Year 10

2025 Mathematics (L2FM) 2026

Unit 18 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Year 10

2025 **Mathematics (L2FM)** 2026

Unit 18 Booklet – Part 2

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Contents Page

1 Advanced Data Handling

Quartiles

Worked Example	Your Turn
Find the lower quartile, upper quartile and interquartile range for:	Find the lower quartile, upper quartile and interquartile range for
1 2 2 3 3 4 5 5	1 2 8 8 9 9 9 10
6 6 8 8 12 13 16 16	10 10 10 12 13 14 14 14
16 17 18 18 18 19 20	15 16 17 17 18 18 18 18
	18 18 20

Dr Frost 396a and 396b

Fill in the Gaps — Medians and Quartiles

Data Set											Median	Lower Quartile	Upper Quartile	Interquartile Range
3	3	4	5	5	6	6	8	10	10	10	6	4	10	
6	6	6	7	7	8	10	10	11	11	12	8	6		
1	2	3	3	5	7	7	7	8	9	9				
4	4	4	4	5	5	5	5	6	6	6				
0.2	0.4	0.5	0.5	0.5	0.6	0.7	0.8	0.8	0.9	1.1				
7	8	8	9	10	10	11	12	12				8	11.5	
14	14	16	17	17	18	18	19	20						
1.7	1.8	2.0	2.1	2.1	2.2	2.3	2.5	2.5						
5	6		7	9		10	11		12	12	10	6	11	
5		7	7	8		8		9	10				9	4
18	18		22			23		27			23	19	24	
8.2		8.4	8.5		8.7	8.7		9.1			8.5	8.3	8.75	

Mode of Ungrouped Data

	Worked Example	Your Turn

Worked Example

Determine the modal score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the modal score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Dr Frost 237b

Range of Ungrouped Data

Worked Example	Your Turn
Determine the range of the scores:	Determine the range of the scores:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Dr Frost 237c

Median of Ungrouped Data

Fluency Practice

	Number of pieces of data:	Position of the median:

(a)   4, 10, 11, 12, 12, 15, 20

(b)   4, 10, 11, 12, 12, 15

(c)   10, 11, 12, 12, 15

(d)   10, 11, 12, 12

(e)   1, 3, 6, 8, 9, 12

Number of pieces of data:	Position of the median:
7	
11	
10	
41	
24	
	8
	3.5
	40
	21.5

Fluency Practice

Number of pets	Frequency	Which pieces of data are in this category?
0	3	1st 2nd 3rd
1	2	4th 5th
2	4	

Number of pets	Frequency	Which pieces of data are in this category?
0		1st 2nd
1		3rd
2		4th 5th 6th 7th 8th
3		9th 10th
4		11th 12th 13th

Number of pets	Frequency	Which pieces of data are in this category?
0	8	
1	9	
2	13	
3	12	
4	9	

Number of pets	Frequency	Which pieces of data are in this category?
0	5	
1	1	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0		1st
1		2nd 3rd 4th
2		5th 6th 7th 8th
3		9th 10th
4		11th 12th

Number of pets	Frequency	Which pieces of data are in this category?
0		1st to 13th
1		14th to 29th
2		30th to 59th
3		60th to 80th
4		81st to 92nd

Number of pets	Frequency	Which pieces of data are in this category?
0	2	
1	1	
2	5	

Number of pets	Frequency	Which pieces of data are in this category?
0	21	1st to 21st
1	15	22nd to…
2	18	
3	25	
4	32	

Number of pets	Frequency	Which pieces of data are in this category?
0	1	
1	3	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0	10	
1	12	
2	15	
3	20	
4	5	

Worked Example	Your Turn
Calculate the median score:	Calculate the median score:

Worked Example table:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	7

Your Turn table:

Score	Frequency
0	9
1	6
2	2
3	4
4	4
5	8

	Worked Example	Your Turn
	Calculate the median score:	Calculate the median score:

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Calculate the median score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Worked Example	Your Turn
Calculate the median score:	Calculate the median score:

Worked Example:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	2

Your Turn:

Score	Frequency
0	8
1	6
2	2
3	4
4	4
5	8

Dr Frost 237a

Quartiles of Ungrouped Data

Worked Example	Your Turn
Calculate the lower quartile and upper quartile score:	Calculate the lower quartile and upper quartile score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	9

Score	Frequency
0	7
1	6
2	2
3	4
4	4
5	8

Dr Frost 396g

Mean of Ungrouped Data

Worked Example	Your Turn
Calculate the total score:	Calculate the total score:

Calculate the total score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Calculate the total score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Dr Frost 235c

	Worked Example	Your Turn

Calculate the mean score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Calculate the mean score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Dr Frost 236a

Fill in the Blanks — Mean from a Frequency Table

Data Set

7	7	7	7
7			8
8	8	8	8
8			
9	9	9	9

Frequency Table

Value	Frequency	Value × Frequency
7	6	42
8	9	72
9	5	45
Totals	20	159

Mean 159 ÷ 20 = 7.95

Data Set

4	4	4	4

Frequency Table

Value	Frequency	Value × Frequency
4	13	
5	2	
6	5	
Totals		

Mean ÷ =

Data Set

2	2	2	2
2	3	3	3
3	3	3	3
4	4	4	4
4	4	4	4

Frequency Table

Value	Frequency	Value × Frequency
2		
3		
4		
Totals	20	

Mean ÷ 20 =

Data Set

12	12	12	12
12	12	12	12
12	12	12	12
12	14	14	14
14	14	14	14

Frequency Table

Value	Frequency	Value × Frequency
12		
13		
14		
Totals		

Mean ÷ =

Fill in the Gaps

Top Left

Data Set			
4	5	5	5
5	6	6	6
6	6	6	7
7	7	7	7
7	7	7	7

Frequency Table

Value	Frequency	Value × Frequency
4		
5		
6		
7		
Totals		

Mean ÷ =

Top Right

Data Set			

Frequency Table

Value	Frequency	Value × Frequency
2.5	8	
2.6	3	
2.7		
2.8	5	
Totals	20	

Mean ÷ =

Bottom Left

Data Set			

Frequency Table

Value	Frequency	Value × Frequency
8	5	
9	8	
10		
11	6	
Totals	24	

Mean ÷ =

Bottom Right

Data Set			

Frequency Table

Value	Frequency	Value × Frequency
3	2	
4		
5		
6	12	
Totals	24	

Mean ÷ = 5.25

Worked Example

The table gives information about the numbers of badges gained by the girls in a Guide group.
a)   Write down the mode.
b)   Find the range.
c)   Work out the median
d)   Calculate the mean.

Number of badges	Frequency
0	2
1	8
2	4
3	3
4	5
5	3

Fill in the Gaps

Class			Total Frequency (class size)	Most common age (modal age)	Oldest student	Youngest student	Range of ages	Total of all their ages	Mean age	Median age
A	**Age** **Frequency** 5 — 3 6 — 7									
B	**Age** **Frequency** 7 — 3 8 — 9 — 9		20							
C	**Age** **Frequency** 10 — 11 — 14 12 — 13 —		30	13		10	3			
D	**Age** **Frequency** (blank) (blank) (blank) (blank)			10 and 11	12		2	108	10.8	

Modal Class of Grouped Data

	Worked Example		Your Turn

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Range of Grouped Data

	Worked Example	Your Turn

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$0 < x \le 10$	5
$10 < x \le 20$	3
$20 < x \le 40$	2
$40 < x \le 46$	6
$46 < x \le 50$	7

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$10 < x \le 20$	5
$20 < x \le 30$	3
$30 < x \le 50$	2
$50 < x \le 56$	6
$56 < x \le 60$	7

	Worked Example	Your Turn

Worked Example

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Median of Grouped Data

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \leq 40$	4
$40 < x \leq 45$	9
$45 < x \leq 50$	26
$50 < x \leq 55$	13
$55 < x \leq 60$	8
$60 < x \leq 65$	20

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

Quartiles of Grouped Data

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the lower quartile and upper quartile.
Give your answer correct to 1 decimal place.

Your Turn

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \leq 40$	4
$40 < x \leq 45$	9
$45 < x \leq 50$	26
$50 < x \leq 55$	13
$55 < x \leq 60$	8
$60 < x \leq 65$	20

Use interpolation to estimate the lower quartile and upper quartile.
Give your answer correct to 1 decimal place.

Estimated Mean of Grouped Data

	Worked Example	Your Turn

Calculate an estimate for the total mass:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	6
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Calculate an estimate for the total mass:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	0
$16 < x \leq 24$	7
$24 < x \leq 32$	4

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	Worked Example	Your Turn

Worked Example

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \le 8$	3
$8 < x \le 16$	6
$16 < x \le 24$	7
$24 < x \le 32$	4

Your Turn

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \le 8$	3
$8 < x \le 16$	0
$16 < x \le 24$	7
$24 < x \le 32$	4

Dr Frost 312c

Fill in the Blanks — Mean from a Grouped Frequency Table

Value	Frequency	Midpoint	Midpoint × Freq
$10 \leq x < 12$	6	11	66
$12 \leq x < 14$	9	13	117
$14 \leq x < 16$	5	15	75
Totals	20		258
Estimate of Mean	258 ÷ 20 =		

Value	Frequency	Midpoint	Midpoint × Freq
$20 \leq x < 30$	9	25	225
$30 \leq x < 40$	7	35	
$40 \leq x < 50$	4	45	
Totals	20		
Estimate of Mean	÷ 20 =		

Value	Frequency	Midpoint	Midpoint × Freq
$20 \leq x < 24$	6	22	
$24 \leq x < 28$	10		
$28 \leq x < 32$	5		
$32 \leq x < 36$	4		
Totals	25		
Estimate of Mean	÷ =		

Value	Frequency	Midpoint	Midpoint × Freq
$5 \leq x < 10$	7		
$10 \leq x < 15$	7		
$15 \leq x < 20$	8		
$20 \leq x < 25$	3		
Totals	25		
Estimate of Mean	÷ =		

Fill in the Gaps

Value	Frequency	Midpoint	Midpoint × Freq
100 ≤ x < 120	7		
120 ≤ x < 140	12		
140 ≤ x < 160			
160 ≤ x < 180	2		
Totals	30		
Estimate of Mean	÷	=	

Value	Frequency	Midpoint	Midpoint × Freq
0 ≤ x < 2			3
2 ≤ x < 4			9
4 ≤ x < 6			40
6 ≤ x < 8			42
Totals	20		
Estimate of Mean	÷ 20	=	

Value	Frequency	Midpoint	Midpoint × Freq
0 ≤ x <			40
≤ x <			240
≤ x <			275
≤ x < 40			
Totals			
Estimate of Mean	÷ 40	= 18.25	

Value	Frequency	Midpoint	Midpoint × Freq
≤ x <			
≤ x <			
≤ x <		55	1430
≤ x <		65	780
Totals			
Estimate of Mean	÷ 50	= 54.2	

In each table, values are grouped into classes of equal width.

Worked Example

Bob asked each of 40 friends how many minutes they took to get to work. The table shows some information about his results.
a)   Write down the modal class.
b)   Work out the upper and lower bounds for the range.
c)   Work out the class in which the median lies.
d)   Calculate an estimate for the median.
e)   Calculate an estimate for the mean.

Time taken (m minutes)	Frequency
$0 < m \leq 10$	3
$10 < m \leq 20$	8
$20 < m \leq 30$	11
$30 < m \leq 40$	9
$40 < m \leq 50$	9

Extra Notes

Worked Example	Your Turn
Expand and simplify: $(x + 2)(x - 3)(x - 4)$	Expand and simplify: $(x + 4)(x - 3)(x - 2)$

Worked Example	Your Turn
Expand and simplify: $2x(7x - 3)(x - 4)$	Expand and simplify: $4x(7x - 3)(x - 2)$

Worked Example	Your Turn
Expand and simplify: $(5x + 2)(7x - 3)(x - 4)$	Expand and simplify: $(5x + 4)(7x - 3)(x - 2)$

Dr Frost 300b and 300c

Expanding Three Brackets

Question	Multiply first two brackets			Simplify first two brackets	Multiply by third bracket				Simplified Answer
$(x+2)(x+1)(x+5)$	$\times$	x	$+2$	x^2+3x+2	$\times$	x^2	$+3x$	$+2$	$x^3+8x^2+17x+10$
	x	x^2	$+2x$		x	x^3	$+3x^2$	$+2x$	
	$+1$	$+x$	$+2$		$+5$	$+5x^2$	$+15x$	$+10$	
$(y+4)(y-2)(y+3)$	$\times$	y	$+4$		$\times$				
	y				y				
	-2				$+3$				
$(b-3)(b-5)(b-6)$	$\times$				$\times$				
$(2x+3)(x-4)(x+2)$	$\times$				$\times$				
$(2a-1)(3a+1)(a-2)$	$\times$				$\times$				

Worked Example	**Your Turn**
Expand and simplify: $(3x - 2)^3$	Expand and simplify: $(4x - 3)^3$

Worked Example	Your Turn
Expand and simplify: $(2x - 3)^2(1 - 3x)$	Expand and simplify: $(3x - 2)^2(1 - 2x)$

Question	Multiply first two brackets			Simplify first two brackets	Multiply by third bracket				Simplified Answer
$(x+3)(x-2)^2$ $= (x+3)(x-2)(x-2)$	$\times$	x	$+3$		$\times$	x^2	$+x$	-6	
	x	x^2	$+3x$	x^2+x-6	x				
	-2	$-2x$	-6		-2				
$(y-5)(y+1)^2$	$\times$				$\times$				
$(t+6)(3t-2)^2$	$\times$				$\times$				
$(x+4)^3$	$\times$				$\times$				
$(2y-3)^3$	$\times$				$\times$				

Fill in the Gaps

Expanded Expression	Factorised Expression
$2x + 8$	$2(x+4)$
	$3(x-2)$
	$x(x+7)$
$5x + 35$	
$8x - 12$	
	$2x(x-5)$
$x^2 - x$	
	$5x(3-x)$
$10x^2 + 2x$	
$6x + 9y$	
	$4xy(x+2)$
$6xy - 4y^2$	
	$(x+2)(x+3)$
	$(x+5)(x-3)$
$x^2 + 8x + 15$	
$x^2 + 3x + 2$	

Expanded Expression	Factorised Expression
$x^2 - 7x + 10$	
	$(x-6)(x+4)$
	$(x+7)(x-7)$
$x^2 + x - 20$	
$x^2 - 25$	
	$(2x+1)(x+5)$
$x^2 - x - 6$	
$x^2 + 3x$	
	$(3x-1)(x-2)$
$4x^2 - 25$	
	$(x+5)^2$
$7x^2 + 10x + 3$	
	$(3x-1)^2$
$4x^2 + 4x + 1$	
$5x^2 - 14x - 3$	
	$(x-2)^3$

Worked Example	**Your Turn**
Expand and simplify: $(3x - 2)^3 - (6x - 5)^2$	Expand and simplify: $(4x - 3)^3 - (5x - 6)^2$

Extra Notes

3 Solving Quadratics

Multiplication by Zero

$(a - 3) \times 2 = 0$ $a = \underline{\hspace{2cm}}$

$(a - 7) \times a = 0$ $a = \underline{\hspace{2cm}}$

$a \times a = 0$ $a = \underline{\hspace{2cm}}$

$(a + 5)(a - 3) = 0$ $a = \underline{\hspace{2cm}}$

$a^2 + 6a + 8 = 0$ $a = \underline{\hspace{2cm}}$

$a^2 + 8a + 16 = 0$ $a = \underline{\hspace{2cm}}$

Worked Example	Your Turn
What values of x satisfy the equation $x(x - 9) = 0$?	What values of x satisfy the equation $(x + 6)x = 0$?

Worked Example	Your Turn
What values of x satisfy the equation $(x-9)(x+5) = 0$?	What values of x satisfy the equation $(x+6)(x-5) = 0$?

Worked Example	Your Turn
Solve the equation $(2x - 3)(3x + 1) = 0$	Solve the equation $(3x + 2)(2x - 1) = 0$

Worked Example	Your Turn
Solve the equation $x^2 + 2x - 8 = 0$	Solve the equation $x^2 + 2x - 15 = 0$

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Worked Example	Your Turn
Solve the equation $x^2 - 49 = 0$	Solve the equation $x^2 - 64 = 0$

Worked Example	Your Turn
Solve the equation $3x^2 + 2x = 0$	Solve the equation $2x^2 - 3x = 0$

Worked Example	Your Turn
Solve the equation $x^2 - 4x + 4 = 0$	Solve the equation $x^2 + 14x + 49 = 0$

Fill in the Gaps

	Quadratic	Factorised	Equations	Solutions
1	$x^2 + 6x + 5 = 0$	$(x+5)(x+1) = 0$	$x+5=0$ $x+1=0$	
2	$x^2 + 7x + 10 = 0$	$(x+5)(x+2) = 0$	$x+5=0$ $x+\ =0$	$x=-5$ $x=$
3	$x^2 + 8x + 15 = 0$	$(x+5)(x+3) = 0$		
4	$x^2 + 7x + 12 = 0$	$(x+4)(x+\ ) = 0$		
5	$x^2 + 8x + 16 = 0$	$(x+4)(x+\ ) = 0$		
6	$x^2 + 10x + 24 = 0$			
7	$x^2 + 14x + 24 = 0$			
8	$x^2 + 14x + 40 = 0$			
9		$(x+\ )(x+\ ) = 0$	$x+8=0$ $x+5=0$	
10				$x=-40$ $x=-1$
11	$x^2 + 4x - 5 = 0$	$(x+5)(x-1) = 0$	$x+5=0$ $x-1=0$	
12	$x^2 - 4x - 5 = 0$	$(x-5)(x+1) = 0$	$x-5=0$ $x+1=0$	
13	$x^2 + 4x - 12 = 0$	$(x+6)(x-2) = 0$	$x+6=0$ $x-2=0$	
14	$x^2 + x - 12 = 0$	$(x+4)(x-3) = 0$		
15	$x^2 - 11x - 12 = 0$	$(x-12)(x\ \ ) = 0$		
16	$x^2 + 2x - 8 = 0$	$(x+4)(x\ \ ) = 0$		
17	$x^2 - 2x - 8 = 0$			
18	$x^2 + 7x - 8 = 0$			
19	$x^2 - 7x - 8 = 0$			
20		$(x-5)(x-1) = 0$		
21				$x=5$ $x=2$
22				$x=5$ $x=5$
23				$x=-5$ $x=5$

Solving Quadratics by Factorising

Quadratic Equation	Factorised Equation	1st Equation	2nd Equation	1st Solution	2nd Solution
$x^2 + 8x + 15 = 0$	$(x + 5)(x + 3) = 0$	$x + 5 = 0$	$x + 3 = 0$	$x = -5$	$x = -3$
$x^2 - 8x - 20 = 0$					
$x^2 - x - 20 = 0$					
$x^2 - x - 6 = 0$					
$x^2 - 5x + 6 = 0$					
$x^2 - 5x + 4 = 0$					
$x^2 - 4x + 4 = 0$					
	$(x - 7)(x + 3) = 0$				
		$x - 8 = 0$	$x + 5 = 0$		
		$x + 6 = 0$			$x = -7$
				$x = 3$	$x = -5$

Worked Example	Your Turn
Solve the equation $5x^2 + 13x - 6 = 0$	Solve the equation $5x^2 + 7x - 6 = 0$

Worked Example	Your Turn
Solve the equation $4x^2 - 9 = 0$	Solve the equation $16x^2 - 81 = 0$

Worked Example	Your Turn
Solve the equation $x^2 - x = 12$	Solve the equation $x^2 = 2x + 3$

Worked Example	Your Turn
Solve the equation $12x^2 + 10x - 12 = 0$	Solve the equation $18x^2 - 15x - 18 = 0$

Worked Example	Your Turn
Solve the equation $x(x - 2) = 15$	Solve the equation $(x - 3)(x + 2) = 6$

Worked Example	Your Turn
a) A quadratic equation has solutions of $x = -9$ and $x = 6$ Find the original equation, giving your answer in the form $x^2 + bx + c = 0$, where b and c are whole numbers.	a) A quadratic equation has solutions of $x = -8$ and $x = 2$ Find the original equation, giving your answer in the form $x^2 + bx + c = 0$, where b and c are whole numbers.
b) A quadratic equation has solutions of $x = -\frac{3}{2}$ and $x = \frac{1}{4}$ Find the original equation, giving your answer in the form $8x^2 + bx + c = 0$	b) A quadratic equation has solutions of $x = -\frac{3}{5}$ and $x = \frac{1}{2}$ Find the original equation, giving your answer in the form $10x^2 + bx + c = 0$

Worked Example	**Your Turn**
Solve $5x - \dfrac{3}{x} = 14$	Solve $3x + \dfrac{14}{x} = 13$

Worked Example	Your Turn
Solve the following equation: $x + 2 = -\dfrac{4}{3x - 7}$	Solve the following equation: $-\dfrac{7}{3x + 4} = x - 6$

Worked Example	Your Turn
The width of a rectangular field is 3 m shorter than its length. The area of the field is 126 m². Show that $x^2 + bx + c = 0$ where x is the length of the field and b and c are constants to be found.	The length of a rectangular swimming pool is 6 m longer than its width. The area of the swimming pool is 144 m². Show that $y^2 + by + c = 0$ where y is the width of the swimming pool and b and c are constants to be found.

Worked Example	Your Turn
The area of the triangle shown below is 15 cm²	The area of the triangle shown below is 30 cm²

The area of the triangle shown below is 15 cm²

$2x - 1$

$3x + 4$
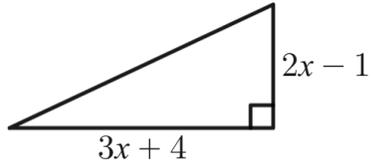
All lengths are in centimetres.

Show that $6x^2 + bx + c = 0$, where b and c are integers to be found.

The area of the triangle shown below is 30 cm²

$2x - 5$

$2x - 1$
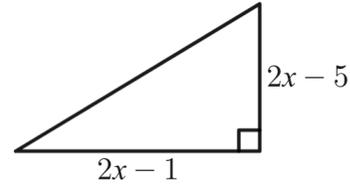
All lengths are in centimetres.

Show that $4x^2 + bx + c = 0$, where b and c are integers to be found.

Dr Frost 418b

The area of the triangle shown below is 82 cm²
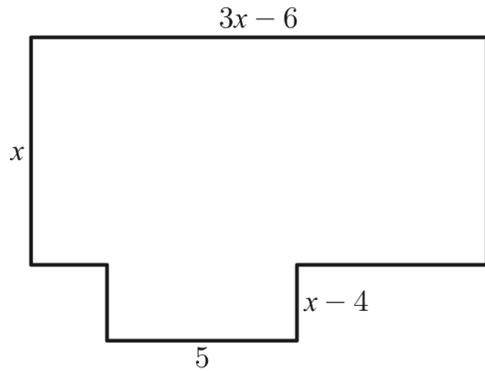
$3x - 6$

x

$x - 4$

5

All lengths are in centimetres.

Show that $3x^2 + bx + c = 0$, where b and c are integers to be found.

The area of the triangle shown below is 62 cm²

$3x + 5$

6

$2x + 6$

$4x$
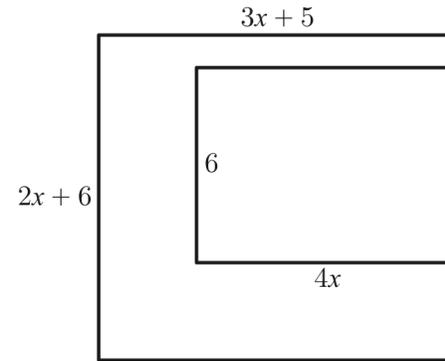
All lengths are in centimetres.

Show that $3x^2 + bx + c = 0$, where b and c are integers to be found.

Worked Example	**Your Turn**
The area of the trapezium is 219 mm². All measurements are in mm. Show that $6y^2 + by + c = 0$, where b and c are integers to be determined.	The area of the trapezium is 216 mm². All measurements are in mm. Show that $2x^2 + bx + c = 0$, where b and c are integers to be determined.
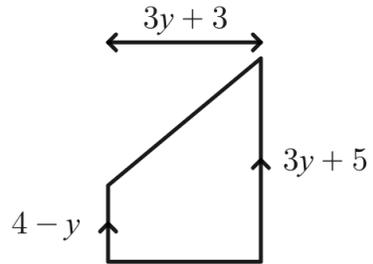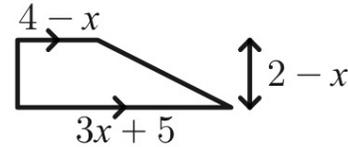
Worked Example diagram labels: $3y + 3$, $3y + 5$, $4 - y$

Your Turn diagram labels: $4 - x$, $2 - x$, $3x + 5$

Dr Frost 418d

Worked Example	Your Turn
In the diagram below, the area of the rectangle is three times the area of the triangle.	In the diagram below, the area of the rectangle is twice the area of the trapezium.
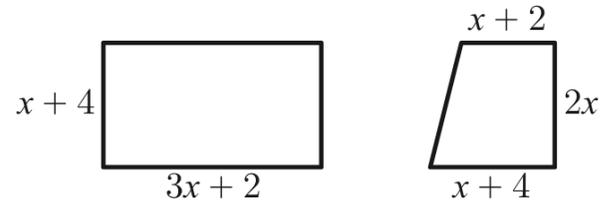
Worked Example side:

$2x - 7$ (rectangle, width $x + 3$)

triangle with height x and base $x - 1$

All measurements are in centimetres.
Determine the value of x

Your Turn side:

$x + 4$ (rectangle, width $3x + 2$)

trapezium with top $x + 2$, side $2x$, bottom $x + 4$

All measurements are in centimetres.
Determine the value of x

Worked Example	Your Turn
The rectangle in the diagram below has a perimeter of 26 cm	The rectangle in the diagram below has a perimeter of 10 cm
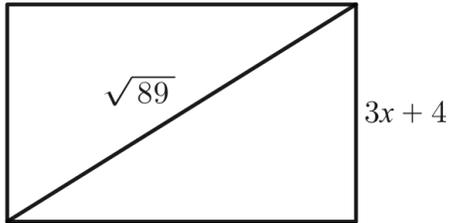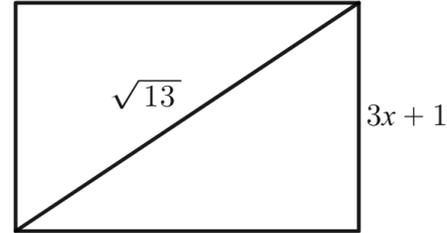
<table>
<tr><td>

The rectangle in the diagram below has a perimeter of 26 cm

$\sqrt{89}$ $3x + 4$

The width of the rectangle is $(3x + 4)$ cm
The length of the diagonal of the rectangle is $\sqrt{89}$ cm

Determine the possible values of x

</td><td>

The rectangle in the diagram below has a perimeter of 10 cm

$\sqrt{13}$ $3x + 1$

The width of the rectangle is $(3x + 1)$ cm
The length of the diagonal of the rectangle is $\sqrt{13}$ cm

Determine the possible values of x

</td></tr>
</table>

Dr Frost 418h

Worked Example	Your Turn
The diagram below shows a cuboid and a cube. The volume of the cuboid is 27 cm³ less than the volume of the cube.	The diagram below shows a cuboid and a cube. The volume of the cuboid is 12 cm³ less than the volume of the cube.

Worked Example

The diagram below shows a cuboid and a cube.
The volume of the cuboid is 27 cm³ less than the volume of the cube.

$x + 5$ $x - 3$ $x - 1$ x
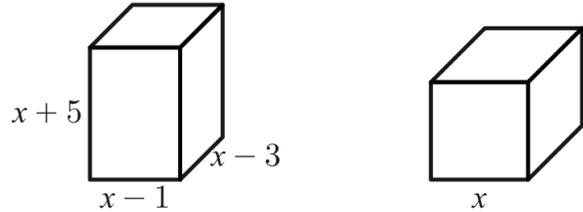
All measurements are in centimetres.
Determine the value of x

Your Turn

The diagram below shows a cuboid and a cube.
The volume of the cuboid is 12 cm³ less than the volume of the cube.

$x - 3$ $x + 4$ $x + 1$ x
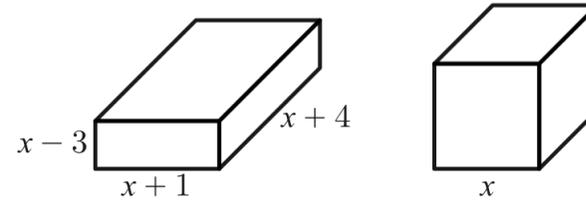
All measurements are in centimetres.
Determine the value of x

Worked Example	Your Turn
The cuboid shown below has a total surface area of 202 cm²	The cuboid shown below has a total surface area of 148 cm²
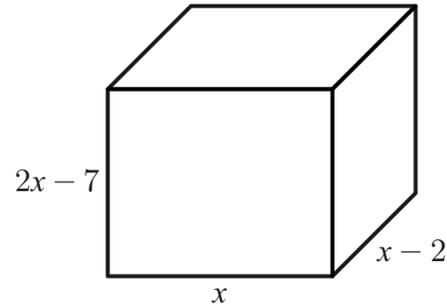
$x + 2$

$x + 3$

$2x + 5$

$2x - 7$

$x - 2$

x

All measurements are in centimetres.
Determine the value of x

All measurements are in centimetres.
Determine the value of x

Worked Example	Your Turn
The prism shown below has a volume of 39 cm³ Diagram NOT drawn to scale. All measurements are given in centimetres. Determine the value of x	The prism shown below has a volume of 15 cm³ Diagram NOT drawn to scale. All measurements are given in centimetres. Determine the value of x
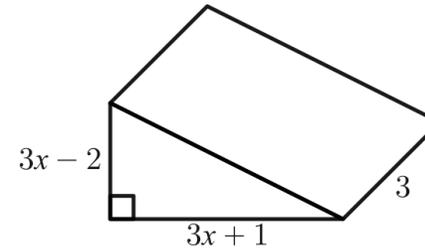
In the Worked Example diagram: $4x - 1$, 3, $2x - 5$

In the Your Turn diagram: $3x - 2$, 3, $3x + 1$

Worked Example	**Your Turn**
Solve: a) $\quad x^2 - 28 = 53$ b) $\quad 5\sqrt[3]{x} = 100$	Solve: a) $\quad \dfrac{\sqrt{x}}{3} = 4$ b) $\quad 24 + x^3 = 88$

Worked Example	Your Turn
Solve: a) $\sqrt{19 + 5x^2} = 8$ b) $\sqrt{13 + \sqrt{4y + 1}} = 4$	Solve: a) $\sqrt{53 - \sqrt{3z + 1}} = 7$ b) $\sqrt{37 + 3x^2} = 8$

Quadratic Formula

Worked Example	Your Turn
Write down the values of a, b and c in: a) $5x^2 + 2x - 3 = 0$ b) $x^2 + 2x - 3 = 0$ c) $x^2 + 2x = 4x - 3$	Write down the values of a, b and c in: a) $5x^2 - 2x + 3 = 0$ b) $x^2 - 2x + 3 = 0$ c) $x^2 - 2x = -4x + 3$

Intelligent Practice

Questions	a	b	c
$3x^2 + 5x + 1 = 0$			
$0 = 3x^2 + 5x + 1$			
$0 = 3x^2 + 5x + 2$			
$3x^2 + 4x + 2 = 0$			
$0 = 3x^2 + 4x - 2$			
$3x^2 - 4x + 2 = 0$			
$x^2 - 4x + 2 = 0$			
$x^2 + 2 - 4x = 0$			
$1 + 2x - 4x^2 = 0$			
$1 + 2x = 4x^2$			

Intelligent Practice

Questions	a	b	c
$2x = 4x^2 + 1$			
$1 = 4x^2 + 2$			
$4x^2 + 2x = 0$			
$4x^2 + 2 = 0$			
$2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 2x$			
$-2(2x^2 + 1) = 2x + 2$			
$-2(2x^2 + 1) = x^2 + 2x + 2$			
$-2(2x^2 + x + 1) = x^2 + 2x + 2$			

Discriminant

Worked Example	Your Turn
Calculate the discriminant of $5x^2 + 6x - 7$	Calculate the discriminant of $-6x^2 + 7x + 8$

Dr Frost 492a

Worked Example	Your Turn
By calculating the discriminant, work out the number of real solutions to the equation $4x^2 - 2x - 9 = 0$	By calculating the discriminant, work out the number of real solutions to the equation $-4x^2 - 7x - 4 = 0$

Dr Frost 492b

Worked Example	Your Turn
Solve the following quadratic equation, leaving your answer in exact form: $x^2 - 9x - 2 = 0$	Solve the following quadratic equation, leaving your answer in exact form: $x^2 - 10x + 3 = 0$

Dr Frost 417a

Worked Example	Your Turn
Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $3x^2 + 7x - 2 = 0$	Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $3x^2 - 9x + 2 = 0$

Dr Frost 417b

Worked Example	Your Turn
Solve the equation $4x^2 = 7x + 1$ giving your answers correct to two decimal places.	Solve the equation $7x^2 = 4x + 1$ giving your answers correct to two decimal places.

Worked Example	Your Turn
Catherine is using the quadratic formula to solve a quadratic equation. She substitutes values into the formula and correctly gets $x = \frac{4 \pm \sqrt{16+60}}{6}$	Brigid is using the quadratic formula to solve a quadratic equation. She substitutes values into the formula and correctly gets $x = \frac{-3 \pm \sqrt{9+64}}{4}$
The original equation was in the form $ax^2 + bx + c = 0$, where a, b, and c are integers which cannot be simplified.	The original equation was in the form $ax^2 + bx + c = 0$, where a, b, and c are integers which cannot be simplified.
Determine the values of a, b, and c.	Determine the values of a, b, and c.

Using the Quadratic Formula

Quadratic Equation	a, b and c	$b^2 - 4ac$	$x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$	$x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$	Solutions to 3sf
$x^2 + 5x + 1 = 0$	$a = 1, b = 5, c = 1$	$5^2 - 4 \times 1 \times 1$ $= 21$	$x = \dfrac{-5 + \sqrt{21}}{2}$	$x = \dfrac{-5 - \sqrt{21}}{2}$	
$2x^2 + 5x + 1 = 0$	$a = 2, b = 5, c = 1$	$5^2 - 4 \times 2 \times 1$ $= 17$			
$2x^2 - 5x + 1 = 0$	$a = 2, b = -5, c = 1$	$(-5)^2 - 4 \times 2 \times 1$ $= 17$	$x = \dfrac{5 + \sqrt{17}}{4}$		
$x^2 - 7x + 3 = 0$					
$2x^2 - 7x + 3 = 0$					
$5x^2 + x - 2 = 0$					
	$a = 3, b = 5, c = 2$				
			$x = \dfrac{-9 + \sqrt{89}}{4}$	$x = \dfrac{-9 - \sqrt{89}}{4}$	

Fill in the Gaps

1) Identify a, b & c and substitute into the quadratic formula.
2) **Remember!** The square root of the discriminant can be positive or negative –
    You must complete the formula for both!
3) Give your answers to 2 decimal places

$x^2 + 5x + 3 = 0$	$a = 1$ $b = 5$ $c = 3$	$x = \dfrac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \dfrac{-(\ ) \pm \sqrt{(\qquad)}}{(\qquad)}$	$x = \dfrac{-(\ ) - (\ )}{(\ )}$ $x = \dfrac{-(\ ) + (\ )}{(\ )}$	$x = -0.70$ or $x =$
$x^2 + 6x - 2 = 0$	$a = 1$ $b =$ $c =$	$x = \dfrac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \dfrac{-(\ ) \pm \sqrt{(\qquad)}}{(\qquad)}$	$x = \dfrac{-(\ ) - (\ )}{(\ )}$ $x = \dfrac{-(\ ) + (\ )}{(\ )}$	$x = 0.32$ or $x =$
$2x^2 + 8x + 3 = 0$	$a =$ $b =$ $c =$	$x = \dfrac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \dfrac{-(\ ) \pm \sqrt{(\qquad)}}{(\qquad)}$		
$4x^2 + 3x - 6 = 0$		$x = \dfrac{-(\ ) \pm \sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$			
$5x^2 - 4x - 8 = 0$					

2) Use the quadratic formula to solve these equations. Give your answers to 2 decimal places.

a) $2x^2 + 9x = -3$

b) $4x^2 - 3 = 7x$

c) $5x = -8 + 5x^2$

Fill in the Gaps

1 Complete the table below.

Q	Equation	a	b	c	Substitution	Simplification	Exact Answer	Answers to 3sf
Ex	$x^2 + 5x + 1 = 0$	1	5	1	$x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(1)}}{2(1)}$	$x = \dfrac{-5 \pm \sqrt{25-4}}{2}$	$x = \dfrac{-5 \pm \sqrt{21}}{2}$	x = -4.79 & x = -0.209
1	$x^2 + 6x + 1 = 0$							
2	$x^2 + 6x + 2 = 0$							
3	$x^2 + 7x + 2 = 0$							
4	$x^2 + 7x - 2 = 0$							
5	$x^2 - 7x - 2 = 0$							
6	$-x^2 - 7x - 2 = 0$							
7	$-x^2 - 7x + 2 = 0$							
8	$2 - 7x - x^2 = 0$							

Fill in the Gaps

#	Equation							
9		1	7	3				
10		1	7	4				
11		1	-7	4				
12					$x=\dfrac{-(-7)\pm\sqrt{(-7)^2-4(-1)(4)}}{2(-1)}$			
13						$x=\dfrac{-(-8)\pm\sqrt{64+16}}{2(-1)}$		
14		-1					$x=\dfrac{-8\pm\sqrt{84}}{-2}$	
15	$\dfrac{1}{2}x^2+4x+5=0$	$\dfrac{1}{2}$						
16	$\dfrac{1}{2}(x^2+4x+5)=0$							
17	$\dfrac{1}{3}(x^2+4x+5)=0$							
18	$\dfrac{2x^2+4x+5}{3}=0$							

2 Complete the table below.

Q	Equation	a	b	c	Substitution	Simplification	Exact Answer	Answers to 3sf
Ex	$2x^2 + 5x + 1 = 0$	2	5	1	$x = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{2(2)}$	$x = \dfrac{-5 \pm \sqrt{25 - 8}}{4}$	$x = \dfrac{-5 \pm \sqrt{17}}{4}$	x = -2.28 & x = -0.219
1	$2x^2 + 6x + 1 = 0$							
2	$2x^2 + 6x + 2 = 0$							
3	$2x^2 + 6x - 2 = 0$							
4	$2x^2 - 6x + 2 = 0$							
5	$2x^2 - 6x - 2 = 0$							
6	$3x^2 - 6x - 2 = 0$							
7	$4x^2 - 6x - 2 = 0$							
8	$4x^2 - 6x + 2 = 0$							

Fill in the Gaps

9	$2 - 6x + 4x^2 = 0$						
10	$2 - 6x - 4x^2 = 0$						
11	$1 - 3x - 2x^2 = 0$						
12	$\dfrac{1}{2} - \dfrac{3}{2}x - x^2 = 0$						
13	$\dfrac{1}{6} - \dfrac{1}{2}x - \dfrac{1}{3}x^2 = 0$						
14	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = 0$						
15	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = 1$						
16	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = x$						
17	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = x^2$						
18	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = -x^2$						

Worked Example	Your Turn
Solve the following equation: $$\frac{x+5}{x+6} = -\frac{6}{5x-2}$$ Give your answer in exact form.	Solve the following equation: $$-\frac{3x}{3x+5} = \frac{x-3}{x+2}$$ Give your answer in exact form.

Worked Example	Your Turn
A right-angled triangle has sides $(x + 2)$ cm, $(2x - 1)$ cm and $(2x + 3)$ cm.	A right-angled triangle has sides $(2y - 4)$ cm, $(2y - 1)$ cm and $(y + 6)$ cm.

Worked Example:

$(2x + 3)$ cm

$(x + 2)$ cm

$(2x - 1)$ cm
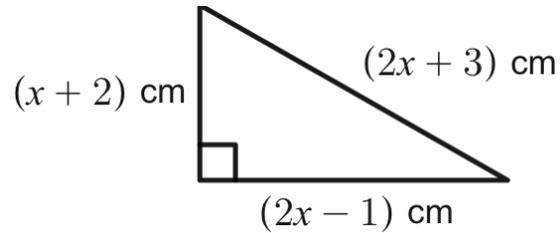
Find the value of x.
Give your answer correct to 2 decimal places.

Your Turn:

$(2y - 1)$ cm

$(2y - 4)$ cm

$(y + 6)$ cm
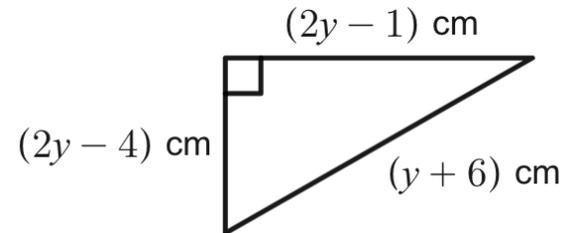
Find the value of y.
Give your answer correct to 2 decimal places.

Worked Example	Your Turn
The surface area of the cylinder is 1158 cm².	The surface area of the cylinder is 1457 cm².

13.9 cm

x

Find the value of x.

Give your answer correct to 1 decimal place.

19.2 cm

x
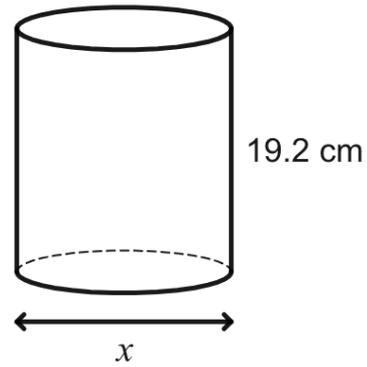
Find the value of x.

Give your answer correct to 1 decimal place.

Extra Notes

Worked Example	Your Turn

Worked Example

a) Find the ratio of Green to Blue
b) Write green as a fraction of the whole bar
c) Write blue as a fraction of the whole bar
d) Write green as a fraction of blue
e) Write blue as a fraction of green
f) Form a linear equation linking green and blue
   Green =
   Blue =

Your Turn

a) Find the ratio of Green to Blue
b) Write green as a fraction of the whole bar
c) Write blue as a fraction of the whole bar
d) Write green as a fraction of blue
e) Write blue as a fraction of green
f) Form a linear equation linking green and blue
   Green =
   Blue =

Fill in the Gaps

x : y	Visual representation	x as a fraction of whole	y as a fraction of a whole	x as a fraction of y	y as a fraction of x	Linear equation	x = ...	y = ...
	☐☐☐☐☐ : ☐☐							
	☐☐☐ : ☐☐☐☐							
1 : 7								
		$\dfrac{3}{8}$						
				$\dfrac{3}{2}$				
						$3x = 7y$		
							$x = \dfrac{10}{3}$	
								$y = \dfrac{2}{9}$

Worked Example	Your Turn
Given that $3y = 7x$, work out the ratio $x : y$	Given that $9q = 4p$, work out the ratio $p : q$

Worked Example	Your Turn
The ratio of x to y is $4 : 5$ Write a linear function involving x and y	The ratio of p to q is $5 : 3$ Write a linear function involving p and q

Dr Frost 224k

Worked Example	Your Turn
The ratio $5x + 3 : 2y - 1$ is equal to $5 : 4$ Express x in terms of y	The ratio $a + 1 : 2b + 5$ is equal to $5 : 7$ Express a in terms of b

Dr Frost 464a

Worked Example	Your Turn
Given that $3x - 10 : 9x - 51 = 2 : 3$ Find the value of x	Given that $9a - 4 : 7a + 21 = 7 : 2$ Find the value of a

Worked Example	**Your Turn**
Given that $7x - 6 : 4x + 12 = 5x - 2 : 5x + 10$ Find the possible values of x.	Given that $6a + 11 : 3a + 3 = 5a + 8 : 2a + 4$ Find the possible values of a.

Worked Example	Your Turn
$2a^2 - 5ab = 12b^2$, where $a > 0$ and $b > 0$ Find the ratio $a : b$ in its simplest form.	$5q^2 - 16pq = 16p^2$, where $p > 0$ and $q > 0$ Find the ratio $p : q$ in its simplest form.

Worked Example	Your Turn
There are blue counters and white counters in a bag in the ratio 4 : 3 10 blue counters are added, and the ratio becomes 2 : 1 Work out how many white counters there are in the bag.	There are black counters and red counters in a bag in the ratio 3 : 4 20 black counters are removed, and the ratio becomes 1 : 3 Work out how many red counters there are in the bag.

Dr Frost 464d

Worked Example	Your Turn
There are black counters and red counters in a bag in the ratio 3 : 7 5 black counters are removed, and 10 red counters are added to the bag, and the ratio becomes 2 : 5 Work out the original number of red counters in the bag.	There are white counters and red counters in a bag in the ratio 3 : 4 10 white counters are removed, and 1 red counter is added to the bag, and the ratio becomes 2 : 3 Work out the original number of red counters in the bag.

Worked Example	Your Turn
a, b, c and d are integers with no common factors. $a : b = 4 : 3$ $c : d = 1 : 6$ $2a = 3d$ Find $a : b : c : d$	a, b, c and d are integers with no common factors. $4a = 7b$ $c : d = 3 : 2$ $a : d = 4 : 7$ Find $a : b : c : d$

Worked Example	Your Turn
Green shapes and purple shapes are used in a game.	Blue shapes and red shapes are used in a game.
Some of the shapes are triangles.	Some of the shapes are circles.
All the other shapes are hexagons.	All the other shapes are squares.
The ratio of triangles to hexagons is $5 : 2$	The ratio of circles to squares is $4 : 5$
The ratio of green triangles to purple triangles is $3 : 5$	The ratio of blue circles to red circles is $3 : 2$
Work out the fraction of shapes that are green triangles.	Work out the fraction of shapes that are red circles.

Dr Frost 227i

Worked Example	Your Turn
White shapes and black shapes are used in a game. Some of the shapes are circles. All of the other shapes are squares. The ratio of the number of white shapes to the number of black shapes is $4:5$ The ratio of the number of white circles to the number of white squares is $3:4$ The ratio of the number of black circles to the number of black squares is $2:1$ Work out what fraction of all the shapes are circles.	Blue shapes and red shapes are used in a game. Some of the shapes are circles. All of the other shapes are squares. The ratio of the number of blue shapes to the number of red shapes is $4:1$ The ratio of the number of blue circles to the number of blue squares is $3:4$ The ratio of the number of red circles to the number of red squares is $3:2$ Work out what fraction of all the shapes are circles.

Dr Frost 227j

Worked Example	Your Turn
A drink seller has two types of drink mix in stock. Drink A is made from apple juice and orange juice in the ratio $8:3$ Drink B is made from apple juice and orange juice in the ratio $2:9$ Both drinks come in the same size cartons. Drink A and Drink B can be combined to produce Drink C. The vendor wants Drink C to contain apple juice and orange juice in the ratio $7:6$ If the seller is to use whole cartons only, find the least number of cartons that she needs of each type.	A drink seller has two types of drink mix in stock. Drink A is made from apple juice and orange juice in the ratio $9:5$ Drink B is made from apple juice and orange juice in the ratio $6:1$ Both drinks come in the same size cartons. Drink A and Drink B can be combined to produce Drink C. The vendor wants Drink C to contain apple juice and orange juice in the ratio $7:2$ If the seller is to use whole cartons only, find the least number of cartons that she needs of each type.

Dr Frost 464g

Extra Notes