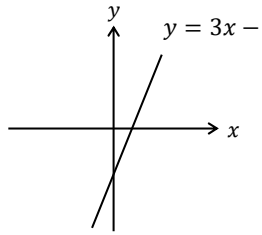
Gradient = _____

y-intercept = _____

$2y + x = 7$

Gradient = _____

y-intercept = _____

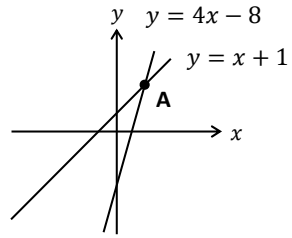
③ 

Which coordinates are on this graph?

(4, 8) (0, 4) (−2, −2)

Complete these coordinates for the graph $2y = 5x + 2$

$(4, y)$ $(x, 6)$ $(x, -9)$

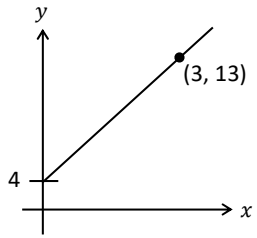
④ 

Find the coordinates of point **A**.

Find the coordinates of the point where the graphs $y = 3x + 9$ & $y = \frac{x}{2} + 4$ intersect.

⑤ Line N is parallel to the graph $y = 5x - 7$ & passes through (4, 22). Find the equation of Line N

Line O is parallel to the graph $y = 5 - 2x$ & passes through (5, −13). Find the equation of Line O

⑥ 

Find the equation of this line.

Line P passes through (1, 3) & (3, 11). Find the equation of this line.

Line Q passes through (−2, 10) & (5, −4). Find the equation of this line.

⑦ $y = 2x + 5$

Find the gradient of a line perpendicular to this graph.

$y = 10 - 4x$

Find the gradient of a line perpendicular to this graph.

$3y + 2x = 7$

Find the gradient of a line perpendicular to this graph.

⑧ Line R is perpendicular to the graph $y = 2x + 4$ & passes through (8, 2). Find the equation of Line R

Line S passes through (4, 6) & (8, 8) Line T is perpendicular to this line & passes through its midpoint. Find the equation of Line T.

Harder Coordinate Geometry		
(a)	**(b)**	**(c)**
Find an equation of the line that passes through the points $(4, 2)$ and $(-8, 11)$. Give your answer in the form $ax + by = c$ where a, b and c are integers.	The straight line L has equation $5x - 3y = 18$. Find an equation of the line that is parallel to L and crosses the x-axis at $(4, 0)$.	The straight line L_1 has equation $x + 2y - 7 = 0$. The straight line L_2 passes through the points $(-2, -6)$ and $(5, 8)$. Show that the lines L_1 and L_2 are perpendicular to each other.
(d)	**(e)**	**(f)**
The straight line L passes through the points $(1, -1)$ and $(5, 9)$. Find an equation of the line that is parallel to L and passes through the point $(2, 4)$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers.	The straight line L_1 has equation $2x - 3y = 4$. The straight line L_2 is perpendicular to L_1 and passes through the point $(1, 2)$. Find the equation of the line L_2 and the coordinates of the point where it crosses the x-axis.	ABC is a triangle, where $\widehat{BAC} = 90°$. The point C has coordinates $(9, 5)$ and points A and B lie on the line with equation $2x + 3y = 7$. Find the equation of the line that passes through A and C, giving your answer in the form $ax + by = c$ where a, b and c are integers.

Find the midpoints of the line segments joining:

(a) (4, 5) and (8, 1)

(b) (6, 0) and (12, 4)

(c) (8, -2) and (4, -10)

(d) (3, -1) and (-5, 1)

(e) (4, 7) and (3, 3)

(f) (9, -1) and (6, 3)

(g) (0, 5) and (-4, 8)

(h) (-2, -3) and (4, -4)

(i) (1.5, 3) and (7.5, 2.5)

(j) (-3.5, 9) and (-2.5, 4)

Find the lengths of the line segments joining:

(a) (1, 1) and (4, 5)

(b) (8, 4) and (2, -4)

(c) (-2, 5) and (3, 17)

(d) (6, 3) and (5, -4)

(e) (4, 7) and (3, 3)

(f) (9, -1) and (6, 3)

(g) (0, 5) and (-4, 8)

(h) (-2, -3) and (4, -4)

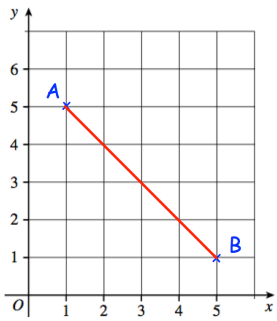
The line segment AB has the midpoint (7, 5). If point A is (3, 4), what are the coordinates of point B?

The line segment CD has the midpoint (-2, 4). If point D is (5, -1), what are the coordinates of point C?

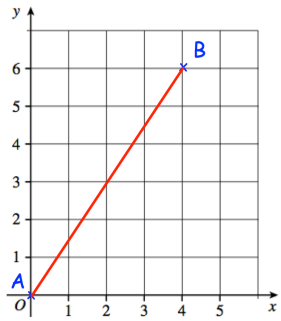
The line segment AB has length 10. If point A is (8, 11), find as many possible positions for point B as you can.

Question 1: Find the coordinates of the midpoints of the following line segments.

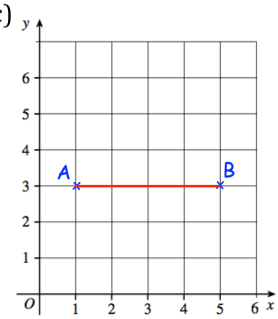
(a)



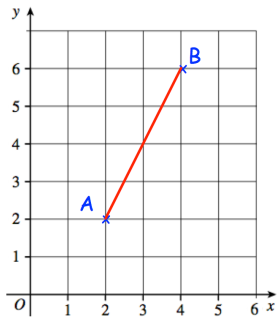
(b)



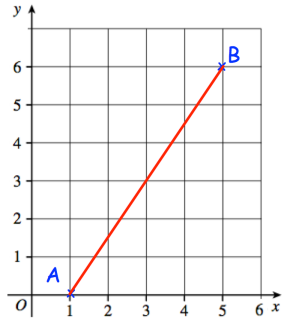
(c)



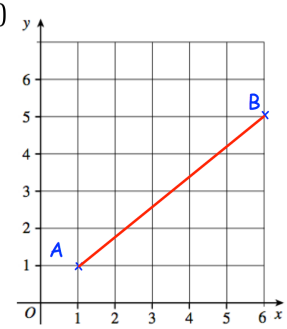
(d)



(e)

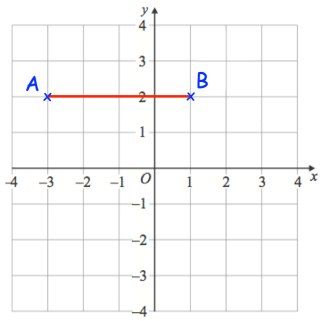


(f)

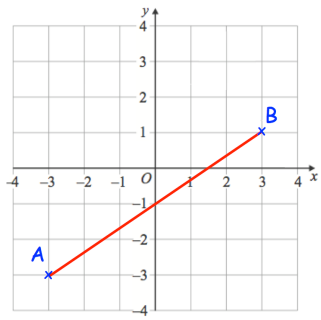


Question 2: Find the coordinates of the midpoints of the following line segments.

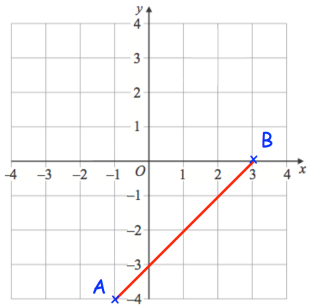
(a)



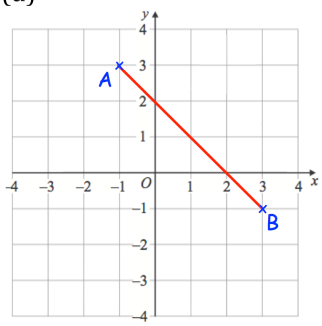
(b)



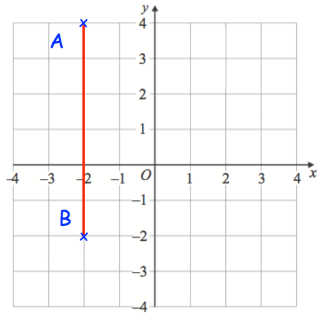
(c)



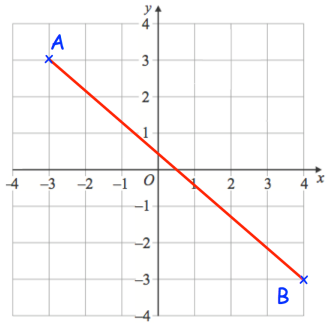
(d)



(e)



(f)



Question 3: Find the midpoint of the line joining these pairs of points

(a) (2, 4) and (6, 10)

(b) (1, 4) and (9, 12)

(c) (0, 7) and (6, 1)

(d) (−5, 2) and (5, −4)

(e) (−3, 9) and (7, −1)

(f) (0, −4) and (9, 0)

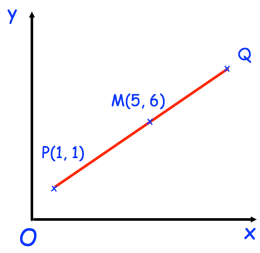
(g) (−10, −6) and (−2, 8)

(h) (0, 5) and (−11, −10)

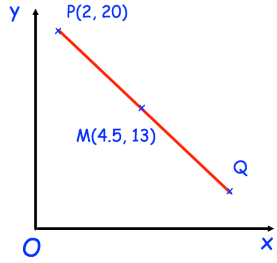
(i) (9, 8) and (4, 8)

Question 4: M is the midpoint of PQ in each diagram below.
Find the coordinates of Q in each diagram.

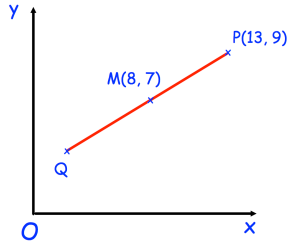
(a)



(b)

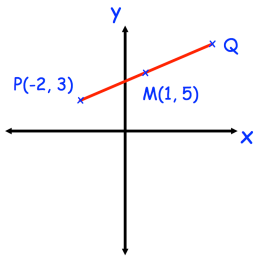


(c)

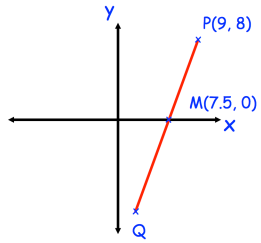


Question 5: M is the midpoint of PQ in each diagram below.
Find the coordinates of Q in each diagram.

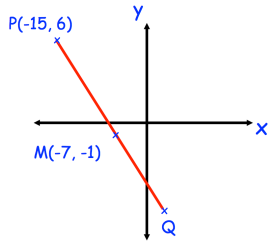
(a)



(b)

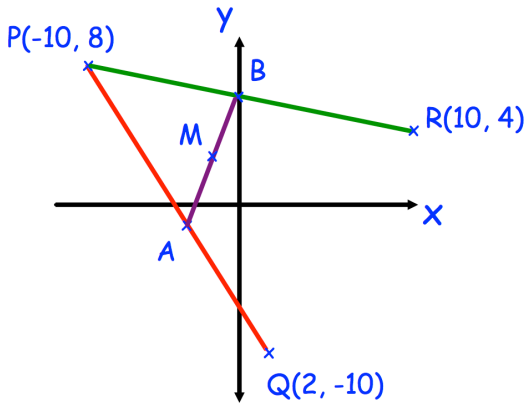


(c)



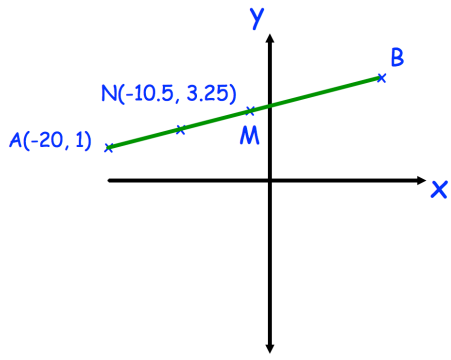
Apply

Question 1: A is the midpoint of the line PQ.
B is the midpoint of the line PR.
M is the midpoint of the line AB.
Find the coordinates of the point M



Question 2: M is the midpoint of the line AB.
The coordinates of the point M are (7, 2)
The coordinates of the point B are (11, 8)
The coordinates of the point C are (7, −4)
Find the area of triangle ACM.

Question 3: M is the midpoint of AB.
N is the midpoint of AM.
Find the coordinates of the point B.



Question 1: Write down the equation of a line parallel to each of the following

(a) $y = 2x + 3$ (b) $y = 5x - 3$ (c) $y = -3x + 1$ (d) $y = x - 7$

(e) $y = -7x - 10$ (f) $y = -x$ (g) $y = 10x$ (h) $y = 4$

(i) $x + y = 5$ (j) $2x + y - 1 = 0$ (k) $x - 2y + 5 = 0$ (l) $3x - 4y - 9 = 0$

Question 2: Write down the equation of each of the following lines

(a) Parallel to $y = 3x + 5$ and passing through $(0, 2)$

(b) Parallel to $y = 4x - 1$ and passing through $(0, 6)$

(c) Parallel to $y = 5x$ and passing through $(0, -3)$

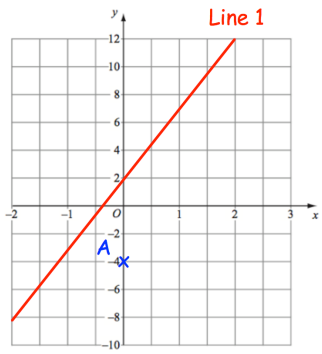
(d) Parallel to $y = -2x + 10$ and passing through the origin

(e) Parallel to $x + y = 8$ and passing through $(0, -4)$

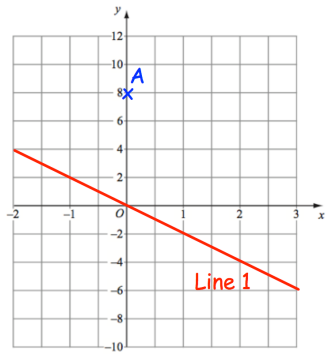
(f) Parallel to $x - 2y + 3 = 0$ and passing through $(0, 5)$

Question 3: Write down the equation of the line parallel to Line 1 and passing through A.

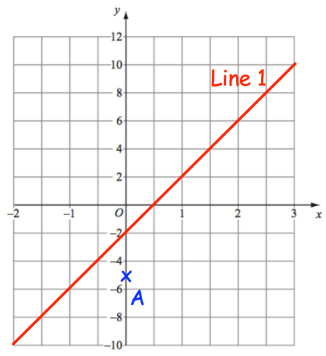
(a)



(b)



(c)



Question 4: Write down the equation of each of the following lines

(a) Parallel to $y = 2x - 1$ and passing through (1, 8)

(b) Parallel to $y = 3x + 2$ and passing through (1, 1)

(c) Parallel to $y = 5x - 4$ and passing through (2, 9)

(d) Parallel to $y = 3x - 7$ and passing through (4, 15)

(e) Parallel to $y = 4x$ and passing through (−1, 3)

(f) Parallel to $y = -2x + 5$ and passing through (−3, 0)

(g) Parallel to $y = 6x + 3$ and passing through (10, 5)

(h) Parallel to $y = -\frac{1}{2}x + 1$ and passing through (3, 0)

(i) Parallel to $x + y = 10$ and passing through (4, 0)

(j) Parallel to $x - 3y - 6 = 0$ and passing through (−9, −2)

Question 5: Write down the negative reciprocal of each number below.

(a) 4 (b) $\frac{2}{3}$ (c) -6 (d) 8 (e) $\frac{1}{2}$ (f) 1

(g) $-\frac{1}{3}$ (h) $-\frac{2}{5}$ (i) $\frac{4}{7}$ (j) $1\frac{1}{2}$ (k) -1 (l) $-1\frac{3}{4}$

Question 6: Write down the equation of a line perpendicular to each of the following

(a) $y = 4x + 2$ (b) $y = 2x - 7$ (c) $y = -5x + 2$ (d) $y = x - 3$

(e) $y = -x + 1$ (f) $y = \frac{1}{2}x + 3$ (g) $y = \frac{3}{4}x - 2$ (h) $y = -\frac{1}{5}x + 1$

(i) $y = -\frac{2}{3}x - 5$ (j) $x + y = 12$ (k) $x - 2y + 8 = 0$ (l) $5x - 3y - 3 = 0$

Question 7: Write down the equation of each of the following lines

(a) Perpendicular to $y = 2x + 4$ and passing through (0, 3)

(b) Perpendicular to $y = -3x - 8$ and passing through (0, −2)

(c) Perpendicular to $x + y = 6$ and passing through (0, 1)

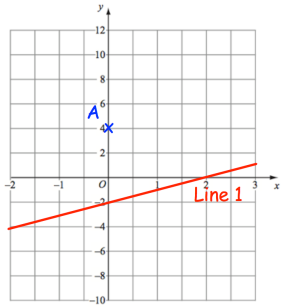
(d) Perpendicular to $y = \frac{1}{3}x - 2$ and passing through the origin

(e) Perpendicular to $y = -\frac{1}{5}x + 8$ and passing through (0, −2)

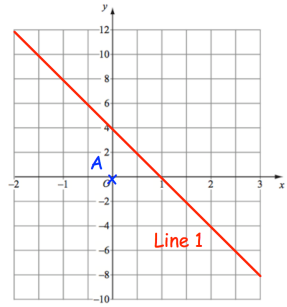
(f) Perpendicular to $y = -\frac{2}{9}x - 10$ and passing through (0, 6)

Question 8: Write down the equation of the line perpendicular to Line 1 & passing through A.

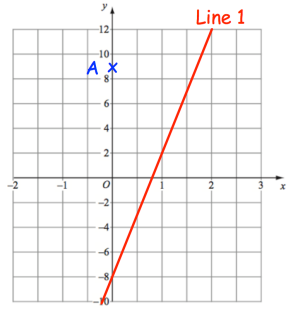
(a)



(b)



(c)



Question 9: Write down the equation of each of the following lines

(a) Perpendicular to y = 2x − 1 and passing through (4, 0)

(b) Perpendicular to y = −3x + 4 and passing through (6, 1)

(c) Perpendicular to y = 4x and passing through (−12, 5)

(d) Perpendicular to y = −½x + 1 and passing through (3, −7)

(e) Perpendicular to y = ⅔x + 4 and passing through (−6, −4)

(f) Perpendicular to y = −⅗x − 2 and passing through (9, 9)

(g) Perpendicular to x + 4y − 6 = 0 and passing through (1, 8)

Apply

Question 1: Write down the equations of the lines, from the box, that are:

y = 2x	y = ½x + 1
y = 3x + 2	y = -5x
y = 5x - 4	y = -2x + 3
	y = 3x - 2

(a) parallel (b) perpendicular (c) cross the y−axis at (0, 3)

(d) pass through the origin (e) pass through the point (1, 1)

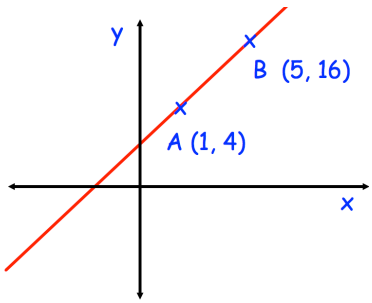
Question 2: Are the lines $2x + y = 8$ and $y = 2x + 5$ parallel?

Question 3: Are the lines $4x - y - 5 = 0$ and $x + 4y + 1 = 0$ perpendicular?

Question 4: A straight line passes through the points A(1, 4) and B(5, 16).

(a) Find the equation of the line parallel to AB that passes through (1, 7)

(b) Find the equation of the line perpendicular to AB that passes through the midpoint of AB.



Question 5: The line L has equation $y = 2x + 8$
The line L crosses the x–axis at the point A.
The line M is perpendicular to Line L and passes through the point A

(a) Find the coordinates of the point A.
(b) Find equation of the Line M.

Question 6: The point A has coordinates (−12, −7) and the point B has coordinates (−8, 1)

Find the equation of the line parallel to AB and passing through (2, 5)

Question 7: The line L passes through the points (−2, 1) and (2, 3).
The line N passes through the points (4, 7) and (12, 11).

Bryan says that the lines L and N are parallel.
Is Bryan correct? Explain your answer.

Question 8: The point C has coordinates (2, −3) and the point D has coordinates (4, 6).

Find the equation of the line perpendicular to CD and passing through D.

Question 9: The line Q passes through the points (−10, −2) and (−8, −8)
The line R passes through the points (1, 2) and (10, a)

The lines Q and R are perpendicular.

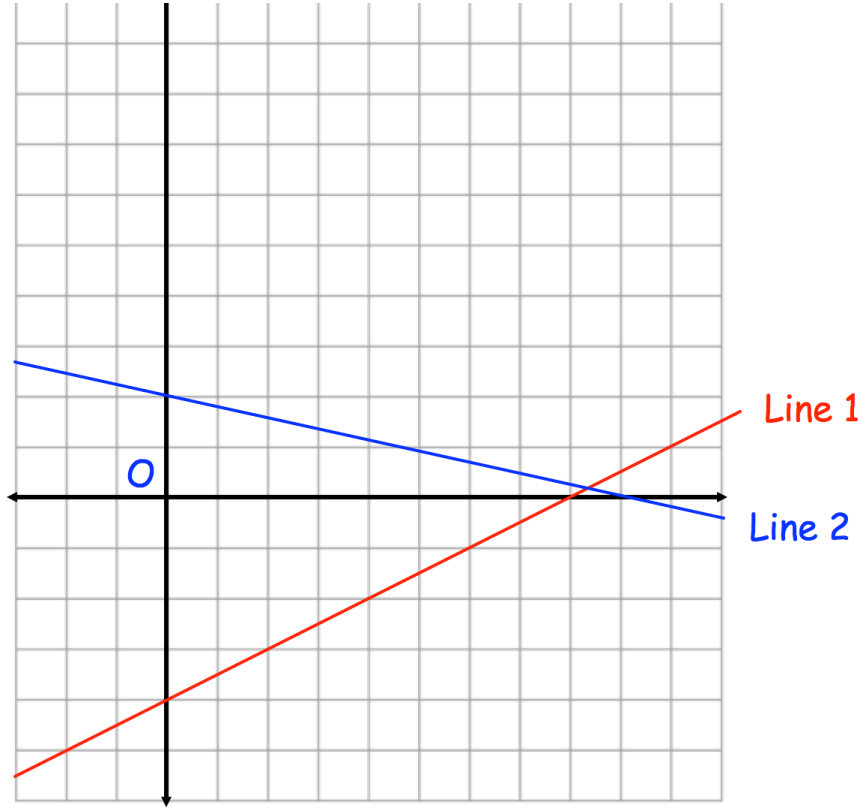
Find a.

Question 10: Two straight lines are shown.

Line 1 has equation $y = \frac{3}{2}x - 24$

(a) Find the equation of Line 2

(b) Are the lines perpendicular?



Purposeful Practice

perpendiculars

based on CBSE(10) exam questions

(1) prove that the triangle with vertices:
A (3, 0), B (6, 4) and C (– 1, 3)
is right–angled
show that it is also isosceles

(2) triangle ABC is isosceles with $AC = BC$
if A = (3, – 2) and B = (5, 2)
and the coordinates of C are (12, k)
a) find the value of k
b) find the area of triangle ABC

(3) triangle ABC has vertices:
A = (3, 3), B = (3, – 7) and C = (6, – 6)
a) show that the triangle is right-angled
b) show that the centre of the circle that passes through A, B and C lies on AB
c) find the area of triangle ABC

(4) a) which point on the y–axis is equidistant from (5, – 3) and (– 4, 6) ?

b) which point on the x–axis is equidistant from (2, – 5) and (– 2, 9) ?

(5) the point P (x, y) is equidistant from
A (5, 1) and C (– 1, 5)
a) prove that $2y = 3x$
b) find the other two coordinates of the square $ABCD$

(6) the point P (x, y) is equidistant from
E (4, 3) and G (– 3, 4)
a) prove that $y = 7x$
b) find the other two coordinates of the square $EFGH$
c) which square, $ABCD$ (above) or $EFGH$ has the larger area?

(7) the point P (x, y) is equidistant from
A $(a + b, b - a)$ and
B $(a - b, a + b)$

prove that $ay = bx$

(8) the triangle with vertices:
A (4, 7), B (t, 6) and C (8, 1)
is right–angled at B

find the values of t

Purposeful Practice

lines and perpendiculars

based on GCSE exam questions

(1) the straight line L_1 passes through the points with coordinates (4, 8) and (12, 4)
the straight line L_2 passes through the origin (O) and has gradient – 3

the two lines meet at point P
find the coordinates of P

the perpendicular to L_1 from the origin meets L_1 at Q

establish that $\triangle OPQ$ is an isosceles right angled triangle

what is its area?

(2) the point $P = (3, 4)$ and $Q = (a, b)$

a line perpendicular to PQ is: $3x + 2y = 7$

find an expression for b in terms of a
and give four possible coordinates for Q
[other than (3, 4)]

(3) PQR is a triangle with vertices P, Q and R
$P = (– 3, – 6)$, $Q = (1, 4)$ and R = (5, – 2)
M is the midpoint of PQ
N is the midpoint of QR

prove that MN is parallel to PR
establish that the length $MN = ½ PR$

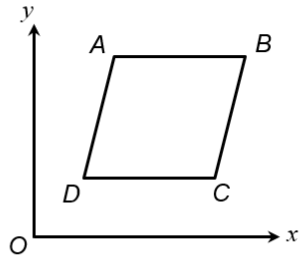
(4) $ABCD$ is a rhombus with $A = (5, 11)$
the equation of the diagonal DB is $y = ½ x + 6$

find an equation of the diagonal AC

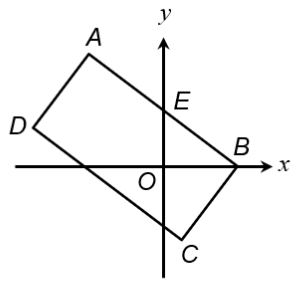
what are the coordinates of the centre of the rhombus?

what are the coordinates of (i) B (ii) D (iii) C ?

DC is parallel to the x axis



(5)



B is on the x axis

$ABCD$ is a rectangle
the equation of the line AB is $x + 2y = 12$

$AE = EB$

find an equation for AD

find an equation for DC

what are the coordinates of (i) A (ii) D (iii) C ?

Give an Example	Straight Lines

A	The equation of a straight line with a y-intercept of $(0, 3)$	
B	The equation of a straight line with a gradient of 4	
C	A pair of coordinates that join to make a line with midpoint $(4, -3)$	
D	The equation of a straight line that is parallel to the line $y = -2x + 9$	
E	A pair of coordinates that join to make a line of length 5	
F	The equation of a straight line that is steeper than the line $y = 1 + \frac{3}{2}x$	
G	A pair of coordinates that join to make a line with gradient $-\frac{1}{2}$	
H	The equation of a straight line that is perpendicular to the line $y = 2x - 5$	
I	The equation of a line with a positive gradient that passes through $(4, 1)$	
J	The equation of a line with a negative gradient that passes through $(5, -2)$	
K	A pair of coordinates that join to make a line with gradient $-\frac{3}{4}$ and length 10	
L	A pair of coordinates that join to make a line with a midpoint of $(-3, 6)$ which is perpendicular to the line $3x + y = 2$	

Problem Solving

	All Sides Equal	Has 4 Right Angles	Diagonals Cross at Right Angles
Forms a Parallelogram			
Diagonals not Equal in Length			

Put these cards into the correct part of the table.

$y = 2x - 1$ $y = 2x - 11$ $2y = 3 - x$ $4y = -(2x - 16)$	$y = x + 3$ $y = x - 5$ $7y = -35 - 49x$ $7x = 35 - y$	**IT'S IMPOSSIBLE!**

There are 3 missing spaces. Create sets of equations for them.

Q1. $y = 5x - 4$ is the equation of a straight line.

(a) Write down the gradient of the line $y = 5x - 4$

Answer .....

(1)

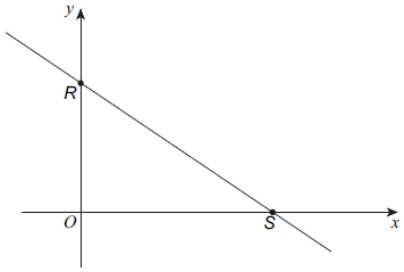
(b) Write down the coordinates of the y-intercept of the line $y = 5x - 4$

Answer (..... , ...)

(1)
(Total 2 marks)

Q2.

A sketch of $2x + 3y = 12$ is shown.



(a) Work out the coordinates of R.

Answer (....., ...)

(1)

(b) Work out the coordinates of S.

Answer (....., ...)

(2)
(Total 3 marks)

Q3.

(a) Write down the equation of a straight line that is parallel to $y = 5x + 6$

.....

(1)

(b) Find an equation of the line that is perpendicular to the line $y = 5x + 6$ and passes through the point (–2, 5).

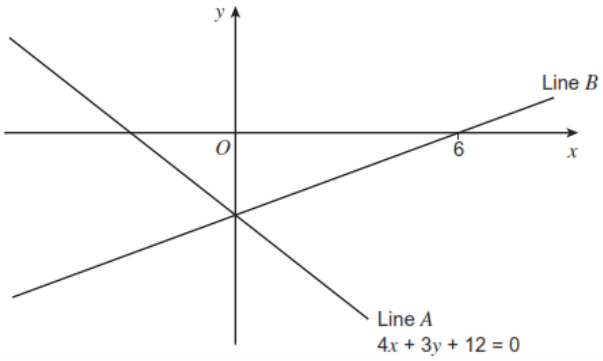
.....

(3)
(Total 4 marks)

Q4. Lines, A and B, intersect on the y-axis.

Line B intersects the x-axis at the point (6, 0).

The equation of line A is $4x + 3y + 12 = 0$



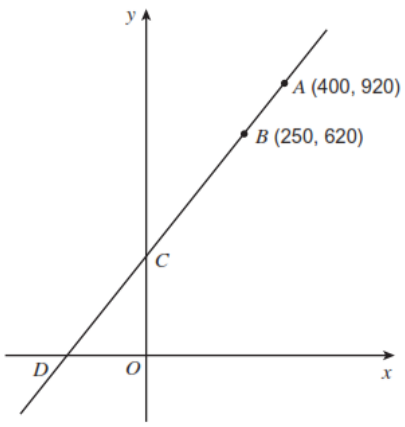
Work out the equation of line B.

Answer .....

(Total 4 marks)

Q5. The diagram shows a line $ABCD$.
A is the point (400, 920).
B is the point (250, 620).
The line cuts the y-axis at C and the x-axis at D.

Not drawn accurately



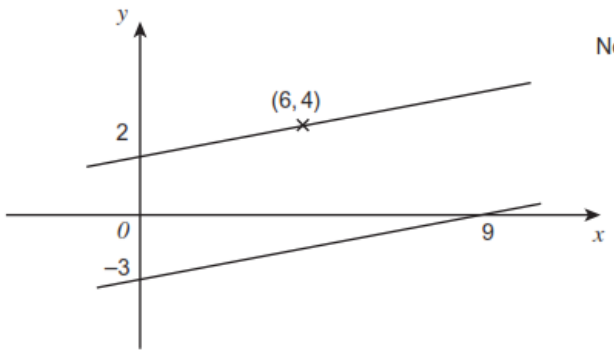
Work out the coordinates of C and D.

C (..... , ...)

D (..... , ...)

(Total 4 marks)

Q6. Two straight lines are shown.



Not drawn accurately

Prove that the lines never meet.

(Total 3 marks)

Q7.

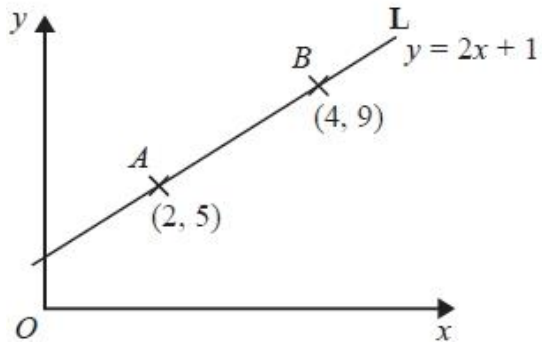


Diagram **NOT** accurately drawn

The point *A* has coordinates (2, 5).
The point *B* has coordinates (4, 9).

The line **L** passes through the points *A* and *B*.
The equation of line **L** is $y = 2x + 1$

M is the midpoint of the line segment *AB*.

Find an equation of the line that is perpendicular to line **L** and passes through *M*.

(Total 4 marks)

***Q8.** *A* is the point with coordinates (1, 3)
B is the point with coordinates (4, −1)
The straight line *L* goes through both *A* and *B*.

Is the line with equation $2y = 3x - 4$ perpendicular to line *L*?
You must show how you got your answer.

(Total 4 marks)

Q9.

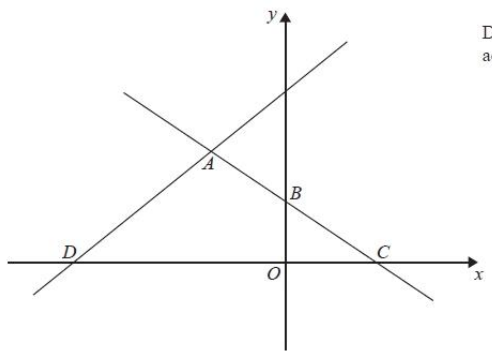


Diagram **NOT** accurately drawn

In the diagram, *ABC* is the line with equation $y = -\frac{1}{2}x + 5$

AB = *BC*
D is the point with coordinates (−13, 0)

Find an equation of the line through *A* and *D*.

(a) A line segment joins point A (1, 2) to point B (4, 8). Point C divides the line segment in the ratio AC : CB = 2 : 1. Find the coordinates of point C.
(b) A line segment joins point A (3, 1) to point B (11, 5). Point C divides the line segment in the ratio AC : CB = 3 : 1. Find the coordinates of point C.
(c) A line segment joins point A (3, 2) to point B (8, 12). Point C divides the line segment in the ratio AC : CB = 1 : 4. Find the coordinates of point C.

(a) A line segment joins point A (2, -3) to point B (5, 9). Point C divides the line segment in the ratio AC : CB = 2 : 1. Find the coordinates of point C.
(b) A line segment joins point A (6, -4) to point B (11, 6). Point C divides the line segment in the ratio AC : CB = 3 : 2. Find the coordinates of point C.
(c) A line segment joins point A (-8, -7) to point B (12, 3). Point C divides the line segment in the ratio AC : CB = 2 : 3. Find the coordinates of point C.

(a) A line segment joins point A (9, -1) to point B (5, 7). Point C divides the line segment in the ratio AC : CB = 1 : 7. Find the coordinates of point C.
(b) A line segment joins point A (1, 5) to point B (-5, 2). Point C divides the line segment in the ratio AC : CB = 5 : 1. Find the coordinates of point C.

A line segment joins point A (5, 2) with point B. Point C with coordinates (4, 4) divides the line segment in the ratio AC : CB = 1 : 3. Find the coordinates of B.

Fluency Practice

dividing a line segment in a ratio

(1) A is the point (4, 3)
C is the point (22, 27)

B divides the line segment AC in the ratio 1 : 2
find the coordinates of B

(2) P is the point (1, 3)
R is the point (11, 18)

Q divides the line segment PR in the ratio 1 : 1½
find the coordinates of Q

(3) Q is the point (5, 2)
R is the point (9, 10)

Q divides the line segment PR in the ratio 1 : 2
find the coordinates of P

(4) P is the point (– 5, 11)
R is the point (16, – 3)

Q divides the line segment PR in the ratio 3 : 4
find the coordinates of Q

(5) A is the point (– 1, – 7)
C is the point (24, 23)

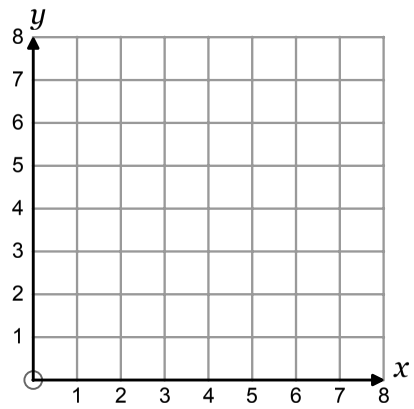
B divides the line segment AC in the ratio 2 : 3
find the coordinates of B

(6) A is the point (– 2, – 4)
C is the point (22, 4)

B divides the line segment AC in the ratio 5 : 3
find the coordinates of B

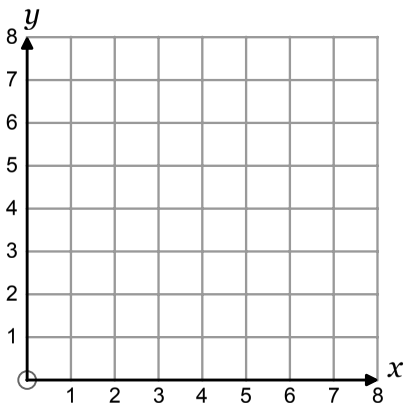
3 Graphical Inequalities

1.



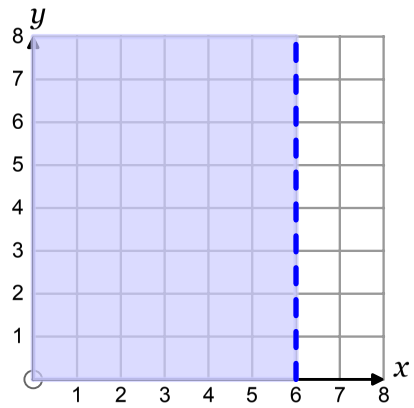
Shade the region: $x \geq 3$.

2.



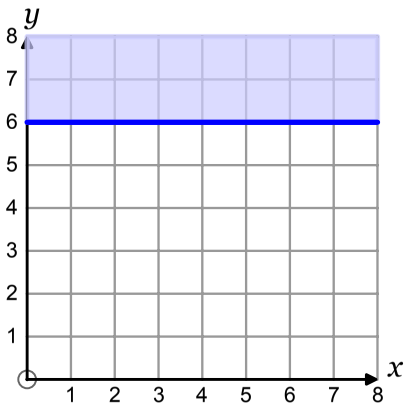
Shade the region: $y < 5$.

3.



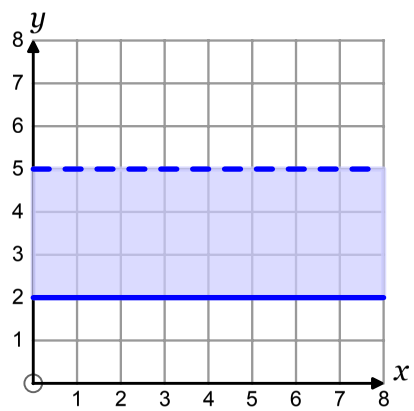
State the inequality: _____

4.



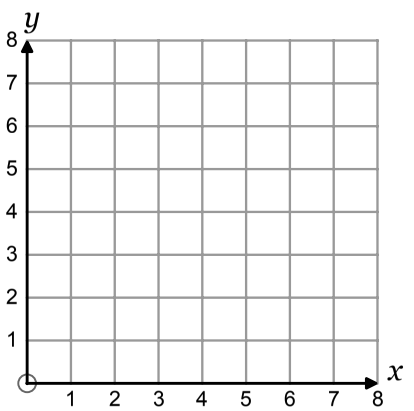
State the inequality: _____

5.



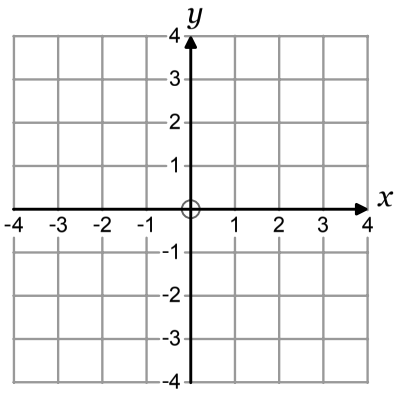
State the inequality: _____

6.



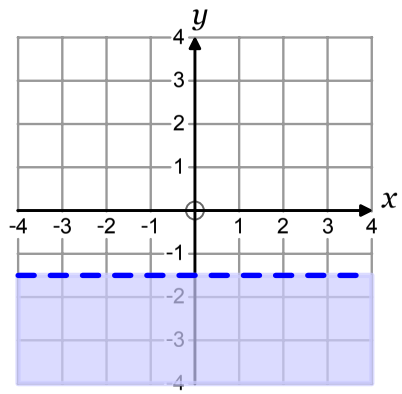
Shade the region: $1 < x < 4$.

7.



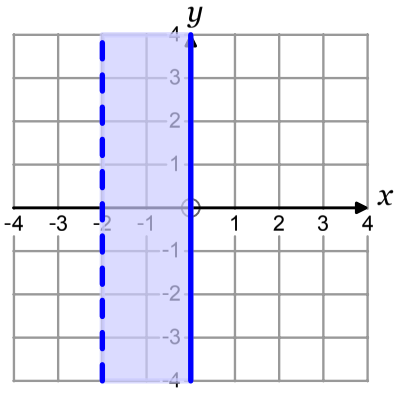
Shade the region: $y > -1$.

8.



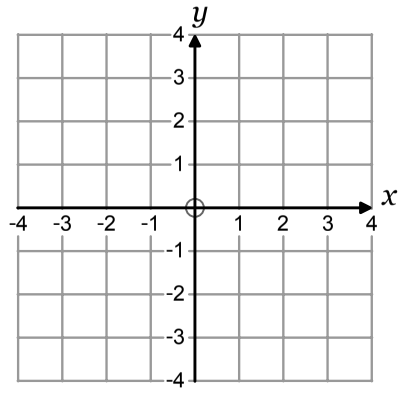
State the inequality: _____

9.



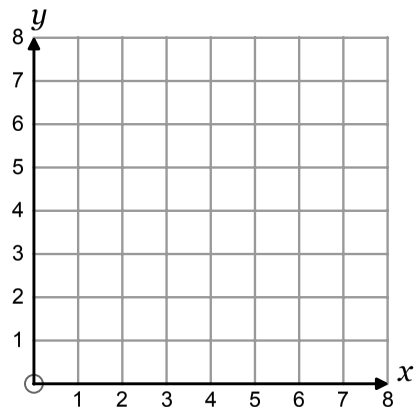
State the inequality: _____

10.



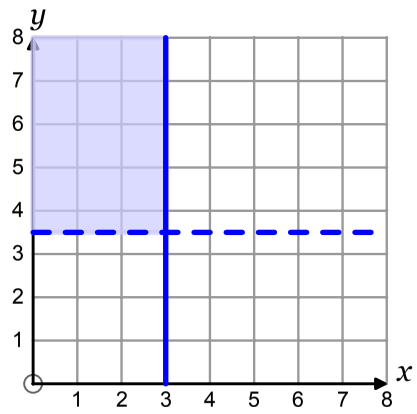
Shade the region: $-3 \leq y \leq -1$.

11.



Shade the region of points that satisfy **both**: $2 < x < 7$ **and** $2 < y < 5$.

12.

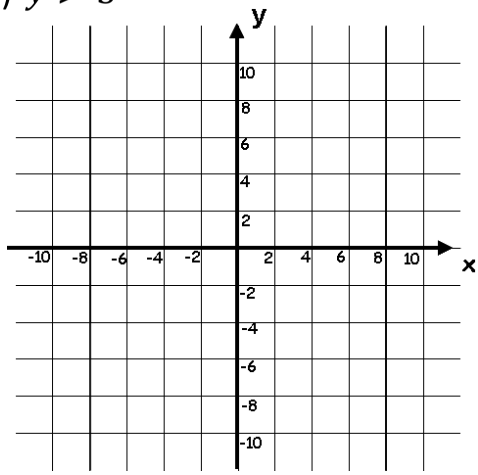


State two inequalites to describe the shaded region:

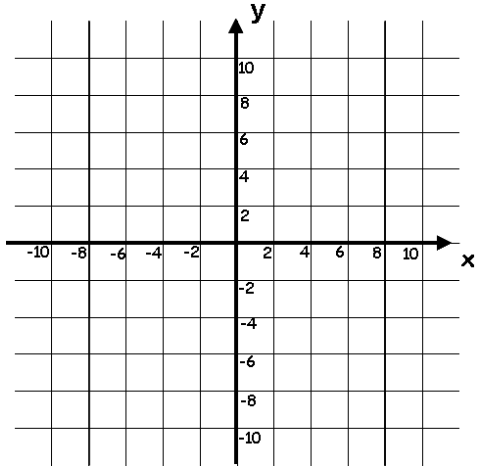
_____ ***and*** _____

Show the regions that satisfy the inequality(ies) given on the set of axes.

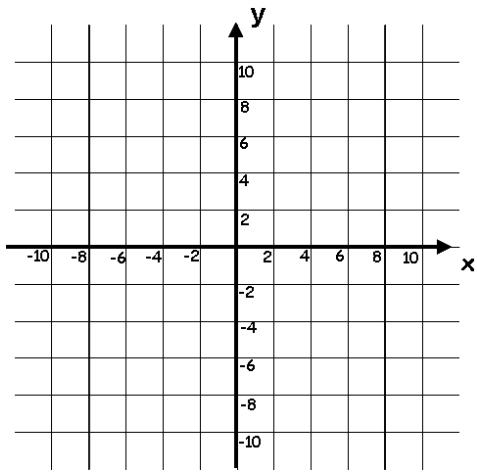
a) $y > 3$



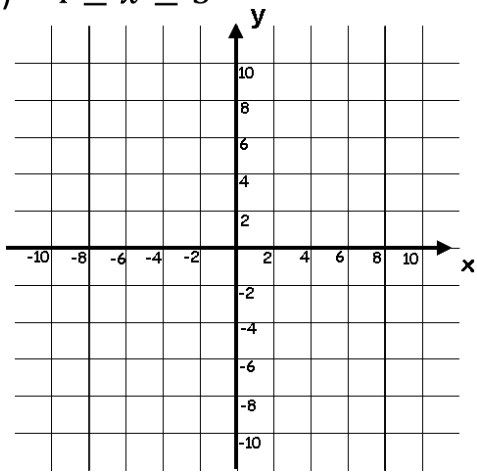
b) $x \leq -4$



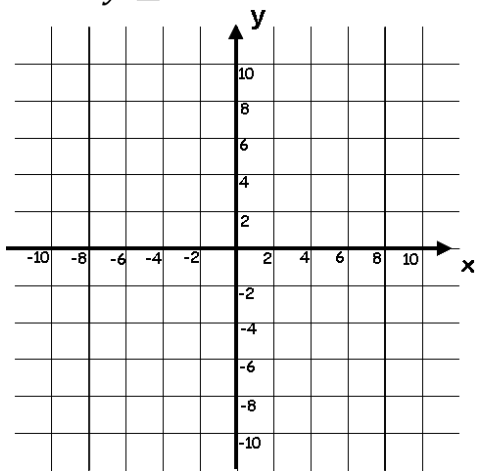
c) $y > 2$ *and* $x < 0$



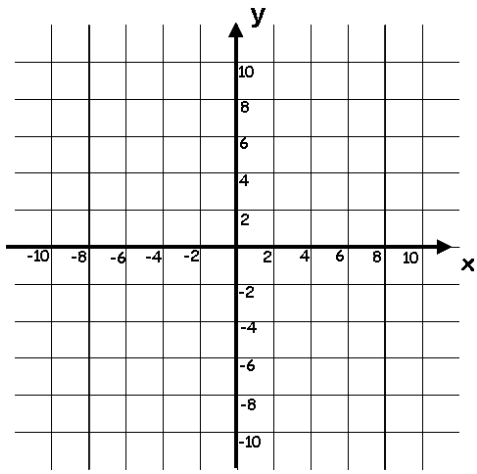
d) $-4 \leq x \leq 5$



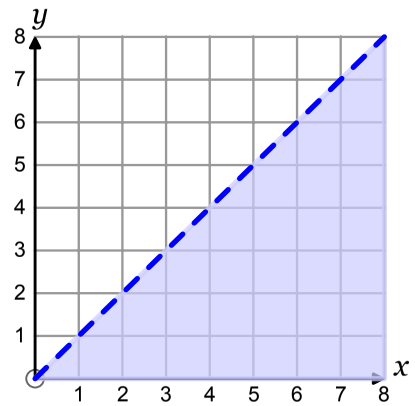
e) $-6 < y \leq 2$



f) $-4 \leq x < 4$ and $-2 < y < 8$

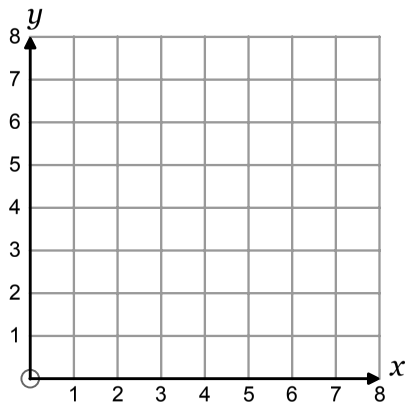


1.



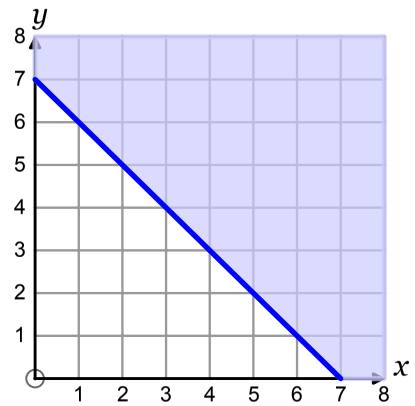
State the inequality: _____

2.



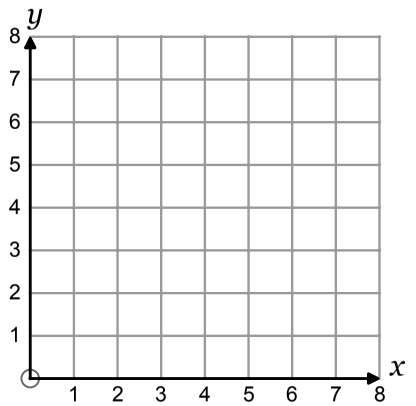
Shade the region: $y \geq x + 2$.

3.



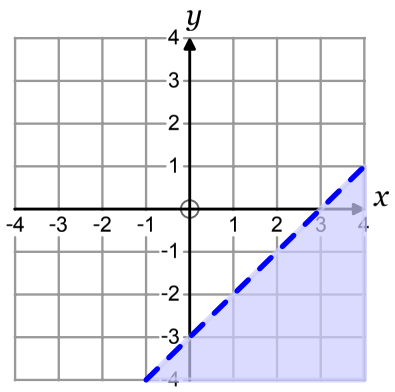
State the inequality: _____

4.



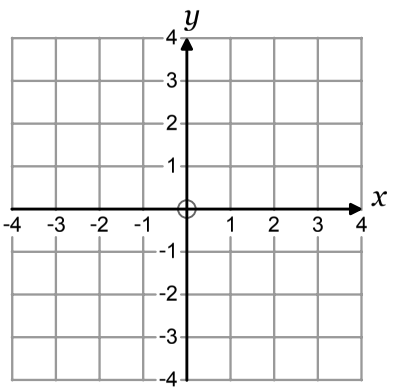
Shade the region: $y > 2x + 2$.

5.



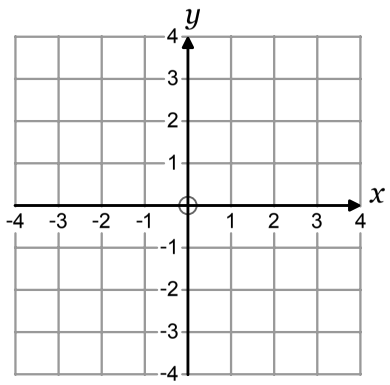
State the inequality: _____

6.



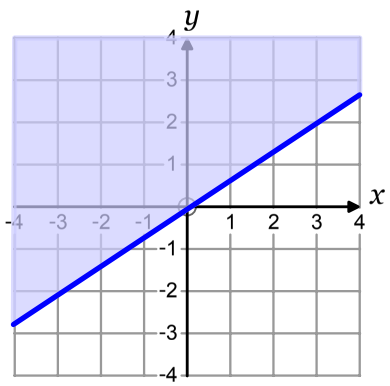
Shade the region: $y \geq \frac{1}{2}x$.

7.



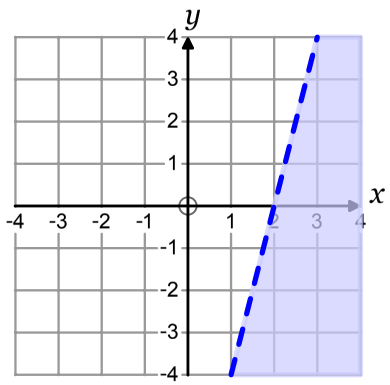
Shade the region: $y < -3x - 2$.

8.



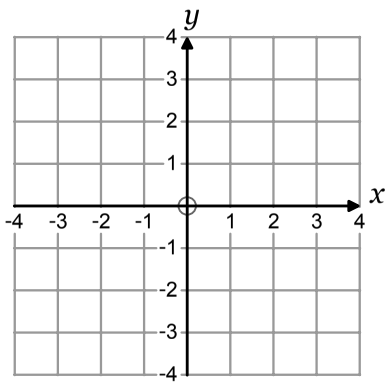
State the inequality: _____

9.



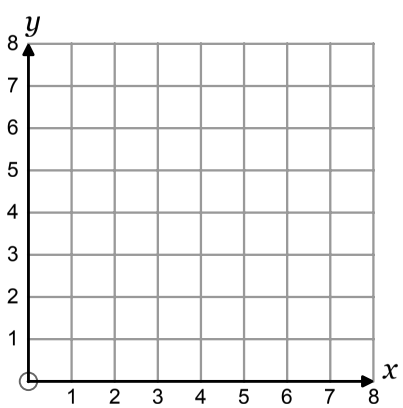
State the inequality: _____

10.



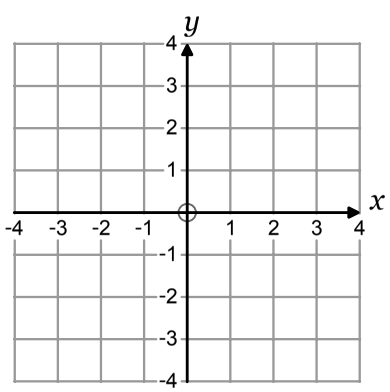
Shade the region: $x + y \leq 3$.

11.



Shade the region: $2x + 6y > 12$

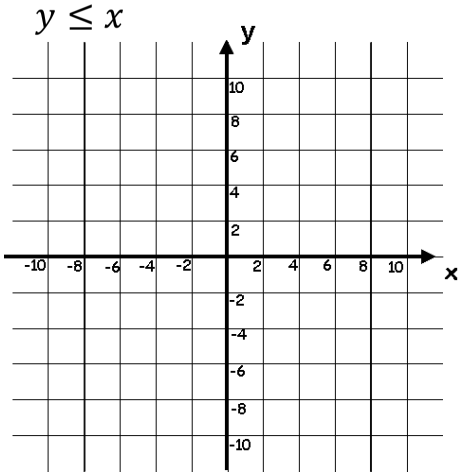
12.



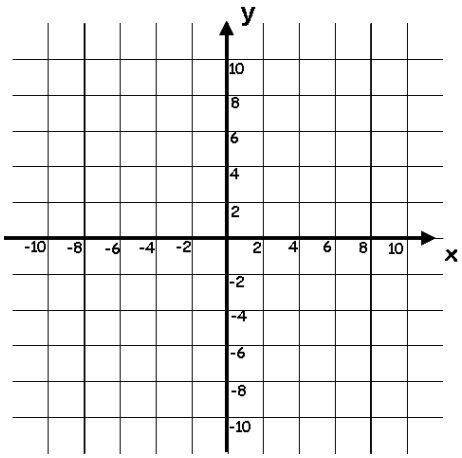
Shade the region: $2x - 3y \leq 6$

Show the regions that satisfy the inequality(ies) given on the set of axes.

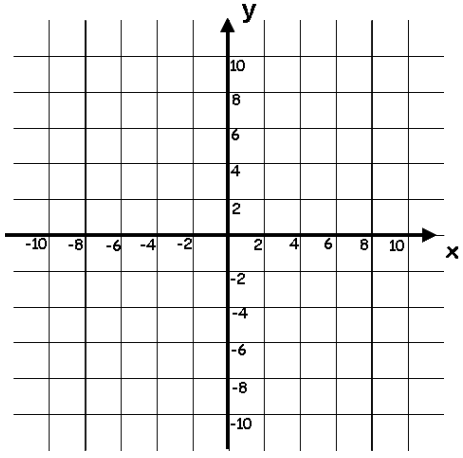
a) $y \leq x$



b) $y > x \text{ and } x \geq 5$



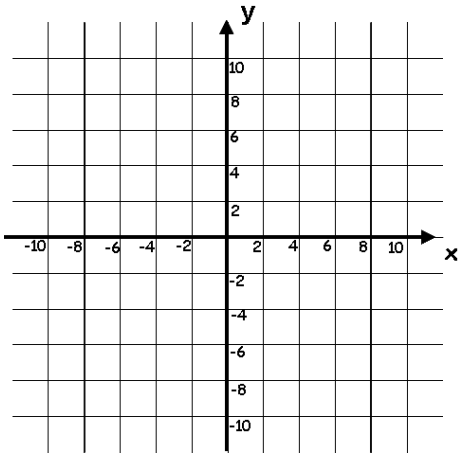
c) $y < x, y > 2 \text{ and } x < 6$



d) $y \geq x - 2 \text{ and } y < x + 1$

x	0	1	2
y			

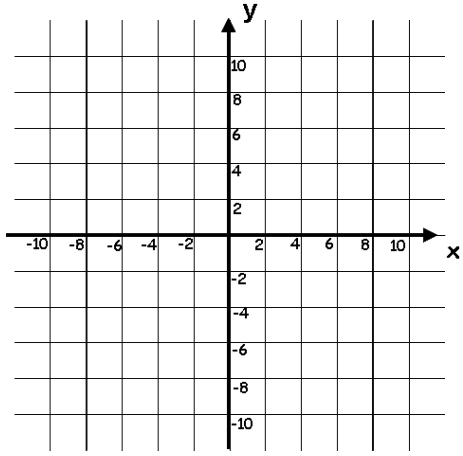
x	0	1	2
y			



e) $y > 2x \text{ and } y > \frac{1}{2}x$

x	0	1	2
y			

x	0	1	2
y			



Fluency Practice

f) $y > 3x \text{ and } y < 2 - x$

x	0	1	2	
y				

x	0	1	2	
y				

h) $y \geq 2x + 1 \text{ and } y < x + 1$

x	0	1	2	
y				

x	0	1	2	
y				

g) $y \geq x - 4 \text{ and } y < 4 - 2x$

x	0	1	2	
y				

x	0	1	2	
y				

i) $y > 4x + 1 \text{and } y > -2 - x$

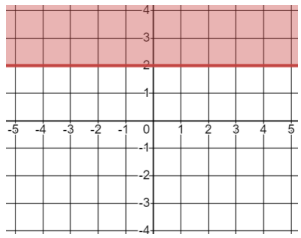
x	0	1	2	
y				

x	0	1	2	
y				

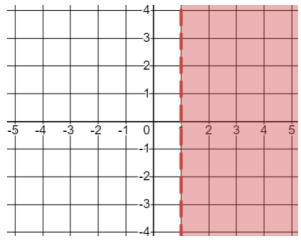
Describing Graphical Inequalities

Write down the inequality or inequalities that define the shaded region.

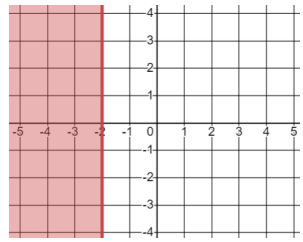
(a)



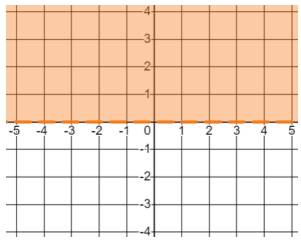
(b)



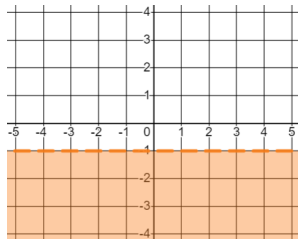
(c)



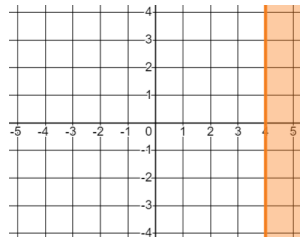
(d)



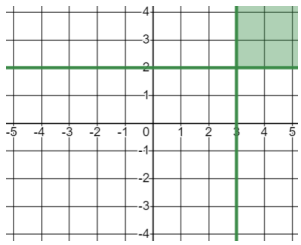
(e)



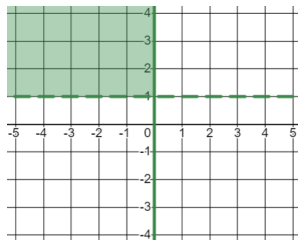
(f)



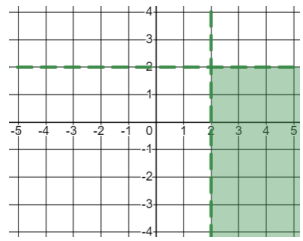
(g)



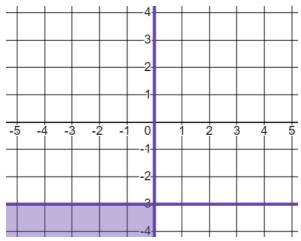
(h)



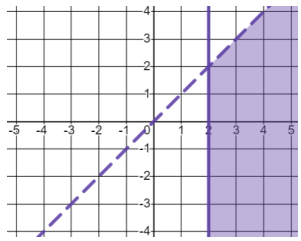
(i)



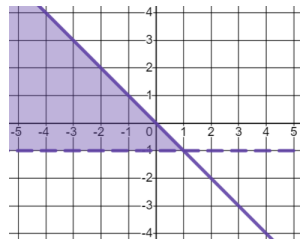
(j)



(k)



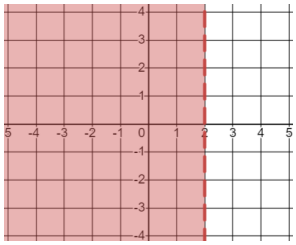
(l)



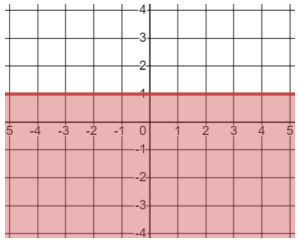
Describing Harder Graphical Inequalities

Write down the inequality or inequalities that define the shaded region.

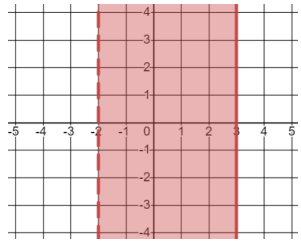
(a)



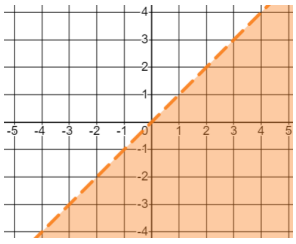
(b)



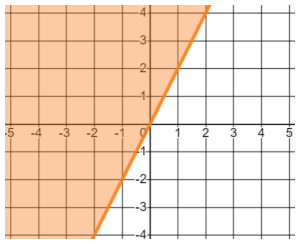
(c)



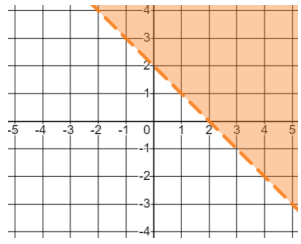
(d)



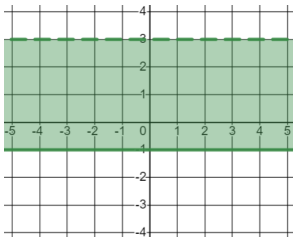
(e)



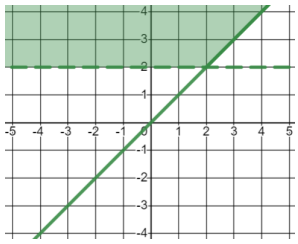
(f)



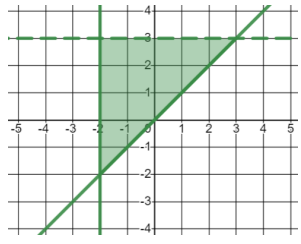
(g)



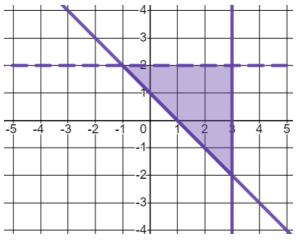
(h)



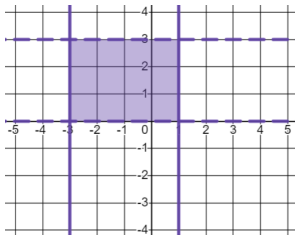
(i)



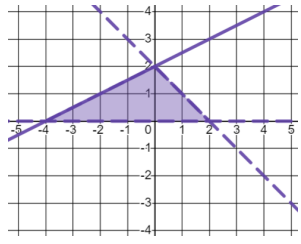
(j)



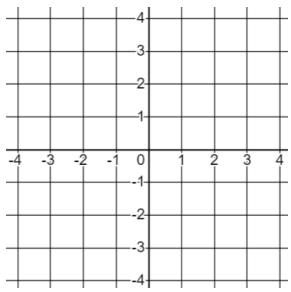
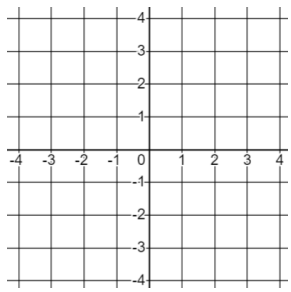
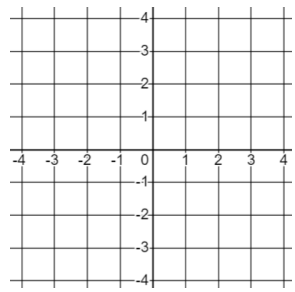
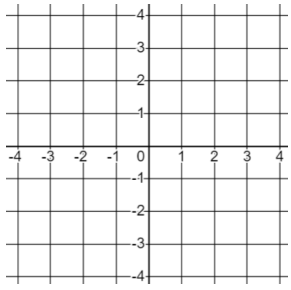
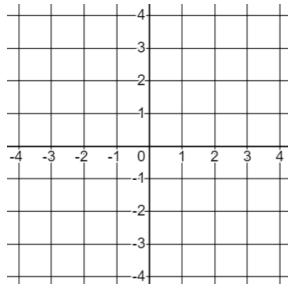
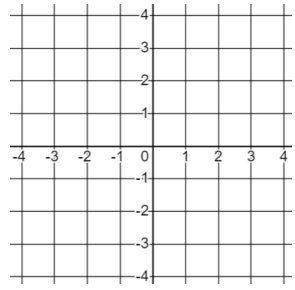
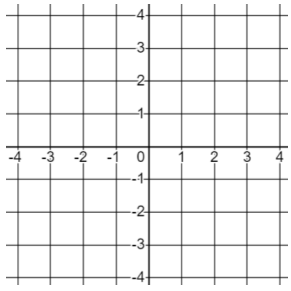
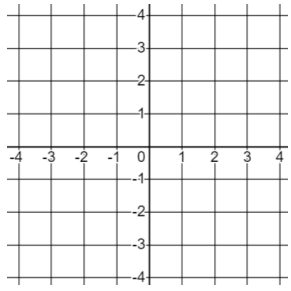
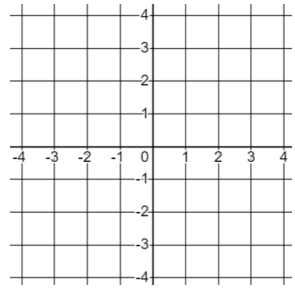
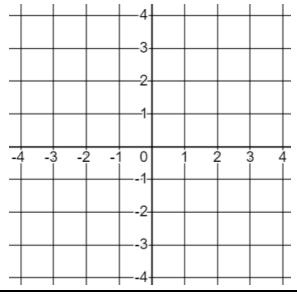
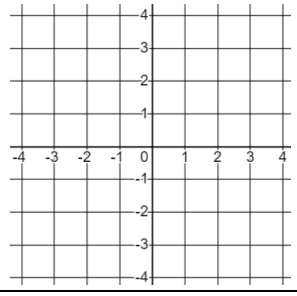
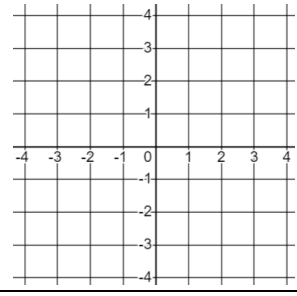
(k)



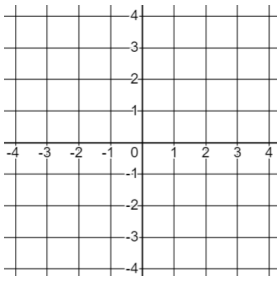
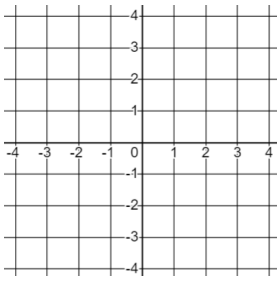
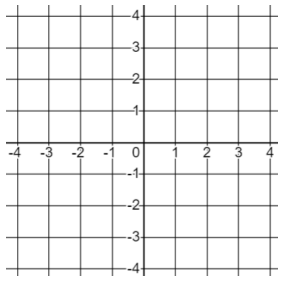
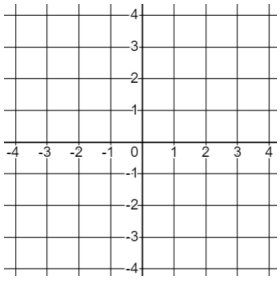
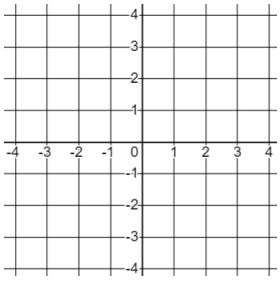
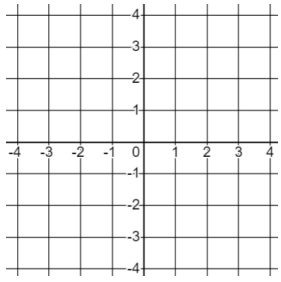
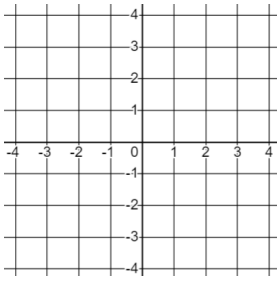
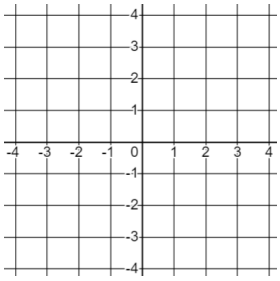
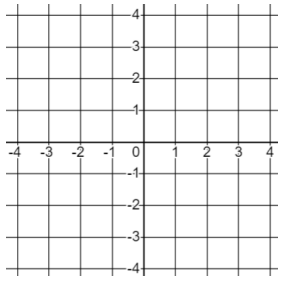
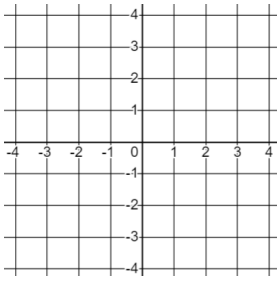
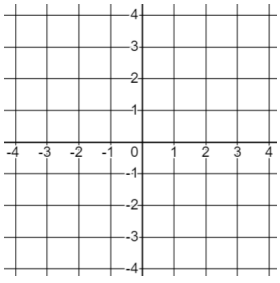
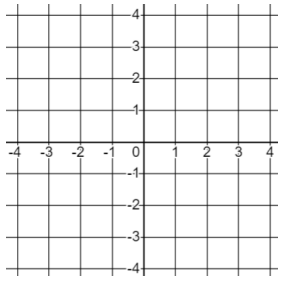
(l)



Shading Graphical Inequalities

(a)	(b)	(c)
$y > 1$ 	$y \leq -2$ 	$x < 0$ 
(d)	**(e)**	**(f)**
$x \geq 3$ 	$y \geq 0$ 	$x \leq -3$ 
(g)	**(h)**	**(i)**
$x \geq 2$ *and* $y < 1$ 	$x < -1$ *and* $y > 3$ 	$x \leq 0$ *and* $y < 0$ 
(j)	**(k)**	**(l)**
$x \geq -1$ *and* $y \geq 3$ 	$y \geq x$ *and* $x < 2$ 	$y < -x$ *and* $y \leq -1$ 

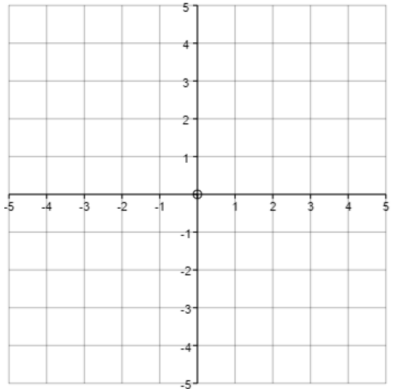
Shading Harder Graphical Inequalities

(a)	(b)	(c)
$y \geq -1$ 	$x < 2$ 	$y > x$ 
(d)	**(e)**	**(f)**
$y \geq 0.5x - 1$ 	$x + y \leq 3$ 	$-3 \leq y < 2$ 
(g)	**(h)**	**(i)**
$-2 \leq x < 3$ 	$x \geq 0 \text{ and } y < x$ 	$x \geq -1, y > 0 \text{ and } x + y < 2$ 
(j)	**(k)**	**(l)**
$2 < x < 4 \text{ and } -1 \leq y \leq 1$ 	$y < 2x, x + y \leq 4 \text{ and } y > 0$ 	$y < x, y \geq -2 \text{ and } 2x + y < 4$ 

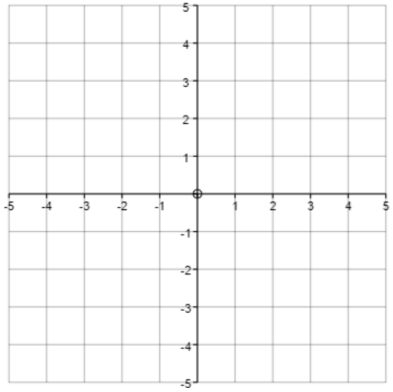
Fluency Practice

Shading and Describing Harder Graphical Inequalities

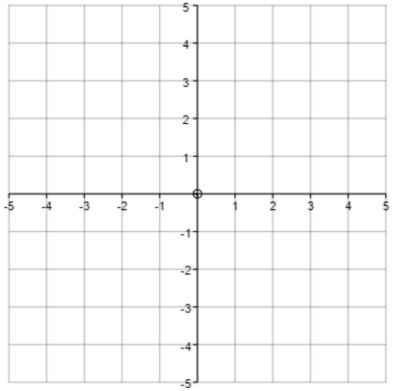
Shade the region that satisfies the inequalities $x \leq 4 \quad y \geq -1 \quad y \leq x$



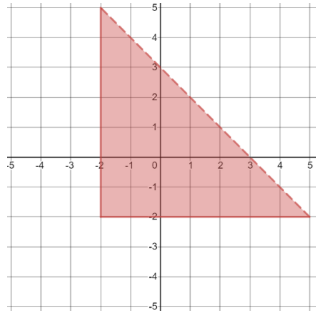
Shade the region that satisfies the inequalities $x > -1 \quad y \geq 0 \quad x + y \leq 3$



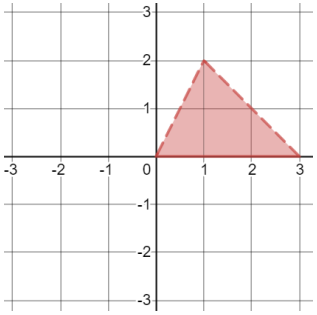
Shade the region that satisfies the inequalities $2x + 3y < 6 \quad y \leq x + 2 \quad y > -1$



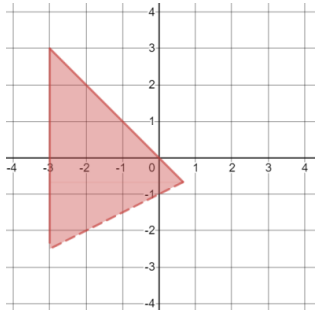
Write down the inequalities which fully describe the shaded region.



Write down the inequalities which fully describe the shaded region.



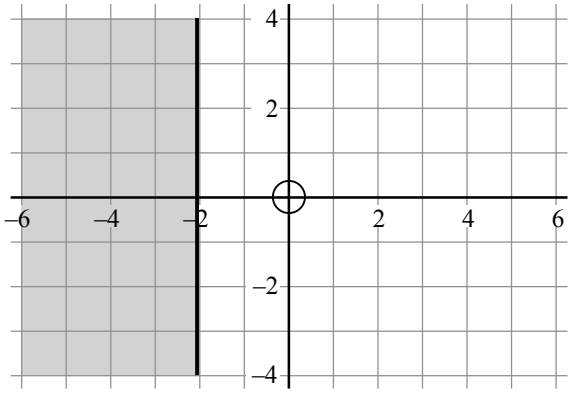
Write down the inequalities which fully describe the shaded region.



Fluency Practice

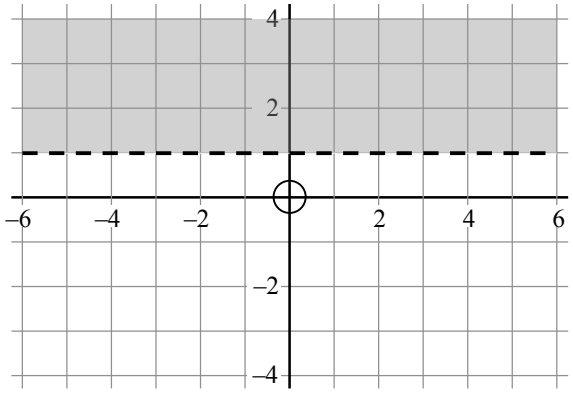
A1

Write down the inequality which defines the unshaded region.



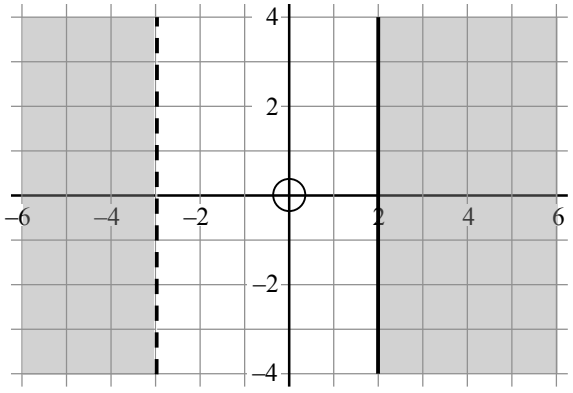
A2

Write down the inequality which defines the unshaded region.



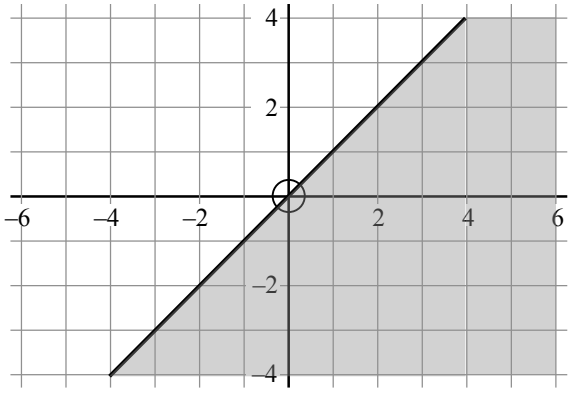
A3

Write down the inequalities which define the unshaded region.



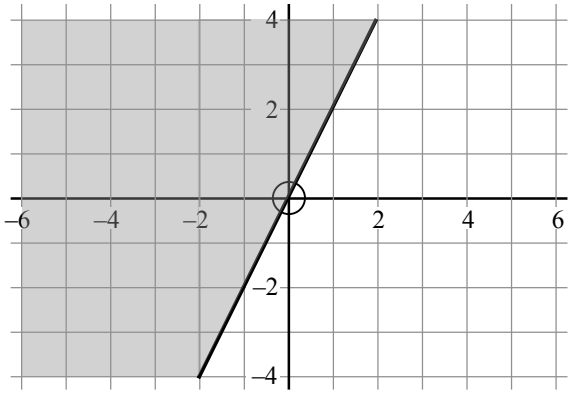
B1

Write down the inequality which defines the unshaded region.



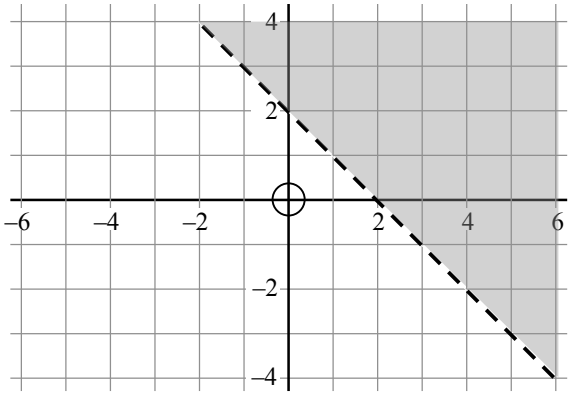
B2

Write down the inequality which defines the unshaded region.



B3

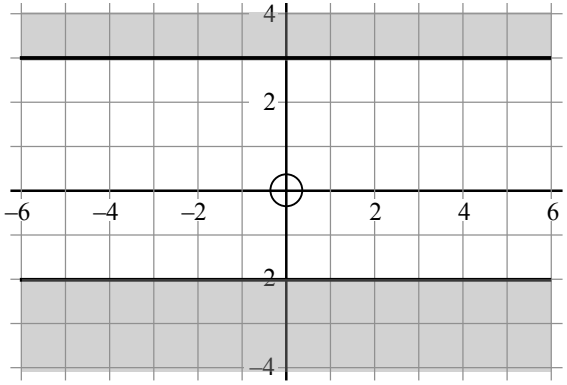
Write down the inequality which defines the unshaded region.



Fluency Practice

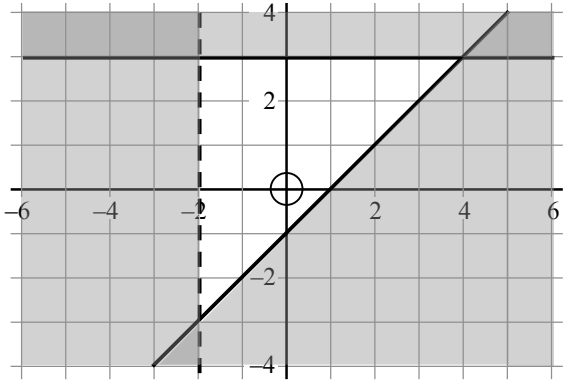
A1

Write down the inequalities which fully define the unshaded region.



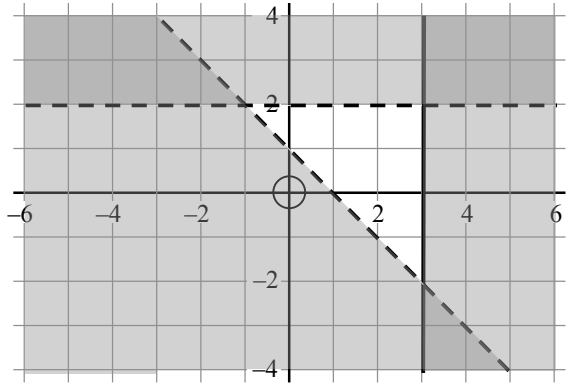
A2

Write down the inequalities which fully define the unshaded region.



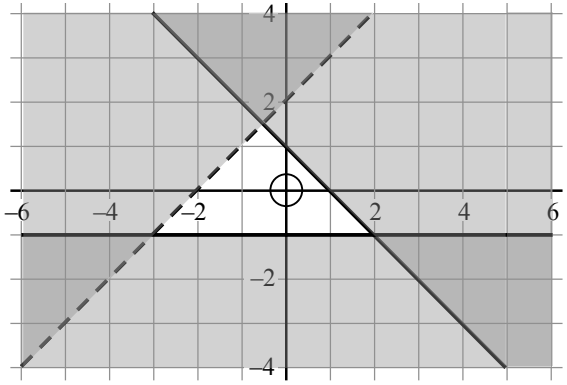
A3

Write down the inequalities which fully define the unshaded region.



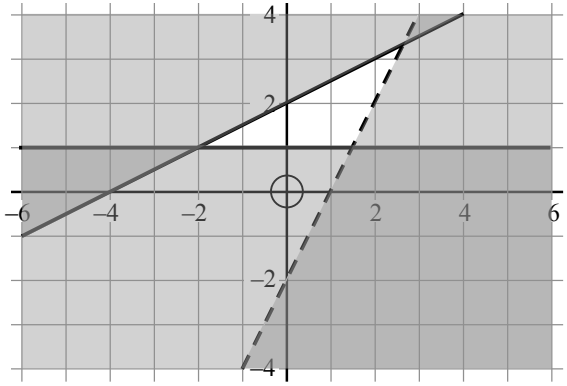
B1

Write down the inequalities which fully define the unshaded region.



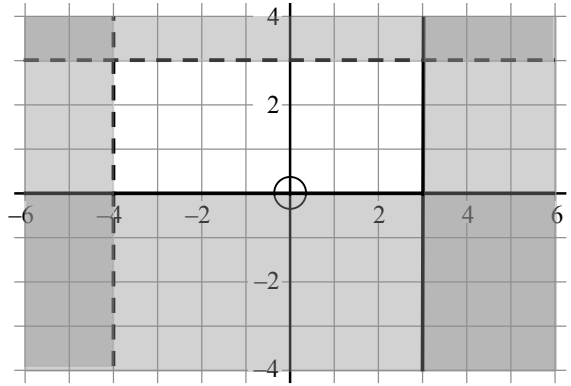
B2

Write down the inequalities which fully define the unshaded region.



B3

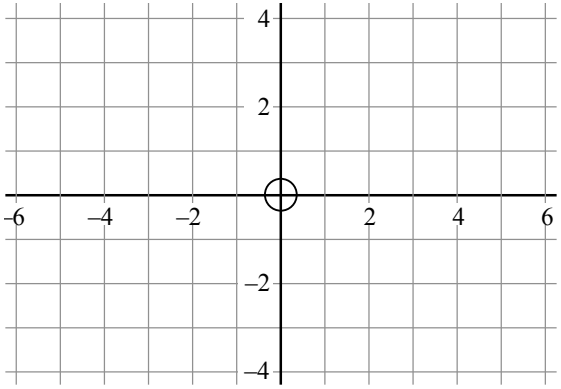
Write down the inequalities which fully define the unshaded region.



Fluency Practice

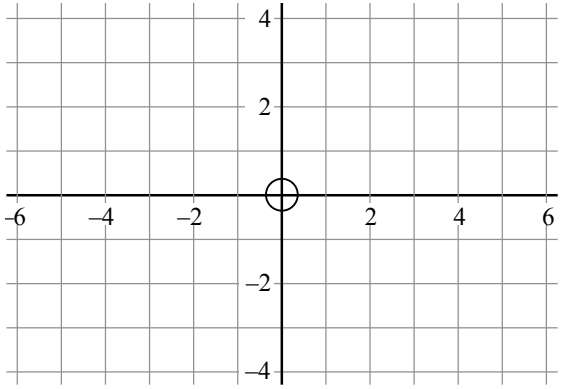
A1 Show, by shading on the grid, the region defined by $y \geq -1$

Label your region **R**.



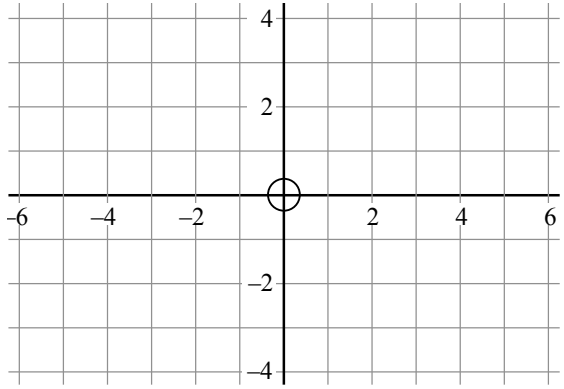
A2 Show, by shading on the grid, the region defined by $x < 3$

Label your region **R**.



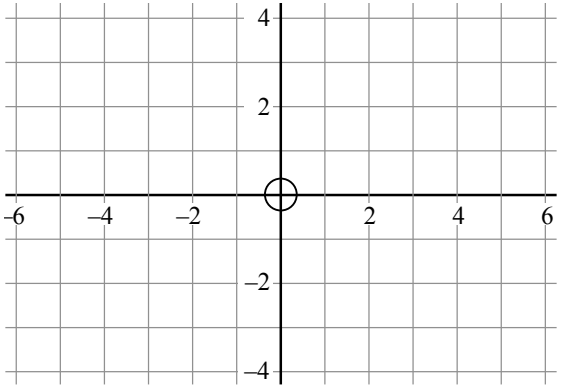
A3 Show, by shading on the grid, the region defined by $y < x$

Label your region **R**.



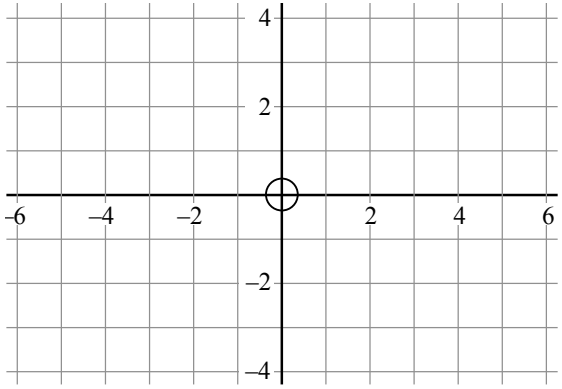
B1 Show, by shading on the grid, the region defined by $y \geq 0.5x - 1$

Label your region **R**.



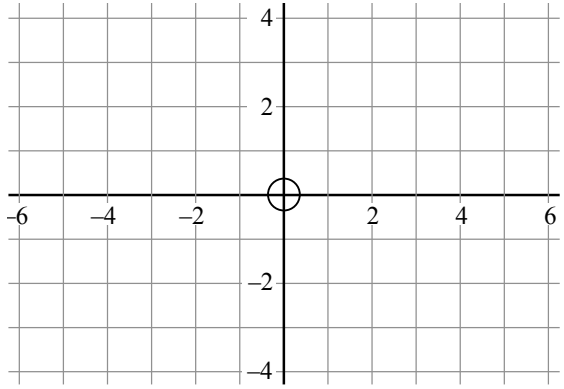
B2 Show, by shading on the grid, the region defined by $x + y \leq 3$

Label your region **R**.



B3 Show, by shading on the grid, the region defined by $-3 \leq y < 2$

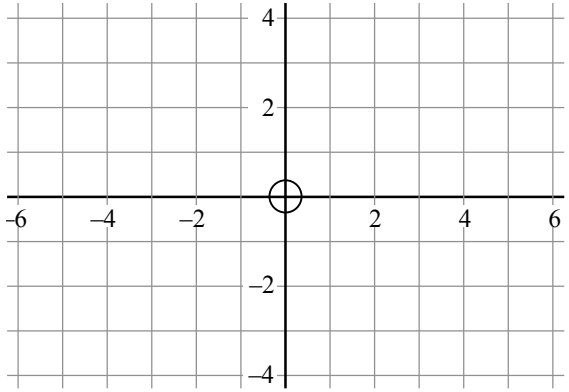
Label your region **R**.



Fluency Practice

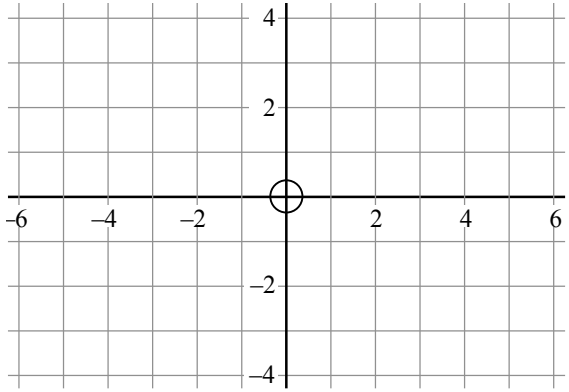
A1 Show, by shading on the grid, the region defined by $-4 \leq x \leq 3$

Label your region **R**.



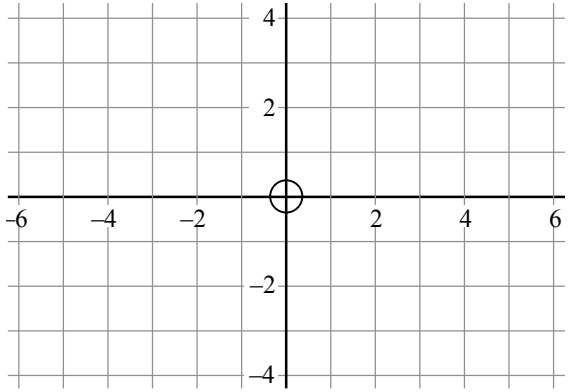
A2 Show by shading on the grid the region defined by $x \geq -3$, $y < 2$ **and** $y > x$

Label your region **R**.



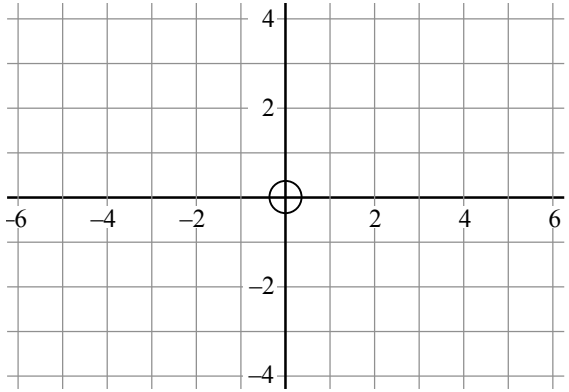
A3 Show by shading on the grid the region defined by $x + y \leq -1$, $x \geq -4$ **and** $y \geq -3$

Label your region **R**.



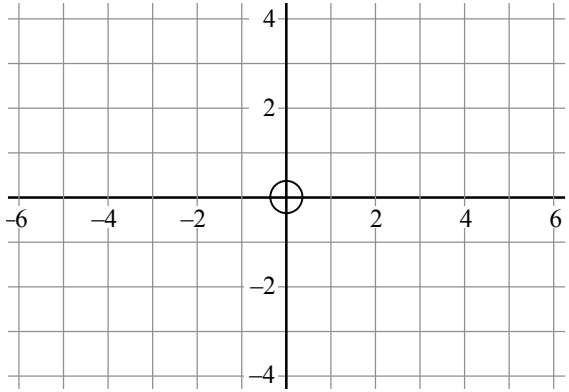
B1 Show by shading on the grid the region defined by $-2 < y \leq 3$ **and** $-3 \leq x < 5$

Label your region **R**.



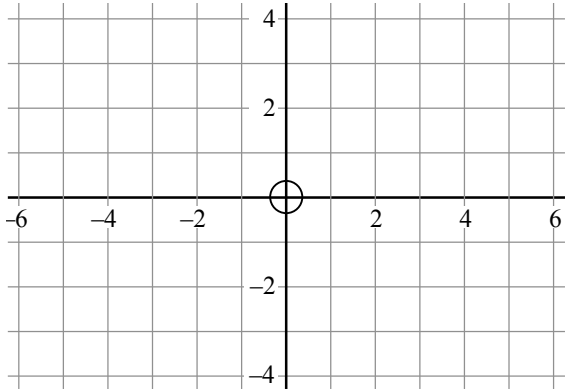
B2 Mark with a cross (x) a point on the grid which satisfies both the inequalities

$x > 1$ **and** $x - 3y > 3$



B3 Show by shading on the grid the region defined by $y > x + 3$, $x \geq -3$ **and** $2y - x \leq 4$

Label your region **R**.



Purposeful Practice

linear inequalities: regions

(1) $y \leq 2x + 3$

$x \leq 3$

$y \geq 5$

(2) $x + 2y \geq 8$

$x \leq 6$

$y \leq 3$

(3) $x + y \geq 9$

$x \leq 4$

$y \leq x + 5$

(4) $y \geq 2x - 3$

$y \geq 3$

$y \leq x + 2$

(5) $y \leq \frac{1}{3}x + 6$

$y \geq x$

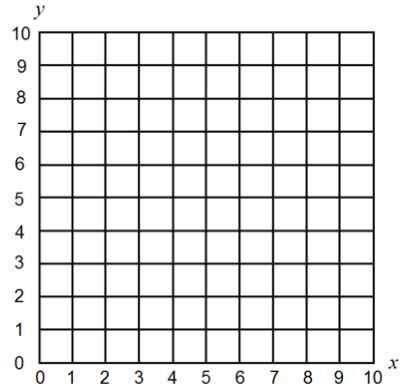
$y \geq 7$

(6) $x + 2y \geq 8$

$x \leq 6$

$2y \leq 3x - 8$

find each of these regions on a 10 by 10 grid:

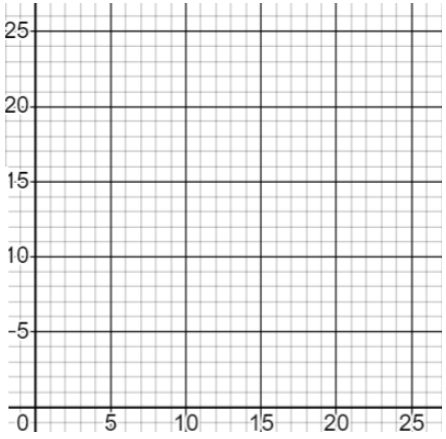
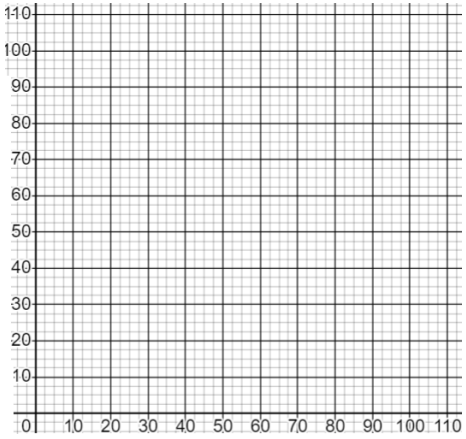
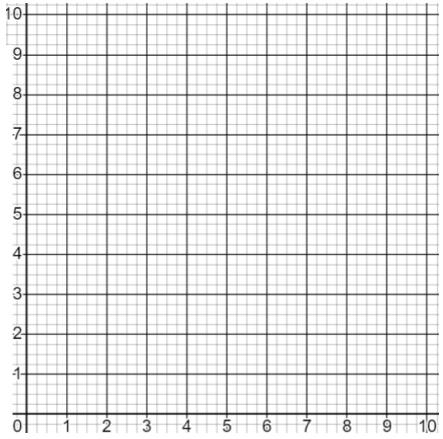


the region for all questions should be a triangle with an area of 4 squares

establish that this is the case

Purposeful Practice

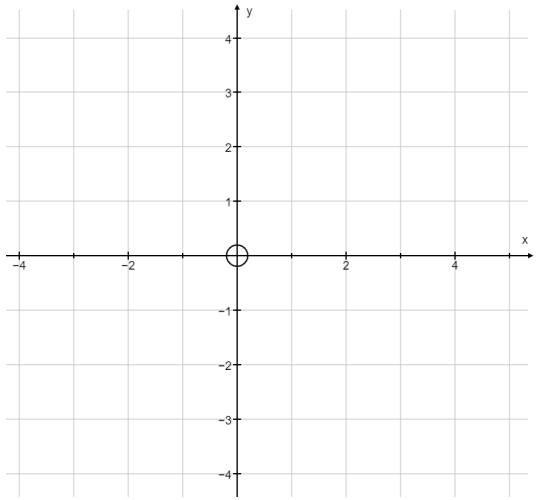
Graphical Inequalities Worded Problems

(a)	(b)	(c)
Junior's pencil case contains pens and pencils. The total number of pens and pencils is less than 25. The number of pens is greater than 5. The number of pencils is between 4 and 16. (a) By letting x represent the number of pens and y the number of pencils, write inequalities to represent this problem. (b) Represent this problem graphically, shading the region which satisfies all the inequalities. 	A factory manufactures beds and sofas. Each week it makes at least 30 beds and between 40 and 100 sofas. The factory always manufactures more sofas than beds. (a) By letting x represent the number of beds and y the number of sofas, write inequalities to represent this problem. (b) Represent this problem graphically, shading the region which satisfies all the inequalities. 	Maya is baking cakes and brownies. Each cake needs 50g of sugar and 20g of flour. Each cookie needs 30g of sugar and 50g of flour. She has 300g of sugar and 200g of flour and wants to make at least 2 cakes. (a) By letting x represent the number of cakes and y the number of cookies, write inequalities to represent this problem. (b) Represent this problem graphically, shading the region which satisfies all the inequalities. 

Purposeful Practice

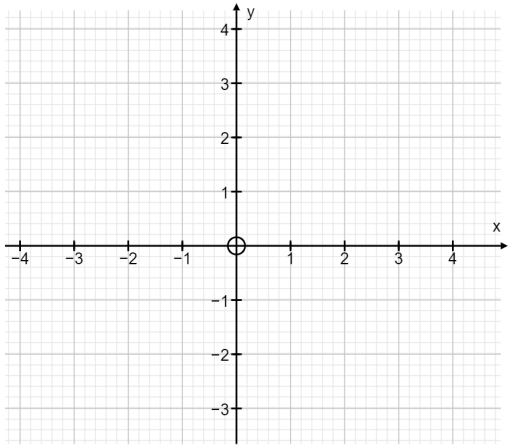
Identify the region described by the inequalities and calculate the perimeter of the region.

$$y \geq 1$$
$$x \geq 2$$
$$2y + x \leq 6$$



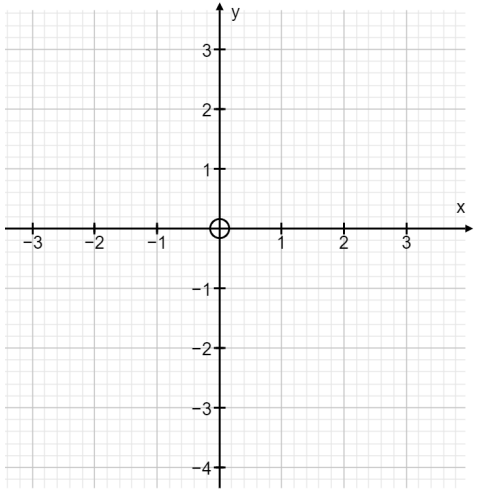
Perimeter:

$$y \geq 0$$
$$y \leq 1.5x$$
$$y \geq 3x - 9$$
$$y \leq 3$$



Perimeter:

$$y \leq 4x + 11$$
$$5y + 4x \leq 7$$
$$y \geq -1$$

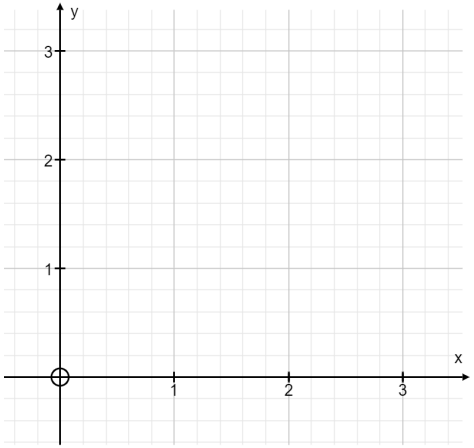


Perimeter:

Purposeful Practice

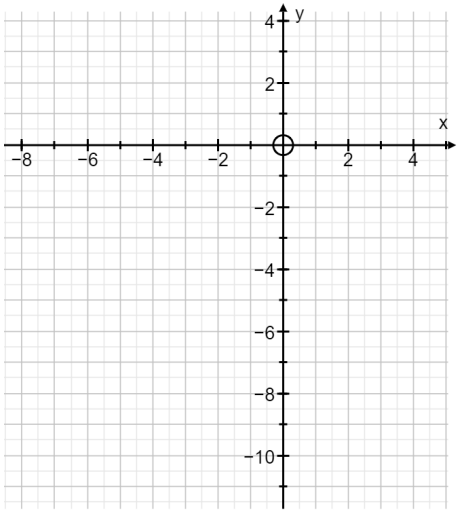
Identify the region described by the inequalities and calculate the perimeter of the region.

$$y \geq \frac{2}{3}x$$
$$y \leq 1.5x$$
$$x \leq 3$$
$$y \leq 3$$



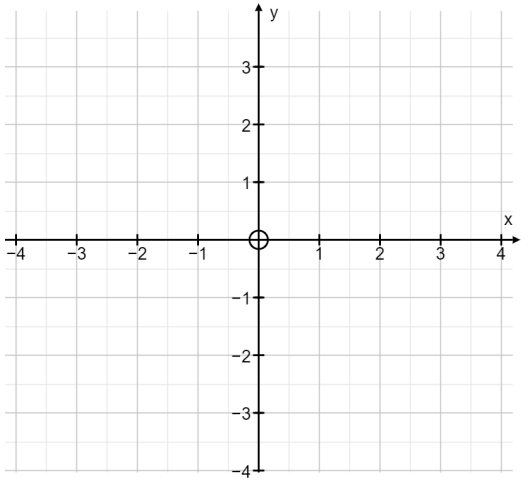
Perimeter:

$$2y + 2x \leq 7$$
$$2x - 2y \leq 7$$
$$2y - 4x \geq 7$$



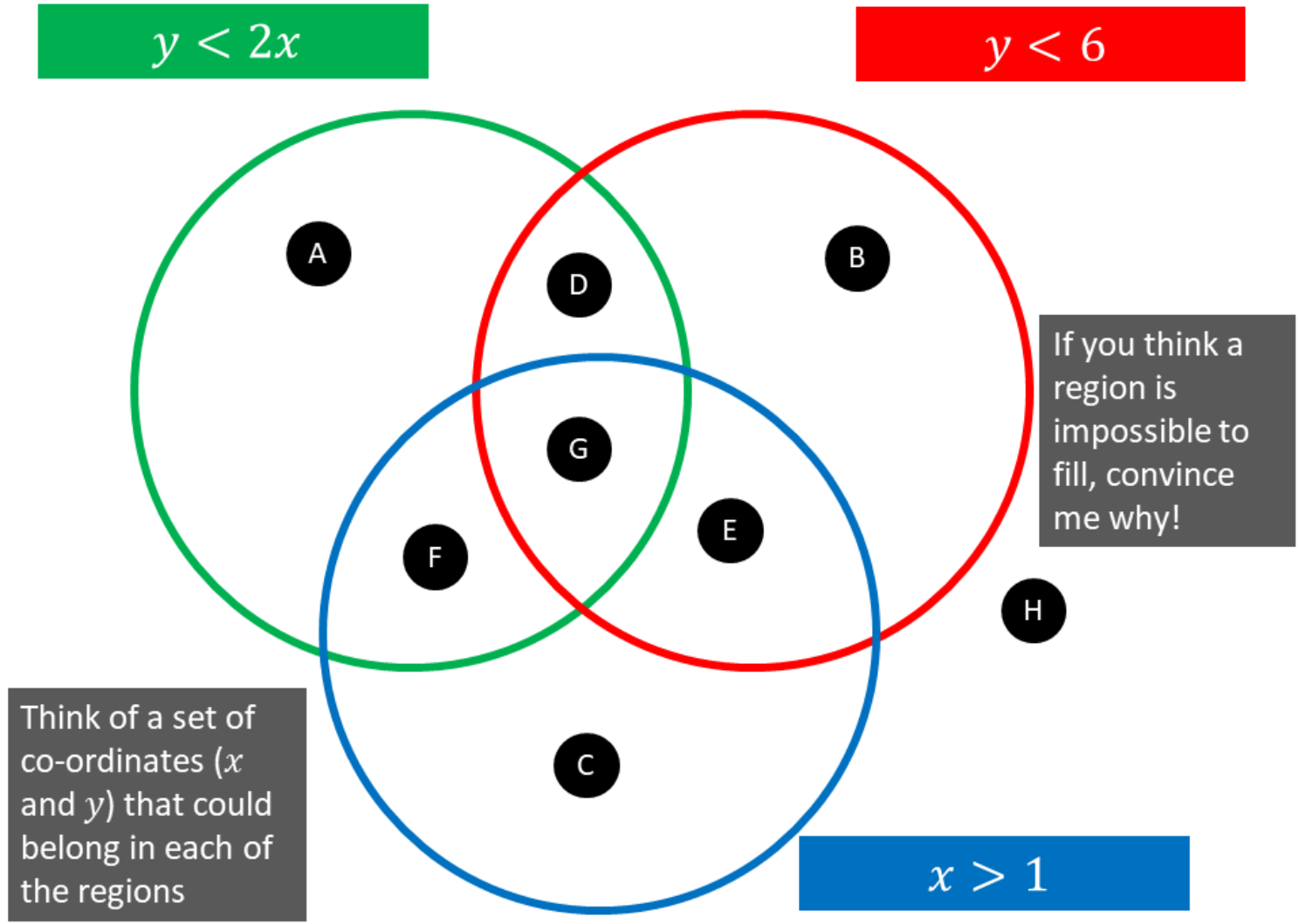
Perimeter:

$$3y + x \leq 6$$
$$4x - 3y \leq 9$$
$$3y - x \leq 6$$
$$4x + 3y \geq -9$$

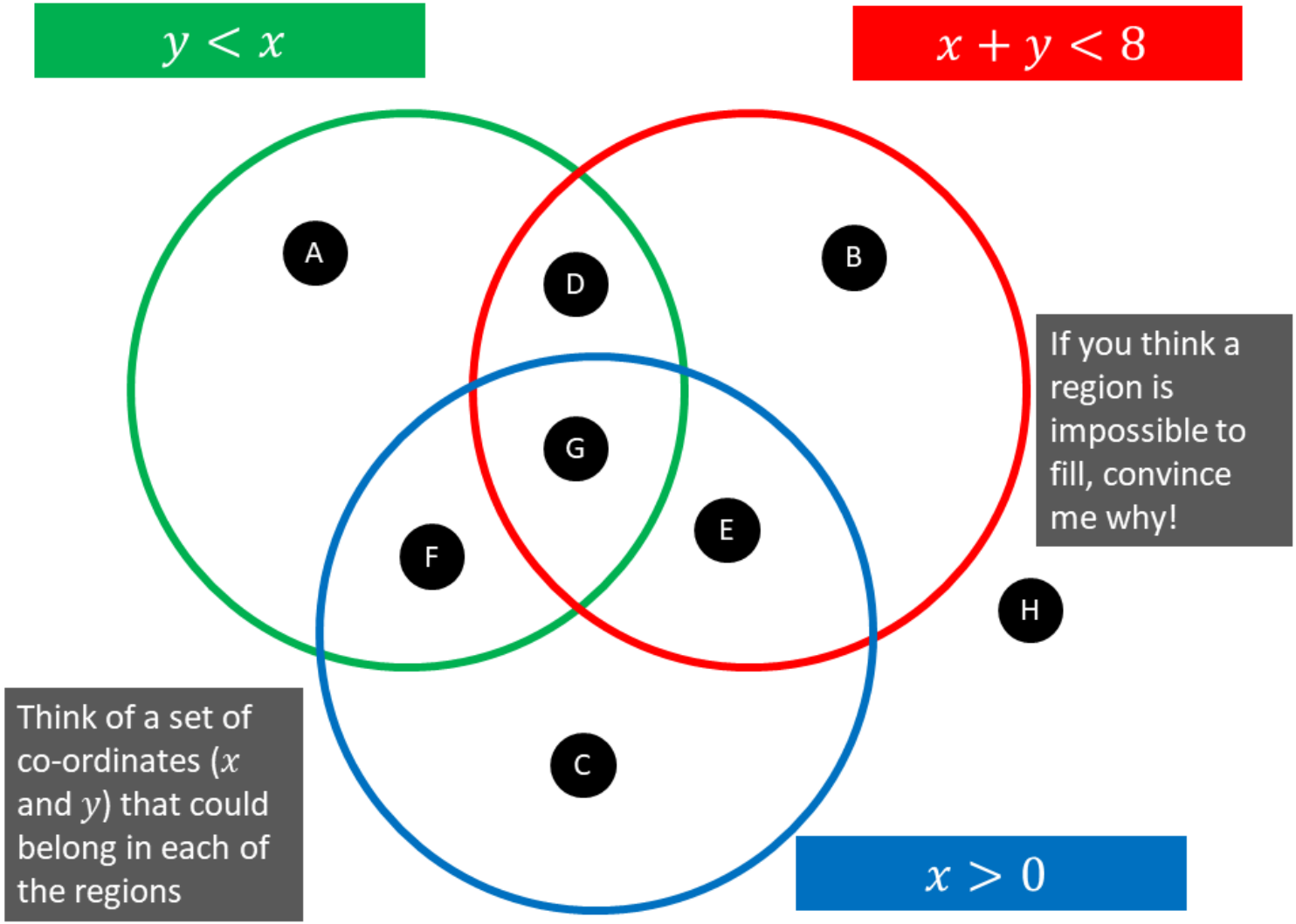


Perimeter:

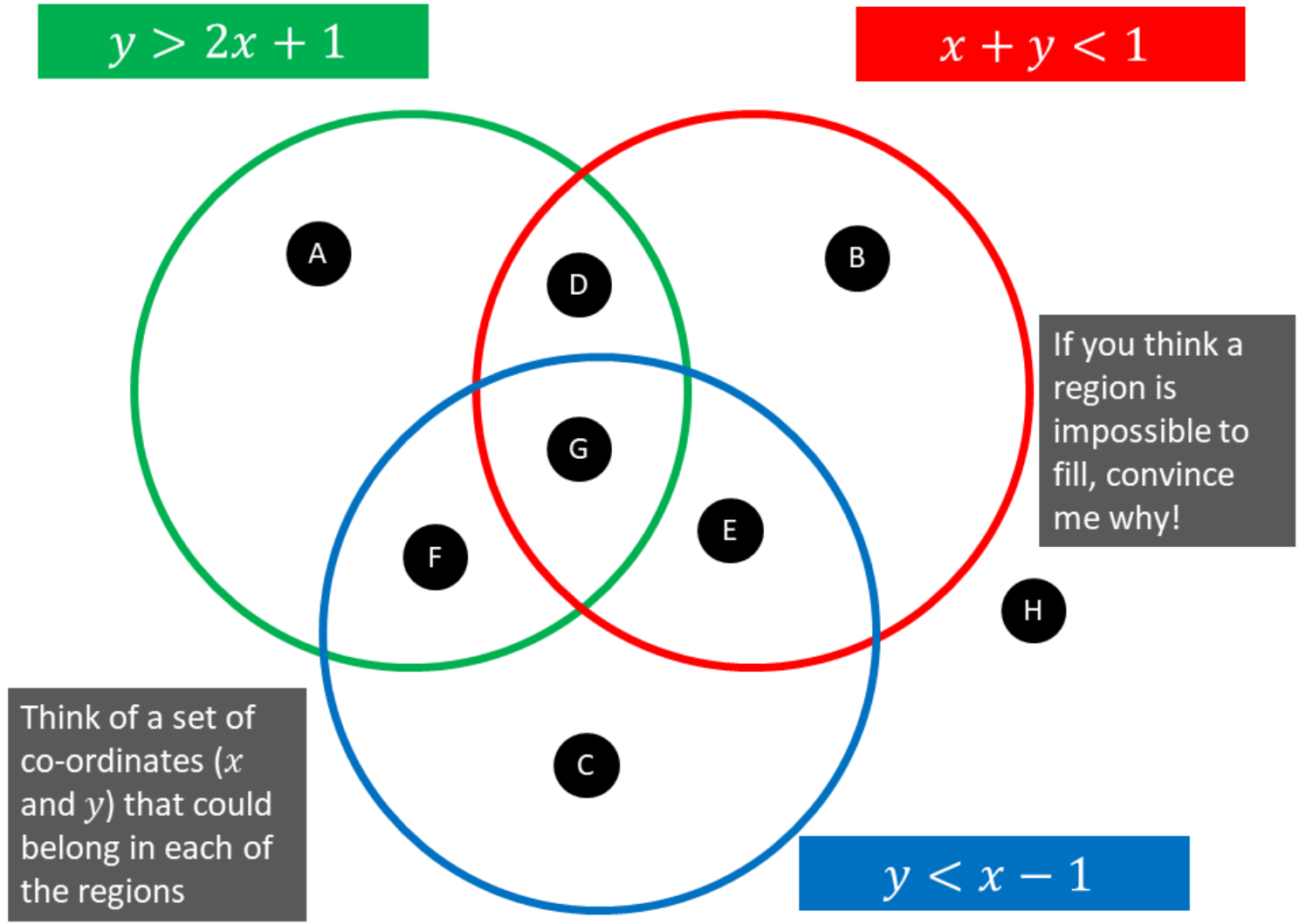
Problem Solving



Problem Solving



Problem Solving



4 Graphical Simultaneous Equations

Fluency Practice

Solving Simultaneous Equations Graphically

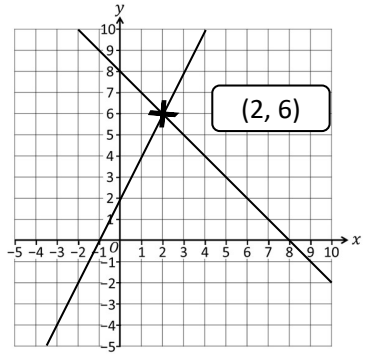
Plot each equation on the same grid.

You may want to rearrange the equations first.

The intersection shows the values (x, y) that satisfy both equations.

Check the solution by substituting values back into the equations.

$y = 2x + 2$ $\quad$ $y + x = 8$



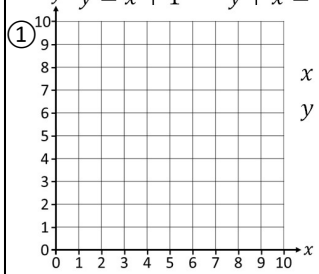
Solution: $x = 2$ $\quad$ $y = 6$

Substitute to check:

$y = 2x + 2$ $\quad$ 6 = 2(2) + 2 ✓

$y + x = 8$ $\quad$ 6 + 2 = 8 ✓

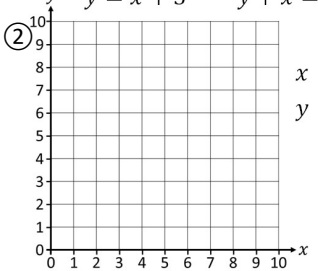
① $y = x + 1$ $\quad$ $y + x = 9$



$x =$

$y =$

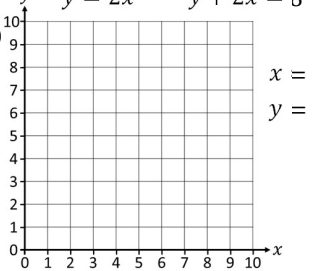
② $y = x + 3$ $\quad$ $y + x = 7$



$x =$

$y =$

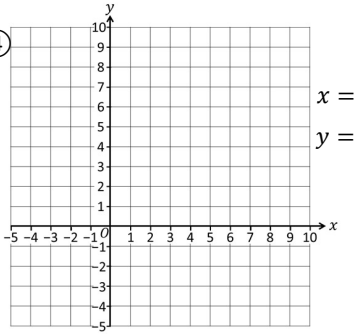
③ $y = 2x$ $\quad$ $y + 2x = 8$



$x =$

$y =$

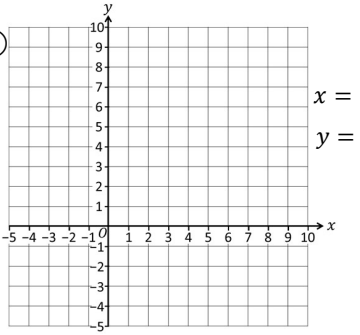
④ $y = 3x - 3$ $\quad$ $2y + 3x = 12$



$x =$

$y =$

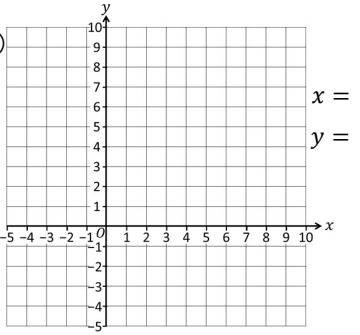
⑤ $y = 5 - 2x$ $\quad$ $y = 0.5x - 5$



$x =$

$y =$

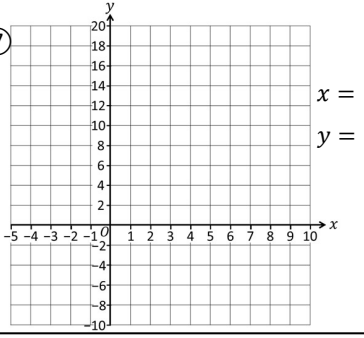
⑥ $y - x = -3$ $\quad$ $2y - 3x = -9$



$x =$

$y =$

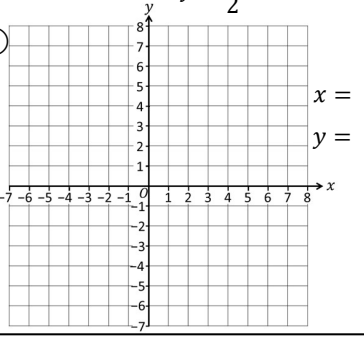
⑦ $2y + 5x = 30$ $\quad$ $y = 6x - 2$



$x =$

$y =$

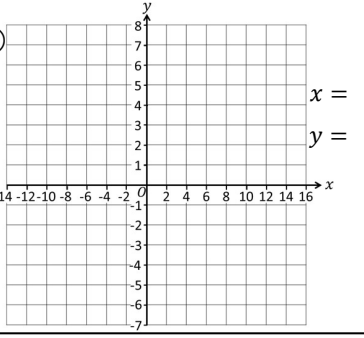
⑧ $4y + 5x = -20$ $\quad$ $y = \frac{1}{2}x + 5.5$



$x =$

$y =$

⑨ $2x = -24 - 6y$ $\quad$ $y - 10 = 2x$



$x =$

$y =$

Plot both lines on the same graph. The point of intersection represents the solution.

1. $y = 3x - 1$

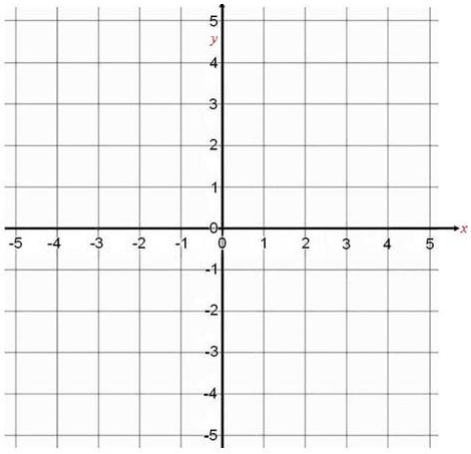
$y = 2x$

$y = 3x - 1$

x	-3	-2	-1	0	1	2	3
y							

$y = 2x$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

2. $y = 2x - 1$

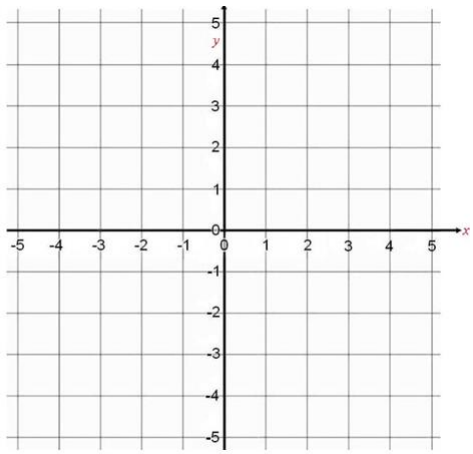
$y = x$

$y = 2x - 1$

x	-3	-2	-1	0	1	2	3
y							

$y = x$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

3. $y = 3x - 2$

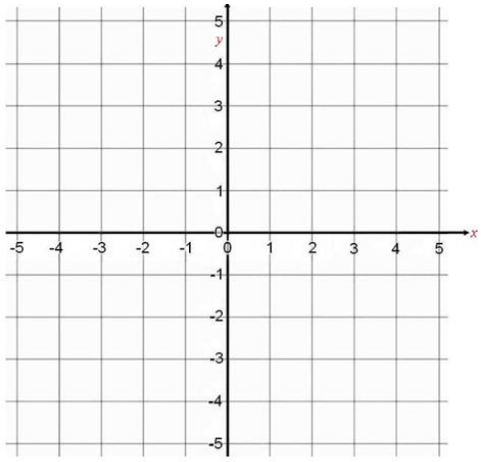
$y = x - 2$

$y = 3x - 2$

x	-3	-2	-1	0	1	2	3
y							

$y = x - 2$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

4. $y = 5 - x$

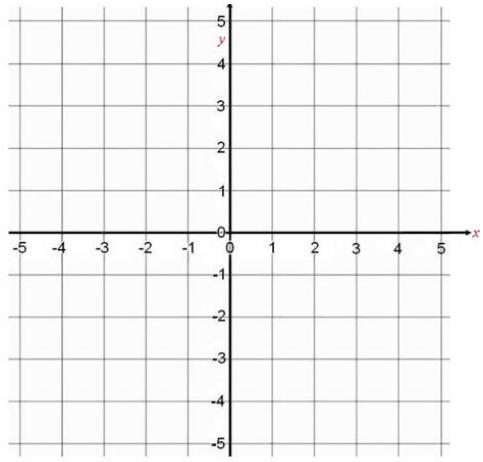
$y = 2x - 1$

$y = 5 - x$

x	-3	-2	-1	0	1	2	3
y							

$y = 2x - 1$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

learn by heart

A function is a type of equation, because it contains an equals sign.

When we plot the graph of a function we can see all the pairs of values that make it true. These are also called the solutions of the equation.

Every function has an infinite number of solutions, but if we take any two functions it is possible that there is just one pair of values that solves both of them. To find this solution we can plot graphs of both functions and see where they cross. This is called solving the equations simultaneously - at the same time.

example

By plotting graphs, find estimates for the solution to the simultaneous equations:
$y = 2x + 4$ and $y = 5 - 3x$

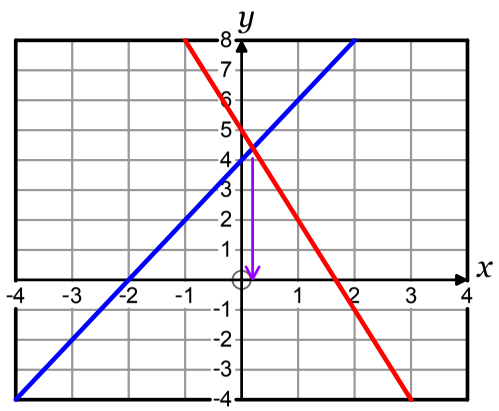
First create tables of values and plot each function:

$y = 2x + 4$

x	y
-1	2
0	4
1	6
2	8

$y = 5 - 3x$

x	y
-1	8
0	5
1	2
2	-1



They cross when x is approximately 0.2 and y is approximately 4.2, so $x \approx 0.2$, $y \approx 4.2$ is an estimate of the solution.

exercise 7o

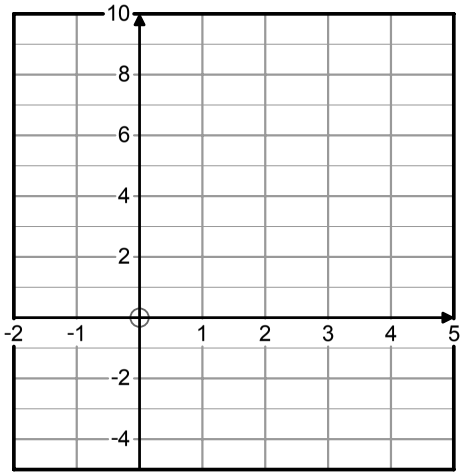
1. By plotting graphs, estimate the solution to the simultaneous equations:
   $y = 4x - 5$ and $y = 6 - 2x$

$y = 4x - 5$

x	y
0	
1	
2	
3	

$y = 6 - 2x$

x	y
-1	
0	
1	
2	



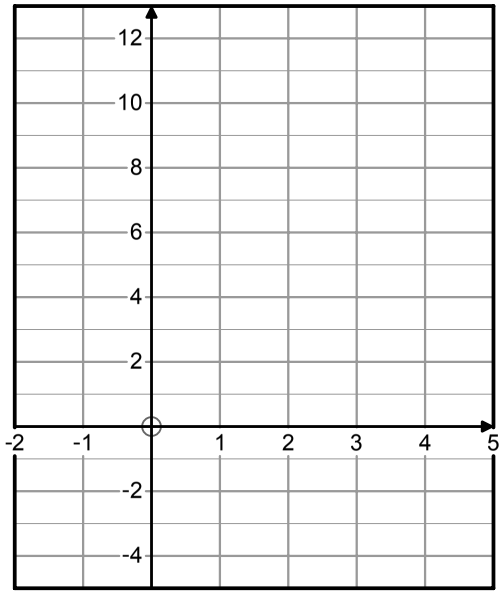
2. By plotting graphs, estimate the solution to the simultaneous equations:
$y = 2x - 3$ and $y = 10 - x$

$y = 2x - 3$

x	y
-1	
0	
1	
2	

$y = 10 - x$

x	y
-1	
0	
1	
2	



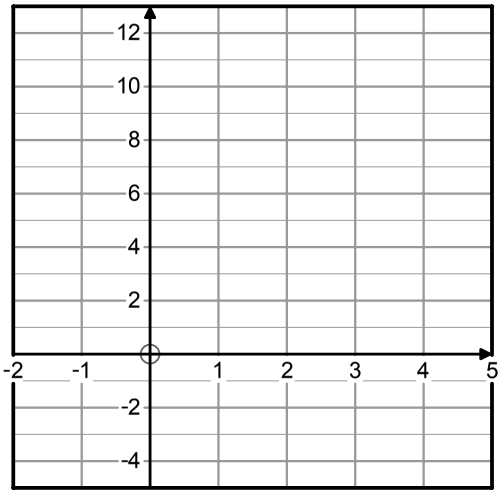
3. By plotting graphs, estimate the solution to the simultaneous equations:
$y = 4x - 5$ and $y = 5 - 2x$

$y = 4x - 5$

x	y
0	
1	
2	
3	

$y = 5 - 2x$

x	y
-1	
0	
1	
2	



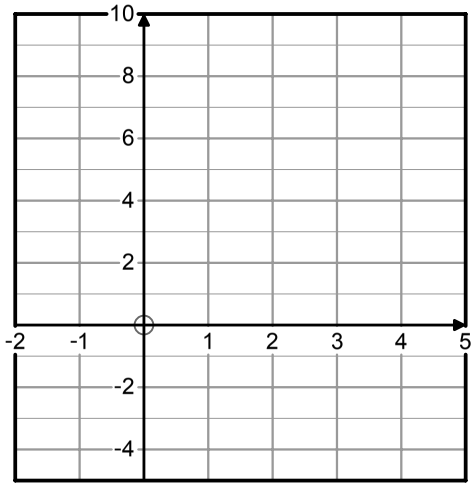
4. By plotting graphs, estimate the solution to the simultaneous equations:
$y = 8 - x$ and $y = 3x - 1$

$y = 8 - x$

x	y
0	
1	
2	
3	

$y = 3x - 1$

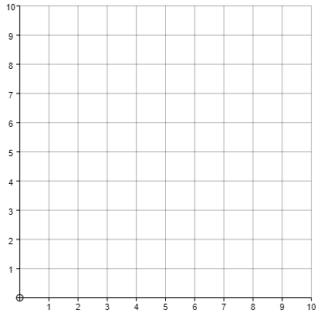
x	y
-1	
0	
1	
2	



Solving Simultaneous Equations Graphically

(a)

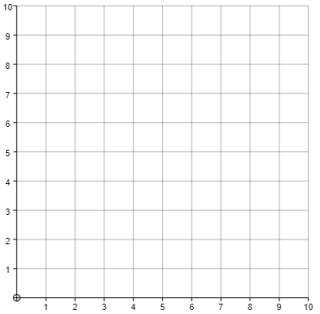
Plot the graphs of
$x + y = 5$ and $x + 3y = 9$



Find the solution to the simultaneous equations $x + y = 5$ and $x + 3y = 9$

(b)

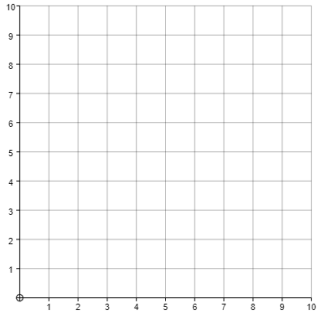
Plot the graphs of
$x + y = 7$ and $5x + 2y = 20$



Find the solution to the simultaneous equations $x + y = 7$ and $5x + 2y = 20$

(c)

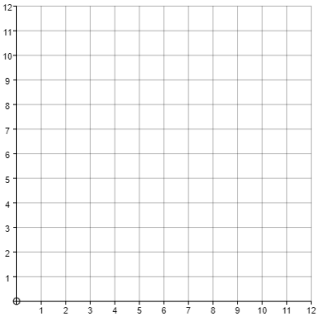
Plot the graphs of
$y = x + 1$ and $x + y = 9$



Find the solution to the simultaneous equations $y = x + 1$ and $x + y = 9$

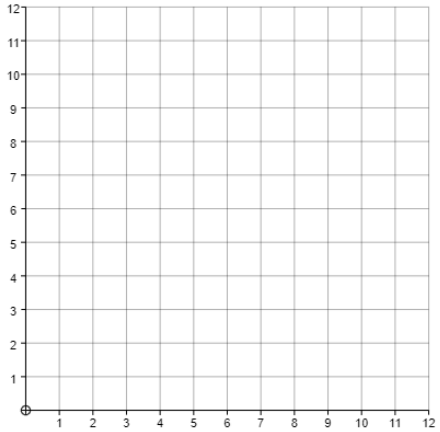
(d)

Plot the graphs of
$y = 2x$ and $2x + y = 12$



Find the solution to the simultaneous equations $y = 2x$ and $2x + y = 12$

(e)



Plot the graphs of $x + y = 8$, $y = x$, $y = \frac{1}{2}x + 5$ and $x + y = 11$

Use the graphs to find the solutions of the simultaneous equations:

(i) $x + y = 8$ and $y = x$

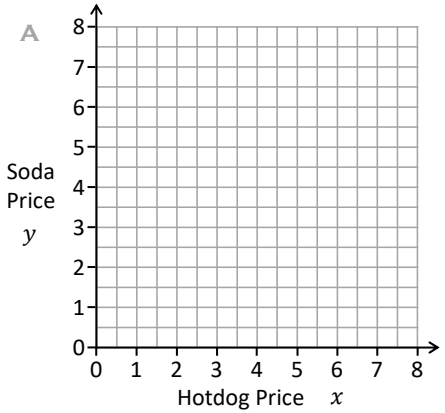
(ii) $x + y = 8$ and $y = \frac{1}{2}x + 5$

(iii) $y = x$ and $x + y = 11$

Purposeful Practice

Simultaneous Equations

A



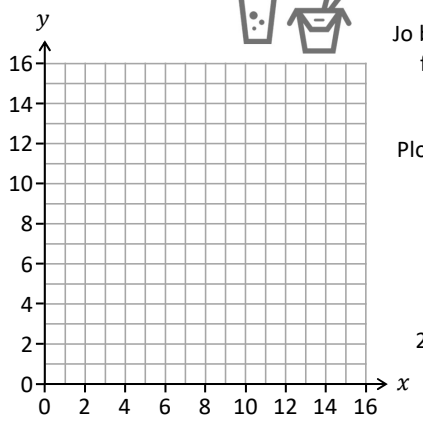
1 hotdog & 1 soda costs £6

1 hotdogs & 2 sodas costs £8

Write 2 equations in x & y to represent these purchases.

Plot both of the equations on the grid.
What do the coordinates of the intersection tell us?

B

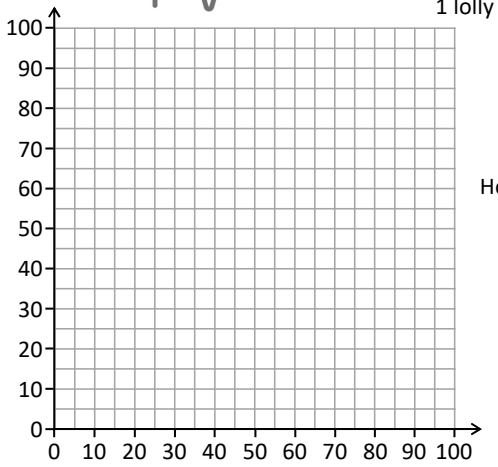


Jo buys 4 sodas and some noodles for £16 and Tye buys 2 sodas and some noodles for £12.

Plot and solve these purchases as simultaneous equations.

Hugh wants to buy 2 sodas and 3 lots of noodles. How much will it cost?

C



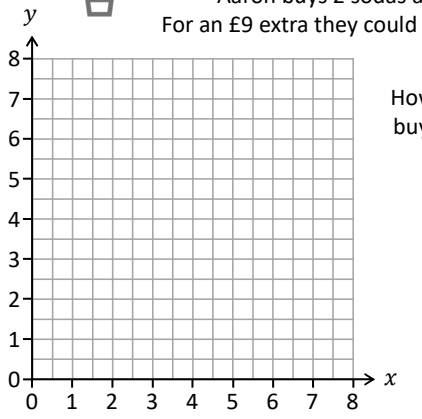
3 lollies and 1 cone costs 90p

1 lolly and 2 cones costs 80p

How much will 5 lollies & 4 cones cost?

D

Aaron buys 2 sodas and 2 tacos for £15.
For an £9 extra they could buy 4 sodas and 3 tacos.

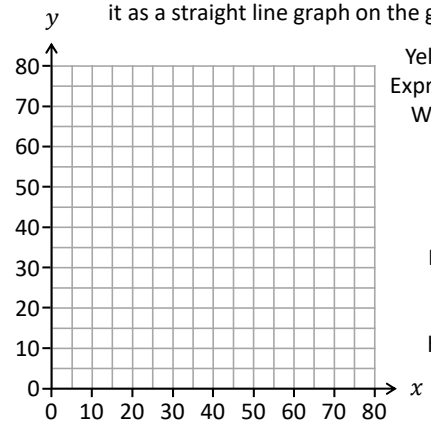


How much does it cost to buy 3 sodas and 4 tacos?

Purposeful Practice

Creating Simultaneous Equations

1) Xavier (x) and Yeliah (y) will share £60.
Create an equation to express this and plot it as a straight line graph on the grid.



Yeliah wants 5 times Xavier's share.
Express this as an equation and plot it.
What does the intersection show?

Instead, Yeliah's share is half the size of Xavier's share.
Plot the graph and find how much they should both get.

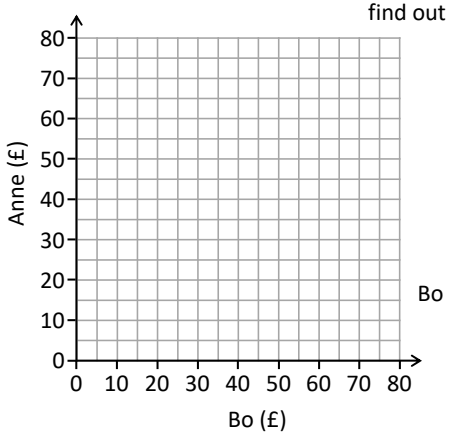
2) Anne and Bo will share £80.
Show this relationship on the grid.

Express and plot these equations & find out how much each person would get.

Anne gets £20 more than Bo.

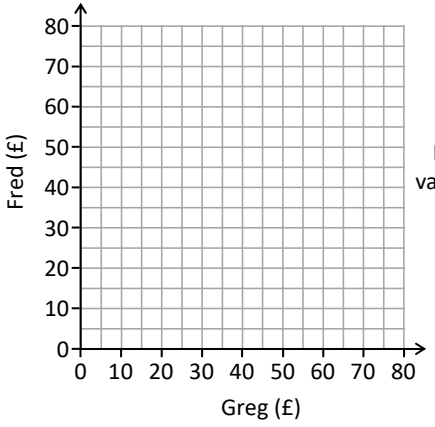
Bo gets £25.

Bo gets 3 times the money Anne gets.



3) Fred and Greg will share £70.
Fred will get £10 more than double what Greg gets.

Express this problem as a pair of simultaneous equations.



Plot both equations to find the values that satisfy both equations.

If they shared £55 how much would each person get?

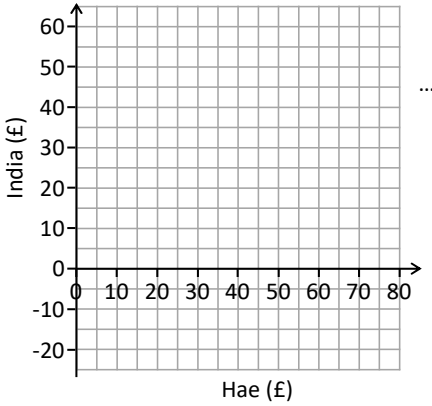
4) Hae and India will share £60.

How much do they get each if…

…India gets £15 more than Hae.

…Hae gets a third of what India gets.

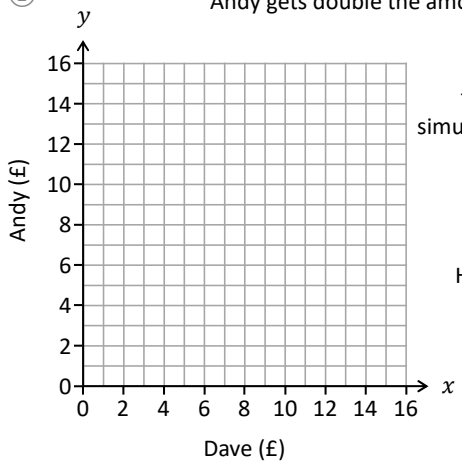
…India gets £15 less than quadruple what Hae gets.



Purposeful Practice

How much!?

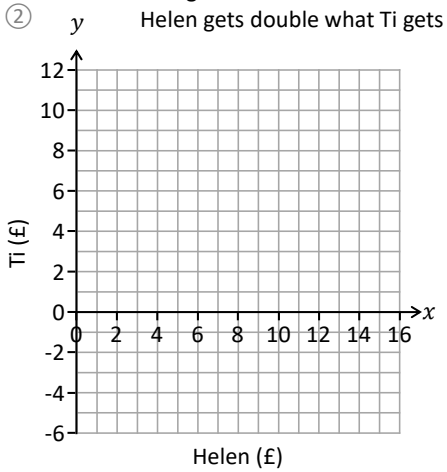
① Andy gets £4 more than Dave.
Andy gets double the amount Dave gets



Represent these two statements as a pair of simultaneous equations on the grid.

How much do they both get?
How much was shared?

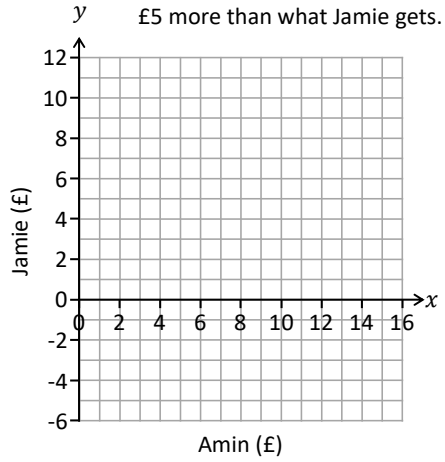
② Ti gets £5 less than Helen.
Helen gets double what Ti gets



Represent these two statements as a pair of simultaneous equations on the grid.

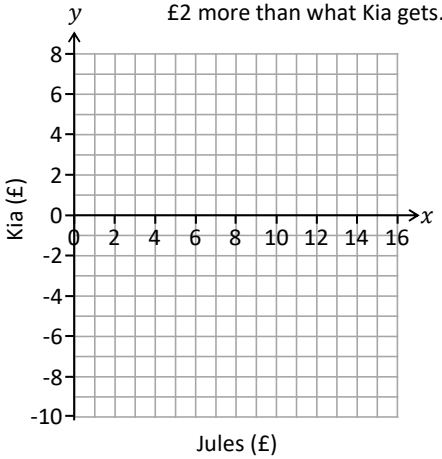
How much do they both get?
How much was shared?

③ Jamie gets £2 more than Amin.
If you double the amount Amin gets, it is £5 more than what Jamie gets.



How much do they both get?
How much was shared?

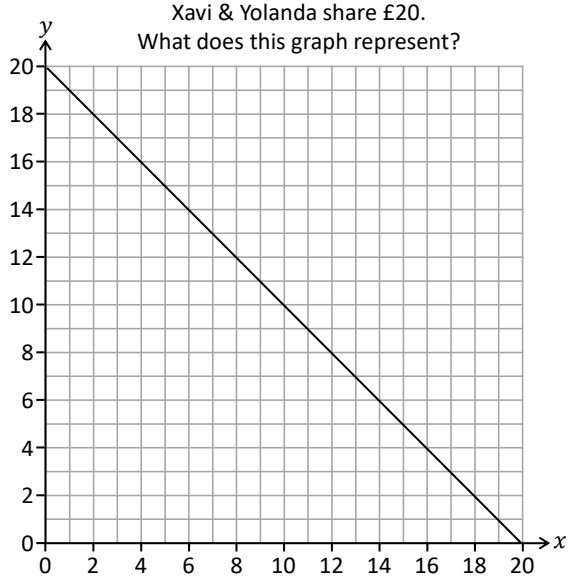
④ Kia gets £8 less than Jules.
If you halve the amount Jules gets, it is still £2 more than what Kia gets.



How much do they both get?
How much was shared?

Purposeful Practice

Xavi & Yolanda share £20.
What does this graph represent?



The money can be split in different ways.
Express and plot each split as an equation.
How much does each person get?

Xavi gets £6.

Xavi =

Yolanda =

Yolanda gets 4 times what Xavi gets.

Xavi =

Yolanda =

Yolanda gets £2 more than Xavi.

Xavi =

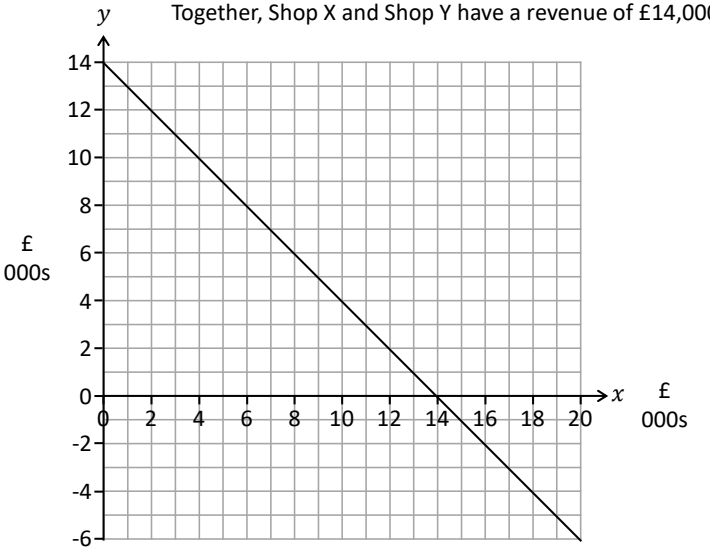
Yolanda =

Yolanda gets a quarter of what Xavi gets.

Xavi =

Yolanda =

Together, Shop X and Shop Y have a revenue of £14,000.



Express each different split of the revenue as an equation on the grid.
Use intersections to solve the simultaneous equations.

Shop Y earnt £6000 less than Shop X.

X =

Y =

Shop X earnt a third of what Shop Y earnt.

X =

Y =

If you halve the amount Shop X earnt,
it is £5000 less than Shop Y.

X =

Y =

Shop X's revenue is £6000 greater than
triple Shop Y's revenue.

X =

Y =

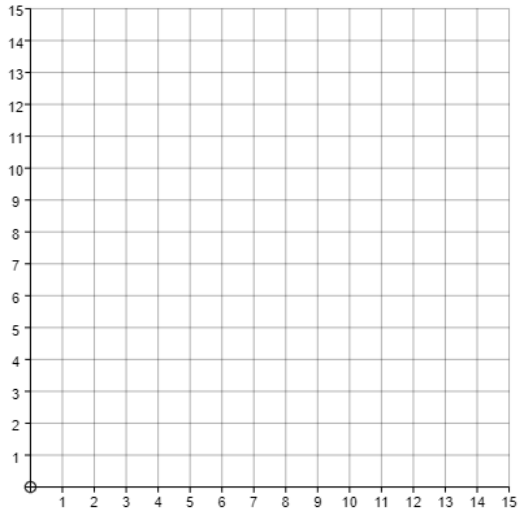
Fluency Practice

Investigating Simultaneous Equations

(a)

Solve the simultaneous equations $x + 2y = 10$, $2x + y = 14$

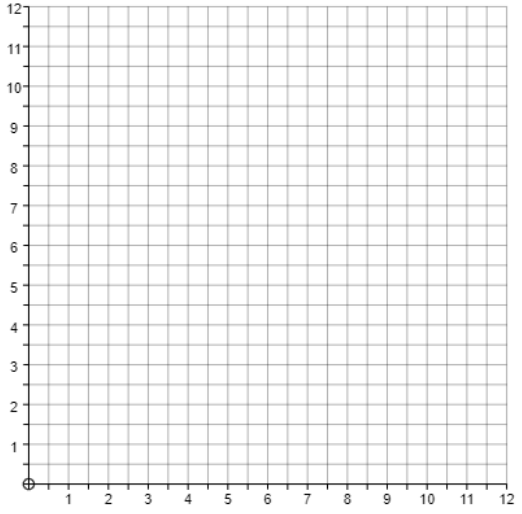
Plot the equations of the lines $x + 2y = 10$ and $2x + y = 14$ on the same axes below. What do you notice?



(b)

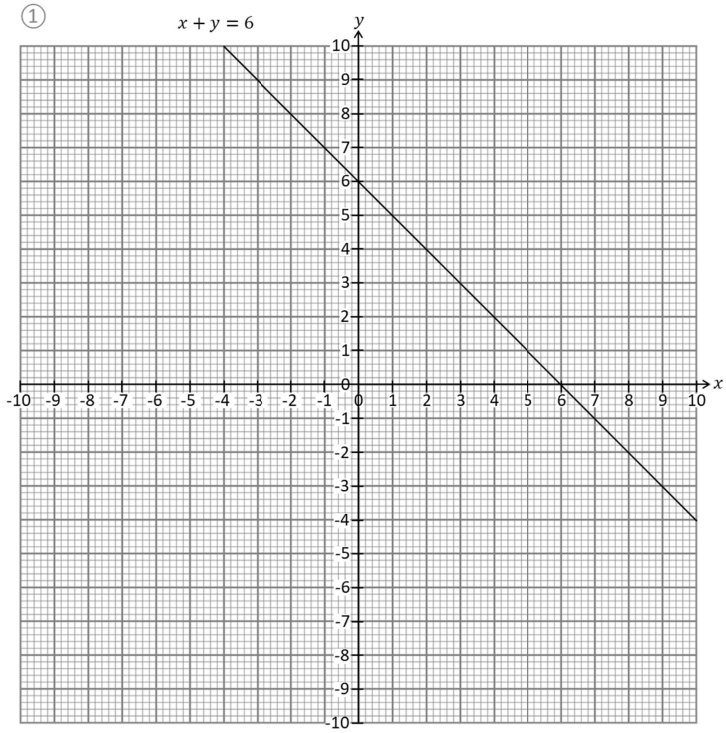
Solve the simultaneous equations $4x + 3y = 24$, $x + 2y = 11$

Plot the equations of the lines $4x + 3y = 24$ and $x + 2y = 11$ on the same axes below. What do you notice?



Plotting Simultaneous Equations

①



Plot & Solve...

$x + y = 6$
$y = 1$

$x =$
$y =$

$x + y = 6$
$y = 2x$

$x =$
$y =$

$x + y = 6$
$y = 2x - 9$

$x =$
$y =$

$y = 2x - 9$
$x + y = -3$

$x =$
$y =$

$x + y = -3$
$y = 0.5x + 3$

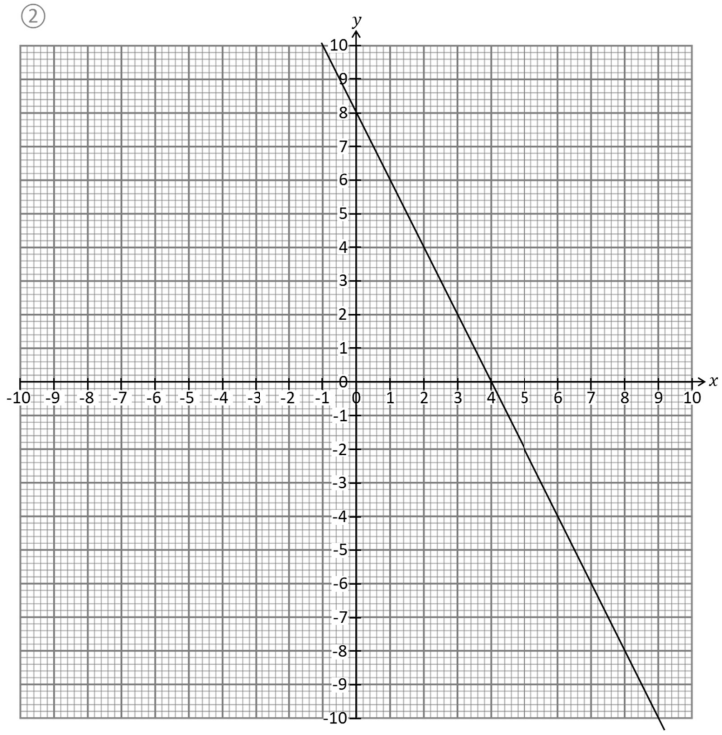
$x =$
$y =$

$x + y = -3$
$y + 2x = 2$

$x =$
$y =$

Plotting Simultaneous Equations

②



Plot & Solve...

$2x + y = 8$
$y = 2x - 4$

$x =$
$y =$

$y = 2x - 4$
$y + 2x = 4$

$x =$
$y =$

$y + 2x = 4$
$y = 0.5x + 9$

$x =$
$y =$

$2y = x + 18$
$3x + y = -5$

$x =$
$y =$

$y - 2x + 4 = 0$
$2x + 3y = -12$

$x =$
$y =$

$2x + 3y = -12$
$3x - 2y = -4$

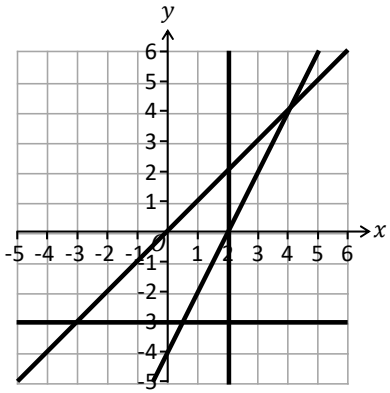
$x =$
$y =$

Purposeful Practice

Identifying Simultaneous Equations

① The grid shows 4 equations:

$y = x$ $\quad x = 2$ $\quad y = -3$ $\quad y = 2x - 4$



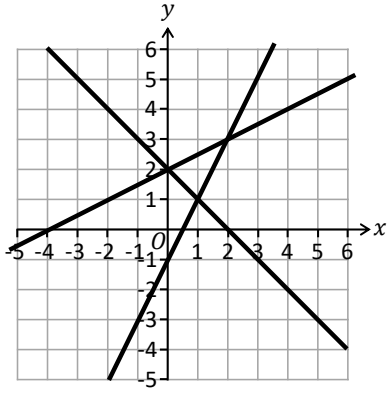
(2, 2) is the solution to which pair of simultaneous equations?

(0.5, −3) is the solution to which pair of simultaneous equations?

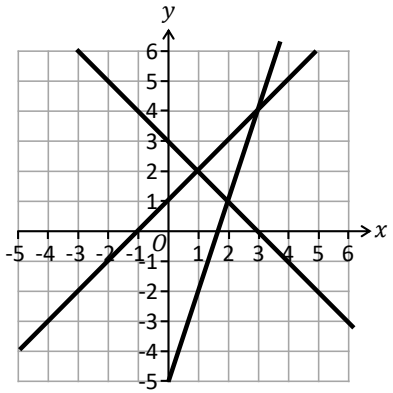
(4, 4) is the solution to which pair of simultaneous equations?

② The grid shows 3 equations. $y = 2x - 1$ $\quad y = \frac{x}{2} + 2$ $\quad y + x = 2$

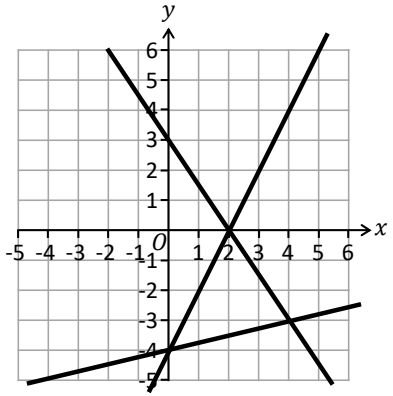
Write down the 3 pairs of simultaneous equations, and their solutions, shown on the grid.



③ What pairs of simultaneous equations, and their solutions, are on the grid?



④ What pairs of simultaneous equations, and their solutions, are on the grid?

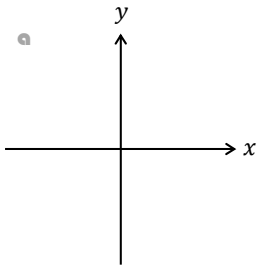


Fluency Practice

Solution Sketching

Sketch each pair of simultaneous equations on the grid. What method can you use to find the solutions?

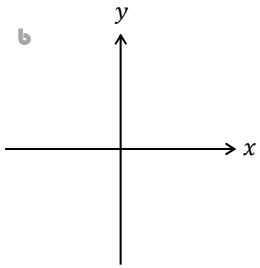
a



$y = x$

$x = 4$

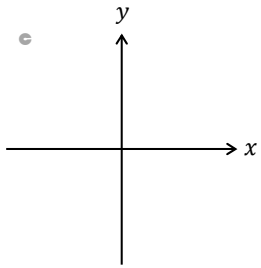
b



$y = 2x$

$y = -4$

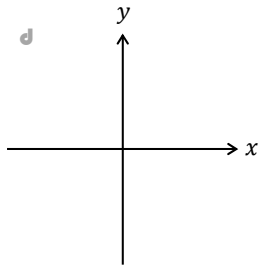
c



$y = 6$

$y = x + 4$

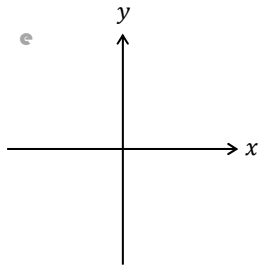
d



$y = x$

$y = x - 2$

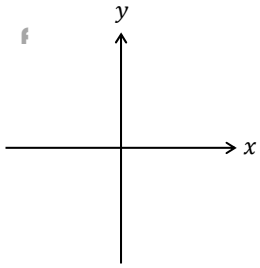
e



$y = -x$

$y = x - 6$

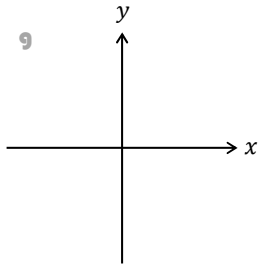
f



$x + y = 8$

$y = 3x$

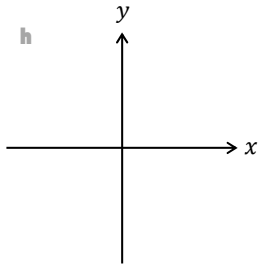
g



$y = -2x$

$x + y = 2$

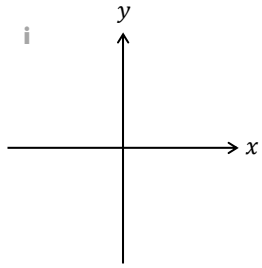
h



$2y - 2x = 10$

$y = x + 5$

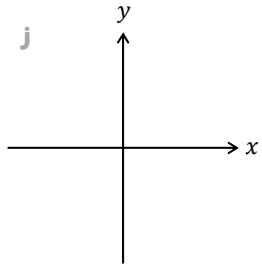
i



$x + y = -5$

$y = 2x - 2$

j

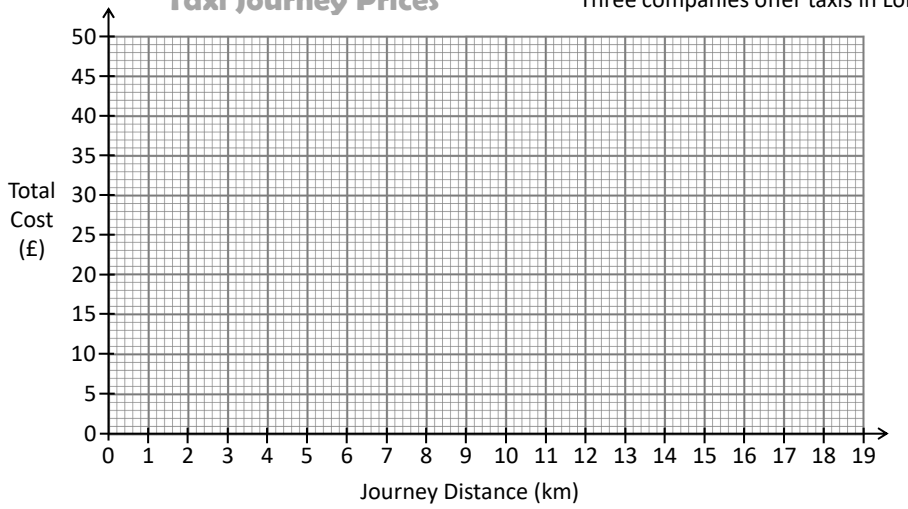


$2x + y = 10$

$y = 3x - 15$

Purposeful Practice

Taxi Journey Prices



Three companies offer taxis in London:

Quick Dash: £3 per mile
Zooom: £2 per mile with a £5 pick-up fee
Kar: 50p per mile with a £20 pick-up fee

Sketch these cost-relationships on the grid.

At what distance is…

… Zooom and Quick Dash the same price?

… Zooom and Kar the same price?

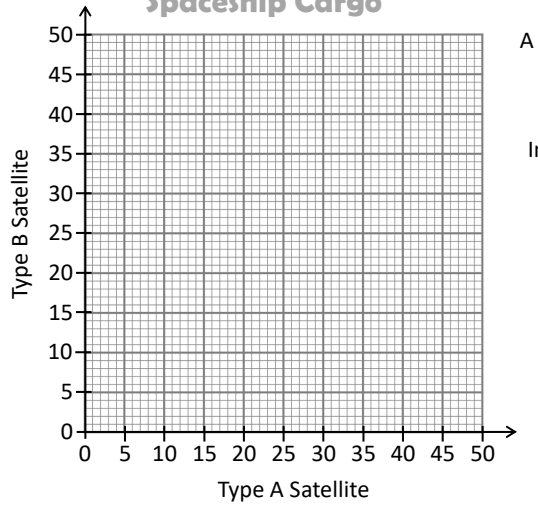
… Quick Dash and Kar the same price?

When is Zooom the cheapest option?

Uri has £22 to spend. Draw this graph and explain any intersections.

Lee wants to travel 13.5 km. *Estimate* the prices they could pay.

Spaceship Cargo



A spaceship will be loaded with 2 different types of mini-satellites (A & B).
In total the spaceship can carry 45 satellites.

Show this limit as a graph on the grid.

In discussion…

… Scientist G argues for an equal amount of A & B satellites.

… Scientist I argues that there should be 15 more B satellites than As onboard.

… Scientist H says there should be double the amount of A satellites compared to B satellites.

Draw each scientist's proposition as a graph and find solutions.

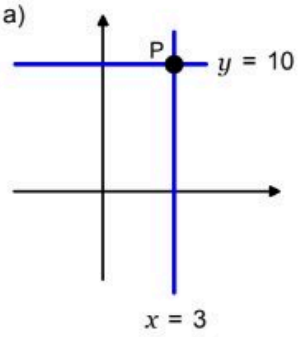
Before launch it is discovered the spaceship can only carry 39 satellites.
Show & find solutions for the new cargo possibilities.

Using the lines above, complete the table below, give your solutions to one decimal place where necessary.

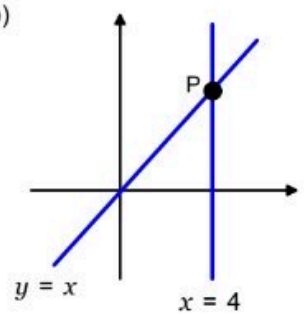
Line:	A	B	C	D
Equation:			$x + 2y = 4$	$x + 4y = 4$
Gradient				
Solution with A:				
Solution with B:				
Solution with C:				
Solution with D:				

10. For each graph, work out the co-ordinates of the point marked P.

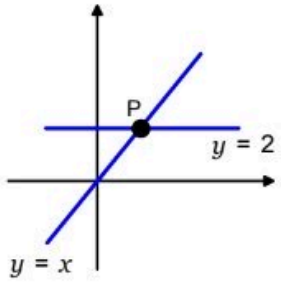
a)



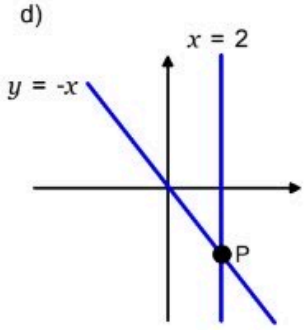
b)



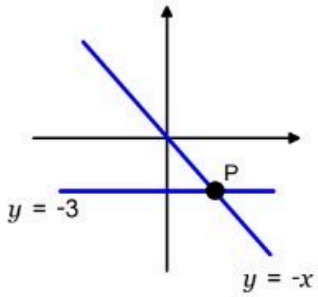
c)



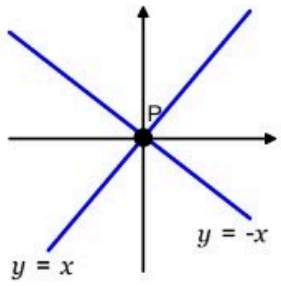
d)



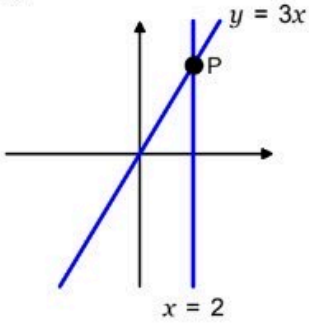
e)



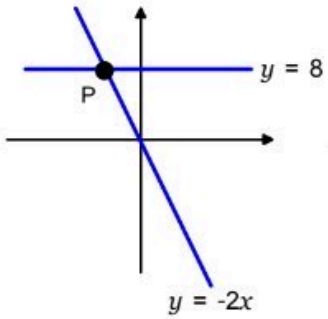
f)



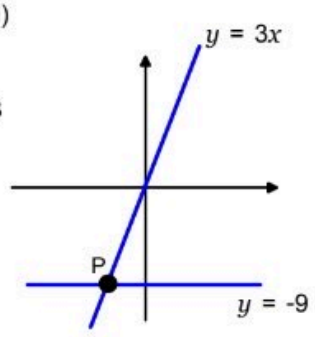
g)



h)



i)



5 Linear Simultaneous Equations

For each statement, circle the correct response.

1	$x = 2$ and $y = 3$ are the solutions to the simultaneous equations $x + y = 5$ and $x - y = 1$	True	False
2	$x = 5$ and $y = 2$ are the solutions to the simultaneous equations $2x + y = 12$ and $x + 5y = 15$	True	False
3	$x = 4$ and $y = 1$ are the solutions to the simultaneous equations $3x + 4y = 16$ and $x - y = 3$	True	False
4	$x = 6$ and $y = 3$ are the solutions to the simultaneous equations $2x - 3y = 6$ and $4x - y = 21$	True	False
5	$x = 3$ and $y = -2$ are the solutions to the simultaneous equations $x + y = 1$ and $x - y = 5$	True	False
6	$x = 4$ and $y = -5$ are the solutions to the simultaneous equations $2x + y = 13$ and $x - 2y = 14$	True	False
7	$x = 1.5$ and $y = -1$ are the solutions to the simultaneous equations $6x - 2y = 11$ and $4x + 3y = 9$	True	False
8	$x = \frac{1}{2}$ and $y = \frac{3}{2}$ are the solutions to the simultaneous equations $x + y = 2$ and $7x - y = 2$	True	False
9	$x = -0.5$ and $y = -4$ are the solutions to the simultaneous equations $2x - y = 3$ and $2x + 3y = -11$	True	False
10	$x = \frac{2}{3}$ and $y = 5$ are the solutions to the simultaneous equations $3x - y = -3$ and $\frac{3}{2}x + 2y = 11$	True	False

1. Solve:

a) $4a + 3b = 44$
$3a + 3b = 39$

b) $2a + b = 23$
$a + b = 16$

c) $5a + 2b = 27$
$5a + 4b = 49$

d) $3a + 2b = 16$
$4a + 6b = 28$

e) $2a + b = 14$
$4a + 6b = 36$

f) $6a + 5b = 39$
$2a + 3b = 17$

2. Explain why it is not possible to solve the pair of simultaneous equations:

$$x + y = 9$$
$$2x + 2y = 18$$

3. Select the correct pair of solutions to the simultaneous equations shown in the box:

$$p + q = 7$$
$$5p + 6q = 36$$

a) $p = 3$, $q = 4$

b) $p = 6$, $q = 1$

c) $p = 1$, $q = 6$

4. Solve:

a) $2a + 5b = 25$
$3a + 4b = 27$

b) $3a + 5b = 17$
$5a + 2b = 22$

c) $4a + 3b = 33$
$6a + 5b = 51$

d) $2a + 3b = 5$
$2a + 5b = 11$

e) $3a + 2b = -11$
$5a + 4b = -21$

f) $3a + 7b = -6$
$2a + 3b = 1$

5. Match pairs of equations with their solutions:

A $3x + y = 8$, $3x + 4y = 14$	B $2x + y = -2$, $3x + 2y = -1$	P $x = 3$, $y = 7$	Q $x = 1$, $y = 4$
C $3x + 2y = 11$, $2x + 3y = 14$	D $4x + 3y = -7$, $2x + 2y = -6$	R $x = 4$, $y = 2$	S $x = 2$, $y = 2$
E $6x + 5y = 34$, $4x + 3y = 22$	F $2x + 4y = -12$, $x + 3y = -9$	T $x = -3$, $y = 4$	U $x = -2$, $y = -1$
G $x + y = 10$, $2x + 5y = 41$	H $x + 2y = -4$, $2x + 3y = -7$	V $x = 2$, $y = -5$	W $x = 0$, $y = -3$

1. Solve:

a) $4p - 3q = 10$
$3p + 6q = 24$

b) $8t - 3u = 61$
$t - u = 7$

c) $2k + 3p = 30$
$4k - 5p = 16$

d) $-x + 3y = -11$
$4x + 2y = 16$

e) $-5c + 2d = 21$
$2c - 3d = -15$

f) $-6x + 4y = -42$
$-4x - 5y = -5$

2. Select the correct pair of solutions to the simultaneous equations shown in the box:

$-2g + 3h = 0$
$3g - h = 7$

a) $g = 2$
$h = -1$

b) $g = -6$
$h = 4$

c) $g = 3$
$h = 2$

3. Jake and Lucy are trying to solve the simultaneous equations shown.

$4x + 3y = 20$
$3y - 3x = 6$

Jake ***adds*** the two equations.
Lucy ***subtracts*** the two equations.

Who has correctly eliminated one unknown?

4. Solve:

a) $3a + 2b = -13$
$2a - 3b = -39$

b) $2c + 3d = 7$
$-c + 6d = -26$

c) $g + 3h = -3$
$h - 2g = -8$

d) $-3x = y + 8$
$2x - y = -7$

e) $4u + 3w - 3 = 0$
$3u - 2w = 32$

f) $-2x + 5y = -14$
$-2y + 5x = -7$

5. Spot the mistake!

a) $4a + 3b = 18$
$5a - 3b = 9$

$4a + 3b = 18$
$5a - 3b = 9$ −
$a = 9$

$36 + 3b = 18$

$a = 9, b = -6$

b) $2a + 3b = 4$
$4a + 5b = 10$

$4a + 6b = 4$
$4a + 5b = 10$ −
$b = -6$

$2a - 18 = 4$

$a = 11, b = -6$

c) $-a + 3b = 3$
$a - b = 1$

$-a + 3b = 3$
$a - b = 1$ +
$4b = 4$
$b = 1$

$a - 1 = 1$

$a = 2, b = 1$

Spot The Error*

*one error is the use of an inefficient method.

Solve **A**

3x - 2y = 8

x + y = 2

Solution

$3x - 2y = 8$

$2x + 2y = 2$ +

$5x = 10$

$x = 2$

$y = 0$

Solve **B**

2x + y = 17

x - 2y = -1

Solution

$2x + y = 17$

$2x - 4y = -2$ -

$-3y = 15$

$y = -5$

$x = 11$

Solve **C**

3x - 5y = 27

2x + 5y = 8

Solution

$2x = 8 - 5y$

$3x = 1.5(8 - 5y)$

$1.5(8 - 5y) - 5y = 27$

$12 - 7.5y - 5y = 27$

$12 - 27 = 12.5y$

$-1.2 = y$

$2x - 6 = 8$; $x = 7$

Solve **D**

3x + y = 10

5x - 4y = 12

Solution

$y = 10 - 3x$

$5x - 4(10 - 3x) = 12$

$5x - 40 - 3x = 12$

$2x = 52$, $x = 26$

$130 - 4y = 12$

$130 - 12 = 4y$

$y = 29.5$

When you have finished, work out the correct answer for each pair of equations.

Fluency Practice

Simultaneous Equations

$2x = y \qquad y = x + 8$

$2x = y = x + 8$

$2x = x + 8$

$x = 8$

$2x = y \qquad 2(8) = 16$

$x \longrightarrow (8, 16) \longleftarrow y$

a) $2x = y \qquad y = x + 4$

$2x = x + 4$

b) $3x = y \qquad y = x + 12$

c) $3x = y \qquad y = x - 4$

d) $x + 3 = y$

$y = 2x - 2$

e) $x + 7 = y$

$y = 3x - 9$

f) $x + 10 = y$

$y = 3x + 18$

g) $2x + 9 = y$

$y = x + 12$

h) $2x + 6 = y$

$y = 3x + 4$

i) $y = 2x + 8$

$y = 4x - 6$

j) $y = 2x + 11$

$y = 5 - x$

k) $y = \frac{x}{2}$

$y = x - 6$

l) $y = \frac{x}{2} + 3$

$y = x - 1$

m) $y = \frac{x}{3} + 2$

$y = 2x - 3$

n) $2y = 3x + 3$

$y = x + 4$

o) $2y = 2x + 10$

$y = x + 5$

p) $y = x - 6$

$3y = 2x - 16$

q) $2y = 2x - 22$

$y = \frac{x}{2} - 7$

r) $x + 3 = y$

$y = \frac{x + 11}{2}$

s) $2y + 10 = 2x$

$y = 2x - 11$

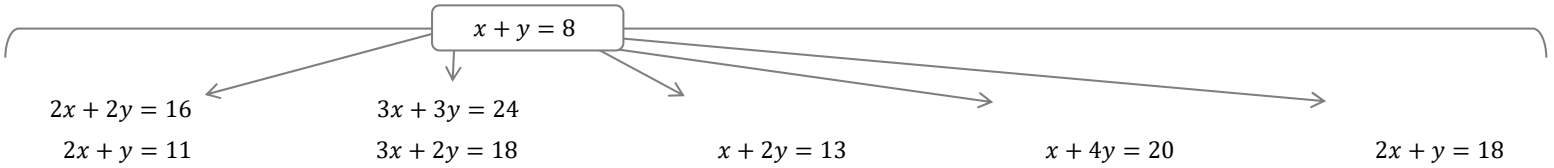
t) $2y = 3x - 2$

$3y = 2x + 7$

Fluency Practice

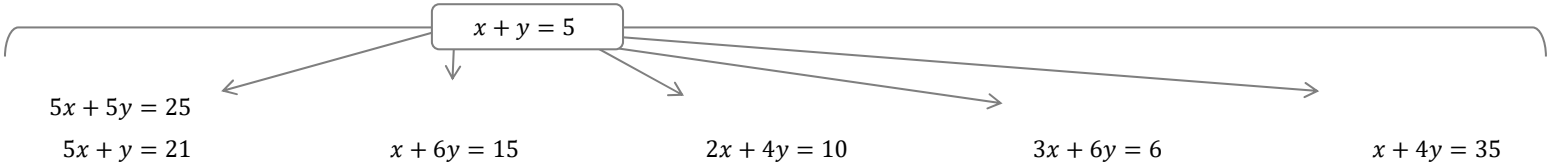
Manipulating Equations to Solve

Use $x + y$ to solve the other equations (the variable values will be different each time).



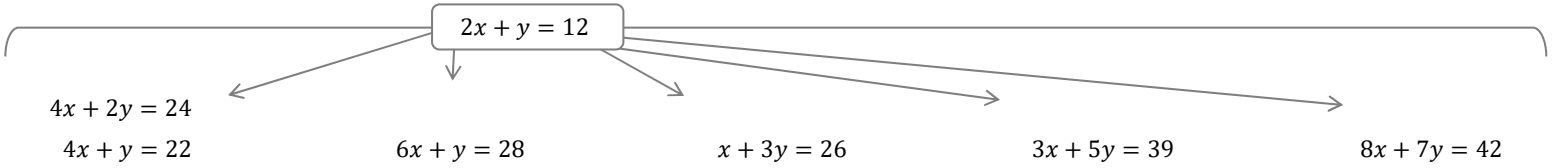
$x =$ $x =$ $x =$ $x =$ $x =$

$y =$ $y =$ $y =$ $y =$ $y =$



$x =$ $x =$ $x =$ $x =$ $x =$

$y =$ $y =$ $y =$ $y =$ $y =$



$x =$ $x =$ $x =$ $x =$ $x =$

$y =$ $y =$ $y =$ $y =$ $y =$

Fluency Practice

Substitution Solving

a

If $3x + 2 = 14$

and $y + x = 9$

$y = ?$

b

If $2x - 5 = 7$

and $y + 2x = 15$

$y = ?$

c

If $5x - 8 = 7$

and $2y + 2x = 18$

$y = ?$

d

If $8 - x = 3$

and $3y + 3x = 27$

$y = ?$

e

If $19 - 2x = 7$

and $2x + 2y = 16$

$y = ?$

f

If $3(x + 5) = 21$

and $6y + 5x = 10$

$y = ?$

g

If $3(2x + 5) = 33$

and $3(y + x) = 36$

$y = ?$

h

If $4x + 13 = 5$

and $3y + 2x = 26$

$y = ?$

i

If $\frac{3x + 1}{2} = 11$

and $4x - 2y = 22$

$y = ?$

j

If $2(5 - 2x) = 26$

and $8 - 3x = 4y$

$y = ?$

Fluency Practice

Mixed Written Methods to Solve Simultaneous Equations

Do you prefer a method?

Eliminate (subtract)

$3x + y = 19$
$x + y = 9$

$x =$ $y =$

$4x + 2y = 18$
$4x - y = 3$

Eliminate (add)

$3x - y = 7$
$x + y = 13$

$x =$ $y =$

$5x + 3y = 32$
$2x - 3y = -4$

Balance & Eliminate

$3x + y = 9$
$x + 2y = 13$

$x =$ $y =$

$x - 2y = 6$
$3x + y = 32$

Substitute

$x = 2y + 3$
$x + 2y = 19$

$x =$ $y =$

$3x + 2y = 14$
$y = 3x - 2$

Rearrange & Substitute

$x - y = 6$
$x + 3y = 14$

$x =$ $y =$

$x = 9 - y$
$2x + 3y = 21$

$3x + 2y = 15$
$3x + y = 12$

$x + y = 8$
$3x + 2y = 18$

$3x + 5y = 28$
$2x + 5y = 22$

$x = 3y - 2$
$3x + 2y = 16$

$x + 3y = 2$
$2x + y = 9$

$2x + 3y = 23$
$6x + y = 29$

$x - y = 8$
$4x + 2y = 44$

$x + 4y = 22$
$3x - 4y = 18$

Fluency Practice

Solving Simultaneous Equations using Substitution

Ex)

$$x + y = 12$$
$$y = x + 2$$

$$x + (\boldsymbol{x} + \mathbf{2}) = 12$$
$$2x + 2 = 12$$
$$2x = 10$$
$$x = 5$$

$$y = (\mathbf{5}) + 2$$
$$y = 7$$

a) $x + y = 10$, $y = x + 4$

b) $x + y = 10$, $y = x - 6$

c) $2x + y = 9$, $y = x - 3$

d) $x + y = 13$, $y = 2x + 4$

---

e) $x + 2y = 14$, $y = x + 1$

f) $x + 3y = 14$, $y = x + 2$

g) $2x - y = 7$, $y = x - 2$

h) $x + 2y = 17$, $y = 2x + 1$

i) $x + 3y = 33$, $y = 2x - 3$

---

j) $2x + 3y = 30$, $y = 3x - 1$

k) $2x + 3y = 28$, $x = 4y + 3$

l) $3x + 2y = 15$, $y = 9 - 2x$

m) $4x + 2y = 32$, $y - 4x = 4$

n) $5x - y = 12$, $2y - 6x = -8$

Fluency Practice

Substitute & Solve

1) $a = b + 1$
$a + b = 5$ $a =$ $b =$

2) $a = b + 3$
$a + b = 9$ $a =$ $b =$

3) $a = 3b$
$a + b = 8$ $a =$ $b =$

4) $a = b - 4$
$a + b = 10$ $a =$ $b =$

5) $x = 2y$
$2x + y = 20$ $x =$ $y =$

6) $a = 2b + 1$
$a + b = 13$ $a =$ $b =$

7) $a = 3b + 2$
$a + b = 6$ $a =$ $b =$

8) $g = h + 3$
$2g + h = 21$ $g =$ $h =$

9) $b = 2a + 3$
$a + 3b = 23$ $a =$ $b =$

10) $a - b = 4$
$a + 3b = 12$ $a =$ $b =$

11) $a - b = 4$
$2a + b = 17$ $a =$ $b =$

12) $m - n = 2$
$2m - n = 10$ $m =$ $n =$

13) $a + b = 6$
$a + 2b = 11$ $a =$ $b =$

14) $k + p = 6$
$2k + p = 9$ $k =$ $p =$

15) $a + b = 10$
$3a + 2b = 22$ $a =$ $b =$

16) $a + 2b = 13$
$2a + b = 11$ $a =$ $b =$

17) $x - 2y = -2$
$3x - y = 9$ $x =$ $y =$

18) $2a + 3b = 23$
$3a + 2b = 17$ $a =$ $b =$

Purposeful Practice

1) In each question say which variable we want to eliminate

a) $5x + 3y = 22$
$2x + 3y = 10$

b) $5x + 3y = 22$
$5x + 2y = 10$

c) $5x + 3y = 22$
$-5x + 2y = 10$

d) $4x - y = 25$
$x + y = 10$

e) $-3x - y = 25$
$-3x + 2y = 10$

2) For each part in question 1, would we add or subtract?

a) b) c) d) e)

3) Tom says that we need to add these to eliminate the x.
Mark says you need to subtract them to eliminate the y.
Who do you agree with? Why?

$$3x - y = 9$$
$$-3x + y = -9$$

4) Tarquin says, *"you add the equations because the equations have different signs in the middle."*

$$11x + 4y = 19$$
$$11x - 3y = 13$$

5) Now solve each pair of equations in question one.

(1) Complete this table, to solve each pair of simultaneous equations.

Equations	Solved for 1st Variable	Solved for 2nd Variable
$3x - y = 1$ $x + y = 3$	$3x - y = 1$ $+ \quad x + y = 3$ $4x \quad = 4$ $x \quad = 1$	Sub $x = 1$ into either equation: $1 + y = 3$ $y = 2$
$2x - y = 2$ $x + y = 7$		
$4x + y = 9$ $2x - y = 3$		
$-x + 2y = 13$ $x + y = 8$		
$2x + y = 7$ $x + y = 4$		
$3x + y = 9$ $2x + y = 7$		
$5x - 2y = 13$ $3x + 2y = 3$		

(2) Complete this table, to solve each pair of simultaneous equations. One of the equations will need to be 'scaled'.

Equations	Scaled	Solved for 1st Variable	Solved for 2nd Variable
$5x + 2y = 11$ $3x - 4y = 4$	$\times 2$ → $10x + 4y = 22$ $3x - 4y = 4$	$10x + 4y = 22$ $+ \quad 3x - 4y = 4$ $13x = 26$ $x = 2$	Sub $x = 2$ into any equation. $5(2) + 2y = 11$ $10 + 2y = 11$ $2y = 1$ $y = \frac{1}{2}$
$3x + 2y = 8$ $2x - y = 3$			
$3x + 2y = 11$ $2x - y = -3$			
$2x + 3y = 11$ $3x + y = 13$			
$2x - 3y = 8$ $x - 5y = 11$			
$x + 6y = 0$ $3x - 2y = -10$			
$5x - 4y = 24$ $2x = y + 9$			

③ Complete this table, to solve each pair of simultaneous equations. One of the equations will need to be 'scaled'.

Equations	Scaled	Solved for 1st Variable	Solved for 2nd Variable
$2x + 3y = 5$ $5x - 2y = -16$	$\times 2$: $4x + 6y = 10$ $\times 3$: $15x - 6y = -48$	$4x + 6y = 10$ $+ \quad 15x - 6y = -48$ $19x = -38$ $x = -2$	Sub $x = -2$ into any equation. $2(-2) + 3y = 5$ $-4 + 3y = 5$ $3y = 9$ $y = 3$
$2x + 5y = 24$ $4x + 3y = 20$			
$2x + 3y = 11$ $3x + 4y = 15$			
$3x + 8y = 27$ $-4x - 3y = -13$			
$2x + 3y = 14$ $8x - 5y = 5$			
$9x - 4y = -20$ $5x - 6y = -13$			
$2y = 3x - 5$ $1 = 5x - 4y$			

Fluency Practice

A1 Solve $x+y=1$ $x-y=5$	**A2** Solve $5x+y=17$ $x+y=3$	**A3** Solve $6x-5y=9$ $6x+3y=33$	**A4** Solve $x+5y=-13$ $4x-5y=48$
B1 Solve $8x-y=7$ $12x-8y=6$	**B2** Solve $3x-2y=13$ $x-y=5$	**B3** Solve $2x-3y=3$ $3x+6y=1$	**B4** Solve $3x+5y=7$ $9x+11y=13$
C1 Solve $5x+3y=14$ $2x+2y=4$	**C2** Solve $4x+5y=13$ $3x-2y=27$	**C3** Solve $2x+7y=12$ $3x+8y=13$	**C4** Solve $3x+2y=7$ $4x-3y=15$
D1 Solve $y=4x-8$ $y=3x-5$	**D2** Solve $y=4x-2$ $y=9x-12$	**D3** Solve $3x+2y=61$ $y=2x-22$	**D4** Solve $3x-2y=-24$ $y=5-2x$

Fluency Practice

Q	Solve the simultaneous equations:	Answers
1	$7x + 2y = 29$ and $3x + 2y = 17$	
2	$7x + 2y = 29$ and $y = 2x - 2$	
3	$7x + 2y = 13$ and $3x + 2y = 1$	
4	$7x + 2y = 13$ and $y = 2x - 10$	
5	$7x + 2y = -20$ and $3x + 2y = -12$	
6	$7x + 2y = -20$ and $x = y + 1$	
7	$7x - 2y = 29$ and $3x - 2y = 17$	
8	$7x - 2y = 29$ and $y = x - 7$	
9	$7x - 2y = -20$ and $3x - 2y = -12$	
10	$7x - 2y = -20$ and $y = x + 5$	
11	$-7x - 2y = 29$ and $-3x - 2y = 17$	
12	$-7x - 2y = 29$ and $x = 1 - y$	

Q	Solve the simultaneous equations:	Answers
13	$3x + 4y = 11$ and $-3x + y = 14$	
14	$3x + 4y = 11$ and $y = 3x + 14$	
15	$5x - 0.7y = 46$ and $6x + 0.7y = 9$	
16	$5x - 0.7y = 46$ and $y = -6x$	
17	$2x + 3y = 9$ and $5x + 7y = 23$	
18	$2x + 3y = 9$ and $x = 5 - y$	
19	$3x - 2y = -5$ and $2x - 4y = 2$	
20	$3x - 2y = -5$ and $x = 2y + 1$	
21	$3x + 2y = 7$ and $4x - 3y = 15$	
22	$3x + 2y = 7$ and $y = 2x - 7$	
23	$ax + by = c$ and $ax + dy = e$	
24	$ax + by = c$ and $y = dx + e$	

(a) Solve $4x - y = 17$
$x = y + 2$

(b) Solve $2x + y = 6$
$y = 4x + 3$

(c) Solve $3x + 7y = 13$
$y = x - 11$

(a) Solve $4x - 3y = 7$
$3y = x + 5$

(b) Solve $y + 1 = 3x$
$2x - 3y = 24$

(c) Solve $3x + 5y = 29$
$y + 11 = 5x$

(a) Solve $4x + 6y = 74$
$11 - y = 2x$

(b) Solve $y - 8 = 6x$
$4x + 5y + 28 = 0$

(c) Solve $8 - x = 3y$
$10 - 3x = 5y$

(a) Given that $7x = 2y + 34$ and $3x + 5y + 3 = 0$, find the value of $x^2 + y^2$

(b) Solve $\frac{3x+1}{2} = y$
$5y - 4x = 13$

(c) Find the coordinates of intersection of the straight lines with equations
$y = 3x - 2$
$x + 3y = 19$

Question 1: Solve the following simultaneous equations by using elimination.

(a) $6x + y = 18$, $4x + y = 14$

(b) $4x + 2y = 10$, $x + 2y = 7$

(c) $9x - 4y = 19$, $4x + 4y = 20$

(d) $2x + y = 36$, $x - y = 9$

(e) $6x - 3y = 12$, $4x - 3y = 2$

(f) $3x - 6y = 6$, $2x - 6y = 3$

(g) $8x + 7y = 39$, $8x + 2y = 34$

(h) $x + 3y = 38$, $x + 6y = 53$

(i) $6x + 3y = 48$, $6x + y = 26$

(j) $2x - 4y = 10$, $2x + 3y = 24$

(k) $5x - 2y = 120$, $5x + y = 165$

(l) $x - 2y = 8$, $x - 3y = 3$

(m) $3x + 2y = 54$, $2x - 2y = 16$

(n) $7x - 4y = 80$, $3x - 4y = -80$

(o) $5x - 2y = -23$, $5x - 6y = -39$

(p) $6x + 2y = -26$, $2x + 2y = -10$

(q) $x - 5y = 65$, $2x - 5y = 85$

(r) $10x - 10y = -40$, $10x + 4y = 16$

Question 2: Solve the following simultaneous equations by using elimination.

(a) $3x + 2y = 23$, $2x - y = 6$

(b) $3x - 3y = 9$, $2x + y = 12$

(c) $4x + 2y = 34$, $3x + y = 21$

(d) $9x - 4y = 59$, $2x - y = 12$

(e) $2x + 8y = 43$, $x + 3y = 18$

(f) $6x + 3y = 45$, $2x - 2y = 12$

(g) $5x + 4y = 130$, $x + 6y = 130$

(h) $10x - 15y = 25$, $x - 2y = 1$

(i) $3x + 8y = 97$, $2x + 4y = 58$

(j) $3x - y = 4$, $5x + 4y = 52$

(k) $4x + 9y = 10$, $2x + 3y = 2$

(l) $5x - 3y = 33$, $3x - 9y = 63$

(m) $2x + 4y = -2$, $4x + 2y = -10$

(n) $8x + 4y = -28$, $3x - 12y = 30$

(o) $15x - 4y = 82$, $5x - 9y = 12$

(p) $12x + 3y = 9$, $2x + 11y = -9$

(q) $9x - 7y = 111$, $x - 2y = 16$

(r) $8x - y = 4$, $3x + 8y = -166$

Question 3: Solve the following simultaneous equations by using elimination.

(a) $2x + 2y = 14$, $5x - 3y = 19$

(b) $2x + 3y = 1$, $7x + 2y = -22$

(c) $5x + 3y = 22$, $2x + 4y = 20$

(d) $5x - 6y = 28$, $4x - 4y = 24$

(e) $3x + 2y = 7$, $2x + 9y = 43$

(f) $3x + 3y = -6$, $4x - 4y = -24$

(g) $3x + 8y = 31$, $5x + 3y = 31$

(h) $7x - 15y = 2.5$, $3x - 2y = 5.5$

(i) $3x + 2y = 53$, $2x + 5y = 72$

(j) $5x - 3y = 18$, $2x + 4y = 54$

(k) $2x + 9y = 11$, $9x + 3y = -63$

(l) $2x - 4y = 4$, $5x - 3y = 24$

(m) $3x + 3y = 42$, $2x + 4y = 38$

(n) $6x + 2y = -2$, $4x - 3y = 29$

(o) $4x - 4y = 8$, $5x - 3y = 18$

(p) $4x + 3y = 9$, $5x + 2y = 13$

(q) $4x - 2y = 18$, $2x - 3y = 15$

(r) $5x + 2y = 38$, $2x - 3y = 19$

Question 4: Solve the following simultaneous equations by rearranging and then using elimination.

(a) $x = 10 - y$, $2x + y = 17$

(b) $x - 4 = y$, $x + 3y = 12$

(c) $2x + 6y = 4$, $x = 12 + 2y$

(d) $3x = 10 + 5y$, $3y = 52 - 4x$

(e) $2x + y - 18 = 0$, $3y = 7x + 80$

(f) $6x + 2y + 6 = 0$, $7x - 5y - 93 = 10$

Apply

Question 1: The cost of buying a coffee and a tea in a cafe is £4.
The cost of buying a coffee and three teas in a cafe is £7.
Work out the cost of buying a coffee and the cost of buying a tea.

Question 2: The sum of Rosemary's age and Hannah's age is 102 years.
The difference between Rosemary's age and Hannah's age is 52 years.
Rosemary is older than Hannah.
Find the age of each woman by using simultaneous equations.

Question 3: Five adult tickets and three child tickets for a movie cost £58.
Two adult tickets and eight child tickets for a movie cost £47.
Find the cost of each type of ticket.

Question 4: Four chairs and two tables cost £218.
Six chairs and seven tables cost £587.
Find the total cost of buying twenty chairs and five tables.

Question 5: A plumber charges a price for each hour, £h, and a fixed charge, £c.
A 5 hour job costs £155 in total.
A 8 hour job costs £230 in total.
How much would a job that lasts 2 hours cost?

Question 6: Barry buys 200 pieces of stationery for £76.
Of the 200 pieces of stationery, x of them are rulers that cost 50p each and y of them are pens that cost 20p each.
Find how many rulers Barry buys and how many pens he buys.

Question 7: In a greengrocers, 4kg of bananas and 3kg of apples costs £7.50
In the same greengrocers, 3kg of bananas and 5kg of apples costs £8.10
How much would 2kg of bananas and 2kg of apples cost?

Question 8: Can you spot any mistakes in the question below?

Solve the simultaneous equations

$3x + 5y = 1$ x2
$2x - 3y = 7$ x3

Do not use trial and improvement

$6x + 10y = 2$
$6x - 9y = 21$

$19y = 23$
$y = 1.21$

$3x + (5 \times 1.21) = 1$
$3x + 6.05 = 1$
$3x = -5.05$
$x = -1.68$

x = -1.68 ..... y = 1.21 .....

(4)

2. $x = 3y + 9$
   $x = 5y + 13$

3. $x = 3y + 9$
   $3x - 5y = 10$

4. $6x + 5y - 8 = 0$
   $4x - 3y - 18 = 0$

5. $y = 2x$
   $3x + 2y = 21$

6. $y = 6x - 3$
   $8 - 5x = y$

7. $9x + 12y = -6$
   $6x - 5y = -17$

8. $x = 5$
   $3x - 2y = 13$

9. $5x + 3y = 8$
   $5x - 4y = 7$

10. $y = \frac{2}{5}(x - 3)$
    $y = \frac{3}{4}x + 1$

11. $y = 2x + 1$
    $3x + \frac{5y}{2} = 8$

12. Make up and solve an example in simultaneous equations which is solved more readily by the method of comparison than by either of the other two methods of elimination.

13. Make up and solve an example in simultaneous equations which is solved more readily by the method of substitution than by either of the other two methods.

14. Make up and solve an example solved more readily by the method of addition and subtraction than by the other two methods.

Solve the following pairs of simultaneous equations:

1. $x + 4y = 1,$
   $x + 7y = 4.$
2. $x + 2y = 1,$
   $3x + 5y = 1.$
3. $5x + 3y = 1,$
   $7x + y = 11.$
4. $x - y = 5,$
   $3x - 4y = 16.$
5. $5x + 4y = 3,$
   $2x + 3y = 4.$
6. $3x + 4y = 1.$
   $5x + 3y = 9.$
7. $\frac{x}{3} + \frac{y}{2} = 4, x - \frac{y}{2} = 4.$
8. $\frac{x}{3} + \frac{y}{6} = 5, \frac{x}{2} - \frac{y}{9} = 1$
9. $2x + y = 9,$
   $3x + 2y = 9.$
10. $7x - 5y + 5 = 0,$
    $3x + 2y - 2 = 0.$
11. $\frac{2}{x} + \frac{3}{y} = 2, \frac{4}{x} + \frac{3}{y} = 3.$
12. $\frac{6}{x} + \frac{1}{y} = 2, \frac{4}{x} + \frac{2}{y} = 2.$
13. $\frac{9}{x} - \frac{4}{y} = 5, \frac{3}{x} + \frac{2}{y} = 0.$
14. $\frac{5}{x} - \frac{2}{y} = 3, \frac{6}{x} + \frac{7}{y} = 13.$
15. $5/x - 1/y = 1,$
    $3/x + 2/y = 11.$
16. $\frac{2}{x} - \frac{1}{y} = 7, \frac{3}{x} + \frac{2}{y} = 28.$
17. $\frac{2}{x} + \frac{1}{y} = 1, \frac{5}{x} - \frac{3}{y} = 19.$
18. $x + \frac{3}{y} = 5, 3x - \frac{2}{y} = 4.$
19. $5x + \frac{2}{y} = 4, 13x + \frac{4}{y} = 11.$
20. $\frac{2}{x} - \frac{4}{y} = 2, \frac{8}{x} + \frac{12}{y} = 1.$
21. $\frac{8}{x} - 2y = 10, \frac{6}{x} - y = 6.$

Solve the following pairs of simultaneous equations by substituting the value of x given by the first equation into the second equation.

22. $x = y - 1,$
    $x + 2y = 8.$
23. $x = 3y - 2,$
    $x + 5y = 14..$
24. $x = y - 2,$
    $5x - y = 6.$
25. $x = 3y - 17,$
    $x + 2y = 8.$
26. $x = 3y + 5,$
    $x + y = 1.$
27. $x = 5y + 13,$
    $2x + y = 4.$

Solve the following pairs of simultaneous equations:

28. $x + y = 5 = 3x - 2y.$
29. $3x - y = 7 = 4x + y.$
30. $x = 5 - x - y = y - 2x.$
31. $3x + 2y = 5x - y = 13.$
32. $4x - 3y = 6 - 2x = y.$

33. If $3x - 2y = 11, x - 5y = 8,$ find the values of $2x + y$ and $x - 2y$

34. If $2x + 5y = 19, 3x + 4y = 11,$ find the values of $x + 3y$ and $3x - y$

35. If $\frac{3}{x} + \frac{4}{y} = 3, \frac{9}{x} - \frac{4}{y} = 1,$ find the values of $\frac{1}{x} + \frac{1}{y}$ and $\frac{1}{x} - \frac{1}{y}$.

Solve the following pairs of simultaneous equations:

36. $\frac{x}{y} = \frac{3}{4}, x + y = 14.$

37. $\frac{x+y}{2x+y} = \frac{3}{5}, x - y = 1.$

38. $\frac{x+2}{y} = 2, x + y = 10.$

39. $\frac{x+1}{x+y} = \frac{1}{2}, \frac{x}{y} = \frac{1}{2}.$

40. $\frac{x-3}{y} = 1, x + y = 13.$

Patrons are reminded to try to use the substitution method where possible. Look for an x or y to isolate from one equation. Then substitute this into the *other* equation. For example solve

$$\begin{matrix} 3x+y=7 \\ 2x+5y=2 \end{matrix}.$$

From the first we can see that $y=7-3x$. Substituting this into the second we find $2x+5(7-3x)=2$ which solves to $x=\frac{33}{13}$. We then place this value into $y=7-3x$ to discover $y=-\frac{8}{13}$. So $(x,y)=(\frac{33}{13},-\frac{8}{13})$

Present answers in the form $(x,y)=(-1,\frac{1}{2})$. (Don't forget the brackets!)

1. $\begin{matrix} x+y=3 \\ x-y=2 \end{matrix}$.
2. $\begin{matrix} x-y=4 \\ x+y=7 \end{matrix}$.
3. $\begin{matrix} 2x+y=3 \\ x-2y=2 \end{matrix}$.
4. $\begin{matrix} x+2y=4 \\ 3x-2y=-7 \end{matrix}$.
5. $\begin{matrix} 5x-y=4 \\ 4x-5y=0 \end{matrix}$.
6. $\begin{matrix} 2x-3y=5 \\ 3x+2y=2 \end{matrix}$.
7. $\begin{matrix} 4x-2y=-9 \\ 3x+5y=3 \end{matrix}$.
8. $\begin{matrix} x-2y=2 \\ 3x+2y=1 \end{matrix}$.
9. $\begin{matrix} 3x-y=3 \\ 2x+3y=-1 \end{matrix}$.
10. $\begin{matrix} y-2x=5 \\ 5x-7y=2 \end{matrix}$.
11. $\begin{matrix} x+3y=4 \\ 5x-2y=6 \end{matrix}$.
12. $\begin{matrix} 4x-y=2 \\ 3x+4y=1 \end{matrix}$.
13. $\begin{matrix} 5x+4y=1 \\ x-3y=0 \end{matrix}$.
14. $\begin{matrix} x+y=-7 \\ 2x+3y=4 \end{matrix}$.
15. $\begin{matrix} 2x+y=2 \\ x-6y=1 \end{matrix}$.
16. $\begin{matrix} 3x+2y=1 \\ y-2x=-3 \end{matrix}$.
17. $\begin{matrix} a-b=2 \\ 3a-2b=-4 \end{matrix}$.
18. $\begin{matrix} 4x+7y=10 \\ 3x-y=-2 \end{matrix}$.
19. $\begin{matrix} 2x+3y=1 \\ 3x-4y=2 \end{matrix}$.
20. $\begin{matrix} 5x+y=7 \\ 4x-\frac{1}{2}y=2 \end{matrix}$.
21. $\begin{matrix} x+5y=0 \\ 3x+4y=-1 \end{matrix}$.
22. $\begin{matrix} \frac{x+y}{2}-\frac{x-y}{3}=1 \\ x-2y=2 \end{matrix}$.
23. $\begin{matrix} \frac{x+2y}{3}-\frac{x-3y}{7}=x \\ x+3y=2 \end{matrix}$.
24. $\begin{matrix} x+ay=0 \\ 2x+3y=-1 \end{matrix}$.

Fluency Practice

25. $\begin{matrix} kx + y = 4 \\ 2x - 3y = 2 \end{matrix}$.

26. $\begin{matrix} ax + 4y = 6 \\ bx - y = 5 \end{matrix}$.

27. $\begin{matrix} kx + y = 1 \\ 5x - ky = m \end{matrix}$.

28. $\begin{matrix} x + y = 1 \\ ax + by = 1 \end{matrix}$.

29. $\begin{matrix} x + ay = 3 \\ ax + by = 4 \end{matrix}$.

30. $\begin{matrix} 4x + y = 3 \\ ax + by = c \end{matrix}$.

31. $\begin{matrix} x + by = 2 \\ \frac{x + ay}{2} - \frac{x - y}{3} = 1 \end{matrix}$.

32. $\begin{matrix} \frac{ax + 1}{2} + \frac{by + 2}{3} = 1 \\ \frac{5x + 1}{3} + \frac{ay + 1}{2} = 1 \end{matrix}$.

33. $\begin{matrix} 23x + 21y = 1 \\ 21x + 23y = -1 \end{matrix}$.

Answers

25. $\begin{matrix} kx+y=4 \\ 2x-3y=2 \end{matrix}$. $\boxed{(x,y)=(\frac{14}{2+3k},\frac{8-2k}{2+3k})}$

26. $\begin{matrix} ax+4y=6 \\ bx-y=5 \end{matrix}$. $\boxed{(x,y)=(\frac{26}{a+4b},\frac{6b-5a}{a+4b})}$

27. $\begin{matrix} kx+y=1 \\ 5x-ky=m \end{matrix}$. $\boxed{(x,y)=(\frac{m+k}{5+k^2},\frac{5-km}{5+k^2})}$

28. $\begin{matrix} x+y=1 \\ ax+by=1 \end{matrix}$. $\boxed{(x,y)=(\frac{1-b}{a-b},\frac{a-1}{a-b})}$

29. $\begin{matrix} x+ay=3 \\ ax+by=4 \end{matrix}$. $\boxed{(x,y)=(\frac{3b-4a}{b-a^2},\frac{4-3a}{b-a^2})}$

30. $\begin{matrix} 4x+y=3 \\ ax+by=c \end{matrix}$. $\boxed{(x,y)=(\frac{c-3b}{a-4b},\frac{3a-4c}{a-4b})}$

31. $\begin{matrix} x+by=2 \\ \frac{x+ay}{2}-\frac{x-y}{3}=1 \end{matrix}$. $\boxed{(x,y)=(\frac{6a+4-6b}{3a+2-b},\frac{4}{3a+2-b})}$

32. $\begin{matrix} \frac{ax+1}{2}+\frac{by+2}{3}=1 \\ \frac{5x+1}{3}+\frac{ay+1}{2}=1 \end{matrix}$. $\boxed{(x,y)=(\frac{3a+2b}{20b-9a^2},\frac{3a+10}{9a^2-20b})}$

33. $\begin{matrix} 23x+21y=1 \\ 21x+23y=-1 \end{matrix}$. $\boxed{(x,y)=(\frac{1}{2},-\frac{1}{2})}$

Solve and check each result:

1. $3x + 4y = 2a$
   $5x + 6y = 4a$

2. $2ax + 3by = 4ab$
   $5ax + 4by = 3ab$

3. $ax + by = 1$
   $a'x + b'y = 1$

4. $x - y = 2n$
   $mx - ny = m^2 + n^2$

5. $2bx + ay = 4b + a$
   $abx - 2aby = 4b + a$

6. $ax - by = a^2 + b^2$
   $bx + ay = 2(a^2 + b^2)$

7. $ax + by = c$
   $mx + ny = d$

8. $bx + ay = a + b$
   $ab(x - y) = a^2 - b^2$

9. $c^2x - d^2y = c - d$
   $cd(2dx - cy) = 2d^2 - c^2$

10. $\dfrac{x + m}{y - n} = \dfrac{n}{m}$
    $x + y = 2n$

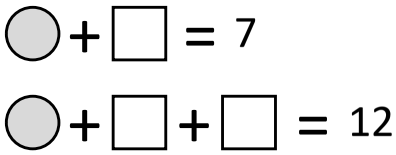
11. $(a + 1)x - by = a + 2$
    $(a - 1)x + 3by = 9a$

12. $(a - b)x - (a + b)y = a^2 + b^2$
    $bx + ay = 0$

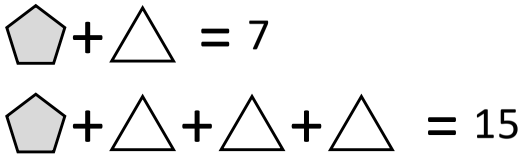
Simultaneous Equations

Find the value of each shape in each pair of equations.

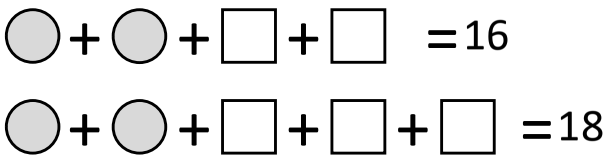
a)



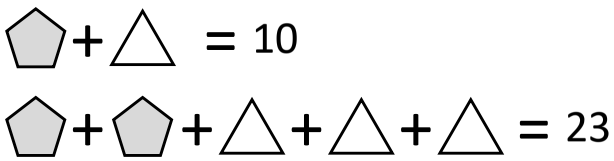
b)



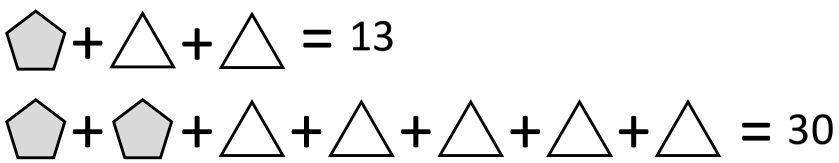
c)



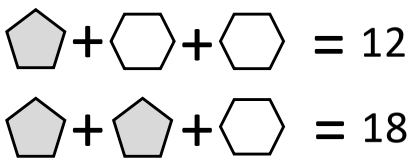
d)



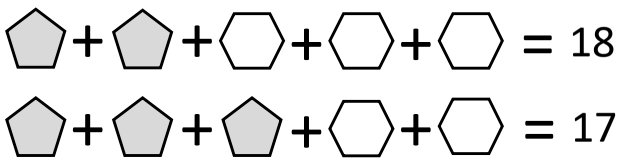
e)



f)

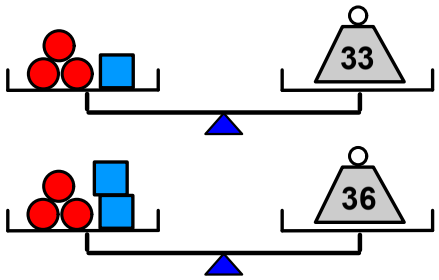


g)

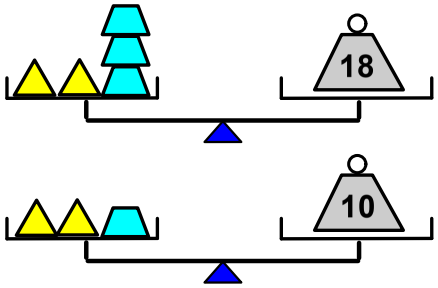


Work out the value of each shape.

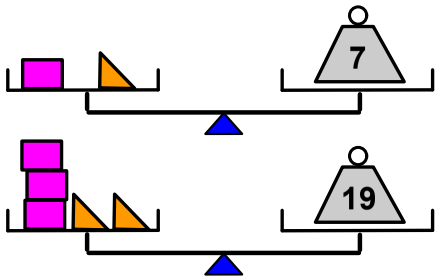
a)



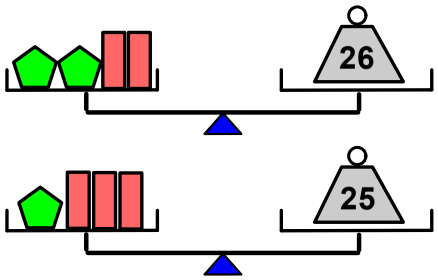
b)



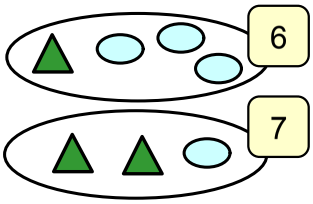
c)



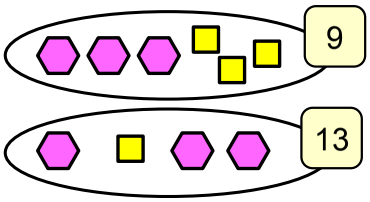
d)



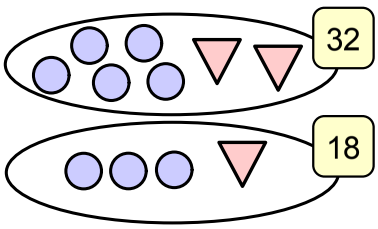
e)



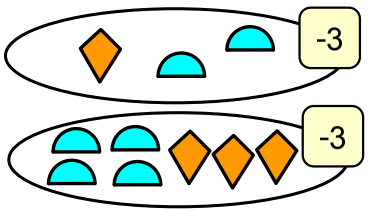
f)



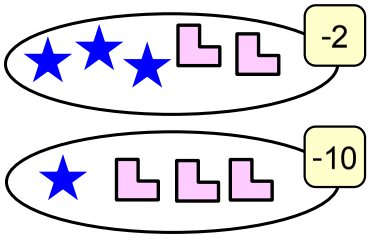
g)



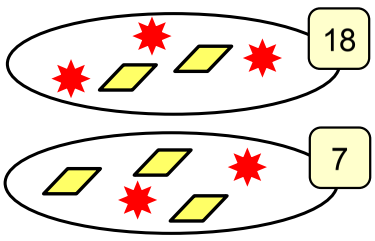
h)



i)



j)



Fluency Practice

a	b	c	15
a	b	c	24
d	a	a	14
d	d	d	20

☺	↑	⌐	32
☺	↑	⌐	22
↑	↑	⌐	30
↑	☺	⌐	34

◀	◀	▼	18
◀	▶	▲	11
▶	▲	▲	18
▶	▲	▲	21

♥	♣	♣	14
♠	♥	♥	8
♠	♣	♥	8
♠	♠	♠	16

☺	☺	☺	12
☺	☺	♡	14
☺	♡	☐	13
♡	◁	△	11

£	€	$	¥
€	£	$	¥
€	£	£	¥
€	£	£	17

¥	£	£	€
¥	£	£	€
¥	£	£	€
¥	£	£	26

☺	↑	⌐	32
☺	⌐	⌐	22
⌐	⌐	⌐	30
⌐	☺	⌐	34

◀	◀	▼	18
◀	◀	▼	11
◀	▶	▲	18
▶	▲	▲	21

♥	♣	♣	14
♠	♥	♥	8
♠	♣	♥	8
♠	♠	♠	16

☺	☺	☺	12
☺	☺	♡	14
☺	♡	☐	13
♡	◁	△	11

Purposeful Practice

Forming & Solving Simultaneous Equations using Substitution

① 1 t-shirt and 1 jumper cost £25.
The jumper costs £5 more than the t-shirt.

② 1 scarf and 1 hat cost £16.
The hat costs £2 more than the scarf.

③ 1 dress and 1 skirt cost £35.
The skirt is £5 cheaper than the dress.

④ 1 coffee and 2 teas cost £11.
A tea is £1 more expensive than a coffee.

⑤ 2 cupcakes and 1 muffin cost £8.
A muffin is double the price of a cupcake.

⑥ 3 notebooks and 2 books cost £33.
A book costs £4 more than a notebook.

⑦ 3 t-shirts and 2 hoodies cost £81.
For the cost of 1 hoodie you
could buy 3 t-shirts

⑧ 2 pairs of sweatpants and 3 jumpers cost £89.
For the cost of 1 jumper you could
buy 2 pairs of sweatpants and
have £3 left over.

⑨ 5 litres of washing up liquid and
2 litres of professional detergent cost £80.
The cost of 1 litre of detergent is £4 less than
the price of 3 litres of washing up liquid.

(a) Maryam buys 5 pears and 2 kiwi fruits for £3.90. At the same grocers, Dave buys 3 pears and 1 kiwi fruit for £2.20. Find the cost of one pear and the cost of one kiwi fruit.
(b) Peter buys 2 pencils and 3 pens for £1.70. Bola buys 4 pencils and 9 pens for £4.60. Find the cost of one pencil and the cost of one pen.

(c) Yusuf visits a café and orders 2 coffees and 3 cakes for £10.35. Maria orders 5 coffees and 2 cakes for £15.70 at the same café. How much would Tia pay for 3 coffees and 2 cakes at the café?
(d) Two numbers have a sum of 57 and a difference of 36. Find the two numbers.

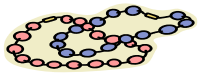
(e) Cards are 65p each and gift wrap sheets are 45p each at a gift shop. Owen buys 21 items from the shop and spends £11.85. How many cards and how many gift wrap sheets did Owen buy?
(f) Terri has a loose change jar that contains only 5p and 10p coins. There are 134 coins in the jar and their total value is £9. Find the number of 5p coins and the number of 10p coins in the jar.

(g) Flour comes in bags of 1.5 kg or 2 kg. A baker has thirty bags of flour and their total weight is 51.5 kg. Find the number of 1.5 kg bags and the number of 2 kg bags the baker has.
(h) A farmer keeps ducks and pigs in the same field. The total number of animals in the field is 54. The difference between the number of pig's feet and the number of duck's feet is 72. Find the number of pigs and the number of ducks in the field.

1. The sum of two numbers is 130, while their difference is 38. Find the two numbers.

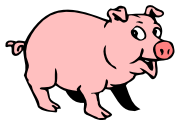
2. Becky and Peter's ages add to 53. If Becky is 3 years younger than Peter, what are their ages?

3. A necklace is made from 164 purple and blue beads. There are 8 more purple beads than blue beads. How many of each colour bead are there?



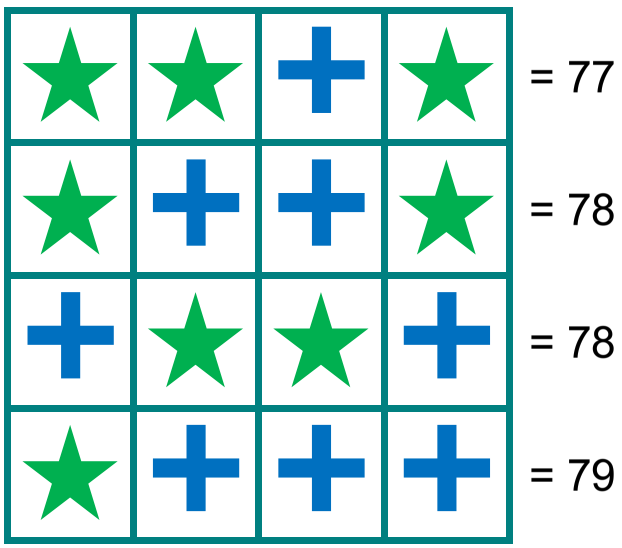
4. Ten Choco bars and six nutty bars cost £5. Three Choco bars and one Nutty bar cost £1.30. Find the cost of each chocolate bar.

5. Some chickens and pigs are in a field. How many of each animal are there if there are 32 heads and 80 legs in total?



6. Peter bought a mixture of large postcards (35p) and small postcards (29p). He bought 20 postcards in total, costing £6.28. How many of each size of postcard did he buy?

7. Find the value of each of the symbols in the grid:



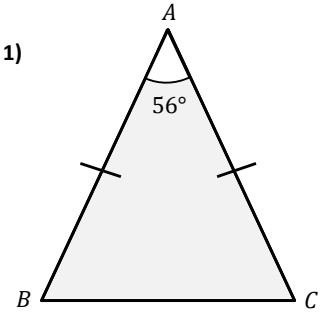
8. Five apples and three pears cost 86 pence. Seven apples and five pears cost £1.30. Find the cost of each piece of fruit.

9. Two adults and three children went to the cinema, and the total cost was £32.40. Three adults and five children cost £51.20. What were the individual prices of adult and child tickets?

10. A slot machine takes only 20p and 50p coins. The machine contains a total of 140 coins worth £45.10. How many of each type of coin are in the machine?

Purposeful Practice

1)

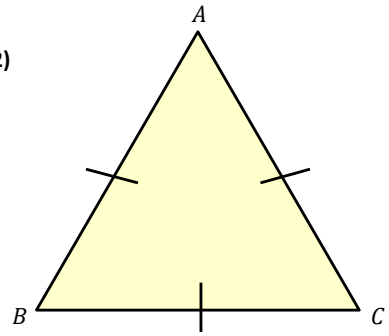


$\angle ABC = 2x + 5y + 4$

$\angle ACB = 5x + 6y - 5$

What are the values of x and y?

2)



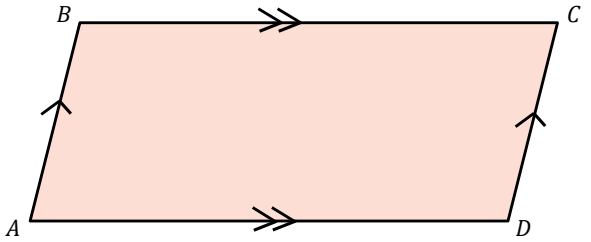
$\angle ABC = 5x + 4y$

$\angle ACB = 8x - y + 1$

$\angle BAC = 4x + 3y + 13$

What are the values of x and y?

3)



$\angle ABC = 9x + 7y + 7$

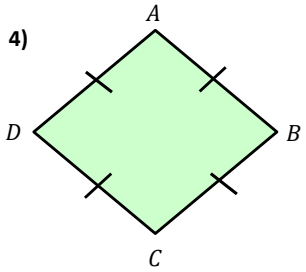
$\angle BCD = 4x + y + 7$

$\angle CDA = 6x + 4y + 58$

$\angle DAB = 3x + 2y + 2$

What is the size of each angle?

4)



$AB = 4x + y + 5$

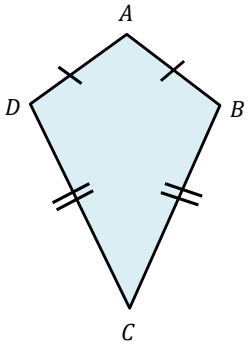
$BC = 3x + 3y$

$CD = x + 4y + 2$

$DA = 2x + 4y - 1$

How long is each side?

5)



$AB = 4x + y - 7$

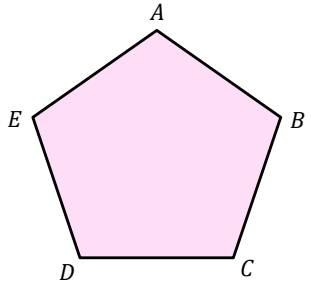
$BC = 5x + 3y - 1$

$CD = 8x + 2y - 5$

$DA = 2x + y + 1$

How long is each side?

6)

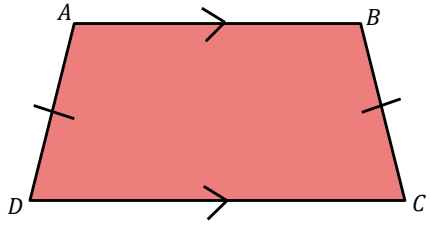


$\angle EAB = 5x + 7y + 14$

$AB = x + 2y$

This is a regular pentagon. Its perimeter is 115cm. What are the values of x and y?

7)



$\angle BCD = 15x + 6y + 16$

$\angle CDA = 10x + 7y + 22$

$AD = 3x + 3y$

$BC = 13x - 2y$

What are the values of the given angles and sides?

Purposeful Practice

subsections

a total of **60** people were observed

of these,

25% of the men had blue eyes

40% of the women had blue eyes

altogether there were **14** people with blue eyes

how many males and how many females were observed?

98 people were interviewed

of these,

30% of the women were born in London

33⅓% of the men were born in London

altogether **31** people were born in London

how many females and how many males gave information in the interviews?

41 people bought a newspaper

of these,

25% of the men were over 60

40% of the women were over 60

altogether **14** people were over 60

how many females and how many males bought newspapers?

a total of **76** people were asked about driving

of these,

20% of the women did not own a car

37½% of the men did not own a car

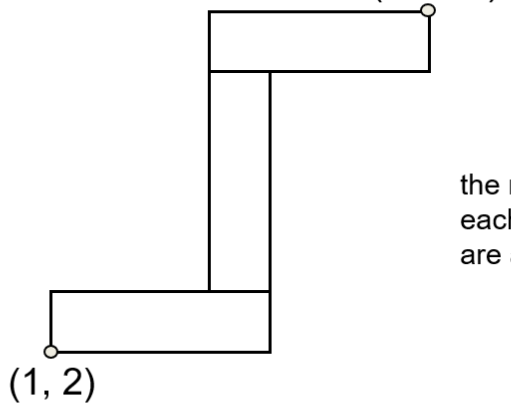
altogether, **25** people did not own a car

how many males and how many females were asked?

Fluency Practice

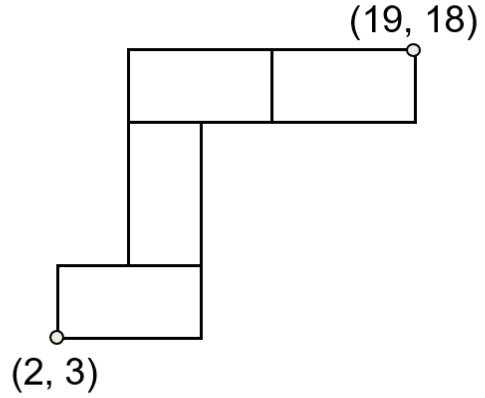
rectangles on a grid how long and wide are each of the rectangles?

(1)

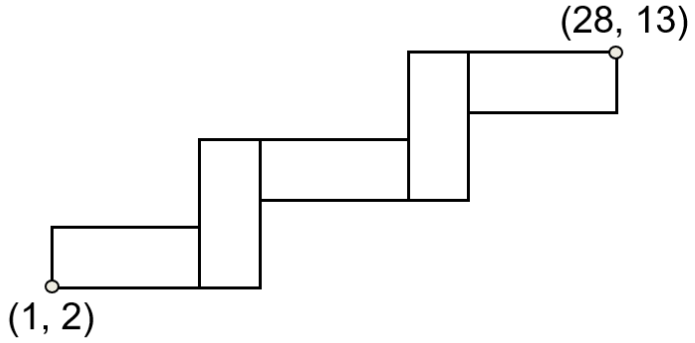


the rectangles in each question are all congruent

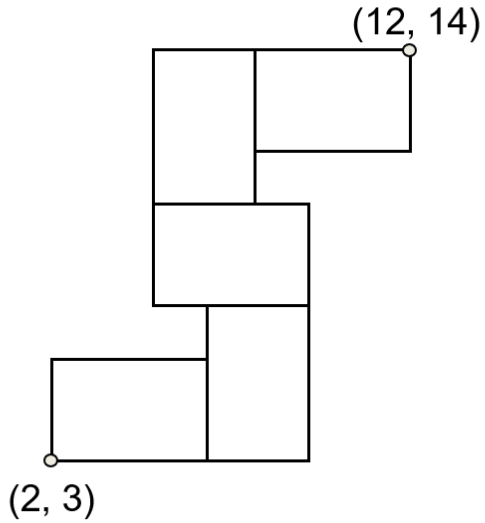
(2)



(3)



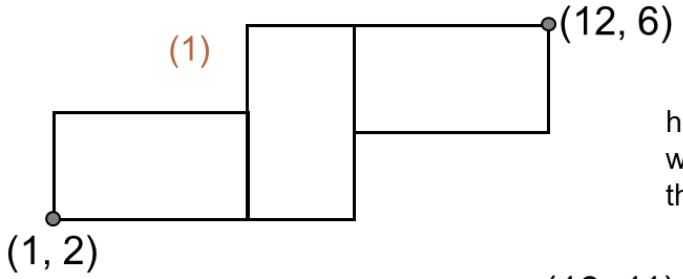
(4)



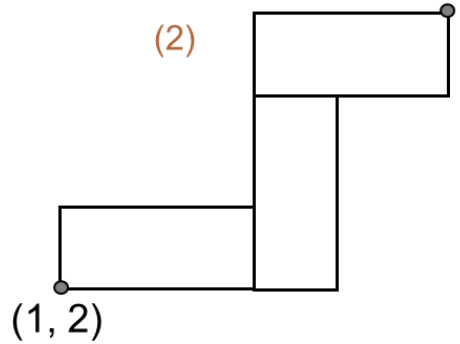
Fluency Practice

rectangles on a grid

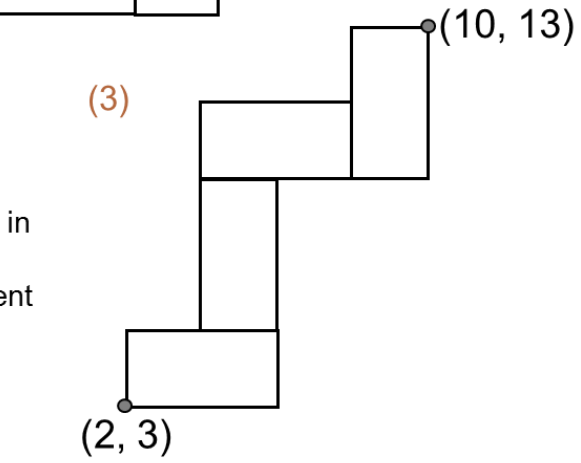
(1)



(2)

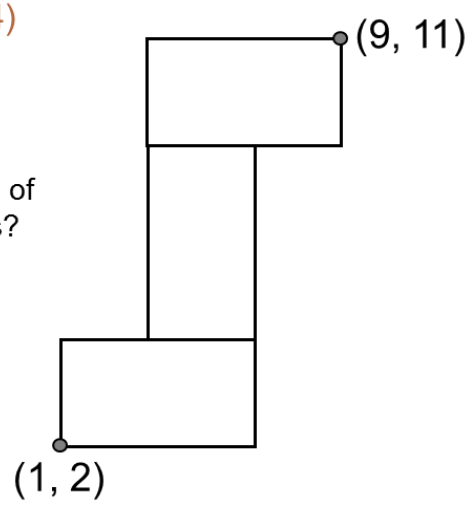


(3)



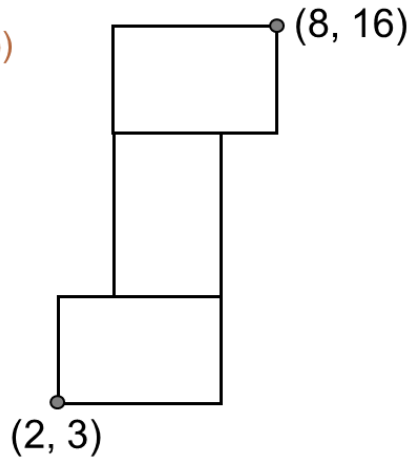
the rectangles in each question are all congruent

(4)



how long and wide are each of the rectangles?

(5)



Solve:

(a) $2x + y = 10$
$x + y = 7$

(b) $4x + y = 9$
$x + y = 3$

(c) $3x + 2y = 22$
$x + 2y = 10$

(d) $5x + 2y = 17$
$3x + 2y = 11$

Solve:

(a) $2x + y = 14$
$x - y = 1$

(b) $3x - y = 1$
$x + y = 7$

(c) $3x + 2y = 25$
$x - 2y = 3$

(d) $x - 3y = 2$
$2x + 3y = 22$

Solve:

(a) $5x + y = 5$
$3x + y = 7$

(b) $6x - y = 9$
$5x - y = 7$

(c) $4x + 2y = 22$
$3x - 2y = 6$

(d) $x - 3y = 4$
$4x + 3y = 1$

(e) $x + y = 0$
$x - y = 6$

(f) $5x + 2y = 13$
$x + 2y = 9$

(g) $3x + 2y = 16$
$x - 2y = 4$

(h) $3x - y = 9$
$5x + y = 11$

(i) $4x + y = 8$
$2x + y = 7$

(j) $5x - 2y = 7$
$4x + 2y = 11$

David buys 5 biscuits and 3 cakes for £3.95. Samira buys 8 biscuits and 3 cakes for £5.15. Find the cost of one biscuit and the cost of one cake.

Jim is thinking of two numbers. When he adds them together he gets 20. When he subtracts one from the other he gets 30. What are Jim's two numbers?

Solve:

(a) $x + 2y = 7$
$3x + y = 6$

(b) $x + 3y = 11$
$4x + y = 22$

(c) $3x - y = 7$
$5x - 2y = 10$

(d) $x - y = 5$
$2x - 5y = 4$

Solve:

(a) $x + y = 5$
$3x - 2y = 5$

(b) $7x - y = 1$
$x + 3y = 19$

(c) $2x + 5y = 24$
$3x - y = 2$

(d) $4x - 2y = 14$
$3x + y = 23$

Solve:

(a) $x + 2y = 13$
$2x + 3y = 20$

(b) $7x - 4y = 5$
$x + 2y = 11$

(c) $2x + 5y = 5$
$3x - 2y = 17$

(d) $4x - 2y = 14$
$x - 3y = -4$

Solve:

(a) $5x + 2y = 31$
$x - 4y = 4$

(b) $4x + y = 5$
$2x + 3y = 10$

(c) $2x - 3y = 16$
$7x - 2y = 39$

(d) $x - 5y = 6$
$3x + 2y = 1$

(e) $3x + 4y = 14$
$x - y = -7$

(f) $x + 7y = 15.5$
$2x - 5y = -7$

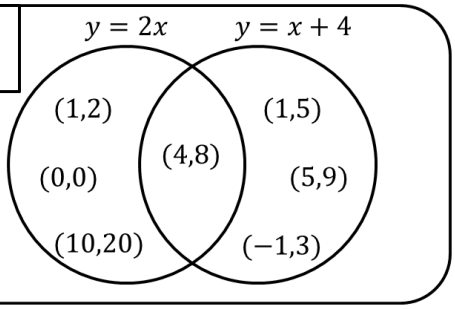
Three apples and two oranges costs £2.53. Five apples and three oranges costs £4.12. Find the cost of one apple and the cost of one orange.

Fluency Practice

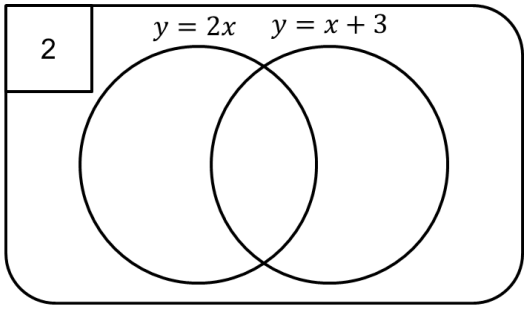
(a)	(b)	(c)	(d)
Solve $2x + 3y = 14$ $x + 3y = 10$	Solve $5x + 2y = 37$ $3x - 2y = 3$	Solve $x + 4y = 17$ $3x + 4y = 19$	Solve $2x + y = 4$ $3x + 2y = 5$
(e)	**(f)**	**(g)**	**(h)**
Solve $5x - y = 36$ $x + 3y = 4$	Solve $7x + 4y = 6$ $3x + 2y = 4$	Solve $6x - y = 4$ $2x - 4y = 5$	Solve $4x + 2y = 19$ $x + 3y = 16$
(i)	**(j)**	**(k)**	**(l)**
Solve $2x - 3y = 20$ $3x + 5y = 11$	4 burgers and 2 sausages costs £4.70. 3 burgers and 5 sausages costs £5.80. Find the cost of one burger and one sausage.	The sum of two numbers is 10.3. The difference between the same two numbers is 2.84. Find the two numbers.	Find the coordinates of the point where the lines $2x + 3y = 21$ and $3x - y = 4$ meet.

1. Write some **sets** of coordinates that fit the equations.
   Place sets that fit both in the middle. The first is done for you.

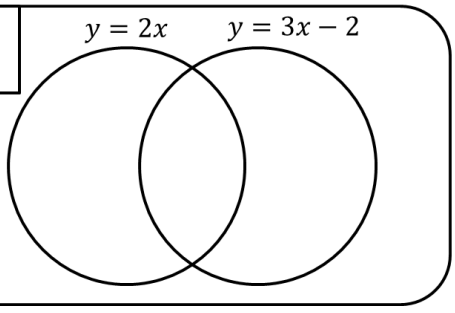
1



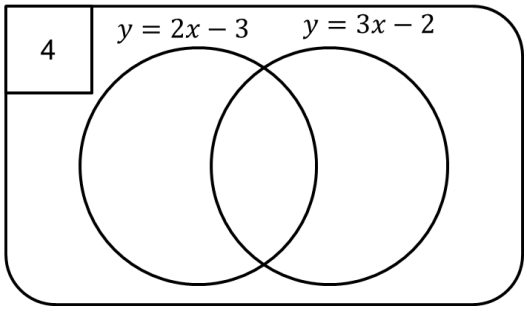
2



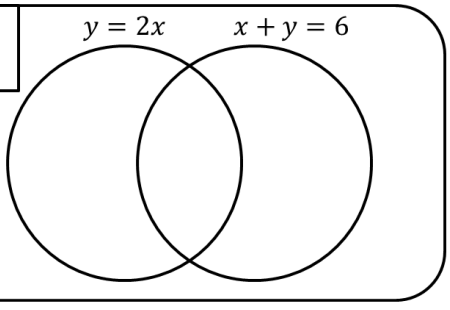
3



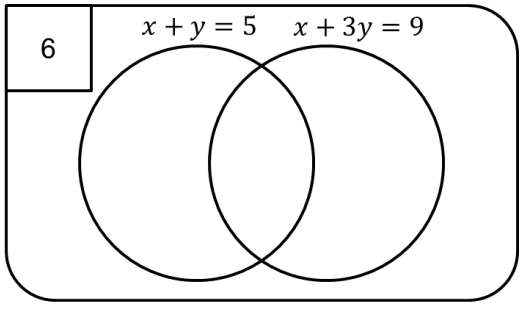
4



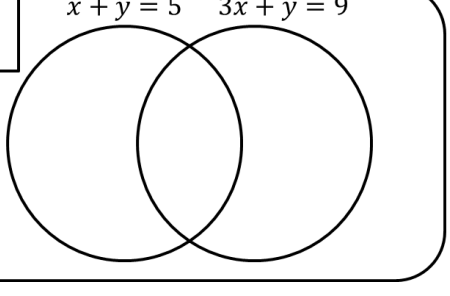
5



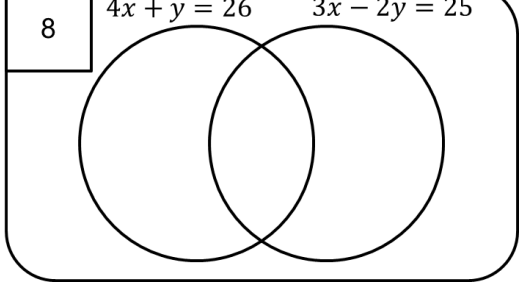
6



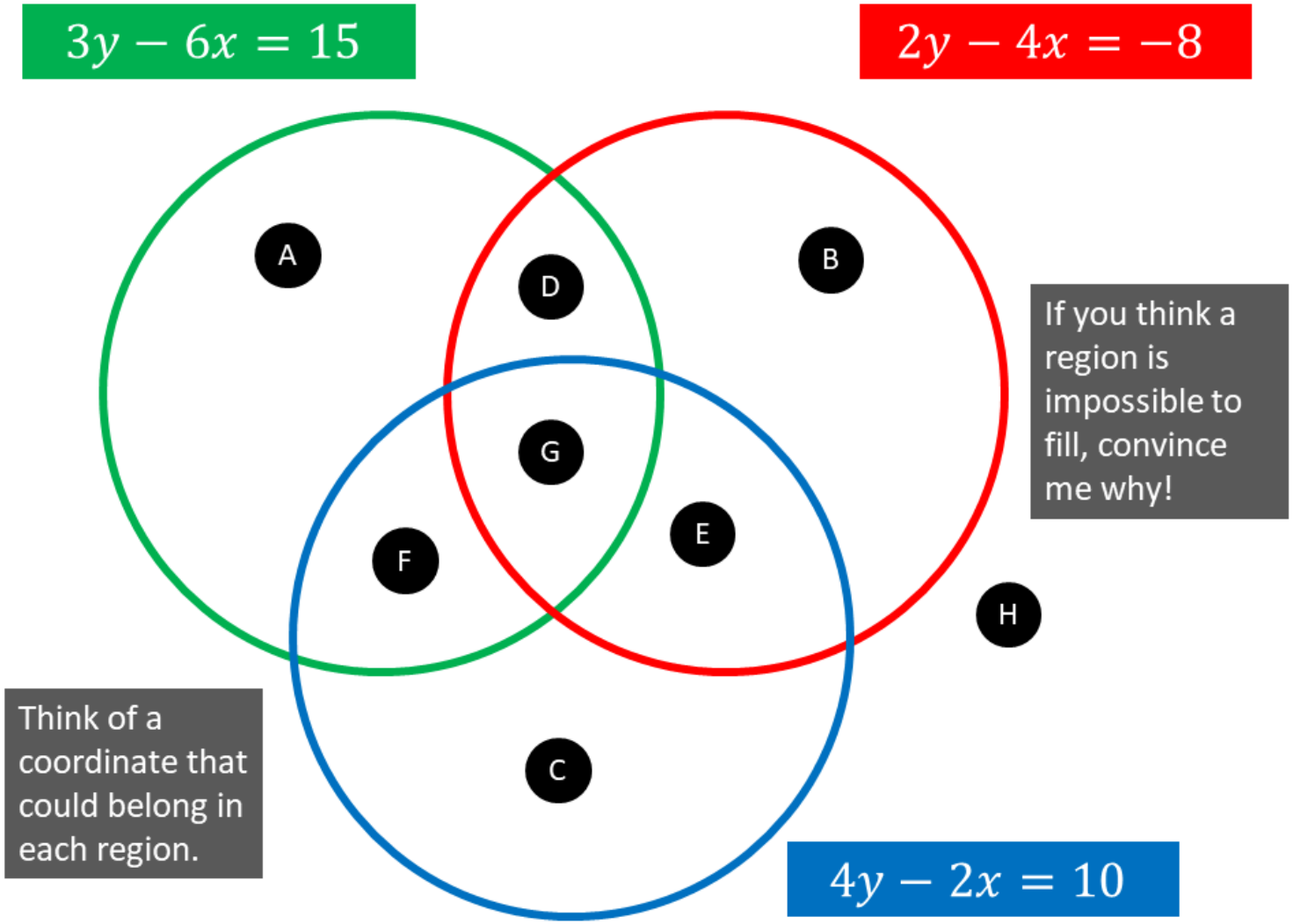
7



8



Problem Solving

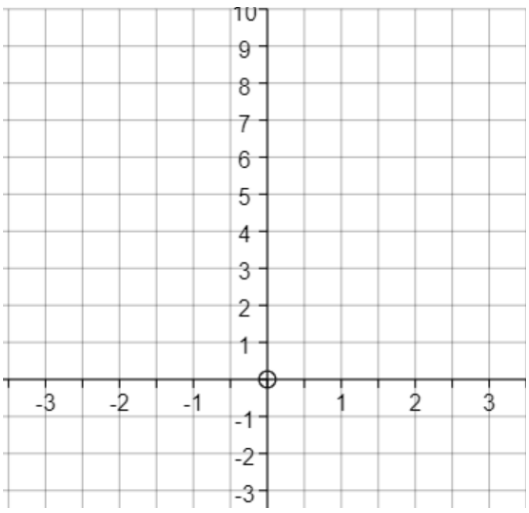


6 Non-Linear Graphs

Plotting Quadratic Graphs

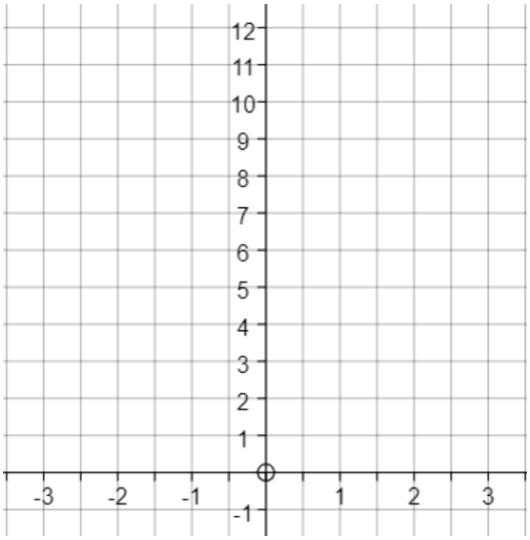
(a) $y = x^2$

x	-3	-2	-1	0	1	2	3
y							



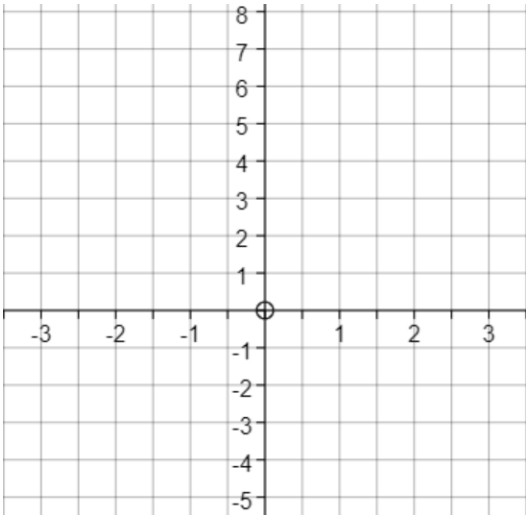
(b) $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							



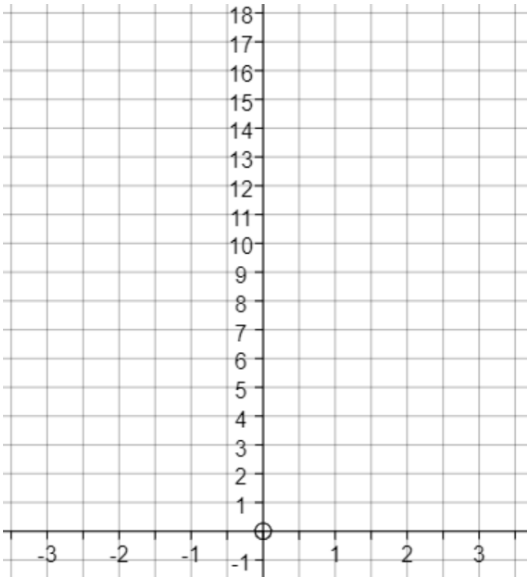
(c) $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y							



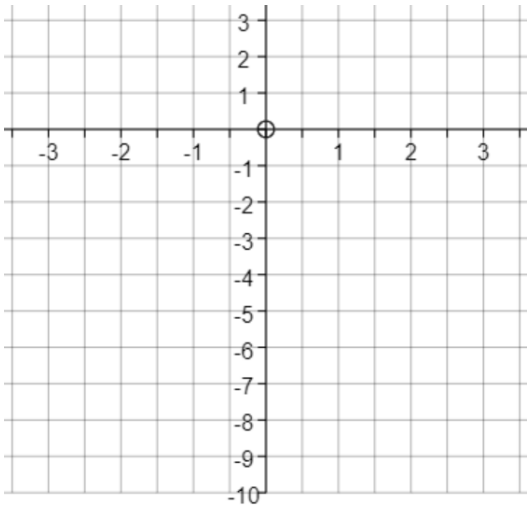
(d) $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y							



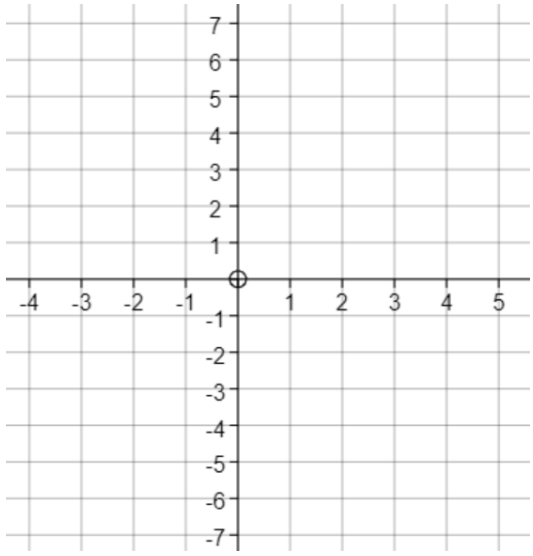
(e) $y = -x^2$

x	-3	-2	-1	0	1	2	3
y							



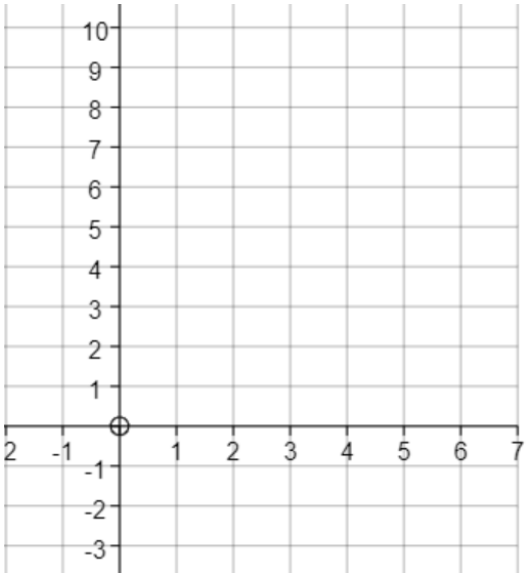
(f) $y = x^2 - x - 6$

x	-3	-2	-1	0	1	2	3	4
y								



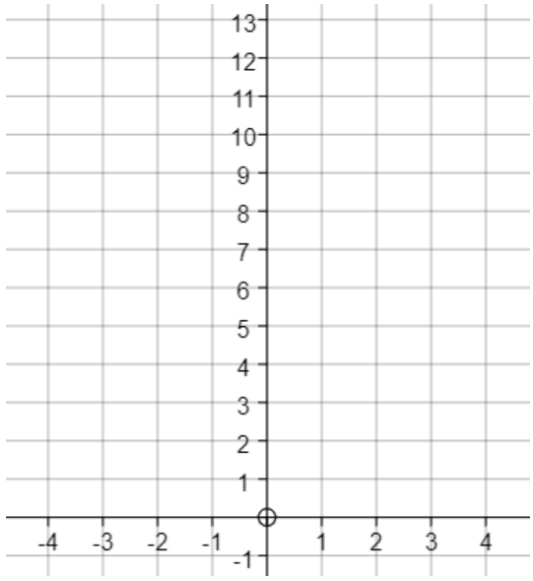
(g)

Plot the graph of $y = x^2 - 5x + 4$ for $-1 \leq x \leq 6$



(h)

Plot the graph of $y = 12 + x - x^2$ for $-3 \leq x \leq 4$

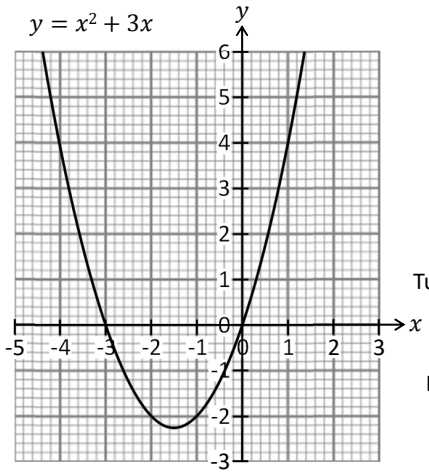


Fluency Practice

Reading the Significant Points of a Quadratic Curve

Use each graph to *estimate* each piece of information.

$y = x^2 + 3x$



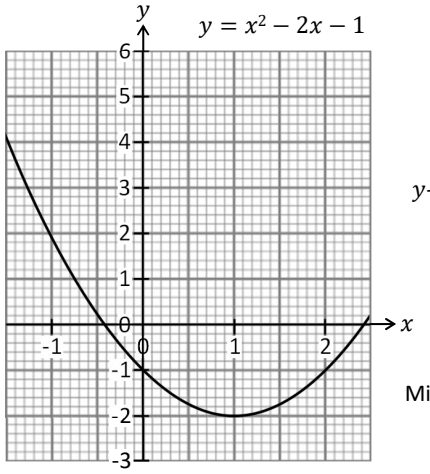
Roots: _____

y-intercept: _____

Turning Point: _____

Min or Max? _____

$y = x^2 - 2x - 1$



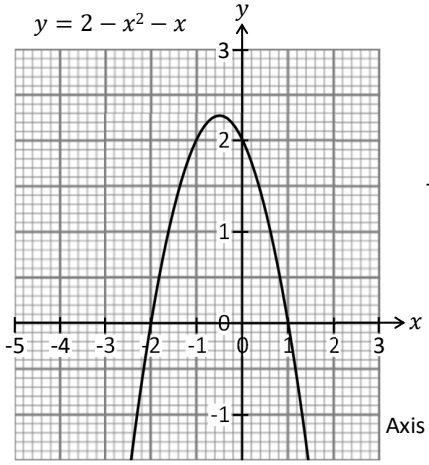
Solutions: _____

y-intercept: _____

Vertex: _____

Min or Max? _____

$y = 2 - x^2 - x$



x-intercepts: _____

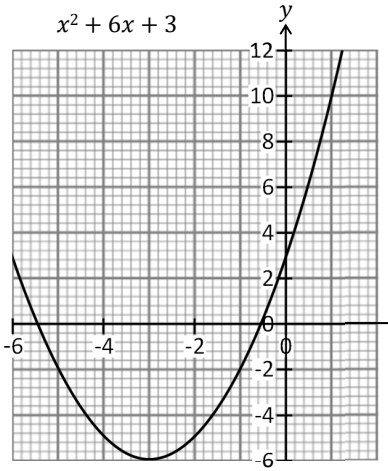
y-intercept: _____

Turning Point: _____

Min or Max? _____

Parabola's Axis of Symmetry: _____

$x^2 + 6x + 3$



Roots: _____

y-intercept: _____

Turning Point : _____

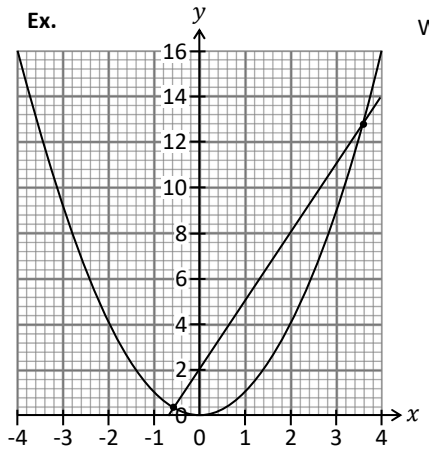
Min or Max? _____

Fluency Practice

Solving Equations using $y = x^2$

The graph $y = x^2$ has been drawn on the grid.

Ex.



We will use it to solve this equation

$x^2 - 3x - 2 = 0$

rearrange ↓

$x^2 = 3x + 2$

Where the curve x^2 intersects the line $3x + 2$, the equation is true.

$x =$ −0.6

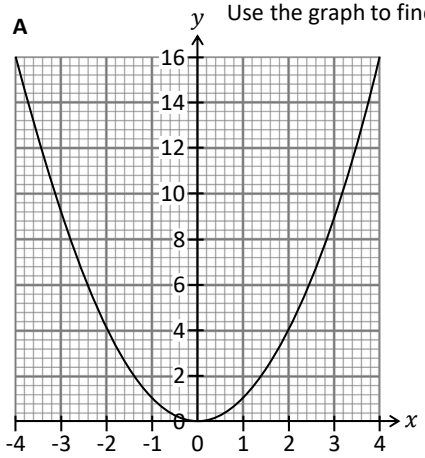
or

$x =$ _____

These are *estimates*.

A

Use the graph to find estimates for the solutions to:



$x^2 - x - 4 = 0$

rearrange ↓

$x^2 = x + 4$

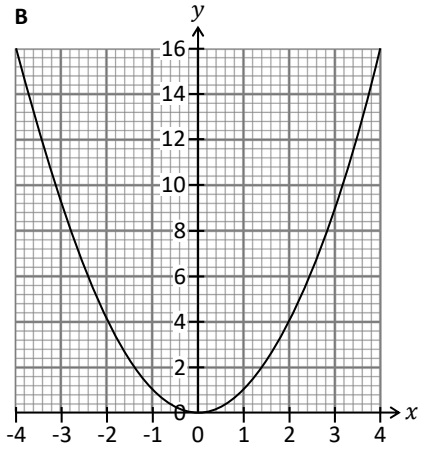
$x =$ _____

or

$x =$ _____

Use the graph to find estimates for the solutions of this equation.

B



$x^2 + 2x - 4 = 0$

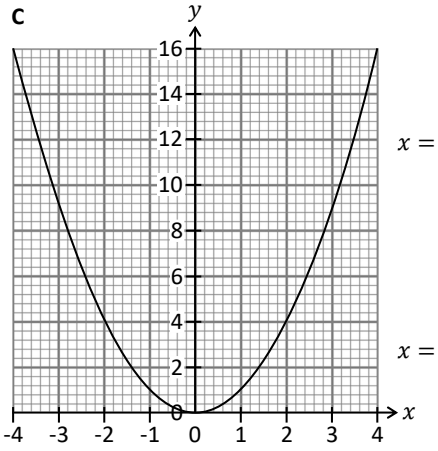
$x =$ _____

or

$x =$ _____

Use the graph to find estimates for the solutions to these equations.

C



$x^2 - 2x - 7 = 0$

$x =$ _____

$x^2 - 3x + 1 = 0$

$x =$ _____

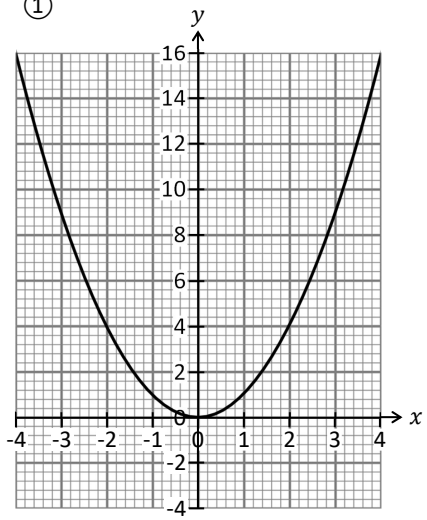
Fluency Practice

Solving Quadratic Equations: Intersection

① $y = x^2$ has been drawn on the grid.

Use the graph to solve the equation:

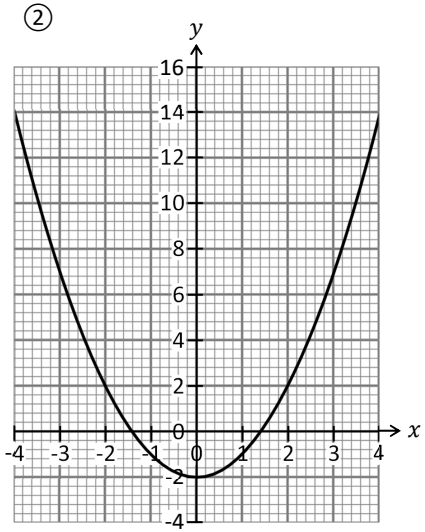
$x^2 - 3x - 2 = 0$



② $y = x^2 - 2$ has been drawn on the grid.

Use the graph to solve the equation:

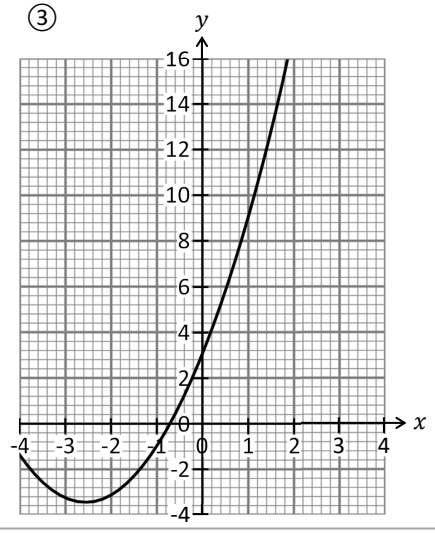
$x^2 - x - 5 = 0$



③ $y = x^2 + 5x + 3$ has been drawn on the grid.

Use the graph to solve the equation:

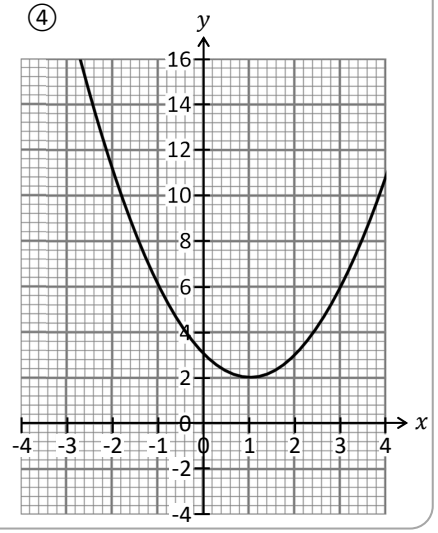
$x^2 + 3x - 1 = 0$



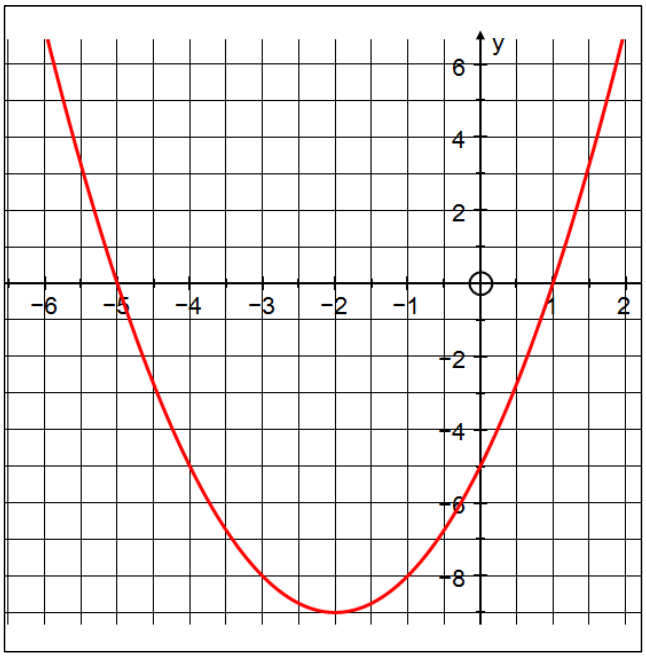
④ $y = x^2 - 2x + 3$ has been drawn on the grid.

Use the graph to solve the equation:

$x^2 + 2x - 2 = 0$



1) Here is the graph of $y = x^2 + 4x - 5$



Use the graph to solve the following equations:

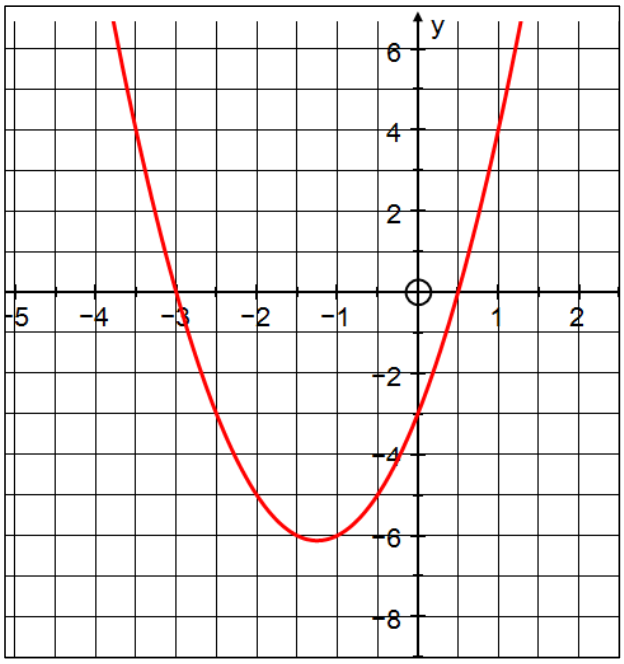
(a) $x^2 + 4x - 5 = 0$

(b) $x^2 + 4x - 5 = 4$

(c) $x^2 + 4x - 5 = -6$

(d) $x^2 + 4x - 5 = x$

2) Here is the graph of $y = 2x^2 + 5x - 3$



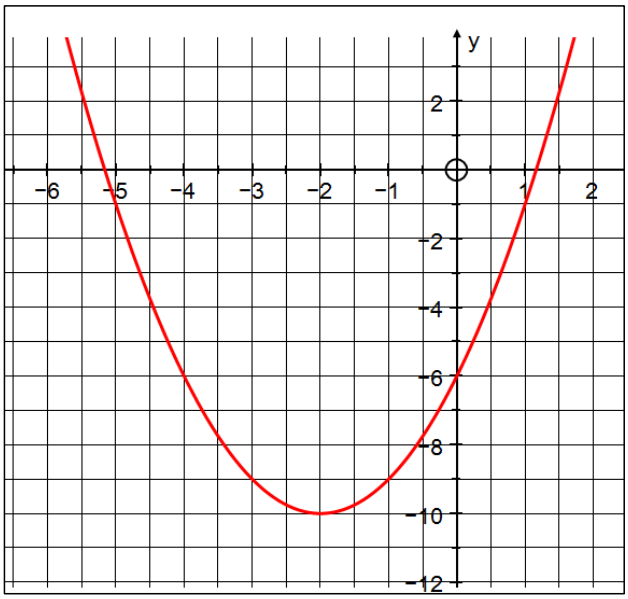
Use the graph to solve the following equations:

(a) $2x^2 + 5x - 3 = 2x + 1$

(b) $2x^2 + 5x = 5$

(c) $2x^2 + 5x + 2 = 0$

3) Here is the graph of $y = x^2 + 4x - 6$



Use the graph to solve the following equations:

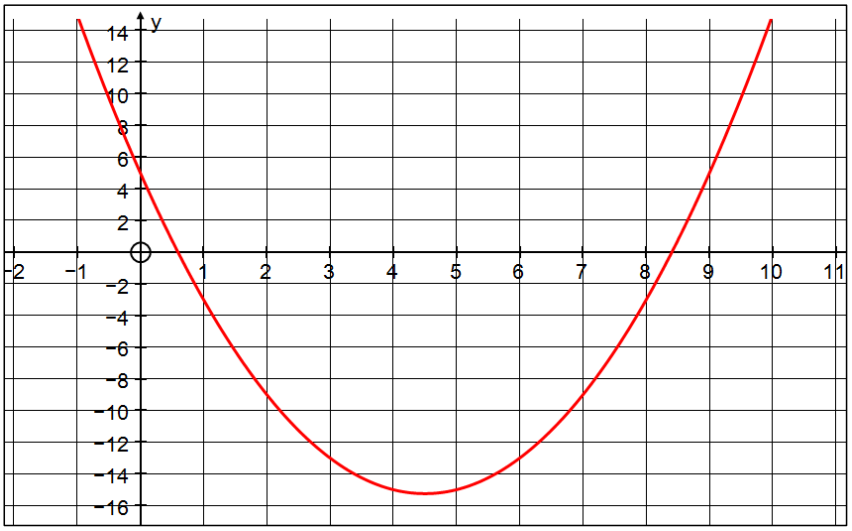
(a) $x^2 + 4x - 6 = -10$

(b) $x^2 + 4x - 6 = -11$

(c) $x^2 + 3x - 6 = 0$

(d) $x^2 + 5x - 2 = 0$

4) This is the graph of $y = x^2 - 9x + 5$



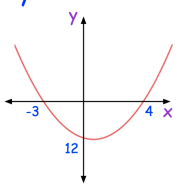
Use the graph to solve:

(a) $x^2 - 11x + 12 = 0$

(b) $x^2 - 6x + 3 = 0$

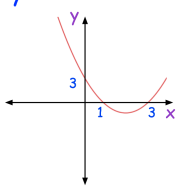
Question 1: Using the graphs below, solve each equation.

$y = x^2 - x - 12$



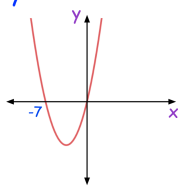
(a) Solve $x^2 - x - 12 = 0$

$y = x^2 - 4x + 3$



(b) Solve $x^2 - 4x + 3 = 0$

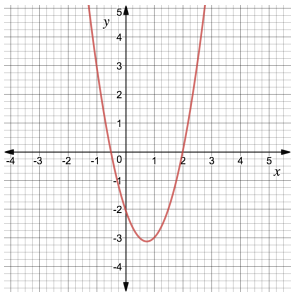
$y = x^2 + 7x$



(c) Solve $x^2 + 7x = 0$

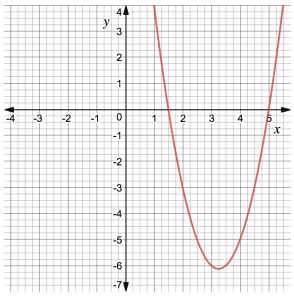
Question 2: Using the graphs below, solve each equation

$y = 2x^2 - 3x - 2$



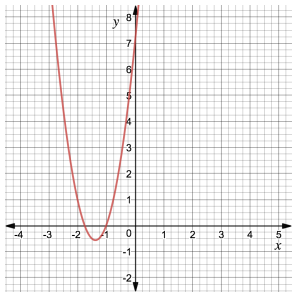
(a) Solve $2x^2 - 3x - 2 = 0$

$y = 2x^2 - 13x + 15$



(b) Solve $2x^2 - 13x + 15 = 0$

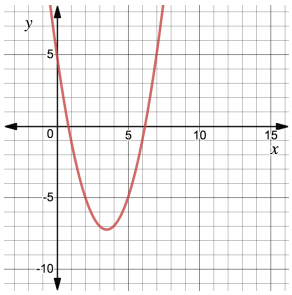
$y = 4x^2 + 11x + 7$



(c) Solve $4x^2 + 11x + 7 = 0$

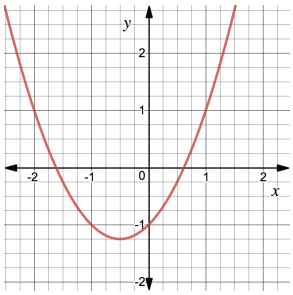
Question 3: Using the graphs, find estimates of the solutions to the following equations

$y = x^2 - 7x + 5$



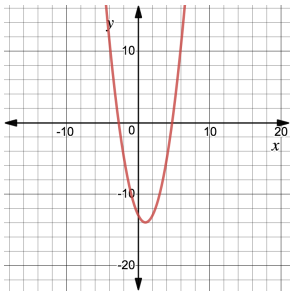
(a) $x^2 - 7x + 5 = 0$

$y = x^2 + x - 1$



(b) $x^2 + x - 1 = 0$

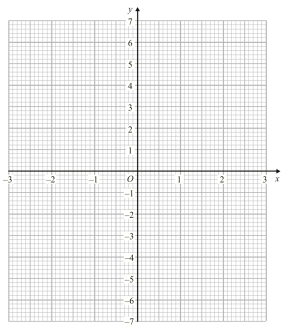
$y = x^2 - 2x - 13$



(c) $x^2 - 2x - 13 = 0$

Question 4: (a) Complete the table of values of $y = x^2 - x - 5$

x	-3	-2	-1	0	1	2	3
y			-3				1



(b) On a copy of the grid, draw the graph of $y = x^2 - x - 5$ for the value of x from -3 to 3

(c) Use your graph to find estimates of the solutions to the equation $x^2 - x - 5 = 0$

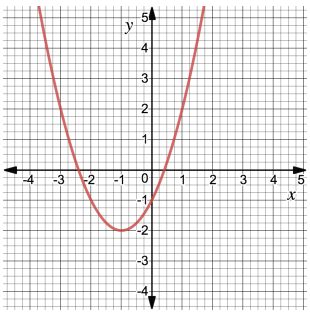
Question 5: Solve each of the following equations graphically

(a) $x^2 - 3x - 3 = 0$ (b) $x^2 + 8x + 5 = 0$ (c) $x^2 - 2x - 1 = 0$

(d) $x^2 - 5x - 8 = 0$ (e) $x^2 + 4x - 10 = 0$ (f) $2x^2 + 3x - 6 = 0$

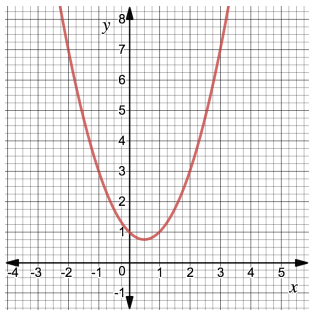
Question 6: Using the graphs below, solve each equation

$y = x^2 + 2x - 1$



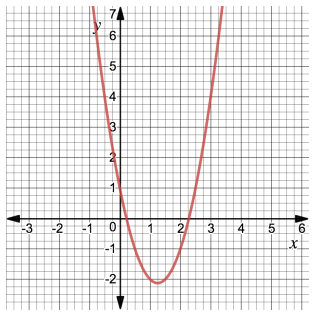
(a) Solve $x^2 + 2x - 1 = 2$

$y = x^2 - x + 1$



(b) Solve $x^2 - x + 1 = 7$

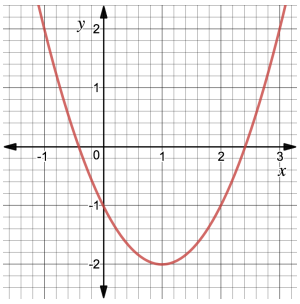
$y = 2x^2 - 5x + 1$



(c) Solve $2x^2 - 5x + 1 = 1$

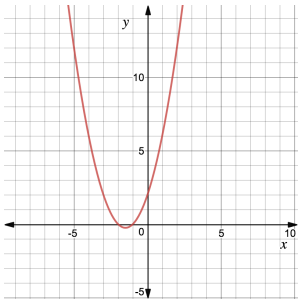
Question 7: Using the graphs, find estimates of the solutions to the following equations

$y = x^2 - 2x - 1$



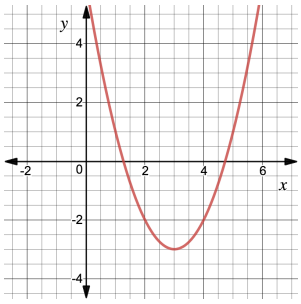
(a) $x^2 - 2x - 1 = 1$

$y = x^2 + 3x + 2$



(b) $x^2 + 3x + 2 = 11$

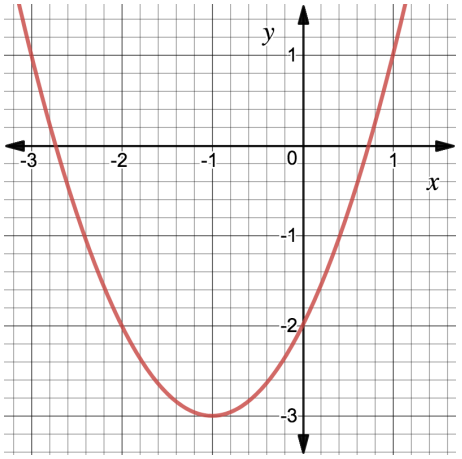
$y = x^2 - 6x + 6$



(c) $x^2 - 6x + 6 = -1$

Apply

Question 1: The graph of $y = f(x)$ is drawn on the grid.

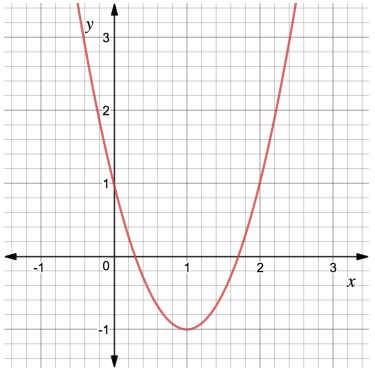


(a) Write down estimates for the roots of $f(x) = 0$

(b) Use the graph to find an estimates for the roots of $f(x) = -1$

(c) Write down the coordinates of the turning point of the graph

Question 2: The grid below shows the graph of $y = 2x^2 - 4x + 1$



The graph of $2x^2 - 4x + 1 = k$ has exactly one solution.

Use the graph to find the value of k

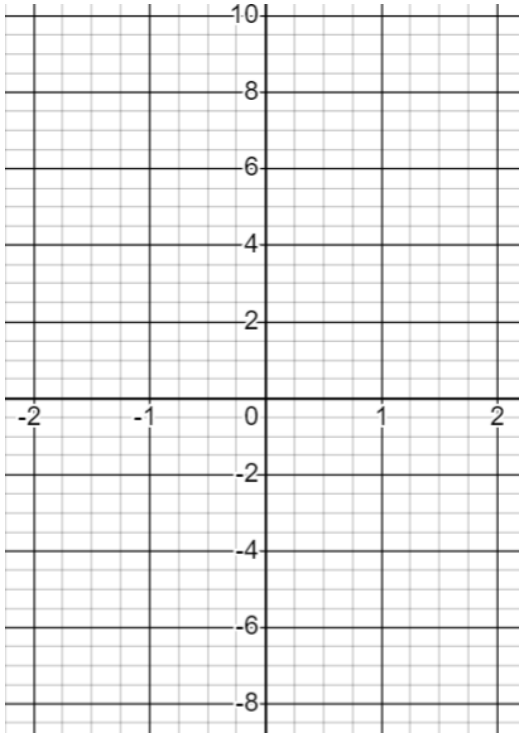
Fluency Practice

Plotting Cubic Graphs

(a)

Plot the graph of $y = x^3 + 1$ from $x = -2$ to $x = 2$

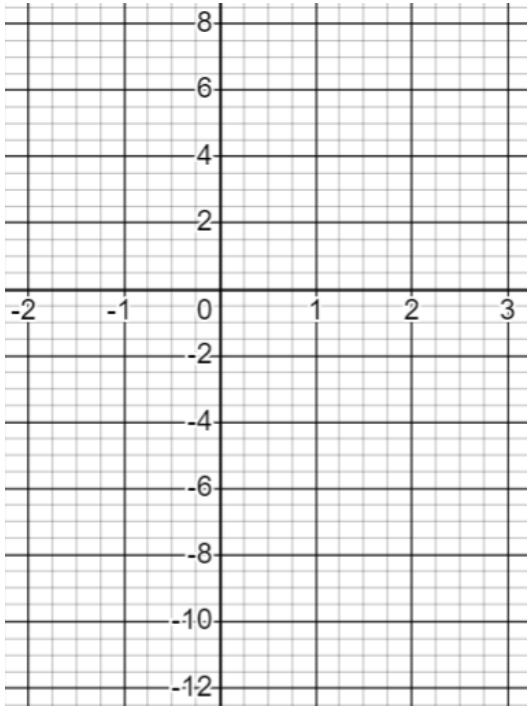
x	-2	-1	0	1	2
y					



(b)

Plot the graph of $y = x^3 - 2x^2 - x + 2$ from $x = -2$ to $x = 3$

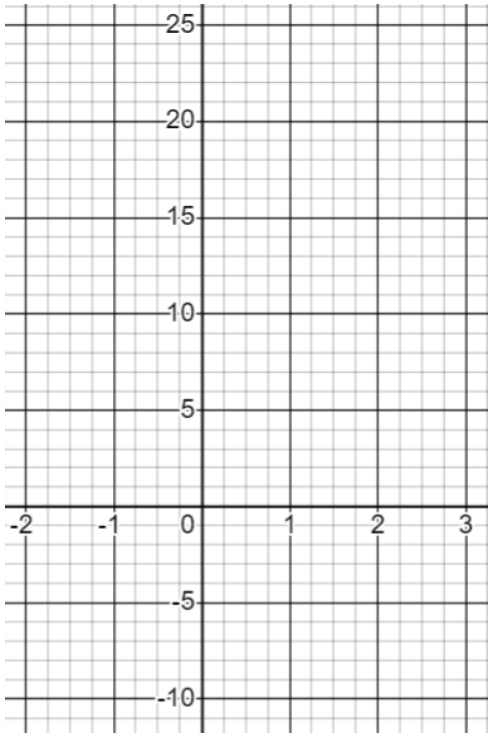
x	-2	-1	0	1	2	3
y						



(c)

Plot the graph of $y = 3 - x + 2x^2 - x^3$ from $x = -2$ to $x = 3$

x	-2	-1	0	1	2	3
y						



Fluency Practice

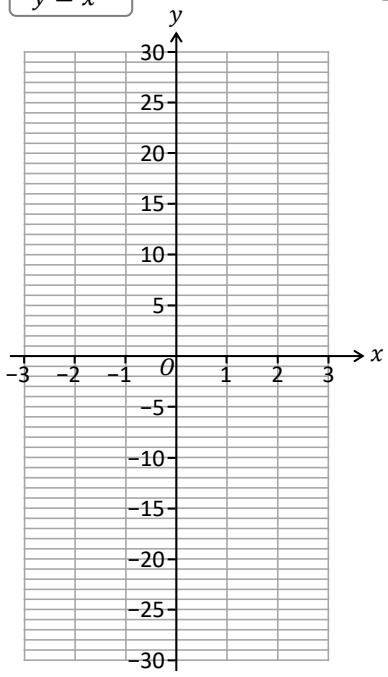
Plotting Cubics

Use a table of values to plot each graph.
What effect does each term in the equation have on the cubic shape?
What comments can you make about the changing gradient of each curve?

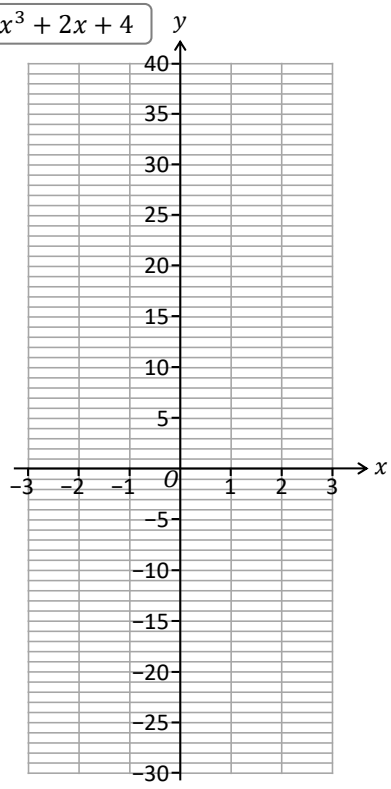
x	−3	−2	−1	0	1	2	3
y							

x	−3	−2	−1	0	1	2	3
y							

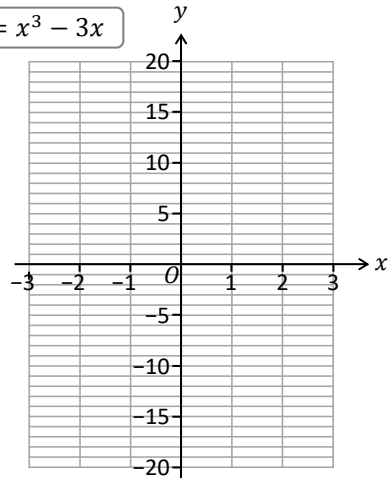
$y = x^3$



$y = x^3 + 2x + 4$

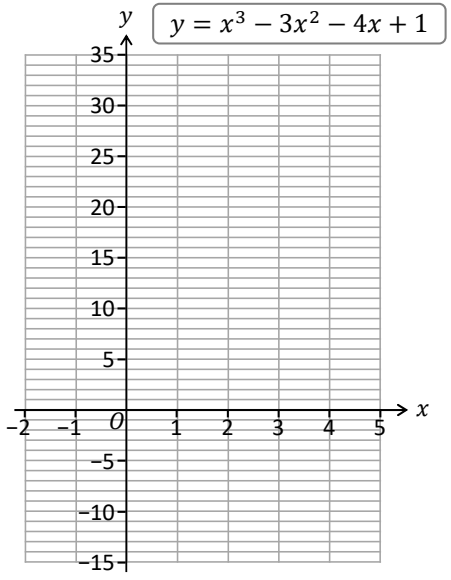


$y = x^3 - 3x$



x	y
−3	
−2	
−1	
0	
1	
2	
3	

$y = x^3 - 3x^2 - 4x + 1$



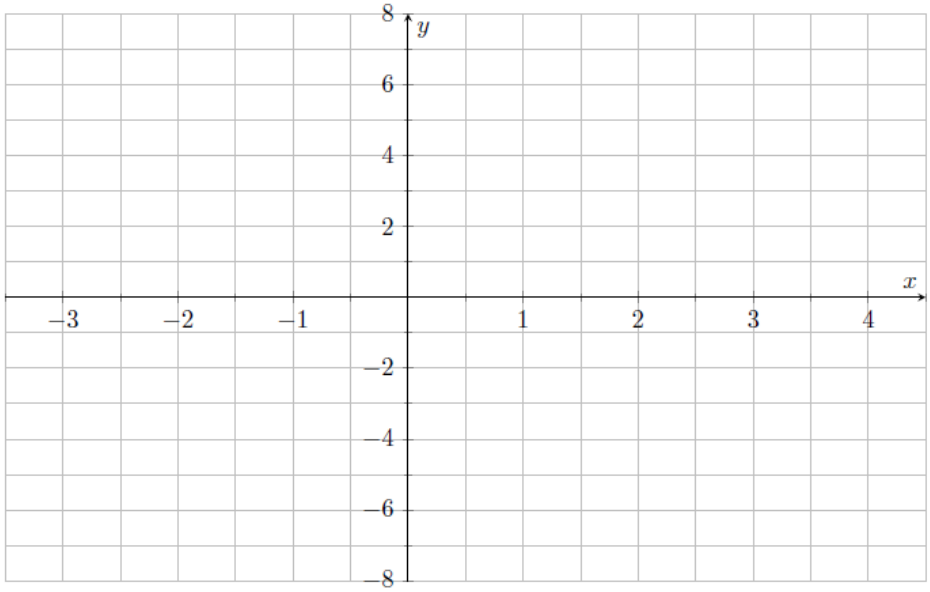
x	y
−3	
−2	
−1	
0	
1	
2	
3	

Solving Equations Graphically

(a)

Plot the graph of $y = 6 + x - x^2$. Use the graph to find the solutions to equation

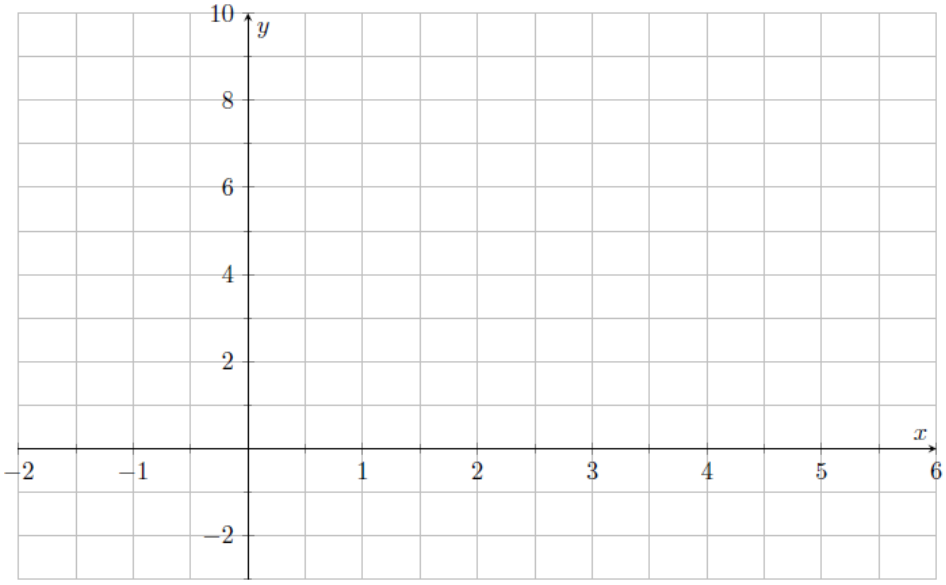
(a) $6 + x - x^2 = 0$ (b) $6 + x - x^2 = -6$



(b)

Plot the graph of $y = x^2 - 4x + 3$. Use the graph to find the solutions to equation

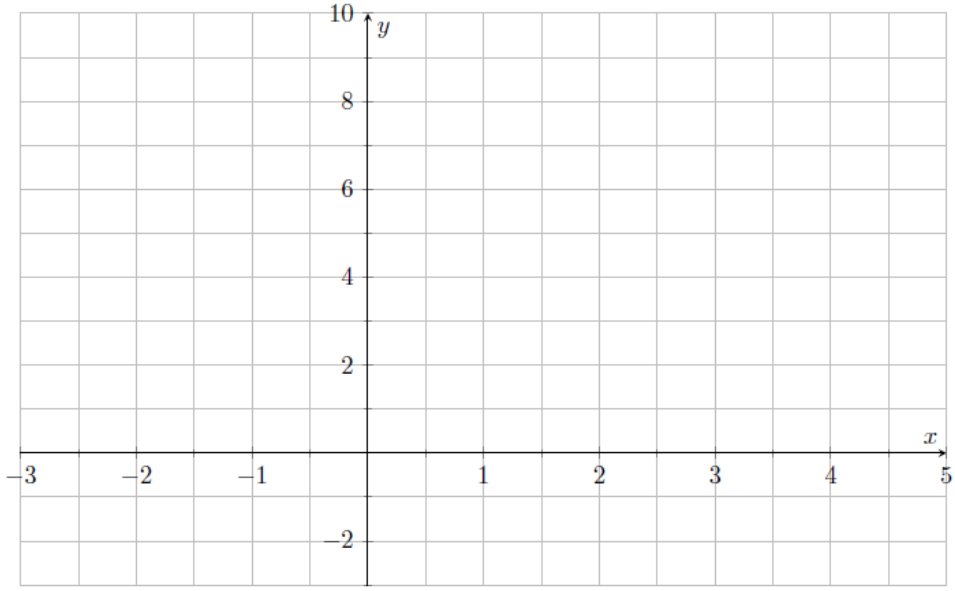
(a) $x^2 - 4x + 3 = 0$ (b) $x^2 - 4x + 3 = 8$



(c)

Plot the graph of $y = 8 + 2x - x^2$. Use the graph to find the solutions to the equation

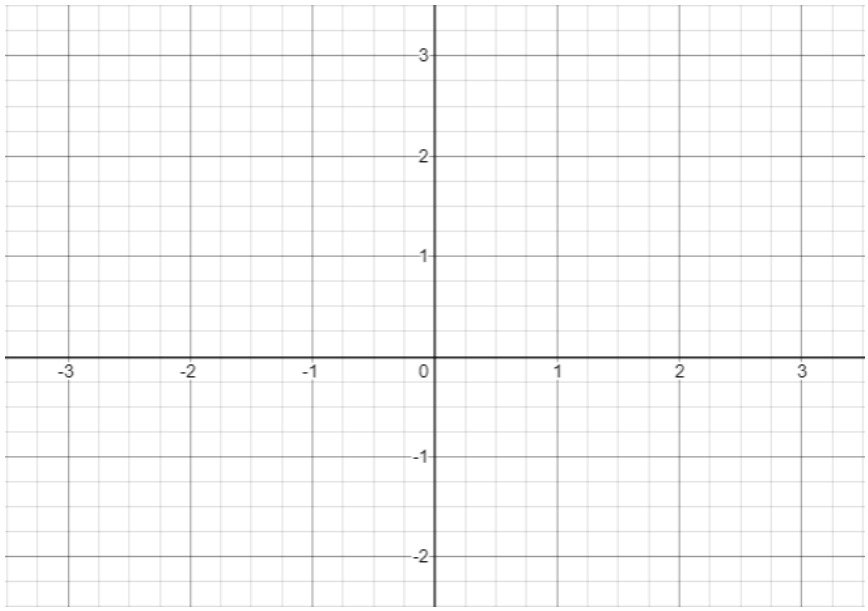
(a) $8 + 2x - x^2 = 5$ (b) $8 + 2x - x^2 = 8 - x$



(d)

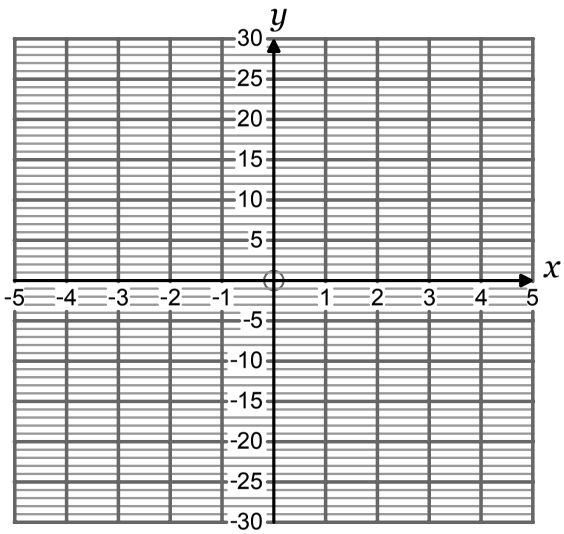
Plot the graph of $y = x^3 - 3x$. Use the graph to estimate the solutions to the equation

(a) $x^3 - 3x = 1$ (b) $x^3 - 3x = x + 1$



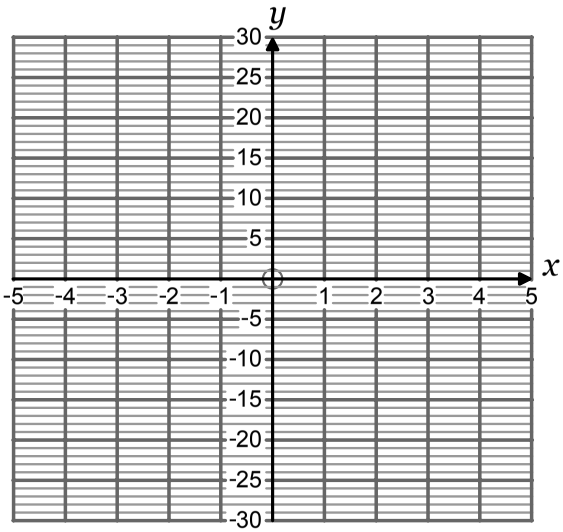
1. Plot the graph of $y = x^3$

x	y
-3	
-2	
-1	
0	
1	
2	
3	



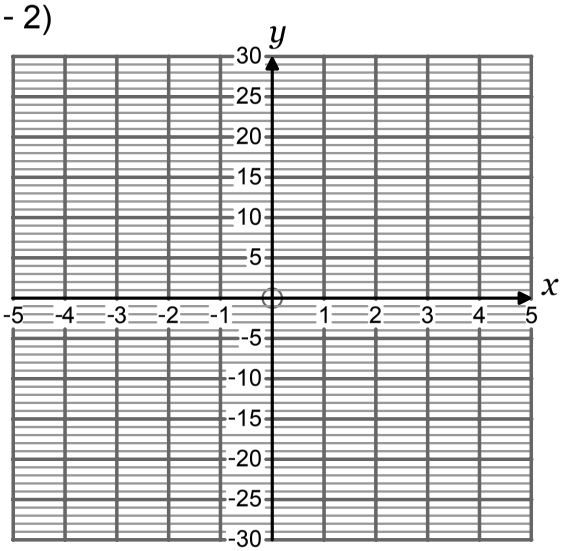
2. Plot the graph of $y = 9x - x^3$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



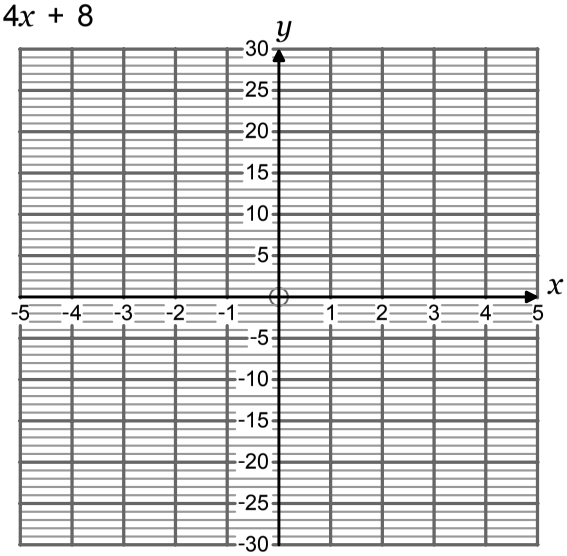
3. Plot the graph of $y = x(x+3)(x-2)$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	



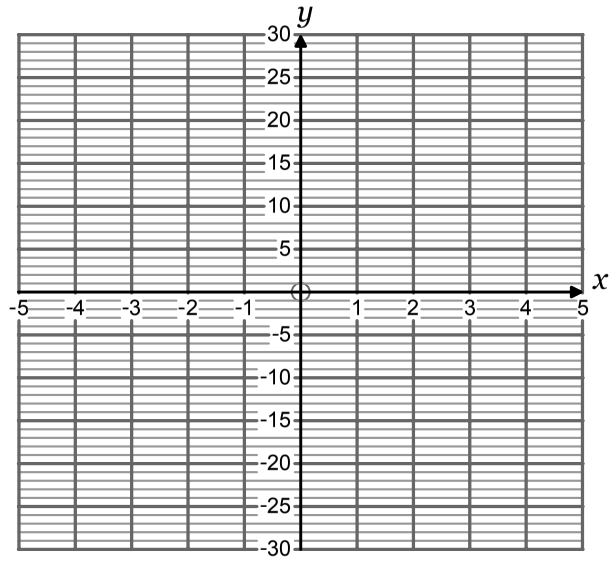
4. Plot the graph of $y = x^3 - 2x^2 - 4x + 8$

x	y
-3	
-2	
-1	
0	
1	
2	
3	
4	



5. Plot the graph of $y = -(x+1)^3$

x	y
-4	
-3	
-2	
-1	
0	
1	
2	



6. True or false?

a) All cubic graphs pass through the origin.

b) All cubic graphs have a line of symmetry.

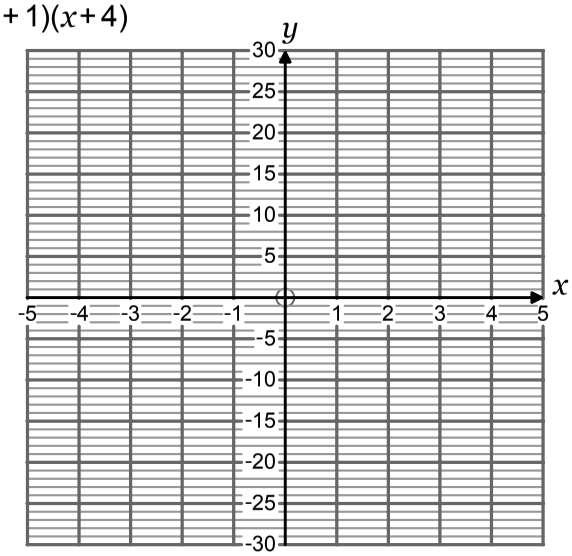
c) All cubic graphs intersect the x-axis at least once.

d) The coefficient of x^3 determines the shape of the graph.

e) Cubic graphs have asymptotes.

7. a) Plot the graph of $y = (x - 3)(x+1)(x+4)$

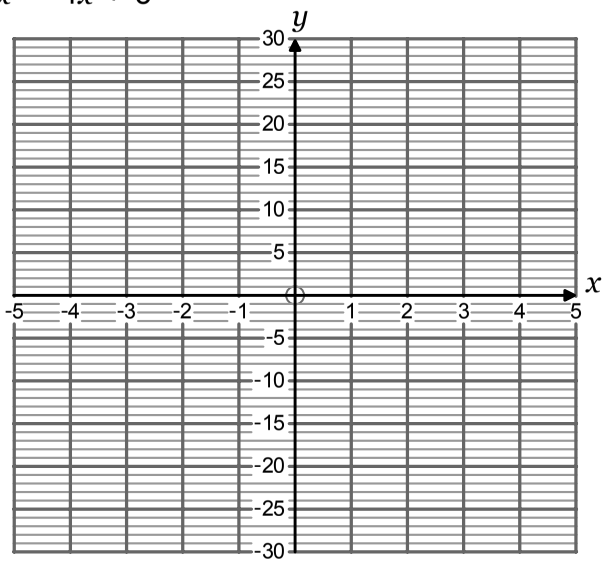
x	y
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	



b) Use your graph to estimate the solutions of: $(x - 3)(x+1)(x+4) = 5$

8. a) Plot the graph of $y = -x^3 + x^2 - 4x + 5$

x	y
-2	
-1	
0	
1	
2	
3	



b) Use your graph to estimate the solution of: $-x^3 + x^2 - 4x + 5 = -15$

9. Work out the y-intercept of the graphs of each of these cubic functions.

a) $y = x^3 + 4$ b) $y = 2x^3 + x - 1$ c) $y = -x^3$

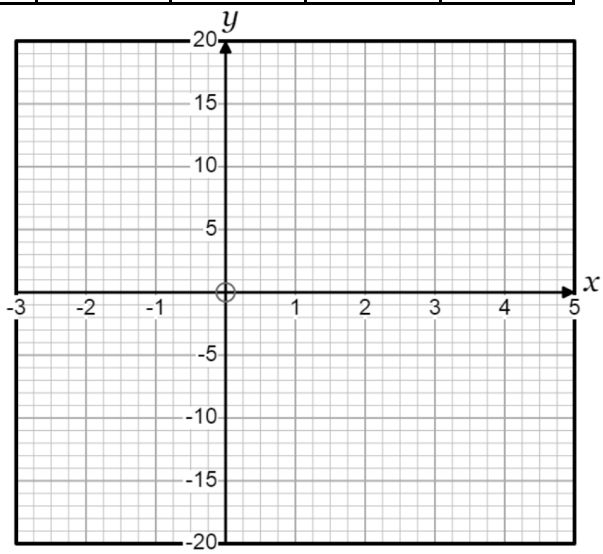
d) $y = x(x^2 + 4)$ e) $y = (x + 3)^3$ f) $y = (x - 2)(x - 3)^2$

1. Draw the graph of $y = x^3 - 4x^2 + 3$

x	-2	-1	0	1	2	3	4
y							

2. Estimate the roots of $y = x^3 - 4x^2 + 3$

3. Estimate the co-ordinates of the turning points of $y = x^3 - 4x^2 + 3$

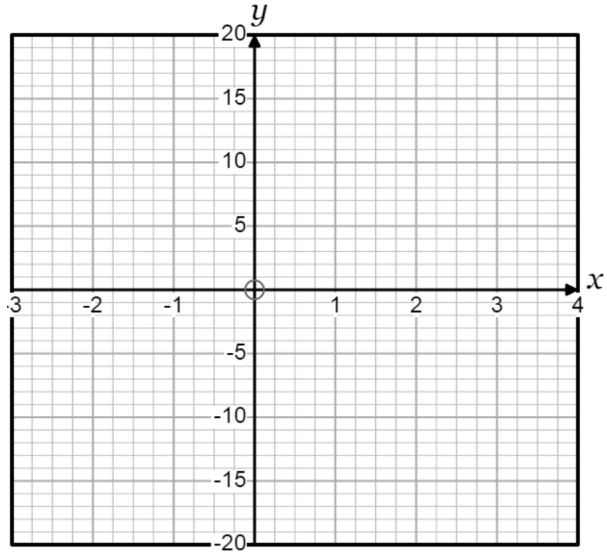


4. Draw the graph of $y = 2x(x-2)(x+2)$

x	-2.5	-2	-1	0	1	2	2.5
y							

5. What are the roots of $y = 2x(x-2)(x+2)$?

6. Estimate the co-ordinates of the turning points of $y = 2x(x-2)(x+2)$



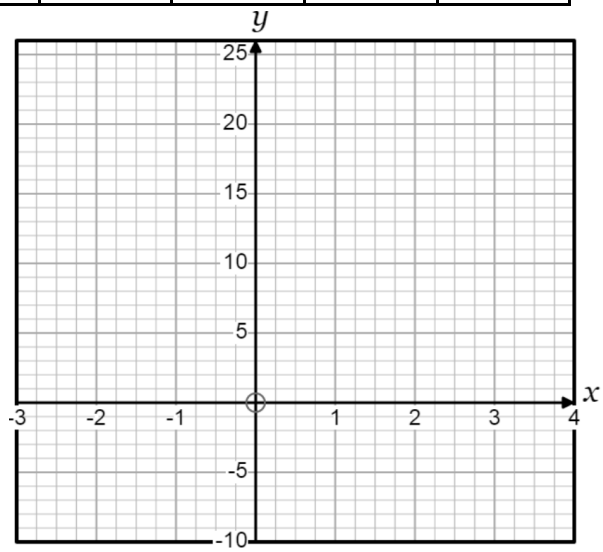
7. Draw the graph of $y = 5 - x^3 + 3x^2$

x	-2	-1	0	1	2	3	4
y							

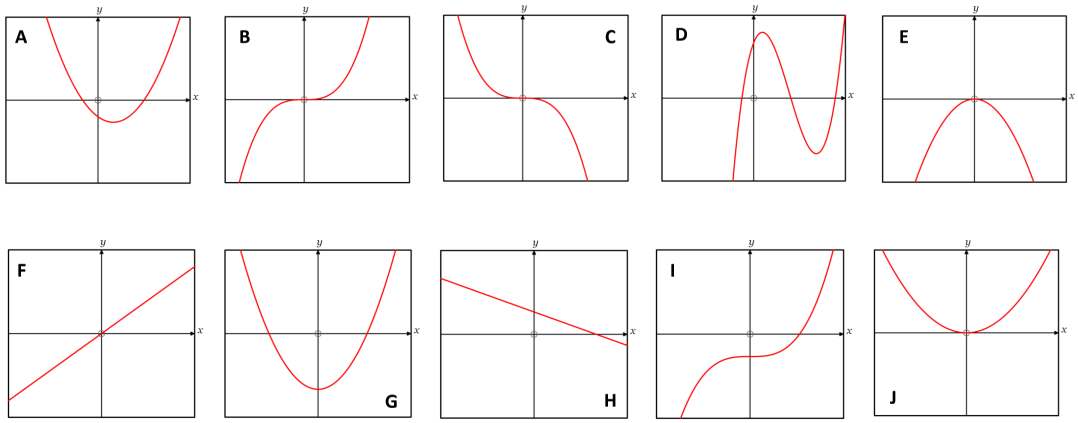
8. Estimate the roots of $y = 5 - x^3 + 2x^2$

9. Estimate the co-ordinates of the turning points of $y = 5 - x^3 + 2x^2$

10. Why is this cubic graph 'upside down'?



11. Match the graphs to their functions:



$y = -x^2$ | $y = 2x$ | $y = (x-3)(x+1)$ | $y = x^2 - 10$ | $y = -x^3$

$y = 4 - x$ | $y = 0.5x^2$ | $y = x^3$ | $y = x^3 - 3$ | $y = (x-5)(x-3)(x+1)$

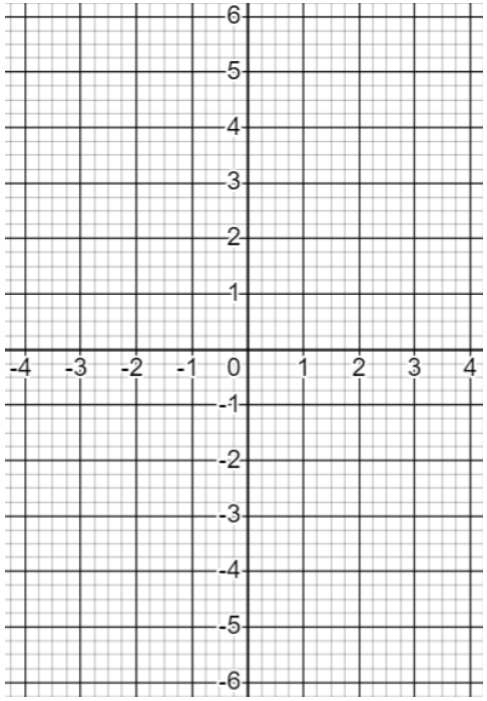
Fluency Practice

Plotting Reciprocal Graphs

(a)

Plot the graph of $y = \frac{2}{x}$ from $x = -4$ to $x = 4$

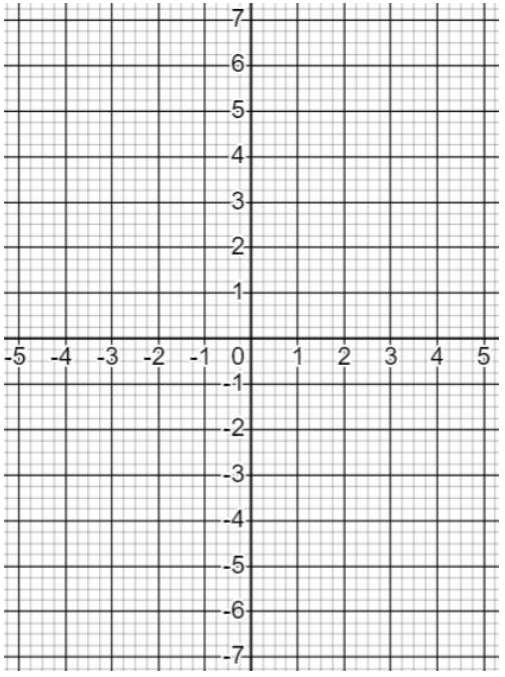
x	-4	-2	-1	0	1	2	4
y							



(b)

Plot the graph of $y = -\frac{5}{x}$ from $x = -5$ to $x = 5$

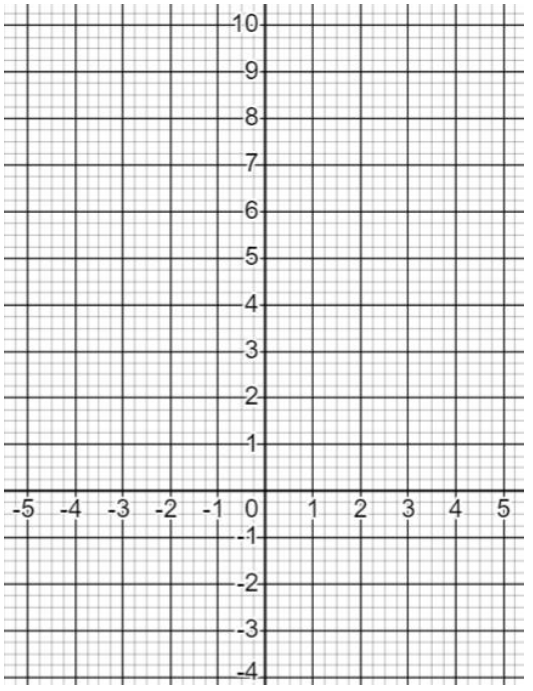
x	-5	-2	-1	0	1	2	5
y							



(c)

Plot the graph of $y = \frac{10}{x^2}$ from $x = -2$ to $x = 3$

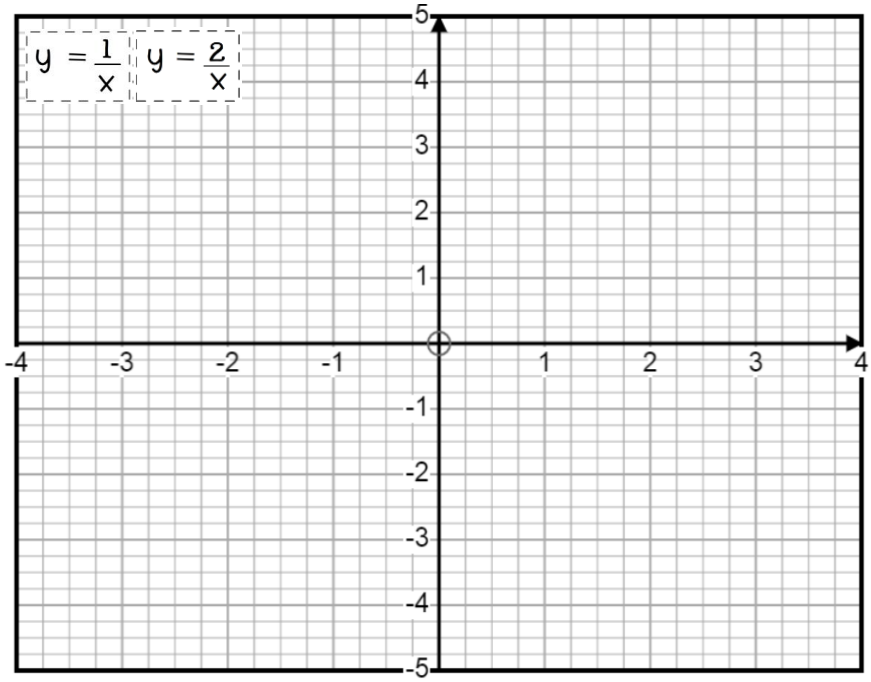
x	-5	-2	-1	0	1	2	5
y							



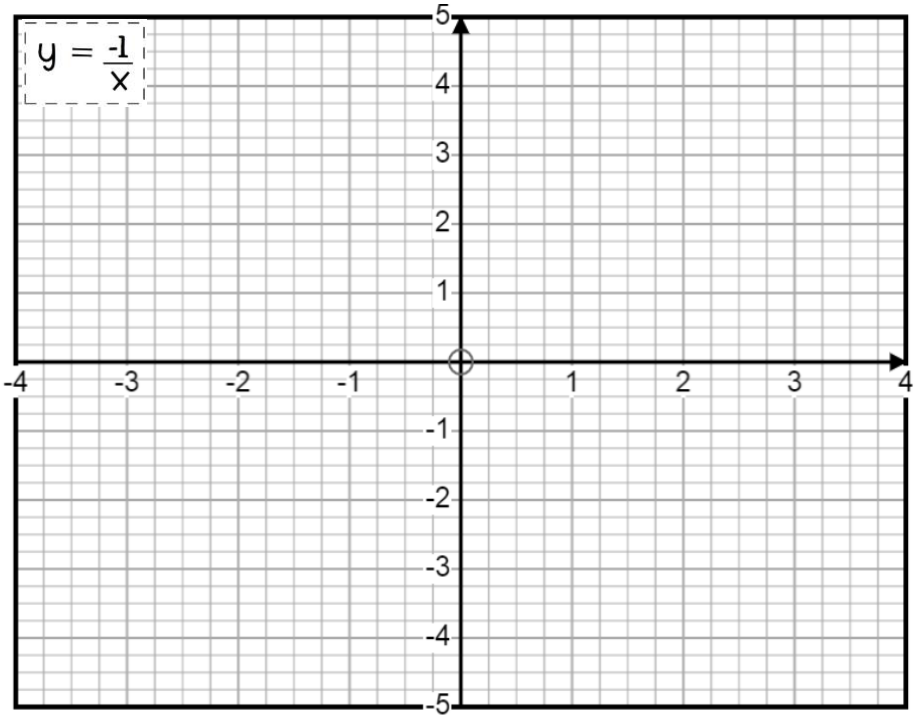
reciprocal functions

These functions involve **dividing by x**.
What shape do their graphs make?
Draw two graphs on the first set of axes.

x	$y=\frac{1}{x}$	$y=\frac{2}{x}$
-4		
-3		
-2		
-1		
-0.5		
-0.25		
0		
0.25		
0.5		
1		
2		
3		
4		



x	y
-4	
-3	
-2	
-1	
-0.5	
-0.25	
0	
0.25	
0.5	
1	
2	
3	
4	

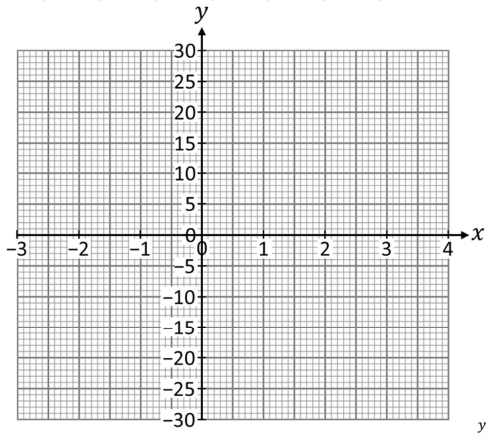


Fluency Practice

Plotting Cubic & Reciprocal Graphs

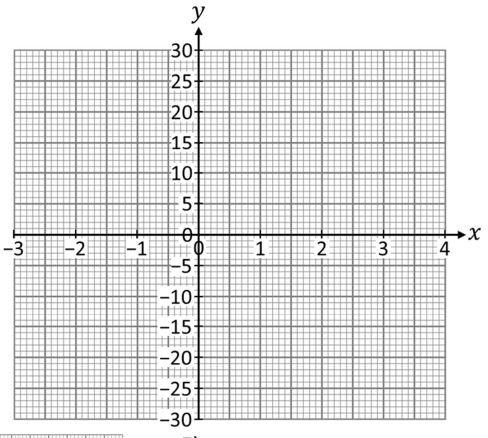
A) Plot the graph of $y = x^3$ for values of x from −3 to 3

x	−3	−2	−1	0	1	2	3
y	−27						



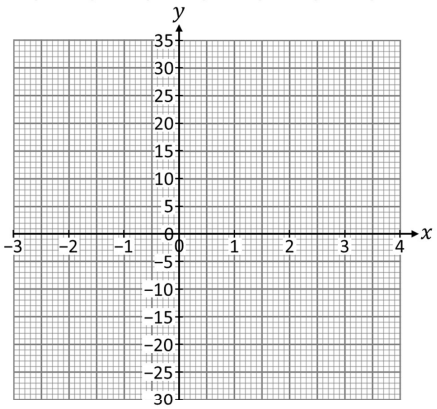
B) Plot the graph of $y = x^3 - 4$ for $-2 \le x \le 3$

x	−2	−1	0	1	2	3
y						



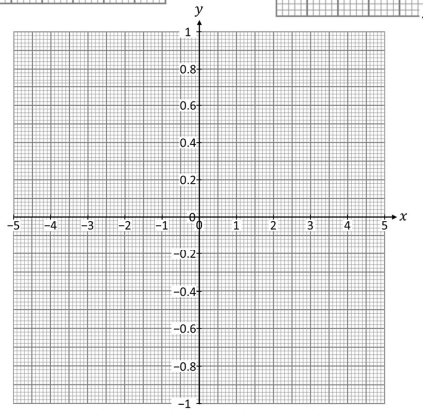
C) Plot the graph of $y = 5 - x^3$ for $-3 \le x \le 3$

x	−3	−2	−1	0	1	2	3
x^3							
y							



D) Plot the graph of $y = \frac{1}{x}$ for $-5 \le x \le 5$

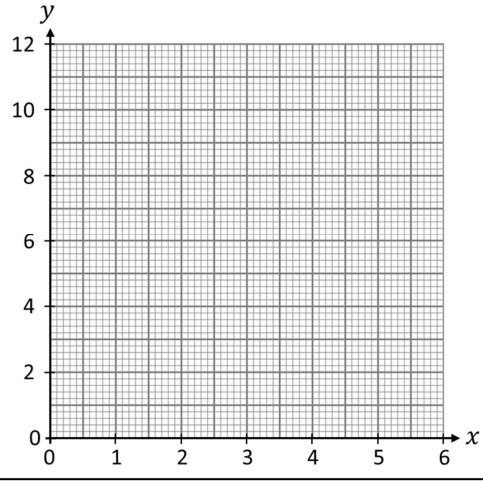
Why is there no value when $x = 0$?



x	−5	−4	−2	−1	0	1	2	4	5
y									

E) Plot the graph of $y = \frac{12}{x}$ for $0 \le x \le 6$

For your table of values, choose values for x that produce integer answers.



exponential functions

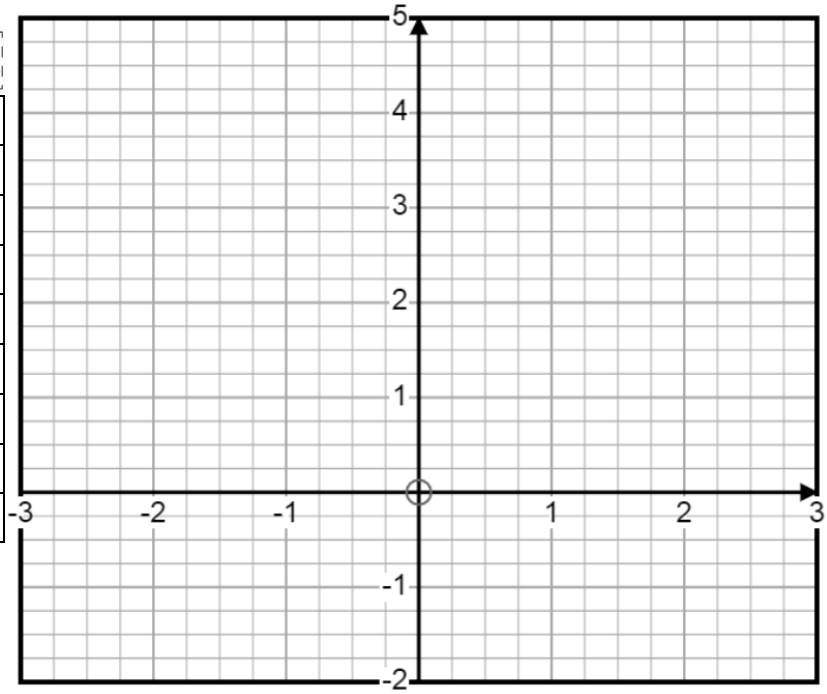
These functions have **x as a power.**
What shape do their graphs make?
Draw two functions on each set of axes.

$y = 2^x$

x	y
-3	
-2	
-1	
-0.5	
0	
0.5	
1	
2	

$y = 4^x$

x	y
-3	
-2	
-1	
-0.5	
0	
0.5	
1	
2	

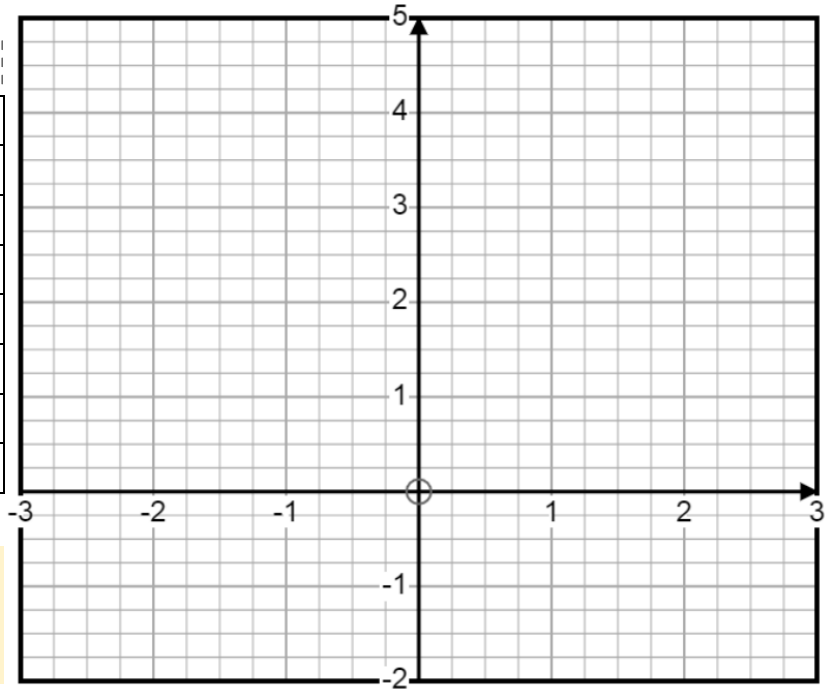


$y = 0.5^x$

x	y
-2	
-1	
-0.5	
0	
0.5	
1	
2	

$y = 4^{-x}$

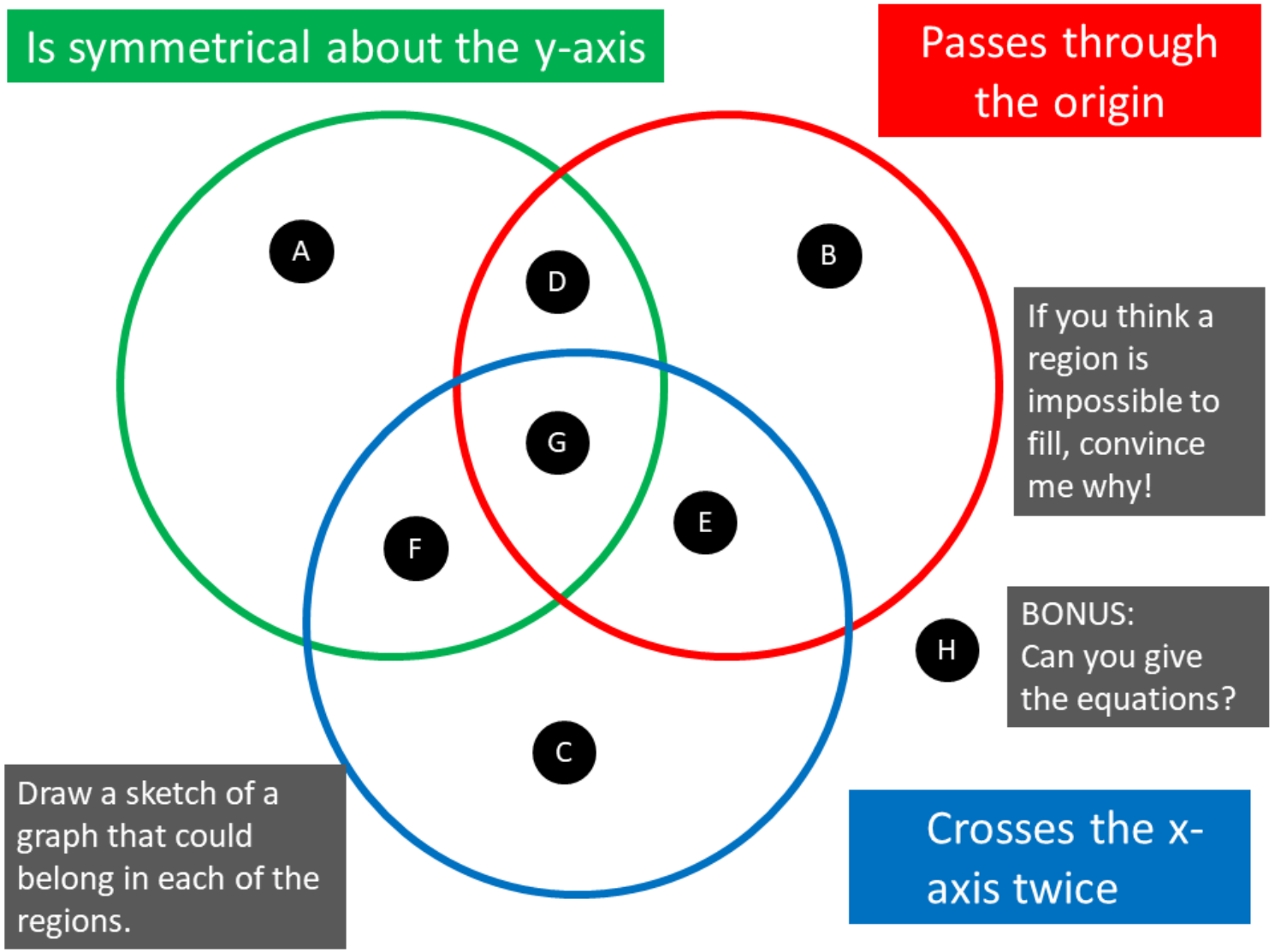
x	y
-2	
-1	
-0.5	
0	
0.5	
1	
2	



extension

All of these graphs pass through (0,1). Why?

Problem Solving

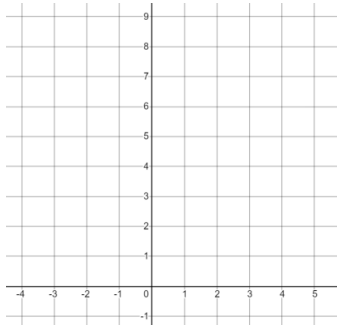


Fluency Practice

(a)

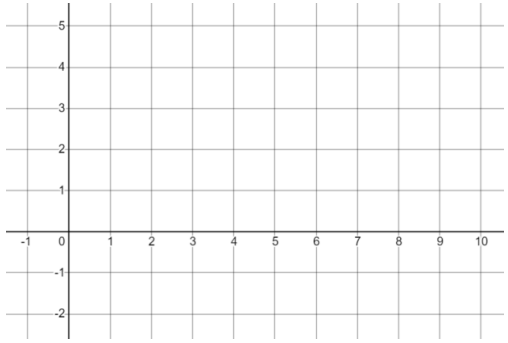
Plot the graph of the equation $y = 2x + 3$

x	-2	-1	0	1	2	3
y						



(b)

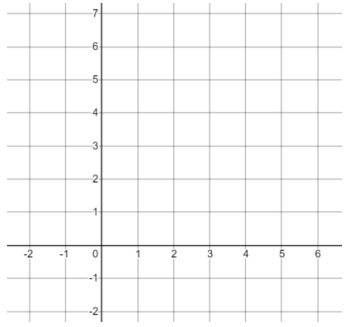
On the grid, plot the graph of $2x - 3y = 6$ from $x = 0$ to $x = 9$



(c)

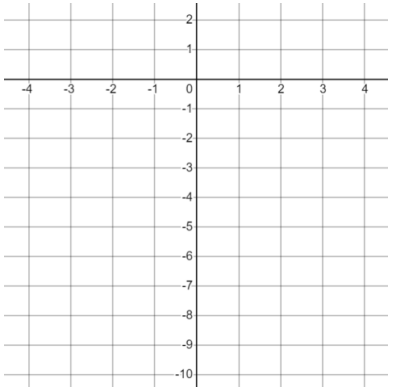
Plot the graph of $y = x^2 - 4x + 2$

x	-1	0	1	2	3	4	5
y							



(d)

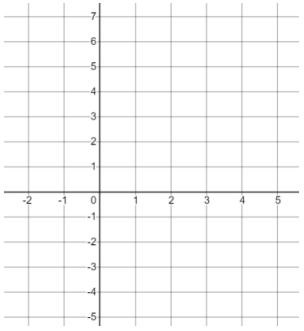
Plot the graph of $y = 2 + x - x^2$ for $x = -3$ to $x = 3$



(e)

Plot the graph of $y = x^3 - 4x^2 + 5$

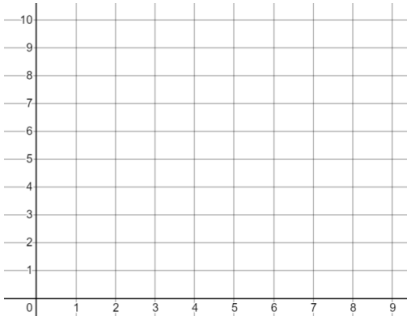
x	-1	0	1	2	3	4
y						



(f)

Plot the graph of $y = x + \frac{8}{x}$

x	1	2	3	4	6	8
y						



Fluency Practice

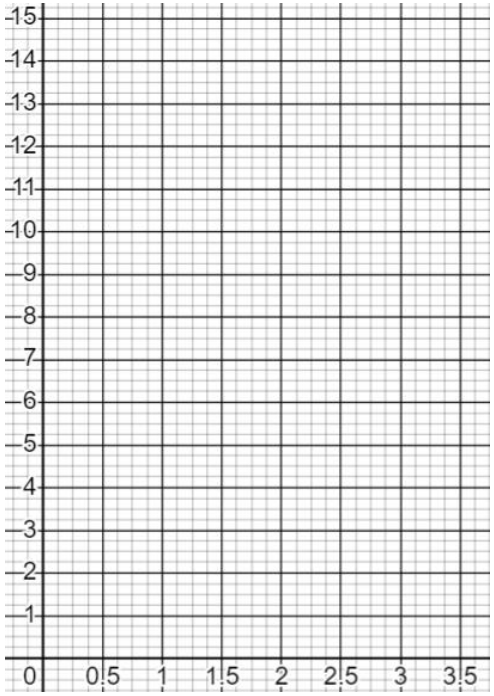
Plotting Harder Non-Linear Graphs

(a)

Plot the graph of $y = x^2 + \frac{2}{x^2}$

between $x = 0.5$ and $x = 3.5$

x	0.5	1	1.5	2	2.5	3	3.5
y							

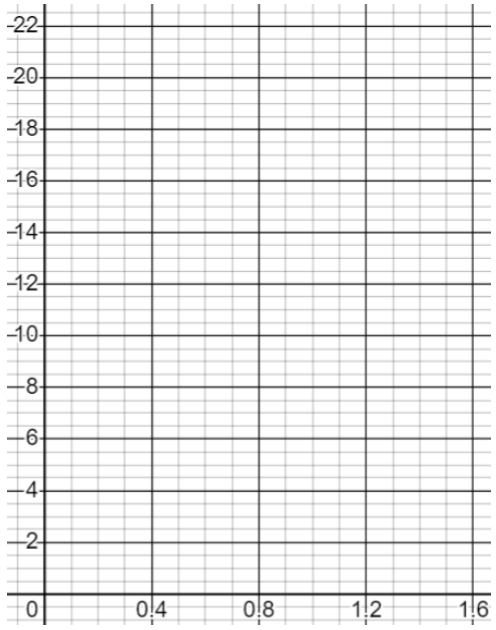


(b)

Plot the graph of $y = x^3 + 3x + \frac{2}{x}$

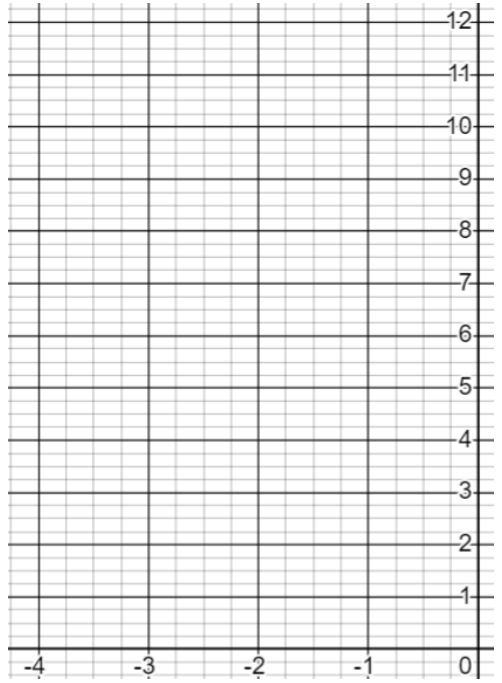
from $x = 0.1$ to $x = 1.5$

x	0.1	0.2	0.4	0.7	1	1.2	1.5
y							



(c)

Plot the graph of $y = x\left(\frac{x}{2} - \frac{1}{x^2}\right)$

from $x = -4$ to $x = -0.5$

x	-4	-3	-2	-1	-0.5	-0.25
y