



Year 11 2024 Mathematics 2025 Unit 21 Booklet – Part 1

HGS Maths





Dr Frost Course



Name:

Class:





Year 11 2024 Mathematics 2025 Unit 21 Booklet – Part 2

HGS Maths





Dr Frost Course



Name:

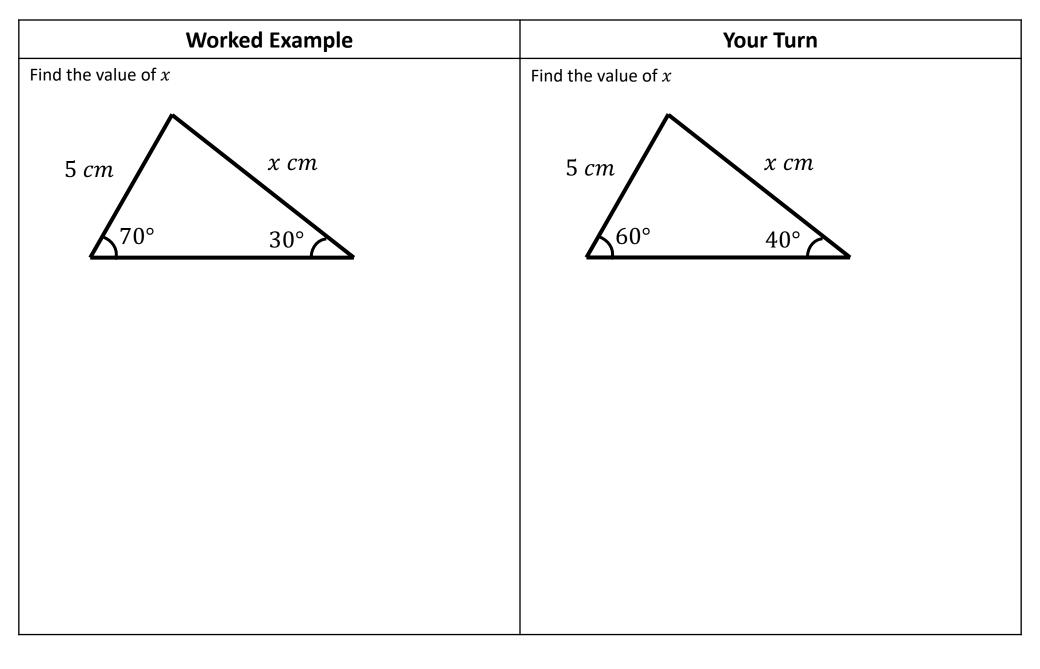
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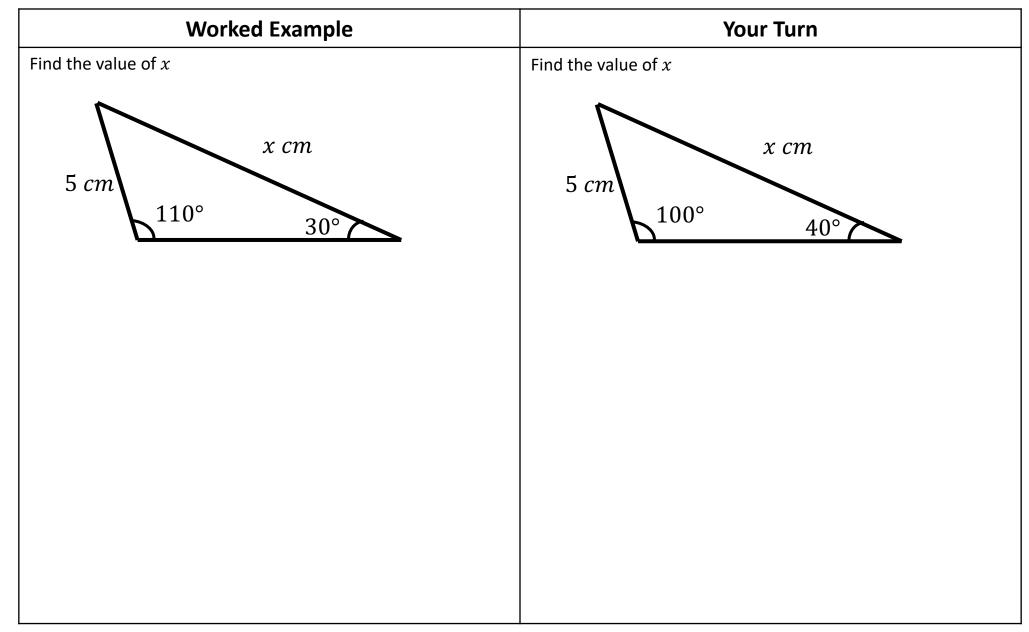
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1 Advanced Trigonometry

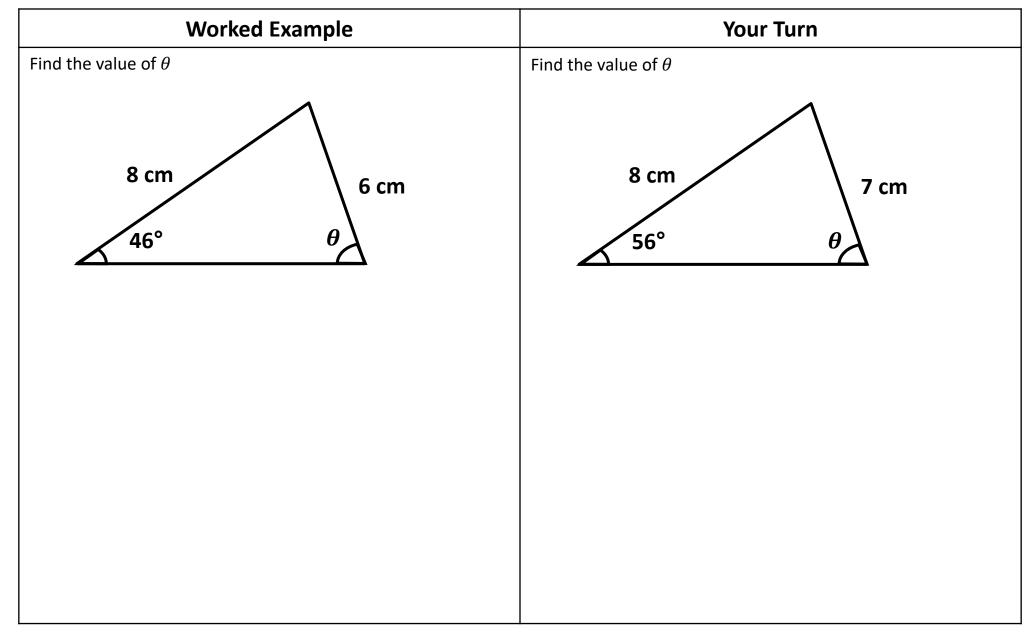
Sine Rule



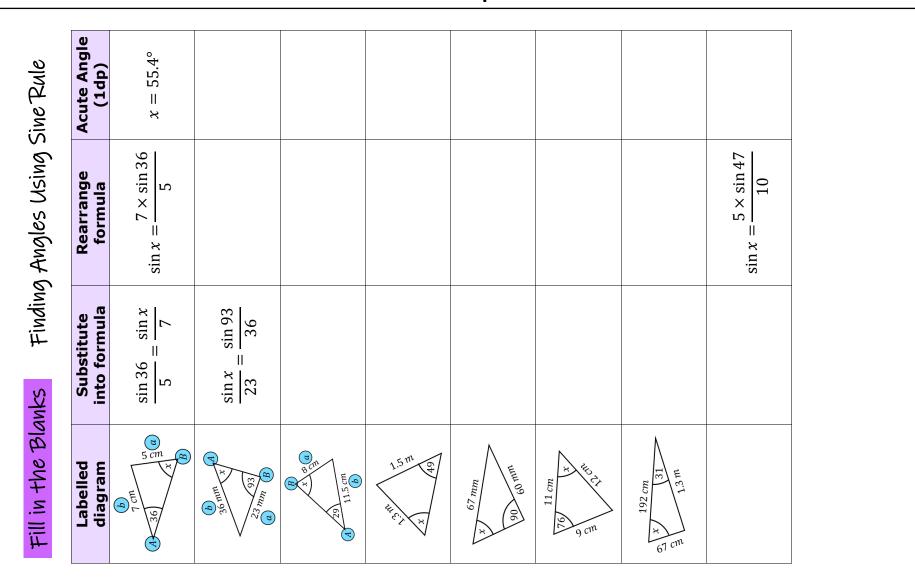


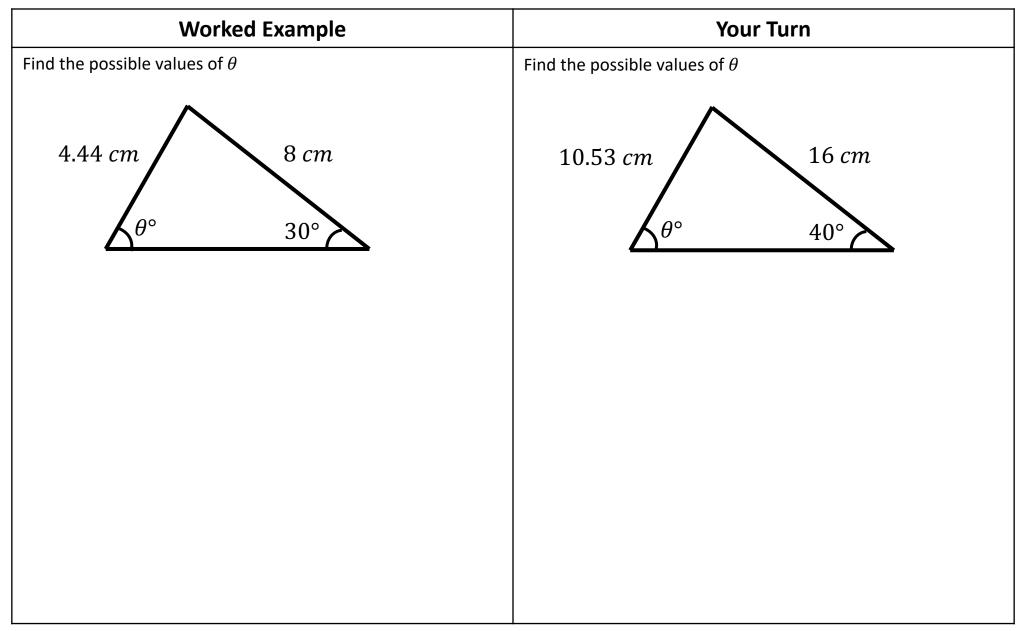
Question	Label the triangle and calculate any angles	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing length.
95 8.1 cm 41 x	$\begin{array}{c} B \\ 95 \\ 44 \\ 41 \\ x \\ b \\ x \\ x$	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{8.1}{\sin 41} = \frac{x}{\sin 95} = \frac{c}{\sin 44}$	$x = \sin 95 \times \frac{8.1}{\sin 41}$	$x = 12.3 \ cm$
x 7.3 cm 55	$\begin{array}{c} & B & a \\ \hline 77 & 7.3 \ cm \\ x & 55 \ c \\ \hline 48 & b \\ \end{array}$	$\frac{7.3}{\sin 48} = \frac{b}{\sin 77} = \frac{x}{\sin 55}$		
11 <i>cm</i> y 57 80	11 <i>cm</i> 57 80			
47 a	105 7.5 cm 47 a			
55 10 mm b	55 10 mm b			

			I	ill in the	Gaps			
Length (1dp)								
Rearrange formula	$x = \frac{9 \times \sin 44}{\sin 59}$							$x = \frac{3.5 \times \sin 36}{\sin 68}$
Substitute into formula	$\frac{x}{\sin 44} = \frac{9}{\sin 59}$	$\frac{x}{\sin 63} = \frac{12}{\sin 48}$					$\frac{x}{\sin 65} = \frac{13}{\sin 76}$	
Labelled diagram	B 4 6 2 6 6 6 6 6 7 4 4 4 4 4 4 4 4 4 4 4 4 4	e 12 cm 6 3 3 4 8 4 8 6 3 3 4 8 6 3 3 4 8 8 6 3 3 4 8 8 6 8 3 8 1 2 6 1 1 2 6 1 1 2 6 1 1 2 6 1 1 2 6 1 1 2 6 1 1 2 1 1 2 1 1 2 1 2	a fi + b	x 34	4	40 10 10 10 10 10 10 10 10 10 1	6E 92	

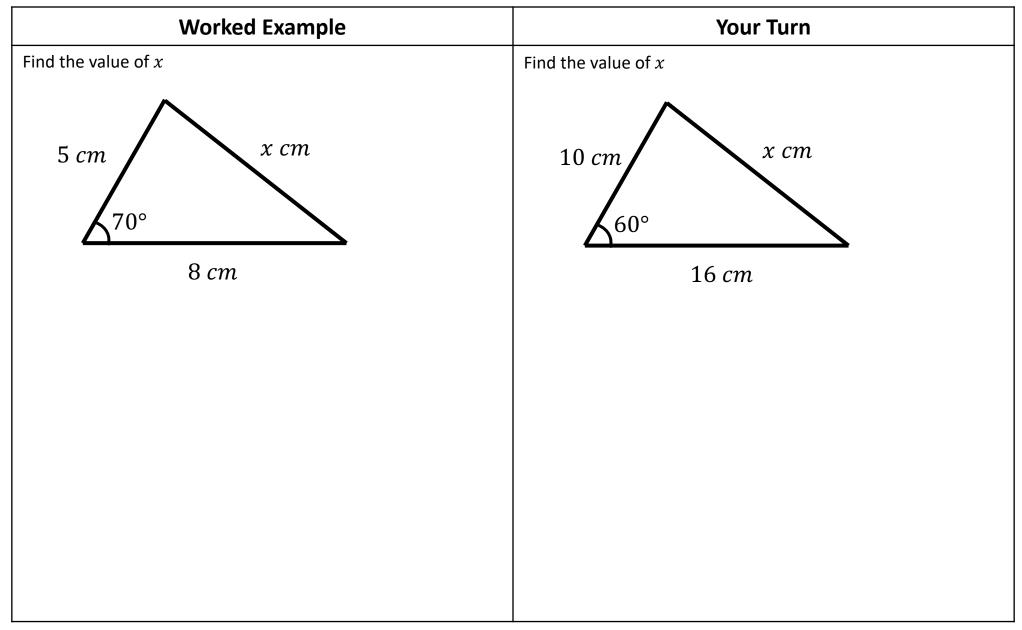


Question	Label the triangle	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing angle.
17.5 cm 6.9 cm 134	$B \xrightarrow{c} C$	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\frac{\sin 134}{17.5} = \frac{\sin B}{b} = \frac{\sin x}{6.9}$	$\sin x = 6.9 \times \frac{\sin 134}{17.5}$	$x = sin^{-1}(0.2836)$ $x = 16.5^{\circ}$
4.8 cm	$ \begin{array}{c} B \\ a \\ 4.8 \ cm \\ C \\ A \end{array} $	$\frac{\sin A}{a} = \frac{\sin d}{6.2} = \frac{\sin 43}{4.8}$		
8 cm 5 cm 61	8 cm e 5 cm 61			
f 9.1 cm 67 7.9 cm	f 9.1 cm 67 7.9 cm			
20 mm 87 46 mm g	20 mm 87 46 mm g			

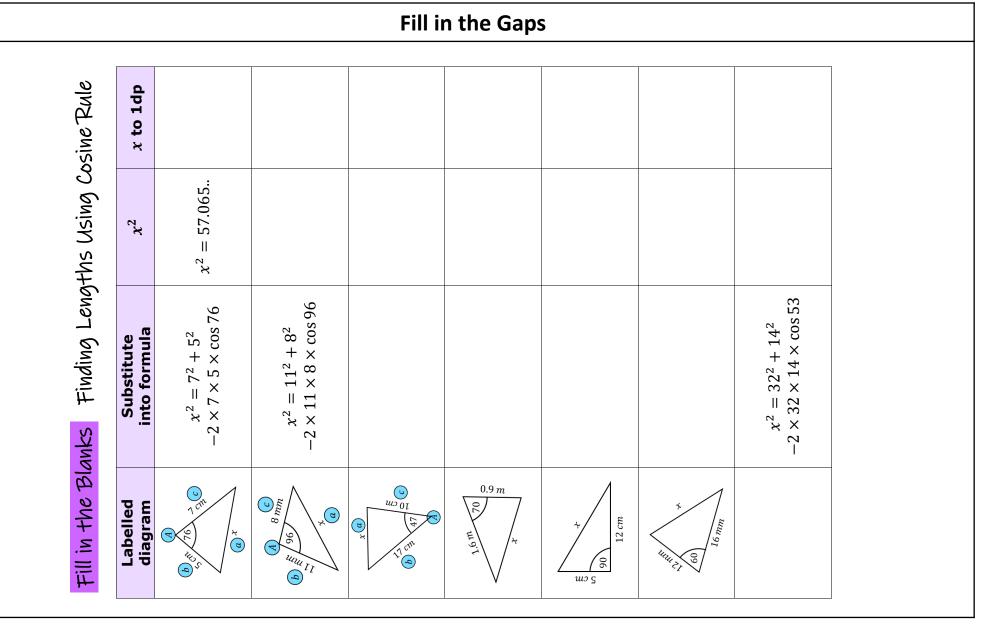


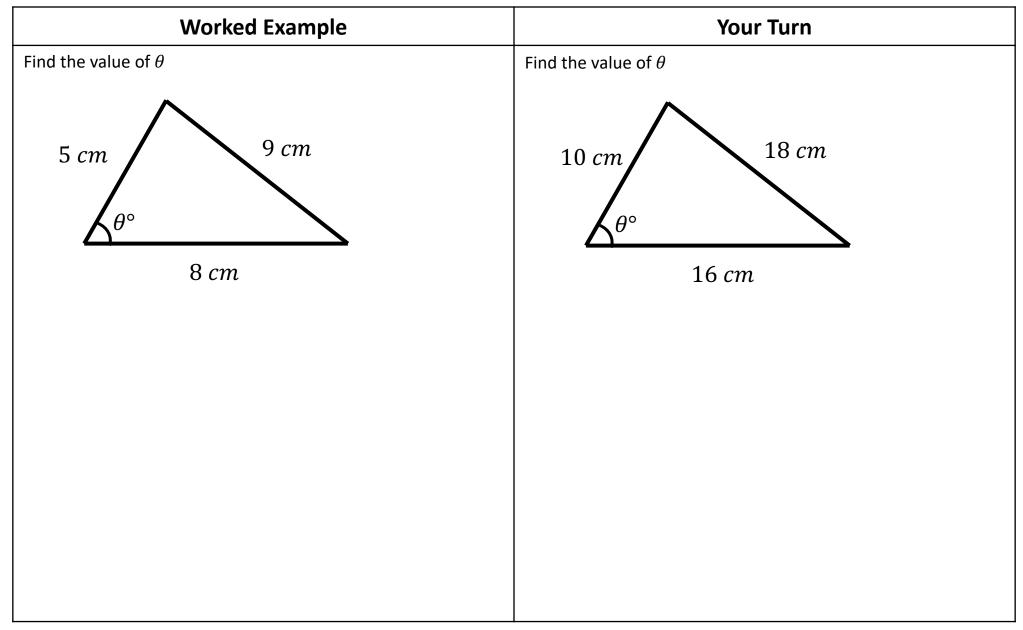


Cosine Rule



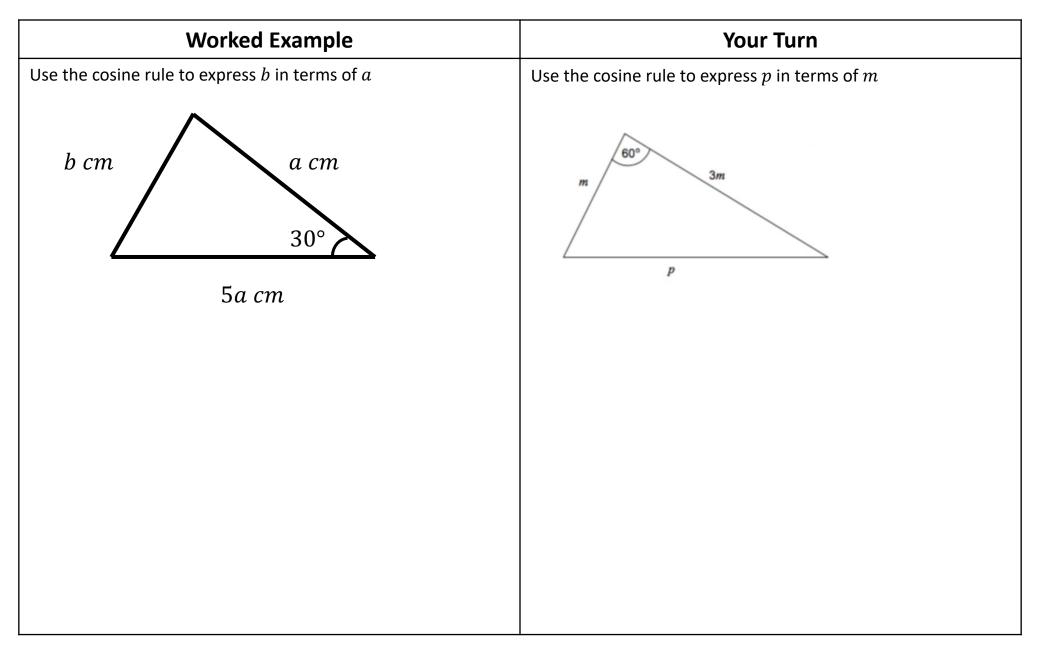
Question	Label the triangle with the angle being used as A	Fill into the formula	Use calculator to find missing length.
62 14.7 cm 17.9 cm x	$\begin{array}{c} A \\ 62 \\ 14.7 \ cm \\ 17.9 \ cm \\ B \end{array}$	$a^{2} = b^{2} + c^{2} - 2bc \cos A$ $x^{2} = 14.7^{2} + 17.9^{2} - 2 \times 14.7 \times 17.9 \cos 62$	$x^2 = 289.436$ $x = 17.0 \ cm \ (1 \ dp)$
8 cm d	$ \begin{array}{c} A & b \\ 6 & cm \\ 8 & cm \\ B \\ B \\ \end{array} $	$a^{2} = b^{2} + c^{2} - 2bc \cos A$ $x^{2} = 6^{2} + 8^{2} - 2 \times 6 \times 8 \times \cos 88$	
e 15 cm 32	e 15 cm 32		
65 mm 67 85 mm	65 mm 67 85 mm		
5.2 cm 48 5.5 cm	5.2 cm 48 5.5 cm		





Question	Label the triangle with the angle being found as A	Fill into the formula	Use calculator to find missing angle
11.4 cm 13.2 cm	$ \begin{array}{c} A \\ y \\ 10.7 \ cm \\ a \\ 13.2 \ cm \\ B \end{array} $	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \frac{10.7^2 + 11.4^2 - 13.2^2}{2 \times 10.7 \times 11.4}$	$\cos A = 0.2878$ $A = \cos^{-1}(0.2878)$ $A = 73.3^{\circ}$
7 cm 6 cm 6 cm	$B = \begin{bmatrix} c & m & b \\ 7 & cm & 6 & cm \\ 8 & cm & c \\ a \end{bmatrix}$	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos m = \frac{6^2 + 7^2 - 8^2}{2 \times 6 \times 7}$	
2.4 cm p 5.8 cm 6.2 cm	2.4 cm p 5.8 cm 6.2 cm		
9 cm 7 cm 11 cm r	9 cm 7 cm 11 cm r		
18 cm 25 cm	18 cm 25 cm		

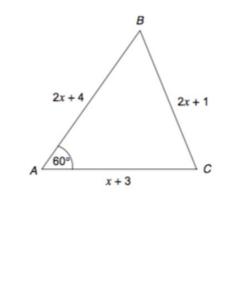
	Fill in the Gaps								
ne Rule	Angle (1dp)	$x = 76.7^{\circ}$							
Finding Angles Using Cosine Rule	Rearrange formula	$\cos x = \frac{9^2 + 13^2 - 14^2}{2 \times 9 \times 13}$	$\cos x = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$					$\cos x = \frac{6^2 + 5^2 - 3^2}{2 \times 6 \times 5}$	
	Substitute into formula	$14^{2} = 9^{2} + 13^{2}$ $-2 \times 9 \times 13 \times \cos x$	$15^{2} = 10^{2} + 7^{2}$ $-2 \times 10 \times 7 \times \cos x$	$4^{2} = 7^{2} + 8^{2}$ $-2 \times 7 \times 8 \times \cos x$					
Fill in the Blanks	Labelled diagram	Contraction of the second seco	b b b c m x c f c m a c f c f c m f c f c m	Bmm 8	A Transformed and the second s	1. Chi X. M. 2. Chi X. A. Chi A. S.	90 cm		



Worked Example	Your Turn
Worked Example A clock's hands are 5 cm and 3.5 cm. Find the distance between the tips of the hands at 4 o'clock	Your Turn A clock's hands are 10 cm and 7 cm. Find the distance between the tips of the hands at 5 o'clock

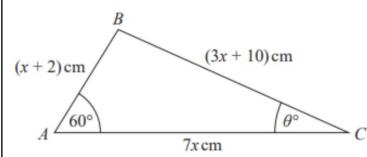
Worked Example

Use the cosine rule to find the exact value of *x*

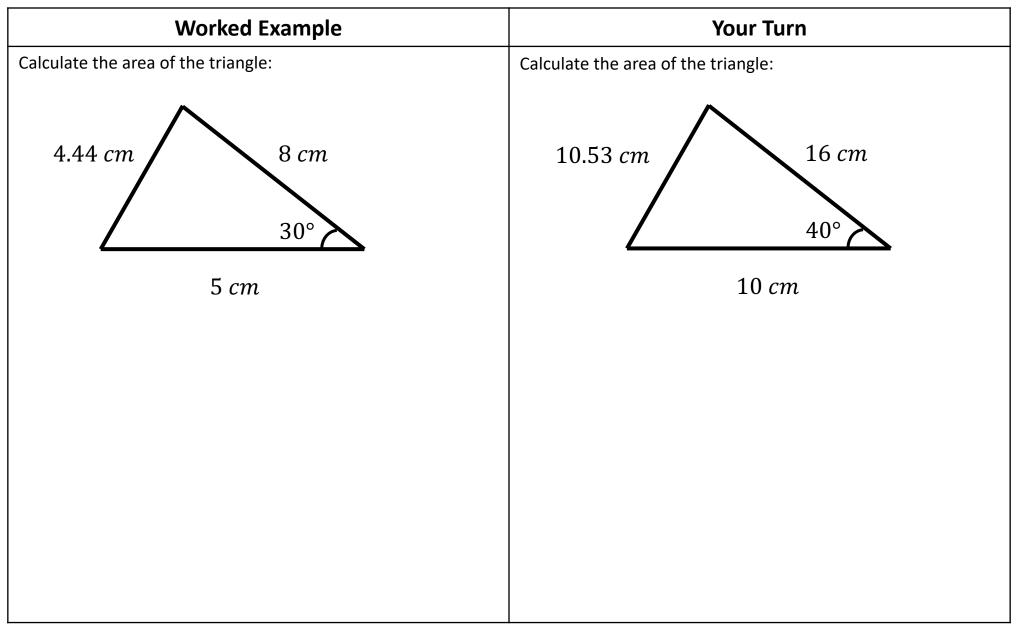


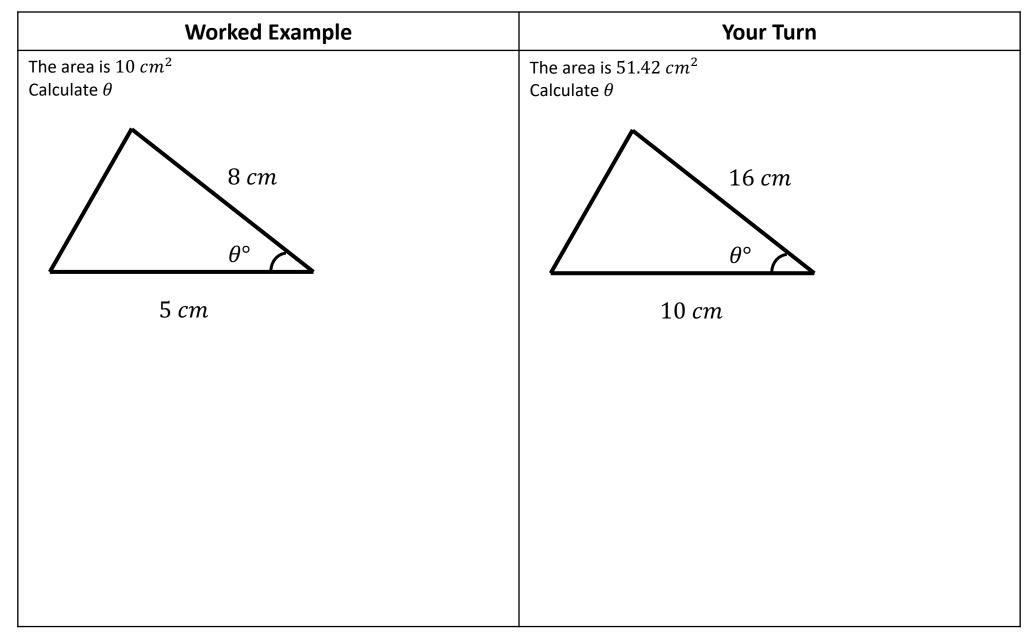
Your Turn

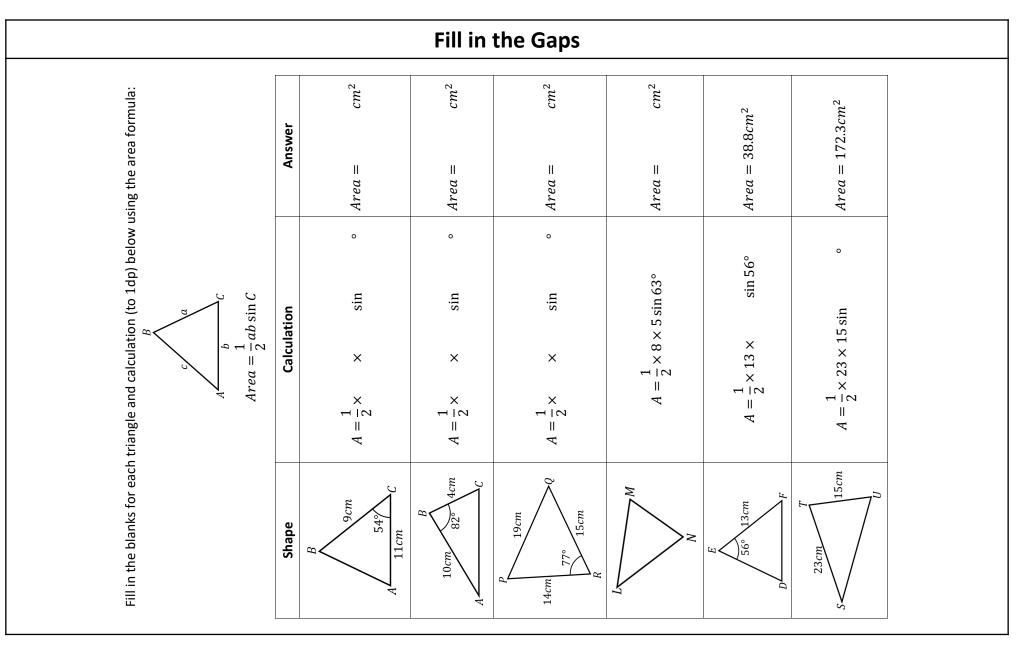
Use the cosine rule to find the exact value of *x*



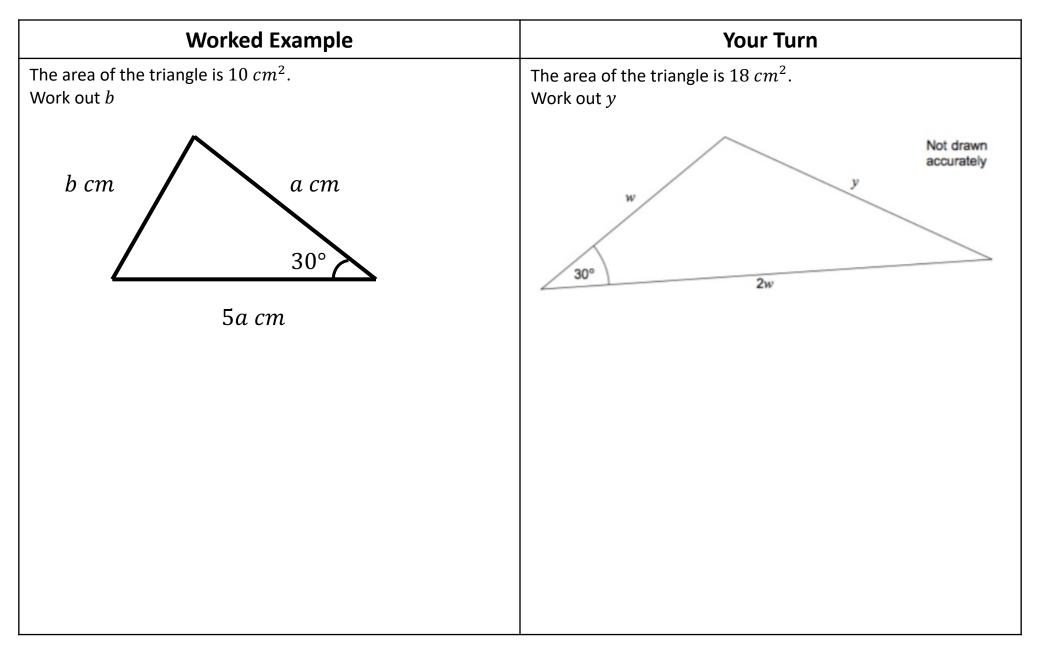
Area of Triangles

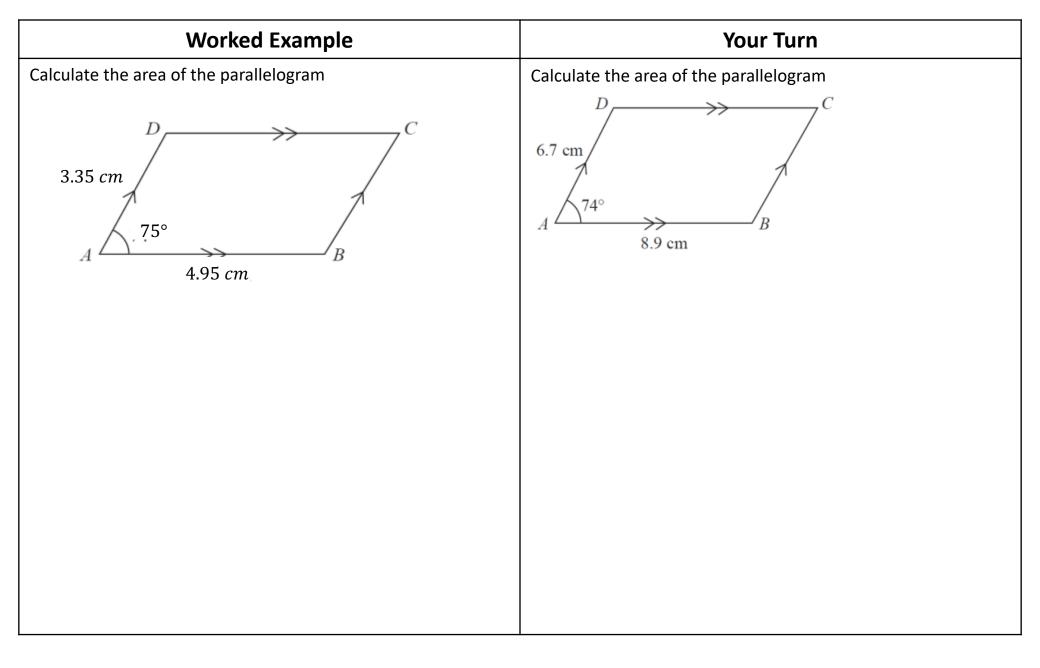


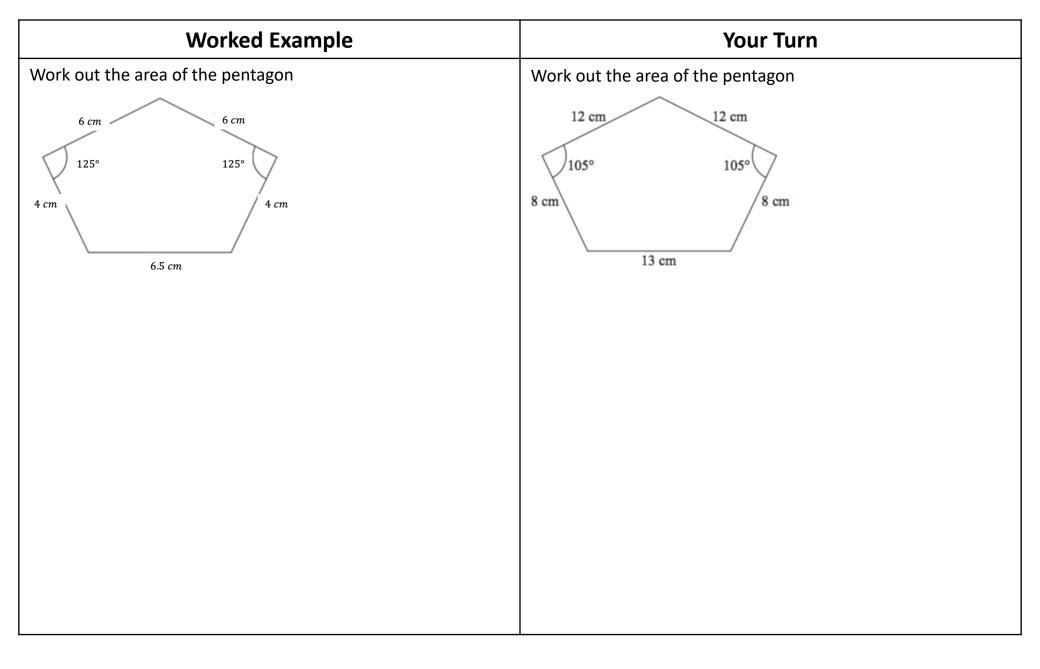


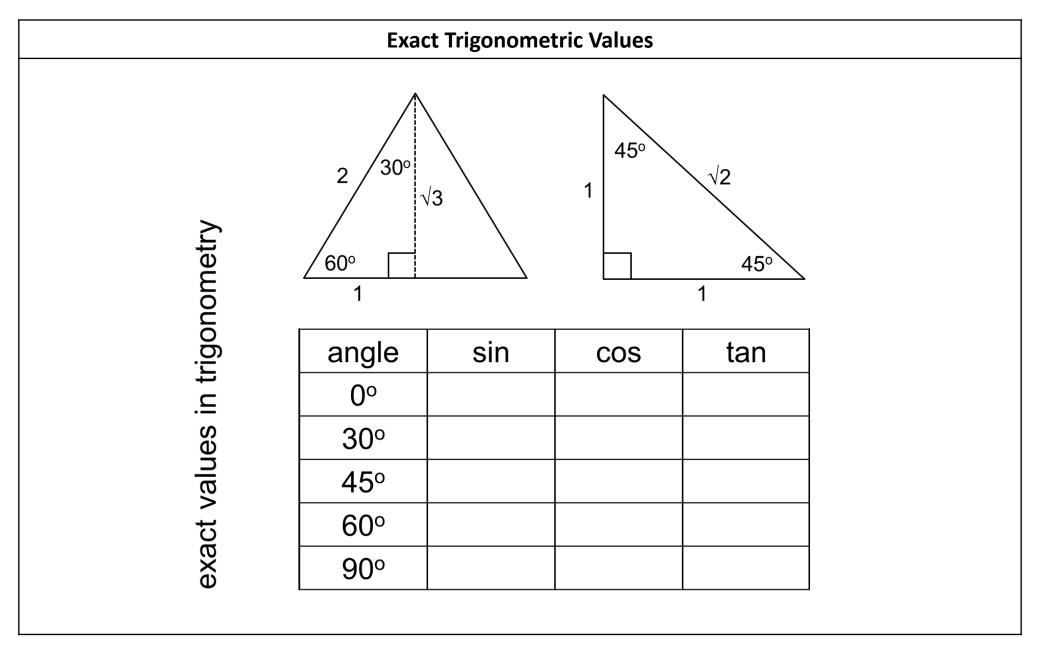


Worked Example	Your Turn
A triangle has sides 5.1 <i>cm</i> , 3.4 <i>cm</i> and 2.85 <i>cm</i> . Work out the area of the triangle.	A triangle has sides 10.2 <i>cm</i> , 6.8 <i>cm</i> and 5.7 <i>cm</i> . Work out the area of the triangle.

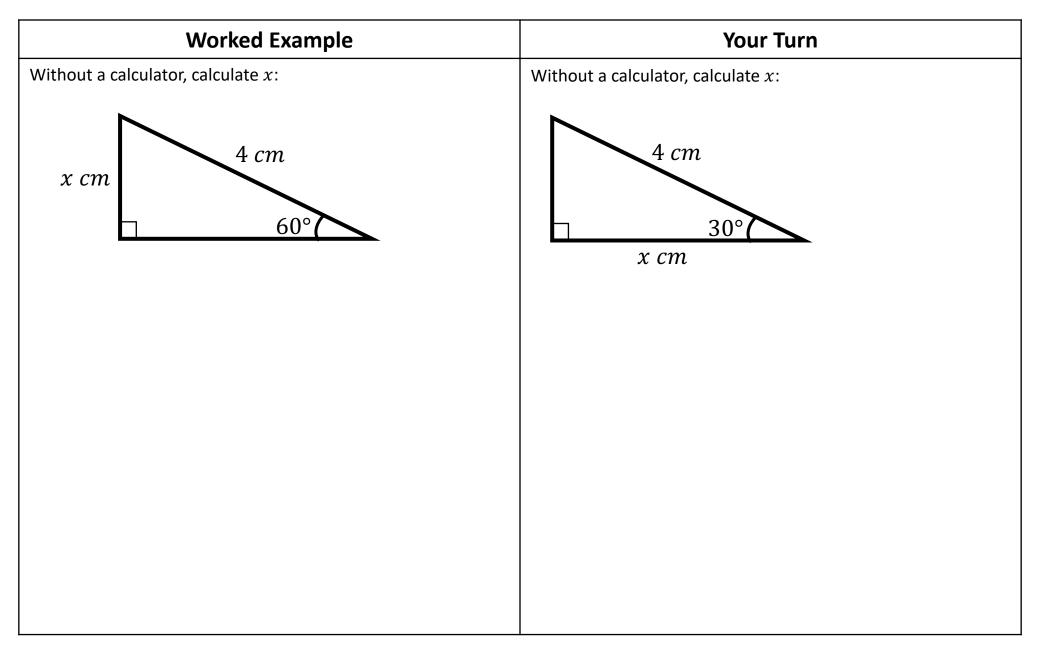


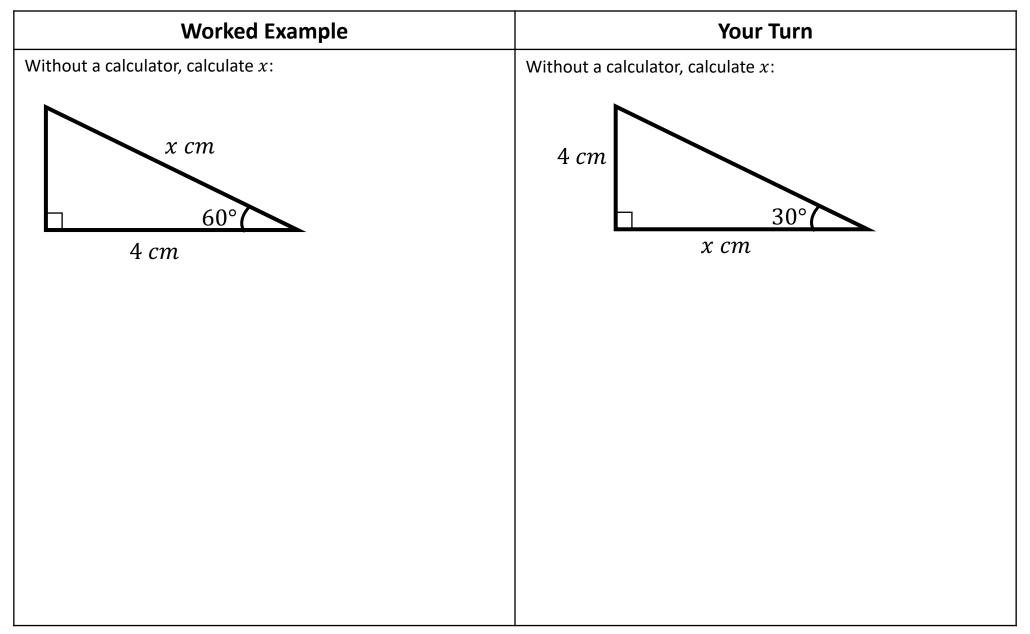






Worked Example	Your Turn
Show that $5 \sin 30^\circ \times \cos 30^\circ \times 8 \tan 30^\circ$ is an integer	Show that $2 \sin 60^\circ \times 5 \cos 60^\circ \times 6 \tan 60^\circ$ is an integer



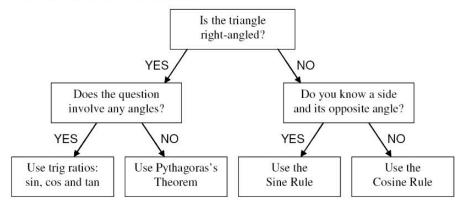


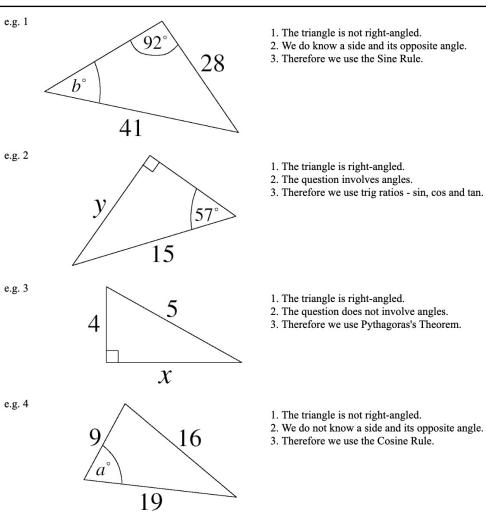
Worked Fxample	Your Turn
Worked Example The diagram shows two right-angled triangles <i>KLN</i> and <i>LMN</i> . $KL = 10 \text{ m}$, $KN = 9 \text{ m}$ and $\angle LMN = 30^{\circ}$. $M \longrightarrow L \longrightarrow $	Your Turn The diagram shows two right-angled triangles <i>IJL</i> and <i>JKL</i> . $\angle JIL = 60^{\circ}$, $IL = 10$ cm and $JK = 23$ cm. I = 10 cm I = 23 cm. Without using a calculator, work out the length of <i>KL</i> .

Review

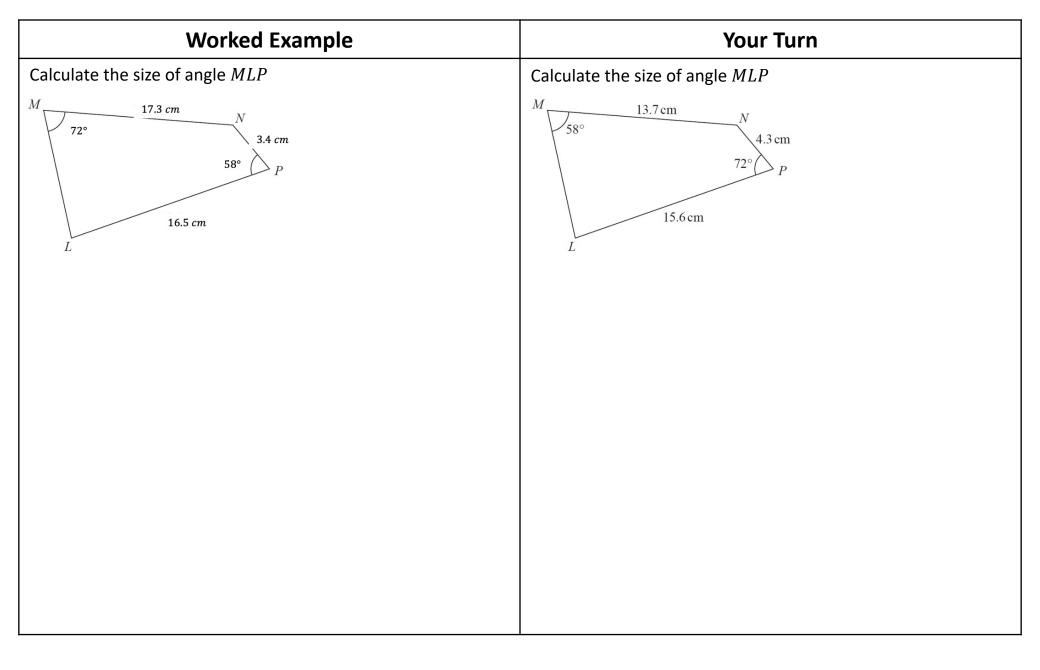
Choosing The Appropriate Technique

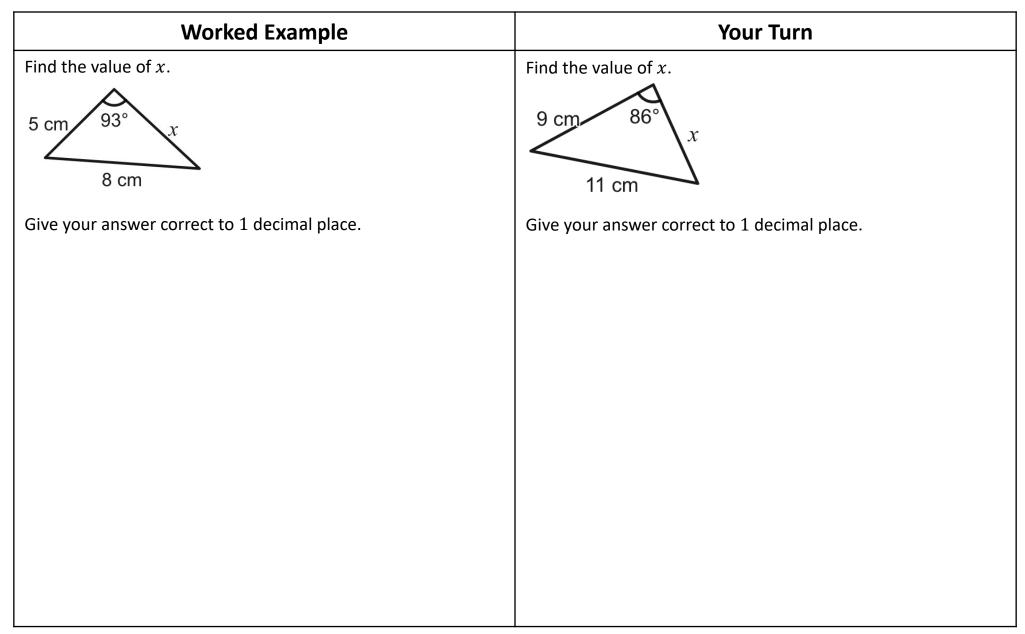
Sometimes more than one technique from the formula table at the top of this page can be used to solve a trig problem, but you will want to choose the most efficient and easiest method to save time. The flowchart below shows how to decide which method to use:





Your Turn Worked Example Find the value of *x*. Find the value of *x*. 63° 62° 46° 53° 5.7 cm 4.5 cm 55° **4**0° 8.7 cm 4.3 cm Give your answer correct to 2 decimal places. Give your answer correct to 2 decimal places.



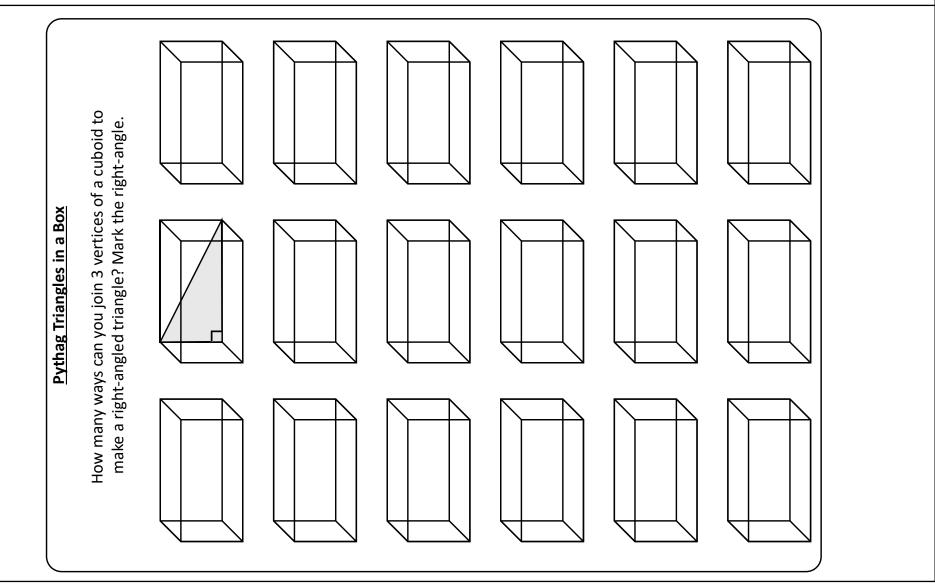


Extra Notes	

2 3D Pythagoras' Theorem and Trigonometry

3D Pythagoras' Theorem

Fluency Practice



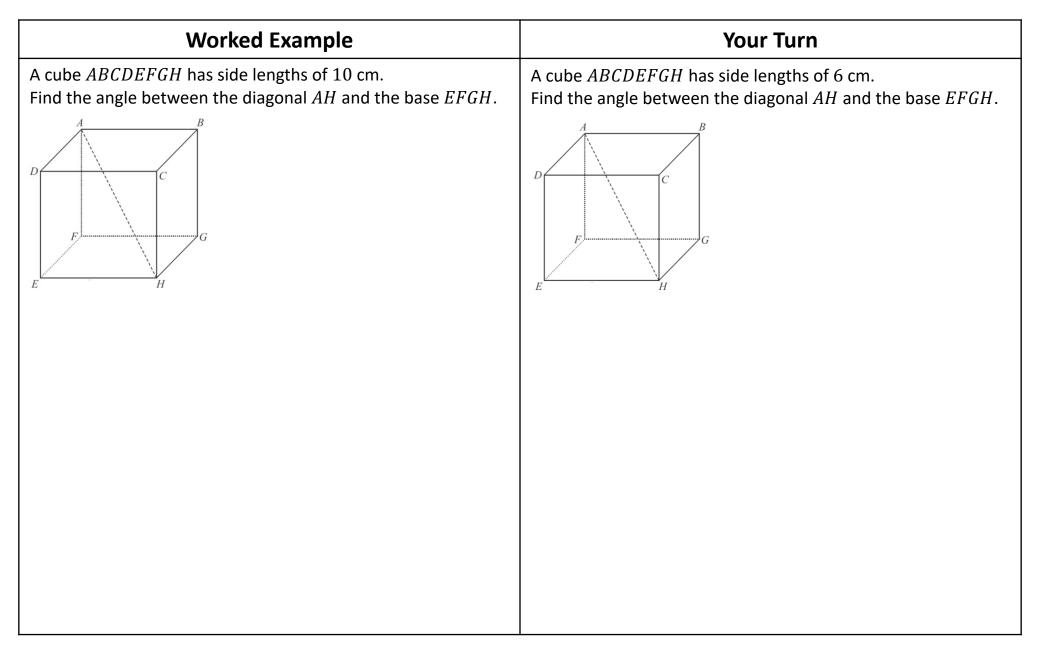
Worked Example	Your Turn
 Shown below is a cube. a) Calculate the length AC. b) Calculate the length AG. 	 Shown below is a cube. a) Calculate the length <i>BD</i>. b) Calculate the length <i>BH</i>.
b) Calculate the length AG.	b) Calculate the length <i>BH</i> .

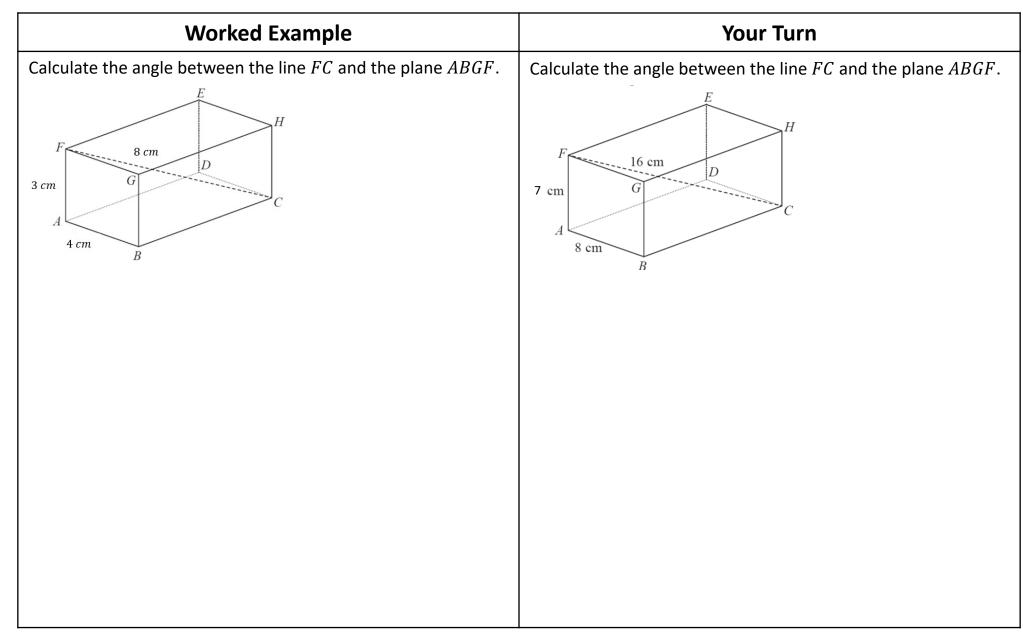
Worked Example	Your Turn
 Shown below is a cuboid. a) Calculate the length AC. b) Calculate the length AG. 	 Shown below is a cuboid. a) Calculate the length AC. b) Calculate the length AG.
F A B C B C B C B C B C B C B C B C B C B	F C A 6 c B 2 cm

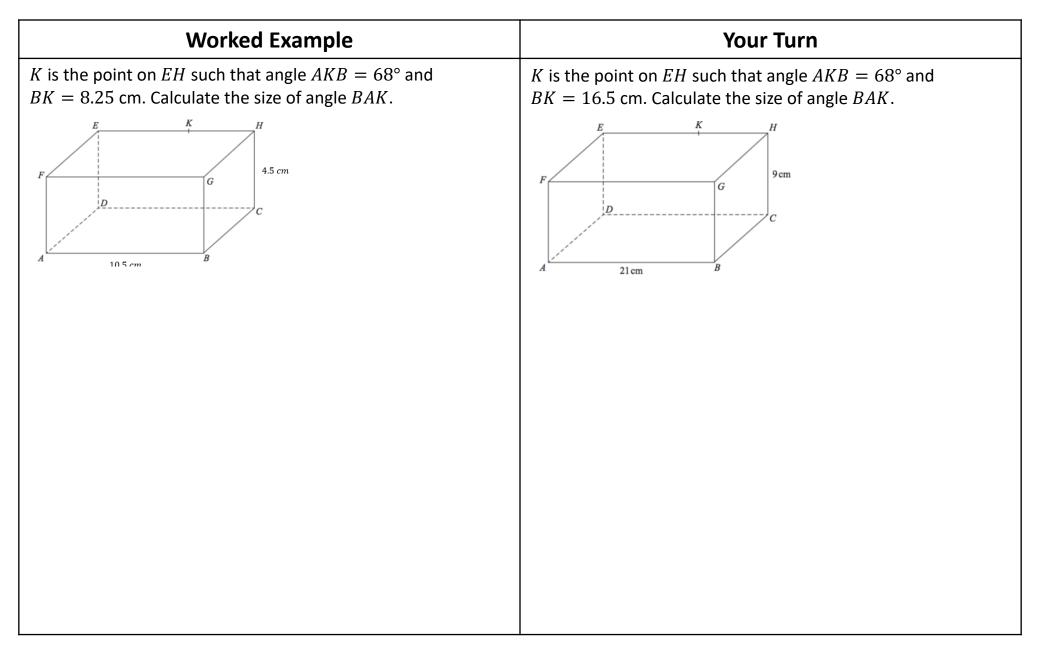
Worked Example	Your Turn
LMNPQR is a triangular prism. LM = 11 cm, QP = 15 cm and LQ = 9 cm. Angle $LMN = 90^{\circ}$	ABCDEF is a triangular prism. AB = 9 m, BC = 10 m and AE = 13 m. Angle $ABC = 90^{\circ}$.
9 cm L 11 cm R M N P	$ \begin{array}{c} E & 13 \text{ m} \\ F & & & & & \\ F & & & & & \\ D & & & & & \\ \end{array} $
Find the length of the line <i>MP</i> .	Find the length of the line <i>CE</i> .
Give your answer correct to 1 decimal place.	Give your answer correct to 1 decimal place.

Worked Example	Your Turn
 Shown below is a square based pyramid. a) Find the length <i>BD</i>. b) Find the length <i>EM</i>. c) Find the length <i>EF</i>. 	 Shown below is a square based pyramid. a) Find the length <i>BD</i>. b) Find the length <i>EM</i>. c) Find the length <i>EF</i>.
A 6cm D F	A Acm D F

3D Trigonometry

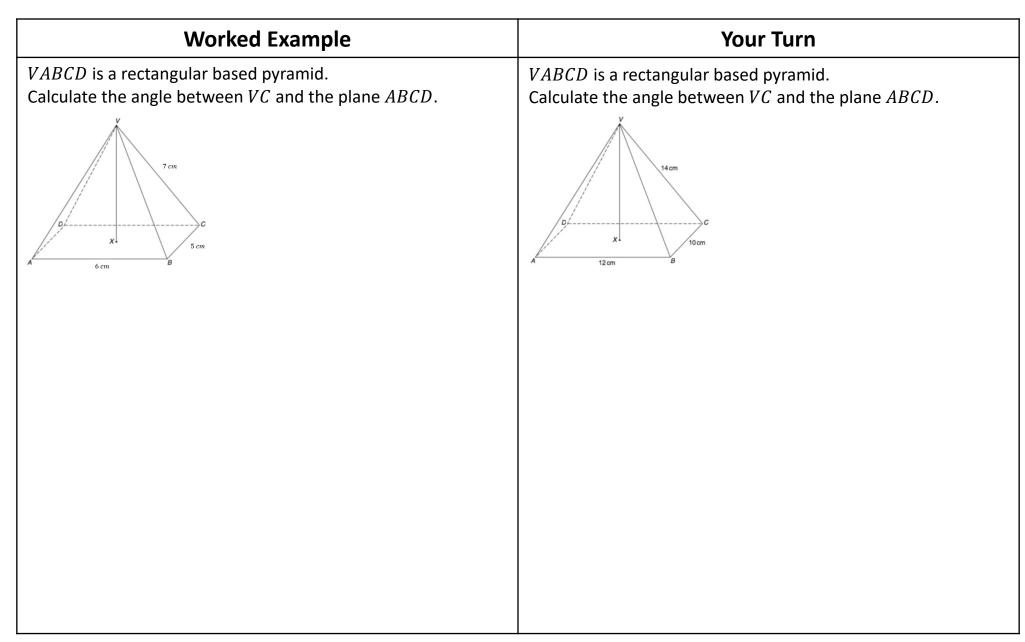


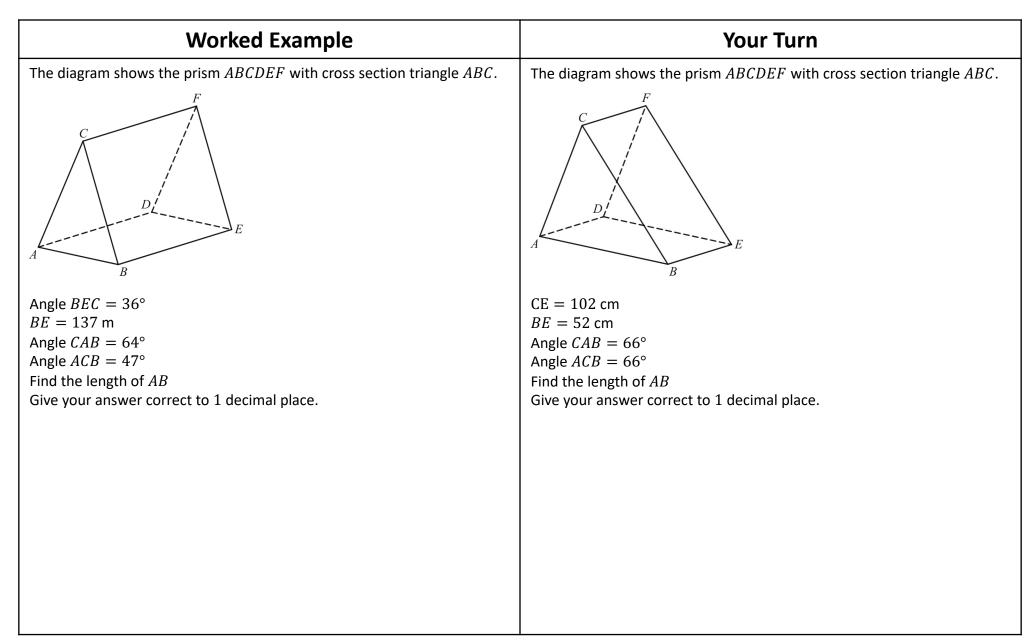




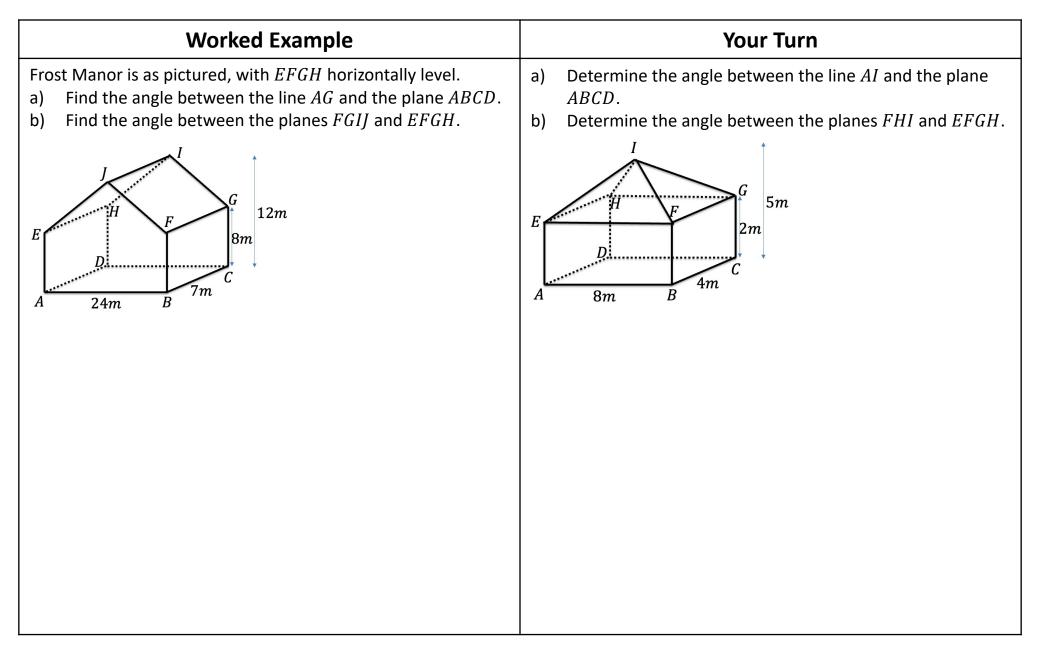
Worked Example	Your Turn
There is a rod in a cylindrical tin.	There is a rod in a cylindrical tin.
AC is a diameter of the base.	AC is a diameter of the base.
BC is vertical.	BC is vertical.
The radius of the base is 2.5 cm.	The radius of the base is 5 cm.
The volume of the tin is 589 cm ³ .	The volume of the tin is 598 cm ³ .
Find the angle between the rod and the base of the tin.	Find the angle between the rod and the base of the tin.

Worked Example	Your Turn
<i>M</i> is the midpoint of <i>PR</i> .	<i>M</i> is the midpoint of <i>PR</i> .
Calculate the size of the angle between <i>TP</i> and the base <i>PQRS</i> .	Calculate the size of the angle between <i>TP</i> and the base <i>PQRS</i> .
Diagram NOT accurately drawn	Diagram NOT accurately drawn s b b b c c c c c c c c c c c c c c c c





Worked Example	Your Turn
Calculate the size of the angle between the line <i>AD</i> and the plane <i>ABC</i> .	Calculate the size of the angle between the line <i>AD</i> and the plane <i>ABC</i> .

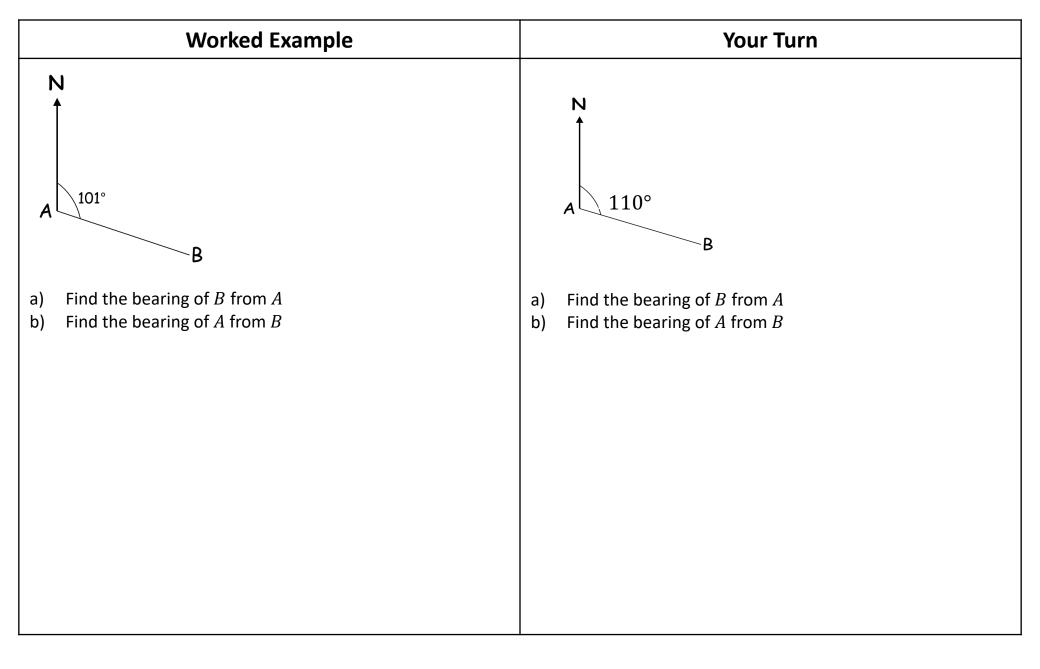


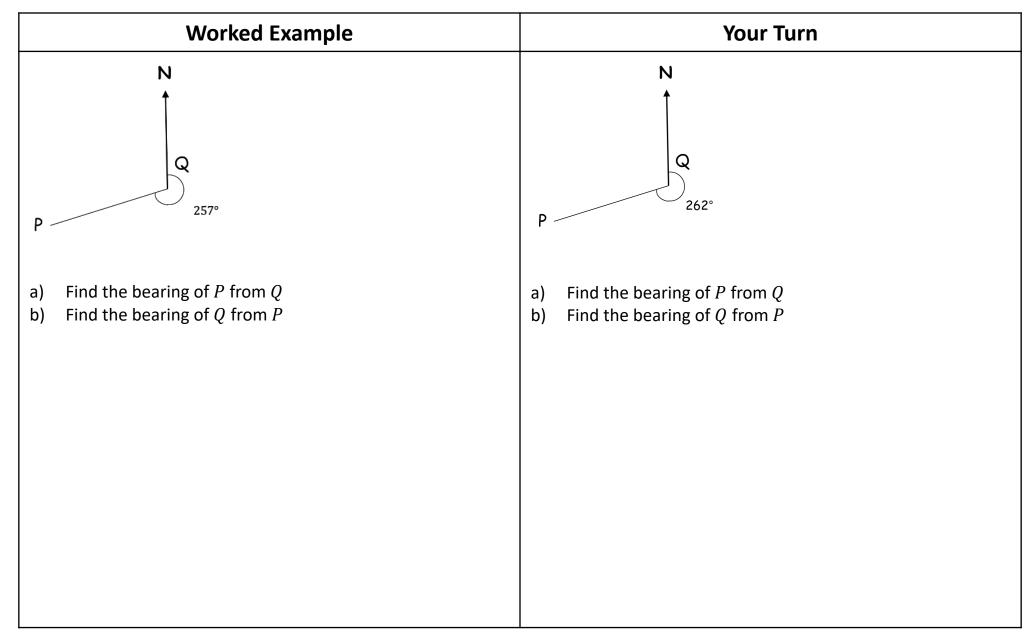
Extra Notes	

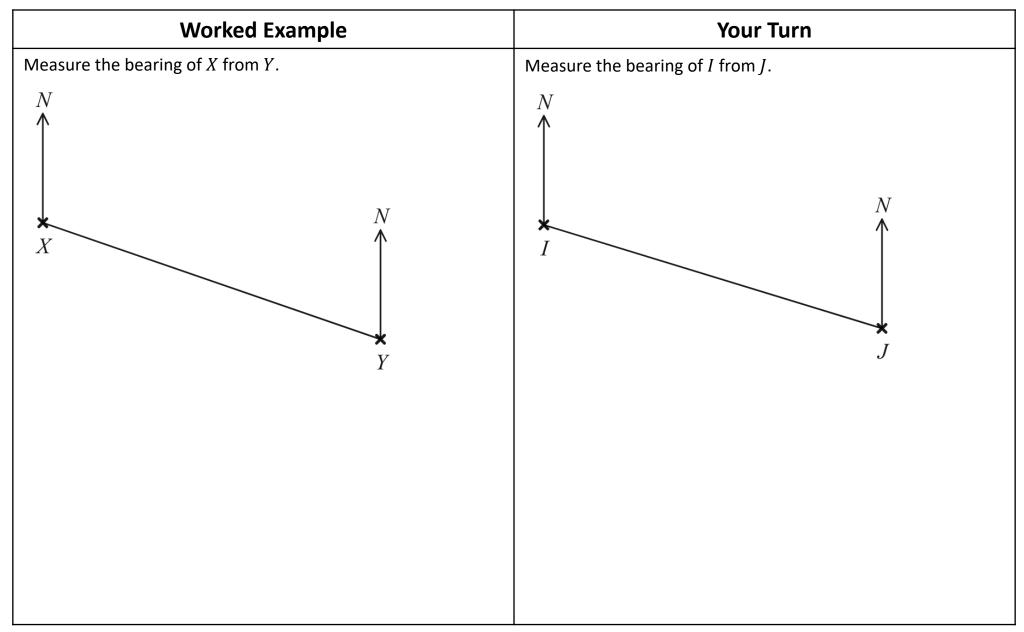
3 Bearings		

Can this be a Bearing?		
	040	Yes / No
	90	Yes / No
	90.5	Yes / No
	158.50	Yes / No
	58.5	Yes / No

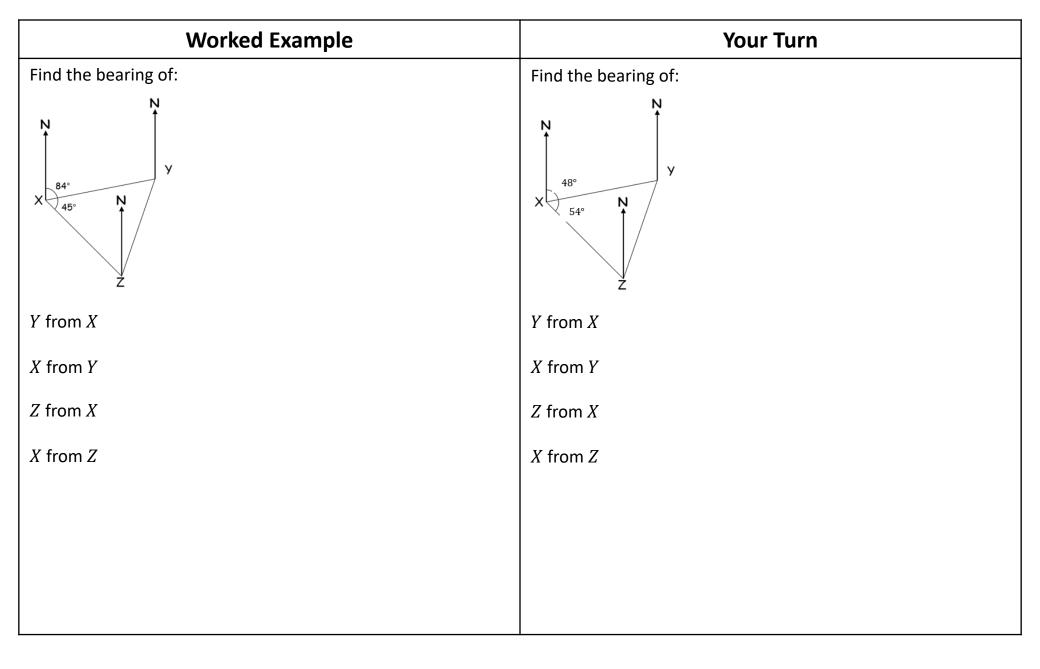
Intelligent Practice										
1)	045	Yes /	No	14)	-049	Yes	/	Νο		
2)	090	Yes /	No	15)	049.5	Yes	1	Νο		
3)	45	Yes /	Νο		0180	Yes	,	No		
4)	360	Yes /	No							
5)	361	Yes /	No	17)	045	Yes	/	Νο		
6)		Yes /		18)	145	Yes	/	Νο		
				19)	-260	Yes	/	Νο		
7)	30	Yes /	Νο	20)	0100	Yes	1	Νο		
8)	030	Yes /	Νο	21)		Yes				
9)	-145	Yes /	No							
10)	260	Yes /	No	22)	080	Yes	/	Νο		
				23)	0005	Yes	/	Νο		
		Yes /		24)	000.5	Yes	/	Νο		
12)	180	Yes /	Νο	25)	100.005	Yes	/	Νο		
13)	27	Yes /	Νο	,			•			

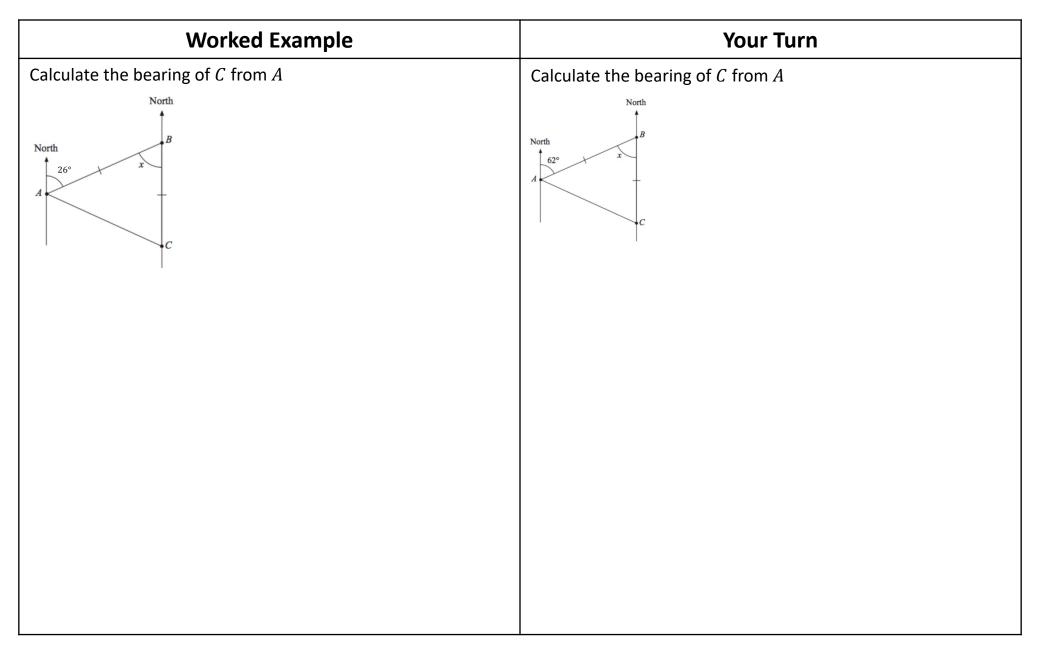




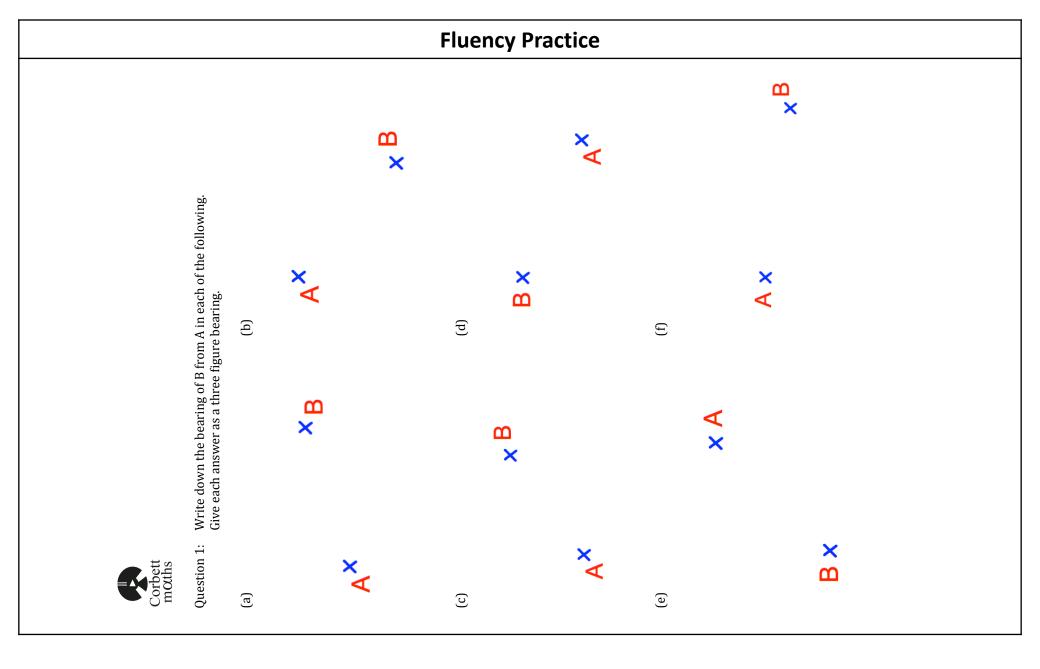


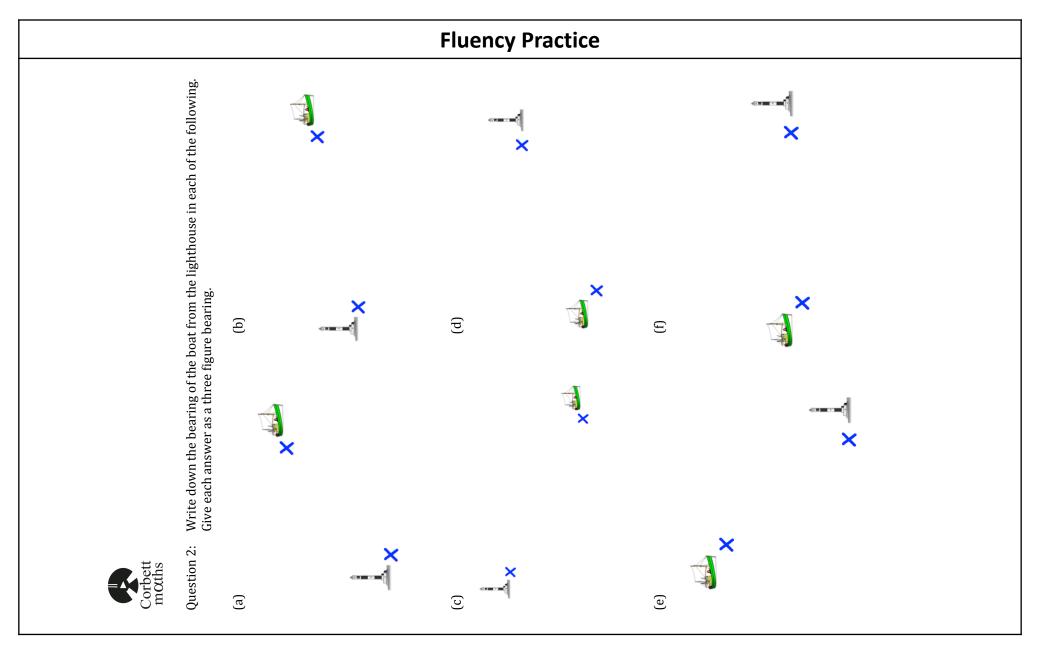
	Worked Example		Your Turn
a)	The bearing of <i>B</i> from <i>A</i> is 030° . What is the bearing of <i>A</i> from <i>B</i> ?	a)	The bearing of <i>B</i> from <i>A</i> is 250° . What is the bearing of <i>A</i> from <i>B</i> ?
b)	The bearing of <i>B</i> from <i>A</i> is 130° . What is the bearing of <i>A</i> from <i>B</i> ?	b)	The bearing of <i>B</i> from <i>A</i> is 050° . What is the bearing of <i>A</i> from <i>B</i> ?
c)	The bearing of <i>B</i> from <i>A</i> is 230° . What is the bearing of <i>A</i> from <i>B</i> ?	c)	The bearing of <i>B</i> from <i>A</i> is 150° . What is the bearing of <i>A</i> from <i>B</i> ?

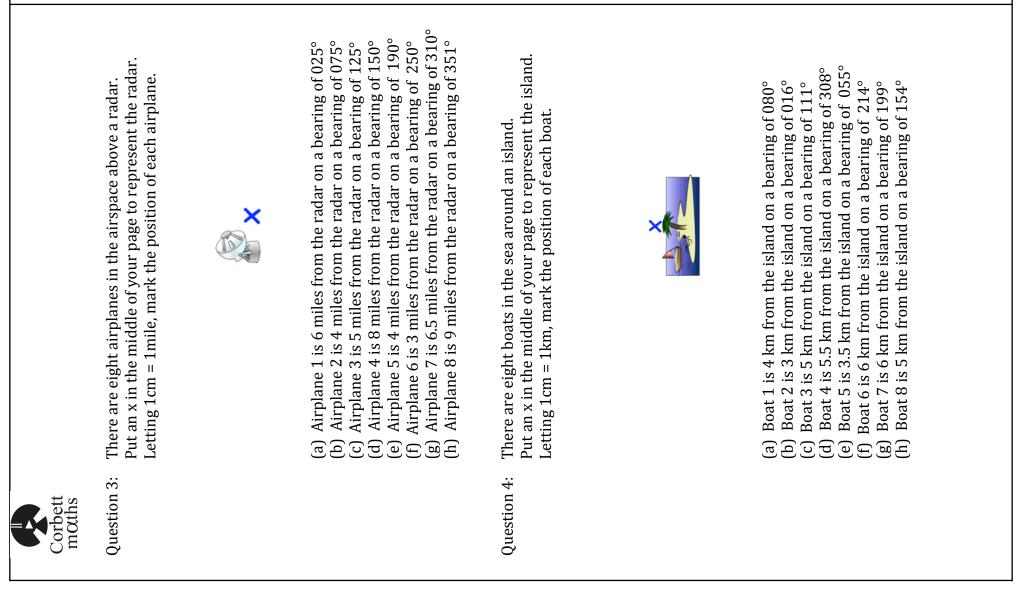




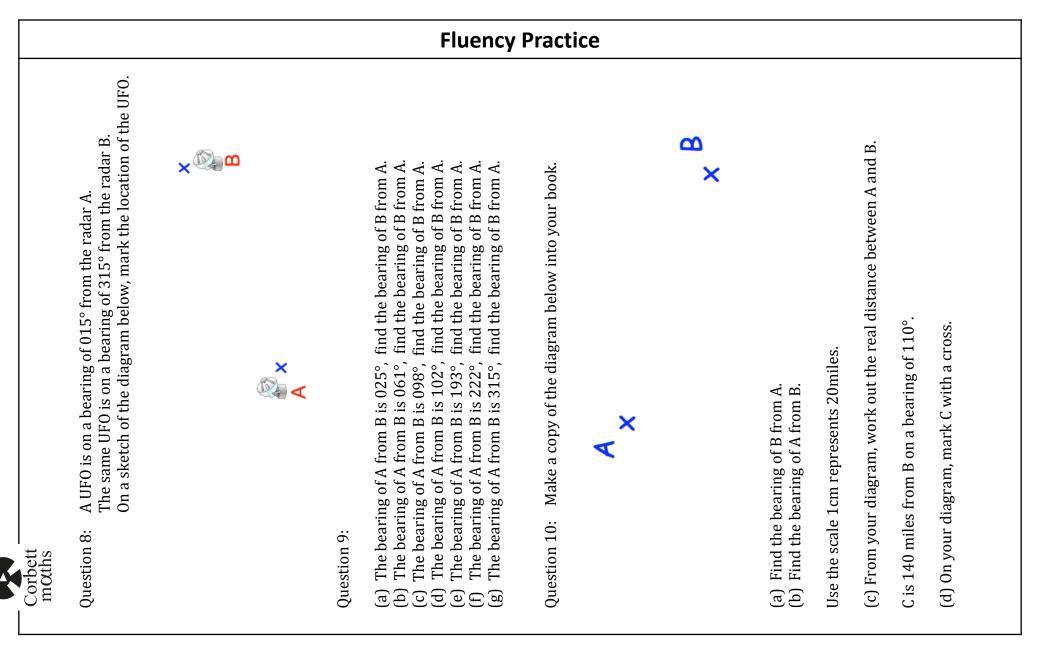
Worked Example	Your Turn
The diagram shows three points, <i>G</i> , <i>H</i> and <i>I</i> . The bearing of <i>H</i> from <i>G</i> is 251°. The bearing of <i>I</i> from <i>H</i> is 040°. HI = IG \int_{1}^{N} \int_{1}^{1} \int_{1}^{1} $\int_{251^{\circ}}^{251^{\circ}}$ Find the bearing of <i>I</i> from <i>G</i> .	The diagram shows three points, <i>G</i> , <i>H</i> and <i>I</i> . The bearing of <i>I</i> from <i>H</i> is 051°. The bearing of <i>H</i> from <i>G</i> is 287°. HI = IG $\int_{I}^{N} \int_{I}^{I} \int_{I}^$

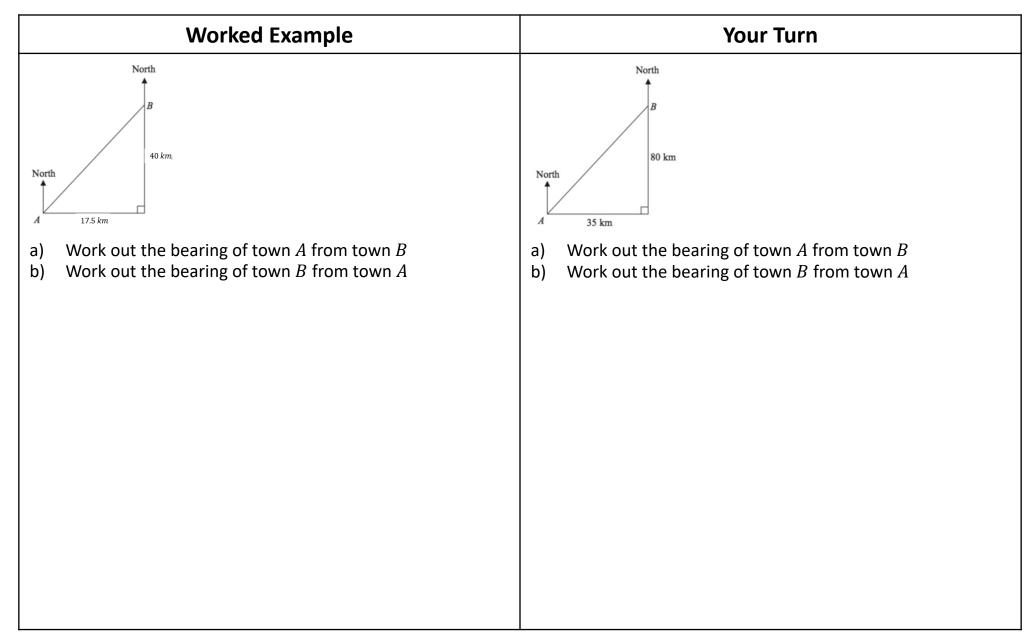


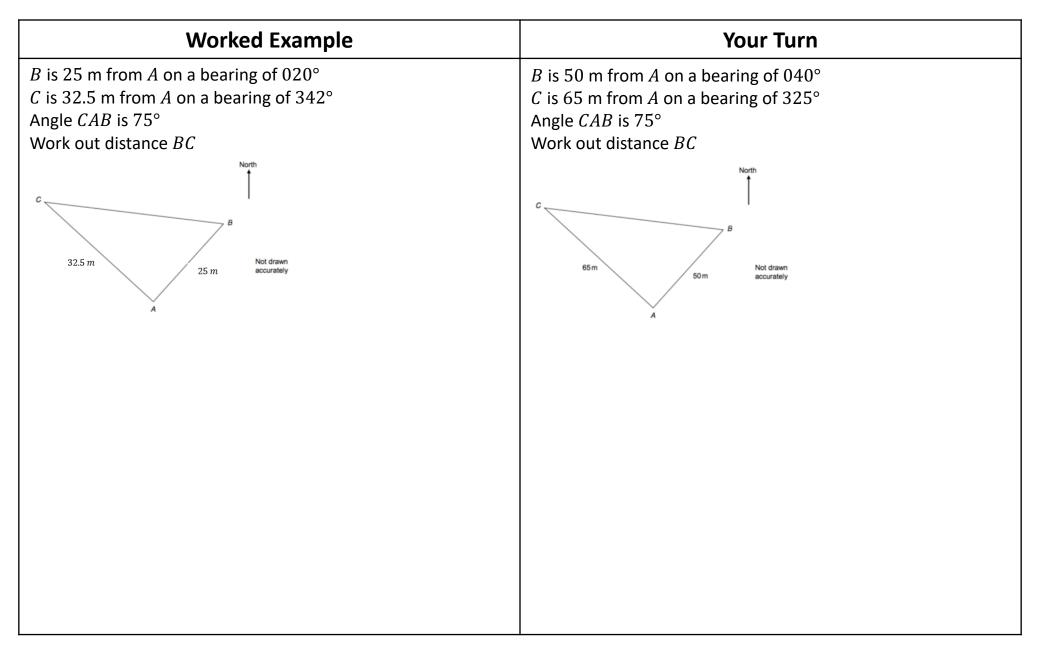


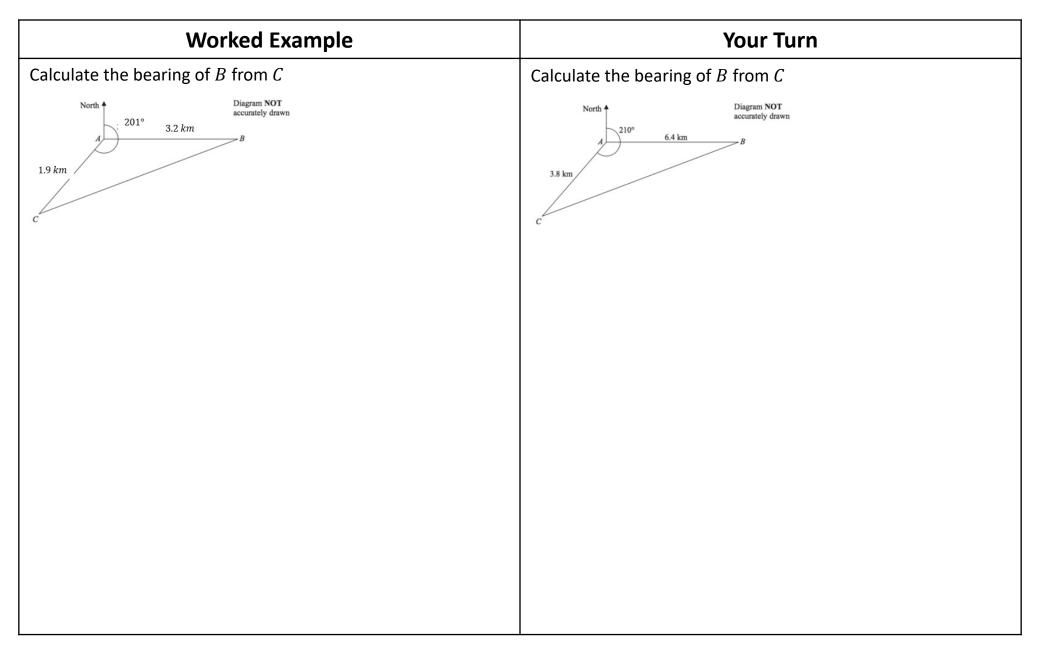


	Fluency Practice							
	,ure bearings	(d) North-east	(h) North-west	A dolphin is on a bearing of 100° from the island. The same dolphin is on a bearing of 015° from the lighthouse. On a sketch of the diagram below, mark the location of the dolphin.	×	A hot-air balloon is on a bearing of 140° from the radar A. The same hot-air balloon is on a bearing of 065° from the radar B. On a sketch of the diagram below, mark the location of the hot-air balloon.		
	avel as three fig.	(c) West	(g) South	of 100° from the bearing of 015°. n below, mark th X		earing of 140° fi is on a bearing c n below, mark th	× ©a ≺	×
	Give these directions of travel as three figure bearings	(b) South-east	(f) South-west	A dolphin is on a bearing of 100° from the island. The same dolphin is on a bearing of 015° from the lighthouse. On a sketch of the diagram below, mark the location of the dol		A hot-air balloon is on a bearing of 140° from the radar A. The same hot-air balloon is on a bearing of 065° from the radar B. On a sketch of the diagram below, mark the location of the hot-air		
Corbett	Question 5:	(a) North	(e) East	Question 6:		Question 7:		







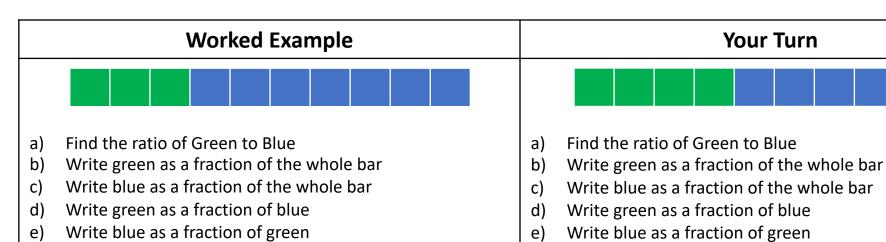


Worked Example	Your Turn
Worked Example A, B and C are three points.The bearing of A from B is 045°.The bearing of C from A is 135°. $AB = 10$ km and $AC = 6$ km.Find the distance BC and the bearing of C from B .	Your Turn A, B and C are three points. The bearing of A from B is 054°. The bearing of C from A is 153°. AB = 6 km and AC = 10 km. Find the distance BC and the bearing of C from B.

Worked Example	Your Turn
The diagram shows the position of three radio towers, <i>A</i> , <i>B</i> and <i>C</i> .	The diagram shows the position of three radio towers, <i>A</i> , <i>B</i> and <i>C</i> .
The bearing of <i>A</i> from <i>B</i> is 195°.	The bearing of <i>B</i> from <i>A</i> is 167°.
The bearing of <i>B</i> from <i>C</i> is 64°.	The bearing of <i>A</i> from <i>C</i> is 65°.
The distance between <i>B</i> and <i>A</i> is 51 km.	The distance between <i>A</i> and <i>B</i> is 29 km.
The distance between <i>B</i> and <i>C</i> is 55 km.	The distance between <i>A</i> and <i>C</i> is 30 km.
$N = \frac{1}{64^{\circ}} = \frac{55 \text{ km}}{51 \text{ km}} = \frac{55 \text{ km}}{51 \text{ km}}$	$N = \frac{167^{\circ}}{C} = \frac{30 \text{ km}}{C} = \frac{167^{\circ}}{29 \text{ km}}$
Calculate the area of triangle <i>ABC</i> .	Calculate the area of triangle <i>ABC</i> .
Give your answer correct to 1 decimal place.	Give your answer correct to 1 decimal place.

Extra Notes

4 Advanced Ratio



f) Form a linear equation linking green and blue Green =

Blue =

f)

Green =

Blue =

Form a linear equation linking green and blue

	Fill in the Gaps							
x : y	Visual representation	x as a fraction of whole	y as a fraction of a whole	x as a fraction of y	y as a fraction of x	Linear equation	× =	y =
1:7								
		$\frac{3}{8}$						
				$\frac{3}{2}$				
						3x = 7y		
							$x = \frac{10}{3}$	
								$y = \frac{2}{9}$

Worked Example	Your Turn
Given that $3y = 7x$, work out the ratio $x : y$	Given that $9q = 4p$, work out the ratio $p : q$

Worked Example	Your Turn	
The ratio $5x + 3 : 2y - 1$ is equal to $5 : 4$ Express x in terms of y	The ratio $a + 1 : 2b + 5$ is equal to $5 : 7$ Express a in terms of b	

Worked Example	Your Turn	
Given that $3x - 10 : 9x - 51 = 2 : 3$ Find the value of x.	Given that $9a - 4 : 7a + 21 = 7 : 2$ Find the value of a .	

Worked Example	Your Turn
Worked ExampleGiven that $7x - 6 : 4x + 12 = 5x - 2 : 5x + 10$ Find the possible values of x .	Your TurnGiven that $6a + 11 : 3a + 3 = 5a + 8 : 2a + 4$ Find the possible values of a .

Your Turn
Your Turn a, b, c and d are integers with no common factors. $4a = 7b$ $c: d = 3: 2$ $a: d = 4: 7$ Find $a: b: c: d$

Worked Example	Your Turn
Worked Example There are blue counters and white counters in a bag in the ratio 4 : 3 10 blue counters are added, and the ratio becomes 2 : 1 Work out how many white counters there are in the bag.	Your Turn There are black counters and red counters in a bag in the ratio 3 : 4 20 black counters are removed, and the ratio becomes 1 : 3 Work out how many red counters there are in the bag.

Worked Example	Your Turn
Worked ExampleThere are black counters and red counters in a bag in the ratio3 : 75 black counters are removed, and 10 red counters are addedto the bag, and the ratio becomes 2 : 5Work out the original number of red counters in the bag.	Your Turn There are white counters and red counters in a bag in the ratio 3:4 10 white counters are removed, and 1 red counter is added to the bag, and the ratio becomes 2:3 Work out the original number of red counters in the bag.

Worked Example	Your Turn
Green shapes and purple shapes are used in a game.BlueSome of the shapes are triangles.SoAll the other shapes are hexagons.AllThe ratio of triangles to hexagons is $5:2$ ThThe ratio of green triangles to purple triangles is $3:5$ Th	Blue shapes and red shapes are used in a game. Some of the shapes are circles. All the other shapes are squares. The ratio of circles to squares is 4 : 5 The ratio of blue circles to red circles is 3 : 2 Work out the fraction of shapes that are red circles.

Your Turn
Blue shapes and red shapes are used in a game. Some of the shapes are circles. All of the other shapes are squares. The ratio of the number of blue shapes to the number of red shapes is 4 : 1 The ratio of the number of blue circles to the number of blue squares is 3 : 4 The ratio of the number of red circles to the number of red
squares is 3 : 2 Work out what fraction of all the shapes are circles.

Extra Notes	