KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2025 Mathematics (L2FM) 2026

Unit 20 Tasks – Part 1

DO NOT WRITE INSIDE

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Unit 20 Tasks – Part 2

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1 Advanced Simultaneous Equations

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $xy = 12$
$y = x + 1$

2. $xy = 12$
$y = x - 1$

3. $xy = 12$
$y = x - 4$

4. $xy = 12$
$y = x + 4$

5. $xy = 12$
$y = 2x + 5$

6. $xy = 12$
$y = 2x - 5$

7. $xy = 12$
$2y = x + 5$

8. $xy = 12$
$2y = x - 5$

9. $4xy = 12$
$y = x - 2$

10. $4xy = 12$
$y = x + 2$

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $x^2 + y^2 = 25$
$y = x + 5$

2. $x^2 + y^2 = 25$
$y = x - 5$

3. $x^2 + y^2 = 25$
$y = 2x + 5$

4. $x^2 + y^2 = 25$
$y = 2x - 5$

5. $x^2 + y^2 = 17$
$y = x + 5$

6. $x^2 + y^2 = 17$
$y = x - 5$

7. $2x^2 + y^2 = 17$
$y = x + 5$

8. $2x^2 + y^2 = 17$
$y = x - 5$

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $y = x^2 + 5x - 2$
$y = x + 3$

2. $y = x^2 - 3x - 2$
$y = x + 3$

3. $y = x^2 - 5x - 2$
$y = -x + 3$

4. $y = x^2 + 3x - 2$
$y = -x + 3$

5. $y = x^2 - 3x - 2$
$y = 2x - 8$

6. $y = x^2 - 3$
$y = x - 3$

7. $y = x^2 - 3$
$y = x + 3$

Fluency Practice

① Solve:

$$y = x^2 + 3x - 1$$
$$y = 2x + 1$$

② Solve:

$$y = x^2 + 5x - 7$$
$$y = 3x + 8$$

③ Solve:

$$xy = 3$$
$$y = x - 2$$

④ Solve:

$$xy = -8$$
$$y = x + 6$$

⑤ Solve:

$$x^2 + y^2 = 13$$
$$y = x + 5$$

⑥ Solve:

$$x^2 + y^2 = 17$$
$$y = x - 3$$

⑦ Solve:

$$x^2 + y^2 = 45$$
$$y = x - 9$$

⑧ Solve:

$$x^2 + y^2 = 5$$
$$y = 2x + 3$$

Solve these simultaneous equations.

(a) $y = x^2 - 4$

$y = 3x$

(b) $y = x^2 + 5x$

$y = 2x + 10$

(c) $y = 2x^2 + x - 3$

$y = 3x + 1$

Solve these simultaneous equations.

(a) $x^2 + 8y = 13$

$x + 2y = 2$

(b) $y = 2x^2$

$3x + y = 20$

(c) $y = 3x^2 - 4$

$y = 2x - 3$

Solve these simultaneous equations.

(a) $x^2 + y^2 = 25$

$x + y = 7$

(b) $x^2 + y^2 = 9$

$y = x + 3$

(c) $x^2 + y^2 = 5$

$y = 3x + 5$

A netball court has an area of 224 m^2. If the length were decreased by 1 m and the width increased by 1 m, the area would be increased by 1 m^2. Find the dimensions of the court.

(a) Solve algebraically $xy = 4$
$y = x + 3$

(b) Solve algebraically $xy = -8$
$x + y = 2$

(c) Solve algebraically $xy + 12 = 0$
$x = 2 - 2y$

(a) Solve algebraically $y = \frac{3}{x}$
$2x = y - 1$

(b) Solve algebraically $y = \frac{4}{x} - 2$
$x = 4y + 8$

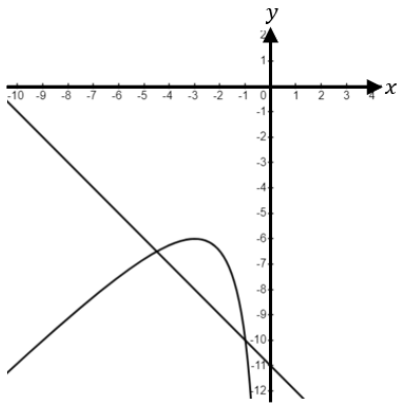
(c) Solve algebraically $y = \frac{4}{x} + x$
$x + y = 6$

(a) Solve algebraically $x^2 + xy = 20$
$x = 4y$

(b) Solve $x^2 + y^2 - xy = 16$
$x + y = 4$

(c) Solve $x^2 - y^2 = 4 + xy$
$y = 2x - 6$

The diagram shows the graphs of $y = x + \frac{9}{x}$ and $x + y + 11 = 0$. Find the coordinates of the points of intersection.



Fluency Practice

Solve the following simultaneous equations.

1. $\begin{matrix} x+y=3 \\ x^2+y^2=5 \end{matrix}$.
2. $\begin{matrix} x-4y=2 \\ 3=5x-2y \end{matrix}$.
3. $\begin{matrix} 2x+y=1 \\ x^2+y^2=2 \end{matrix}$.
4. $\begin{matrix} x+2y=1 \\ x^2+2y^2=3 \end{matrix}$.
5. $\begin{matrix} x-3y=2 \\ 4x-y=1 \end{matrix}$.
6. $\begin{matrix} x-3y=2 \\ y^2-x^2=-4 \end{matrix}$.
7. $\begin{matrix} 2x+y=3 \\ 4x^2+y^2=5 \end{matrix}$.
8. $\begin{matrix} 2x-3y=2 \\ 3x+y=1 \end{matrix}$.
9. $\begin{matrix} x+2y=3 \\ 3x^2+4y=6 \end{matrix}$.
10. $\begin{matrix} 2x-y=4 \\ x^2+2y^2=17 \end{matrix}$.
11. $\begin{matrix} x+3y=7 \\ x^2+x+y=21 \end{matrix}$.

Fluency Practice

Solving Non-Linear Simultaneous Equations			
(a)	**(b)**	**(c)**	**(d)**
Solve $y = x^2 + 2$ $y = 3x$	Solve $y = x^2$ $y = 10 - 3x$	Solve $y = 5 + x^2$ $y = x + 11$	Solve $y = x^2 + 9x - 6$ $y = 5x + 6$
(e)	**(f)**	**(g)**	**(h)**
Solve $y = x^2 - 4$ $x + y = 8$	Solve $y = 6x - x^2$ $y + 5x = 24$	Solve $y = 2x^2 - 1$ $x + y = 5$	Solve $y = x - 3x^2$ $x - y = 3$

Fluency Practice

Q	Solve the simultaneous equations:	Answers
1	$7x + 2y = 29$ and $y = 2x - 2$	
2	$y = x^2 - x - 2$ and $y = 2x - 2$	
3	$x^2 + y^2 = 25$ and $y = 2x - 2$	
4	$7x - 2y = -20$ and $y = x + 5$	
5	$y = x^2 + x + 1$ and $y = x + 5$	
6	$x^2 + y^2 = 13$ and $y = x + 5$	
7	$3x + 4y = 11$ and $y = 3x + 14$	
8	$y = 2x^2 + 3x - 4$ and $y = 3x + 14$	
9	$x^2 + y^2 = 34$ and $y = 3x + 14$	
10	$3x - 2y = -5$ and $y = \frac{1}{2}x - \frac{1}{2}$	
11	$y = -2x^2 - 5x + 1$ and$y = \frac{1}{2}x - \frac{1}{2}$	
12	$(x - 1)^2 + (y + 3)^2 = 17$ and $y = \frac{1}{2}x - \frac{1}{2}$	
13	$ax + by = c$ and $y = dx + e$	
14	$y = ax^2 + bx + c$ and $y = dx + e$	
15	$x^2 + y^2 = c$ and $y = dx + e$	

Fluency Practice

A1 Solve $y = x^2$ $y = 11x - 10$	**A2** Solve $y = x^2$ $y + 3x = 4$	**A3** Solve $y = 3x^2$ $y = 5x + 8$	**A4** Solve $y + 7x = 15$ $y = 2x^2$
B1 Solve $x^2 + y^2 = 29$ $y = 3x + 1$	**B2** Solve $x^2 + y^2 = 41$ $y = 2x - 3$	**B3** Solve $x^2 + y^2 = 17$ $y + 2x = 7$	**B4** Solve $3x = 4 - y$ $x^2 + y^2 = 8$
C1 Solve $y = (x+1)^2$ $y = 5x - 1$	**C2** Solve $y = x^2 + 7x + 3$ $y = 9x + 2$	**C3** Solve $x + y = 11$ $xy = 30$	**C4** Solve $xy = 6$ $2x + y = 7$
D1 Solve $3x - y = 4$ $x^2 + y^2 = 34$	**D2** Solve $x^2 - y^2 = 8$ $3x = y + 8$	**D3** Solve $2x^2 - xy = 12$ $x = 4 + y$	**D4** Solve $3y^2 + 2x^2 = 21$ $y - 2x = -7$

Question 1: Solve the following simultaneous equations

(a) $y = x + 3$
$y = x^2 + 5x - 2$

(b) $y = x^2 + x - 14$
$y = x - 5$

(c) $y = 2x - 1$
$y = x^2 - 2x + 2$

(d) $y = 2x^2 + 9x + 1$
$y = 3x + 9$

(e) $y = 2x^2 + x + 1$
$y = x^2 - 5x - 7$

(f) $y = -x^2 + 5x + 2$
$y = 3x^2 - x - 2$

Question 2: Solve the following simultaneous equations

(a) $x + y = 4$
$y = x^2 + 3x - 1$

(b) $x + y = 7$
$xy = 10$

(c) $x^2 + y^2 = 13$
$x + y = 5$

(d) $2x - y + 4 = 0$
$y = x^2 + x - 2$

(e) $x^2 + y^2 = 29$
$7 + x + y = 0$

(f) $xy = -6$
$x + 2y = -4$

(g) $y = x^2 - 3x + 3$
$y = 10x - 39$

(h) $x^2 + y^2 = 1$
$x + 2y = 1$

(i) $5x + y = 5$
$2x^2 - 9x - y = 11$

(j) $y = 9x^2 + 11x + 3$
$5x - y + 2 = 0$

(k) $2x + y = 5$
$2x^2 + y^2 = 11$

(l) $x - 4y + 1 = 0$
$x^2 - 4xy + y^2 = 13$

Question 3: Solve the following simultaneous equations

(a) $2x + y = 7$
$x^2 - y^2 = 8$

(b) $x^2 + y^2 = 20$
$y = x + 3$

(c) $y = x^2 - 9x - 3$
$y = x$

(d) $2x^2 + y^2 = 10$
$2x - y = 5$

(e) $y = x^2 + x - 7$
$4x + 2y + 1 = 0$

(f) $y = x - 2$
$2x^2 - xy = 11$

Apply

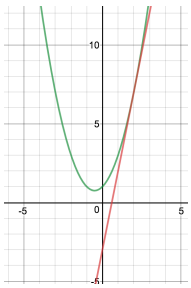
Question 1: Find the coordinates where the line $x + y = 3$ and the curve $x^2 + 3y = 27$ intersect

Question 2: How many points of intersection does the curve $y = (x - 3)(x + 4)$ have with the line $y = x - 8$?

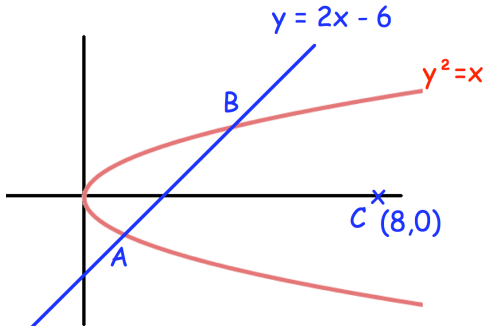
Question 3: The line $y = x + 4$ and the curve $y = x^2 + 3x + 4$ intersect at the points A and B. Find the distance between the points A and B.

Question 4: Find the coordinates of the points where the line $x + 5y = 37$ and the curve $y = x^2 + x + 2$ meet.

Question 5: Show, by using simultaneous equations, that the line $y = 5x - 3$ is a tangent to the curve $y = x^2 + x + 1$

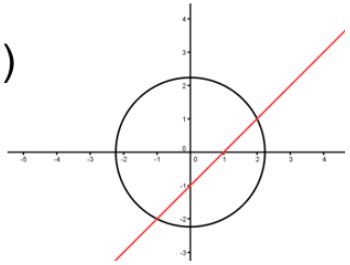


Question 6: Shown below is the curve $y^2 = x$ and the line $y = 2x - 6$
The curve and the line meet at the points A and B
The point C is (8, 0)
Show ABC is a right angled triangle.



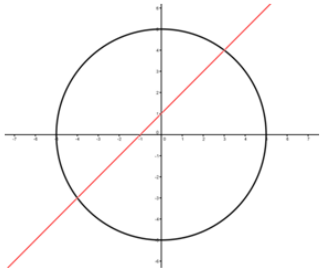
Purposeful Practice

solving simultaneous equations: one linear, one quadratic (i)
find the coordinates of the points of intersection

(1) 

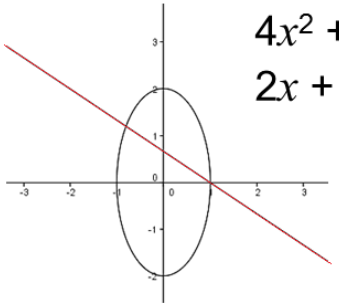
$x^2 + y^2 = 5$
$x - y = 1$

(2) $x^2 + y^2 = 58$
$y - x = 4$

(3) 

$x^2 + y^2 = 25$
$-x + y = 1$

(4) $x^2 + y^2 = 2$
$x + 3y = 2$

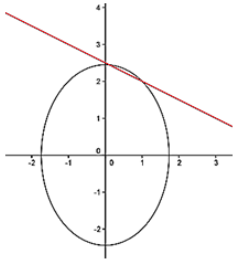
(5) 

$4x^2 + y^2 = 4$
$2x + 3y = 2$

(6) $2x^2 + xy + 1 = 0$
$y - x = 4$

(7) $4x^2 + y^2 = 37$
$2y + x = 1$

(8) $2x^2 + y^2 = 6$
$x + 2y = 5$



factorise any quadratic equations …

Purposeful Practice

solving simultaneous equations: one linear, one quadratic (ii)

find the coordinates of the points of intersection

(1) $y = 3x^2 - 14x - 5$
$y = 4x - 32$

(2) $x^2 + 8y = 13$
$2y + x = 2$

(3) $4x^2 + y^2 = 10$
$2x - y = 4$

(4) $2x^2 + y^2 = 57$
$2y + x = 6$

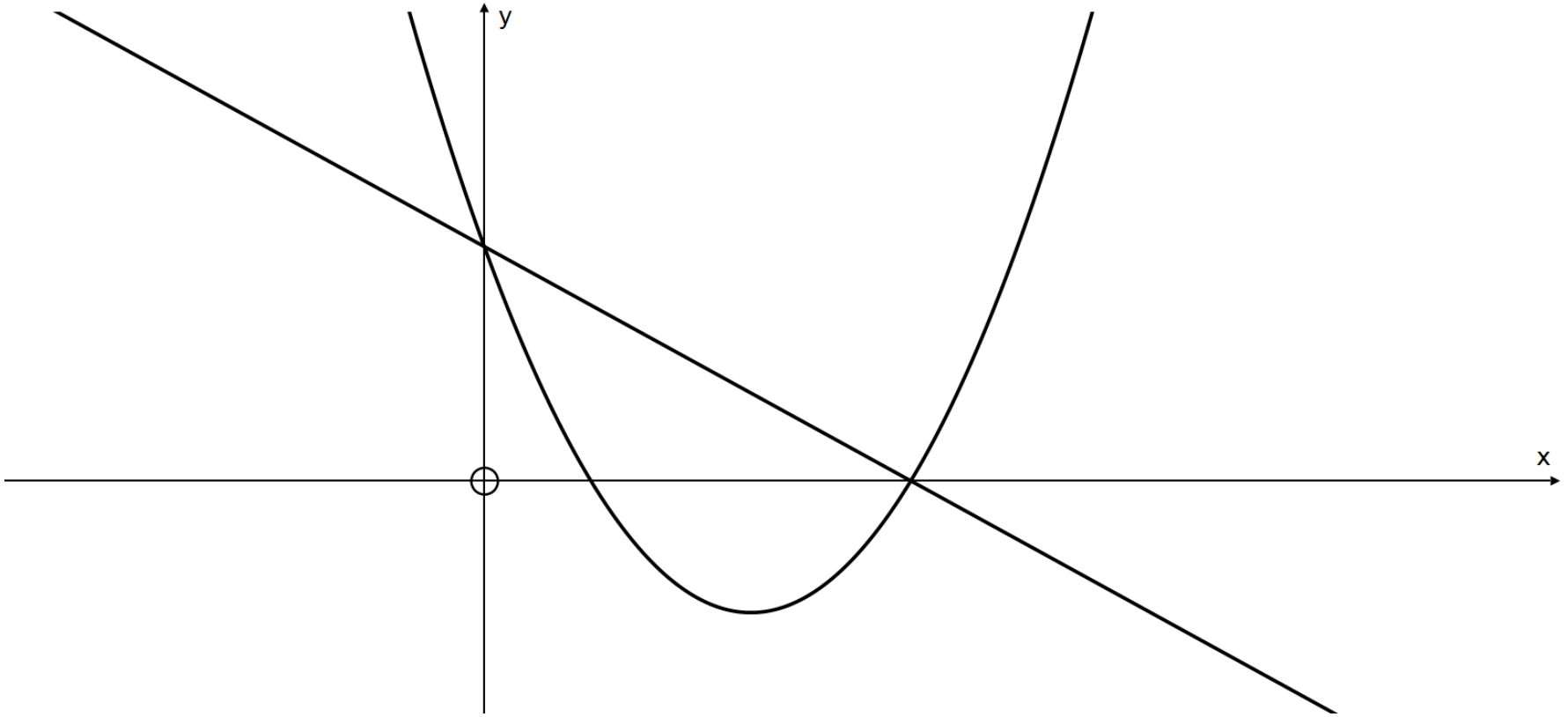
(5) $2y^2 + x^2 = 48$
$y + x = 8$

(6) $x^2 + y^2 + 2x - 4y = 8$
$x + y = 6$

(7) $x^2 + y^2 + 6x - 4y = 4$
$y = 3x + 4$

(8) $2x^2 - y^2 + xy = 14$
$4x + 5y = 0$

Problem Solving



Suggest possible equations for the line and the parabola.

Justify your answer.

Generalise your answer.

Fluency Practice

Solving Linear & Non-Linear Simultaneous Equations Graphically

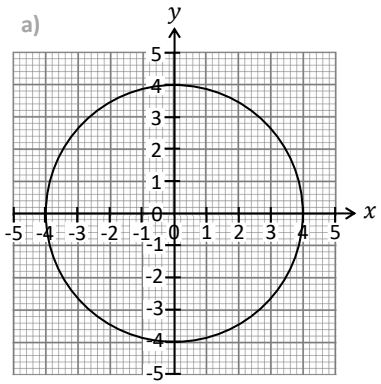
Estimate solutions to each pair of simultaneous equations.

Substitute the linear equation into the non-linear equation to express a quadratic equation in the general form: $ax^2 + bx + c = 0$

a)

$x^2 + y^2 = 16$

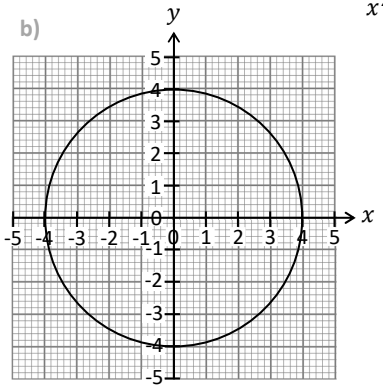
$y = 2x$



b)

$x^2 + y^2 = 16$

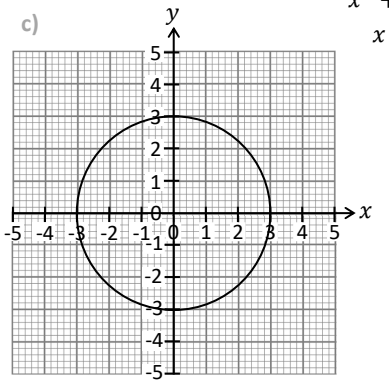
$y = x + 1$



c)

$x^2 + y^2 = 9$

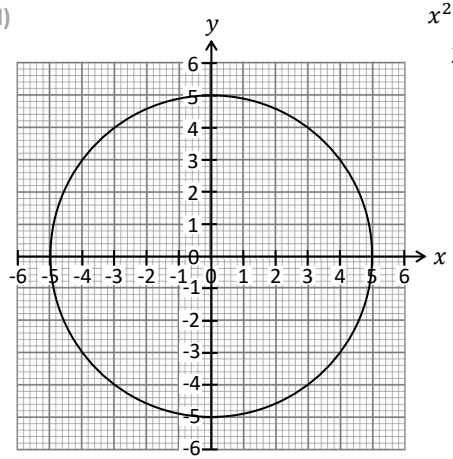
$x + y = 1$



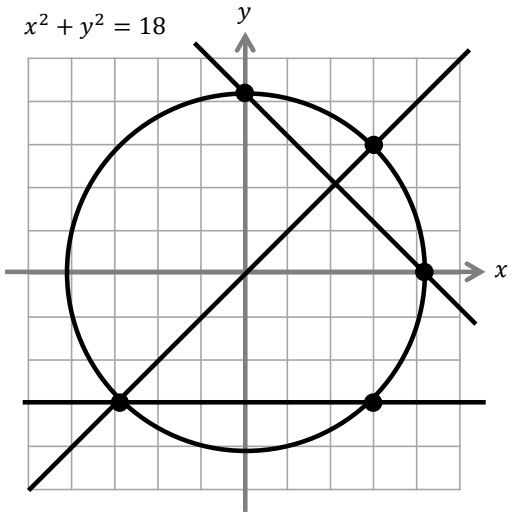
d)

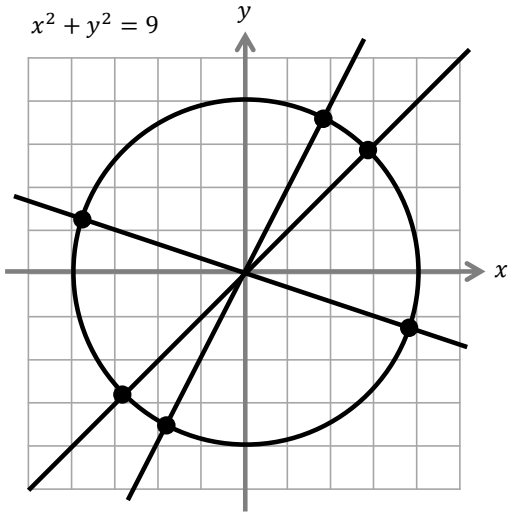
$x^2 + y^2 = 25$

$y = 3x - 2$



Fluency Practice

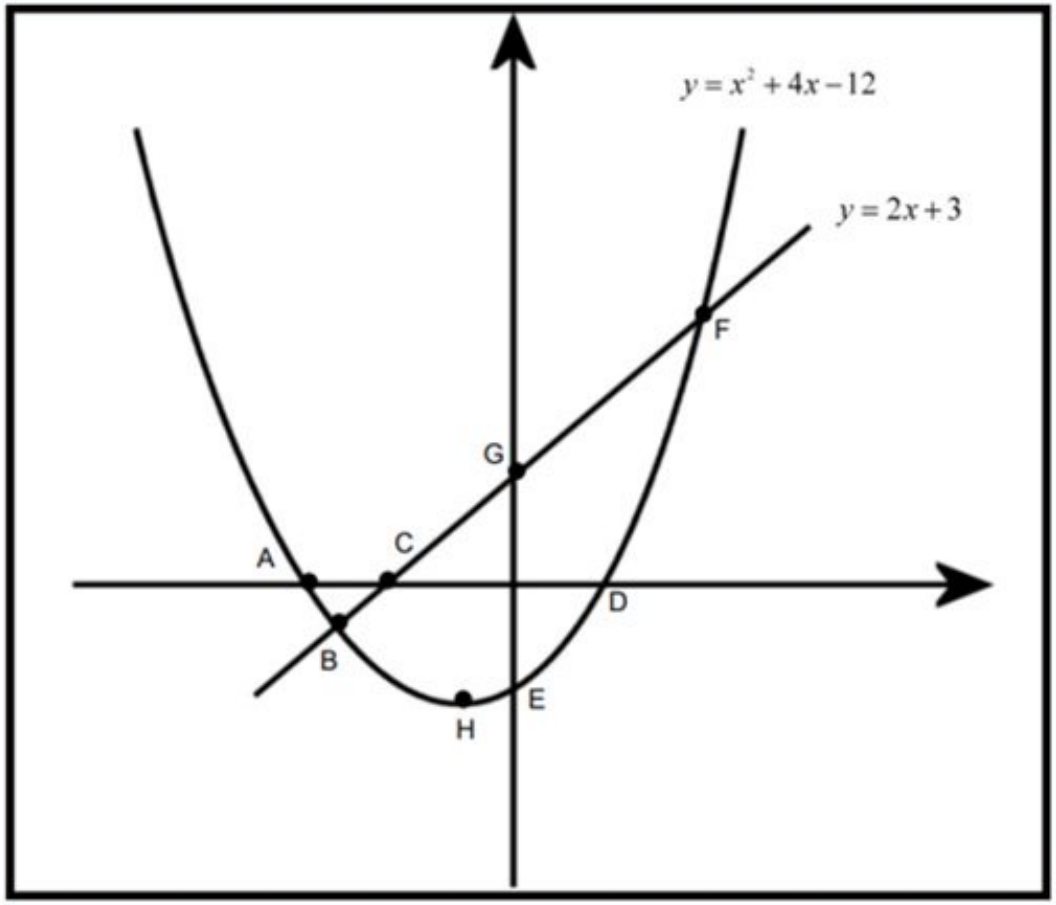




Each grid shows **3 simultaneous equations** involving **1 linear graph & 1 non-linear graph**.
Calculate **exact solutions** for each. Can you substitute & simplify the equations?

Problem Solving

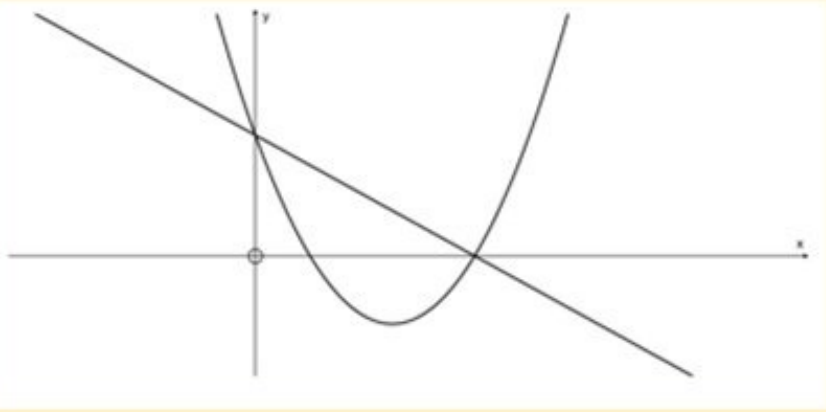
Find the coordinates of the points A, B, C, D, E, F, G and H



TASK

CHALLENGE

When you finish the task on the left, look at the diagram below. Suggest possible equations for the line and parabola. Justify your answer. Generalise your answer.



Purposeful Practice

①

Max wants to spend £100 on plants.
But each plant costs £5 more than expected, so
Max has to buy 10 less than planned.

How many plants did Max plan to buy?

②

Changing Prices

Hae wants to spend £90 on plant pots.
But each pot costs £3 more than expected, so
Hae has to buy 5 less than planned.

How many pots did Hae plan to buy?

③

Jia wants to spend £160 on calculators.
But each calculator costs £2 more than expected, so
Jia can afford 4 less than they planned.

How many calculators can Jia now buy?

④

A rectangle has an area of 84 m^2.
The length is increased by 1 metre.
The width is increased by 1 metre.
The new area is 104 m^2.

What was the width of the original rectangle?

⑤

Changing Values

A rectangle has an area of 60 m^2.
The length is decreased by 2 metres.
The width is increased by 3 metres.
The new area is 72 m^2.

What was the length of the original rectangle?

⑥

With £360 a shop can buy q t-shirts.
With a price decrease of £8, the shop can
now buy 12 more t-shirts.

How many t-shirts can the shop now buy?

Purposeful Practice

Speed Change

Tim runs 300 metres in t seconds.
By increasing their speed (s) by 1 m/s,
Tim runs the 300 metres 10 seconds faster.

What was Tim's original 300 m time?

$$s = \frac{d}{t} \qquad st = d$$

Original: $st = 300$

New: $(s + 1)(t - 10) = 300$

Jan runs 200 metres in t seconds.
By increasing their speed by 3 m/s,
Jan runs the 200 metres 15 seconds faster.

What was Jan's original 200 m time?

A cyclist travels 1 km in t seconds.
Their speed decreases by 5 m/s and they
travel the next kilometre 10 seconds slower.

How long did it take the cyclist to cover the 2 km?

Fluency Practice

Harder Simultaneous Equations			
(a)	**(b)**	**(c)**	**(d)**
Solve $2x + 3y - 18 = 0$ $x = y + 4$	Solve $3x + 4y = 8$ $6 - x = 2y$	Solve $y = x^2 - 2x + 6$ $y = x + 4$	Solve $x^2 + y^2 = 50$ $y = x - 8$
(e)	**(f)**	**(g)**	**(h)**
Solve $x^2 - 5x = y - 5$ $2x + y = 5$	Solve $x^2 + 2y^2 = 22$ $3x = 2y$	Solve $x^2 + y^2 + xy = 12$ $x = 6 - 2y$	Solve $y = x^2 + 3x - 5$ $x - y = 4$
(i)	**(j)**	**(k)**	**(l)**
Find the coordinates of the points where the curve $y = 2x^2 - 3x - 4$ intersects with the line $y = 2x - 1$	Solve $xy = 16$ $x + y = 10$	Solve $x + 2y = 5$ $(x - 1)^2 + (y - 2)^2 = 20$	Find the length of the line joining the points of intersection of $y = \frac{x}{2} + 1$ and $x^2 + y^2 = xy + 4$

Fluency Practice

More Quadratic Equations and Inequalities Revision

(a)	(b)	(c)	(d)
Solve $x^2 + x - 20 = 0$	Solve, by completing the square, $x^2 + 6x + 3 = 0$ giving answers in surd form	Solve $(x + 3)(x - 5) > 0$	Solve $3x^2 - 11x - 7$ giving your answers to 3 significant figures

(e)	(f)	(g)	(h)
Solve $x^2 \leq 25$	Solve $5x^2 + 18x = 8$	Solve $y = x + 1$ $y = x^2 + 5x - 11$	Solve $2x^2 - 13x + 10 < 0$

(i)	(j)	(k)
The area of a triangle with base $(x + 9)$ cm and height $(2x + 1)$ cm is $21\ cm^2$. Find the value of x.	A rectangle has a length of $(x - 6)$ cm and a width of $4x$ cm. The area of the rectangle is less than $13\ cm^2$. Find the range of possible values of x.	Solve $x + 2y = 7$ $x^2 + y^2 = 10$

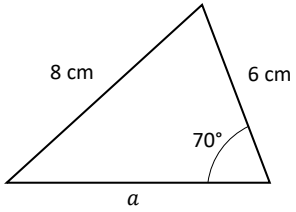
2 Advanced Trigonometry

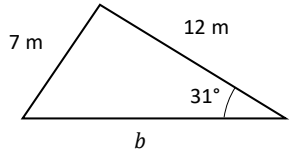
Fluency Practice

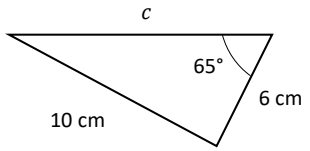
Sine Rule ×2

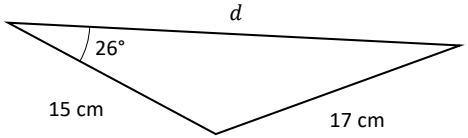
To find each missing length in these triangles, we will need to use the sin rule twice.
How can we maintain accuracy?

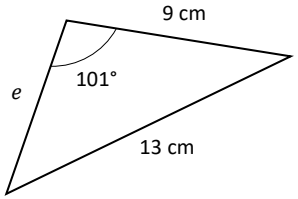
NOT TO SCALE

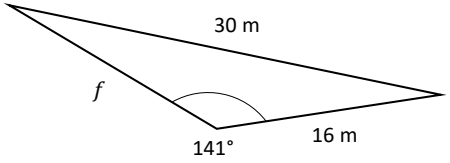


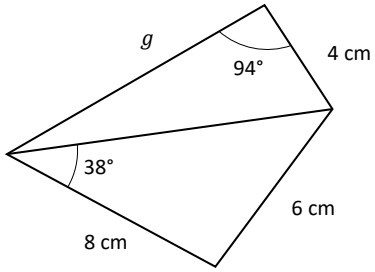


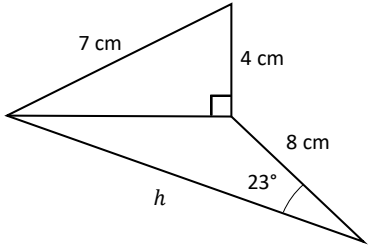


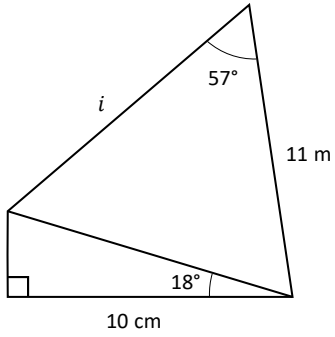






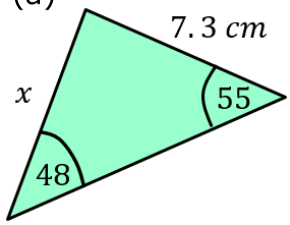




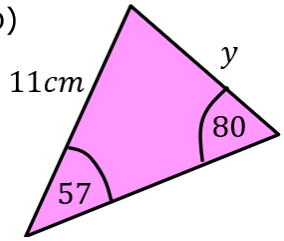


Find the missing length.

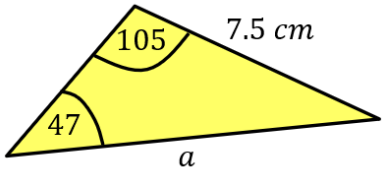
(a)



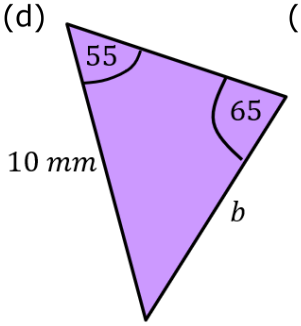
(b)



(c)



(d)



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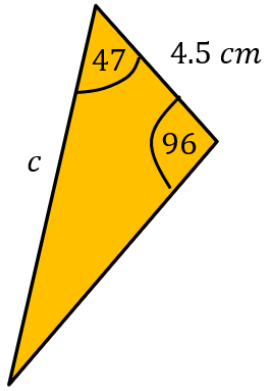
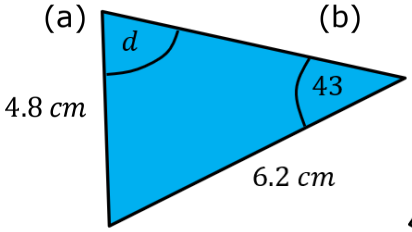


Find the missing angle.

(a)



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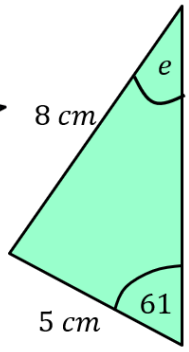

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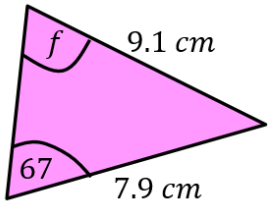

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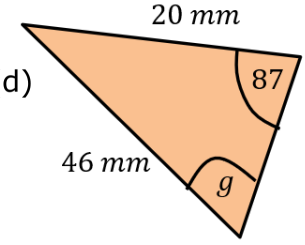

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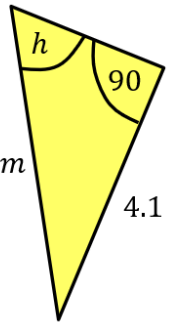
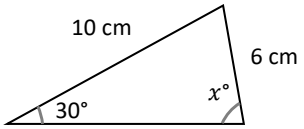


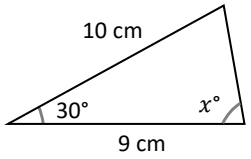
Purposeful Practice

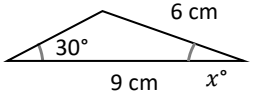
Ambiguous or **Not?**

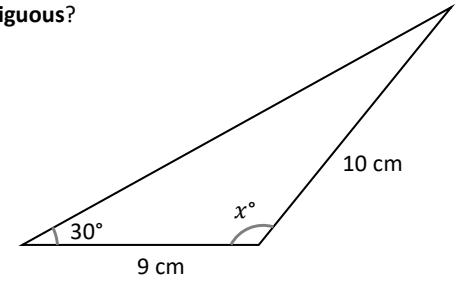
The measurements for these different scalene triangles are correct, but they are **not to scale** and their **shapes are not drawn accurately**!

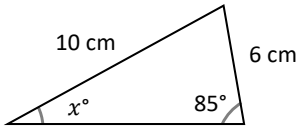
Which triangles can have two values for x… which are **ambiguous**?

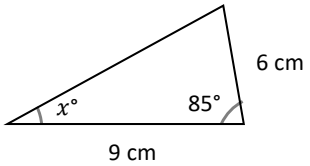


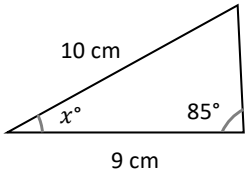


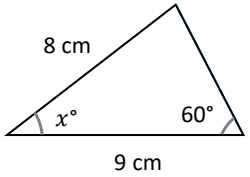


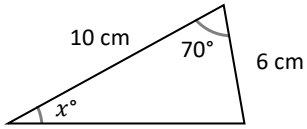


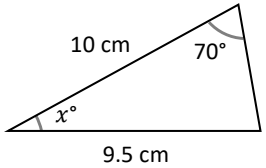


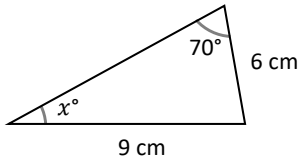


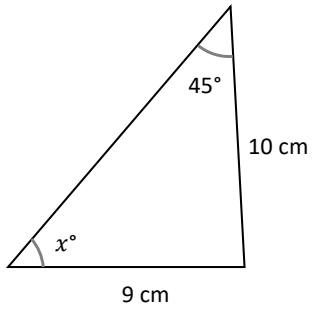








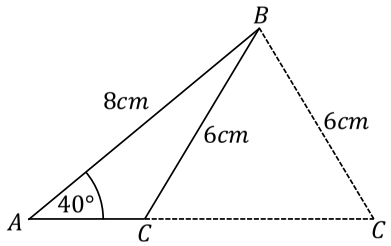




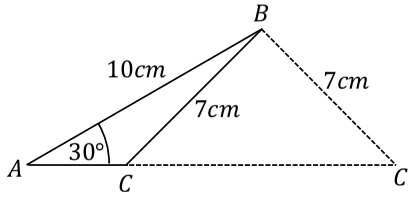
Can you draw an ambiguous triangle with a known obtuse angle?

What are the **conditions** for ambiguity? Sketch 3 triangles where x is ambiguous.

1. In triangle ABC, $AB = 8cm$, $BC = 6cm$ and angle $CAB = 40°$ (see diagram)
   Find the two possible sizes of angle BCA.



2. In triangle ABC, $AB = 10cm$, $BC = 7cm$ and angle $CAB = 30°$ (see diagram)
   Find the two possible sizes of angle BCA.



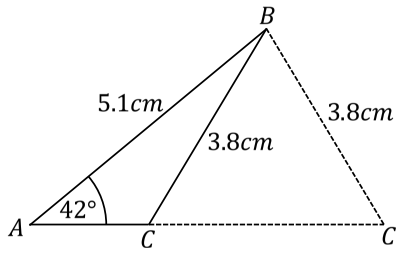
3. In triangle ABC, $AB = 12cm$, $BC = 9cm$ and angle $CAB = 47°$
   Find the two possible sizes of angle BCA.

4. In triangle ABC, $AB = 13cm$, $BC = 11cm$ and angle $CAB = 53°$

   Find the two possible sizes of angle BCA.

5. In triangle ABC, $AB = 5.1cm$, $BC = 3.8cm$ and angle $CAB = 42°$ (see diagram)

   (i) Find the two possible sizes of angle BCA.



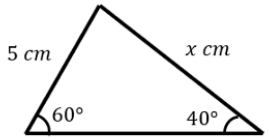
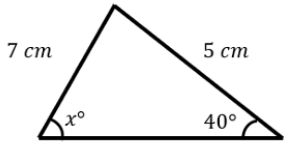
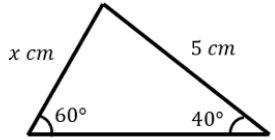
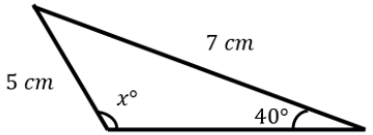
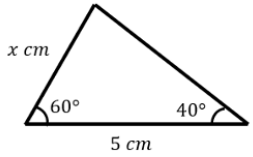
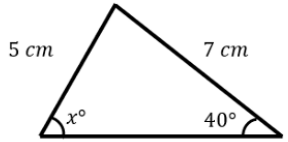
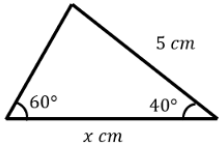
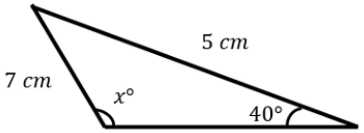
   (ii) Find the two possible lengths of AC.

6. In triangle ABC, $AB = 17.4cm$, $BC = 10.7cm$ and angle $CAB = 23°$

   (i) Find the two possible sizes of angle BCA.

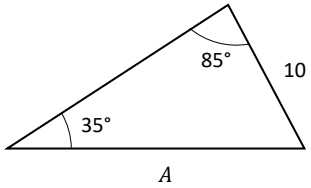
   (ii) Find the two possible lengths of AC.

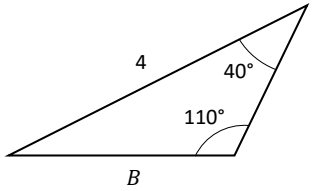
Fluency Practice

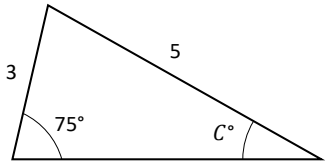
Q	Work out the missing length	Answers	Q	Work out the missing angle	Answers
1a			1b	x acute 	
2a			2b	x obtuse 	
3a			3b	x acute 	
4a			4b	x obtuse 	

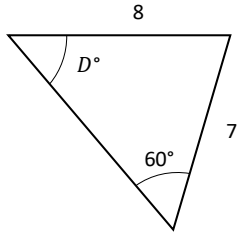
Fluency Practice

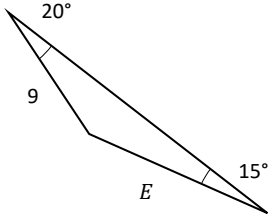
Sine Rule Lengths are centimetres. not to scale

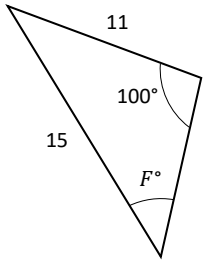


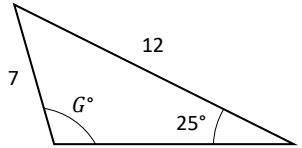


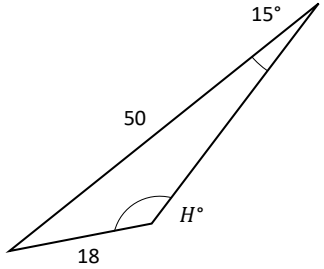


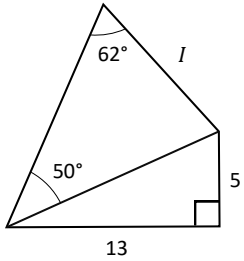


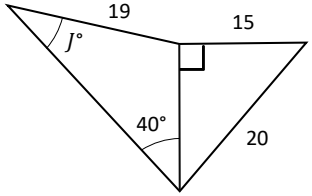


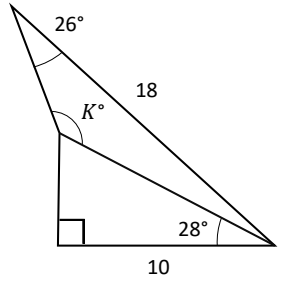


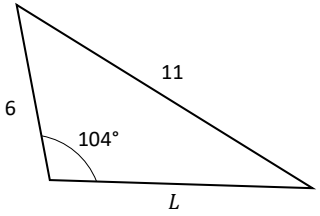








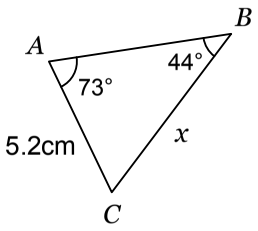




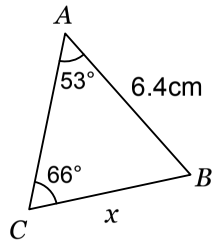
10 Answers	46.2	49.3	17.4	133.6	35.4
	2.7	12.1	26.6	11.9	134.0

1. Work out the lengths of the sides marked x. Round your answers to the nearest 0.1cm.

a)



b)



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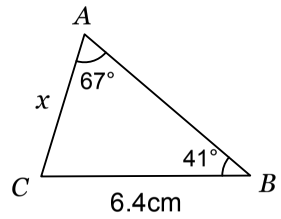

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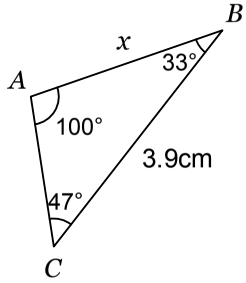

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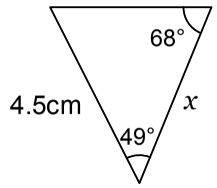
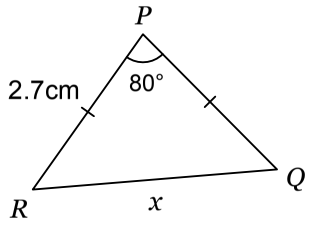
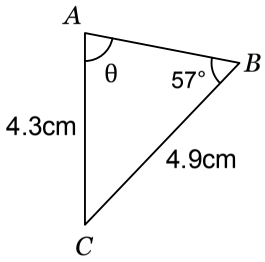


f)



2. Work out the acute angles marked θ. Round your answers to the nearest degree.

a)



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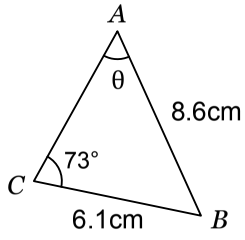

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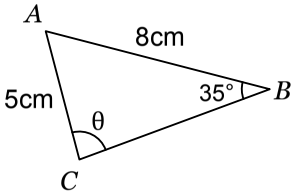

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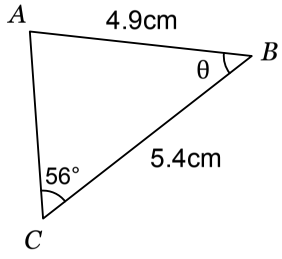

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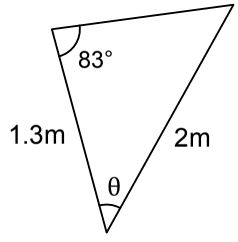

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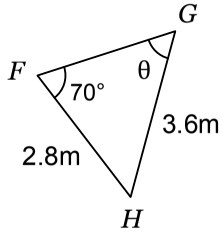
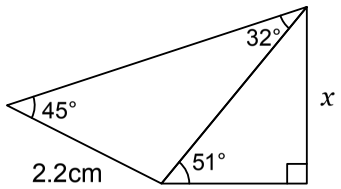
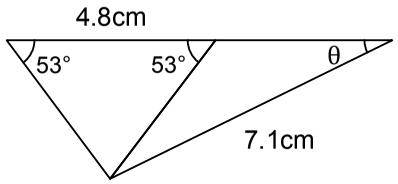


3. Triangle ABC is such that AB = 4.5cm, $A\hat{B}C$ = 37° and $B\hat{C}A$ = 78°.
Work out the length AC, correct to the nearest millimetre.

4. Work out the length marked x.
Give your answer correct to the nearest 0.1cm.



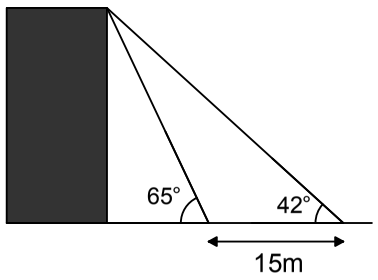
5. Work out the acute angle marked θ.
Give your answer correct to the nearest degree.



6. Hardeep is trying to work out the height of a building.

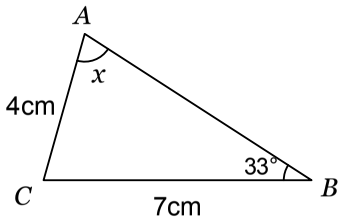
He measures the angle of elevation to the top of the building to be 42°. He walks 15m towards the building and the angle of elevation increases to 65°.

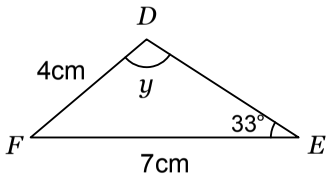
Work out the height of the building correct to the nearest metre.



7. Triangles ABC and DEF each have side lengths of 4cm and 7cm, and an angle of 33°, but they are not congruent. Angle x is **acute** and angle y is **obtuse**.

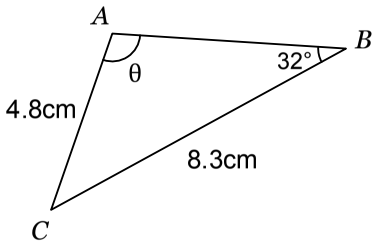
Work out the values of x and y, correct to the nearest degree.





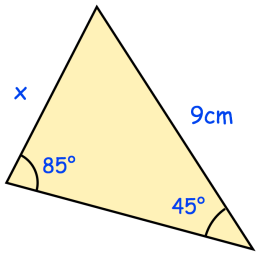
8. a) Use the sine rule to work out the value of $\sin\theta$.

b) Given that θ is obtuse, work out the value of θ correct to the nearest degree.

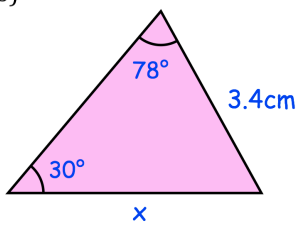


Question 1: Find x for each of the triangles below.

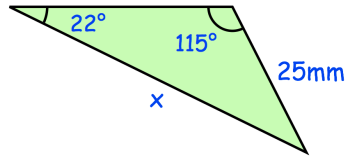
(a)



(b)

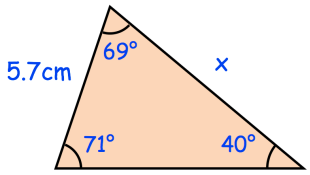


(c)



Question 2: Find x for each of the triangles below.

(a)



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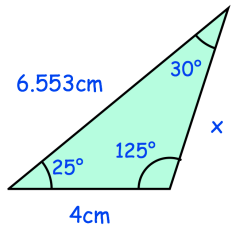

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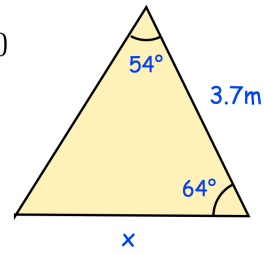
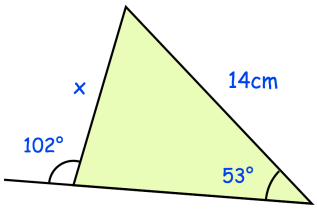
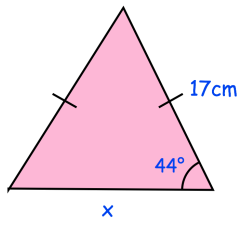


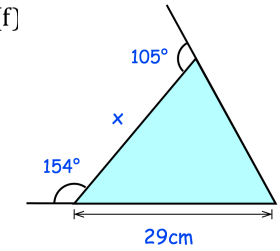
(d)



(e)

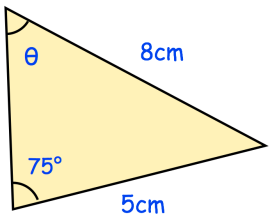


(f)

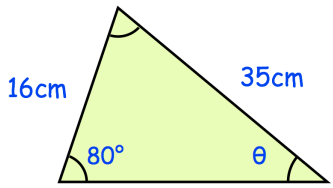


Question 3: Find the size of θ for each of these triangles.

(a)



(b)



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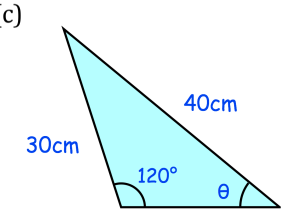
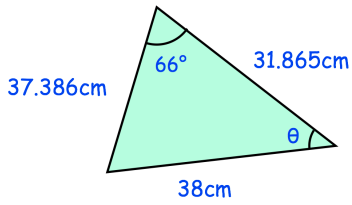


Question 4: Find the size of θ for each of these triangles.

(a)



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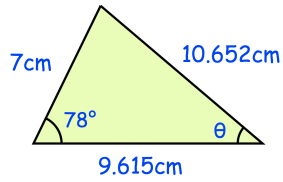

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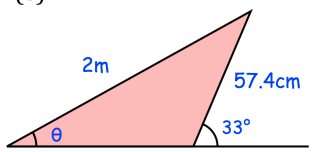
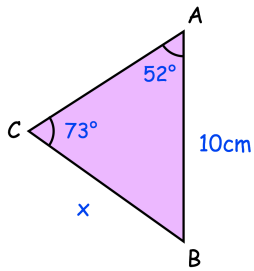


Question 5: Find the missing angle θ or side, x, for each of the triangles below.

(a)



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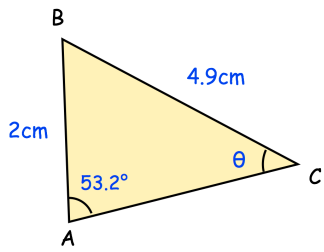

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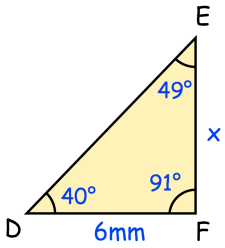
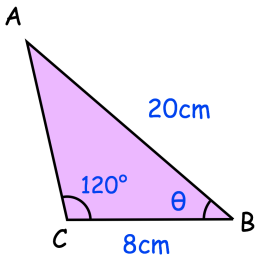
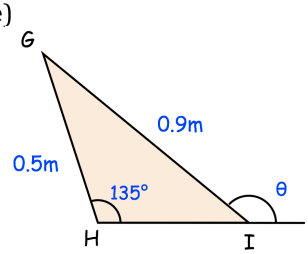


(d)



(e)



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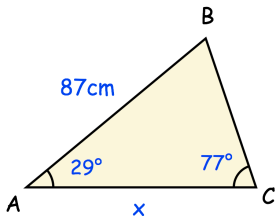
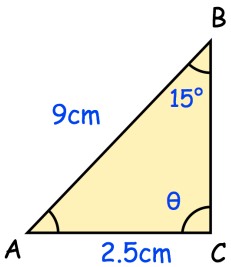
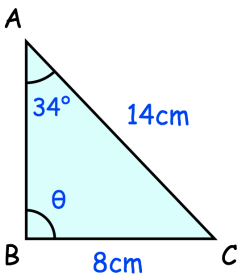


Question 6: The triangles below have two possible values for the angle θ.
Find the size of these possible angles.

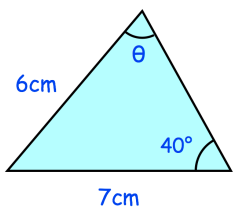
(a)



(b)



(c)



Apply

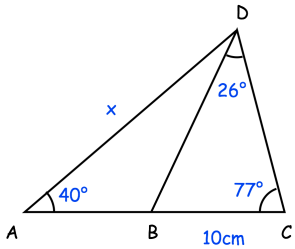
Question 1: ACD is a triangle and B is a point on AC.
BC = 10cm
Angle ACD = 77° Angle BDC = 26°

Angle DAC = 40°

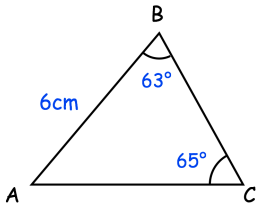
(a) Find the length of BD

(b) Find the size of angle ABD

(c) Find the length of AD

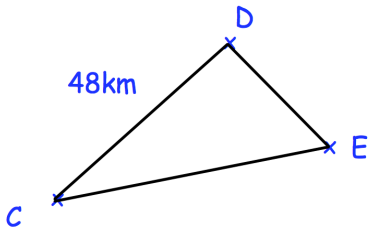


Question 2: Calculate the perimeter of triangle ABC



Question 3: Donhampton is 48km from Castletown on a bearing of 050°.
Eastville is on a bearing of 075° from Castletown and on a bearing of 150° from Donhampton.

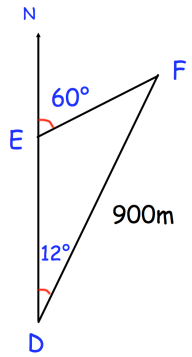
Calculate the distance of Eastville from Castletown.



Question 4: The diagram shows 3 helicopters D, E and F.
Helicopter D is due South of helicopter E.
The bearing of helicopter F from helicopter E is 060°
The bearing of helicopter F from helicopter D is 012°
The distance of helicopter F from helicopter D is 900m

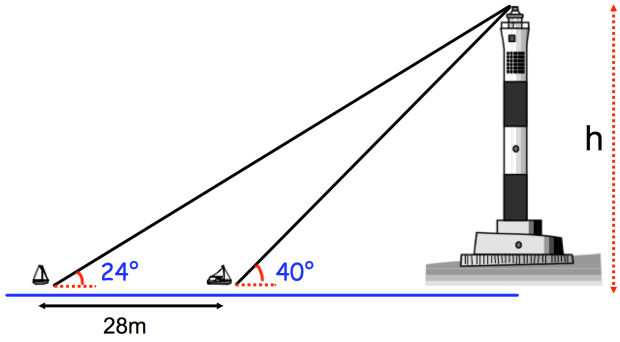
(a) Find the distance of helicopter F from helicopter E

(b) Find the distance of helicopter D from helicopter E.



Question 5: Two small boats are 28m apart.
The angle of elevation of the boats to the top of the lighthouses are 24° and 40° respectively.

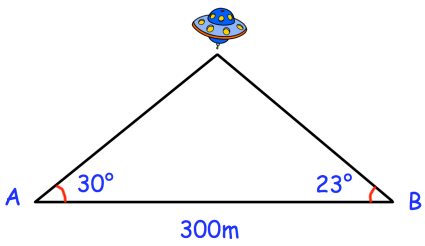
Calculate the height of the lighthouse.



Question 6: There are two possible triangles DEF, where DE = 6.2cm, EF = 7.9cm and ∠DFE = 30°

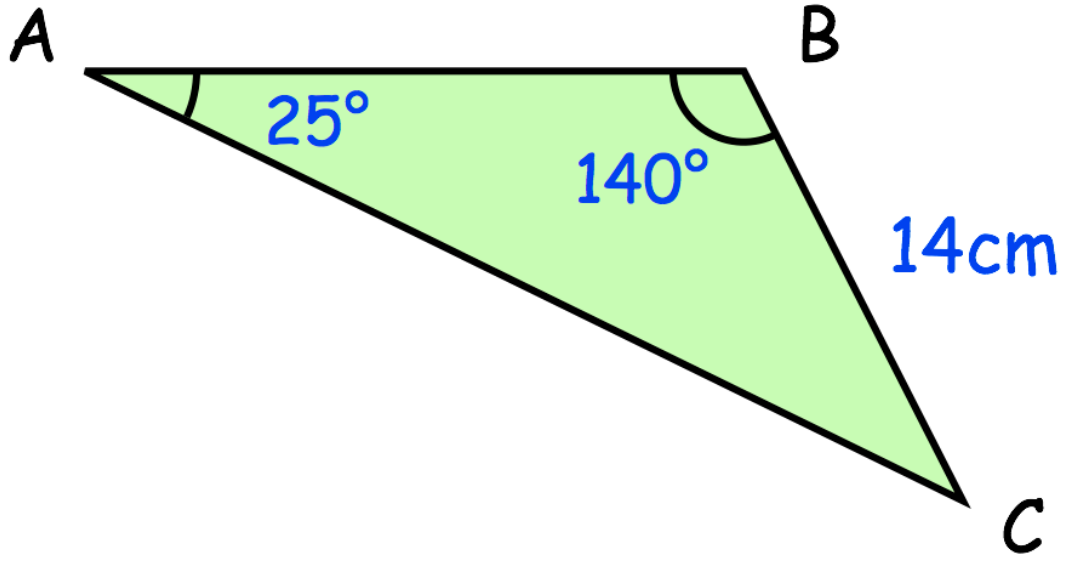
Sketch both possible triangles with all sides and angles found.

Question 7: A UFO is flying above two people, standing on the ground at points A and B.
A and B are 300m apart.
The angle of elevation of the UFO from A is 30°
The angle of elevation of the UFO from B is 23°
Find the height of the UFO above the ground.



Question 8: Two ships, A and B, leave a port at midday.
A travels on a bearing of 085° at a speed of 18km/h.
B travels on a bearing of 113° at a speed of y km/h.
At 14:00 the distance between A and B is 30km.
Boat B was travelling at a slower speed than boat A
Work out y, the speed of boat B.

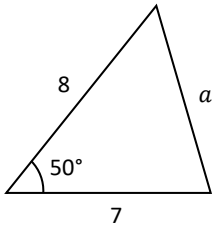
Question 9: Shown below is triangle ABC.
All values are correct to 2 significant figures.
Calculate the upper bound for the length of AC.

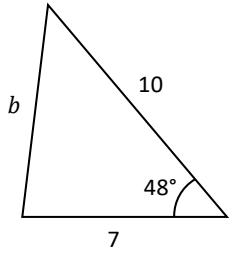


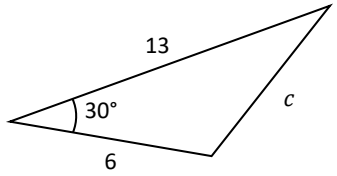
Question 10: Prove the Sine Rule.

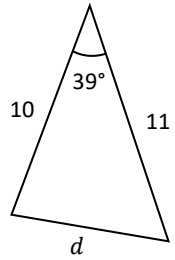
Fluency Practice

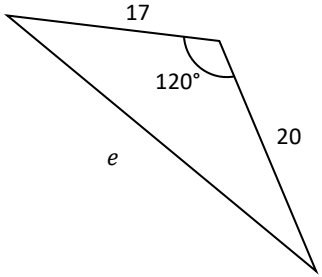
Cosine Rule: Lengths Lengths are in centimetres. not to scale

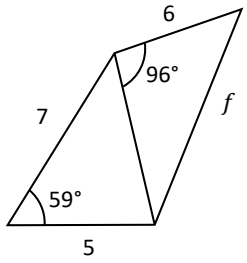


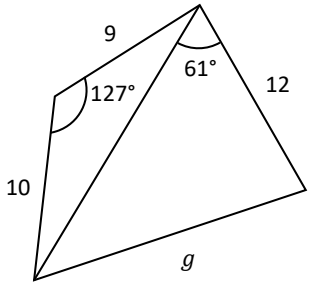


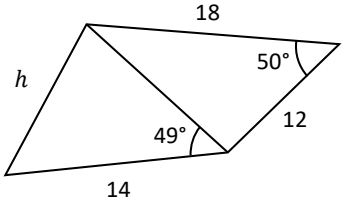


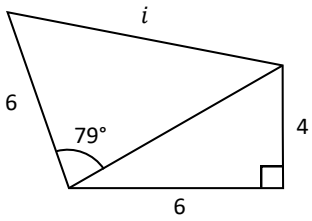


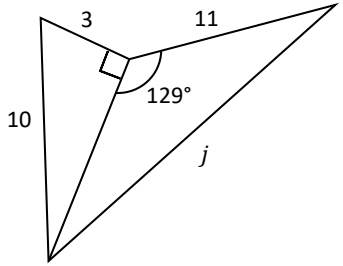


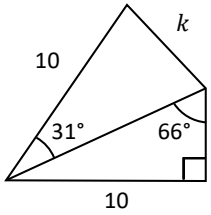


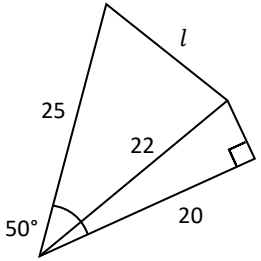








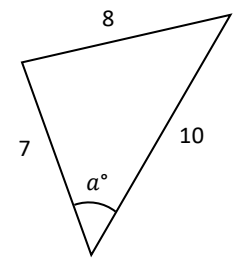


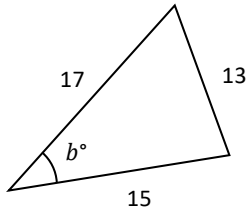


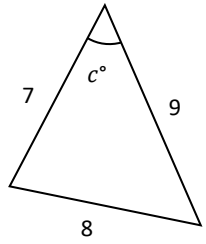
10 Answers	7.4	8.5	6.4	32.1	11.5
	15.3	7.1	9.0	18.5	8.4

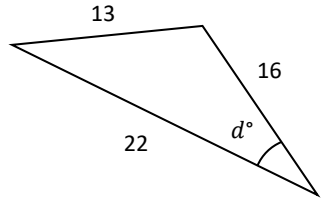
Fluency Practice

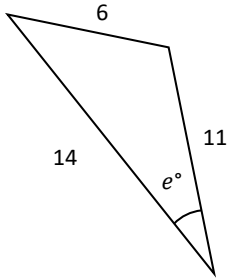
Cosine Rule: Angles Lengths are metres. not to scale

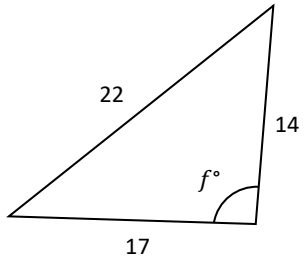


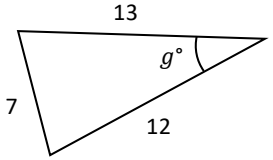


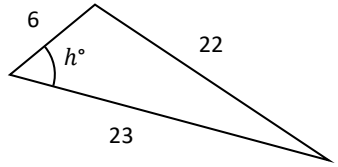


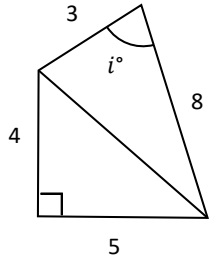


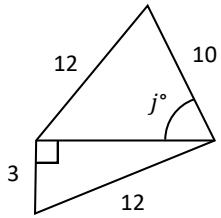


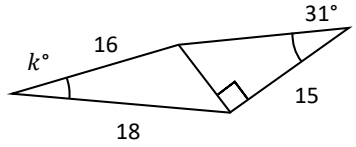


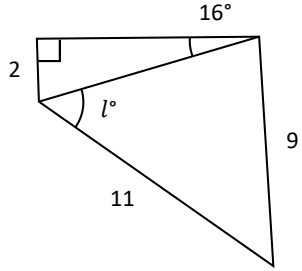








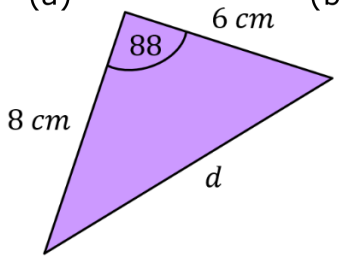




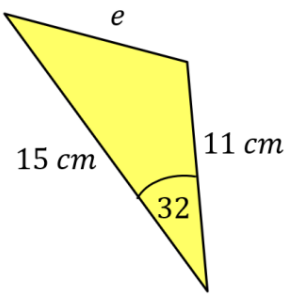
10 Answers	32.2	35.8	66.9	72.9	58.4
	47.4	90.0	52.6	24.2	48.2

Find the missing length.

(a)



(b)



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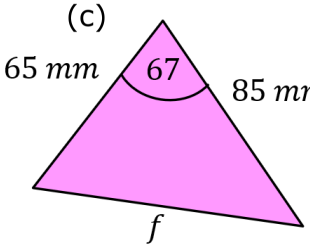

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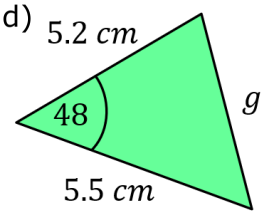

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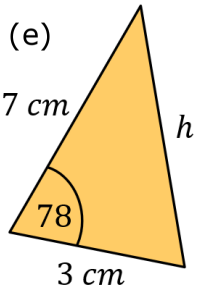
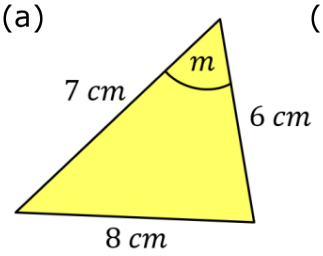


Find the missing angle.

(a)



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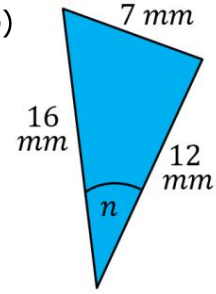

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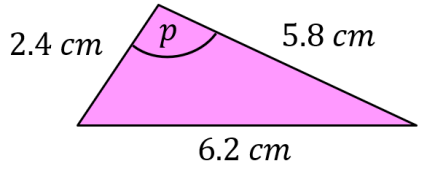

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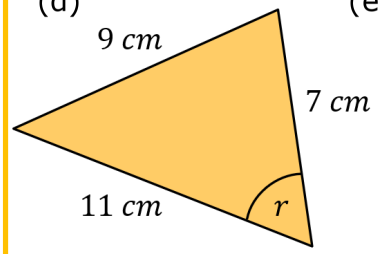

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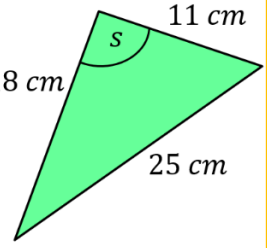
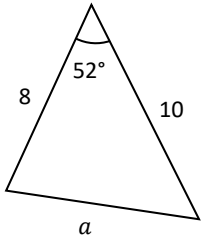
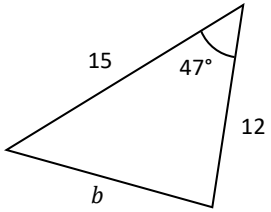
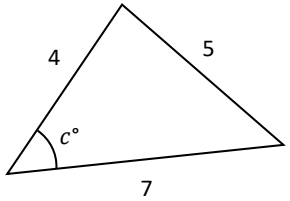


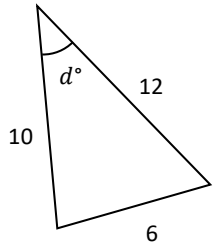
Fluency Practice

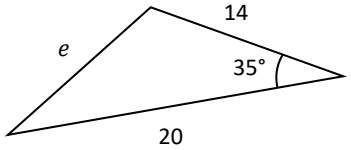
① **Cosine Rule** Lengths are centimetres. not to scale

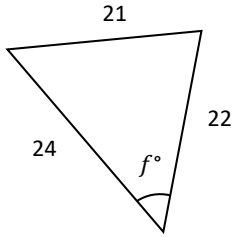


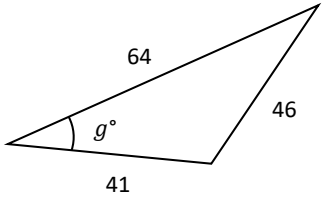


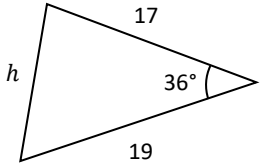


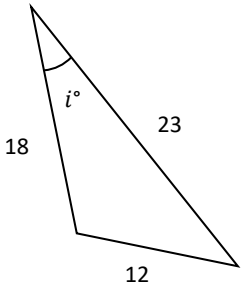


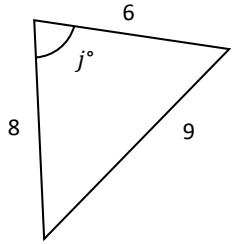


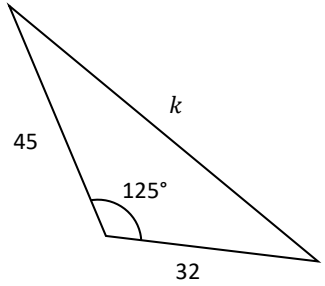


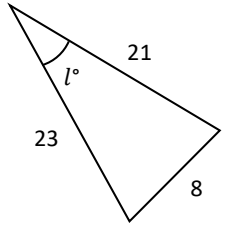








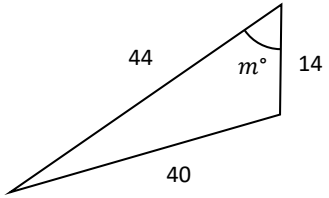


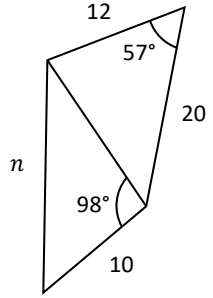


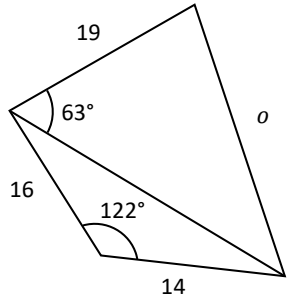
10 Answers	29.9	45.8	8.1	11.7	44.4
	31.1	11.1	54.1	78.6	11.3

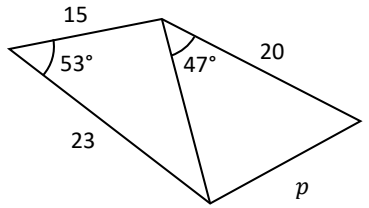
Fluency Practice

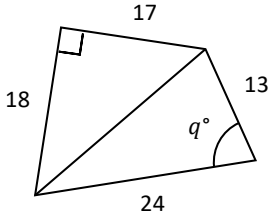
② **Cosine Rule** Lengths are centimetres. not to scale

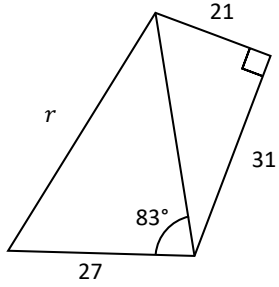


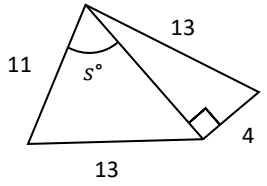


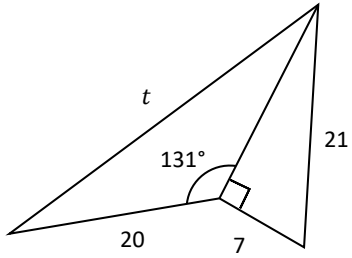


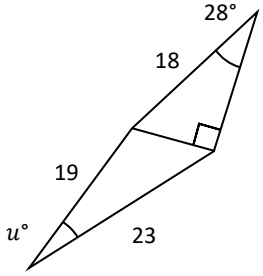


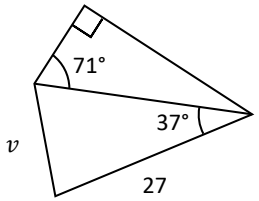


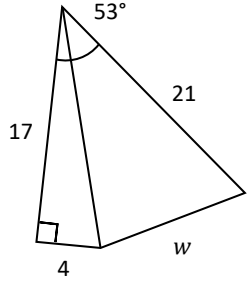


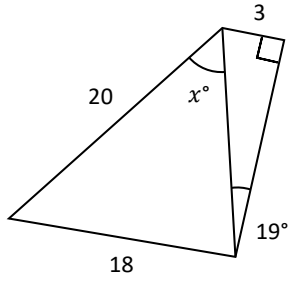






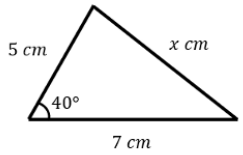
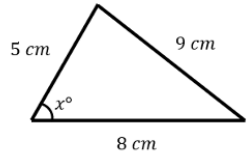
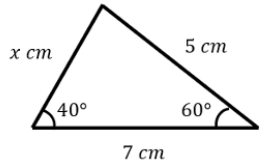
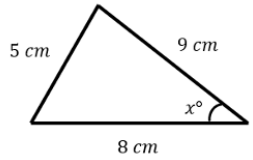
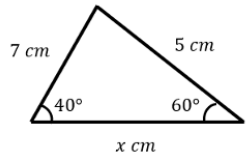
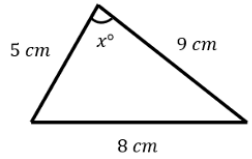
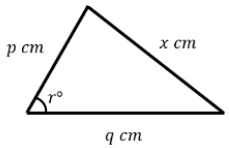
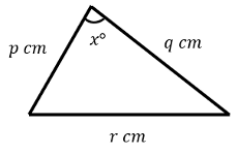






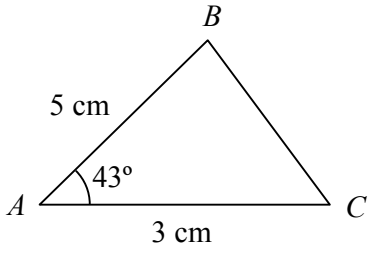
10 Answers	36.2	77.8	64.4	16.5	24.4
	20.7	20.5	43.4	15.4	67.3

Fluency Practice

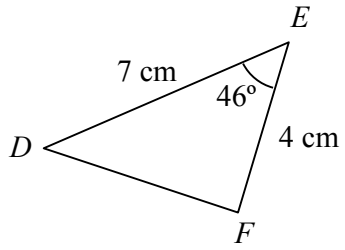
Q	Work out the missing length	Answers	Q	Work out the missing angle	Answers
1a			1b		
2a			2b		
3a			3b		
4a			4b		

Fluency Practice

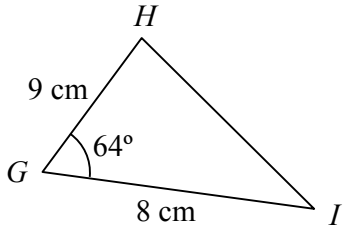
A1 Find the length of BC



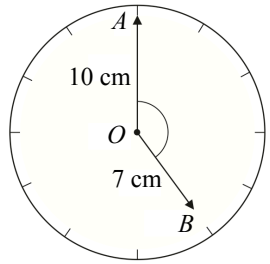
A2 Find the length of DF



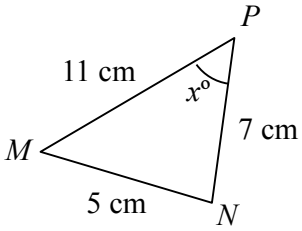
A3 Find the length of HI



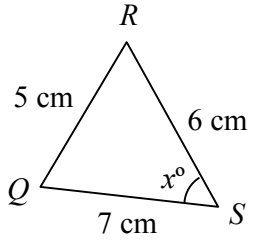
A4 Find the distance AB



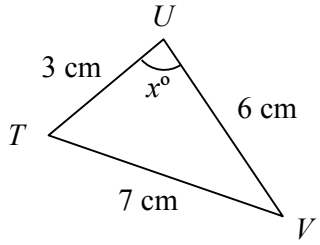
B1 Find angle MPN



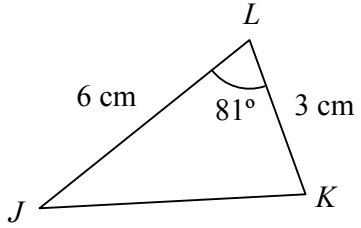
B2 Find angle QSR



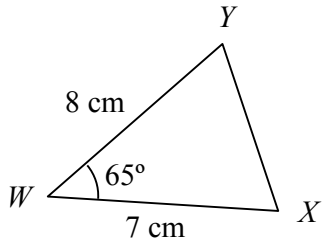
B3 Find angle TUV



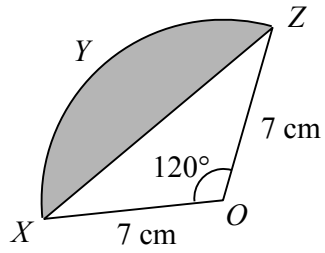
B4 Find angle JKL



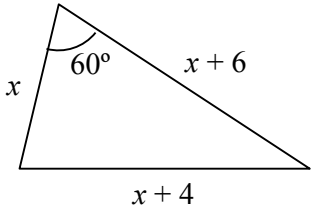
C1 Find angle XYW



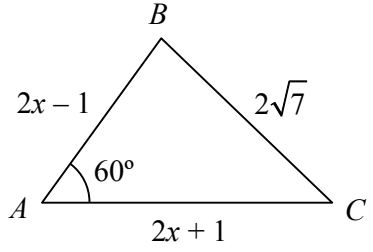
C2 Find the perimeter of XYZ



C3 Find x

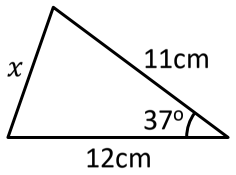


C4 Find x

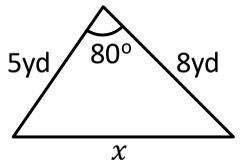


Q1 Work out the missing side, x, of each of the following triangles to one decimal place.

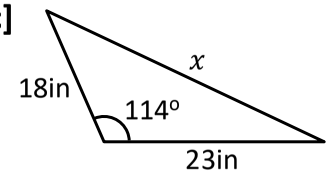
[a]



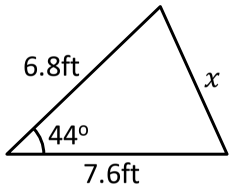
[b]



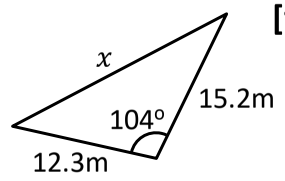
[c]



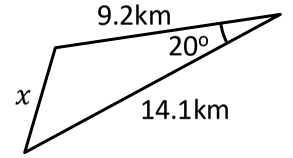
[d]



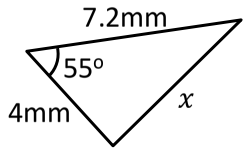
[e]



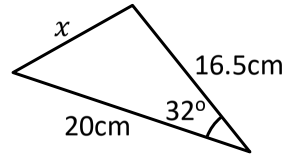
[f]



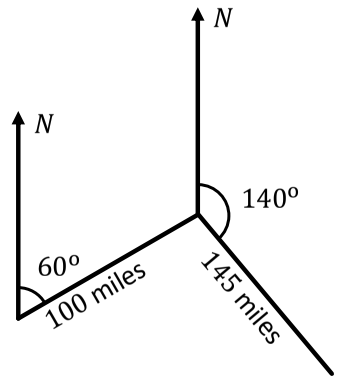
[g]



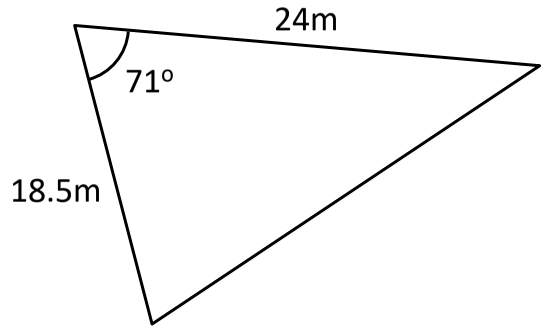
[h]



Q2 A ship sails from port A on a bearing of 060° for 100 miles and then on a bearing of 140° for 145 miles. Calculate the distance of the ship from port A to the nearest mile.

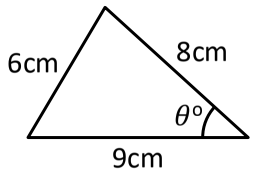


Q3 Find the perimeter of the following triangle.

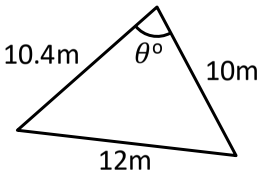


Q1 Work out the missing angle, θ^o, in each of the following triangles to one decimal place.

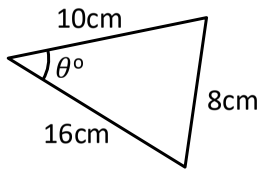
[a]



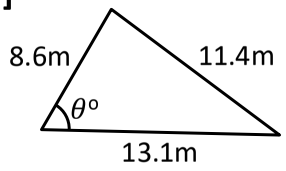
[b]



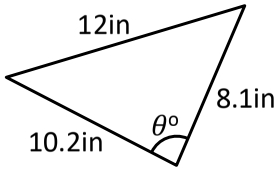
[c]



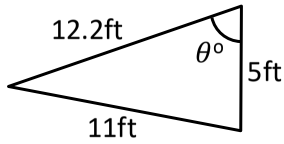
[d]



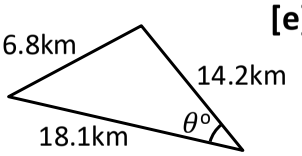
[e]



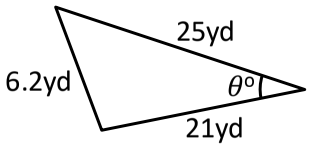
[f]



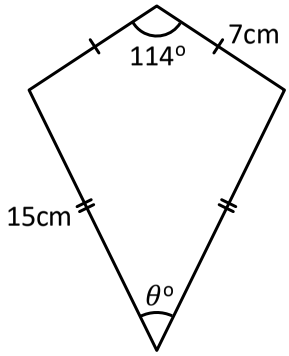
[d]



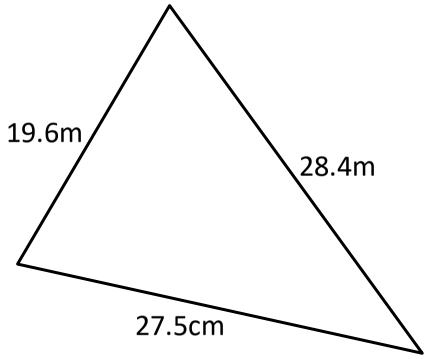
[e]



Q2 The diagram shows a kite.
Find the size of the missing angle, θ^o, correct to the nearest one decimal place.

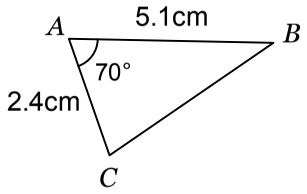


Q3 Find the size of the smallest angle in the following triangle.

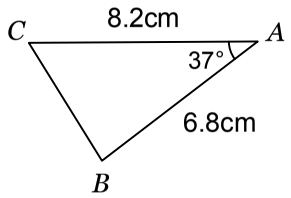


1. Work out the missing side lengths in each triangle, correct to the nearest 0.1cm:

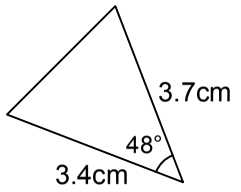
a)



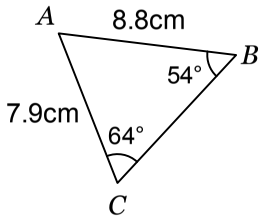
b)



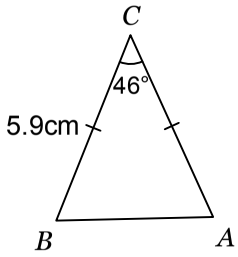
c)



d)



e)



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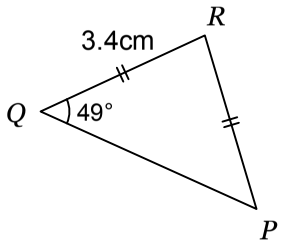
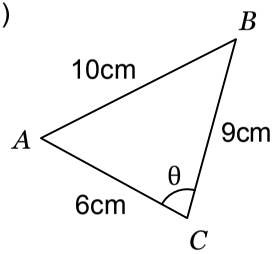
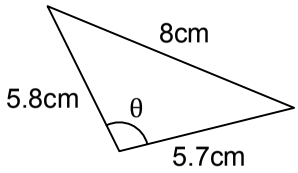


2. Work out the size of the angle marked θ in each triangle, correct to the nearest degree:

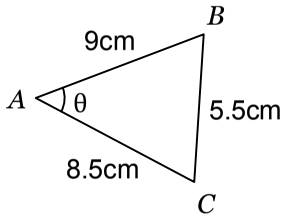
a)



b)



c)



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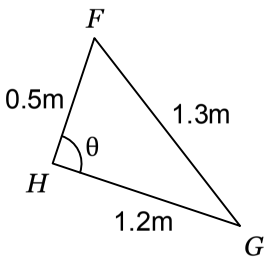
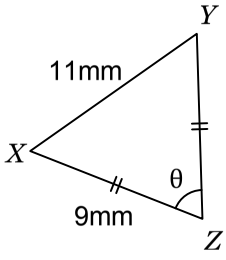


e)



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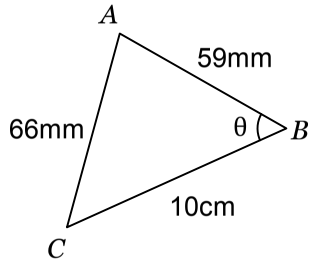
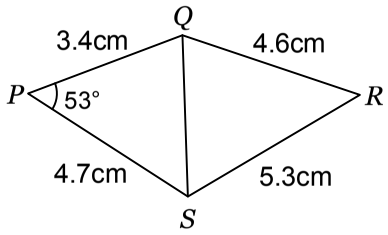


3. Triangle ABC is such that AB = 6.1cm, AC = 7cm and $B\hat{A}C$ = 65°.
Work out the length BC to the nearest 0.1cm.

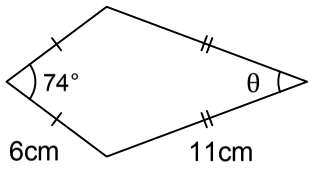
4. Triangle ABC is such that AB = 6m, BC = 5m and AC = 8m.
Work out the size of $B\hat{A}C$ to the nearest degree.

5. a) Work out the length QS, correct to the nearest 0.1cm.



b) Work out the angle $Q\hat{R}S$.

6. The driagram shows a kite.
Work out the angle marked θ, correct to the nearest degree.

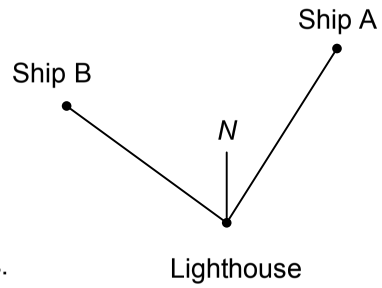


7. Two ships are being tracked from a lighthouse.

Ship A is 2.3km from the lighthouse on a bearing of 029°.

Ship B is 2.5km from the lighthouse on a bearing of 306°.

Work out the straight-line distance between the ships.



8. Two aircraft, flying at the same height, are being tracked from a control centre.
Aircraft A is 4.1km from the control centre on a bearing of 078°.
Aircraft B is 1.6km from the control centre on a bearing of 163°.

Work out the straight-line distance between the aircraft.

9. Justin is going to buy lots of identical tiles in the shape of an isosceles triangle, as shown in *figure 1*.

He wants to arrange them in a pattern around a point, as shown in *figure 2*.

Work out how many of the tiles are needed to fit completely around a point.

figure 1

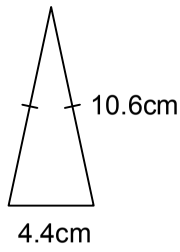
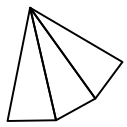
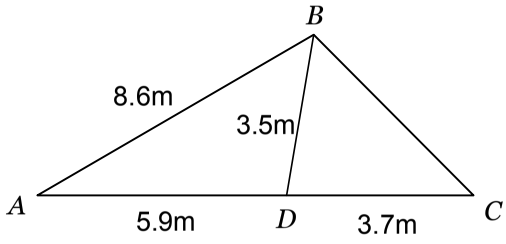


figure 2

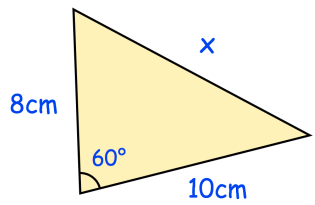


10. In the diagram, ADC is a straight line.
Work out the length BC, correct to the nearest 0.1m.

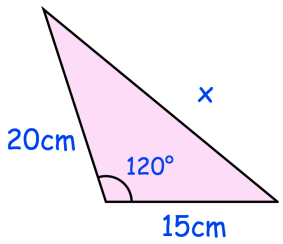


Question 1: Find x for each of the triangles below.

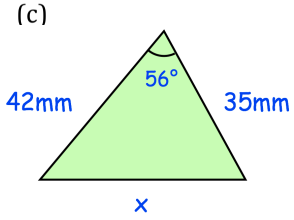
(a)



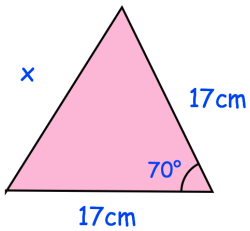
(b)



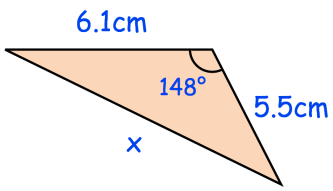
(c)



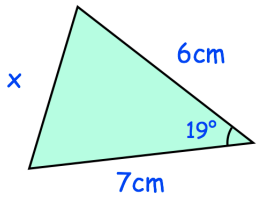
(d)



(e)

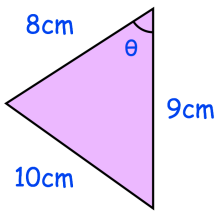


(f)

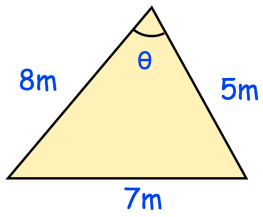


Question 2: Find the size of θ for each of these triangles.

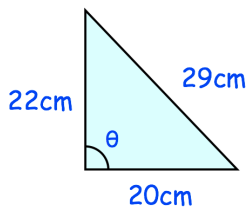
(a)



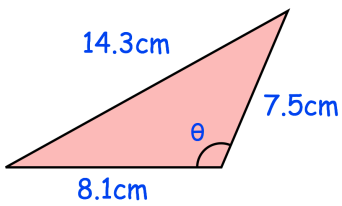
(b)



(c)



(d)



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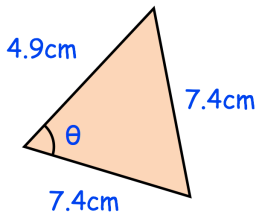

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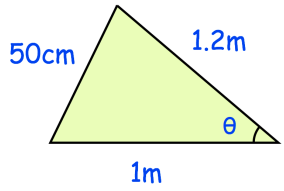


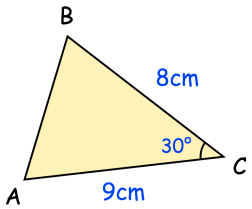
Question 3: In triangle ABC, the side AB = 6cm, the side BC = 8cm and angle ABC = 100°
Find the length of side AC.
Give your answer to 1 decimal place.

Question 4: In triangle DEF, the side DE = 30cm, the side DF = 40cm and the side EF = 45cm.
Find the size of angle DFE.
Give your answer to 3 significant figures.

Question 5: In triangle GHI, the side GH = 3cm, the side HI = 18cm and the side GI = 20.5cm.
Find the size of angle HGI.
Give your answer to 3 significant figures.

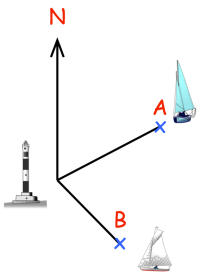
Apply

Question 1: Calculate the perimeter of triangle ABC

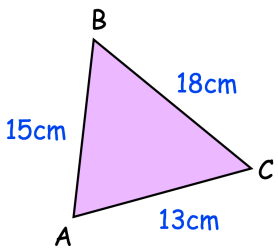


Question 2: Boat A is 16km from a lighthouse on a bearing of 055°
Boat B is 11km from the same lighthouse on a bearing of 152°

Calculate the distance between the two boats.



Question 3: Find the size of the smallest angle in this triangle.

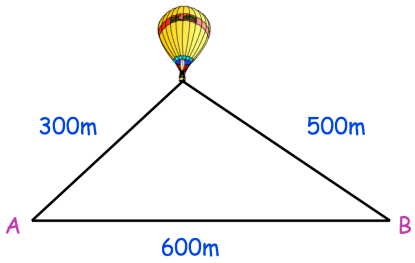


Question 4: A hot air balloon is flying above two point, standing on the ground at points A and B, 600m apart.

The hot air balloon is 300m from A and 500m from B.

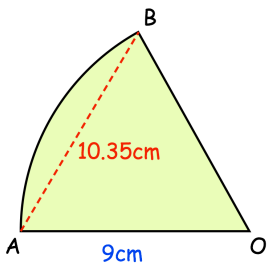
(a) Work out the angle of elevation from point B

(b) How high is the hot air balloon from the ground?



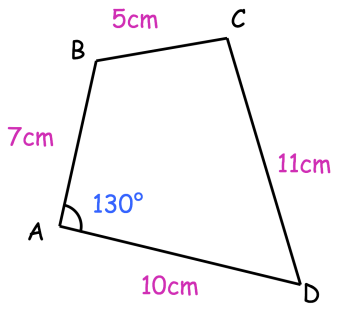
Question 5: Shown is sector OAB.
O is the centre of the circle with radius 9cm
A and B are points on the circle.
The length of the chord AB is 10.35cm

Find the area of sector OAB



Question 6: ABCD is a quadrilateral.
AB = 7cm BC = 5cm BC = 11cm AD = 10cm Angle BAD = 130°

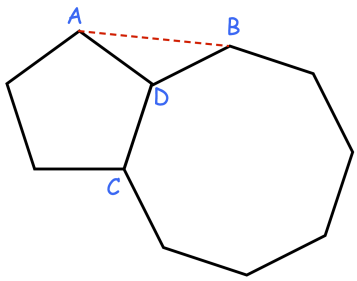
Work out the size of angle BCD.



Question 7: A is a vertex of a regular pentagon.
B is a vertex of a regular octagon.
C and D are vertices of both polygons.

The perimeter of the octagon is 40cm.

Work out the length AB



Question 8: A clock has two hands.
A minute hand which is 5.5cm long and an hour hand which is 4cm long.

Find the distance between the tips of the two hands at 7:15am

Question 9: Two ships, A and B, leave a port at 10:30

Ship A travels on a bearing of 196° at a speed of 30km/h.
Ship B travels on a bearing of 244° at a speed of 24km/h.

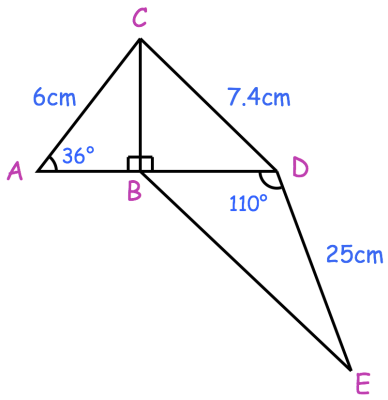
(a) Work out the distance between A and B at 14:00

(b) Work out the bearing of B from A at 14:00

Question 10: In the diagram:

ABD is a straight line.
AC = 6cm CD = 7.4cm DE = 25cm
Angle BAC = 36° Angle BDE = 110°

Calculate the length of BE

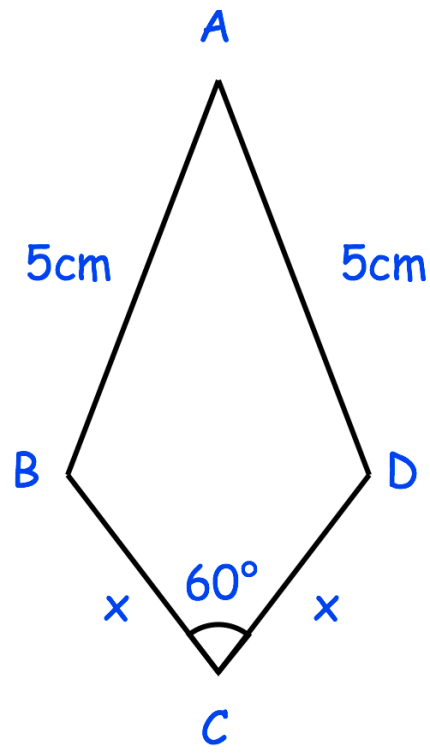


Question 11: The Cosine Rule is $a^2 = b^2 + c^2 - 2bcCosA$

Make CosA the subject.

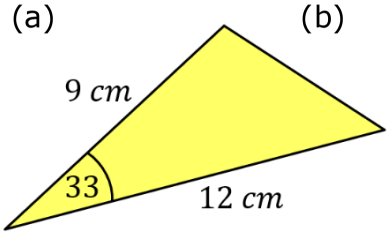
Question 12: Shown is kite ABCD

Prove $CosBAD = 1 - \frac{x^2}{50}$

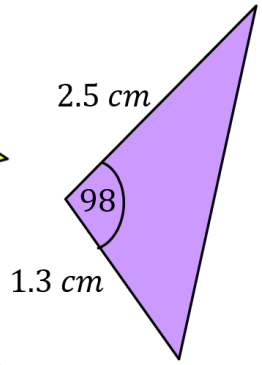


Find the area of the triangle.

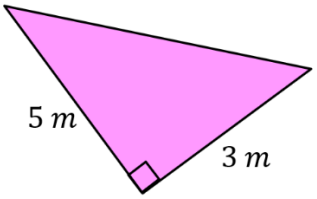
(a)



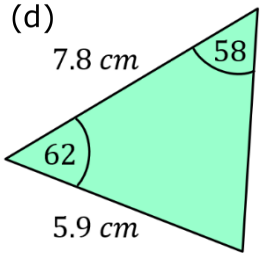
(b)



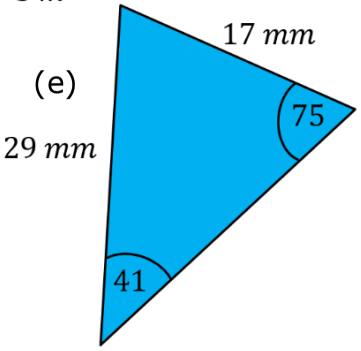
(c)



(d)

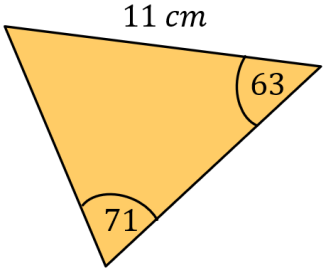


(e)

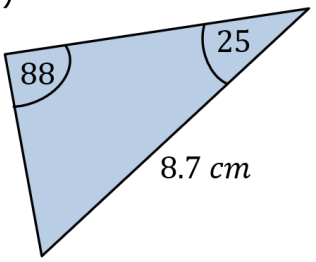


Find the area of the triangle.

(a)

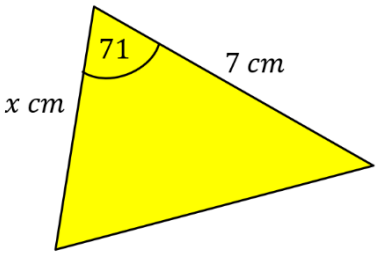


(b)

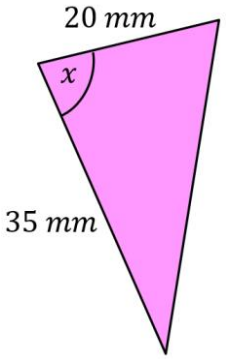


Find the missing length or angle.

(a) $Area = 32\ cm^2$



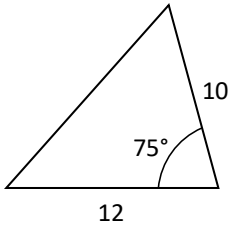
(b) $Area = 300\ mm^2$



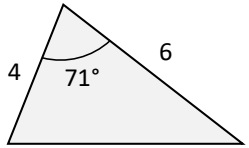
Fluency Practice

Finding the Area of a Triangle using the Area Rule (lengths are cm) not drawn accurately

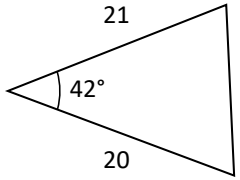
A



B



C



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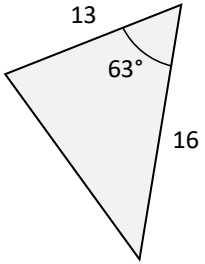

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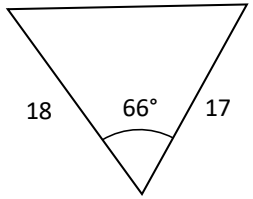

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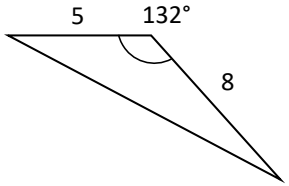

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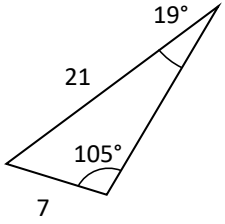

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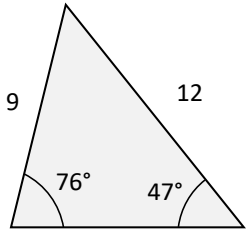

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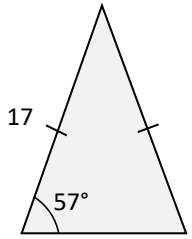

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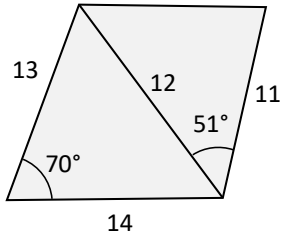

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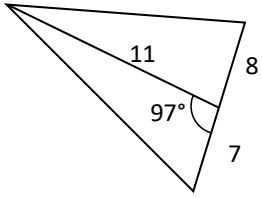

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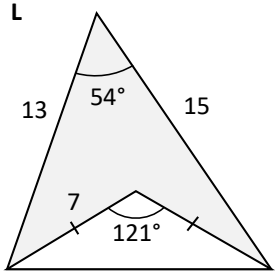

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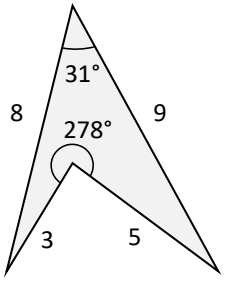

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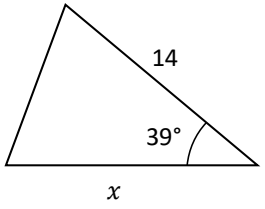
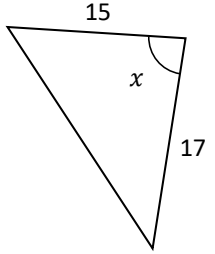


Area = 57 cm^2

O

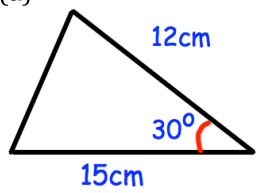


Area = 125 cm^2

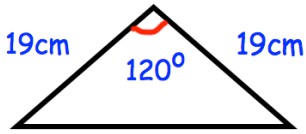
14 Answers							
	14.9	136.8	139.8	58.0	140.5	45.3	57.9
	11.1	11.3	132.0	60.9	81.9	12.9	92.7

Question 1: Find the area of each of these triangles.

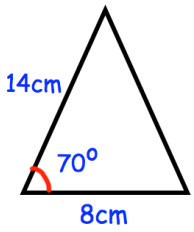
(a)



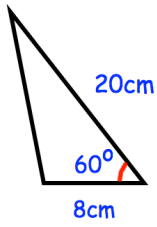
(b)



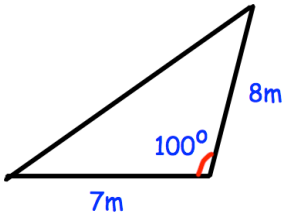
(c)



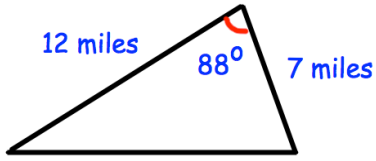
(d)



(e)

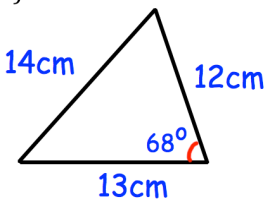


(f)

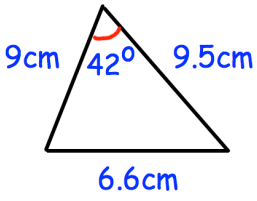


Question 2: Find the area of each of these triangles.

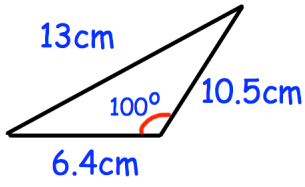
(a)



(b)



(c)



Question 3: Find the area of each of these triangles

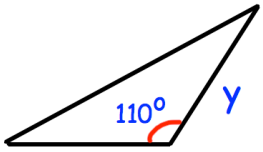
(a) ABC with AB = 10cm, BC = 9cm and angle ABC = 44°.

(b) DEF with EF = 28cm, DF = 34cm and angle DFE = 81°.

(c) XYZ with YZ = 9mm, XY = 13mm and angle XYZ = 121°.

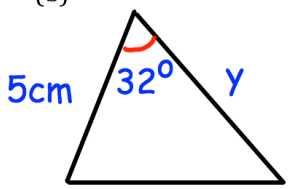
Question 4: Find the length of the missing side in each of these triangles.
Give each answer to one decimal place.

(a)



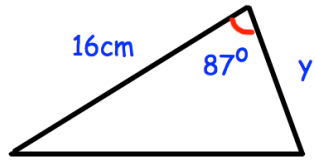
area = $24cm^2$

(b)



area = $9.22cm^2$

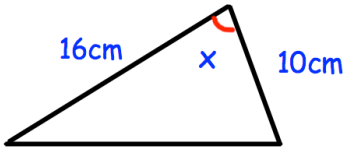
(c)



area = $63.5cm^2$

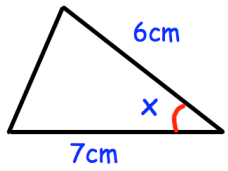
Question 5: Find the size of the missing acute angles below.
Give each answer to one decimal place.

(a)



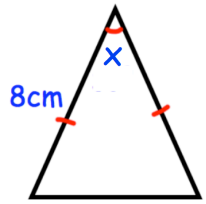
area = $78.5cm^2$

(b)



area = $11cm^2$

(c)

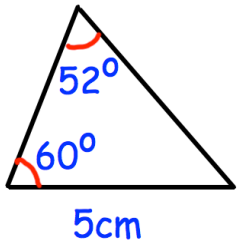


area = $16cm^2$

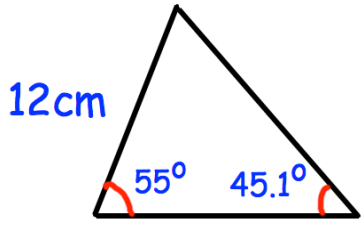
Apply

Question 1: Find the area of these triangles.

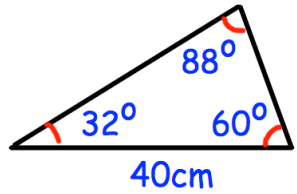
(a)



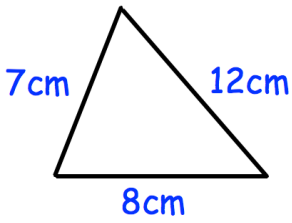
(b)



(c)

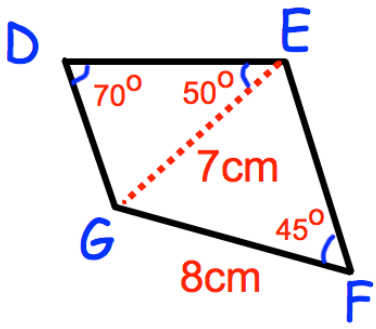


Question 2: Find the area of this triangle.



Question 3: A triangular field has three fences. One is 40m long, another is 50m and the other is 60m.
Find the area of the field.

Question 4: Find the area of quadrilateral DEFG.

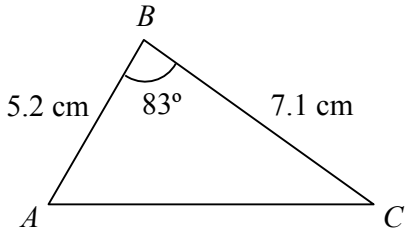


Question 5: Parallelogram ABCD has side lengths AB = 5cm and BC = 9cm.
Angle BCD = 55°.
Find the area of the parallelogram.

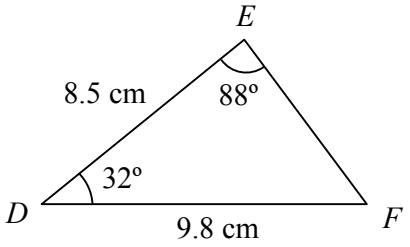
Question 6: Prove the area of any triangle is given by the formula A = ½abSinC.

Fluency Practice

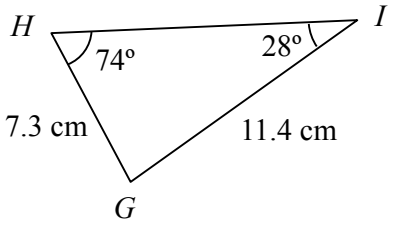
A1 Find the area of triangle *ABC*



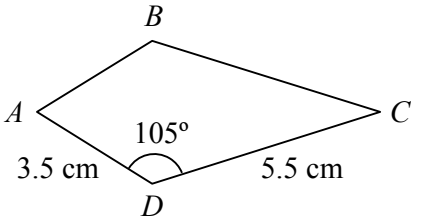
A2 Find the area of triangle *DEF*



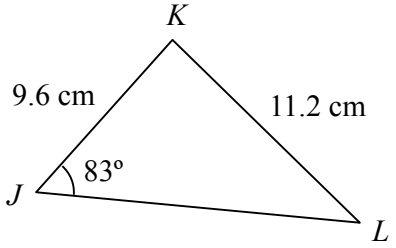
A3 Find the area of triangle *GHI*



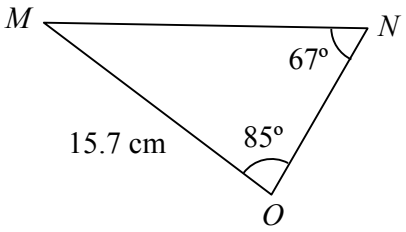
A4 Find the area of the kite



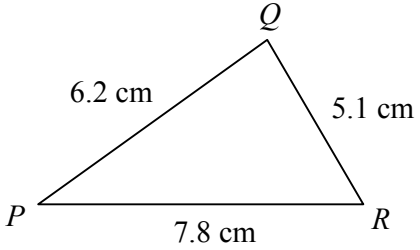
B1 Find the area of triangle *JKL*



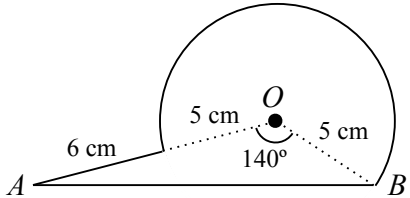
B2 Find the area of triangle *MNO*



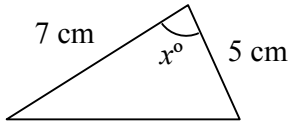
B3 Find the area of triangle *PQR*



B4 Find the area of the shape

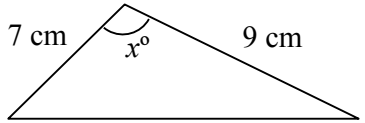


C1 The area of the triangle is 16.5 cm^2.



The angle $x°$ is **acute**.
Find the value of x.

C2 The area of the triangle is 20 cm^2.



The angle $x°$ is **obtuse**.
Find the value of x.

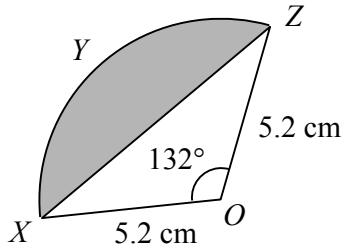
C3

ABC is a triangle.
AB = 11 cm
AC = 7 cm

The area of triangle *ABC* is 32 cm^2

Find, in degrees, the **two** possible sizes of angle *BAC*.

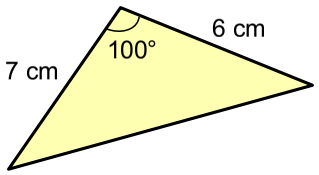
C4 Find the **shaded** area



Section A

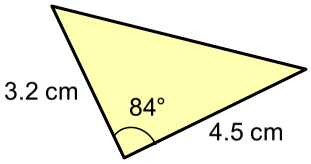
Questions 1 - 5 find the area of the triangles, for question 6 find the missing side x.

1)



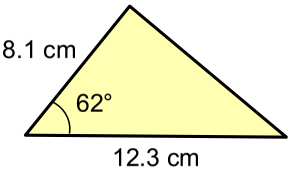
Area =

2)



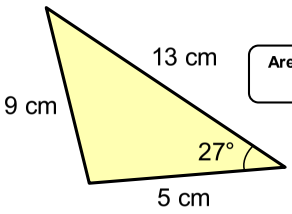
Area =

3)



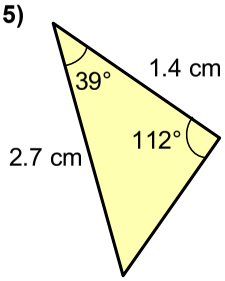
Area =

4)



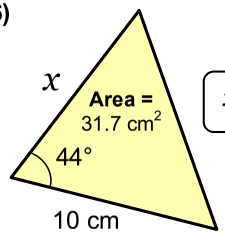
Area =

5)



Area =

6)

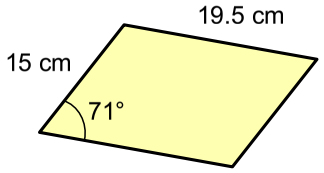


x =

Section B

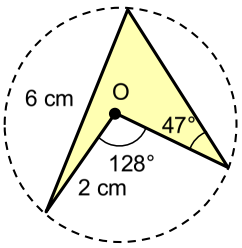
Work out the area of the parallelogram and irregular quadrilaterals.

1)



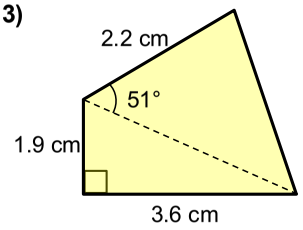
Area =

2)



Area =

3)



Area =

Section C

1) *A regular hexagon has sides lengths 10 cm.*
Calculate the area of the hexagon.

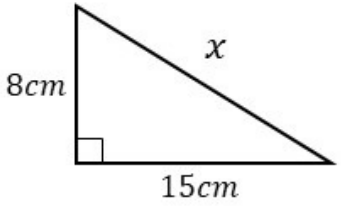
2) *The area of triangle ABC is 19.6 cm².*
AB = 5.9 cm, AC = 8.7 cm.
Calculate the two possible sizes of angle A.

Extension

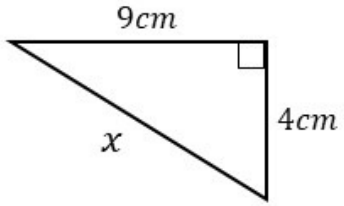
The points X, Y and Z are are on the circumference of a circle, with centre O and radius 5 cm.
XY = 7 cm and YZ = 4.5 cm.
Calculate the area of quadrilateral OXYZ.

Fluency Practice

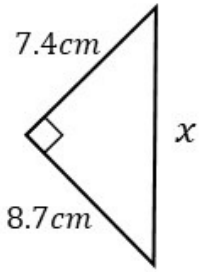
1 Calculate x



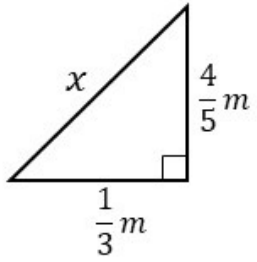
2 Calculate x to 2 d.p.



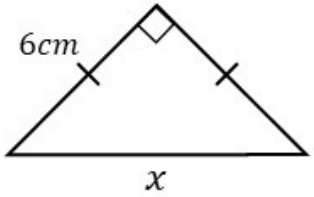
3 Calculate x to 1 d.p.



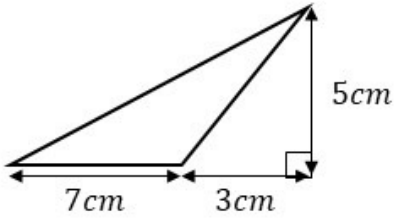
4 Calculate x, leave as a fraction



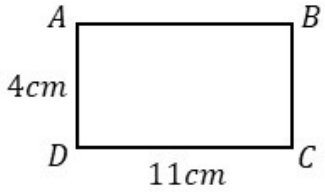
5 Find x leave in exact form



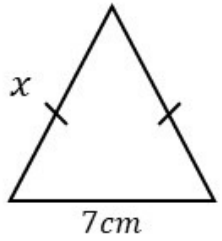
6 Calculate the perimeter to 1 d.p.



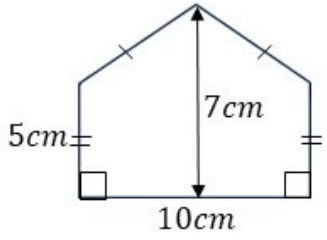
7 Calculate the length BD of the rectangle to 3 s.f.



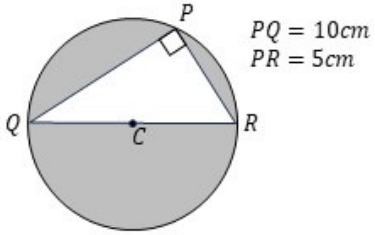
8 The area of the triangle is $63cm^2$, calculate x to 2 d.p.



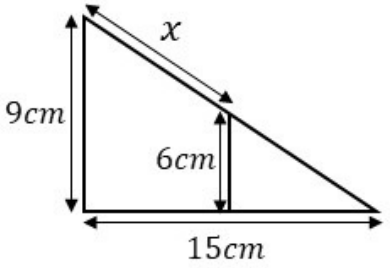
9 Calculate the perimeter to 1 d.p.



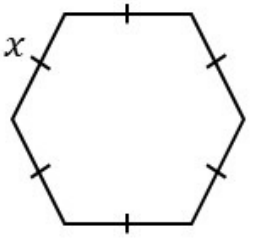
10 Calculate the shaded area to 2 d.p.



11 Calculate x to 2 d.p.



12 The area of the hexagon is $24cm^2$, calculate x to 2 d.p.

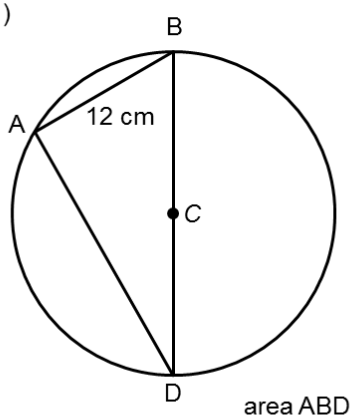


Fluency Practice

circle geometry and areas

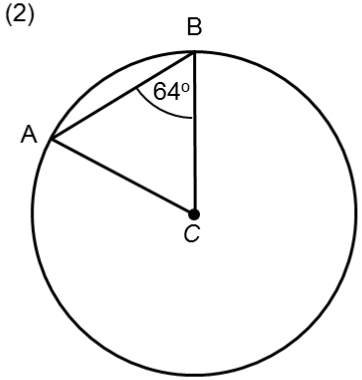
C is the centre of a circle, radius 10 cm

(1)



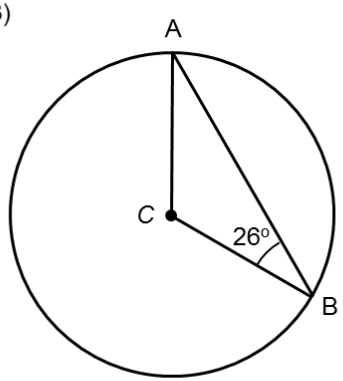
area ABD ?

(2)



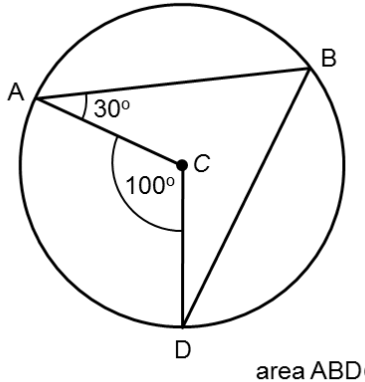
area ABC ?

(3)



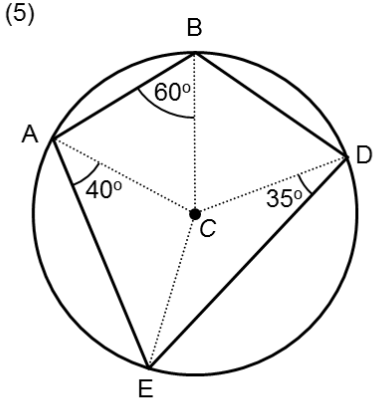
area ABC ?

(4)



area ABD*C* ?

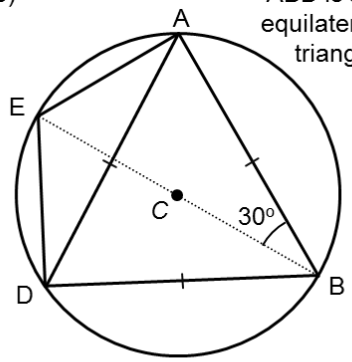
(5)



area ABDE ?

(6)

ABD is an equilateral triangle



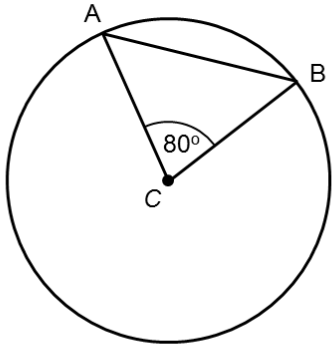
area ABDE ?

Fluency Practice

circle geometry and areas

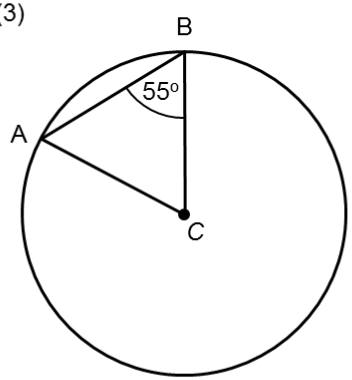
C is the centre of a circle, radius 10 cm

(1)



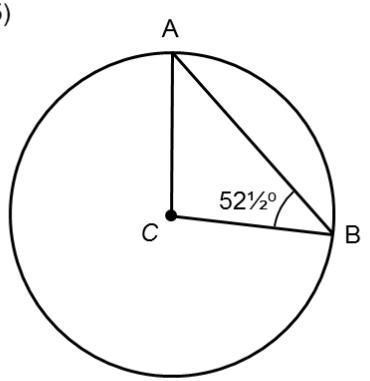
area ABC ?

(3)



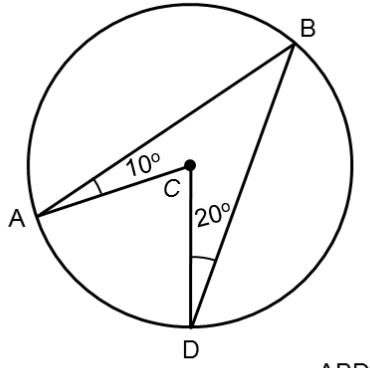
area ABC ?

(5)



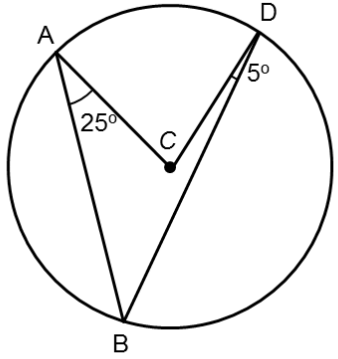
area ABC ?

(2)



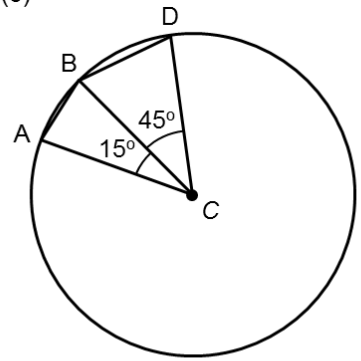
area ABDC ?

(4)



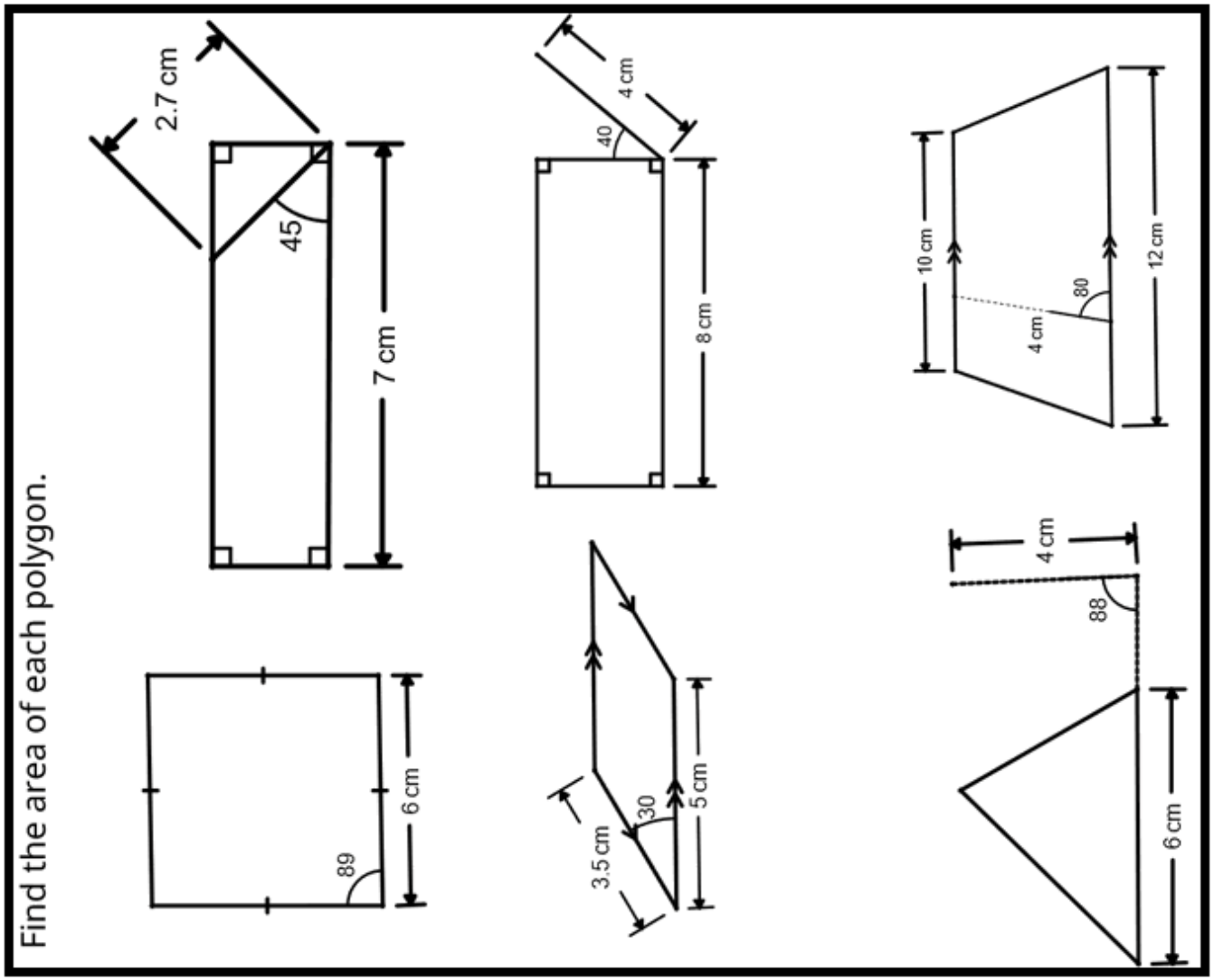
area ABDC ?

(6)

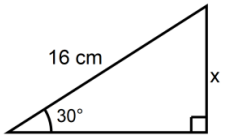
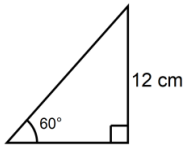
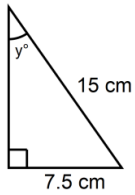
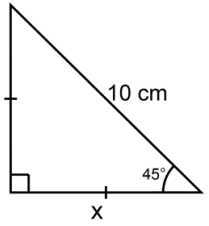
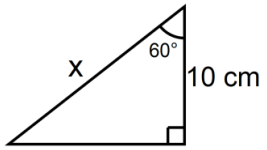
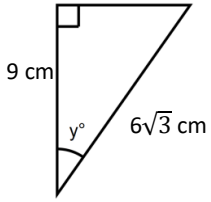
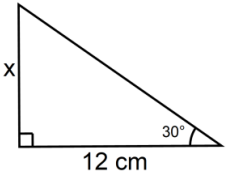
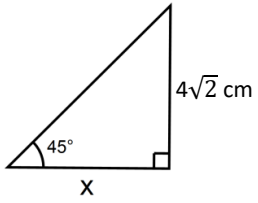
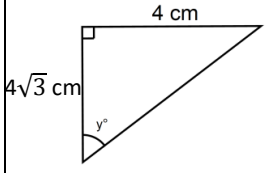
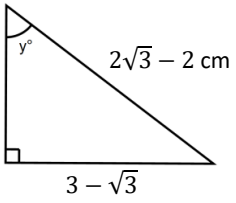
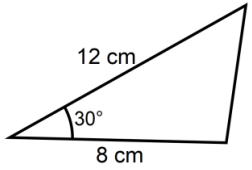
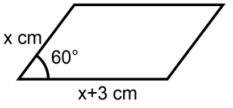


area ABDC ?

Fluency Practice

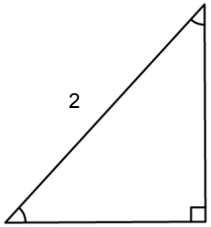


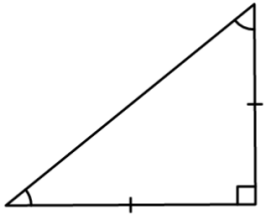
Fluency Practice

Sine	State the value of sine of 0^o | 30^o | 45^o | 60^o | 90^o	Work out the exact value of x 	Work out the exact value of x 	Work out the size of angle y 
Cosine	State the value of cosine for 0^o | 30^o | 45^o | 60^o | 90^o	Work out the exact value of x 	Work out the exact value of x 	Work out the size of angle y 
Tangent	State the value of tan for 0^o | 30^o | 45^o | 60^o	Work out the exact value of x 	Work out the exact value of x 	Work out the size of angle y 
Challenge Questions	Show that 20 cos 30° + 4 sin 60° - 2 tan 60° can be written in the form $\sqrt{k}$ where k is an integer	Work out the size of angle y 	Work out the area of the triangle 	The parallelogram has an area of $20\sqrt{3}$ cm^2. Find the value of x. 

exact trig values

Complete the missing sides and angles for these special triangles:





Complete the table to show the exact values of sin, cos and tan for these angles:

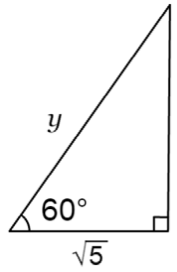
	30°	45°	60°
sin			
cos			
tan			

In this exercise, assume θ is an acute angle and rationalise the denominator of your answers, where appropriate.

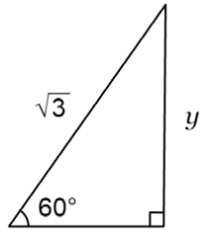
1. Given that $\sin(\theta) = \frac{1}{\sqrt{2}}$, what is θ?
2. Given that $\tan(\theta) = \sqrt{3}$, what is $\cos(\theta)$?

3. Calculate the exact length of y in each triangle:

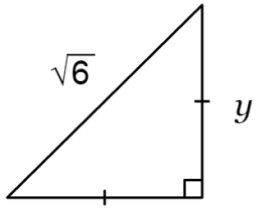
a)



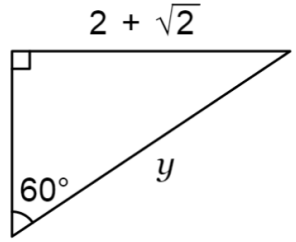
b)



c)

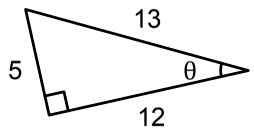


d)



4. Simplify $4\sin(60°)\cos(60°)$
5. Simplify $2\sin(45°)\cos(45°)$
6. Write $6\tan(60°) + 3\tan(30°)$ in the form $a\sqrt{b}$
7. Show that $\tan(45°) = \frac{\sin(45°)}{\cos(45°)}$
8. Show that $\tan(30°) = \frac{\sin(30°)}{\cos(30°)}$

1. From the triangle, write in fractional form, the values of:



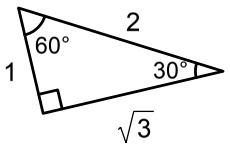
a) $\sin(\theta)$ b) $\tan(\theta)$ c) $\cos(\theta)$

2. Which of the following are impossible?

a) $\sin(\theta) = 1.8$ b) $\tan(\theta) = 2.5$ c) $\cos(\theta) = 1.6$ d) $\sin(\theta) = 0.25$

3. Use triangle A to work out the exact value of cos(30°).

Triangle A



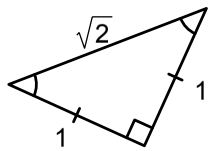
a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{\sqrt{3}}{2}$

4. Use triangle A to work out the exact value of:

a) sin(30°) b) tan(30°) c) sin(60°)

5. Use triangle B to work out the exact value of:

Triangle B



a) sin(45°) b) cos(45°) c) tan(45°)

6. Complete the table of exact values:

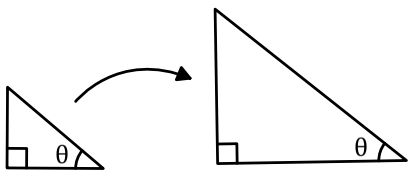
	30°	45°	60°
sin(θ)			
cos(θ)			
tan(θ)			

7. Use the table above to work out the exact values of:

a) $8\sin(30)$ b) $\cos^2(60)$ c) $\sin(30) + \cos(60)$

8. True or false?

If you double the side lengths of a right angled triangle, the value of tan(θ) remains the same.



Fluency Practice

using exact trigonometric values for some angles

what are the values of these:

(1) $5\sin30^\circ - 11\cos60^\circ + 4\tan45^\circ$

(2) $\sin 45^\circ.\cos 45^\circ + \cos 30^\circ.\tan 30^\circ$

(3) $\sin 30^\circ.\tan 45^\circ + \tan 30^\circ.\sin 60^\circ$

(4) $2\sin 45^\circ.\cos 45^\circ$

(5) $\cos^2 45^\circ + \sin^2 45^\circ$

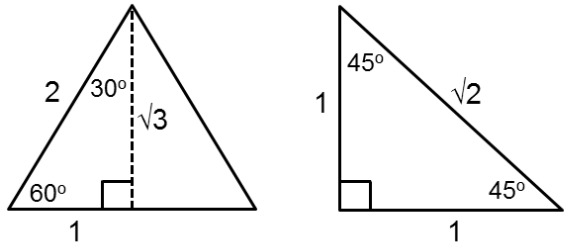
(6) $\cos 30^\circ.\sin 60^\circ + \sin 30^\circ.\cos 60^\circ$

(7) $\sin^2 30^\circ + \cos^2 30^\circ$

(8) $3\cos^2 30^\circ - 5\sin^2 30^\circ$

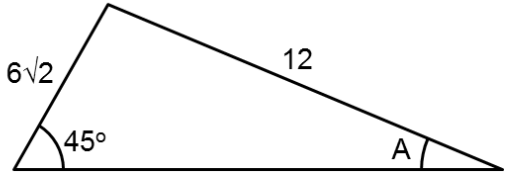
(9) $\dfrac{1}{\cos^2 30^\circ} - \tan^2 30^\circ$

(10) $\dfrac{2\tan^2 45^\circ}{\tan^2 60^\circ} + 2\tan^2 30^\circ$

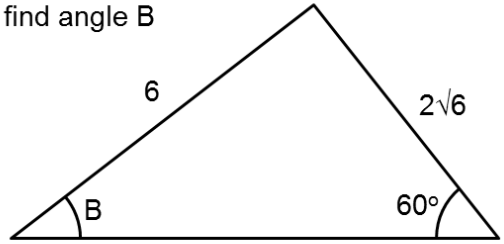


note that $\sin^2 30^\circ$ is $(\sin30^\circ)^2$

(11) find angle A



(12) find angle B



Question 1: Write down the exact values of each of the following

(a) $\sin 30°$ (b) $\cos 0°$ (c) $\tan 45°$ (d) $\sin 90°$ (e) $\sin 0°$

(f) $\cos 60°$ (g) $\tan 0°$ (h) $\sin 45°$ (i) $\cos 30°$ (j) $\tan 60°$

(k) $\cos 90°$ (l) $\sin 60°$ (m) $\cos 45°$ (n) $\tan 30°$

Question 2: Write down the exact values of each of the following

(a) $\cos 60° + \sin 30°$ (b) $\cos 0° + \tan 45° + \sin 90°$

(c) $\sin 30° + \sin 90°$ (d) $\sin 45° + \cos 45°$

Question 3: Write down the exact values of each of the following

(a) $\sin 45° + \cos 45°$ (b) $\tan 30° + \tan 60°$ (c) $\cos 30° + \sin 60°$

Question 4: Write down the values of each of the following

(a) $\sin 180°$ (b) $\cos 270°$ (c) $\tan 360°$ (d) $\sin 360°$

(e) $\sin 270°$ (f) $\cos 180°$ (g) $\cos 360°$ (h) $\cos 450°$

(i) $\sin 720°$ (j) $\sin 900°$ (k) $\cos 990°$ (l) $\sin 1710°$

Question 5: Write down the exact values of each of the following

(a) $\sin 120°$ (b) $\cos 150°$ (c) $\tan 135°$ (d) $\cos 210°$

(e) $\tan 120°$ (f) $\sin 240°$ (g) $\sin 225°$ (h) $\cos 315°$

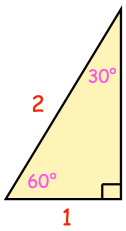
(i) $\sin 150°$ (j) $\tan 405°$ (k) $\sin 495°$ (l) $\cos 870°$

Apply

Question 1: Using the triangle below, explain each of the following.

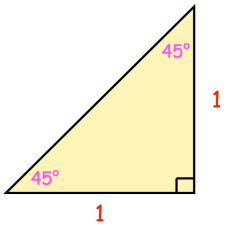
(a) $sin(30°) = \frac{1}{2}$ (b) $cos(30°) = \frac{\sqrt{3}}{2}$ (c) $tan(30°) = \frac{\sqrt{3}}{3}$

(d) $sin(60°) = \frac{\sqrt{3}}{2}$ (e) $cos(60°) = \frac{1}{2}$ (f) $tan(60°) = \sqrt{3}$



Question 2: Using the triangle below, explain each of the following.

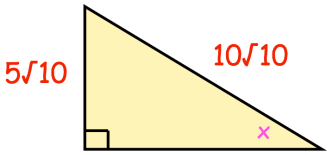
(a) $tan(45°) = 1$ (b) $sin(45°) = \frac{\sqrt{2}}{2}$ (c) $cos(45°) = \frac{\sqrt{2}}{2}$



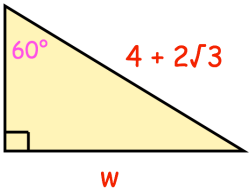
Question 3: Conor says that $cos(45°) = \frac{1}{\sqrt{2}}$

Is he correct?

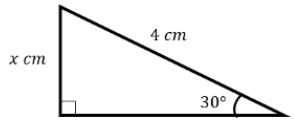
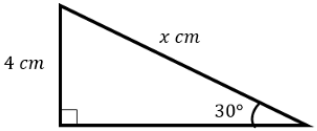
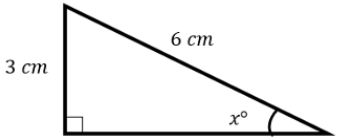
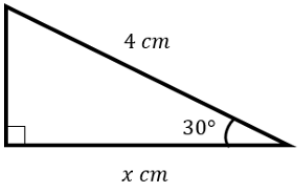
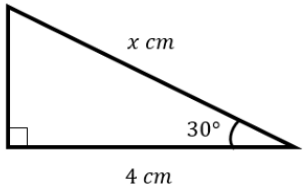
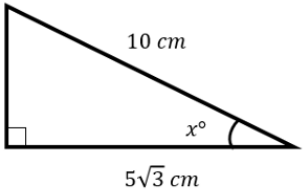
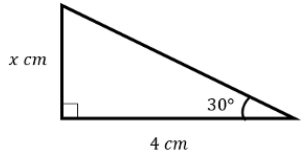
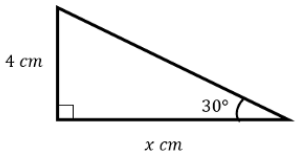
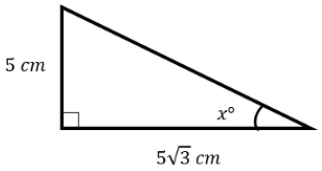
Question 4: Find the size of the angle labeled x.



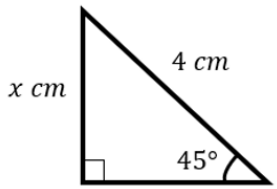
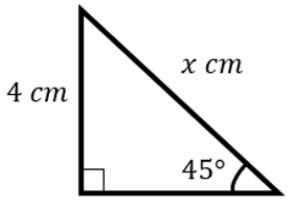
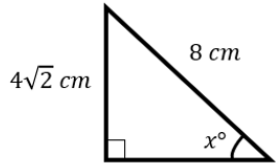
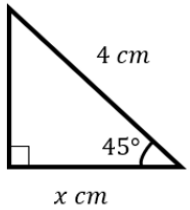
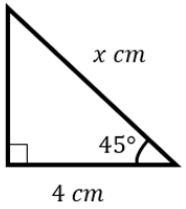
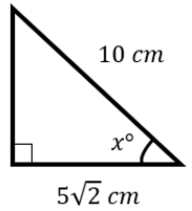
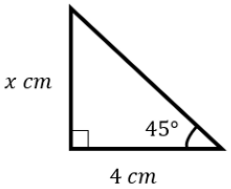
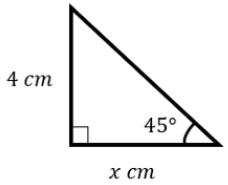
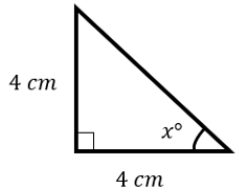
Question 5: Find the exact length of the side labelled w.



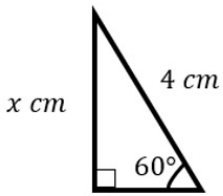
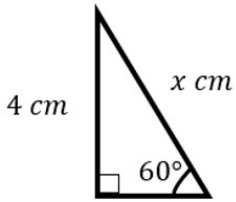
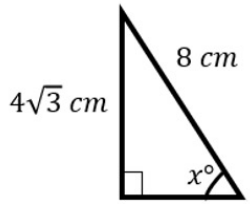
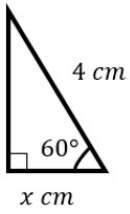
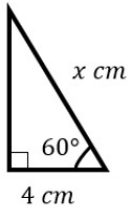
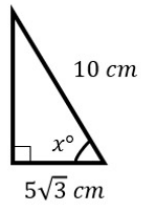
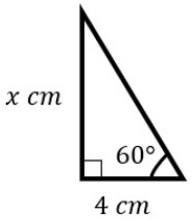
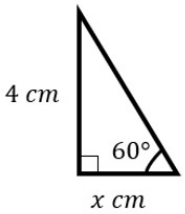
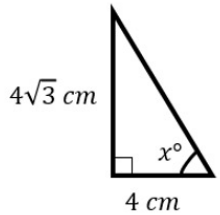
Fluency Practice

Q	Find the length	Answers	Q	Find the length	Answers	Q	Find the angle	Answers
1a			**1b**			**1c**		
2a			**2b**			**2c**		
3a			**3b**			**3c**		

Fluency Practice

Q	Find the length	Answers	Q	Find the length	Answers	Q	Find the angle	Answers
1a			**1b**			**1c**		
2a			**2b**			**2c**		
3a			**3b**			**3c**		

Fluency Practice

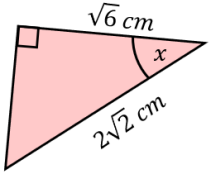
Q	Find the length	Answers	Q	Find the length	Answers	Q	Find the angle	Answers
1a			**1b**			**1c**		
2a			**2b**			**2c**		
3a			**3b**			**3c**		

Fluency Practice

Exact Trigonometric Values in Right-Angled Triangles

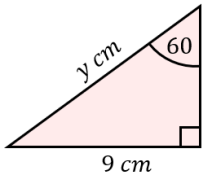
(a)

Find the exact value of x.



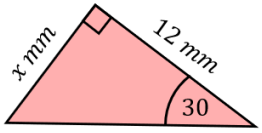
(b)

Find the exact length of y.



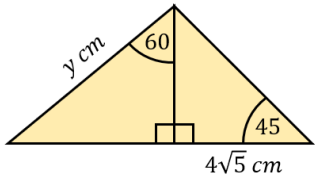
(c)

Find the exact length of x.



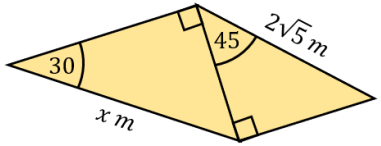
(d)

Find the exact length of y.



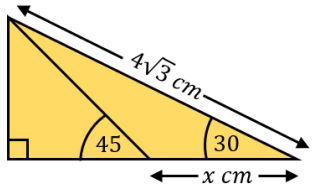
(e)

Find the exact length of x.



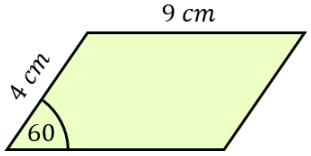
(f)

Find the exact length of x.



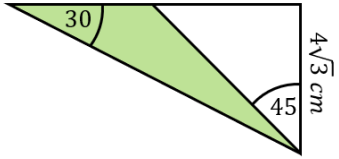
(g)

Find the area of the parallelogram.



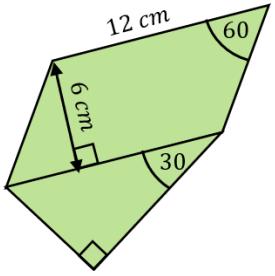
(h)

Find the exact value of the shaded area.



(i)

A triangle is joined to a parallelogram. Find the exact perimeter of the shape.



Fluency Practice

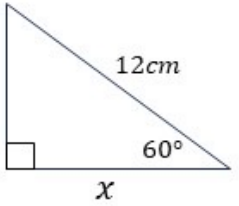
1 What is the exact solution of:

$$\sin(60)$$

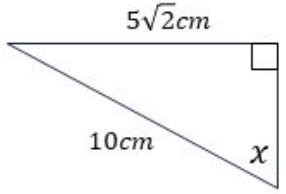
2 What is the exact solution of:

$$tan(90)$$

3 Calculate the value of x. Leave your answer in exact form.



4 Calculate the value of x. Leave your answer in exact form.



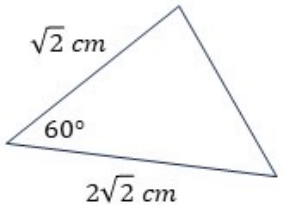
5 $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$

Find the exact value of $\cot(45)$

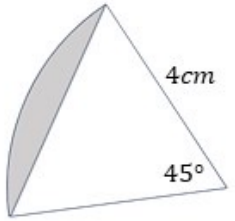
6 $\sec^2(x) = 1 + \tan^2(x)$

Find the exact value of $\sec(30)$

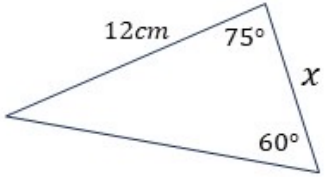
7 Calculate the exact area of the triangle.



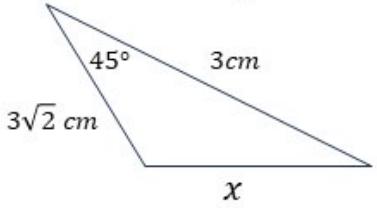
8 Calculate the area of the segment. Leave your answer in terms of pi.



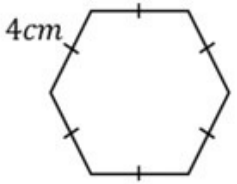
9 Calculate the value of x.
Leave your answer in the form $k\sqrt{a}$.



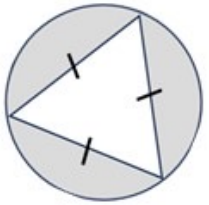
10 Calculate the value of x. Leave your answer as an integer.



11 Calculate the area of the regular hexagon. Leave your answer in the form $k\sqrt{a}$.



12 The following is a cyclic equilateral triangle. The radius of the circle is 5cm.

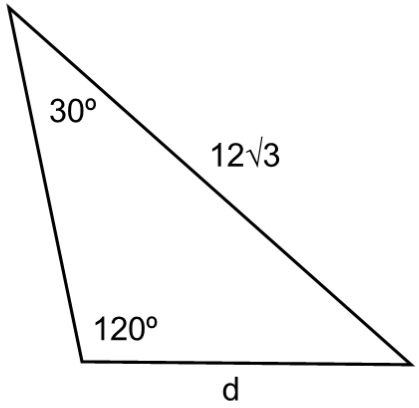


Calculate the shaded area in terms of π.

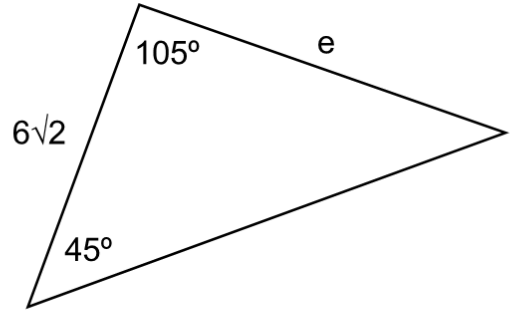
Fluency Practice

find the exact values of the sides indicated, without using a calculator

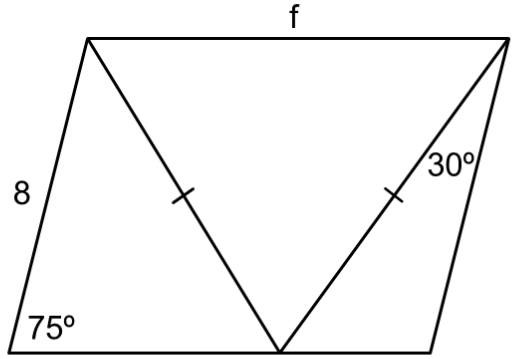
(1)



(2)

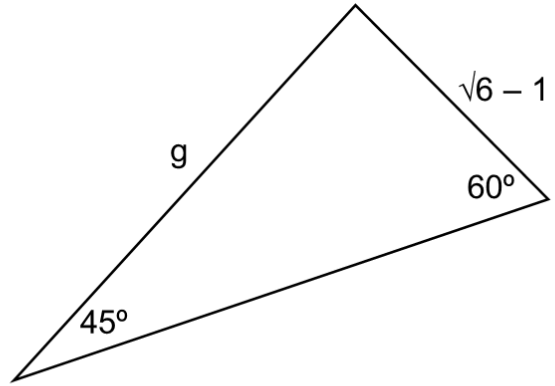


(3)



a parallelogram with an isosceles triangle

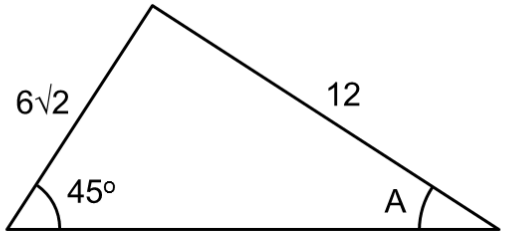
(4)



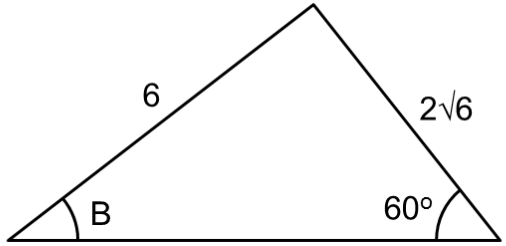
Fluency Practice

find the values of the angles, without using a calculator

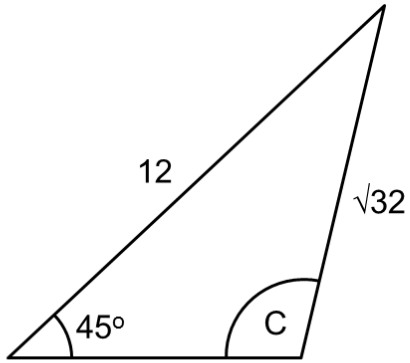
(5) find angle A



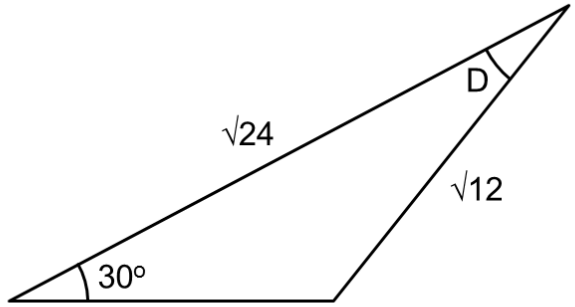
(6) find angle B



(7) find angle C



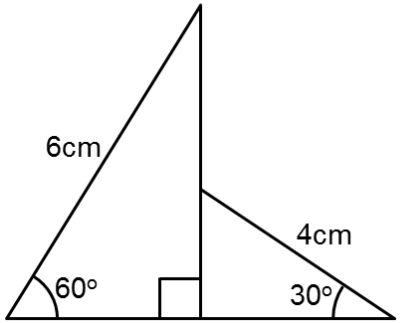
(8) find angle D



Fluency Practice

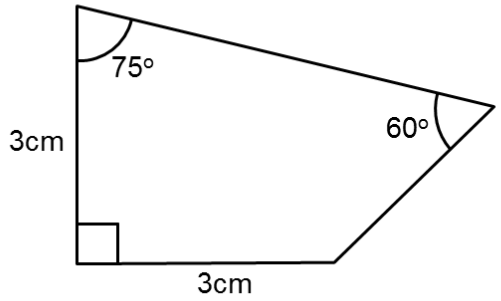
exact trigonometric forms

(5)



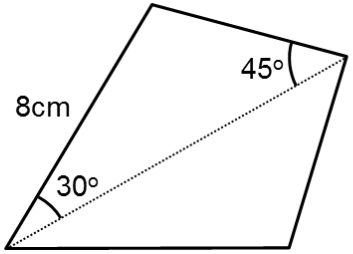
show that the perimeter of the quadrilateral is $11 + 5\sqrt{3}$

(6)



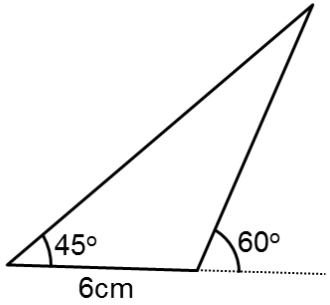
what is an exact value for
(i) the perimeter (ii) the area?
give your answers in the form $a + b\sqrt{c}$

(7)



what is an exact value for the area of the (symmetrical) kite?
give your answer in the form $a + a\sqrt{b}$

(8)

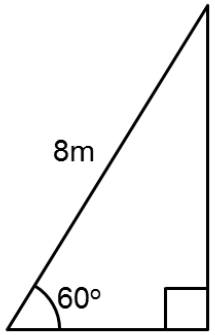


what is an exact value for the area of the triangle?
give your answer in the form $a + b\sqrt{c}$

Fluency Practice

exact trigonometric forms

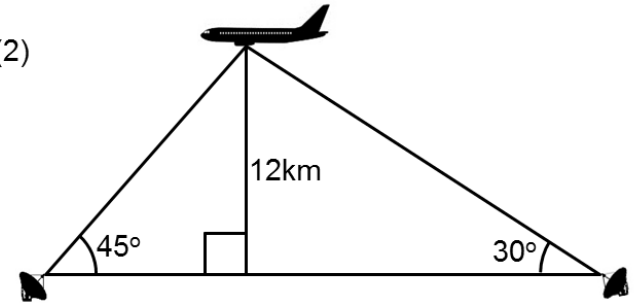
(1)



what is an exact value for the perimeter of the right angled triangle?

give your answer in the form a + b√c

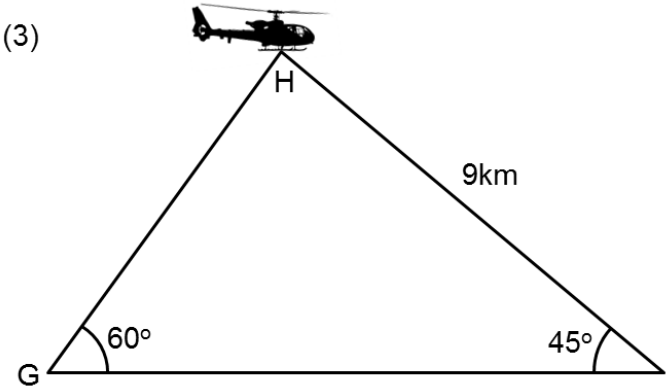
(2)



what is an exact value for the distance between the radar sites?

give your answer in the form a + b√c

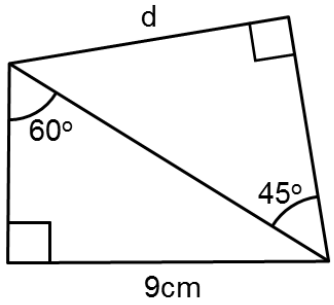
(3)



what is an exact value for the distance HG?

give your answer in the form a√b

(4)



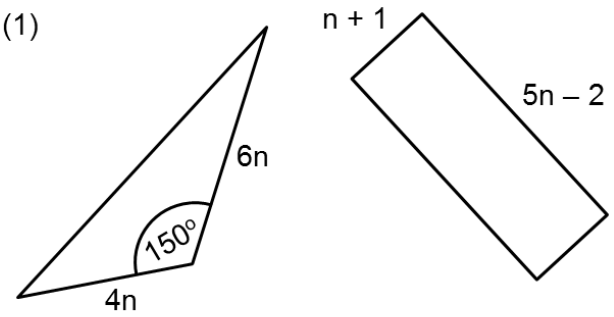
what is an exact value for the distance 'd' ?

give your answer in the form a√b

Purposeful Practice

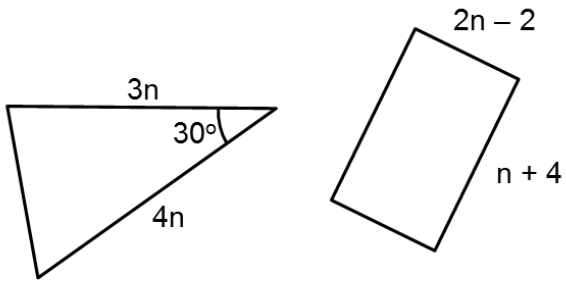
using exact trigonometric values for some angles

(1)



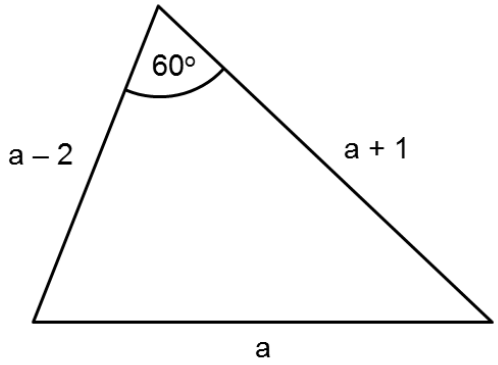
what values of 'n' make the area of the triangle and the area of the rectangle equal?

(2)



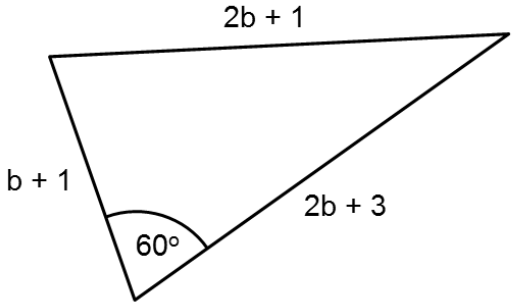
what values of 'n' make the area of the triangle and the area of the rectangle equal?

(3)



find the value of 'a'

(4)



find the value of 'b'

$\tan 15^\circ + \tan 75^\circ = 4$

$\tan 15^\circ \times \tan 75^\circ = 1$

find $\tan 15^\circ$
and $\tan 75^\circ$

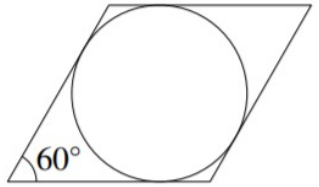
in surd
form

$$\cos(45^\circ) \times \cos(46^\circ) \times \cos(47^\circ) \times \ldots \times \cos(135^\circ) = ?$$

A circle with radius 1 touches the sides of a rhombus, as shown. Each of the smaller angles between the sides of the rhombus is 60°.

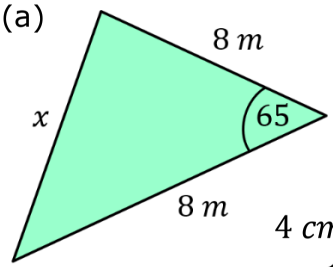
What is the area of the rhombus?

A 6 B 4 C $2\sqrt{3}$ D $3\sqrt{3}$ E $\frac{8\sqrt{3}}{3}$

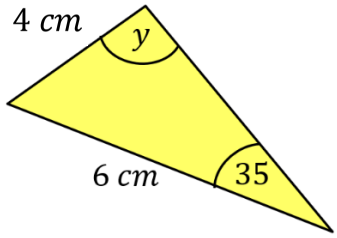


Find the missing length or angle.

(a)

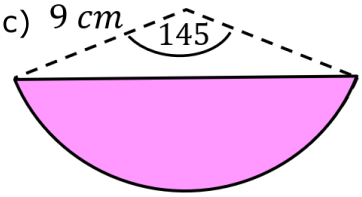


(b)

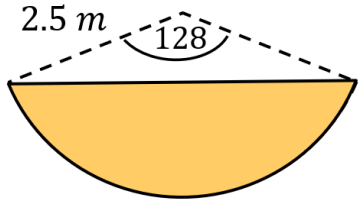


Find the area of the segment.

(c)

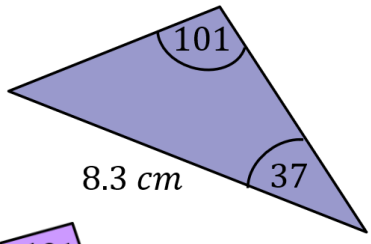


(d)

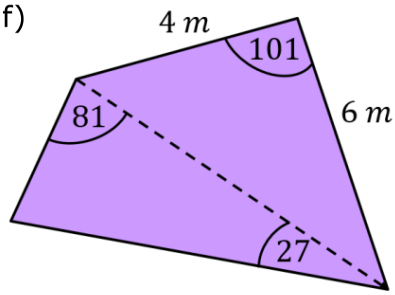


Find the perimeter of these shapes.

(e)



(f)



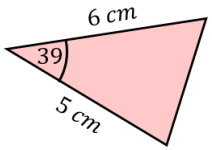
(g) The hands of a clock have lengths 8cm and 10 cm. The time on the clock is 5 o'clock. Find the distance between the tips of the two hands.

Fluency Practice

Sine and Cosine Rules Revision

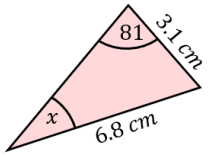
(a)

Find the area of the triangle.



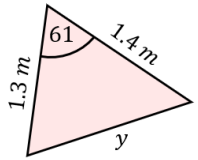
(b)

Find x to 1 decimal place.



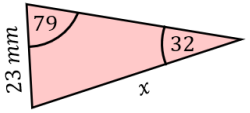
(c)

Find y to 2 decimal places.



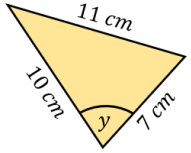
(d)

Find x to 3 significant figures.



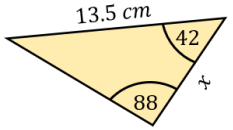
(e)

Find y to 1 decimal place.



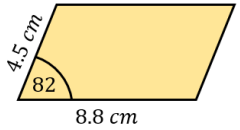
(f)

Find x to 1 decimal place.



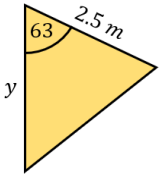
(g)

Find the area of the parallelogram.



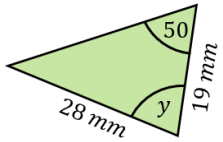
(h)

The area of the triangle is 4.6 m^2. Find y to 1 decimal place.



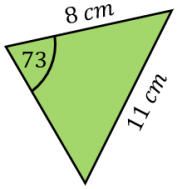
(i)

Find y to 3 significant figures.



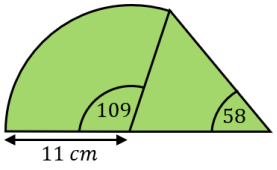
(j)

Find the area of the triangle.



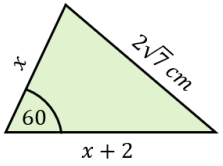
(k)

Find the area of the compound shape.

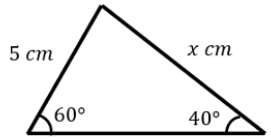
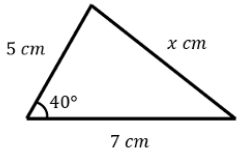
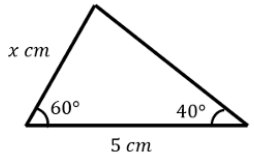
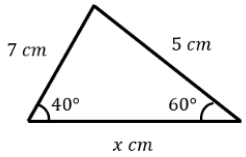


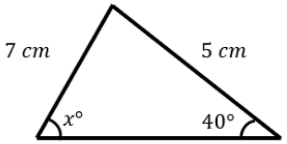
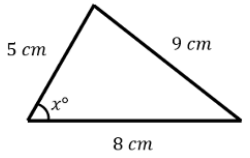
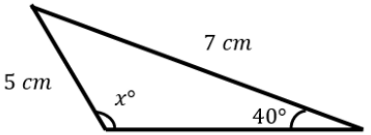
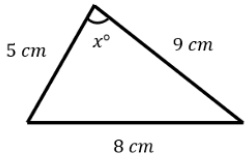
(l)

Find the value of x.



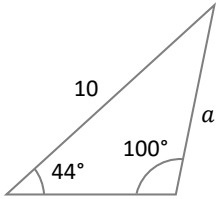
Fluency Practice

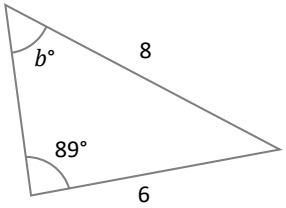
Q	Work out the missing length	Answers
1a		
2a		
3a		
4a		

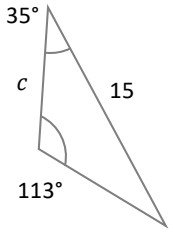
Q	Work out the missing angle	Answers
1b	x acute 	
2b		
3b	x obtuse 	
4b		

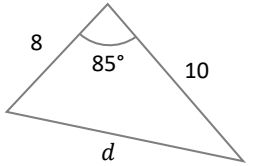
Fluency Practice

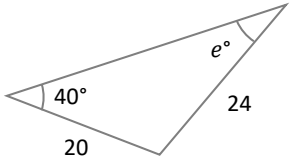
Sine & Cosine Rules Lengths are centimetres. not to scale

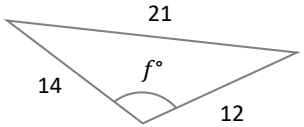


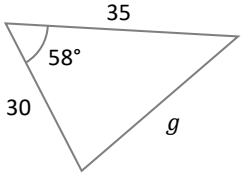


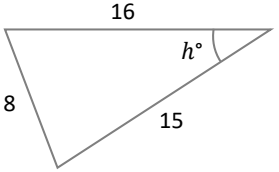


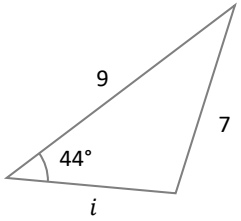


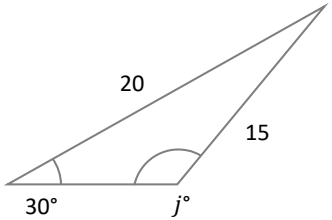


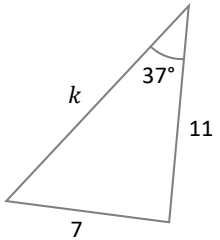


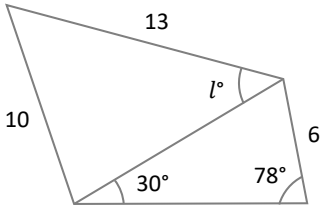








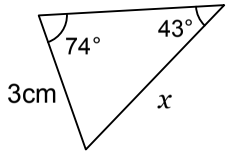




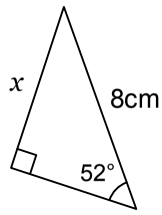
10 Answers	8.6	31.8	138.2	12.2	107.5
	9.6	48.6	32.4	29.7	7.1

1. Work out the value of x in each triangle. Give your answers correct to 2 significant figures.

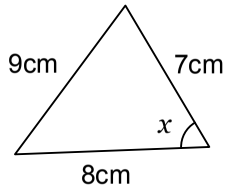
a)



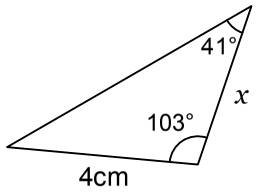
b)



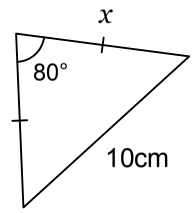
c)



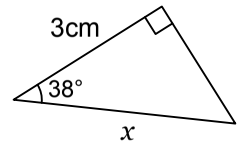
d)



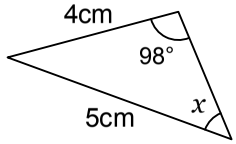
e)



f)



g)



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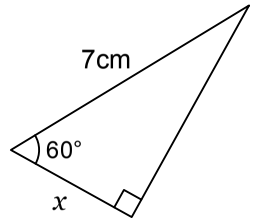
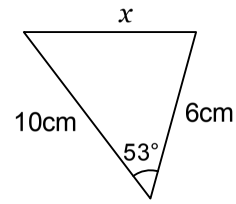
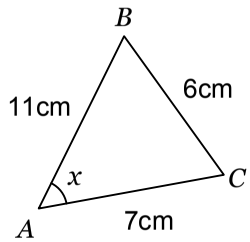


i)



2. Work out the value of x in each triangle. Give your answers correct to 2 significant figures.

a)



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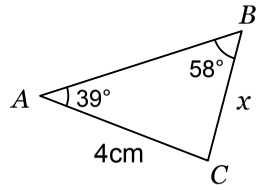

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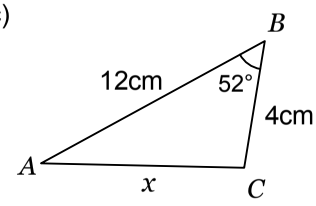
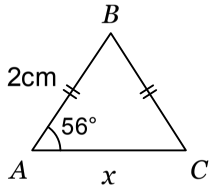
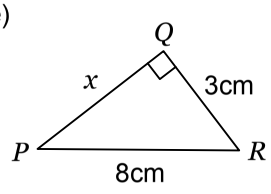


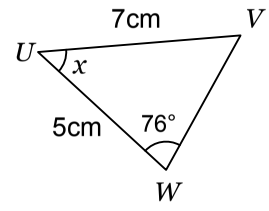
d)



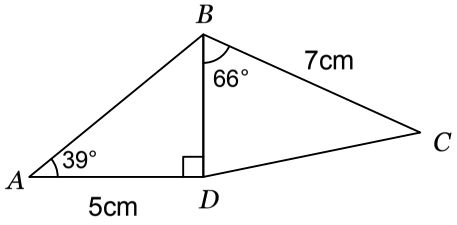
e)



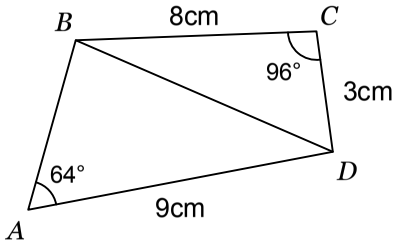
f)



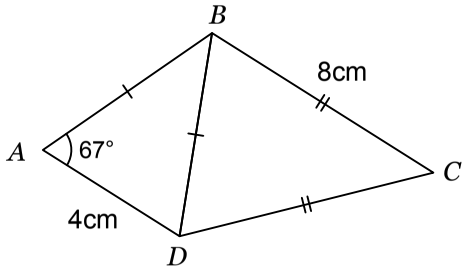
3. Work out the length CD. Round your answer to the nearest 0.1cm.



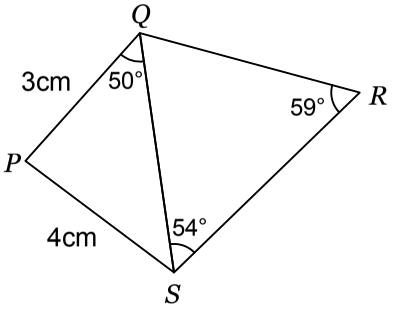
4. Work out $A\hat{B}D$. Round your answer to the nearest degree.



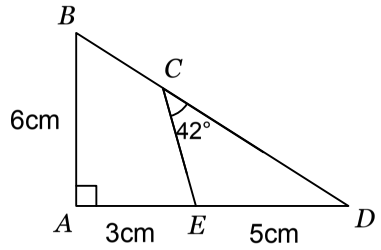
5. Work out the $B\hat{C}D$ correct to the nearest degree.



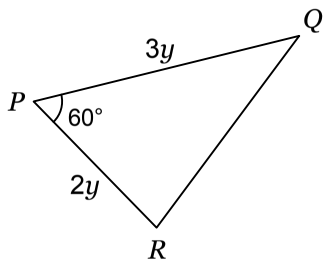
6. Work out RS. Round your answer to the nearest 0.1cm.



7. BCD and AED are straight lines.
AE = 3cm and ED = 5cm.
Work out CE correct to 2 significant figures.

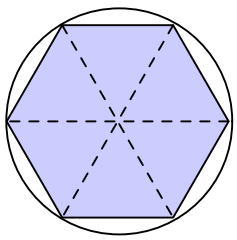


8. Show that $QR = y\sqrt{7}$.

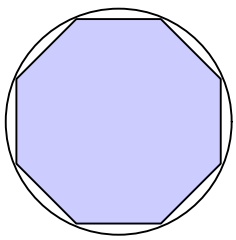


9. Each regular polygon is inscribed in a circle with radius 1cm. Work out the area of each polygon, correct to the nearest 0.01cm^2.

a)



b)



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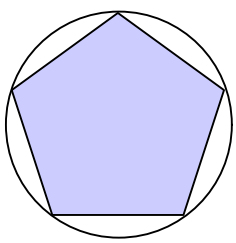
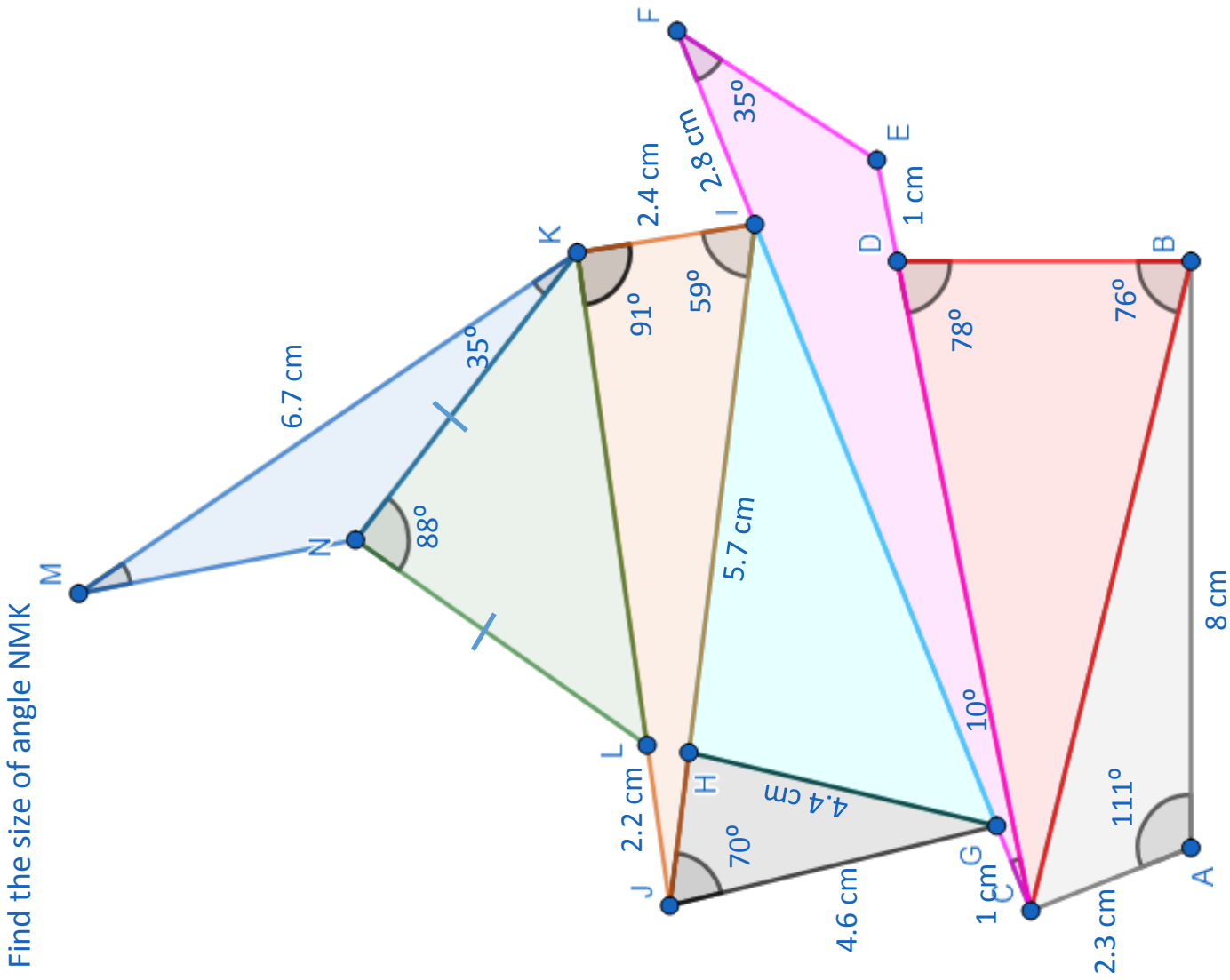


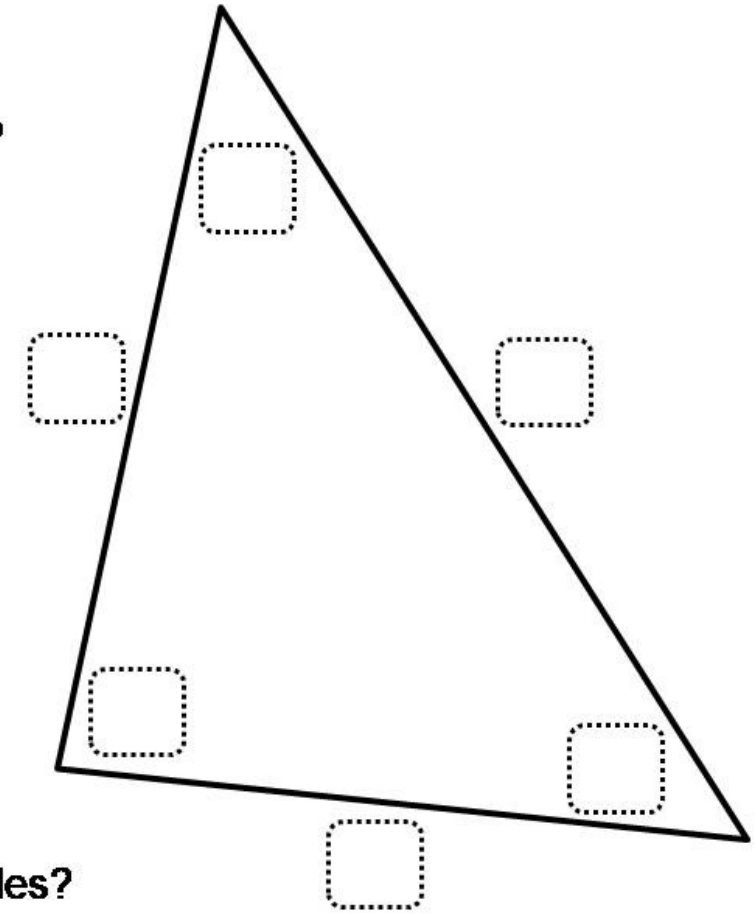
Purposeful Practice



Problem Solving

Position the three numbers as either:
- a side length in cm
- an angle size in degrees.

90 40 50



Can you always work out
the other three measurements?

Try to create scenarios
that require one or more of:

- SOHCAHTOA
- Sine rule
- Cosine rule
- Pythagoras' theorem

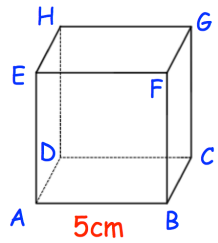
Are there any impossible triangles?

3 3D Pythagoras' Theorem and Trigonometry

Question 1: ABCDEFGH is a cube with side length 5cm.

(a) Work out the length of AC

(b) Work out the length of AG



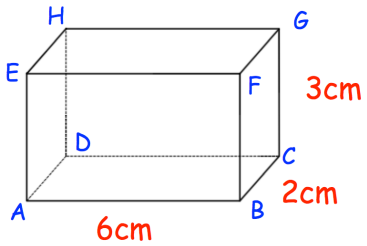
Question 2: ABCDEFGH is a cuboid.
AB = 6cm, BC = 2cm and CG = 3cm.

(a) Work out the length of BG

(b) Work out the length of BD

(c) Work out the length of HC

(d) Work out the length of AG

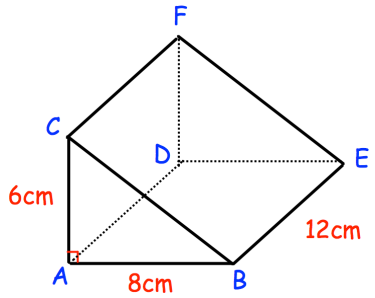


Question 3: Shown is a triangular prism.
Triangle ABC is a right angle triangle.

(a) Work out the length of BC

(b) Work out the length of CD

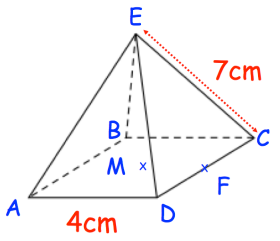
(c) Work out the length of BF



Question 4: Shown is a square based pyramid ABCDE.
F is the midpoint of CD.
M is the point on the base directly below the vertex E.
AD = 4cm and CE = 7cm

Calculate the length of

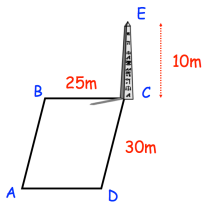
(a) AC (b) AM (c) EM (d) EF



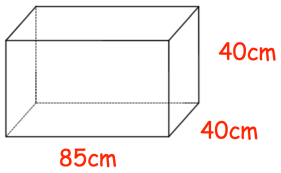
Apply

Question 1: A rectangular field is 30m long and 25m wide.
An obelisk is situated in the corner of the field.

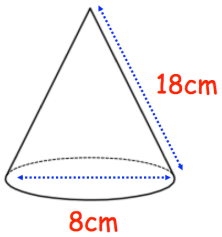
Calculate the length AE.



Question 2: Can a 1m rod fit inside this box?
Explain your answer.



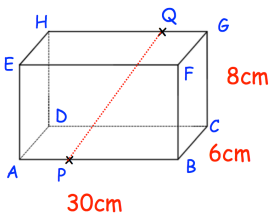
Question 3: Calculate the volume of this cone.



Question 4: ABCDEFGH is a cuboid

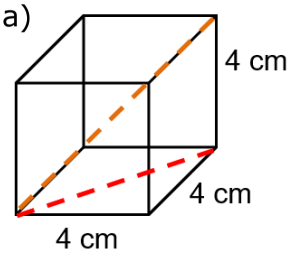
P is a point on AB, such that AP:PB is 1:2
Q is a point on GH, such that GQ:QH is 2:3

Calculate the distance PQ.

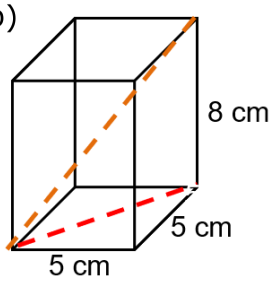


Find the length of the diagonal to 1dp.

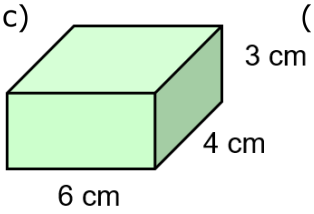
(a)



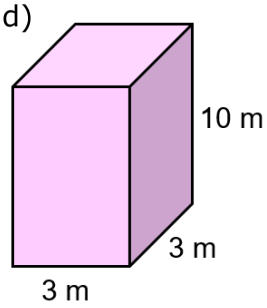
(b)



(c)

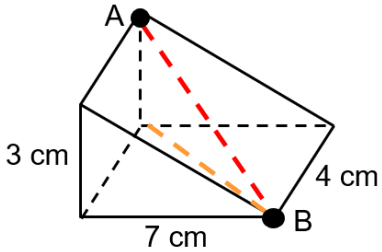


(d)

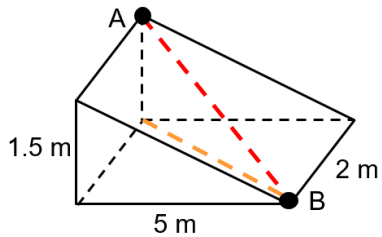


Find the distance AB to 1dp.

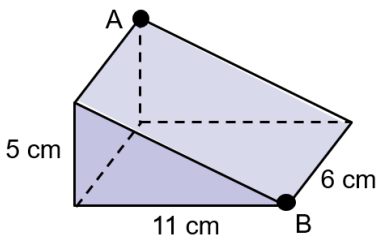
(a)



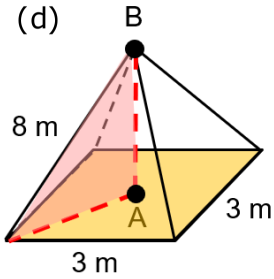
(b)



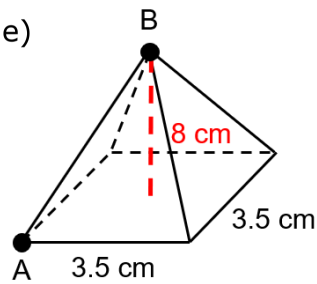
(c)



(d)



(e)

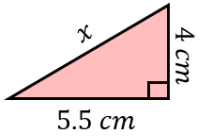


Purposeful Practice

Pythagoras' Theorem Revision

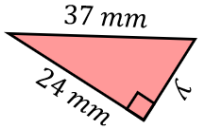
(a)

Find the value of x to 1 decimal place.



(b)

Find the value of y to 3 significant figures.

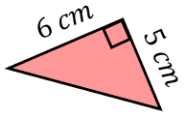


(c)

A triangle has sides of lengths $10.5\ cm$, $12\ cm$ and $16.5\ cm$. Is the triangle right-angled? Explain your answer.

(d)

Find the perimeter of the triangle to 3 significant figures.

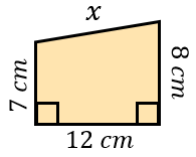


(e)

Find the distance between the coordinates $(1, 6)$ and $(3, 2)$, giving an exact answer.

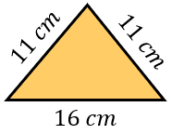
(f)

Find the value of x to 3 significant figures.



(g)

Find the area of the isosceles triangle to 1 decimal place.

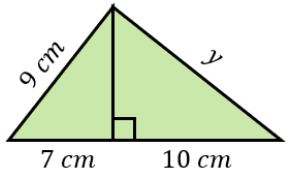


(h)

From point A, a boat sails 80 km east. It then turns and sails 110 km south to point B. Find the distance AB to the nearest km.

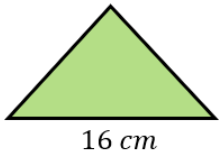
(i)

Find the value of y to 3 significant figures.



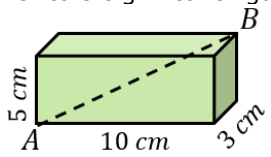
(j)

The area of the isosceles triangle is $40\ cm^2$. Find the perimeter of the triangle, to 1 decimal place.



(k)

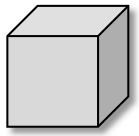
Find the length of the line AB, giving your answer to 3 significant figures.



Purposeful Practice

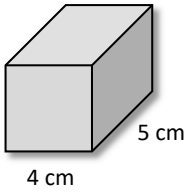
3D-Pythag

1)



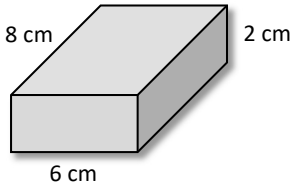
Find the diagonal length within this cube.

2)



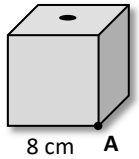
Find the diagonal length within this square-based prism.

3)



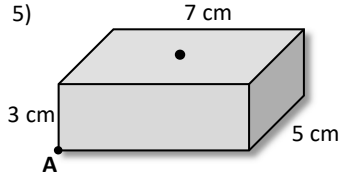
Find the diagonal length within this cuboid.

4)



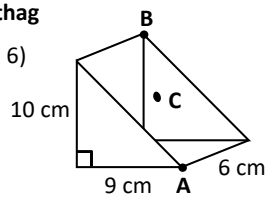
The centre of the top face of this cube has been marked.
Find the distance from **A** to this point.

5)



Find the distance from **A** to the centre of the top face.

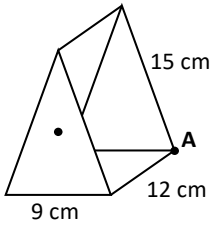
6)



Point **C** is equidistant between **A** & **B** on the surface of this prism.

Find distance **AC**.

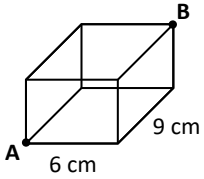
7)



The point halfway up the front face of this isosceles-triangle-based prism has been marked.

Find its distance from **A**.

8)



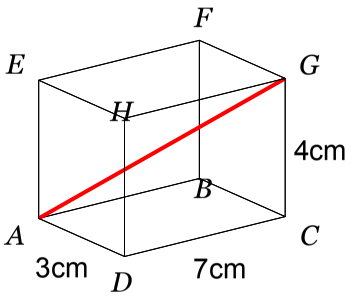
Length **AB** is 12 cm.
Find the volume of the cuboid.

9)

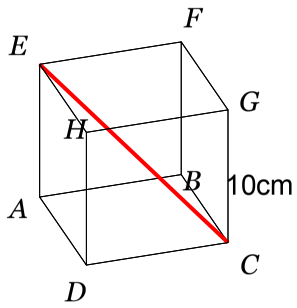
The diagonal within this cube is 12 cm.
Find the volume of the shape.

10) The sides of a cuboid are in the ratio 1:2:3
The diagonal within the shape is 38 cm long.
Find the volume of the cuboid.

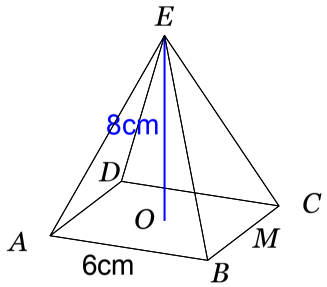
1. $ABCDEFGH$ is a cuboid.
Work out the length of AG.



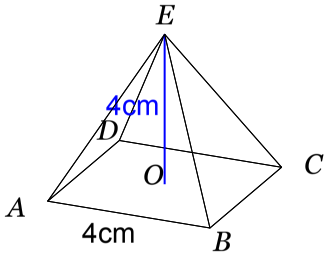
2. $ABCDEFGH$ is a cube.
Work out the length of CE.



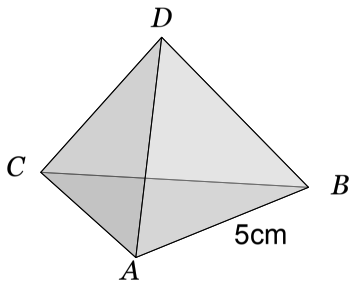
3. $ABCDE$ is a square-based pyramid with perpendicular height 8cm.
The apex of the pyramid is directly above the centre of the square, O.
M is the midpoint of BC. Work out the length of EM.



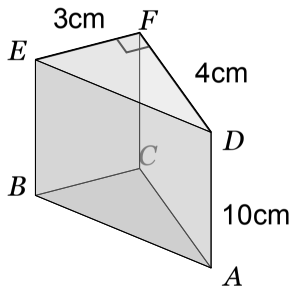
4. $ABCDE$ is a square-based pyramid with perpendicular height 4cm.
The apex of the pyramid is directly above the centre of the square, O.
Work out the length of EC.



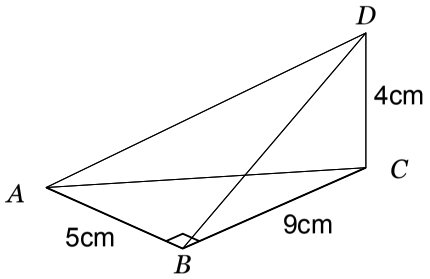
5. $ABCD$ is a regular tetrahedron with each edge length equal to 5cm.
M is the midpoint of AC. Work out the exact length of DM.



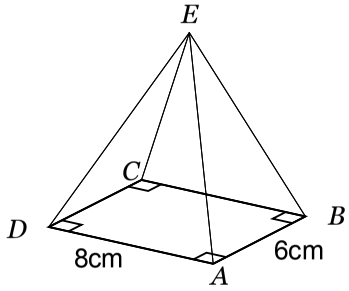
6. A triangular prism is shown.
Work out the length AE.



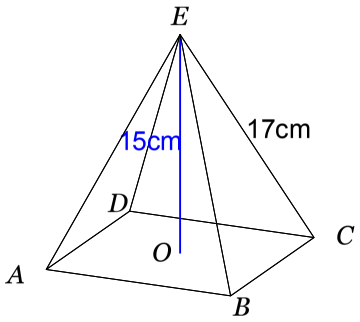
7. $ABCD$ is a skew pyramid.
D is directly above C.
$A\hat{B}C = 90°$.
Work out the length of AD.



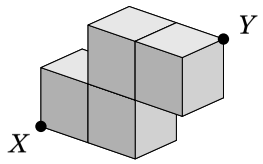
8. A rectangular-based pyramid has a perpendicular height 9cm. The apex of the pyramid is directly above the centre of the rectangle. Work out the length BE.



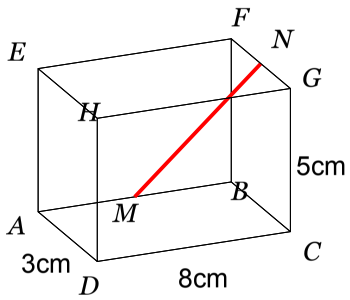
9. Square-based pyramid $ABCDE$ has a perpendicular height of 15cm. The apex of the pyramid is directly above the centre of the square. CE = 17cm.
Work out the length AB.



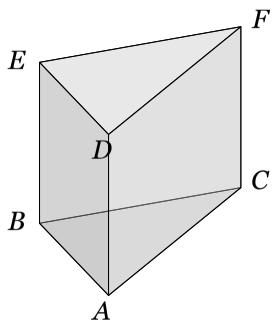
10. Four cubes, each with edge lengths of 3cm are arranged as shown. Work out the straight-line distance from X to Y.



11. In the cuboid, M is the midpoint of AB and N is the midpoint of FG.
Work out the length MN.

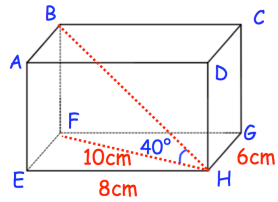


12. In the triangular prism, AC = 8cm, BC = 8cm, ED = 4cm, BE = 7cm and M is the midpoint of AB.
Work out the length MF.



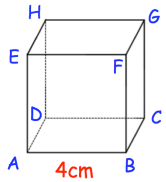
Question 1: Here is a cuboid.

EH = 8cm, GH = 6cm, FH = 10cm and ∠FHB = 40°



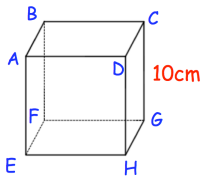
(a) Work out the length BF
(b) Work out the length BH
(c) Work out the size of angle FBH

Question 2: Here is a cube with side length 4cm.



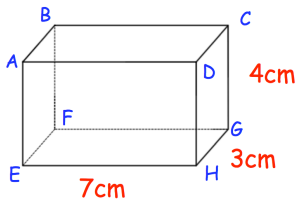
(a) Calculate the length AC
(b) Calculate the size of angle CAG

Question 3: Shown is a cube with side length 10cm



(a) Calculate the length of FH.
(b) Calculate the size of angle DFH

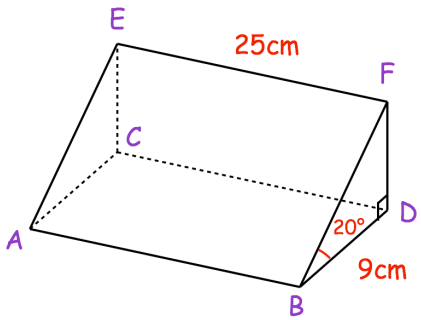
Question 4: Shown is a cuboid.



(a) Work out the size of angle GEH
(b) Work out the size of angle CHG
(c) Work out the length EG
(d) Work out the size of angle CEG

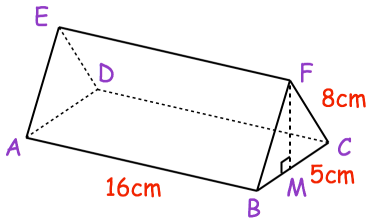
Question 5: ABCDEF is a triangular prism

BD = 9cm, EF = 25cm, ∠BDF = 90° and ∠DBF = 20°



(a) Work out the length DF
(b) Work out the length BF
(c) Work out the length BC
(d) Work out the size of angle CBE

Question 6: Shown is a triangular prism.



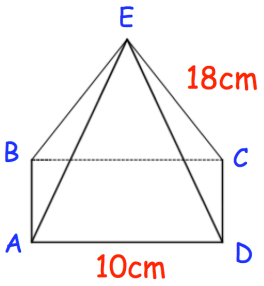
M is the midpoint of BC.
AB = 16cm, CM = 5cm, CF = 8cm and ∠BMF = 90°

Calculate

(a) Length FM (b) Length BE (c) Length BD

(d) Angle CBD (e) Angle ABE (f) Angle DMF

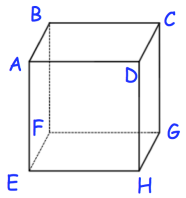
Question 7: Shown is a square-based pyramid.
The apex E is directly over the centre of the base.



AD = 10cm and CE = 18cm

(a) Work out the length of AC
(b) Calculate angle CAE
(c) Work out the height of the pyramid.

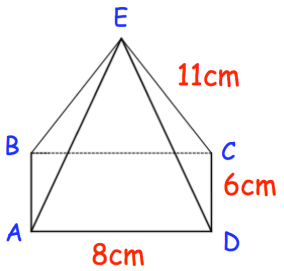
Question 8: Shown is a cuboid.



FG = 5.5cm DH = 6.2cm Angle FHG = 47°

Calculate the angle between DF and the plane EFGH.

Question 9: Shown below is a rectangular-based pyramid.
The apex E is directly over the centre of the base.



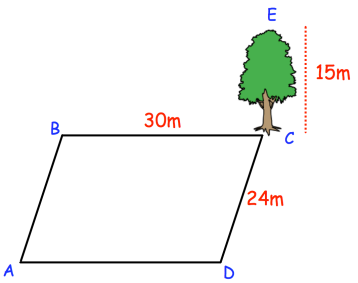
AD = 8cm CD = 6cm CE = 11cm

(a) Calculate the height of the pyramid.
(b) Calculate the angle between face ABE and the base ABCD.

Apply

Question 1: A tree is located in the corner of a rectangular field.
The field is 30 metres long and 24 metres wide.
The tree is 15 metres tall.

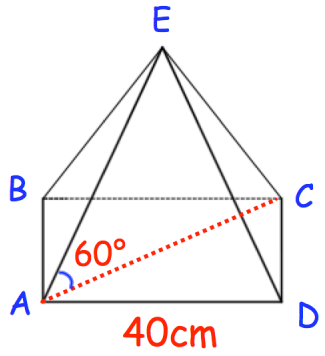
Calculate angle CAE.



Purposeful Practice

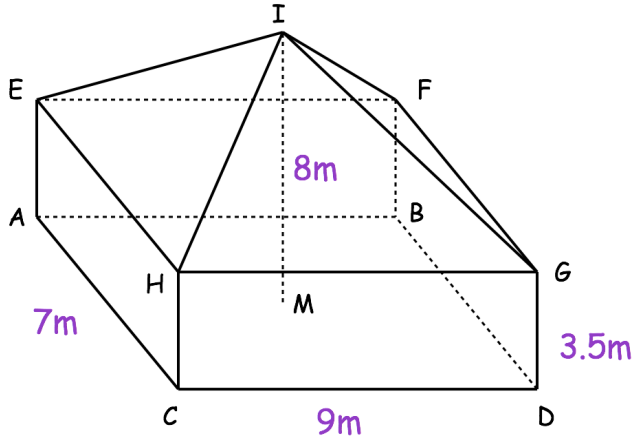
Question 2: Here is a square-based pyramid
The apex E is directly over the centre of the base.

Calculate the volume of the pyramid.



Question 3: The diagram shows a cuboid and a pyramid.
The apex of the pyramid, I, is directly above the centre, M, of ABCD.

AC = 7m CD = 9m DG = 3.5m
IM = 8m



(a) Calculate the angle between EI and the plane EFGH.

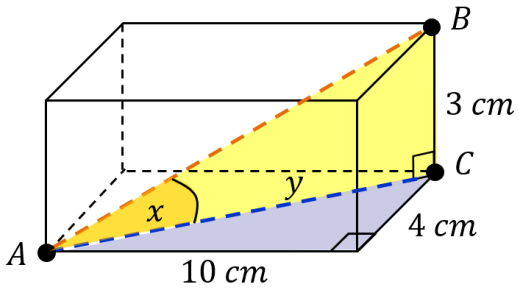
(b) Calculate the angle between AI and plane ABCD.

(c) Calculate the angle between the planes FGI and EFGH

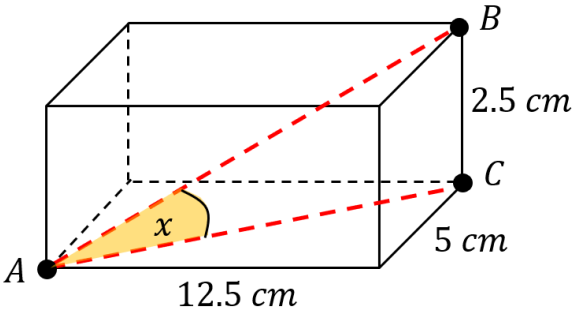
(d) Calculate the angle between the planes EHI and ACEH

Find the angle $\widehat{BAC}$

(a)

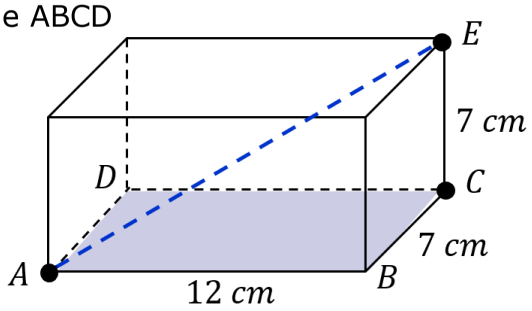


(b)

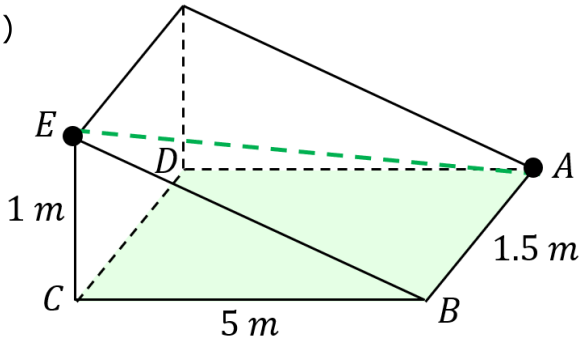


Find the angle between the line EA and the plane ABCD

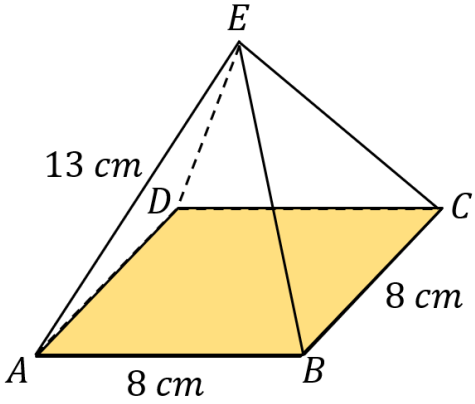
(a)



(b)



(c)

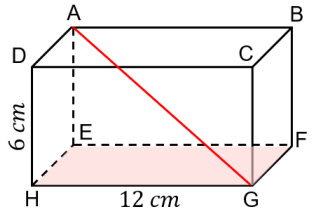


Purposeful Practice

3D Pythagoras and Trigonometry

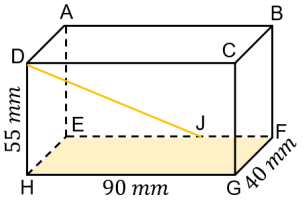
(a)

The volume of the cuboid ABCDEFGH is $360\ cm^3$. Find:
(a) the length of AD
(b) the length of AG
(c) the angle between AG and the plane EFGH



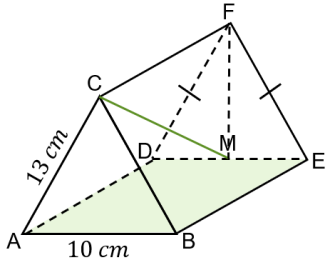
(b)

The point J splits the edge EF in the ratio $2:1$. Find:
(a) the length HJ
(b) the length DJ
(c) the angle HDJ
(d) The angle between DJ and the plane EFGH



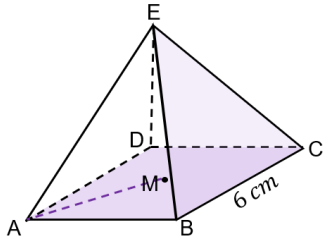
(c)

ABCDEF is a triangular prism with a surface area of $660\ cm^2$.
M is the midpoint of DE. Find:
(a) the length of MF
(b) the length of BE
(c) the length of CM
(d) the angle between CM and the plane ABED

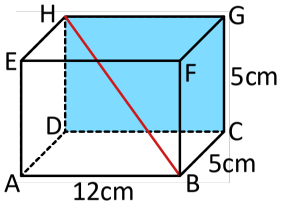


(d)

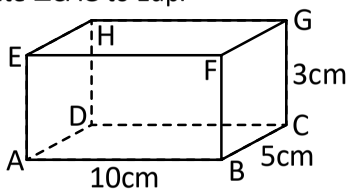
The volume of the square-based pyramid ABCDE is $180\ cm^3$.
M is the centre of the base and is vertically below E. Find:
(a) the height of the pyramid ME
(b) the length of AE
(c) the angle EAM
(d) the angle between the planes BCE and ABCD



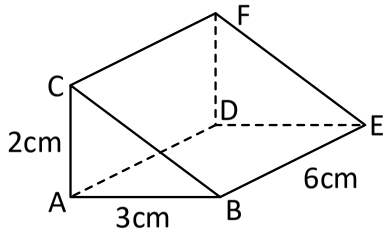
Q1 ABCDEFGH is a cuboid. AB = 12cm and BC = CG = 5cm.
Calculate the angle between:
[a] the line BH and the plane DCGH.
[b] the line BH and the plane ADHE.



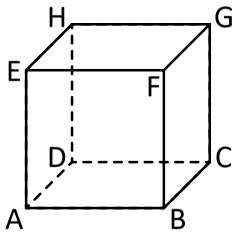
Q2 ABCDEFGH is a cuboid.
AB = 10cm, BC = 5cm and CG = 3cm.
[a] Calculate ∠FEG to 1dp.
[b] Calculate ∠DAH to 1dp.
[c] Calculate ∠DCH to 1dp.
[d] Calculate ∠CAG to 1dp.



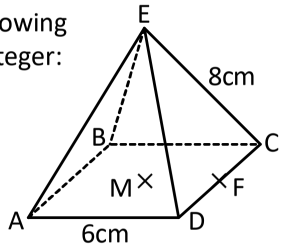
Q3 Shown is a triangular prism.
Triangle ABC is a right angle triangle.
[a] Calculate ∠ACB to 1dp.
[b] Calculate ∠EBF to 1dp.
[c] Calculate ∠AEC to 1dp.



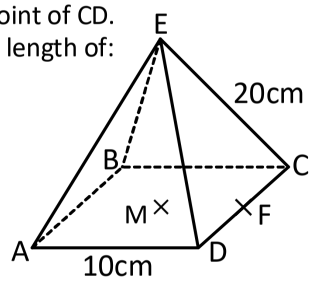
Q4 Ruth says that because a cube has all equal sides, ∠CAG = 45°. Show that Ruth is incorrect.



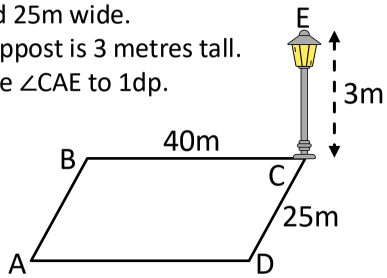
Q5 Shown is a square based pyramid, ABCDE.
F is the midpoint of CD. M is the point directly below the vertex E.
Calculate the following to the nearest integer:
[a] ∠MAE
[b] ∠MEC
[c] ∠AEC
[d] ∠BFC



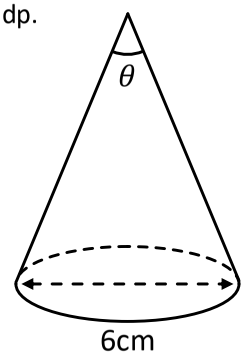
Q6 Shown is a square based pyramid, ABCDE.
F is the midpoint of CD.
Calculate the length of:
[a] ∠BED
[b] ∠FBC
[c] ∠BFD



Q7 A lamppost is located in the corner of a rectangular car park. The car park is 40m long and 25m wide.
The lamppost is 3 metres tall.
Calculate ∠CAE to 1dp.

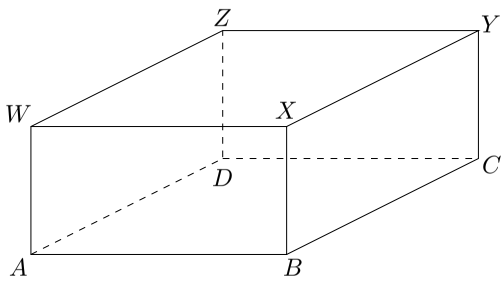


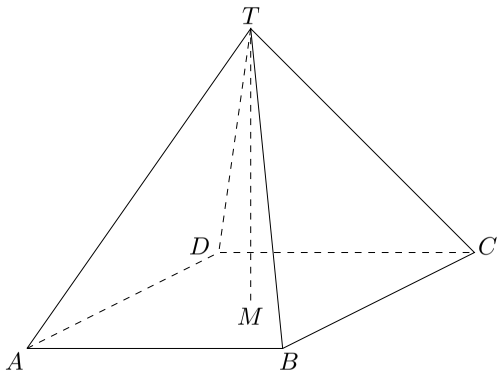
Q8 Shown is a cone with volume $30\pi cm^3$.
The diameter of the base is 6cm.
Calculate the angle, θ, made at the apex to 1dp.

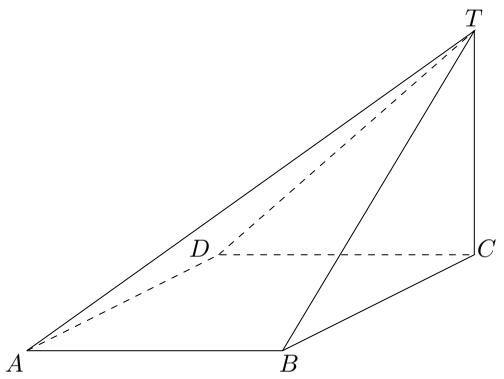


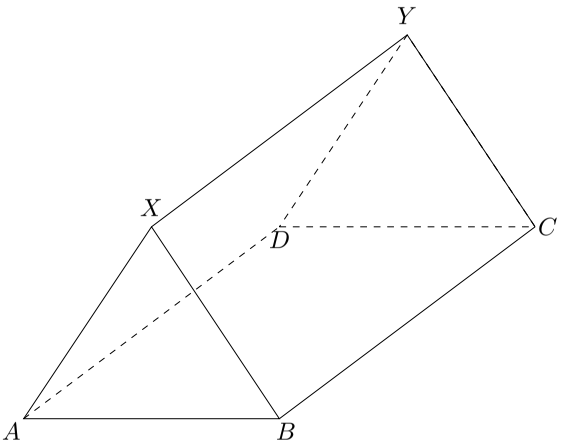
Patrons are given questions on four types of shapes:

1. A CUBOID.
2. A PYRAMID. (A rectangular base such that the vertex lies above the mid-point of the rectangle.)
3. A YANGMA. (A rectangular based pyramid such that the vertex lies above one of the rectangle's corners.)
4. A PRISM. (A prism with triangular cross-section.)









Please give all non-exact answers to three significant figures.

Questions

1. CUBOID. $AB = 7$ $BC = 6$ $CY = 4$.
   (a) Find the length AY.
   (b) Find the angle the diagonal AY makes with the plane $ABCD$.
   (c) Find the angle the diagonal AY makes with the plane $ADZW$.
2. PYRAMID. $AB = 8$ $BC = 6$ $CT = 9$.
   (a) Find the length AC.
   (b) Find the length MT.
   (c) Find the angle CT makes with the plane $ABCD$.
   (d) Find the angle $A\hat{T}C$.
   (e) Find the angle between the planes $ABCD$ and ADT.

3. Yangma. $AB = 11$ $BC = 11$ $CT = 5$.
   (a) Find the length BT.
   (b) Find the length AT.
   (c) Find the angle AT makes with the plane $ABCD$.
   (d) Find the angle between the planes ABT and $ABCD$.

4. Prism. $AB = BX = XA = 8$ $BC = 12$.
   (a) Find the length XC.
   (b) Find the angle $C\hat{X}A$.
   (c) Find the angle that XC makes with the plane $ABCD$.

5. Cuboid. $AB = 9$ $BC = 12$ $AYC = 65°$.
   (a) Find the length CY.
   (b) Find the length AY.
   (c) Find the angle the diagonal WC makes with the plane $BCYX$.

6. Pyramid. $AB = 10$ $CAB = 50°$ $TAM = 40°$.
   (a) Find the length BC.
   (b) Find the length MT.
   (c) Find the length DT.
   (d) Find the angle between the planes $ABCD$ and ADT.

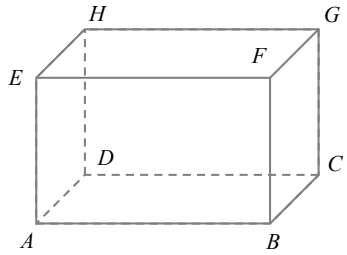
7. Yangma. $AB = 12$ $BC = 15$ $TAC = 28°$.
   (a) Find the length TC.
   (b) Find the length AT.
   (c) Find the angle BT makes with the plane $ABCD$.
   (d) Find the angle between the planes ADT and $ABCD$.

8. Prism. $AB = 6$ $BX = 7$ $ABX = 50°$ $BCX = 22°$.
   (a) Find the length BC.
   (b) Find the length XC.
   (c) Find the angle $A\hat{Y}B$.
   (d) Find the angle AY makes with the plane $ABCD$.

Fluency Practice

① *ABCDEFG* is a cuboid.

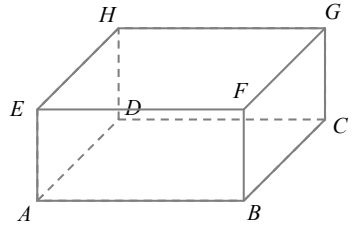
not to scale



GH = 5 cm
AE = 3 cm

Calculate angle *ABE*.

② *ABCDEFG* is a cuboid.



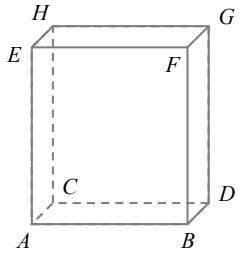
FB = 2 cm
EG = 9 cm
DC = 7 cm

Calculate angle *AEB*.
Calculate angle *EGH*.

Give answers correct to 2 decimal places.

ABCDEFGH

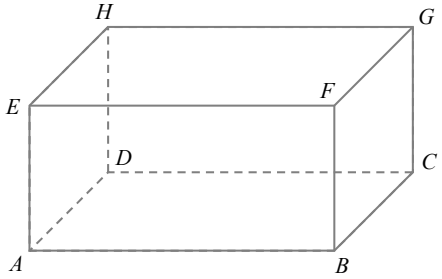
③ *ABCDEFG* is a cuboid.



HE = 1 cm
AB = 4 cm
Angle *ABE* = 50°

Calculate the length *AE*.
Calculate angle *GBD*.

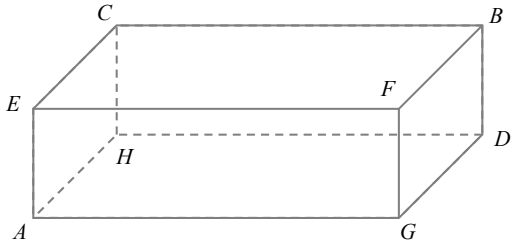
④ *ABCDEFG* is a cuboid.



GC = 4 cm
Angle *ABD* = 22°
CB = 5 cm

Calculate the length *DB*.
Calculate angle *DBH*.

⑤ *ABCDEFG* is a cuboid.

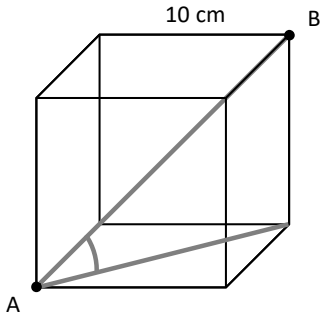


CF = 21 cm
GD = 6 cm
Angle *HGC* = 12°

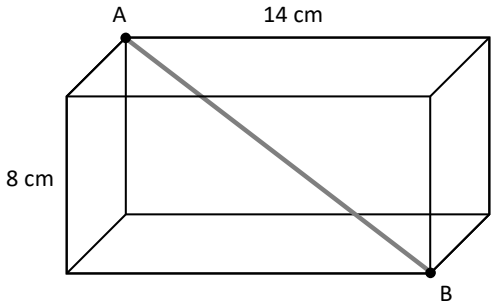
Calculate angle *GCD*.

Purposeful Practice

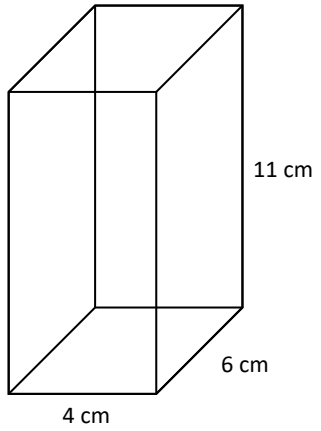
Angles of Elevation & Depression



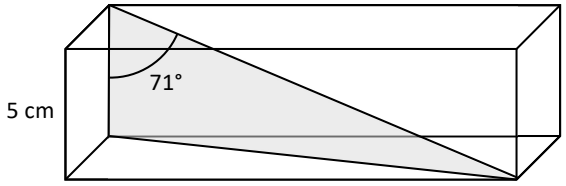
Find the angle of elevation of
the diagonal (A to B) within this cube.



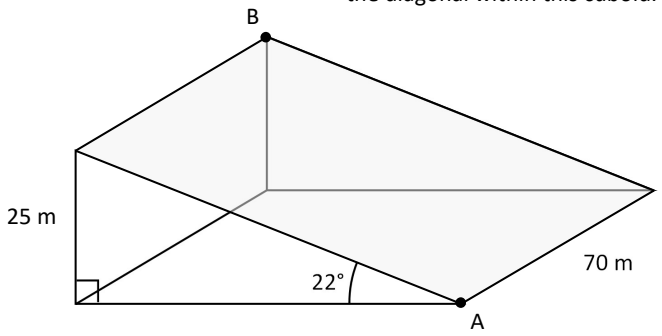
This is a square-based prism.
Find the angle of depression from A to B.



Find the angle of elevation of
the diagonal within this cuboid.



Find the volume of
this square-based prism.

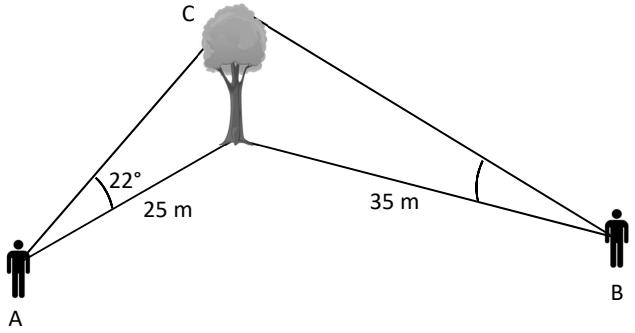


This triangular prism represents a hill with a 22° incline.
John walks diagonally up the hill from A to B.
At what angle of elevation does he walk?
Explain why this gradient is smaller than that of the hill.

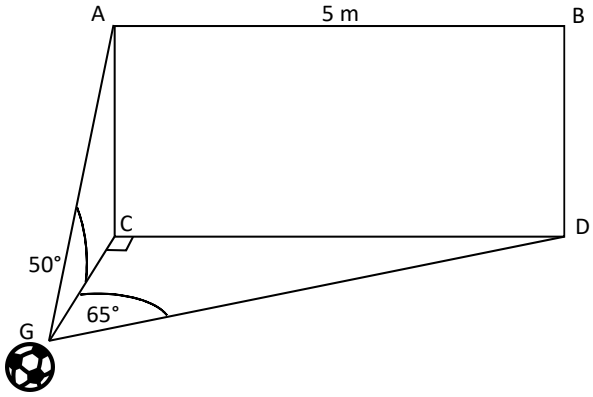
Purposeful Practice

3D Trigonometry

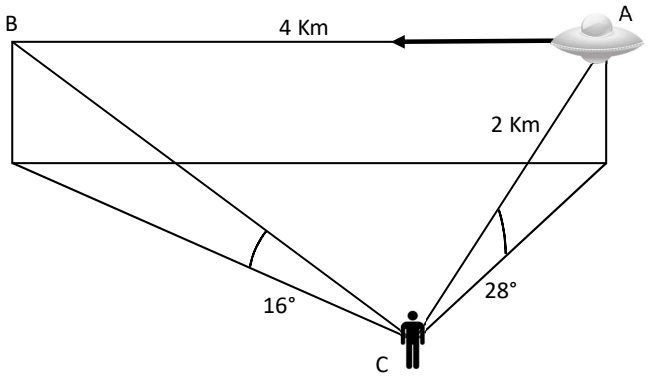
1) Ahmed is 25 metres from a tree.
The angle of elevation to the top of the tree (C) is 22°
Billy is 35 metres from the tree.
What is the angle of elevation to the top of the tree from where he is standing?



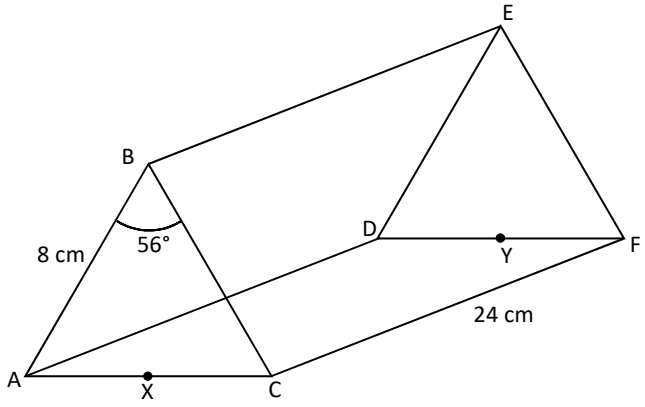
2) ABCD is a football goal 5 metres wide.
The ball (G) is directly in front of the left post.
Calculate the height of the goal



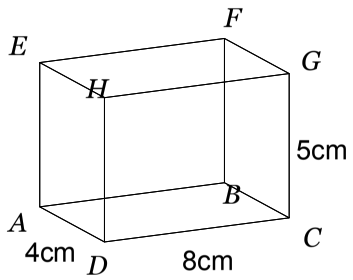
3) A UFO is flying at the same height for 4 km.
At A, it is 2 km away from C
How far away is the UFO from C when it is at B?



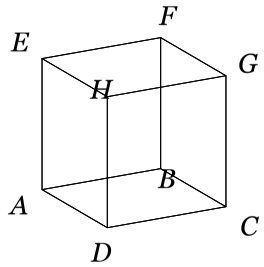
4) ABCDEF is an isosceles-triangle-based prism.
X is the midpoint of AC. Y is the midpoint of DF.
Find the angle of depression from B to Y



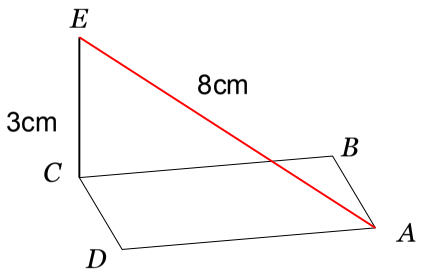
1. $ABCDEFGH$ is a cuboid.
Work out the value of $C\hat{A}G$.



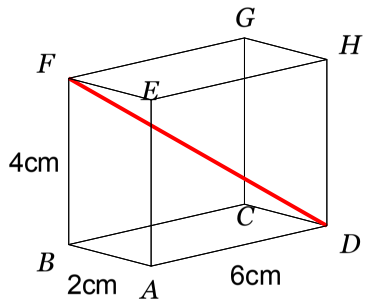
2. $ABCDEFGH$ is a cube.
Thomas says: "Angle ECA is 45°."
Annabel says: "No, it is less than that."
Work out who is correct.



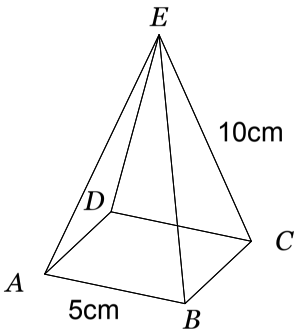
3. Point E is vertically above point C.
CE = 3cm and AE = 8cm.
Work out the angle between the line AE and the horizontal plane $ABCD$.



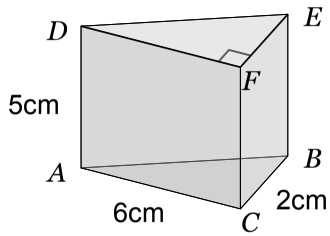
4. $ABCDEFGH$ is a cuboid.
Work out the angle between the line DF and the plane $DCGH$.



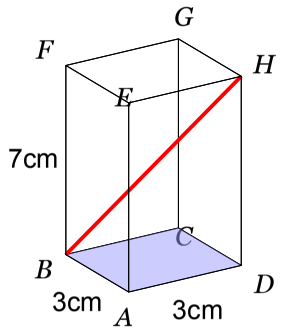
5. $ABCDE$ is a square-based pyramid.
The apex of the pyramid is directly above the centre of the square.
EC = 10cm. Work out the angle $A\hat{E}C$.



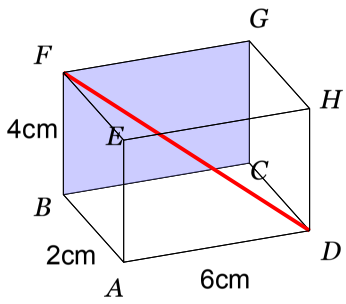
6. A triangular prism is shown.
Work out $D\hat{E}A$.



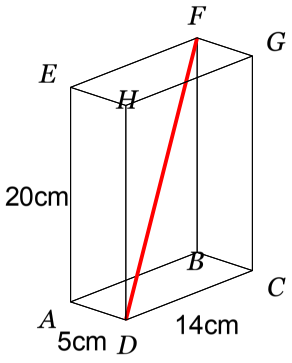
7. $ABCDEFGH$ is a cuboid.
Work out the angle between the line BH and the plane $ABCD$.



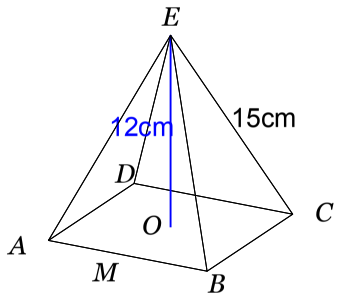
8. $ABCDEFGH$ is a cuboid.
Work out the angle between the line FD and the plane $BCGF$.



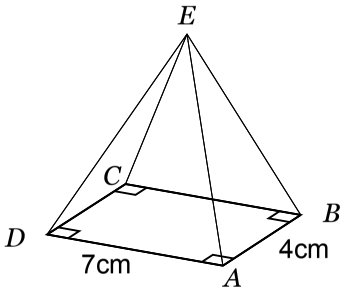
9. $ABCDEFGH$ is a cuboid.
Work out the angle between the line FD and the plane $ADEH$.



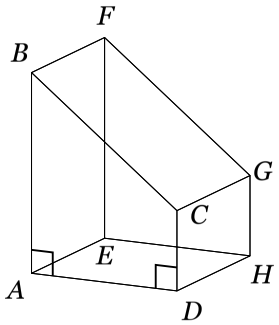
10. $ABCDE$ is a square based pyramid.
O is the centre of the square.
E is directly above O. M is the midpoint of AB, EO = 12cm and EC = 15cm.
Work out the angle $E\hat{M}O$.

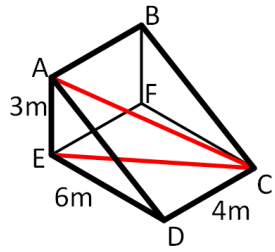
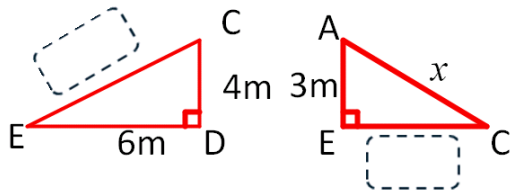
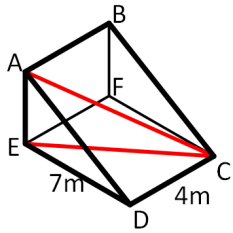
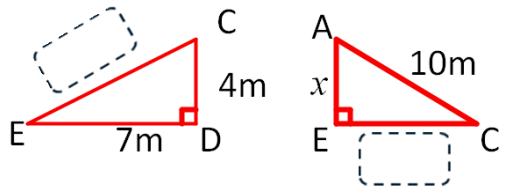
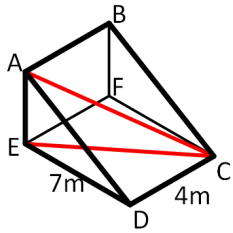
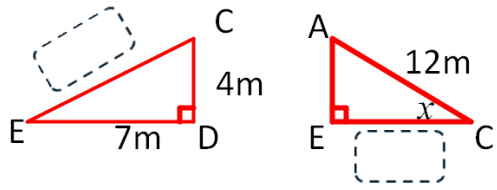
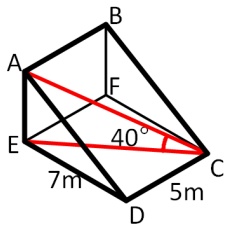
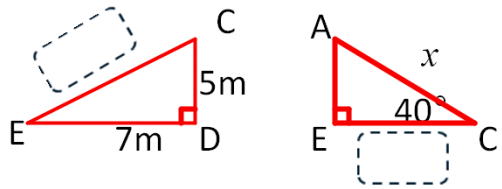
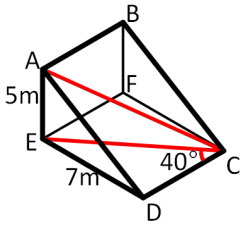
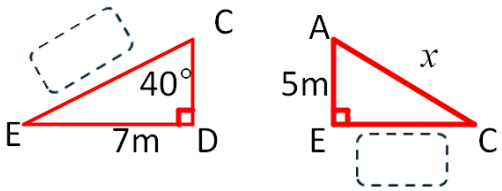


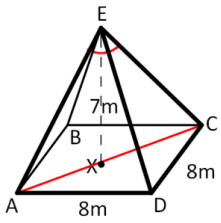
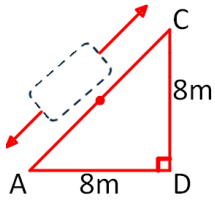
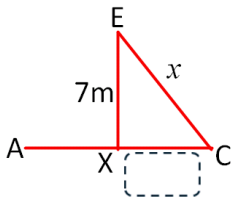
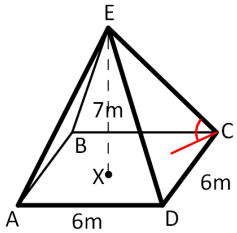
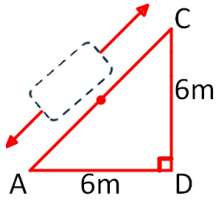
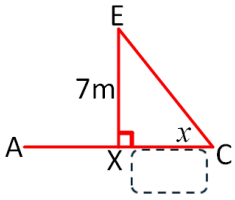
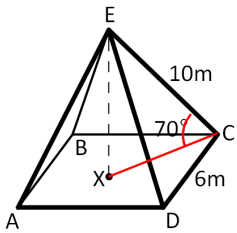
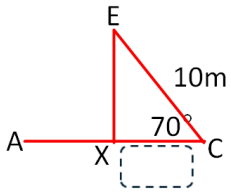
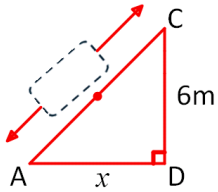
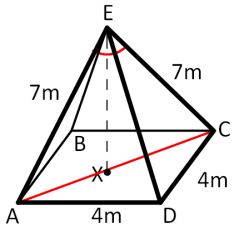
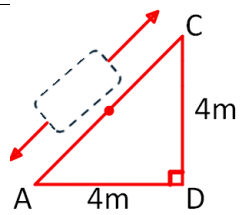
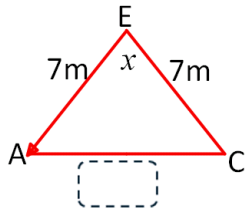
11. A rectangular-based pyramid has a perpendicular height 8cm. The apex of the pyramid is directly above the centre of the rectangle. Work out $D\hat{B}E$.



12. In the triapezoidal prism, AD = 6cm, AB = 9cm, CD = 2cm and DH = 4cm.
Work out the angle between the line that passes through the points B and G and the infinite plane that includes $ADHE$.



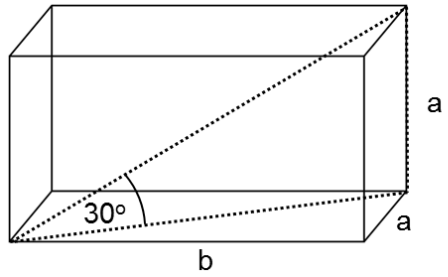
1	$AE = 3m$ $ED = 6m$ $DC = 4m$ Find the length AC.		
2	$AC = 10m$ $ED = 7m$ $DC = 4m$ Find the length AE.		
3	$AC = 12m$ $ED = 7m$ $DC = 4m$ Find the angle $\angle ACE$.		
4	Angle $\angle ACE = 40°$ $ED = 7m$ $DC = 5m$ Find the length AC.		
5	Angle $\angle DCE = 40°$ $ED = 7m$ $AE = 5m$ Find the length AC.		

6	$AD = 8m$ $DC = 8m$ E is $7m$ above the midpoint of AC. Find the length EC.		 
7	$AD = 6m$ $DC = 6m$ E is $7m$ above the midpoint of AC. Find the angle $\angle ACX$.		 
8	E is above the midpoint of AC. Angle $\angle ECX = 70°$ $DC = 6m$ $EC = 10m$ Find the length AD.		 
9	E is above the midpoint of AC. $AD = DC = 4m$ $EC = AE = 7m$ Find the angle $\angle AEC$. HINT: COSINE RULE $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$		 

1	2	3	4	5	6	7	8	9
7.81	5.92	47.8°	11.2m	12.0m	9m	58.8°	3.28	47.7°

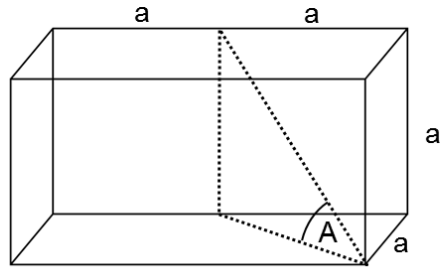
Purposeful Practice

the angle that the space diagonal makes with the base is 30°



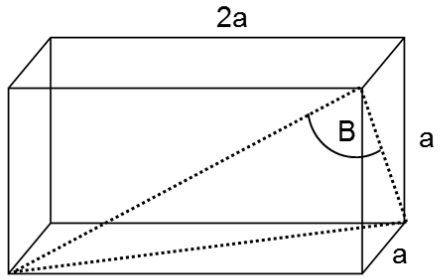
what is the relationship between 'a' and 'b'?

a cuboid with a square cross section is twice as long as it is wide



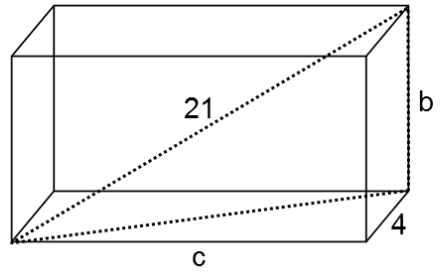
what is angle A (to 2 dp)?

a cuboid with a square cross section is twice as long as it is wide



what is angle B (to 2 dp)?

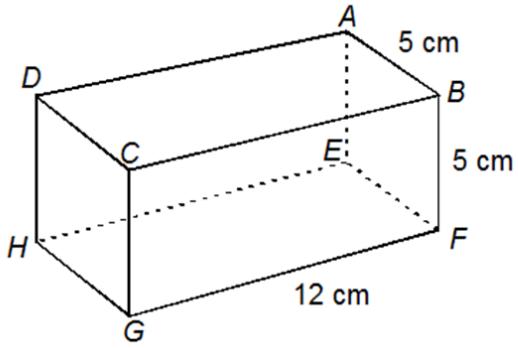
the cuboid has a space diagonal of length 21
the width of the cuboid is 4



what are b and c if:
(i) b + c = 25 (ii) b + c = 27 (iii) b + c = 29

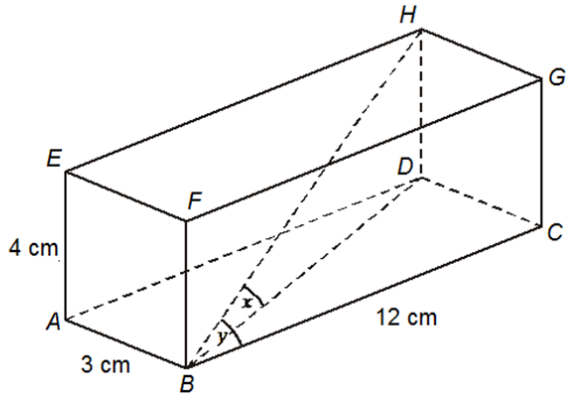
Purposeful Practice

(1)



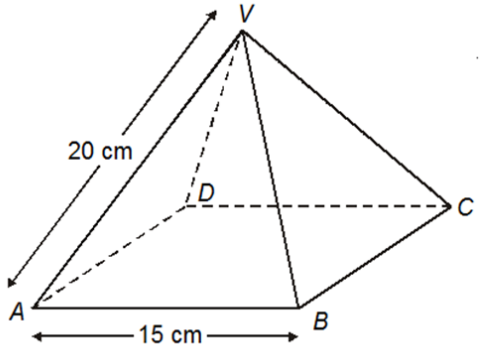
calculate angle *DFH* for a cuboid
with sides of 5 cm, 5 cm and l2 cm

(2)



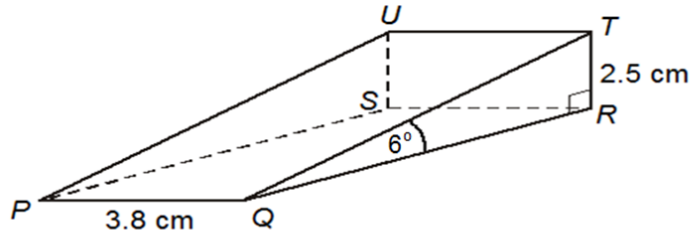
what are the sizes of the angles marked x (angle *HBD*) and y (angle *HBC*) for a cuboid
with sides of 3 cm, 4 cm and l2 cm ?

(3)



for the right, square-based pyramid shown,
calculate the angle between
the edge *VA* and the base, *ABCD*

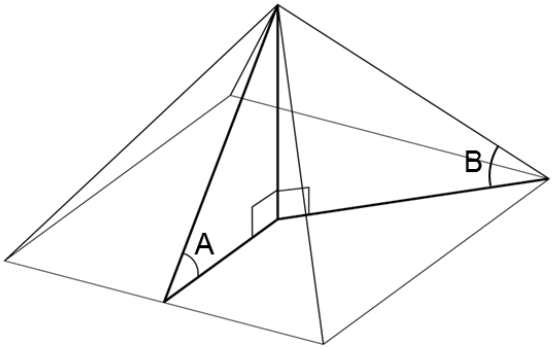
(4)



for the door wedge shape (triangular prism)
calculate the length of the diagonal *PT*

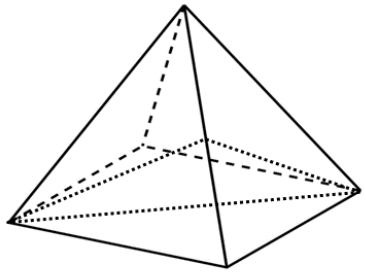
Purposeful Practice

a square base pyramid has a height of 6 and a base length of 16



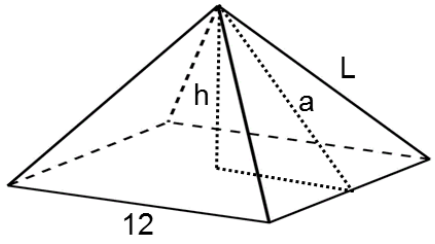
show that A – B is roughly 9°

a special square base pyramid with four equilateral triangles



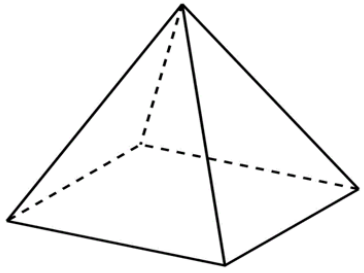
show that the obtuse angle between two of the triangular faces is 109.57°

square base length = 12, height = h
apothem = a, lateral edge = L



create a formula for L in terms of h
find L if (i) h = 3 (ii) h = 7 (iii) h = 16

a special square base pyramid with four equilateral triangles



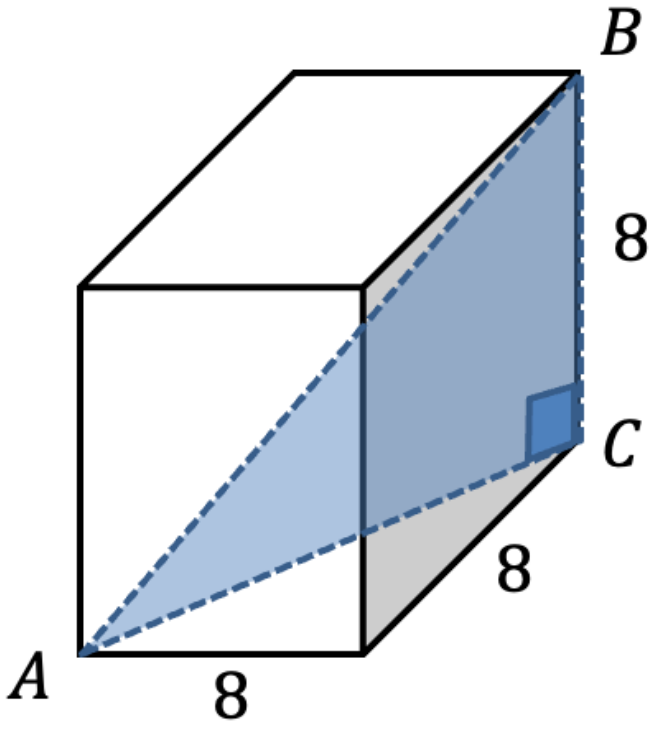
create formulas for (i) the total surface area (ii) the volume

Fluency Practice

1 A cube has sides 8cm. Find:

a) The length AB. ?

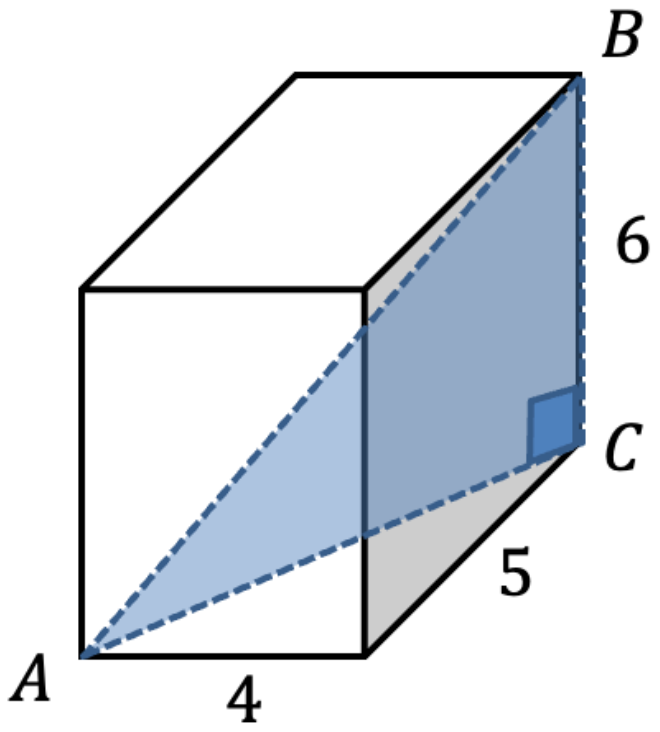
b) The angle between AB and the horizontal plane. ?



2 For the following cuboid, find:

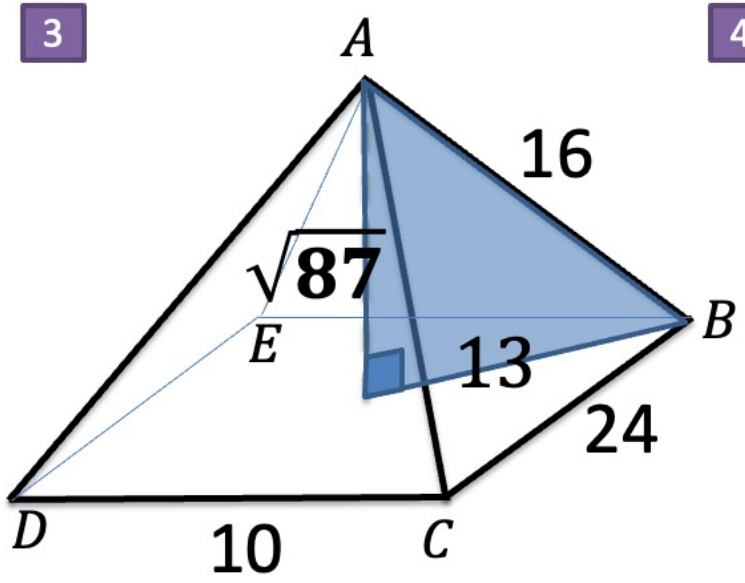
a) The length AB. ?

b) The angle between AB and the horizontal plane. ?



Fluency Practice

3



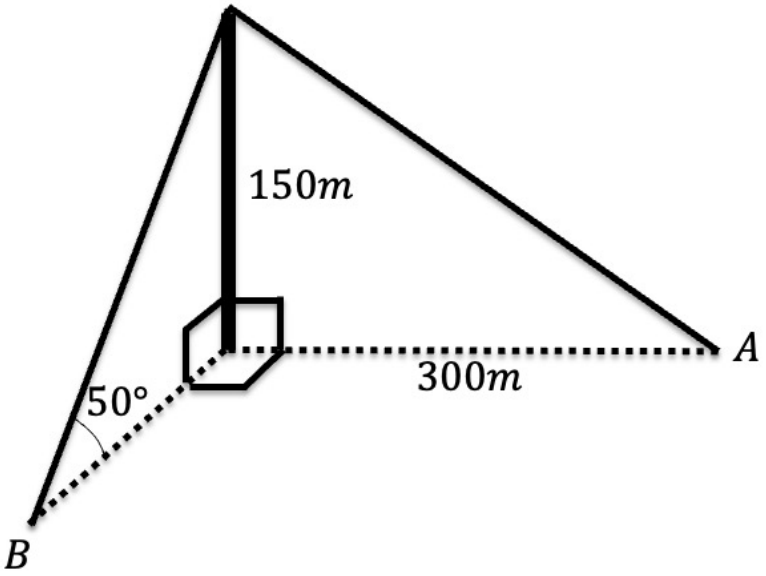
a) Determine the height of the pyramid.

?

b) Determine the angle between the line AB and the base $BCDE$ of the pyramid.

?

4 A radio tower 150m tall has two support cables running 300m due East and some distance due South, anchored at A and B. The angle of inclination to the horizontal of the latter cable is $50°$ as indicated.



a) Find the angle between the cable attached to A and the horizontal plane.

?

b) Find the distance between A and B.

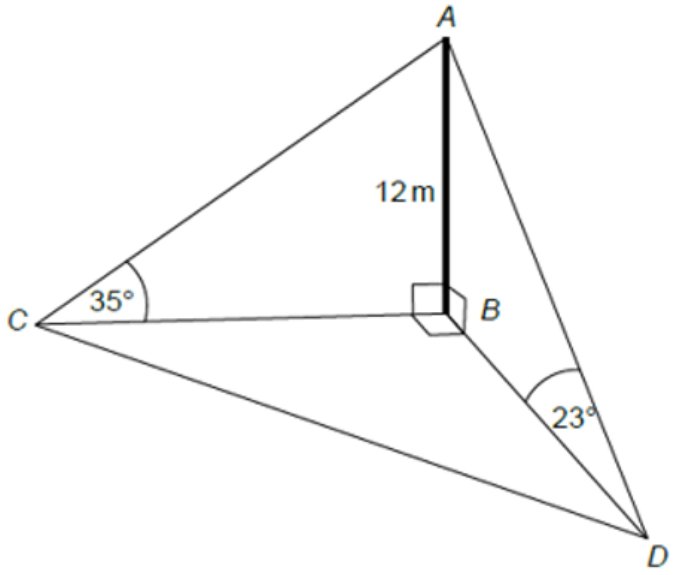
?

5 *[AQA FM Mock Set 1 Paper 2]* The diagram shows a vertical mast, AB, 12 metres high. Points B, C, D are on a horizontal plane.

Point C is due West of B.

a) Calculate CD

b) Calculate the bearing of D from C to the nearest degree.



a)

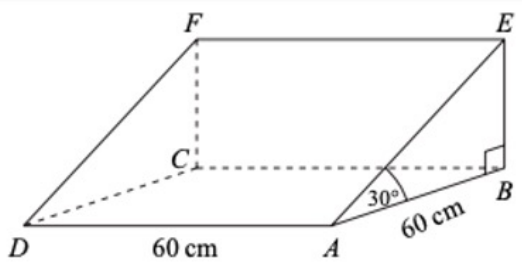
?

b)

?

Fluency Practice

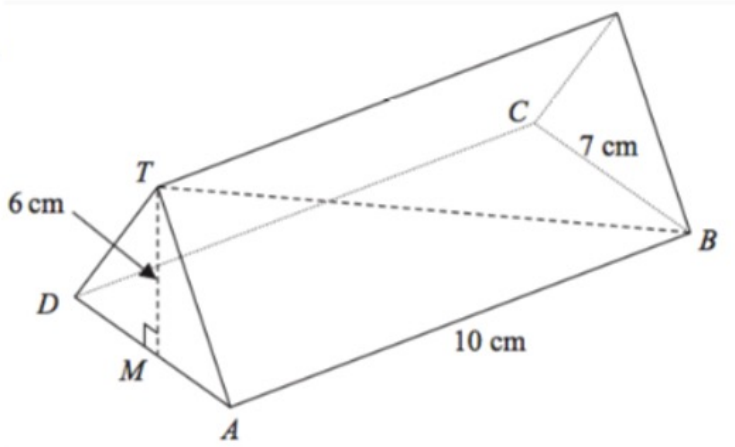
6



[Edexcel GCSE June 2004 6H Q18] The diagram represents a prism.
$AEFD$ is a rectangle. $ABCD$ is a square.
EB and FC are perpendicular to plane $ABCD$.
$AB = 60$ cm. $AD = 60$ cm. Angle $ABE = 90°$.
Angle $BAE = 30°$.
Calculate the size of the angle that the line DE makes with the plane $ABCD$.
Give your answer correct to 1 decimal place.

?

7

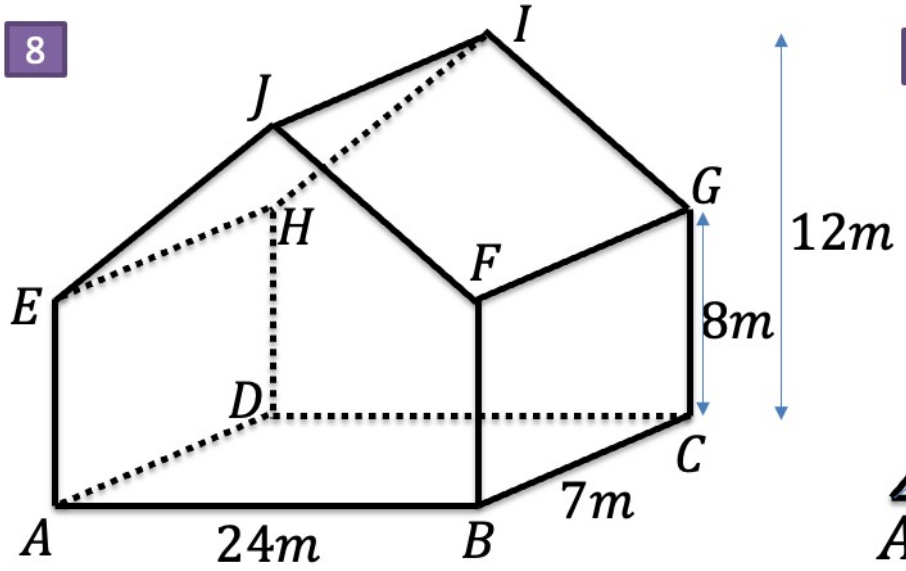


[Edexcel IGCSE May2013-4H Q22]
The diagram shows a triangular prism with a horizontal rectangular base *ABCD*.
AB = 10 cm. *BC* = 7 cm.
M is the midpoint of *AD*.
The vertex *T* is vertically above *M*.
MT = 6 cm. Calculate the size of the angle between *TB* and the base *ABCD*.
Give your answer correct to 1 decimal place.

?

Fluency Practice

8

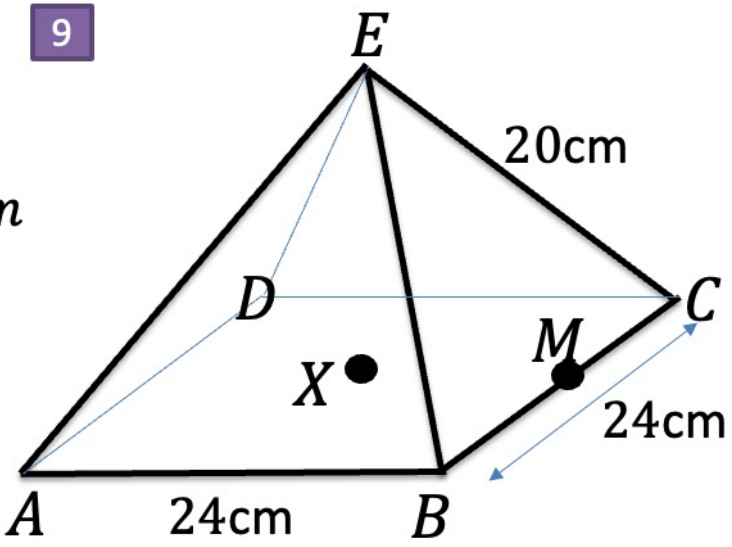


Frost Manor is as pictured, with $EFGH$ horizontally level.

a) Find the angle between the line AG and the plane $ABCD$. ?

b) Find the angle between the planes $FGIJ$ and $EFGH$. ?

9



A school buys a set of new 'extra comfort' chairs with its seats pyramid in shape. X is at the centre of the base of the pyramid, and M is the midpoint of BC.

a) By considering the triangle EBC, find the length EM. ?

b) Hence determine the angle between the triangle EBC and the plane $ABCD$. ?

10 [AQA FM June 2013 Paper 2] $ABCDEFGH$ is a cuboid. M is the midpoint of HG. N is the midpoint of DC.

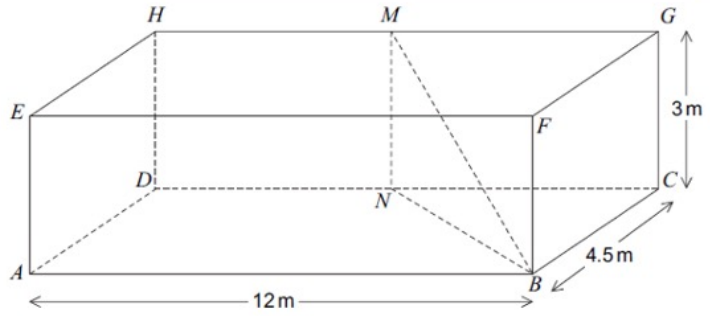
a) Show that $BN = 7.5$m

b) Work out the angle between the line MB and the plane $ABCD$.

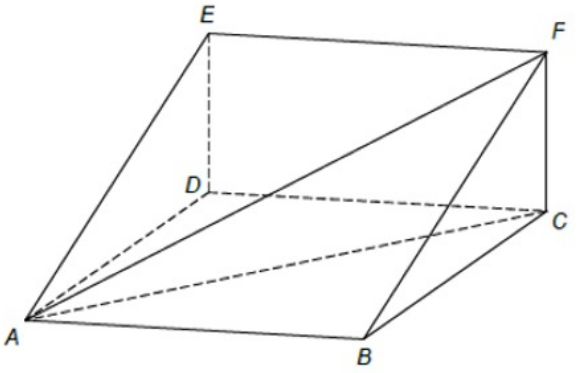
?

c) Work out the **obtuse** angle between the planes MNB and $CDHG$.

?



11 [AQA FM Set 3 Paper 2] The diagram shows part of a skate ramp, modelled as a triangular prism. $ABCD$ represents horizontal ground. The vertical rise of the ramp, CF, is 7 feet. The distance $BC = 24$ feet.



You are given that $gradient = \frac{vertical\ rise}{horizontal\ distance}$

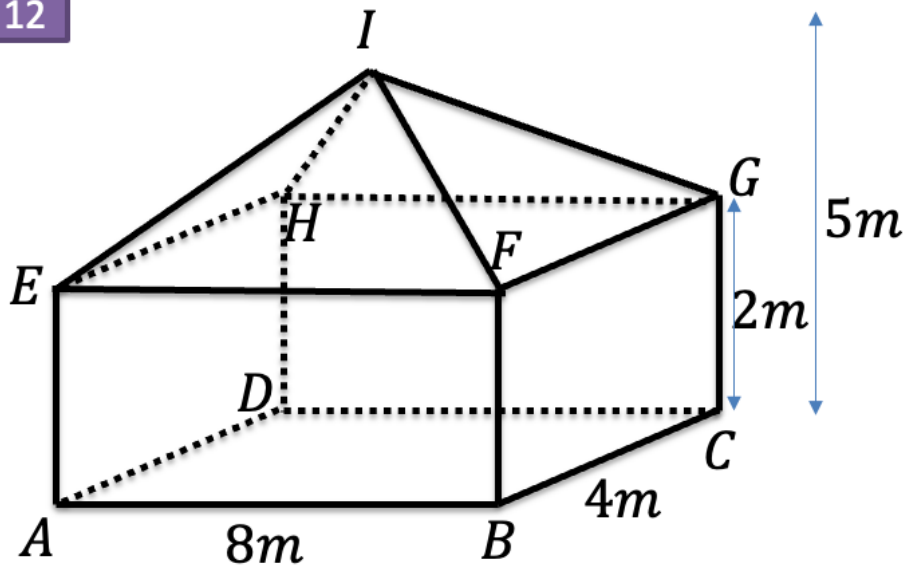
a) The gradient of BF is twice the gradient of AF. Write down the distance AC. ?

b) Greg skates down the ramp along FB. How much further would he travel if he had skated along FA.

?

Fluency Practice

12



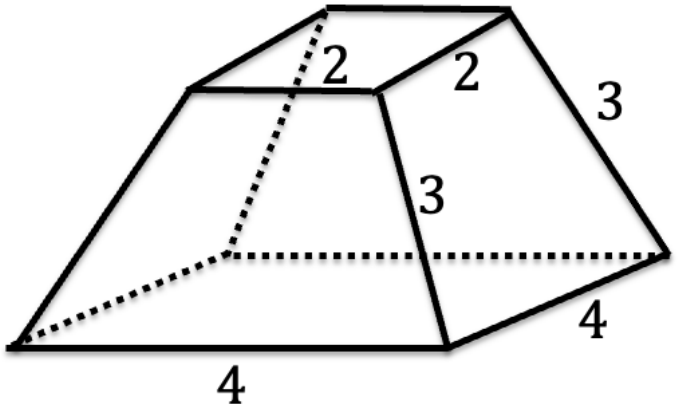
a) Determine the angle between the line AI and the plane $ABCD$.

?

b) Determine the angle between the planes FHI and $EFGH$.

?

13

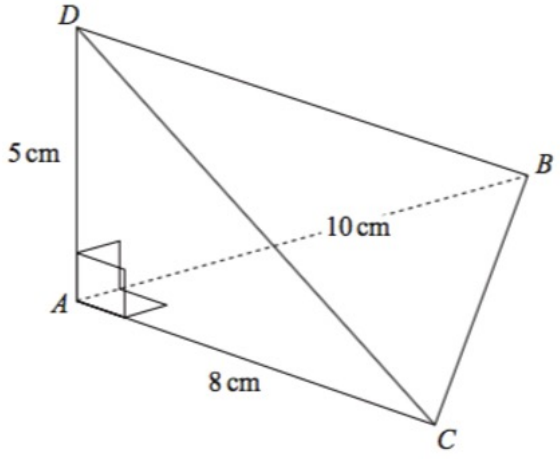


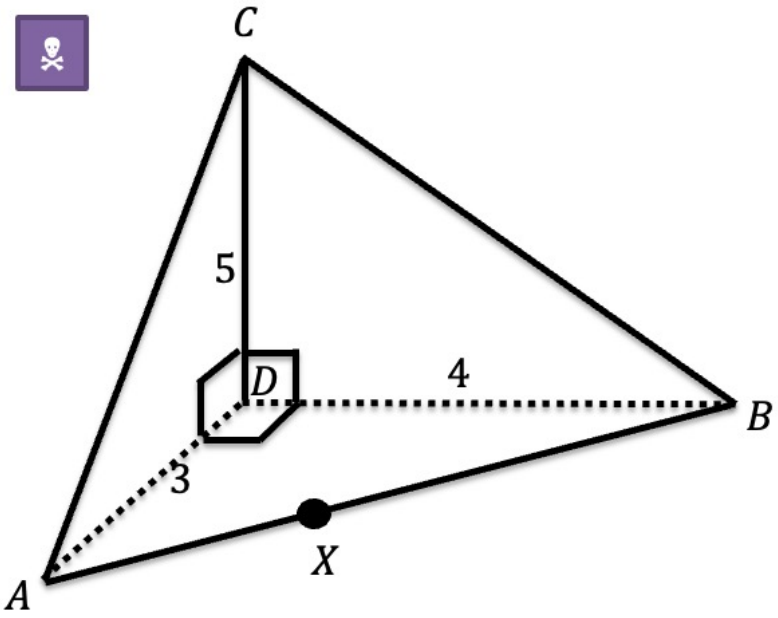
A 'truncated pyramid' is formed by slicing off the top of a square-based pyramid, as shown. The top and bottom are two squares of sides 2 and 4 respectively and the slope height 3. Find the angle between the sloped faces with the bottom face.

?

Fluency Practice

14 *(Knowledge of sine/cosine rules required)*
[Edexcel GCSE Nov2007-6H Q25] The diagram shows a tetrahedron.
AD is perpendicular to both AB and AC.
AB = 10 cm.
AC = 8 cm.
AD = 5 cm.
Angle BAC = 90°.
Calculate the size of angle BDC.
Give your answer correct to 1 decimal place.



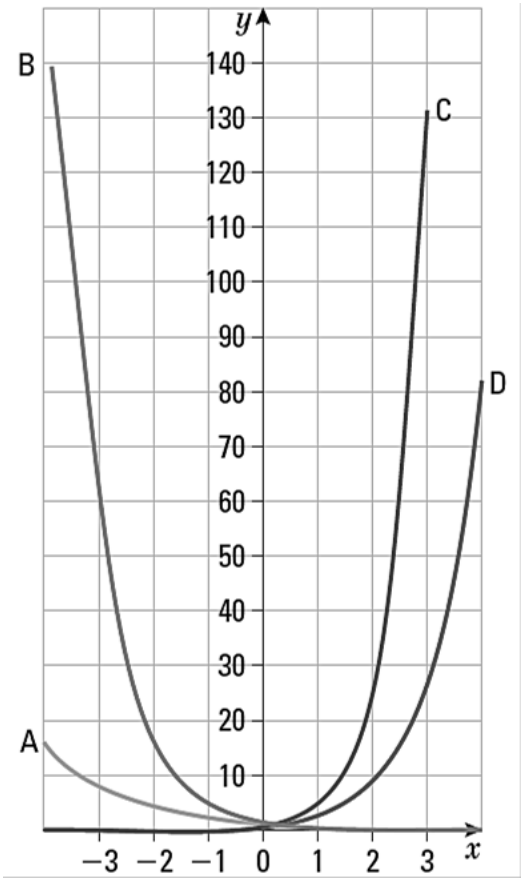


Determine the angle between the planes ABC and ABD.

?

4 Exponential and Trigonometric Graphs

3. The diagram shows the graphs of $y = 3^x$, $y = 2^{-x}$, $y = 5^x$ and $y = \left(\frac{1}{4}\right)^x$.



Match each graph to its equation.

4. The number of rabbits, n, in a particular population grows at a rate given by the equation $n = 5 \times 2^y$ where y is the number of years.

   a) How many rabbits were there initially (when $y = 0$)?

   b) How many rabbits are there after 6 years?

   c) How many years will it take for the rabbit population to exceed 5000?

(a) Sketch the graph of $y = 2^x$, marking the coordinates of any points that cross the axes.
(b) Sketch the graph of $y = 0.5^x$, marking the coordinates of any points that cross the axes.

The exponential growth of a bat population can be described by the equation $P = 20 \times 1.2^t$, where P is the population at time t in months.
(a) What is the initial bat population?
(b) Calculate the population of bats after 6 months.
(c) What is the percentage increase in the bat population per month?
(d) Sketch the graph of the bat population over time, marking the coordinates of any points where the graph crosses the axes.

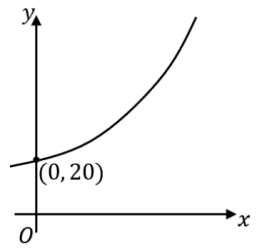
A radioactive element decays according to the equation $m = 500 \times 0.5^t$ where m is the mass of the element in kg and t is the time in days.
(a) What is the initial mass of the radioactive element?
(b) What is the mass of the element after 2 days?
(c) What is the mass of the element after 15 days? Give your answer in grams to 1 decimal place.
(d) What is the half-life of the element? The half-life is the time it takes to decay to half its original mass.
(e) Sketch the graph of the mass against time.

Fluency Practice

Finding the Equation of Exponential Graphs

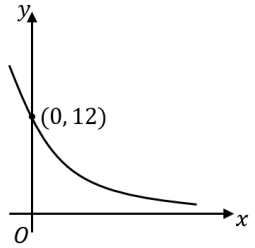
(a)

The curve has equation $y = ab^x$. Write down the value of a.



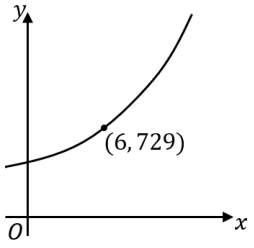
(b)

The curve has equation $y = ab^{-x}$. Write down the value of a.



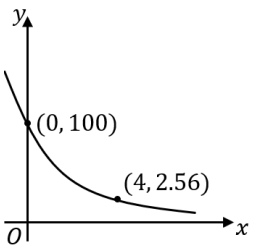
(c)

The curve has equation $y = k^x$. Find the value of k.



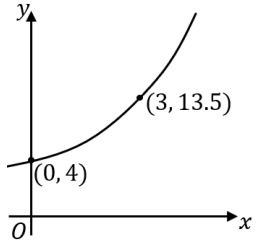
(d)

The curve has equation $y = ab^{-x}$. Find the values of a and b, where $b > 0$



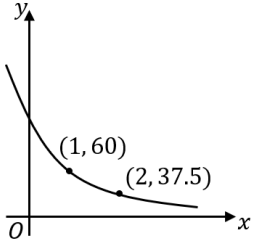
(e)

The curve has equation $y = ab^x$. Find the values of a and b.



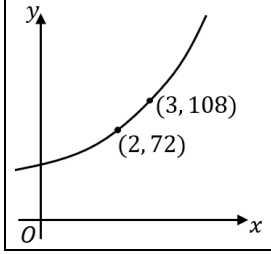
(f)

The curve has equation $y = ab^{-x}$. Find the values of a and b.



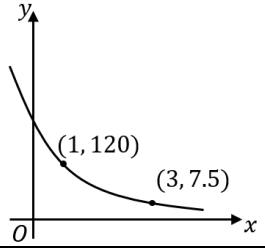
(g)

The curve has equation $y = ab^x$. Find the values of a and b.



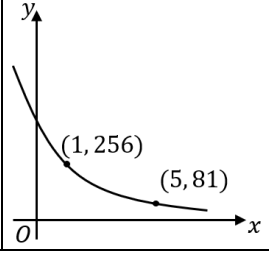
(h)

The curve has equation $y = ab^{-x}$. Find the values of a and b, where $b > 0$



(i)

The curve has equation $y = ab^{-x}$. Find the values of a and b, where $b > 0$

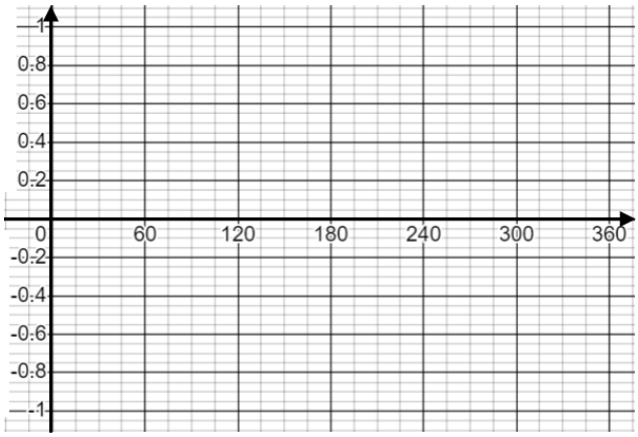


Plotting Trigonometric Graphs

$y = \sin x$

x	y
0	
30	
60	
90	
120	
150	
180	

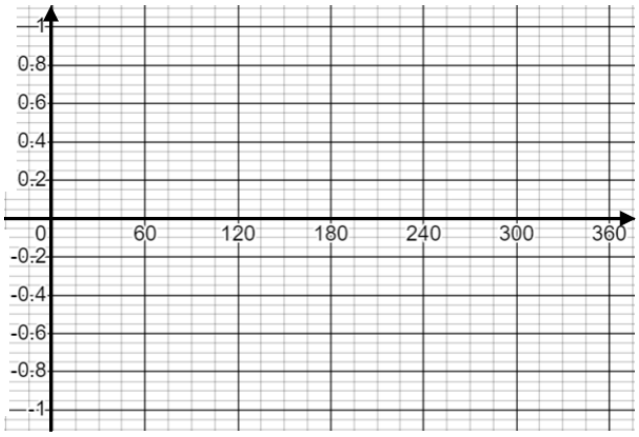
x	y
210	
240	
270	
300	
330	
360	



$y = \cos x$

x	y
0	
30	
60	
90	
120	
150	
180	

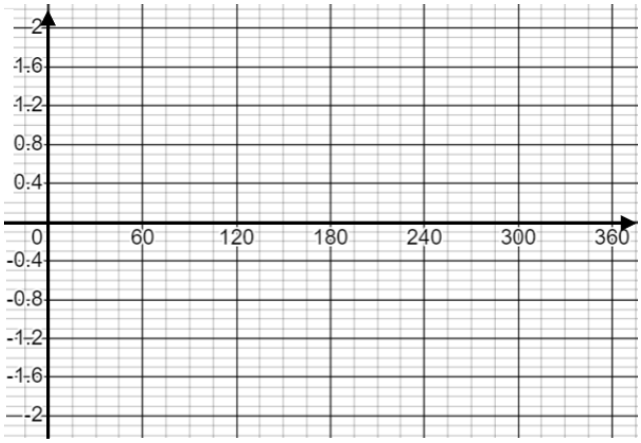
x	y
210	
240	
270	
300	
330	
360	



$y = \tan x$

x	y
0	
30	
60	
90	
120	
150	
180	

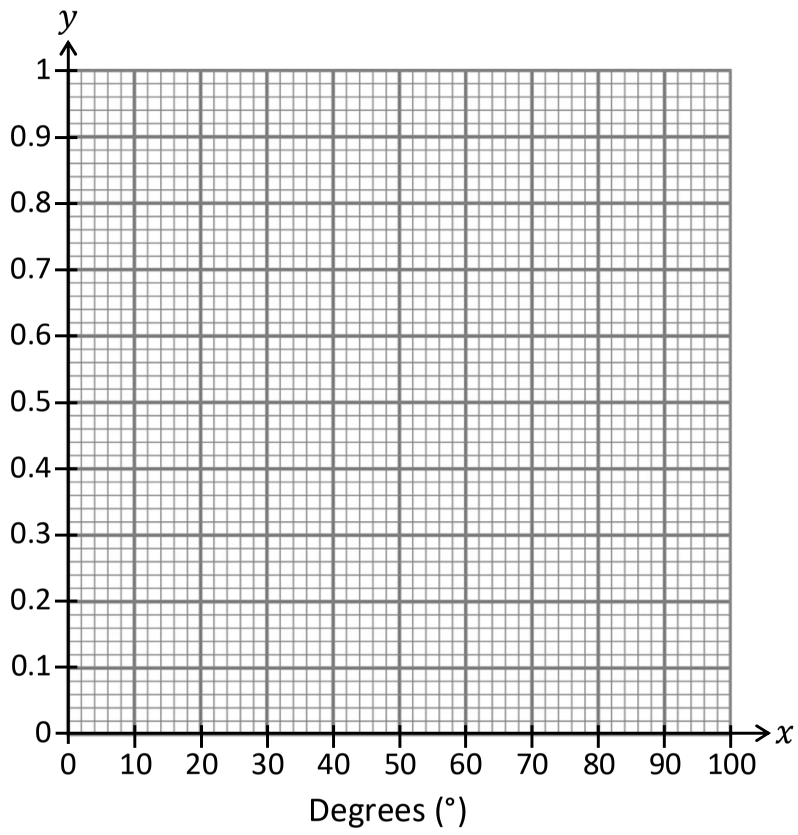
x	y
210	
240	
270	
300	
330	
360	



Graphing the Sine & Cosine Trigonometric Functions

Angle, x	$\sin(x)$
0°	
10°	
20°	
30°	
40°	
50°	
60°	
70°	
80°	
90°	
100°	

Angle, x	$\cos(x)$
0°	
10°	
20°	
30°	
40°	
50°	
60°	
70°	
80°	
90°	
100°	



Sketch & label the graphs:

$y = \sin x$

&

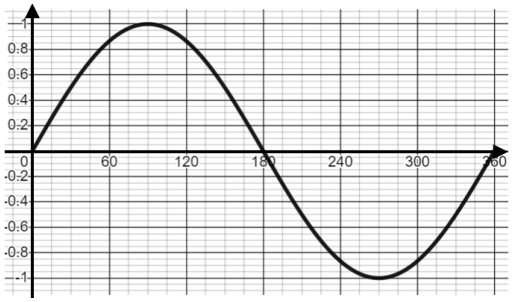
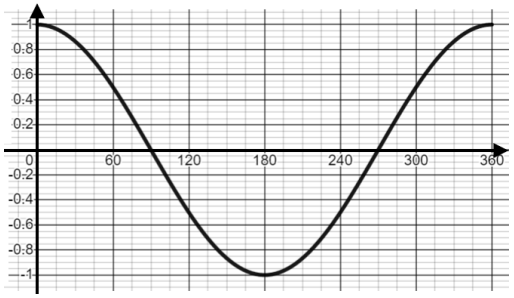
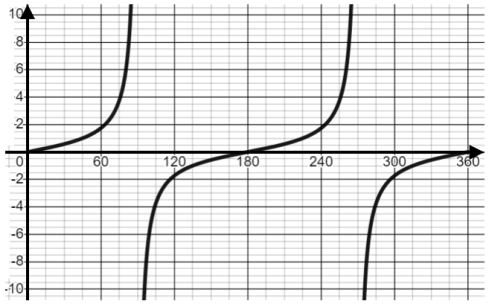
$y = cos\, x$

When does $\sin x = \cos x$? Why?

Fluency Practice

Solving Trigonometric Equations Using Graphs

Use your calculator and the trigonometric graphs to find all values of the angle between 0° and 360°

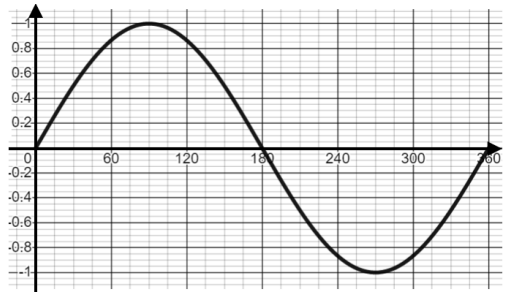
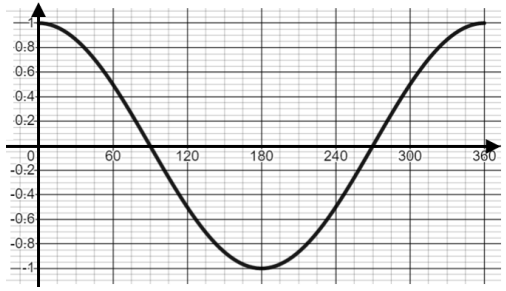
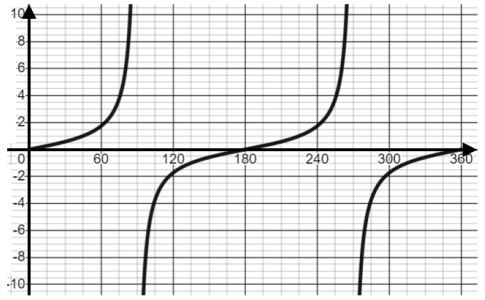
$y = \sin x$	$y = \cos x$	$y = \tan x$
		

$\sin x = 0.3$	$\cos x = 0.9$	$\tan x = 3.2$	$\cos x = 0.6$
$\sin x = 0.8$	$\cos x = 0.15$	$\tan x = 8$	$\sin x = 0.43$
$\cos x = 0.37$	$\sin x = 0.285$	$\tan x = 0$	$\cos x = 1$
$\sin x = 0.5$	$\tan x = 1$	$\sin x = 1$	$\cos x = 0$

Fluency Practice

More Solving Trigonometric Equations Using Graphs

Use your calculator and the trigonometric graphs to find all values of the angle between 0° and 360°

$y = \sin x$	$y = \cos x$	$y = \tan x$
		

$\sin x = -0.4$	$\cos x = -0.1$	$\tan x = -3.7$	$\cos x = -0.65$
$\sin x = -0.88$	$\cos x = -0.25$	$\tan x = -6$	$\sin x = -0.97$
$\cos x = -0.31$	$\sin x = -0.745$	$\tan x = -2.3$	$\cos x = -0.523$
$\sin x = -0.5$	$\tan x = -1$	$\sin x = -1$	$\cos x = -1$

1 Solve the following in the range $0 \leq x \leq 360$

a $\sin(x) = 0.5 \rightarrow x =$?

b $\cos(x) = \frac{\sqrt{3}}{2} \rightarrow x =$?

c $\tan(x) = \sqrt{3} \rightarrow x =$?

d $\sin(x) = 0.1 \rightarrow x =$?

e $4\cos(x) = 3 \rightarrow$?

f $6\tan(x) = 5 \rightarrow$?

g $\sin(x) = 0.4 \rightarrow$?

2 Solve the following in the range $0 \leq x \leq 360$

a $\sin(\theta) = -0.5 \rightarrow$?

b $\cos(\theta) = -0.5 \rightarrow$?

c $\tan(\theta) = -1 \rightarrow$?

d $\sin(\theta) = -0.4 \rightarrow$?

e $\cos(\theta) = -0.7 \rightarrow$?

f $\tan(\theta) = -0.2 \rightarrow$?

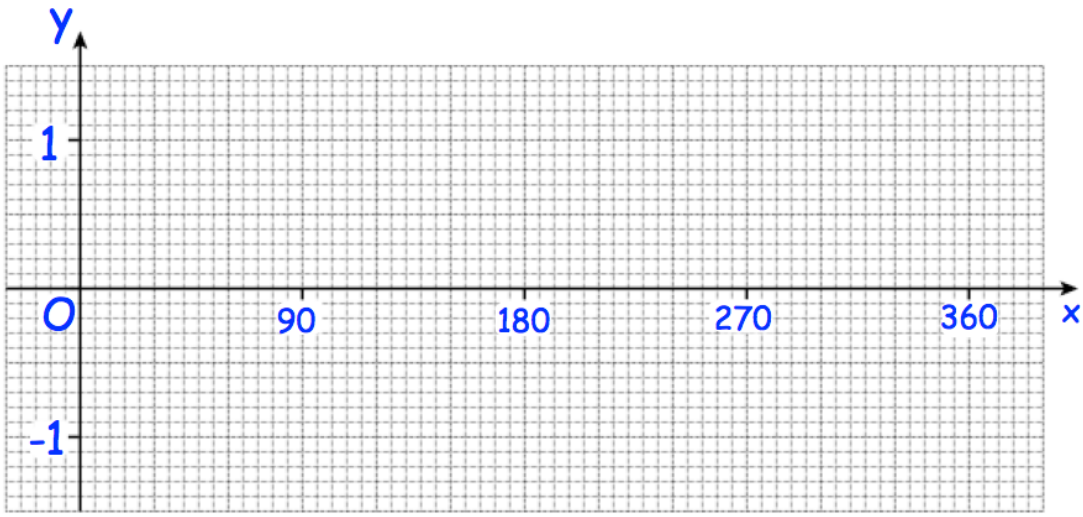
Question 1: (a) Complete the tables below for $y = sin(x)$

x	0°	30°	45°	60°	90°	120°
y						

x	135°	150°	180°	210°	225°	240°
y						

x	270°	300°	315°	330°	360°
y					

(b) Plot the points and draw the graph of $y = sin(x)$



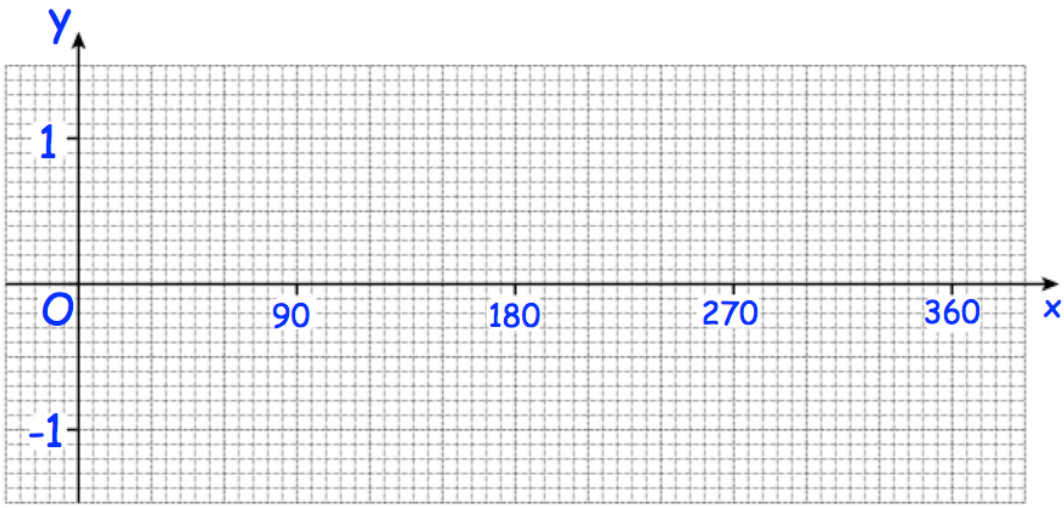
Question 2: (a) Complete the tables below for $y = cos(x)$

x	0°	30°	45°	60°	90°	120°
y						

x	135°	150°	180°	210°	225°	240°
y						

x	270°	300°	315°	330°	360°
y					

(b) Plot the points and draw the graph of $y = cos(x)$



Question 3: (a) Complete the tables below for $y = tan(x)$

x	0°	1°	15°	30°	45°	60°
y						

x	75°	89°	90°	91°	105°	120°
y						

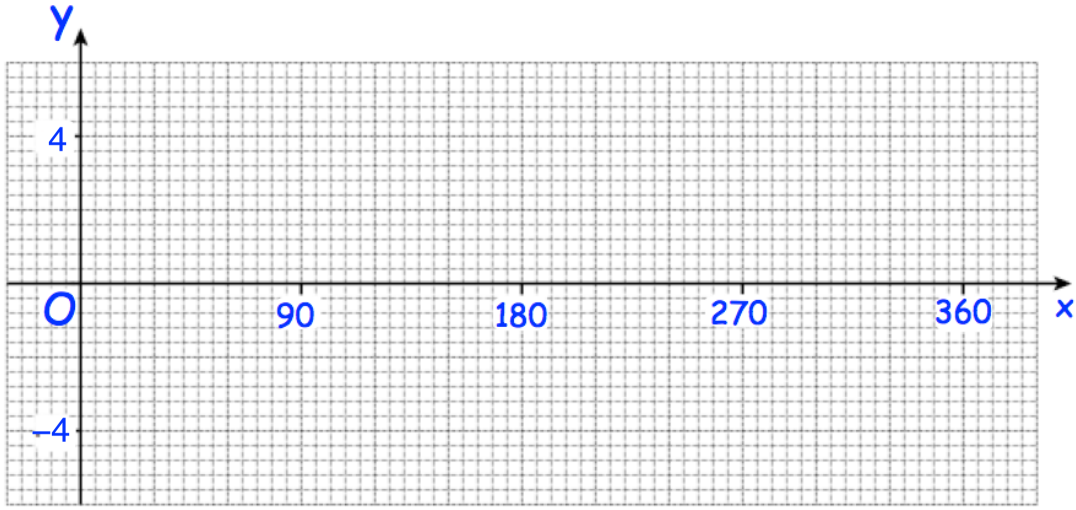
x	135°	150°	165°	179°	180°	181°
y						

x	195°	210°	225°	240°	255°	269°
y						

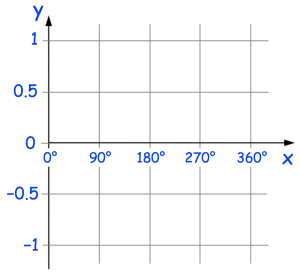
x	270°	271°	285°	300°	315°	330°
y						

x	345°	360°
y		

(b) Plot the points and draw the graph of $y = tan(x)$

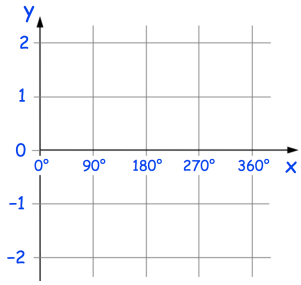


Question 4: Sketch the graph of $y = cos(x)$ for $0^\circ \leq x \leq 360^\circ$



Question 5: Sketch the graph of $y = sin(x)$ for $0^\circ \leq x \leq 360^\circ$

Question 6: Sketch the graph of $y = tan(x)$ for $0^\circ \leq x \leq 360^\circ$



Apply

Question 1: Which of these values cannot be the sine of an angle?

0 −0.9 ⅔ 1.2

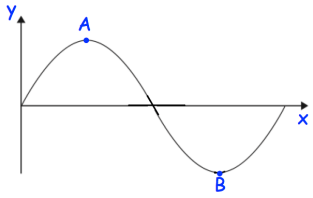
Question 2: Which of these values cannot be the cosine of an angle?

−1 3 0.7 −0.04

Question 3: Here is part of the curve $y = sin(x)$

(a) Write down the coordinates of the point A

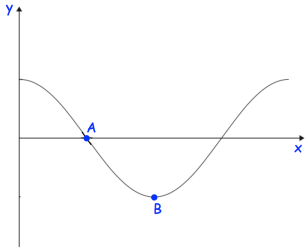
(b) Write down the coordinates of the point B



Question 4: Here is part of the curve $y = cos(x)$

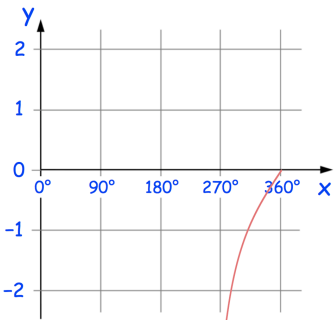
(a) Write down the coordinates of the point A

(b) Write down the coordinates of the point B

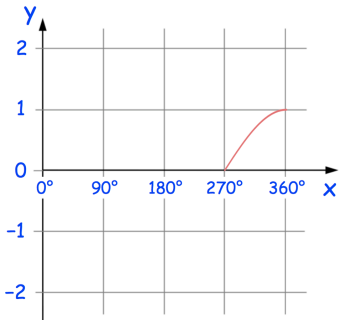


Question 5: Here are three graphs for $270^\circ \leq x \leq 360^\circ$

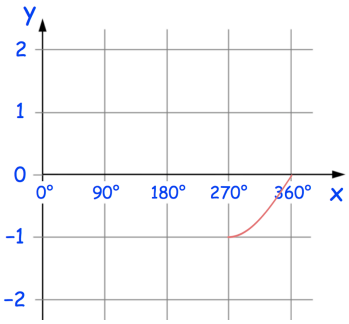
Graph 1



Graph 2



Graph 3



(a) Which graph is $y = sin(x)$?

(b) Which graph is $y = cos(x)$?

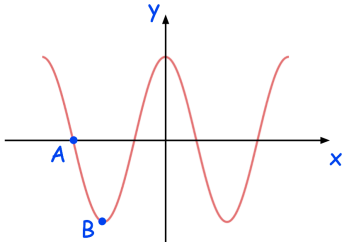
(c) Which graph is $y = tan(x)$?

Question 6: Write down the coordinates of the maximum point of $y = sin(x)$ for $180^\circ \leq x \leq 540^\circ$

Question 7: Write down the coordinates of the minimum point of $y = sin(x)$ for $360^\circ \leq x \leq 720^\circ$

Question 8: Write down the coordinates of the minimum point of $y = cos(x)$ for $360^\circ \leq x \leq 720^\circ$

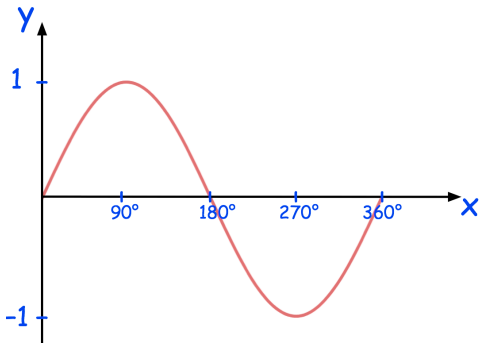
Question 9: Here is a sketch of $y = cos(x)$ for $-360^\circ \leq x \leq 360^\circ$



(a) Write down the coordinates of the point A

(b) Write down the coordinates of the point B

Question 10: Here is the graph of $y = sin(x)$ for $0^\circ \leq x \leq 360^\circ$

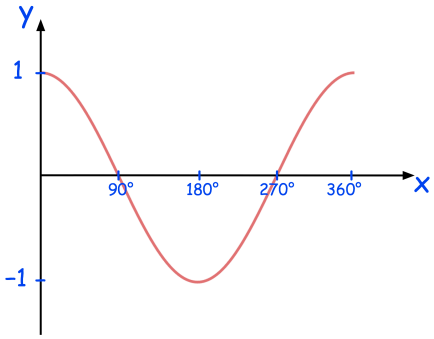


One solution of $sin(x^\circ) = -0.5$ is $x = 210^\circ$

(a) Find another solution of $sin(x^\circ) = -0.5$ for $0^\circ \leq x \leq 360^\circ$

(b) Find the solutions of $sin(x^\circ) = 0.5$ for $0^\circ \leq x \leq 360^\circ$

Question 11: Here is a sketch of $y = cos(x)$ for $0^\circ \leq x \leq 360^\circ$



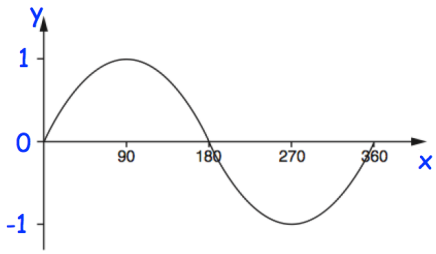
$cos(x) = cos(30^\circ)$

(a) Work out the value of x when $90^\circ \leq x \leq 360^\circ$

$cos(x) = -cos(30^\circ)$

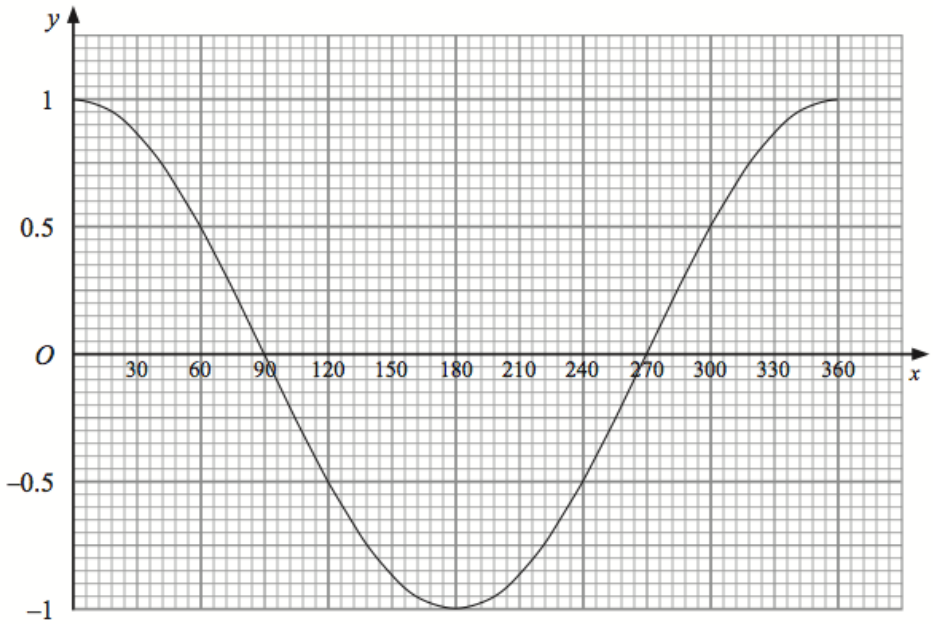
(b) Find the two values of x for $0^\circ \leq x \leq 360^\circ$

Question 12: Here is the graph of $y = sin(x)$ for $0 \leq x \leq 360$



One solution of $sinx = -0.5$ is $x = 330^{\circ}$
Find another solution of $sinx = -0.5$

Question 13: Here is the graph of $y = cos(x)$



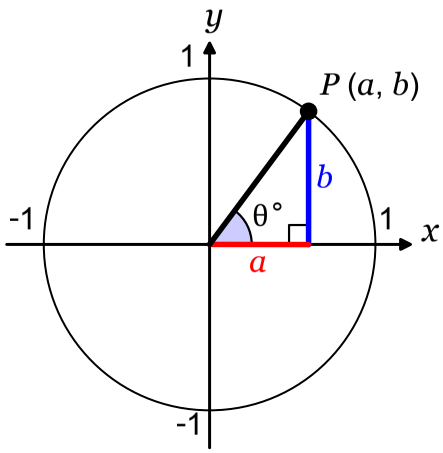
(a) Use the graph to solve $cos(x) = 0.75$

(b) Use the graph to solve $cos(x) = -0.75$

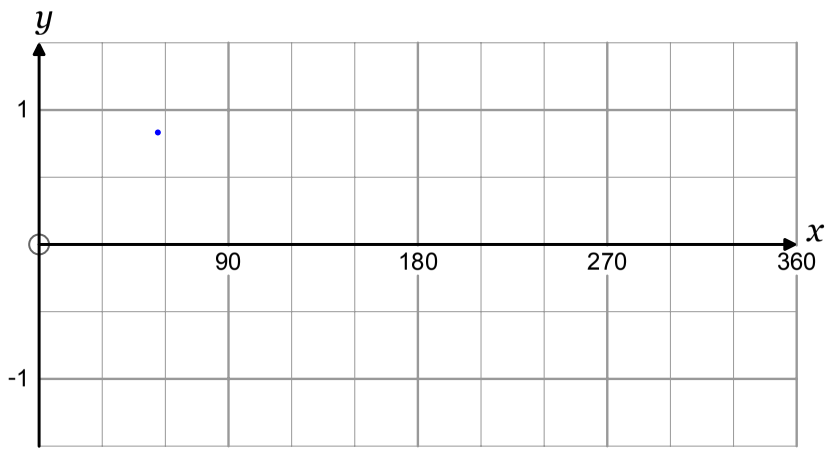
trigonometric graphs

The unit circle is centered on the origin and has a radius of 1.

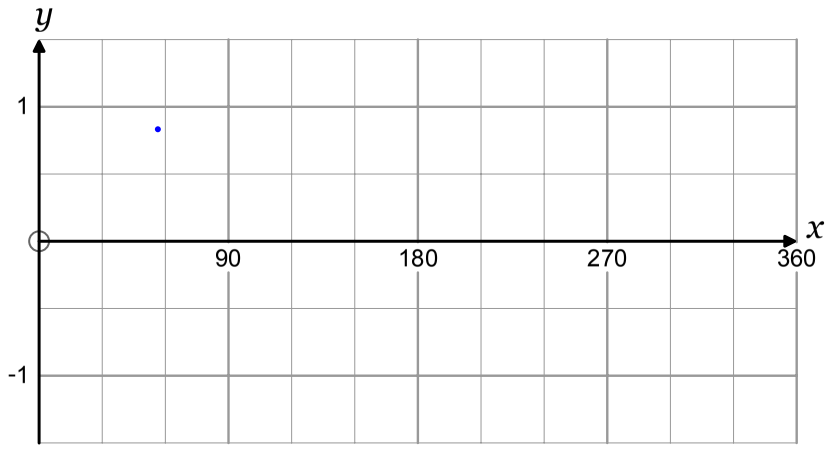
1. Work out a and b in terms of θ.



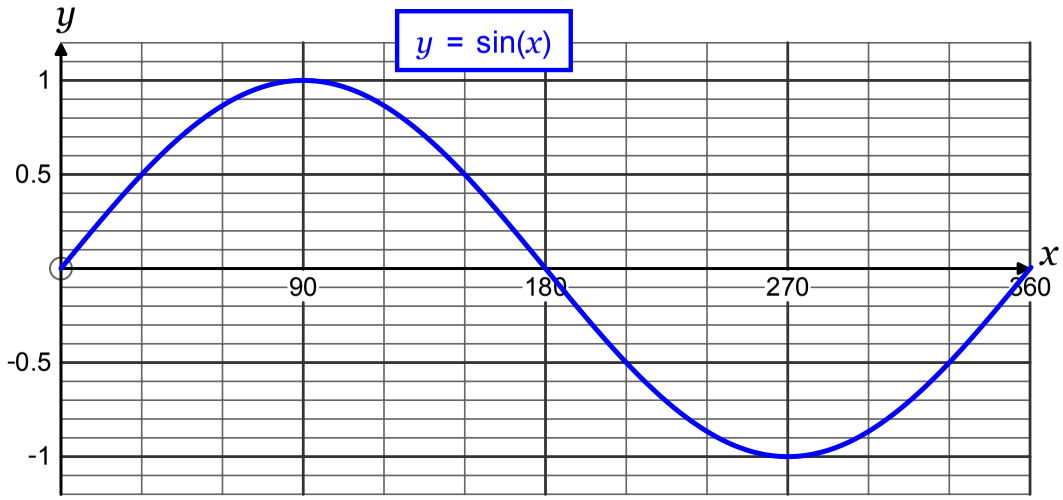
2. Sketch the graph of $y = \sin(x)$ for $0 \leq x \leq 360°$.



3. Sketch the graph of $y = \cos(x)$ for $0 \leq x \leq 360°$.



the sine graph



1. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place.

   a) $\sin(x) = 0.7$ b) $\sin(x) = 0.4$

   c) $\sin(x) = -0.3$ d) $\sin(x) = -0.8$

2. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place where necessary.

   a) $\sin(x) = 0.55$ b) $\sin(x) = -0.9$ c) $\sin(x) = -0.5$

   d) $\sin(x) = 1$ e) $\sin(x) = \frac{\sqrt{3}}{2}$ f) $\sin(x) = 0$

   g) $2\sin(x) = 1$ h) $\sin(x) = 0.95$ i) $\sin(x) = -\frac{1}{3}$

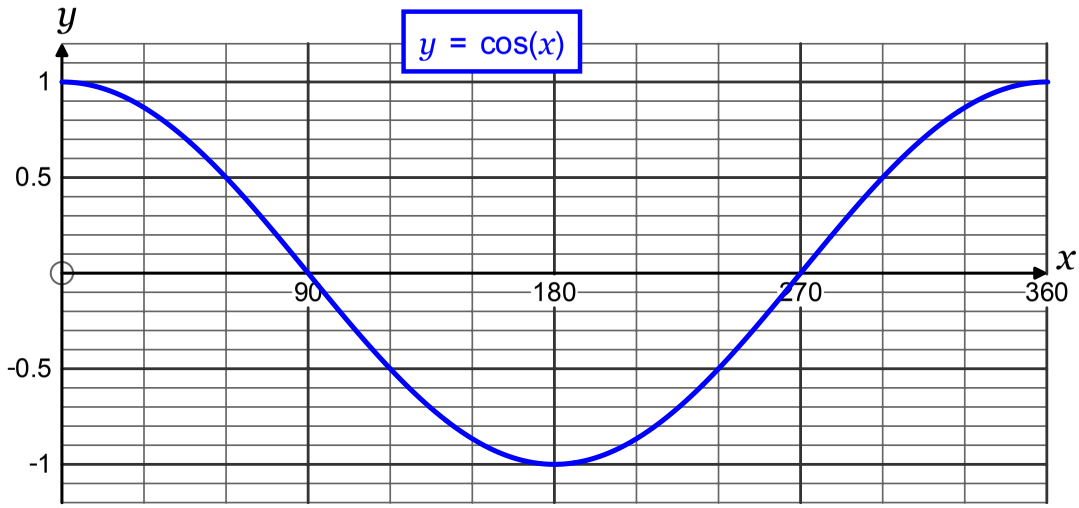
3. a) Given $\sin(40°) = 0.643$, complete: $\sin(140°) =$ _____

   b) Given $\sin(25°) = 0.423$, complete: $\sin(205°) =$ _____

   c) Given $\sin(165°) = 0.259$, complete: $\sin(15°) =$ _____

   d) Given $\sin(315°) = -0.707$, complete: $\sin(45°) =$ _____

the cosine graph



1. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place.

   a) $\cos(x) = 0.75$ b) $\cos(x) = 0.2$

   c) $\cos(x) = -0.6$ d) $\cos(x) = -0.35$

2. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place where necessary.

   a) $\cos(x) = 0.45$ b) $\cos(x) = -0.08$ c) $\cos(x) = 0.6$

   d) $\cos(x) = -1$ e) $\cos(x) = \frac{1}{2}$ f) $\cos(x) = 0$

   g) $4\cos(x) = 1$ h) $\cos(x) = -0.65$ i) $\cos(x) = \frac{7}{8}$

3. a) Given $\cos(25°) = 0.906$, complete: $\cos(335°) =$ _____

   b) Given $\cos(80°) = 0.174$, complete: $\cos(100°) =$ _____

   c) Given $\cos(160°) = -0.940$, complete: $\cos(20°) =$ _____

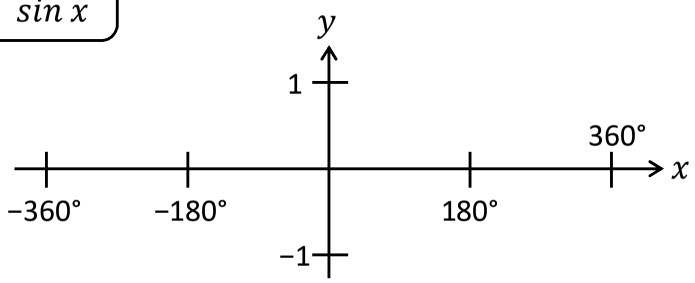
   d) Given $\cos(235°) = -0.574$, complete: $\cos(125°) =$ _____

Trigonometry: Angles Between −360° and 360°

Input the values into your calculator to find coordinates and sketch these graphs.

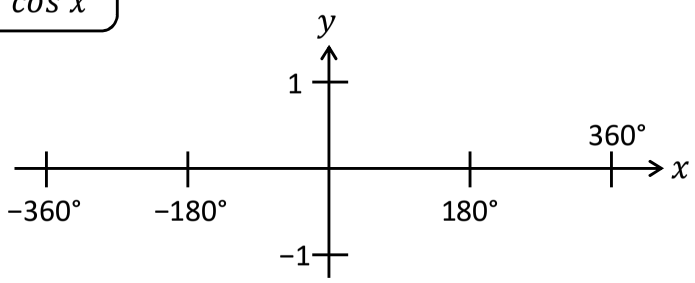
$y = \sin x$

1) sin 0° =
2) sin 90° =
3) sin 180° =
4) sin 270° =
5) sin 360° =
6) Use your own values to sketch the graph for $-360° \leqslant x \leqslant 0°$



$y = \cos x$

1) cos 0° =
2) cos 90° =
3) cos 180° =
4) cos 270° =
5) cos 360° =
6) Use your own values to sketch the graph for $-360° \leqslant x \leqslant 0°$



Using the graphs, and **without** using a calculator, answer these questions.

A) sin 30° = 0.5

For what 3 other values of x, between −360° and 360°, does y = 0.5?
(Sketch y = 0.5 on your graph. Find where it crosses the sin curve.)

1) _____ 2) _____ 3) _____

B) sin −20° = −0.34

For what 3 other values of x, between −360° and 360°, does y = −0.34?

1) _____ 2) _____ 3) _____

3) Use the cosine graph to complete this table.

y = 0.5			cos 60°	
y = 0.77				cos 320°
y = −0.94		cos −160°		
y = −0.09	cos −265°			

In each box, cross off pairs that are **equal in value**. Angles are in degrees.
Circle the value that is left over.

A

sin90	$\frac{1}{\sqrt{3}}$	sin30
tan30	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$
1	cos45	0

B

$\sqrt{3}$	tan60	1
cos0	sin60	0.5
0	$\frac{\sqrt{3}}{2}$	tan180

C

$\frac{1}{\sqrt{2}}$	tan45	sin60
cos60	$\frac{1}{2}$	1
cos30	sin45	tan90

D

sin50	sin40	sin100
sin150	sin60	sin130
sin140	sin80	sin30

E

cos50	cos260	cos10
cos250	cos110	cos130
cos310	cos100	cos350

F

tan15	tan275	tan165
tan105	tan195	tan135
tan315	tan95	tan285

G

sin150	1	$-\frac{1}{2}$
cos240	-1	$\frac{1}{2}$
cos180	$-\frac{\sqrt{3}}{2}$	tan225

H

$-\frac{1}{2}$	$\frac{1}{2}$	sin210
sin60	$-\frac{\sqrt{3}}{2}$	tan135
-1	cos210	$\frac{\sqrt{3}}{2}$

I

sin90	$-\frac{\sqrt{3}}{2}$	sin45
$\frac{\sqrt{3}}{2}$	tan45	cos120
cos45	$-\frac{1}{2}$	sin300

J

$-\frac{\sqrt{3}}{2}$	cos(-45)	$\frac{1}{2}$
cos135	cos45	cos(-40)
cos210	cos40	cos(-60)

K

sin(-30)	sin270	-sin45
$-\frac{\sqrt{3}}{2}$	sin(-60)	sin30
sin(-45)	sin150	sin(-90)

L

tan(-45)	-tan(-75)	tan45
tan(-15)	tan330	tan(-30)
tan75	-tan15	tan135

Explain how we can know if these are true or false.

Use the circle and your graphs to help you decide.

1. $cos\emptyset = \cos(360 - \emptyset)$
2. $sin\emptyset = \sin(360 - \emptyset)$
3. $tan\emptyset = \tan(360 - \emptyset)$

4. $cos\emptyset = \cos(180 - \emptyset)$
5. $sin\emptyset = \sin(180 - \emptyset)$
6. $tan\emptyset = \tan(180 - \emptyset)$

7. $cos\emptyset = \cos(180 + \emptyset)$
8. $sin\emptyset = \sin(180 + \emptyset)$
9. $tan\emptyset = \tan(180 + \emptyset)$

10. $cos\emptyset = \cos(-\emptyset)$
11. $sin\emptyset = \sin(-\emptyset)$
12. $tan\emptyset = \tan(-\emptyset)$

13. $cos\emptyset = -\cos(-\emptyset)$
14. $sin\emptyset = -\sin(-\emptyset)$
15. $tan\emptyset = -\tan(-\emptyset)$

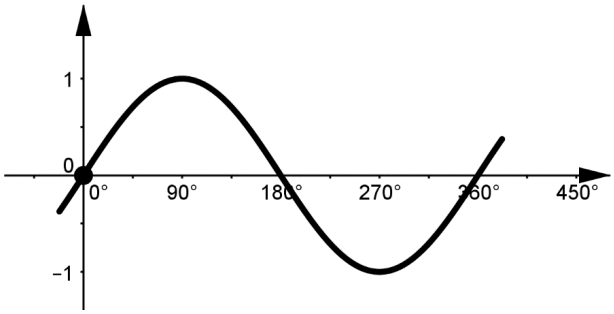
16. $\sin(\emptyset + 90) = \cos(\emptyset)$
17. $\sin(\emptyset - 90) = \cos(\emptyset)$
18. $\sin(90 - \emptyset) = \cos(\emptyset)$

Purposeful Practice

Symmetry in Trigonometric Graphs

NON-CALCULATOR TRIGONOMETRY

1. The graph shows part of the function $y = \sin x$. Decide whether each statement is true or false, explaining your thinking by referring to the graph.



a) $\sin(450°) = 1$ b) $\sin(900°) = 2$ c) $\sin(280°) > \sin(80°)$

d) $\sin(20°) = \sin(160°)$ e) $\sin(45°) = -\sin(315°)$ f) $\sin(-10) < \sin(190)$

2. a) Sketch the graph of $y = \cos x$ for $0 \leq x \leq 360$.
   b) Use the graph to help you list these values in order, starting with the smallest:

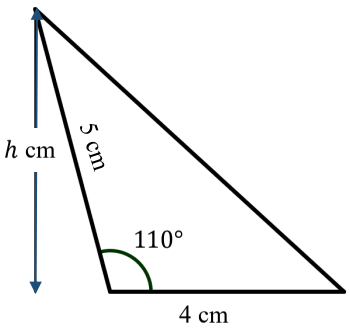
$\cos(75°)$ $\cos(90°)$ $\cos(180°)$ $\cos(330°)$

3. Given that $\sin(30°) = \frac{1}{2}$ and $\cos(30°) = \frac{\sqrt{3}}{2}$, state the exact values of

a) $\sin(150°)$ b) $\sin(330°)$ c) $\cos(390°)$

d) $\sin(-90°)$ e) $\cos(210°)$ f) $\cos(-210°)$

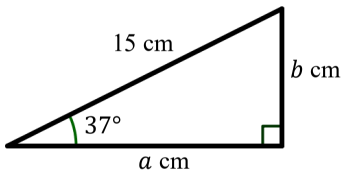
4. a) Which of these expressions can be used to calculate the height h cm of this triangle?



   i. $5\sin(70°)$
   ii. $\sqrt{5^2 + 4^2}$
   iii. $5\sin(110°)$
   iv. $5\cos(20°)$
   iv. $4\tan(35°)$

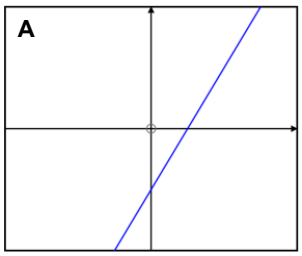
   b) Hence or otherwise find the area of the triangle given that $\sin(70°)$ to two decimal places is 0.94.

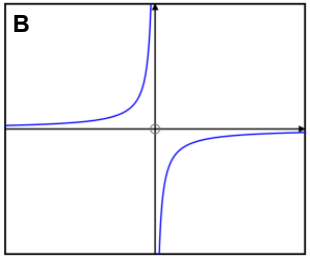
5. Given that the hypotenuse of this right-angled triangle is 15 centimetres and $\cos(143°) = -0.80$ (to two significant figures), find the lengths of the other two sides.

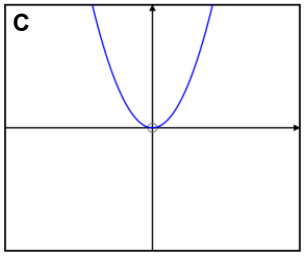


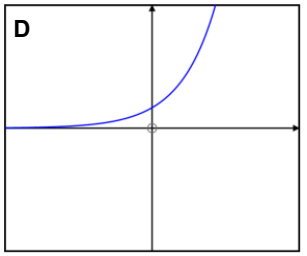
recognising graphs

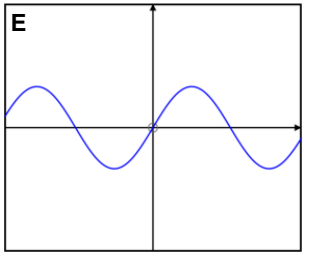
Match these graphs to their equations at the bottom.

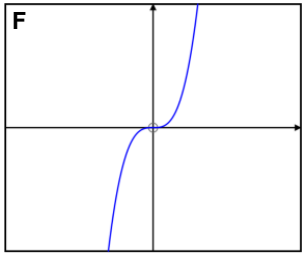


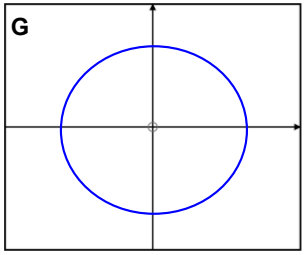


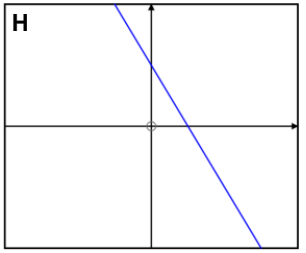


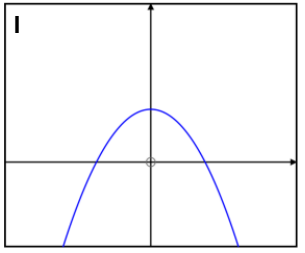


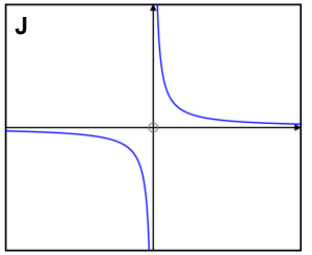


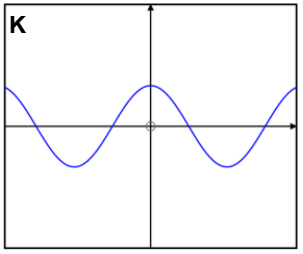


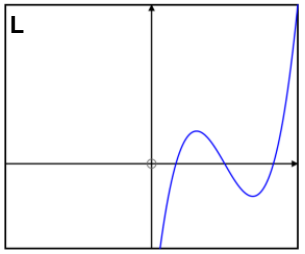


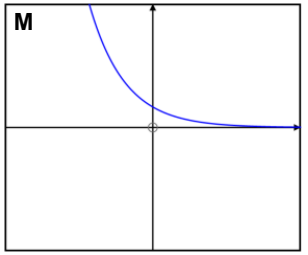


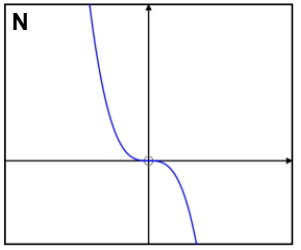


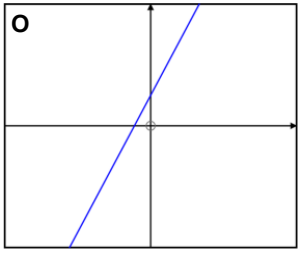


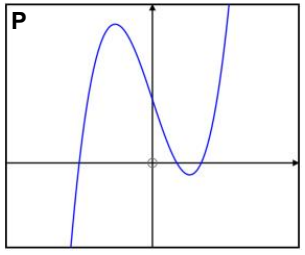






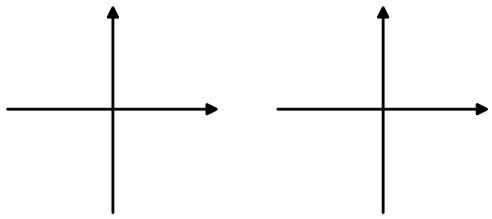




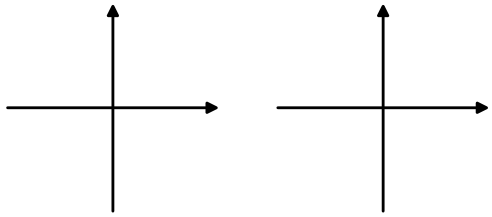


$y = 3x + 2$	$y = x^3$	$y = 3 - 2x$	$y = (x-2)(x-1)(x+3)$	
$y = x^2$	$y = 0.5^x$	$y = 5 - x^2$	$y = (x-5)(x-3)(x-1)$	
$y = \cos(x)$	$y = -x^3$	$y = \frac{-1}{x}$	$y = \frac{1}{x}$	$y = 2x - 3$
$y = \sin(x)$	$y = 2^x$			$x^2 + y^2 = 16$

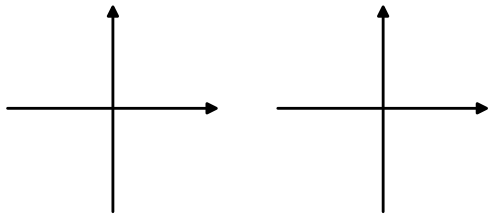
1. Type:

Shapes:	Equation forms:
	

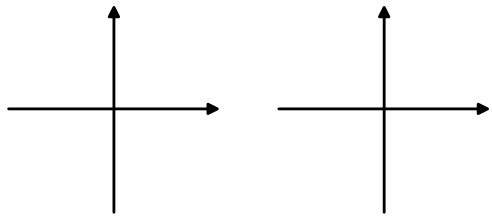
2. Type:

Shapes:	Equation forms:
	

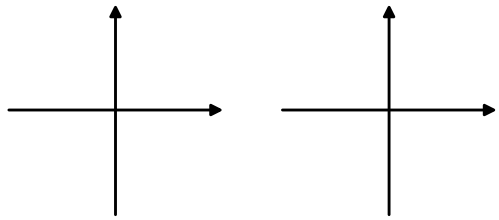
3. Type:

Shapes:	Equation forms:
	

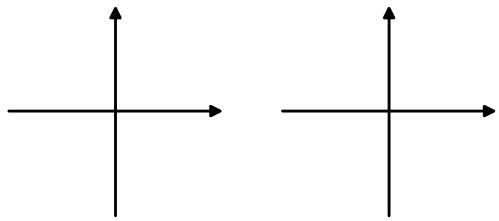
4. Type:

Shapes:	Equation forms:
	

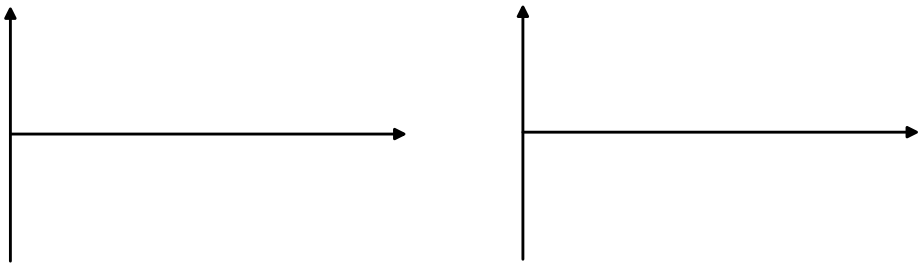
5. Type:

Shapes:	Equation forms:
	

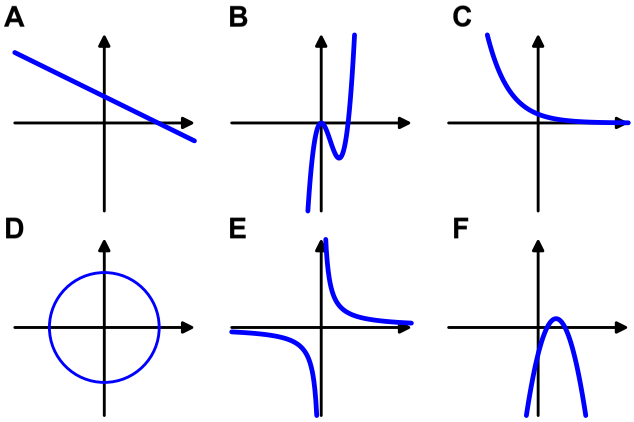
6. Type:

Shapes:	Equation forms:
	

7. Type:



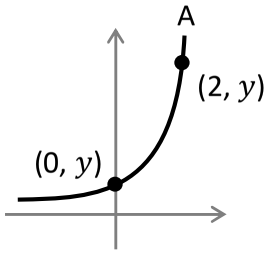
8. Match the graphs with the equations.

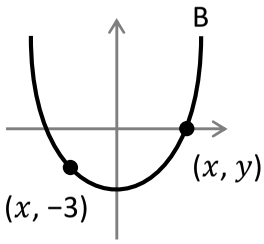


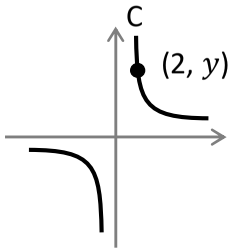
P	$y = x^2(x-3)$
Q	$y = (x-3)(1-x)$
R	$x^2 + y^2 = 49$
S	$y = \frac{5}{x}$
T	$y = 3 - \frac{1}{2}x$
U	$y = 3^{-x}$

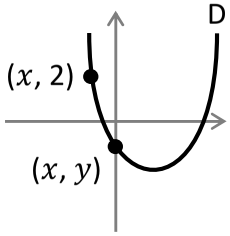
Non-Linear Graph Matchup

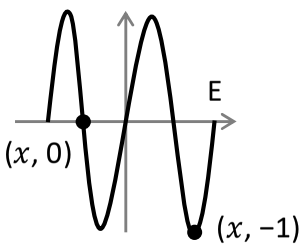
Match each graph to its equation.
Complete the missing coordinates on each graph.











$y = x^2 + 3x$

$y = x^2 - x - 4$

$y = \frac{5}{x}$

$y = \cos x$

$y = 2^x$

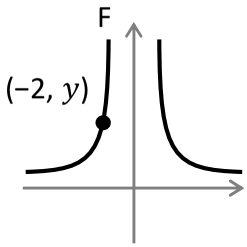
$y = x^3 + 2x$

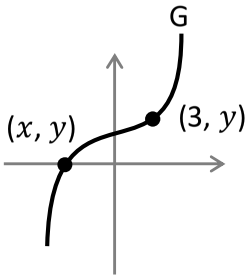
$y = \sin x$

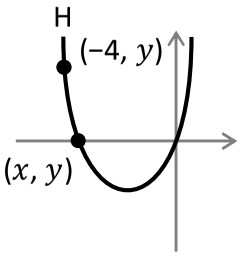
$y = x^2 - 4$

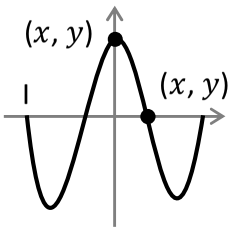
$y = x^3 + 2$

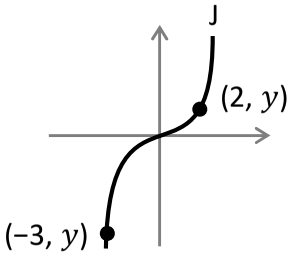
$y = \frac{12}{x^2}$







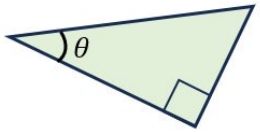




Purposeful Practice

1. Factual Recall

a. Label the triangle below Opposite (O), Adjacent (A), and Hypotenuse (H) with respect to the angle θ



b. Complete the three trigonometric ratios below:

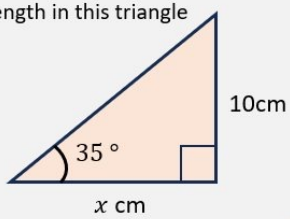
$\sin\theta = \frac{opposite}{h}$

$\cos\theta = \frac{a}{_____}$

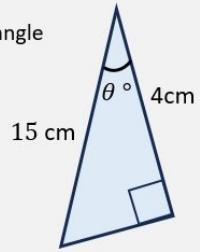
$____\theta = \frac{opposite}{adjacent}$

2. Carry out a routine procedure

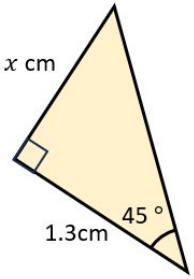
a. Find the missing side length in this triangle



b. Find the missing angle in this triangle

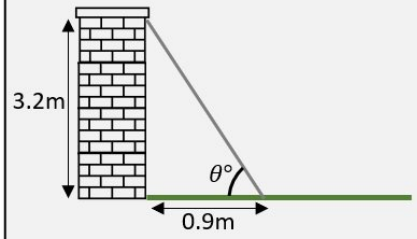


3. Classify a mathematical object



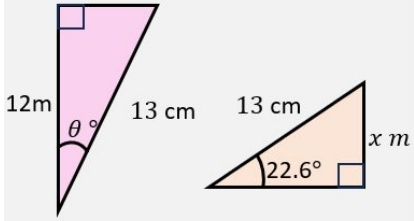
By calculation or otherwise, what type of triangle is this (other than a right angled triangle!) ?

4. Interpret a situation or answer



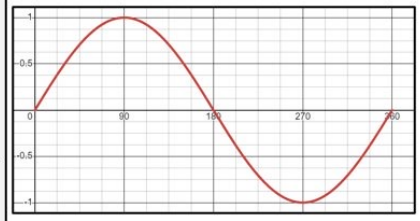
A ladder is placed 0.9m away from the base of a wall. The ladder reaches the top of the 3.2m tall wall.
The angle the ladder makes with the ground needs to be 75° or more to be safe. Is this ladder safe?

5. Prove, Show, or Justify



These two triangles are **congruent.** Explain why we are able to make this claim.

6. Extend a concept

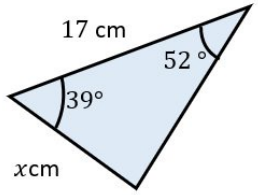


This is the graph of $y = \sin x$.
If you type $\sin(180)$ into your calculator, you will get a result of 0.
Using the graph, can you identify what other values you can put in to $\sin\theta$ to get a result of 0?

7. Construct a scenario

A helicopter leaves the heli-port from the point **H** , and flies 80 miles due East. It then flies 20 miles due North to reach its destination, **D**.
Construct a diagram for this situation, and find the **bearing** of **D** from **H**.

8. Criticise a fallacy



Jamie wants to use SOHCAHTOA to find the missing side length in this triangle. Why can't they?