KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2025 Mathematics (L2FM) 2026

Unit 18 Tasks – Part 1

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Unit 18 Tasks – Part 2

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Unit 18 Tasks – Part 3

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Contents Page

1 Advanced Data Handling

Find the lower quartile, upper quartile and interquartile range for each of these sets of data:

a) 4, 5, 7, 8, 8, 8, 9, 10, 10

b) 8, 9, 11, 12, 12, 12, 14, 14, 15, 17

c) 7.1, 7.2, 7.4, 7.4, 7.5

d) 19, 29, 31, 21, 28, 27, 24

e) 0.6, 0.23, 0.2, 0.7, 0.14, 0.1, 0.68

f) 20, 31, 25, 45, 46, 20, 34, 31

Find the median, lower quartile, upper quartile and interquartile range for each of these sets of data.

(a) $5, 7, 7, 8, 10, 12, 13$

(b) $18, 14, 20, 13, 17, 16, 16$

(c) $7, 4, 6, 7, 9, 5, 4, 8, 3, 10, 5$

(d) $3.5, 7.1, 2.7, 6.3, 4.9, 7.0, 3.7$

(e) $4, 2, 2, 3, 1, 5, 4, 4, 3, 6, 3$

(f) $6, 9, -2, 0, -3, 1, 6, 4, -5, 8, 3$

The number of goals scored in each of Bury United's last eleven matches is:

$$0, 2, 0, 1, 1, 4, 0, 2, 3, 1, 2$$

Find the median number of goals scored and the interquartile range.
Over Wigan United's last seven matches their median number of goals scored was 1 and their interquartile range of goals scored was 3. Make two comparisons between the two sets of data.

The English test results for 11 students in each of classes 8A and 8B are shown below.

8A	64	37	52	87	57
61	89	49	68	55	39

8B	71	92	63	83	81
62	55	48	84	90	73

(a) By finding the median and interquartile range, compare the two sets of results.
(b) The teacher of 8B has marked one question incorrectly, and all student marks in 8B go up by 1. How does this affect the median and interquartile range?

Problem Solving

For each statement, decide if its possible and suggest 7 values that fit the description	
The interquartile range is equal to the range	
The interquartile range is equal to the median	
The interquartile range is larger than the range	
The upper quartile is equal to the median	
The median is 0 and the interquartile range is 5	
The range is negative	
The upper quartile is 1 and the range is 5	

Fluency Practice

Favourite subject	Frequency
Maths	7
English	7
Science	
History	3
Geography	3
	24

In a survey, 24 students say what their favourite subject is. The results are shown in the frequency table.

(a) How many students said Geography was their favourite subject?

(b) Work out how many students said Science was their favourite subject. Add this information to the table.

Items purchased	Frequency
1	
2	3
3	5
4	4
5	3
6	3
7	1
	25

The number of items purchased one day by 25 customers in a shop is shown in the table.

(a) Six customers bought exactly one item. Add this information to the table.

(b) How many customers bought exactly three items?

(c) How many customers bought five or more items?

Score	Frequency
0	0
1	1
2	0
3	4
4	14
5	36
6	48
7	
8	
9	17
10	8
	200

200 students sat a test. The results are shown in the table.

(a) 42 students scored 7 marks. Add this information to the table.

(b) How many students scored 8 marks?

(c) How many students scored *less than* 5 marks?

Score	Frequency
0	0
1	0
2	0
3	
4	6
5	4
6	0
7	3
8	5
9	2
10	1
	23

Twenty-three students sat a test. Their scores are shown in the table.

A score of 9 or 10 gets an A grade. A score of 7 or 8 gets a B grade, and a score of 5 or 6 gets a C grade.

a) How many students scored 3 marks?

b) How many students got an A grade?

c) How many students got at least a B grade?

Frequency Tables

① George asked 20 students about the pets they have.
He wrote the results like this.


cat, dog, cat, hamster, dog, cat, hamster, fish, dog,
fish, hamster, cat, dog, bird, hamster, dog,
cat, dog, fish, bird,

Can you help George and simplify the results in a tally table?
When you have complete the tallies, write the total in the frequency column.

Pet	Tally	Frequency
Dog		
Cat		
Fish		
Bird		
Hamster		

What is the most popular pet?

② Anne recorded the favourite subject for some students.
Complete a tally chart for this information.

Maths, P.E., Science, History, English, Science, English, Science, P.E., Drama, Science, Science, P.E., Maths, Science, History, English, Science, History, Maths, Geography, P.E., Maths, P.E., Drama, Science, Maths, P.E., Maths, P.E., History, Geography, English, Science, Science, Drama, History, Geography, P.E., Science,

Subject	Tally	Frequency
Maths		
English		
Science		
Drama		
Geography		
History		
P.E.		
	Total	

How many students were asked in total?

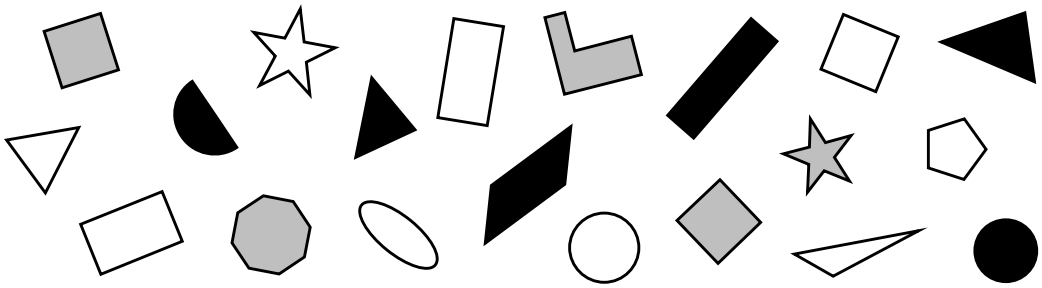
Which was the favourite subject?

What fraction of students said P.E their favourite?

① SHAPE SORTING

Here is a **set** of 20 shapes.

Complete each tally chart to **summarise** the data set.



Colour	Tally	Total
Black		
White		
Grey		

Shape	Tally	Total
Circle		
Triangle		
Square		
Rectangle		
Other		

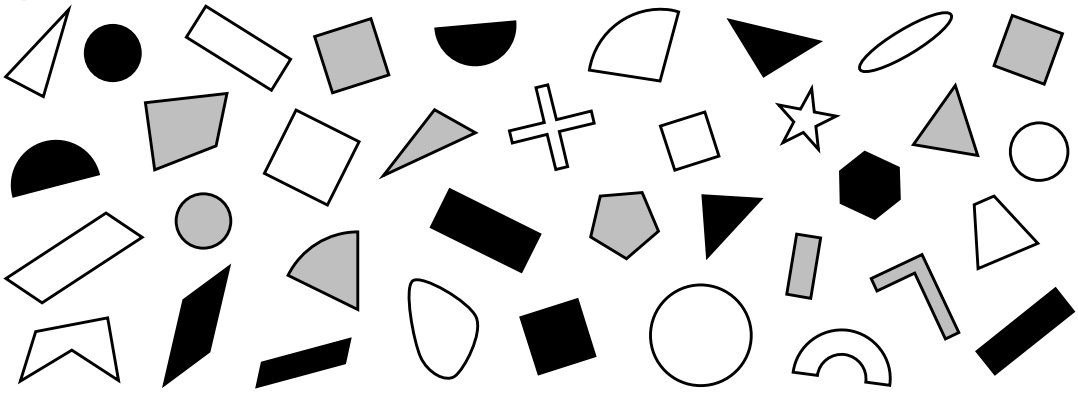
Sides	Tally	Total
1		
2		
3		
4		
5 or more		

a) What is the most common colour?

b) What is the least common shape?

c) What is the most common number of sides?

② **SHAPE SORTING** **Summarise** this set of shapes in the **tally charts**.



Colour	Tally	Total
Black		
White		
Grey		

Shape	Tally	Total
Circle		
Triangle		
Square		
Rectangle		
Other		

Sides	Tally	Total
1		
2		
3		
4		
5 or more		

What fraction of the shapes…

…are grey?

…are not white?

…are circles?

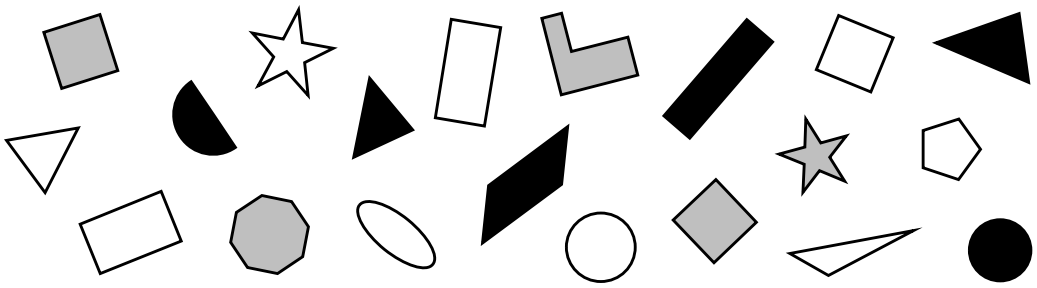
…are not triangles or squares?

…have 5 or more sides?

…have more than 2 sides?

① SHAPE SORTING

Here is a **set** of 20 shapes.
Complete each frequency table to **summarise** the data set.



Colour	Frequency
Black	
White	
Grey	

What is the most common colour?

What is the least common colour?

What is the least common shape?

How much more common are triangles compared to circles?

Shape	Frequency
Circle	
Triangle	
Square	
Rectangle	
Other	

Sides	Frequency
1	
2	
3	
4	
5 or more	

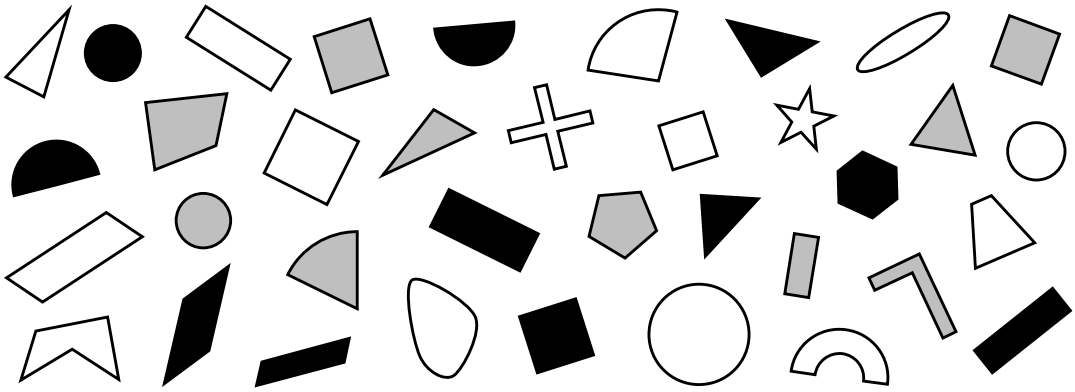
What is the most common number of sides?

Which shape is unique to all the others?

What fraction of the shapes have more than 4 sides?

② SHAPE SORTING

Summarise this set of shapes in the **frequency tables**.



As a fraction, express the proportion of shapes that…

Colour	Frequency
Black	
White	
Grey	

…are grey.

…are white.

…are not white.

…are rectangles.

…are not triangles.

…are not triangles or squares.

Shape	Frequency
Circle	
Triangle	
Square	
Rectangle	
Other	

Sides	Frequency
1	
2	
3	
4	
5 or more	

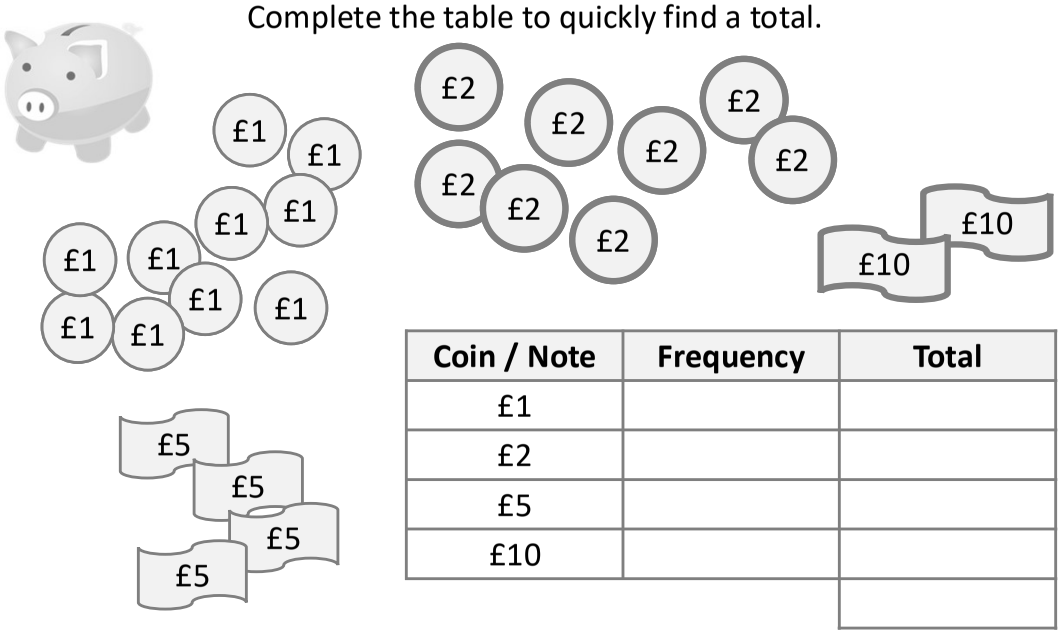
…have 1 side.

…have 4 or more sides.

…have less than 3 sides.

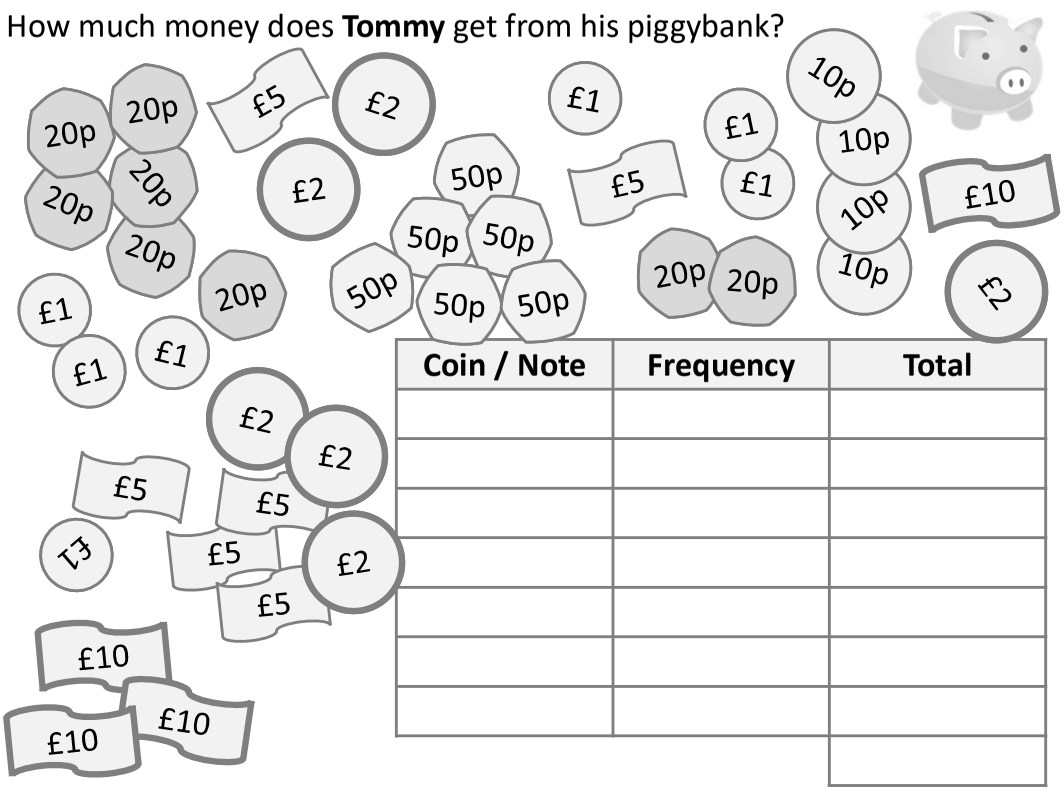
Counting CASH

Janine opens her piggybank to see how much money she has saved.
Complete the table to quickly find a total.



Coin / Note	Frequency	Total
£1		
£2		
£5		
£10		

How much money does **Tommy** get from his piggybank?



Coin / Note	Frequency	Total

Fluency Practice

Making Money

These companies recorded sales over one day.
Which made the most **profit**?

Revenue – the total money a business brings in
Profit – the money a business keeps after costs

Vicky's Vegan Snax

Product	Price, £	Sales	Subtotal
Wrap	1.50	62	
Bowl	2.50	28	

Costs: Ingredients: £21, Wages: £35

Mr. Milkshake

Product	Price, £	Sales	Subtotal
Small	2	17	
Medium	3	33	
Large	4	12	

Total Costs: £62

Jando Mobile Phones

Product	Customer Price, £	Cost, £		Sales	Subtotal
A12	120	100		5	
A12-X	190	118		3	
A14	245	152		2	

Costs: Store Rent: £73, Wages: £262, Advertising: £64

BillsTravelBlog.com

Advert	Revenue, £	Clicks	Subtotal
Train Ticket	20p	46	
Flight Ticket	50p	10	

Total Costs: £2.60

Divine Soaps Online

Product	Profit, £	Sales	Subtotal
Cherry	1.20	7	
Vanilla	2.30	9	
Orange	1.50	6	
Mint	75p	3	

Costs:
Postage for 12 packages at £2.40 per package,
Website Hosting, £3.50

Fluency Practice

Items purchased	Frequency
1	6
2	3
3	5
4	4
5	3
6	3
7	1
	25

The number of items purchased one day by 25 customers in a shop is shown in the table. What is the modal number of items purchased?

Animal	Frequency
Cat	7
Dog	6
Hamster	3
Guinea pig	2
Mouse	1
Rabbit	1
Snake	1

Shaun asks everyone in his class how many pets they have, and what type of animal they are. He records the number of pets in the table shown.

(a) What is the mode?

(b) There are 28 students in Shaun's class. Explain why the frequencies **don't** add up to 28.

Score	Frequency
0	0
1	1
2	0
3	4
4	14
5	36
6	48
7	42
8	30
9	17
10	8
	200

200 students sat a test. The results are shown in the table.

What was the modal score achieved by the students?

This table shows the number of goals scored by a hockey team in their first 9 matches of a tournament. In their tenth and final match of the tournament, the team scores 3 goals. What is the modal number of goals scored by the team in the tournament?

Number of goals scored	Frequency
0	3
1	4
2	1
3	0
4	1

Fluency Practice

Cars sold	Frequency
0	1
1	9
2	3
3	5
4	4
5	3
6	3
7	2
	30

Information about the number of cars sold by a dealer each day in June is shown in the table. What is the range in number of cars sold?

Score	Frequency
0	0
1	1
2	0
3	4
4	14
5	36
6	48
7	42
8	30
9	17
10	8
	200

200 students sat a test. The results are shown in the table.

What was the range of scores achieved by the students?

Number of crisps	Frequency
14	6
15	7
16	5
17	1
18	1
	20

The frequency table shows the number of crisps found in 20 packs of crisps.

Micah looks at the table and says:
The range in number of crisps is 19 because 20 is the highest and 1 is the lowest.

Niamh disagrees with Micah and says:
The range is 6 because 7 is the highest and 1 is the lowest. You can't use 20 because that's just a total.

Both Micah and Niamh haven't found the correct range in number of crisps. What is the correct range?

This table shows the number of goals scored by a hockey team in their first 9 matches of a tournament. In their tenth and final match of the tournament, the team scores 3 goals. What is the range in number of goals scored by the team in the tournament?

Number of goals scored	Frequency
0	3
1	4
2	1
3	0
4	1

Question 1: Work out the median from each of the frequency tables.

(a)

Age	Frequency
18	2
19	3
20	13
21	1

(b)

Shoe Size	Frequency
5	2
6	11
7	5
8	4
9	1

(c)

Number of TVs	Frequency
0	3
1	15
2	9
3	11
4	1

(d)

Days absent	Frequency
0	31
1	8
2	3
3	4
4	1
5	3

(e)

Age	Frequency
5	12
6	20
7	23
8	65

(f)

Goals Scored	Frequency
0	2
1	4
2	5
3	8
4	0
5	1

Question 2: Work out which class interval contains the median for each table below.

(a)

Time taken	Frequency
$0 < t \leq 5$	5
$5 < t \leq 10$	14
$10 < t \leq 15$	10
$15 < t \leq 20$	1

(b)

Lifetime (months)	Frequency
$0 < t \leq 12$	1
$12 < t \leq 24$	9
$24 < t \leq 36$	13
$36 < t \leq 48$	56
$48 < t \leq 60$	21

(c)

Mass (Kg)	Frequency
$50 < m \leq 60$	41
$60 < m \leq 70$	39
$70 < m \leq 80$	28
$80 < m \leq 90$	6
$90 < m \leq 100$	2

(d)

Mass (Kg)	Frequency
$0 < m \leq 100$	123
$100 < m \leq 200$	290
$200 < m \leq 300$	2009
$300 < m \leq 400$	1817
$400 < m \leq 500$	584
$500 < m \leq 600$	177

Apply

Question 1: There are 30 students in a class.
Miss Williamson knows that the median shoe size is 5.
Fill in the frequency table with two possible values.

Shoe Size	Frequency
4	4
5	
6	
7	10

Question 2: The frequency table shows the piano grade of 17 students in a class.

Grade	Frequency
2	3
3	3
4	4
5	3
6	2
7	2

3 new students, who are all Grade 6, join the class.
The teacher says the median piano grade will increase.
Is she correct?

Question 1: Work out the mean for each of these frequency tables.
You may not use a calculator

(a)

Age	Frequency
5	2
6	2
7	5
8	1

(b)

Number of phones	Frequency
0	1
1	3
2	2
3	0
4	4
5	0

(c)

Number of pets	Frequency
0	13
1	28
2	50
3	9

(d)

Money Withdrawn	Frequency
£10	16
£20	19
£30	4
£40	3
£50	6
£60	2

(e)

Number of bedrooms	Frequency
1	34
2	275
3	512
4	179

(f)

Level	Frequency
3	1
4	9
5	7
6	2
7	1

Question 2: Work out the mean for each of these frequency tables.
You may use a calculator

(a)

Age	Frequency
16	28
17	7
18	3
19	2

(b)

Grade	Frequency
3	16
4	27
5	45
6	49
7	50
8	13

(c)

Siblings	Frequency
0	71
1	25
2	14

(d)

Pocket Money	Frequency
£1	5
£2	34
£3	86
£4	19
£5	3
£6	3

(e)

Star rating	Frequency
0	9
1	12
2	17
3	19
4	21
5	8

(f)

Times visited	Frequency
0	131
1	873
2	599
3	205

Question 1: A teacher asked his class how long they spent revising for a test, to the nearest hour. By calculating the mean, compare the amount of time the boys and girls spent revising.

Boys

Hours	Frequency
0	0
1	2
2	3
3	4
4	5
5	1

Girls

Hours	Frequency
0	2
1	7
2	2
3	2
4	1
5	1

Question 2: Aidan plays 50 games in an arcade. The table shows how many tickets he won in each game.

(a) Work out the missing frequency

(b) Work out the total number of tickets won

(c) Work out the mean number of tickets won per game.

Aidan wants to exchange his ticket for a prize that costs 800 tickets.

Tickets won	Frequency
0	4
1	3
2	5
3	
4	11
5	6
6	10
7	2
8	3

(d) How many more games do you expect Aidan would have to play?

Question 3: Max rolls a dice 80 times. The table shows the results.

(a) Find the value of x

(b) Work out the mean score

Number	Frequency
1	4
2	6
3	x + 5
4	x
5	2x
6	5

Purposeful Practice

In a questionnaire, 35 pupils are asked how many pets they own.

No of pets	Frequency
0	
1	
2	
3	
4	
5	

Input the following frequencies into the chart:

0 , 2 , 5 , 5 , 10 , 13

Your target is to get as close as possible to a mean of 3.5 pets per household.

Explain your choices

How would you change your values if the target mean was 1.5 pets per household?

What is the largest possible mean?

How do you know it is the largest?

Fluency Practice

Vicky counts the number of birds in her garden at 5 pm on each of 20 days. She records the information in a frequency table.

Number of birds	Frequency
0	3
1	2
2	3
3	4
4	5
5	3

Jim asked each person in his class how many cars their family have. The frequency table shows the results.

Number of cars	Frequency
0	2
1	12
2	8
3	6
4	2

Marta asked some students how many cans of drink they each drank yesterday. The table shows her results.

Number of cans	Frequency
0	6
1	9
2	7
3	3
4	2
5	1

Chris works in a cafe. At noon one day, he records the number of customers sitting at each table in the cafe. Here are his results.

Number of customers sitting at a table	Number of tables
0	4
1	5
2	10
3	7
4	3
5	1

(a) Find the mean, median, mode and range of test marks from the table.

Test Mark	Frequency
7	6
8	7
9	5
10	2

(b) Find the mean, median, mode and range of number of goals scored.

Number of goals	Frequency
0	4
1	8
2	5
3	3

(c) Find the mean, median, mode and range of the age of the students.

Age (y)	Frequency
11	6
12	7
13	8
14	4

(d) Find the mean, median, mode and range number of pets.

Number of pets	Frequency
0	11
1	15
2	3
3	1

example

The ages of a group of children are shown in the frequency table.

Age, x	Frequency, f	xf
11	4	44
12	9	108
13	7	91
14	7	98
15	5	75

a) How many children are in the group? sum of f column = 32

b) What is the modal age?

age with highest freq. = 12

c) Work out the median age.

$(32+1) \div 2 \rightarrow 16.5^{th}$ position
16^{th} and 17^{th} positions are both 13, so median = 13

d) Work out the mean age.

$\text{mean} = \frac{\text{sum of xf column}}{32} = \frac{416}{32} = 13$

exercise

1. A number of people were asked how many hot drinks they had in a day. The results are shown in the frequency table.

Number of hot drinks, x	Frequency, f
0	4
1	9
2	8
3	12
4	7
5	1

a) How many people were asked?

b) Which of these is the mode?

no mode | 3 | 4 | 1 & 4

c) Work out the median.

d) Work out the mean. Round your answer to 3 significant figures.

2. A group of people were surveyed about the number of pets they owned. The results are shown in the frequency table.

Number of pets, x	0	1	2	3
Frequency, f	5	12	6	2

a) State the modal number of pets.

b) Work out the median.

c) Work out the mean.

d) Work out the range.

3. Adam and Tilly played mini golf and recorded the number of shots they took on each hole. The results are shown in the frequency tables.

Adam's results

Shots, x	Frequency, f
2	2
3	0
4	9
5	3
6	4

Tilly's results

Shots, x	Frequency, f
2	1
3	5
4	7
5	5

a) True or false? The modal number of shots is the same for both players.

b) Work out the mean for each player, correct to 1 decimal place.

c) Work out the median for each player.

d) Work out the range for each player.

e) Compare the two players' performances.

4. The frequency table shows the number of merit points earned by pupils in class 8A on a given day.

Merit points, x	0	1	2	3	4
Frequency, f	6	13	4	2	4

a) Work out the mean number of merit points earned by pupils in 8A.

b) Work out the range of merit points earned by pupils in 8A.

c) On the same day, the number of merit points earned by pupils in class 8B had a mean of 1.9 and a range of 7.
Compare the number of merit points earned by the two classes.

5. The table shows the number of days that 30 pupils were absent in a term.

Days absent, x	0	1	2	3	4	5
Frequency, f	14	a	3	b	1	2

Given that the mean number of days absent was 1.4, work out the values of a and b.

Fluency Practice

1.

John recorded the goals scored by his favourite football team.

Complete the table and calculate the Mean, Mode and Median.

Score	Tally	Games	Total Goals
0	𝍸		
1	𝍸		
2	𝍸 |		
3	|		
Total			

Mean = _____ Mode = _____ Median = _____

2.

Score	Frequency	Score x Frequency
1	3	
2	5	
3		
4	1	
5	0	
6	4	
Total		

Anna rolled a dice 20 times and recorded the results.

Complete the table and calculate the Mean, Mode and Median.

Mean = _____ Mode = _____ Median = _____

3.

The mean of this data is 1.4. Complete the table and find the median.

Median = _____

Score	Frequency	Score x Frequency
0		0
1		9
2		12
3	7	
Total	30	

4.

Josh and Jane played mini-golf and recorded their scores.

JOSH

Score	Frequency	S x F
2	5	
3	2	
4	4	
5	0	
6	1	
Total		

JANE

Score	Frequency	S x F
2	4	
3	3	
4	2	
5	1	
6	0	
7	2	
Total		

Calculate the mean, median and range for each player.

Who is the better player?

5.

Hannah recorded the merit points of students in his class over a week.

Points	0	1	2	3	4	5
Frequency	3	0	4	6	3	2

Another class had a mean of 2.3 and a range of 4. Which class did better?

Fluency Practice

Frequency Tables: Averages & Range

① 

Kimmy asks some people to record how many sweets they eat over 3 days.

Complete the frequency table to help calculate how many people she asked and how many sweets were eaten in total.

No. of Sweets	Frequency	Subtotals
0	2	
1	3	
2	1	
3	2	
4	2	
9	1	
Totals		

Complete this ordered list of quantities of sweets.

0		1		1		3			4	

Calculate: Mean = Median =

Mode = Range =

Which of these measures do you think are useful to describe the data?

② 

Henry askes some friends:

"Approximately how many hours did you play computer games on Saturday?"

Approx. Hours	Freq.	
0	4	
1	2	
2	0	
3	3	
4	5	

Calculate: Mean = Median =

Mode = Range =

Why is this data difficult to represent with an average?

③ Sasha asks some gym buddies approximately how much time they spend at the gym every week.

Approx. Hours	1	2	3	4	5	6	7	8	9	10	
Freq.	4	5	2	6	0	0	0	4	4	2	

How can we find the median without writing an ordered list?

Mean = Median =

Mode = Range =

Is the approximate mean value actually in the data?

④ Abigail asked 10 children and 16 adults about how many pieces of fruit they each ate yesterday. 

Complete both frequency tables then calculate the averages & range for both data sets.

Pieces of Fruit	F	Subtotals
0	2	
1		5
2		4
3		

Mean = Median =

Mode = Range =

Pieces of Fruit	F	Subtotals
0	0	
1	4	
2	4	
3		
5	3	

Mean = Median =

Mode = Range =

What comments can you make about the two groups?

Problem Solving

9

Age	Frequency	
5	3	
6		
7		
8	3	
Total	**20**	

Mean = 6.5

Mode =

Median =

Range =

10

Age	Frequency	
5	3	
6	2	
7		
8		
Total	**20**	

Mean =

Mode =

Median = 7.5

Range =

11

Age	Frequency	
5		
6		
7	6	
8		
Total	**20**	

Mean =

Mode = 5 and 6

Median =

Range =

12

Age	Frequency	
5		
6		
7		
8		
Total	**20**	

Mean = 6.75

Mode =

Median =

Range = 1

Fluency Practice

Height, h cm	Frequency
$120 \leq h < 130$	1
$130 \leq h < 140$	4
$140 \leq h < 150$	
$150 \leq h < 160$	16
$160 \leq h < 170$	20
$170 \leq h < 180$	23
$180 \leq h < 190$	8
	80

The heights of 80 people are measured. The results are shown in the table.

(a) Eight people have a height that is at least 140 cm but less than 150 cm. Add this information to the table.

(b) How many people measured were at least 180 cm tall?

(c) How many people measure were under 120 cm tall?

Bill total, £x	Frequency
$0 < x \leq 5$	19
$5 < x \leq 10$	33
$10 < x \leq 15$	27
$15 < x \leq 20$	17
$20 < x \leq 25$	4
	100

The table shows information about the shopping bills of 100 customers at a shop one day.

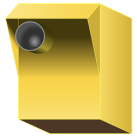
How many of the customers spent more than £10?

Speed, s mph	Frequency
$20 < s \leq 25$	5
$25 < s \leq 30$	8
$30 < s \leq 35$	7
$35 < s \leq 40$	4
$40 < s \leq 45$	1
	25

The speeds of 25 drivers in a 30 mph zone were measured and recorded in the table shown.

(a) How many drivers were not exceeding the speed limit?

(b) How many drivers exceeded the speed limit by over 10 mph?



Fluency Practice

① **Sort & Summarise**

The ages of visitors to **Jump NOW!** on the 16th of July.

What was the most common age group?

15	27	10
14	15	19
7	17	1
6	13	25
26	12	20

Age (years)	**Tally**	**Frequency**
0 to 6		
7 to 13		
14 to 20		
21 to 27		

② Weights of penguins in the colony at **Hempstead Zoo**.

How many penguins weighed above 50 kg?

27	34	17	21	31
37	42	40	36	50
20	33	30	47	11
30	24	58	32	22

Weight (kg)	**Tally**	**Frequency**
11 to 20		
21 to 30		
31 to 40		
41 to 50		
51 to 60		

③

Javelin distances (metres) for members of **Gately Athletics Club**.

How many people threw over 40 metres?

17	60	20.1	44	40	0.9	27
54	19.9	32.3	72	60.5	57	80
75.56	40.08	48.5	20	24	39.8	52.68

Distance, d (metres)	**Tally**	**Frequency**
$0 < m \leqslant 20$		
$20 < m \leqslant 40$		
$40 < m \leqslant 60$		
$60 < m \leqslant 80$		

④ **Green Minster Running Club** recorded race times.

How many runners took less than 90 seconds?

2 minutes	1.5 minutes	30.5 seconds
53 seconds	100 seconds	72 seconds
94 seconds	78 seconds	0.5 minutes
1 minute	30 seconds	1.02 minutes
1 min 2 seconds	0.75 minutes	1 min 43 seconds
2.5 minutes	1.1 minutes	90 seconds

Time, t (seconds)	**Tally**	**Frequency**
$0 < s \leqslant 30$		
$30 < s \leqslant 60$		
$60 < s \leqslant 90$		
$90 < s \leqslant 120$		
$120 < s \leqslant 150$		

Fluency Practice

80 people take part in a survey. Their ages are shown in the frequency table. What is the **modal class**?

Age range	Frequency
$20 \leq \text{age} < 30$	8
$30 \leq \text{age} < 40$	14
$40 \leq \text{age} < 50$	12
$50 \leq \text{age} < 60$	16
$60 \leq \text{age} < 70$	11
$70 \leq \text{age} < 80$	10
$80 \leq \text{age} < 90$	9
	80

Bill total, £x	Frequency
$0 < x \leq 5$	5
$5 < x \leq 10$	8
$10 < x \leq 15$	7
$15 < x \leq 20$	4
$20 < x \leq 25$	1
	25

The table shows information about the shopping bills of 25 customers at a shop one day. What is the modal class?

Height, h cm	Frequency
$120 \leq h < 130$	1
$130 \leq h < 140$	4
$140 \leq h < 150$	8
$150 \leq h < 160$	16
$160 \leq h < 170$	20
$170 \leq h < 180$	23
$180 \leq h < 190$	8
	80

The heights of 80 people are measured. The results are shown in the table.

What is the modal class?

Fluency Practice

Scientists measured the lengths of 80 turtles on a beach. Their lengths are shown in the frequency table. Find upper and lower bounds for the range of lengths.

Length, cm	Frequency
$20 \leq \text{length} < 30$	8
$30 \leq \text{length} < 40$	14
$40 \leq \text{length} < 50$	12
$50 \leq \text{length} < 60$	16
$60 \leq \text{length} < 70$	11
$70 \leq \text{length} < 80$	10
$80 \leq \text{length} < 90$	9
	80

Time spent in the shop	Frequency
$0 < x \leq 5$	5
$5 < x \leq 10$	8
$10 < x \leq 15$	7
$15 < x \leq 20$	4
$20 < x \leq 25$	1
	25

The table shows information about the time spent by 25 people in a shop one day. Find the upper bound and lower bound for the range of times spent in the shop.

Height, h cm	Frequency
$120 \leq h < 130$	1
$130 \leq h < 140$	4
$140 \leq h < 150$	8
$150 \leq h < 160$	16
$160 \leq h < 170$	20
$170 \leq h < 180$	23
$180 \leq h < 190$	8
	80

The heights of 80 people are measured. The results are shown in the table.

Find upper and lower bounds for the range of heights.

Question 1: Calculate an estimate of the median for each of the following

(a)

Length (x cm)	Frequency
$10 < x \leq 20$	17
$20 < x \leq 30$	26
$30 < x \leq 40$	11
$40 < x \leq 50$	6

(b)

Time (t seconds)	Frequency
$0 < t \leq 20$	4
$20 < t \leq 40$	12
$40 < t \leq 60$	19
$60 < t \leq 80$	60
$80 < t \leq 100$	5

(c)

Mass (m kg)	Frequency
$40 < m \leq 45$	64
$45 < m \leq 50$	74
$50 < m \leq 55$	155
$55 < m \leq 60$	80
$60 < m \leq 65$	26
$65 < m \leq 70$	1

(d)

Height (h cm)	Frequency
$0 < h \leq 40$	6
$40 < h \leq 80$	14
$80 < h \leq 120$	20
$120 < h \leq 160$	30
$160 < h \leq 200$	15
$200 < h \leq 240$	15

(e)

Cost (p pounds)	Frequency
$0 < p \leq 2$	40
$2 < p \leq 4$	90
$4 < p \leq 5$	80
$5 < p \leq 8$	100
$10 < p \leq 20$	120

(f)

Length (l cm)	Frequency
$0 < l \leq 50$	87
$50 < l \leq 75$	91
$75 < l \leq 100$	43
$100 < l \leq 150$	25

Question 2: For each data set above, calculate:

(a) the lower quartile
(b) the upper quartile
(c) the interquartile range

Apply

Question 1: The table below shows information about the salaries of 120 workers from a small company.

Salary (£1000s)	Frequency
$0 < s \leq 10$	8
$10 < s \leq 20$	48
$20 < s \leq 30$	50
$30 < s \leq 50$	11
$50 < s \leq 200$	3

(a) Calculate an estimate of the mean salary

(b) Calculate an estimate of the median salary

(c) State whether the mean or the median is a better representation of the average salary a worker in the company receives.

Question 2: A shop sells two different types of lightbulb, Xtra Brite and Bright Bulbs. The lifetimes of 200 Xtra Brite bulbs, to the nearest month is shown below.

Lifetime (months)	Frequency
$0 < t \leq 12$	19
$12 < t \leq 24$	53
$24 < t \leq 36$	74
$36 < t \leq 48$	42
$48 < t \leq 120$	12

(a) Calculate an estimate of the median

(b) Calculate an estimate of the percentage of lightbulbs that last longer than 5 years

The median lifetimes of the Bright Bulbs is 30 months.
20% of the Bright Bulbs last longer than 5 years.
10% of the Bright Bulbs last less than 1 year.

(c) By comparing their lifetimes, decide which bulb is best.

Question 3: A professor believed that second year university students spent longer revising than first year university students.

Compare the time spent revising by the 1st year and 2nd year university students.
Use an estimate of the means, an estimate of the medians and an estimate of the interquartile ranges.

Time (hours)	1st Year Frequency	2nd Year Frequency
$0 < h \leq 5$	18	0
$5 < h \leq 10$	20	7
$10 < h \leq 20$	41	63
$20 < h \leq 40$	30	54
$40 < h \leq 60$	16	9
$60 < h \leq 100$	9	1

Question 1: Work out an estimate of the mean for each of these frequency tables.

(a)

Length	Frequency	Midpoint	
$0 < L \leq 10$	6		
$10 < L \leq 20$	7		
$20 < L \leq 30$	5		
$30 < L \leq 40$	1		
$40 < L \leq 50$	1		

(b)

Cost	Frequency	Midpoint	
$0 < c \leq 4$	2		
$4 < c \leq 8$	3		
$8 < c \leq 12$	5		
$12 < c \leq 16$	12		
$16 < c \leq 20$	3		

(c)

Length	Frequency	Midpoint	
$0 < t \leq 5$	11		
$5 < t \leq 10$	37		
$10 < t \leq 15$	43		
$15 < t \leq 20$	9		

(d)

Mass	Frequency	Midpoint	
$50 < m \leq 55$	3		
$55 < m \leq 60$	5		
$60 < m \leq 65$	10		
$65 < m \leq 70$	12		
$70 < m \leq 75$	10		

Question 2: Work out an estimate of the mean for each of these frequency tables.

(a)

Duration (years)	Frequency
$0 \leq d < 10$	9
$10 \leq d < 20$	13
$20 \leq d < 30$	16
$30 \leq d < 40$	2

(b)

Length (cm)	Frequency
$0 \leq L < 30$	8
$30 \leq L < 60$	43
$60 \leq L < 90$	25
$90 \leq L < 120$	4

(c)

Mass	Frequency
$20 < m \leq 25$	12
$25 < m \leq 30$	24
$30 < m \leq 35$	17
$35 < m \leq 40$	15
$40 < m \leq 45$	4

(d)

Height	Frequency
$120 < h \leq 130$	51
$130 < h \leq 140$	120
$140 < h \leq 150$	66
$150 < h \leq 160$	59
$160 < h \leq 170$	4

Apply

Question 1: Sally is raising money for charity for a fun run.
The table below has been given to her from the website.

Sally says the average donation is £10.
By calculating the estimated mean, decide if you agree with Sally.

Donation	Frequency
$0 < d \leq 5$	44
$5 < d \leq 10$	35
$10 < d \leq 20$	16
$20 < d \leq 50$	3
$50 < d \leq 100$	2

Question 2: Nathan delivers pizzas.
The table below shows information about his delivery times.
The pizza company has a promotion that if the delivery time is over 30 minutes, the customer gets their meal for free

(a) Calculate an estimate for the mean delivery time

(b) What percentage of deliveries took over 30 minutes?

Nathan's manager thinks that the promotion should be changed to 40 minutes

(c) Do you agree? Explain your answer.

Delivery Time	Frequency
$0 < t \leq 10$	3
$10 < t \leq 20$	10
$20 < t \leq 30$	14
$30 < t \leq 40$	19
$40 < t \leq 50$	4

Question 3: The manager of a small company is calculating the mean salary for his workers.
He has calculated this to be £568,500 per year.
Can you spot any mistakes?

Salary	Frequency	Midpoint	fx
$0 < s \leq 15000$	2	7500	15000
$15000 < s \leq 30000$	15	22500	337500
$30000 < s \leq 45000$	6	37500	2250000
$45000 < s \leq 60000$	2	52500	105000
$60000 < s \leq 100000$	2	67500	135000
			2842500

Mean salary = 2842500 ÷ 5 = £568500

Find an estimate of the mean messages.

Number of messages	Frequency
0 - 4	5
5 - 9	8
10 - 14	4
15 - 19	3

Find an estimate of the mean weight.

Weight (g)	Frequency
$0 < w \leq 10$	2
$10 < w \leq 20$	4
$20 < w \leq 30$	3
$30 < w \leq 40$	1

Find an estimate of the mean time.

Time (min)	Frequency
$0 < t \leq 2$	4
$2 < t \leq 4$	9
$4 < t \leq 6$	0
$6 < t \leq 8$	7

Find an estimate of the mean height.

Height (cm)	Frequency
$100 < h \leq 120$	6
$120 < h \leq 140$	6
$140 < h \leq 160$	6
$160 < h \leq 180$	2

Intelligent Practice

1.

Mass, x (kg)	Frequency
$0 < x \leq 10$	1
$10 < x \leq 20$	2
$20 < x \leq 30$	4
$30 < x \leq 40$	3

2.

Mass, x (kg)	Frequency
$0 < x \leq 10$	2
$10 < x \leq 20$	4
$20 < x \leq 30$	8
$30 < x \leq 40$	6

3.

Mass, x (kg)	Frequency
$0 < x \leq 20$	2
$20 < x \leq 40$	4
$40 < x \leq 60$	8
$60 < x \leq 80$	6

4.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	4
$60 < x \leq 80$	2

5.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	4
$60 < x \leq 80$	20

6.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	20
$60 < x \leq 80$	20

7.

Mass, x (kg)	Frequency
$0 < x \leq 20$	6
$20 < x \leq 40$	8
$40 < x \leq 60$	0
$60 < x \leq 80$	20

8.

Mass, x (kg)	Frequency
$1 < x \leq 21$	6
$21 < x \leq 41$	8
$41 < x \leq 61$	0
$61 < x \leq 81$	20

9.

Mass, x (kg)	Frequency
$1 < x \leq 21$	6
$21 < x \leq 41$	8
$41 < x \leq 61$	0
$61 < x \leq 66$	5
$66 < x \leq 81$	15

10.

Mass, x (kg)	Frequency
$1 < x \leq 21$	6
$21 < x \leq 41$	8
$41 < x \leq 81$	20

Fluency Practice

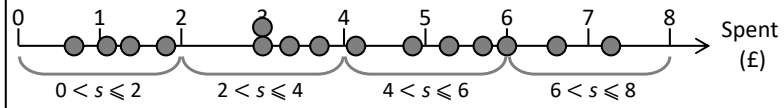
Q	Find the mean	Answers
1	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>0</td><td>3</td></tr><tr><td>1</td><td>11</td></tr><tr><td>2</td><td>4</td></tr><tr><td>3</td><td>2</td></tr></table>	
3	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>0</td><td>2</td></tr><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>0</td></tr><tr><td>3</td><td>3</td></tr></table>	
5	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>1</td><td>1</td></tr><tr><td>2</td><td>0</td></tr><tr><td>3</td><td>7</td></tr><tr><td>4</td><td>2</td></tr></table>	
7	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>1</td><td>a</td></tr><tr><td>2</td><td>b</td></tr><tr><td>3</td><td>c</td></tr><tr><td>4</td><td>d</td></tr></table>	

Q	Estimate the mean	Answers
2	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>$0 \leq x < 10$</td><td>3</td></tr><tr><td>$10 \leq x < 15$</td><td>11</td></tr><tr><td>$15 \leq x < 30$</td><td>4</td></tr><tr><td>$30 \leq x < 50$</td><td>2</td></tr></table>	
4	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>$0 \leq x < 20$</td><td>2</td></tr><tr><td>$20 \leq x < 25$</td><td>5</td></tr><tr><td>$25 \leq x < 40$</td><td>0</td></tr><tr><td>$40 \leq x < 50$</td><td>3</td></tr></table>	
6	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>$0 \leq x < 5$</td><td>1</td></tr><tr><td>$5 \leq x < 30$</td><td>0</td></tr><tr><td>$30 \leq x < 85$</td><td>7</td></tr><tr><td>$85 \leq x < 100$</td><td>2</td></tr></table>	
8	<table><tr><th>Score, x</th><th>Frequency</th></tr><tr><td>$0 \leq x < 10$</td><td>a</td></tr><tr><td>$10 \leq x < 30$</td><td>b</td></tr><tr><td>$30 \leq x < 60$</td><td>c</td></tr><tr><td>$60 \leq x < 100$</td><td>d</td></tr></table>	

Fluency Practice

Mean from a Grouped Frequency Tables

1) A bakery recorded how much each of its first 15 customers spent.
Each customer is represented by a dot on the scale.



To process the data, we can be put customers into 4 groups.

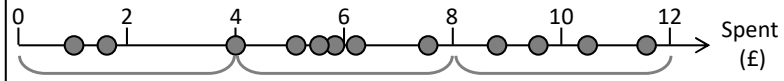
Spent, s (£)	Frequency (f)	Midpoint Value (m)	Estimated Group Spending ($f \times m$)
$0 < s \leqslant 2$	4	1	4
$2 < s \leqslant 4$	4	3	12
$4 < s \leqslant 6$			
$6 < s \leqslant 8$			
Totals			

To calculate an average, we assume each customer spent the group midpoint value.

Using midpoints & frequencies we can estimate the spending for each group.

a) Sum the spending estimates for each group to find a total.
b) Divide this by the number of customers to find an estimate for the mean.
c) Where is the median? Which group is it in?
d) Which group is the modal group (the highest frequency group)?
e) What is the actual mode?

2) Complete the **Grouped Frequency Table** using this data.

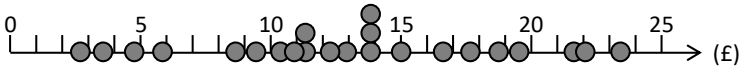


Spent, s (£)	Frequency (f)	Midpoint Value (m)	Estimated Group Spending ($f \times m$)
$0 < s \leqslant 4$			
Totals			

a) Use the table to estimate the mean.

3) A t-shirt shop recorded customer purchases during one day.
a) Decide how to group the data. Complete a grouped frequency table & calculate an estimate for the mean.

(Groups should be the same width. Think about easy-to-calculate midpoints).



Spent, s (£)	Frequency (f)	(m)	($f \times m$)

Estimate for the Mean =

b) Which group is the median within?
c) What is the modal group?

4) A takeaway records its purchases in the morning. Start with a tally chart to convert the raw data into a grouped frequency table.
Use this to **estimate** a mean.

9.30	10.54	8.65	22.12	7.82
24.78	12.03	5.80	8.23	7.61
17.40	23.61	17.80	9.45	16.70
22.12	12.24	19.81	22.30	17.20
13.22	14.28	16	6.60	8.13

Spent, s (£)	Tally			

Estimate for the Mean =

Fluency Practice

estimating the mean from grouped data

(1) the table shows the Flesch scores for some articles in tabloid newspapers
estimate the mean Flesch score for the articles

Flesch score	frequency
$30 \le Y < 40$	5
$40 \le Y < 50$	10
$50 \le Y < 60$	14
$60 \le Y < 70$	5
$70 \le Y < 80$	1
$80 \le Y < 90$	0

(2) someone with nothing better to do in their life looked at some passages in a book and recorded the number of words that there were in 100 randomly chosen sentences
estimate the mean number of words per sentence

number of words	frequency
$1 \le W < 7$	15
$7 \le W < 13$	33
$13 \le W < 19$	27
$19 \le W < 25$	14
$25 \le W < 31$	11

(3) the heights (H, in cm) of a group of Y11 girls
estimate their mean height

height (cms)	frequency
$140 \le H < 145$	3
$145 \le H < 150$	1
$150 \le H < 155$	11
$155 \le H < 160$	19
$160 \le H < 165$	16
$165 \le H < 170$	34
$170 \le H < 175$	16
$175 \le H < 180$	5

(4) the following data is the pulse rate for a group of 30 pupils:

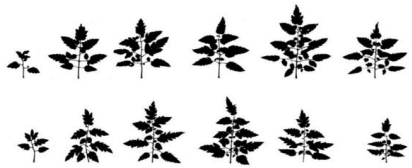
62, 65, 67, 67, 71, 73, 73, 73, 74, 74, 75, 75, 75, 76, 76, 77, 77, 78, 78, 79, 80, 80, 82, 83, 83, 84, 86, 87, 87, 92.

(a) find the actual mean
(b) find an estimate of the mean by grouping the data into 3 blocks
(c) find an estimate of the mean by grouping the data into 6 blocks

Comment on the accuracy of the estimated means.

Fluency Practice

tomato plant heights



the heights of 50 tomato plants were measured in metres, to the nearest cm:

1.52	2.05	2.39	2.14	1.84	1.65	1.91	2.34	1.04	2.95
1.72	2.28	2.32	2.00	2.11	1.66	1.74	1.97	2.21	1.43
2.08	1.76	2.68	1.91	2.07	1.85	2.19	2.14	1.99	1.57
2.06	2.45	1.82	1.11	2.68	1.86	2.19	1.56	2.78	1.23
2.83	2.01	2.44	2.04	2.63	1.90	2.21	1.37	2.57	2.54

how could you organise the data into groups to get an idea of the shape of the data?

10 groups could have these intervals

$1.0 < h \leq 1.2$
$1.2 < h \leq 1.4$
$1.4 < h \leq 1.6$
$1.6 < h \leq 1.8$
$1.8 < h \leq 2.0$
$2.0 < h \leq 2.2$
$2.2 < h \leq 2.4$
$2.4 < h \leq 2.6$
$2.6 < h \leq 2.8$
$2.8 < h \leq 3.0$

7 groups could have these intervals

$1.0 < h \leq 1.3$
$1.3 < h \leq 1.6$
$1.6 < h \leq 1.9$
$1.9 < h \leq 1.2$
$2.2 < h \leq 2.5$
$2.5 < h \leq 2.8$
$2.8 < h \leq 3.1$

5 groups could have these intervals

$1.0 < h \leq 1.4$
$1.4 < h \leq 1.8$
$1.8 < h \leq 2.2$
$2.2 < h \leq 2.6$
$2.6 < h \leq 3.0$

work out an estimate for the mean by grouping the data into
(i) 10 groups (ii) 7 groups and (iii) 5 groups

compare the estimate of the mean with the actual mean: $101.59 \div 50 = 2.0318$

comment on the accuracy of the estimates

Fluency Practice

Algebraic Frequency Tables

① The table shows the fish Tabby caught (and released).
There is one unknown frequency and x has been used to find totals.

Weight, w (kg)	Frequency	Midpoint	f × m
$0 < w \leqslant 2$	9	1	9
$2 < w \leqslant 4$	3	3	9
$4 < w \leqslant 6$	x	5	$5x$
Totals	$x + 12$		$5x + 18$

The estimated mean is 2 kg.

Describe how these two equations were formed. Solve one to find x.

$$\frac{5x + 18}{x + 12} = 2 \qquad 2(x + 12) = 5x + 18$$

② This table shows the fish Jamie caught.
If the estimated mean is 3 kg find x.

Weight, w (kg)	Frequency	Midpoint	f × m
$0 < w \leqslant 2$	7	1	7
$2 < w \leqslant 4$	6	3	18
$4 < w \leqslant 6$	x	5	$5x$
Totals			

③ This table shows the fish Jamie caught.
If the estimated mean is 6 kg find x.

Weight, w (kg)	Frequency	Midpoint	
$0 < w \leqslant 4$	4	2	
$4 < w \leqslant 8$	7	6	
$8 < w \leqslant 12$	x	10	
Totals			

④ If the estimated mean is 4 kg find x.

Weight, w (kg)	Frequency	Midpoint	
$0 < w \leqslant 2$	9		
$2 < w \leqslant 4$	2		
$4 < w \leqslant 6$	5		
$6 < w \leqslant 8$	x		
Totals			

⑤ If the estimated mean is 5 kg find x.

Weight, w (kg)	Frequency		
$0 < w \leqslant 3$	x		
$3 < w \leqslant 6$	7		
$6 < w \leqslant 9$	4		
$9 < w \leqslant 12$	2		
Totals			

⑥ If the estimated mean is 3 kg find x.

Weight, w (kg)	Frequency		
$0 < w \leqslant 2$	5		
$2 < w \leqslant 4$	14		
$4 < w \leqslant 6$	$2x - 11$		
Totals			

Fluency Practice

The table gives some information about the lengths of time, in hours, that some adults watched TV last week.

Length of time (t hours)	Frequency
$0 \leq t < 10$	8
$10 \leq t < 15$	15
$15 \leq t < 20$	11
$20 \leq t < 30$	10
$30 \leq t < 50$	6

The table shows some information about the times, in minutes, 60 people took to get to work.

Time (x minutes)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 30$	11
$30 < x \leq 50$	23
$50 < x \leq 80$	13
$80 < x \leq 100$	8

The table shows some information about the prices of 64 second-hand cars that are for sale.

Price (£x)	Frequency
$0 < x \leq 2000$	8
$2000 < x \leq 4000$	14
$4000 < x \leq 6000$	28
$6000 < x \leq 8000$	10
$8000 < x \leq 10000$	4

The table shows information about the ages of 90 employees in a factory.

Age (a years)	Frequency
$15 < a \leq 25$	12
$25 < a \leq 35$	27
$35 < a \leq 45$	18
$45 < a \leq 55$	23
$55 < a \leq 65$	10

Fluency Practice

Averages from Grouped Frequency Tables

Anne's tallest friend is 140 cm tall. Her shortest friend is 120 cm tall.
Anne and her four other friends are between these heights.
What is a good **estimate of the mean height** of Anne's **group** of friends?

When we use data that is in **groups**, we must use the **midpoint** of each group to **estimate** averages.

The frequency table below shows the height of everyone in Anne's class.

1) Which groups are these students in?
Alice is 132 cm tall. Arjun is 130 cm tall. Andy is 140 cm tall.

Height, h (cm)	Frequency	Group Midpoint	Estimated Group Value (Frequency × Midpoint)
$110 < h \leqslant 120$	4	115	
$120 < h \leqslant 130$	2		
$130 < h \leqslant 140$	4		
$140 < h \leqslant 150$	5		
$150 < h \leqslant 160$	2		
Totals			Class Total =

We can use this data to find **an estimated mean**.

2) Find the **midpoint** of each group and complete that column.
3) Find the **total value of each group** (multiply the frequency by the midpoint).
4) Find the **class total** by adding all the estimated group values.

$$\text{Estimated Mean} = \frac{\text{Total data}}{\text{Total Frequency}}$$

5) Use this formula to find the estimated mean.

Estimated Mean = _____

6) How many people are in Anne's class?
7) How could we describe where the median is?

This table shows data for a different class.
8) Complete the table and find the estimated mean and the group with the median.

Height, h (cm)	Frequency	Group Midpoint	Estimated Group Value (Frequency × Midpoint)
$120 < h \leqslant 130$	2		
$130 < h \leqslant 140$	6		
$140 < h \leqslant 150$	4		
$150 < h \leqslant 160$	3		
Totals			

Estimated Mean = _____ Group including Median = _____

Mr Higgins collected the data from his Geography test.
9) Complete the table and find the estimated mean and the median group.

Score, s (cm)	Frequency		
$0 < s \leqslant 20$	7		
$20 < s \leqslant 40$	11		
$40 < s \leqslant 60$	13		
Totals			

Estimated Mean = _____ Group including Median = _____

10) The pass mark was 30. How many people do you think passed the test?

11) Complete this grouped frequency table and find an estimate for the mean and median.

Score, s (cm)	Frequency		
$0 < s \leqslant 30$	5		
$30 < s \leqslant 60$			
$60 < s \leqslant 90$			675
$90 < s \leqslant 120$			420
Totals	25		

Estimated Mean = _____ Group including Median = _____

Grouped Frequency Tables

① Mrs Jenson collected the results from 20 maths tests.
She wrote the results like this.

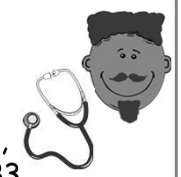
21, 27, 31, 6, 44, 26, 18, 5, 17, 25,
43, 22, 19, 11, 10, 20, 31, 41, 0, 7

Simplify the results and group them in the frequency table.

Mark	Tally	Frequency
0 to 9		
10 to 19		
20 to 29		
30 to 39		
40 to 49		

What is the modal group?

The pass mark was 25. How many students do you think passed?



② Dr Lewis recorded the heights of 27 patients (in cm).

128, 65, 82, 110, 126, 108, 90, 88, 71, 122, 99, 80, 120,
125, 111, 82, 61, 128, 130, 70, 61, 74, 82, 66, 121, 115, 83

He decided to put the data into 7 groups.
Which group would someone with a height of 120 cm go in to?
Complete the tally chart for this information.

Height, h (cm)	Tally	Frequency
$60 < h \leqslant 70$		
$70 < h \leqslant 80$		
$80 < h \leqslant 90$		
$90 < h \leqslant 100$		
$100 < h \leqslant 110$		
$110 < h \leqslant 120$		
$120 < h \leqslant 130$		
	Total	

What is the modal group?

Using **only** the table,
how could Dr Lewis estimate
a median and a mean?

Fluency Practice

(a)

Find the mode and range of the set of numbers:

$6, 2, -3, 6, 8, 7, 4$

(b)

Find the mean and median of the set of numbers:

$4.4, 2.7, 8.1, 3.6, 7.6, 4.8$

(c)

The mean of the following set of numbers is 6.

$7, 5, 9, x, 2, 2, 5, 7$

Find the value of x.

(d)

Three positive integers have a mean of 4 and a range of 7.

Find the three numbers.

(e)

Kai got the following scores in his tests:

$9, 13, 7, 15, 14, 11, 8$

Find the interquartile range.

(f)

In a class of 30 students, the mean height of the 12 girls is 164 cm and the mean height of the 18 boys is 166 cm. Find the mean height of the whole class.

(g)

The mean of five numbers is 3.6. The number 6 is added to these numbers. Find the mean of all six numbers.

(h)

Find the modal number of pets from the frequency table.

Number of Pets	Frequency
0	6
1	7
2	5
3	2

(i)

The table shows the number of goals scored in 40 games.

Number of goals	Frequency
0	11
1	17
2	8
3	4

Find

(a) the median number of goals

(b) the mean number of goals

(c) the range of the goals

(j)

The table shows the distance in km to school for 50 students.

Distance	Frequency
$0 < d \leq 5$	14
$5 < d \leq 10$	20
$10 < d \leq 15$	10
$15 < d \leq 20$	6

Find

(a) an estimate of the mean distance

(b) the modal class

exercise

1. The table shows how a group of pupils travel to school. Which of the following is the **mode**?

Transport	Car	Walk	Bus	Cycle
Frequency	9	9	11	3

a) 9 b) 11 c) Bus d) Car *and* Walk

2. Select the correct value for the mode, mean and range of each data set:

a) The number of pets owned by a group of people.

Pets	Frequency
0	6
1	8
2	3
3	3

Mode:	1	2	4	8
Mean:	1.15	3.5	4.25	5
Range:	3	4	5	8

b) The number of books read by some children in a month.

Books	Frequency
2	7
3	8
4	6
5	5
6	5

Mode:	2	3	5	5 *and* 6
Mean:	1.6	2	3.77	6.2
Range:	3	4	6	8

c) The number of points scored by players of a game.

Points	Frequency
6	3
7	3
8	1
9	0
10	3

Mode:	3	6	7	*no mode*
Mean:	2	4.4	7.7	15.4
Range:	3	4	6	10

d) The number of pupils in classes at a school.

Pupils	Frequency
30	9
31	12
32	9

Mode:	9	12	31	*no mode*
Mean:	12	20.5	30	31
Range:	2	3	20	22

3. The table shows the ages of some children attending an after-school club.

Age	Frequency
5	9
6	11
7	10
8	10

a) Graham says that there are 4 children in total. Graham is wrong. How many children are there?

b) Work out the mean age of a child at the club.

c) State the modal age of a child at the club.

4. The table shows the number of goals scored by a football team in some matches.

Goals	Frequency
0	6
1	8
2	4
3	0
4	2

a) How many matches were played by the team?

b) How many goals were scored in total?

c) Work out the mean number of goals per match.

d) State the modal number of goals per match.

e) Work out the range of goals per match.

5. Some children measured how long they could hold their breath.
The grouped frequency table shows the results.

Time (t secs)	Frequency	Mid-interval value
$10 < t \leq 20$	3	15
$20 < t \leq 30$	8	
$30 < t \leq 40$	7	
$40 < t \leq 50$	4	

a) Complete the mid-interval values.

b) Use the mid-interval values to work out an estimate for the mean time.

c) State the modal group.

6. The grouped frequency table shows the distances some children walked for charity.

Distance (d km)	Frequency	Mid-interval value
$0 < d \leq 3$	9	
$3 < d \leq 5$	7	
$5 < d \leq 10$	2	

a) Complete the mid-interval values.

b) Use the mid-interval values to work out an estimate for the mean time.

c) State the modal group.

example

The times taken by 35 people to travel to work are shown in the frequency table.

Time, x mins	Freq. f	Mid-value, m	$m \times f$
$0 \leq x < 20$	8	10	80
$20 \leq x < 30$	12	25	300
$30 \leq x < 40$	8	35	280
$40 \leq x < 50$	5	45	225
$50 \leq x < 60$	2	55	110

a) State the modal class.

$20 \leq x < 30$

b) Work out the class in which the median lies.

$(35+1) \div 2 \rightarrow 18^{th}$ position
median lies in: $20 \leq x < 30$

c) Work out an estimate for the mean.

estimated mean = $\frac{\text{sum of mf column}}{35}$

$= \frac{995}{35}$

$= 28.4$ mins

exercise

1. The frequency table shows the weights of 43 cats.

Weight, x kg	Freq. f	Mid-value, m	$m \times f$
$2.5 \leq x < 3.0$	1		
$3.0 \leq x < 3.5$	8		
$3.5 \leq x < 4.0$	16		
$4.0 \leq x < 4.5$	14		
$4.5 \leq x < 5.5$	4		

a) State the modal class.

b) Work out the class in which the median lies.

c) Work out an estimate for the mean.

2. A travel website sells weekend mini-breaks. The costs of the mini-breaks are summarised in the table.

Cost, x (£)	Freq. f
$160 \leq x < 200$	4
$200 \leq x < 240$	7
$240 \leq x < 280$	8
$280 \leq x < 320$	5
$320 \leq x < 360$	9

a) State the modal class.

b) Work out the class in which the median lies.

c) Work out an estimate for the mean.

3. Here is some data about the number of pupils that attend some schools.

No. of pupils	$0 \leq x < 500$	$500 \leq x < 1000$	$1000 \leq x < 1500$	$1500 \leq x < 2500$
Frequency	6	11	14	6

a) Which of these could be the actual median number of pupils?

12.5 | 954 | 1243 | 1512

b) True or false? The mode of the data is 6.

c) Is the data discrete or continuous?

d) One of the schools is to be picked at random. Work out the probability that the chosen school will have fewer than 1000 pupils.

4. Shade the true statements for a group of people's scores in a quiz.

Score	Frequency
11	3
12	3
13	2
14	3
15	1

A The data set contains exactly 10 values.

B The median is 13 becuase it's in the middle of: 11, 12, 13, 14, 15.

C The mean can't be 12.7 because only whole numbers could be scored.

D The mean score is 2.4.

E The data is discrete.

F The modal value is 3.

G There are 8 different values in the data set.

H The range of the scores is 4.

I The combined total scores of all the people is 152.

J The number 3 does not appear in the data set.

5. Shade the true statements for the heights of London's tallest structures.

Height h (m)	Freq.
$100 < h \leq 120$	23
$120 < h \leq 140$	16
$140 < h \leq 160$	11
$160 < h \leq 200$	6
$200 < h \leq 250$	6
$250 < h \leq 350$	1

London's tallest structures (2016)

K The data is continuous.

L The class widths are all equal.

M The modal value is 6.

N There are 63 values in the data set.

O It could be true that 7 structures are taller than 225m.

P 39 structures are less than or equal to 140m tall.

Q It is possible to work out the exact mean height from the table.

R At least 10 of the structures must be less than 110m tall.

S One of the structures must be at least 3 times as tall as some of the others.

frequency tables - averages

Round answers to 2 significant figures where necessary.

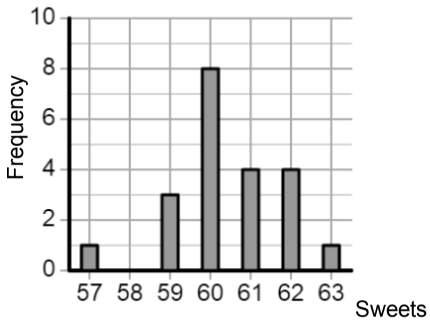
A. A survey of the number of people in cars on a particular road.

Mode: ☐ Mean: ☐ Median: ☐

People in car	Frequency
1	17
2	12
3	5
4	3
5	2

B. The number of sweets in packets that state 'average contents: 60'.

Mode: ☐ Mean: ☐ Median: ☐



C. 30 children were asked how many brothers and sisters they have.

Mode: ☐ Mean: ☐ Median: ☐

Siblings	Frequency
0	8
1	13
2	___
3	2

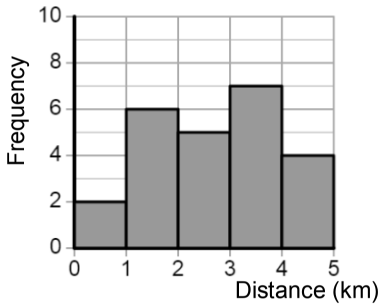
D. The weights of cats.

Modal class: ☐ Estimated Mean: ☐

Weight w (kg)	Frequency
$3 < w \leq 3.5$	3
$3.5 < w \leq 4$	19
$4 < w \leq 4.5$	12
$4.5 < w \leq 5$	2

E. The distance a person walked each day for 24 days.

Modal class: ☐ Estimated Mean: ☐



F. The number of late trains per day on a particular route, recorded over 30 days.

Mode: ☐ Mean: **0.8** Median: ☐

Late trains	Frequency
0	19
1	4
2	___
3	___
4	1

Manipulating the Data

Sales are recorded in an online App Store.
Use this data to complete the two different frequency tables &
calculate a mean from each. How & why are the mean averages different?

3	3	4	3
4	3	5	4
3	4	3	3
6	3	4	3
5	4	6	3

Amount Spent, £	Frequency	m	m × f
$2 < s \leqslant 4$			
$4 < s \leqslant 6$			
Totals			

Mean =

Amount Spent, £	Frequency	m	m × f
$0 < s \leqslant 3$			
$3 < s \leqslant 6$			
Totals			

Mean =

A mobile gaming company records in-game purchases.
Which table would the company prefer to show investors?

Spent	$0 < s \leqslant 5$	$5 < s \leqslant 10$	Totals
Freq.			

Mean =

2	1	4	3	1	4
1	5	3	1	6	1
6	4	1	3	1	2
6	1	9	6	2	1

Spent	$0 < s \leqslant 2$	$2 < s \leqslant 4$	$4 < s \leqslant 6$	$6 < s \leqslant 8$	$8 < s \leqslant 10$	Totals
Freq.						

Mean =

Which mean is more accurate?
What are the advantages & disadvantages of smaller/larger tables?

JAM IT! JEANZ recorded how much each customer spent in their shop (rounded to the nearest dollar). ①

Key

1 | 5

= $15

0	9 9 9 9 9 9
1	1 1 2 2 3 4 4 5 5
2	1 1 5 5 5 6 7 8 8 9
3	1 2 2 4 4 4

Find the **mode** and the **exact median**.
Estimate the mean using the frequency table.

Spent, s, $	Frequency	Midpoint	FM
$0 < s \leqslant 10$			
$10 < s \leqslant 20$			
$20 < s \leqslant 30$			
$30 < s \leqslant 40$			

By looking at the raw data,
do you expect the estimated mean to be higher or lower than the actual mean?

Eppington Clothing Company Ltd ②

The stem & leaf diagram shows spending by customers on a Saturday.
(Rounded to the nearest $)

Group the data into a frequency table to **estimate** a mean.
Using inequality symbols, what groups could you use?

Key

6 | 3 = $63

2	00111555558888889
3	22222444677889999
4	0225555556778888
5	022223444445555555
6	3334444556777788
7	02222444455556666789

How much larger (%) is
the median amount spent?

Spent, s, $			

2 Expanding Triple Brackets

Intelligent Practice

1) $(x+1)(x+2)(x+3)$
2) $(x+1)(x+2)(x+4)$
3) $(x+1)(x+2)(x+5)$
4) $(x+1)(x+2)(x+6)$
5) $(x+6)(x+1)(x+2)$
6) $(x+6)(x+1)(x-2)$
7) $(x+6)(x+1)(x-3)$
8) $(x+6)(x+3)(x-3)$
9) $(x+6)(x-3)(x-3)$
10) $(x+6)(x-4)(x-4)$
11) $(x+4)(x-4)^2$
12) $(x+4)^3$
13) $(x-4)^3$
14) $(2x+1)(x-4)^2$
15) $(2x+1)(x+4)(x-4)$
16) $(2x+1)(x+3)(x-5)$
17) $(2x+1)(4x+3)(x-5)$
18) $(2x+1)(4x+3)(6x-5)$
19) $(2x+1)(4x+3)(5-6x)$
20) $(2x+1)(3-4x)(5-6x)$
21) $(2x+1)(3-4x)^2$
22) $(2x+1)(4x-3)^2$
23) $2x(4x-3)^2$
24) $(4x-3)^3$
25) $(3-4x)^3$

Question 1: Expand and simplify

(a) $(x + 3)(x + 2)(x + 1)$ (b) $(x + 2)(x + 2)(x + 5)$ (c) $(x + 3)(x - 2)(x + 1)$

(d) $(x - 1)(x - 2)(x + 7)$ (e) $(x - 2)(x - 3)(x - 4)$ (f) $(x - 6)(x + 1)(x - 2)$

(g) $(2x + 1)(x + 3)(x + 1)$ (h) $(3x - 2)(x + 5)(x - 1)$ (i) $(5x + 3)(x - 1)(x + 2)$

(j) $x(x - 3)(2x + 5)$ (k) $(3x + 5)(3x + 2)(x - 10)$

Question 2: Expand and simplify

(a) $(x + 2)^3$ (b) $(x + 5)^3$ (c) $(x - 3)^3$

(d) $(x - 5)^3$ (e) $(x + 1)(x + 3)^2$ (f) $(x - 5)(x - 4)^2$

(g) $(2x + 3)^3$ (h) $(4 - x)^3$ (i) $(5 - 2x)^3$

(j) $(x + 2)(3 - x)^2$ (k) $x(x + 6)^2$

Question 3: Expand and simplify

(a) $(3x + 2)(x + 1)(x + 5) + (x + 3)^3$ (b) $(2x - 3)^3 - (x - 4)^3$

Apply

Question 1: Given $(x + 3)(x + a)(x + 7) = x^3 + 15x^2 + 71x + 105$, find a.

Question 2: Given $(ax + 1)(x - 3)(x + b) = 2x^3 - 3x^2 - 8x - 3$, find a and b.

Question 3: Given $(x + a)^2(x - 2) = x^3 + bx^2 + 12x - 72$, find a and b

Expand and simplify:

(a) $x(x^2 + 4x + 2)$

(b) $(x + 2)(x^2 + x + 1)$

(c) $(x - 1)(x^2 - x - 1)$

(d) $(3x - 2)(x^2 - x - 1)$

(e) $(2x - 1)(2x^2 - 3x + 5)$

(f) $(2x + 3)(x^2 - 6x - 3)$

Expand and simplify:

(a) $x(x + 1)(x + 2)$

(b) $(x + 1)(x + 2)(x + 3)$

(c) $(x + 4)(x - 1)(x + 1)$

(d) $(x - 2)(x - 3)(x + 1)$

(e) $(x + 1)(2x + 1)(x + 2)$

(f) $(2x + 1)(x - 3)(3x - 1)$

Expand and simplify:

(a) $(x + 2)(x + 1)^2$

(b) $(x + 2)(2x - 1)^2$

(c) $(2x + 3)(x - 2)^2$

Expand and simplify:

(a) $(x + 1)^3$

(b) $(2x - 1)^3$

(c) $(3x + 2)^3$

Expand and simplify:

(a) $(2x + y)(x - y)$

(b) $(x - 3y)(2x - y)$

(c) $(4a + 3b)(2a + b)$

(d) $(2x - y)(x + 5y)$

(e) $(3b + 2c)^2$

Expand and simplify:

(a) $(x + y)(x + y + 1)$

(b) $(2a + b + 3c)(a - b)$

(c) $(5x - y)(5 - 2x + y)$

(d) $(a - 2b - c)(2b + 3c)$

Expand and simplify:

(a) $xy(x + y)(x + 2y)$

(b) $ax(x + a)(x - b)$

(c) $2ab(2a - b)(2b - a)$

(d) $xy(3x + 2y)^2$

Expand and simplify:

(a) $(x + y)(x + 2y)(x + 3y)$

(b) $(a + b)(b + c)(c + a)$

(c) $(2x + y)(x - 3y)(y - x)$

(d) $(a - b)(3a + 2b)^2$

(e) $(a - 2b)^2(5a + b)$

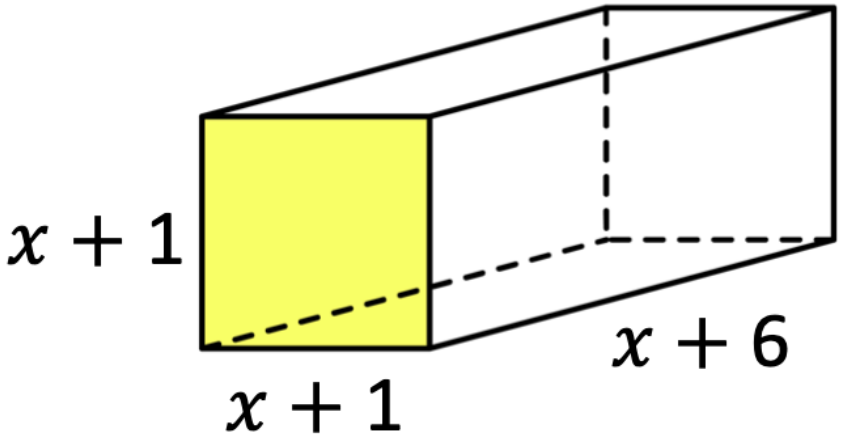
(f) $(x + 2y)^3$

(g) $(a - 4b)^3$

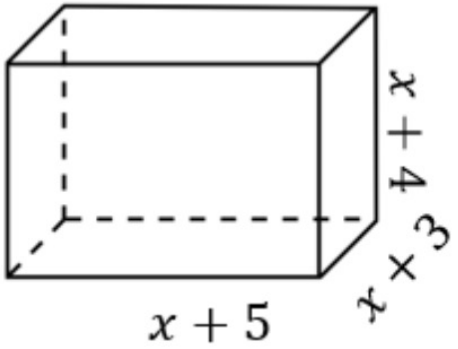
Fluency Practice

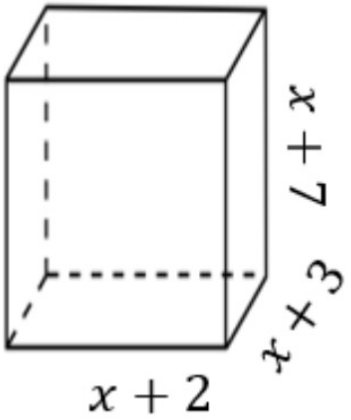
A1 Expand and simplify $x(x + 2)(x + 3)$	**A2** Expand and simplify $(x + 1)(x + 3)(x – 4)$	**A3** Expand and simplify $(x + 3)(x – 4)(x – 2)$
B1 Expand and simplify $(x + 4)^3$	**B2** Expand and simplify $(x + 5)^2(x + 3)$	**B3** Expand and simplify $(x – 3)(x – 1)^2$
C1 Expand and simplify $(2x + 1)(x + 2)(x + 3)$	**C2** Expand and simplify $(3x – 1)(x + 3)(x – 3)$	**C3** Expand and simplify $(2x – 3)^2(4 – x)$
D1 If $(x + k)^2(x + 2) = x^3 + 14x^2 + 60x + 72$ Find the value of k.	**D2** If $(x + p)(x + q)(x + 5) = x^3 + 8x^2 – 3x – 90$ Find the values of p and q.	**D3** If $(ax + b)^2(x + c) = 4x^3 + dx^2 – 55x – 100$ Find the values of the integers a, b, c and d.

Write a simplified expression for the volume of the following cuboid.



Which has the bigger volume?





Fluency Practice

(a)	(b)	(c)	(d)
Expand $7(x-3)$	Expand $x(5+2x)$	Expand $5y(3y-1)$	Expand $-6(2x+3)$
(e)	**(f)**	**(g)**	**(h)**
Expand $x^2(9-2x)$	Expand and simplify $5(x+3)+2(x-4)$	Expand and simplify $4(2x-3)-2(x-1)$	Expand and simplify $7-3(4x-1)$
(i)	**(j)**	**(k)**	**(l)**
Expand and simplify $(x+3)(x+7)$	Expand and simplify $(x-5)(x+1)$	Expand and simplify $(y-8)(y-7)$	Expand and simplify $(5x+1)(x-4)$
(m)	**(n)**	**(o)**	**(p)**
Expand and simplify $(2x-3y)(x-2y)$	Expand and simplify $(x+3)^3$	Expand and simplify $(2x+3)(x-1)(x+5)$	$(3x-1)(x+a)^2$ $\equiv 3x^3-19x^2+bx-9$ Find the values of a and b.

Intelligent Practice

1) $3(x+4)$
2) $3(x-4)$
3) $x(x-4)$
4) $(x+1)(x-4)$
5) $(x-1)(x-4)$
6) $2(x-1)(x-4)$
7) $3(x-1)(x-4)$
8) $3(x-1)(x+4)$
9) $x(x-1)(x+4)$
10) $x(x-2)(x+4)$
11) $(x+1)(x-2)(x+4)$
12) $(x-1)(x-2)(x+4)$
13) $(2x-1)(x-2)(x+4)$
14) $4(2x-1)(x-2)$
15) $x(2x-1)(x-2)$
16) $(2x-1)(x-2)$
17) $(2x+1)(x-2)$
18) $(2x+1)(2x-2)$
19) $(2x+1)(2x-1)$
20) $(2x+1)(1-2x)$
21) $(2x+1)(1-2x^2)$
22) $(2x+1)(1-2x)^2$
23) $(2x+1)(2x-1)^2$
24) $(2x-1)^3$
25) $(2x-y)^3$

3 Solving Quadratics

Fluency Practice

$3\times0=a$ $a=$

$10\times0=a$ $a=$

$\mathrm{a}\times0=0$ $a=$

$4\times a=0$ $a=$

$b\times0=a$ $a=$

$(a-2)\times3=0$ $a=$

$(0-2)\times a=0$ $a=$

$(a-2)\times a=0$ $a=$

$(a-3)\times a=0$ $a=$

$(a+3)\times a=0$ $a=$

$(a+3)(a+1)=0$ $a=$

$a^2+4a+3=0$ $a=$

Find the value of $(x-3)(x-7)$ if
a) $x = 8$ b) $x = 7$ c) $x = 3$

a) If $x = 8$ $(x-3)(x-7) = (8-3)(8-7)$
$= (5)(1)$
$= 5$

b) If $x = 7$ $(x-3)(x-7) = (4)(0)$
$= 0$

c) If $x = 3$ $(x-3)(x-7) = (0)(-4)$
$= 0$

1. Find the value of $(x-4)(x-2)$ if
a) $x = 6$ b) $x = 4$ c) $x = 2$

2. Find the value of $(x-5)(x-9)$ if
a) $x = 5$ b) $x = 10$ c) $x = 9$

3. Find the value of $(x-7)(x-1)$ if
a) $x = 1$ b) $x = 8$ c) $x = 7$

4. Find the value of $(x-4)(x-6)$ if
a) $x = 4$ b) $x = 6$ c) $x = 3$

5. Find the value of $(x-6)(x-7)$ if
a) $x = 2$ b) $x = 6$ c) $x = 9$

Find the value of $(x-2)(x+4)$ if
a) $x = 2$ b) $x = 4$ c) $x = -4$

a) If $x = 2$ $(x-2)(x+4) = (0)(6)$
$= 0$

b) If $x = 4$ $(x-2)(x+4) = (2)(8)$
$= 16$

c) If $x = -4$ $(x-2)(x+4) = (-6)(0)$
$= 0$

6. Find the value of $(x-3)(x+5)$ if
a) $x = 6$ b) $x = 3$ c) $x = -5$

7. Find the value of $(x-4)(x+6)$ if
a) $x = 0$ b) $x = -6$ c) $x = 4$

8. Find the value of $(x-7)(x+2)$ if
a) $x = -7$ b) $x = -2$ c) $x = 7$

9. Find the value of $(x+4)(x+5)$ if
a) $x = -4$ b) $x = -5$ c) $x = 0$

10. Find the value of $(x+7)(x+1)$ if
a) $x = -4$ b) $x = -1$ c) $x = -7$

The results of this exercise show that if the product of two factors is 0, then either one or both of these factors must be 0

In general we can say

if	$A \times B = 0$		
then either	$A = 0$	or/and	$B = 0$

In questions 1 to 12 find, if possible, the value or values of A. Note that if $A \times 0 = 0$ then A can have any value.

1. $A \times 6 = 0$

2. $A \times 7 = 0$

3. $A \times 4 = 0$

4. $A \times 0 = 0$

5. $3 \times A = 12$

6. $8 \times A = 8$

7. $A \times 10 = 0$

8. $A \times 9 = 18$

9. $A \times 20 = 0$

10. $A \times 3 = 21$

11. $0 \times A = 0$

12. $4 \times A = 0$

13. If $AB = 0$ find a) A if $B = 2$ b) B if $A = 10$

14. If $AB = 0$ find a) A if $B = 5$ b) B if $A = 5$

15. If $AB = 0$ find a) A if $B = 10$ b) B if $A = 3$

16. If $AB = 0$ find a) B if $A = 6$ b) A if $B = 0$

Find a and b if $a(b - 3) = 0$

Either $a = 0$ or/and $b - 3 = 0$

i.e., either $a = 0$ or/and $b = 3$

Find a and b if:

17. $a(b - 1) = 0$

18. $a(b - 5) = 0$

19. $a(b - 2) = 0$

20. $(a - 3)b = 0$

21. $(a - 9)b = 0$

22. $a(b - 4) = 0$

23. $a(b - 10) = 0$

24. $(a - 1)b = 0$

25. $(a - 7)b = 0$

26. $(a - 12)b = 0$

Fluency Practice

What values of x satisfy the following equations?

1. $x(x-3)=0$
2. $x(x-5)=0$
3. $(x-3)x=0$
4. $x(x+4)=0$
5. $(x+5)x=0$
6. $x(x-6)=0$
7. $x(x-10)=0$
8. $(x-7)x=0$
9. $x(x+7)=0$
10. $(x+9)x=0$

Fluency Practice

What values of x satisfy the following equations?

11. $(x-1)(x-2)=0$

12. $(x-5)(x-9)=0$

13. $(x-10)(x-7)=0$

14. $(x-4)(x-7)=0$

15. $(x-6)(x-1)=0$

16. $(x-8)(x+11)=0$

17. $(x-3)(x+5)=0$

18. $(x+7)(x-2)=0$

19. $(x+2)(x+3)=0$

20. $(x+4)(x+9)=0$

21. $(x+1)(x+8)=0$

22. $(x-p)(x-q)=0$

23. $(x+a)(x+b)=0$

24. $(x-4)(x+1)=0$

25. $(x+9)(x-8)=0$

26. $(x+6)(x+7)=0$

27. $(x+10)(x+11)=0$

28. $(x-a)(x-b)=0$

29. $(x+a)(x-b)=0$

30. $(x-c)(x+d)=0$

Fluency Practice

Solve the following equations:

1. $(2x - 5)(x - 1) = 0$
2. $(x - 4)(3x - 2) = 0$
3. $(5x - 4)(4x - 3) = 0$
4. $x(4x - 5) = 0$
5. $x(10x - 3) = 0$
6. $(5x + 2)(x - 7) = 0$
7. $(6x + 5)(3x - 2) = 0$
8. $(8x - 3)(2x + 5) = 0$
9. $(7x - 8)(4x + 15) = 0$
10. $(4x + 3)(2x + 3) = 0$
11. $(3x - 7)(x - 2) = 0$
12. $(3x - 5)(2x - 1) = 0$
13. $x(3x - 1) = 0$
14. $x(7x - 3) = 0$
15. $(2x + 3)(x - 3) = 0$
16. $(4x + 3)(2x - 5) = 0$
17. $(10x + 9)(5x - 4) = 0$
18. $(3x - 2)(4x + 9) = 0$
19. $(5x - 12)(2x + 7) = 0$
20. $(5x + 8)(4x + 3) = 0$

Solve the equations:

1. $x^2 - 3x + 2 = 0$
2. $x^2 - 8x + 7 = 0$
3. $x^2 - 5x + 6 = 0$
4. $x^2 - 7x + 10 = 0$
5. $x^2 - 7x + 12 = 0$
6. $x^2 - 6x + 5 = 0$
7. $x^2 - 12x + 11 = 0$
8. $x^2 - 6x + 8 = 0$
9. $x^2 - 8x + 12 = 0$
10. $x^2 - 13x + 12 = 0$

Solve the equations:

11. $x^2 + 6x - 7 = 0$
12. $x^2 - 2x - 8 = 0$
13. $x^2 + x - 12 = 0$
14. $x^2 - 2x - 15 = 0$
15. $x^2 + 7x - 18 = 0$
16. $x^2 - 12x - 13 = 0$
17. $x^2 + x - 6 = 0$
18. $x^2 - 4x - 12 = 0$
19. $x^2 + x - 20 = 0$
20. $x^2 - 5x - 24 = 0$

Solve the equations:

21. $x^2 + 3x + 2 = 0$
22. $x^2 + 8x + 7 = 0$
23. $x^2 + 8x + 15 = 0$
24. $x^2 + 8x + 12 = 0$
25. $x^2 + 11x + 18 = 0$
26. $x^2 + 7x + 6 = 0$
27. $x^2 + 7x + 10 = 0$
28. $x^2 + 14x + 13 = 0$
29. $x^2 + 16x + 15 = 0$
30. $x^2 + 9x + 18 = 0$

Solve the equations:

31. $x^2 - 1 = 0$

32. $x^2 - 9 = 0$

33. $x^2 - 16 = 0$

34. $x^2 - 81 = 0$

35. $x^2 - 169 = 0$

36. $x^2 - 4 = 0$

37. $x^2 - 25 = 0$

38. $x^2 - 100 = 0$

39. $x^2 - 144 = 0$

40. $x^2 - 36 = 0$

Fluency Practice

Solve the equations:

1. $x^2 - 2x = 0$
2. $x^2 - 10x = 0$
3. $x^2 + 8x = 0$
4. $2x^2 - x = 0$
5. $4x^2 - 5x = 0$
6. $x^2 - 5x = 0$
7. $x^2 + 3x = 0$
8. $x^2 + x = 0$
9. $3x^2 - 5x = 0$
10. $5x^2 - 7x = 0$
11. $2x^2 + 3x = 0$
12. $8x^2 + 5x = 0$
13. $x^2 - 7x = 0$
14. $3x^2 + 5x = 0$
15. $7x^2 - 12x = 0$
16. $6x^2 + 7x = 0$
17. $12x^2 + 7x = 0$
18. $x^2 + 4x = 0$
19. $7x^2 - 2x = 0$
20. $14x^2 + 3x = 0$

Fluency Practice

Solve the equations:

1. $x^2 - 2x + 1 = 0$
2. $x^2 - 10x + 25 = 0$
3. $x^2 - 20x + 100 = 0$
4. $x^2 + 8x + 16 = 0$
5. $x^2 + 6x + 9 = 0$
6. $x^2 - 6x + 9 = 0$
7. $x^2 - 8x + 16 = 0$
8. $x^2 - 18x + 81 = 0$
9. $x^2 + 2x + 1 = 0$
10. $x^2 + 20x + 100 = 0$
11. $x^2 + 18x + 81 = 0$
12. $x^2 - 14x + 49 = 0$
13. $x^2 - 22x + 121 = 0$
14. $x^2 + 12x + 36 = 0$
15. $x^2 - x + \frac{1}{4} = 0$
16. $x^2 + 10x + 25 = 0$
17. $x^2 - 12x + 36 = 0$
18. $x^2 - 40x + 400 = 0$
19. $x^2 - 16x + 64 = 0$
20. $x^2 + \frac{4}{3}x + \frac{4}{9} = 0$

Fluency Practice

Solve the equations:

1. $2x^2 - 5x + 2 = 0$
2. $2x^2 - 11x + 12 = 0$
3. $2x^2 - 13x + 20 = 0$
4. $3x^2 + 5x + 2 = 0$
5. $2x^2 + 9x - 35 = 0$
6. $3x^2 - 11x + 6 = 0$
7. $3x^2 - 7x + 2 = 0$
8. $2x^2 + 5x - 12 = 0$
9. $3x^2 + 11x + 6 = 0$
10. $5x^2 + 27x + 10 = 0$
11. $6x^2 - x - 2 = 0$
12. $15x^2 + 14x - 8 = 0$
13. $12x^2 - 7x + 1 = 0$
14. $6x^2 - 13x - 5 = 0$
15. $20x^2 + 19x + 3 = 0$
16. $8x^2 - 18x + 9 = 0$
17. $12x^2 - 20x - 25 = 0$
18. $4x^2 + 8x + 3 = 0$
19. $12x^2 + 17x + 6 = 0$
20. $10x^2 - 29x - 21 = 0$

Solve the equations:

21. $16x^2 - 25 = 0$

22. $100x^2 - 81 = 0$

23. $4x^2 - 25 = 0$

24. $9x^2 - 16 = 0$

25. $25x^2 - 144 = 0$

26. $9x^2 - 4 = 0$

27. $81x^2 - 25 = 0$

28. $25x^2 - 4 = 0$

29. $36x^2 - 25 = 0$

30. $4x^2 - 81 = 0$

Solve the equations:

1. $x^2 - x = 30$
2. $x^2 - 6x = 16$
3. $x^2 + 9x = 36$
4. $3x^2 + 4x = 4$
5. $x^2 - x = 6$
6. $x^2 + 6x = 7$
7. $2x^2 + 5x = 3$
8. $5x^2 - 12x = 9$

Solve the equations:

9. $x^2 = 2x + 8$
10. $x^2 = 2x + 24$
11. $x^2 = 12x - 35$
12. $10x^2 = 13x + 3$
13. $x^2 = 3x + 10$
14. $x^2 = 6x - 8$
15. $6x^2 = x + 1$
16. $3x^2 = 13x - 4$

17. $10 = 7x - x^2$
18. $7 = 8x - x^2$
19. $8 = 6x - x^2$
20. $21 = 10x - x^2$
21. $12 = 8x - x^2$
22. $20 = 9x - x^2$
23. $35 = 12x - x^2$
24. $15 = 8x - x^2$

Fluency Practice

Solve the equations:

25. $8x^2 - 4x = 0$

26. $2x^2 - 10x + 12 = 0$

27. $3x^2 - 24x + 36 = 0$

28. $12x^2 + 20x + 8 = 0$

29. $8x^2 + 20x = 12$

30. $3x^2 - 9x = 0$

31. $5x^2 - 15x + 10 = 0$

32. $6x^2 + 18x + 12 = 0$

33. $15x^2 - 35x + 10 = 0$

34. $30x^2 = 39x + 9$

Fluency Practice

Solve the equations:

1. $x^2 - x - 20 = 0$
2. $x^2 = 4x - 4$
3. $9x^2 - 1 = 0$
4. $2x^2 + 7x = 0$
5. $x^2 + 13x + 12 = 0$
6. $1 - 16x^2 = 0$
7. $x^2 - 6x = 0$
8. $x^2 = 2x + 35$
9. $2x^2 + 3x - 14 = 0$
10. $2x^2 + 12x + 18 = 0$
11. $x^2 = 7 - 6x$
12. $4 = 25x^2$
13. $4x^2 = 25$
14. $x^2 + 11x + 18 = 0$
15. $2 - x = 6x^2$
16. $5x - 2x^2 = 0$
17. $5x = 3x^2 - 2$
18. $4 + 11x + 6x^2 = 0$
19. $7x = 4x^2$
20. $14x - 2 = 24x^2$
21. $6x^2 + 13x - 5 = 0$
22. $5x + 2 = 3x^2$
23. $3 + 8x + 4x^2 = 0$
24. $3 - 12x^2 = 0$

Fluency Practice

Solve the equations:

25. $x(x+1) = 12$

26. $x(x-1) = x+3$

27. $3x(2x+1) = 4x+1$

28. $5x(x-1) = 4x^2 - 4$

29. $x(x-5) = 24$

30. $x(x+3) = 5(3x-7)$

31. $3x(x+3) = 5x+4$

32. $2x(2x-1) = x^2 + 3x + 2$

Solve the equations:

33. $(x+2)(x+3) = 56$

34. $(x+9)(x-6) = 34$

35. $(x-2)(x+6) = 33$

36. $(x+3)(x-8) + 10 = 0$

37. $(x-5)(x+2) = 18$

38. $(x+8)(x-2) = 39$

39. $(x+1)(x+8) + 12 = 0$

40. $(x-1)(x+10) + 30 = 0$

Fluency Practice

Q	Solve the quadratic equation:	Answers	Q	Solve the quadratic equation:	Answers
1	$x^2 + 7x + 6 = 0$		13	$x^2 + x = 0$	
2	$2x^2 + 7x + 6 = 0$		14	$5x^2 + x = 0$	
3	$x^2 - 5x + 6 = 0$		15	$x^2 - 5x = 0$	
4	$3x^2 - 11x + 6 = 0$		16	$7x^2 - 5x = 0$	
5	$x^2 - 5x - 6 = 0$		17	$x^2 - x = 0$	
6	$2x^2 - 11x - 6 = 0$		18	$3x^2 - x = 0$	
7	$x^2 + 5x - 6 = 0$		19	$x^2 + 8x = 0$	
8	$3x^2 + 17x - 6 = 0$		20	$2x^2 + 8x = 0$	
9	$x^2 - x - 2 = 0$		21	$x^2 - 6x = 0$	
10	$3x^2 - x - 2 = 0$		22	$12x^2 - 6x = 0$	
11	$x^2 + x - 12 = 0$		23	$x^2 - 64 = 0$	
12	$2x^2 + x - 15 = 0$		24	$4x^2 - 64 = 0$	

Fluency Practice

Q	Solve the quadratic equation:	Answers	Q	Solve the quadratic equation:	Answers
1	$x^2 + 5x + 6 = 0$		13	$2x^2 + 7x + 6 = 0$	
2	$x^2 + 5x + 5 = 0$		14	$2x^2 + 7x + 4 = 0$	
3	$x^2 + 5x + 4 = 0$		15	$2x^2 + 7x + 5 = 0$	
4	$x^2 + 5x + 3 = 0$		16	$2x^2 + 7x + 8 = 0$	
5	$x^2 - 5x - 6 = 0$		17	$2x^2 - 11x - 6 = 0$	
6	$x^2 - 5x - 5 = 0$		18	$2x^2 - 11x - 5 = 0$	
7	$x^2 - 3x - 4 = 0$		19	$3x^2 + 17x - 6 = 0$	
8	$x^2 - 3x - 2 = 0$		20	$3x^2 + 17x - 6 = 0$	
9	$x^2 - 6x + 9 = 0$		21	$-3x^2 - x + 2 = 0$	
10	$x^2 - 6x + 7 = 0$		22	$-3x^2 - x + 3 = 0$	
11	$x^2 - 6x + 8 = 0$		23	$-3x^2 - x + 4 = 0$	
12	$x^2 - 6x + 10 = 0$		24	$-3x^2 - x + 5 = 0$	

Intelligent Practice

A	B	C
1) $(x+2)(x-3)=0$	1) $x^2-x-6=0$	1) $2x^2-8x+6=0$
2) $(x-3)(x+2)=0$	2) $x^2+x-6=0$	2) $2x^2-7x+6=0$
3) $(x+3)(x-2)=0$	3) $(x+1)(x-6)=0$	3) $4x^2-10x+6=0$
4) $(x-3)(x-2)=0$	4) $x^2+x-20=0$	4) $2x^2-5x+3=0$
5) $(x-3)(2x-2)=0$	5) $x^2-x-20=0$	5) $x^2-10x+7=0$
6) $(2x-3)(x-2)=0$	6) $(x-1)(x-20)=0$	6) $6x^2-10x+4=0$
7) $(2x-3)(2x-2)=0$	7) $(2x-1)(2x-20)=0$	7) $3x^2-5x+2=0$
8) $(3x-2)(2x-2)=0$	8) $x^2-5x+6=0$	8) $x^2-3x-4=0$
9) $(3x+2)(2x-2)=0$	9) $x^2-7x+6=0$	9) $6x^2-2x-4=0$
10) $\left(\frac{1}{2}x+2\right)(2x-2)=0$	10) $x^2+7x+6=0$	10) $12x^2-36x-48=0$

Solve these quadratic equations.

(a) $(x-4)(x-1)=0$

(b) $(x-4)(x+1)=0$

(c) $(x+4)(x+1)=0$

(d) $(x+4)(x-6)=0$

Solve these quadratic equations.

(a) $x^2+5x+4=0$

(b) $x^2+5x+6=0$

(c) $x^2+7x+6=0$

(d) $x^2+10x+16=0$

(e) $x^2+10x+21=0$

Solve these quadratic equations.

(a) $x^2-7x+6=0$

(b) $x^2-6x+8=0$

(c) $x^2-9x+8=0$

(d) $x^2-9x+18=0$

(e) $x^2-11x+18=0$

Solve these quadratic equations.

(a) $x^2+3x-4=0$

(b) $x^2+5x-6=0$

(c) $x^2+x-6=0$

(d) $x^2-2x-8=0$

(e) $x^2-6x-16=0$

Solve these quadratic equations.

(a) $x^2=24+2x$

(b) $30+x^2=13x$

(c) $x^2+9=6x$

(d) $44=x^2-7x$

(e) $x^2-x=72$

(f) $(x+1)(x-2)=4$

Solve these quadratic equations.

(a) $2x^2 + 11x + 5 = 0$

(b) $2x^2 + 5x - 3 = 0$

(c) $3x^2 - 7x + 2 = 0$

(d) $2x^2 + x - 15 = 0$

(e) $2x^2 - 3x - 5 = 0$

(f) $2x^2 + 5x + 3 = 0$

Solve these quadratic equations.

(a) $x^2 - 5x = 0$

(b) $x^2 + 7x = 0$

(c) $2x^2 + 14x = 0$

(d) $2x^2 - 7x = 0$

Solve these quadratic equations.

(a) $x^2 - 25 = 0$

(b) $x^2 - 144 = 0$

(c) $2x^2 - 32 = 0$

(d) $5x^2 - 45 = 0$

Solve these quadratic equations.

(a) $x^2 = 24 + 2x$

(b) $30 + x^2 = 13x$

(c) $2x^2 = 3 - x$

(d) $7x^2 + 13x = 10 - 20x$

(e) $15 + 2x = 2x^2 + 3x$

(f) $x^2 + 5x + 56 = 20x$

Solve by factorisation:

(a) $x^2 + 4x = 5$

(b) $12 = x^2 - 4x$

(c) $x = x^2 - 20$

(d) $x^2 = 6x - 8$

(e) $9x = 22 - x^2$

Solve by factorisation:

(a) $x^2 + 4x = 2x + 15$

(b) $11 + x^2 + 3x = 5 - 2x$

(c) $7 - 4x = x^2 - 10x$

(d) $2x^2 + 30 = x^2 + 13x$

(e) $4 + 5x - x^2 = 34 - 6x$

Solve by factorisation:

(a) $x(x - 5) = 14$

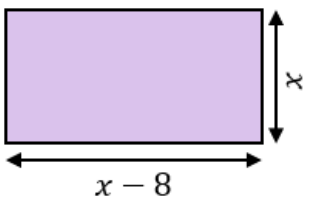
(b) $25 + x = x(x + 1)$

(c) $3(x + 4) = x^2 + 2x$

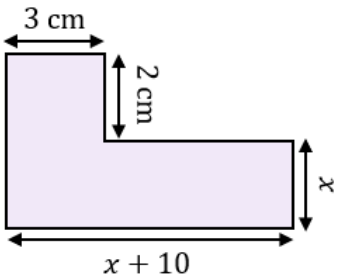
(d) $x(x - 1) = 5(x + 2) + 6$

(e) $(x + 2)(x - 3) = 5x + 12$

(a) Given that the area of the rectangle is $65\ cm^2$, form a quadratic equation and solve it to find x.



(b) Given that the area of the compound shape is $45\ cm^2$, form a quadratic equation and solve it to find x.



solve these quadratic equations

1) $n(n + 2) + n + 2 = 12$

2) $n^2 + (n + 1)^2 = 13$

3) $n + (n + 1)(n + 2) = 14$

4) $(3n – 1)(2n – 1) = 15$

5) $n(n – 2) + 2n(n + 2) = 16$

6) $2n(n + 2) + 3n(n – 2) + 1 = 17$

7) $n(n + 1) + (n + 1)(n + 2) = 18$

8) $(2n + 4)(2n – 1) – 1 = 19$

9) $3n(2n – 1) + 2(n – 1)(3n – 5) = 20$

10) $2(n – 1)(2n – 1) + (3n – 1)(n + 1) = 21$

Fluency Practice

Write the solution to each equation:

1. $x^3 = 8$
2. $x^2 = 16$
3. $(x - 5)(x + 4) = 0$
4. $x(x - 5)(x + 4) = 0$
5. $3(x - 5)(x + 4) = 0$
6. $3x(x - 5)(x + 4) = 0$
7. $(4x + 1)(2x - 6) = 0$
8. $x(4x + 1)(2x - 6) = 0$
9. $5(4x + 1)(2x - 6) = 0$
10. $5x(4x + 1)(2x - 6) = 0$
11. $(x - 2)^2 = 0$
12. $3(x - 2)^2 = 0$
13. $3(4x - 5)(x - 2)^2 = 0$
14. $3x^2(4x - 5)(x - 2)^2 = 0$

1. The square of a number x is 16 more than six times the number. Form an equation in x and solve it.

2. When five times a number x is subtracted from the square of the same number, the answer is 14. Form an equation in x and solve.

3. I think of a number x. If I square it and add it to the number I first thought of the total is 42. Find the number I first thought of.

4. Peter had x marbles. The number of marbles Fred had was six fewer than the square of the number Peter had. Together they had 66 marbles. Form an equation in x and solve it. How many marbles did Fred have?

5. Ahmed is x years old and his father is x^2 years old. If the sum of their ages is 56 years, form an equation in x and solve it to find the age of each.

6. Kathryn is x years old. If her mother's age is two years more than the square of Kathryn's age, and the sum of their ages is 44 years, form an equation in x and solve it to find the ages of Kathryn and her mother.

7. Peter is x years old and his sister is 5 years older. If the product of their ages is 84, form an equation in x and solve it to find Peter's age.

8. Sally is x years old and her sister Ann is 4 years younger. If the product of their ages is 140, form an equation in x and solve it to find Ann's age.

9. A rectangle is x cm wide and is 3 cm longer than it is wide. If its area is $28\,cm^2$, form an equation in x and solve it to find the dimensions of the rectangle.

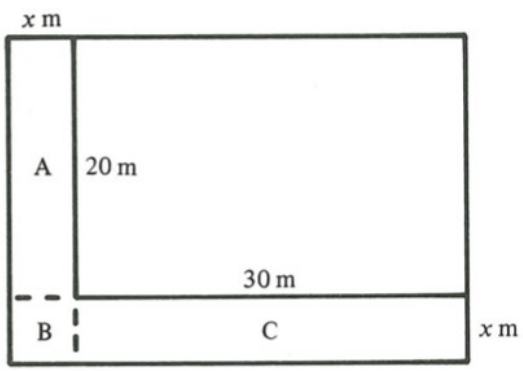
10. A rectangle is 5 cm longer than it is wide. If its width is x cm and its area is $66\,cm^2$ form an equation in x and solve it. Hence find the dimensions of the rectangle.

11. The base of a triangle is x cm long and its perpendicular height is half the length of its base. If the triangle has an area of $25\,cm^2$, form an equation in x and solve it. What is the height of the triangle?

12. A rectangular lawn measuring 30 m by 20 m is bordered on two adjacent sides by a uniform path of width x m as shown in the diagram.

a) Express in terms of x each of the areas denoted by the letters A, B and C.

b) If the area of the path is $104\,m^2$ form an equation in x and solve it to find the width of the path.



The following problems lead to quadratic equations that factorise.

1. The sum of two numbers is 13 and the sum of their squares is 85. Find them.

2. The difference between two positive numbers is 2 and the sum of their squares is 20. Find the numbers.

3. The sum of the squares of two consecutive positive numbers is 61. Find two numbers.

4. One side of a rectangle is 4 cm longer than the other. Find the sides if the area of the rectangle is $45\,cm^2$.

5. The perimeter of a rectangle is 26 cm and its area is $40\,cm^2$. Find the sides.

6. Two positive whole numbers differ by 3, and the sum of their squares is 89. If the smaller number is x form an equation in x and solve it to find the numbers.

7. The sides of a right-angled triangle are x cm, $(x+7)$ cm and $(x+8)$ cm. Find them.

8. A rectangle is 6 cm longer than it is wide. If its area is the same as that of a square of side 4 cm find its dimensions.

9. The sides of a right-angled triangle are x cm, $(x-2)$ cm and $(x-4)$ cm. Find them.

10. The hypotenuse of a right-angled triangle is 10 cm. Find the other two sides if their sum is 14 cm.

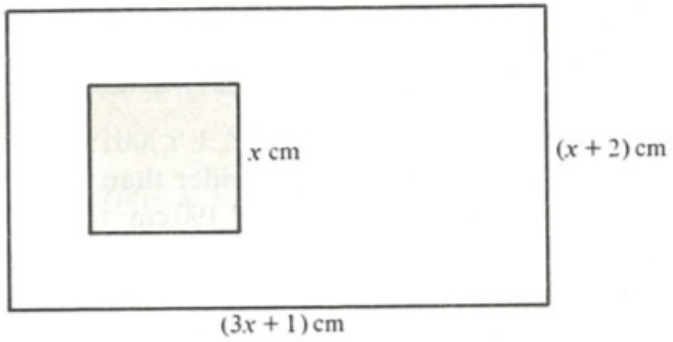
11. The product of two numbers is 84. If these numbers differ by 5, find them.

12. One number is 3 more than another. If their product is 88, find them.

13. The length of a rectangle is 5 cm more than its width. If the area of the rectangle is $36\,cm^2$ find its dimensions.

14. The base of a triangle is 5 cm more than its perpendicular height. If the area of the triangle is $42\,cm^2$ find
    a) the length of its base
    b) its perpendicular height.

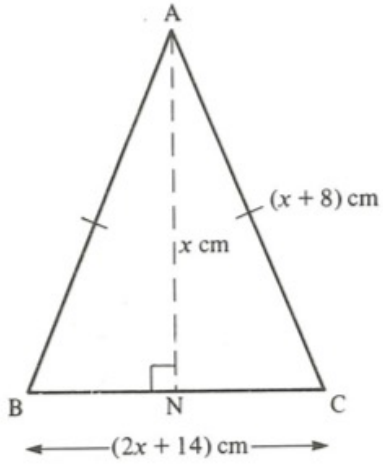
15.



A square of side x cm is removed from a rectangular piece of cardboard measuring $(3x + 1)$ cm by $(x + 2)$ cm. If the area of card remaining is $62\,\text{cm}^2$ form an equation in x and solve it to find the dimensions of the original card.

16.

N is the midpoint of the base BC of a triangle ABC. If AB = AC, AN = x cm, BC = $(2x + 14)$ cm and AC = $(x + 8)$ cm form an equation in x and solve it. Hence find the length of the base and height of the triangle ABC.



Part 5

Some quadratic equations can be solved by using common factors or 'the difference of two squares'. Solve these equations.

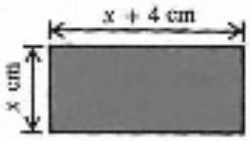
1	$x^2 - 25 = 0$	11	$x^2 - 8x = 0$	21	$x^2 - 81x = 0$
2	$x^2 - 4 = 0$	12	$x^2 + 8x = 0$	22	$x^2 + 81x = 0$
3	$x^2 - 5x = 0$	13	$x^2 + 7x = 0$	23	$x^2 - 1 = 0$
4	$x^2 - 7x = 0$	14	$x^2 - 36 = 0$	24	$x^2 - \frac{1}{4} = 0$
5	$x^2 - 9 = 0$	15	$x^2 + 2x = 0$	25	$4x^2 - 25 = 0$
6	$x^2 - 9x = 0$	16	$x^2 - 100 = 0$	26	$4x^2 - 49 = 0$
7	$x^2 - 4x = 0$	17	$x^2 + 3x = 0$	27	$4x^2 - 8x = 0$
8	$x^2 - 16 = 0$	18	$x^2 + x = 0$	28	$5x^2 + 15x = 0$
9	$x^2 - 16x = 0$	19	$x^2 - x = 0$	29	$9x^2 - 16 = 0$
10	$x^2 - 49 = 0$	20	$x^2 - 81 = 0$	30	$16x^2 - 49 = 0$

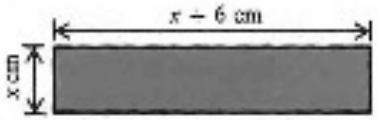
Part 6

1 When a number x is added to its square, the total is 20. Find the values of x.

2 When a number x is subtracted from its square, the answer is 30. Find the values of x.

3 This rectangle has a width of x cm and its length is 4 cm longer than this. If its area is 21 cm^2, find the value of x.

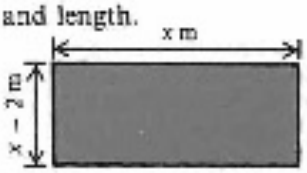


4 

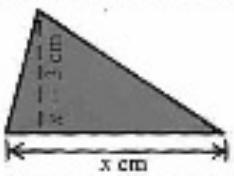
This rectangle has its short sides x cm long. Its long sides are 6 cm longer than this. If its area is 16 cm^2, find the value of x.

5 The area of a rectangle is 15 cm^2. Its width is x cm, and its length is 2 cm longer than its width.
- a Sketch the rectangle, marking on its width and length.
- b Calculate its width.

6 This room is x metres long, and its width is 2 metres less than its length. If its area is 24 m^2, find the value of x.

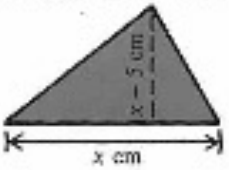


7 A carpet is x metres long, and its width is one metre less than its length. If its area is 12m^2, find the length of the carpet.

8 

This triangle has a base x cm long, and its height is 3 cm shorter than its base.
- a Write down an expression for its area in terms of x and $x - 3$.
- b If its area is 20 cm^2, find the value of x.

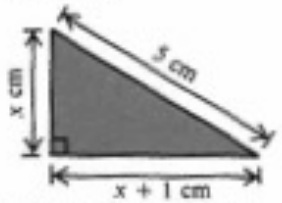
9 Another triangle has a base of x cm and a height 5 cm shorter than its base.
- a Write down an expression for its area.
- b If its area is 18 cm^2, find the value of x.



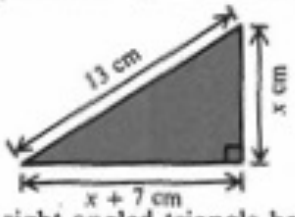
10 A triangle has a height of x cm, and its base is 2 cm longer than its height.
- a Sketch the triangle and mark on your sketch its height and base.
- b Write down an expression for the area of the triangle.
- c If its area is 12 cm^2, find its height.

11 Another triangle has a height of x cm, and its base is 11 cm shorter than its height.
- a Sketch the triangle and mark on your sketch its height and base.
- b Write down an expression for its area.
- c If its area is 13 cm^2, calculate both its height and its base.

12 This right-angled triangle has sides x cm, $x + 1$ cm and 5 cm long.
Use Pythagoras' Theorem to write down an equation in x, and solve it to find the value of x.



13 This right-angled triangle has sides of x cm, $x + 7$ cm and 13 cm.
Use Pythagoras' Theorem to write down an equation in x, and solve it to find the value of x.



14 A right-angled triangle has a hypotenuse 10 cm long and a base x cm long.
- a Sketch the triangle and mark on it these two lengths.
- b If the perimeter of the triangle is 24 cm, write down an expression for the height of the triangle in terms of x.
- c If the area of the triangle is 24 cm^2, write down an equation and solve it to find the value of x.

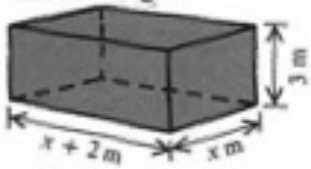
15 Another right-angled triangle has a hypotenuse 26 cm long and a base x cm long.
- a Sketch the triangle and mark on it the two lengths given above.
- b If the triangle has a perimeter of 60 cm, write down an expression for the height of the triangle.
- c The area of the triangle is 120 cm^2. Write an equation for x, and solve it to find the base and height.

16 This rectangle has a perimeter of 30 cm and a width of x cm.
- a Write down an expression for its length in terms of x.
- b If its area is 50 cm^2, construct an equation in x and solve it to find the value of x.

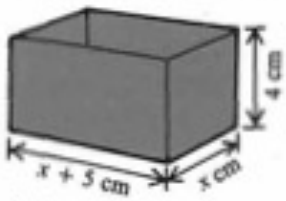
17 Another rectangle has a perimeter of 22 cm and a width of x cm.
- a Write down an expression for the length of the rectangle in terms of x.
- b If the area of the rectangle is 28 cm^2, construct an equation and then solve it to find the width and the length of the rectangle.

18 A rectangular carpet has a perimeter of 26 metres.
- a If its width is x m, write down its length in terms of x.
- b The area of the carpet is 40 m^2. Calculate its width and its length.

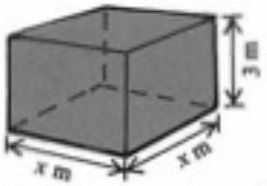
19 This room is 3 metres high and x metres wide.
Its length is 2 metres more than its width.
If its volume is 72 m^3, calculate the value of x.



20 This cardboard box is 4 cm tall and x cm wide.
It is 5 cm longer than it is wide.
If its volume is 96 cm^3, calculate the value of x.



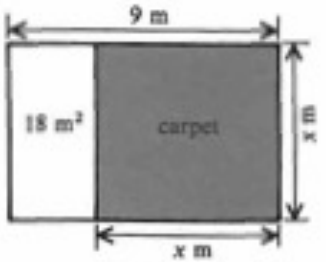
21 This room has a square floor of side x metres. Its height is 3 metres.



Write down an expression for
- a the area of its floor
- b the area of one side wall.
- c If the total area of its floor, ceiling and four walls is 80 m^2, form an equation and solve it to find the value of x.

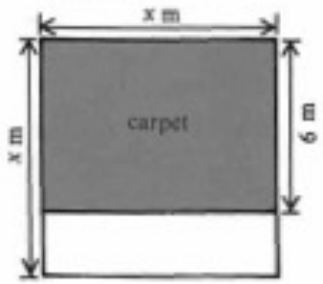
22 Another room is as high as it is wide. Its length is 8 metres.
- a Sketch the room and, taking x metres as its width, write the dimensions on the sketch.
- b Write the area of its floor in terms of x.
- c Write the area of each of its walls in terms of x.
- d If the total area of its four walls is 96 m^2, form an equation and solve it to find the value of x.
- e What is the volume of the room in m^3?

23 A rectangular floor is 9 metres by x metres, and a square carpet of side x metres is placed as shown over part of the floor.



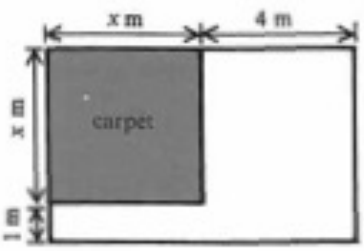
- a Write an expression for the area of the floor.
- b Write an expression for the area of the carpet.
- c If the area of floor **not** covered by the carpet is 18 m^2, form an equation and solve it to find the value of x.

24



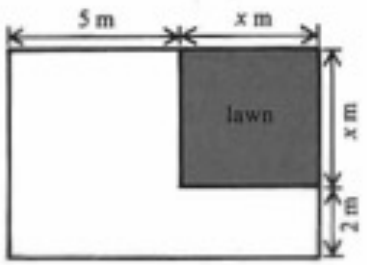
A square floor of side x metres is partly covered by a rectangular carpet 6 metres wide and x metres long as shown in the diagram.
- a Write an expression for the area of the whole floor.
- b Write an expression for the area of the carpet.
- c If the area of the floor **not** covered with carpet is 16 m^2, construct an equation and solve it to find the value of x.

25 A square carpet of side x metres is laid in the corner of a room as shown, leaving gaps of 1 metre and 4 metres along the edges of the room.



- a Write expressions for the width and the length of the room.
- b If the area of the whole floor is 28 m^2, construct an equation in x and solve it to find the value of x.
- c What are the width and the length of the room?

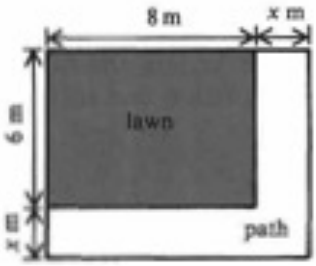
26 A rectangular garden has a square lawn in one corner, leaving an L-shaped flower bed with 2 metre and 5 metre lengths as shown.



a Write expressions for the length and width of the whole garden.

b If the area of the whole garden is 54 m^2, form an equation and solve it for x.

c What is the perimeter of the garden?



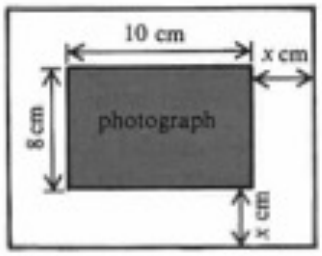
27 This rectangular garden has a lawn 8 m by 6 m with a path x metres wide along two sides of the lawn.

a Write expressions for the total width and total length of the garden.

b If the garden has an area of 80 m^2, write an equation involving x, and solve it.

c What is the width and the length of the whole garden?

28 A photograph 8 cm by 10 cm has a border x cm wide all the way round it.



a Write expressions for the total width and the total length of the display.

b If the total area is 168 cm^2, write an equation and solve it to find the value of x.

c Calculate the area of the border.

29 A rectangular garden 16 m by 10 m has a central rectangular lawn surrounded by a path.

a Draw a diagram of the garden and label the lawn and the path.

b If the path is x metres wide, write expressions for the width and the length of the lawn.

c If the area of the lawn is 112 m^2, form an equation and solve it to find the value of x.

d Calculate the area of the path.

30 A square has sides x cm long. A larger square has sides 2 cm longer than this. Write, in terms of x,

a the length of one side of the larger square

b the area of the larger square

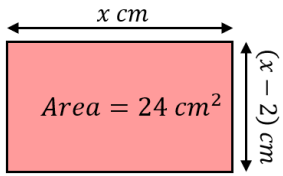
c the area of the smaller square.

d If the total area of both squares is 34 cm^2, find the value of x.

Purposeful Practice

Solving Quadratic Equations Problems

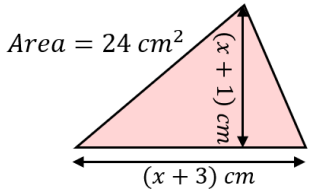
(a)



(i) Show that $x^2 - 2x - 24 = 0$

(ii) Hence find the length and width of the rectangle.

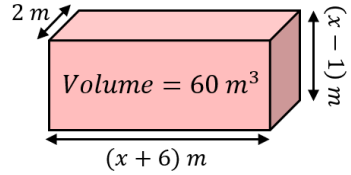
(b)



(i) Show that $x^2 + 4x - 45 = 0$

(ii) Hence find the width and height of the triangle.

(c)

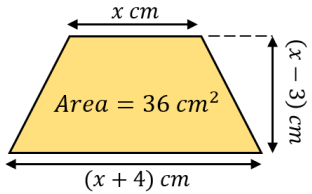


(i) Show that $x^2 + 5x - 36 = 0$

(ii) Hence find the dimensions of the cuboid.

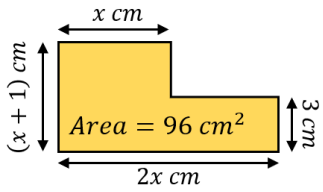
(d)

Find the value of x and hence the dimensions of the trapezium. Show clear algebraic working.



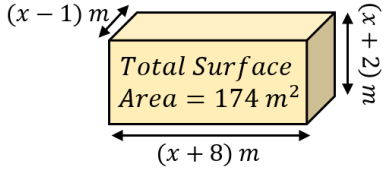
(e)

Find the value of x and hence the dimensions of the compound shape. Show clear algebraic working.



(f)

Find the value of x and hence the dimensions of the cuboid. Show clear algebraic working.

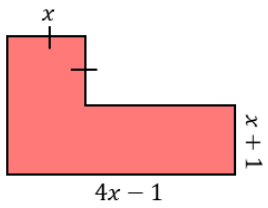


Purposeful Practice

Solving Harder Quadratic Equations in Context

(a)

The shape shown has an area of $13\ cm^2$. All lengths on the diagram are in cm.

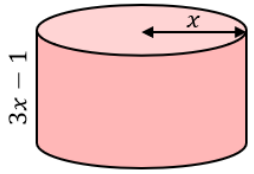


(i) Show that $5x^2 + 3x - 14 = 0$

(ii) Hence, find the value of x and the dimensions of the shape.

(b)

The cylinder shown has a total surface area of $78\pi\ cm^2$. All lengths on the diagram are in centimetres.

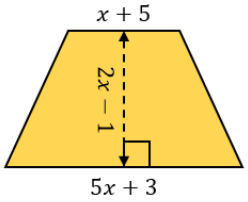


(i) Show that $4x^2 - x - 39 = 0$

(ii) Hence, find the value of x and the height of the cylinder.

(c)

The trapezium shown has an area of $30\ cm^2$. All lengths on the diagram are in centimetres.

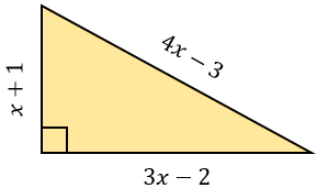


(i) Show that $6x^2 + 5x - 34 = 0$

(ii) Hence, find the value of x and the dimensions of the trapezium.

(d)

A right-angled triangle has sides of lengths $(x+1)\ cm, (3x-2)\ cm$ and $(4x-3)\ cm$ as shown.

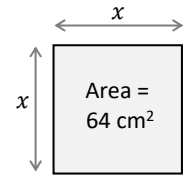


(i) Show that $3x^2 - 7x + 2 = 0$

(ii) Hence find the value of x and the length of the hypotenuse.

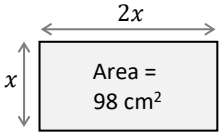
Purposeful Practice

Forming & Solving Quadratic Equations



Form an equation in terms of x.
Solve to find possible values of x.

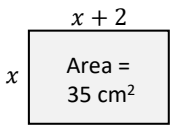
A rectangle has a width of x.
The rectangle's length is twice its width.
The area of the rectangle is 98 cm².
Form & solve an equation to find x.



Can we have a rectangle with
negative side lengths?

A 432 m² field has a length triple its width.
Form then solve an equation to find x
& calculate the length of the field.

The length of this rectangle is
2 cm longer than its width.
We can form & solve an equation to find x.



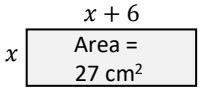
$$x(x+2) = 35$$
$$x^2 + 2x = 35$$
$$x^2 + 2x - 35 = 0$$

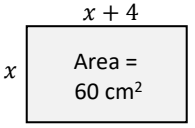
Factorise the expression into two brackets to
find 2 possible values for x.

Which value of x makes sense in real-life?

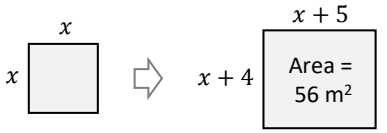
Form & solve equations to
find the dimensions of these rectangles.

Which value for x would we use in real life?





A square is extended by 5 metres on one side and
4 metres on another to make a rectangle with
an area of 56 m².

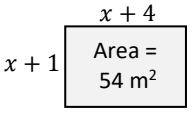


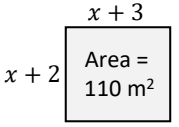
$$(x+4)(x+5) = 56$$

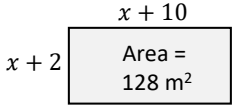
Expand this, then create
an equation you can factorise & solve.

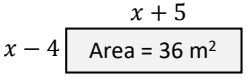
If we think about the original square,
which value for x *makes sense*?

Find the values of x for these rectangles.
Which value makes sense in real-life?

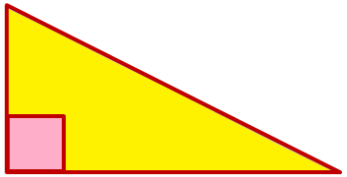








work out the lengths of the sides of the right angled triangle if they are:



(1) n + 2 , n + 4 and n + 6

(2) x + 4 , 5x – 6 and 5x – 4

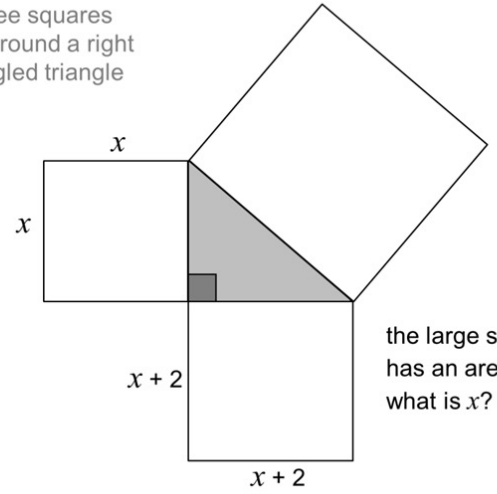
(3) n + 2 , 3n + 3 and 5n – 2

(4) x + 2 , 5x – 1 and 4x + 5

re-do questions (1) and (2) using the difference of two squares (D.O.T.S)
$a^2 - b^2 = (a + b)(a - b)$

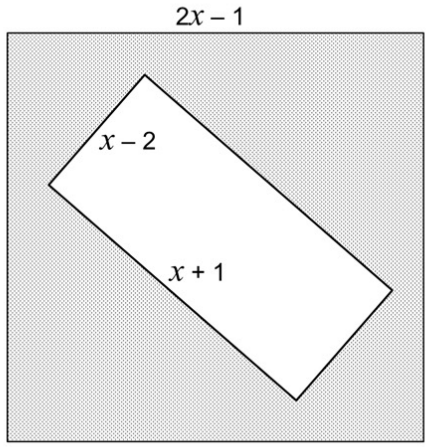
Purposeful Practice

(1) three squares surround a right angled triangle



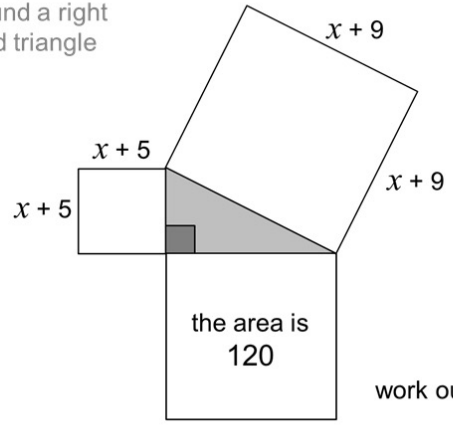
the large square has an area of 34
what is x?

(2)



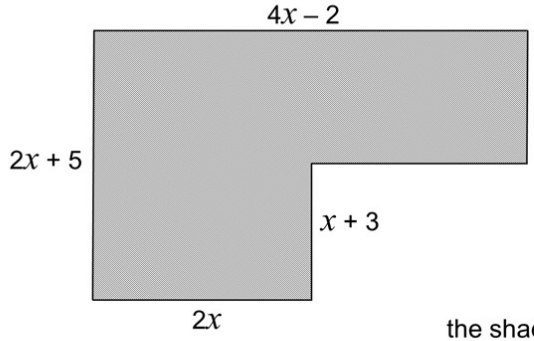
the shaded area is 39
what is x?

(3) three squares surround a right angled triangle



work out what x is

(4)



the shaded area is 206
what is x?

Question 1: Solve each of the equations below

(a) $(x - 1)(x - 3) = 0$ (b) $(y - 4)(y - 9) = 0$ (c) $(m + 1)(m + 6) = 0$

(d) $(x - 3)(x + 2) = 0$ (e) $(t + 7)(t - 3) = 0$ (f) $(k - 10)(k + 9) = 0$

(g) $(w + 5)(w + 11) = 0$ (h) $(y - 8)(y - 2) = 0$ (i) $(x + 3)(x - 9) = 0$

Question 2: Solve each of the equations below

(a) $x^2 + 6x + 8 = 0$ (b) $x^2 + 7x + 12 = 0$ (c) $y^2 + 7y + 10 = 0$

(d) $y^2 + 3y - 4 = 0$ (e) $x^2 - 2x - 8 = 0$ (f) $m^2 - 7m + 12 = 0$

(g) $y^2 - 10y + 25 = 0$ (h) $y^2 - 4y - 45 = 0$ (i) $x^2 - x - 56 = 0$

(j) $y^2 + 10y + 24 = 0$ (k) $x^2 + 9x + 18 = 0$ (l) $x^2 + 23x + 22 = 0$

(m) $y^2 - 13y + 22 = 0$ (n) $x^2 + x - 12 = 0$ (o) $m^2 - 6m - 27 = 0$

(p) $x^2 - 11x + 18 = 0$ (q) $y^2 - 14y + 48 = 0$ (r) $x^2 - 15x + 56 = 0$

(s) $m^2 - m - 56 = 0$ (t) $y^2 + 22y + 96 = 0$ (u) $k^2 - 18k - 88 = 0$

(v) $x^2 - 38x + 72 = 0$ (w) $x^2 + 14x - 51 = 0$ (x) $y^2 + 32y + 240 = 0$

(y) $g^2 - 12g - 64 = 0$ (z) $y^2 + 22y + 121 = 0$

Question 3: Solve each of the equations below

(a) $(y - 5)(y + 5) = 0$ (b) $(x + 2)(x - 2) = 0$ (c) $(m - 9)(m + 9) = 0$

Question 4: Solve each of the equations below

(a) $x^2 - 9 = 0$ (b) $y^2 - 100 = 0$ (c) $w^2 - 1 = 0$

(d) $k^2 - 144 = 0$ (e) $x^2 - 64 = 0$ (f) $c^2 - 0.25 = 0$

Question 5: Solve each of the equations below

(a) $x^2 + 2x = -1$ (b) $y^2 + 8y + 10 = 3$ (c) $x^2 = 7x - 12$

(d) $y^2 + 6y + 15 = 3 - 7y$ (e) $x^2 - x - 8 = 2x + 2$ (f) $2x^2 - 14x + 49 = x^2$

(g) $-2x^2 + x - 1 = -x^2 - 5x + 8$ (h) $11x^2 - 105 = 10x^2 + x + 105$

Question 6: Solve each of the equations below

(a) $\dfrac{3}{x-4} = x - 2$ (b) $\dfrac{x+3}{4} = \dfrac{3}{x-1}$ (c) $\dfrac{45}{x^2} - \dfrac{4}{x} - 1 = 0$

Apply

Question 1: Alex is w years old.
His sister Claudia is three years younger than Alex.
The product of their ages is 180.

(a) Set up an equation to represent this information.
(b) Solve your equation from (a) to find Alex's age.

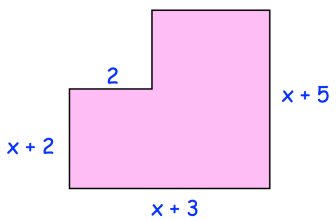
Question 2: A rectangular field is 10m longer than wide.
The area of the field is 2000m^2.
Find the perimeter of the field.

Question 3: A triangle has an area of 85cm^2.
The height of the triangle is 7cm longer than the base of the triangle.

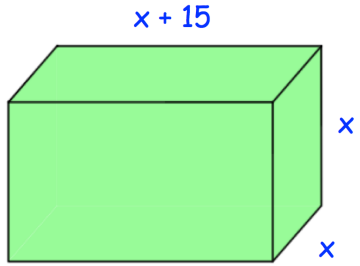
Find the lengths of the height and the base of the triangle.

Question 4: Two positive numbers, which have a difference of 3, are squared.
The difference in the results is 81.
Find the two numbers.

Question 5: The area of the shape is 74cm^2.
Find the perimeter of the shape.



Question 6: The surface area of this cuboid is $3600cm^2$



(a) Show $x^2 + 10x - 600 = 0$

(b) Find x

(c) Find the volume of the cuboid.

Question 7: $(2^{x-8})^{x-10} = 8$

Find the possible values of x

Question 8: There are x apples in a crate.
4 of the apples are bad.

Joanne chooses two apples from the crate, without replacement.
The probability she selects two bad apples is $^1/_{11}$

(a) Prove $x^2 - x - 132 = 0$
(b) Find x, the number of apples in the crate.

Question 1: Solve each of the equations below

(a) $(2y - 1)(y - 2) = 0$ (b) $(4x - 3)(x + 1) = 0$ (c) $(2y + 3)(2y - 5) = 0$

(d) $(5m - 4)(m + 2) = 0$ (e) $(h + 9)(3h - 1) = 0$ (f) $(2x - 3)(3x + 7) = 0$

(g) $(7y + 4)(2y + 1) = 0$ (h) $(8w - 5)(w - 11) = 0$ (i) $(5x + 6)(3x - 4) = 0$

Question 2: Solve each of the equations below

(a) $2x^2 + 5x + 2 = 0$ (b) $2x^2 + 7x + 5 = 0$ (c) $5x^2 + 7x + 2 = 0$

(d) $2x^2 + 17x + 36 = 0$ (e) $5x^2 + 23x + 12 = 0$ (f) $3x^2 + 7x + 2 = 0$

(g) $3x^2 + 4x + 1 = 0$ (h) $2x^2 + 7x - 4 = 0$ (i) $2x^2 - x - 6 = 0$

(j) $7x^2 + 23x + 6 = 0$ (k) $3x^2 - x - 2 = 0$ (l) $5x^2 - 16x + 3 = 0$

(m) $3x^2 + x - 4 = 0$ (n) $2x^2 - 13x + 15 = 0$ (o) $7x^2 - 22x + 16 = 0$

(p) $2x^2 + 15x - 38 = 0$ (q) $5x^2 - 31x + 30 = 0$ (r) $3x^2 - 10x - 48 = 0$

Question 3: Solve each of the equations below

(a) $4x^2 + 8x + 3 = 0$ (b) $4x^2 + 12x - 7 = 0$ (c) $4x^2 - 11x + 6 = 0$

(d) $6x^2 + 31x + 5 = 0$ (e) $4x^2 - 16x - 9 = 0$ (f) $8x^2 - 10x - 3 = 0$

(g) $10x^2 - 11x + 1 = 0$ (h) $6x^2 + 31x + 18 = 0$ (i) $9x^2 - 6x - 8 = 0$

(j) $4x^2 - 4x - 35 = 0$ (k) $12x^2 + 25x + 12 = 0$ (l) $14x^2 + 23x - 10 = 0$

(m) $6x^2 + 13x - 5 = 0$ (n) $6x^2 - 11x - 7 = 0$ (o) $16x^2 - 30x + 9 = 0$

Question 4: Solve each of the equations below

(a) $4x^2 - 9 = 0$ (b) $4x^2 - 121 = 0$ (c) $16x^2 - 25 = 0$ (d) $36x^2 - 1 = 0$

(e) $9x^2 - 196 = 0$ (f) $100x^2 - 49 = 0$ (g) $4x^2 - 900 = 0$ (h) $64x^2 - 169 = 0$

Question 5: Solve each of the equations below

(a) $2x^2 + 5x = 0$ (b) $2x^2 - 9x = 0$ (c) $3x^2 + x = 0$ (d) $4x^2 + 15x = 0$

(e) $5x^2 - x = 0$ (f) $6x + 3x^2 = 0$ (g) $15x - 2x^2 = 0$ (h) $16x^2 - 20x = 0$

Question 6: Solve each of the equations below

(a) $5x^2 - 9x + 6 = 2$ (b) $2m^2 + 6m + 2 = m + 5$ (c) $10x^2 + 26x - 3 = x^2$

(d) $3x^2 + 9x + 8 = x^2 + 2x + 3$ (e) $6y^2 + 4 = 13 - 3y + 4y^2$ (f) $3x^2 + x + 2 = 3(x + 1)$

(g) $(4x + 3)(x + 2) = 3(x + 1)$

Question 7: Solve each of the equations below

(a) $\dfrac{3}{2x-1} = x - 3$ (b) $\dfrac{2x-1}{4} = \dfrac{1}{2x-1}$ (c) $\dfrac{2}{x^2} + \dfrac{13}{x} + 6 = 0$

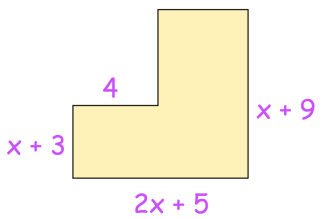
(d) $\dfrac{3}{x^2} - \dfrac{5}{x} - 12 = 0$

Apply

Question 1: A rectangular field has a width of x metres.
The length of the field is 25 metres greater than twice the width of the field.
The area of the field is $450m^2$
Work out the length of the field.

Question 2: The nth term of a sequence is $3n^2 - n + 10$ where n is a positive integer.
Which term in the sequence is equal to 54?

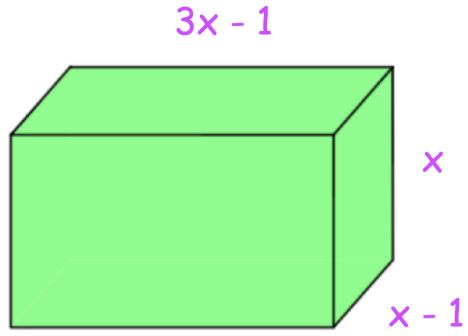
Question 3: The area of this shape is $75cm^2$
Find the perimeter of the shape.



Purposeful Practice

Question 4: The surface area of this cuboid is $92cm^2$

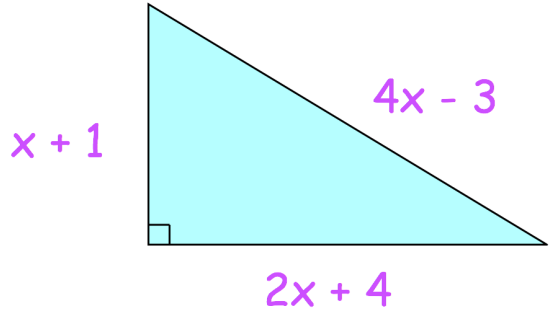
Calculate the volume of the cuboid.



Question 5: Shown is a right angled triangle.

(a) Show that $11x^2 - 42x - 8 = 0$

(b) Find the value of x



Question 6: The numbers 2, 3x and (x + 16) are the first three terms of a geometric sequence.

(a) Find the possible values of x
(b) Find the possible values of the 4th term of the geometric sequence.

Solve these equations

1. $\sqrt{x} = 9$
2. $\sqrt{x} + 5 = 9$
3. $\sqrt{x + 5} = 9$
4. $\sqrt{x} - 1 = 9$
5. $\sqrt{x - 1} = 9$
6. $2\sqrt{x} = 8$
7. $\sqrt{2x} = 8$
8. $2\sqrt{x} + 2 = 8$
9. $2\sqrt{x + 2} = 8$
10. $\sqrt{2x + 2} = 8$

Intelligent Practice

Work out the value of $b^2 - 4ac$ when:

1) $a = 3, b = 7, c = 2$

2) $a = 3, b = -7, c = 2$

3) $a = 6, b = -14, c = 4$

4) $a = 6, b = -14, c = -4$

5) $a = -4, b = -14, c = 6$

6) $a = -4, b = -14, c = -6$

7) $a = -4, b = -5, c = -6$

8) $a = -4, b = -5, c = -1$

9) $a = -6, b = -5, c = -1$

10) $a = 1, b = -5, c = 6$

11) $a = 1, b = -6, c = 5$

12) $a = 1, b = -6, c = 9$

13) $a = 1, b = 0, c = 9$

14) $a = 1, b = -6, c = 0$

Use the formula to solve the following quadratic equations.

1. $x^2+6x+3=0$
2. $x^2+7x+4=0$
3. $x^2+5x+5=0$
4. $x^2+7x-2=0$
5. $x^2+4x-3=0$
6. $x^2+9x+12=0$
7. $x^2+8x+13=0$
8. $x^2+10x-15=0$
9. $x^2+6x-6=0$
10. $x^2+9x-1=0$
11. $x^2+3x-5=0$
12. $x^2+4x-7=0$
13. $x^2-4x+2=0$
14. $x^2-7x+3=0$
15. $x^2-6x+6=0$
16. $x^2-4x-3=0$
17. $x^2-5x-5=0$
18. $x^2-5x+2=0$
19. $x^2-3x+1=0$
20. $x^2-7x-3=0$
21. $x^2-9x-2=0$
22. $x^2-4x-9=0$
23. $x^2+7x-2=0$
24. $x^2+8x+5=0$

Fluency Practice

25. $2x^2 + 7x + 2 = 0$

26. $2x^2 + 7x + 4 = 0$

27. $3x^2 + 7x + 3 = 0$

28. $4x^2 + 7x + 1 = 0$

29. $5x^2 + 9x + 2 = 0$

30. $2x^2 - 7x + 4 = 0$

31. $4x^2 - 7x + 1 = 0$

32. $5x^2 - 9x + 2 = 0$

33. $3x^2 + 5x - 3 = 0$

34. $3x^2 + 9x - 1 = 0$

Fluency Practice

Question 1: Solve the following equations using the quadratic formula.
Give your answers to 1 decimal place.

(a) $x^2 + 5x + 1 = 0$

(b) $2x^2 + 7x + 2 = 0$

(c) $4x^2 + 8x + 3 = 0$

(d) $x^2 + 2x - 4 = 0$

(e) $3x^2 + 4x - 5 = 0$

(f) $2x^2 + 5x - 10 = 0$

(g) $x^2 - 4x + 2 = 0$

(h) $7x^2 - 6x + 1 = 0$

(i) $3x^2 - 10x + 4 = 0$

(j) $x^2 - x - 11 = 0$

(k) $x^2 - 6x - 20 = 0$

(l) $2x^2 - x - 9 = 0$

(m) $9x^2 - 12x + 2 = 0$

(n) $4x^2 + 4x + 1 = 0$

(o) $8x^2 - 8x - 9 = 0$

(p) $2x^2 + 3x - 100 = 0$

(q) $3x^2 - 23x - 67 = 0$

(r) $2x^2 + 16x + 1 = 0$

Fluency Practice

1. $2x^2 = 8x + 11$
2. $4x^2 = 8x + 3$
3. $3x^2 = 3 - 5x$
4. $5x^2 = x + 3$
5. $4x^2 + 2 = 7x$
6. $3x^2 = 12x + 2$
7. $2x^2 = 3x + 1$
8. $4x^2 = 5 - 3x$
9. $3x^2 + 2 = 9x$
10. $6x^2 - 9x = 4$
11. $2x^2 = 5x + 5$
12. $3x^2 + 4x = 1$
13. $4x^2 = 4x + 1$
14. $3x^2 + 7x = 2$
15. $5x^2 = 5x - 1$
16. $8x^2 = x + 1$

Fluency Practice

Question 2: Solve the following equations using the quadratic formula.
Give your answers to 2 decimal places.

(a) $x^2 + 7x = 20$

(b) $2x^2 = 9x + 40$

(c) $3x^2 = 10 - 2x$

(d) $x^2 - 8 = x$

(e) $7x = 13 - x^2$

(f) $4x^2 - 9 = 2x^2 + 4x$

Fluency Practice

In this exercise try the method of factorising first. If factors cannot be found use the formula.

1. $2x^2 + 3x - 2 = 0$

2. $3x^2 + 6x + 2 = 0$

3. $6x^2 + 7x + 2 = 0$

4. $2x^2 + 3x - 3 = 0$

5. $3x^2 - 8x + 2 = 0$

6. $3x^2 - 8x - 3 = 0$

7. $2x^2 - 3x - 3 = 0$

8. $8x^2 + 10x - 3 = 0$

9. $6x^2 + 7x - 2 = 0$

10. $4x^2 - 3x - 2 = 0$

11. $7x^2 + 8x - 2 = 0$

12. $5x^2 - 3x - 1 = 0$

13. $3x^2 = 7x - 2$

14. $11x^2 + 12x + 3 = 0$

15. $20x^2 = 3 - 11x$

16. $3x^2 - 14x + 15 = 0$

17. $5x^2 + 8x + 2 = 0$

18. $2x^2 = 7x + 3$

19. $2x^2 + 9x = 5$

20. $6x^2 = 5x + 2$

Solve these quadratic equations, giving your answers to 2 decimal places.

(a) $x^2 + 5x + 1 = 0$

(b) $x^2 - 5x + 1 = 0$

(c) $2x^2 + 5x + 1 = 0$

(d) $2x^2 - 7x - 6 = 0$

(e) $4x^2 + x - 6 = 0$

(f) $4x^2 + 9x - 2 = 0$

Solve these quadratic equations, giving your answers to 2 decimal places.

(a) $2x^2 = 5x + 6$

(b) $x^2 + 7x = 2$

(c) $5x^2 = 11x + 3$

(d) $2x^2 = 3 - 5x$

Solve these quadratic equations, leaving your answers in surd form.

(a) $4x^2 - 9x + 4 = 0$

(b) $7x^2 + 3x = 2$

(c) $x^2 + 3x - 6 = 0$

(d) $7x^2 + 12x + 2 = 0$

(a) The answers to a quadratic equation are $x = \frac{3 \pm \sqrt{37}}{2}$.

What is the quadratic equation?

(b) Solve the equation

$$x + \frac{3}{x} = 7$$

Give your answers correct to 2 decimal places.

(c) What is special about the solutions to the equation

$$4x^2 - 4x + 1 = 0$$

Purposeful Practice

3 Amir is trying to solve the equation $2x^2 - 7x - 5 = 0$, his working is shown below.
Identify the mistakes that Amir has made and write the correct solution in the right-hand box.

Amir's Working	Amir's Mistakes – There are a few!!	Correct Working
$2x^2-7x-5=0$ $a=2,\ b=-7,\ c=5$ $x=\frac{-7\pm\sqrt{7^2-4(2)(5)}}{2}$ $x=\frac{-7\pm\sqrt{9}}{2}$ $x=\frac{-7\pm3}{2}$ **$x=-5$ and $x=-2$**		

solving quadratic equations

apply the quadratic formula to obtain two solutions:

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(1) $6x^2 - 5kx + k^2 = 0$

(2) $2x^2 + tx - t^2 = 0$

(3) $6x^2 + mx - m^2 = 0$

(4) $6x^2 + 11wx - 2w^2 = 0$

(5) $abx^2 + (a^2 + b^2)x + ab = 0$

(6) $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

based on CBSE (India) class 10 problems

Purposeful Practice

The following questions may lead to quadratic equations that do not factorise. Always check whether a quadratic equation will factorise before using the formula. If an answer is not exact give it correct to 3 s.f.

1. The sum of two numbers is 10 and the sum of their squares is 80. Find them.

2. The sum of two numbers is 9 and the difference between their squares is 60. Find them.

3. Find a number such that the sum of the number and its reciprocal is 20. In this case give the answers correct to 2 decimal places.

4. One side of a rectangle is 3 cm longer than another. Find the sides if the area of the rectangle is $20\,\text{cm}^2$.

5. Find the length of the hypotenuse of a right-angled triangle whose sides are x cm, $(x+1)$ cm and $(x+3)$ cm.

6. The parallel sides of a trapezium are $(x-2)$ cm and $(x+4)$ cm long. If the distance between the parallel sides is x cm and the area of the trapezium is $42\,\text{cm}^2$ find its dimensions.

7. A rectangular block is 2 cm wider than it is high and twice as long as it is wide. If its total surface area is $190\,\text{cm}^2$ find its dimensions.

8. Sally is x years old. Her mother's age is (x^2-4) years and her father is 6 years older than her mother. If the combined age of all three is 76 years form an equation in x and solve it. How old is her father?

Question 1: Solve the following equations using the quadratic formula.
Give your answers to 1 decimal place.

(a) $x^2 + 5x + 1 = 0$ (b) $2x^2 + 7x + 2 = 0$ (c) $4x^2 + 8x + 3 = 0$

(d) $x^2 + 2x - 4 = 0$ (e) $3x^2 + 4x - 5 = 0$ (f) $2x^2 + 5x - 10 = 0$

(g) $x^2 - 4x + 2 = 0$ (h) $7x^2 - 6x + 1 = 0$ (i) $3x^2 - 10x + 4 = 0$

(j) $x^2 - x - 11 = 0$ (k) $x^2 - 6x - 20 = 0$ (l) $2x^2 - x - 9 = 0$

(m) $9x^2 - 12x + 2 = 0$ (n) $4x^2 + 4x + 1 = 0$ (o) $8x^2 - 8x - 9 = 0$

(p) $2x^2 + 3x - 100 = 0$ (q) $3x^2 - 23x - 67 = 0$ (r) $2x^2 + 16x + 1 = 0$

Question 2: Solve the following equations using the quadratic formula.
Give your answers to 2 decimal places.

(a) $x^2 + 7x = 20$ (b) $2x^2 = 9x + 40$ (c) $3x^2 = 10 - 2x$

(d) $x^2 - 8 = x$ (e) $7x = 13 - x^2$ (f) $4x^2 - 9 = 2x^2 + 4x$

Apply

Question 1: A rectangular garden is 5m longer than it is wide.
The area of the garden is $600m^2$
Calculate the width and length of the garden.

Question 2: A rectangular field is 30m longer than it is wide. The area of the field is $5000m^2$
Calculate the width and length of the field.

Question 3: A rectangular playground is 10m longer than it is wide.
The area of the playground is $1400m^2$
Calculate the width and length of the playground.

Question 4: A field has width x and length 2x + 1.
The area of the field is $600m^2$
Find the width and length of the field.

Question 5: James is solving a quadratic equation in the form $ax^2 + bx + c = 0$
He has got to this point in his working out.

$$x = \frac{-6 \pm \sqrt{12}}{4}$$

Find the values of a, b and c for the equation James is solving.

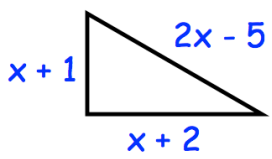
Question 6: Hannah is solving a quadratic equation in the form $ax^2 + bx + c = 0$
She has got to this point in her working out.

$$x = \frac{3 \pm \sqrt{29}}{2}$$

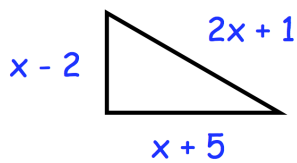
Find the values of a, b and c for the equation Hannah is solving.

Question 7: Below are three right angled triangles.
For each, find the possible values for x.

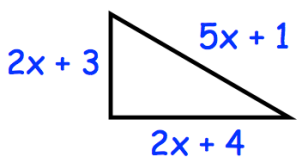
(a)



(b)

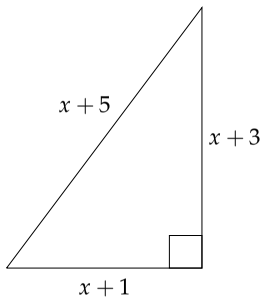


(c)

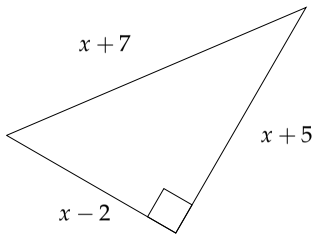


Find the value(s) of x for each question. **All of the quadratics can be factorised**.

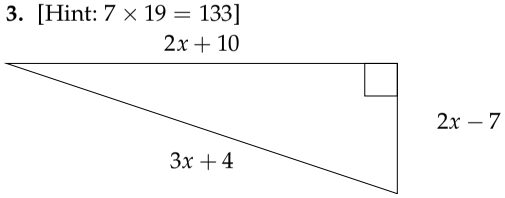
1.



2.



3. [Hint: $7 \times 19 = 133$]



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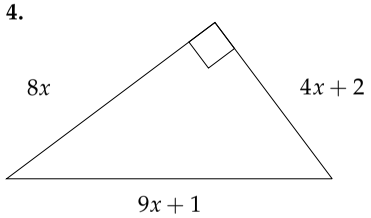

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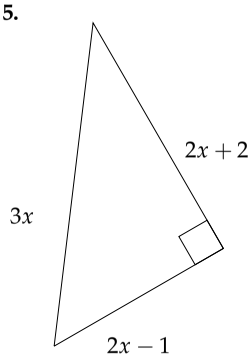

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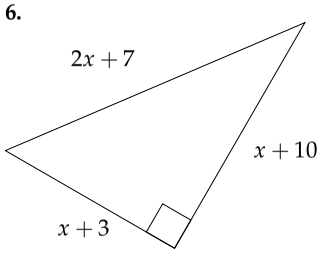

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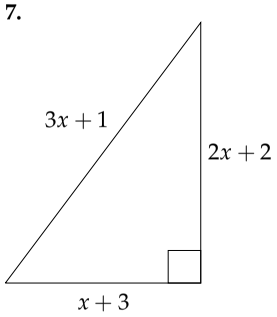
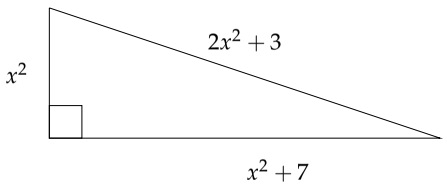
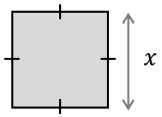


8.



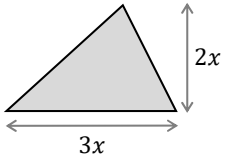
Purposeful Practice

A



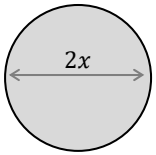
Area = 80 cm²

B



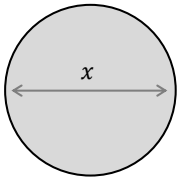
Area = 33 cm²

C



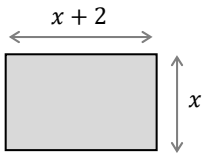
Area = 60 cm²

D



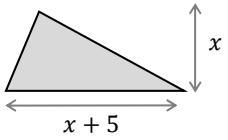
Area = 90 cm²

E



Area = 15 cm²

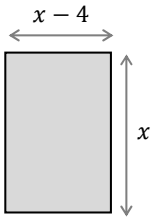
F



Area = 18 cm²

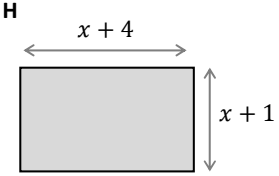
Forming & Solving Quadratic Equations

G



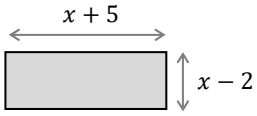
Area = 32 cm²

H



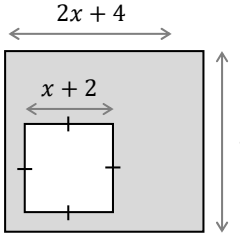
Area = 54 cm²

I



Area = 18 cm²

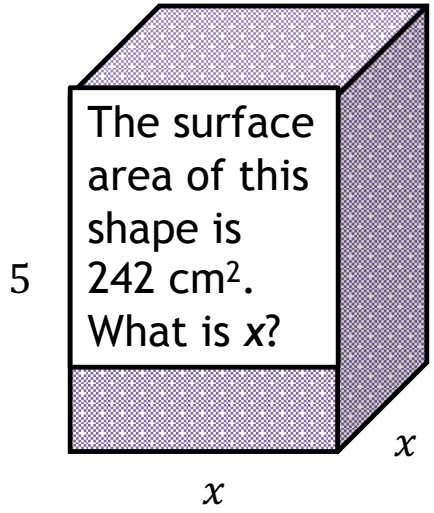
J



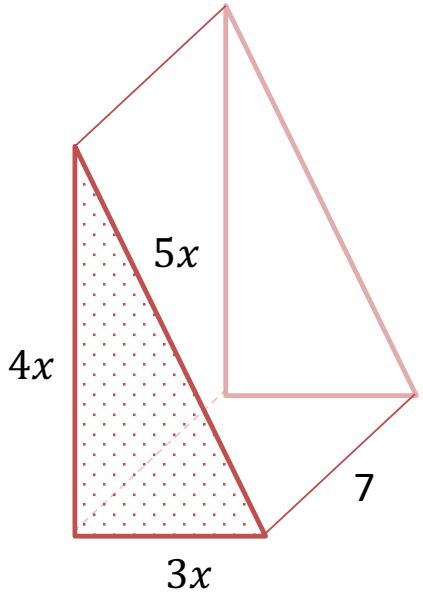
Area = 65 cm²

Purposeful Practice

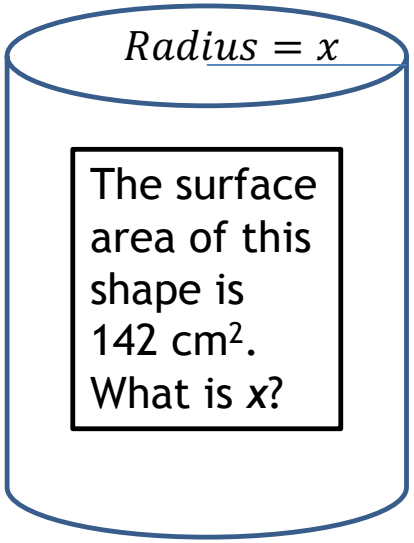
Find to two decimal places. Diagrams not even close to being drawn to scale.



The surface area of this shape is 242 cm^2. What is x?



The surface area of this shape is 100 cm^2. What is x?



The surface area of this shape is 142 cm^2. What is x?

Fluency Practice

Solve each of the following quadratic equations using an appropriate method. For non-integer solutions, give answers to 3sf.				
$x^2 - 9x + 14 = 0$	$2x^2 + 3x + 1 = 0$	$x^2 + 2x - 8 = 0$	$2x^2 + 6x + 1 = 0$	$x^2 + 10x + 21 = 0$
$x^2 - x - 6 = 0$	$x^2 - 9x = 0$	$x^2 - 7x + 2 = 0$	$x^2 + 2x + 1 = 0$	$x^2 - 11x - 60 = 0$
$x^2 - 9 = 0$	$2x^2 - 9x = 0$	$5x^2 - 2x - 80 = 0$	$5x^2 - 80 = 0$	$5x^2 + 42x - 80 = 0$
Find a quadratic equation that satisfies each of the following conditions. Can you generalise?				
There are two integer solutions, one positive and one negative.	There are two solutions, but the equation cannot be solved by factorising.	There are two solutions and one of them is zero.	There are two fractional solutions, where one is twice the other.	The equation cannot be solved by any method (I know so far…)

Fluency Practice

(a)	(b)	(c)	(d)
Solve $x^2 = 9$	Solve $x^2 - 7x + 10 = 0$	Solve $x^2 - 5x + 6 = 0$	Solve $x^2 + 8x + 12 = 0$
(e)	**(f)**	**(g)**	**(h)**
Solve $x^2 + 2x - 8 = 0$	Solve $x^2 + 10x + 21 = 0$	Solve $x^2 - 3x - 18 = 0$	Solve $x^2 - 1 = x + 5$
(i)	**(j)**	**(k)**	**(l)**
Solve $3x^2 - 7x + 2 = 0$	Solve $x^2 + 5x + 2 = 0$, giving your solutions to 3 significant figures.	Solve $x^2 + 3x - 8 = 0$, giving your solutions to 3 significant figures.	Solve $3x^2 + 2x - 9 = 0$, giving your solutions in surd form.

Fluency Practice

A1 Solve $3x^2 + 8x + 2 = 0$ Give your answers correct to 3 significant figures.	**A2** Solve $2x^2 + 5x - 4 = 0$ Give your answers correct to 3 significant figures.	**A3** Solve $4x^2 - 7x + 1 = 0$ Give your answers correct to 3 significant figures.	**A4** Solve $2x^2 - 4x - 9 = 0$ Give your answers correct to 3 significant figures.
B1 Solve $5x^2 + 8x - 1 = 4$ Give your answers correct to 3 significant figures.	**B2** Solve $5x^2 + 7x + 3 = x^2$ Give your answers correct to 3 significant figures.	**B3** Solve $x^2 - 4x + 3 = 4x + 8$ Give your answers correct to 3 significant figures.	**B4** Solve $5 + 9x + 4x^2 = 4$ Give your answers correct to 3 significant figures.
C1 Calculate the discriminant and state the number of solutions to: $x^2 + 4x + 2$	**C2** Calculate the discriminant and state the number of solutions to: $2x^2 + 4x + 5$	**C3** Calculate the discriminant and state the number of solutions to: $3x^2 - 7x - 2$	**C4** Calculate the discriminant and state the number of solutions to: $x^2 - 6x + 9$
D1 Write down an equation, which leads to the calculation $x = \frac{-3 \pm \sqrt{9+12}}{2}$	**D2** Write down an equation, which leads to the calculation $x = \frac{11 \pm \sqrt{121-40}}{4}$	**D3** Write down an equation, which leads to the calculation $x = \frac{-4 \pm \sqrt{16+20}}{10}$	**D4** Write down an equation, which leads to: $x = \frac{-8 \pm \sqrt{64-48}}{6}$

The following exercise is a mix of linear and quadratic equations.
Solve each equation using an appropriate technique.

1. $4x + 3 = 2x + 9$
2. $5x^2 = 3x$
3. $50 - x^2 = 25 - x - x^2$
4. $x^2 - 11x - 12 = 0$
5. $x^2 + 10x = 0$
6. $3(3x + 5) + 6 = 3$
7. $(y + 2)^2 = y^2 + 13$
8. $8x^2 - 2x - 3 = 0$
9. $3x^2 + 27x + 42 = 0$
10. $2x - 9 = \frac{x}{4}$
11. $(a + 2)(a - 4) = (a + 3)^2$
12. $6x^2 = x + 2$
13. $(x - 3)(x + 5) = -16$
14. $(b + 5)^2 = (b + 1)^2$
15. $x^3 - 6x^2 + 8x = 0$
16. $4x(x + 1) = 3$
17. $(2y + 3)^2 = (y + 1)(y - 3) + 3y^2$
18. $-2x^2 = -8x + 6$
19. $(5m + 2)^2 - 1 = (m - 3)(m + 5) + 24m^2$

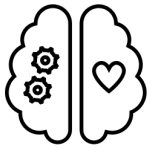
Fluency Practice

Solve $x^2 + 5x + 3 = 0$

Give your solutions correct to 2 decimal places.

Solve $x^2 + 2x - 7 = 0$

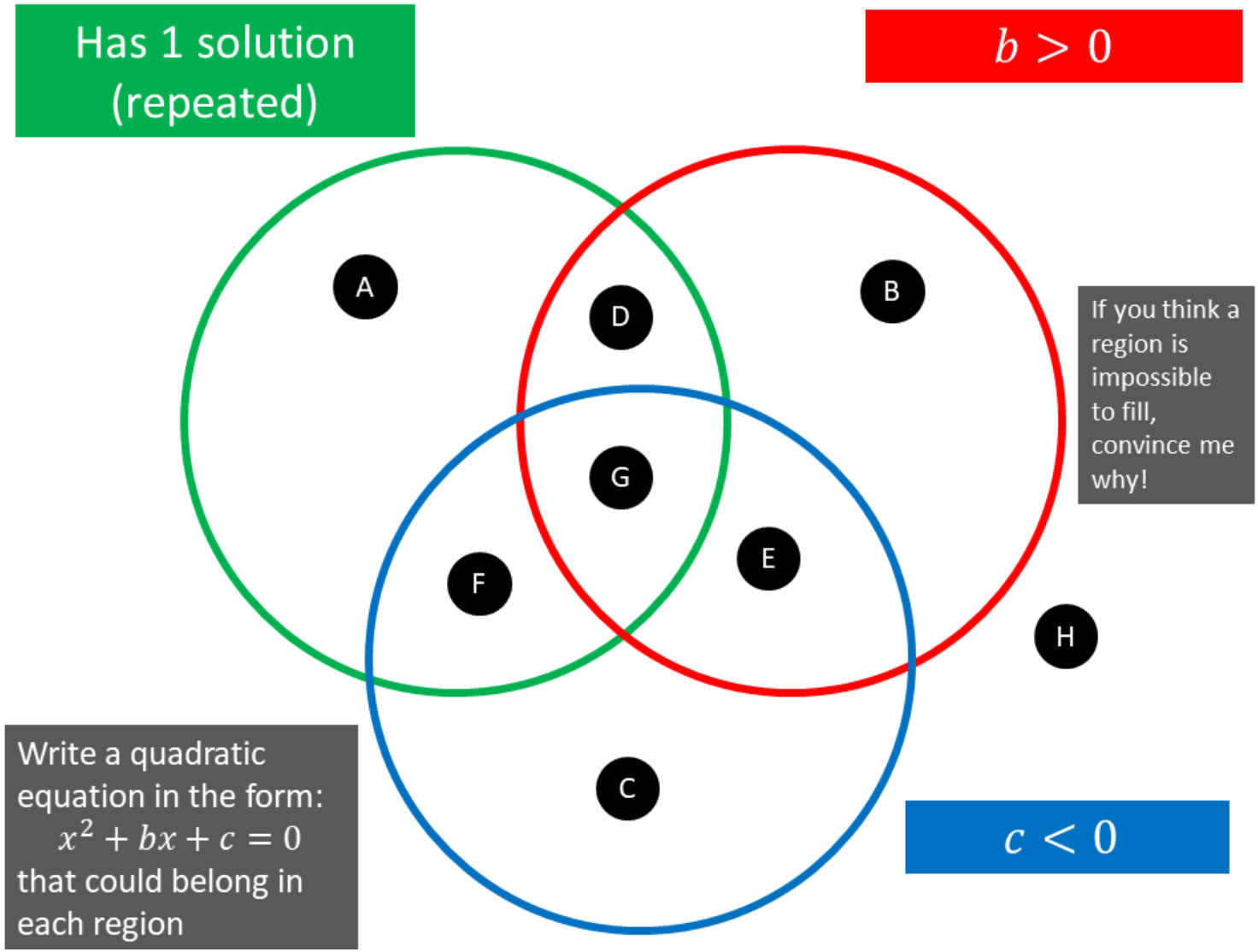
Give your answers in the form $a \pm b\sqrt{c}$.



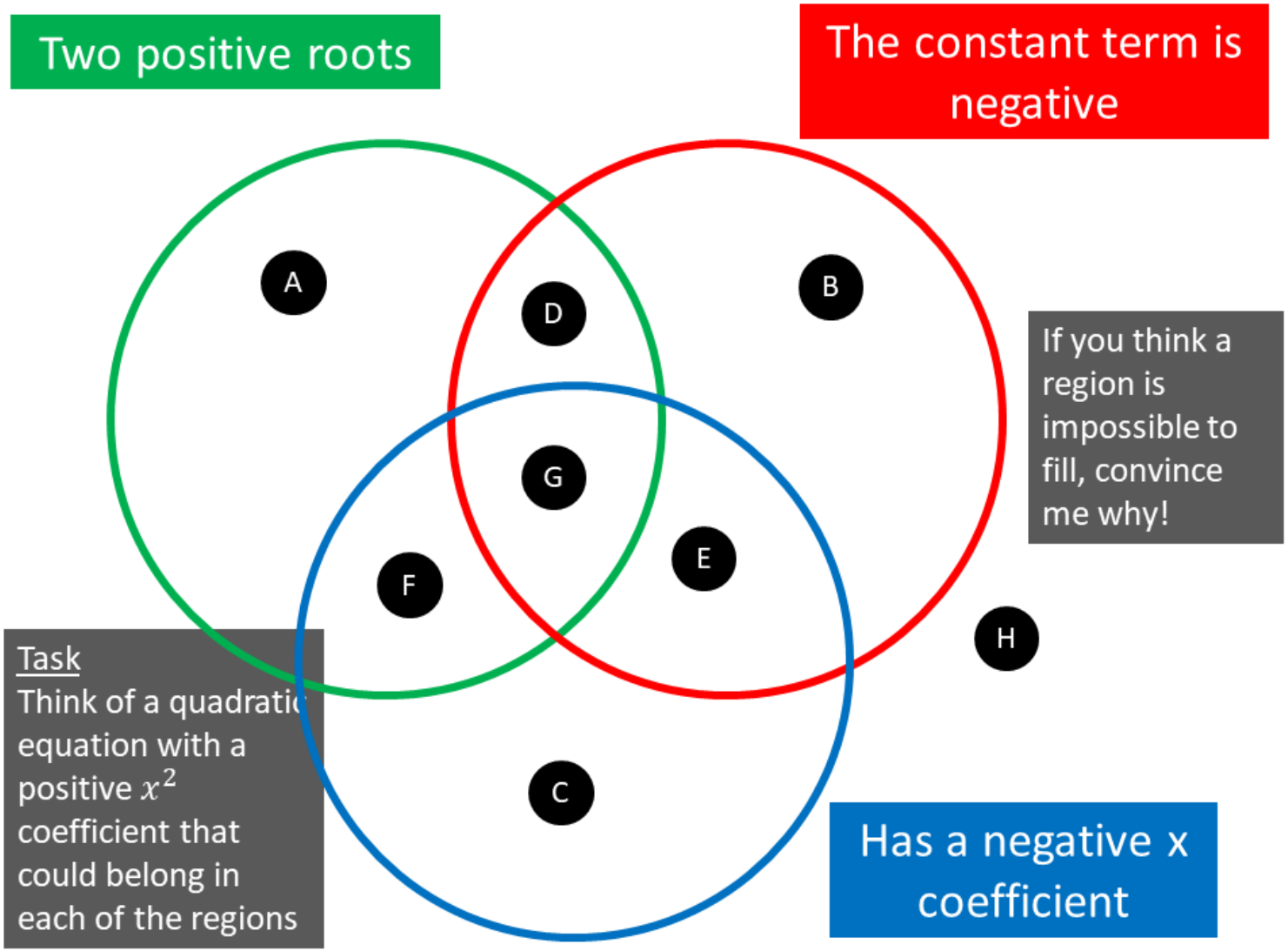
Solve $5x^2 = 6x + 3$

Give your solutions correct to 3 significant figures.

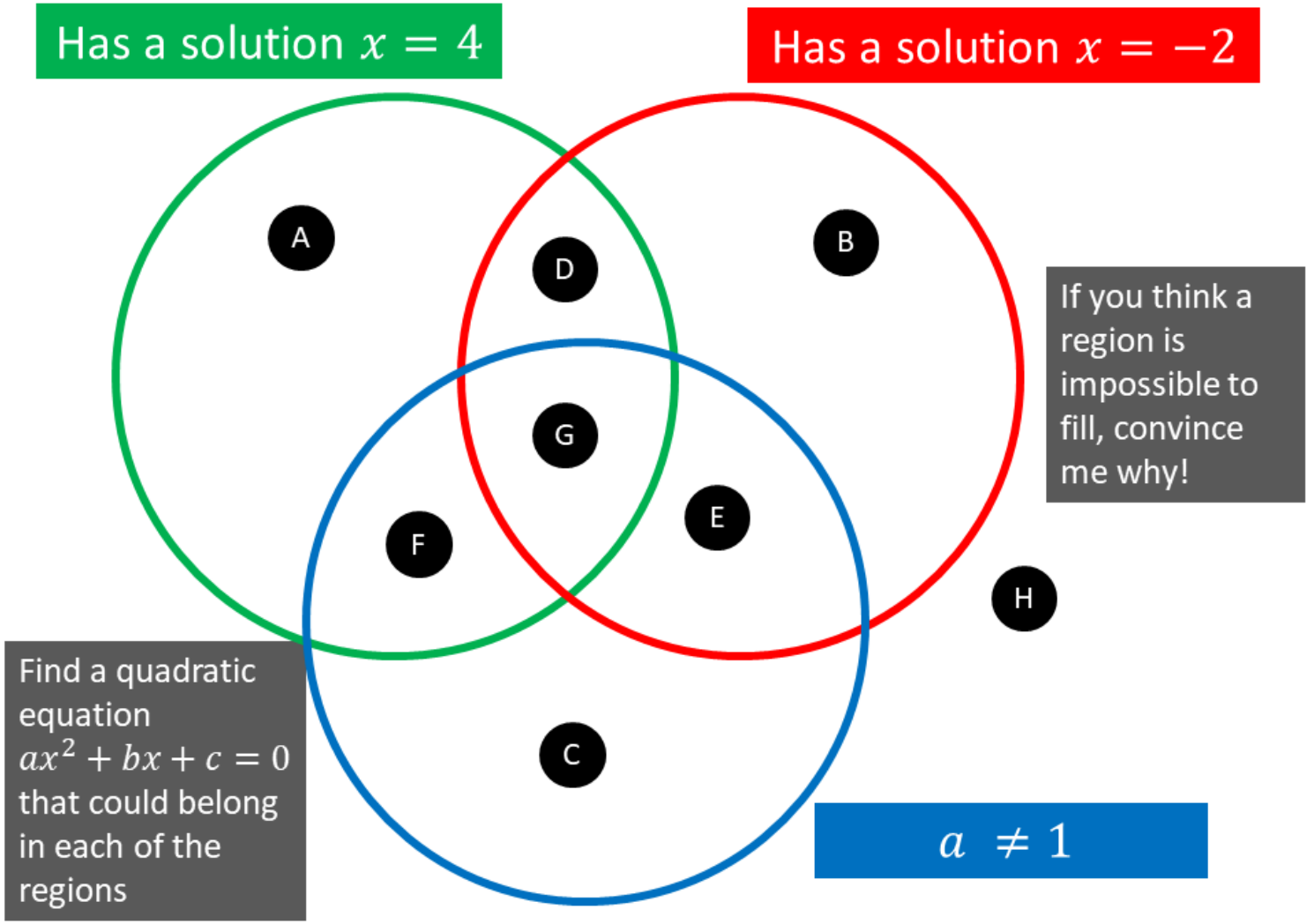
Problem Solving



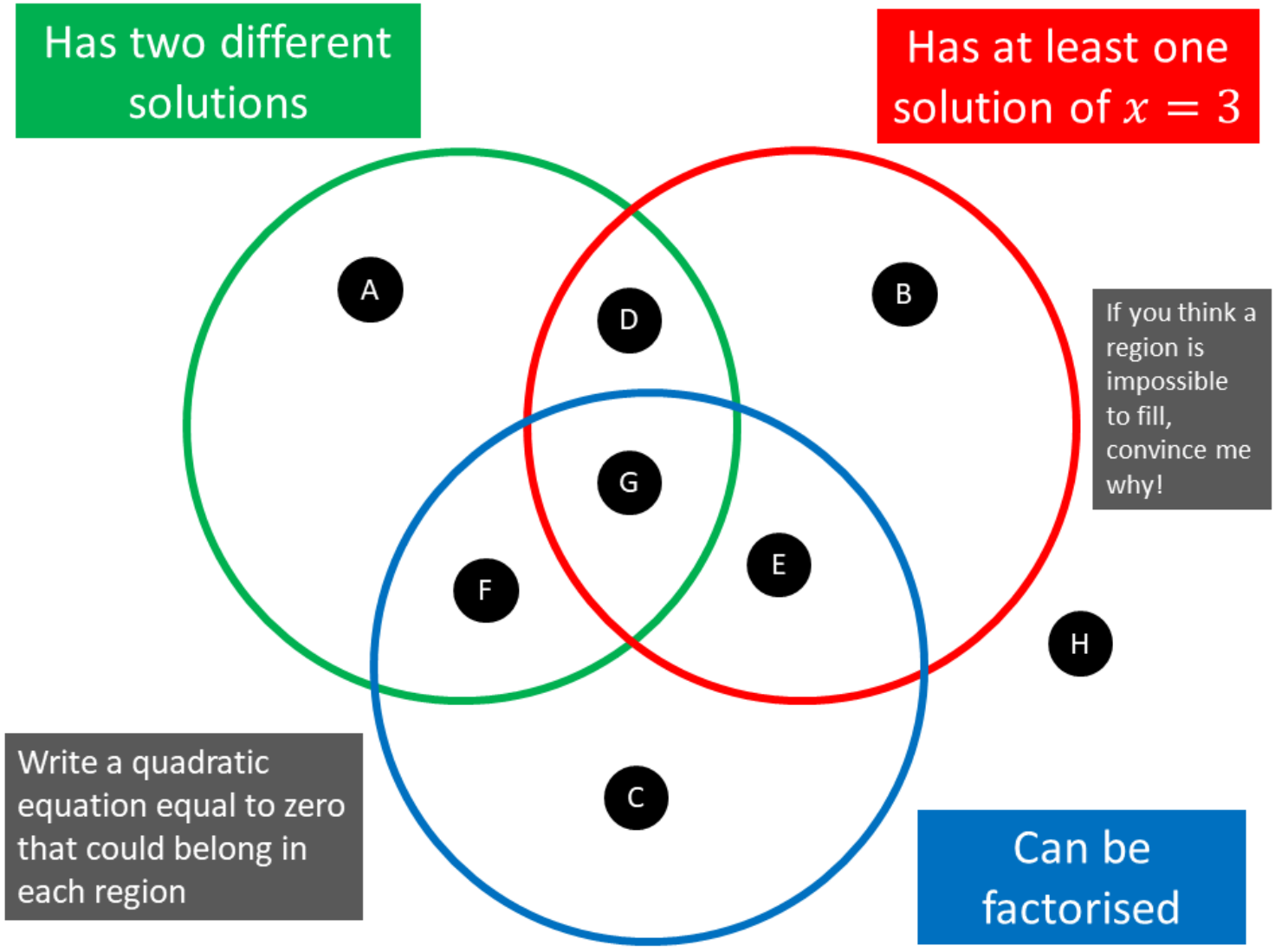
Problem Solving



Problem Solving



Problem Solving



4 Advanced Ratio

Fluency Practice

Ratio & Formulae

$a : b = 3 : 4$

$\frac{a}{b} = \frac{3}{4}$ → $a = \frac{3}{4}b$

$\frac{b}{a} = \frac{4}{3}$ → $b = \frac{4}{3}a$

① $a : b = 2 : 5$

$\frac{a}{b} =$ → $a =$

$\frac{b}{a} =$ → $b =$

② $a : b = 1 : 3$

$\frac{a}{b} =$ → $a =$

$\frac{b}{a} = \frac{1}{1}$ → $b =$

③ $a : b = 6 : 5$

$\frac{a}{b} =$ → $a =$

$\frac{b}{a} =$ → $b =$

④ $a : 4 = 3 : b$

$a =$ $\quad$ $= b$

⑥ $a : 2 = b : 9$

$\frac{a}{\quad} =$

$a =$ $\quad$ $= b$

⑦ $3 : a = 4 : b$

$a =$ $\quad$ $= b$

⑧ $a : 2 = 3 : 7$

$a =$

⑨ $5 : a = 2 : 3$

$a =$

⑩ $4 : 7 = 3 : 5a$

$= \frac{\quad}{3}$

$= a$

⑪ If, $a = \frac{4}{5}b$

$a : b =$

⑫ If, $b = \frac{8}{7}a$

$a : b =$

⑬ If, $a = 1.25b$

$a : b =$

⑭ If, $b = 1.8a$

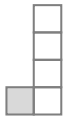
$a : b =$

Fluency Practice

Comparing Ratios For each ratio, express a in terms of b… and b in terms of a. Complete each sentence without using fractions.

① $a : b = 1 : 4$

$a : b$



$\frac{a}{b} = \frac{1}{\quad}$ $\frac{b}{a} = \frac{4}{\quad}$

$a = \frac{1}{\quad}b$ $b = \quad a$

b is _____ times the size of a.

② $a : b =$

$a : b$

$\frac{a}{b} = \frac{1}{\quad}$ $\frac{b}{a} = \frac{\quad}{1}$

$a = \quad b$ $b =$

a is _____ times the size of b.

③ $a : b =$

$a : b$

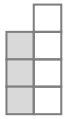
$\frac{a}{b} =$ $\frac{b}{a} = \frac{\quad}{2}$

$a =$ $b =$

b is _____ times the size of a.

④ $a : b =$

$a : b$



$\frac{a}{b} =$ $\frac{b}{a} =$

$a =$ $b =$

a is _____ times the size of b.

⑤ $a : b =$

$a : b$

$\frac{a}{b} =$ $\frac{b}{a} = \frac{\quad}{4}$

$a =$ $b =$

b is _____ times the size of a.

a is _____ times the size of b.

⑥ $a : b =$

$a : b$

$\frac{a}{b} =$

$a = \frac{4}{\quad}b$ $b =$

b is _____ % larger than a.

⑦ $a : b = 5 : 2$

$a : b$

$a =$ $b =$

a is _____ times the size of b.

b is _____ % smaller than a.

⑧ $a : b =$

$a : b$

$a = \frac{5}{7}b$ $b =$

a is _____ times the size of b.

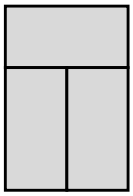
⑨ $a : b$ $a : b =$

$a =$ $b =$

b is $2\frac{2}{3}$ times the size of a.

a is _____ % smaller than b.

Ratios & Formulas

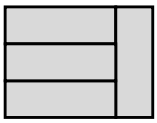


This compound shape is made from 3 congruent rectangles.

For one small rectangle…

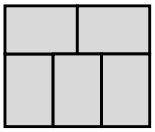
a) What is the formula for the length in terms of width?

b) What is the ratio **length : width** ?

1) 

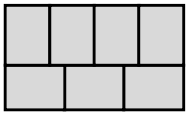
Formula _____

Ratio _____

2) 

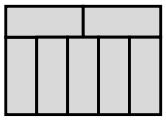
Formula for length ⇨

length : width = ⇨

3) 

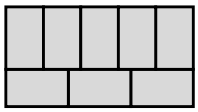
Formula _____

Ratio _____

4) 

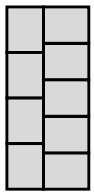
Formula _____

Ratio _____

5) 

Formula _____

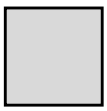
Ratio _____

6) 

Formula _____

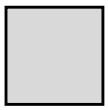
Ratio _____

For these **squares**, write a formula for **a** in terms of **b**.
Convert the formula into the ratio **a : b**

7) a  2b

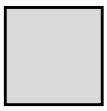
Formula _____

a : b _____

8) 2a  3b

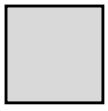
Formula _____

a : b _____

9) 4a  3b

Formula _____

a : b _____

10) 8a  10b

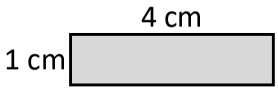
Formula _____

a : b _____

Ratios & Formulas

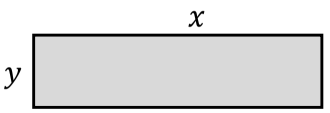
These rectangles are **similar**.

What can we **deduce** about the **relationship** between x and y?



Length : Width

4 : 1



Length : Width

$x : y$

The length and width are in the same ratio.

$4 : 1 = x : y$

Therefore, $\frac{4}{1} = \frac{x}{y}$ and $\frac{1}{4} = \frac{y}{x}$

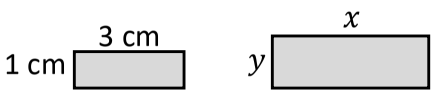
We can rearrange these equations to express x in terms of y and vice-versa.

$x = 4y$ $\quad y = \frac{x}{4}$

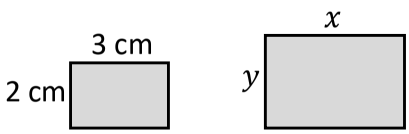
For each similar pair, express their equal ratios and convert this into a formula.

Rearrange the formula to express x in terms of y.

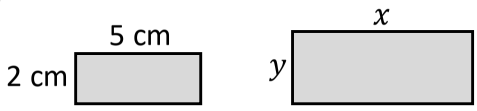
A)



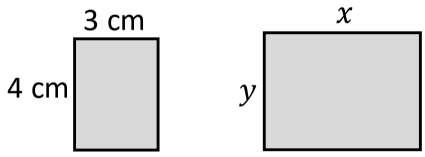
B)



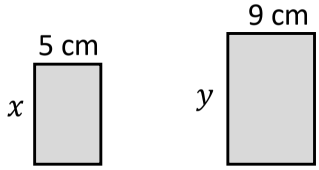
C)



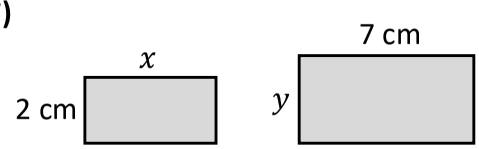
D)



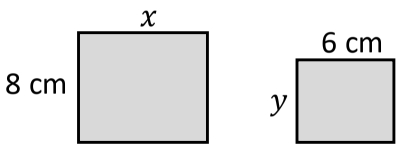
E)



F)

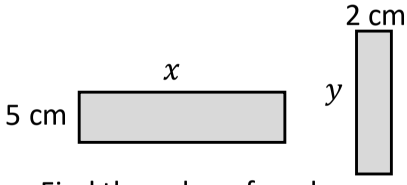


G)



Find the value of x when $y = 12$

H)



Find the value of y when $x = 8$

Show each of the formulas using the equivalent ratios below.

(y+x) : (y-x) is equivalent to k : 1

Show that $y = \frac{x(k+1)}{k-1}$

(b+m) : (b-m) is equivalent to k : 1

Show that $b = \frac{m(k+1)}{k-1}$

(p+y) : (p-y) is equivalent to k : 3

Show that $p = \frac{y(k+3)}{k-3}$

(p+q) : (p-q) is equivalent to k : 5

Show that $p = \frac{q(k+5)}{k-5}$

(3x+y) : (x-3y) is equivalent to k : 2

Show that $x = \frac{y(3k+2)}{k-6}$

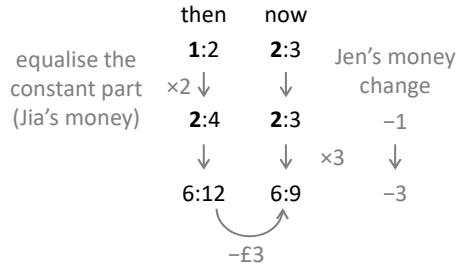
(x+2y) : (3x-4y) is equivalent to k : 2

Show that $x = \frac{4y(k+1)}{(3k-2)}$

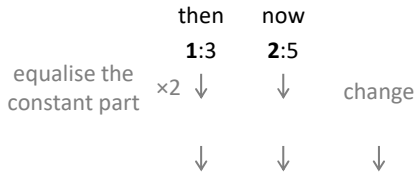
Fluency Practice

Changing Ratios — scaling

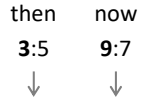
Jia and Jen share some money in the ratio 1:2
Jen spends £3. The money is now in the ratio 2:3



① Rosh and Roe share some money in the ratio 1:3
Roe spends £5. The money is now in the ratio 2:5
How much money does Roe have now?



② Pen and Pol share some money in the ratio 3:5
Pol spends £16. The money is now in the ratio 9:7
How much money is there in total now?



③ Mia and May share some money in the ratio 3:4
May spends £20. The money is now in the ratio 2:1
How much money does May have now?

3:4 **2**:1

④ Ken and Kel share some money in the ratio 4:3
Kel spends £3. The money is now in the ratio 3:2
How much money did they start with?

⑤ Ann and Ai share some money in the ratio 3:2
Ann spends £18. The money is now in the ratio 3:5
How much money did Ann start with?

⑥ A bag contains red and blue disks in the ratio 4:5
After 4 blue disks are removed the ratio is now 2:3
How many disks are there now?

⑦ In a store there are 50% more bananas than apples.
15 bananas are added so the apple:banana ratio = 3:7
How many apples are there?

⑧ A bag has 25% more triangles than circles.
After 15 triangles are added the circle:triangle ratio is 5:7
How many triangles are there now?

1. The ratio of boys to girls in a group is 7 : 5.
   15 more boys join the group.
   The ratio is now 12 : 5.
   How many students are there in total?

2. The ratio of green pens to red pens in a drawer is 5 : 1.
   15 red pens are added to the drawer.
   The ratio becomes 5 : 4.
   How many pens of each colour are now in the drawer?

3. In an animal sanctuary the ratio of dogs : cats : donkeys is 5: 4 : 2
   8 more cats arrive at the sanctuary.
   The ratio is now 5 : 8 : 2.
   How many donkeys are there?

4. Jim likes to buy shirts of particular colours.
   The ratio of blue, black and white shirts is 1 : 3 : 4.
   There are 6 more black shirts than blue shirts.
   How many shirts are there in total?

5. Julia keeps pigs, sheep and goats in the ratio of 2 : 3 : 5.
   She has 9 more goats than pigs.
   How many sheep does she have?

6. Bill buys sweets for his friends, Tom, John and Kevin.
   He gives out the sweets in the ratio 4 : 5 : 2.
   John has 12 more sweets than Kevin.
   How many sweets does each friend have?

7. Green, white and yellow counters are placed in a bag in the ratio of 4 : 3 : 2.
   20 green counters are removed and the ratio becomes 2 : 3 : 2.
   How many of each counter are left in the bag?

8. Burgers, hot-dogs, and sausage rolls are sold in the ratio of 5 : 6 : 7.
   10 more hot-dogs than burgers were sold.
   How many items were sold in total?

Fluency Practice

Person A and Person B share money in a ratio. A gives B some money. Now they have the same amount …

A	B	Ratio	A gives B	Question
James	Jessie	5 : 3	$4	How much did Jessie have to start?
Jessie	Brock	9 : 7	$2	How much did Jessie have to start?
Brock	Ash	3 : 1	$8	How much did they start with altogether?
Ash	Misty	5 : 1	$6	How much did Ash start with?
Misty	Brock	3 : 1	$1	How much do they both have now?
Ash	James	9 : 5	$1	How much did Ash start with?
Brock	Jessie	3 : 1	$0.50	How much do they both have now?

Purposeful Practice

A bag contains black and white cubes in the ratio 1:2
3 black cubes are added and the ratio becomes 3:5

then	$x : 2x$	$= 1 : 2$	$15 : 30$
now	$x + 3 : 2x$	$= 3 : 5$	$18 : 30$

$$\frac{x+3}{2x} = \frac{3}{5}$$
$$5(x+3) = 2x(3)$$
$$5x + 15 = 6x$$
$$15 = x$$

Changing Ratios using algebra

① A bag contains black and white cubes in the ratio 1:2
4 black cubes are added and the ratio becomes 3:5
How many white cubes are there?

then	$x : 2x$	$= 1 : 2$
now		$= 3 : 5$

② In a bag, black cubes : grey cubes = 1:3
2 black cubes are added and
the ratio becomes 2:5
How many black cubes are there now?

$x : 3x$

③ In a bag, red cubes : grey cubes = 2:3
10 red cubes are added and
the ratio becomes 3:2
How many red cubes are there now?

④ In a bag, red cubes : blue cubes = 3:4
6 red cubes are added and
the ratio becomes 4:5
How many red cubes are there now?

⑤ In a bag, green : blue cubes = 4:5
2 green cubes are **removed** and
the ratio becomes 3:4
How many cubes are there now?

$4x - 2 :$

⑥ In a bag, grey : blue cubes = 3:7
4 grey cubes are removed and
the ratio becomes 2:5
How many blue cubes are there?

⑦ In a sports club, adults : teens = 5:8
2 adults join **and** 8 teens join and
the ratio becomes 1:2
How many adults are there now?

$5x + 2 :$

⑧ In a sports club, adults : teens = 2:3
2 adults join and 4 teens **leave** and
the ratio becomes 5:4
How many people are in the club now?

⑨ There are 80% more adults than teens in a sports club.
9 adults leave and 2 teens join and
the adult:teen ratio becomes 3:2
Now 3 teens join the club. As a percentage,
how many more adults than teens are there?

(a) The ratio of white socks to black socks is 1 : 4 in a drawer. Four white socks are added to the drawer and the ratio of white socks to black socks becomes 5 : 12. Find the number of white socks and black socks there were initially.

(b) Lily and Mary have marbles in the ratio 5 : 6. Lily gets two more marbles and now the ratio is 7 : 8. How many marbles did each girl have initially?

(c) The ratio of the number of boys to girls at a party is 3 : 4 Six boys leave the party. The ratio of the number of boys to girls at the party is now 5 : 8 Work out the number of girls at the party.

(d) The ratio of pigeons to ducks in a park is 3 : 2. When 5 pigeons fly away, the ratio of pigeons to ducks becomes 5 : 4. How many ducks and pigeons were there originally?

(e) Bill and Chuck share some sweets in the ratio 7 : 3. Bill gives 3 sweets to Chuck and now the ratio is 5 : 3. How many sweets did each have initially?

(f) There are two bags containing counters, bag A and bag B. The ratio of counters in bag A to bag B is 3 : 4. Twelve counters are taken from bag B and added to bag A and the number of counters in each bag is now the same. How many counters were there originally in each bag?

A A bag contains red and blue counters in the ratio 1 : 3.

When four red counters are added to the bag, the ratio changes to 1 : 2.

Work out the number of blue counters in the bag.

B A fruit bowl contains apples and bananas in the ratio 4 : 5.

Two apples are removed, changing the ratio to 2 : 3.

Work out the total number of pieces of fruit that remain in the bowl.

C In a group of people, the ratio of right-handed to left-handed people is 8 : 1.

Two right-handed people join the group. This changes the ratio to 25 : 3.

Work out the total number of people that are now in the group.

D At a company, 75% of the employees were men.

After six women joined the company, the ratio of men to women was 2 : 1.

Work out how many men work at the company.

E After a certain number of matches, the ratio of wins, draws and losses for a football team is 3 : 1 : 2.

If the team win both of their next two matches, they will have won 55% of their matches.

Work out how many matches the team has played so far.

F The colours of cars in a car park are shown in the table.

	Red	Blue	Grey	White	Black
Freq.	3	5	2	1	4

Work out the smallest number of cars that would need to enter the car park (without any cars leaving) for the ratio of blue to white cars to change to 2 : 5.

G There are 108 people in a hall. The ratio of adults to children is 5 : 4.

How many adults must leave to hall for the ratio to change to 1 : 1 ?

H The ratio of red pencils to green pencils in a box is 4 : 1.

What is the smallest possible number of pencils that would have to be added to change the ratio to 5 : 2 ?

I A box contains gold, silver and bronze tokens in the ratio 5 : 7 : 6.

Six gold tokens are added.

A token is then selected at random from the box.

The probability that it is gold is $\frac{7}{20}$.

Work out the probabilty that it is silver.

J Helen has 1.5 litres of pink paint, made from a mixture of red and white paint in the ratio 1 : 4.

She adds enough red paint to change the ratio to 1 : 3.

How much white paint must she add to change the ratio back to 1 : 4 ?

A In a company, the ratio of the number of men to the number of women is 2 : 3.

$\frac{1}{4}$ of the men are part-time.

$\frac{1}{3}$ of the women are part-time.

There are 12 part-time workers at the company. Work out how many workers there are in total.

B On a bookshelf, the ratio of fiction to non-fiction books is 2 : 5.

The ratio of paperback books to hardback books on the shelf is 3 : 7.

Work out the smallest possible number of books on the shelf.

C At a gallery, there is a photography exhibition. The ratio of portrait to landscape photographs on show is 5 : 3.

60% of the portrait photographs are in colour. 24 of the portrait photographs are not in colour.

Work out the total number of photographs at the exhibition.

D The ratio of men to women in an orchestra is 3 : 2.

35% of the men play a stringed instrument.
45% of the women play a stringed instrument.

What percentage of all of the members of the orchestra play a stringed instrument?

E The table shows the prices for hiring boats at a lake.

	Rowing boat	Pedal boat
30 mins	£10	£12
1 hour	£16	£18

One day, the ratio of rowing boats to pedal boats hired was 2 : 3.

$\frac{5}{8}$ of the rowing boats hired were for 1 hour.

$\frac{5}{9}$ of the pedal boats hired were for 1 hour.

Given that there were 16 pedal boats hired for 30 minutes, work out the total sales income for the day.

F A box contains a number of tokens, of 4 different types. Each token is either blue or white, and either round or square.

The ratio of blue to white tokens in the box is 3 : 4.

The ratio of round blue tokens to square blue tokens is 1 : 2.

The ratio of round white tokens to square white tokens is 3 : 1.

What fraction of all the tokens are round?

G Pupils at a school all study exactly one foreign language.
The ratio of pupils who study French, German and Spanish is 4 : 3 : 2.
One of the French classes has 24 pupils, which is $\frac{2}{9}$ of all the pupils who study French.

Work out the number of pupils who study Spanish at the school.

Purposeful Practice

ratio problems (i)

(1) the ratio of shares owned by Asterix and Cleopatra is in the ratio 3 : 5

if Asterix gives Cleopatra 2 of his shares then the ratio will be 1 : 3

how many shares do they each have initially?

(2) Jan and Kim own numbers of marbles that are in the ratio 5 : 6

Jan gains 2 more marbles and the ratio is now 7 : 8

how many marbles do each own initially?

(3) the ratio of Ann's age to Bob's age is 3 : 4

in 7 years time this ratio will be 4 : 5

(i) what are their ages now?
(ii) after how many years (from now) will the ratio be 5 : 6?

(4) the ratio of cockles to winkles in a bag of sea shells is 1 : 2

when 6 winkles are accidentally spilt from the bag the ratio is 3 : 5

how many of each shell were there initially?

(5) the ratio of geese to ducks on a lake is 4 : 9

when 10 extra geese land on the lake (and there are no duck changes) the ratio is 2 : 3

how many of each were there initially?

(6) the ratio of the width to depth (when they are measured in metres) of a dam is 5 : 7

when 10 extra metres is added to the width of the dam (and the depth remains the same) the ratio is 5 : 6

how wide and deep was the dam initially?

Purposeful Practice

ratio problems (ii)

(1)

bag A contains 7n counters
bag B contains 3n counters

3 counters are taken from bag A and put in bag B

the ratio of counters in bag A to bag B is now 5 : 3

how many counters were in the bags initially?

(2)

box C contains 5k pens
box D contains 2k pens

8 pens are taken from box C and placed in box D

the ratio of pens in box C to box D is now 3 : 2

how many pens were there in each box initially?

(3)

the ratio of pairs of white socks to pairs of red socks is 9 : 5

9 pairs of the white socks are dyed red

the ratio of pairs of white socks to pairs of red socks is now 3 : 2

how many pairs of each colour were there initially?

(4)

Sapna and Rob have many paperweights, in the ratio of 8 : 3

Sapna, being a generous person, gives Rob 14 of her paperweights

the ratio is now 6 : 5

how many paperweights did each have initially?

explore what other ratios could have been created

Purposeful Practice

ratio problems (iii) (simplest form involves integers)

(1) the ratio **a : b** = 3 : 4

what is the ratio

(i) b : 3a – b in its simplest form?

(ii) 2a + b : 4a – b in its simplest form?

(2) the ratio **a : b** = 5 : 6

the ratio **b : c** = 9 : 10

what is the ratio
(i) a : c in its simplest form?

(ii) a + b : 9b – 2a in its simplest form?

(3) the ratio **a : b** = 4 : 5

what is the ratio

(i) a + 2b : 3a – 2b in its simplest form?

(ii) 3b – 2a : a + 9b in its simplest form?

(4) the ratio **a : b** = 4 : 3

the ratio **a : c** = 2 : 5

what is the ratio
(i) b : c in its simplest form?

(ii) 2a – b : 2b + a in its simplest form?

(5) the ratio **a : b** = 2 : 5

the ratio **d : e** = 10a : 3b

what is the ratio
(i) d : e in its simplest form?

(ii) a + 2b : 2a + b in its simplest form?

(6) the ratio **a : b** = 3 : 2

the ratio **b : c** = 5 : 6

what is the ratio
(i) a : c in its simplest form?

(ii) 2a + b : 2b + c in its simplest form?

Purposeful Practice

ratio problems (iv)

(1)

the ratio

(a number add 2 : the number add 4)

is the same as the ratio

(the same number add 6 : this number add 9)

what is the number?

(2)

the ratio

(a number take 1 : the number add 3)

is the same as the ratio

(the same number add 1 : this same number add 6)

what is the number?

(3)

the ratio

(twice a number add 1 : this number take 1)

is the same as the ratio

(twice the same number add 8 : this same number add 2)

what is the number?

(4)

the ratio

(a number add 1 : the number add 4)

is the same as the ratio

(twice the same number take 5 : this same number add 20)

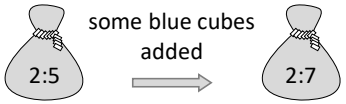
what is the number?

[find two solutions if you can]

Purposeful Practice

Cube Ratios

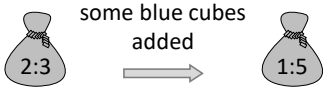
① red : blue



What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were added?

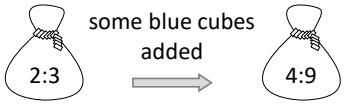
② red : blue



What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were added?

③ red : blue

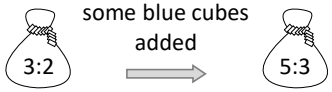


What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were added?

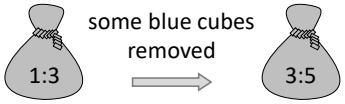
If 6 cubes were added, how many are there now?

④ red : blue



Why is this impossible?

⑤ red : blue

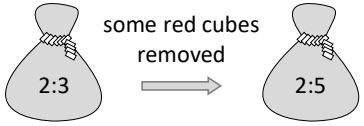


What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were removed?

If 12 cubes were removed, how many are there now?

⑥ red : blue

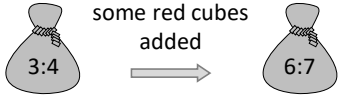


What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were removed?

If 20 cubes were removed, how many are there now?

⑦ red : blue

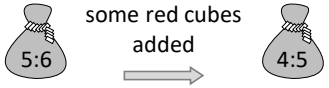


What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were added?

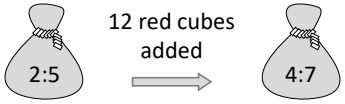
If 30 cubes were added, how many were there?

⑧ red : blue



Why is this impossible?

⑨ red : blue



How many cubes are there now?

ratio problems involving algebra

(1) if $a : 7 = b : 4$ then $(a + b) : 11 = (a – b) : 3$

(2) if $a : b = c : d$ then $a : c = (a + b) : (c + d)$

(3) if $a : b = c : d$ then $(a + b) : (a – b) = (c + d) : (c – d)$

(4) if $3a : 5b : 6c = p : q : r$ then $a : b : c = 10p : 6q : 5r$

(5) if $a : b = b : c$ then $(b + c) : c = (a – c) : (b – c)$

(6) if $a : b = b : c$ then $(a + b) : a = (a – c) : (a – b)$

Purposeful Practice

Ratio Worded Problems			
(a)	**(b)**	**(c)**	**(d)**
A pencil case contains 5 blue pencils, 3 red pencils and 7 black pencils. What fraction of the pencils are red? Give your answer in its simplest form.	In a florist shop, the ratio of roses to tulips to gerberas is 7 : 2 : 3. What fraction of the flowers in the shop are roses or tulips? Give your answer in its simplest form.	The ratio of the number of boys to the number of girls in a school is 4 : 7. There are 91 girls in the school. Work out the number of boys in the school.	The books in a library are fiction or non-fiction in the ratio 3 : 2. If there are 573 fiction books, how many books are there in the library in total?
(e)	**(f)**	**(g)**	**(h)**
A map has a scale of 1 : 500. Martha measures a distance of 5 cm on the map. What actual distance in m does this correspond to?	Madeira cake mixes flour, sugar and butter in the ratio 5 : 3 : 2. How much flour and butter is required to mix with 75 g sugar?	In a car park there are twice as many white cars as black cars, and three times as many red cars as white cars. Write the ratio of cars red : white : black in its simplest form.	A recipe to make 8 pancakes needs 200 ml milk and 120 g flour. How much milk and flour is need to make 20 pancakes?
(i)	**(j)**	**(k)**	**(l)**
The angles in a quadrilateral are in the ratio 5 : 7 : 4 : 2. Find the size of each of the angles.	120 ml of orange drink is made with cordial and water in the ratio 1 : 4. To make the drink stronger 10 ml more of cordial is added. What is the ratio of cordial to water for the new drink?	Ishaq builds a model rocket. The height of the model rocket is 16 cm, and the actual height of the model is 48 m. Find the ratio of the model rocket to the real rocket.	Jim mixes 1 litre of red and white paint in the ratio 1 : 3. How many ml of red paint should be added so that the ratio becomes 8 : 15 ?

Purposeful Practice

Ratio 1

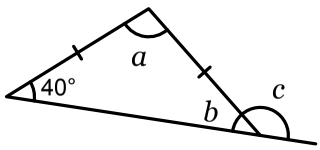
① Some money is shared in the ratio 2 : 3 : 5 The largest share is £45 larger than the smallest. How much money is there in total?	② Trees in a forest, evergreen : deciduous = 5 : 2 For ever-green trees, healthy : unhealthy = 4 : 7 There were 35 unhealthy evergreen trees. How many deciduous trees were there?	③ Write the ratio 9 : 4 in the form n : 1 where n Is a decimal $a : b = 5 : 3$ How many times larger is a than b?	④ The value of y is 50% more than the value of x. Express the ratio $x : y$ using integers. a : b = 3 : 2 a : c = 4 : 7 Express the ratio a : b : c
⑤ There are triangles, hexagons & circles in a bag. triangles to hexagons = 2:3 hexagons to circles = 6:7 If there are less than 100 shapes in the bag, what is the greatest number of hexagons in the bag?	⑥ Jan has some money. Caleb has £6 more than four times the money Jan has. In total they have £51. Express this as a ratio Jan : Caleb	⑦ $\frac{1}{7} : \frac{3}{5} = x : 1$ Calculate the value of x.	⑧ The points A, B, C and D lie in order on a straight line. AB : BD = 1:3 AC : CD = 5:7 Work out AB : BC : CD
⑨ Greg scored 20 more than Kim. Tan scored triple Kim's score. In total they scored 80 points at archery. Find the ratio Kim's score : Greg's score	⑩ a : b = 2 : 5 and 4b = 5c Work out a : c in its simplest form.	⑪ b is one-quarter of c $2a = 3c$ Express the ratio $a : b : c$ using integers.	⑫ Ann has £28 in savings. Bill has no savings. Each week from now, Ann saves £6 & Bill saves £4. How many weeks will it take for Ann & Bill to have savings in the ratio 5 : 2?

Purposeful Practice

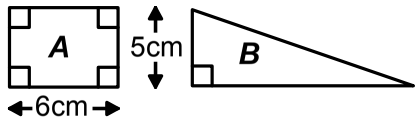
Ratio 1

① A distance is split in the ratio 5 : 7 : 2 The smallest distance is 3.5 km smaller than the largest. What is the total distance?	② Electricity generation, renewable : non-renewable = 9 : 5 For renewable energy, solar : wind = 11 : 8 18 MW of power was generated from wind power. How much energy was from non-renewable sources?	③ Write the ratio 4 : 2.5 in the form n : 1 where n is a decimal $a : b = 3 : \frac{2}{3}$ How many times larger is a than b?	④ The value of y is 20% more than the value of x. Express the ratio $x : y$ using integers. a : b = 2 : 3 b : c = 7 : 8 Express the ratio a : c
⑤ There are octagons, pentagons & squares in a bag. octagons to pentagons = 4:9 octagons to squares = 5:8 If there are less than 1000 shapes in the bag, what is the greatest number of squares in the bag?	⑥ On a test, Will scored double the lowest score. Dan scored 140 marks more than the lowest score. Will's score : Dans score = 1:4 What was the lowest score?	⑦ $\frac{2}{5} : \frac{6}{7} = x : 1$ Calculate the value of x.	⑧ The points A, B, C and D lie in order on a straight line. AB : BD = 1:2 AC : CD = 3:1 Work out AB : BC : CD
⑨ Wai has £8 less than Toby. Cindy has 4 times the money Wai has. In total they have £260. Find the ratio of money Toby : Cindy	⑩ a : b = 4 : 5 and 3b = 10c Work out a : c in its simplest form.	⑪ b is two-fifths of c $3a = 7c$ Express the ratio $a : b : c$ using integers.	⑫ Cane has £27 in savings. Dale has £28 savings. Each week from now, Cane saves £6 & Dale saves £3. How many weeks will it take for Cane & Dale to have savings in the ratio 3 : 2?

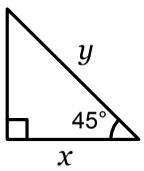
A. Work out the ratio $a : b : c$ in the simplest form.



B. Shapes A and B have the same height and the same area. Work out the ratio *perimeter of A : perimeter of B* in the simplest form.



C. Work out the ratio $x : y$ in the form $1 : n$.



D. Given that:

$$2x - 9 : x - 4 = x : 4$$

Work out the value of x.

E. A shop sells a type of cereal in two packet sizes. The larger packets cost 40% more than the smaller packets.

Work out the ratio of the cost of a smaller packet to the cost of a larger packet in the form $1 : n$.

F. Jason sells red, blue and green raffle tickets. He starts with 120 of each colour.

Jason managed to sell red, blue and green tickets in the ratio 3 : 2 : 1.
168 of the tickets remained **unsold**.

Work out how many of each colour ticket **were sold**.

G. Penny has a small block of iron and a small block of silver.

The iron is in the shape of a cuboid, with dimensions 31mm, 25mm and 43mm.

The silver has a mass of 324g and has a density of 10.5g/cm^3.

Work out the ratio of the volume of iron to the volume of silver in the form *1 : n*.

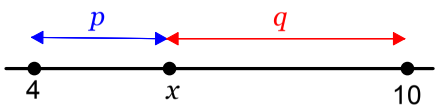
H. The prices of two games are in the ratio $a : b$.

If both prices were decreased by £10, the ratio would become 3 : 5.

If both prices were increased by £5, the ratio would become 3 : 4.

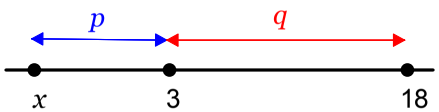
Work out the ratio $a : b$ in the simplest form.

I. In the diagram, the dots represent points on a number line.



Given that $p : q = 1 : 3$, work out x.

J. In the diagram, the dots represent points on a number line.



Given that $p : q = 2 : 5$, work out x.

Question 1: The table below shows information about the number of medals won by 6 countries in the 2016 Olympic Games.

Country	Gold	Silver	Bronze	Total
Spain	7	4	6	17
France	10	18	14	42
Germany	17	10	15	42
Italy	8	12	8	28
Japan	12	8	21	41
Australia	8	11	10	29

(a) Write down the ratio of the total number of medals won by Italy to the total number of medals won by France.
Give your answer in its simplest form.

(b) Write down the ratio of the total number of gold medals won to the total number of bronze medals won.
Give your answer in its simplest form.

Question 2: There are between 30 and 40 sweets in a bag.
Ben and Daisy share the sweets in the ratio 4:5.
There are no sweets remaining in the bag.

How many sweets were in the bag to begin with?

Question 3: George buys some first class and second class stamps to put on some letters.
Each first class stamp costs 67p
Each second class stamp costs 58p
The ratio of first class stamps to second class stamps that he buys is 1:4
George bought 20 stamps.

How much did the stamps cost George?

Question 4: 1,935 people visit a library during one week.
The ratio children : adults is 1 : 4
How many more adults than children visited the library?

Question 5: The sizes of the interior angles of a triangle are in the ratio 1:3:8
Calculate the difference in size between the largest and smallest angles.

Question 6: Mr Rodgers is organising a trip to a football match for 84 children.
The ratio of teachers on the trip to children on the trip is 1:6
Each bus holds 30 passengers.
Show four buses are needed.

Question 7: Isaac and Victoria share money in the ratio 2:5
Victoria receives £6.60
Work out the difference between how much money Isaac and Victoria receive.

Question 8: Charlotte and Melissa booked theatre tickets costing £180.
They have a voucher that entitles them to 20% off the total price.
Charlotte and Melissa share the total cost of the tickets in the ratio 5:7.

Work out how much more Melissa pays than Charlotte.

Question 9: Barnaby has 288 counters in a bag.
The counters are red, yellow and white.

$\frac{3}{8}$ of the counters are red.

The other counters are yellow and white in the ratio 1:4

Work out how many counters of each colour there are.

Question 10: The sizes of the interior angles of a pentagon are in the ratio 1:2:5:5:7
Calculate the size of the largest

Question 11: Jack is 10 years old.
Kylie is 17 years old.
Vanessa is 23 years old.
Kylie and Vanessa share £16 in the ratio of their ages.
Kylie gives 20% of her share to Jack.
Vanessa gives a quarter of her share to Jack.
How much money does Jack receive?

Question 12: Two numbers are in the ratio 3:7
One of the numbers is 42
There are two possible values for the other number.
What are the two possible values?

Question 13: Alex is 12 years older than George.
Carl is three times older than Alex
The sum of their ages is 68
Find the ratio of George's age to Carl's age to Alex's age.

Question 14: Felicity has two boxes of counters, each with an equal ratio of black and white beads.
In box A, 54 of the beads of black and 36 are white.
There are 162 white beads in box B.

How many beads are there in the two boxes in total?

Question 15: Sophie is making 400 scones.
She uses butter, sugar and flour in the ratio 2:1:9
Here are the costs of those ingredients.

Butter	£2.20 per 500g
Sugar	£1.60 per kilogram
Flour	60p per 1.5kg

The total mass of the butter, sugar and flour in each scone is 30g

Work out the total cost of these ingredients for the 400 scones.

Question 16: Copper costs £5 per kilogram.
Zinc costs £3.20 per kilogram.
Copper and zinc are mixed in the ratio 4:1 to make brass.

Work out the cost of 7 kilograms of brass.

Question 17: Geraint has 2p and 50p coins in the ratio 20 : 3

Write the ratio of the value of the 2p coins to the value of 50p coins in its simplest form.

Question 18: Mrs Chambers is organising a school trip to a museum for year 7 and year 8.
She needs to work out the total cost of the museum tickets and bus hire.
The table below shows the museum ticket prices.

Visitor Age	Price
0 - 3	free
4 - 12	£4.50
13 - 17	£6.50
18+	£11.50

Each bus has 51 seats and costs £125

Altogether 300 students want to go on the trip.
The ratio of the number of students to the number of teachers is 25:1
The ratio of the number of students in year 7 to the number of students in year 8 is 8:7

At the time of the trip, all of the students in year 7 are 11 or 12 years old.
Of year 8 students, the ratio of number of 12 year olds to 13 year olds is 2:3.
Work out the total price of the school trip.

Question 19: Alannah, Colin and Hannah visit the park and collect conkers and acorns.

In total they collect the same number of conkers and acorns.

Alannah, Colin and Hannah collect conkers in the ratio 4:7:9
Alannah, Colin and Hannah collect acorns in the ratio 3:1:5

(a) Did Hannah collect more conkers or acorns?

(b) What is the lowest possible number of conkers they could have collected?

Question 20: A bag contains red, yellow and blue beads.

The ratio of red beads to yellow beads is 2:3
The ratio of yellow beads to blue beads is 5:4

Work out what fraction of the beads are red.

Question 21: In a box

the number of blue counters and the number green counters are in the ratio 7:4
the number of green counters and the number of red counters are in the ratio 3:1

The total number of counters in the bag is 444.

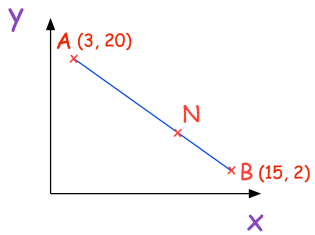
How many green counters are in the bag?

Question 22: The ratio of the red cards to black cards in a deck is 3:10
2 more red cards are added to the deck.
The ratio of red cards to black cards is now 1:3

Work out the number of black cards in the deck.

Question 23: A is the point with coordinates (3, 20)
B is the point with coordinates (15, 2)
N is a point of the line AB such that AN : NB = 2 : 1

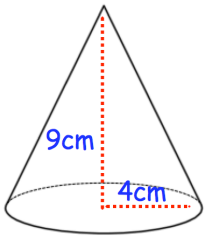
Find the coordinates of the point N.

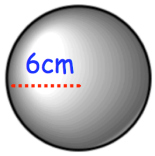


Question 24: Bag A contains 5x coins.
Bag B contains 3x coins.
8 coins are taken from Bag B and put into Bag A
The ratio of coins in Bag A to Bag B is now 11:5

Work out the total number of coins.

Question 25: A cone has radius 4cm and perpendicular height of 9cm.
A sphere has a radius of 6cm





Volume of a sphere	$\frac{4}{3}\pi r^3$
Volume of a cone	$\frac{1}{3}\pi r^2 h$

Calculate the ratio of the volume of the cone to the volume of the sphere.

Question 26: On 1st March 2001, the ratio of Freddie's age to his mother's age was 1:11
On 1st March 2018, the ratio of Freddie's age to his mother's age was 2:5
Write the ratio of Freddie's age to his mother's age on 1st March 2030

Question 27: The ratio 25 000 000 000 : 500 can be written in the form $n : 1$
Work out the value of n

Give your answer in standard form

Question 28: The distance of Mercury from the Sun is 5.7×10^7 km
The distance of Neptune from the Sun is 4.3×10^9 km

Work out the ratio of the distance of Mercury from the Sun to the distance of Neptune from the Sun.

Question 29: Class 10D make some cakes using milk chocolate, dark chocolate or white chocolate.
Some of the cakes contain nuts and the rest do not.

The ratio of the number of milk chocolate cakes to dark chocolate cakes is 10:3
The ratio of the number of white chocolate cakes to milk chocolate cakes is 1:6

Of the milk chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 1:8

Of the dark chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 3:2

Of the white chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 2:5

What percentage of the cakes contain nuts?

Purposeful Practice

gcse ratio problems (based on Edexcel)

(1) Riley and Dan share some jelly beans
Dan gets 40% of the jelly beans
write down the ratio of the number of jelly beans
Riley gets to the number that Dan gets

(2) there are 20 boxes of eggs with 6 eggs in each box
write, as a ratio, the number of eggs in four of the boxes to the number of eggs that there are altogether
what does this ratio simplify to?

(3) Harry, Ryan and Kara share £405 in the ratio 3 : 2 : 4
how much money does Kara get?

(4) Arnold and Lara share some money in the ratio 7 : 13
what percentage of the money does Lara get?

(5) a path is made of white tiles and grey and
¼ of the tiles are white
(i) what is the ratio of white tiles to grey tiles ?

there are 72 tiles altogether
(*ii)* work out the number of grey tiles

(6) there are only red buttons, yellow buttons and orange buttons in a jar
these are in the ratio 2 : 4 : 9 respectively

what percentage of buttons in the jar that are orange?

(7) 2.5 kg of plums cost £5.10
work out the cost of 3.5 kg of plums at the same price

(8) there are only red, orange and green tennis balls in a bag
there are three times as many orange as red
and four times as many green as orange

Hannah takes a tennis ball at random from the bag
work out the probability that Hannah picks a green one

(9) the ratio of the cost of one metre of cotton to the cost of one metre of silk (same widths) is 2 : 7

if 2m of cotton cost £8,
how much does 9m of silk cost?

(10) this recipe for flapjacks makes 12 pieces

120 g butter
140 g light brown sugar
250 g porridge oats
2 tablespoons syrup

how much of each ingredient is needed to make 18 pieces?

(11) Kim is 6 years older than Jim
Tim is twice as old as Kim
the sum of their three ages is 90 years

find the ratio of Jim's age : Kim's age : Tim's age
in the simplest possible form

Purposeful Practice

gcse ratio problems (based on Edexcel)

(12) one Saturday, some adults and some children were in a the Grand theatre, Wolverhampton

the ratio of the number of adults to the number of children was 7 : 4

each person sat in the circle or the stalls

¾ of the children had seats in the stalls
68 children had seats in the circle

there are exactly 1200 seats in the theatre

on this particular Saturday, were there people on more than 60% of the seats?

(13) Jay bakes 280 cakes,

four different kinds of cake are baked:
vanilla, banana, lemon and chocolate

- 2 out of every 7 of the cakes are vanilla cakes
- 35% of the cakes are banana cakes
- the ratio of the number of lemon cakes to the number of chocolate cakes is 1 : 5

are there more chocolate cakes than vanilla cakes?

you must show your working to support your answer

(14) it normally takes 5 cleaners 4½ hours to clean all of the rooms in a hotel

today there are only 3 cleaners to clean all of the rooms

each cleaner is paid at a rate of £8.10 per hour (or part of an hour) that he/she works

how much will **each** cleaner be paid today?

(15) Theo drove 54 km from Liverpool to Manchester
he then drove 62 km from Manchester to Sheffield

Theo's average speed from Liverpool to Manchester was 72 km/h

Theo then took 75 minutes to drive from Manchester to Sheffield

work out Theo's average speed for his total drive from Liverpool to Sheffield

(16) a gold bullion bar has a mass of 12.4 kg

the density of gold is 19.32 g/cm^3

work out the volume of the gold bar

give your answer correct to 1 decimal place

gcse ratio problems (based on Edexcel)

(17) the density of apple juice is 1.04 grams per cm^3
the density of fruit syrup is 1.4 grams per cm^3
the density of carbonated water is 0.99 grams per cm^3

30 cm^3 of apple juice are mixed with 10 cm^3 of fruit syrup and 280 cm^3 of carbonated water to make a drink with a volume of 320 cm^3

work out the density of the drink

give your answer correct to 2 decimal places

(18) there are only blue, black and red pens in a box

the ratio of the number of blue to the number of black is 2 : 5

the ratio of the number of black to the number of red is 4 : 1

there are less than 100 pens altogether in the box

what is the greatest possible number of red pens in the box?

(19) Jack and Jill cycled along the same 40 km route
Jack took 2½ hours to cycle the 40 km

Jill started to cycle 10 minutes after Jack started to cycle
Jill caught up with Jack when they had both cycled 24 km
Jack and Jill both cycled at constant speeds

work out Jill's speed

(20) the points *A*, *B*, *C* and *D* lie in order along a straight line

$AB : BD = 2 : 5$
$AC : CD = 11 : 10$

work out the ratio $AB : BC : CD$

(21) white shapes and black shapes are used in a game

some shapes are circles and the other shapes are squares

the ratio of white shapes to black shapes is 3 : 7
the ratio of white circles to white squares is 4 : 5
the ratio of black circles to black squares is 2 : 5

work out what fraction of all the shapes are circles

1 **(a)** A box contains only red pens and blue pens.

The ratio red : blue = 3 : 2

There are 24 **red** pens.

Work out the number of pens in the box.

[2 marks]

(b) Another box contains only green pens and black pens.

$\frac{3}{4}$ of the pens are **green**.

What is the ratio of green pens to black pens?

[1 mark]

2 Amy has some red beads and some blue beads.
She uses the beads to make two necklaces, A and B.

Necklace A
24 beads
3 times as many red beads as blue beads

Necklace B
35 beads
number of red beads : number of blue beads = 3 : 2

How many **more** red beads than blue beads does she use?

[4 marks]

3 There are 60 balls in a bag.
The balls are either blue, red or yellow.

The ratio of blue balls to red balls is 5 : 3
The ratio of red balls to yellow balls is 1 : 4

Work out how many **red** balls are in the bag.

[3 marks]

4 Ahmed, Ben and Carla share some money.

Ahmed gets $\frac{3}{7}$ of the money.

Ben and Carla share the rest of the money in the ratio 3 : 2

Carla gets £64

Work out how much Ahmed gets.

[5 marks]

5 Jack sells packs of envelopes.
There are three types of pack, Economy (E), Standard (S) and Luxury (L).

One week he sells 270 packs in the ratio

E : S : L = 5 : 3 : 2

He makes a profit of

12p a pack on Economy packs
15p a pack on Standard packs
20p a pack on Luxury packs.

Work out the **total** profit he makes selling envelopes.

[3 marks]

6 Three of the notes on a piano keyboard are C, F and G.
Each note has a frequency, measured in Hertz (Hz)

frequency of C : frequency of F = 3 : 4

frequency of F : frequency of G = 8 : 9

(a) Show that frequency of C : frequency of G = 2 : 3

[2 marks]

(b) The frequency of C is 261.6 Hz

Work out the frequency of G.

[2 marks]

7 A blend of tea contains Darjeeling tea and Assam tea in the ratio

Darjeeling : Assam = 4 : 1

Darjeeling tea costs £14 per kilogram.

Assam tea costs £42 per kilogram.

Work out the cost of 250 grams of this blend.

[5 marks]

8 A, B and C are such that

A : B = 1 : 5

B is $\frac{3}{4}$ of C

Work out the ratio A : C

[3 marks]

9 $0.4a = 0.75b$

Work out the ratio $a : b$

Give your answer in its simplest form.

[3 marks]

10 Bob and Kai share £x in the ratio 2 : 3

How much money does Kai receive?
Circle the correct expression.

[1 mark]

£$\frac{2}{3}x$ £$\frac{2}{5}x$ £$\frac{3}{5}x$ £$\frac{5}{8}x$

Bob gives Kai £8 from his share.
The amount of money they each have now is in the ratio 2 : 5

Work out the value of x.
You **must** show your working.
Do **not** use Trial and Improvement.

[4 marks]