



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2025 Mathematics 2026

Unit 19 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



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1 Advanced Indices

Indices Recap

Multiplication Law:

$$a^m \times a^n = a^{m+n}$$

Division Law:

$$a^m \div a^n = a^{m-n} \quad (a \neq 0)$$

Power Law:

$$(a^m)^n = a^{mn}$$

Power of Zero Law:

$$a^0 = 1 \quad (a \neq 0)$$

Other Laws:

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$$

Negative Indices

Worked Example

Evaluate:

- a) 3^{-2}
- b) -3^{-2}
- c) $(-3)^{-2}$

Your Turn

Evaluate:

- a) 5^{-3}
- b) -5^{-3}
- c) $(-5)^{-3}$

Worked Example

Write $\frac{1}{4^2}$ in index form

Your Turn

Write $\frac{1}{5^3}$ in index form

Fill in the Gaps



Negative Indices

| | | | | | | | | | | | | | |
|----------------------------|-----------------|---------------|----------|----------|----------------|-------|----------|---------------|-----------------|----------|----------------|----------|----------|
| | 2^{-1} | 2^{-2} | 3^{-1} | 3^{-2} | | 4^0 | 2^{-3} | | | 2^{-4} | | 4^{-3} | 1^{-4} |
| As a Fraction | $\frac{1}{2^1}$ | | | | | | | | $\frac{1}{5^3}$ | | | | |
| Simplified Fraction | $\frac{1}{2}$ | $\frac{1}{4}$ | | | $\frac{1}{25}$ | | | $\frac{1}{7}$ | | | $\frac{1}{27}$ | | |

Worked Example

Simplify:

a) $\left(\frac{3}{10}\right)^{-2}$

b) $\left(-\frac{3}{10}\right)^{-2}$

Your Turn

Simplify:

a) $\left(\frac{2}{5}\right)^{-3}$

b) $\left(-\frac{2}{5}\right)^{-3}$

Worked Example

Calculate 0.08^{-1}

Leave your answer as a decimal.

Your Turn

Calculate 0.016^{-1}

Leave your answer as a decimal.

Worked Example

Simplify:

a) $(5x^8)^{-3}$

b) $\left(\frac{6p^2q^9}{18p^5q^7}\right)^{-3}$

Your Turn

Simplify:

a) $(4d^6)^{-3}$

b) $\left(\frac{2f^9g^5}{6f^8g^7}\right)^{-3}$

Worked Example

Write the following in the form ax^b in its simplest form.

a) $\frac{1}{x^5}$

b) $\frac{3}{x^5}$

c) $\frac{3}{7x^5}$

d) $\frac{3}{7x^{-5}}$

Your Turn

Write the following in the form ax^b in its simplest form.

a) $\frac{1}{d^{10}}$

b) $\frac{9}{d^{10}}$

c) $\frac{9}{13d^{10}}$

d) $\frac{9}{13d^{-10}}$

Expanding Brackets with Indices

Worked Example

Simplify:

a) $2a^3(3a^2 + 5a^{-4})$

b) $p^{\frac{1}{2}}(2p^{\frac{1}{2}} - p^{-\frac{3}{2}})$

c) $x^2(x^{\frac{1}{3}} - x^{\frac{1}{4}})$

Your Turn

Simplify:

a) $3a^{-2}(4a^5 + 2a)$

b) $2p^{\frac{1}{3}}(3p^{\frac{2}{3}} - p^{-\frac{1}{3}})$

c) $n^{\frac{3}{5}}(n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}})$

Worked Example

Simplify:

$$(2m^9 - m^{-2})(6m^{-3} + m^5)$$

Your Turn

Simplify:

$$(7x^3 - x^{-4})(4x^{-2} + x^9)$$

Fractional Indices

Worked Example

Evaluate:

a) $64^{\frac{1}{2}}$

b) $64^{-\frac{1}{2}}$

c) $\left(\frac{81}{16}\right)^{\frac{1}{4}}$

d) $\left(\frac{81}{16}\right)^{-\frac{1}{4}}$

Your Turn

Evaluate:

a) $64^{\frac{1}{3}}$

b) $64^{-\frac{1}{3}}$

c) $\left(\frac{81}{16}\right)^{\frac{1}{2}}$

d) $\left(\frac{81}{16}\right)^{-\frac{1}{2}}$

Worked Example

Given that $5^{-n} = 0.4$, find the value of $(5^3)^n$

Your Turn

Given that $3^{-n} = 0.2$, find the value of $(3^4)^n$

Fractional Indices

Worked Example

Evaluate:

a) $25^{\frac{3}{2}}$

b) $25^{-\frac{3}{2}}$

c) $\left(\frac{36}{25}\right)^{\frac{3}{2}}$

d) $\left(\frac{36}{25}\right)^{-\frac{3}{2}}$

Your Turn

Evaluate:

a) $81^{\frac{3}{4}}$

b) $81^{-\frac{3}{4}}$

c) $\left(\frac{81}{256}\right)^{\frac{3}{4}}$

d) $\left(\frac{81}{256}\right)^{-\frac{3}{4}}$

Fill in the Gaps



Fractional Indices ①

| | | | | | | | | | | | | | | |
|------------------|-------------------|-------------------|---------------|-------------|--------------------|--------------------|-------------------|----------------------|------------|---------------------|-----------------------|-------------------------|-------------|------------|
| | $4^{\frac{1}{2}}$ | $9^{\frac{1}{2}}$ | | | $27^{\frac{1}{3}}$ | $12^{\frac{1}{2}}$ | | $(-8)^{\frac{1}{3}}$ | $64^{0.5}$ | $125^{\frac{1}{3}}$ | $(-27)^{\frac{1}{3}}$ | $100,000^{\frac{1}{5}}$ | $81^{0.25}$ | $32^{0.2}$ |
| As a Root | $\sqrt[2]{4}$ | | $\sqrt[3]{8}$ | $\sqrt{36}$ | | | $\sqrt[2]{\quad}$ | | | | | | | |
| Integer | | | | | | | 7 | | | | | | | |



Fractional Indices ②

| | | | | | | | | | | | |
|------------------|---|---|-------------------|--------------------|--|-----------------------------------|-------------------|-------------------|-------------|-------------|------------|
| | $8^{\frac{2}{3}}$ | $4^{\frac{3}{2}}$ | $8^{\frac{4}{3}}$ | $16^{\frac{3}{4}}$ | | $32^{\frac{2}{5}}$ | $8^{\frac{5}{3}}$ | $4^{\frac{5}{2}}$ | $81^{0.75}$ | $243^{0.4}$ | $32^{1.2}$ |
| Expanded | $8^{\left(\frac{1}{3} \times 2\right)}$ | $4^{\left(\frac{1}{2} \times 3\right)}$ | | | | | | | | | |
| 2 Indices | $\left(8^{\frac{1}{3}}\right)^2$ | | | | | $\left(27^{\frac{1}{3}}\right)^5$ | | | | | |
| 1 Index | $(2)^2$ | | | | | | | | | | |
| Integer | 4 | | | | | 243 | | | | | |

Worked Example

Simplify:

a) $(81a^{16}b^8)^{\frac{1}{2}}$

b) $(8m^{21}n^{24})^{\frac{2}{3}}$

Your Turn

Simplify:

a) $(100a^8b^{16})^{\frac{1}{2}}$

b) $(25p^2q^6)^{\frac{3}{2}}$

Worked Example

Simplify:

a) $(27p^{18}q^6)^{-\frac{4}{3}}$

b) $\left(\frac{64a^{13}b^6}{27a^4}\right)^{-\frac{2}{3}}$

Your Turn

Simplify:

a) $(32m^{40}n^{10})^{-\frac{2}{5}}$

b) $\left(\frac{8p^{14}q^{15}}{125p^5}\right)^{-\frac{2}{3}}$

Worked Example

Given that $x^n = \sqrt{x} \times \frac{1}{x^4 \sqrt{x^3}}$

Find the value of n .

Your Turn

Given that $a^n = a^2 \times \frac{1}{\sqrt[3]{a^2} \times \sqrt{a}}$

Find the value of n .

Worked Example

Expand and simplify

$$\left(\frac{3\sqrt{x}}{2} + \frac{2}{\sqrt[3]{x}}\right)^2$$

expressing all terms in the form of ax^b

Your Turn

Expand and simplify

$$\left(\frac{2\sqrt[3]{x}}{3} + \frac{3}{4\sqrt[3]{x}}\right)^2$$

expressing all terms in the form of ax^b

Worked Example

Write

$$10\sqrt[3]{x} + \frac{7}{\sqrt{x}} + \frac{2}{7(\sqrt[3]{x})^2} + \frac{1}{x^7}$$

as an expression where all terms are in the form ax^b

Your Turn

Write

$$\frac{4}{3\sqrt[3]{x}} + \frac{9}{\sqrt{x}} + \sqrt[3]{x} - \frac{2}{x^3}$$

as an expression where all terms are in the form ax^b

Worked Example

Full simplify $\frac{\sqrt[3]{27p^2q^7}}{5p^4}$,

writing your answer in the form ap^bq^c .

Your Turn

Full simplify $\frac{\sqrt[3]{125x^3y^7}}{7x^6}$,

writing your answer in the form ax^by^c .

Worked Example

Write

$$\frac{5m^{\frac{3}{2}}n^3 + 2m^{\frac{1}{2}}}{10m^{\frac{3}{2}}}$$

in the form $an^c + bm^d$, where a, b, c and d are constants to be found.

Your Turn

Write

$$\frac{6x^{\frac{8}{5}} - 2x^{\frac{13}{5}}y^2}{3x^{\frac{3}{5}}}$$

in the form $ax + bx^c y^d$, where a, b, c and d are constants to be found.

Worked Example

Express

$$\frac{10x^5 + 6x - 8x^{\frac{2}{3}} + 2x^4}{4\sqrt{x}}$$

in the form of $ax^b + cx^d + ex^f + gx^h$

Your Turn

Express

$$\frac{6x^3 - 3x^{\frac{2}{3}} + 3x + 3x^2}{2\sqrt{x}}$$

in the form of $ax^b + cx^d + ex^f + gx^h$

Change of Base

Worked Example

- a) Write 27 as a power of 3
- b) Write 27^4 as a power of 3
- c) Write 27^x as a power of 3
- d) Write 8^{2x} as a power of 2

Your Turn

- a) Write 8 as a power of 2
- b) Write 8^3 as a power of 2
- c) Write 8^x as a power of 2
- d) Write 8^{3x} as a power of 2

Worked Example

Express 25^{2x-7} in the form 5^y , stating y in terms of x .

Your Turn

Express 8^{8x-6} in the form 2^y , stating y in terms of x .

Worked Example

Write $64^7 \times 4^5 \times 16^{11}$ in the form 4^k where k is an integer.

Your Turn

Write $2^3 \times 4^{19} \times 8^2$ in the form 2^k where k is an integer.

Worked Example

Given that $\frac{12 \times (\sqrt{32})^{3y-1}}{3 \times 4^{2y+3}}$ can be written in the form 2^p , find an expression for p in terms of y

Your Turn

Given that $\frac{12 \times (\sqrt{8})^{2y+2}}{6 \times 4^{2y+1}}$ can be written in the form 2^p , find an expression for p in terms of y

Equations with Indices

Worked Example

- a) Given that $125^q = \frac{1}{25}$ find the value of q
- b) Given that $z^{-4} = \frac{1}{16}$ find the possible values of z

Your Turn

- a) Given that $27^z = \frac{1}{9}$ find the value of z
- b) Given that $q^{-2} = \frac{1}{25}$ find the possible values of q

Worked Example

- a) Given that $(5^5)^z = 0.008$ find the value of z
- b) Given that $(y^{-2})^3 = 0.000001$ find the possible values of y

Your Turn

- a) Given that $(5^{-4})^p = 0.04$ find the value of p
- b) Given that $(z^{-2})^2 = 0.0625$ find the possible values of z

Worked Example

Find the value of x that satisfies:

a) $2^x \times 2^{x-3} = 32$

b) $2^{2x} \div 2^{x-3} = 32$

Your Turn

Find the value of x that satisfies:

a) $3^x \times 3^{x-2} = 81$

b) $3^{3x} \div 3^{x-2} = 81$

Worked Example

Find the value of x that satisfies:

$$125^{\frac{1}{4}} \times 5^{2x+3} = 25^{\frac{2}{3}}$$

Your Turn

Find the value of x that satisfies:

$$64^{\frac{1}{4}} \times 4^{3x+1} = 16^{\frac{2}{3}}$$

Worked Example

Solve

a) $k^{\frac{2}{3}} = 9$

b) $m^{\frac{2}{3}} = 45$

Your Turn

Solve

a) $m^{\frac{4}{3}} = 16$

b) $a^{\frac{2}{5}} = 8$

Worked Example

Solve
 $10x = 810\sqrt[5]{x}$

Your Turn

Solve
 $7x = 63\sqrt[3]{x}$

Worked Example

Solve the equation

$$2x^{\frac{5}{3}} + 5x^{\frac{2}{3}} = 3x^{-\frac{1}{3}}$$

where $x > 0$

Your Turn

Solve the equation

$$2x^{\frac{3}{2}} - 3x^{\frac{1}{2}} = 2x^{-\frac{1}{2}}$$

where $x > 0$

Extra Notes

2 Calculating with Surds

Multiplying Surds

Worked Example

Simplify:

- a) $5 \times \sqrt{6}$
- b) $\sqrt{5} \times \sqrt{6}$
- c) $2\sqrt{5} \times 3\sqrt{6}$

Your Turn

Simplify:

- a) $\sqrt{5} \times \sqrt{7}$
- b) $\sqrt{7} \times 5$
- c) $3\sqrt{5} \times 2\sqrt{7}$

Worked Example

Simplify:

a) $\sqrt{3} \times \sqrt{6}$

b) $4\sqrt{3} \times 5\sqrt{6}$

Your Turn

Simplify:

a) $\sqrt{3} \times \sqrt{8}$

b) $7\sqrt{3} \times 2\sqrt{8}$

Worked Example

Simplify

a) $\sqrt{6} \times \sqrt{6}$

b) $(\sqrt{6})^2$

c) $(2\sqrt{6})^2$

d) $2(\sqrt{6})^2$

e) $2(\sqrt{6})^3$

Your Turn

Simplify

a) $\sqrt{7} \times \sqrt{7}$

b) $(\sqrt{7})^2$

c) $(2\sqrt{7})^2$

d) $2(\sqrt{7})^2$

e) $2(\sqrt{7})^3$

Dividing Surds

Worked Example

Simplify:

a) $\sqrt{60} \div \sqrt{2}$

b) $6\sqrt{60} \div 3\sqrt{2}$

Your Turn

Simplify:

a) $\sqrt{90} \div \sqrt{6}$

b) $8\sqrt{90} \div 2\sqrt{6}$

Worked Example

Simplify:

a) $\sqrt{60} \div \sqrt{3}$

b) $12\sqrt{60} \div 2\sqrt{3}$

Your Turn

Simplify:

a) $\sqrt{90} \div \sqrt{2}$

b) $12\sqrt{90} \div 3\sqrt{2}$

Worked Example

Simplify fully

$$(\sqrt{10})x^2 \div \sqrt{\frac{160}{x^6}}$$

Your Turn

Simplify fully

$$(\sqrt{11})x^3 \div \sqrt{\frac{99}{x^4}}$$

Adding and Subtracting Surds

Worked Example

Simplify:

- a) $2\sqrt{5} + 5\sqrt{5}$
- b) $2\sqrt{20} + 5\sqrt{5}$
- c) $2\sqrt{20} + 5\sqrt{10}$

Your Turn

Simplify:

- a) $2\sqrt{6} + 5\sqrt{6}$
- b) $2\sqrt{54} + 5\sqrt{6}$
- c) $2\sqrt{20} + 5\sqrt{15}$

Worked Example

Simplify:

$$\frac{2\sqrt{20} + 5\sqrt{5}}{\sqrt{5}}$$

Your Turn

Simplify:

$$\frac{2\sqrt{54} - 5\sqrt{6}}{\sqrt{6}}$$

Expanding Brackets with Surds

Worked Example

Expand and simplify:

- a) $2(5 + \sqrt{3})$
- b) $-\sqrt{3}(5 + \sqrt{3})$
- c) $\sqrt{12}(5 + \sqrt{3})$
- d) $\sqrt{12}(\sqrt{5} + \sqrt{3})$

Your Turn

Expand and simplify:

- a) $-2(\sqrt{3} - 5)$
- b) $\sqrt{3}(\sqrt{3} - 5)$
- c) $\sqrt{27}(\sqrt{3} - 5)$
- d) $-\sqrt{27}(\sqrt{3} - \sqrt{5})$

Worked Example

Expand and simplify:

a) $(2 - \sqrt{3})(4 + \sqrt{3})$

b) $(2 - 5\sqrt{3})^2$

Your Turn

Expand and simplify:

a) $(\sqrt{3} - 2)(\sqrt{3} + 4)$

b) $(5\sqrt{3} - 2)^2$

Worked Example

Expand and simplify:

a) $(2 - \sqrt{20})(4 + \sqrt{5})$

b) $(2 - 2\sqrt{20})(4 + 5\sqrt{5})$

Your Turn

Expand and simplify:

a) $(\sqrt{54} - 2)(\sqrt{6} + 4)$

b) $(2\sqrt{54} - 2)(5\sqrt{6} + 4)$

Worked Example

Expand and simplify:

$$(3 + \sqrt{2})^2 - (3 - \sqrt{2})^2$$

Your Turn

Expand and simplify:

$$(2 + \sqrt{3})^2 - (2 - \sqrt{3})^2$$

Worked Example

Expand and simplify:

$$(\sqrt{6} - 1)(4 + \sqrt{6})(5 - \sqrt{6})$$

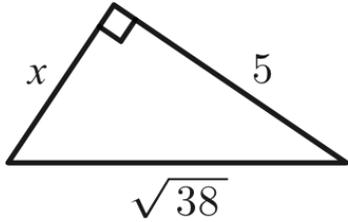
Your Turn

Expand and simplify:

$$(\sqrt{11} - 2)(3 - \sqrt{11})(1 - \sqrt{11})$$

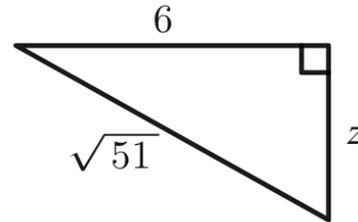
Worked Example

Given that all lengths are in centimetres, find the exact value of x .



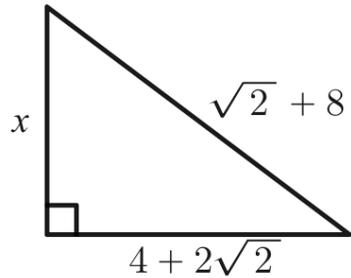
Your Turn

Given that all lengths are in centimetres, find the exact value of z .



Worked Example

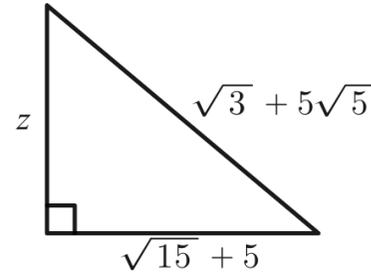
Given that all lengths are in centimetres, find the exact value of x .



Give your answer in simplified surd form.

Your Turn

Given that all lengths are in centimetres, find the exact value of z .



Give your answer in simplified surd form.

Worked Example

Express b and c in terms of a :

$$(a + \sqrt{12})^2 = b + c\sqrt{3}$$

Your Turn

Express b and c in terms of a :

$$(a + \sqrt{8})^2 = b + c\sqrt{2}$$

Worked Example

Find the value of a and b :

$$(a - 3\sqrt{5})^2 = b - 42\sqrt{5}$$

Your Turn

Find the value of a and b :

$$(a - 2\sqrt{3})^2 = b - 20\sqrt{3}$$

Rationalising Surds

Worked Example

Rationalise and simplify if possible:

a) $\frac{3}{\sqrt{7}}$

b) $\frac{3}{\sqrt{6}}$

c) $\frac{3}{2\sqrt{6}}$

d) $\frac{3\sqrt{5}}{2\sqrt{6}}$

Your Turn

Rationalise and simplify if possible:

a) $\frac{10}{\sqrt{11}}$

b) $\frac{10}{\sqrt{5}}$

c) $\frac{10}{4\sqrt{5}}$

d) $\frac{10\sqrt{3}}{4\sqrt{5}}$

Worked Example

Express

$$\frac{30}{\sqrt{6}} - \sqrt{24}$$

in the form $a\sqrt{6}$ where a is an integer to be found.

Your Turn

Express

$$\frac{21}{\sqrt{7}} + \sqrt{28}$$

in the form $a\sqrt{7}$ where a is an integer to be found.

Worked Example

Express

$$\frac{4 - \sqrt{32}}{\sqrt{2}}$$

in the form $a + b\sqrt{2}$ where a and b are constants to be found.

Your Turn

Express

$$\frac{14 - \sqrt{28}}{\sqrt{7}}$$

in the form $a + b\sqrt{7}$ where a and b are constants to be found.

Worked Example

Express

$$\left(\frac{2}{\sqrt{6}}\right)^5$$

in the form $\frac{a\sqrt{6}}{27}$ where a is an integer.

Your Turn

Express

$$\left(\frac{2}{\sqrt{3}}\right)^7$$

in the form $\frac{a\sqrt{3}}{81}$ where a is an integer.

Worked Example

Solve the equation:

$$2\sqrt{2}x = 3$$

Give your answer in the form $\frac{a\sqrt{b}}{c}$ where a , b , and c are integers to be found.

Your Turn

Solve the equation:

$$5\sqrt{3}x = 8\sqrt{5}$$

Give your answer in the form $\frac{a\sqrt{b}}{c}$ where a , b , and c are integers to be found.

Worked Example

A rectangle has area 64 cm^2 and a width of $\sqrt{32} \text{ cm}$. Find the length of the rectangle in the form $a\sqrt{b}$

Your Turn

A rectangle has area 60 cm^2 and a width of $\sqrt{12} \text{ cm}$. Find the length of the rectangle in the form $a\sqrt{b}$

Conjugates

Is $\sqrt{3} - 1$ the conjugate of $\sqrt{3} + 1$?

Is $-\sqrt{3} + 1$ the conjugate of $\sqrt{3} + 1$?

Is $-\sqrt{3} + 1$ the conjugate of $1 + \sqrt{3}$?

Is $1 - \sqrt{3}$ the conjugate of $1 + \sqrt{3}$?

Is $-1 - \sqrt{3}$ the conjugate of $1 - \sqrt{3}$?

Is $1 + \sqrt{3}$ the conjugate of $1 - \sqrt{3}$?

Is $1 + \sqrt{5}$ the conjugate of $1 - \sqrt{5}$?

Is $1 - 3\sqrt{5}$ the conjugate of $1 + 3\sqrt{5}$?

Is $3\sqrt{5} - 1$ the conjugate of $1 + 3\sqrt{5}$?

Is $3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} + 1$?

Is $-3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} + 1$?

Is $-3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} - 1$?

Worked Example

Rationalise the denominator of

$$\frac{6}{\sqrt{3} + 1}$$

giving your answer in the form $a + b\sqrt{3}$

Your Turn

Rationalise the denominator of

$$\frac{5}{\sqrt{6} - 1}$$

giving your answer in the form $a + b\sqrt{6}$

Worked Example

Rationalise the denominator of

$$\frac{8\sqrt{3}}{3 - \sqrt{5}}$$

giving your answer in the form $a\sqrt{3} + b\sqrt{15}$

Your Turn

Rationalise the denominator of

$$\frac{3\sqrt{3}}{\sqrt{7} - 2}$$

giving your answer in the form $a\sqrt{3} + b\sqrt{21}$

Worked Example

Rationalise the denominator of
 $\frac{8}{\sqrt{7} + \sqrt{3}}$

$$\frac{8}{\sqrt{7} + \sqrt{3}}$$

Give your answer in the form $a\sqrt{7} + b\sqrt{3}$

Your Turn

Rationalise the denominator of
 $\frac{4}{\sqrt{6} - \sqrt{2}}$

$$\frac{4}{\sqrt{6} - \sqrt{2}}$$

Give your answer in the form $a\sqrt{6} + b\sqrt{2}$

Worked Example

Rationalise the denominator of

$$\frac{4}{2\sqrt{5} + 4}$$

Give your answer in the form $a + b\sqrt{5}$

Your Turn

Rationalise the denominator of

$$\frac{12\sqrt{2}}{9 + 5\sqrt{3}}$$

Give your answer in the form $a\sqrt{2} + b\sqrt{6}$

Worked Example

Rationalise the denominator of

$$\frac{2 + \sqrt{5}}{\sqrt{5} - 2}$$

Give your answer in the form $a + b\sqrt{5}$

Your Turn

Rationalise the denominator of

$$\frac{9 + \sqrt{6}}{\sqrt{6} - 1}$$

Give your answer in the form $a + b\sqrt{6}$

Worked Example

Rationalise the denominator of

$$\frac{3\sqrt{5} + 5}{5 - 2\sqrt{5}}$$

Give your answer in the form $a + b\sqrt{5}$

Your Turn

Rationalise the denominator of

$$\frac{4\sqrt{2} - 3}{3 - 2\sqrt{2}}$$

Give your answer in the form $a + b\sqrt{2}$

Fill in the Gaps

| Question | Working | | Answer |
|--|------------------------------------|--|-------------------------------------|
| $\frac{5}{\sqrt{3}}$ | $\times \frac{\sqrt{3}}{\sqrt{3}}$ | $= \frac{5\sqrt{3}}{\sqrt{9}}$ | $= \frac{5\sqrt{3}}{3}$ |
| $\frac{\sqrt{3}}{\sqrt{7}}$ | $\times \frac{\sqrt{7}}{\sqrt{7}}$ | | |
| $\frac{5\sqrt{5}}{\sqrt{6}}$ | | | |
| $\frac{2 + \sqrt{3}}{\sqrt{5}}$ | $\times \frac{\sqrt{5}}{\sqrt{5}}$ | $= \frac{\sqrt{5}(2 + \sqrt{3})}{\sqrt{25}}$ | $= \frac{2\sqrt{5} + \sqrt{15}}{5}$ |
| $\frac{3 - \sqrt{5}}{\sqrt{2}}$ | | | |
| $\frac{1 + \sqrt{2}}{2\sqrt{3}}$ | | | |
| $\frac{\sqrt{2} - 3\sqrt{5}}{5\sqrt{2}}$ | | | |

| Question | Working | | Answer |
|-------------------------------------|--|---|-------------------------------|
| $\frac{3}{2 + \sqrt{2}}$ | $\times \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$ | $= \frac{3(2 - \sqrt{2})}{4 - \sqrt{4}}$ | $= \frac{6 - 3\sqrt{2}}{2}$ |
| $\frac{8}{4 - \sqrt{3}}$ | | | |
| $\frac{\sqrt{5}}{6 + \sqrt{5}}$ | | | |
| $\frac{3\sqrt{5}}{3 - \sqrt{7}}$ | | | |
| $\frac{7 + \sqrt{2}}{3 - \sqrt{2}}$ | $\times \frac{3 + \sqrt{2}}{3 + \sqrt{2}}$ | $= \frac{(7 + \sqrt{2})(3 + \sqrt{2})}{9 - \sqrt{4}}$ | $= \frac{23 + 10\sqrt{2}}{7}$ |
| $\frac{1 - \sqrt{8}}{5 + \sqrt{2}}$ | | | |
| $\frac{a + \sqrt{b}}{a\sqrt{b}}$ | | | |

Worked Example

Show that

$$\frac{3 - \sqrt{54}}{3 + \sqrt{6}}$$

$$\frac{3 - \sqrt{54}}{3 + \sqrt{6}}$$

can be written in the form $a + b\sqrt{6}$ where a and b are integers

Your Turn

Show that

$$\frac{9 - \sqrt{75}}{2 - \sqrt{3}}$$

$$\frac{9 - \sqrt{75}}{2 - \sqrt{3}}$$

can be written in the form $a + b\sqrt{3}$ where a and b are integers

Worked Example

Show that

$$\frac{4 + 2\sqrt{54}}{\sqrt{6} - 1}$$

can be written in the form $a + b\sqrt{6}$ where a and b are integers

Your Turn

Show that

$$\frac{8 + 5\sqrt{12}}{\sqrt{3} + 1}$$

can be written in the form $a + b\sqrt{3}$ where a and b are integers

Worked Example

Show that

$$\frac{3 - \sqrt{24}}{(1 - \sqrt{6})^2}$$

can be written in the form $\frac{a+b\sqrt{6}}{c}$ where a , b and c are integers in their simplest form

Your Turn

Show that

$$\frac{\sqrt{150} - 3}{(1 + \sqrt{6})^2}$$

can be written in the form $\frac{a+b\sqrt{6}}{c}$ where a , b and c are integers in their simplest form

Worked Example

Solve

$$(2 - \sqrt{2})x = 6$$

giving your answer in the form $a + b\sqrt{2}$

Your Turn

Solve

$$(\sqrt{5} + 2)x = 2$$

giving your answer in the form $a + b\sqrt{5}$

Worked Example

Solve

$$3x - \sqrt{45} = \sqrt{6}x$$

giving your answer in the form $a\sqrt{5} + b\sqrt{30}$

Your Turn

Solve

$$\sqrt{6}x - \sqrt{20} = 2x$$

giving your answer in the form $a\sqrt{30} + b\sqrt{5}$

Worked Example

A rectangle has an area of $(2 + \sqrt{2}) \text{ cm}^2$ and a width of $(3\sqrt{2} - 4) \text{ cm}$. Find the length of the rectangle in the form $a + b\sqrt{2}$

Your Turn

A rectangle has an area of $(15 - 6\sqrt{3}) \text{ cm}^2$ and a width of $(2\sqrt{3} - 3) \text{ cm}$. Find the length of the rectangle in the form $a + b\sqrt{3}$

Worked Example

Rationalise:

$$\frac{4}{\frac{1}{\sqrt{3}} + \sqrt{3}}$$

Your Turn

Rationalise:

$$\frac{3}{\sqrt{2} + \frac{1}{\sqrt{2}}}$$

Extra Notes

3 Algebraic Fractions

Simplifying Algebraic Fractions

Worked Example

Simplify
$$\frac{6xy^2}{10x^2y}$$

Your Turn

Simplify
$$\frac{8x^2y}{12xy^3}$$

Fill in the Gaps

| Question | Write Each Term Separately | Simplify Each Variable | Answer |
|-----------------------|--|------------------------|--------|
| $\frac{10a^2}{2a}$ | $\frac{10 \times a^2}{2 \times a}$ | $5 \times a$ | $5a$ |
| $\frac{9b}{3b}$ | $\frac{9 \times b}{3 \times b}$ | | |
| $\frac{2a}{4b}$ | $\frac{2 \times a}{4 \times b}$ | | |
| $\frac{12a^3}{3a}$ | | | |
| $\frac{5ab}{a}$ | | | |
| $\frac{15a^2}{5ab}$ | | | |
| $\frac{24ab}{4bc}$ | $\frac{24 \times a \times b}{4 \times b \times c}$ | | |
| $\frac{6ab^2}{3a^2b}$ | | | |

| Question | Write Each Term Separately | Simplify Each Variable | Answer |
|----------------------------|---|------------------------|--------|
| $\frac{4a^7}{2a^3}$ | $\frac{4 \times a^7}{2 \times a^3}$ | $2 \times a^4$ | |
| $\frac{16b^5}{2b^2}$ | | | |
| $\frac{2a^9}{10a^2}$ | | | |
| $\frac{5a^6}{2a}$ | | | |
| $\frac{25ab^7}{5b^2}$ | $\frac{25 \times a \times b^7}{5 \times b^2}$ | | |
| $\frac{14a^4b^8}{2a^3b^6}$ | | | |
| $\frac{9ab^5}{3a^2b^3}$ | | | |
| $\frac{15a^4b}{25a^2b^2}$ | | | |

Worked Example

Simplify

a) $\frac{8(2z-1)}{7z(2z-1)}$

b) $\frac{(4z-7)(8z-3)}{(7z-1)(4z-7)}$

Your Turn

Simplify

a) $\frac{7x(5x-1)}{9(5x-1)}$

b) $\frac{(6x-4)(3x-7)}{(3x-7)(6x-9)}$

Worked Example

Simplify

a) $\frac{x+3}{2x^2+6x}$

b) $\frac{5x+25}{4x^2+20x}$

Your Turn

Simplify

a) $\frac{x+4}{4x^3+16x^2}$

b) $\frac{6x^2+9x}{6x^4+9x^3}$

Worked Example

Simplify

a) $\frac{x+5}{x^2+x-20}$

b) $\frac{x^2-x-2}{x^2-4}$

Your Turn

Simplify

a) $\frac{x+3}{x^2-x-12}$

b) $\frac{x^2-9}{x^2+6x+9}$

Worked Example

Simplify

$$\frac{2x^2 + 14x + 24}{3x^2 - 15x - 108}$$

Your Turn

Simplify

$$\frac{2x^2 + 14x + 24}{3x^2 + 15x + 18}$$

Worked Example

Simplify

a) $\frac{x+1}{2x^2+7x+5}$

b) $\frac{2x^2+5x-25}{2x^2+x-15}$

Your Turn

Simplify

a) $\frac{4x+3}{4x^2+7x+3}$

b) $\frac{4x^2+21x+20}{4x^2-3x-10}$

Worked Example

Simplify

$$\frac{7x - 3x^2 + 20}{3x^2 - 16x + 16}$$

Your Turn

Simplify

$$\frac{2x^2 - x - 6}{x - 3x^2 + 10}$$

Multiplying and Dividing Algebraic Fractions

Worked Example

Simplify:

a) $\frac{6x}{2y^2} \times \frac{4y}{5x}$

b) $\frac{6x}{2y^2} \div \frac{4y}{5x}$

Your Turn

Simplify:

a) $\frac{5a^2}{2b} \times \frac{5b}{30a^3}$

b) $\frac{5a^2}{2b} \div \frac{5b}{30a^3}$

Fill in the Gaps

| Question | Write as a Single Fraction | Simplify Numerator and Denominator | Simplified Answer (where possible) |
|---|--|------------------------------------|---------------------------------------|
| $\frac{x}{4} \times \frac{2x}{3}$ | $\frac{x \times 2x}{4 \times 3}$ | $\frac{2x^2}{12}$ | $\frac{x^2}{6}$ |
| $x \times \frac{4y}{6}$ | $\frac{x \times 4y}{6 \times 5}$ | $\frac{4xy}{30}$ | |
| $2 \times \frac{3xy}{x}$ | $\frac{2 \times 3xy}{x \times 5}$ | | |
| $\frac{4x}{3y} \times \frac{2y}{x}$ | | | |
| $2 \times \frac{3xy}{7x}$ | | | |
| $\frac{x^2}{8} \times \frac{4y}{x}$ | | | |
| $\frac{2y}{x} \times \frac{9x^2y}{4}$ | | | |
| $\frac{10y}{x^2} \times \frac{3xy^2}{5}$ | | | |
| $\frac{4yz}{3} \times \frac{x^2}{6y^3}$ | | | |
| $\frac{2x^3}{15yz} \times \frac{5x^2y^2}{z^3}$ | | | |
| $\frac{\square}{5y^2} \times \frac{4x^2y}{\square}$ | $\frac{\square \times 4x^2y}{5y^2 \times \square}$ | $\frac{24x^3y}{15y^2}$ | |
| $\frac{12x}{5yz} \times \frac{\square}{\square}$ | | | $\frac{6x^3y}{25z}$ |

Fill in the Gaps

| Question | Write as a Multiplication | Simplify Numerator and Denominator | Simplified Answer (where possible) |
|---|---|------------------------------------|---------------------------------------|
| $\frac{x}{4} \div \frac{2y}{2y}$ | $\frac{x}{4} \times \frac{2y}{x}$ | $\frac{2xy}{4x}$ | $\frac{y}{2}$ |
| $\frac{3x}{y} \div \frac{1}{xy}$ | $\frac{3x}{y} \times \frac{xy}{1}$ | $\frac{3x^2y}{y}$ | |
| $\frac{2y}{3} \div \frac{4}{y}$ | $\frac{2y}{3} \times \frac{y}{4}$ | | |
| $\frac{xy}{5} \div \frac{3y}{10}$ | | | |
| $\frac{4y}{3x} \div \frac{xy}{6}$ | | | |
| $\frac{x^2}{8} \div \frac{3x}{4y}$ | | | |
| $\frac{6xy}{5} \div \frac{x}{y}$ | | | |
| $\frac{5x}{3} \div \frac{10}{xy}$ | | | |
| $\frac{2x}{y} \div \frac{4xy}{9}$ | | | |
| $\frac{4}{5xy} \div \frac{2y^2}{x^2}$ | | | |
| $\frac{\square}{y} \div \frac{\square}{3y^2}$ | $\frac{\square}{y} \times \frac{3y^2}{\square}$ | $\frac{6xy^2}{8y}$ | |
| $\frac{5x}{2y} \div \frac{\square}{\square}$ | | | 10y |

Worked Example

Simplify:

a) $\frac{8(p-2)^4}{3q^2} \times \frac{5p^4}{8(p-2)^3}$

b) $\frac{2(p+2)^4}{7q} \div \frac{4(p+2)^2}{9q}$

Your Turn

Simplify:

a) $\frac{4x^3}{7(y-1)^4} \times \frac{5(y-1)^2}{7x^4}$

b) $\frac{3(p+4)^4}{10q^3} \div \frac{7(p+4)^2}{4q^2}$

Worked Example

a) Express

$$\frac{2x-4}{6x+12} \times \frac{3x+12}{10x-20}$$

as a single fraction in its simplest form.

b) Express

$$\frac{20x-80}{9x+36} \div \frac{10x-30}{3x+12}$$

as a single fraction in its simplest form.

Your Turn

a) Express

$$\frac{5x-5}{4x+16} \times \frac{8x+32}{15x+45}$$

as a single fraction in its simplest form.

b) Express

$$\frac{20x+60}{9x+36} \div \frac{10x+30}{3x+6}$$

as a single fraction in its simplest form.

Worked Example

Simplify fully:

$$\frac{x^2 - 9}{x^2 - x - 12} \times \frac{x - 4}{2x - 6}$$

Your Turn

Simplify fully:

$$\frac{x^3 - 4x^2}{x^2 - 9x + 20} \times \frac{x^2 - 25}{5x}$$

Worked Example

Simplify fully:

$$\frac{2x + 6}{x^2 + 2x - 3} \div \frac{10x + 50}{x^2 - 1}$$

Your Turn

Simplify fully:

$$\frac{5x - 10}{x^2 - 25} \times \frac{20x - 40}{x^2 + x - 20}$$

Worked Example

Simplify fully:

$$\frac{2x^2 + 7x - 15}{x^2 - 36} \times \frac{2x + 12}{2x^3 - 3x^2}$$

Your Turn

Simplify fully:

$$\frac{2x^2 - 17x + 21}{x^2 - 49} \times \frac{5x^2 + 15x}{2x^2 - 3x}$$

Worked Example

Simplify fully:

$$\frac{3x^2 + 8x + 5}{x^2 - 25} \div \frac{3x^2 + 5x}{5x^2 - 25x}$$

Your Turn

Simplify fully:

$$\frac{3x^2 - x - 14}{9x^2 - 4} \div \frac{x + 2}{3x^2 + 2x}$$

Adding and Subtracting Algebraic Fractions

Fill in the Gaps

| Question | With a Common Denominator | Unsimplified Answer | Simplified Answer (where possible) |
|----------------------------------|---|---------------------|---------------------------------------|
| $\frac{x}{4} + \frac{7x}{20}$ | $\frac{5x}{20} + \frac{7x}{20}$ | $\frac{12x}{20}$ | |
| $\frac{7x}{18} - \frac{2x}{9}$ | $\frac{7x}{18} - \frac{4x}{18}$ | | |
| $\frac{2x}{3} + \frac{x}{4}$ | $\frac{\square}{12} + \frac{\square}{12}$ | | |
| $\frac{17x}{30} + \frac{x}{10}$ | $\frac{17x}{\square} + \frac{3x}{\square}$ | | |
| $\frac{x}{6} + \frac{11x}{24}$ | | | |
| $\frac{3x}{4} - \frac{7x}{36}$ | | | |
| $\frac{7}{2x} + \frac{3}{x}$ | $\frac{\square}{2x} + \frac{\square}{2x}$ | | |
| $\frac{6}{5x} - \frac{9}{20x}$ | | | |
| $\frac{5}{x} + \frac{2}{x^2}$ | $\frac{\square}{x^2} + \frac{2}{x^2}$ | | |
| | $\frac{\square}{xy} - \frac{\square}{xy}$ | $\frac{3-y}{xy}$ | |
| | $\frac{\square}{4x^2} + \frac{\square}{4x^2}$ | $\frac{7x+6}{4x^2}$ | |
| $\frac{3}{10xy} - \frac{2}{x^2}$ | | | |

Worked Example

Simplify

a) $\frac{5x+2}{3} + \frac{4x-3}{2}$

b) $\frac{5x+2}{3} - \frac{4x-3}{2}$

Your Turn

Simplify

a) $\frac{4x-5}{2} + \frac{6x-1}{3}$

b) $\frac{4x-5}{2} - \frac{6x-1}{3}$

Worked Example

Simplify

$$\frac{7}{4x} + \frac{8}{5x}$$

Your Turn

Simplify

$$\frac{4}{3x} - \frac{5}{8x}$$

Worked Example

Simplify

a) $\frac{3}{5x-3} + \frac{8}{4x+1}$

b) $\frac{3}{5x-3} - \frac{8}{4x+1}$

Your Turn

Simplify

a) $\frac{5}{3x+4} + \frac{2}{7x-4}$

b) $\frac{5}{3x+4} - \frac{2}{7x-4}$

Worked Example

Express

$$\frac{5}{8x + 12} - \frac{1}{6x + 9}$$

as a single fraction in the form $\frac{A}{B(Cx+D)}$ where A , B , C , and D are integers to be found.

Your Turn

Express

$$\frac{x}{30x + 5} - \frac{3}{12x + 2}$$

as a single fraction in the form $\frac{Ax+B}{C(Dx+E)}$ where A , B , C , D , and E are integers to be found.

Worked Example

Simplify

$$6 - \frac{2x^2 - 13x + 20}{(2x - 5)(3x + 2)}$$

giving your answer in the form $\frac{ax+b}{cx+d}$

Your Turn

Simplify

$$1 - \frac{x - 4}{(2x + 5)(3x + 5)}$$

giving your answer in the form $\frac{ax^2+bx+c}{(dx+e)(fx+g)}$

Worked Example

Simplify

$$\frac{x}{x+1} - \frac{4x+3}{(2x+3)(x+1)}$$

giving your answer in the form $\frac{ax+b}{cx+d}$

Your Turn

Simplify

$$\frac{3x}{3x+2} - \frac{5x-4}{(3x+2)(2x+5)}$$

giving your answer in the form $\frac{a(x+b)}{cx+d}$

Worked Example

Simplify

$$\frac{10}{x^2 - 14x + 24} - \frac{1}{x - 12}$$

Leave the denominator in factorised form when applicable.

Your Turn

Simplify

$$\frac{7}{y^2 + 6y - 40} - \frac{6}{y - 4}$$

Leave the denominator in factorised form when applicable.

Worked Example

Simplify

$$\frac{2}{a^2 - 36} - \frac{3}{a^2 + 7a + 6}$$

Leave the denominator in factorised form when applicable.

Your Turn

Simplify

$$\frac{2}{y^2 - 9} - \frac{5}{y^2 + 7y + 12}$$

Leave the denominator in factorised form when applicable.

Worked Example

Simplify

$$\frac{1}{2x+1} - \frac{x-4}{20x^2-8x} + \frac{2}{8x^2+4x}$$

Give your answer in the form $\frac{A}{B(Cx+D)}$ where A , B , C and D are integers to be found.

Your Turn

Simplify

$$\frac{1}{3y-1} + \frac{y-2}{30y^2+6y} - \frac{2}{18y^2-6y}$$

Give your answer in the form $\frac{A}{B(Cy+D)}$ where A , B , C and D are integers to be found.

Worked Example

Express

$$\frac{x}{x-2} \div \left(\frac{5}{x-2} + \frac{1}{x+1} \right)$$

as a single fraction in the form $\frac{x(x+b)}{a(cx+d)}$

Your Turn

Express

$$\left(\frac{11}{2x+3} + \frac{7}{x-1} \right) \div \frac{x+3}{x-1}$$

as a single fraction in the form $\frac{a(bx+c)}{(dx+e)(x+f)}$

Worked Example

Write as a single simplified fraction:

$$3 - (x - 4) \div \frac{x^2 - 16}{x - 5}$$

Your Turn

Write as a single simplified fraction:

$$5 - (x - 2) \div \frac{x^2 - 4}{x + 3}$$

Worked Example

Express

$$x - 1 - \frac{5x^2 - 16x + 3}{2x + 5} \div \frac{5x^2 - 21x + 4}{x^2 - 9x + 20}$$

as a single fraction in the form $\frac{ax^2+bx+c}{dx+e}$

Your Turn

Express

$$x - 3 - \frac{3x^2 + 7x}{3x^2 + x - 14} \div \frac{2x + 5}{3x^2 - 8x + 4}$$

as a single fraction in the form $\frac{ax^2+bx+c}{dx+e}$

Solving Equations with Algebraic Fractions

Worked Example

Solve

$$\frac{x+4}{2} + \frac{x+1}{5} = 5$$

Your Turn

Solve

$$\frac{x-4}{2} + \frac{x-1}{5} = 2$$

Worked Example

Solve

$$\frac{x+1}{3} - \frac{x-3}{5} = 1$$

Your Turn

Solve

$$\frac{x+2}{3} - \frac{x-6}{5} = 2$$

Worked Example

Solve

$$\frac{3m + 6}{2} - \frac{m - 4}{4} = 3m - 6$$

Your Turn

Solve

$$\frac{4x - 2}{2} - \frac{4x - 1}{3} = x + 4$$

Worked Example

Solve

$$\frac{3m - 5}{5} = \frac{3m - 2}{2} - 2m + 3$$

Your Turn

Solve

$$\frac{3x + 4}{2} = \frac{4x - 1}{2} - 3x + 1$$

Worked Example

Solve

$$\frac{4}{x+6} + \frac{5}{x+8} = 1$$

Your Turn

Solve

$$\frac{4}{x+3} + \frac{5}{x+4} = 2$$

Worked Example

Solve

$$\frac{3}{x-6} + \frac{4}{x-9} = 1$$

Your Turn

Solve

$$\frac{3}{x-2} + \frac{4}{x-3} = 3$$

Worked Example

Solve the following equation:

$$\frac{4}{3x^2 + x - 14} - \frac{3}{x - 2} = 2$$

Give your answer in exact form.

Your Turn

Solve the following equation:

$$\frac{4}{x - 2} - \frac{3}{2x^2 - x - 6} = 2$$

Give your answer in exact form.

Rearranging Formulae with Algebraic Fractions

Worked Example

Make x the subject:

$$\frac{y}{a} + \frac{3y}{x-2} = 5$$

Your Turn

Make x the subject:

$$\frac{5p}{x+3} + \frac{p}{b} = 2$$

Worked Example

Make x the subject:

$$\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$$

Your Turn

Make p the subject:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$$

Worked Example

Make x the subject:

$$\frac{5x}{A} - \frac{b}{c} = \frac{4x - d}{a}$$

Your Turn

Make x the subject:

$$\frac{3x}{E} - \frac{f}{g} = \frac{5x - h}{F}$$

Extra Notes