Year 10

2025   Mathematics   2026

Unit 19 Tasks – Part 1

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Year 10
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Unit 19 Tasks – Part 2

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Year 10

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Unit 19 Tasks – Part 3

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1 Advanced Indices

For the following terms,

    a) Write with a positive exponent
    b) Evaluate:

1.	26^{-1}	2.	2^{-1}	3.	10^{-2}	4.	2^{-2}
5.	-26^{-1}	6.	-2^{-1}	7.	-10^{-2}	8.	-2^{-2}
9.	$(-26)^{-1}$	10.	$(-2)^{-1}$	11.	$(-10)^{-2}$	12.	$(-2)^{-2}$
13.	2^{-5}	12.	-7^{-3}	13.	$(-8)^{-2}$	14.	$(-10)^{-5}$
15.	$(-4)^{-3}$	16.	9^{-4}	17.	-11^{-2}	18.	$(-3)^{-3}$
19.	-3^{-4}	20.	25^{-2}	21.	$(-2)^{-6}$	22.	15^{-2}

Decide if there are mistakes in the following and explain how to fix the answer:

a) $4^{-2} = -16$ b) $10^{-3} = \frac{1}{30}$

1. Write in index form:

   a) $\frac{1}{5^2}$ b) $\frac{1}{3^4}$ c) $\frac{1}{8^3}$ d) $\frac{1}{4^5}$ e) $\frac{1}{10^3}$ f) $\frac{1}{2^6}$

2. Write in the form 2^n:

   a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) $\frac{1}{32}$ e) $\frac{1}{64}$ f) $\frac{1}{256}$

3. Write in the form 5^n:

   a) $\frac{1}{5}$ b) $\frac{1}{125}$ c) $\frac{1}{625}$ d) $\frac{1}{3125}$ e) $\frac{1}{78,125}$ f) $\frac{1}{390,625}$

4. Arrange in ascending order:

$$\frac{1}{50} \qquad 5^{-2} \qquad \frac{3}{10} \qquad 2^{-3}$$

Fluency Practice

Simplify the following:

1. $\left(\frac{1}{5}\right)^{-1}$

2. $\left(\frac{2}{5}\right)^{-1}$

3. $\left(\frac{3}{5}\right)^{-1}$

4. $\left(\frac{4}{5}\right)^{-1}$

5. $\left(\frac{1}{4}\right)^{-1}$

6. $\left(\frac{3}{4}\right)^{-1}$

7. $\left(\frac{1}{3}\right)^{-2}$

8. $\left(\frac{2}{3}\right)^{-2}$

9. $\left(\frac{1}{5}\right)^{-2}$

10. $\left(\frac{2}{5}\right)^{-2}$

11. $\left(\frac{3}{5}\right)^{-2}$

12. $\left(\frac{4}{5}\right)^{-2}$

13. $\left(\frac{2}{3}\right)^{-3}$

14. $\left(\frac{4}{3}\right)^{-3}$

15. $\left(\frac{7}{8}\right)^{-2}$

16. $\left(-\frac{1}{10}\right)^{-4}$

17. $\left(-\frac{4}{9}\right)^{-3}$

18. $\left(-\frac{9}{10}\right)^{-2}$

19. $\left(-\frac{5}{3}\right)^{-3}$

20. $\left(-\frac{3}{2}\right)^{-4}$

21. $\left(-\frac{3}{10}\right)^{-3}$

22. $\left(-\frac{3}{2}\right)^{-3}$

23. $\left(-\frac{8}{5}\right)^{-3}$

24. $\left(-\frac{3}{8}\right)^{-2}$

25. $\left(-\frac{6}{5}\right)^{-2}$

Fluency Practice

Write down the reciprocal of:

(a) 4 (b) 10 (c) 15

(d) -3 (e) 0.4 (f) 1

(g) $\dfrac{3}{4}$ (h) $\dfrac{1}{8}$ (i) $\dfrac{2}{7}$

Evaluate:

(a) 4^{-1} (b) 10^{-1}

(c) 15^{-1} (d) $(-3)^{-1}$

(e) a^{-1} (f) $\left(\dfrac{2}{3}\right)^{-1}$

(g) $\left(\dfrac{8}{5}\right)^{-1}$ (h) $\left(1\dfrac{3}{7}\right)^{-1}$

Evaluate:

(a) 4^{-2} (b) 10^{-3}

(c) 15^{-2} (d) $(-3)^{-2}$

(e) b^{-3} (f) $\left(\dfrac{1}{3}\right)^{-2}$

(g) $\left(\dfrac{2}{5}\right)^{-3}$ (h) 0.1^{-2}

Find x.

(a) $\dfrac{1}{10} = 10^{x}$ (b) $\dfrac{1}{1000} = 10^{x}$

(c) $\dfrac{1}{2} = 2^{x}$ (d) $\dfrac{1}{8} = 2^{x}$

(e) $\dfrac{1}{16} = 2^{x}$ (f) $\dfrac{1}{256} = 2^{x}$

Fluency Practice

1 Complete each collection of equivalent expressions.

Example

5^{-2}

$\dfrac{1}{25}$

$\dfrac{1}{5} \times \dfrac{1}{5}$

$1 \div 5 \div 5$

25^{-1}

$\dfrac{1}{5^2}$

(a)

$\dfrac{1}{64}$

(b)

2^{-3}

(c)

$1 \div 3 \div 3 \div 3 \div 3$

(d)

$\dfrac{1}{4} \times \dfrac{1}{4}$

(e)

$\dfrac{3}{16}$

(f)

$\dfrac{4}{16}$

(g)

$4^{-2} \times 3^{-1}$

Fluency Practice

Positive, Negative & Zero Indices

Match these calculations to their answers, which are jumbled up below. Record your answers in the table at the bottom.

A	B	C	D	E
$3^2 \times 2^0$	$3^{-1} \times 3$	$(-2)^2$	$6^2 \div 6^1$	30×3^{-1}

F	G	H	I	J
2×3^{-1}	$2^{-1} \times 6^2$	$5^{-1} \times 5^{-1}$	10×5^{-1}	$-2^1 \times 2^1$

K	L	M	N	O
$2^2 \times 2^3$	$25 \div 5^0$	11×2^0	$3^1 \times 3^0$	$5^{-1} \times 2^{-2}$

P	Q	R	S	T
$2^{-1} \div 2^1$	$(3^2)^{-1}$	$2^{-1} \times 2 \times 2^3$	$(-2 \times -3)^{-1}$	$2^{-2} \times 3$

U	V	W	X	Y
$2 \times 5^2 \times 10^{-1}$	63×3^{-2}	$12 \div 6^0$	$5^{-1} + 5^{-1}$	$(4^{-2} \times 4^1)^2$

$\frac{2}{3}$ 1 $\frac{1}{25}$ 6 32 $\frac{1}{6}$ 18 $\frac{1}{9}$ 7 9 $\frac{1}{16}$ 10 25 8 3 $\frac{1}{20}$ $\frac{3}{4}$ 5 -4 4 2 12 $\frac{1}{4}$ $\frac{2}{5}$

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Fluency Practice

(a) $$\left(\frac{3}{4}\right)^{-1} + 5^{-1}$$	(d) $$\left(\frac{2}{3}+\frac{4}{5}\right)^{-1}$$	(g) $$6^{-2} + 3^{-3}$$
(b) $$\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{12}\right)^{-2}$$	(e) $$\left(2\frac{1}{3}\right)^{-2} - \left(1\frac{3}{4}\right)^{-2}$$	(h) $$\left(\frac{2}{x}\right)^{-3} + \left(\frac{5}{x}\right)^{-1}$$
(c) $$\left(\frac{4}{7}\right)^{-2} + \left(1\frac{5}{11}\right)^{-1}$$	(f) $$0.4^{-1} + 0.75^{-2}$$	(i) $$\left(\frac{1}{3x}\right)^{-2} + \left(\frac{3}{2x}\right)^{-3}$$

Fluency Practice

(a)
$$\left(\frac{3}{4}\right)^{-1} + 5^{-1} = 1\frac{8}{15}$$

(d)
$$\left(\frac{2}{3} + \frac{4}{5}\right)^{-1} = \frac{15}{22}$$

(g)
$$6^{-2} + 3^{-3} = \frac{7}{108}$$

(b)
$$\left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{12}\right)^{-2} = 152$$

(e)
$$\left(2\frac{1}{3}\right)^{-2} - \left(1\frac{3}{4}\right)^{-2} = -\frac{1}{7}$$

(h)
$$\left(\frac{2}{x}\right)^{-3} + \left(\frac{5}{x}\right)^{-1} = \frac{5x^3 + 8x}{40}$$

(c)
$$\left(\frac{4}{7}\right)^{-2} + \left(1\frac{5}{11}\right)^{-1} = 3\frac{3}{4}$$

(f)
$$0.4^{-1} + 0.75^{-2} = 4\frac{5}{18}$$

(i)
$$\left(\frac{1}{3x}\right)^{-2} + \left(\frac{3}{2x}\right)^{-3} = \frac{8x^3 + 243x^2}{27}$$

Fluency Practice

Rewrite the following with positive indices:

1. x^{-6} 2. x^{-7} 3. a^{-8} 4. a^{-10} 5. p^{-11}

6. k^{-14} 7. $2s^{-3}$ 8. $7z^{-3}$ 9. $3d^{-4}$ 10. $3d^{-7}$

11. $14x^{-1}$ 12. $\frac{12}{3}x^{-2}$ 13. $\frac{16x^{-5}}{4}$ 14. $\frac{a^{-9}}{2}$ 15. $\frac{e^{-5}}{6}$

16. $\frac{y^{-8}}{3}$ 17. $\frac{g^{-7}}{10}$ 18. $\frac{2x^{-4}}{3}$ 19. $\frac{4f^{-5}}{7}$ 20. $\frac{5x^{-8}}{9}$

21. $(3x)^{-3}$ 22. $(-3x)^{-3}$ 23. $(5x)^{-2}$ 24. $(-5x)^{-2}$ 25. $(-7a)^{-3}$

Fluency Practice

Task 1

Rewrite the following with negative indices:

1. $\dfrac{1}{x^4}$ 2. $\dfrac{1}{x}$ 3. $\dfrac{1}{d^{10}}$ 4. $\dfrac{1}{b^{13}}$ 5. $\dfrac{1}{y^x}$

6. $\dfrac{1}{m^n}$ 7. $\dfrac{3}{x^{13}}$ 8. $\dfrac{5}{x^9}$ 9. $\dfrac{7}{y^{10}}$ 10. $\dfrac{3}{x^{11}}$

11. $\dfrac{4}{x^3}$ 12. $\dfrac{12}{3x^2}$ 13. $\dfrac{a}{x^n}$ 14. $\dfrac{1}{2x^4}$ 15. $\dfrac{1}{3x^7}$

16. $\dfrac{1}{8a^5}$ 17. $\dfrac{1}{7b^6}$ 18. $\dfrac{3}{4x^5}$ 19. $\dfrac{2}{5x^{12}}$ 20. $\dfrac{1}{8a^5}$

Task 2

Match the equivalent pairs of expressions

$(3x)^{-1}$ $(3x)^{-2}$ $3x^{-1}$ x^3 x^{-3} $3x^{-2}$

$\dfrac{1}{x^3}$ $\dfrac{1}{x^{-3}}$ $\dfrac{1}{9x^2}$ $\dfrac{3}{x}$ $\dfrac{1}{3x}$ $\dfrac{3}{x^2}$

Fluency Practice

Simplify each of the following. Assume that all variables represent positive numbers. Present your answer without negative exponents.

1. 3^{-2}

2. $\dfrac{1}{2^{-3}}$

3. m^{-4}

4. $\dfrac{1}{x^{-5}}$

5. $a^8 \cdot a^{-1}$

6. $p^3 \left(p^{-7}\right) p^8$

7. $\dfrac{x^{-4}}{x^{-9}}$

8. $\dfrac{50a^{12}}{10a^{-3}}$

9. $\dfrac{t^{-3}}{t^4}$

10. x^0

11. $-x^0$

12. $(-x)^0$

13. $\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)$

14. $\dfrac{1}{\left(b^{-5}\right)\left(b^2\right)\left(b^{-1}\right)}$

15. $\dfrac{m^{-2}}{m^{-5}}$

16. $\dfrac{x^3 y^{-5}}{z^{-4}}$

17. $\dfrac{18q^3}{6q^{-3}}$

18. $\left(\dfrac{2}{3}\right)^{-3}$

19. $2y^{-3}$

20. $(2y)^{-3}$

21. $\left(-\dfrac{3}{5}\right)^{-2}$

22. $\dfrac{a^3 b^{-5}}{a^{-2} b^3}$

23. $\left(3m^3\right)^{-2}$

24. $\left(-2ab^{-3}\right)^{-3}$

25. $\dfrac{\left(k^3\right)^{-3}}{\left(k^{-5}\right)^2}$

26. $\left(\dfrac{2a^{-3}b^5}{-3a^3 b^{-2}}\right)^{-2}\left(a^3 b^{-5}\right)^{-3}$

27. $\left(-2a^{-3}\right)\left(-2a^{-2}b\right)^{-4}$

28. $\dfrac{\left(-3p^3 q^5\right)^2}{\left(2q^0 p^3\right)^{-1}}$

29. $\left(\dfrac{2a^{-2}b^3}{-2^2\left(a^{-1}b\right)^{-3}}\right)^{-2}$

30. $\left(-\dfrac{x^3 y^0 x^{-5}}{y^{-3}}\right)^{-2}$

31. $\left(-\dfrac{x^3 y^7 x^{-5}}{y^{-3}}\right)^0$

32. $\dfrac{x^{-1}+y^{-1}}{x^{-2}-y^{-2}}$

33. $\dfrac{\left(-2a^{-2}\right)^{-2} b^3 a^0 \left(-aba^{-2}b^{-2}\right)^{-3}}{2a^2 \left(-2a^{-2}b\right)^{-2} ab^0}$

34. $\left(\dfrac{-a^2\left(b^{-1}a\right)^{-5}}{b^7\left(-ab^2\right)^{-3}}\right)^{-2}$

35. $\dfrac{\left(x^{-2}\right)^{-2} y^3 x^0 \left(-2yx^0 y^{-2} x^{-2}\right)^0}{yx^5 \left(y^{-2}x\right)^{-3}\left(2x^{-1}yx^3\right)^{-1}}$

36. Suppose that $x = 8.5 \cdot 10^{-12}$ and $y = 7.5 \cdot 10^7$. Perform each of the following operations. Present your answer using scientific notation.

   a) xy b) x^3 c) xy^2 d) $\dfrac{x}{y}$ e) $\dfrac{y}{x^5}$

Fluency Practice

Task 1

Expand and fully simplify:

1. $x(x^3 - 4)$

2. $x(x^2 - x^{-2})$

3. $2y^{-1}(3y^2 - 2y^3)$

4. $2y^2(3y^4 + 5y^{-2})$

5. $m(2m^{-1} - 4m^{-4})$

6. $3c(4c^3 - 6c^{-4})$

7. $4a^2(2a^{-1} + 3a^{-2})$

8. $t^{-2}(3t^{-2} - t^2)$

9. $4d^{\frac{1}{2}}(3d^{\frac{1}{2}} - d^{-\frac{1}{2}})$

10. $a^{-2}(a + a^{-1})$

11. $2w^5\left(\frac{1}{w} + 4w^{-2}\right)$

12. $x^2\left(x^{\frac{1}{2}} + x^{\frac{1}{3}}\right)$

13. $x^{\frac{1}{2}}(x^4 + x^3)$

14. $u^{\frac{1}{2}}(3u + u^3)$

15. $x^{-\frac{1}{4}}(x^8 + x^6)$

16. $3m^{\frac{3}{2}}\left(m^2 + \frac{3}{m^{\frac{1}{2}}}\right)$

17. $n^{\frac{1}{3}}\left(2n^{-\frac{4}{3}} - \frac{1}{n^{\frac{2}{3}}}\right)$

18. $b^{\frac{1}{3}}(b^2 + 2b^{-1})$

19. $x^{-\frac{1}{3}}(x^{-2} + x^4)$

20. $x^{\frac{1}{2}}\left(\frac{3}{x^4} - \frac{2}{x^{\frac{1}{5}}}\right)$

21. $p^3(p^{-2} + p^3)$

22. $x^{-3}(x^5 + x^2)$

23. $5x^{\frac{1}{2}}\left(2x^{\frac{1}{2}} + 3x^{\frac{3}{2}}\right)$

24. $3a^{-1}(4a^3 + 2a)$

25. $2u^{-5}(u + 2u^5)$

26. $3m^2(2m^2 + 7m^{-4})$

27. $a^{\frac{1}{2}}\left(a^{\frac{1}{2}} + a^{-\frac{1}{2}}\right)$

28. $p^{\frac{1}{3}}\left(p^{\frac{2}{3}} + p^{-\frac{1}{3}}\right)$

29. $e^{-\frac{2}{3}}\left(e^{\frac{7}{3}} - 2e^{\frac{2}{3}}\right)$

30. $5n^4\left(n^{-2} + \frac{2}{n^3}\right)$

31. $p^4\left(3p^{-4} - \frac{2}{p^3}\right)$

32. $3a\left(a^{\frac{1}{2}} + 2a^{-2}\right)$

33. $x^{\frac{1}{2}}(2x - 3)$

34. $2p^{\frac{1}{4}}\left(p^{\frac{1}{4}} - p\right)$

35. $x^{\frac{2}{3}}\left(x^{\frac{1}{2}} + x^{\frac{1}{4}}\right)$

36. $x^{\frac{1}{5}}\left(2x^{\frac{9}{5}} + \frac{3}{x^{\frac{1}{5}}}\right)$

37. $2a^{\frac{5}{3}}\left(\frac{1}{a^{\frac{2}{3}}} - 4a^{\frac{4}{3}}\right)$

38. $x^{-4}\left(x^{-\frac{1}{2}} - x^2\right)$

39. $x^2\left(\frac{1}{x^{\frac{1}{2}}} + \frac{1}{x^3}\right)$

40. $x^{-\frac{1}{2}}\left(\frac{1}{x^{\frac{1}{4}}} - \frac{3}{x^{\frac{1}{6}}}\right)$

Task 2

Lauren writes down the following statement:

$$p^{\frac{1}{3}}\left(p^{\frac{2}{3}} - p^{-\frac{1}{3}}\right) = p - 1$$

Is this statement true? Justify your answer.

Fluency Practice

Expand and fully simplify:

1. $(m^{-2} + m^3)(m^{-2} + m)$

2. $(x^{-3} + x^{-6})(x^7 + x^{-2})$

3. $(x + x^{-1})^2$

4. $(b^{\frac{1}{2}} + 1)^2$

5. $(3k^{\frac{1}{2}} - 2)^2$

6. $\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{2}}}\right)\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{2}}}\right)$

7. $\left(c^{\frac{1}{2}} + \frac{1}{c^{\frac{1}{2}}}\right)^2$

8. $\left(x^{\frac{1}{2}} + \frac{1}{x^{\frac{1}{4}}}\right)\left(x^{\frac{1}{2}} - \frac{1}{x^{\frac{1}{4}}}\right)$

9. $\left(x^{\frac{2}{3}} + y^{\frac{1}{2}}\right)\left(x^{\frac{1}{3}} - y^{\frac{1}{2}}\right)$

10. $\left(5c^{\frac{3}{4}} + 9c^{\frac{1}{2}}\right)\left(6c^{\frac{1}{3}} - 2c^{\frac{1}{4}}\right)$

Expand and simplify

a $x(x^2 - x^{-1})$

b $2x^3(x^{-1} + 3)$

c $x^{-1}(3x - x^3)$

d $4x^{-2}(3x^5 + 2x^3)$

e $\frac{1}{2}x^2(6x + 4x^{-1})$

f $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

g $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

h $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

i $(x^2 + 1)(x^4 - 3)$

j $(2x^5 + x)(x^4 + 3)$

k $(x^2 - 2x^{-1})(x - x^{-2})$

l $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

Fluency Practice

Task 1

Rewrite the following using the radical sign:

1. $x^{\frac{1}{5}}$
2. $x^{\frac{1}{6}}$
3. $x^{\frac{1}{7}}$
4. $x^{\frac{1}{8}}$
5. $x^{\frac{1}{9}}$
6. $x^{\frac{1}{10}}$
7. $x^{\frac{1}{m}}$
8. $x^{-\frac{1}{2}}$
9. $x^{-\frac{1}{3}}$
10. $x^{-\frac{1}{4}}$
11. $x^{-\frac{1}{5}}$
12. $x^{-\frac{1}{6}}$
13. $x^{-\frac{1}{7}}$
14. $x^{-\frac{1}{a}}$

Task 2

Rewrite the following using the radical sign:

1. $4x^{\frac{1}{3}}$
2. $4x^{-\frac{1}{3}}$
3. $\frac{1}{4}x^{\frac{1}{3}}$
4. $\frac{1}{4}x^{-\frac{1}{3}}$
5. $15y^{\frac{1}{10}}$
6. $15y^{-\frac{1}{10}}$
7. $\frac{1}{15}y^{\frac{1}{10}}$
8. $\frac{1}{15}y^{-\frac{1}{10}}$
9. $-14x^{\frac{1}{8}}$
10. $-14x^{-\frac{1}{8}}$
11. $-\frac{1}{14}x^{\frac{1}{8}}$
12. $-\frac{1}{14}x^{-\frac{1}{8}}$
13. $18c^{\frac{1}{2}}$
14. $18c^{-\frac{1}{2}}$
15. $\frac{1}{18}c^{\frac{1}{2}}$
16. $\frac{1}{18}c^{-\frac{1}{2}}$
17. $-20k^{\frac{1}{9}}$
18. $-20k^{-\frac{1}{9}}$
19. $-\frac{k^{\frac{1}{9}}}{20}$
20. $-\frac{k^{-\frac{1}{9}}}{20}$
21. $3c^{\frac{1}{7}}$
22. $3c^{-\frac{1}{7}}$
23. $\frac{c^{\frac{1}{7}}}{3}$
24. $\frac{1}{3}c^{-\frac{1}{7}}$

Task 3

Rewrite the following using fractional exponents:

1. $\sqrt[9]{a}$
2. $\sqrt[4]{k}$
3. $\sqrt[5]{p}$
4. $\sqrt[8]{x}$
5. $\sqrt[3]{m}$
6. $\sqrt[7]{z}$
7. $\sqrt[10]{w}$
8. $\sqrt[14]{t}$
9. $\frac{1}{\sqrt[9]{a}}$
10. $\frac{1}{\sqrt[4]{k}}$
11. $\frac{1}{\sqrt[10]{l}}$
12. $\frac{1}{\sqrt[7]{h}}$
13. $\frac{1}{\sqrt[13]{p}}$
14. $\frac{1}{\sqrt[7]{m}}$
15. $\frac{1}{\sqrt[16]{b}}$

Fluency Practice

Task 1

Evaluate

1. $25^{\frac{1}{2}}$
2. $25^{-\frac{1}{2}}$
3. $16^{\frac{1}{2}}$
4. $16^{-\frac{1}{2}}$
5. $9^{\frac{1}{2}}$
6. $9^{-\frac{1}{2}}$
7. $100^{\frac{1}{2}}$
8. $100^{-\frac{1}{2}}$
9. $36^{\frac{1}{2}}$
10. $36^{-\frac{1}{2}}$
11. $8^{\frac{1}{3}}$
12. $8^{-\frac{1}{3}}$
13. $125^{\frac{1}{3}}$
14. $125^{-\frac{1}{3}}$
15. $1000^{\frac{1}{3}}$
16. $1000^{-\frac{1}{3}}$
17. $16^{\frac{1}{4}}$
18. $16^{-\frac{1}{4}}$
19. $32^{\frac{1}{5}}$
20. $32^{-\frac{1}{5}}$
21. $216^{\frac{1}{3}}$
22. $216^{-\frac{1}{3}}$
23. $512^{\frac{1}{3}}$
24. $512^{-\frac{1}{3}}$
25. $\left(\frac{25}{49}\right)^{\frac{1}{2}}$
26. $\left(\frac{25}{49}\right)^{-\frac{1}{2}}$
27. $\left(\frac{36}{121}\right)^{\frac{1}{2}}$
28. $\left(\frac{36}{121}\right)^{-\frac{1}{2}}$
29. $\left(\frac{625}{1296}\right)^{\frac{1}{4}}$
30. $\left(\frac{625}{1296}\right)^{-\frac{1}{4}}$
31. $\left(\frac{216}{343}\right)^{\frac{1}{3}}$
32. $\left(\frac{216}{343}\right)^{-\frac{1}{3}}$
33. $\left(\frac{343}{1000}\right)^{\frac{1}{3}}$
34. $\left(\frac{343}{1000}\right)^{-\frac{1}{3}}$
35. $\left(\frac{125}{729}\right)^{\frac{1}{3}}$
36. $\left(\frac{125}{729}\right)^{-\frac{1}{3}}$

Task 2

a) Given that $y = x^{\frac{1}{3}}$, find y when $x = 64$.

b) Given that $y = x^{\frac{1}{2}}$, find y when $x = 16$.

c) Given that $y = x^{\frac{1}{4}}$, find y when $x = 81$.

d) Given that $y = x^{\frac{1}{3}}$, find y when $x = 125$.

Purposeful Practice

Task 3

For $a > 1$, put the following in order of size from smallest to largest:

$$a^0, a^2, a, a^{-2}, a^{\frac{1}{2}}$$

Task 4

Solve for a:

$$\frac{x \times x^4 \times x^a}{\sqrt{x}} = \sqrt{\frac{(x^a)^8}{\left(\frac{1}{x^4}\right)}}$$

Task 5

The statement $x > x^{\frac{1}{2}}$ is:

Always True

Sometimes True

Never True

Fluency Practice

Find the value of:

(a) $4^{1/2}$

(b) $100^{1/2}$

(c) $196^{1/2}$

(d) $49^{1/2}$

(e) $64^{1/3}$

(f) $512^{1/3}$

(g) $1000^{1/3}$

(h) $16^{1/4}$

Evaluate:

(a) $\left(\dfrac{4}{9}\right)^{1/2}$

(b) $\left(\dfrac{1}{36}\right)^{1/2}$

(c) $\left(\dfrac{225}{16}\right)^{1/2}$

(d) $\left(\dfrac{25}{169}\right)^{1/2}$

(e) $\left(\dfrac{64}{27}\right)^{1/3}$

(f) $\left(\dfrac{8}{729}\right)^{1/3}$

(g) $\left(\dfrac{1}{1000}\right)^{1/3}$

(h) $\left(\dfrac{16}{10000}\right)^{1/4}$

Evaluate:

(a) $9^{-1/2}$

(b) $225^{-1/2}$

(c) $25^{-1/2}$

(d) $512^{-1/3}$

(e) $343^{-1/3}$

(f) $8^{-1/3}$

(g) $\left(\dfrac{8}{343}\right)^{-1/3}$

(h) $\left(\dfrac{25}{9}\right)^{-1/2}$

Solve these equations.

(a) $x^{1/2} = 9$

(b) $x^{1/2} = 11$

(c) $x^{1/3} = 5$

(d) $x^{-1/2} = \dfrac{1}{4}$

Which is bigger?

$$10^{\frac{1}{2}} \quad \text{or} \quad 20^{\frac{1}{3}}$$

$$20^{\frac{1}{2}} \quad \text{or} \quad 40^{\frac{1}{3}}$$

$$40^{\frac{1}{2}} \quad \text{or} \quad 80^{\frac{1}{3}}$$

$$5^{\frac{1}{2}} \quad \text{or} \quad 10^{\frac{1}{3}}$$

Fluency Practice

Task 1

Write in index form:

1. $\sqrt{a^3}$

2. $5\sqrt{a^3}$

3. $\dfrac{1}{\sqrt{a^3}}$

4. $\dfrac{5}{\sqrt{a^3}}$

5. $\sqrt{w^5}$

6. $4\sqrt{w^5}$

7. $\dfrac{1}{\sqrt{w^5}}$

8. $\dfrac{4}{\sqrt{w^5}}$

9. $\sqrt[3]{x^2}$

10. $9\sqrt[3]{x^2}$

11. $\dfrac{1}{\sqrt[3]{x^2}}$

12. $\dfrac{9}{\sqrt[3]{x^2}}$

13. $\sqrt[3]{w^4}$

14. $-6\sqrt[3]{w^4}$

15. $\dfrac{1}{\sqrt[3]{w^4}}$

16. $-\dfrac{6}{\sqrt[3]{w^4}}$

17. $\sqrt[5]{m^2}$

18. $-2\sqrt[5]{m^2}$

19. $\dfrac{1}{\sqrt[5]{m^2}}$

20. $-\dfrac{2}{\sqrt[5]{m^2}}$

21. $\sqrt[9]{k^4}$

22. $-12\sqrt[9]{k^4}$

23. $\dfrac{1}{\sqrt[9]{k^4}}$

24. $-\dfrac{12}{\sqrt[9]{k^4}}$

25. $\sqrt[6]{x^5 y^7}$

26. $\sqrt[4]{a^7 b^3 c}$

27. $\sqrt[7]{p^2 q^3 r^4}$

28. $\sqrt[7]{p^2 q^3 r^4}$

Fluency Practice

Task 2

Write with a radical sign in simplest form:

1. $x^{\frac{3}{4}}$ 2. $m^{\frac{7}{5}}$ 3. $p^{\frac{3}{2}}$ 4. $y^{\frac{5}{8}}$ 5. $c^{\frac{4}{3}}$

6. $4\left(x^{\frac{3}{4}}\right)$ 7. $9\left(m^{\frac{7}{5}}\right)$ 8. $16\left(p^{\frac{3}{2}}\right)$ 9. $5\left(y^{\frac{5}{8}}\right)$ 10. $10\left(c^{\frac{4}{3}}\right)$

11. $x^{-\frac{3}{4}}$ 12. $m^{-\frac{7}{5}}$ 13. $p^{-\frac{3}{2}}$ 14. $y^{-\frac{5}{8}}$ 15. $c^{-\frac{4}{3}}$

16. $4\left(x^{-\frac{3}{4}}\right)$ 17. $9\left(m^{-\frac{7}{5}}\right)$ 18. $16\left(p^{-\frac{3}{2}}\right)$ 19. $5\left(y^{-\frac{5}{8}}\right)$ 20. $10\left(c^{-\frac{4}{3}}\right)$

Task 3

Find equivalent terms from the following:

x^{-3}	x^3	x^2	$\sqrt{x}$	$\sqrt[3]{x}$	x^{-1}	$2x^{-1}$	$2\sqrt{x}$
$x^{\frac{3}{2}}$	$x^{\frac{1}{2}}$	$\sqrt{x^3}$	$x\sqrt{x}$	$x^{-\frac{1}{3}}$	x^{-2}	$\dfrac{1}{2x^2}$	$x^{\frac{2}{3}}$
$x^{-\frac{1}{2}}$	$2x^{-2}$	$\dfrac{\sqrt{x}}{x}$	x^4	x^{-4}	$\dfrac{1}{2\sqrt{x}}$	x^0	$x^{-\frac{3}{2}}$
$\dfrac{1}{2}x^{-\frac{1}{2}}$	$\dfrac{1}{2}x^{-2}$	$x^{\frac{1}{3}}$	x	$\dfrac{2}{\sqrt{x}}$	$\dfrac{1}{x}$	$\dfrac{1}{2x}$	$\sqrt[3]{x^2}$

Fluency Practice

Task 1

Evaluate:

1. $8^{\frac{4}{3}}$
2. $27^{\frac{5}{3}}$
3. $64^{\frac{5}{6}}$
4. $81^{\frac{3}{2}}$
5. $625^{\frac{3}{4}}$

6. $9^{\frac{3}{2}}$
7. $100^{\frac{3}{2}}$
8. $3125^{\frac{2}{5}}$
9. $8^{\frac{5}{3}}$
10. $4^{\frac{5}{2}}$

11. $81^{\frac{3}{4}}$
12. $125^{-\frac{2}{3}}$
13. $243^{-\frac{2}{5}}$
14. $32^{-\frac{2}{5}}$
15. $16^{-\frac{3}{4}}$

16. $4^{-\frac{5}{2}}$
17. $64^{-\frac{2}{3}}$
18. $8^{-\frac{4}{3}}$
19. $32^{-\frac{4}{5}}$
20. $125^{-\frac{2}{3}}$

21. $1000^{-\frac{2}{3}}$
22. $81^{-\frac{3}{4}}$
23. $625^{-\frac{3}{4}}$
24. $(-8)^{\frac{4}{3}}$
25. $(-27)^{\frac{5}{3}}$

26. $(-3125)^{\frac{2}{5}}$
27. $(-8)^{\frac{5}{3}}$
28. $(-125)^{-\frac{2}{3}}$
29. $(-243)^{-\frac{2}{5}}$
30. $(-32)^{-\frac{2}{5}}$

31. $(-64)^{-\frac{2}{3}}$
32. $(-8)^{-\frac{4}{3}}$
33. $(-32)^{-\frac{4}{5}}$
34. $(-125)^{-\frac{2}{3}}$
35. $(-1000)^{-\frac{2}{3}}$

36. $\left(\frac{4}{25}\right)^{\frac{3}{2}}$
37. $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$
38. $\left(\frac{8}{125}\right)^{\frac{2}{3}}$
39. $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$
40. $\left(\frac{49}{100}\right)^{\frac{3}{2}}$

41. $\left(\frac{49}{100}\right)^{-\frac{3}{2}}$
42. $\left(\frac{8}{125}\right)^{\frac{2}{3}}$
43. $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$
44. $\left(\frac{16}{100}\right)^{\frac{5}{2}}$
45. $\left(\frac{16}{100}\right)^{-\frac{5}{2}}$

46. $\left(\frac{64}{1000}\right)^{\frac{4}{3}}$
47. $\left(\frac{64}{1000}\right)^{-\frac{4}{3}}$
48. $\left(\frac{16}{25}\right)^{\frac{3}{2}}$
49. $\left(\frac{16}{25}\right)^{-\frac{3}{2}}$
50. $\left(\frac{32}{243}\right)^{\frac{6}{5}}$

Task 2

a) Arrange the following in ascending order:

$$25^{\frac{1}{2}}, 8^{\frac{2}{3}}, 27^{\frac{1}{3}}$$

b) Which is the odd one out? Explain your answer.

$$64^{\frac{1}{2}}, 16^{\frac{3}{4}}, 9^{\frac{2}{3}}, 4^{\frac{3}{2}}$$

Task 3

Gina has completed her homework.

Can you spot any mistakes?

Question 1

Work out $9^{\frac{1}{2}}$

4.5

Question 2

Work out $27^{\frac{2}{3}}$

18

Task 4

a) Given that $y = x^{\frac{3}{5}}$, find y when $x = 32$.

b) Given that $y = x^{\frac{3}{2}}$, find y when $x = 49$.

c) Given that $y = x^{\frac{2}{3}}$, find y when $x = 1000$.

d) Given that $y = x^{\frac{3}{4}}$, find y when $x = 81$.

e) Given that $y = x^{\frac{5}{6}}$, find y when $x = 64$.

f) Given that $y = x^{\frac{3}{4}}$, find y when $x = 10,000$.

Task 5

Find equivalent pairs from the following:

$8^{\frac{2}{3}}$	$(-1)^{\frac{1}{3}}$	$9^{\frac{3}{2}}$	$16^{\frac{1}{4}}$
$\left(\frac{1}{2}\right)^{-3}$	$\left(\frac{2}{5}\right)^{-2}$	$\left(\frac{1}{9}\right)^{-\frac{1}{2}}$	$36^{-\frac{1}{2}}$
3^{-2}	$\left(\frac{16}{25}\right)^{\frac{3}{2}}$	3^{-1}	$4^{-\frac{3}{2}}$
$8^{-\frac{1}{3}}$	$(-1)^{-2}$	$\left(\frac{1}{5}\right)^{-1}$	$\left(\frac{4}{9}\right)^{\frac{1}{2}}$

$\frac{2}{3}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{64}{125}$
$\frac{1}{3}$	4	1	$\frac{1}{2}$
8	-1	$\frac{1}{9}$	3
5	$\frac{25}{4}$	27	2

Task 6

1. $(4x)^{\frac{3}{4}}$ 2. $(9m)^{\frac{7}{5}}$ 3. $(16p)^{\frac{3}{2}}$ 4. $(5y)^{\frac{5}{8}}$ 5. $(10c)^{\frac{4}{3}}$

6. $(4x)^{-\frac{3}{4}}$ 7. $(9m)^{-\frac{7}{5}}$ 8. $(16p)^{-\frac{3}{2}}$ 9. $(5y)^{-\frac{5}{8}}$ 10. $(10c)^{-\frac{4}{3}}$

11. $\left(\frac{4x^6}{9y^4}\right)^{\frac{5}{2}}$ 12. $\left(\frac{243x^5}{32y^{20}}\right)^{\frac{3}{5}}$ 13. $\left(\frac{64x^3}{27y^6}\right)^{\frac{2}{3}}$

Task 7

Match up each expression from the first column with its partner from the second column, and write the answers in the table below

First column		Second column			
1	$\sqrt{x}$	A	$x^{\frac{-3}{2}}$	K	$x^{\frac{-7}{2}}$
2	$\dfrac{1}{x}$	B	$\dfrac{1}{2}x^{-3}$	L	$x^{\frac{9}{2}}$
3	$\dfrac{1}{\sqrt{x}}$	C	$\dfrac{1}{2}x^{-1}$	M	x^{-2}
4	$\sqrt[3]{x}$	D	x^{-4}	N	$x^{\frac{3}{2}}$
5	$\dfrac{1}{x^2}$	E	x^{-1}	O	$2x^{-1}$
6	$\dfrac{1}{2x^3}$	F	$\dfrac{1}{2}x$	P	$x^{\frac{1}{2}}$
7	$\dfrac{2}{x^3}$	G	x^4	Q	x^{-5}
8	$x\sqrt{x}$	H	x^2	R	$x^{\frac{2}{3}}$
9	$\dfrac{\sqrt{x}}{x^2}$	I	$2x$	S	$2x^{-3}$
10	$\dfrac{1}{x^{-2}}$	J	$x^{\frac{1}{3}}$	T	$x^{\frac{7}{2}}$
11	$\dfrac{1}{x^2} \times \dfrac{1}{x^3}$				
12	$\left(\sqrt[3]{x}\right)^2$				
13	$\sqrt{x^7}$				
14	$\sqrt{\left(\dfrac{1}{x^8}\right)}$				
15	$\dfrac{1}{\sqrt{x^{-8}}}$				
16	$x \times \sqrt{x \times x^6}$				
17	$\sqrt{\dfrac{4}{x^2}}$				
18	$\sqrt{\dfrac{1}{4x^2}}$				
19	$\sqrt{\dfrac{x^2}{4}}$				
20	$\sqrt{4x^2}$				

Table for Answers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

Box 1 (center 16):
- $2^{\square}$
- $\square^2$
- $\square^{\frac{3}{2}}$ (green)
- $\square_{\square}^2$ (yellow)
- center: 16

Box 2 (center 8):
- $\square\square^{\frac{3}{}}$ (red)
- $\square^3$
- $\square\square^{\frac{3}{}}$ (green)
- $\square\square^{\frac{3}{\square}}$ (yellow)
- center: 8

Box 3 (center 16):
- $2^{\square}$
- $\square^2$
- $\square\square^{\frac{3}{2}}$ (green)
- $\square\square^{\frac{2}{\square}}$ (yellow)
- center: 16

Box 4 (center 8):
- $\square\square^{\frac{3}{\square}}$ (red)
- $\square^3$
- $\square\square^{\frac{3}{\square}}$ (green)
- $\square\square^{\frac{3}{\square}}$ (yellow)
- center: 8

Box 5 (center 27):
- $3^{\square}$
- $\square^3$
- $\square\square^{\frac{3}{\square}}$ (green)
- $\square\square^{\frac{3}{\square}}$ (yellow)
- center: 27

Box 6 (center 4):
- $4^{\square}$
- $\square^2$
- $\square\square^{\frac{3}{2}}$ (green)
- $\square\square^{\frac{2}{\square}}$ (yellow)
- center: 4

Box 7 (center 27):
- $3^{\square}$
- $\square^3$
- $\square\square^{\frac{3}{\square}}$ (green)
- $\square\square^{\frac{3}{\square}}$ (yellow)
- center: 27

Box 8 (center 10,000):
- $10^{\square}$
- $\square^2$
- $\square\square^{\frac{3}{3}}$ (green)
- $\square\square^{\frac{4}{\square}}$ (yellow)
- center: 10,000

Fluency Practice

Find the value of each of the following.

$$\left(\frac{125}{1000}\right)^{-\frac{2}{3}}$$

$$\left(\frac{216}{512}\right)^{\frac{2}{3}}$$

$$\left(3\frac{3}{8}\right)^{\frac{1}{3}}$$

$$\left(\frac{25}{81}\right)^{-\frac{1}{2}}$$

$$\left(\frac{16}{81}\right)^{-\frac{3}{4}}$$

$$\left(2\frac{1}{4}\right)^{-\frac{1}{2}}$$

1. Fill in the blanks

CHALLENGE   EXAM

1. $100^{\frac{3}{2}} = \left(\sqrt[\square]{\square}\right)^{\square} = 10^{\square} = ?$

2. $16^{\frac{5}{4}} = \left(\sqrt[\square]{\square}\right)^{\square} = \square^5 = ?$

3. $64^{\frac{\square}{\square}} = \left(\sqrt[3]{64}\right)^4 = \square^{\square} = ?$

4. $\square^{\frac{\square}{5}} = \left(\sqrt[\square]{32}\right)^{\square} = \square^{\square} = 8$

5. $\square^{\frac{\square}{\square}} = \left(\sqrt[3]{\square}\right)^{\square} = 3^5 = ?$

6. $\square^{\frac{3}{2}} = \left(\sqrt[\square]{\square}\right)^{\square} = \square^{\square} = 216$

[Edexcel GCSE June2003-6H Q17b]
Let $x = 2^p$, $y = 2^q$,

If $xy = 32$ and $2xy^2 = 32$, find the value of p and the value of q.

Fluency Practice

Evaluate:

(a) $27^{2/3}$

(b) $125^{2/3}$

(c) $8^{2/3}$

(d) $(-8)^{2/3}$

(e) $49^{3/2}$

(f) $100^{3/2}$

(g) $16^{3/4}$

(h) $4^{5/2}$

Evaluate:

(a) $64^{-2/3}$

(b) $343^{-2/3}$

(c) $64^{-3/2}$

(d) $1000^{-2/3}$

(e) $25^{-3/2}$

(f) $81^{-3/2}$

(g) $10000^{-3/4}$

(h) $32^{-2/5}$

Evaluate:

(a) $\left(\dfrac{8}{343}\right)^{-2/3}$

(b) $\left(\dfrac{1}{125}\right)^{-2/3}$

(c) $\left(\dfrac{343}{64}\right)^{-2/3}$

(d) $\left(\dfrac{4}{9}\right)^{-3/2}$

(e) $\left(\dfrac{100}{9}\right)^{-3/2}$

(f) $\left(\dfrac{27}{8}\right)^{-4/3}$

(g) $\left(\dfrac{25}{4}\right)^{-3/2}$

(h) $\left(\dfrac{16}{10000}\right)^{-3/4}$

Find x.

(a) $x^{2/3} = 4$

(b) $x^{3/2} = 64$

(c) $y^x \times y^{2/3} = y^3$

Fluency Practice

Fractional Indices

Patrons are reminded that $\sqrt{x} \equiv x^{\frac{1}{2}}$ and $\sqrt[n]{x} \equiv x^{\frac{1}{n}}$.
Also note that $\sqrt{4} = 2$. ($\sqrt{4} \neq \pm 2$.)

1. Without a calculator evaluate the following:

(a) $4^{\frac{1}{2}}$.

(b) $16^{\frac{1}{2}}$.

(c) $9^{\frac{1}{2}}$.

(d) $8^{\frac{1}{3}}$.

(e) $64^{\frac{1}{6}}$.

(f) $125^{\frac{1}{3}}$.

(g) $16^{\frac{1}{4}}$.

(h) $81^{\frac{1}{4}}$.

(i) $32^{\frac{1}{5}}$.

(j) $27^{\frac{1}{3}}$.

(k) $4^{\frac{3}{2}}$.

(l) $8^{\frac{2}{3}}$.

(m) $4^{-\frac{1}{2}}$.

(n) $4^{\frac{5}{2}}$.

(o) $9^{-\frac{3}{2}}$.

(p) $8^{-\frac{2}{3}}$.

(q) $32^{\frac{4}{5}}$.

(r) $27^{-\frac{5}{3}}$.

(s) $\left(\frac{1}{4}\right)^{\frac{1}{2}}$.

(t) $\left(\frac{1}{27}\right)^{-\frac{1}{3}}$.

(u) $\left(\frac{1}{64}\right)^{\frac{1}{3}}$.

(v) $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$.

(w) $\left(\frac{64}{9}\right)^{\frac{3}{2}}$.

(x) $\left(\frac{27}{8}\right)^{-\frac{5}{3}}$.

(y) $\left(\frac{4}{9}\right)^{-\frac{1}{2}}$.

(z) $\left(64 \times 5^{-3}\right)^{\frac{1}{3}}$.

2. Evaluate:

(a) $\dfrac{1}{\left(\frac{2}{3}\right)^2 + \left(\frac{4}{9}\right)^{\frac{1}{2}}}$.

(b) $\dfrac{2}{\left(\frac{4}{9}\right)^{\frac{3}{2}} + \left(\frac{2}{3}\right)^3}$.

(c) $\dfrac{1}{\left(\frac{8}{27}\right)^{\frac{2}{3}} - \left(\frac{81}{4}\right)^{-\frac{1}{2}}}$.

(d) $\dfrac{4}{\left(\frac{81}{16}\right)^{-\frac{3}{4}} + \left(\frac{4}{3}\right)^3}$.

(e) $\dfrac{\frac{2}{3} + \left(\frac{4}{9}\right)^{\frac{1}{2}}}{1 + \left(\frac{8}{27}\right)^{-\frac{1}{3}}}$.

(f) $\dfrac{4 + \left(\frac{9}{4}\right)^{\frac{3}{2}}}{2 + \left(\frac{1}{9}\right)^{\frac{1}{2}}}$.

(g) $\dfrac{\left(\frac{27}{8}\right)^{-\frac{1}{3}} + \frac{1}{6}}{\left(\frac{1}{4}\right)^{\frac{1}{2}} - 9^{-\frac{1}{2}}}$.

(h) $\dfrac{\left(\frac{9}{4}\right)^{\frac{1}{2}} - \left(\frac{9}{16}\right)^{-\frac{1}{2}}}{3 - \left(\frac{27}{8}\right)^{\frac{1}{3}}}$.

3. Simplify the following:

(a) $\dfrac{x}{\sqrt{x}}$.

(b) $\dfrac{x^2 \times \sqrt{x}}{x^{-1}}$.

(c) $x^{-2} \times x^{\frac{1}{2}}$.

(d) $\dfrac{x^2 \times x^{0.5}}{\sqrt[3]{x}}$.

Fluency Practice

Simplify each of the following expressions.

1. $9^{1/2}$

2. $9^{-1/2}$

3. $-9^{1/2}$

4. $-9^{-1/2}$

5. $(-9)^{1/2}$

6. $9^{3/2}$

7. $8^{1/3}$

8. $8^{-1/3}$

9. $(-8)^{1/3}$

10. $16^{-3/4}$

11. $5^{1/2}$

12. $-4^{-5/2}$

13. $(-32)^{1/5}$

14. $32^{2/5}$

15. $36^{1/2}$

16. $36^{-1/2}$

17. $-36^{1/2}$

18. $(-36)^{1/2}$

19. $(-36)^{-1/2}$

20. $36^{-3/2}$

21. $-4^{-3/2}$

22. $(-4)^{-3/2}$

23. $(-8)^{1/3}$

24. $(-8)^{2/3}$

25. $-16^{1/2}$

26. $(-16)^{1/2}$

27. $8^{-2/3}$

28. $(32)^{-4/5}$

29. $(-32)^{-4/5}$

30. $81^{-3/4}$

Simplify each of the following. Present your final answers using positive exponents only. For example, $x^{-2/3} = \dfrac{1}{\sqrt[3]{x^2}}$.
Assume that all variables represent positive numbers.

31. $\left(x^{1/3}\right)^6$

32. $\dfrac{x^{2/5}x^{-3/4}}{x^{-1/2}}$

33. $\left(a^{-1/2}\right)^{-4/5}$

34. $\dfrac{\left(m^{1/2}\right)\left(m^{3/2}\right)}{m^{-3}}$

35. $\sqrt[3]{\dfrac{x^2 y^{-1}}{\left(-x^4 y^{-2}\right)^2}}$

36. $\left(-8a^6 b^{-24}\right)^{1/3}$

37. $\left(\dfrac{16x^4 y^{\frac{2}{3}}}{36x^2 y^2}\right)^{\frac{3}{2}}$

38. $\left(-3x^{-1/2}y^{3/8}x^{-7/2}\right)^0$

39. $\left(A^{2/3}\right)^{3/2}$

40. $\dfrac{\left(y^{-3/4}\right)^{-8}}{\left(y^{-1/2}\right)^{10}}$

Use exponential notation to simplify each of the following. Present your answer using positive integers only.

41. $\sqrt[8]{x^6}$

42. $\left(\sqrt[6]{y}\right)^9$

43. $\left(\sqrt[6]{2}\right)^{18}$

44. $\left(\sqrt{x}\right)\left(\sqrt[3]{x}\right)$

45. $\dfrac{\sqrt[5]{x}\sqrt[3]{x^2}}{x\sqrt[6]{x}}$

46. $\sqrt{\sqrt{\sqrt{\sqrt{x}}}}$

always, sometimes, never true?

$$4a^{\frac{1}{2}} = 2\sqrt{a}$$

$$a^{\frac{1}{2}} \geqslant a^{\frac{1}{4}}$$

$$a^{\frac{4}{3}} = a^{\frac{2}{3}} \times a^{\frac{2}{3}}$$

$$2a^{\frac{1}{2}} \geqslant a^{\frac{1}{2}}$$

$$a^{\frac{1}{4}} = \sqrt{a^{\frac{1}{2}}}$$

$$\left(a^{\frac{1}{6}}\right)^{\frac{1}{6}} = a^{\frac{1}{12}}$$

$$\left(a^{\frac{1}{2}}\right)^{2} = 1$$

$$a \neq 0$$
$$a > a^{\frac{1}{2}}$$

$$a^{\frac{1}{2}} > (a+1)^{\frac{1}{2}}$$

$$a < 0$$
$$a^{\frac{1}{2}} = $$
undefined

$$a^{1.25} = \sqrt[4]{a^{5}}$$

$$a^{3} \geqslant a^{2}$$

Fluency Practice

A1 State the value of 5^0	**A2** State the value of 7^1	**A3** Evaluate 4^{-3} Give your answer as a fraction in its simplest terms.	**A4** Evaluate $\sqrt[3]{125}$
B1 Evaluate $49^{\frac{1}{2}}$	**B2** Evaluate $27^{\frac{1}{3}}$	**B3** Evaluate $64^{\frac{2}{3}}$	**B4** Evaluate $16^{\frac{3}{4}}$
C1 Evaluate $8^{-\frac{1}{3}}$ Give your answer as a fraction in its simplest terms.	**C2** Evaluate $36^{-\frac{1}{2}}$ Give your answer as a fraction in its simplest terms.	**C3** Evaluate $81^{-\frac{3}{4}}$ Give your answer as a fraction in its simplest terms.	**C4** Evaluate $216^{-\frac{2}{3}}$ Give your answer as a fraction in its simplest terms.
D1 Evaluate $\left(\dfrac{9}{16}\right)^{\frac{1}{2}}$ Give your answer as a fraction in its simplest terms.	**D2** Evaluate $\left(\dfrac{27}{343}\right)^{\frac{2}{3}}$ Give your answer as a fraction in its simplest terms.	**D3** Evaluate $\left(\dfrac{125}{8}\right)^{-\frac{1}{3}}$ Give your answer as a fraction in its simplest terms.	**D4** Evaluate $\left(\dfrac{49}{36}\right)^{-\frac{3}{2}}$ Give your answer as a fraction in its simplest terms.

Fluency Practice

square and cube roots: give your answer as a fraction

(1) $\sqrt{0.36}$

(2) $\sqrt{1\frac{24}{25}}$

(3) $\sqrt{2\frac{14}{25}}$

(4) $\sqrt{3.24}$

(5) $\sqrt{4.84}$

(6) $\sqrt{5\frac{19}{25}}$

(7) $\sqrt[3]{1\frac{91}{125}}$

(8) $\left(1\frac{61}{64}\right)^{-\frac{1}{3}}$

(1) $\sqrt{2\frac{1}{4}}$

(2) $\sqrt{1\frac{15}{49}}$

(3) $\sqrt{1\frac{11}{25}}$

(4) $\sqrt[3]{1\frac{271}{729}}$

(5) $\frac{1}{2}\sqrt[3]{15.625}$

(6) $2\left(2\frac{46}{49}\right)^{-\frac{1}{2}}$

(7) $\sqrt{\frac{2}{7} \times 3\frac{3}{8} \times 1\frac{5}{16}}$

(8) $3(3.375)^{-\frac{1}{3}}$

no calculator

$$(2\tfrac{1}{4})^{-\frac{1}{2}} \times (3\tfrac{3}{8})^{\frac{1}{3}}$$

$$(1.25)^{-\frac{1}{3}} \times (10)^{\frac{1}{3}}$$

$$(1.5)^{-2} \times (16)^{-\frac{1}{2}} \times (81)^{\frac{3}{4}}$$

$$(48)^{\frac{1}{2}} \times (9)^{\frac{1}{6}} \times (3)^{-\frac{5}{6}}$$

$$(20)^{\frac{2}{3}} \div (100)^{\frac{1}{6}} \times \left(\frac{5}{2\sqrt{2}}\right)^{\frac{2}{3}}$$

$$(12)^{\frac{1}{3}} \div (81)^{\frac{1}{6}} \times (162)^{\frac{1}{3}}$$

$$1 \times 10^{\square}$$

$$\square^{3}$$

$$2^{\square} \times 5^{\square}$$

$$\sqrt{10^{\square}}$$

$$= 1 \text{ Million}$$

$$\frac{1}{10^{\square}}$$

$$0.1 \times 10^{\square}$$

$$\square \times 10^{-1}$$

$$\left(\square \times 10^{\square}\right)^{2}$$
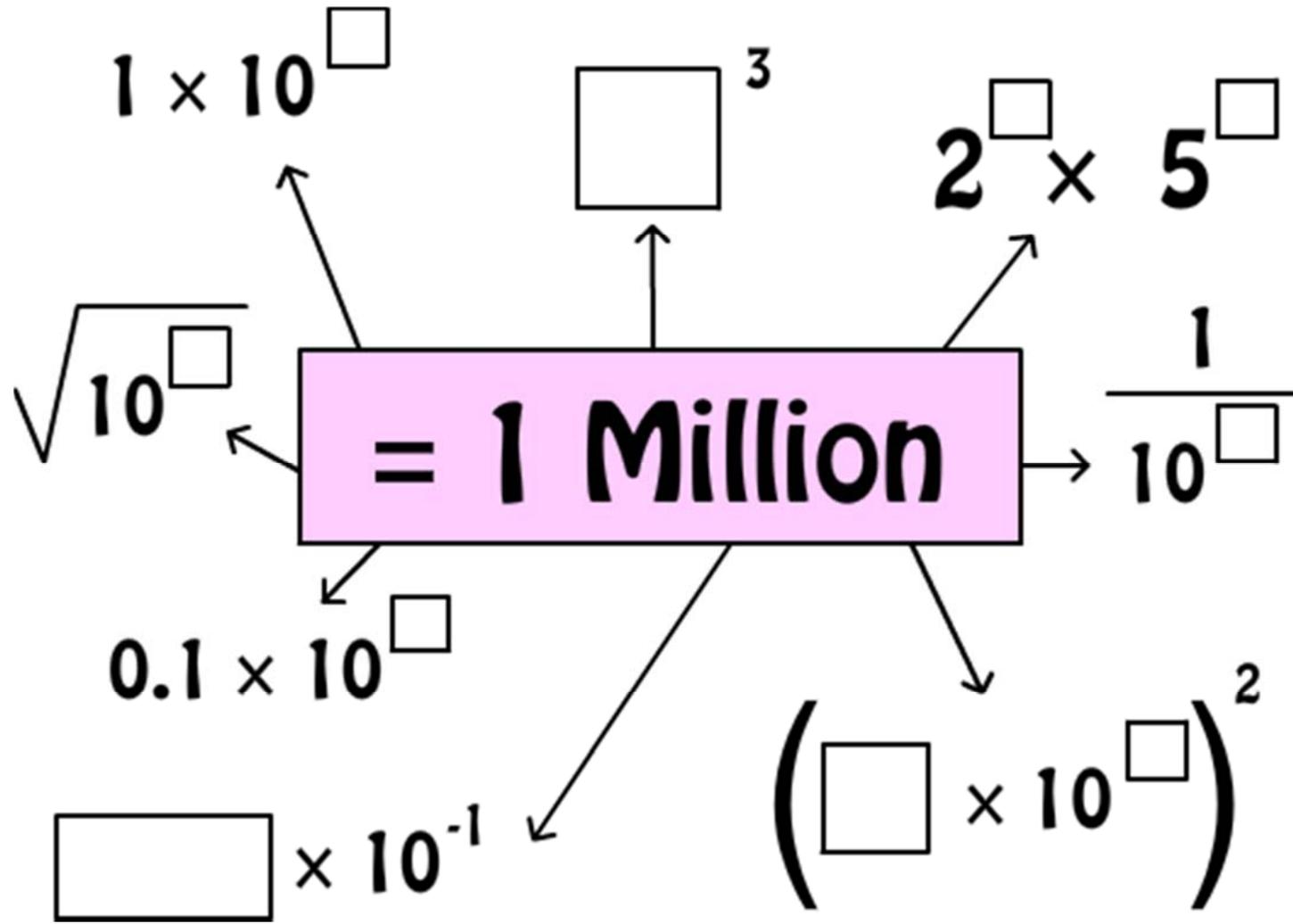
Fluency Practice

Write the following without negative or fraction powers.

(1) x^{-3} **(2)** $3x^{-1}$ **(3)** $5x^{-2}$ **(4)** $\frac{2}{3}x^{-4}$ **(5)** $\frac{4}{5}x^{-3}$ **(6)** $\frac{1}{5}x^{-3}$

(7) $\frac{x^{-3}}{5}$ **(8)** $x^{\frac{1}{2}}$ **(9)** $x^{\frac{3}{2}}$ **(10)** $3x^{\frac{5}{2}}$ **(11)** $x^{-\frac{1}{2}}$ **(12)** $x^{-\frac{3}{4}}$

(13) $3x^{-\frac{2}{3}}$ **(14)** $\frac{1}{6}x^{-\frac{3}{4}}$

Write the following without roots and, if the x is on the denominator, with a negative power:

(1) $\frac{4}{x}$ **(2)** $\frac{4}{x^2}$ **(3)** $\frac{3}{4x^2}$ **(4)** $\frac{1}{4x^2}$ **(5)** $\frac{4}{3x^2}$ **(6)** $\frac{1}{5x}$

(7) $\frac{5}{x^3}$ **(8)** $\sqrt{x}$ **(9)** $\sqrt[3]{x}$ **(10)** $\sqrt[3]{x^2}$ **(11)** $5\sqrt{x^3}$ **(12)** $\frac{1}{\sqrt{x}}$

(13) $\frac{1}{\sqrt[4]{x}}$ **(14)** $\frac{3}{\sqrt[3]{x^2}}$ **(15)** $\frac{2}{3\sqrt{x}}$ **(16)** $\frac{1}{3\sqrt[3]{x^2}}$ **(17)** $\frac{4}{5\sqrt{x^3}}$ **(18)** $\frac{1}{4\sqrt{x}}$

Fluency Practice

Q	Evaluate, without a calculator:	Answers	Q	Evaluate, without a calculator:	Answers
1	9^{-1}		13	$49^{\frac{1}{2}}$	
2	$9^{\frac{1}{2}}$		14	$9^{-\frac{1}{2}}$	
3	16^{-1}		15	$\left(\frac{1}{81}\right)^{-1}$	
4	$16^{\frac{1}{2}}$		16	$\left(\frac{1}{81}\right)^{\frac{1}{2}}$	
5	5^{-2}		17	$\left(\frac{1}{81}\right)^{-\frac{1}{2}}$	
6	$25^{\frac{1}{2}}$		18	$\left(\frac{1}{64}\right)^{-2}$	
7	8^{-2}		19	$\left(\frac{1}{64}\right)^{\frac{2}{3}}$	
8	$64^{\frac{1}{3}}$		20	$\left(\frac{1}{64}\right)^{-\frac{4}{3}}$	
9	4^{-2}		21	a^{-1}	
10	$16^{\frac{1}{4}}$		22	$b^{\frac{1}{n}}$	
11	32^{-2}		23	$c^{\frac{m}{n}}$	
12	$32^{\frac{1}{5}}$		24	$d^{-\frac{m}{n}}$	

Fluency Practice

One variable $a^2 \times a^3$	$(b^2)^3$	$c^8 \times c^{-2}$	$d \div d^{-3}$
$e^2 \times e^a$	$(f^2)^y$	$g^{3k} \div g^k$	$(h^p)^p$
$(3i^5)^2$	$2j^2 \times 7j^{-3} \times j$	$8k^2 \div 6k^5$	$9l^8 \div 3l$
$\left(m^{\frac{2}{5}}\right)^5$	$\left(5n^{\frac{3}{2}}\right)^2$	$(9p^6)^{\frac{1}{2}}$	$(8q^3)^{\frac{1}{3}}$
$(3rt)^2 \times 3rt^2$	$\dfrac{x^2 \times 12x}{(3x^2)^2}$	$\dfrac{2}{5}t^{\frac{2}{5}} \times \dfrac{3}{4}t^{\frac{3}{4}}$	$\dfrac{4}{5}y^{\frac{4}{5}} \div \dfrac{1}{2}y^{\frac{1}{2}}$

Two variables $4ab^2 \times 5a^3b$	$10c^2d \div 8cd^{-1}$	$\left(\dfrac{1}{2}ef\right)^3 \times \dfrac{1}{2}ef^3$
$\dfrac{6gh^2 \times 4gh^3}{8gh}$	$\dfrac{(5g^2h)^3}{5g^2h}$	$\dfrac{3}{4}j^{\frac{1}{2}}k \times \left(\dfrac{4}{9}jk^3\right)^{\frac{1}{2}}$

Rewriting $\dfrac{1}{a} \times a^3$	$\sqrt{b} \times \sqrt[3]{b}$	$c^2 \div \dfrac{1}{c^5}$	$\dfrac{1}{d} \times \sqrt{d}$
$\sqrt{9e} \times 9\sqrt{e}$	$\dfrac{1}{gf^2} \times \sqrt{4gf^2}$	$\dfrac{1}{\sqrt{h}} \times (8h)^{\frac{1}{3}}$	$\dfrac{2}{j^2} \div \dfrac{3}{\sqrt[3]{j}}$

Write as a single power of x:

(a) $\sqrt{x^3}$

(b) $\left(\sqrt{x}\right)^3$

(c) $\sqrt[3]{x^2}$

(d) $\sqrt[4]{x}$

(e) $\left(\sqrt[3]{x}\right)^4$

(f) $\sqrt{x^5}$

Write as a single power of x:

(a) $\dfrac{1}{\sqrt[3]{x}}$

(b) $\left(\dfrac{1}{\sqrt[3]{x}}\right)^2$

(c) $\dfrac{1}{\sqrt{x^3}}$

(d) $\left(\dfrac{1}{\sqrt{x}}\right)^3$

(e) $\left(\dfrac{1}{\sqrt[3]{x}}\right)^5$

(f) $\dfrac{1}{\sqrt[2]{x^7}}$

Write as a single power of x:

(a) $x^2 \times \sqrt{x}$

(b) $\sqrt[3]{x} \times x$

(c) $\dfrac{x^4}{\sqrt{x}}$

(d) $\dfrac{\sqrt[3]{x}}{x}$

(e) $\sqrt{\dfrac{1}{x^5}}$

(f) $\dfrac{1}{x\sqrt{x}}$

(a) Given that

$$\frac{y^4 \times \sqrt{y}}{\sqrt{y^5}} = y^a$$

find the value of a.

(b) Given that

$$\frac{1}{\sqrt[3]{y^2}} \times \left(y\sqrt{y}\right)^4 = y^b$$

find the value of b.

(c) Given that

$$\left(\sqrt[2]{y^3}\right)^3 \times \frac{1}{y^c} = \left(y^2 \times \sqrt[4]{y^3}\right)^{-2}$$

find the value of c.

Fluency Practice

Question 1: Evaluate each of the following

(a) 5^{-2} (b) 2^{-1} (c) 2^{-3} (d) 4^{-2} (e) 3^{-3} (f) 6^{-1}

(g) 10^{-2} (h) 2^{-4} (i) 9^{-2} (j) 3^{-4} (k) 10^{-1} (l) 7^{-2}

(m) 2^{-5} (n) 5^{-3} (o) 2^{-6} (p) 10^{-4} (q) 6^{-3} (r) 10^{-6}

Question 2: Write each of the following in index form.

(a) $\frac{1}{5^2}$ (b) $\frac{1}{3^4}$ (c) $\frac{1}{8^3}$ (d) $\frac{1}{4^5}$ (e) $\frac{1}{10^3}$ (f) $\frac{1}{2^6}$

Question 3: Write each of the following in the form 2^n

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{32}$ (d) $\frac{1}{8}$ (e) $\frac{1}{64}$ (f) $\frac{1}{256}$

Question 4: Write each of the following in the form 5^n

(a) $\frac{1}{125}$ (b) $\frac{1}{25}$ (c) $\frac{1}{5}$ (d) $\frac{1}{3125}$ (e) $\frac{1}{625}$ (f) $\frac{1}{15625}$

Question 5: Write each of the following as fractions

(a) a^{-2} (b) y^{-1} (c) w^{-4} (d) 2^{-x} (e) 5^{-a} (f) x^{-n}

Question 6: Write each of the following in index form

(a) $\frac{1}{w^2}$ (b) $\frac{1}{2^y}$ (c) $\frac{1}{x^5}$ (d) $\frac{1}{4^x}$ (e) $\frac{1}{y^x}$ (f) $\frac{1}{m^n}$

Question 7: Write each of the following as fractions

(a) $5y^{-2}$ (b) $8c^{-1}$ (c) $(5x)^{-2}$ (d) $(2y)^{-3}$ (e) $2x^{-3}$ (f) $(10w)^{-3}$

Fluency Practice

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Negative Indices
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Question 8: Write each of the following in index form

(a) $\dfrac{3}{x^2}$ (b) $\dfrac{5}{w^8}$ (c) $\dfrac{2}{3y^2}$ (d) $\dfrac{1}{4x^3}$ (e) $\dfrac{6}{y^m}$ (f) $\dfrac{a}{x^n}$

Question 9: Write each of the following as fractions

(a) $100^{-\frac{1}{2}}$ (b) $25^{-\frac{1}{2}}$ (c) $9^{-\frac{1}{2}}$ (d) $8^{-\frac{1}{3}}$ (e) $125^{-\frac{1}{3}}$ (f) $16^{-\frac{1}{4}}$

Question 10: Write each of the following as fractions

(a) $8^{-\frac{2}{3}}$ (b) $25^{-\frac{3}{2}}$ (c) $64^{-\frac{2}{3}}$ (d) $4^{-\frac{5}{2}}$ (e) $81^{-\frac{3}{4}}$ (f) $100000^{-\frac{2}{5}}$

Apply

Question 1: Arrange in order from smallest to largest.

$$\dfrac{1}{50} \qquad 5^{-2} \qquad \dfrac{3}{10} \qquad 2^{-3}$$

Question 2: Work out

(a) $4^{-2} \times 3^2$ (b) $10^{-1} \div 5^{-2}$ (c) $2^{-2} + 3^{-2} \times 2^3$

Question 3: Sally has completed her homework. Can you spot any mistakes?

Question 1 Evaluate

$$4^{-2}$$

$$-16$$

Question 2 Work out

$$10^{-3}$$

$$\dfrac{1}{30}$$

Question 4: Given that $2^m + 2^n = \dfrac{9}{32}$

Work out mn

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Question 5:

$$x^{-2} \quad x^0 \quad x \quad x^3$$

Put the expressions above in order, from smallest to largest, when:

(a) x = 2   (b) x = 1   (c) x = 0.5   (d) x = −0.5

(e) x = −1   (f) x = −2

Answers

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Fluency Practice

Question 1: Evaluate each of the following

(a) $25^{\frac{1}{2}}$ (b) $81^{\frac{1}{2}}$ (c) $4^{\frac{1}{2}}$ (d) $144^{\frac{1}{2}}$ (e) $8^{\frac{1}{3}}$ (f) $125^{\frac{1}{3}}$

(g) $100^{\frac{1}{2}}$ (h) $1000^{\frac{1}{3}}$ (i) $49^{\frac{1}{2}}$ (j) $225^{\frac{1}{2}}$ (k) $64^{\frac{1}{2}}$ (l) $27^{\frac{1}{3}}$

(m) $216^{\frac{1}{3}}$ (n) $64^{\frac{1}{3}}$ (o) $16^{\frac{1}{4}}$ (p) $1^{\frac{1}{4}}$ (q) $81^{\frac{1}{4}}$ (r) $625^{\frac{1}{4}}$

Question 2: Write each of the following in index form

(a) $\sqrt{x}$ (b) $\sqrt{y}$ (c) $\sqrt[3]{a}$ (d) $\sqrt[4]{y}$ (e) $\sqrt[6]{x}$ (f) $\sqrt[8]{c}$

Question 3: Evaluate each of the following

(a) $9^{\frac{3}{2}}$ (b) $4^{\frac{3}{2}}$ (c) $8^{\frac{2}{3}}$ (d) $27^{\frac{2}{3}}$ (e) $125^{\frac{2}{3}}$ (f) $49^{\frac{3}{2}}$

(g) $4^{\frac{5}{2}}$ (h) $64^{\frac{2}{3}}$ (i) $9^{\frac{5}{2}}$ (j) $100^{\frac{3}{2}}$ (k) $16^{\frac{3}{2}}$ (l) $1000^{\frac{2}{3}}$

(m) $100^{\frac{5}{2}}$ (n) $32^{\frac{2}{5}}$ (o) $4^{\frac{7}{2}}$ (p) $8^{\frac{5}{3}}$ (q) $16^{\frac{3}{4}}$ (r) $81^{\frac{3}{4}}$

(s) $32^{\frac{3}{5}}$ (t) $27^{\frac{5}{3}}$ (u) $64^{\frac{5}{6}}$ (v) $10000^{\frac{3}{4}}$

Question 4: Write each of the following in index form

(a) $\sqrt{a^3}$ (b) $\sqrt{w^5}$ (c) $\sqrt[3]{x^2}$ (d) $\sqrt[3]{w^4}$ (e) $\sqrt[5]{m^2}$ (f) $\sqrt[9]{k^4}$

Question 5: Write each of the following in the form 9^n

(a) 81 (b) 3 (c) 27

Question 6: Write each of the following in the form 64^n

(a) 8 (b) 4 (c) 16

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Question 7: Simplify each of the following

(a) $\left(9x^2\right)^{\frac{1}{2}}$ (b) $\left(4x^6\right)^{\frac{1}{2}}$ (c) $\left(25x^8\right)^{\frac{1}{2}}$ (d) $\left(4x^3\right)^{\frac{1}{2}}$ (e) $\left(8x^3\right)^{\frac{1}{3}}$ (f) $\left(125x^6\right)^{\frac{1}{3}}$

Question 8: Evaluate each of the following

(a) $\left(64x^3\right)^{\frac{2}{3}}$ (b) $\left(9x^4\right)^{\frac{3}{2}}$ (c) $\left(27x^6\right)^{\frac{2}{3}}$ (d) $\left(4x^6\right)^{\frac{5}{2}}$ (e) $\left(16x^8\right)^{\frac{3}{4}}$ (f) $\left(32x^{20}\right)^{\frac{3}{5}}$

Question 9: Write each of the following as fractions

(a) $8^{-\frac{2}{3}}$ (b) $25^{-\frac{3}{2}}$ (c) $64^{-\frac{2}{3}}$ (d) $4^{-\frac{5}{2}}$ (e) $81^{-\frac{3}{4}}$ (f) $100000^{-\frac{2}{5}}$

Apply

Question 1: Arrange the following in order, smallest first.
$$25^{\frac{1}{2}} \ , \ 8^{\frac{2}{3}} \ , \ 27^{\frac{1}{3}}$$

Question 2: Which is the odd one out?
Explain your answer.
$$64^{\frac{1}{2}} \ , \ 16^{\frac{3}{4}} \ , \ 9^{\frac{2}{3}} \ , \ 4^{\frac{3}{2}}$$

Question 3: Work out

(a) $64^{\frac{1}{3}} \times 2^3$ (b) $27^{\frac{2}{3}} \div 9^{\frac{3}{2}}$ (c) $\left(8^{\frac{2}{3}}\right)^2$

Question 4: Gina has completed her homework.
Can you spot any mistakes?

Question 1

Work out $9^{\frac{1}{2}}$

4.5

Question 2

Work out $27^{\frac{2}{3}}$

18

Answers

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Intelligent Practice

Write in terms of the given number:

1) 16 as a power of 2

2) 16 as a power of 4

3) 64 as a power of 4

4) 125 as a power of 5

5) 4^2 as a power of 2

6) 4^3 as a power of 2

7) 4^4 as a power of 2

8) 27^3 as a power of 3

9) 27^{3x} as a power of 3

10) 25^{3x} as a power of 5

Intelligent Practice

Solve the following equations:

1) $3^x = 243$

2) $3^x = \dfrac{1}{243}$

3) $4^x = 64$

4) $4^x = \dfrac{1}{64}$

5) $5^x = 625$

6) $5^x = \dfrac{1}{625}$

Intelligent Practice

Solve the following equations:

1) $\left(\frac{1}{3}\right)^x = 81$

2) $\left(\frac{1}{3}\right)^x = \frac{1}{81}$

3) $\left(\frac{1}{3}\right)^x = \frac{1}{27}$

4) $\left(\frac{1}{3}\right)^x = 27$

5) $\left(\frac{1}{4}\right)^x = 64$

6) $\left(\frac{1}{4}\right)^x = 256$

7) $\left(\frac{1}{4}\right)^x = \frac{1}{256}$

8) $\left(\frac{5}{25}\right)^x = 625$

Intelligent Practice

Find the value of x that satisfies:

1) $2^{3x} \times 2^{x-3} = 32$

2) $2^{2x} \times 2^{x+3} = 64$

3) $3^{2x} \times 3^{x-6} = 81$

4) $3^{x+2} \times 3^{x-2} = 243$

5) $2^{4x} \div 2^{x-2} = 32$

6) $3^{6x} \div 3^{x-5} = 243$

Intelligent Practice

Find the value of x that satisfies:

1) $16^{\frac{1}{4}} \times 2^x = 8^{\frac{2}{3}}$

2) $81^{\frac{1}{4}} \times 3^x = 27^{\frac{2}{3}}$

3) $25^{\frac{1}{2}} \times 125^x = 5^{\frac{4}{2}}$

4) $27^{\frac{1}{4}} \times 3^x = 9^{\frac{2}{3}}$

Fluency Practice

Repeat addition Index problems

1) Simplify

a) $3^5 + 3^5 + 3^5$

b) $3^y + 3^y + 3^y$

c) $3^{2y+4} + 3^{2y+4} + 3^{2y+4}$

d) $2^3 + 2^3$

e) $2^3 + 2^3 + 2^3 + 2^3$

f) $2^3 + 2^3 + 3^3 + 3^3 + 3^3$

g) $2^3 + 2^3 + 3^3$

h) $2^x + 2^x + 2^y$

i) $2^x + 2^y + 2^x + 2^y$

j) $2^3 \times 2^3 + 2^6$

k) $2^5 + 2^{10} \div 2^5$

l) $(2^x)^2 + 2^{x^2} + 2^{2x}$

2) Solve

a) $4^5 + 4^5 + 4^5 + 4^5 = 4^x$

b) $4^5 + 4^5 + 4^5 + 4^5 = 4^{3x-5}$

c) $4^x + 4^x + 4^x + 4^x = 4^{10}$

d) $4^{3x+1} + 4^{3x+1} + 4^{3x+1} + 4^{3x+1} = 4^{10}$

e) $2^{3x+1} + 2^{3x+1} + 2^{3x+1} + 2^{3x+1} = 2^{10}$

f) $2^{3x+1} + 2^{3x+1} + 2^{3x+1} + 2^{3x+1} = 4^{10}$

g) $2^{5x-2} + 2^{5x-2} = 4^{10x-1}$

h) $2^{5x+3} + 2^5 \times 2^{5x-2} = 4^{10x-1}$

i) $4^3 + 4^3 = 2^x$

j) $8^3 + 8^3 + 8^3 + 8^3 = 2^x$

j) $8^3 + 8^3 + 8^3 + 8^3 = 4^x$

k) $8^{3x+2} + 8^{3x+2} + 8^{3x+2} + 8^{3x+2} = 2^x$

3) Solve

a) $2^3 + 2^3 = \dfrac{1}{2^x}$

b) $\dfrac{1}{2^3} + \dfrac{1}{2^3} = \dfrac{1}{2^x}$

c) $\dfrac{1}{8} + \dfrac{1}{4} \div 2 = \dfrac{1}{2^x}$

d) $\dfrac{1}{4^{3x}} + \dfrac{1}{4^{3x}} = \dfrac{1}{2^{7(x-2)}}$

e) $\left(\sqrt{2}\right)^3 + \left(\sqrt{2}\right)^3 = 2^x$

f) $\sqrt[3]{2^5} + \sqrt[3]{2^5} = 2^x$

g) $\sqrt[3]{4^5} + \sqrt[3]{4^5} = 8^x$

4) $2^{3x-2} + 2^{3x-2} = 4^{y+1}$ and $3^x \times (3^{x-4})^4 = \dfrac{1}{3^{3y}}$ find the value of x and y.

Fluency Practice

Find the value of m in each question

$16^5 = 2^{2m+4}$

$81^5 = 3^{3m+2}$

$125^7 = 5^{9m+3}$

$8^2 = 2^{2m-4}$

$27^4 = 3^{5m-3}$

$25^6 = 5^{7m-9}$

Fluency Practice

Find the value of x in each question

$5^{-n} = 0.25$

$(5^3)^n = x$

$7^{-n} = 0.1$

$(7^3)^n = x$

$3^{-n} = 0.04$

$(3^{0.5})^n = x$

$3^{-n} = 0.2$

$(3^4)^n = x$

$6^{-n} = 0.5$

$(6^5)^n = x$

$9^{-n} = 0.4$

$(9^2)^n = x$

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Fluency Practice

Red box:

(a) Write 4^5 as a power of 2

(b) Write 8^3 as a power of 2.

(c) Write 16^3 as a power of 4.

(d) Write 27^4 as a power of 3.

(e) Write 125^3 as a power of 5.

Yellow box:

(a) $2^6 \times 2^4 = 4^n$. Find n.

(b) $8^n = 2^3 \times 2^9$. Find n.

(c) $3^n = 3^3 \times 9^2$. Find n.

(d) $2^n = \dfrac{16}{2^3}$. Find n.

(e) $\dfrac{3^8}{81} = 3^n$. Find n.

Green box:

(a) $4^3 \times 16 = 2^n$. Find n.

(b) $243 \times 9^2 = 3^n$. Find n.

(c) $2^n \times 4^3 = 1024$. Find n.

(d) $\dfrac{9^2 \times 81^{1/2}}{3^2} = 3^n$. Find n.

(e) $\dfrac{2^{2n} \times 64}{4^2} = 2^3$. Find n.

Purple box:

(a) $128 = 4^{2x} \times 2^x$. Find x.

(b) $\dfrac{1}{\sqrt[3]{9^4}} = 3^x$. Find x.

(c) $16^{1/5} \times 2^x = 8^{3/4}$. Find x.

(d) $\left(2^{1/2}\right)^x = \dfrac{32}{2^2}$. Find x.

(e) $243 = \dfrac{3^x}{\sqrt[3]{81}}$. Find x.

Change of Base Jumbled Answers

Write each question on the left as a single power of 5. Find your answer on the right.

5^{13}	5^{10}	5^{7}	5^{3}	$5^{1.5}$	$5^{\frac{3}{4}}$	5^{-1}	5^{-3}	5^{-5}	5^{-9}	$5^{\square}$
5^{14}	5^{12}	5^{8}	5^{4}	5^{2}	5^{1}	$5^{\frac{1}{2}}$	5^{-2}	5^{-4}	5^{-6}	5^{-24}

- -

A $\quad 125 = \underline{\hspace{2cm}}$

B $\quad \dfrac{1}{25} = \underline{\hspace{2cm}}$

C $\quad 25^{6} = \underline{\hspace{2cm}}$

D $\quad 125^{-3} = \underline{\hspace{2cm}}$

E $\quad \dfrac{1}{25^{2}} = \underline{\hspace{2cm}}$

F $\quad 25^{\frac{1}{2}} = \underline{\hspace{2cm}}$

G $\quad \left(\dfrac{1}{25}\right)^{3} = \underline{\hspace{2cm}}$

H $\quad (125^{2})^{-4} = \underline{\hspace{2cm}}$

I $\quad 25 \times 25 = \underline{\hspace{2cm}}$

J $\quad 25^{4} \times 25^{3} = \underline{\hspace{2cm}}$

K $\quad 25^{6} \times 5^{-5} = \underline{\hspace{2cm}}$

L $\quad \dfrac{1}{125} \times \dfrac{1}{25} = \underline{\hspace{2cm}}$

M $\quad 25^{7} \times \dfrac{1}{5} = \underline{\hspace{2cm}}$

N $\quad \dfrac{1}{5^{-9}} = \underline{\hspace{2cm}}$

O $\quad 125^{\frac{1}{4}} = \underline{\hspace{2cm}}$

P $\quad \dfrac{1}{25^{-4}} = \underline{\hspace{2cm}}$

more challenging

Q $\quad \sqrt{5} \times 5 = \underline{\hspace{2cm}}$

R $\quad \dfrac{25^{2}}{5^{4}} \times 5^{-3} = \underline{\hspace{2cm}}$

S $\quad \sqrt{25^{10}} = \underline{\hspace{2cm}}$

T $\quad \dfrac{1}{\sqrt[3]{125}} = \underline{\hspace{2cm}}$

U $\quad \dfrac{5}{\sqrt{5}} = \underline{\hspace{2cm}}$

V $\quad \sqrt{625} = \underline{\hspace{2cm}}$

Fluency Practice

$2^1 = 2$	$2^{11} = 2,048$
$2^2 = 4$	$2^{12} = 4,096$
$2^3 = 8$	$2^{13} = 8,192$
$2^4 = 16$	$2^{14} = 16,384$
$2^5 = 32$	$2^{15} = 32,768$
$2^6 = 64$	$2^{16} = 65,536$
$2^7 = 128$	$2^{17} = 131,072$
$2^8 = 256$	$2^{18} = 262,144$
$2^9 = 512$	$2^{19} = 524,288$
$2^{10} = 1024$	$2^{20} = 1,048,576$

$2^{21} = 2,097,152$
$2^{22} = 4,194,304$
$2^{23} = 8,388,608$
$2^{24} = 16,777,216$
$2^{25} = 33,554,432$
$2^{26} = 67,108,864$
$2^{27} = 134,217,728$
$2^{28} = 268,435,456$
$2^{29} = 536,870,912$
$2^{30} = 1,073,741,824$

Without using a calculator or paper, can you mentally compute these expressions?

1. 32×16

2. $4 \times 64 \times 1024$

3. $4 \times 16 \times 32 \times 64$

4. $1024 \div 64$

5. $1,048,576 \div 32,768$

6. $1,073,741,824 \div 64$

7. 128^2

8. $(1,024)^3$

9. $\sqrt{65,536}$

10. $\sqrt[3]{134,217,728}$

11. $\sqrt[5]{1,073,741,824}$

12. $(1,048,576)^{1/2}$

13. $(262,144)^{2/3}$

14. $\dfrac{524,288}{(16,384)^{\frac{1}{2}}}$

15. $\dfrac{(67,108,864)^{1/2}}{16^3 (256)^{1/4}}$

16. $\dfrac{1}{4} \bullet (65,536)$

Equating Powers

Whenever you see 4 you should think 2^2.
Whenever you see 8 you should think 2^3.
Whenever you see 27 you should think 3^3.
Whenever you see 64 you should think 4^3 or 8^2 or 2^6.
Etcetera...

Also, pro tip; when you see 1 you should think 2^0 or 3^0 or 5^0 or ...
All of these problems reduce to

$$k^{\text{fish}} = k^{\text{dog}} \quad \Rightarrow \quad \text{fish} = \text{dog}.$$

Questions

1. Solve $2^{x+1} = 4^x$.

2. Solve $8^{3x} = 2^{x-2}$.

3. Solve $3^{2x-1} = 9^{4x}$.

4. Solve $27^{3-x} = 81^{2x}$.

5. Solve $2 \times 2^{2x+1} = 8^{x-1}$.

6. Solve $5^x \times 25^3 = 5^{2x-1}$.

7. Solve $4^x \times 8^{x-1} = 2 \times 4^{3x+1}$.

8. Solve $8 \times 2^{x-1} = 4^{2x-1}$.

9. Solve $16 \times 8^{2x+1} = 2 \times 16^x$.

10. Solve $32^x = \dfrac{8^x}{2^{x+1}}$.

11. Solve $\dfrac{2^{x+1}}{4} = \dfrac{4^{x+3}}{8^x}$.

12. Solve $\dfrac{9^x}{27^{x+1}} = \dfrac{3^x}{81^{1-x}}$.

13. Solve $\dfrac{5^{-2x+1}}{25^{3-x}} = \dfrac{125^x}{5^{x-4}}$.

14. Solve $8 \times \dfrac{2^{2-x}}{4^{3-2x}} = \dfrac{8^x}{4^{2+x}}$.

15. Solve $1 = 8^2 \times 4^{x-1} \times 2^{x+1}$.

16. Solve $\dfrac{7^x}{49^{6-x}} = \dfrac{343^{1-x}}{7^{2x-3}}$.

17. Solve $8^{ax} = 4^{bx+1}$.

18. Solve $27^{ax+b} = 3^{cx-a}$.

19. Solve $9^{ax} \times 27^x \times 3^{ax} = 1$.

20. Solve $\dfrac{5^{ax-2}}{25^{b-x}} = 125^{cx+d}$.

21. Solve $\dfrac{2^{ax}}{2^{3-bx}} = \dfrac{4^{4+cx}}{8^{k-x}}$.

Only do the following if you've studied
solving quadratics by factorisation.

22. Solve $4^{x+2} = 2^{x^2+5}$.

23. Solve $3^{x^2+2} = 27^x$.

24. Solve $5^{2x^2} = 25^{2x+3}$.

25. Solve $(x^2 + 5x + 5)^{x^2+11x+30} = 1$.

Fluency Practice

Indices

1. Simplify fully:

(a) $x^2 \times x^3 \times x^5$.

(b) $2 \times (x^2)^3$.

(c) $(x^{-3})^{-5}$.

(d) $(3x^2)^3$.

(e) $(12x^9) \div (4x^3)$.

(f) $4x \times (3x^{-2})^2$.

(g) $\dfrac{(3x^2)^3}{3x^{-2}}$.

(h) $\dfrac{12x^8}{6x^4} + 6x^4$.

(i) $2x^5(2x^5 + 2x^5)$.

2. Simplify fully:

(a) $\dfrac{(2x^{-2})^3 \times (2x^3)^2}{6^2 \times x^{-4}}$.

(b) $\dfrac{2x^8 \times (2x)^8}{2x \times (2x)^5}$.

(c) $\dfrac{(3x^{-1})^3 \times (2x^{-2})^4}{(6x^2)^3}$.

(d) $\dfrac{3x^3(2xy)^2}{6x^5y^2}$.

(e) $\dfrac{a^3b^7}{a^{-2}b^4}$.

(f) $\dfrac{64(x^{-3}y^7)^2x^6y}{(4x^4y^2)^2}$.

(g) $\dfrac{7pq^{-1}(2pq^3)^3}{14p(2p^2q)^2}$.

(h) $\dfrac{(4m^7n)^3}{8n^3m^{-7}}$.

(i) $\dfrac{(uv)^2(2u^2v)^3}{(u^3v^8)^2(uv^{-1})^4}$.

(j) $\dfrac{3x(x^4y^8)^3}{x^3(3x^5y)^2}$.

(k) $\dfrac{6u^2v^3}{3u^{-2}v^5} \times \dfrac{4(u^3v^4)^2}{2uv^{-1}}$.

(l) $\dfrac{(2x^3)^3y}{3x^{-2}y^3} \div \dfrac{4(xy)^{-8}}{(3x^{-4})^2}$.

(m) $\dfrac{4x^3y}{7xy^2} \div \dfrac{8x(yz)^2}{14zx^2y^3}$.

(n) $\dfrac{\frac{6x}{y^2}}{(3xy^3)^3}$.

3. Evaluate without a calculator:

(a) 3^{-1}.

(b) 5^{-2}.

(c) 4^{-3}.

(d) 7^0.

(e) $\left(\frac{2}{3}\right)^{-1}$.

(f) $\left(\frac{4}{3}\right)^{-2}$.

(g) $\left(2\frac{1}{3}\right)^{-1}$.

(h) $\left(1\frac{3}{4}\right)^{-3}$.

(i) $\left(\frac{1}{2}\right)^{-2}$.

(j) $(0.6)^{-2}$.

(k) $\left(\frac{2}{3}\right)^{-1} + \left(\frac{5}{2}\right)^{-1}$.

(l) $\left(\frac{3}{2}\right)^{-2} + \left(\frac{3}{2}\right)^{2}$.

(m) $\left(\frac{1}{5}\right)^{-2} - \left(\frac{2}{7}\right)^{-1}$.

(n) $\left(2\frac{1}{2}\right)^{-3} + \left(3\frac{2}{3}\right)^{-1}$.

(o) $(0.4)^{-2} - (0.2)^{-1}$.

4. Evaluate without a calculator:

(a) $\dfrac{2^8 \times 2^{-3}}{2^4}$.

(b) $8^4 \div 4^6$.

(c) $27^3 \div 9^4$.

(d) $\dfrac{64^2}{8 \times 16}$.

5. Solve for x

(a) $2^x = 8$.

(b) $3^x = 81$.

(c) $5^x = \frac{1}{25}$.

(d) $3^{-x} = \frac{1}{27}$.

Fluency Practice

Simplify:

(a) $4^5 \times 4^2$

(b) $4^5 \times 4^3$

(c) $4^3 \times 4^5$

(d) 4×4^5

(e) $4^5 \times 4^{-2}$

(f) $4^0 \times 4^3$

(g) $4^{2.5} \times 4^{0.5}$

(h) $4^5 \times 5^2$

(i) $5^{-3} \times 5^2$

(j) $5^{-5} \times 5^{-3}$

Simplify:

(a) $4^5 \div 4^2$

(b) $4^5 \div 4^3$

(c) $4^2 \div 4^5$

(d) $4^5 \div 4$

(e) $4^2 \div 4^0$

(f) $4^5 \div 4^{-2}$

(g) $4^5 \div 5^4$

(h) $5^{2.5} \div 5^{0.5}$

(i) $\dfrac{5^7}{5^2}$

(j) $\dfrac{(-5)^7}{(-5)^2}$

Simplify:

(a) $(3^4)^5$

(b) $(3^5)^4$

(c) $(3^2)^5$

(d) $(3^{-2})^5$

(e) $(3^4)^1$

(f) $3^3 \times (3^4)^{0.5}$

Simplify:

(a) $\dfrac{2^3 \times 2^8}{2^5}$

(b) $\dfrac{2^{-3} \times 2^8}{2^1}$

Find x:

(a) $5^x \times 5^4 = 5^7$

(b) $\dfrac{3^x \times 3^{-2}}{3^4} = 3^{10}$

(c) $10^2 \times 10^x = 1000000$

Purposeful Practice

(a)	(b)	(c)	(d)
Write down the cube root of 27	Work out $3^5 - \sqrt{441}$	Write down the value of 5^0	Simplify $y^5 \times y^4$

(e)	(f)	(g)	(h)
Simplify $(x^{-3})^5$	Write as a power of 2 $$\frac{2^{12}}{2^3}$$	Simplify $(3a^2b^4)^3$	Simplify $\dfrac{a^5 \times a^2}{a^{-3}}$

(i)	(j)	(k)	(l)
Write $2\sqrt{2}$ as a single power of 2	Evaluate $\left(\dfrac{4}{9}\right)^{3/2}$	Evaluate $8^{-4/3}$	$\dfrac{4^{10} \times 4^x}{4^6} = 4^{-1}$ Find the value of x.

(m)	(n)	(o)	(p)
$$\frac{2^{10}}{64} = 2^n$$ Find the value of n.	Write $\dfrac{1}{\sqrt[3]{4}}$ as a single power of 2	$4^a = 16 \times 8^{2a}$ Find the value of a.	Given that $9^x = (27^a)^{1/2} \times 3^b$ find an expression for x in terms of a and b.

A1 Write 5^8 as a power of 25	**A2** Write 8^6 as a power of 2	**A3** Write 8^6 as a power of 4	**A4** Write $8^2 \times 4^3$ as a power of 2
B1 Write 2 as a power of 8	**B2** Write 4 as a power of 8	**B3** Write $\sqrt{8}$ as a power of 2	**B4** Express $3\sqrt{3}$ as a power of 9
C1 Express $\dfrac{1}{81}$ as a single power of 3	**C2** Express $\dfrac{1}{\sqrt{3}}$ as a single power of 9	**C3** Express $\dfrac{1}{4\sqrt{2}}$ as a single power of 2	**C4** Express $3^7 + 9^4 + 15 \times 27^2$ as a power of 3
D1 Solve: $32^x = \dfrac{1}{16}$	**D2** Solve: $2^x = 8^{\frac{1}{4}} \times 16^{\frac{1}{3}}$	**D3** Find the value of x if: $9^{\frac{3}{4}} \times 27^x = 81^{\frac{2}{3}}$	**D4** Find the value of m and n if: $6 \times 12^m = 9^4 \times 2^n$

Fluency Practice

Section a:

Simplify the following, giving your answer in index notation, or as a fraction or integer where possible:

A) $3^4 \times 3^5$

B) $4^4 \times 4^{-2}$

C) $y^0 \times y^{-3}$

D) $(-2)^3 \times (-2)^2$

E) $3^{2a} \times 3^{4a}$

F) $(a^2)^5$

G) $(2^2)^3$

H) $(2c^2)^3$

I) $(a^4 b^2)^{-2}$

J) $(2b^2)^3 \times (2b^2)^4$

K) $(5^2)^a \times (5^a)^4$

L) $z^3 \div z^2$

M) $8^3 \div 8^{-3}$

N) $z^3 \div z^{-4}$

O) $6a^3 \div 2a$

P) $15m^3 k^3 \div 3m^2 k^5$

Q) $(ab^2)^3 \div ab^2$

R) $c^2 \times c^0 \div c$

S) $27c^2 \times c^{-1} \div 9c^4$

T) $3ab^4 \times 6a^{-5}b^2$

U) $-2x^3 y^0 \times -2x^2 y^{-5}$

Section b:

Simplify the following, giving your answer in index notation

A) $12ab^3 \div 6a^2 b^2$

B) $\dfrac{6x^3 y}{3x^2}$

C) $\dfrac{12a^2 b}{4ab^2}$

D) $\dfrac{2a^4 b}{(3a^2)^2}$

E) $\dfrac{1}{a^2} \times \dfrac{2}{ab^2}$

F) $\dfrac{2b^3 c}{3ab} \times \dfrac{4ab}{2c}$

G) $\dfrac{(2xy)^3}{(6x^3 y)^2} \times \dfrac{2xy}{3x^2}$

H) $\left(\dfrac{2}{x^2}\right)^3$

I) $\left(\dfrac{ab^2}{8a}\right)^2$

Section c (hard):

Determine the value of the letters:

A) $2^a = 4^2$

B) $2^b = 8^2$

C) $2^c \times 4 = 2^5$

D) $2^{3d} = 64$

E) $2^e \times 2 = 32$

F) $3^f \div 3^2 = 3^8$

G) $2^g \div 2^3 = 16$

H) $(3^h)^2 = 3^6$

I) $26^i = 1$

J) $(-4j)^3 = -64$

K) $(2k)^2 = 100$

L) $\dfrac{2^3}{2^{-1}} = 2^l$

M) $\dfrac{2^m}{(2^m)^2} = \dfrac{1}{4}$

N) $\dfrac{2 \times 3^{2n}}{2^2 \times 3^n} = \dfrac{18}{4}$

Fluency Practice

1. Find the value of ...

a. $144^{\frac{1}{2}}$ b. $625^{\frac{1}{4}}$ c. $27^{\frac{1}{3}}$ d. $\left(\frac{1}{16}\right)^{0.25}$ e. $0.01^{\frac{1}{2}}$ f. $\left(\frac{125}{64}\right)^{\frac{1}{3}}$ g. $\left(\frac{1}{64}\right)^{-\frac{1}{6}}$ h. $81^{-\frac{1}{4}}$

2. Find the value of ...

a. $16^{\frac{3}{2}}$ b. $1000^{\frac{2}{3}}$ c. $32^{\frac{3}{5}}$ d. $\left(\frac{1}{4}\right)^{\frac{5}{2}}$ e. $\left(\frac{1}{81}\right)^{\frac{3}{4}}$ f. $\left(\frac{27}{125}\right)^{\frac{2}{3}}$ g. $\left(\frac{3125}{243}\right)^{\frac{4}{5}}$ h. $625^{-\frac{3}{4}}$

3. Find the missing values

a. $16^{\frac{3}{\square}} = 64$ b. $\square^{\frac{2}{5}} = 25$ c. $\left(\frac{\square}{81}\right)^{\frac{3}{4}} = \frac{8}{\square}$ d. $\left(\frac{100}{\square}\right)^{\frac{\square}{2}} = \frac{1000}{27}$ e. $\left(\frac{\square}{125}\right)^{\frac{2}{\square}} = 25$

4. Write in the form 9^n

a. 81 b. 3 c. 27 d. 243

5. Write in the form 64^n

a. 8 b. 4 c. 16 d. 512

6. Find the value of x

a. $9^{\frac{x}{4}} = 3$ b. $16^{\frac{x+2}{4}} = 2$ c. $125^{\frac{2x-1}{6}} = 5$ d. $27^{\frac{5x-7}{6}} = 81$

e. $32^{\frac{2x-3}{10}} = 4$ f. $\left(\frac{4}{25}\right)^{5x-2} = \frac{2}{5}$ g. $2 \times 9^{\frac{3x-7}{4}} = 6$ h. $3 \times 4^{2x+8} = 24$

Fluency Practice

Part One - Consolidate

1) $5^6 \times 5^3 = 5^?$

2) $5^{-3} \div 5^3 = 5^?$

3) $5^2 \times 5^{-4} = 5^?$

4) $b^6 \times b^{-3} = b^?$

5) $h^{-4} \div h^{-2} = h^?$

6) $y^{\frac{1}{2}} \times y^2 = y^?$

Part Two – Develop

1) $y^5 \times y^? = y^3$

2) $6^2 \div ?^2 = 9$

3) $5^0 \times 5^? = 5$

4) $y^{-3} \div y^? = 1$

5) $h^2 \times h = 1$

6) $y^{\frac{1}{2}} \times y^? = y^2$

Part Three – Enhance

1) $\frac{1}{36} \times \sqrt{6} = 6^?$

2) $8^x \div 8^{3x} = 8^4$

3) $\sqrt[3]{125} \times 5^x = 25$

4) $(\frac{1}{b})^2 \times \sqrt[3]{b} = b^?$

5) $(\sqrt[3]{(h)}^2)^{\frac{3}{2}} = h^?$

6) $y^x \times y^{\frac{2}{3}} = y^3$

Part Four - Investigate

$$x^2 \qquad x^{-3} \qquad \sqrt{x}^3 \qquad x^{-\frac{1}{2}} \qquad x^{\frac{2}{3}} \qquad \left(\frac{1}{x}\right)^2 \qquad x\sqrt{x}$$

Here are seven cards: Assume that x is a positive integer greater than 1

Find:

Two different pairs of cards that when multiplied, equal x
A matching pair of cards
Two cards which multiply and give an answer that is on a different card also in the list!
The two cards who give the largest product

Finally – create one more card similar to above, and use three more from the list, so that you can complete this sum:

$\boxed{}$ x $\boxed{}$ = $\boxed{}$ ÷ $\boxed{}$

Fluency Practice

shifting bases (i)

use a calculator to work out the missing powers

(1) $8 = 2^? = 4^? = 64^?$

(2) $64 = 2^? = 4^? = 8^?$

(3) $729 = 3^? = 9^? = 27^?$

(4) $4096 = 2^? = 4^? = 16^?$

(5) $15625 = 5^? = 25^? = 125^?$

shifting bases (ii)

(1) $9^2 = 3^?$

(2) $2^? = 8^2$

(3) $16^2 = 4^?$

(4) $81^3 = 3^?$

(5) $5^? = 625^2$

(6) $243^2 = 3^?$

(7) $78125^2 = 5^?$

(8) $3^? = 531441$

(9) $9^? = 43046721$

(10) $32^? = 33554432$

Fluency Practice

multiple writings

(a) use a calculator to work out the missing powers:

1.) $9^2 = 3^?$

6.) $243^2 = 9^?$

2.) $2^? = 8^2$

7.) $78125^2 = 5^?$

3.) $16^2 = 4^?$

8.) $3^? = 531441$

4.) $81^3 = 3^?$

9.) $9^? = 43046721$

5.) $5^? = 625^2$

10.) $32^? = 33554432$

example:
$25^2 = 5^?$
$625 = 5^4$

(b) write these as powers of two different base numbers:

1.) 81

6.) 4096

2.) 16

7.) 59049

3.) 10,000

8.) 0.001

4.) 2401

9.) 390625

5.) 512

10.) 1771561

example:
$625 = 5^4$
$625 = 25^2$

(c) write these as a power of a power:

1.) 4096

2.) 729

3.) 10,000,000

4.) 15625

5.) 117649

examples:
$46656 = (6^3)^2$
$531441 = (3^4)^3$

Fluency Practice

Complete the tables below! The first row has been completed for you.

2^x	Evaluate
2^3	8
2^0	
	32
2^4	
2^{-1}	$\sqrt{2}$
	2
$2^{\frac{3}{2}}$	$\frac{1}{8}$
2^{-6}	
	$\sqrt{128}$
	$\frac{1}{\sqrt[3]{2}}$
$2^{-\frac{1}{2}}$	
	$8\sqrt{2}$
$2^{-\frac{5}{2}}$	

3^x	Evaluate
3^2	9
	1
3^4	
	729
3^{-3}	
	$\sqrt[3]{3}$
	$\frac{1}{3}$
$3^{0.5}$	
	$9\sqrt{3}$
$3^{0.2}$	
	$\sqrt{27}$
$3^{-1.5}$	
	$\frac{1}{243}$
	$\frac{81}{\sqrt{3}}$
	$27\sqrt[5]{3}$

HENCH MATHS

Exam Questions

Complete the following exam style questions.

1) Find the value of the missing constant in the following equations.

a) $3^a = \frac{1}{27}$

b) $5^b = \frac{1}{\sqrt{5}}$

c) $7^c = \sqrt[3]{49}$

(3)

2) Solve the following equations to find x.

a) $\frac{8}{2^{3x}} = \sqrt{2}$

b) $4^x = \frac{\sqrt[3]{2}}{32}$

c) $125 = \frac{\left(\sqrt{5}\right)^3}{25^{1-x}}$

(9)

3) Solve the equations below to find x and y.

$\frac{16^x}{8^y} = \frac{1}{4}$

$4^x 2^y = 16$

(6)

Problem Solving

$x\sqrt{x}$ $(x-2)$ $(x+3)$ $(x+5)$ $(x+2)$

$(x-3)$ $(x+1)$ $(x-1)$ x^2 $\sqrt{x}$

$x^{-\frac{3}{2}}$ $\dfrac{1}{x}$ $x^2 + 2x + 4$

Select expressions from above which multiply to given the answers listed below.

You may not use an expression more than once in each product.

a) $x^2 + 8x + 15$

b) $x^3 + 2x^2$

c) $x^2 - 9$

d) x^2

e) $x^{\frac{3}{2}} - 3x^{\frac{1}{2}}$

f) 1

g) $\sqrt{x}$

h) $\dfrac{4}{x} + x + 2$

i) $x^2 + 6x + 5$

j) $x^3 + 2x^2 - 5x - 6$

k) $x^3 - 2x^2 - 5x + 6$

Problem Solving

The power (q) is negative

The base number (p) is a fraction

A

D

B

If you think a region is impossible to fill, convince me why!

G

F

E

H

Think of a number in the form p^q that could belong in each of the regions

C

The answer is positive

Problem Solving

The power (q) is a fraction

The base number (p) is a fraction

A

D

B

If you think a region is impossible to fill, convince me why!

G

F

E

H

Think of a number in the form p^q that could belong in each of the regions

C

The answer is an integer

Problem Solving

The power (q) is negative

The power (q) cannot be written as a unit fraction

A

D

B

G

E

F

C

H
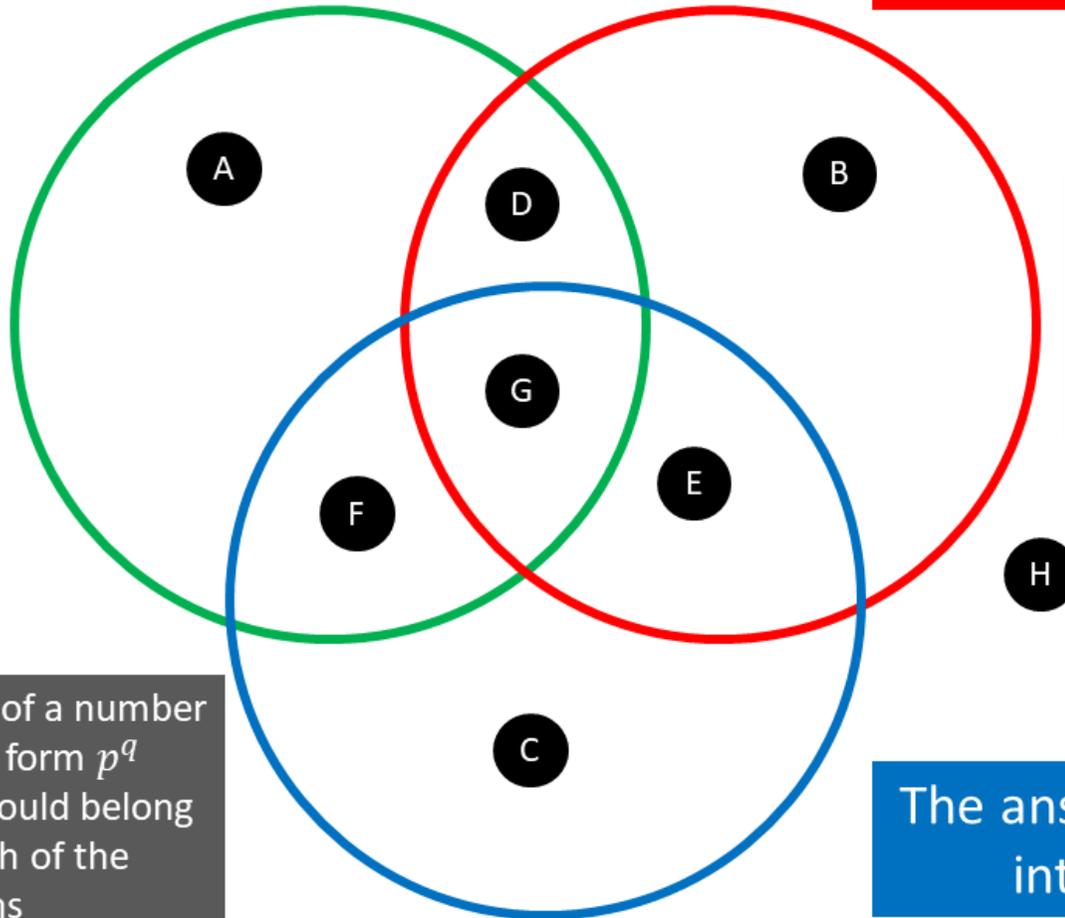
If you think a region is impossible to fill, convince me why!

Think of a number in the form p^q that could belong in each of the regions

p^q is a positive integer
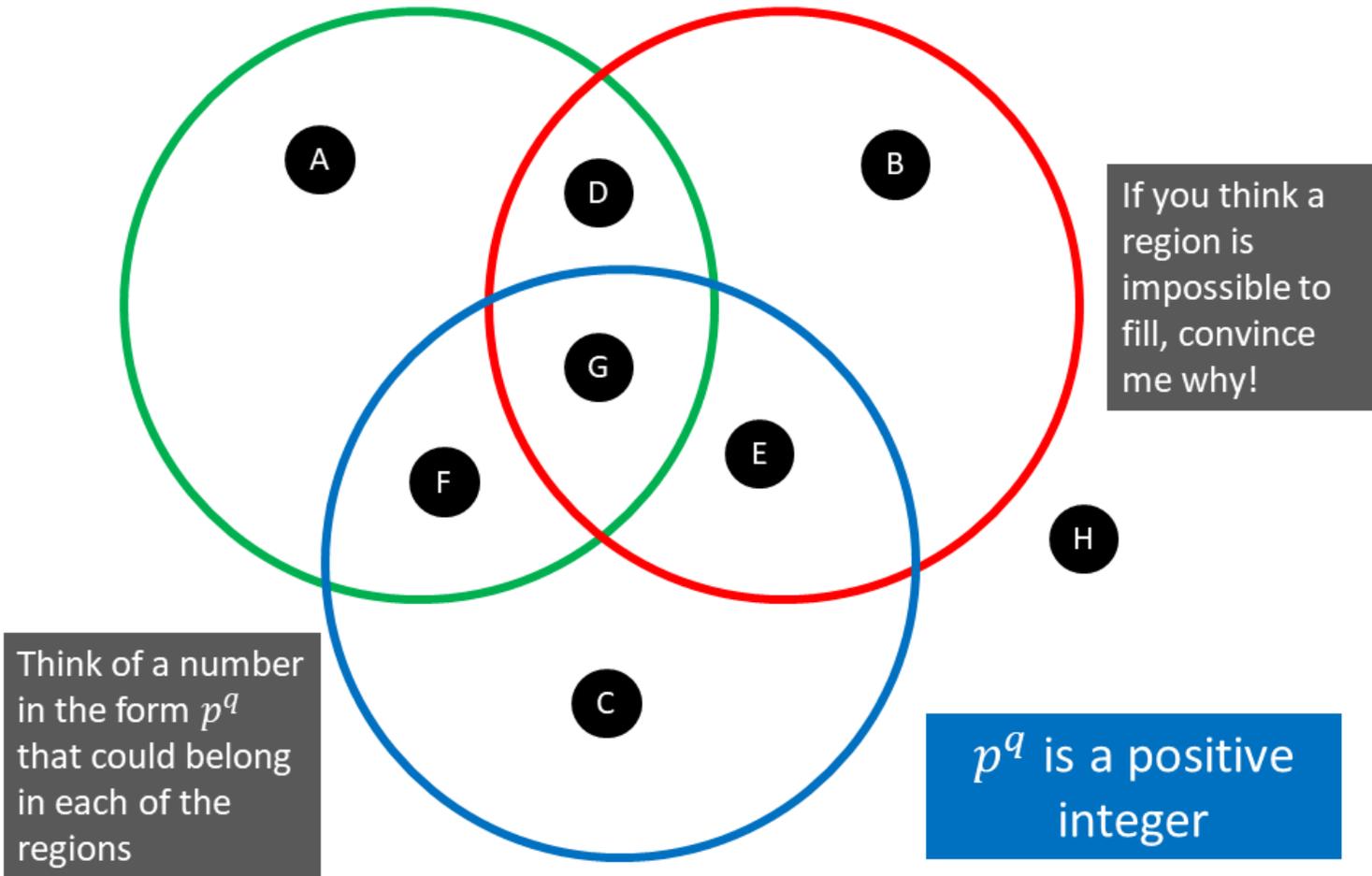
Problem Solving

$2^3 + 2^3 + 3^4 + 3^4 + 3^4 = ?$

- ◯ $2^3 + 3^4$
- ◯ 4^5
- ◯ $2^4 + 3^5$
- ◯ $2^6 + 3^{12}$

$$\sqrt{9^{16x^2}}$$

- ◯ 9^{4x}
- ◯ 9^{8x^2}
- ◯ 3^{4x}
- ◯ 3^{8x^2}

$$\frac{6^{2007} - 6^{2006}}{30} = ?$$

- ◯ 30
- ◯ 0.2
- ◯ 6^{2005}
- ◯ 6^{2007}
- ◯ 36

$$27^{-\frac{x}{3}} + 81^{\frac{1-x}{4}}$$

If the expression above can be stated in the form of $\dfrac{a}{b^x}$ for positive integers a and b, what is the value of $a + b$?

$$\begin{cases} ab = a^b \\ \dfrac{a}{b} = a^{3b} \end{cases}$$

a and b are real numbers, such that $a > 1$ and $b \neq 0$, satisfying the above system.

Find b^{-a}.

Intelligent Practice

Simplify:

1) $\sqrt{2} \times \sqrt{3}$

2) $\sqrt{4} \times \sqrt{3}$

3) $\sqrt{5} \times \sqrt{3}$

4) $\sqrt{6} \times \sqrt{3}$

5) $\sqrt{24} \times \sqrt{3}$

6) $\sqrt{12} \times \sqrt{3}$

7) $\sqrt{12} \times \sqrt{4}$

8) $\sqrt{11} \times \sqrt{4}$

9) $\sqrt{10} \times \sqrt{4}$

10) $\sqrt{10} \times \sqrt{8}$

11) $\sqrt{20} \times \sqrt{4}$

12) $\sqrt{20} \times \sqrt{5}$

13) $\sqrt{20} \times \sqrt{10}$

14) $\sqrt{20} \times \sqrt{20}$

15) $\sqrt{20} \times \sqrt{40}$

16) $\sqrt{20} \times \sqrt{40} \times \sqrt{2}$

17) $\sqrt{20} \times \sqrt{40} \times \sqrt{3}$

18) $\left(\sqrt{20} \times \sqrt{40} \times \sqrt{3}\right)^2$

Intelligent Practice

Simplify:

1) $2\sqrt{2} \times \sqrt{3}$

2) $2\sqrt{4} \times \sqrt{3}$

3) $\sqrt{5} \times 2\sqrt{3}$

4) $\sqrt{6} \times 2\sqrt{3}$

5) $\sqrt{24} \times 3\sqrt{3}$

6) $2\sqrt{12} \times 3\sqrt{3}$

7) $2\sqrt{12} \times 3\sqrt{4}$

8) $2\sqrt{11} \times 3\sqrt{4}$

9) $\sqrt{10} \times 2\sqrt{4}$

10) $\sqrt{10} \times 2\sqrt{8}$

11) $\sqrt{20} \times 2\sqrt{4}$

12) $5\sqrt{20} \times 3\sqrt{5}$

13) $5\sqrt{20} \times 3\sqrt{10}$

14) $5\sqrt{20} \times 3\sqrt{20}$

15) $2\sqrt{20} \times 3\sqrt{40}$

16) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{2}$

17) $2\sqrt{20} \times 2\sqrt{40} \times \sqrt{8}$

18) $2\sqrt{20} \times 2\sqrt{40} \times 2\sqrt{8}$

Intelligent Practice

Simplify:

1) $\sqrt{6} \div \sqrt{3}$

2) $\sqrt{6} \div \sqrt{2}$

3) $\sqrt{12} \div \sqrt{2}$

4) $\sqrt{12} \div \sqrt{3}$

5) $\sqrt{15} \div \sqrt{3}$

6) $\sqrt{18} \div \sqrt{3}$

7) $\sqrt{72} \div \sqrt{3}$

8) $\sqrt{36} \div \sqrt{3}$

9) $\sqrt{36} \div \sqrt{4}$

10) $\sqrt{40} \div \sqrt{4}$

11) $\sqrt{40} \div \sqrt{40}$

12) $\sqrt{40} \div \sqrt{5}$

13) $\sqrt{100} \div \sqrt{5}$

14) $\sqrt{200} \div \sqrt{10}$

15) $\sqrt{400} \div \sqrt{20}$

16) $\sqrt{800} \div \sqrt{40}$

17) $\sqrt{800} \div \sqrt{40} \div \sqrt{2}$

18) $\sqrt{800} \div \sqrt{40} \div \sqrt{20}$

Intelligent Practice

Simplify:

1) $2\sqrt{6} \div \sqrt{3}$

2) $4\sqrt{6} \div \sqrt{2}$

3) $4\sqrt{12} \div 2\sqrt{2}$

4) $4\sqrt{12} \div 2\sqrt{3}$

5) $6\sqrt{15} \div 3\sqrt{3}$

6) $6\sqrt{18} \div 3\sqrt{3}$

7) $24\sqrt{72} \div 3\sqrt{3}$

8) $12\sqrt{36} \div 3\sqrt{3}$

9) $12\sqrt{36} \div 4\sqrt{4}$

10) $20\sqrt{40} \div 4\sqrt{4}$

11) $20\sqrt{40} \div 20\sqrt{40}$

12) $20\sqrt{40} \div 5\sqrt{5}$

13) $10\sqrt{100} \div 5\sqrt{5}$

14) $20\sqrt{200} \div 10\sqrt{10}$

15) $40\sqrt{400} \div 20\sqrt{20}$

16) $80\sqrt{800} \div 40\sqrt{40}$

17) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{2}$

18) $80\sqrt{800} \div 40\sqrt{40} \div 2\sqrt{20}$

Fluency Practice

Multiplication Madness

Think of this like a normal multiplication table, just with terms missing everywhere. Fill in all blanks.

	$-\sqrt{5}$	$\sqrt{2}$		$-\sqrt{6}$		$2\sqrt{3}$	$-\sqrt{2}$				-2
3											
		$\sqrt{10}$			$3\sqrt{5}$	$2\sqrt{15}$					
			$16\sqrt{2}$								$8\sqrt{2}$
											4
$\sqrt{7}$	$-\sqrt{35}$					$2\sqrt{14}$					
							$2\sqrt{6}$				
		6			$9\sqrt{2}$						
				$-9\sqrt{2}$							
			$4\sqrt{6}$			$8\sqrt{3}$					
$-2\sqrt{5}$									-20		
$\sqrt{2}$			$2\sqrt{3}$	$-2\sqrt{3}$			$-\sqrt{6}$				
		4									

Surds: Multiplication Squares

Can you fill in the missing numbers in these multiplication squares?

×	√10	√6
√8	√	2√
√6	2√	

×	√6	√3
√12	_√8	
√7	√3 × √	√

×	3√6	2√8
√6		_√48
√3	_√2	4√

×	√9	2√3
2√3	_√	
3√3	_√	

×	√	√
√6	√48	√30
	√72	√45

×		2√5
	√60	10
	4√6	4√

Challenge: Make up one of your own!

Page 123

Fluency Practice

Multiplying, Dividing and Simplifying Surds

(a)	(b)	(c)	(d)	(e)
Simplify $\sqrt{5} \times \sqrt{7}$	Simplify $\sqrt{3} \times \sqrt{27}$	Simplify $\sqrt{10} \div \sqrt{2}$	Simplify $\dfrac{\sqrt{27}}{\sqrt{3}}$	Simplify $\sqrt{2} \times \sqrt{10} \times \sqrt{5}$
(f)	**(g)**	**(h)**	**(i)**	**(j)**
Simplify $2\sqrt{3} \times 4\sqrt{7}$	Simplify $\sqrt{5} \times 8\sqrt{2}$	Simplify $3\sqrt{5} \times 4\sqrt{5}$	Simplify $\dfrac{9\sqrt{6}}{3\sqrt{2}}$	Simplify $\dfrac{\sqrt{48}}{\sqrt{3}}$
(k)	**(l)**	**(m)**	**(n)**	**(o)**
Simplify $\dfrac{10\sqrt{24}}{5\sqrt{6}}$	Simplify $\sqrt{12}$	Simplify $\sqrt{28}$	Simplify $\sqrt{35}$	Simplify $\sqrt{108}$
(p)	**(q)**	**(r)**	**(s)**	**(t)**
Express $\sqrt{90}$ in the form $a\sqrt{10}$ where a is an integer	Express $\sqrt{75}$ in the form $a\sqrt{3}$ where a is an integer	Express $\sqrt{125}$ in the form $a\sqrt{5}$ where a is an integer	Simplify $\dfrac{\sqrt{200}}{5}$	Simplify $\dfrac{\sqrt{112}}{2}$

Intelligent Practice

Simplify:

1) $2\sqrt{3} + 5\sqrt{3}$

2) $2\sqrt{3} + 5\sqrt{6}$

3) $2\sqrt{3} + 5\sqrt{12}$

4) $2\sqrt{3} + 5\sqrt{27}$

5) $2\sqrt{3} + 6\sqrt{27}$

6) $2\sqrt{30} + 6\sqrt{27}$

7) $2\sqrt{30} + 6\sqrt{120}$

8) $6\sqrt{30} + 2\sqrt{120}$

9) $6\sqrt{60} + 2\sqrt{240}$

10) $6\sqrt{6} + 2\sqrt{24}$

11) $6\sqrt{6} + 2\sqrt{24} + 5\sqrt{36}$

12) $6\sqrt{16} + 2\sqrt{24} + 5\sqrt{36}$

Intelligent Practice

Simplify:

1) $\dfrac{2\sqrt{3}+5\sqrt{3}}{\sqrt{3}}$

2) $\dfrac{2\sqrt{3}+5\sqrt{12}}{\sqrt{3}}$

3) $\dfrac{2\sqrt{3}+5\sqrt{27}}{\sqrt{3}}$

4) $\dfrac{5\sqrt{27}-2\sqrt{3}}{\sqrt{3}}$

5) $\dfrac{10\sqrt{27}-4\sqrt{3}}{2\sqrt{3}}$

6) $\dfrac{10\sqrt{27}-30\sqrt{3}}{2\sqrt{3}}$

7) $\dfrac{10\sqrt{270}+30\sqrt{30}}{2\sqrt{30}}$

8) $\dfrac{10\sqrt{120}+30\sqrt{30}}{2\sqrt{30}}$

9) $\dfrac{10\sqrt{120}+30\sqrt{30}}{50\sqrt{30}}$

Work out:

(a) $4\sqrt{2} + 3\sqrt{2}$ (b) $10\sqrt{3} - 2\sqrt{3}$

(c) $-3\sqrt{5} + 7\sqrt{5}$ (d) $6\sqrt{2} - 8\sqrt{2}$

(e) $4\sqrt{3} + 7\sqrt{3} - \sqrt{3}$

(f) $2\sqrt{7} + 5\sqrt{7} - 8\sqrt{7}$

(g) $\frac{3}{2}\sqrt{5} + \frac{7}{2}\sqrt{5} - \frac{1}{2}\sqrt{5}$

Express as a single surd:

(a) $3\sqrt{2} + \sqrt{8}$ (b) $\sqrt{40} + \sqrt{10}$

(c) $6\sqrt{5} - \sqrt{20}$ (d) $-2\sqrt{3} + \sqrt{48}$

(e) $\sqrt{8} + \sqrt{32} - 10\sqrt{2}$

(f) $5\sqrt{3} - \sqrt{3} + 2\sqrt{12}$

(g) $-3\sqrt{10} - \sqrt{90} - 2\sqrt{160}$

Simplify:

(a) $5 + 2\sqrt{3} + 13 + 5\sqrt{3}$

(b) $5\sqrt{2} - 3 - 2\sqrt{2} + 11$

(c) $2\sqrt{3} + 3\sqrt{2} + 6\sqrt{3} - \sqrt{2}$

(d) $3\sqrt{5} - \sqrt{10} - 6\sqrt{10} - \sqrt{5}$

(e) $\sqrt{8} + \sqrt{20} + 6\sqrt{2} + 3\sqrt{5}$

(f) $\sqrt{200} - 3\sqrt{6} + 6\sqrt{2} - \sqrt{486}$

Find the values of x and y and the perimeter of the compound shape in the form $a\sqrt{3} + b\sqrt{5}$.

Fluency Practice
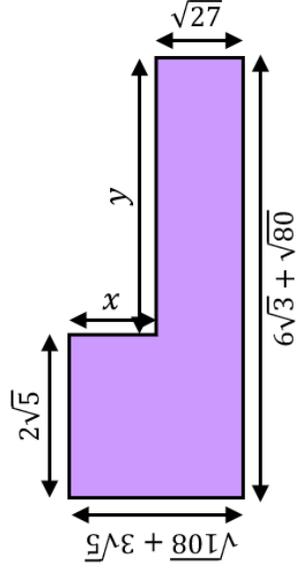
Adding & Subtracting Surds

1) True or False?   Are these calculations correct?

a) $\sqrt{8} + \sqrt{18} =$

$2\sqrt{2}$ + $3\sqrt{2}$ $= 5\sqrt{2}$

b) $\sqrt{8} + \sqrt{50} =$

$2\sqrt{2}$ + $5\sqrt{2}$ $= 7\sqrt{2}$

c) $\sqrt{12} + \sqrt{27} =$

$2\sqrt{3}$ + $4\sqrt{3}$ $= 6\sqrt{3}$

d) $\sqrt{90} - \sqrt{40} =$

$3\sqrt{10}$ $-$ $2\sqrt{10} = \sqrt{10}$

e) $\sqrt{80} - \sqrt{20} =$

$8\sqrt{10}$ $-$ $2\sqrt{10} = 6\sqrt{10}$

f) $\sqrt{125} + \sqrt{45} =$

$5\sqrt{5}$ + $3\sqrt{5}$ $= 8\sqrt{5}$

g) $\sqrt{63} + \sqrt{28} =$

$3\sqrt{7}$ + $4\sqrt{7}$ $= 7\sqrt{7}$

h) $\sqrt{150} - \sqrt{54} =$

$5\sqrt{6}$ $-$ $3\sqrt{6}$ $= 2\sqrt{6}$

i) $\sqrt{75} - \sqrt{108} =$

$5\sqrt{3}$ $-$ $6\sqrt{3}$ $= \sqrt{3}$

j) $\sqrt{180} + \sqrt{18} =$

$10\sqrt{18}$ + $\sqrt{18} = 11\sqrt{18}$

k) $\sqrt{72} + 2\sqrt{8} =$

$6\sqrt{2}$ + $4\sqrt{2} = 10\sqrt{2}$

l) $2\sqrt{18} - \sqrt{32} =$

$5\sqrt{2}$ $-$ $4\sqrt{2}$ $= \sqrt{2}$

2) Free Practice

a) $\sqrt{48} + \sqrt{27} =$

b) $\sqrt{360} + \sqrt{40} =$

c) $\sqrt{150} - \sqrt{96} =$

d) $\sqrt{200} - \sqrt{32} =$

e) $\sqrt{20} + \sqrt{40} =$

f) $\sqrt{48} - \sqrt{12} + \sqrt{27} =$

g) $\sqrt{8} + \sqrt{8} =$

h) $\sqrt{45} + \sqrt{45} =$

i) $\sqrt{18} + \sqrt{18} + \sqrt{32} =$

j) $\sqrt{90} + (2 \times \sqrt{40}) =$

k) $\sqrt{18} + 2\sqrt{18} =$

l) $3\sqrt{20} - \sqrt{180} =$

3) Extension

a) $2\sqrt{24} + \sqrt{54} =$

b) $2\sqrt{28} - \sqrt{63} =$

c) $2\sqrt{32} + 2\sqrt{18} =$

d) $2\sqrt{48} - 2\sqrt{117} =$

e) $3\sqrt{50} - \sqrt{200} =$

f) $2\sqrt{180} - 2\sqrt{125} =$

g) $3\sqrt{175} + 4\sqrt{28} =$

h) $3\sqrt{500} - 3\sqrt{245} =$

i) $2\sqrt{192} - 4\sqrt{75} =$

j) $2\sqrt{448} + 3\sqrt{640} =$

Fluency Practice

surd simplification

1.) $\sqrt{2} + \sqrt{32}$

2.) $\sqrt{3} + \sqrt{12}$

3.) $\sqrt{20} + \sqrt{125}$

4.) $\sqrt{18} + \sqrt{72}$

5.) $\sqrt{45} + \sqrt{20}$

6.) $\sqrt{24} + \sqrt{54}$

7.) $\sqrt{243} - \sqrt{48}$

8.) $\sqrt{176} - \sqrt{99}$

9.) $\sqrt{539} - \sqrt{275}$

10.) $\sqrt{800} - \sqrt{450}$

11.) $\sqrt{80} + \sqrt{125} + \sqrt{180}$

12.) $\sqrt{112} - \sqrt{63} + \sqrt{28}$

13.) $\sqrt{245} - \sqrt{125} + \sqrt{20}$

14.) $\sqrt{50} - 2\sqrt{72} + \sqrt{98}$

15.) $\sqrt{135} - \sqrt{540} + \sqrt{240}$

16.) $\sqrt{104} - \sqrt{234} + \sqrt{650}$

17.) $3\sqrt{50} - \sqrt{200} + 4\sqrt{50}$

18.) $5\sqrt{24} - 2\sqrt{54} + 3\sqrt{150}$

19.) $10\sqrt{20} - 3\sqrt{80} + 2\sqrt{605}$

20.) $\sqrt{198} - \sqrt{1078} + \sqrt{352}$

Pyramid II

Cut out the bricks below. Place the bricks in a pyramid shape so that each brick is the sum of the two bricks beneath it.

$7\sqrt{2} + \sqrt{147}$	$\sqrt{2} - \sqrt{75}$	$\sqrt{3}$
$2\sqrt{2} - \sqrt{432}$	$-5\sqrt{3}$	$\sqrt{3} + \sqrt{8}$
$-9\sqrt{3}$	$\sqrt{18} + \sqrt{48}$	$-3\sqrt{3} + \sqrt{722}$
$2\sqrt{2}$	$-\sqrt{2} - \sqrt{48}$	$12\sqrt{2} + 8\sqrt{3}$
$5\sqrt{2} + \sqrt{3}$	$2(\sqrt{3} + \sqrt{2})$	$-\sqrt{2} + \sqrt{3}$
$\sqrt{98} - \sqrt{363}$	$2\sqrt{3}$	$\sqrt{2} + \sqrt{12}$
$\sqrt{2}$	$\sqrt{8} - \sqrt{27}$	$\sqrt{27} + 4\sqrt{2}$
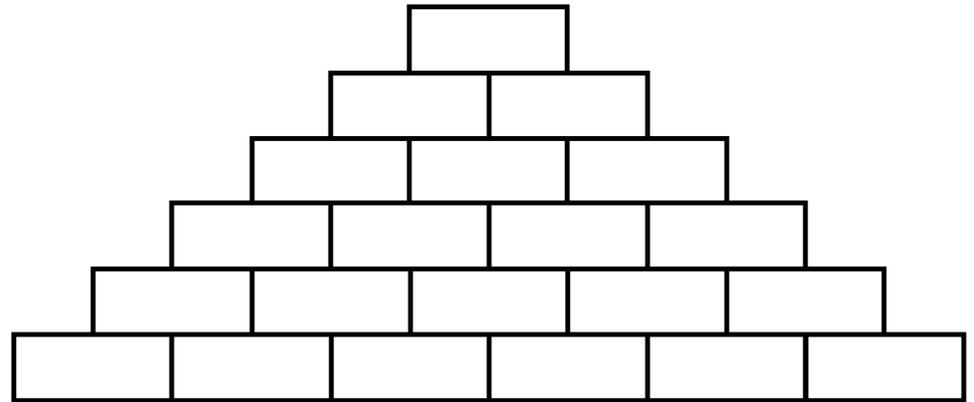
Surd Pyramids

Each brick is the two bricks below it <u>added together</u>.

A)

$6\sqrt{2}$ $2\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$

B)

$9\sqrt{2}$ $\sqrt{32}$ $\sqrt{8}$ $\sqrt{18}$

C)

$\sqrt{300}$ $\sqrt{75}$ $\sqrt{27}$ $\sqrt{12}$

D)

$13\sqrt{5}$ $\sqrt{45}$ $\sqrt{80}$ $\sqrt{20}$

E)

$\sqrt{72}$ $\sqrt{98}$ $\sqrt{32}$ $\sqrt{8}$ $\sqrt{18}$

F)

$22\sqrt{3}$ $\sqrt{108}$ $\sqrt{75}$ $\sqrt{48}$ $\sqrt{27}$ $\sqrt{27}$ $\sqrt{12}$ $\sqrt{3}$

②

Surd Pyramids

Each brick is the two bricks below it <u>added together</u>.

A)

$11\sqrt{2}$	
$\sqrt{8}$	$\sqrt{18}$
$\sqrt{32}$		

B)

	$13\sqrt{3}$
$\sqrt{48}$	$\sqrt{27}$
$\sqrt{12}$		

C)

$13\sqrt{5}$	
$\sqrt{45}$	$\sqrt{125}$
$\sqrt{20}$		

D)

$\sqrt{28}$	$\sqrt{63}$
$\sqrt{112}$		

E)

$\sqrt{200}$	$\sqrt{32}$
$\sqrt{98}$	$\sqrt{72}$

F)

$\sqrt{300}$	
$\sqrt{75}$	$\sqrt{108}$
$\sqrt{48} - \sqrt{3}$	$\sqrt{27}$	$\sqrt{12}$
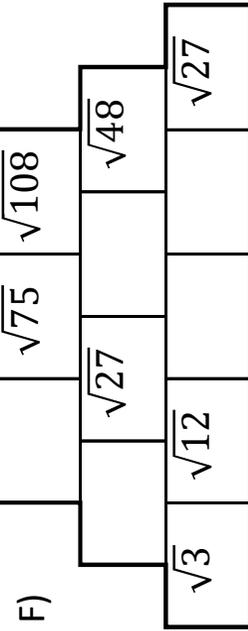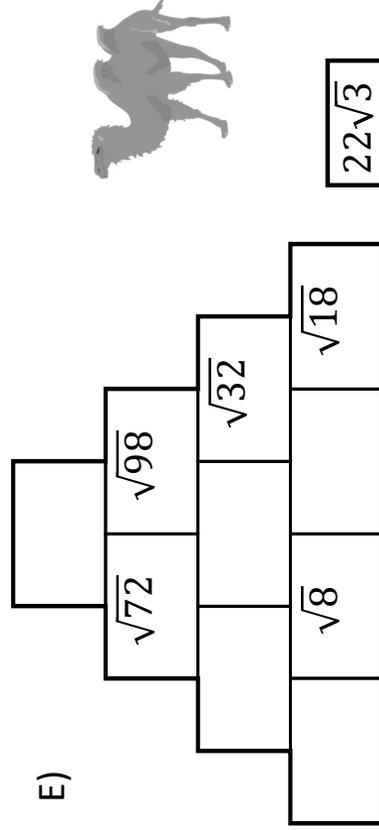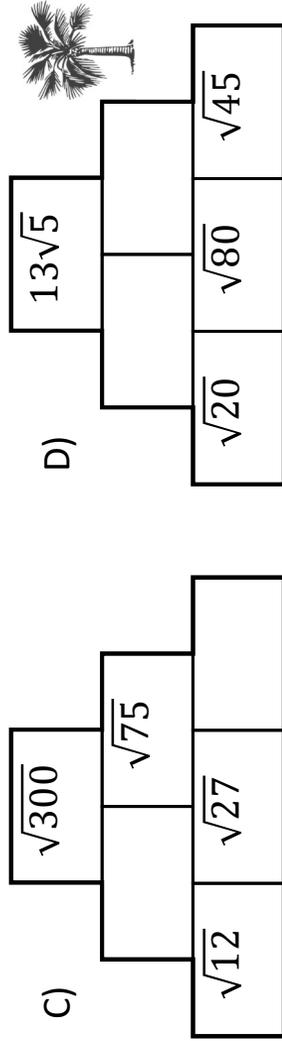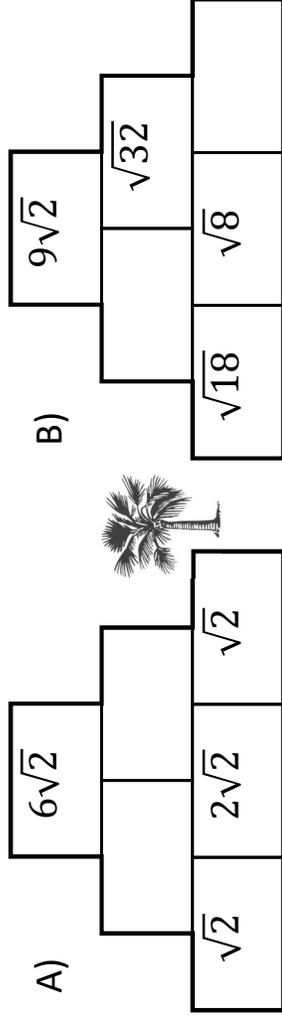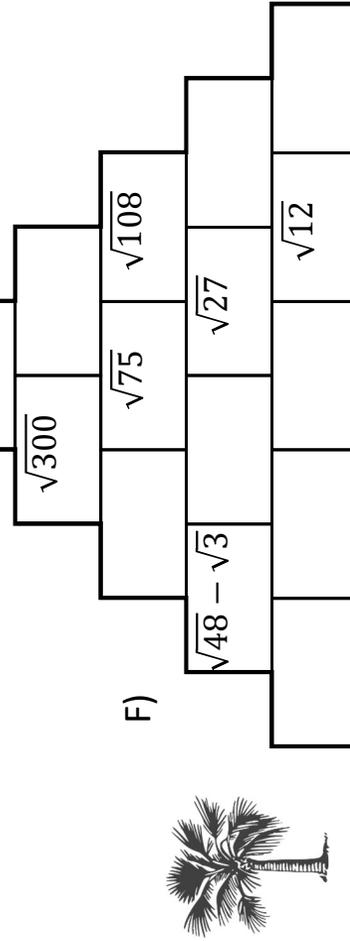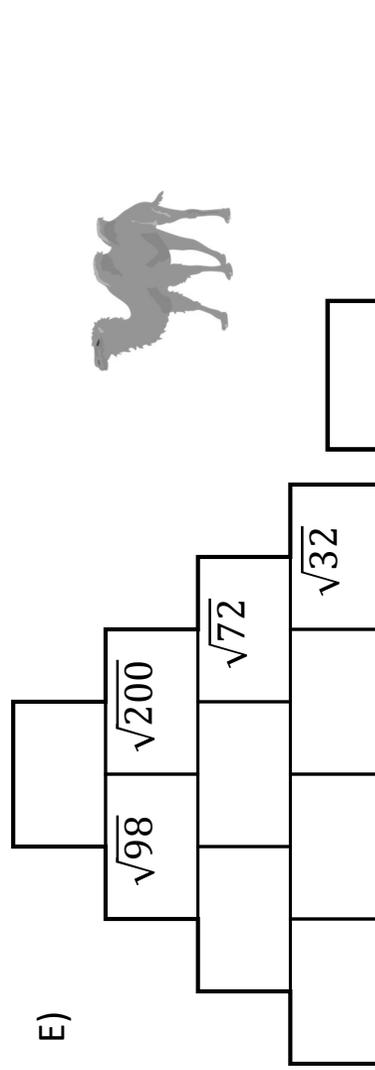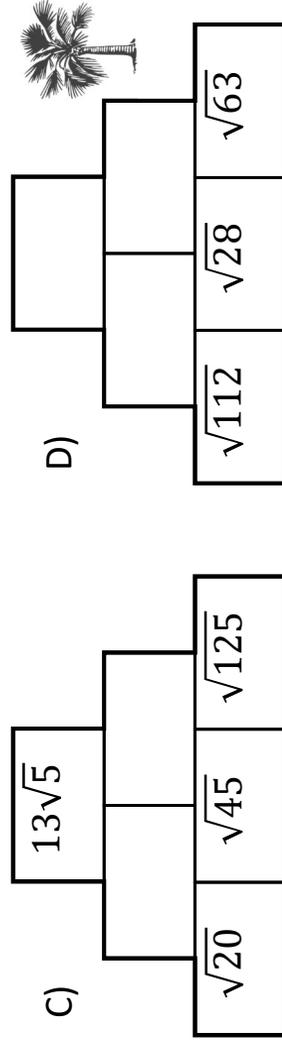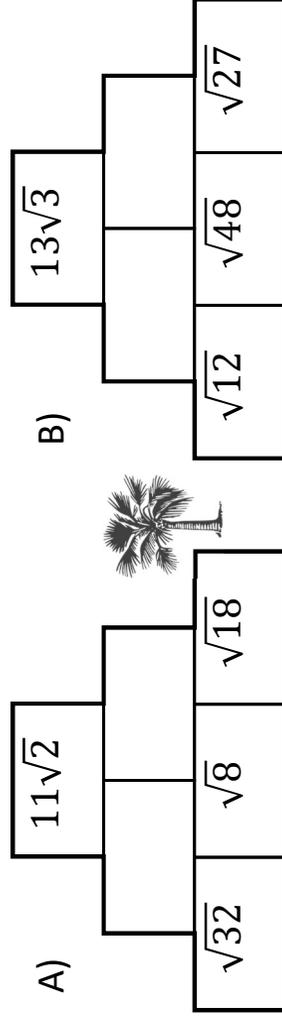
Problem Solving

surds puzzles

A

Tiles: $\sqrt{2}$, $\sqrt{6}$, 3, $3\sqrt{5}$, $\sqrt{5}$, $4\sqrt{2}$, $\sqrt{20}$, $3\sqrt{2}$

Operations: $\sqrt{2}$, $\times\sqrt{3}$, $\times\sqrt{3}$, $+\sqrt{2}$, $\div 4$, $\times\sqrt{10}$, $\div 2$, $\times 3$, $\div\sqrt{5}$

B

Tiles: $3\sqrt{2}$, $4\sqrt{3}$, $4\sqrt{6}$, $2\sqrt{3}$, $8\sqrt{6}$, $2\sqrt{3}$, $\sqrt{2}$, $8\sqrt{3}$

Operations: $5\sqrt{2}$, $-2\sqrt{2}$, $\div 3$, $\times\sqrt{6}$, $\times 4\sqrt{2}$, double, $\div 4$, $\times\sqrt{2}$, $\div 2$

C

Tiles: $2\sqrt{3}$, $12\sqrt{3}$, $4\sqrt{3}$, $6\sqrt{2}$, 3, $2\sqrt{6}$, $3\sqrt{3}$, 12

Operations: $\sqrt{8}$, $\times\sqrt{3}$, $\times\sqrt{2}$, half, square, $\times\sqrt{3}$, $\div 4$, $\div\sqrt{3}$, $\times\sqrt{8}$
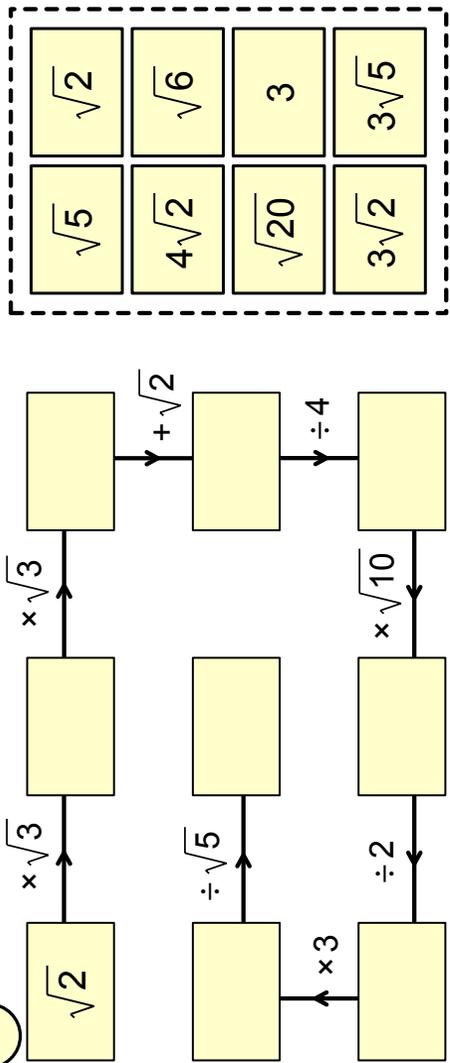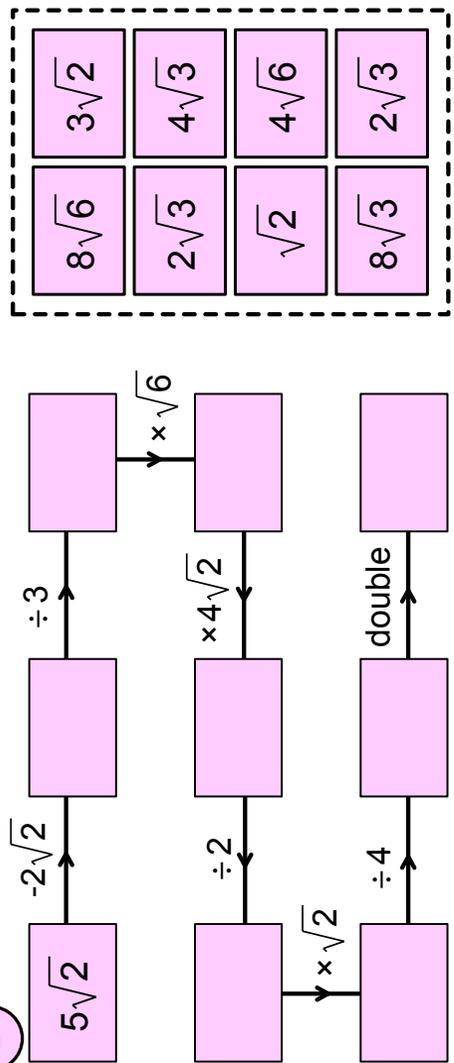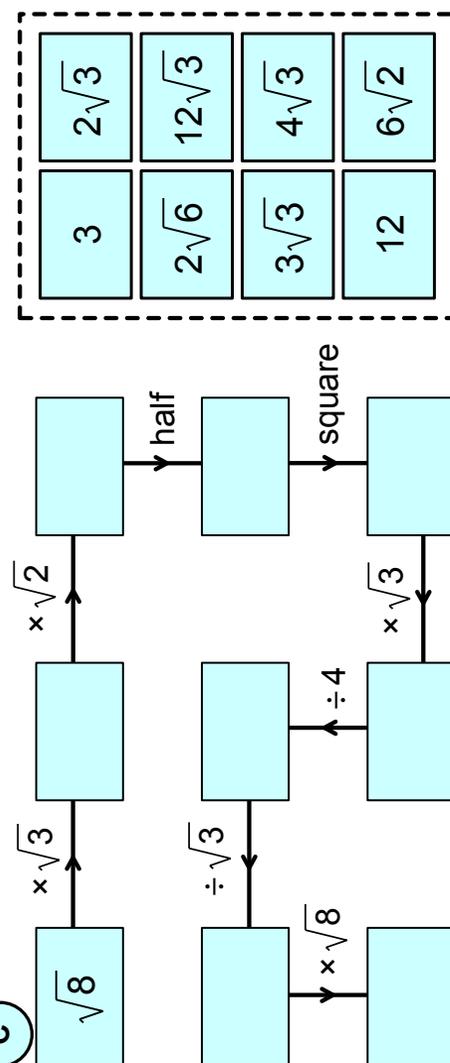
D *(hint: start in the middle)*

Tiles: 5, $2\sqrt{3}$, 36, $\sqrt{2}$, $5\sqrt{2}$, 18, $4\sqrt{3}$, 50, $5\sqrt{2}$, $\sqrt{10}$, $3\sqrt{2}$, $6\sqrt{3}$

Operations/values: $+2\sqrt{3}$, $\times 2$, $\times\sqrt{5}$, $\div\sqrt{3}$, square root, 6, $\times\sqrt{2}$, $\div\sqrt{3}$, $\sqrt{6}$, $\times\sqrt{5}$, 5, $\div\sqrt{2}$, $\times 2$, square, $\times\sqrt{3}$, $\times 5$, square, $\div 10$

E

Tiles: $\sqrt{18}$, $\sqrt{3}$, $\sqrt{2}$, 4, $2\sqrt{6}$, $\sqrt{6}$, $2\sqrt{3}$, 6, $4\sqrt{3}$, $5\sqrt{3}$

Operations/values: $\div 5$, $+$, $\times\sqrt{3}$, $\times 2\sqrt{3}$, square, $\div\sqrt{3}$, $\times 3$, $\times\sqrt{8}$, $\times\sqrt{2}$, $\div\sqrt{2}$, $\times\sqrt{3}$, $\times 2$, $\sqrt{8}$, $\sqrt{3}$
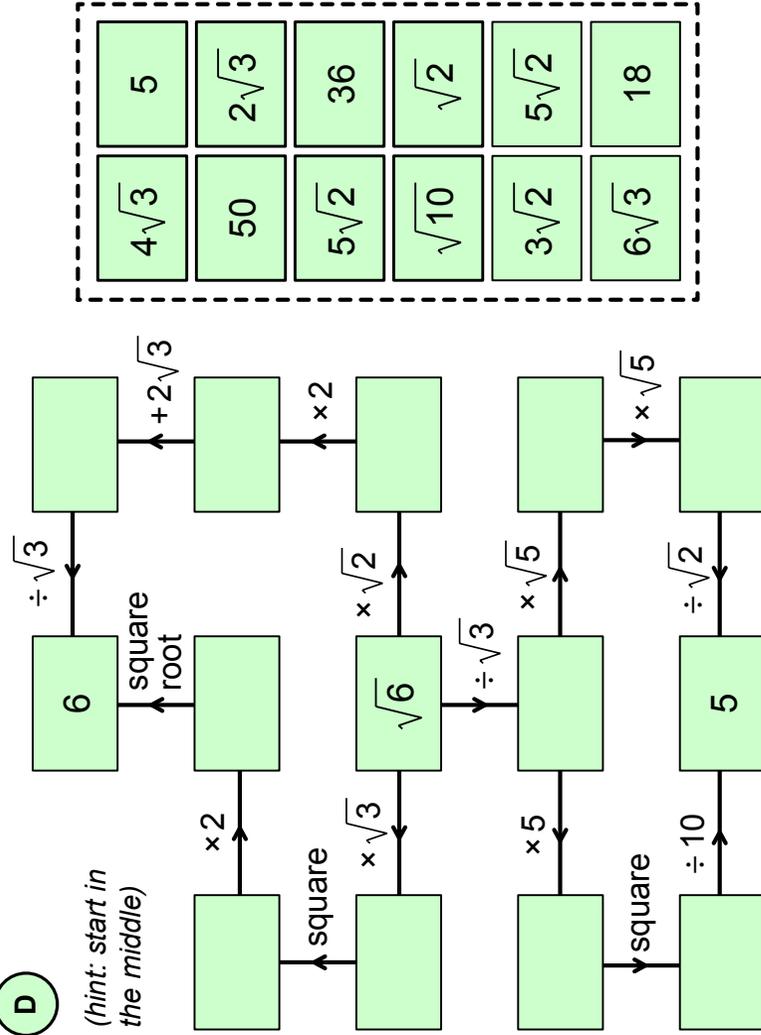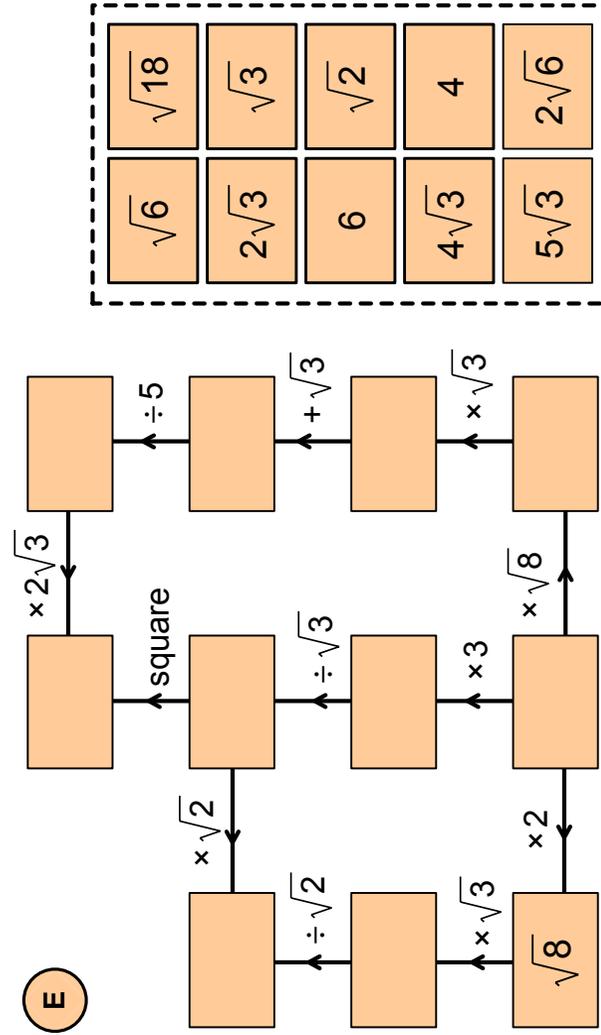
Fluency Practice

Find the missing lengths & simplify your answers.   **Pythagoras' Theorem with Surds**   Diagrams not drawn accurately.
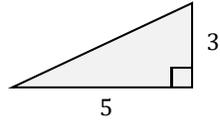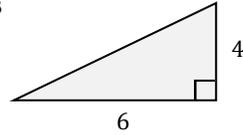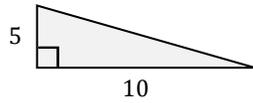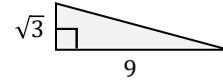
A

3
5

B

4
6

C

5
10

D

$\sqrt{3}$
9

E

$\sqrt{6}$
$\sqrt{12}$

F

$\sqrt{63}$
3

G

$\sqrt{123}$
5

H

$\sqrt{15}$ $\sqrt{48}$

I

$\sqrt{140}$
$\sqrt{110}$

J

$\sqrt{250}$ $\sqrt{70}$

K

$3\sqrt{5}$ $\sqrt{30}$

L

$4\sqrt{6}$
$4\sqrt{5}$

M

$5\sqrt{3}$

N

10 $8\sqrt{3}$

O

$2\sqrt{43}$
$4\sqrt{2}$

Calculate the area.
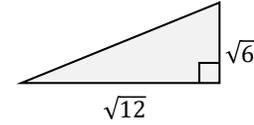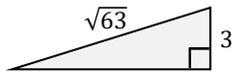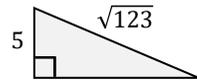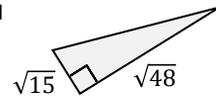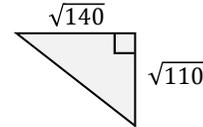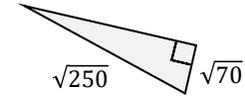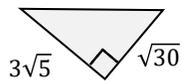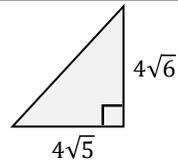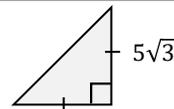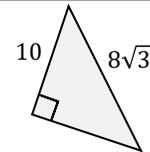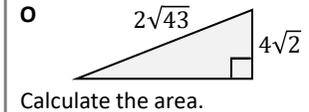
14 Answers	$3\sqrt{7}$	$3\sqrt{2}$	$5\sqrt{6}$	$\sqrt{34}$	$2\sqrt{23}$	$5\sqrt{5}$	$5\sqrt{3}$
	$2\sqrt{13}$	$4\sqrt{11}$	$2\sqrt{21}$	$5\sqrt{10}$	$7\sqrt{2}$	$6\sqrt{5}$	$3\sqrt{6}$

Averages with... **Surds**

For each set of numbers find: (i) the mean, (ii) the median, (iii) the range.

Leave your answers in their simplest form.

a) $5\sqrt{3}$ $4\sqrt{3}$ $6\sqrt{3}$

b) $\sqrt{3}$ $\sqrt{108}$ $\sqrt{12}$

c) $2\sqrt{12}$ $4\sqrt{12}$ $3\sqrt{27}$

d) $2\sqrt{27}$ $3\sqrt{12}$ $\sqrt{243}$

e) $2\sqrt{75}$ $\sqrt{48}$ $5\sqrt{12}$

f) $\sqrt{243}$ $5\sqrt{27}$ $3\sqrt{75}$

g) $\sqrt{20}$ $\sqrt{20}$ $\sqrt{80}$ $\sqrt{320}$

h) $\sqrt{28}$ $\sqrt{63}$ $3\sqrt{63}$ $4\sqrt{7}$

i) $-\sqrt{99}$ $2\sqrt{44}$ $-\sqrt{11}$ $4\sqrt{44}$

j) 0 $-2\sqrt{44}$ $-4\sqrt{44}$ $\sqrt{44}$

k) $\sqrt{2}$ $-\sqrt{18}$ $\sqrt{48}$ $\sqrt{8}$

l) $2\sqrt{150}$ $-2\sqrt{54}$ $2\sqrt{24}$ $-2\sqrt{96}$

Intelligent Practice

Expand and simplify:

1) $3(4 + \sqrt{5})$

2) $3(\sqrt{5} + 4)$

3) $6(\sqrt{5} + 4)$

4) $6(\sqrt{5} - 4)$

5) $6(4 - \sqrt{5})$

6) $-6(4 - \sqrt{5})$

7) $-6(-4 - \sqrt{5})$

8) $6(4 + \sqrt{5})$

9) $\sqrt{5}(4 + \sqrt{5})$

10) $\sqrt{5}(4 - \sqrt{5})$

11) $2\sqrt{5}(4 - \sqrt{5})$

12) $\sqrt{20}(4 - \sqrt{5})$

13) $\sqrt{20}(\sqrt{4} - \sqrt{5})$

14) $\sqrt{20}(\sqrt{6} - \sqrt{5})$

15) $\sqrt{20}(\sqrt{6} - \sqrt{10})$

16) $\sqrt{20}(\sqrt{12} - \sqrt{10})$

17) $\sqrt{10}(\sqrt{6} - \sqrt{5})$

18) $\sqrt{10}(\sqrt{6} - \sqrt{5} + \sqrt{24})$

Intelligent Practice

Expand and simplify:

1) $(\sqrt{5} + 2)(\sqrt{5} + 1)$

2) $(2 + \sqrt{5})(1 + \sqrt{5})$

3) $(2 - \sqrt{5})(1 + \sqrt{5})$

4) $(2 - \sqrt{5})(1 - \sqrt{5})$

5) $(2 + 2\sqrt{5})(1 + \sqrt{5})$

6) $(\sqrt{5} + 2)(\sqrt{5} - 1)$

7) $(\sqrt{7} + 2)(\sqrt{7} - 1)$

8) $(2\sqrt{7} + 2)(3\sqrt{7} - 1)$

9) $(2\sqrt{7} + 2)(3\sqrt{7} - 6)$

10) $(\sqrt{7} + 6)(\sqrt{7} - 6)$

11) $2(\sqrt{7} + 6)(\sqrt{7} - 6)$

12) $(\sqrt{7} - 6)(\sqrt{7} - 6)$

13) $(\sqrt{7} - 6)^2$

14) $(\sqrt{7} - 6)^3$

Intelligent Practice

Expand and simplify:

1) $(2 + \sqrt{3})(4 + \sqrt{12})$

2) $(2 + \sqrt{3})(4 - \sqrt{12})$

3) $(2 - \sqrt{3})(4 + \sqrt{12})$

4) $(2 - \sqrt{3})(4 - \sqrt{12})$

5) $(\sqrt{3} - 2)(\sqrt{12} - 4)$

6) $(\sqrt{12} - 2)(\sqrt{3} - 4)$

7) $(\sqrt{27} - 2)(\sqrt{3} - 4)$

8) $(\sqrt{27} - 3)(\sqrt{3} - 5)$

9) $(\sqrt{54} - 3)(\sqrt{6} - 5)$

10) $(2\sqrt{54} - 3)(\sqrt{6} - 5)$

11) $(2\sqrt{54} - 3)(4\sqrt{6} - 5)$

12) $(2\sqrt{44} - 3)(4\sqrt{11} - 5)$

13) $(3\sqrt{44} - 2)(5\sqrt{11} - 4)$

14) $(3\sqrt{44} - 2)^2$

Fluency Practice

1) 5×5

2) $5 \times \sqrt{5}$

3) $\sqrt{5} \times \sqrt{5}$

4) $5 \times 5\sqrt{5}$

5) $\sqrt{5} \times 5\sqrt{5}$

6) $5\sqrt{5} \times 5\sqrt{5}$

7) $5(5 + \sqrt{5})$

8) $\sqrt{5}(5 + \sqrt{5})$

9) $5(5 + 5\sqrt{5})$

10) $5\sqrt{5}(5 + 5\sqrt{5})$

11) $5\sqrt{5}(\sqrt{5} + 5\sqrt{5})$

12) $(5 + \sqrt{5})(5 + \sqrt{5})$

13) $(5 + \sqrt{5})(5 - \sqrt{5})$

14) $(5 + \sqrt{5})(5 + 5\sqrt{5})$

15) $(5 + 5\sqrt{5})(5 + 5\sqrt{5})$

16) $(\sqrt{5} + 5\sqrt{5})(\sqrt{5} + 5\sqrt{5})$

Fluency Practice

(a)

Simplify $2\sqrt{5} + 3\sqrt{5}$

(b)

Simplify $6\sqrt{5} + 5\sqrt{5}$

(c)

Simplify $2\sqrt{5} - 3\sqrt{5}$

(d)

Simplify $7\sqrt{3} - 3\sqrt{3}$

(e)

Simplify $2\sqrt{3} + 3\sqrt{2}$

(f)

Simplify $7\sqrt{10} + 3\sqrt{10} - \sqrt{10}$

(g)

Expand $3(6 + \sqrt{5})$

(h)

Expand $\sqrt{2}(7 - \sqrt{3})$

(i)

Expand $5\sqrt{2}(3 - \sqrt{7})$

(j)

Expand and simplify
$(3 + \sqrt{2})(6 + \sqrt{2})$

(k)

Expand and simplify
$(3 + \sqrt{7})(5 - \sqrt{7})$

(l)

Expand and simplify
$(4 + 3\sqrt{5})(2 - \sqrt{5})$

Fluency Practice

Simplify

(a) $4\sqrt{3} + 2\sqrt{3}$

(b) $4\sqrt{3} - 2\sqrt{3}$

(c) $2\sqrt{3} - 4\sqrt{3}$

(d) $2\sqrt{3} - 4\sqrt{5}$

(e) $-4\sqrt{2} + 2\sqrt{2}$

(f) $\sqrt{2} - 2\sqrt{2}$

(g) $6\sqrt{5} + 2\sqrt{5} - 3\sqrt{5}$

(h) $\sqrt{3} - 2\sqrt{3} + 7\sqrt{3}$

Expand and simplify where possible

(a) $5(2 + \sqrt{3})$

(b) $\sqrt{5}(2 + \sqrt{3})$

(c) $\sqrt{5}(\sqrt{2} - \sqrt{3})$

(d) $5(\sqrt{2} - \sqrt{3})$

(e) $\sqrt{3}(\sqrt{3} - 7)$

(f) $\sqrt{3}(2 + \sqrt{3})$

(g) $5\sqrt{2}(2 + \sqrt{3})$

(h) $\sqrt{5}(2\sqrt{3} + \sqrt{5})$

(i) $\sqrt{5}(\sqrt{2} + 2\sqrt{3})$

(j) $3\sqrt{5}(2\sqrt{2} + 3\sqrt{3})$

Expand and simplify where possible

(a) $(2 + \sqrt{3})(1 + \sqrt{3})$

(b) $(2 + \sqrt{3})(4 + \sqrt{3})$

(c) $(5 - \sqrt{5})(4 + \sqrt{5})$

(d) $(2 + \sqrt{5})(4 - \sqrt{5})$

(e) $(1 + 2\sqrt{3})(4 - \sqrt{3})$

(f) $(2 + 3\sqrt{5})(4 - 2\sqrt{5})$

Calculate the areas of these shapes, giving answers in their simplest form

(a) A square with side length $2 + \sqrt{5}$ cm

(b) A rectangle width length $\sqrt{7}$ cm and width $1 + \sqrt{3}$ cm

(c) A triangle with base $\sqrt{8}$ cm and height $2\sqrt{8}$ cm

Purposeful Practice

(a) Show that $(5 - \sqrt{12})(2 + \sqrt{3})$ can be written in the form $a + \sqrt{b}$ where a and b are integers.

(b) Show that $(6 + \sqrt{2})(\sqrt{8} - 4)$ can be written in the form $a\sqrt{2} + b$ where a and b are integers.

(c) Show that $\sqrt{3}(\sqrt{12} - 2)^2$ can be written in the form $a\sqrt{3} + b$ where a and b are integers.

(d) Show that $(5 + \sqrt{5})(\sqrt{20} - 2)(3 - \sqrt{5})$ can be written in the form $a\sqrt{5} + b$ where a and b are integers.

(e) Show that $(1 + \sqrt{8})^3$ can be written in the form $p + \sqrt{q}$ where p and q are integers.

(f) Given that

$$(a + \sqrt{6})(10 - \sqrt{6}) = 24 + b\sqrt{6}$$

find the values of the integers a and b.

(g) Given that

$$(5 - \sqrt{8})(3 - a\sqrt{2}) = b - 21\sqrt{2}$$

find the values of the integers a and b.

(h) Given that

$$\left(p + 2\sqrt{q}\right)^2 = 40 + 16\sqrt{q}$$

find the values of the integers p and q.

(i) Express $(6 + \sqrt{3})(a - 2\sqrt{3})(4 - \sqrt{12})$ in the form $b - 76\sqrt{3}$ where a and b are integers to be found.

(j) Given that

$$(p + \sqrt{q})(p - 3\sqrt{q}) = 13 - 14\sqrt{q}$$

find the values of the integers p and q.

Find the area and perimeter of these rectangles.

(a) $\sqrt{3}$, $2\sqrt{3}$

(b) $\sqrt{5}$, $7 + \sqrt{5}$

(c) $4 + \sqrt{2}$, $6 - 2\sqrt{2}$
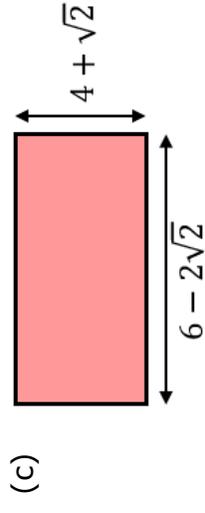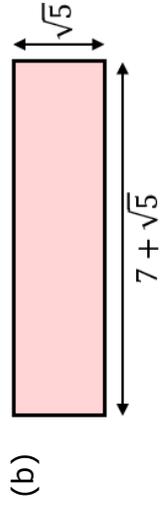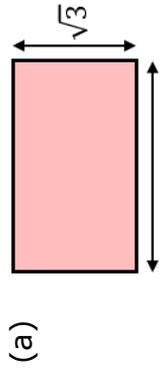
Find the area of these triangles.

(a) $\sqrt{6}$, $\sqrt{12}$

(b) $2\sqrt{2}$, $5 - \sqrt{2}$
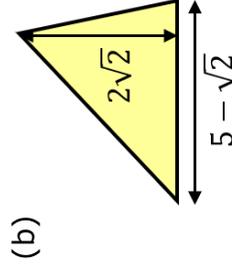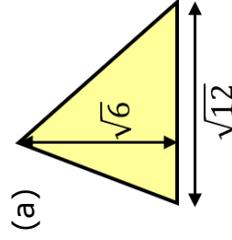
The perimeter of this rectangle is $8\sqrt{2}$. Find the area of the rectangle.

$\sqrt{2} - 1$
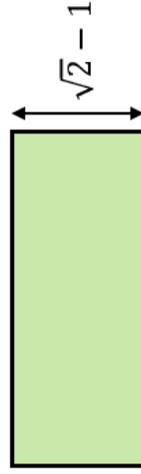
(a) The volume of the cuboid shown is $15 + 5\sqrt{3}$. Find the height of the cuboid.

$2 + \sqrt{3}$, $5\sqrt{3}$
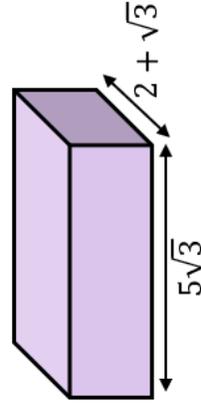
(b) The volume of a cylinder is $40\pi + 20\pi\sqrt{2}$. If the height of the cylinder is $2 + \sqrt{2}$, find the exact radius of the cylinder.

Purposeful Practice

generalising pythagoras surd families (i)

(1)
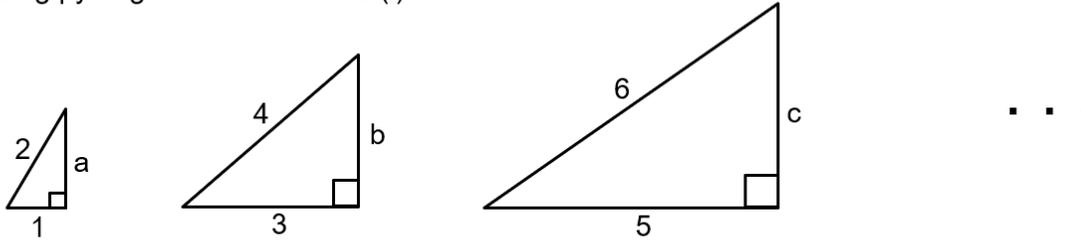
(2)
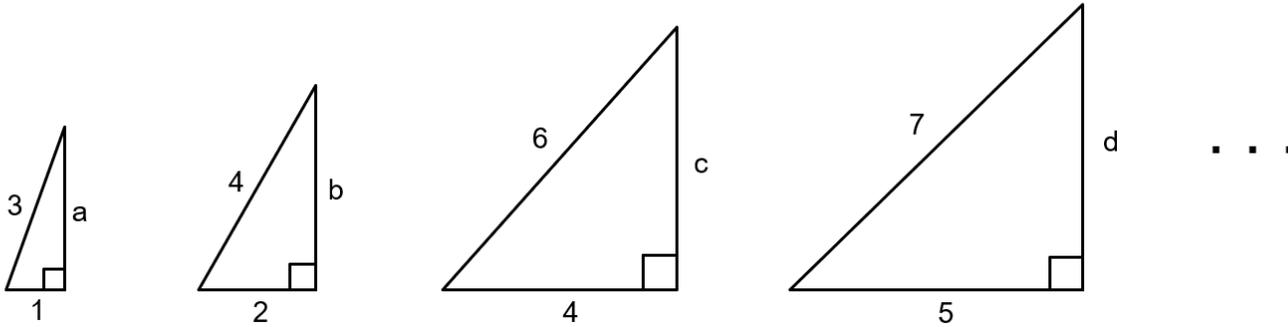
(3)

Purposeful Practice

generalising pythagoras surd families (ii)

(1)

4√6 6√2 a

10√15 15√5 b

14√21 21√7 c

. . .

(2)

a √6 − √2 √6 + √2

b √5 + √3 √5 − √3

c √7 − 1 √7 + 1

. . .

(3)

a √10 − √8 √10 + √8

b √14 + 2 √14 − 2

c 4 − √2 √2 + 4

. . .
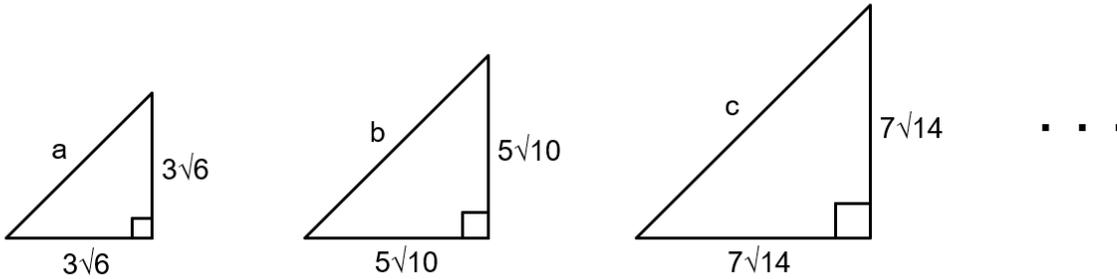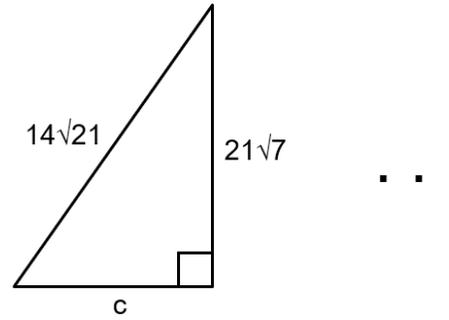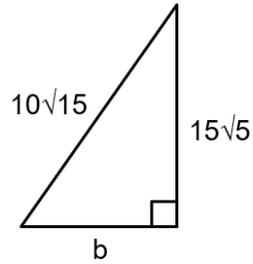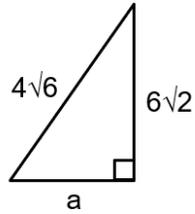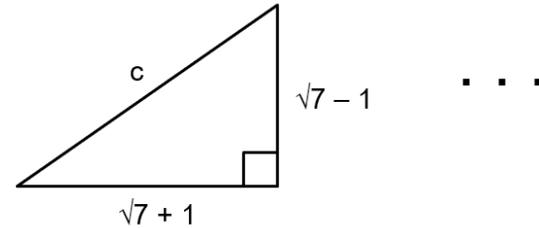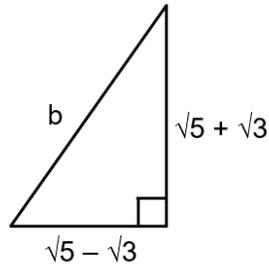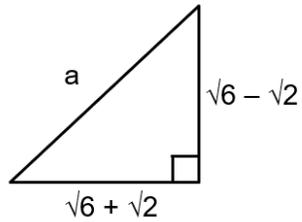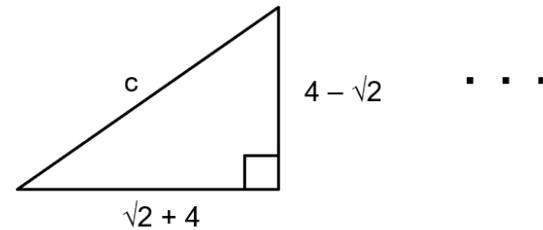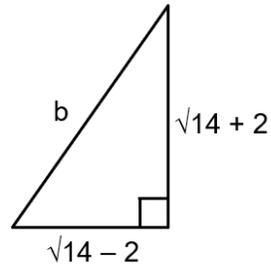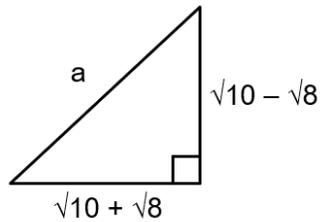
Purposeful Practice

find the missing length

(1)

a

$5 + \sqrt{7}$

$5 - \sqrt{7}$

(2)

b

$2 + 2\sqrt{7}$

$2\sqrt{7} - 2$

(3)

c

$6 + \sqrt{14}$

$6 - \sqrt{14}$

(4)

d

$4 + \sqrt{34}$

$\sqrt{34} - 4$

(5)

$2a\sqrt{5a}$

$4a\sqrt{a}$

b

(6)

$2c\sqrt{14c}$

$7c\sqrt{c}$

d

(7)

$6e\sqrt{2e}$

$8e\sqrt{e}$

f

(8)

$3g\sqrt{10g}$

$9g\sqrt{g}$

h
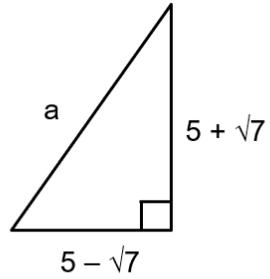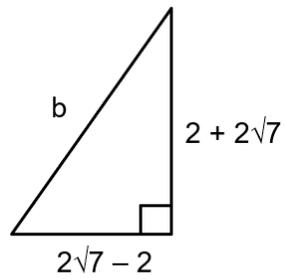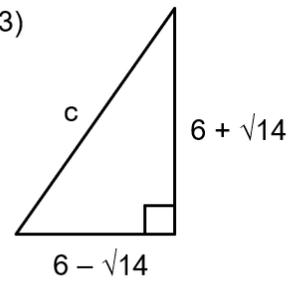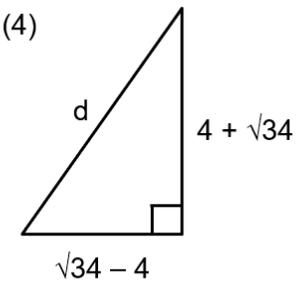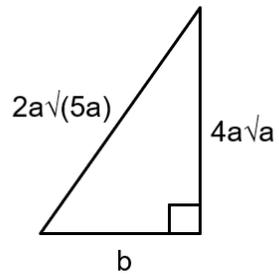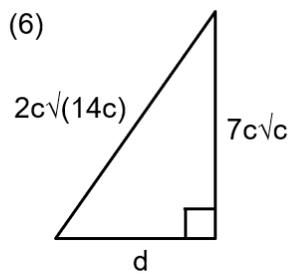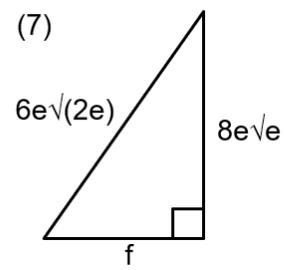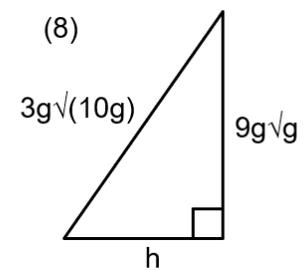
Problem Solving

making numbers by multiplying two surds

(A)

(1) $(3 + \sqrt{7})(3 - \sqrt{7}) = ?$

(2) $(a + \sqrt{b})(a - \sqrt{b}) = 3$

(3) $(a + \sqrt{b})(a - \sqrt{b}) = 1$

(4) $(a + b\sqrt{c})(a - b\sqrt{c}) = 4$ b and c cannot be 1 or 0

(5) $(a + b\sqrt{c})(a - b\sqrt{c}) = 5$

can you make the other digits in this way?

in more than one way?

(B)

(1) $(3 + b\sqrt{c})(3 - b\sqrt{c}) = 1$

(2) $(8 + b\sqrt{c})(8 - b\sqrt{c}) = 1$

(3) $(a + b\sqrt{3})(a - b\sqrt{3}) = 1$

(4) $(a + b\sqrt{5})(a - b\sqrt{5}) = 1$ b cannot be 0

(5) $(a + b\sqrt{11})(a - b\sqrt{11}) = 1$

can you make 1 in at least three other ways?

(C)

(1) $(b + \sqrt{b})(b - \sqrt{b}) = 6$

(2) $(a + \sqrt{a})(a - \sqrt{a}) = 42$

(3) $= 90$

(4) $= 110$

(5) $= 420$

(D)

(1) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 5$

(2) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 13$

(3) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 29$

(4) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 38$

(5) $(a\sqrt{b} + \sqrt{c})(a\sqrt{b} - \sqrt{c}) = 43$

Intelligent Practice

Rationalise:

1) $\dfrac{5}{\sqrt{2}}$

2) $\dfrac{6}{\sqrt{2}}$

3) $\dfrac{6}{\sqrt{3}}$

4) $\dfrac{4}{\sqrt{3}}$

5) $\dfrac{4}{\sqrt{6}}$

6) $\dfrac{5}{\sqrt{6}}$

7) $\dfrac{5}{2\sqrt{6}}$

8) $\dfrac{5}{3\sqrt{6}}$

9) $\dfrac{5}{10\sqrt{6}}$

10) $\dfrac{5}{10\sqrt{5}}$

11) $\dfrac{5}{\sqrt{5}}$

12) $\dfrac{8}{\sqrt{8}}$

13) $\dfrac{10}{\sqrt{8}}$

14) $\dfrac{10}{\sqrt{12}}$

15) $\dfrac{10}{2\sqrt{3}}$

16) $\dfrac{10+\sqrt{3}}{2\sqrt{3}}$

17) $\dfrac{10-\sqrt{3}}{2\sqrt{3}}$

18) $\dfrac{12-\sqrt{3}}{2\sqrt{3}}$

Fluency Practice

Simplify each fraction, leaving the denominator as a rational number.   Any surds in the numerator should also be fully simplified.

$\dfrac{2}{\sqrt{2}} =$	$\dfrac{3}{\sqrt{2}} =$	$\dfrac{4}{\sqrt{2}} =$	$\dfrac{5}{\sqrt{2}} =$	$\dfrac{6}{\sqrt{2}} =$	$\dfrac{7}{\sqrt{2}} =$	$\dfrac{8}{\sqrt{2}} =$
$\dfrac{\sqrt{2}}{\sqrt{2}} =$	$\dfrac{\sqrt{3}}{\sqrt{2}} =$	$\dfrac{\sqrt{4}}{\sqrt{2}} =$	$\dfrac{\sqrt{5}}{\sqrt{2}} =$	$\dfrac{\sqrt{6}}{\sqrt{2}} =$	$\dfrac{\sqrt{7}}{\sqrt{2}} =$	$\dfrac{\sqrt{8}}{\sqrt{2}} =$
$\dfrac{2}{2\sqrt{2}} =$	$\dfrac{3}{2\sqrt{2}} =$	$\dfrac{4}{2\sqrt{2}} =$	$\dfrac{5}{2\sqrt{2}} =$	$\dfrac{6}{2\sqrt{2}} =$	$\dfrac{7}{2\sqrt{2}} =$	$\dfrac{8}{2\sqrt{2}} =$
$\dfrac{2}{\sqrt{8}} =$	$\dfrac{3}{\sqrt{8}} =$	$\dfrac{4}{\sqrt{8}} =$	$\dfrac{5}{\sqrt{8}} =$	$\dfrac{6}{\sqrt{8}} =$	$\dfrac{7}{\sqrt{8}} =$	$\dfrac{8}{\sqrt{8}} =$
$\dfrac{2\sqrt{6}}{2\sqrt{2}} =$	$\dfrac{3\sqrt{6}}{2\sqrt{2}} =$	$\dfrac{4\sqrt{6}}{2\sqrt{2}} =$	$\dfrac{5\sqrt{6}}{2\sqrt{2}} =$	$\dfrac{6\sqrt{6}}{2\sqrt{2}} =$	$\dfrac{7\sqrt{6}}{2\sqrt{2}} =$	$\dfrac{8\sqrt{6}}{2\sqrt{2}} =$
$\dfrac{2\sqrt{6}}{2\sqrt{3}} =$	$\dfrac{3\sqrt{6}}{2\sqrt{3}} =$	$\dfrac{4\sqrt{6}}{2\sqrt{3}} =$	$\dfrac{5\sqrt{6}}{2\sqrt{3}} =$	$\dfrac{6\sqrt{6}}{2\sqrt{3}} =$	$\dfrac{7\sqrt{6}}{2\sqrt{3}} =$	$\dfrac{8\sqrt{6}}{2\sqrt{3}} =$

Fluency Practice

Express each fraction below as either:

- a rational number
- a fraction with a rational denominator, or
- a surd in simplest form

$$\frac{\sqrt{8}}{\sqrt{1}} \quad \frac{\sqrt{8}}{\sqrt{2}} \quad \frac{\sqrt{8}}{\sqrt{3}} \quad \frac{\sqrt{8}}{\sqrt{4}} \quad \frac{\sqrt{8}}{\sqrt{5}} \quad \frac{\sqrt{8}}{\sqrt{6}} \quad \frac{\sqrt{8}}{\sqrt{7}} \quad \frac{\sqrt{8}}{\sqrt{8}} \quad \frac{\sqrt{8}}{\sqrt{9}} \quad \frac{\sqrt{8}}{\sqrt{10}} \quad \frac{\sqrt{8}}{\sqrt{11}} \quad \frac{\sqrt{8}}{\sqrt{12}}$$

Intelligent Practice

Rationalise:

1) $\dfrac{2}{\sqrt{2}+3}$

2) $\dfrac{2}{3+\sqrt{2}}$

3) $\dfrac{2}{3-\sqrt{2}}$

4) $\dfrac{7}{3-\sqrt{2}}$

5) $\dfrac{7}{\sqrt{2}-3}$

6) $\dfrac{7}{\sqrt{7}-3}$

7) $\dfrac{\sqrt{7}}{\sqrt{7}-3}$

8) $\dfrac{\sqrt{5}}{\sqrt{5}-3}$

9) $\dfrac{\sqrt{5}}{5-\sqrt{3}}$

10) $\dfrac{2\sqrt{5}}{5-\sqrt{3}}$

11) $\dfrac{4\sqrt{5}}{5-\sqrt{3}}$

12) $\dfrac{4\sqrt{5}}{5-2\sqrt{3}}$

13) $\dfrac{4\sqrt{5}}{5-3\sqrt{3}}$

14) $\dfrac{4\sqrt{3}}{5-3\sqrt{3}}$

Fluency Practice

Q	Rationalise the denominator for:	Answers		Rationalise the denominator for:	Answers
1	$\frac{5}{\sqrt{3}}$		13	$\frac{5+\sqrt{3}}{\sqrt{3}}$	
2	$\frac{3}{\sqrt{5}}$		14	$\frac{3-\sqrt{5}}{\sqrt{5}}$	
3	$\frac{5}{2\sqrt{3}}$		15	$\frac{3-\sqrt{5}}{\sqrt{5}-2}$	
4	$\frac{3}{2\sqrt{5}}$		16	$\frac{3+\sqrt{5}}{\sqrt{5}+2}$	
5	$\frac{3}{5+\sqrt{2}}$		17	$\frac{5+\sqrt{3}}{7+2\sqrt{3}}$	
6	$\frac{3}{5-\sqrt{2}}$		18	$\frac{5-\sqrt{2}}{3\sqrt{2}+7}$	
7	$\frac{7}{\sqrt{5}}$		19	$\frac{5+\sqrt{2}}{7-\sqrt{18}}$	
8	$\frac{5}{\sqrt{7}}$		20	$\frac{4}{\frac{1}{\sqrt{3}}+\sqrt{3}}$	
9	$\frac{5}{3\sqrt{7}}$		21	$\frac{3}{\frac{1}{\sqrt{5}}-\sqrt{5}}$	
10	$\frac{5}{7\sqrt{3}}$		22	$\frac{a+\sqrt{b}}{\sqrt{b}}$	
11	$\frac{5}{\sqrt{7}+3}$		23	$\frac{a-\sqrt{b}}{\sqrt{c}}$	
12	$\frac{5}{\sqrt{7}-3}$		24	$\frac{a+\sqrt{b}}{c+\sqrt{b}}$	
13	$\frac{a}{b\sqrt{c}}$		25	$\frac{a+d\sqrt{b}}{c+\sqrt{b}}$	
14	$\frac{a}{b+\sqrt{c}}$		26	$\frac{a+d\sqrt{b}}{c+e\sqrt{b}}$	

Fluency Practice

Rationalise the denominator.

(a) $\dfrac{1}{\sqrt{5}}$

(b) $\dfrac{2}{\sqrt{5}}$

(c) $\dfrac{2}{\sqrt{7}}$

(d) $\dfrac{\sqrt{2}}{\sqrt{7}}$

(e) $\dfrac{2\sqrt{2}}{\sqrt{5}}$

(f) $\dfrac{2}{3\sqrt{5}}$

Rationalise the denominator.

(a) $\dfrac{6\sqrt{2}}{\sqrt{6}}$

(b) $\dfrac{5\sqrt{2}}{\sqrt{10}}$

(c) $\dfrac{5\sqrt{3}}{\sqrt{6}}$

(d) $\dfrac{2\sqrt{6}}{\sqrt{15}}$

Rationalise the denominator.

(a) $\dfrac{1+\sqrt{2}}{\sqrt{5}}$

(b) $\dfrac{2-\sqrt{2}}{\sqrt{5}}$

(c) $\dfrac{2-3\sqrt{2}}{\sqrt{7}}$

(d) $\dfrac{5+\sqrt{5}}{\sqrt{3}}$

(e) $\dfrac{10-\sqrt{2}}{\sqrt{3}}$

(f) $\dfrac{2+\sqrt{3}}{3\sqrt{5}}$

Rationalise the denominator.

(a) $\dfrac{1}{2+\sqrt{5}}$

(b) $\dfrac{2}{2-\sqrt{5}}$

(c) $\dfrac{\sqrt{2}}{3+\sqrt{7}}$

(d) $\dfrac{2\sqrt{5}}{3-\sqrt{7}}$

(e) $\dfrac{10\sqrt{2}}{5+\sqrt{3}}$

(f) $\dfrac{2+\sqrt{3}}{3-\sqrt{5}}$

(g) $\dfrac{2+\sqrt{3}}{5+\sqrt{3}}$

(h) $\dfrac{2+3\sqrt{3}}{3-4\sqrt{5}}$

Fluency Practice

surds: rationalise the denominators

a) simple rationalisation

(1) $\dfrac{12}{\sqrt{6}}$

(2) $\dfrac{\sqrt{24}}{\sqrt{6}}$

(3) $\dfrac{10\sqrt{2}}{\sqrt{5}}$

(4) $\dfrac{15}{5\sqrt{3}}$

(5) $\dfrac{6\sqrt{3}}{\sqrt{12}}$

(6) $\dfrac{\sqrt{8}}{\sqrt{72}}$

b) harder rationalisation

(1) $\dfrac{4}{3+\sqrt{5}}$

(2) $\dfrac{8}{1+\sqrt{3}}$

(3) $\dfrac{12}{6+\sqrt{12}}$

(4) $\dfrac{5}{2-\sqrt{3}}$

(5) $\dfrac{3}{\sqrt{5}-2}$

(6) $\dfrac{7}{3-\sqrt{2}}$

Fluency Practice

rationalise the denominators

(1) $\dfrac{3 - \sqrt{3}}{2 - \sqrt{3}}$

(2) $\dfrac{5 + \sqrt{7}}{3 + \sqrt{7}}$

(3) $\dfrac{5 + \sqrt{7}}{4 - \sqrt{7}}$

(4) $\dfrac{7 - \sqrt{5}}{3 - \sqrt{5}}$

(5) $\dfrac{9 - \sqrt{11}}{4 - \sqrt{11}}$

(6) $\dfrac{\sqrt{13} + 1}{\sqrt{13} - 3}$

(7) $\dfrac{4 + \sqrt{10}}{\sqrt{10} - 2}$

(8) $\dfrac{7 - 3\sqrt{5}}{3 + \sqrt{5}}$

(9) $\dfrac{5 + 3\sqrt{5}}{5 - 2\sqrt{5}}$

(10) $\dfrac{1 + 4\sqrt{11}}{2 + \sqrt{11}}$

(11) $\dfrac{8 + 4\sqrt{10}}{\sqrt{10} + 2}$

(12) $\dfrac{\sqrt{3} + 2}{\sqrt{12} + 4}$

Fluency Practice

simplifying surds
rationalising the denominator

Write each expression in the form $a + b\sqrt{2}$, where $a, b \in \mathbb{Q}$.

Answer tiles:

$\sqrt{2}-1$ | $\frac{1}{2}+\frac{1}{2}\sqrt{2}$ | $1+\sqrt{2}$ | $\sqrt{2}-2$ | $2+\sqrt{2}$ | $\frac{3}{2}+\sqrt{2}$ | $\frac{1}{2}+\frac{1}{4}\sqrt{2}$

$\frac{1}{2}+\frac{5}{2}\sqrt{2}$ | $2+3\sqrt{2}$ | $\frac{1}{2}+\sqrt{2}$ | $\frac{1}{4}\sqrt{2}$ | $1+\frac{1}{2}\sqrt{2}$ | $\frac{1}{8}\sqrt{2}$ | 1

$$\frac{\sqrt{50}-\sqrt{18}}{\sqrt{8}} \qquad \frac{\sqrt{8}-\sqrt{18}}{1+\sqrt{2}}$$

Questions:

$\dfrac{2+\sqrt{2}}{4}$	$\dfrac{2+\sqrt{8}}{4}$	$\dfrac{2+2\sqrt{8}}{4}$	$\dfrac{2(2+\sqrt{200})}{8}$	$\dfrac{4+\sqrt{32}}{4}$	$\dfrac{(2+\sqrt{2})^2}{4}$	$\dfrac{\sqrt{50}}{20}$	$\dfrac{1}{4\sqrt{2}}$	$\dfrac{1}{1+\sqrt{2}}$	$\dfrac{1}{2-\sqrt{2}}$	$\dfrac{4}{4-\sqrt{8}}$	$\dfrac{4-\sqrt{2}}{\sqrt{2}-1}$

Fluency Practice

Yay or Nay?

Each fraction has been standardised by **rationalising the denominator.**

Which have been completed incorrectly?

a) $\dfrac{1}{4+\sqrt{2}} \times \dfrac{4-\sqrt{2}}{4-\sqrt{2}} = \dfrac{1(4-\sqrt{2})}{16+4\sqrt{2}-4\sqrt{2}-2} = \dfrac{4-\sqrt{2}}{14}$

b) $\dfrac{2}{3+\sqrt{3}} \times \dfrac{3-\sqrt{3}}{3-\sqrt{3}} = \dfrac{2(3-\sqrt{3})}{9+3\sqrt{3}-3\sqrt{3}-3} = \dfrac{6-\sqrt{6}}{6}$

c) $\dfrac{3}{6-\sqrt{5}} \times \dfrac{6+\sqrt{5}}{6+\sqrt{5}} = \dfrac{3(6+\sqrt{5})}{36-6\sqrt{5}+6\sqrt{5}-5} = \dfrac{18-3\sqrt{5}}{30}$

d) $\dfrac{\sqrt{5}}{2-\sqrt{2}} \times \dfrac{2+\sqrt{2}}{2+\sqrt{2}} = \dfrac{\sqrt{5}(2+\sqrt{2})}{4-2\sqrt{2}+2\sqrt{2}-2} = \dfrac{2\sqrt{5}+\sqrt{10}}{2}$

e) $\dfrac{\sqrt{3}}{4+\sqrt{3}} \times \dfrac{4-\sqrt{3}}{4-\sqrt{3}} = \dfrac{\sqrt{3}(4-\sqrt{3})}{16+4\sqrt{3}-4\sqrt{3}-3} = \dfrac{3\sqrt{4}-3}{13}$

f) $\dfrac{\sqrt{3}}{5+\sqrt{6}} \times \dfrac{5-\sqrt{6}}{5-\sqrt{6}} = \dfrac{\sqrt{3}(5-\sqrt{6})}{25-5\sqrt{6}+5\sqrt{6}-6} = \dfrac{5\sqrt{3}-\sqrt{18}}{19}$

g) $\dfrac{\sqrt{7}}{6-\sqrt{7}} \times \dfrac{6+\sqrt{7}}{6+\sqrt{7}} = \dfrac{\sqrt{7}(6+\sqrt{7})}{36-6\sqrt{7}+6\sqrt{7}-7} = \dfrac{6\sqrt{7}+7}{29}$

h) $\dfrac{2+\sqrt{2}}{3+\sqrt{2}} \times \dfrac{3-\sqrt{2}}{3-\sqrt{2}} = \dfrac{(2+\sqrt{2})(3-\sqrt{2})}{9+3\sqrt{2}-3\sqrt{2}-2} = \dfrac{4+\sqrt{2}}{7}$

i) $\dfrac{4-\sqrt{5}}{3-\sqrt{5}} \times \dfrac{3+\sqrt{5}}{3+\sqrt{5}} = \dfrac{(4-\sqrt{5})(3+\sqrt{5})}{9-3\sqrt{5}+3\sqrt{5}-5} = \dfrac{7-\sqrt{5}}{4}$

Fluency Practice

Rationalising the Denominator

To rationalise these denominators we will multiply by one term.

A) $\dfrac{2 + \sqrt{3}}{\sqrt{3}}$

B) $\dfrac{5 + \sqrt{2}}{\sqrt{2}}$

C) $\dfrac{\sqrt{5} + 3}{\sqrt{5}}$

D) $\dfrac{6 + \sqrt{2}}{\sqrt{3}}$

E) $\dfrac{\sqrt{5} + 8}{\sqrt{2}}$

F) $\dfrac{\sqrt{3} - 2}{\sqrt{3}}$

G) $\dfrac{3 + 2\sqrt{5}}{\sqrt{5}}$

H) $\dfrac{\sqrt{10} + 3}{\sqrt{2}}$

I) $\dfrac{3\sqrt{6} - 5}{\sqrt{3}}$

J) $\dfrac{4 - 3\sqrt{14}}{\sqrt{2}}$

To rationalise these denominators we will multiply by two terms within a bracket.

K) $\dfrac{5}{\sqrt{2} + 3}$

L) $\dfrac{8}{1 + \sqrt{3}}$

M) $\dfrac{1}{\sqrt{5} - 2}$

N) $\dfrac{\sqrt{2}}{-3 + \sqrt{2}}$

O) $\dfrac{2\sqrt{3}}{\sqrt{5} - 1}$

P) $\dfrac{5\sqrt{2}}{2\sqrt{5} - 3}$

Q) $\dfrac{3\sqrt{7}}{5\sqrt{3} - 4}$

R) $\dfrac{\sqrt{3} + 2}{\sqrt{3} - 1}$

S) $\dfrac{\sqrt{5} + 2\sqrt{3}}{4\sqrt{2} + 3}$

T) $\dfrac{5\sqrt{5} - 5}{-1 + 2\sqrt{3}}$

U) $\dfrac{5}{\sqrt{2}} + \dfrac{2}{3}$

V) $\dfrac{3}{4} + \dfrac{5}{\sqrt{3}}$

W) $\dfrac{\sqrt{2}}{\sqrt{5}} - \dfrac{3\sqrt{5}}{2}$

X) Work out the missing length.

?

$\sqrt{3} - 1$ | Area $= 2\sqrt{5} + 5\sqrt{3}$

Fluency Practice

(a) Show that $\dfrac{5+\sqrt{3}}{2-\sqrt{3}}$ can be written in the form $a + b\sqrt{3}$, where a and b are integers to be found.

(b) Show that $\dfrac{8-\sqrt{2}}{2+\sqrt{2}}$ can be written in the form $a + b\sqrt{2}$, where a and b are integers to be found.

(c) Show that $\dfrac{4\sqrt{2}-1}{3+\sqrt{8}}$ can be written in the form $a\sqrt{2} + b$, where a and b are integers to be found.

(d) Show that $\dfrac{2+\sqrt{20}}{3-\sqrt{5}} \times 3\sqrt{5}$ can be written in the form $a + b\sqrt{5}$, where a and b are integers to be found.

(e) Show that $\dfrac{(2+2\sqrt{7})^2}{3-\sqrt{7}}$ can be written in the form $a + b\sqrt{7}$, where a and b are integers to be found.

(f) Show that $\dfrac{\sqrt{8}(4-3\sqrt{2})}{\sqrt{2}+1} + 5(4 - \sqrt{2})$ can be written in the form $a + b\sqrt{2}$, where a and b are integers to be found.

(g) Given that $\dfrac{a+\sqrt{12}}{2-\sqrt{3}} = b + 10\sqrt{3}$ find the values of a and b.

(h) Given that $\dfrac{a}{(1+\sqrt{3})^2} + (a\sqrt{3})^3 = 2 + b\sqrt{3}$ find the values of a and b.

Fluency Practice

(a)	(b)	(c)	(d)
Write $\sqrt{108}$ in the form $k\sqrt{3}$	Write $\sqrt{45} + \sqrt{20}$ in the form $k\sqrt{5}$	Write $\sqrt{96} - \sqrt{24}$ in the form $k\sqrt{6}$	Expand $\sqrt{2}(5 + \sqrt{8})$

(e)	(f)	(g)	(h)
Expand and simplify $(7 + \sqrt{3})(4 - \sqrt{3})$	Expand and simplify $(5 + 2\sqrt{2})(6 - \sqrt{2})$	Expand and simplify $(5 + 3\sqrt{2})^2$	Rationalise the denominator and simplify fully $\dfrac{15}{\sqrt{18}}$

(i)	(j)	(k)	(l)
Rationalise the denominator and simplify fully $\dfrac{5 + 4\sqrt{3}}{\sqrt{3}}$	Express $\dfrac{\sqrt{3}+\sqrt{27}}{\sqrt{2}}$ as a single surd.	Rationalise the denominator and simplify fully $\dfrac{\sqrt{3} + 5}{2 - \sqrt{3}}$	$(4 + \sqrt{a})(7 - \sqrt{a}) = 23 + k\sqrt{a}$ Find the values of the positive integers a and k.

Fluency Practice

A1 Express $\sqrt{48}$ in the form $a\sqrt{b}$ where b is a prime number.	**A2** Express $4\sqrt{5}$ in the form $\sqrt{a}$ where a is an integer.	**A3** Express $\sqrt{175}-\sqrt{63}$ in the form $a\sqrt{7}$ where a is an integer.	**A4** Express $\sqrt{75}+\sqrt{27}$ in the form $a\sqrt{b}$ where a and b are integers
B1 Expand $(3+\sqrt{5})(2+\sqrt{5})$ Give your answer in the form $a+b\sqrt{5}$ where a and b are integers	**B2** Expand $(2+3\sqrt{7})^2$ Give your answer in the form $a+b\sqrt{7}$ where a and b are integers	**B3** Show that $(4+2\sqrt{3})(5-\sqrt{3})=14+6\sqrt{3}$ Show clear working out.	**B4** Show that $(2+\sqrt{8})(9-3\sqrt{2})=6+12\sqrt{2}$ Show clear working out.
C1 Show that $\dfrac{12}{\sqrt{45}}=\dfrac{4\sqrt{5}}{5}$ Show clear working out.	**C2** Rationalise the denominator and simplify fully $\dfrac{9+2\sqrt{3}}{\sqrt{3}}$	**C3** Simplify $\dfrac{2+\sqrt{20}}{\sqrt{5}-1}$ giving your answer in the form $a+b\sqrt{5}$	**C4** A rectangle has an area of $\sqrt{80}$ cm^2 and a width of $1+\sqrt{5}$ cm. Calculate the exact height of the rectangle, leaving your answer as a simplified surd.
D1 Given that $(a+\sqrt{b})^2=49+12\sqrt{b}$ Find the value of a and the value of b.	**D2** Given that $(1+\sqrt{e})(3+\sqrt{e})=f+4\sqrt{5}$ Find the value of e and the value of f.	**D3** Given that $(5+3\sqrt{2})^2=p+\dfrac{q}{\sqrt{8}}$ Find the value of p and the value of q.	**D4** Given that $(\sqrt{x}+\sqrt{8x})^2=54+y\sqrt{2}$ Find the value of x and the value of y.

Worksheets, Videos, Interactive Quizzes and Exam Solutions

Fluency Practice

Question 1: Work out each of the following

(a) $\sqrt{3} \times \sqrt{5}$ (b) $\sqrt{7} \times \sqrt{2}$ (c) $\sqrt{11} \times \sqrt{6}$ (d) $\sqrt{2} \times \sqrt{3}$

(e) $\sqrt{8} \times \sqrt{2}$ (f) $\sqrt{3} \times \sqrt{3}$ (g) $\sqrt{5} \times \sqrt{6}$ (h) $\sqrt{5} \times \sqrt{2}$

(i) $\sqrt{6} \times \sqrt{6}$ (j) $\sqrt{10} \times \sqrt{3}$ (k) $\sqrt{5} \times \sqrt{20}$ (l) $\sqrt{11} \times \sqrt{10}$

(m) $\sqrt{2} \times \sqrt{5} \times \sqrt{3}$ (n) $\sqrt{2} \times \sqrt{6} \times \sqrt{3}$ (o) $\sqrt{10} \times \sqrt{7} \times \sqrt{3}$

(p) $\sqrt{2} \times \sqrt{2} \times \sqrt{2}$ (q) $\sqrt{2} \times \sqrt{3} \times \sqrt{2} \times \sqrt{3}$

Question 2: Work out each of the following

(a) $(\sqrt{3})^2$ (b) $(\sqrt{7})^2$ (c) $(\sqrt{10})^2$ (d) $(\sqrt{5})^2$

Question 3: Work out each of the following

(a) $(\sqrt{2})^3$ (b) $(\sqrt{6})^3$ (c) $(\sqrt{10})^3$ (d) $(\sqrt{3})^3$

(e) $(\sqrt{3})^4$ (f) $(\sqrt{5})^4$ (g) $(\sqrt{10})^4$ (h) $(\sqrt{2})^5$

Question 4: Work out each of the following

(a) $2\sqrt{3} \times 3\sqrt{5}$ (b) $7\sqrt{2} \times 4\sqrt{11}$ (c) $2\sqrt{13} \times 2\sqrt{2}$ (d) $10\sqrt{2} \times 5\sqrt{3}$

(e) $\sqrt{3} \times 6\sqrt{5}$ (f) $2\sqrt{2} \times \sqrt{7}$ (g) $4\sqrt{3} \times 2\sqrt{3}$ (h) $2\sqrt{2} \times 3\sqrt{8}$

(i) $2\sqrt{2} \times \sqrt{5} \times 4\sqrt{3}$ (j) $3\sqrt{2} \times 2\sqrt{6} \times 2\sqrt{3}$

Question 5: Work out each of the following

(a) $\sqrt{10} \div \sqrt{5}$ (b) $\sqrt{21} \div \sqrt{7}$ (c) $\sqrt{30} \div \sqrt{6}$ (d) $\sqrt{8} \div \sqrt{2}$

(e) $\sqrt{80} \div \sqrt{5}$ (f) $\sqrt{56} \div \sqrt{7}$ (g) $\sqrt{15} \div \sqrt{3}$ (h) $\sqrt{72} \div \sqrt{8}$

(i) $\sqrt{7} \div \sqrt{7}$ (j) $\sqrt{26} \div \sqrt{13}$ (k) $\sqrt{48} \div \sqrt{12}$ (l) $\sqrt{48} \div \sqrt{8}$

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Fluency Practice

Question 6: Work out each of the following

(a) $4\sqrt{6} \div 2\sqrt{2}$ (b) $12\sqrt{10} \div 3\sqrt{5}$ (c) $9\sqrt{15} \div 3\sqrt{3}$ (d) $7\sqrt{30} \div \sqrt{2}$

(e) $20\sqrt{8} \div 5\sqrt{2}$ (f) $40\sqrt{80} \div 20\sqrt{5}$ (g) $51\sqrt{7} \div 3\sqrt{7}$ (h) $36\sqrt{54} \div 18\sqrt{6}$

Question 7: Work out each of the following

(a) $\dfrac{\sqrt{6}}{\sqrt{3}}$ (b) $\dfrac{\sqrt{18}}{\sqrt{2}}$ (c) $\dfrac{15\sqrt{14}}{3\sqrt{2}}$ (d) $\dfrac{24\sqrt{72}}{3\sqrt{2}}$

Question 8: Simplify the following

(a) $\sqrt{8}$ (b) $\sqrt{75}$ (c) $\sqrt{20}$ (d) $\sqrt{32}$ (e) $\sqrt{48}$ (f) $\sqrt{200}$

(g) $\sqrt{300}$ (h) $\sqrt{80}$ (i) $\sqrt{50}$ (j) $\sqrt{98}$ (k) $\sqrt{63}$ (l) $\sqrt{800}$

(m) $\sqrt{180}$ (n) $\sqrt{220}$ (o) $\sqrt{96}$ (p) $\sqrt{175}$ (q) $\sqrt{1000}$ (r) $\sqrt{60}$

Question 9: Simplify the following

(a) $5\sqrt{8}$ (b) $2\sqrt{20}$ (c) $4\sqrt{50}$ (d) $3\sqrt{98}$ (e) $15\sqrt{32}$ (f) $10\sqrt{75}$

Question 10: Work out each of the following. Simplify each answer.

(a) $\sqrt{6} \times \sqrt{10}$ (b) $\sqrt{15} \times \sqrt{10}$ (c) $\sqrt{30} \times \sqrt{6}$

(d) $\sqrt{22} \times \sqrt{10}$ (e) $3\sqrt{2} \times \sqrt{6}$ (f) $\sqrt{14} \times 4\sqrt{2}$

(g) $4\sqrt{6} \times 3\sqrt{15}$ (h) $2\sqrt{70} \times 3\sqrt{10}$ (i) $5\sqrt{10} \times 2\sqrt{30}$

Question 11: Work out the following additions/subtractions

(a) $\sqrt{8} + \sqrt{18}$ (b) $\sqrt{50} + \sqrt{8}$ (c) $\sqrt{75} + \sqrt{27}$ (d) $\sqrt{200} - \sqrt{32}$

(e) $\sqrt{8} + \sqrt{2} + \sqrt{72}$ (f) $\sqrt{300} - \sqrt{48}$ (g) $\sqrt{1000} + \sqrt{90}$ (h) $\sqrt{28} + \sqrt{63}$

Question 12: Work out the following additions/subtractions

(a) $3\sqrt{8} + \sqrt{2}$ (b) $4\sqrt{27} - \sqrt{75}$ (c) $2\sqrt{50} + 5\sqrt{32}$ (d) $\sqrt{200} - 3\sqrt{18}$

(e) $4\sqrt{80} + 3\sqrt{45}$ (f) $6\sqrt{75} - 2\sqrt{12}$ (g) $10\sqrt{7} + 2\sqrt{175}$

Fluency Practice

Question 13: Expand each of the following. Simplify if possible.

(a) $\sqrt{2}(\sqrt{3}+5)$ (b) $\sqrt{3}(\sqrt{5}+\sqrt{2})$ (c) $\sqrt{6}(2-\sqrt{3})$ (d) $\sqrt{10}(5+\sqrt{10})$

(e) $\sqrt{2}(\sqrt{18}-\sqrt{2})$ (f) $\sqrt{5}(3\sqrt{2}-\sqrt{5})$ (g) $2\sqrt{3}(3\sqrt{2}+\sqrt{3})$ (h) $4\sqrt{11}(5\sqrt{2}+2\sqrt{11})$

(i) $\sqrt{27}(\sqrt{2}+\sqrt{3})$ (j) $\sqrt{12}(7-\sqrt{3})$

Question 14: Expand each of the following. Simplify if possible.

(a) $(2+\sqrt{3})(1+\sqrt{3})$ (b) $(\sqrt{2}+5)(1+\sqrt{2})$ (c) $(\sqrt{3}+1)(\sqrt{3}+4)$

(d) $(3+\sqrt{5})(4-\sqrt{5})$ (e) $(\sqrt{7}-1)(\sqrt{7}-1)$ (f) $(5-\sqrt{3})(5+\sqrt{3})$

(g) $(3+\sqrt{2})(1+\sqrt{3})$ (h) $(\sqrt{12}+\sqrt{3})(\sqrt{3}+2)$ (i) $(4-\sqrt{2})(3+\sqrt{8})$

(j) $(\sqrt{7}+\sqrt{2})(\sqrt{8}+\sqrt{7})$ (k) $(1+2\sqrt{2})(2-\sqrt{2})$ (l) $(3\sqrt{5}+7)(2\sqrt{5}+1)$

(m) $(3+\sqrt{2})^2$ (n) $(1+\sqrt{5})^2$ (o) $(10-\sqrt{2})^2$

(p) $(\sqrt{5}+\sqrt{2})^2$ (q) $(2\sqrt{3}-1)^2$ (r) $(5\sqrt{2}+3\sqrt{3})^2$

Question 15: Expand each of the following. Simplify if possible.

(a) $(4+\sqrt{3})(4-\sqrt{3})$ (b) $(1-\sqrt{6})(1+\sqrt{6})$ (c) $(\sqrt{10}+1)(\sqrt{10}-1)$

(d) $(\sqrt{7}+\sqrt{5})(\sqrt{7}-\sqrt{5})$ (e) $(2\sqrt{3}-1)(2\sqrt{3}+1)$ (f) $(5-4\sqrt{3})(5+4\sqrt{3})$

Question 16: Rationalise the denominators for each of the following

(a) $\dfrac{2}{\sqrt{3}}$ (b) $\dfrac{5}{\sqrt{2}}$ (c) $\dfrac{7}{\sqrt{6}}$ (d) $\dfrac{1}{\sqrt{10}}$

(e) $\dfrac{4}{\sqrt{2}}$ (f) $\dfrac{9}{\sqrt{6}}$ (g) $\dfrac{\sqrt{2}}{\sqrt{3}}$ (h) $\dfrac{3}{2\sqrt{5}}$

(i) $\dfrac{\sqrt{5}}{\sqrt{80}}$ (j) $\dfrac{5\sqrt{5}}{\sqrt{20}}$ (k) $\dfrac{\sqrt{2}+1}{\sqrt{5}}$ (l) $\dfrac{2-\sqrt{3}}{\sqrt{3}}$

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Question 17: Rationalise the denominators for each of the following

(a) $\dfrac{6}{3 - \sqrt{2}}$

(b) $\dfrac{4}{1 + \sqrt{5}}$

(c) $\dfrac{8}{\sqrt{3} - \sqrt{2}}$

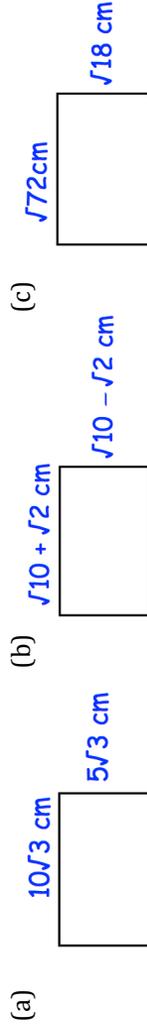(d) $\dfrac{\sqrt{2}}{\sqrt{10} + \sqrt{8}}$

(e) $\dfrac{3\sqrt{6}}{\sqrt{6} - 3}$

(f) $\dfrac{1}{2\sqrt{3} + \sqrt{5}}$

(g) $\dfrac{1 + \sqrt{5}}{1 - \sqrt{5}}$

(h) $\dfrac{\sqrt{11} - \sqrt{3}}{\sqrt{11} + \sqrt{3}}$

Apply

Question 1:   Find the area of each of these rectangles

(a) √10 cm   √5 cm

(b) √12 cm   √3 cm

(c) 9√2 cm   2√2 cm

Question 2:   Find the perimeter of each of these rectangles

(a) 10√3 cm   5√3 cm

(b) √10 + √2 cm   √10 − √2 cm

(c) √72 cm   √18 cm

Question 3:   Calculate x for each of the following right angled triangles.
Give your answer as a simplified surd (or integer).

(a) √12 cm   x   √30 cm

(b) √300 cm   x   √200 cm

(c) √66 cm   x   √17 cm

(d) 2√3 cm   x   5√2 cm

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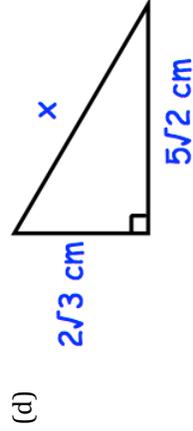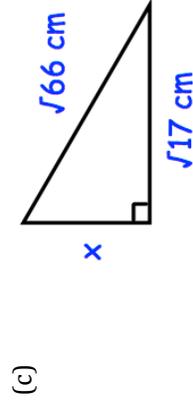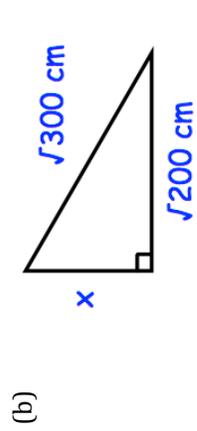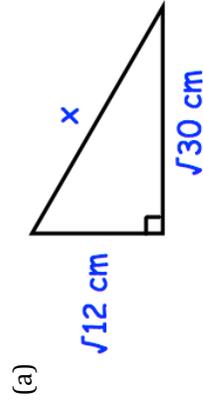
**Question 4:** A shed has dimensions, in metres, of height $\sqrt{5}$, width $\sqrt{6}$ and length $\sqrt{10}$
Find the volume of the shed.
Give your answer as a simplified surd.

Question 5: Mrs Jenkins is making decorations for a wedding.
She needs $18\sqrt{5}$ metres of ribbon in total.
Mrs Jenkins has 40 metres of ribbon.
Does she have enough ribbon?

Question 6: The midpoints of the sides of a square of side 16cm are joined to form another square. This process is then repeated to create the shaded square.

A   C   B

E

D
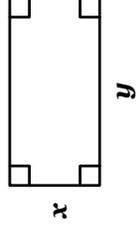
16cm
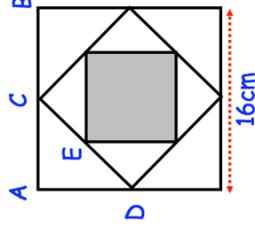
Find the area of the shaded square.

Question 7: The area of a rectangle is $\sqrt{125}$ cm^2
The length of the rectangle is $(2 + \sqrt{5})$ cm.
Calculate the width of the rectangle.
Express your answer in the form a + b$\sqrt{5}$, where a and b are integers.

Question 8: The triangle below has an area of $2\sqrt{6}$ cm^2.
Find the height of the triangle, x.
Give your answer as a simplified surd.

x

$\sqrt{12}$ cm
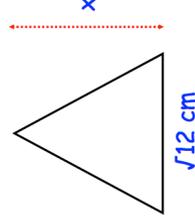
Purposeful Practice

Surds & rectangles 1

Complete each row with the side lengths x and y, the perimeter and area of a rectangle.

	Side length x	Side length y	Perimeter	Area
A	$3\sqrt{2}$	$\sqrt{2}$		
B	$\sqrt{3}$	$\sqrt{48}$		
C	$\sqrt{5}$		$6\sqrt{5}$	
D		$\sqrt{8}$	$\sqrt{200}$	
E		$2\sqrt{3}$		$8\sqrt{3}$
F	3			$\sqrt{27}$
G	$\sqrt{2}$	$\sqrt{2} + 3$		
H	$2\sqrt{5}$	$\sqrt{5} - 2$		
I		$\sqrt{3} + 2$	$4(\sqrt{3} + 1)$	
J	$2\sqrt{2}$		$6\sqrt{2} - 2$	
K	2			$4\sqrt{3} + 6$
L		$\sqrt{3}$		$\sqrt{75}$
M	$\sqrt{5}$			15
N		$2\sqrt{2}$		12
O	$\sqrt{3}$			$2\sqrt{6}$
P		$3\sqrt{3}$		18

Surds & rectangles 2

Complete each row with the side lengths x and y, the perimeter and area of a rectangle.
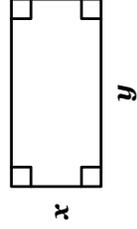
Write your answers in the simplest form, without fractions.

	Side length x	Side length y	Perimeter	Area
A	$\sqrt{5} + \sqrt{2}$	$\sqrt{2}$		
B	$3\sqrt{2}$	$2\sqrt{3} - \sqrt{2}$		
C	$\sqrt{3} + 2\sqrt{2}$		$2\sqrt{3} + 8\sqrt{2}$	
D		$2\sqrt{7} - \sqrt{5}$	$4\sqrt{7}$	
E	$\sqrt{2} + 5$	$\sqrt{2} + 1$		
F	$2 + \sqrt{3}$	$2 - \sqrt{3}$		
G	$\sqrt{3} + 4$		$6\sqrt{3} + 6$	
H	$5 - 2\sqrt{2}$		$2(5 + \sqrt{2})$	
I		$\sqrt{3}$		$\sqrt{3} + 6$
J	$\sqrt{5}$			$4\sqrt{5} - 5$
K		$4\sqrt{3} - 2$	$16\sqrt{3}$	
L	$\sqrt{7} - 2$			3
M		$2 - \sqrt{2}$		4
N	$\sqrt{3}$			$3 + \sqrt{3}$
O		$3 + \sqrt{3}$		$2\sqrt{3}$
P	$\sqrt{5} + 3$			$2 + 2\sqrt{5}$

Fluency Practice

Given a choice between $\sqrt{8}$ and $2\sqrt{2}$ patrons should prefer $2\sqrt{2}$. Also square roots should never be left on the denominator of a fraction in a final answer.

1. Write the following in the form $a\sqrt{b}$ where b is as small as it can be:

(a) $\sqrt{8}$.

(b) $\sqrt{27}$.

(c) $\sqrt{12}$.

(d) $\sqrt{45}$.

(e) $\sqrt{450}$.

(f) $\sqrt{2}\sqrt{24}$.

(g) $\sqrt{a^4 b^2 c}$.

(h) $\sqrt{ab^4 c^3 d^{100}}$.

(i) $\sqrt[3]{54}$.

(j) $\sqrt{8} + \sqrt{18}$.

(k) $\sqrt{12} + \sqrt{75} - \sqrt{48}$.

(l) $\sqrt{20} + \sqrt{5} - \sqrt{45}$.

(m) $\sqrt{200} - \sqrt{288} + \sqrt{128}$.

2. Write the following in the form $\sqrt{a}$:

(a) $5\sqrt{2}$.

(b) $4\sqrt{3}$.

(c) $2\sqrt{5}$.

(d) $7\sqrt{6}$.

(e) $\dagger^2\sqrt{\bullet}$.

(f) $3\sqrt[3]{x} + 1$.

(g) $a\sqrt[n]{k}$.

(h) $\sqrt{12} + 3\sqrt{3}$.

(i) $\sqrt{8} + \sqrt{2}$.

(j) $\sqrt{45} - \sqrt{20}$.

(k) $\sqrt{108} + \sqrt{12} - \sqrt{3}$.

(l) $\sqrt{44} - \sqrt{99} + \sqrt{176}$.

(m) $\sqrt{3a^2} + \sqrt{12b^2} - \sqrt{3a^2} + 3b^2 - 6ab$.

3. Expand and simplify fully:

(a) $(2\sqrt{3})^3$.

(b) $(\sqrt{18})^3$.

(c) $(2 - \sqrt{3})^2$.

(d) $(1 - \sqrt{8})^2$.

(e) $(4 + \sqrt{32})^2$.

(f) $(3 + \sqrt{20})^3$.

4. Rationalise the denominator and simplify fully:

(a) $\dfrac{4}{\sqrt{2}}$.

(b) $\dfrac{1}{\sqrt{3}}$.

(c) $\dfrac{25}{2\sqrt{5}}$.

(d) $\dfrac{21}{6\sqrt{7}}$.

(e) $\dfrac{ab}{c\sqrt{b}}$.

(f) $\left(\dfrac{2}{\sqrt{3}}\right)^3$.

(g) $\dfrac{2}{\sqrt{2}} + \dfrac{\sqrt{2}}{3}$.

(h) $\dfrac{3}{\sqrt{5}} - 7\sqrt{5}$.

(i) $\dfrac{1}{\sqrt{8}} + \dfrac{1}{\sqrt{2}}$.

(j) $\sqrt{\dfrac{1}{2}} + \sqrt{\dfrac{1}{4}} + \sqrt{\dfrac{1}{8}}$.

(k) $\dfrac{1}{\sqrt{2} - 1}$.

(l) $\dfrac{4}{\sqrt{3} + 1}$.

Fluency Practice

(m) $\dfrac{4 - \sqrt{3}}{2 + \sqrt{3}}.$

(n) $\dfrac{1 - \sqrt{5}}{\sqrt{5} + 2}.$

(o) $\dfrac{\sqrt{2} + 3}{1 - 2\sqrt{2}}.$

(p) $\dfrac{\sqrt{7} + 1}{\sqrt{7} - 1}.$

(q) $\dfrac{a}{b + c\sqrt{k}}.$

(r) $\dfrac{9}{\sqrt{3} - 1} + \dfrac{7}{\sqrt{3} + 1}.$

(s) $\dfrac{\sqrt{2} + 3}{\sqrt{2} - 1} + \dfrac{1}{\sqrt{2}}.$

(t) $\left(\dfrac{1}{1 - \sqrt{2}}\right)^2 + \left(\dfrac{2}{3 + \sqrt{2}}\right)^2.$

(u) $\dfrac{69}{1 + \sqrt{2} + \sqrt{3}}.$

Sequences with... **Surds**

1) Assuming that each pair of numbers is the start of an arithmetic sequence, find:
   (i) the next three terms, (ii) the nth term rule, (iii) the 200th term.

2) Assuming that each pair of numbers is the start of a geometric sequence, find:
   (i) the next three terms, (ii) the ratio between the first and third terms,
   (iii) the ratio between the second and fifth terms.

a) $\sqrt{2},\ 3\sqrt{2}$

b) $\sqrt{2},\ \sqrt{32}$

c) $\sqrt{2},\ 2$

d) $\sqrt{2},\ 1$

e) $\sqrt{2},\ 2 + \sqrt{2}$

f) $\sqrt{27},\ \sqrt{108}$

g) $\sqrt{48},\ \sqrt{108}$

h) $\sqrt{3},\ \sqrt{108}$

i) $6,\ \sqrt{108}$

j) $-\sqrt{75},\ \sqrt{108}$

Show clearly that the mean and median of $\sqrt{3}$, $\sqrt{12}$, $\sqrt{48}$ and $\sqrt{75}$ are equal.
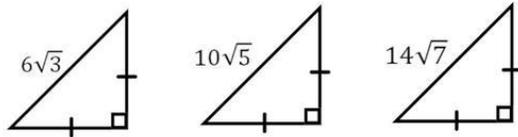
Simplify

$$\sqrt{1+\frac{1}{2}\sqrt{1+\frac{1}{3}\sqrt{1+\frac{1}{4}\sqrt{1+\frac{1}{5}\sqrt{1+\frac{1}{6}\sqrt{1+\frac{1}{7}}}}}}}$$

Complete the multiplication table.

$\times$		
	$2\sqrt{6}$	$\sqrt{6}-2$
	$2\sqrt{6}+6$	1

On the number line shown below, a is a number between 0 and 1, and b is a number between 1 and 2. Mark possible positions on this line for

$$\sqrt{a},\ \sqrt{b},\ a^2,\ b^2 \text{ and } \sqrt{\frac{b}{a}}$$

These triangles all share a rather wonderful property. Find the shorter sides and you'll see!

Can you find other triangles with this property?

Find the sum of

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \text{ and so on up to } + \frac{1}{\sqrt{99}+\sqrt{100}}$$

Fluency Practice

powers of surds

no calculator please

what are the surd square roots of these surds?

(1) $9 + 4\sqrt{5}$

(2) $25 + 4\sqrt{21}$

(3) $7 - 4\sqrt{3}$

(4) $8 + 2\sqrt{7}$

(5) $36 - 10\sqrt{11}$

(6) $36 + 16\sqrt{2}$

(7) $49 + 20\sqrt{6}$

(8) $29 - 12\sqrt{5}$

(9) $52 + 16\sqrt{3}$

(10) $64 - 24\sqrt{7}$

(11) $11 + 4\sqrt{6}$

(12) $8 - 2\sqrt{15}$

(13) $18 - 12\sqrt{2}$

(14) $55 + 30\sqrt{2}$

(15) $42 - 24\sqrt{3}$

what are the reciprocals of these surds?

(1) $2 + \sqrt{3}$

(2) $5 - 2\sqrt{6}$

(3) $3 + 2\sqrt{2}$

(4) $7 - 4\sqrt{3}$

(5) $8 + 3\sqrt{7}$

try to find some other examples

what are the cube roots of these surds?

(1) $26 + 15\sqrt{3}$

(2) $7 - 5\sqrt{2}$

(3) $38 + 17\sqrt{5}$

(4) $54 - 30\sqrt{3}$

(5) $9\sqrt{3} + 11\sqrt{2}$

what are these in surd form?

(1) $(8 + 4\sqrt{3})^{-\frac{1}{2}}$

(2) $(9 + 6\sqrt{2})^{-\frac{1}{2}}$

(3) $(15 - 10\sqrt{2})^{-\frac{1}{2}}$

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Purposeful Practice

Red box:

(a) Given that $(3 + \sqrt{a})(4 + \sqrt{a}) = 17 + b\sqrt{a}$ find the values of a and b.

(b) Given that $(5 - \sqrt{x})^2 = y - 10\sqrt{2}$ find the values of x and y.

(c) Given that $(c - \sqrt{d})^2 = 39 - 12\sqrt{d}$, find the values of c and d.

Yellow box:

(a) Find the nth term of the sequence

$\sqrt{2}$ $\sqrt{8}$ $\sqrt{18}$ $\sqrt{32}$

(b) Find the nth term of the sequence

20 $17 + \sqrt{5}$ $14 + \sqrt{20}$ $11 + \sqrt{45}$

Green box:

(a) Show that $\sqrt{25a^2} + \frac{2ab\sqrt{75}}{\sqrt{3b^2}}$ is always a multiple of 5, given that a and b are integers.

(b) Show that $\sqrt{2}(c\sqrt{8} - d\sqrt{50})$ is always even when c and d are integers.

Purple box:

(a) Given that d is a prime number, rationalise the denominator of $\frac{5\sqrt{d}}{2 + \sqrt{d}}$

(b) Rationalise the denominator of $\frac{2e + \sqrt{f}}{e - \sqrt{f}}$ where e is an integer and f is a prime number.

Blue box:

Solve, giving your answer in its simplest form:

(a) $3\sqrt{3} = \sqrt{3}x + 2$

(b) $3x - \sqrt{5} = \sqrt{5}x + 1$

(c) $x^2 - 2\sqrt{3}x + 3 = 0$

(d) $2x^2 + \sqrt{7}x - 7 = 0$

(e) $3x^2 + 5\sqrt{5}x = 10$

Fluency Practice

$$\frac{8}{\sqrt{3} - \frac{1}{\sqrt{3}}}$$

$$\frac{5\sqrt{2}}{\frac{3}{\sqrt{2}} + \sqrt{2}}$$

$$\frac{4}{\sqrt{3}} + 2\sqrt{3}$$

$$\frac{\sqrt{3} + \frac{2}{\sqrt{3}}}{}$$

$$\frac{6}{\frac{1}{\sqrt{5}} + \sqrt{5}}$$

$$\frac{18}{\frac{2}{\sqrt{7}} + \sqrt{7}}$$

$$\frac{3\sqrt{5}}{\sqrt{5} - \frac{2}{\sqrt{5}}}$$

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Intelligent Practice

Simplify:

1) $\dfrac{3}{12}$

2) $\dfrac{3x}{12}$

3) $\dfrac{3}{12x}$

4) $\dfrac{12x}{3}$

5) $\dfrac{12x}{6}$

6) $\dfrac{12x}{6x}$

7) $\dfrac{12xy}{6x}$

8) $\dfrac{6x}{12xy}$

9) $\dfrac{10x}{12xy}$

10) $\dfrac{10x}{12xy^2}$

11) $\dfrac{10xy}{12xy^2}$

12) $\dfrac{10x^2y}{12xy^2}$

13) $\dfrac{10x^2y^2}{12xy^2}$

14) $\dfrac{10x^2y^3}{12xy^2}$

15) $\dfrac{100x^2y^4}{120xy^2}$

Intelligent Practice

Simplify:

1) $\dfrac{2(x+3)}{3(x+3)}$

2) $\dfrac{2(x+3)}{(x+3)}$

3) $\dfrac{(x+3)}{2(x+3)}$

4) $\dfrac{x+3}{2x+6}$

5) $\dfrac{(x+3)(x+4)}{2x+6}$

6) $\dfrac{(x+3)(x+4)}{2x+8}$

7) $\dfrac{x^2+7x+12}{2x+8}$

8) $\dfrac{x^2+7x+12}{(x+4)(x-9)}$

9) $\dfrac{x^2+7x+12}{x^2-5x-36}$

10) $\dfrac{2x^2+14x+24}{x^2-5x-36}$

11) $\dfrac{2x^2+14x+24}{3x^2-15x-108}$

12) $\dfrac{2x^2+14x+24}{3x^2+4x-15}$

13) $\dfrac{14x-24-2x^2}{3x^2+4x-15}$

14) $\dfrac{14x-24-2x^2}{3x^2-4x-15}$

15) $\dfrac{14x-24-2x^2}{4x+15-3x^2}$

Intelligent Practice

Simplify:

1) $\dfrac{x+2}{(x+2)(x-1)}$

2) $\dfrac{x-1}{(x+2)(x-1)}$

3) $\dfrac{(x-1)^2}{(x+2)(x-1)}$

4) $\dfrac{(x+2)^2}{(x+2)(x-1)}$

5) $\dfrac{x^2-4}{(x+2)(x-1)}$

6) $\dfrac{x^2-4}{(x+2)(2x-5)}$

7) $\dfrac{4x^2-25}{2x^2-x-10}$

8) $\dfrac{4x^2-25}{4x^2+20x+25}$

9) $\dfrac{4x^2+8x-5}{4x^2+20x+25}$

10) $\dfrac{4ax^2+8ax-5a}{4a^2x^2+20a^2x+25a^2}$

11) $\dfrac{14x-24-2x^2}{3x^2+4x-15}$

12) $\dfrac{14x-24-2x^2}{3x^2-4x-15}$

Q1 Simplify the following algebraic fractions.

[a] $\dfrac{6x + 10}{2}$

[e] $\dfrac{21x + 14}{7}$

[i] $\dfrac{12x + 18}{9}$

[b] $\dfrac{8x - 12}{4}$

[f] $\dfrac{8x - 12}{6}$

[j] $\dfrac{8x - 28}{12}$

[c] $\dfrac{5x + 15}{5}$

[g] $\dfrac{5x + 20}{10}$

[k] $\dfrac{22x - 44}{33}$

[d] $\dfrac{20x + 24}{4}$

[h] $\dfrac{6x - 12}{9}$

[l] $\dfrac{18x + 45}{27}$

Q2 Simplify the following algebraic fractions.

[a] $\dfrac{3(x+4)}{x+4}$
[e] $\dfrac{5(x-3)}{x-3}$
[i] $\dfrac{5x+10}{x+2}$
[m] $\dfrac{2x+6}{3x+9}$

[b] $\dfrac{3(x+4)}{5(x+4)}$
[f] $\dfrac{5(x-3)}{6(x-3)}$
[j] $\dfrac{x+2}{5x+10}$
[n] $\dfrac{3x-21}{4x-28}$

[c] $\dfrac{x+4}{3(x+4)}$
[g] $\dfrac{x-3}{5(x-3)}$
[k] $\dfrac{4x+24}{x+6}$
[o] $\dfrac{4x-20}{3x-15}$

[d] $\dfrac{x+4}{3x+12}$
[h] $\dfrac{x-3}{5x-15}$
[l] $\dfrac{x+6}{4x+24}$
[p] $\dfrac{2x+12}{3x+18}$

Q3 Simplify the following algebraic fractions.

[a] $\dfrac{4x + 8}{7x + 14}$

[e] $\dfrac{3x + 12}{2x + 8}$

[i] $\dfrac{2x + 2}{7x + 7}$

[m] $\dfrac{5x - 20}{8x - 32}$

[b] $\dfrac{5x + 25}{9x + 45}$

[f] $\dfrac{2x + 6}{10x + 30}$

[j] $\dfrac{2x - 20}{7x - 70}$

[n] $\dfrac{10x + 40}{9x + 36}$

[c] $\dfrac{5x - 40}{3x - 24}$

[g] $\dfrac{2x - 14}{9x - 63}$

[k] $\dfrac{8x + 24}{10x + 30}$

[o] $\dfrac{8x + 96}{9x + 108}$

[d] $\dfrac{3x - 15}{5x - 25}$

[h] $\dfrac{5x - 30}{7x - 42}$

[k] $\dfrac{10x - 80}{7x - 56}$

[p] $\dfrac{4x - 24}{6x - 36}$

Q4 Simplify the following algebraic fractions.

[a] $\dfrac{6x^2 + 5x}{x}$

[e] $\dfrac{12x + 6}{2x + 1}$

[i] $\dfrac{4x^4 + 8x^5}{12x^3}$

[b] $\dfrac{2x - 7x^3}{x}$

[f] $\dfrac{4x + 8}{x + 2}$

[j] $\dfrac{9x^5 - 6x^3}{18x^3}$

[c] $\dfrac{5t^2 - 4t^4}{t^2}$

[g] $\dfrac{6x + 10}{9x + 15}$

[k] $\dfrac{6x^2 + 15x}{2x + 5}$

[d] $\dfrac{9y^5 + 2y^3}{y^3}$

[h] $\dfrac{6x + 9}{8x + 12}$

[l] $\dfrac{(x - 3)(x + 5)}{(x + 5)}$

Q5 Simplify the following algebraic fractions.

[a] $\dfrac{x^2 + 4x}{2x + 8}$

[e] $\dfrac{2x^2 + 6x^3}{6x + 2}$

[i] $\dfrac{(x + 2)(x + 3)}{4x + 12}$

[b] $\dfrac{6x^3 - x^2}{12x - 2}$

[f] $\dfrac{x^4 - 4x^3}{x^3 - 4x^2}$

[j] $\dfrac{5x - 20}{(x + 5)(x - 4)}$

[c] $\dfrac{x + 3x^2}{5 + 15x}$

[g] $\dfrac{3x^5 - 9x^4}{x^4 - 3x^3}$

[k] $\dfrac{9x + 27}{(x + 3)(x + 2)}$

[d] $\dfrac{x^4 - 5x^3}{6x - 30}$

[h] $\dfrac{4x^3 + 8x^2}{2x^3 + 4x^2}$

[l] $\dfrac{(x - 3)(x + 5)}{(x + 5)(x + 4)}$

Q1 Simplify the following algebraic fractions.

[a] $\dfrac{(x-5)(x+2)}{(x+2)(x+5)}$

[e] $\dfrac{(x+6)(x-8)}{(x-8)(x-2)}$

[b] $\dfrac{(x+4)(x+3)}{(x+5)(x+3)}$

[f] $\dfrac{(x+3)(x-9)}{(x-5)(x+3)}$

[c] $\dfrac{(x-1)(x+1)}{(x+6)(x-1)}$

[g] $\dfrac{(x+4)(x-4)}{(x-4)(x+6)}$

[d] $\dfrac{(x+7)(x+5)}{(x-9)(x+5)}$

[h] $\dfrac{(x+2)(x-4)}{(x-4)(x+3)}$

Q2 Simplify the following algebraic fractions.

[a] $\dfrac{(x-2)^2}{x(x-2)}$

[b] $\dfrac{x^2+4x}{(x+4)(x+3)}$

[c] $\dfrac{(x-1)(x+1)}{(x-1)^2}$

[d] $\dfrac{x(x+6)^2}{x(x-9)(x+6)}$

[e] $\dfrac{x-8}{(x-8)^2(x-2)}$

[f] $\dfrac{(x+3)^3(x+2)}{(x+2)(x+3)^2}$

[g] $\dfrac{(x+4)^2(x-4)}{(x-4)(x+4)^3}$

[h] $\dfrac{(x+2)(x-3)(x+5)}{(x+5)(x-6)(x-3)}$

Q3 Simplify the following algebraic fractions.

[a] $\dfrac{x^2 + 6x + 8}{x + 2}$

[b] $\dfrac{x^2 + 5x - 24}{x + 8}$

[c] $\dfrac{x^2 - 1}{x - 1}$

[d] $\dfrac{x + 4}{x^2 - 2x - 24}$

[e] $\dfrac{x + 2}{x^2 - x - 6}$

[f] $\dfrac{x^2 - 3x - 10}{x - 5}$

[g] $\dfrac{x + 4}{x^2 + 13x + 36}$

[h] $\dfrac{x^2 + 8x - 9}{x + 9}$

Q4 Simplify the following algebraic fractions.

[a] $\dfrac{x^2 + 3x + 2}{x^2 + 5x + 4}$

[b] $\dfrac{x^2 + 4x + 4}{x^2 + 6x + 8}$

[c] $\dfrac{x^2 + 6x + 5}{x^2 + 3x - 10}$

[d] $\dfrac{x^2 + 2x - 3}{x^2 - 6x + 5}$

[e] $\dfrac{x^2 + 4x - 45}{x^2 + 10x + 9}$

[f] $\dfrac{x^2 - 16}{x^2 + 7x + 12}$

[g] $\dfrac{x^2 - x - 20}{x^2 - 7x + 10}$

[h] $\dfrac{x^2 + 15x + 56}{x^2 + 3x - 40}$

Q5 Simplify the following algebraic fractions.

[a] $\dfrac{2x^2 - 18x}{x^2 - 2x - 63}$

[b] $\dfrac{4x^2 + 16x}{x^2 + 12x + 32}$

[c] $\dfrac{3x^2 + 24x}{x^2 + 2x - 48}$

[d] $\dfrac{x^2 + 7x}{2x^2 + 17x + 21}$

[e] $\dfrac{2x^2 + 4x}{2x^2 + 8x + 8}$

[f] $\dfrac{2x^2 + 8x}{2x^2 + 20x + 48}$

[g] $\dfrac{6x^2 + 40x - 14}{6x^2 - 2x}$

[h] $\dfrac{6x^2 + 16x - 32}{6x^2 - 8x}$

Q6 Simplify the following algebraic fractions.

[a] $\dfrac{3x^2 + 6x + 3}{x^2 - 4x - 5}$

[b] $\dfrac{2x^2 + 19x + 24}{x^2 + 4x - 32}$

[c] $\dfrac{2x^2 + 15x + 25}{x^2 - x - 30}$

[d] $\dfrac{3x^2 + 25x + 28}{x^2 - 49}$

[e] $\dfrac{5x^2 + 28x - 12}{x^2 + 4x - 12}$

[f] $\dfrac{3x^2 - 25x + 42}{x^2 - 12x + 36}$

[g] $\dfrac{5x^2 + 7x - 6}{x^2 - 5x - 14}$

[h] $\dfrac{4x^2 + 40x - 64}{x^2 - 64}$

Fluency Practice

A1 Simplify

$$\frac{x+3}{x^2+x-6}$$

A2 Simplify

$$\frac{x+6}{x^2+10x+24}$$

A3 Simplify

$$\frac{2x-18}{x^2-12x+27}$$

A4 Simplify

$$\frac{x^2-7x}{x^2-2x-35}$$

B1 Simplify

$$\frac{x^2-49}{x^2+2x-35}$$

B2 Simplify

$$\frac{x^2-4}{x^2+6x-16}$$

B3 Simplify

$$\frac{x^2-16}{x^2-2x-24}$$

B4 Simplify

$$\frac{x^2-36}{x^2+3x-18}$$

C1 Simplify

$$\frac{x^2+6x-16}{2x^2-x-6}$$

C2 Simplify

$$\frac{x^2-7x+10}{3x^2-14x-5}$$

C3 Simplify

$$\frac{x^2+3x}{5x^2+19x+12}$$

C4 Simplify

$$\frac{x^2+4x-21}{4x^2-17x+15}$$

D1 Simplify

$$\frac{2x^2-5x-12}{2x^2-11x+12}$$

D2 Simplify

$$\frac{5x^2-24x-5}{5x^2-14x-3}$$

D3 Simplify

$$\frac{4x^2-9}{2x^2+11x+12}$$

D4 Simplify

$$\frac{6x^2-11x+4}{4x^2+8x-5}$$

Fluency Practice

a $\dfrac{3x + 6}{4x + 8}$ ➡️

b $\dfrac{12x + 4}{15x + 5}$ ➡️

c $\dfrac{2x^2 + 6}{x^2 + 5x + 6}$ ➡️

d $\dfrac{x^2 + 7x + 12}{x^3 + 4x^2}$ ➡️

e $\dfrac{x^2 - 7x + 10}{2x^3 - 10x^2}$ ➡️

f $\dfrac{x^2 - 2x - 8}{x^2 - 6x + 8}$ ➡️

g $\dfrac{x^2 - 5x - 6}{x^2 + 5x + 4}$ ➡️

h $\dfrac{8x^2 - 24x}{2x^2 + 4x - 30}$ ➡️

i $\dfrac{3x^2 - 3x - 18}{3x^2 - 9x - 30}$ ➡️

j $\dfrac{x^2 - 1}{2x - 2}$ ➡️

k $\dfrac{x^2 - 4}{4x + 8}$ ➡️

l $\dfrac{5x + 25}{x^2 - 25}$ ➡️

m $\dfrac{x^2 - 9}{x^2 - 3x}$ ➡️

n $\dfrac{3x^2 + 6x}{x^2 - 4}$ ➡️

o $\dfrac{12 - 4x}{9 - x^2}$ ➡️

p $\dfrac{x^2 - 16}{3x^3 + 12x^2}$ ➡️

q $\dfrac{2x^2 - 8}{2x^2 - 4x}$ ➡️

r $\dfrac{x^2 + 2x - 3}{x^2 - 1}$ ➡️

s $\dfrac{x^2 - 16}{x^2 - 5x + 4}$ ➡️

t $\dfrac{x^2 + 5x + 6}{x^2 - 9}$ ➡️

u $\dfrac{x^2 + 12x + 36}{x^2 - 36}$ ➡️

v $\dfrac{4x^2 - 1}{6x^2 + 3x}$ ➡️

w $\dfrac{3x^3 + 4x^2}{9x^2 - 16}$ ➡️

x $\dfrac{2x^2 - x - 3}{4x^2 - 9}$ ➡️

y $\dfrac{9x^2 - 4}{3x^2 - 8x + 4}$ ➡️

z $\dfrac{16x^2 - 25}{8x^2 + 2x - 15}$ ➡️

aa $\dfrac{6x^2 - 7x - 20}{9x^2 - 16}$ ➡️

Simplifying Algebraic Fractions
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Examples

Workout

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Question 1: Simplify the following algebraic fractions

(a) $\dfrac{42xyz}{56}$

(b) $\dfrac{45ab}{60abc}$

(c) $\dfrac{16mn}{18n}$

(d) $\dfrac{40x^2y}{32xy}$

(e) $\dfrac{17cf}{34c^3}$

(f) $\dfrac{8x^4}{2x^2}$

(g) $\dfrac{33a^2b^2}{44a^3b}$

(h) $\dfrac{12x^3}{20x^7}$

Question 2: Simplify the following algebraic fractions

(a) $\dfrac{6x+8}{2}$

(b) $\dfrac{9x-12}{3}$

(c) $\dfrac{35x^2+20}{5}$

(d) $\dfrac{7m-70n^3}{7}$

(e) $\dfrac{10c+25}{15}$

(f) $\dfrac{8w+2-4x}{12}$

(g) $\dfrac{9x^2+12x+33}{6}$

(h) $\dfrac{3x^2+5x}{x}$

(i) $\dfrac{3x^3-7x^2}{x}$

(j) $\dfrac{8x^6+x^4+3x}{x}$

(k) $\dfrac{10x^7+15x^5-30x^4}{5x}$

(l) $\dfrac{3c^6-15c^4}{6c}$

(m) $\dfrac{-8x^5-12x^4+2x^3}{-4x}$

(n) $\dfrac{6c^9-12c^3}{3c^2}$

(o) $\dfrac{6c^6+2c^2}{4c^4}$

Fluency Practice

Simplifying Algebraic Fractions
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Question 3: Simplify the following algebraic fractions

(a) $\dfrac{(x+6)(x+3)}{(x+3)}$

(b) $\dfrac{(x-1)(x+1)}{(x-1)}$

(c) $\dfrac{(x-3)}{(x-4)(x-3)}$

(d) $\dfrac{(x+7)^2}{(x+7)}$

(e) $\dfrac{(x-3)(x+2)}{(x+2)(x+9)}$

(f) $\dfrac{(x+2)(x+4)^2}{(x+4)}$

(g) $\dfrac{(x+1)(x+2)(x+3)}{(x+2)(x+3)(x+4)}$

(h) $\dfrac{x(x+3)^2}{x(x+1)(x+3)}$

Question 4: Simplify the following algebraic fractions

(a) $\dfrac{x^2+5x+4}{x^2+4x+3}$

(b) $\dfrac{x^2+6x+9}{x^2-2x-15}$

(c) $\dfrac{x^2-2x}{x^2+2x-8}$

(d) $\dfrac{x^2-7x+10}{x^2+3x-10}$

(e) $\dfrac{x^2+8x+15}{x^2-x-12}$

(f) $\dfrac{x^2+13x+40}{x^2+14x+48}$

(g) $\dfrac{x^2-2x-8}{x^2+6x-40}$

(h) $\dfrac{x^2+10x+24}{x^2-36}$

(i) $\dfrac{x^2+4x-45}{x^2+10x+9}$

(j) $\dfrac{x^2+11x}{x^2-121}$

(k) $\dfrac{x^2-1}{x^2+x}$

(l) $\dfrac{x^2-15x+44}{x^2-16}$

(m) $\dfrac{x^2-x-6}{x^2-2x-3}$

Fluency Practice

Question 5:   Simplify the following algebraic fractions

(a)
$$\frac{3x^2 + 7x + 4}{x^2 + 3x + 2}$$

(b)
$$\frac{x^2 - 2x - 8}{3x^2 + 7x + 2}$$

(c)
$$\frac{5x^2 - 13x - 6}{x^2 - 9}$$

(d)
$$\frac{2x^2 + 3x - 2}{2x^2 - 15x + 7}$$

(e)
$$\frac{9x^2 - 1}{3x^2 - 13x + 4}$$

(f)
$$\frac{x^2 + 17x + 70}{5x^2 + 38x + 21}$$

(g)
$$\frac{3x^2 + 5x - 12}{12x^2 - 19x + 4}$$

(h)
$$\frac{3x^2 + 11x + 6}{9x^2 + 21x + 10}$$

(i)
$$\frac{4x^2 + x - 3}{4x^2 + 9x + 5}$$

(j)
$$\frac{9x^2 - 30x + 25}{6x^2 + 5x - 25}$$

(k)
$$\frac{10x^2 - 23x + 12}{4x^2 + 4x - 15}$$

(l)
$$\frac{20x^2 + 21x + 4}{16x^2 - 1}$$

Answers

Fluency Practice

Simplify

(a) $\dfrac{4x^2}{8x}$

(b) $\dfrac{4xy}{12y^2}$

(c) $\dfrac{10x^2y}{20xy}$

(d) $\dfrac{9x^2yz}{27y^2}$

(e) $\dfrac{15ab}{18a^2}$

(f) $\dfrac{36ab^2c}{6abd}$

Simplify

(a) $\dfrac{x(x+9)}{6x}$

(b) $\dfrac{2x(x-4)}{5(x-4)}$

(c) $\dfrac{x^2+6x}{5x}$

(d) $\dfrac{x^2-5x}{3x-15}$

(e) $\dfrac{3x-x^2}{6-2x}$

(f) $\dfrac{12x^2-6x}{14x-7}$

Simplify

(a) $\dfrac{x(x-5)}{(x+2)(x-5)}$

(b) $\dfrac{2x(x+1)}{4(x+2)(x+1)}$

(c) $\dfrac{x(x+5)}{x^2+7x+10}$

(d) $\dfrac{x^2+2x+1}{(x+1)(x-4)}$

Simplify

(a) $\dfrac{x^2+6x}{x^2+5x-6}$

(b) $\dfrac{x^2-x-6}{x^2+x-12}$

(c) $\dfrac{2x^2+14x}{x^2+10x+21}$

(d) $\dfrac{x^2-4}{x^2+2x-8}$

Simplify

(a) $\dfrac{x^2+7x+6}{2x^2+3x+1}$

(b) $\dfrac{3x^2+7x+2}{x^2+9x+14}$

(c) $\dfrac{5x^2+28x-12}{x^2+4x-12}$

(d) $\dfrac{2x^2+15x+25}{x^2-x-30}$

(e) $\dfrac{5x^2+7x-6}{x^2-5x-14}$

(f) $\dfrac{2x^2-32}{2x^2+9x+4}$

Purposeful Practice

tough algebra questions

(1)   simplify $(a + b)^2 - (a - b)^2$

(2)   expand $(a - b)(a^2 + ab + b^2)$

(3)   expand (i) $(n - 1)(n^2 + n + 1)$
         (ii) $(n - 1)(n^3 + n^2 + n + 1)$

(4)   show that $(n + 1)(n + 2) + (n + 2)^2 - (n + 1)(n + 6)$ is a perfect square

(5)   if $\frac{1}{2}(\frac{2}{3}(2a + b) + a - 3b) = \frac{1}{4}(a + 2b)$ show that $b = \frac{7}{8}a$

(6)   simplify $\dfrac{a^2b + a}{b^2a + b}$

(7)   simplify $\dfrac{k^2 - 7k + 10}{k^2 - 5k}$

(8)   simplify $\dfrac{d^2 - 8d + 15}{2d^2 - 7d - 15}$

(9)   if $a = t - \dfrac{1}{t}$ and $b = t + \dfrac{1}{t}$ show that $b^2 - a^2 = 4$

(10)   if $a = \dfrac{b}{b + c}$ show that $\dfrac{a}{1 - a} = \dfrac{b}{c}$

Problem Solving

Use the digits 0-9 (at most one time each) to fill in the boxes below so that the rational expression on the left simplifies to the rational expression on the right.

$$\frac{x^2 + \boxed{}\, x + \boxed{}\boxed{}}{x^2 - \boxed{}\boxed{}} = \frac{x + \boxed{}}{x - \boxed{}}$$

Problem Solving

Denominator is $x^2 + 6x + 8$

Equivalent to $\frac{1}{x+4}$

A

D

B

If you think a region is impossible to fill, convince me why!

G

F

E

H

Think of an algebraic fraction which simplifies that could belong in each of the regions

C
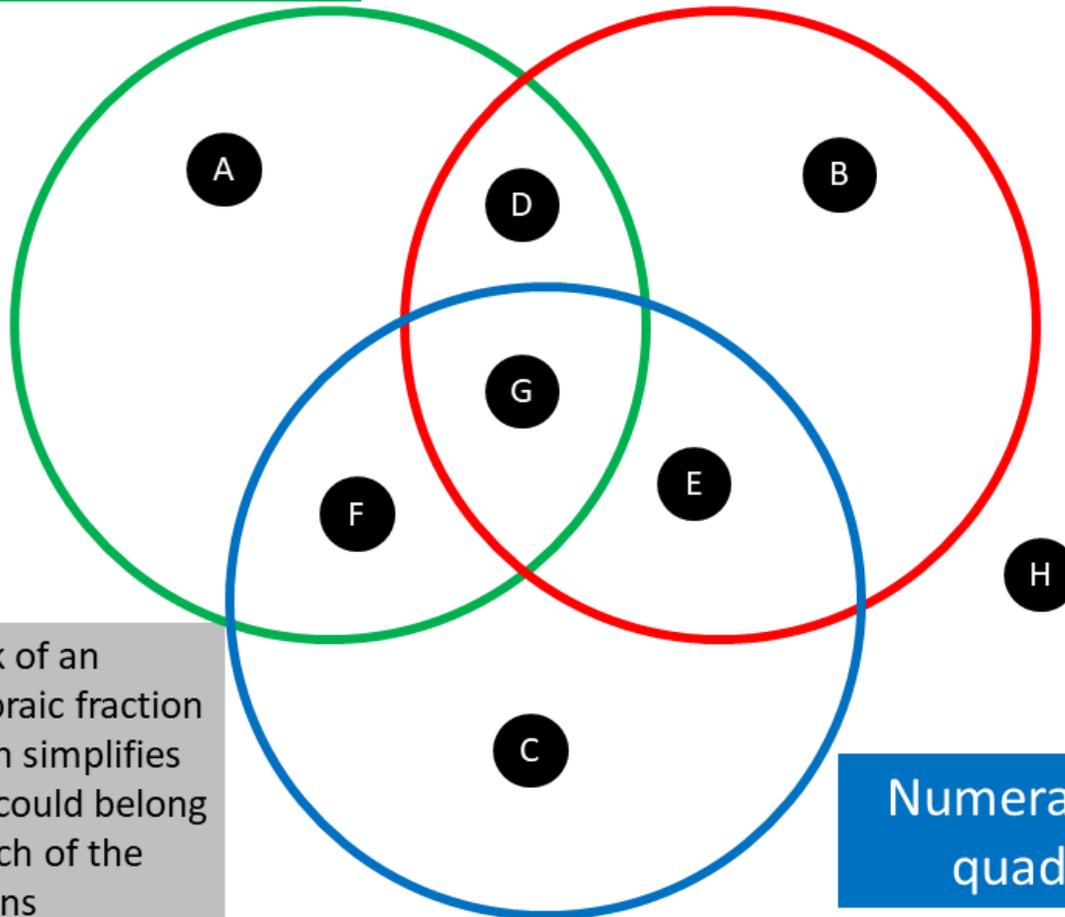
Numerator is a quadratic

Problem Solving

Denominator is $x^2 + 6x + 8$
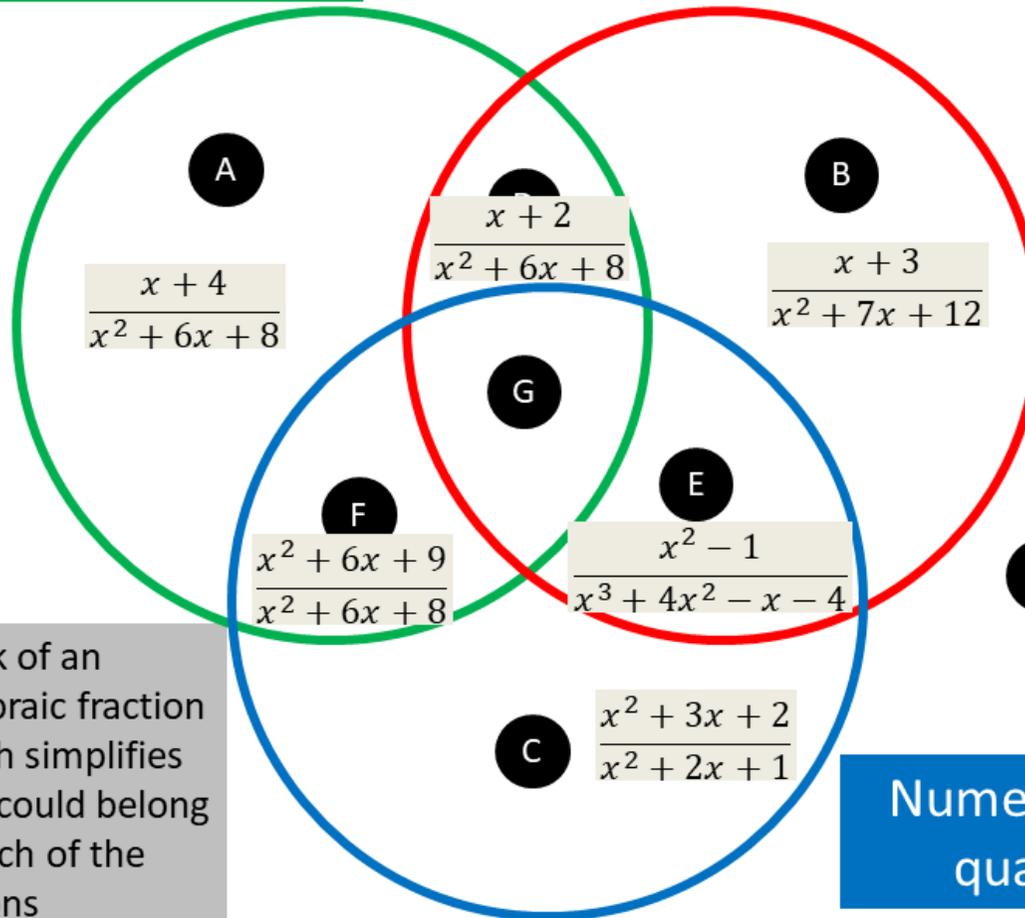
Equivalent to $\frac{1}{x+4}$

A

B

$\frac{x+2}{x^2+6x+8}$

$\frac{x+4}{x^2+6x+8}$

$\frac{x+3}{x^2+7x+12}$

If you think a region is impossible to fill, convince me why!

G

F

E

$\frac{x^2+6x+9}{x^2+6x+8}$

$\frac{x^2-1}{x^3+4x^2-x-4}$

H $\frac{x+1}{x^2+2x+1}$

Think of an algebraic fraction which simplifies that could belong in each of the regions

$\frac{x^2+3x+2}{x^2+2x+1}$

C

Numerator is a quadratic

Intelligent Practice

Simplify:

1) $\dfrac{3}{5} \times \dfrac{x}{y}$

2) $\dfrac{x}{y} \times \dfrac{3}{5}$

3) $\dfrac{x}{y} \times \dfrac{3}{6}$

4) $\dfrac{x}{y} \times \dfrac{3}{6x}$

5) $\dfrac{x}{xy} \times \dfrac{3}{6}$

6) $\dfrac{7x}{xy} \times \dfrac{3}{6}$

7) $\dfrac{7x}{xy} \times \dfrac{3}{6y}$

1) $\dfrac{7x}{xy} \div \dfrac{3}{6y}$

2) $\dfrac{7x}{xy} \div \dfrac{6y}{3}$

3) $\dfrac{7x}{xy} \times \dfrac{6y}{3}$

4) $\dfrac{7x}{xy} \times \dfrac{6y}{3} \times \dfrac{4y}{14}$

5) $\dfrac{7x}{xy} \div \dfrac{6y}{3} \times \dfrac{4y}{14}$

6) $\dfrac{7x}{xy} \div \dfrac{6y}{3} \div \dfrac{4y}{14}$

7) $\dfrac{7x^2}{xy} \div \dfrac{6y}{3} \div \dfrac{4y}{14}$

1) $\dfrac{7x^2}{xy} \div \left(\dfrac{6y}{3} \div \dfrac{4y}{14} \right)$

2) $\dfrac{7x^2}{xy} \div \left(\dfrac{6y}{3} \times \dfrac{4y}{14} \right)$

3) $\dfrac{7x^2}{xy} \div \left(\dfrac{6y}{9} \times \dfrac{4y}{14} \right)$

Fluency Practice

Simplify:

(a) $\dfrac{4x^2}{8x}$

(b) $\dfrac{4xy}{12y^2}$

(c) $\dfrac{9abc}{12bc}$

(d) $\dfrac{5a}{8a^2b}$

(e) $\dfrac{10x^2y}{20xy}$

(f) $\dfrac{9x^2yz}{27y^2}$

Simplify:

(a) $\dfrac{x}{2} \times \dfrac{x}{5}$

(b) $\dfrac{2x}{5} \times \dfrac{10}{3y}$

(c) $\dfrac{2x^2}{7} \times \dfrac{x}{5}$

(d) $\dfrac{5xy}{11} \times \dfrac{33}{2x}$

(e) $\dfrac{4ab}{9} \times \dfrac{3a}{8b^2}$

(f) $\dfrac{x^2}{y^2} \times \dfrac{y}{x}$

(g) $\dfrac{4}{9} \times \dfrac{abc}{8}$

(h) $\dfrac{2x^3}{5y} \times y$

Simplify:

(a) $\dfrac{x}{3} \div \dfrac{x}{7}$

(b) $\dfrac{2x}{5} \div \dfrac{10y}{3}$

(c) $\dfrac{2x^2}{7} \div \dfrac{x}{5}$

(d) $\dfrac{5x}{11y} \div \dfrac{3}{2x}$

(e) $\dfrac{3}{x} \div \dfrac{6y}{11}$

(f) $\dfrac{4}{5ab} \div \dfrac{10b}{3a}$

(g) $\dfrac{8a}{7b} \div 4$

(h) $\dfrac{2x^2y}{5z} \div \dfrac{7x}{2yz}$

Simplify:

(a) $\dfrac{a^3b^4cd^2}{5a} \times \dfrac{10d}{ab^2c^3}$

(b) $\dfrac{20a^2b^5c^3}{15} \times \dfrac{3c^2}{a^5b^2}$

Fluency Practice

Simplify fully:

(a) $\dfrac{3}{x+2} \times \dfrac{2(x+2)}{9}$

(b) $\dfrac{y-1}{5} \times \dfrac{3}{4(y-1)}$

(c) $\dfrac{6(2-x)}{5} \times \dfrac{y}{2-x}$

(d) $\dfrac{x+7}{4y} \div \dfrac{2(x+7)}{3}$

(e) $\dfrac{3}{x+5} \div \dfrac{10}{x(x+5)}$

(f) $\dfrac{2x}{x+1} \div \dfrac{7}{5(x+1)}$

Simplify fully:

(a) $\dfrac{3x+6}{2x} \times \dfrac{4}{x+2}$

(b) $\dfrac{y}{y-1} \times \dfrac{2y-2}{5}$

(c) $\dfrac{4y+4}{7} \times \dfrac{2x}{y+1}$

(d) $\dfrac{5+x}{3} \div \dfrac{10+2x}{7y}$

(e) $\dfrac{1-y}{2x} \div \dfrac{5-5y}{3}$

(f) $\dfrac{5y}{4y-12} \div \dfrac{2}{y-3}$

Simplify fully:

(a) $\dfrac{8+8y}{3y} \times \dfrac{7}{y(1+y)}$

(b) $\dfrac{6y}{x^2+x} \times \dfrac{x+1}{3}$

(c) $\dfrac{y^2-2y}{11x} \times \dfrac{4x}{y-2}$

(d) $\dfrac{x-1}{5x} \div \dfrac{x^2-x}{2y}$

(e) $\dfrac{3y}{x^2-8x} \div \dfrac{5x}{2x-16}$

(f) $\dfrac{3y-6}{7x} \div \dfrac{y^2-2y}{x^2}$

Simplify fully:

(a) $\dfrac{2x^2}{x^2+4x} \times \dfrac{3(x+4)}{5x} \div \dfrac{x+1}{10}$

(b) $\dfrac{x^2}{4y} \times \dfrac{y^2-2y}{6y} \div \dfrac{3y-6}{2y^2}$

(c) $\dfrac{5x+15}{6x^3} \div \dfrac{4}{3x^2} \times \dfrac{2x^2}{x^2+5x}$

(d) $\dfrac{3x-x^2}{5x+10} \div \dfrac{6-2x}{x^5} \times \dfrac{6x+12}{4x}$

Intelligent Practice

Simplify the following as far as possible:

1) $\dfrac{5}{x^2+x-2} \times \dfrac{x-1}{2}$

2) $\dfrac{9}{x^2-4x+3} \times \dfrac{x-3}{2}$

3) $\dfrac{7}{x-2} \div \dfrac{8}{x^2-x-2}$

4) $\dfrac{3}{x-4} \div \dfrac{8}{x^2+x+20}$

Simplify the following as far as possible:

1) $\dfrac{x^2-4x+3}{4x} \times \dfrac{3}{x^2-2x+1}$

2) $\dfrac{x^2+3x-4}{7x} \times \dfrac{3x}{x^2+7x+12}$

3) $\dfrac{x^2-2x-3}{2x} \div \dfrac{x^2-7x+12}{3}$

4) $\dfrac{x^2+4x+3}{3x} \div \dfrac{x^2+3x+2}{5x}$

Simplify the following as far as possible:

1) $\dfrac{x^2-2x+1}{x^2-1} \times \dfrac{x^2+4x+3}{x^2+2x-3}$

2) $\dfrac{x^2+4x-5}{x^2-x-2} \times \dfrac{x^2-x-2}{x^2+6x+5}$

3) $\dfrac{x^2+4x-5}{x^2+x-2} \div \dfrac{x^2+10x+25}{x^2+8x+15}$

4) $\dfrac{x^2+x-2}{x^2-2x-8} \div \dfrac{x^2-4x+3}{x^2-6x+9}$

Multiplying Algebraic Fractions
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Question 1: Express the following as a single fraction.

(a) $\dfrac{2}{g} \times \dfrac{3}{h}$

(b) $\dfrac{3}{c} \times \dfrac{a}{4}$

(c) $\dfrac{w}{x} \times \dfrac{3}{a}$

(d) $\dfrac{3a}{7} \times \dfrac{2c}{g}$

(e) $\dfrac{a}{e} \times \dfrac{f}{b}$

(f) $\dfrac{e}{8} \times \dfrac{d}{8}$

(g) $\dfrac{x}{2} \times \dfrac{x}{5}$

(h) $\dfrac{7}{y} \times \dfrac{2}{y}$

(i) $\dfrac{3}{w} \times \dfrac{x}{4} \times \dfrac{y}{w}$

(j) $\dfrac{2x}{5} \times \dfrac{3x}{7}$

(k) $\dfrac{x}{y} \times \dfrac{x}{y}$

(l) $\dfrac{6a}{7c} \times \dfrac{5a}{c}$

Question 2: Express the following as a single **simplified** fraction.

(a) $\dfrac{2x}{y} \times \dfrac{y}{4}$

(b) $\dfrac{3a}{c} \times \dfrac{5}{6}$

(c) $\dfrac{4}{5a} \times \dfrac{5w}{8}$

(d) $\dfrac{3a}{7} \times \dfrac{2c}{9}$

(e) $\dfrac{10g}{w} \times \dfrac{w}{5}$

(f) $\dfrac{4x}{5y} \times \dfrac{3y}{8x}$

(g) $\dfrac{2y}{3} \times \dfrac{4x}{wy}$

(h) $\dfrac{6x}{5y} \times \dfrac{4x}{3y}$

(i) $\dfrac{x^2}{a} \times \dfrac{a^2}{x^2}$

(j) $\dfrac{ab}{c} \times \dfrac{c}{ae}$

(k) $\dfrac{6c}{w^2} \times \dfrac{15w^3}{2c^2}$

(l) $\dfrac{2a^4}{3b^3} \times \dfrac{6b^2}{5a}$

(m) $\dfrac{2a^3b}{3} \times \dfrac{6}{ab^2}$

(n) $\dfrac{x^4y^4}{z^2} \times \dfrac{z}{x^6y}$

(o) $\dfrac{14a^2bc^3}{9} \times \dfrac{6b^3}{21a^3c}$

Fluency Practice

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Multiplying Algebraic Fractions
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Question 3: Express the following as a single fraction. **Simplify** if possible.

(a) $\dfrac{x}{4} \times \dfrac{x-3}{2}$

(b) $\dfrac{x}{9} \times \dfrac{6}{x+7}$

(c) $\dfrac{x+1}{15} \times \dfrac{5}{x}$

(d) $\dfrac{1}{x+3} \times \dfrac{2}{x+1}$

(e) $\dfrac{3x+2}{3} \times \dfrac{x+1}{3}$

(f) $\dfrac{x+4}{x-4} \times \dfrac{x-2}{x+5}$

(g) $\dfrac{x+1}{x-7} \times \dfrac{x-5}{x+1}$

(h) $\dfrac{7}{2x+8} \times \dfrac{x+4}{14}$

(i) $\dfrac{4}{2x-1} \times \dfrac{6x-3}{x+7}$

(j) $\dfrac{x+8}{15} \times \dfrac{10}{x^3+8x^2}$

(k) $\dfrac{4}{x-2} \times \dfrac{x^2-2x}{8}$

(l) $\dfrac{x^2+5x+6}{4} \times \dfrac{2}{x+2}$

(m) $\dfrac{x^2+2x-8}{x^2+5x+6} \times \dfrac{x+2}{x+4}$

(n) $\dfrac{x^2+x-6}{x^2-25} \times \dfrac{x^2+10x+25}{x^2-4}$

(o) $\dfrac{3x^2+8x-3}{25} \times \dfrac{30}{6x^2+13x-5}$

Answers

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Fluency Practice

Factorise $\Rightarrow$ Cancel $\Rightarrow$ Calculate

To make each calculation easier we can factorise out **binomial expressions** (e.g. $x + 2$) from each fraction.

At which stage should we factorise out any integers or single variables?

a) $\dfrac{3}{2x+2} \times \dfrac{3x+3}{4}$ $\Rightarrow$ $\dfrac{3}{2(x+1)} \times \dfrac{3(x+1)}{4}$ $\Rightarrow$ $\dfrac{3}{2} \times \dfrac{3}{4}$ $\Rightarrow$

b) $\dfrac{5}{3x+6} \times \dfrac{2x+4}{3}$ $\Rightarrow$ $\dfrac{5}{3(\quad)} \times \dfrac{2(\quad)}{3}$ $\Rightarrow$ $\Rightarrow$

c) $\dfrac{4x-12}{5} \times \dfrac{1}{2x-6}$ $\Rightarrow$ $\dfrac{4(\quad)}{5} \times \dfrac{1}{2(\quad)}$ $\Rightarrow$ $\Rightarrow$

d) $\dfrac{2x+8}{7} \times \dfrac{3}{5x+20}$ $\Rightarrow$ $\dfrac{\quad}{7} \times \dfrac{3}{\quad}$ $\Rightarrow$ $\Rightarrow$

e) $\dfrac{2x+5}{6} \times \dfrac{2}{6x+15}$ $\Rightarrow$ $\dfrac{\quad}{6} \times \dfrac{2}{\quad}$ $\Rightarrow$ $\Rightarrow$

f) $\dfrac{3y}{12-8x} \times \dfrac{9-6x}{5y}$ $\Rightarrow$ $\Rightarrow$ $\Rightarrow$

g) $\dfrac{x^2+3x+2}{3} \times \dfrac{8}{3x+6}$ $\Rightarrow$ $\dfrac{(x+1)(\quad)}{\quad}$ $\Rightarrow$ $\Rightarrow$

h) $\dfrac{x^2+3x-10}{10} \times \dfrac{5}{2x+10}$ $\Rightarrow$ $\Rightarrow$ $\Rightarrow$

i) $\dfrac{x^2-9}{7} \times \dfrac{3}{4x-12}$ $\Rightarrow$ $\Rightarrow$ $\Rightarrow$

j) $\dfrac{x^2+4x-5}{3x^2-6x} \times \dfrac{2x^2-14x+20}{x^2-25}$ $\Rightarrow$ $\Rightarrow$ $\Rightarrow$

Fluency Practice

Question 1: Express the following as a single **simplified** fraction.

(a) $\dfrac{x}{2} \div \dfrac{2}{3}$

(b) $\dfrac{a}{c} \div \dfrac{d}{5}$

(c) $\dfrac{3}{w} \div \dfrac{2}{a}$

(d) $\dfrac{c}{4} \div \dfrac{3}{c}$

(e) $\dfrac{3a}{4} \div \dfrac{6c}{7}$

(f) $\dfrac{4x}{9y} \div \dfrac{6x}{7}$

(g) $\dfrac{10x}{3y} \div \dfrac{15x}{y}$

(h) $\dfrac{ab}{3} \div \dfrac{2a}{b}$

(i) $\dfrac{4fg}{h} \div \dfrac{f}{2h}$

Question 2: Express the following as a single fraction. **Simplify** if possible.

(a) $\dfrac{x-4}{8} \div \dfrac{3x-12}{2}$

(b) $\dfrac{x+3}{x+2} \div \dfrac{x+1}{x+2}$

(c) $\dfrac{x+1}{2} \div \dfrac{2x+2}{3}$

(d) $\dfrac{3x+9}{2} \div \dfrac{x+3}{4}$

(e) $\dfrac{4}{x-2} \div \dfrac{3}{x^2-2x}$

(f) $\dfrac{11}{4x^2+2x} \div \dfrac{3}{2x+1}$

(g) $\dfrac{x+3}{x+1} \div \dfrac{x}{(x+1)^2}$

(h) $\dfrac{x^2}{7} \div \dfrac{6x^3+8x^2}{x^2-7x}$

(i) $\dfrac{x}{x+6} \div \dfrac{x+6}{x^2}$

(j) $\dfrac{x^2+7x+10}{2} \div \dfrac{x^2+4x-5}{4}$

(k) $\dfrac{14}{x^2-5x+6} \div \dfrac{7}{x^2+3x-10}$

(l) $\dfrac{x+4}{x+5} \div \dfrac{3x+12}{x^2-25}$

(m) $\dfrac{x^3-x}{x+2} \div \dfrac{x^2-x}{x^2-5x-14}$

Answers

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Fluency Practice

Q7 Simplify the following algebraic fractions.

[a] $\dfrac{1}{2x^2 + x - 10} \div \dfrac{1}{6x^2 + 15x}$

[b] $\dfrac{1}{4x^2 + 11x - 3} \div \dfrac{1}{2x^2 + 6x}$

[c] $\dfrac{1}{2x^2 + 5x + 2} \div \dfrac{1}{x^2 + 2x}$

[d] $\dfrac{1}{3x^2 - 5x + 2} \div \dfrac{1}{6x^2 - 4x}$

[e] $\dfrac{4}{2x^2 + 5x - 3} \div \dfrac{2}{10x^2 - 5x}$

[f] $\dfrac{6}{3x^2 + 10x - 8} \div \dfrac{3}{2x^2 + 8x}$

[g] $\dfrac{4}{5x^2 - 19x - 4} \div \dfrac{1}{3x^2 - 12x}$

[h] $\dfrac{1}{4x^2 + 4x - 15} \div \dfrac{9}{4x^2 + 10x}$

Fluency Practice

Q9 Simplify the following algebraic fractions.

[a] $\dfrac{x^3 + 7x^2 + 12x}{x^2 + 4x}$

[b] $\dfrac{x^3 + 9x^2 + 8x}{x^2 + x}$

[c] $\dfrac{x^2 + 7x}{x^3 + 11x^2 + 28x}$

Q10 Show that,

$$\frac{(x^2 - x - 6)(x^2 - 3x - 4)(x^2 + 5x)}{(x^2 - 2x - 8)(x^2 + 2x - 15)(3x + 3)} = \frac{x}{3}$$

Fill in the Gaps

Fill in the table. One row has multiple possible correct answers.

First Expression	Second Expression	Highest Common Factor	Lowest Common Multiple
$3x^2y^3$	$4x^4$	x^2	
$(x+3)^2(y+1)$	$(x+3)(x+4)(y+1)$		$(x+3)^2(x+4)(y+1)$
$9x^3yz$	$6x^2y^2z^2$		
x^2+7x+6	$(x+6)(x+2)$		
x^2-9		$x-3$	$(x+3)(x-3)$
	$2x^2y$	$2xy$	$14x^2y^3$
$2y^2-7y-4$	$3y^2-11y-4$		
		$5a^2b$	$5a^2b$
		1	$10(x-3)(x+2)(x-1)$
$12x^3y^2$		$6xy$	$36x^3y^2z$
$96ab^3$		24	$288ab^3$

Intelligent Practice

Simplify:

1) $\dfrac{3x}{2} + \dfrac{4x}{5}$

2) $\dfrac{x}{2} + \dfrac{4x}{5}$

3) $\dfrac{x}{2} + \dfrac{x}{5}$

4) $\dfrac{x}{2} - \dfrac{x}{5}$

5) $\dfrac{x}{5} - \dfrac{x}{2}$

6) $\dfrac{5x}{5} - \dfrac{x}{2}$

7) $\dfrac{5}{x} + \dfrac{2}{x}$

8) $\dfrac{5}{x^2} + \dfrac{2}{x}$

9) $\dfrac{5}{2x^2} - \dfrac{2}{x}$

10) $\dfrac{6}{2x^2} - \dfrac{2}{x}$

11) $\dfrac{6}{2xy} + \dfrac{2}{x}$

12) $\dfrac{6}{2xy} + \dfrac{2}{3x}$

13) $\dfrac{6}{2xy} - \dfrac{2}{3x^2}$

14) $\dfrac{6y}{2xy} + \dfrac{2}{3x^2}$

15) $\dfrac{y}{2xy} - \dfrac{2}{3x^3}$

Fluency Practice

Simplify:

(a) $\dfrac{x}{5} + \dfrac{x}{6}$ (b) $\dfrac{x}{3} + \dfrac{x}{8}$

(c) $\dfrac{x}{2} - \dfrac{x}{3}$ (d) $\dfrac{x}{4} - \dfrac{x}{9}$

(e) $\dfrac{2x}{3} + \dfrac{x}{4}$ (f) $\dfrac{3x}{5} + \dfrac{2x}{9}$

(g) $\dfrac{3x}{5} - \dfrac{2x}{7}$ (h) $\dfrac{4x}{3} - \dfrac{5x}{8}$

Simplify:

(a) $\dfrac{3}{x} + \dfrac{5}{x}$ (b) $\dfrac{6}{y} - \dfrac{4}{y}$

(c) $\dfrac{2}{x} + \dfrac{5}{2x}$ (d) $\dfrac{6}{xy} - \dfrac{4}{x}$

(e) $\dfrac{2}{x} + \dfrac{7}{x^2}$ (f) $\dfrac{4}{xy} - \dfrac{2}{3x}$

(g) $\dfrac{5}{4a} - \dfrac{1}{b}$ (h) $\dfrac{7}{ab} - \dfrac{3}{2a}$

Simplify:

(a) $\dfrac{3y}{6x} + \dfrac{5}{9x}$ (b) $\dfrac{5}{xy} - \dfrac{3x}{y}$

(c) $\dfrac{3}{ab} + \dfrac{6b}{5a}$ (d) $\dfrac{7}{x^2} - \dfrac{2}{x}$

Simplify:

(a) $\dfrac{3}{x} + \dfrac{5}{3x} + \dfrac{2}{3}$

(b) $\dfrac{8}{xy} + \dfrac{2y}{x} - \dfrac{x}{y}$

(c) $\dfrac{9}{a^2b} - \dfrac{3}{ab} - \dfrac{1}{ab^2}$

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Fluency Practice

Red box

Simplify

(a) $\dfrac{x+1}{2} + \dfrac{x+4}{3}$

(b) $\dfrac{y}{5} + \dfrac{y+2}{2}$

(c) $\dfrac{a-3}{6} + \dfrac{a+2}{5}$

(d) $\dfrac{x+5}{2} + \dfrac{2x-1}{4}$

(e) $\dfrac{3x-5}{9} + \dfrac{x-2}{4}$

(f) $\dfrac{w+8}{5} + \dfrac{w-2}{3}$

Yellow box

Simplify

(a) $\dfrac{x+5}{3} - \dfrac{x+1}{4}$

(b) $\dfrac{f+7}{2} - \dfrac{f+2}{5}$

(c) $\dfrac{a}{4} - \dfrac{a+5}{8}$

(d) $\dfrac{y+5}{2} - \dfrac{y-1}{3}$

(e) $\dfrac{2x-5}{5} - \dfrac{x-2}{4}$

(f) $\dfrac{9-w}{5} - \dfrac{w+1}{6}$

Green box

Simplify

(a) $\dfrac{5}{x} + \dfrac{3}{x+2}$

(b) $\dfrac{2}{y+1} + \dfrac{1}{y+3}$

(c) $\dfrac{4}{x-1} + \dfrac{2}{3x+4}$

(d) $\dfrac{5}{w-3} + \dfrac{3}{w+2}$

Purple box

Simplify

(a) $\dfrac{4}{a+2} - \dfrac{1}{a+3}$

(b) $\dfrac{5}{x+1} - \dfrac{2}{x-3}$

(c) $\dfrac{2}{y-4} - \dfrac{3}{2y-1}$

(d) $\dfrac{6}{w+5} - \dfrac{4}{w-1}$

Blue box

Simplify

(a) $\dfrac{x+2}{4} + \dfrac{x-1}{2} + \dfrac{3+x}{5}$

(b) $\dfrac{2y-1}{3} + \dfrac{y+5}{2} - \dfrac{y+2}{9}$

(c) $\dfrac{5}{x} + \dfrac{3}{x+2} + \dfrac{4}{3x}$

(d) $\dfrac{7}{a-3} - \dfrac{1}{a+2} + \dfrac{4}{a}$

Fluency Practice

Q1 Write the following as a fraction in its simplest form.

[a] $\dfrac{x}{3} + \dfrac{x}{9}$

[b] $\dfrac{x}{2} - \dfrac{x}{8}$

[c] $\dfrac{x}{4} - \dfrac{x}{6}$

[d] $\dfrac{x}{10} + \dfrac{x}{4}$

[e] $\dfrac{x}{8} + \dfrac{2x}{3}$

[f] $\dfrac{3x}{4} - \dfrac{x}{5}$

[g] $\dfrac{5x}{6} - \dfrac{x}{9}$

[h] $\dfrac{3x}{7} + \dfrac{2x}{5}$

Q2 Write the following as a fraction in its simplest form.

[a] $\dfrac{3}{x} + \dfrac{9}{x}$

[b] $\dfrac{8}{x} - \dfrac{2}{x}$

[c] $\dfrac{6}{x} - \dfrac{4}{x}$

[d] $\dfrac{10}{x} + \dfrac{4}{x}$

[e] $\dfrac{8}{x} + \dfrac{3}{2x}$

[f] $\dfrac{5}{x} - \dfrac{4}{3x}$

[f] $\dfrac{9}{x} - \dfrac{6}{5x}$

[h] $\dfrac{7}{3x} + \dfrac{5}{2x}$

Q3 Work out the following.

[a] $\dfrac{3}{x} + \dfrac{5}{x^2}$

[b] $\dfrac{6}{x^2} + \dfrac{2}{x}$

[c] $\dfrac{4}{xy} - \dfrac{3}{x}$

[d] $\dfrac{10}{xy} - \dfrac{8}{xyz}$

[e] $\dfrac{5}{x^2} - \dfrac{5}{2x^2y}$

[f] $\dfrac{1}{ab^2} - \dfrac{2}{b}$

[g] $\dfrac{2}{3x} + \dfrac{1}{3t^2}$

[h] $\dfrac{8}{p^2} + \dfrac{3}{2pq}$

[i] $\dfrac{5}{3s} + \dfrac{2}{s^2t}$

[j] $\dfrac{3}{u^2v^2} - \dfrac{2}{uv}$

[k] $\dfrac{3}{2r} - \dfrac{2}{3st}$

[l] $\dfrac{2}{5x^3} + \dfrac{3x}{2}$

Fluency Practice

Q4 Work out the following.

[a] $\dfrac{1}{x} + \dfrac{2}{y} + \dfrac{3}{z}$

[b] $\dfrac{5}{m} + \dfrac{4}{n} + \dfrac{1}{p}$

[c] $\dfrac{1}{2r} + \dfrac{2}{s} + \dfrac{3}{2t}$

[d] $\dfrac{1}{2a} + \dfrac{2}{3b} + \dfrac{3}{4c}$

Q5 Gemma says that,

$$\dfrac{2x}{5} + \dfrac{x}{7} = \dfrac{3x}{12}$$

Why is Gemma wrong?
Work out the correct answer.

Q6 Find the value of,

$$\dfrac{x}{2} + \dfrac{x}{3} + \dfrac{x}{6}$$

Q7 Write the following as a single fraction in its simplest form.

[a] $\dfrac{x+2}{7} + \dfrac{x}{3}$

[b] $\dfrac{x+5}{4} + \dfrac{2x}{3}$

[c] $\dfrac{x+5}{2} + \dfrac{x+2}{5}$

[d] $\dfrac{x+1}{6} - \dfrac{x-1}{7}$

[e] $\dfrac{x+1}{2} - \dfrac{x+2}{4}$

[f] $\dfrac{x+8}{2} - \dfrac{x+3}{9}$

[g] $\dfrac{3x+2}{10} + \dfrac{5x-4}{6}$

[h] $\dfrac{3x+1}{6} + \dfrac{4x-1}{4}$

Fluency Practice

Simplify the following as far as possible:

1) $\dfrac{6}{2xy} + \dfrac{2}{x}$

2) $\dfrac{6}{2xy} + \dfrac{2}{3x}$

3) $\dfrac{6}{2xy} - \dfrac{2}{3x^2}$

4) $\dfrac{6y}{2xy} + \dfrac{2}{3x^2}$

5) $\dfrac{y}{2xy} - \dfrac{2}{3x^3}$

Simplify the following as far as possible:

1) $\dfrac{1}{x^2-4} - \dfrac{1}{x-2}$

2) $\dfrac{x}{x^2-4} - \dfrac{1}{x-2}$

3) $\dfrac{x}{x^2-x-2} - \dfrac{1}{x-2}$

4) $\dfrac{x-3}{x^2-x-2} - \dfrac{1}{x-2}$

5) $\dfrac{x-3}{x^2-x-2} - \dfrac{2x}{x-2}$

Simplify the following as far as possible:

1) $\dfrac{2x+1}{x+4} - \dfrac{x-5}{x-2}$

2) $\dfrac{11x+27}{2x^2+11x-6} - \dfrac{3}{x+6}$

Challenge: Express the following as a fraction in its simplest form:

$$\dfrac{3}{x-1} + \dfrac{2}{x+1} + \dfrac{4}{x-3}$$

Fluency Practice

Q	Express as a simplified fraction	Answers
1	$\dfrac{4}{x+3} + \dfrac{2}{x+1}$	
2	$\dfrac{4}{x+3} - \dfrac{2}{x+1}$	
3	$\dfrac{4}{x+3} + \dfrac{2}{x-1}$	
4	$\dfrac{4}{x+3} - \dfrac{2}{x-1}$	
5	$\dfrac{4}{x-3} + \dfrac{2}{x-1}$	
6	$\dfrac{4}{x-3} - \dfrac{2}{x-1}$	
7	$\dfrac{4}{3x-5} + \dfrac{2}{1-7x}$	
8	$\dfrac{4}{3x-5} - \dfrac{2}{1-7x}$	
9	$\dfrac{a}{bx+c} + \dfrac{d}{ex+f}$	
10	$\dfrac{a}{bx+c} - \dfrac{d}{ex+f}$	

Fluency Practice

adding/subtracting algebraic fractions (simplification)

(1) $\dfrac{4m + 1}{4} + \dfrac{1 - 3m}{3}$

(2) $\dfrac{4m - 1}{4} + \dfrac{5 - 6m}{6}$

(3) $\dfrac{3m + 1}{3} - \dfrac{4m - 1}{4}$

(4) $\dfrac{3m + 2}{6} + \dfrac{2 - 4m}{8}$

(5) $\dfrac{3d + 4}{2} - \dfrac{8 + 6d}{4}$

(6) $\dfrac{6d - 4}{8} - \dfrac{3d - 2}{4}$

(7) $\dfrac{6d + 2}{4} - \dfrac{15d + 5}{10}$

(8) $\dfrac{3d - 9}{9} - \dfrac{2d - 6}{6}$

(9) $\dfrac{3k - 4}{5} + \dfrac{2 - k}{2}$

(10) $\dfrac{6k - 3}{4} + \dfrac{3 - 4k}{3}$

(11) $\dfrac{3k + 2}{3} - \dfrac{4k + 2}{5}$

(12) $\dfrac{4k - 7}{6} - \dfrac{4k - 9}{7}$

(13) $\dfrac{3f - 4}{4} + \dfrac{2f - 3}{5}$

(14) $\dfrac{7f - 1}{5} + \dfrac{3f - 6}{4}$

(15) $\dfrac{7f + 6}{5} - \dfrac{3f + 11}{4}$

(16) $\dfrac{7f - 11}{4} - \dfrac{4f + 9}{5}$

Q8 Write the following as a single fraction in its simplest form.

[a] $\dfrac{2}{x+2} + \dfrac{5}{x}$

[b] $\dfrac{1}{x+5} + \dfrac{5}{2x}$

[c] $\dfrac{3}{x+5} + \dfrac{4}{x+2}$

[d] $\dfrac{6}{x+1} - \dfrac{3}{x-1}$

[e] $\dfrac{2}{x+1} - \dfrac{4}{x+2}$

[f] $\dfrac{5}{x+3} - \dfrac{9}{x+11}$

[g] $\dfrac{10}{3x+2} + \dfrac{2}{5x-4}$

[h] $\dfrac{2}{2x+1} + \dfrac{5}{4x-1}$

Q9 Fill in the missing boxes so that each block is the sum of the two below it.

Bottom row: $\dfrac{2x}{5}$, $\dfrac{x}{4}$, $\dfrac{x+5}{5}$
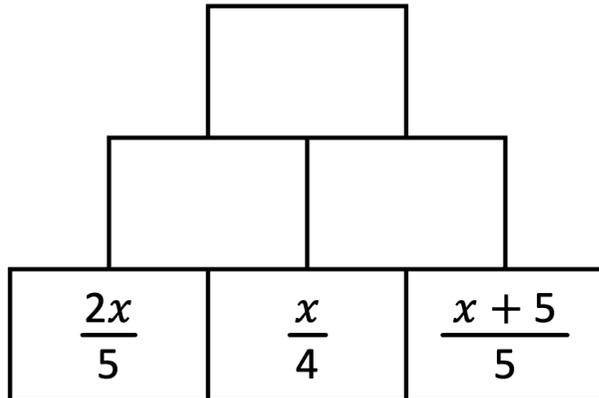
Q10 Fill in the empty squares to make all the equations in this grid correct both horizontally and vertically.

	$+$		$=$	$\dfrac{2x}{3}$
$+$	■	$+$	■	$+$
$\dfrac{2x}{3}$	$+$	$\dfrac{2x}{5}$	$=$	
$=$	■	$=$	■	$=$
	$+$	$\dfrac{14x}{15}$	$=$	

Q8 Simplify the following algebraic fractions.

[a] $\dfrac{1}{3x^2 + 11x - 4} \div \dfrac{1}{3x + 12} + \dfrac{2}{3x + 1}$

[b] $\dfrac{1}{5x^2 + 14x - 3} \div \dfrac{1}{2x + 6} + \dfrac{3}{2x - 5}$

[c] $\dfrac{1}{2x^2 + 5x - 12} \div \dfrac{1}{5x + 20} - \dfrac{1}{4x + 3}$

[d] $\dfrac{1}{6x^2 + x - 1} \div \dfrac{1}{9x - 3} - \dfrac{2}{3x - 2}$

Fluency Practice

A1 Simplify $$\frac{x}{5}+\frac{3x}{4}$$	**A2** Simplify $$\frac{3x}{2}-\frac{5x}{6}$$	**A3** Simplify $$\frac{x+3}{5}+\frac{x-5}{4}$$	**A4** Simplify $$\frac{2x-1}{3}-\frac{2}{5}$$
B1 Simplify $$\frac{4}{x-1}+\frac{3}{x+1}$$	**B2** Simplify $$\frac{3}{x+1}-\frac{6}{2x+3}$$	**B3** Simplify $$\frac{3}{2x-3}-\frac{1}{x+2}$$	**B4** Simplify $$\frac{3}{x-1}-\frac{2}{x}$$
C1 Simplify $$\frac{2}{x-1}+\frac{x-11}{x^2+3x-4}$$	**C2** Simplify $$\frac{2}{x+2}+\frac{x}{x^2+5x+6}$$	**C3** Simplify $$\frac{2}{x-1}+\frac{x}{x^2-4x+3}$$	**C4** Simplify $$\frac{5}{2x-6}-\frac{x+2}{x^2-4x+3}$$
D1 Simplify $$1+\frac{4}{(x+3)(x-2)}$$	**D2** Simplify $$1+\frac{x^2+x-6}{(x+4)(x-2)}$$	**D3** Simplify $$2+\frac{5-x}{x^2-2x-15}$$	**D4** Simplify $$5-(x+2)\div\left(\frac{x^2-4}{x-3}\right)$$

Fluency Practice

1. Combine the following algebraic fractions, fully simplifying your answer.

(a) $\dfrac{2}{x+4} + \dfrac{3}{x+1}$.

(b) $\dfrac{5}{x-2} + \dfrac{4}{x+4}$.

(c) $\dfrac{5}{x-1} - \dfrac{2}{x+2}$.

(d) $\dfrac{3}{x-5} - \dfrac{1}{x-7}$.

(e) $\dfrac{a}{2x+4} - \dfrac{a}{3x+6}$.

(f) $\dfrac{a}{x+b} + \dfrac{c}{x+c}$.

(g) $2 + \dfrac{3}{x-5}$.

(h) $\dfrac{5}{2-x} + 3 + \dfrac{1}{x+1}$.

(i) $\dfrac{a}{x+k} + \dfrac{b}{x+2k}$.

(j) $\dfrac{5}{2x+3} - 2x$.

(k) $\dfrac{1}{(x-1)^2} - \dfrac{2}{x-1}$.

(l) $\dfrac{5}{x^2+3x} + \dfrac{2}{x+3}$.

(m) $\dfrac{x}{x^2-x-6} - \dfrac{3}{2x+4}$.

(n) $\dfrac{x+1}{x^2-4x+3} - \dfrac{x-3}{x^2-1}$.

(o) $\dfrac{2}{x} - \dfrac{3}{x^2-x} - \dfrac{4}{x-1}$.

(p) $\dfrac{1}{x-1} - \dfrac{1}{x^2+x-2} + \dfrac{4}{x+2}$.

(q) $\dfrac{2}{x^2-x} - \dfrac{1}{x^2-1}$.

(r) $\dfrac{3}{2x-3} + \dfrac{1}{x-4} - \dfrac{5x}{2x^2-11x+12}$.

(s) $\dfrac{a}{x+1} - \dfrac{b}{x^2+2x+1} - \dfrac{c}{(x+1)^3}$.

2. Write the following as a single fraction.

(a) $\dfrac{1}{\frac{2}{x-4}-3}$.

(b) $\dfrac{3}{\frac{1}{x-2}+\frac{2}{x}}$.

(c) $\dfrac{\frac{1}{x}}{2-\frac{3}{x}}$.

(d) $\dfrac{\frac{4}{x-1}}{1-\frac{3}{x-1}}$.

(e) $\dfrac{2+\frac{5}{x+3}}{\frac{1}{x+3}-7}$.

(f) $\dfrac{a-\frac{c}{x+b}}{e+\frac{f}{x+b}}$.

(g) $\dfrac{\frac{3}{x-2}+2}{1-\frac{4}{x}}$.

(h) $\dfrac{\frac{4}{3x}}{\frac{1}{x}-\frac{2}{x+1}}$.

(i) $\dfrac{\frac{1}{x-1}+\frac{3}{x-2}}{\frac{1}{x-1}-\frac{2}{x+3}}$.

(j) $\dfrac{1-\frac{2}{x-3}}{\frac{1}{x+2}-1}$.

(k) $\dfrac{\frac{a-b}{b}+\frac{c-b}{c}}{\frac{a}{b}-\frac{b}{c}}$.

(l) $1+\dfrac{1}{1+\frac{1}{1+\frac{1}{x}}}$.

Fluency Practice

(m) $3 - \dfrac{\frac{5}{x-1}}{x - \frac{4}{x-1}}$.

(n) $\dfrac{1}{\frac{1}{\frac{1}{x}+1} + 1} - \dfrac{1}{1 + \frac{1}{1+\frac{1}{x}}}$.

(o) $\dfrac{2}{3 + \frac{4}{5+\frac{6}{x}}} - \dfrac{1}{2 + \frac{3}{4+\frac{5}{x}}}$.

3. Cancel the following to their simplest form.

(a) $\dfrac{x^2 + 3x}{2x + 6}$.

(b) $\dfrac{x^2 - 1}{x^2 + 5x + 4}$.

(c) $\dfrac{2x^2 + 7x - 4}{2x^2 + 9x - 5}$.

(d) $\dfrac{4x^2 - 8x - 12}{2x^2 - 2x - 12}$.

(e) $\dfrac{4\pi x^2 - \pi}{12\pi x^2 - 10\pi x + 2\pi}$.

(f) $\dfrac{2a^2 x^2 + a^2 x - 10a^2}{2ax^2 - 2ax - 4a}$.

(g) $\dfrac{12x^4 + 17x^3 - 5x^2}{9x^3 - 25x}$.

(h) $\dfrac{x^2 + 6x + 8}{x^2 - 4x + 3} \div \dfrac{2x + 8}{x - 3}$.

(i) $\dfrac{\pi x^2 - \pi x - 2\pi}{x^3 + x^2 - 6x} \div \dfrac{2x + 2}{3x + 9}$.

(j) $\dfrac{12x^2 + 10x + 2}{2x - 6} \div \dfrac{18x^2 + 15x + 3}{5x - 15}$.

(k) $\dfrac{2x^2 + 2x - 4}{3x^3 - 3x} \div \dfrac{4x + 8}{9 + 9x}$.

(l) $\dfrac{4x^2 - 1}{2x^2 - 7x - 4} \div \dfrac{2x^2 + 5x - 3}{2x^2 - 2x - 24}$.

(m) $\dfrac{6\pi x^2 + 19\pi x + 10\pi}{3\pi x^3 + 4\pi x^2 - 7\pi x} \div \dfrac{6x + 15}{18x + 42}$.

4. Find the value of the capital letters which make the following identities correct.

(a) $\dfrac{A}{x} + \dfrac{B}{x + 1} \equiv \dfrac{5x + 2}{x(x + 1)}$.

Fluency Practice

Q1
Solve the following equations.

[a] $\dfrac{x+3}{4} + \dfrac{x+4}{5} = 2$

[b] $\dfrac{x-1}{2} + \dfrac{x+3}{6} = 4$

[c] $\dfrac{x-1}{4} - \dfrac{x+1}{6} = 1$

[d] $\dfrac{x+2}{2} + \dfrac{x+3}{3} = 1$

[e] $\dfrac{x+2}{2} + \dfrac{7x-7}{4} = 6$

[f] $\dfrac{x+5}{3} + \dfrac{2x+3}{4} = 2$

[g] $\dfrac{x+3}{6} + \dfrac{3x+8}{5} = 9$

[h] $\dfrac{x+2}{6} + \dfrac{7x+6}{2} = 6$

Q2
Solve the following equations.

[a] $\dfrac{3x+1}{2} + \dfrac{5x+7}{6} = 2$

[b] $\dfrac{6x-1}{3} - \dfrac{3x+1}{7} = 10$

[c] $\dfrac{2x+5}{2} - \dfrac{3x+1}{5} = 2$

[d] $\dfrac{2x+4}{7} + \dfrac{2x-3}{2} = 1$

[e] $\dfrac{8x-1}{5} + \dfrac{4x-5}{2} = 3$

[f] $\dfrac{3x-2}{6} + \dfrac{5x+6}{7} = 7$

[g] $\dfrac{9x+8}{6} + \dfrac{4x+3}{5} = 5$

[h] $\dfrac{2x+7}{6} + \dfrac{2x-8}{3} = 5$

Fluency Practice

Q3

Solve the following equations.

[a] $\dfrac{2}{x+6} + \dfrac{4}{x+8} = 2$

[b] $\dfrac{4}{x-5} + \dfrac{2}{x-8} = 3$

[c] $\dfrac{2}{x-9} + \dfrac{4}{x-7} = 2$

[d] $\dfrac{3}{x-1} + \dfrac{2}{x-3} = 1$

[e] $\dfrac{1}{2x-7} + \dfrac{6}{x+4} = 1$

[f] $\dfrac{1}{2x-5} + \dfrac{4}{x+2} = 1$

[g] $\dfrac{5}{2x-3} + \dfrac{2}{x-6} = 1$

[h] $\dfrac{2}{5x+2} + \dfrac{5}{x+4} = 1$

Q4

Solve the following equations.

[a] $\dfrac{4}{2x-1} + \dfrac{6}{4x-7} = 2$

[b] $\dfrac{5}{2x-6} + \dfrac{3}{2x+6} = 1$

[c] $\dfrac{6}{4x+1} + \dfrac{4}{2x+3} = 2$

[d] $\dfrac{2}{4x-9} + \dfrac{4}{2x-3} = 2$

[e] $\dfrac{6}{2x-3} + \dfrac{4}{3x-7} = 2$

[f] $\dfrac{3}{4x+7} + \dfrac{6}{2x+5} = 3$

[g] $\dfrac{3}{4x+9} + \dfrac{2}{2x+7} = 1$

[h] $\dfrac{3}{2x-1} + \dfrac{5}{2x-7} = 2$

Fluency Practice

Examples

Workout

Question 1: Express the following as a single simplified fraction.

(a) $\dfrac{x}{3} + \dfrac{x}{5}$

(b) $\dfrac{c}{2} + \dfrac{c}{7}$

(c) $\dfrac{w}{3} + \dfrac{w}{9}$

(d) $\dfrac{x}{2} - \dfrac{x}{3}$

(e) $\dfrac{a}{5} - \dfrac{a}{9}$

(f) $\dfrac{m}{2} - \dfrac{m}{8}$

(g) $\dfrac{m}{3} + \dfrac{2m}{7}$

(h) $\dfrac{3x}{5} + \dfrac{x}{2}$

(i) $\dfrac{3c}{4} + \dfrac{5c}{9}$

(j) $\dfrac{m}{2} - \dfrac{2m}{5}$

(k) $\dfrac{3n}{4} - \dfrac{5n}{9}$

(l) $\dfrac{7h}{8} - \dfrac{5h}{12}$

Question 2: Express the following as a single simplified fraction.

(a) $\dfrac{2}{x^2} + \dfrac{5}{x}$

(b) $\dfrac{5}{6x} - \dfrac{1}{3x}$

(c) $\dfrac{2}{fg} - \dfrac{4}{g}$

(d) $\dfrac{6}{ac} + \dfrac{2}{3}$

(e) $\dfrac{9}{w} + \dfrac{wx}{4}$

(f) $\dfrac{d}{3} + \dfrac{2}{d^2}$

(g) $\dfrac{m^2}{6} - \dfrac{9}{4m}$

(h) $\dfrac{3}{4b^2} - \dfrac{1}{2b}$

(i) $\dfrac{ac}{5} + \dfrac{4}{c}$

(j) $\dfrac{x^3}{w} - \dfrac{2}{wx^2}$

(k) $\dfrac{2}{ab^2} - \dfrac{3}{b^3}$

(l) $\dfrac{xy}{5} - \dfrac{1}{x}$

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Question 3: Express the following as a single simplified fraction.

(a) $\dfrac{x+4}{3} + \dfrac{x+1}{2}$

(b) $\dfrac{2x+1}{2} + \dfrac{x+3}{5}$

(c) $\dfrac{5x-2}{3} + \dfrac{2x+7}{4}$

(d) $\dfrac{3x+11}{2} + \dfrac{x-20}{3}$

(e) $\dfrac{5x-4}{2} + \dfrac{x+1}{3}$

(f) $\dfrac{x-4}{8} + \dfrac{2x-3}{2}$

(g) $\dfrac{7x+4}{2} - \dfrac{x+1}{3}$

(h) $\dfrac{9x+1}{5} - \dfrac{x+2}{2}$

(i) $\dfrac{3x+11}{2} - \dfrac{2x-3}{6}$

(j) $\dfrac{x-8}{3} - \dfrac{x-5}{7}$

(k) $\dfrac{4x-7}{10} - \dfrac{2x-9}{3}$

(l) $\dfrac{5-x}{6} - \dfrac{8-3x}{10}$

Question 4: Express the following as a single simplified fraction.

(a) $\dfrac{2}{x+5} + \dfrac{3}{x+1}$

(b) $\dfrac{2}{x+1} + \dfrac{1}{x+3}$

(c) $\dfrac{4}{x+5} - \dfrac{2}{x-1}$

(d) $\dfrac{x+1}{x-2} + \dfrac{x+3}{x+5}$

(e) $\dfrac{x+3}{2x+1} - \dfrac{x-2}{x-1}$

(f) $\dfrac{x}{x+7} + \dfrac{2x+5}{3x+1}$

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$\dfrac{}{} + \dfrac{}{(x+}$

Question 5: Solve the following equations

(a) $\dfrac{x+5}{3} + \dfrac{x+1}{2} = 8$

(b) $\dfrac{x+6}{5} + \dfrac{x-3}{2} = 13$

(c) $\dfrac{2x-1}{4} + \dfrac{x-3}{5} = 3$

(d) $\dfrac{x+5}{10} - \dfrac{x+4}{2} = 1$

(e) $\dfrac{x-3}{4} - \dfrac{x-8}{3} = 4$

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Question 5: Solve the following equations

(a) $\dfrac{x+5}{3} + \dfrac{x+1}{2} = 8$

(b) $\dfrac{x+6}{5} + \dfrac{x-3}{2} = 13$

(c) $\dfrac{2x-1}{4} + \dfrac{x-3}{5} = 3$

(d) $\dfrac{x+5}{10} - \dfrac{x+4}{2} = 1$

(e) $\dfrac{x-3}{4} - \dfrac{x-8}{3} = 4$

Question 6: Solve the following equations

(a) $\dfrac{4}{x+1} + \dfrac{2}{x-2} = 3$

(b) $\dfrac{2}{x-3} + \dfrac{1}{x-4} = 2$

(c) $\dfrac{2}{x-5} - \dfrac{2}{x-4} = 1$

(d) $\dfrac{2}{x+1} - \dfrac{10}{x+4} = -1$

(e) $\dfrac{3}{x-2} + \dfrac{3}{x+2} = 2$

(f) $\dfrac{11}{(x-1)(x+4)} + \dfrac{5}{x-1} = 1$

Answers

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Fluency Practice

Solve

(a) $\dfrac{x-1}{3} = 2$

(b) $\dfrac{2x-3}{5} = 7$

(c) $\dfrac{x+1}{10} = \dfrac{3x}{5}$

(d) $\dfrac{x-8}{6} = \dfrac{3-x}{4}$

Solve

(a) $\dfrac{x}{2} + \dfrac{x}{3} = 5$

(b) $\dfrac{8x}{15} - \dfrac{x}{5} = 6$

(c) $\dfrac{x+3}{8} + \dfrac{x+4}{3} = 4$

(d) $\dfrac{x+1}{3} + \dfrac{x}{4} = 5$

(e) $\dfrac{x+1}{2} + \dfrac{3x-1}{4} = 4$

(f) $\dfrac{x+13}{2} - \dfrac{12-3x}{3} = 1$

Solve

(a) $\dfrac{4}{x+1} + \dfrac{5}{x+2} = 2$

(b) $\dfrac{3}{x+2} + \dfrac{4}{x-3} = 2$

(c) $\dfrac{5}{3x+2} - \dfrac{3}{2x-3} = 4$

(d) $\dfrac{18}{4x-1} - \dfrac{1}{x+1} = 1$

(e) $\dfrac{3}{2x-1} - \dfrac{4}{3x-1} = 1$

Fluency Practice

simplifying and using algebraic fractions

express as a single fraction:

(1) $\dfrac{4}{2x-1} - \dfrac{4}{2x+1}$

(2) $\dfrac{5}{x-1} + \dfrac{5}{x+1}$

(3) $\dfrac{2x+1}{3} + \dfrac{x-1}{4}$

(4) $\dfrac{x}{x+1} - \dfrac{x}{x+7}$

(5) $\dfrac{x}{x-7} - \dfrac{x}{x-1}$

(6) $\dfrac{x+1}{x+2} - \dfrac{x}{x+1}$

solve the equations

(7) $\dfrac{x}{4} + \dfrac{x+5}{3} = 4$

(8) $\dfrac{x+1}{2} + \dfrac{x+2}{5} = 3$

(9) $\dfrac{2x+1}{3} - \dfrac{x-4}{5} = 3$

(10) $\dfrac{5x-2}{7} - \dfrac{3x-4}{4} = \frac{1}{4}$

(11) $\dfrac{3}{2x} + \dfrac{1-x}{x} = 4$

(12) $\dfrac{x-1}{2x} + \dfrac{x+3}{4x} = \frac{5}{8}$

Fluency Practice

solve the algebraic fraction equations (ii)

(1) $\dfrac{n}{12} + \dfrac{1}{4n} = \dfrac{1}{n}$

(2) $\dfrac{n}{20} + \dfrac{1}{5n} = \dfrac{1}{n}$

(3) $\dfrac{2}{n} - \dfrac{n}{5} = \dfrac{n}{20}$

(4) $\dfrac{n}{6} + \dfrac{4}{n} = \dfrac{49}{6n}$

(5) $\dfrac{1}{n+2} + \dfrac{2}{3n} = \dfrac{3}{2n}$

(6) $\dfrac{2}{n-1} - \dfrac{n}{15} = \dfrac{n}{10}$

(7) $\dfrac{1}{n+1} + \dfrac{1}{n} = \dfrac{5}{3n}$

(8) $\dfrac{3}{n+1} - \dfrac{n}{5} = \dfrac{n}{20}$

(9) $\dfrac{2}{n-8} + \dfrac{n}{2} = \dfrac{n}{3}$

(10) $\dfrac{4}{n-3} + \dfrac{n}{10} = \dfrac{n}{2}$

(11) $\dfrac{n+1}{10} + \dfrac{2n}{20} = \dfrac{3}{3n}$

(12) $\dfrac{2}{n} - \dfrac{n}{5} = \dfrac{1}{15}$

Fluency Practice

solve the algebraic fraction equations (iii)

(1) $\dfrac{a}{2} - \dfrac{2}{a+1} = 1$

(2) $\dfrac{4}{b+3} + \dfrac{3(b-6)}{b-2} = 1$

(3) $\dfrac{4}{c+4} + \dfrac{3}{2c+1} = 1$

(4) $\dfrac{3}{d+2} + \dfrac{4}{2d-1} = \dfrac{7}{5}$

(5) $\dfrac{5}{e+2} - \dfrac{3}{e-1} = 8$

(6) $\dfrac{6}{12f+1} + \dfrac{1}{2f+1} = 1$

(7) $\dfrac{6}{g+2} - \dfrac{g-1}{3g-6} = \dfrac{1}{2}$

(8) $\dfrac{3}{h} + \dfrac{4}{h+2} = 1$

(9) $\dfrac{4}{j-1} - \dfrac{2}{j+2} = 24$

(10) $\dfrac{k}{k-1} - \dfrac{k-1}{k} = \dfrac{1}{12}$

(11) $\dfrac{(m+1)^2}{6} + \dfrac{(m+3)^2}{4} = 1$

(12) $\dfrac{24}{n^2+n-6} + \dfrac{6}{n+3} = \dfrac{5}{n-2}$

Fluency Practice

algebraic fractions

solve

express as a single fraction in simplest form

(1) $\dfrac{3x-1}{4} - \dfrac{x+6}{3} = \dfrac{3}{2}$

(6) $\dfrac{7}{x-3} - \dfrac{4}{3x+5}$

(2) $\dfrac{4x-5}{2} + \dfrac{7x-10}{5} = 4$

(7) $\dfrac{7x}{2x+3} + \dfrac{2}{3x-2}$

(3) $\dfrac{8x-5}{3} + \dfrac{4x+5}{4} = \dfrac{149}{12}$

(8) $\dfrac{x}{x-3} - \dfrac{x}{x+6}$

(4) $\dfrac{8x+1}{3} - \dfrac{4x+7}{2} = \dfrac{1}{2}$

(9) prove that

$\dfrac{2x+3}{4} - \dfrac{3x-2}{3} + \dfrac{1}{6} \equiv \dfrac{19-6x}{12}$

(5) $\dfrac{12x-1}{5} - \dfrac{x}{2} = -4$

(10) show that the following identity is true

$\dfrac{3x+2}{5} - \dfrac{5x-2}{4} + \dfrac{7}{10} \equiv \dfrac{32-13x}{20}$

Fluency Practice

(a)	(b)	(c)	(d)
Simplify $\frac{x(x-5)}{x^2}$	Simplify $\frac{x^2+2x}{x^2+7x+10}$	Simplify $\frac{x^2-x-6}{x^2+6x+8}$	Simplify $\frac{3x^2-13x+4}{x^2-16}$

(e)	(f)	(g)	(h)
Simplify $\frac{x^2}{x-2} \times \frac{3x-6}{4x}$	Simplify $\frac{x(x-1)}{x^2+2x} \div \frac{x^2-1}{3x}$	Write as a single fraction $\frac{x}{3}+\frac{2x}{5}$	Write as a single fraction $\frac{5x}{6}-\frac{x}{4}$

(i)	(j)	(k)	(l)
Simplify fully $\frac{5}{x^2}+\frac{4}{3x}$	Simplify fully $\frac{3}{x+2}-\frac{2}{x-1}$	Write as a single fraction $\frac{2x-1}{5}+\frac{x+3}{4}$	Simplify fully $\frac{x}{2x+4}-\frac{3}{x^2+5x+6}$

(m)	(n)	(o)
Solve $\frac{x}{3}+\frac{2x-1}{4}=1$	Solve $\frac{2(x+1)}{5}=\frac{8+x}{6}$	Solve $\frac{5}{x+3}+\frac{3}{x-1}=4$

Fluency Practice

A1 Solve

$$\frac{3x}{5} + 3 = 7$$

A2 Solve

$$\frac{x+4}{3} = 11$$

A3 Solve

$$\frac{5x-11}{3} = x$$

A4 Solve

$$\frac{7x-16}{2} = 2x - 3$$

B1 Solve

$$\frac{x+4}{2} = \frac{x+10}{3}$$

B2 Solve

$$\frac{x-2}{3} = \frac{x+4}{5}$$

B3 Solve

$$\frac{x-10}{10} = \frac{10-x}{3}$$

B4 Solve

$$\frac{2}{5x-2} = \frac{3}{6x+1}$$

C1 Solve

$$\frac{x-1}{2} + \frac{2x+3}{4} = 1$$

C2 Solve

$$\frac{2x-1}{4} + \frac{x-1}{5} = 2$$

C3 Solve

$$\frac{6x-1}{4} - \frac{5-2x}{2} = 1$$

C4 Solve

$$\frac{2x+1}{3} + \frac{x-5}{2} = 4$$

D1 Solve

$$\frac{6}{x-2} - \frac{6}{x+1} = 1$$

D2 Solve

$$\frac{3}{x+2} + \frac{4}{x-3} = 2$$

D3 Solve

$$\frac{3}{x+1} + \frac{2}{2x-3} = 1$$

D4 Solve

$$\frac{4}{2x+1} + \frac{1}{4x^2-1} = 3$$

Fluency Practice

Simplify

(a) $\dfrac{x}{2} \times \dfrac{x}{5}$

(b) $\dfrac{2x}{5} \times \dfrac{10}{3y}$

(c) $\dfrac{2x^2}{7} \times \dfrac{x}{5}$

(d) $\dfrac{5xy}{11} \times \dfrac{33}{2x}$

(e) $\dfrac{x}{3} \div \dfrac{x}{7}$

(f) $\dfrac{2x}{5} \div \dfrac{10y}{3}$

(g) $\dfrac{2x^2}{7} \div \dfrac{x}{5}$

(h) $\dfrac{5x}{11y} \div \dfrac{3}{2x}$

Simplify

(a) $\dfrac{x}{5} + \dfrac{x}{6}$

(b) $\dfrac{3x}{5} - \dfrac{2x}{9}$

(c) $\dfrac{3}{x} + \dfrac{5}{2x}$

(d) $\dfrac{6}{xy} - \dfrac{4}{3x}$

(e) $\dfrac{2}{x} + \dfrac{7}{x^2}$

(f) $\dfrac{x}{2} + \dfrac{x+1}{4}$

(g) $\dfrac{2x}{3} + \dfrac{x+2}{9}$

(h) $\dfrac{x+4}{5} - \dfrac{x}{7}$

Simplify

(a) $\dfrac{x-3}{5} \times \dfrac{15}{2x-6}$

(b) $\dfrac{x}{6} \div \dfrac{2x^2+x}{3}$

(c) $\dfrac{x^2-1}{8} \div \dfrac{x+1}{4}$

(d) $\dfrac{5x+5}{4} \times \dfrac{x^2+2x+1}{2}$

Simplify

(a) $\dfrac{x+2}{2} + \dfrac{x+3}{5}$

(b) $\dfrac{x-3}{3} + \dfrac{x+4}{7}$

(c) $\dfrac{2x+1}{2} - \dfrac{3x+1}{5}$

(d) $\dfrac{7x+4}{2} - \dfrac{2x-1}{7}$

Simplify

(a) $\dfrac{2}{x+1} + \dfrac{3}{x+2}$

(b) $\dfrac{5}{x-3} - \dfrac{3}{x+2}$

(c) $\dfrac{5}{x-3} - \dfrac{3}{2x+1}$

(d) $\dfrac{8}{x-1} + \dfrac{2}{3x-2}$

Intelligent Practice

1) Make r the subject of $\dfrac{6}{p} = \dfrac{7}{q} - \dfrac{6}{r}$

2) Make b the subject of $\dfrac{9}{a} = \dfrac{10}{b} - \dfrac{2}{c}$

3) Make z the subject of $\dfrac{1}{x} = \dfrac{3}{y} - \dfrac{3}{z}$

4) Make b the subject of $\dfrac{1}{a} = \dfrac{10}{b} + \dfrac{6}{c}$

5) Make q the subject of $\dfrac{6}{p} = \dfrac{1}{q} + \dfrac{9}{r}$

1) Make p the subject of $\dfrac{1}{10p} - \dfrac{4}{q} = \dfrac{7}{s}$
   Give your answer in its simplest form.

2) Make x the subject of $\dfrac{1}{9x} - \dfrac{1}{y} = \dfrac{4z}{w}$
   Give your answer in its simplest form.

3) Make a the subject of $\dfrac{4}{2a} - \dfrac{1}{b} = \dfrac{1}{d}$
   Give your answer in its simplest form.

4) Make a the subject of $\dfrac{1}{2a} + \dfrac{5}{b} = \dfrac{9}{d}$
   Give your answer in its simplest form.

5) Make a the subject of $\dfrac{3}{5a} + \dfrac{1}{b} = \dfrac{c}{d}$
   Give your answer in its simplest form.