KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 22 Tasks – Part 1

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KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 22 Tasks – Part 2

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ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 22 Tasks – Part 3

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KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 22 Tasks – Part 4

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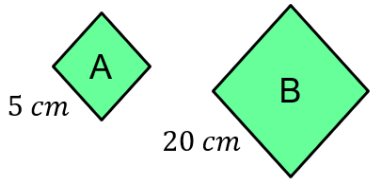
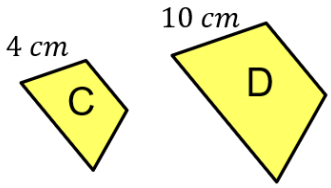
Contents

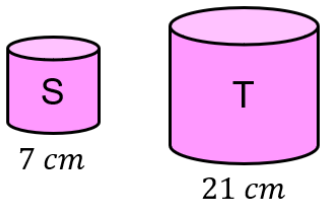
1 Similarity with Area and Volume



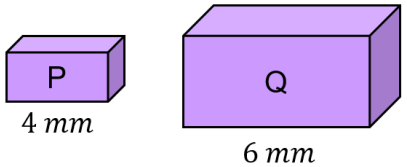
(a) Find the linear scale factor.
(b) Find the area scale factor.
(c) The area of shape A is $15\ cm^2$, find the area of shape B.
(d) The area of shape B is $360\ cm^2$, find the area of shape A.



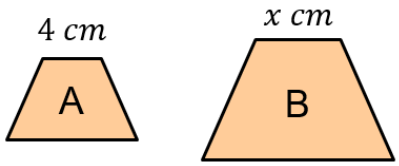
(a) The area of shape C is $12\ cm^2$, find the area of shape D.
(b) The area of shape D is $50\ cm^2$, find the area of shape C.



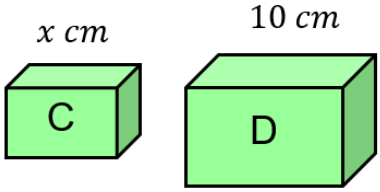
(a) The volume of shape S is $70\ cm^3$, find the volume of shape T.
(b) The volume of shape T is $810\ cm^3$, find the volume of shape S.



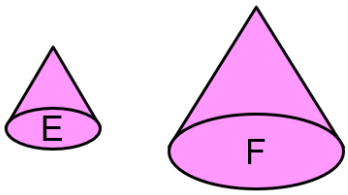
(a) The volume of shape P is $24\ cm^3$, find the volume of shape Q.
(b) The volume of shape Q is $270\ cm^3$, find the volume of shape P.
(c) The surface area of shape P is $110\ cm^2$, find the surface area of shape Q.
(d) The surface area of shape Q is $180\ cm^2$, find the surface area of shape P.



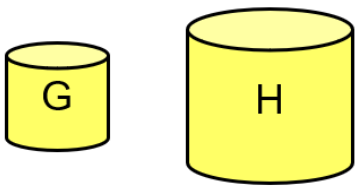
(a) The area of A is $20\ cm^2$ and the area of B is $180\ cm^2$. Find x.



(b) The volume of C is $5\ cm^3$ and the volume of D is $320\ cm^2$. Find x.



(c) The surface area of E is $15\ cm^2$ and the surface area of F is $60\ cm^2$. If the volume of E is $27\ cm^3$, find the volume of F.



(d) The volume of G is $4\ cm^3$ and the volume of H is $171.5\ cm^3$. If the surface area of H is $122.5\ cm^2$, find the surface area of G.

(e) If a painting with area of $220\ cm^2$ has a diagonal length of $21\ cm$, what will be the diagonal length of a similar painting with area $350\ cm^2$?

(f) It takes 5.6 litres of paint to paint a tower that is 3 m high. What is the tallest similar tower that can be painted with 8 litres of paint?

(g) A bronze statue has a mass of 300g and a height of 9 cm. A similar statue has a mass of 2 kg. What is its height?

Remember: If (between two similar shapes) a length is a and the equivalent length is b then the length scale factor k is given by $k = \frac{b}{a}$. Going from the first to the second:
lengths get multiplied by k,
areas get multiplied by k^2,
volumes get multiplied by k^3.

1. Two boxes of cereal are similar in shape. The smaller one has a height of 15 cm. The larger one has a height of 20 cm.
   (a) Find the length scale factor k going from the smaller to the larger. [4/3]
   (b) If the small box has volume 750 cm^3, find the volume of the larger box. [1777.$\dot{7}$]
   (c) If the label on the front of the smaller box has area 20 cm^2, find the area of the label on the larger box. [35.$\dot{5}$]

2. Two children's toys are similar in shape. The smaller one has a height of 20 cm. The larger one has a height of 25 cm.
   (a) Find the length scale factor k going from the smaller to the larger. [5/4]
   (b) If the small toy has surface area 800 cm^2, find the surface area of the larger toy. [1250]
   (c) If the volume of the larger toy is 250 cm^3, find the volume of the smaller toy. [128]

3. Two cones are similar in shape. The height of the larger one is 10 cm. The height of the smaller one is 7 cm.
   (a) Find the length scale factor k going from the larger to the smaller. [0.7]
   (b) If the larger cone has volume 100 cm^3, find the volume of the smaller cone. [34.3]

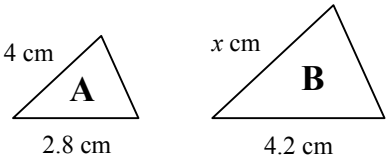
4. Two flasks are similar in shape. The surface area of the larger one is 100 cm^2. The surface area of the smaller one is 64 cm^2.
   (a) Find the length scale factor k going from the larger to the smaller. [0.8]
   (b) If the volume of the larger one is 94 cm^3 find the volume of the smaller one. [48.128]

5. Charlie has two similar sized rocks. The smaller has a volume of 2 cm^3. The larger has a volume of 2.662 cm^3.
   (a) If the larger one has length 1.4 cm, find the length of the smaller one. [14/11]
   (b) If the smaller has a surface area of 4.2 cm^2, find the surface area of the larger one. [5.082]

6. Tiffany is similar in shape to her daughter Candy. Tiffany's back has surface area 1500 cm^2. Candy's is 1215 cm^2.
   (a) If Candy is 130 cm tall, how tall is Tiffany? [144.$\dot{4}$]
   (b) If Tiffany has a volume of 50000 cm^3, what is Candy's volume? [36450]

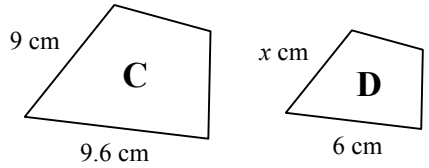
Fluency Practice

A1 Shape **A** is similar to shape **B**



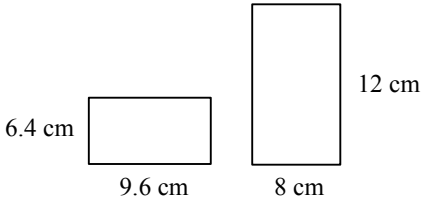
Work out the value of x.

A2 Shape **C** is similar to shape **D**

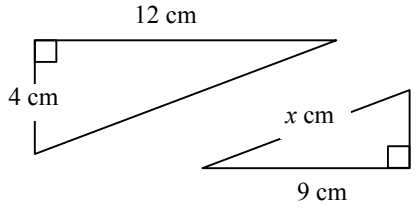


Work out the value of x.

A3 Do some calculations to work out if the rectangles are mathematically similar.



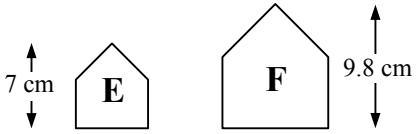
A4 The two triangles are similar



Work out the value of x.

B1 Shape **E** is similar to shape **F**

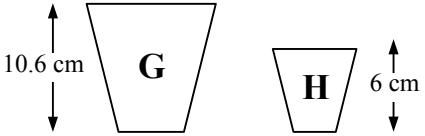
The area of **E** is 30 cm^2



Calculate the area of **F**.

B2 Shape **G** is similar to shape **H**

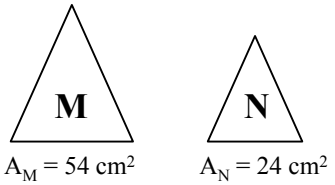
The area of **G** is 210 cm^2



Calculate the area of **H**.

B3 Shape **M** is similar to shape **N**

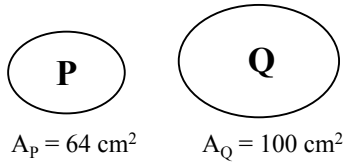
The height of **M** is 12 cm.



Calculate the height of **N**.

B4 Shape **P** is similar to shape **Q**

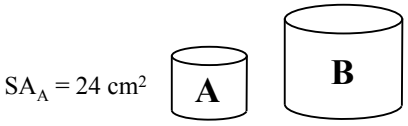
The width of **Q** is 14 cm.



Find the width of **P**.

C1 Cylinders **A** and **B** are similar

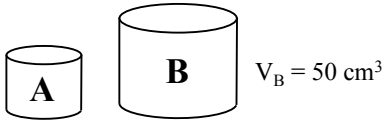
Cylinder **B** is 1.6 times as high as cylinder **A**.



Calculate the surface area of **B**.

C2 Cylinders **A** and **B** are similar

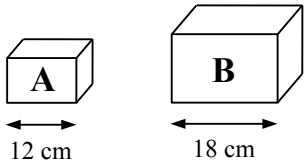
Cylinder **B** is 1.4 times as high as cylinder **A**.



Calculate the volume of **A**.

C3 Cuboids **A** and **B** are similar

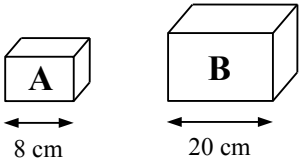
The volume of **A** is 250 cm^3.



Calculate the volume of **B**.

C4 Cuboids **A** and **B** are similar

The surface area of **B** is 1000 cm^2.



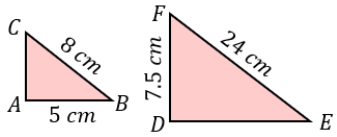
Calculate the surface area of **A**.

Fluency Practice

Similar Shapes Revision

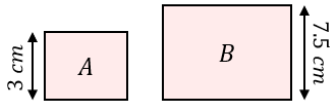
(a)

Triangles ABC and DEF are similar. Calculate the lengths of DE and AC.



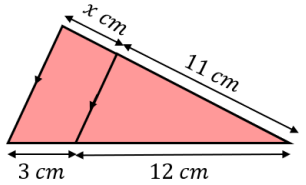
(b)

Rectangles A and B are mathematically similar. The area of A is $40\ cm^2$. Work out the area of rectangle B.



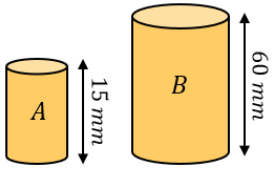
(c)

Find the missing length x.



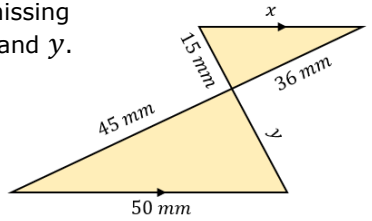
(d)

Cylinders A and B are similar. The volume of cylinder B is $2080\ cm^3$. Find the volume of cylinder A.



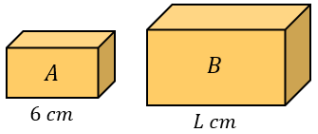
(e)

Find the missing lengths x and y.



(f)

Cuboids A and B are similar. A has a volume of $28\ cm^3$ and B has a volume of $437.5\ cm^3$. Find the length L.

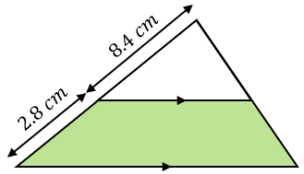


(g)

Cones A and B are mathematically similar. Cone A has a volume of $857.5\ cm^3$ and a surface area of $73.5\ cm^2$. Cone B has a volume of $160\ cm^3$. Find its surface area.

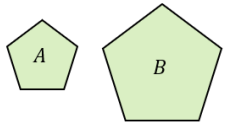
(h)

The area of the white triangle is $18\ cm^2$. Find the area of the shaded region.



(i)

Pentagons A and B are similar. The scale factor of their lengths is x. The area of A is $12\ cm^2$. If the area of B is $(16x + 3)\ cm^2$, find the value of x.

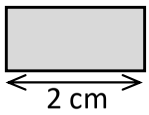


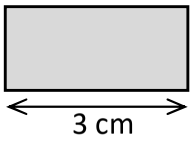
~ similar

Areas of Similar Shapes

Not drawn to scale

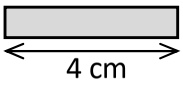
For each pair of similar shapes… 1) Express either the length or area ratio.
2) Use that to find the corresponding ratio.
3) Use the appropriate ratio to find missing areas or lengths (1dp).

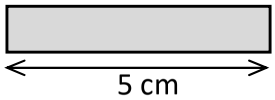
① 

~ 

Area = 10 cm² **Area** = _____ cm²

Length Ratio = 2 : 3
(Square) ↓ ↓
Area Ratio = 4 : 9

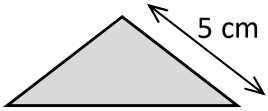
② 

~ 

Area = 9 cm² **Area** = _____ cm²

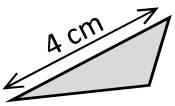
Length Ratio = ____ : ____
Area Ratio = ____ : ____

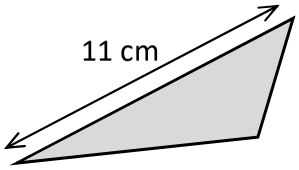
③ 

~ 

Area = _____ cm² **Area** = 8 cm²

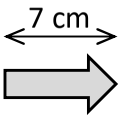
Length Ratio = ____ : ____
Area Ratio = ____ : ____

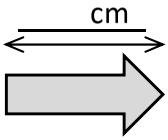
④ 

~ 

Area = _____ cm² **Area** = 45 cm²

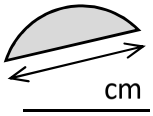
Length Ratio = ____ : ____
Area Ratio = ____ : ____

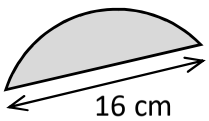
⑤ 

~ 

Area = 16 cm² **Area** = 25 cm²

Length Ratio = ____ : ____
Area Ratio = ____ : ____

⑥ 

~ 

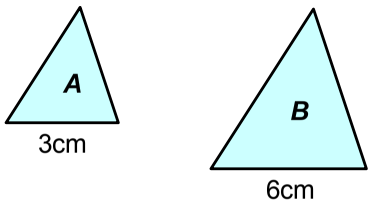
Area = 9 cm² **Area** = 49 cm²

Length Ratio = ____ : ____
Area Ratio = ____ : ____

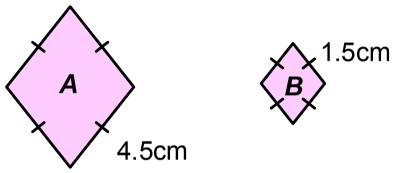
Exercise

1. Work out the ratio ***area of shape A : area of shape B*** for each pair of similar shapes.
Give your answers in the simplest form.

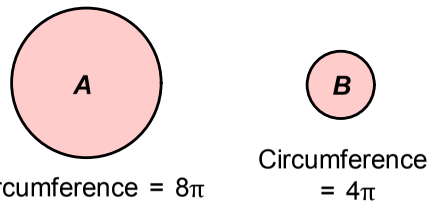
a)



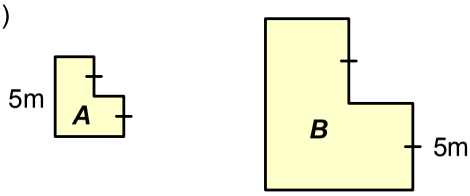
b)



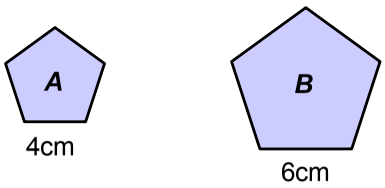
c)



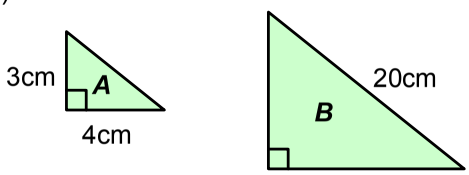
d)



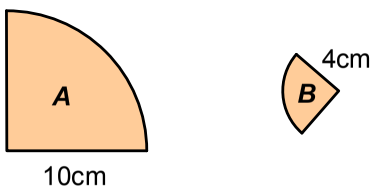
e)



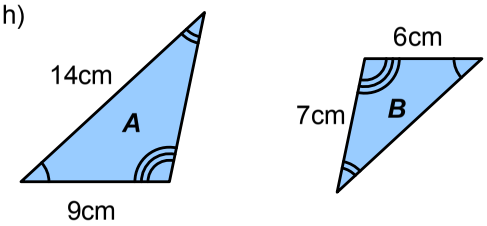
f)



g)

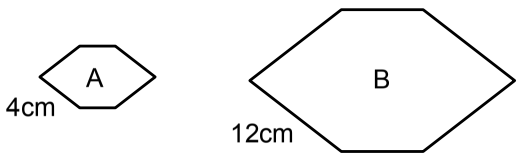


h)



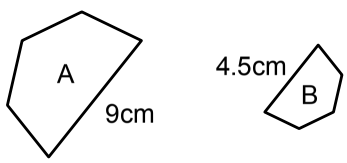
2. Two similar hexagons are shown.
The area of hexagon A is 25cm^2.

Work out the area of hexagon B.

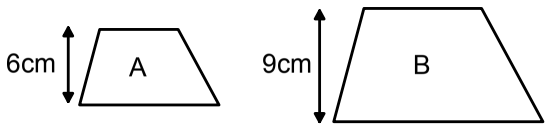


3. Two similar shapes are shown.
The area of shape A is 54cm^2.

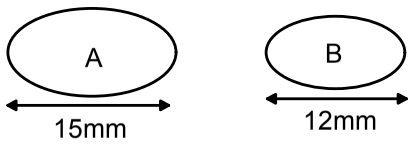
Work out the area of shape B.



4. Two similar trapezia are shown.
The area of trapezium A is 48cm^2.
Work out the area of trapezium B.



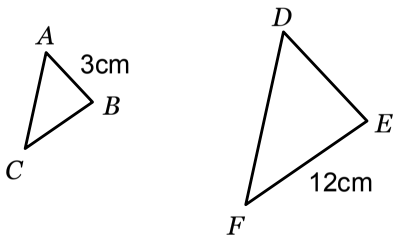
5. Two similar ellipses are shown.
The area of ellipse A is 40cm^2.
Work out the area of ellipse B.



6. ABC and DEF are similar triangles with areas 8cm^2 and 72cm^2 respectively.

a) Work out the length of DE.

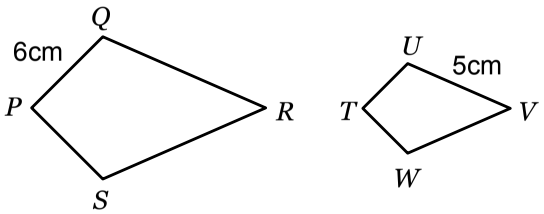
b) Work out the length of BC.



7. $PQRS$ and $TUVW$ are similar kites with areas 96cm^2 and 24cm^2 respectively.

a) Work out the length of TU.

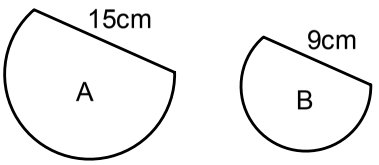
b) Work out the length of QR.



8. Two similar segments are shown.
Work out the ratio

Area of segment A : Area of segment B

Give your answer in the simplest form.



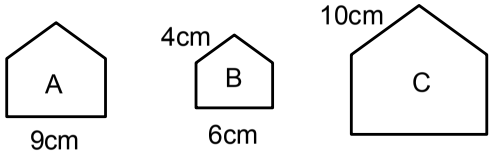
9. A and B are two similar rectangles. The area of B is 44% larger than the area of A.
Rectangle A has a width of 20cm.
Work out the width of rectangle B.

10. A and B are two similar squares, such that Area of A : Area of B = 1 : 8.
Square A has a diagonal length of $3\sqrt{2}$ cm.
Work out the length of a diagonal of square B.

11. A, B and C are three similar shapes.
Work out the ratio

Area of shape A : Area of shape C

Give your answer in the simplest form.

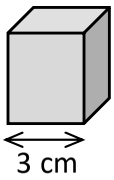
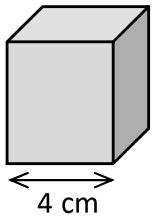


~ similar

Volumes of Similar Shapes

Not drawn to scale

For each pair of similar solids…
1) Express either the length or volume ratio.
2) Use that to find the corresponding ratio.
3) Use the appropriate ratio to find missing lengths or volumes (1dp).

①  ~ 

3 cm

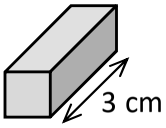
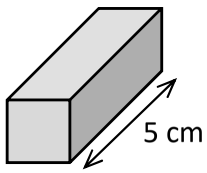
4 cm

Volume = 36 cm³ **Volume** = _____ cm³

Length Ratio = 3 : 4

↓ (Cube) ↓

Volume Ratio = 27 : 64

②  ~ 

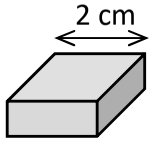
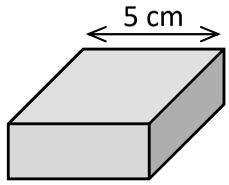
3 cm

5 cm

Volume = 7 cm³ **Volume** = _____ cm³

Length Ratio = ___ : ___

Volume Ratio = ___ : ___

③  ~ 

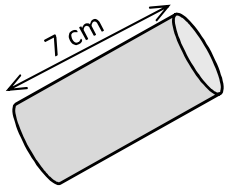
2 cm

5 cm

Volume = _____ cm³ **Volume** = 110 cm³

Length Ratio = ___ : ___

Volume Ratio = ___ : ___

④  ~ 

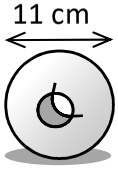
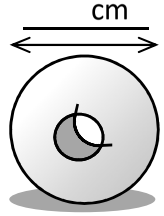
4 cm

7 cm

Volume = _____ cm³ **Volume** = 280 cm³

Length Ratio = ___ : ___

Volume Ratio = ___ : ___

⑤  ~ 

11 cm

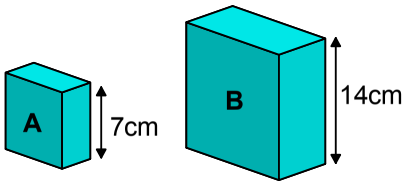
cm

Volume = 54 cm³ **Volume** = 686 cm³

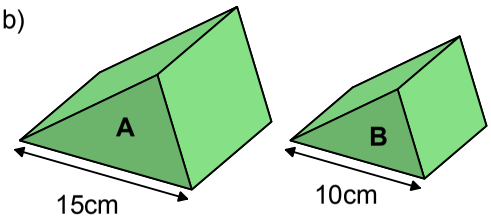
Exercise

1. For each pair of similar shapes, work out **volume of A : volume of B**.
Give your answers in the simplest form.

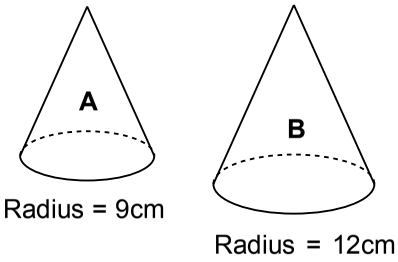
a)



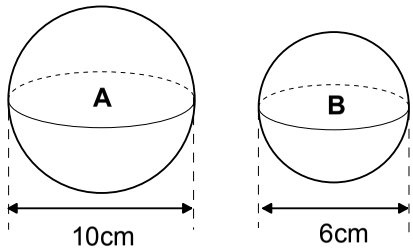
b)



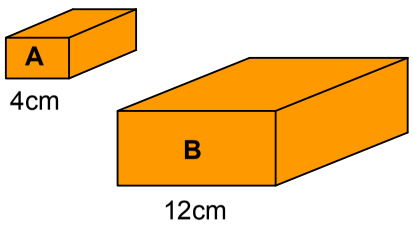
c)



d)



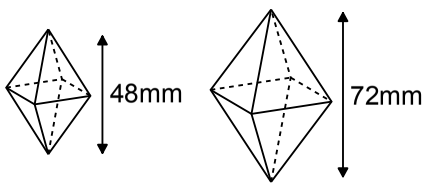
2. Two blocks are cut from the same piece of wood, as shown. The two blocks are similar cuboids.



a) The volume of block A is $50cm^3$
Work out the volume of block B.

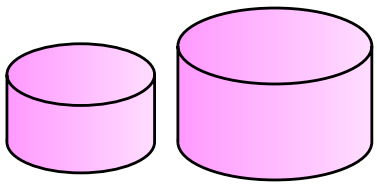
b) The mass of block B is 5.4kg.
Work out the mass of block A.

3. Two similar octahedra are shown.
The smaller octahedron has a volume of $6.4cm^3$.



Work out the volume of the larger octahedron.

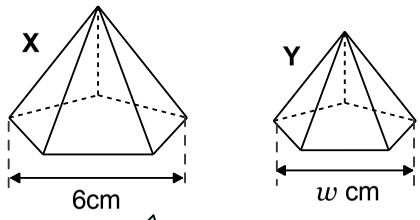
4. Two containers are in the shape of similar cylinders.
The containers have diameters of 12cm and 16cm.
The larger container has a capacity of 1.6 litres.



Work out the capacity of the smaller container.

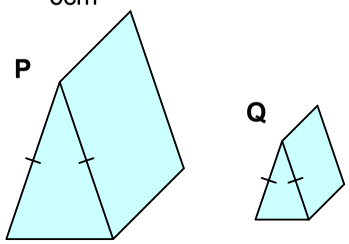
5. X and Y are similar pyramids.
The volume of X is 135cm^3.
The volume of Y is 40cm^3.

Work out the value of w.



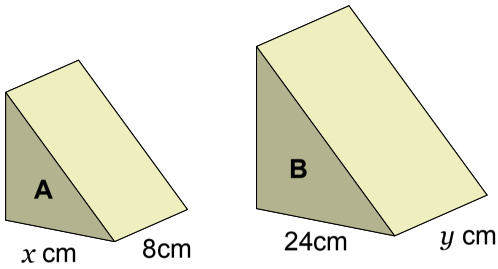
6. P and Q are similar prisms.
The volume of P is 2000cm^3.
The volume of Q is 686cm^3.
The height of P is 20cm.

Work out the height of Q.



7. Shape B is an enlargement of shape A.
The volume of B is 72.8% larger than the volume of A.

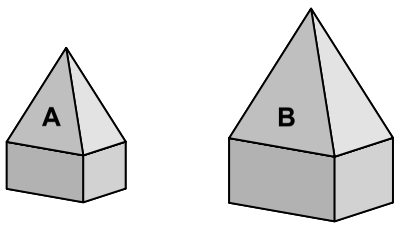
Work out the values of x and y.



8. A and B are two similar 3D shapes.
Surface area of A : surface area of B = 4 : 9.

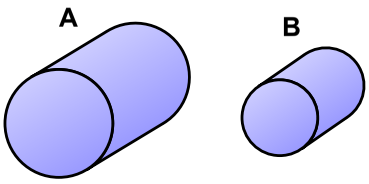
a) Work out the ratio of the volume of A to the volume of B in the simplest form.

b) The volume of A is 76cm^3.
Work out the volume of B.



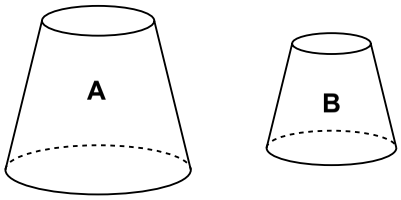
9. A and B are two similar cylinders.
The surface area of A is 150π cm^2.
The surface area of B is $\frac{75}{2}$π cm^2.
The volume of A is 250π cm^3.

Work out the volume of B in terms of π.



10. A and B are two similar frustums.
Volume of A : volume of B = 64 : 27.

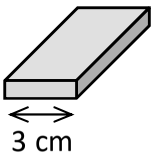
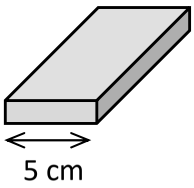
The surface area of A is 72π cm^2.
Work out the surface area of B.



Lengths, Areas & Volumes of Similar Shapes

Not drawn to scale

1) Express the length, area or volume ratio.
2) Use that to find the corresponding ratios.
3) Use the appropriate ratio to find the missing measurements (1dp).

①  ~ 

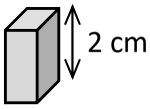
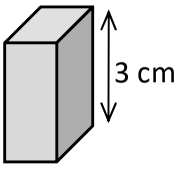
3 cm 5 cm

(Cube) **Length Ratio** = 3 : 5

(Square) **Area Ratio** = 9 : 25

Volume Ratio = 27 : 125

Surface Area = 70 cm² **Surface Area** = ____ cm²

②  ~ 

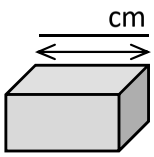
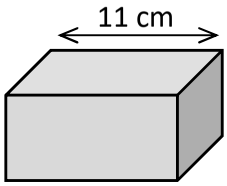
2 cm 3 cm

Length Ratio = ____ : ____

Area Ratio = ____ : ____

Volume Ratio = ____ : ____

Volume = 9 cm³ **Volume** = ____ cm³

③  ~ 

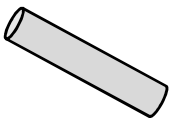
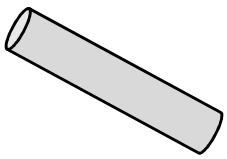
cm 11 cm

Length Ratio = ____ : ____

Area Ratio = ____ : ____

Volume Ratio = ____ : ____

Surface Area = 16 cm² **Surface Area** = 49 cm²

④  ~ 

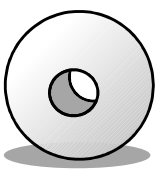
Length Ratio = ____ : ____

Area Ratio = ____ : ____

Volume Ratio = ____ : ____

Surface Area = 50 cm² **Surface Area** = 72 cm²

Volume = 40 cm³ **Volume** = ____ cm³

⑤  ~ 

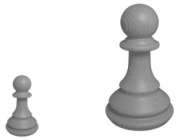
Surface Area = ____ cm² **Surface Area** = 140 cm²

Volume = 81 cm³ **Volume** = 1536 cm³

Fluency Practice

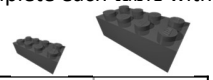
Similar Shapes: Length, Area & Volume Ratios

Here are pairs of shapes – a model & the real-life object (not shown to scale).
Complete each table with measurements & ratios.

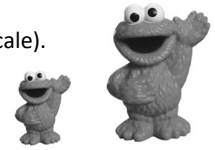


Answers to 1 dp where necessary.

Shape **A**	Shape **B**	Ratios	
1 cm	3 cm	Length	
5 cm^2		Area	
3 cm^3		Volume	



Shape **C**	Shape **D**	Ratios	
2 cm		Length	
8 cm^2	18 cm^2	Area	
	108 cm^3	Volume	



Shape **E**	Shape **F**	Ratios	
6 cm		Length	
20 cm^2		Area	4:25
40 cm^3		Volume	

Shape **G**	Shape **H**	Ratios	
9 cm		Length	
	144 cm^2	Area	
3 cm^3		Volume	1:216

Shape **I**	Shape **J**	Ratios	
	12 cm	Length	
7 cm^2		Area	
15 cm^3		Volume	125:216

Shape **K**	Shape **L**	Ratios	
84 cm	120 cm	Length	
5 m^2		Area	
15 m^3		Volume	

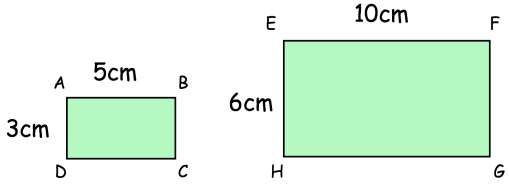
Simplify these ratios first.

Shape **M**	Shape **N**	Ratios	
	2.5 cm	Length	
13 cm^2		Area	18:50
3 cm^3		Volume	

Shape **O**	Shape **P**	Ratios	
24 cm		Length	
45 cm^2		Area	
120 cm^3		Volume	250:686

Shape **Q**	Shape **R**	Ratios	
25 cm		Length	
0.2 m^2		Area	128:242
13 cm^3		Volume	

Question 1: Rectangle EFGH is an enlargement of rectangle ABCD.

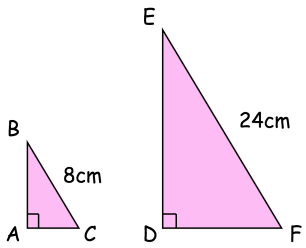


(a) What is the scale factor of enlargement?

(b) How many times larger is the area of EFGH than ABCD?

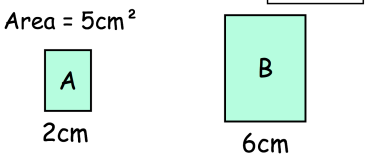
Question 2: Triangles ABC and DEF are similar.

How many times larger is the area of ABC than ABCD?

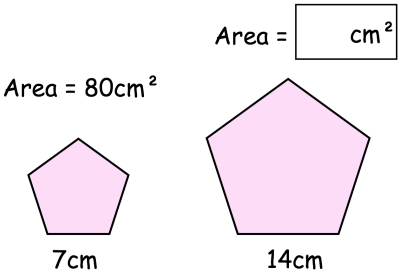


Question 3: Each pair of shapes below are similar.
Find the missing areas.

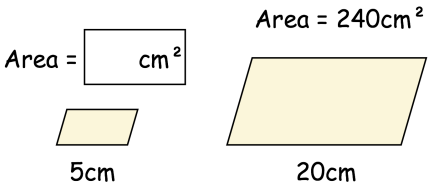
(a)



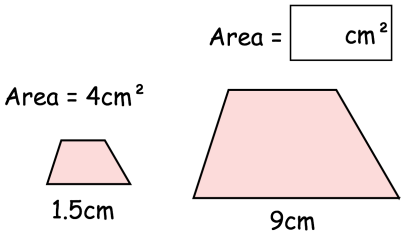
(b)



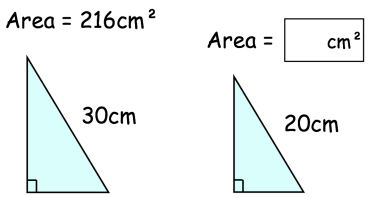
(c)



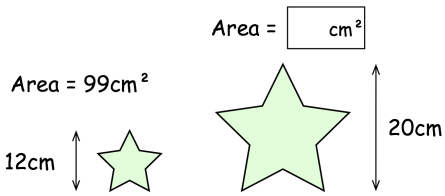
(d)



(e)

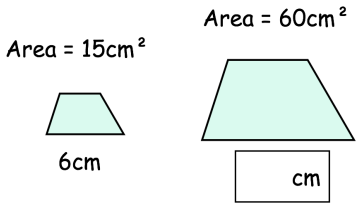


(f)

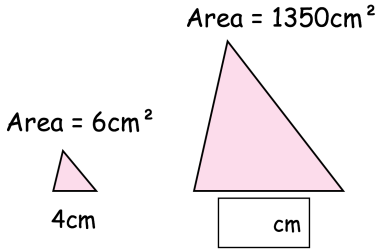


Question 4: Each pair of shapes below are similar.
Find the missing lengths.

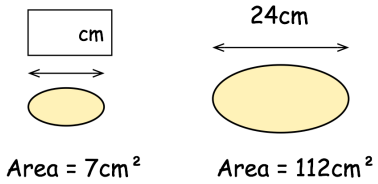
(a)



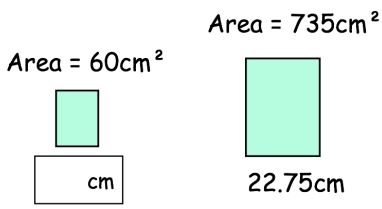
(b)



(c)

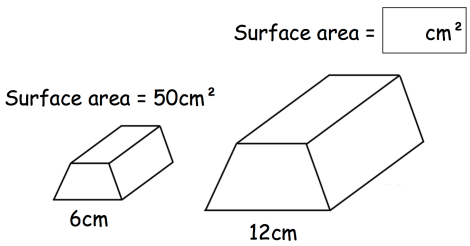


(d)

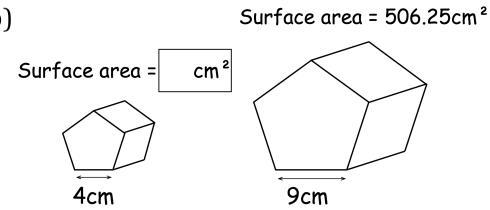


Question 5: The solid shapes below are mathematically similar.
Find the missing surface areas.

(a)

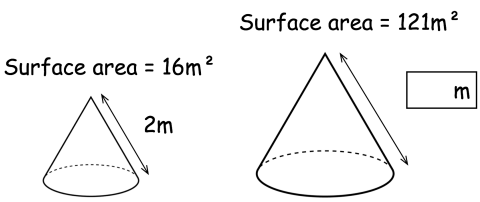


(b)

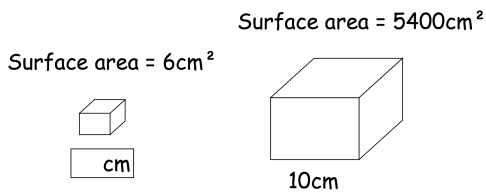


Question 6: The solid shapes below are mathematically similar.
Find the missing lengths.

(a)



(b)

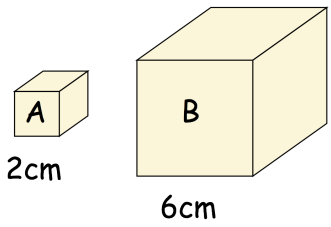


Question 1: Cube B is an enlargement of cube A.

(a) What is the scale factor of enlargement?

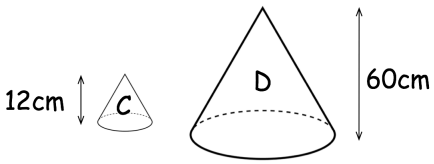
(b) How many times larger is the surface area of B than A?

(c) How many times larger is the volume of cube B than A?



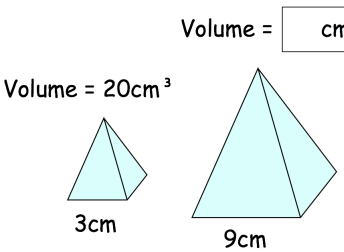
Question 2: Cones C and D are similar.

How many times larger is the volume of D than C?

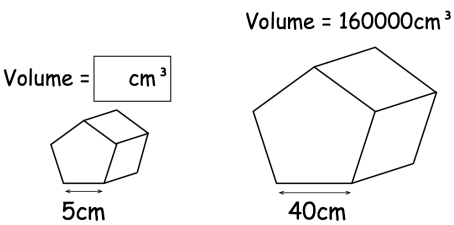


Question 3: The solids below are mathematically similar.
Find the missing volumes.

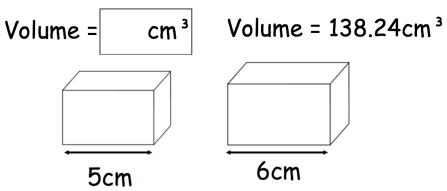
(a)



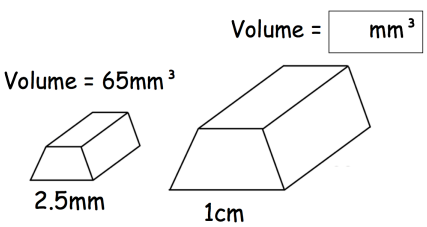
(b)



(c)

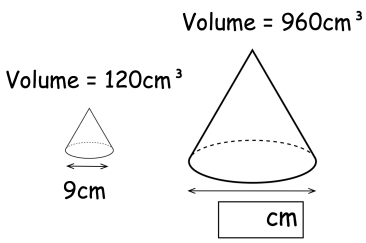


(d)

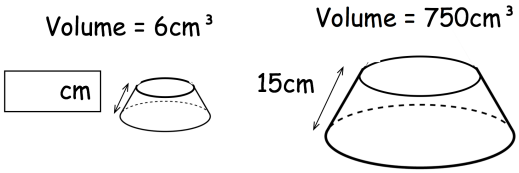


Question 4: The solid shapes below are mathematically similar.
Find the missing lengths.

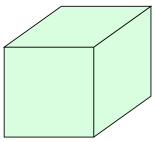
(a)



(b)



(c) Volume = 10000cm³



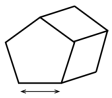
20cm

Volume = 640cm³

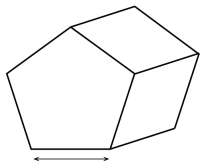
cm

(d) Volume = 14288.4cm³

Volume = 2450cm³



12cm

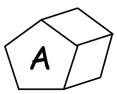


cm

Question 5: The solid shapes below are similar.
Find the missing volumes.

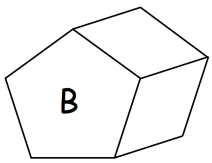
(a)

Surface area = 300cm²



Volume = 600cm³

Surface area = 1200cm²



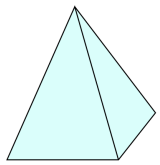
Volume = cm³

(b)

Surface area = 450cm²

Surface area = 18cm²

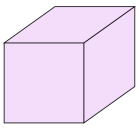
Volume = cm³



Volume = 400cm³

(c)

Surface area = 2124cm²



Volume = cm³

Surface area = 944cm²

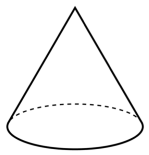
Volume = 1920cm³

(d)

Surface area = 5184π cm²

Surface area = 36π cm²

Volume = 16π cm³



Volume = cm³

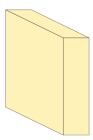
Question 6: The solid shapes below are similar.
Find the missing surface areas.

(a)

Surface area = cm²

Volume = 20cm³

Surface area = 576cm²



Volume = 540cm³

(b)

Surface area = 324π cm²

Volume = 972π cm³

Surface area = cm²

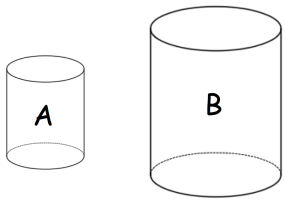
Volume = 2304π cm³

Question 1: A and B are mathematically similar.

The height of A : the height of B = 3 : 5

(a) Find the surface area of A : the surface area of B

(b) Find the volume of A : the volume of B

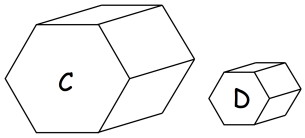


Question 2: Solids C and D are similar.

The length of C : the length of D = 9 : 2

(a) Find the surface area of C : the surface area of D

(b) Find the volume of C : the volume of D

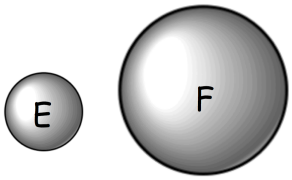


Question 3: Shown are spheres E and F.

The surface area of E : the surface area of F = 4 : 49

(a) Find the diameter of E : the diameter of F

(b) Find the volume of E : the volume of F

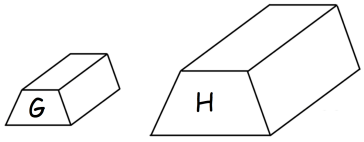


Question 4: Shown are similar solids G and H.

The volume of G : the volume of H = 27 : 1000

(a) Find the height of G : the height of H

(b) Find the surface area of G : the surface area of H



Question 5: The surface areas of two similar shapes are in the ratio $25 : 81$
The length of the smaller shape is 30cm.

Work out the length of the larger shape.

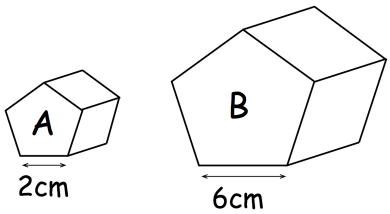
Question 6: The volumes of two similar shapes are in the ratio $1000 : 27$
The surface area of the larger shape is 250cm^2

Work out the surface area of the smaller shape.

Apply

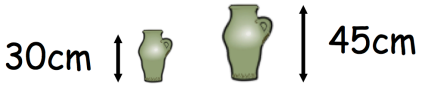
Question 1: Myles says that the surface area of B is three times larger than the surface area of A.

Explain why Myles is incorrect.



Question 2: Two vases are similar.
The smaller vase holds 800ml.

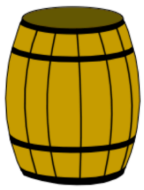
How much does the larger vase hold?



Question 3: Two barrels are mathematically similar.
It takes 90ml of paint to paint the smaller barrel.

How much paint is needed for the larger barrel?


4 Litres


108 Litres

Question 4: A large bottle of cola is 16cm tall.
A small bottle of cola is 12cm tall.
The bottles are mathematically similar.
Fernando claims the small bottle contains three-quarters of the amount of cola than the large bottle.

Show Fernando is wrong.

Question 5: Nina bought a toy that grows when placed in water.
Before placing the toy in water, it was 4cm tall.
After placing the toy in water it grew to a similarly shaped toy that was 11cm tall.

Grows 20 times larger

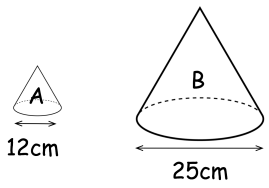
Is the claim that it will "grow 20 times larger" reasonable?

Question 6: Alec makes two solid mathematically similar models of the Titanic.

The smaller model had a length of 10cm
The larger model had a length of 45cm
The mass of the larger model is 7.29kg

Work out the mass of the smaller model.

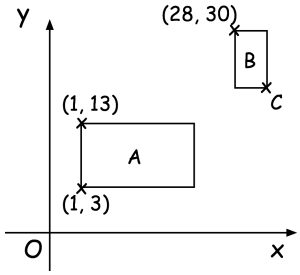
Question 7: Below are two similar cones.



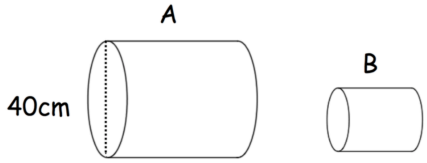
Anna says the volume of A is approximately 11% of the volume of B.
Is Anna correct? Explain your answer.

Question 8: Rectangles A and B are similar.
The area of rectangle A is 240
The area of rectangle B is 15.

Find the coordinates of the point C.



Question 9: Cylinders A and B are similar.



Volume B $\times \frac{512}{27}$ = Volume A

Find the radius of cylinder B.

Question 10: A, B and C are similar.

The volume of A is 729cm^3 and the volume of B is 64cm^3.

The surface area of B is 25cm^2 and the surface area of C is 121cm^2.

Find the ratio length of A : length of B : length of C

Question 11: Shapes A, B and C are similar.
The height of shape A is 8cm.
The height of shape C is 4cm.
The ratio of the surface area of shape B to the surface area of shape C is 25:9

Work out the ratio of the volume of shape A to shape B.

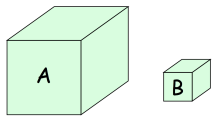
Question 12: Two solid toys, C and D, are similar.
The volume of toy C is $40cm^3$

The surface area of C : surface area of D = 2 : 9

Work out the volume of toy D.

Question 13: Washing powder is sold in two different sizes, a large box A and a smaller box B.
Cuboid boxes A and B are similar.

Surface area of A : Surface area of B = 81 : 4



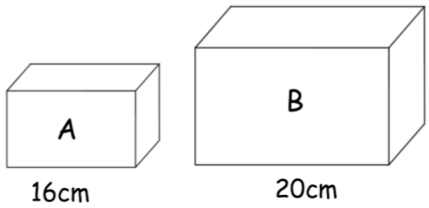
How many smaller boxes, B, can be completely filled using the contents of a full box A?

Question 14: Pyramid A and pyramid B are similar.

The surface area of B is 42.25 times larger than the surface area of A.

Find the ratio of the volume of A to the volume of B.

Question 15: Cuboids A and B are similar but made from different materials.
Both cuboids are placed on a table.



The pressure on the table due to cuboid A is 3.5 newtons/cm^2
Cuboid A exerts a force of 420N on the table.

The pressure on the table due to cuboid B is 4 times larger than cuboid A.

Work out the force exerted by cuboid B on the table.

Question 16: Ornament A and B are mathematically similar.
They are solid and both made from copper and zinc in the ratio 3:2

Ornament A has a height of 5cm and volume of $30cm^3$
Ornament B has a height of 18cm.

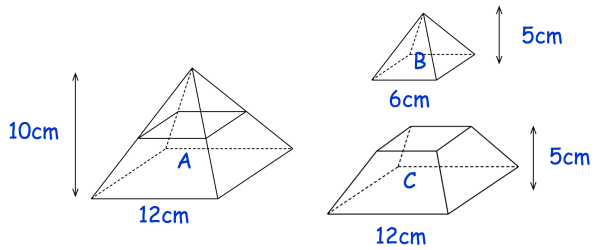
The density of copper is $8.96g/cm^3$
The density of zinc is $7.13g/cm^3$

Work out the difference in mass between ornament A and ornament B.

Question 17: Cylinders A and B are similar.
The height of A is 6cm.
The volume of A is $240cm^3$ to 2 significant figures.
The height of B is 15cm and the volume of B is y.

Work out the error interval of y.

Question 18: The square based pyramid A is divided into Pyramid B and Frustum C.



(a) Express the volume of Pyramid B as a fraction of the volume of Pyramid A.

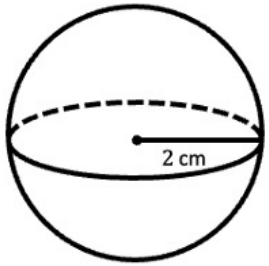
(b) Express the volume of Frustum C as a fraction of the volume of Pyramid A.

2 Volume and Surface Area of Non-Prisms

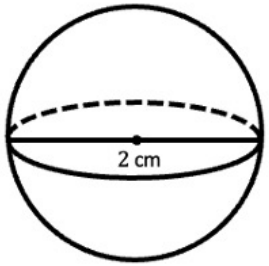
Intelligent Practice

Calculate the volume. Give your answers in terms of π and to 1 decimal place.

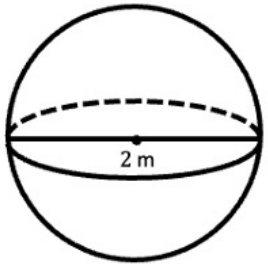
Question 1



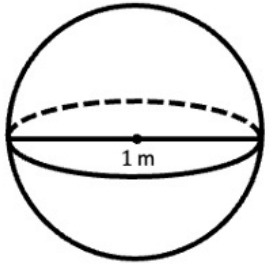
Question 2



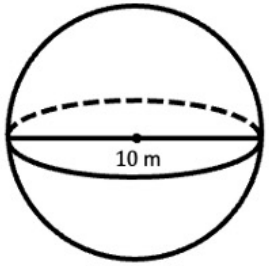
Question 3



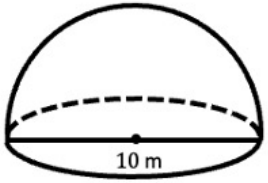
Question 4



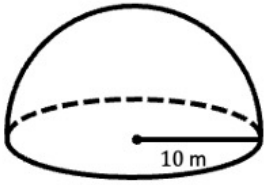
Question 5



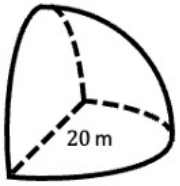
Question 6



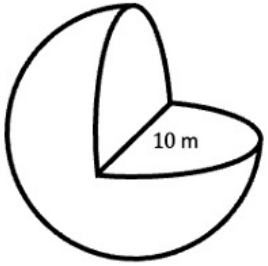
Question 7



Question 8



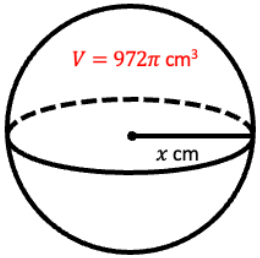
Question 9



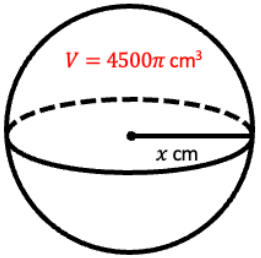
Intelligent Practice

Find x given the volume. Give your answers to 1 decimal place if required.

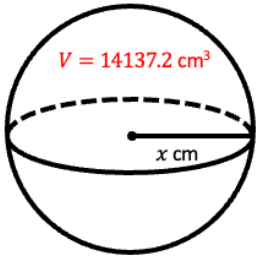
Question 1



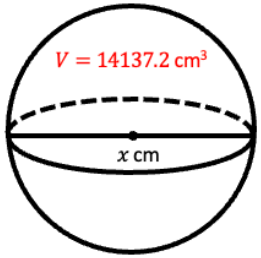
Question 2



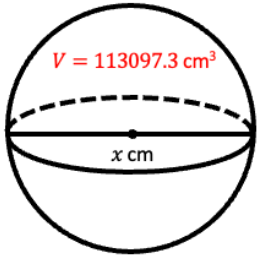
Question 3



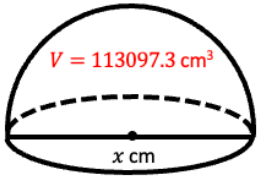
Question 4



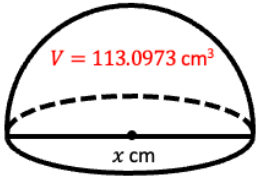
Question 5



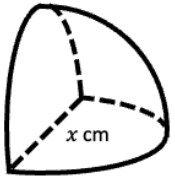
Question 6



Question 7

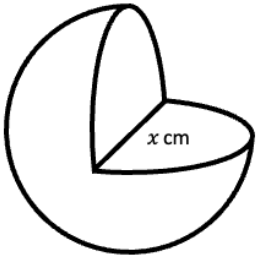


Question 8



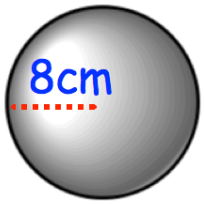
$V = 113.0973$ cm³

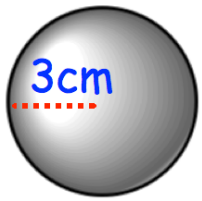
Question 9

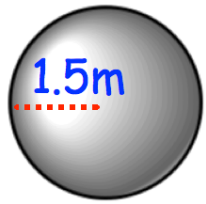


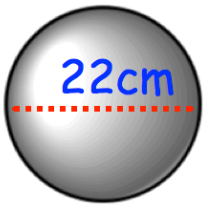
$V = 339.2920$ cm³

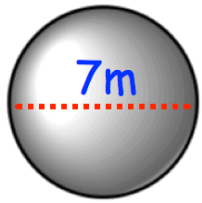
Question 1: Find the volume of each of these spheres.
Give each answer to one decimal place (you may use a calculator)

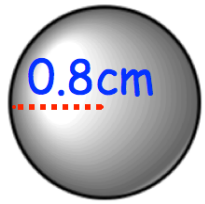
(a) 

(b) 

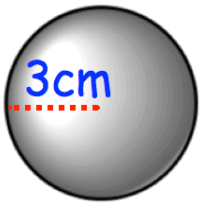
(c) 

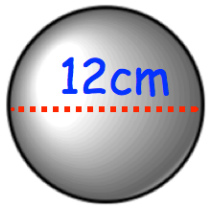
(d) 

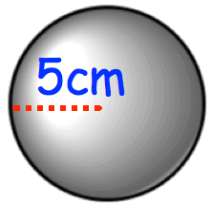
(e) 

(f) 

Question 2: Find the volume of each of these spheres.
Give each answer in terms of π (you may not use a calculator)

(a) 

(b) 

(c) 

Question 3: Find the volume of each of these spheres.
Give your answers to three significant figures (you may use a calculator)

(a) A sphere with radius 9cm

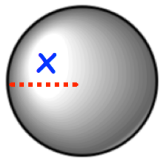
(b) A sphere with diameter 38cm

(c) A sphere with diameter 6.7cm

(d) A sphere with radius 1.25 inches.

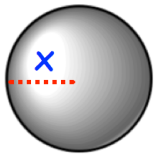
Question 4: Find the size of the radius in each of the spheres below.
Give your answers to one decimal place (you may use a calculator)

(a)



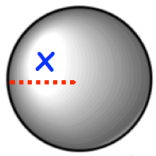
Volume = $200cm^3$

(b)



Volume = $1950cm^3$

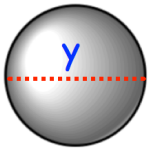
(c)



Volume = $1m^3$

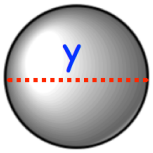
Question 5: Find the size of the diameter in each of the spheres below.
Give your answers to one decimal place (you may use a calculator)

(a)



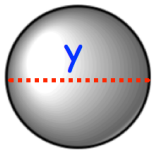
Volume = $50cm^3$

(b)



Volume = $2360cm^3$

(c)

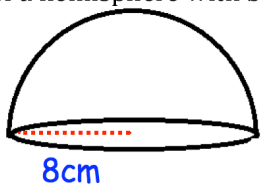


Volume = $0.4m^3$

Apply

Question 1: A metal cuboid measuring 4cm by 5cm by 12cm is melted down and a sphere is made.
Calculate the radius of the sphere.

Question 2: Calculate the volume of a hemisphere with base of radius 8cm.



Question 3: A solid sphere fits perfectly inside of a cube box of side length 10cm.
What percentage of the box is empty?

Question 4: A ball of gold has a radius of 9cm.
The density of gold is $19.3g/cm^3$.
Work out the mass of the ball.

Fluency Practice

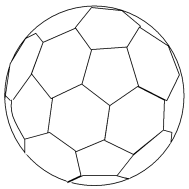
volume of a sphere volume of a sphere = $\frac{4}{3}\pi r^3$

(1)

a standard tennis ball has a diameter of 6.7 cm

what is the volume of the tennis ball?

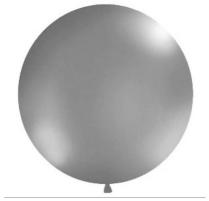
(2)



a regulation football must have a circumference of between 68 cm and 70 cm

what are the limits of the football's volumes?

(3)



how much larger (approximately) is the volume of a 14" (diameter) balloon to an 11" balloon?

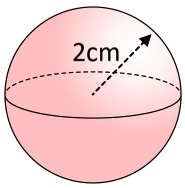
(4)

the Earth has a volume of 1,083,206,916,846 km^3

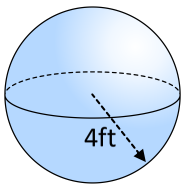
how wide is a time zone if there are 24 of them around the circumference?

Q1 Work out the volume of each of the following spheres.

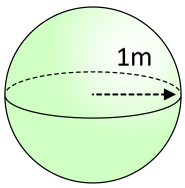
[a]



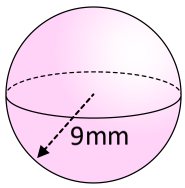
[b]



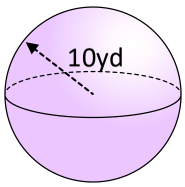
[c]



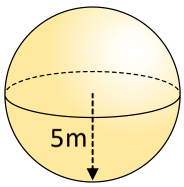
[d]



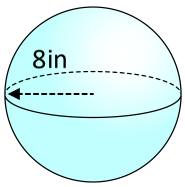
[e]



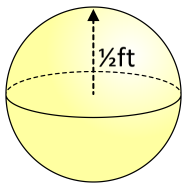
[f]



[g]

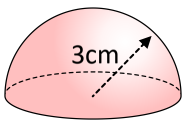


[h]

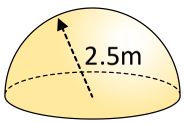


Q2 Work out the volume of each of the following hemispheres to three significant figures.

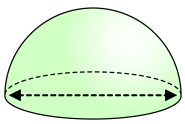
[a]



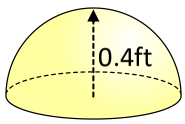
[b]



[c]



[d]



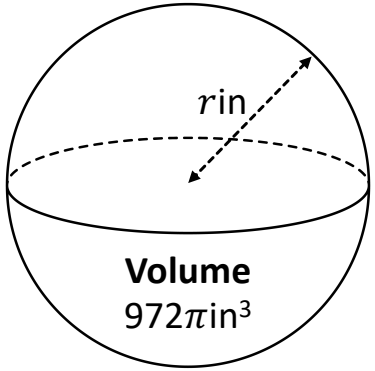
Q3 Work out, correct to four significant figures, the volume of a sphere with:

[a] Radius = 7cm **[b]** Radius = 1.5mm **[c]** Diameter = 4.2yd

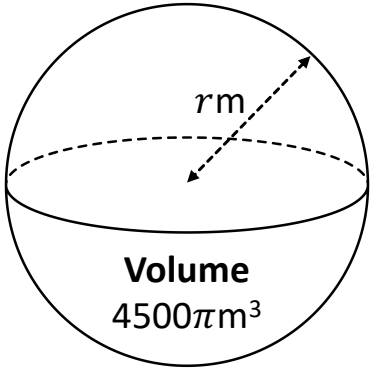
Q4 Given that the radius of Mars is approximately 3.4×10^3 kilometres. Find its volume to three significant figures.

Fluency Practice

Ex3 The following sphere has a volume of $972\pi\text{in}^3$. Find the length of its radius, $r$in.

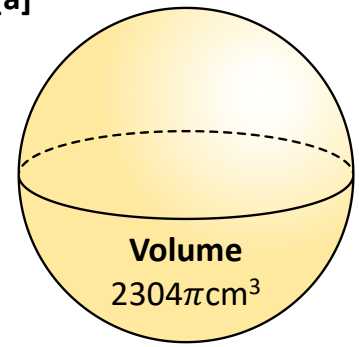


Q5 The following sphere has a volume of $4500\pi\text{m}^3$. Find the length of its radius, rm.

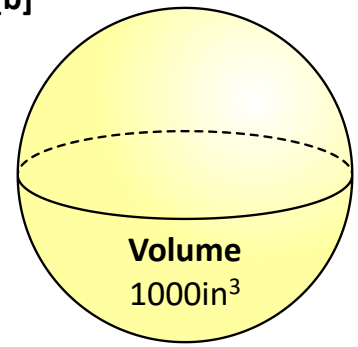


Q6 Work out the radius of the following spheres given their volume.

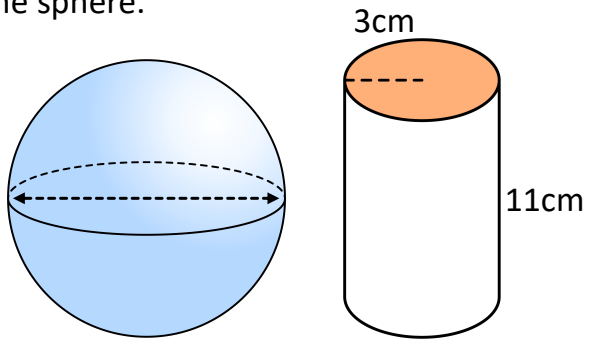
[a]



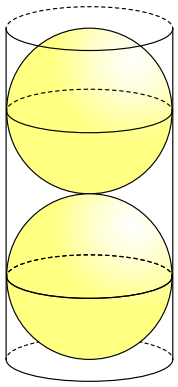
[b]



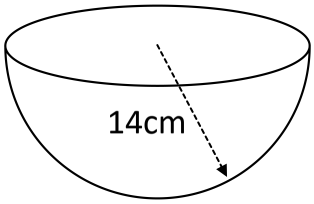
Q7 The following cylinder and sphere have the same volume. Find the diameter of the sphere.



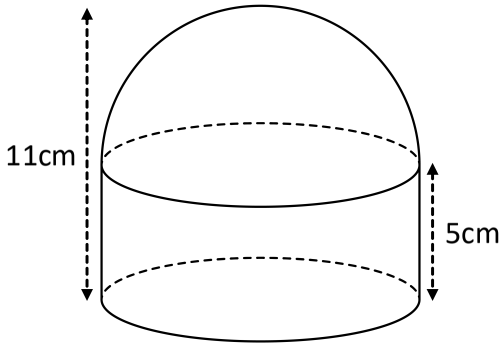
Q1 Craig buys two tennis balls which fit inside a cylindrical container of the same diameter. The tennis ball's diameter is 6cm.What fraction of the container's volume do the tennis balls take up?



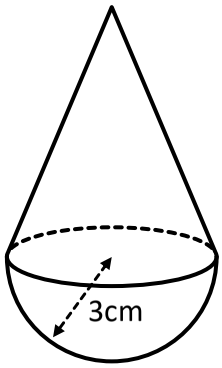
Q2 Marie has some hanging flower baskets in the shape of hemispheres with radius 14cm. Marie is going to fill the flower baskets completely with soil. She has 50 litres of compost. **1 litre $=$ 1000cm^3**. Work out how many flower baskets Marie can fill completely with compost.



Q3 The following composite solid is made by joining a cylinder and hemisphere. Calculate the volume of the solid leaving your answer in terms of π.



Q4 A child's toy is made from joining a cone and a hemisphere. The cone and the hemisphere both have a radius of 3cm. The ratio of the height of the toy to the radius of the hemisphere is 3:1. Calculate the volume of the toy in terms of π.



Gold facts:

Gold has a density of $\mathbf{19.3\ g/cm^3}$.

The price of gold is currently around ***£35 per gram***.

The entire gold reserves of the world currently total around ***165,000 tonnes***.

Task 1

What is the value of one cubic centimetre of gold?

Task 2a

How heavy would a lump of gold worth £1 million be?

Task 2b

What would the volume of this lump of gold be?

Task 2c

If this lump of gold were made into a sphere, what would its diameter be?

Hint: The volume of a sphere is $V = \frac{4}{3}\pi r^3$. *Remember the diameter is twice the radius.*

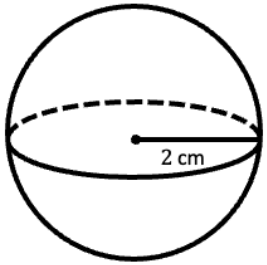
Task 3

If the entire gold reserves of the world were melted down to form a huge sphere, what would its diameter be?

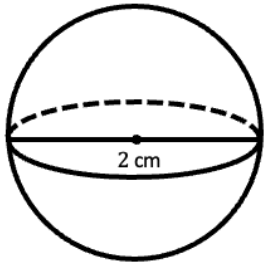
Intelligent Practice

Calculate the total surface area. Give your answers in terms of π and to 1 decimal place.

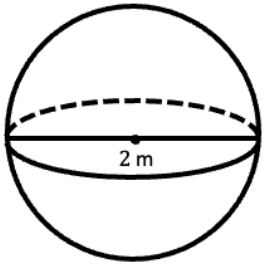
Question 1



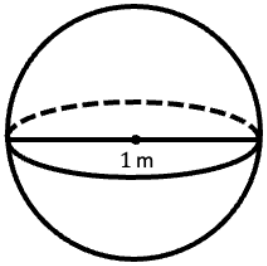
Question 2



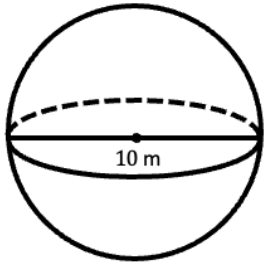
Question 3



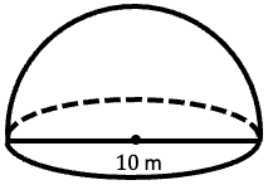
Question 4



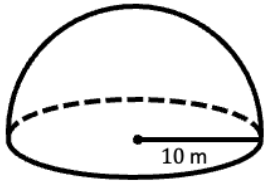
Question 5



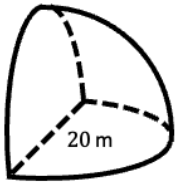
Question 6



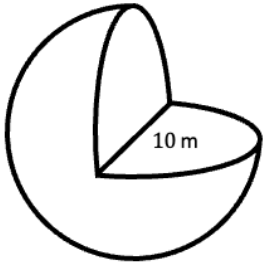
Question 7



Question 8



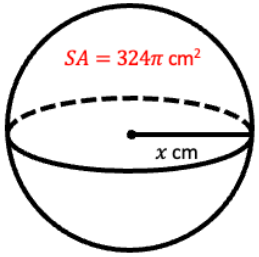
Question 9



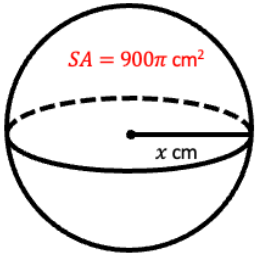
Intelligent Practice

Find x given the total surface area. Give your answers to 1 decimal place if required.

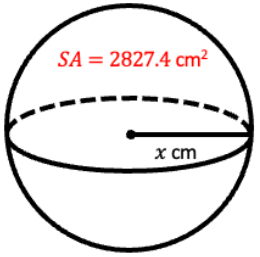
Question 1



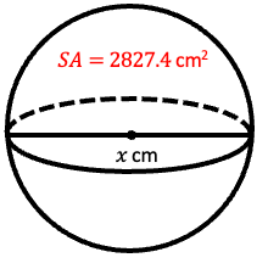
Question 2



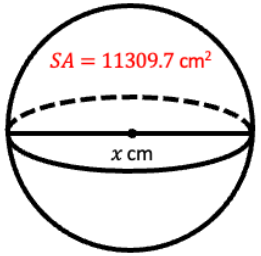
Question 3



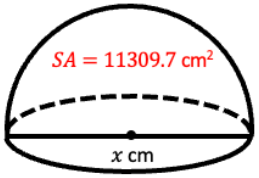
Question 4



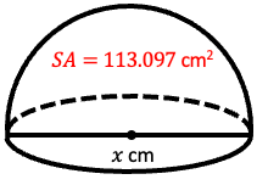
Question 5



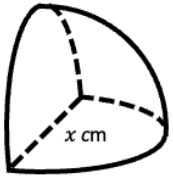
Question 6



Question 7

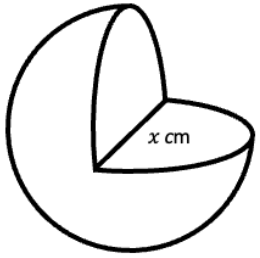


Question 8



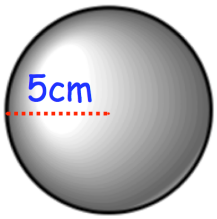
$SA = 113.097$ cm²

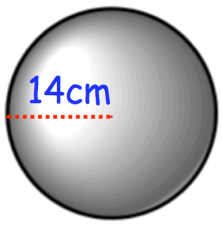
Question 9

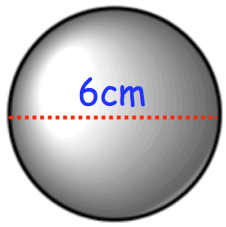


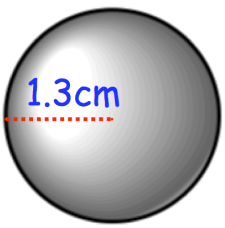
$SA = 226.194$ cm²

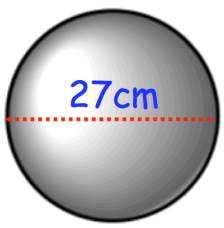
Question 1: Work out the surface area of each of these spheres.
Give each answer to 2 decimal places (you may use a calculator)

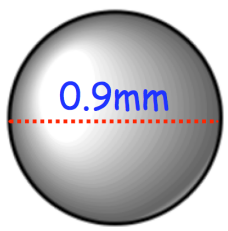
(a) 

(b) 

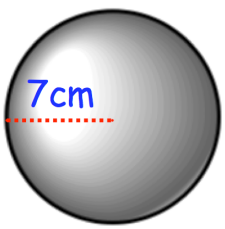
(c) 

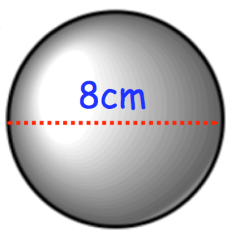
(d) 

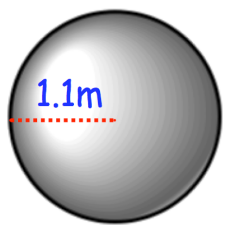
(e) 

(f) 

Question 2: Find the surface area of each of these spheres.
Give each answer in terms of **π** (you may not use a calculator)

(a) 

(b) 

(c) 

Question 3: Find the surface area of each of these spheres.
Give your answer to 3 significant figures (you may use a calculator)

(a) A sphere with diameter 2cm

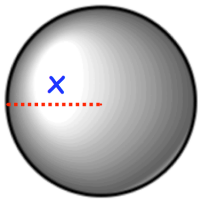
(b) A sphere with radius 36mm

(c) A sphere with radius 0.4m

(d) A sphere with diameter 2.07 inches

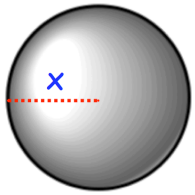
Question 4: Find the size of x in each of the sphere below.
Give your answers to two decimal places (you may use a calculator)

(a)



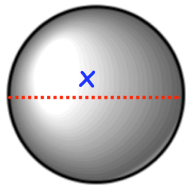
Surface area = $50cm^2$

(b)



Surface area = $940cm^2$

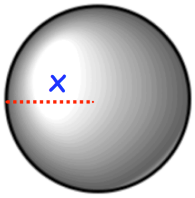
(c)



Surface area = $4800cm^2$

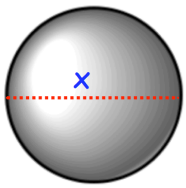
Question 5: Find the size of x in each of the sphere below.
You may not use a calculator

(a)



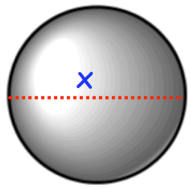
Surface area = $16\pi\ cm^2$

(b)



Surface area = $100\pi\ cm^2$

(c)



Surface area = $3600\pi\ cm^2$

Apply

Question 1: A glass paperweight is shown below.
The paperweight is a hemisphere with diameter 9cm.
Find the surface area of the paperweight

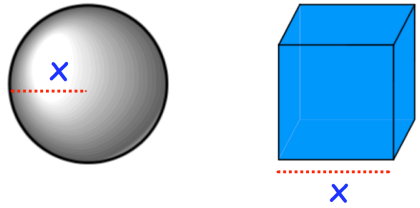
9cm

Question 2: Show the surface area of a sphere with radius 6cm is four times larger than the surface area of a sphere with radius 3cm.

Fluency Practice

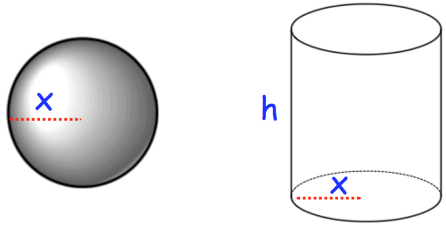
Question 3: The formula for the surface area of a sphere is $A = 4\pi r^2$
Make r the subject of the formula

Question 4: The diameter of a sphere is equal to the side length of a cube.



Peter says the surface area of the sphere is double the surface area of the cube.
Is Peter correct?

Question 5: A sphere has a radius of x.
A cylinder has a radius of x and height h.

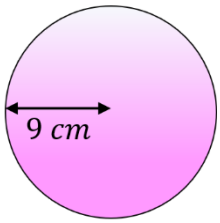


The surface area of the sphere and cylinder are equal.

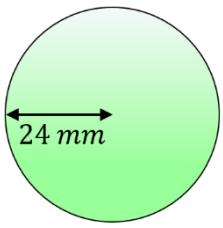
Show h = x

Find the volume and surface area of these spheres.

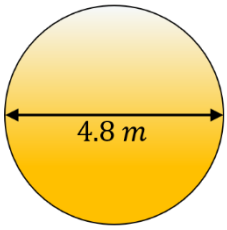
(a)



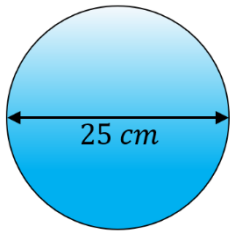
(b)



(c)

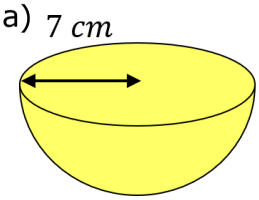


(d)

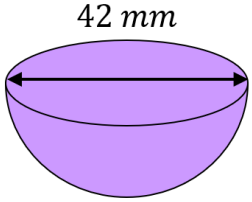


Find the volume and total surface area of these hemispheres.

(a)

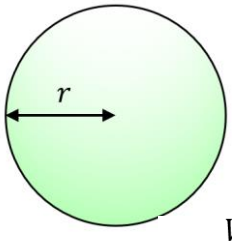


(b)

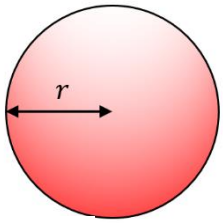


Find the missing lengths.

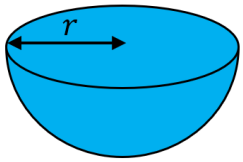
(a) $Volume = 972\pi\ cm^3$



(b) $Surface\ Area = 4072\ cm^2$



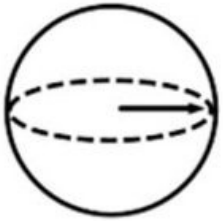
(c) $Volume = 1150\ cm^3$



A container is made up of a hemisphere on top of a cylinder, both with the radius 26 cm. The total volume of the container is $230\,000\ cm^3$. Find the height of the cylinder.

Fluency Practice

Sphere A



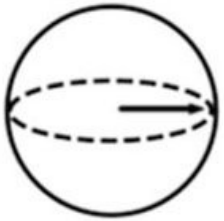
Radius
=3

Surface area
=36π

Volume
=36π

Sphere B

Radius is three times sphere A



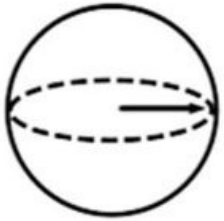
Radius
=

Surface area
=

Volume
=

Sphere C

Surface area is four times sphere A



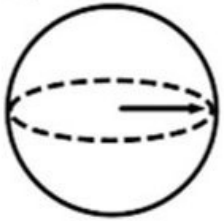
Radius
=

Surface area
=

Volume
=

Sphere D

Volume is sixty-four times sphere A



Radius
=

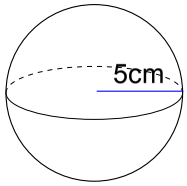
Surface area
=

Volume
=

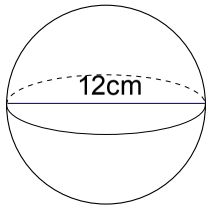
Radius	Surface area	Volume
A : B : C : D	A : B : C : D	A : B : C : D

1. Work out the volume and surface area of each sphere. Give your answers both in terms of π, and correct to 3 significant figures.

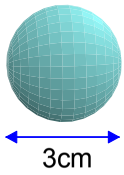
a)



b)

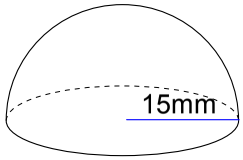


c)



2. A hemisphere has a radius of 15mm.

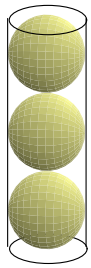
Work out the volume and surface area of the hemisphere, giving your answers in terms of π.



3. A plastic tube in the shape of a cylinder contains three tennis balls.

The tube and each ball have a diameter of 6.7cm and the balls fit snugly into the tube, such that none of them are able to move.

Work out the volume of free space within the tube.



4. The moon has a radius of approximately (1.7×10^3)km.

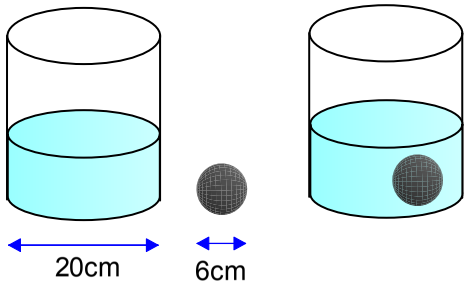
a) Work out the volume of the moon.

b) Work out the surface area of the moon.

5. A cylindrical container with diameter 20cm is partially filled with water.

A metal sphere with diameter 6cm is dropped into the water and is completely submerged.

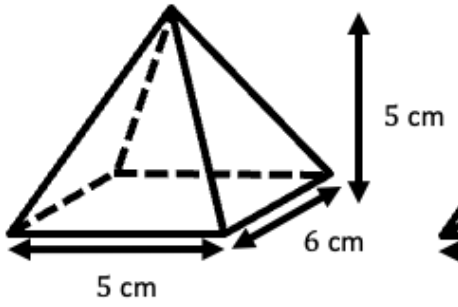
Work out by how much the water level rises.



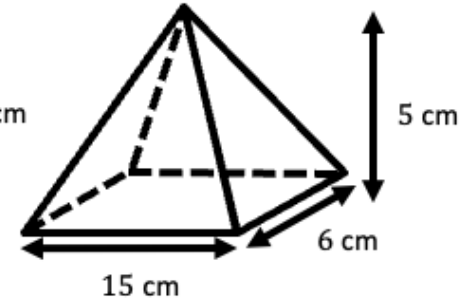
Intelligent Practice

Calculate the volume.

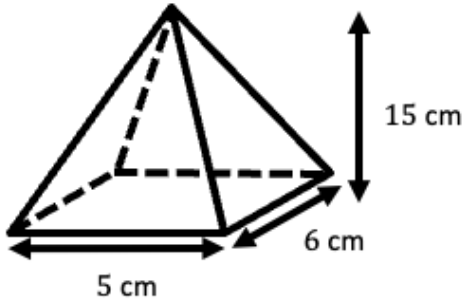
Question 1



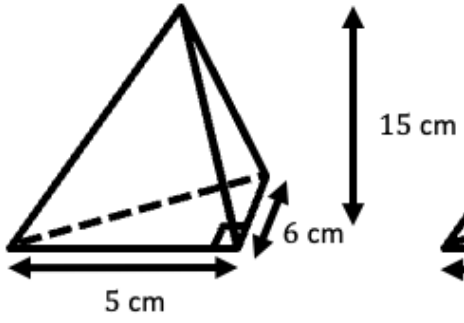
Question 2



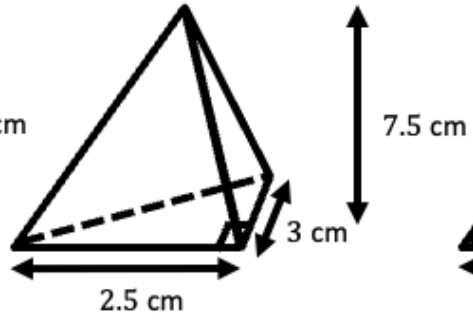
Question 3



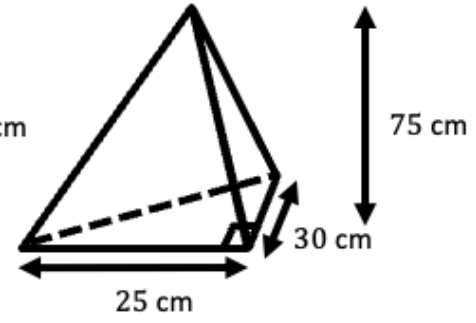
Question 4



Question 5



Question 6

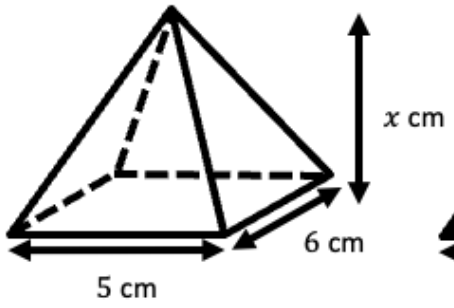


Intelligent Practice

Find x given the volume.

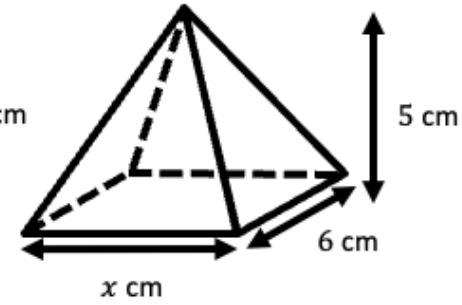
Question 1

$V = 50\ \text{cm}^3$



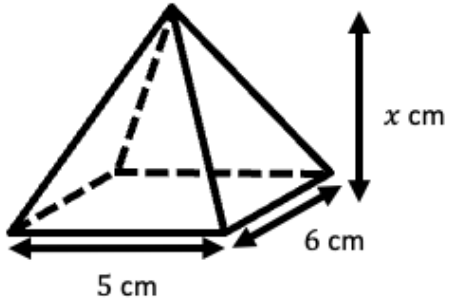
Question 2

$V = 150\ \text{cm}^3$



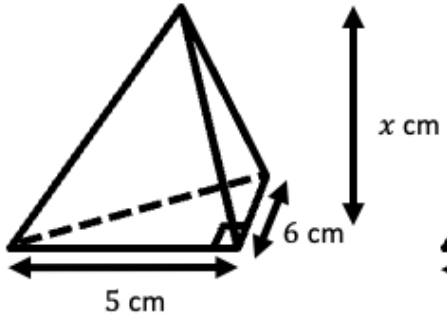
Question 3

$V = 150\ \text{cm}^3$



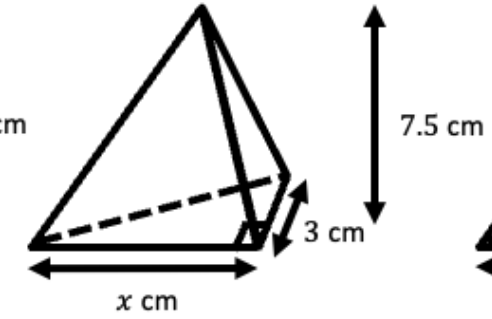
Question 4

$V = 75\ \text{cm}^3$



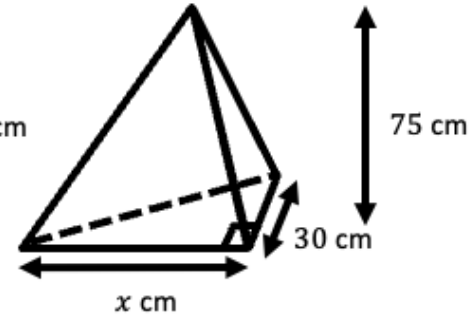
Question 5

$V = 9.375\ \text{cm}^3$



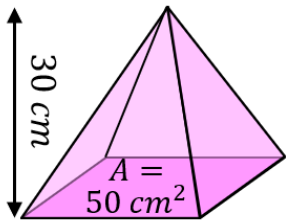
Question 6

$V = 9375\ \text{cm}^3$

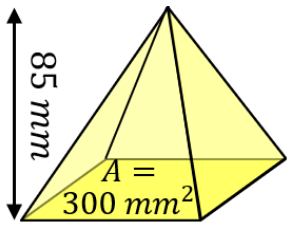


Find the volume of each of these pyramids.

(a)

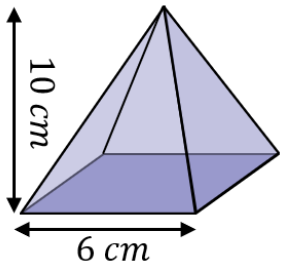


(b)

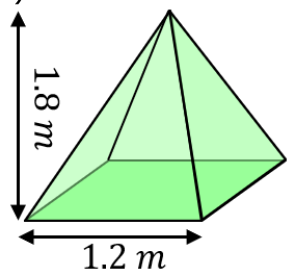


Find the volume of each of these square-based pyramids.

(a)

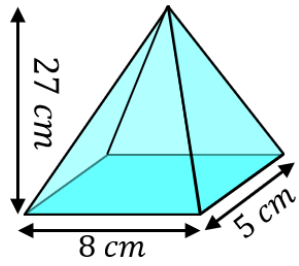


(b)

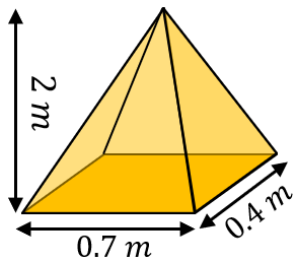


Find the volumes of each of these pyramids.

(a)

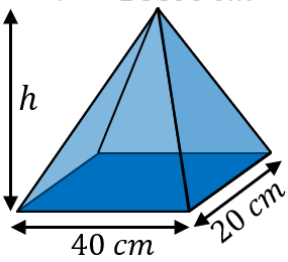


(b)

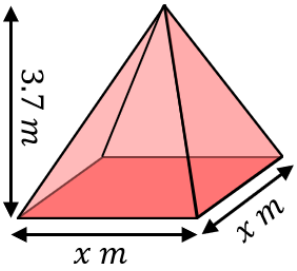


Find the missing measurements from each of these pyramids when given their volumes.

(a) $V = 14400\ cm^3$

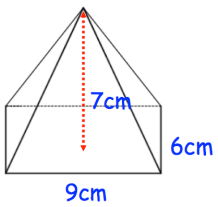


(b) $V = 0.111\ m^3$

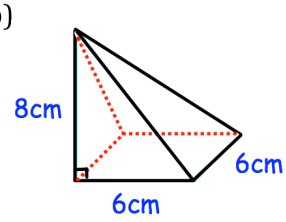


Question 1: Find the volume of each of these pyramids.
Give each answer to one decimal place (you may use a calculator)

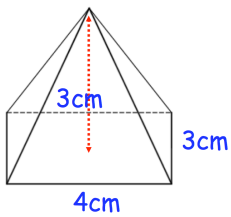
(a)



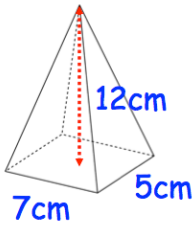
(b)



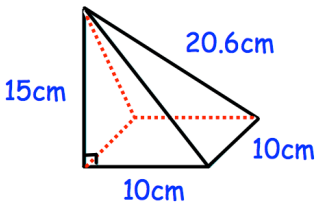
(c)



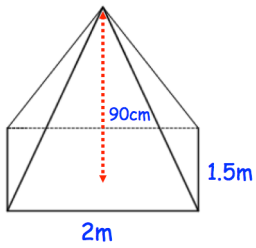
(d)



(e)



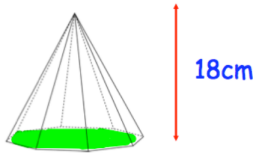
(f)



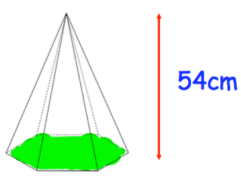
Question 2: A square-based pyramid has a base with side length 8cm.
The height of the pyramid is 11cm.
Calculate the volume of the pyramid.

Question 3: A rectangular-based pyramid has a base with length 12cm and width 6cm.
The height of the pyramid is 8cm.
Calculate the volume of the pyramid.

Question 4: An octagon-based pyramid has a height of 18cm.
The area of the octagon base is $20cm^2$.
Calculate the volume of the pyramid

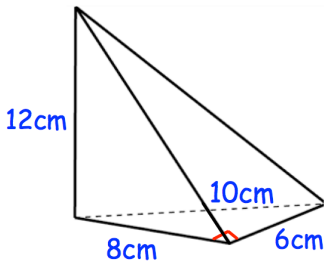


Question 5: A hexagon-based pyramid has a height of 54cm.
The volume of the pyramid is $1080cm^3$.
Calculate the area of the base of the pyramid.

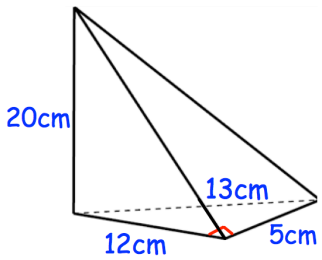


Question 6: Shown below are two triangular-based pyramids.
Find the volume of each.

(a)

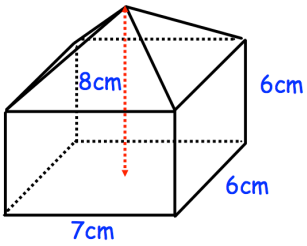


(b)

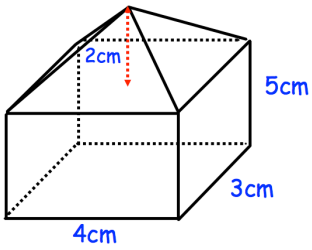


Question 7: Find the volume of each of composite solids.

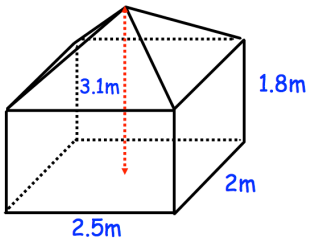
(a)



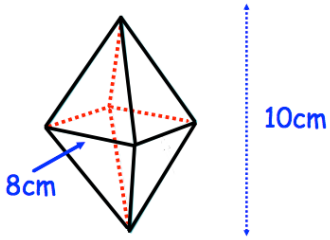
(b)



(c)

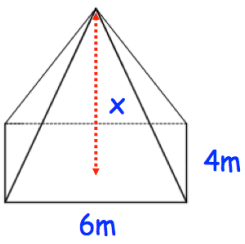


Question 8: A solid shape is created by joining two square based pyramids.
Find the volume of the shape.



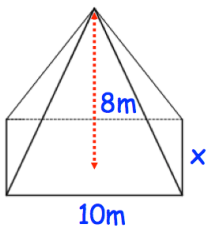
Question 9: Find x for each of these pyramids.
The volume of each is given.

(a)



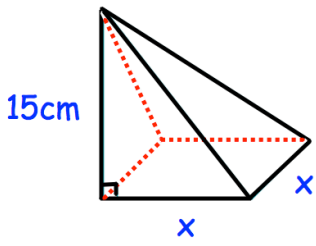
Volume = $36m^3$

(b)



Volume = $256m^3$

(c)



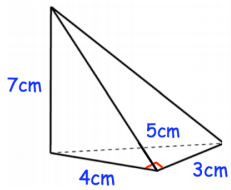
Volume = $605cm^3$

Apply

Question 1: The Great Pyramid of Giza is a square based pyramid.
The base has a side length of 440 cubits.
The height of the pyramid is 280 cubits.
Calculate the volume of the Great Pyramid of Giza.

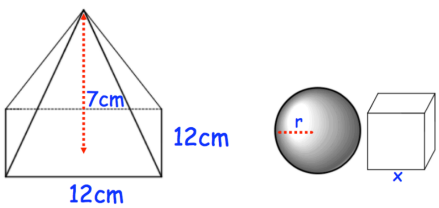
Question 2: A solid rectangular based pyramid has a base with length 28cm and width 20cm.
The height of the pyramid is 16cm.
The pyramid has a mass of 35.84kg
Calculate the density of the material used to make the pyramid, in g/cm^3

Question 3: A solid triangular based pyramid is made from a material which has a density of $7.2g/cm^3$.
The dimensions of the pyramid are shown below.



Calculate the mass of the pyramid.

Question 4: A solid square based pyramid is made out of gold.
The pyramid has base of length 12cm and a height of 7cm.
The pyramid is melted and the gold is used to make a sphere and a cube.
The sphere and cube have the same volume.
Calculate the radius of the sphere, r, and the side length of the cube, x.



Exam Question

The diagram shows a pyramid.

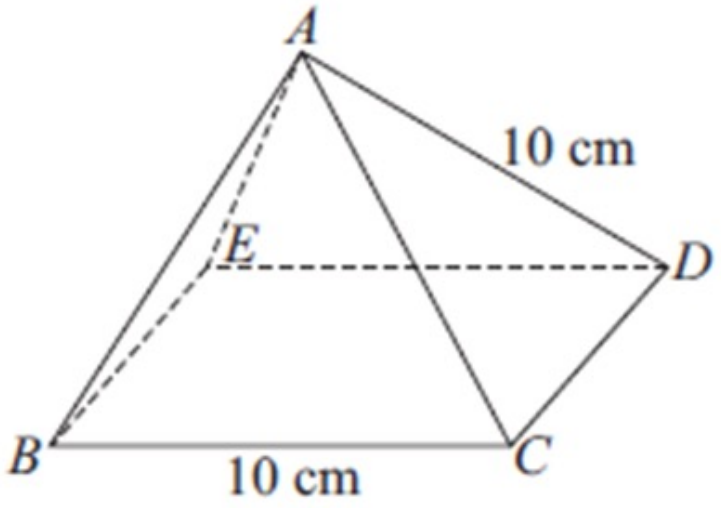


Diagram **NOT** accurately drawn

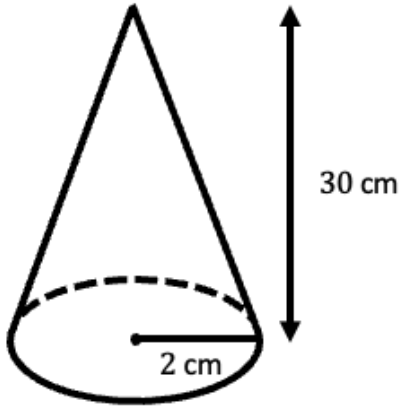
BCDE is a square with sides of length 10 cm.
The other faces of the pyramid are equilateral triangles with sides of length 10 cm.

(a) Calculate the volume of the pyramid.
Give your answer correct to 3 significant figures.

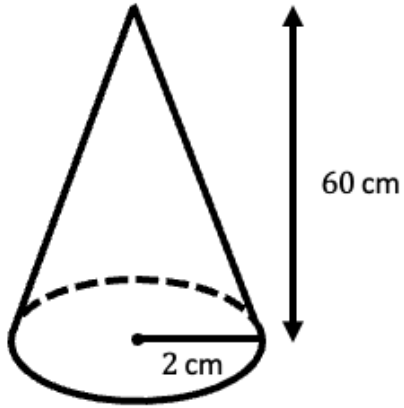
Intelligent Practice

Calculate the volume. Give your answers in terms of π and to 1 decimal place.

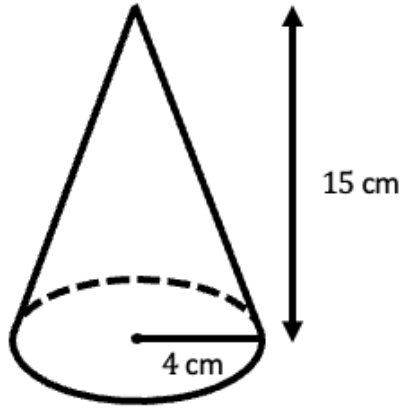
Question 1



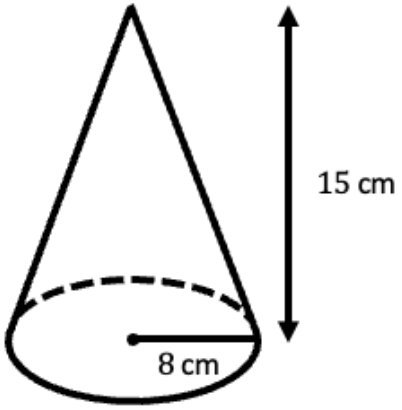
Question 2



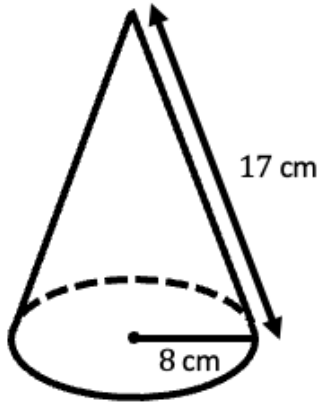
Question 3



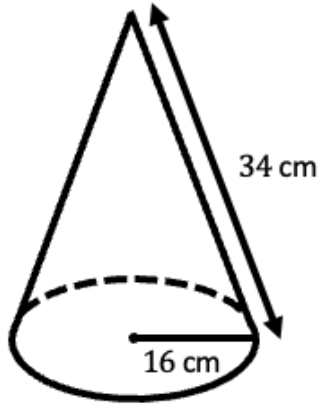
Question 4



Question 5

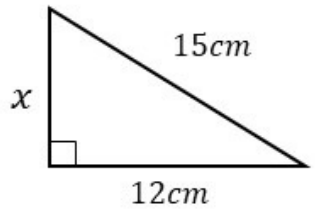


Question 6

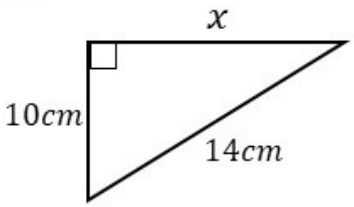


Fluency Practice

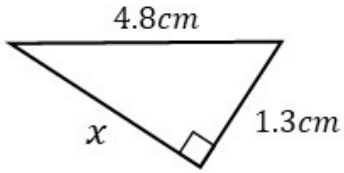
1 Calculate x



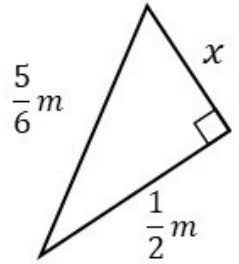
2 Calculate x to 2 d.p.



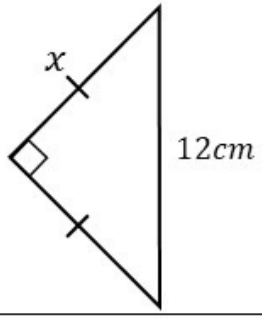
3 Calculate x to 1 d.p.



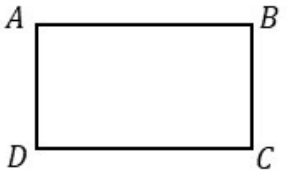
4 Calculate x, leave as a fraction



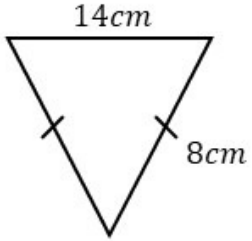
5 Find x leave in exact form



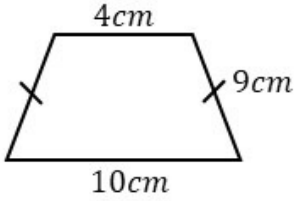
6 AB = 12cm
CB = 18cm
Calculate BD to 1 d.p.



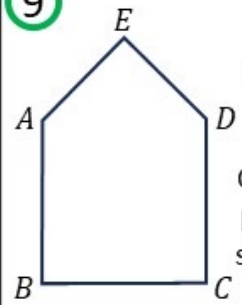
7 Calculate the area to 3 s.f.



8 Calculate the area to 1 d.p.

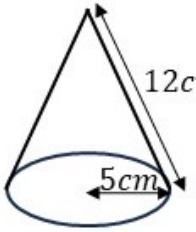


9



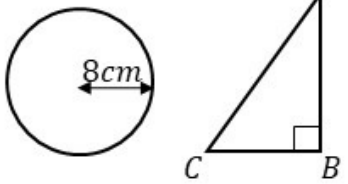
ABCD is a square with an area of $36cm^2$. Calculate the perimeter of shape ABCDE to 2 d.p.

10 Calculate the volume of the cone to 2 d.p.



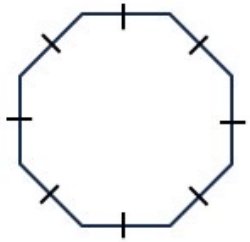
$V_{Cone} = \frac{\pi r^2 h}{3}$
r = radius
h = height

11 Both shapes have the same perimeter



$AC : AB = 9 : 4$
Calculate BC to 2 d.p.

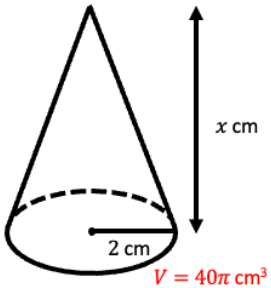
12 The perimeter is $56cm$.
Calculate the area to 2 d.p.



Intelligent Practice

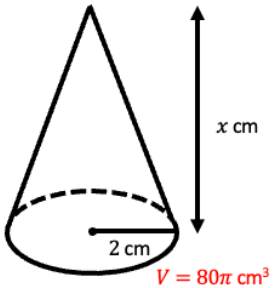
Find x given the volume. Give your answers to 1 decimal place if required.

Question 1



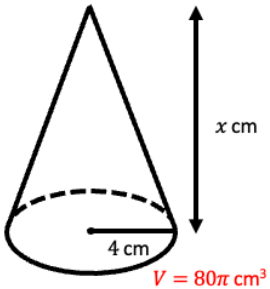
$V = 40\pi$ cm^3

Question 2



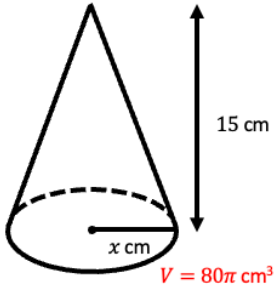
$V = 80\pi$ cm^3

Question 3



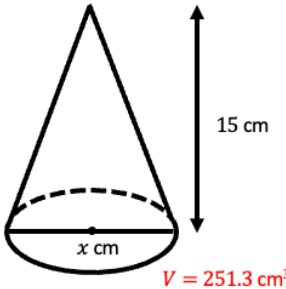
$V = 80\pi$ cm^3

Question 4



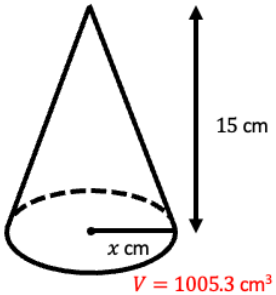
$V = 80\pi$ cm^3

Question 5



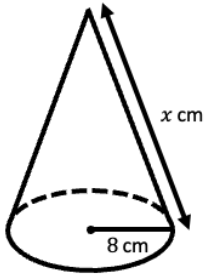
$V = 251.3$ cm^3

Question 6



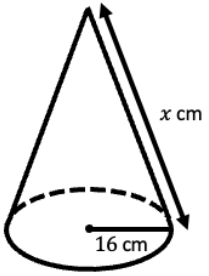
$V = 1005.3$ cm^3

Question 7



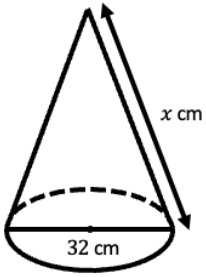
$V = 1005.3$ cm^3

Question 8



$V = 8042.5$ cm^3

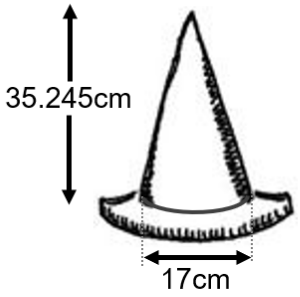
Question 9



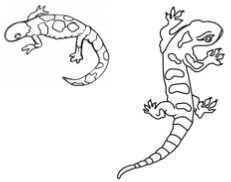
$V = 16889.2$ cm^3

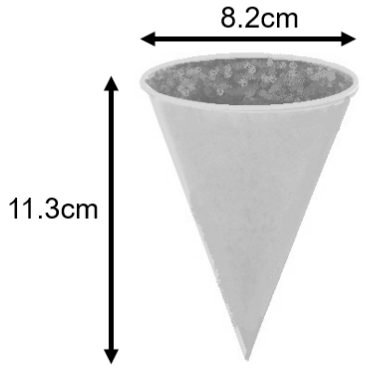
Fluency Practice

volume of a cone



what volume of lizard soup would fit inside the witches' hat?

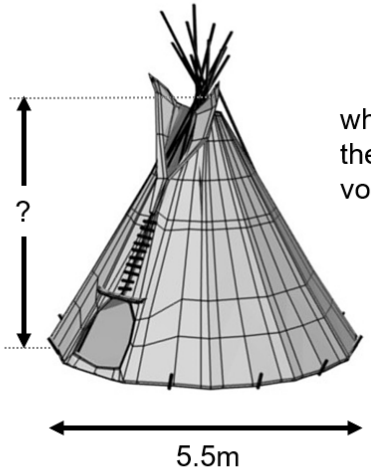




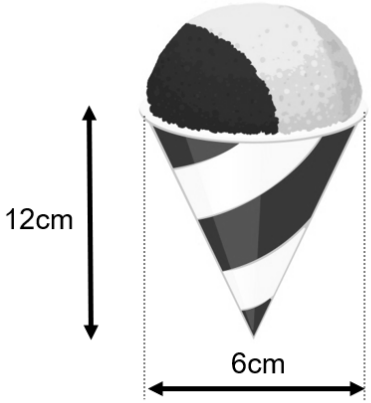
what volume of slush would fit inside the snowkone?

volume of a cone = $\frac{1}{3}\pi r^2 h$

volume of a sphere = $\frac{4}{3}\pi r^3$



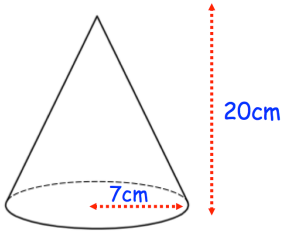
what is the height of the teepee if the volume is $35.6m^3$?



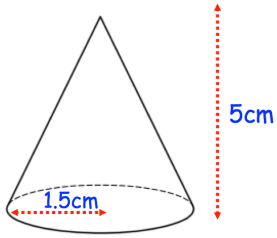
what is the volume of ice cream if the top is a hemisphere (and the cone is full)?

Question 1: Work out the volumes of each of following cones.
Give each answer to one decimal place.

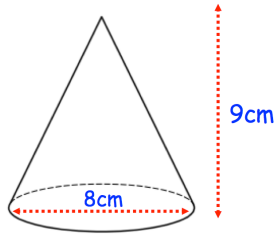
(a)



(b)

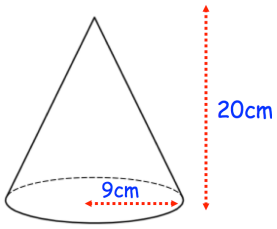


(c)

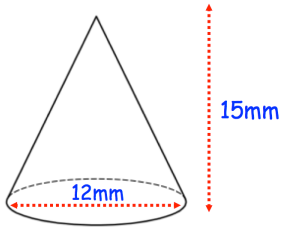


Question 2: Work out the volumes of each of the following cones.
Give each answer in terms of **π**

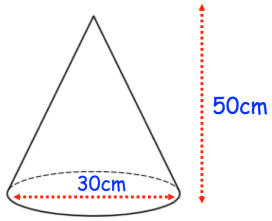
(a)



(b)

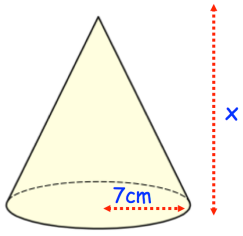


(c)



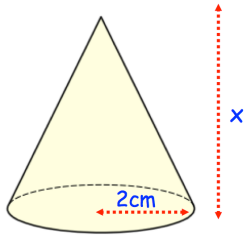
Question 3: Work out the vertical height of each cone.
Give each answer to a suitable degree of accuracy.

(a)



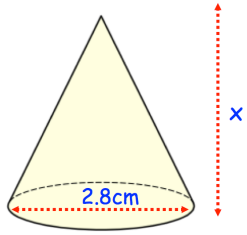
Volume = 1000cm³

(b)



Volume = 22cm³

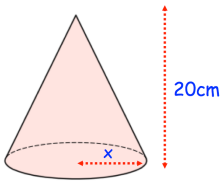
(c)



Volume = 17cm³

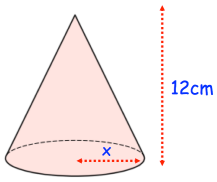
Question 4: Calculate the length of the radius for each of these cones.
Give each answer to a suitable degree of accuracy.

(a)



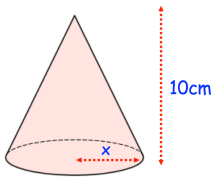
Volume = 195cm^3

(b)



Volume = 300cm^3

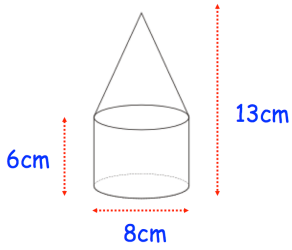
(c)



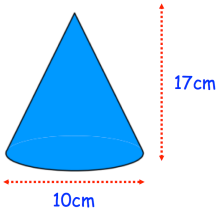
Volume = 880cm^3

Apply

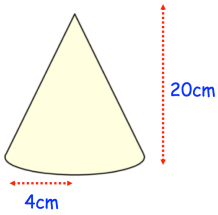
Question 1: A solid is formed from a cylinder and a cone.
Find the volume of the solid.

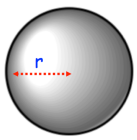


Question 2: A solid cone is made from a material which has a density of 8.7 g/cm^3.
The dimensions of the cone are shown below.
Find the mass of the cone.

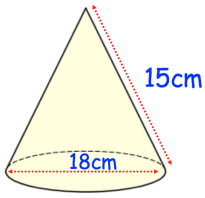


Question 3: The sphere and cone have an equal volume.
Find the radius of the sphere.



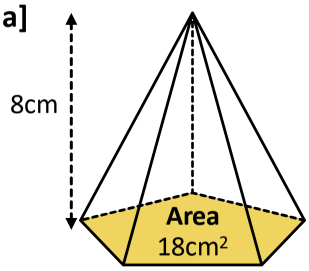


Question 4: Calculate the volume of the cone shown
Give your answer to 1 decimal place.

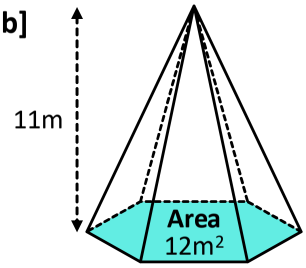


Q1 Work out the volume of each of the following pyramids.

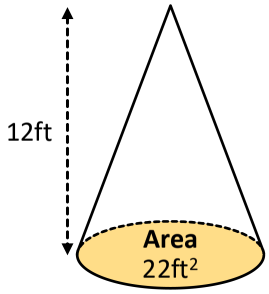
[a]



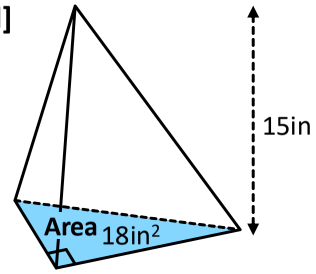
[b]



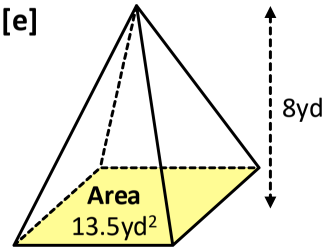
[c]



[d]

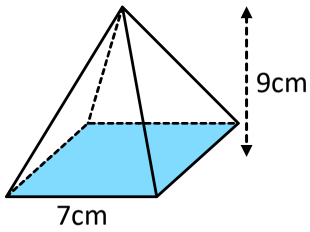


[e]

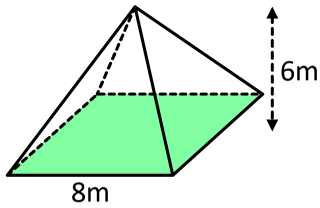


Q2 Work out the volume of each of the following pyramids.

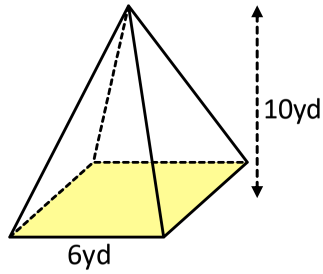
[a] Square-based pyramid



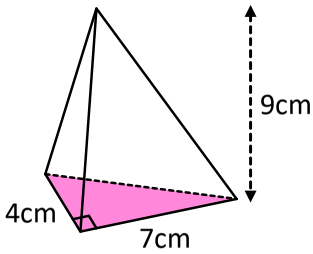
[b] Square-based pyramid



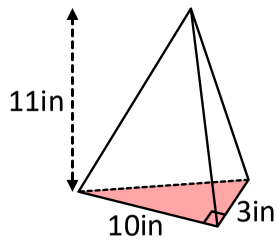
[c] Square-based pyramid



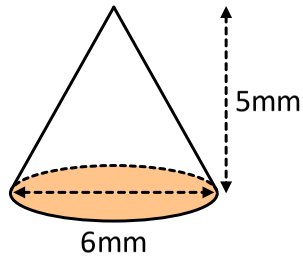
[d] Right-triangular-based pyramid



[e] Right-triangular-based pyramid

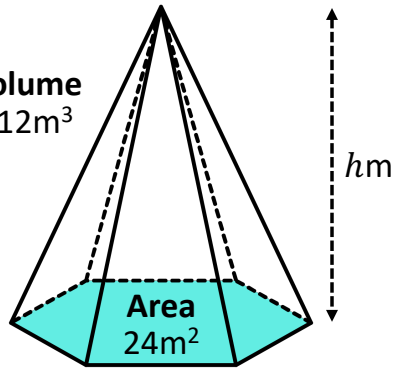


[f] Cone

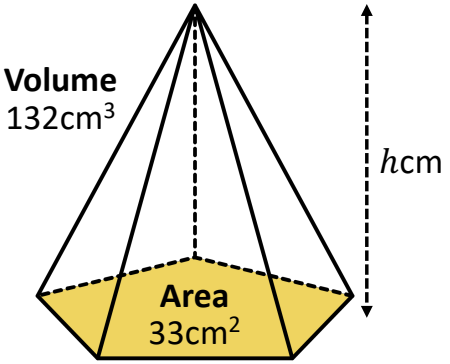


Fluency Practice

Ex3 The following pyramid has a base area of 24m^2. Given that the volume of the pyramid is 112m^3, find its height, h m.

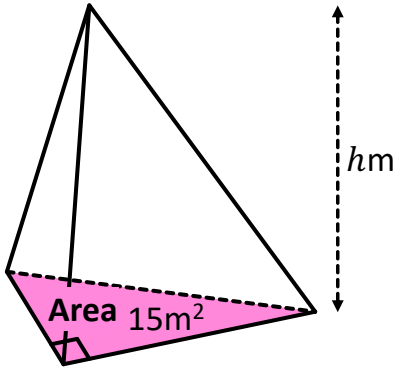


Q3 The following pyramid has a base area of 33cm^2. Given that the volume of the pyramid is 132cm^3, find its height, h cm.

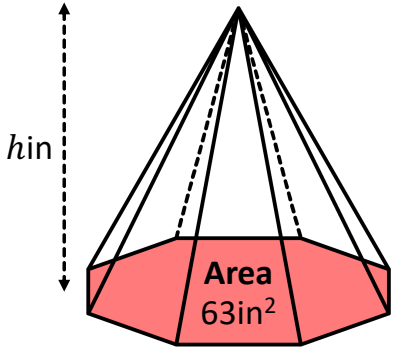


Q4 Work out the height of each of the following pyramids given their volume.

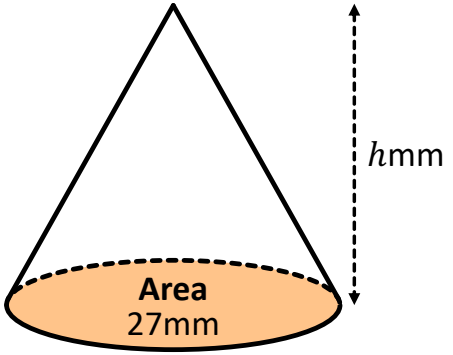
[a] Volume $=$ **105m^3**



[b] Volume $=$ **168in^3**

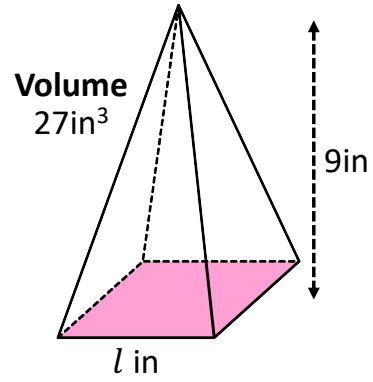


[c] Volume $=$ **108mm^3**

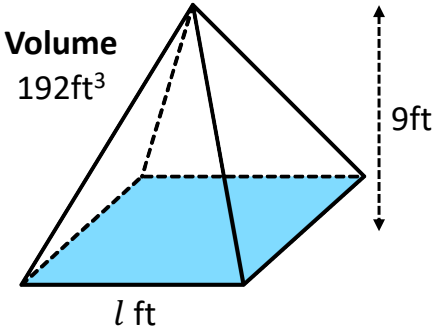


Fluency Practice

Ex4 The following square-based pyramid has a height of 9in. Given that the volume of the pyramid is 27in^3, find the length of the base, l in.

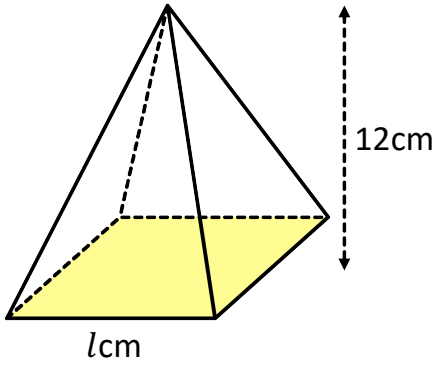


Q5 The following square-based pyramid has a height of 9ft. Given that the volume of the pyramid is 192ft^3, find the length of the base, l ft.

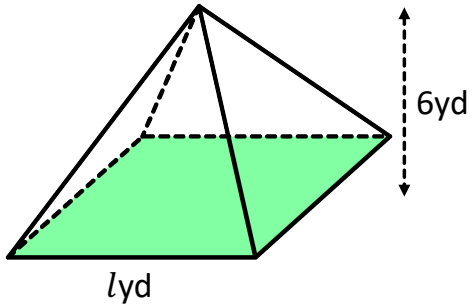


Q6 Work out the missing length of each of the following pyramids given their volume.

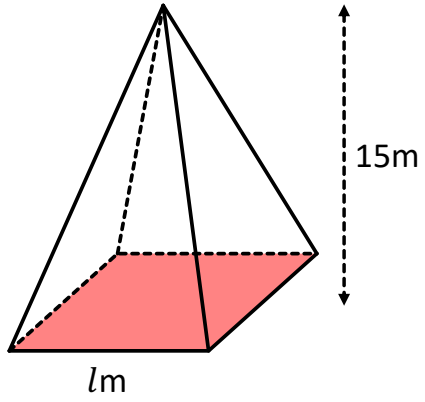
[a] Volume $=$ **100cm^3**



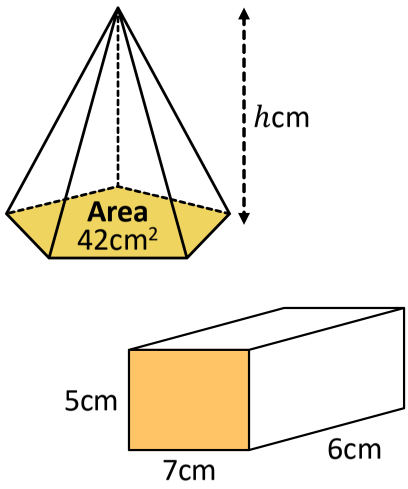
[b] Volume $=$ **128yd^3**



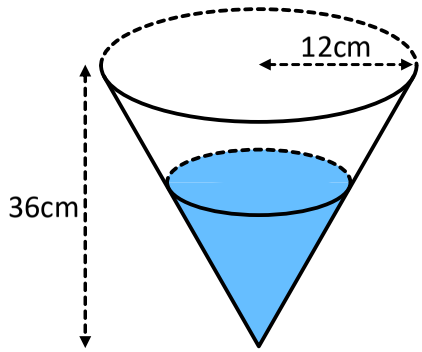
[c] Volume $=$ **500m^3**



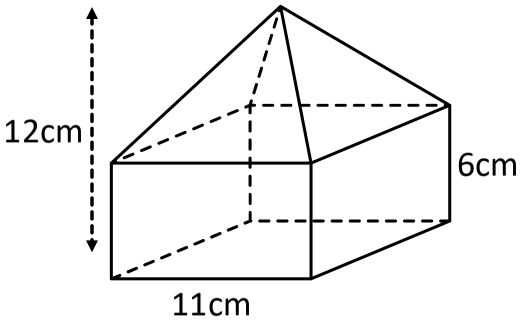
Q1 The pyramid and the cuboid have the same volume. Find the value of h.



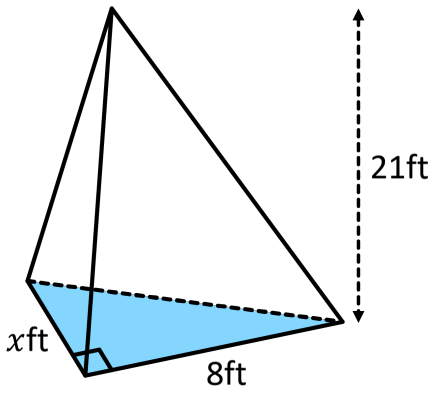
Q2 The following container is in the shape of a cone.
The container is filled with 3L of water. Use **1L = 1000cm³** to find the percentage of the volume of the cone that is taken up by the water to the nearest 1%.



Q3 The following composite solid is made by joining a cuboid and square-based pyramid. Calculate the volume of the solid.



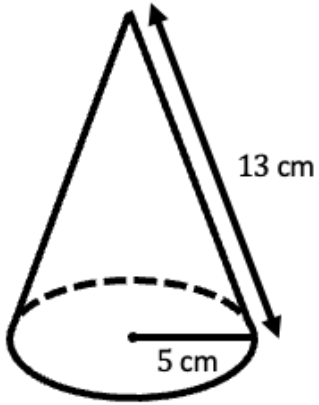
Q4 The following right-triangular-based pyramid has a height of 9ft. Given that the volume of the pyramid is 84ft³, find the missing length, x ft.



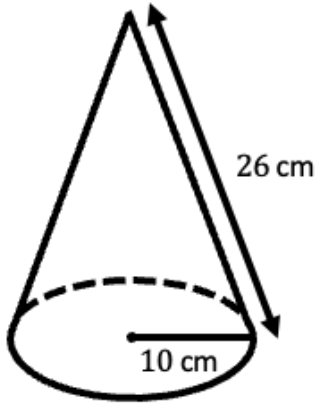
Intelligent Practice

Calculate the total surface area. Give your answers in terms of π and to 1 decimal place.

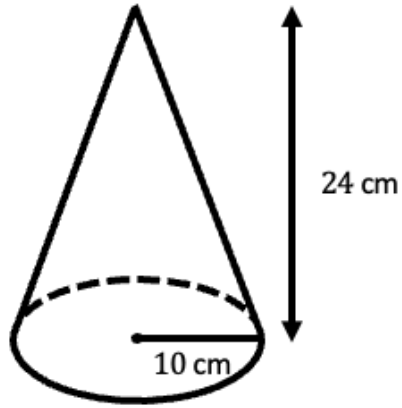
Question 1



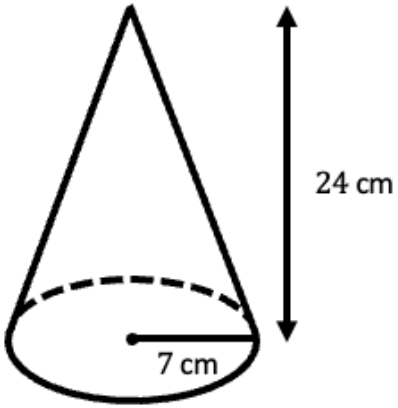
Question 2



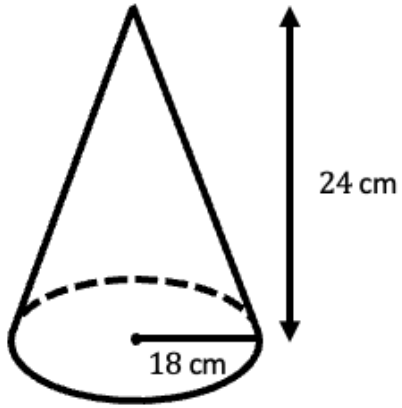
Question 3



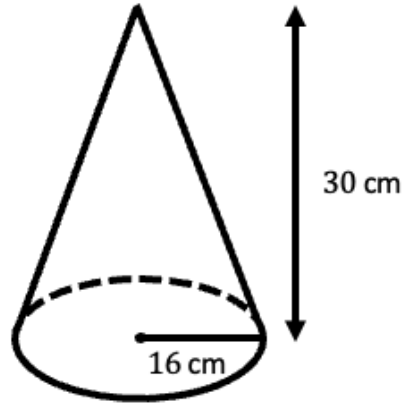
Question 4



Question 5



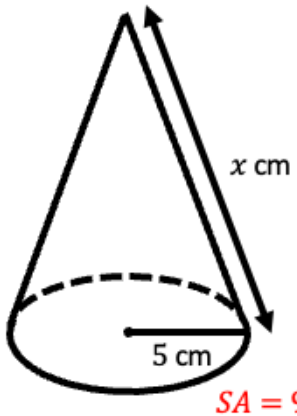
Question 6



Intelligent Practice

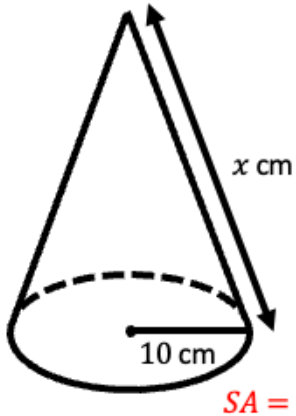
Find x given the total surface area. Give your answers to 1 decimal place if required.

Question 1



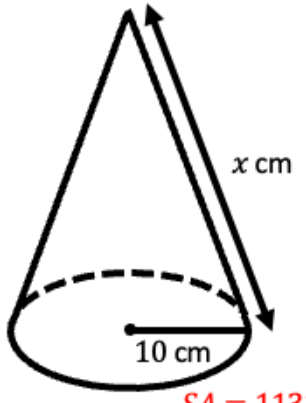
$SA = 90\pi$ cm^2

Question 2



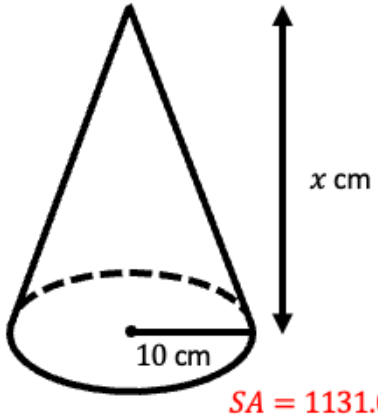
$SA = 360\pi$ cm^2

Question 3



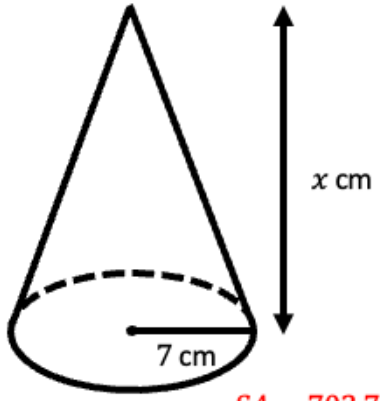
$SA = 1131.0$ cm^2

Question 4



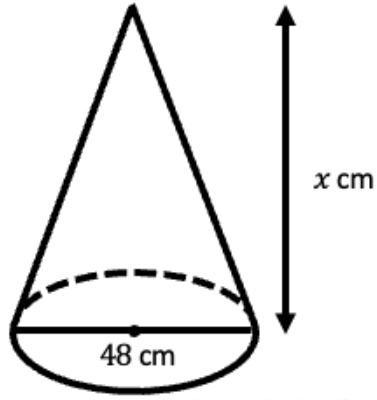
$SA = 1131.0$ cm^2

Question 5



$SA = 703.7$ cm^2

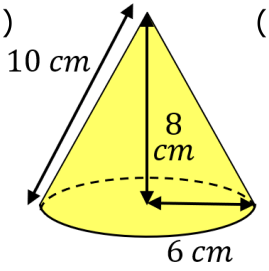
Question 6



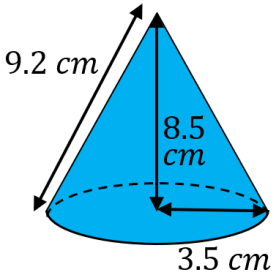
$SA = 3694.5$ cm^2

Find the volume and total surface area of each of these cones.

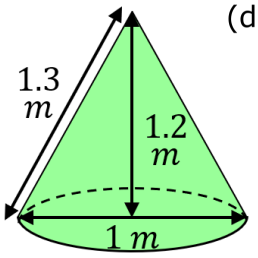
(a)



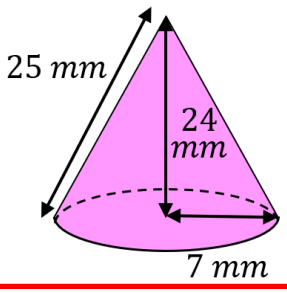
(b)



(c)

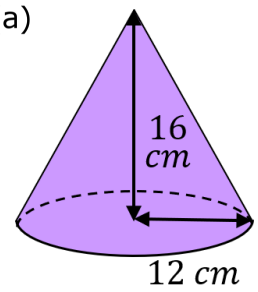


(d)

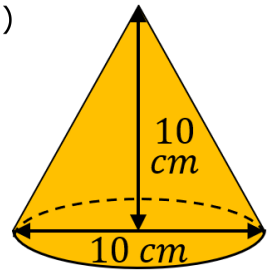


Find the slanted height and curved surface area of these cones.

(a)

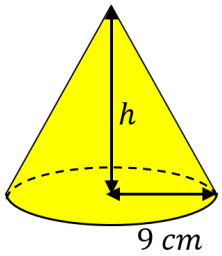


(b)

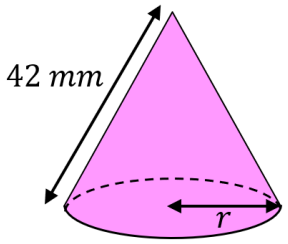


Find the missing lengths.

(a) $Volume = 1103\ cm^3$



(b) $Curved\ Surface\ Area = 3300\ cm^2$

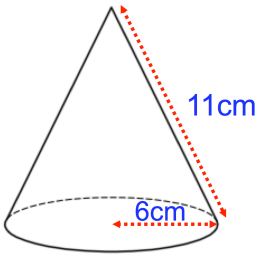


(a) A cone has a slanted height of $26\ cm$ and a curved surface area of $260\pi\ cm^2$. Find its volume.

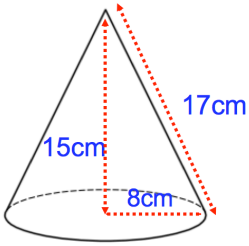
(b) A cone has a radius of $8.5\ cm$ and a volume of $1059\ cm^3$. Find its total surface area.

Question 1: Work out the surface areas of each of the following cones.
Give each answer in terms of **π**

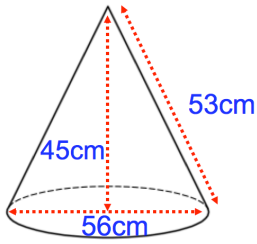
(a)



(b)

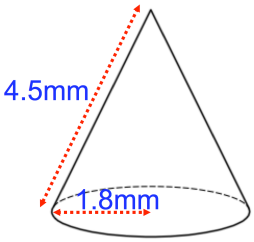


(c)

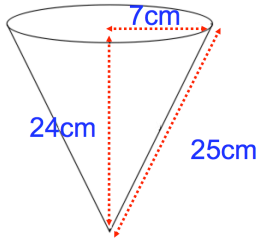


Question 2: Work out the surface areas of each of the following cones.
Give each answer to one decimal place.

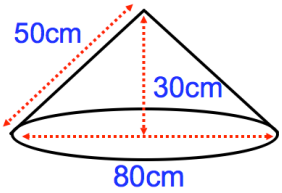
(a)



(b)

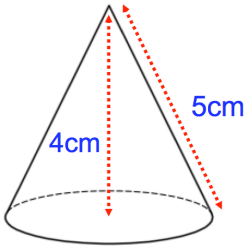


(c)

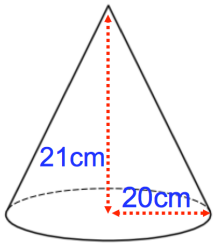


Question 3: Work out the surface areas of each of the following cones.
Give each answer to one decimal place.

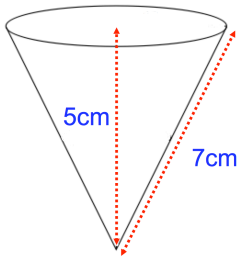
(a)



(b)

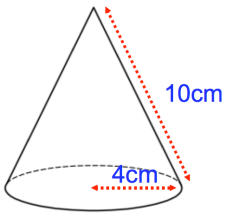


(c)

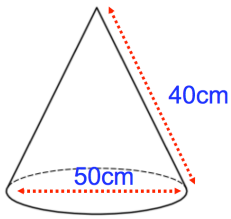


Question 4: Work out the surface area of each of the following cones.
Give each answer in terms of **π**

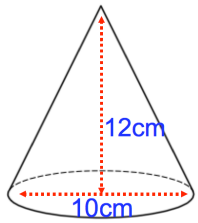
(a)



(b)

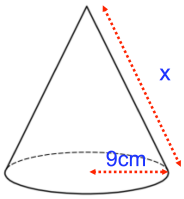


(c)



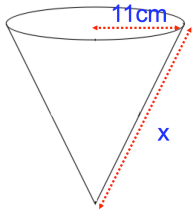
Question 5: Calculate the slant height for each of these cones

(a)



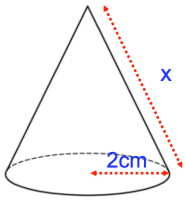
Total surface area = 216π cm²

(b)



Total surface area = 792π cm²

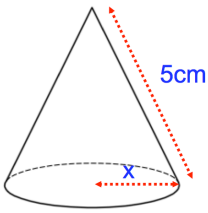
(c)



Total surface area = 100 cm²

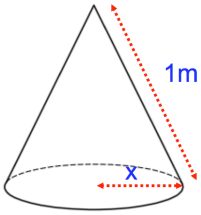
Question 6: Calculate the lengths of the radius for each of these cones

(a)



Total surface area = 36π cm²

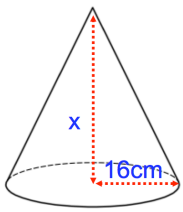
(b)



Total surface area = 9600π cm²

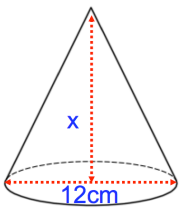
Question 7: Calculate the heights of these cones

(a)



Total surface area = 800π cm²

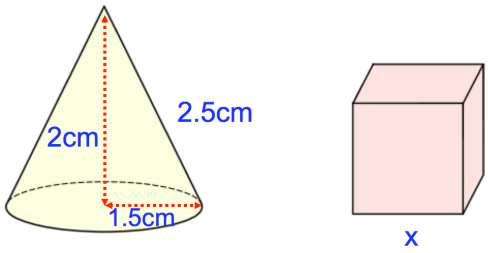
(b)



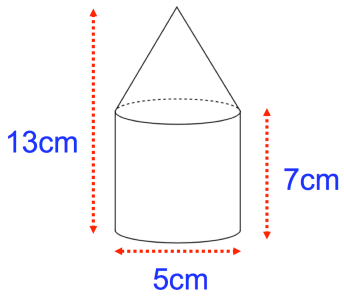
Total surface area = 750 cm²

Apply

Question 1: The cone and cube below have the same surface areas.
Work out the side length of the cube, x.

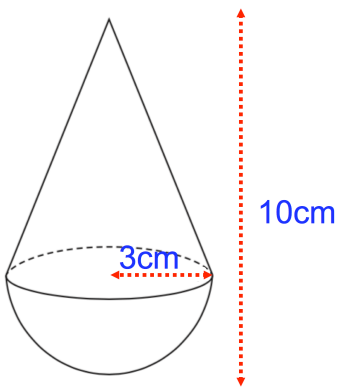


Question 2: The diagram shows a solid shape.
The shape is a cone on top of a cylinder.
Work out the surface area of the shape.
Give your answer correct to 2 significant figures

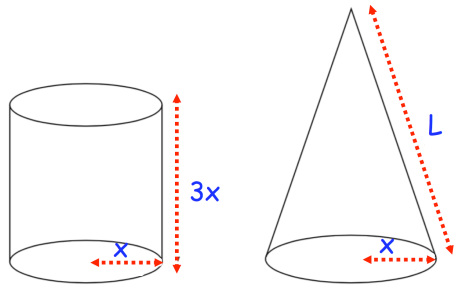


Question 3: A cone has a radius of 9cm.
The surface area of the cone is 450π cm^2
Work out the volume of the cone.
Give your answer in terms of π

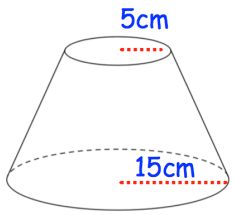
Question 4: The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.
Work out the surface area of the shape.
Give your answer correct to 2 significant figures



Question 5: The cylinder and cone has the same surface area.
Express L in terms of x.

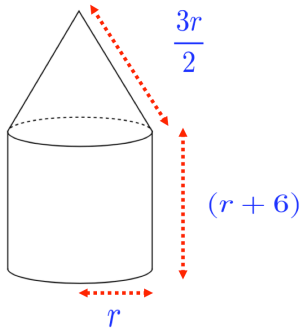


Question 6: A frustum is made from cutting a small cone from the top of a larger cone.
The larger cone was 21cm tall.



Calculate the surface area of the frustum

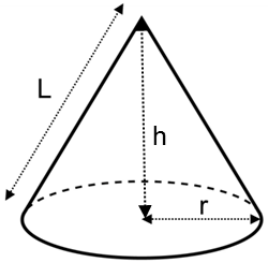
Question 7: A cone and cylinder are joined to make a solid.



Show the total surface area of the solid is $\frac{3\pi r}{2}(3r+8)$

Purposeful Practice

surface area of a cone
leaving π in the answer



(1) find the total surface area, including the circular base, for cones with:

(a) r = 5 , L = 35
(b) r = 4 , L = 46
(c) r = 8 , h = 15

total surface area $\pi rL + \pi r^2$

(2) the total surface area, including the circular base, for cones is 300π

what is L if:

(a) r = 12
(b) r = 10
(c) r = 5
(d) r = 4
(e) r = 1

(3) the total surface area, including the circular base, for cones is 100π

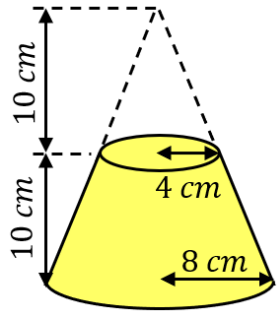
what is r if:

(a) L = 15
(b) L = 21
(c) L = 48
(d) L = 99

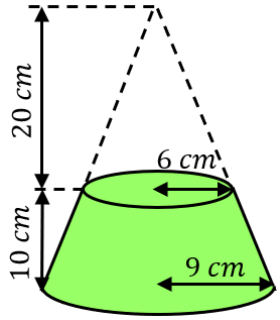
why these numbers for L?

Find the volume of each of these frustums.

(a)

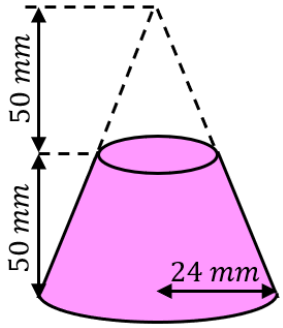


(b)

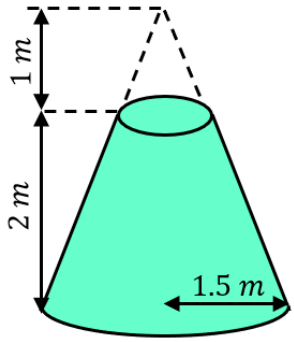


Find the volume of each of these frustums.

(a)

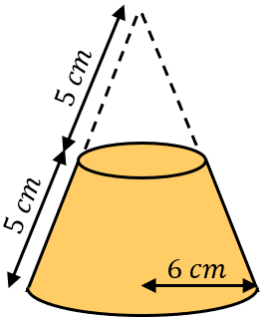


(b)

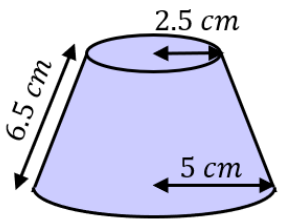


Find the curved surface area and total surface area of each of these frustums.

(a)



(b)

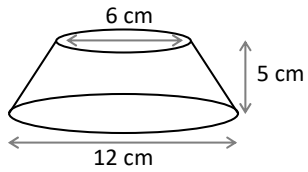


The base diameter of a frustum is $18\ cm$ and the top diameter is $9\ cm$. If the frustum has a volume of $378\pi\ cm^3$, find its height.

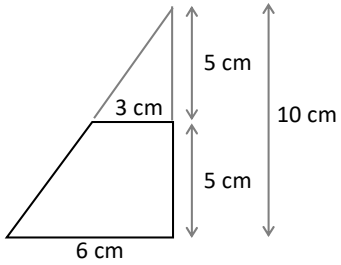
Fluency Practice

Finding the Volume of Frustums using Similar Triangles

① 3D View



Halved Cross-Section

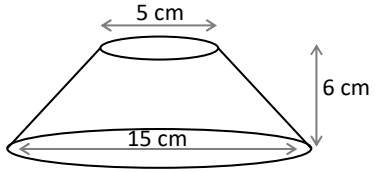


Volume of Large Cone =

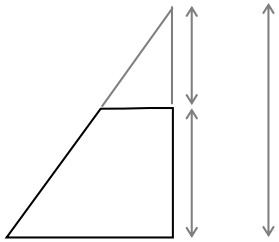
Volume of Small Cone =

Volume of Frustum =

② 3D View



Halved Cross-Section

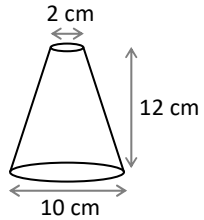


Volume of Large Cone =

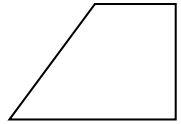
Volume of Small Cone =

Volume of Frustum =

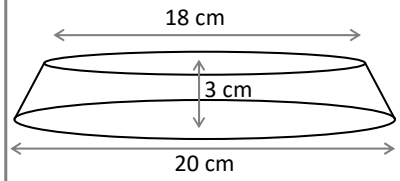
③ 3D View



Halved Cross-Section



④ 3D View

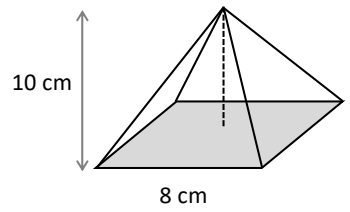


Fluency Practice

Volume of Pyramids

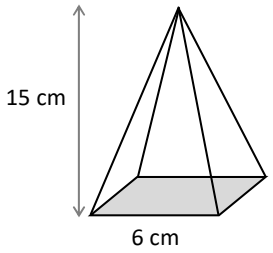
The apex is above the centre of a square base unless stated.

①



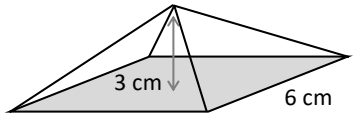
Volume =

②



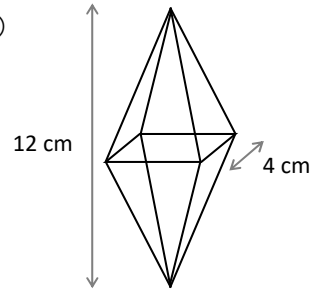
Volume =

③



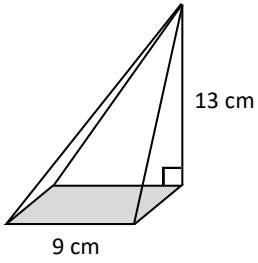
Volume =

④



Volume =

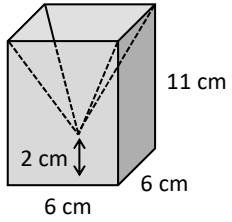
⑤



A pyramid with its apex vertically above a base corner.

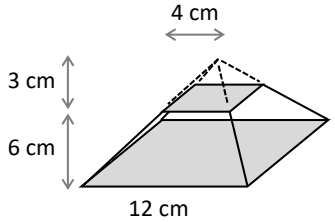
Volume =

⑥



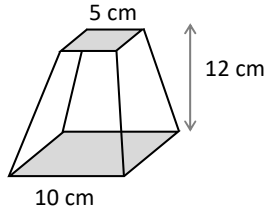
Volume =

⑦



Volume =

⑧

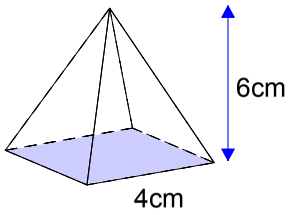


Volume =

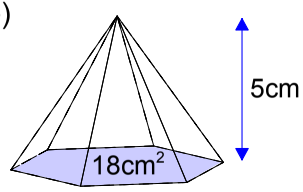
7 Answers	351	180	64	288	213.33	416	36

1. The base of each of these pyramids is a regular polygon. Work out the volume of each pyramid.

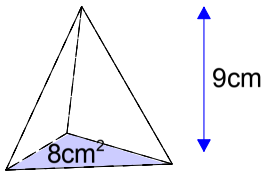
a)



b)

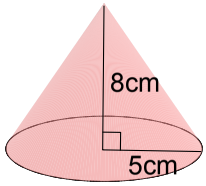


c)

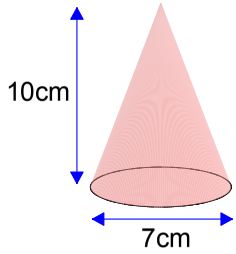


2. Work out the volume of each of these cones, giving your answers both in terms of π, and correct to 3 significant figures.

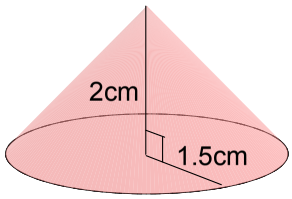
a)



b)



c)

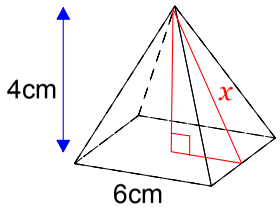


3. Which has a larger volume?
   A - a cone with radius 5cm and height 6cm.
   B - a cone with radius 6cm and height 5cm.

4. A pyramid has a perpendicular height of 4cm and a square base with a side length of 6cm.

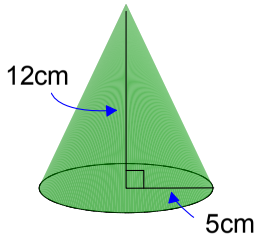
   a) Work out the 'slant height', marked x.

   b) Hence, find the surface area of the pyramid.

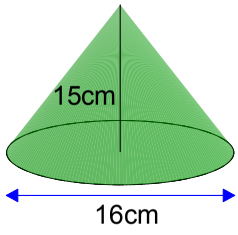


5. Work out the surface area of each cone, giving your answers both in terms of π, and correct to 3 significant figures.

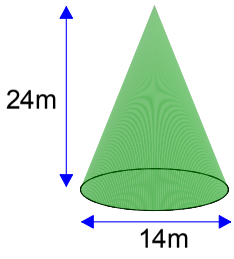
a)



b)

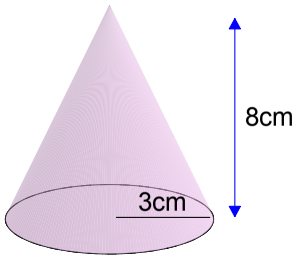


c)

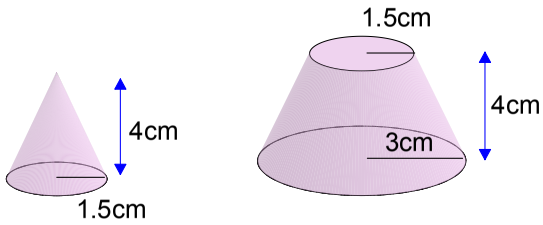


6. The cone in *part a* is cut horizontally to produce the frustum and cone in *b*. Work out the volume of all three shapes in terms of π.

a)

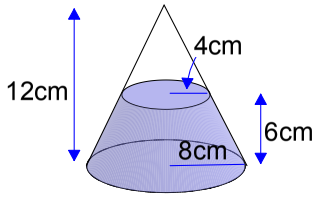


b)

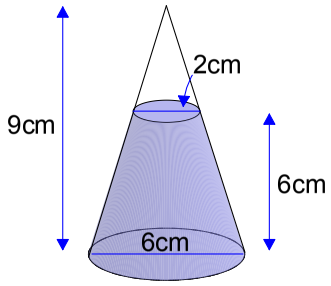


7. Work out the volume of each shaded frustum in terms of π.

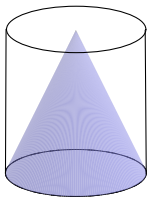
a)



b)

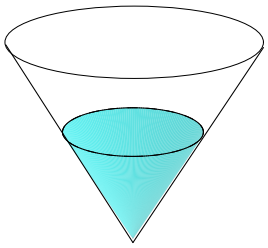


8. A cone sits inside a cylinder with the same radius and same height.



What fraction of the cylinder is taken up by the cone?

9. A container in the shape of a cone is partially filled with water to half of its total height.

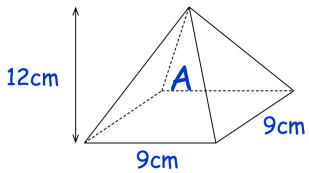


Gemma says that half of the volume of the cone is taken up by the water.

Gemma is wrong. Find the actual fraction of the volume of the cone that is taken up by the water.

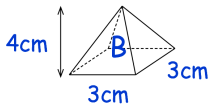
(Hint: since the cones are similar, the radius of the surface of the water is half the radius at the top of the cone.)

Question 1: Shown is a square based pyramid A.



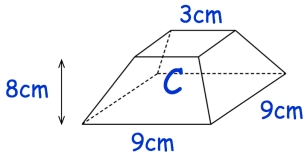
(a) Find the volume of the square based pyramid.

Shown is a smaller square based pyramid B.



(b) Find the volume of the smaller square based pyramid.

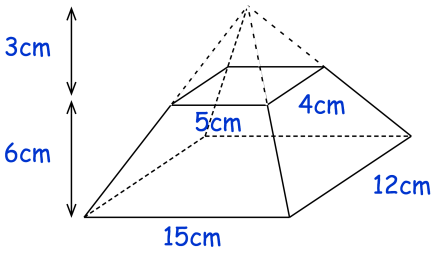
A frustum is created by removing the Pyramid B from the top of Pyramid A.



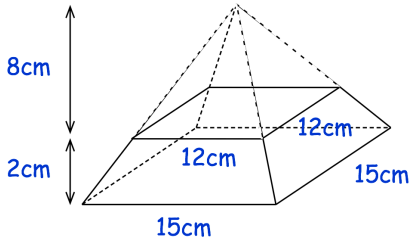
(c) Find the volume of the frustum.

Question 2: Find the volume of the following frustums.

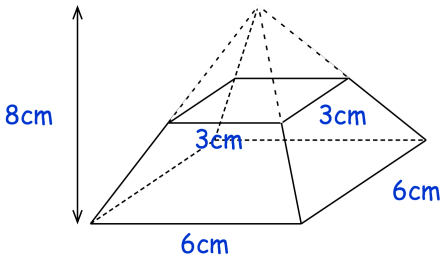
(a)



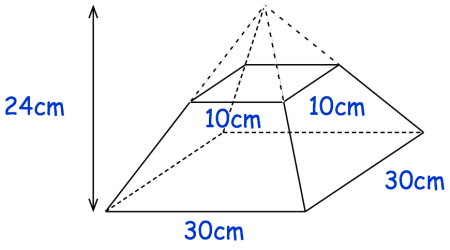
(b)



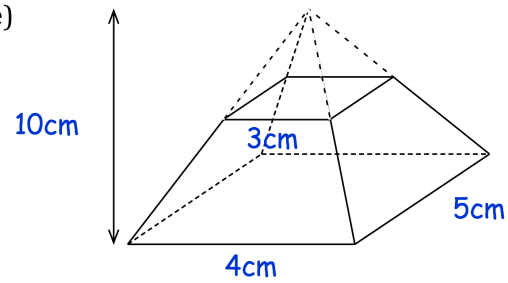
(c)



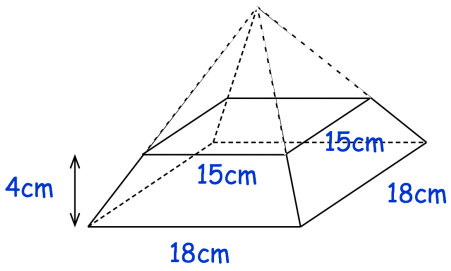
(d)



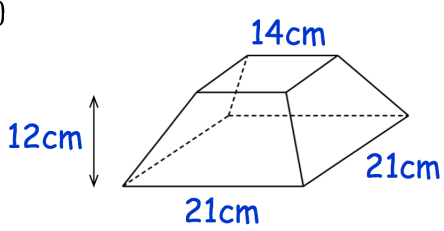
(e)



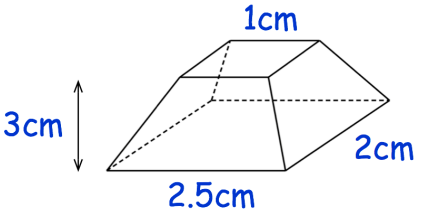
(f)



(g)

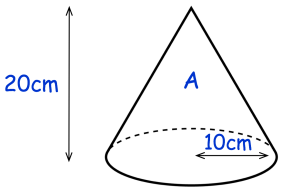


(h)



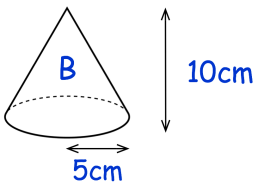
Question 3: Shown is Cone A.

(a) Find the volume of the cone.



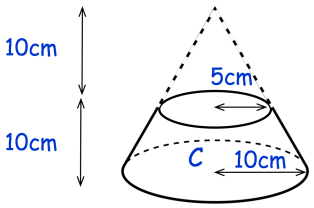
Shown is a smaller Cone, B.

(b) Find the volume of the smaller cone.



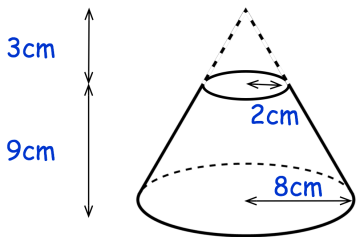
A frustum is created by removing Cone B from the top of Cone A.

(c) Find the volume of the frustum.

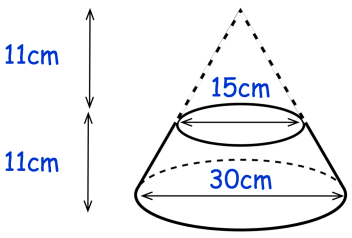


Question 4: Find the volume of the following frustums.

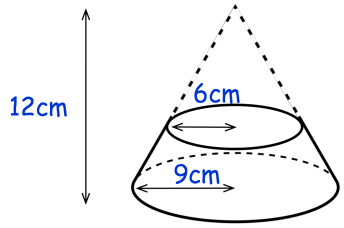
(a)



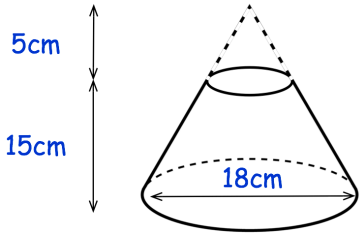
(b)



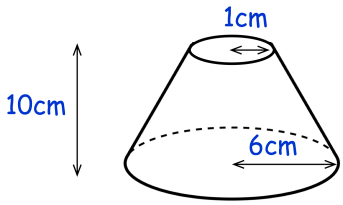
(c)



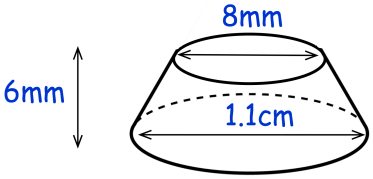
(d)



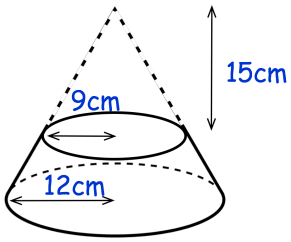
(e)



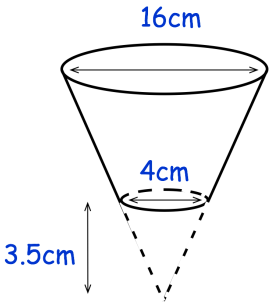
(f)



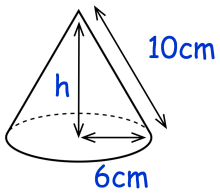
(g)



(h)

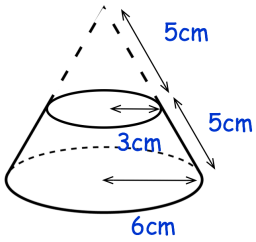


Question 5: Shown below is a cone with a radius of 6cm and a slant height of 10cm.



(a) Find the perpendicular height of the cone, h.

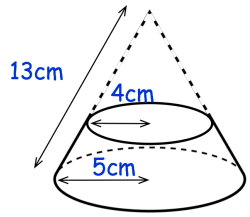
A frustum is made by removing a smaller cone from the top of the cone in (a).



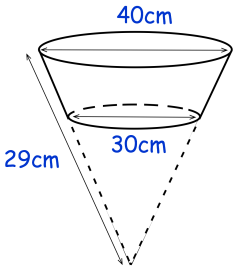
(b) Find the volume of the frustum.

Question 6: Find the volumes of the following frustums.

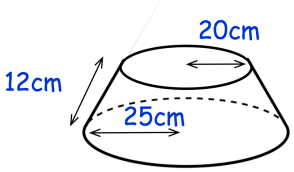
(a)



(b)

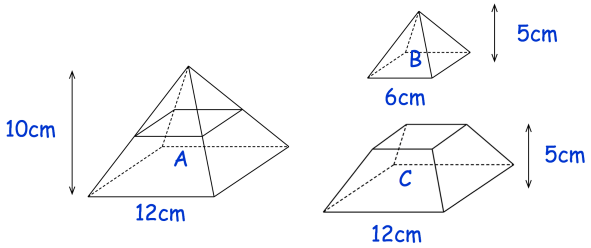


(c)



Apply

Question 1: The square based pyramid A is divided into Pyramid B and Frustum C.



(a) Work out the volume of Pyramid A.

(b) Work out the volume of Pyramid B.

(c) Work out the volume of Frustum C.

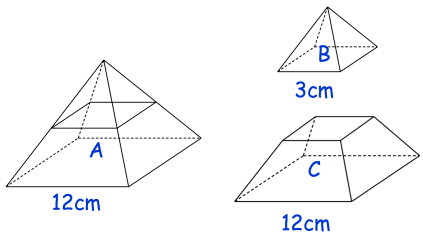
Douglas says that since Pyramid A and Pyramid B are similar, and since the base length of B is half of the base length of A, the volume of B will be one-eighth of the volume of A.

(d) Show that Douglas is correct.

Jack says that means that the volume of Frustum C will be seven-eighths of the volume of Pyramid A.

(e) Show that Jack is correct.

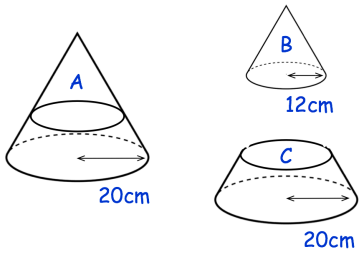
Question 2: The square based Pyramid A is divided in Pyramid B and Frustum C.



(a) Express the volume of Pyramid B as a fraction of the volume of Pyramid A.

(b) Express the volume of Frustum C as a fraction of the volume of Pyramid A.

Question 3: Cone A is divided into Cone B and Frustum C.

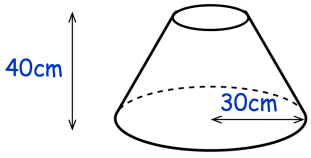


Express the volume of Frustum C as a fraction of the volume of Cone A.
Give your answer in its simplest form.

Question 4: Magda is making a solid glass sculpture.

Magda has a glass cone with base radius of 30cm and perpendicular height 60cm.

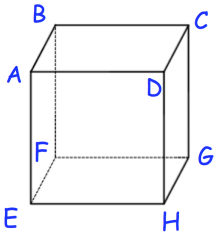
She removes a smaller cone from the top of the cone to leave a frustum.

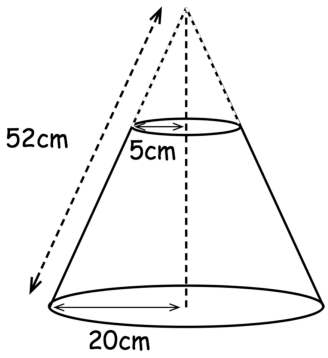


The density of the glass is 2.7g/cm^3

Find the mass of the sculpture.

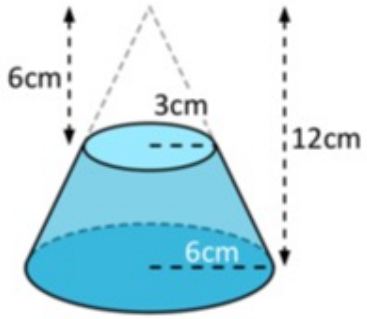
Question 5: The cube and the frustum have the same volume.



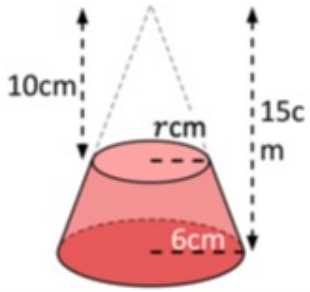


Find the length of AG.

Ex1 Find the volume of the frustum.



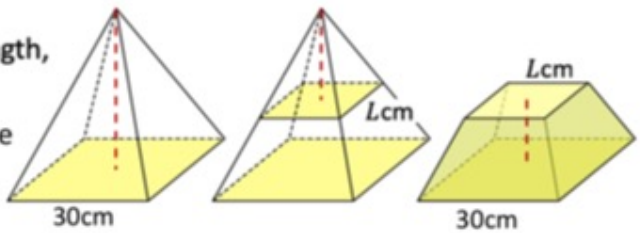
Ex2 Find the missing length of the frustum's top radius and hence find the volume of the frustum in terms of π.



Ex3 Shown is a square-based pyramid with height 50cm and base side length 30cm. A smaller square-based pyramid height 20cm is removed to form a frustum.

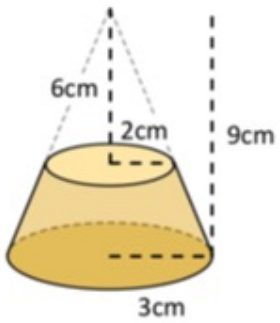
[a] Work out the base side length, L, of the smaller pyramid.

[b] Work out the volume of the frustum.

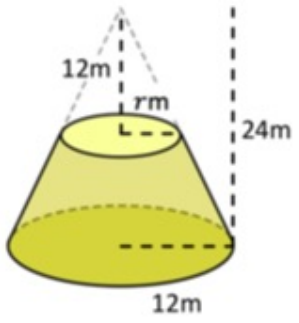


Q1 Work out the volume of each of the following frustums.

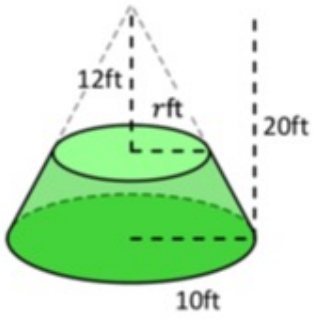
[a]



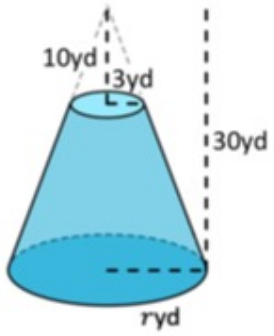
[b]



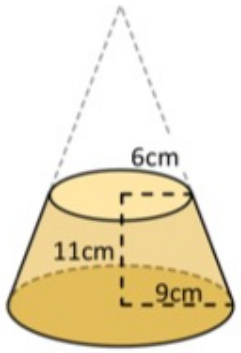
[c]



[d]

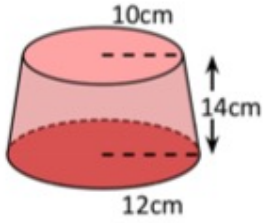


Ex4 Find the missing length of the frustum's top radius and hence find the volume of the frustum in terms of π.

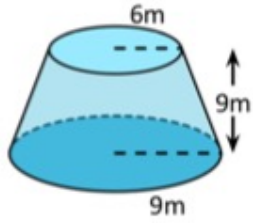


Q2 Work out the volume of each of the following frustums.

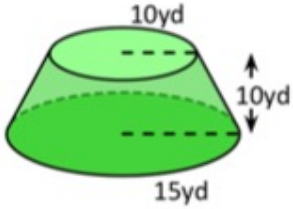
[a]



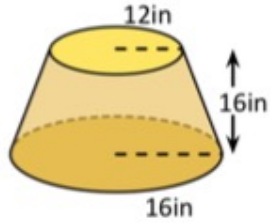
[b]



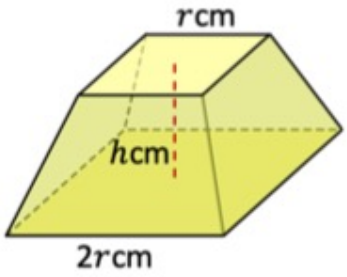
[c]



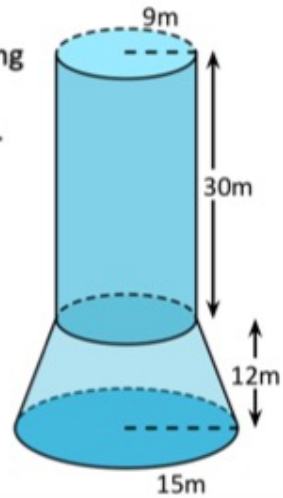
[d]



Q3 The diagram shows a square-based frustum. The base side-length, $2r$ cm is twice the size of the top side-length, r cm. The height of the frustum is h cm. Find an expression for the volume of the frustum.



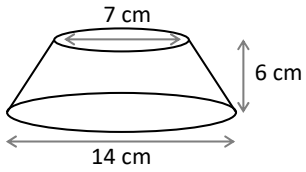
Q4 The chemistry flask consists of a frustum and a cylinder. Calculate its volume leaving your answer in terms of π.



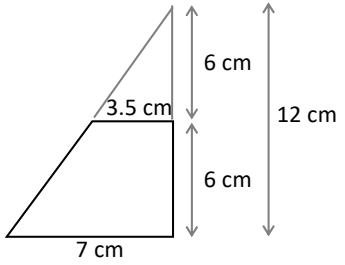
Fluency Practice

Finding the Surface Area of Frustums using Similar Triangles

① 3D View



Halved Cross-Section

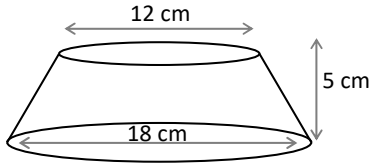


Curved Surface Area of Large Cone =

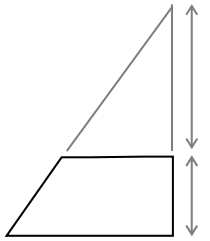
Curved Surface Area of Small Cone =

Curved Surface Area of Frustum
+ Top & Base =

② 3D View



Halved Cross-Section

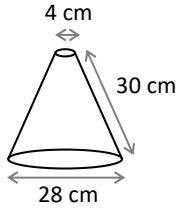


Curved Surface Area of Large Cone =

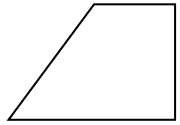
Curved Surface Area of Small Cone =

Curved Surface Area of Frustum
+ Top & Base =

③ 3D View



Halved Cross-Section

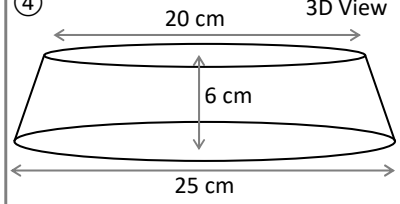


Curved Surface Area of Large Cone =

Curved Surface Area of Small Cone =

Curved Surface Area of Frustum
+ Top & Base =

④ 3D View



Halved Cross-Section

Curved Surface Area of Large Cone =

Curved Surface Area of Small Cone =

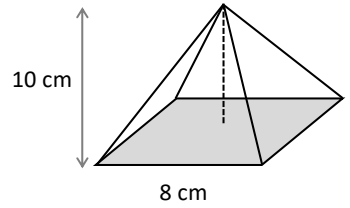
Curved Surface Area of Frustum
+ Top & Base =

Fluency Practice

Surface Area of Pyramids

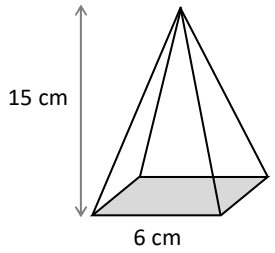
The apex is above the centre of a square base unless stated.

①



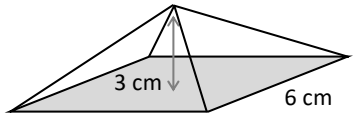
Surface Area =

②



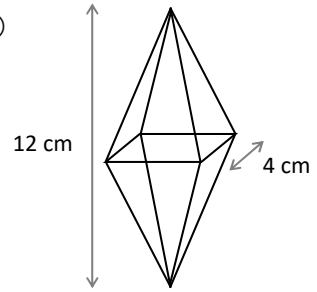
Surface Area =

③



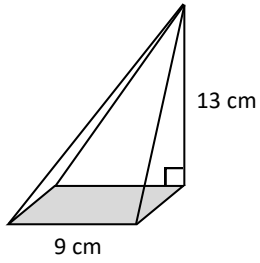
Surface Area =

④



Surface Area =

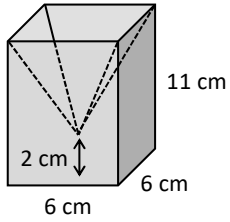
⑤



A pyramid with its apex vertically above a base corner.

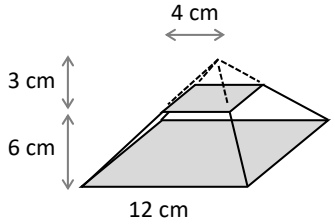
Surface Area =

⑥



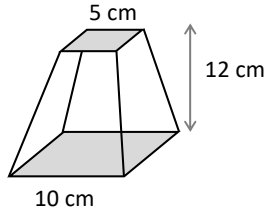
Surface Area =

⑦



Surface Area =

⑧



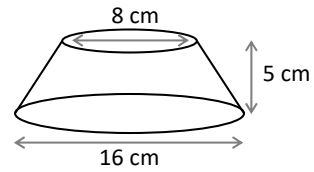
Surface Area =

7 Answers	413.84	219.56	340.30	86.91	236.33	390.75	101.19

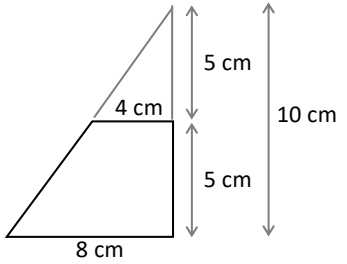
Fluency Practice

Finding the Volume & Surface Area of Frustums using Similar Triangles

① 3D View



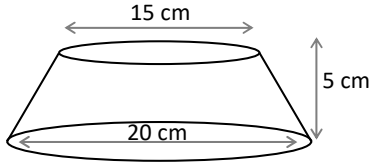
Halved Cross-Section



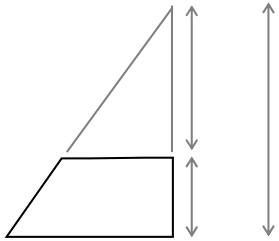
Volume =

Surface Area =

② 3D View



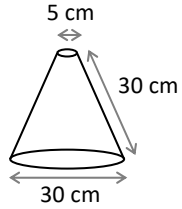
Halved Cross-Section



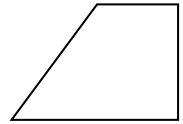
Volume =

Surface Area =

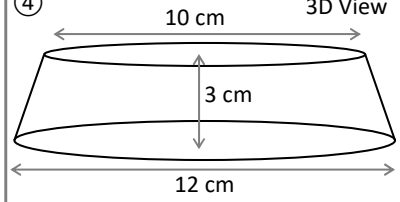
③ 3D View



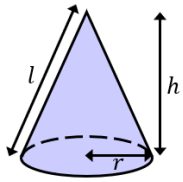
Halved Cross-Section



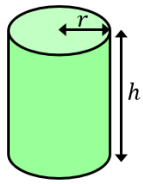
④ 3D View



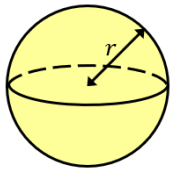
Volume of Cylinders, Cones and Spheres



Volume of Cone $= \frac{1}{3}\pi r^2 h$



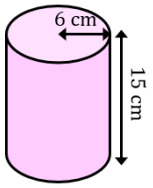
Volume of Cylinder $= \pi r^2 h$



Volume of Sphere $= \frac{4}{3}\pi r^3$

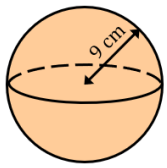
(a)

Find the volume, giving your answer in terms of π



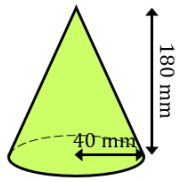
(b)

Find the volume, giving your answer to the nearest cm^3



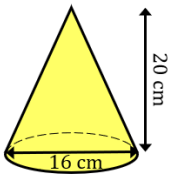
(c)

Find the volume, giving your answer to 3 significant figures



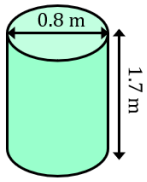
(d)

Find the volume, giving your answer to 3 significant figures



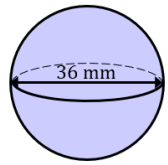
(e)

Find the volume, giving your answer to 2 decimal places



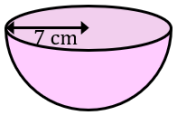
(f)

Find the volume, leaving your answer in terms of π



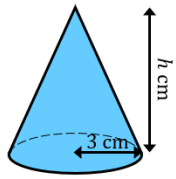
(g)

Find the volume of the hemisphere to the nearest cm^3



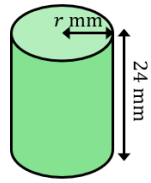
(h)

The cone has a volume of $39\pi\ \text{cm}^3$. Find the height h.



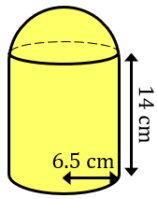
(i)

The cylinder has a volume of $6100\ \text{mm}^2$. Find its radius to the nearest mm.



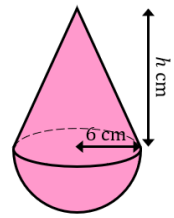
(j)

A shape is made by joining a hemisphere to a cylinder. Both have a radius of $6.5\ \text{cm}$. Find the total volume of the shape.

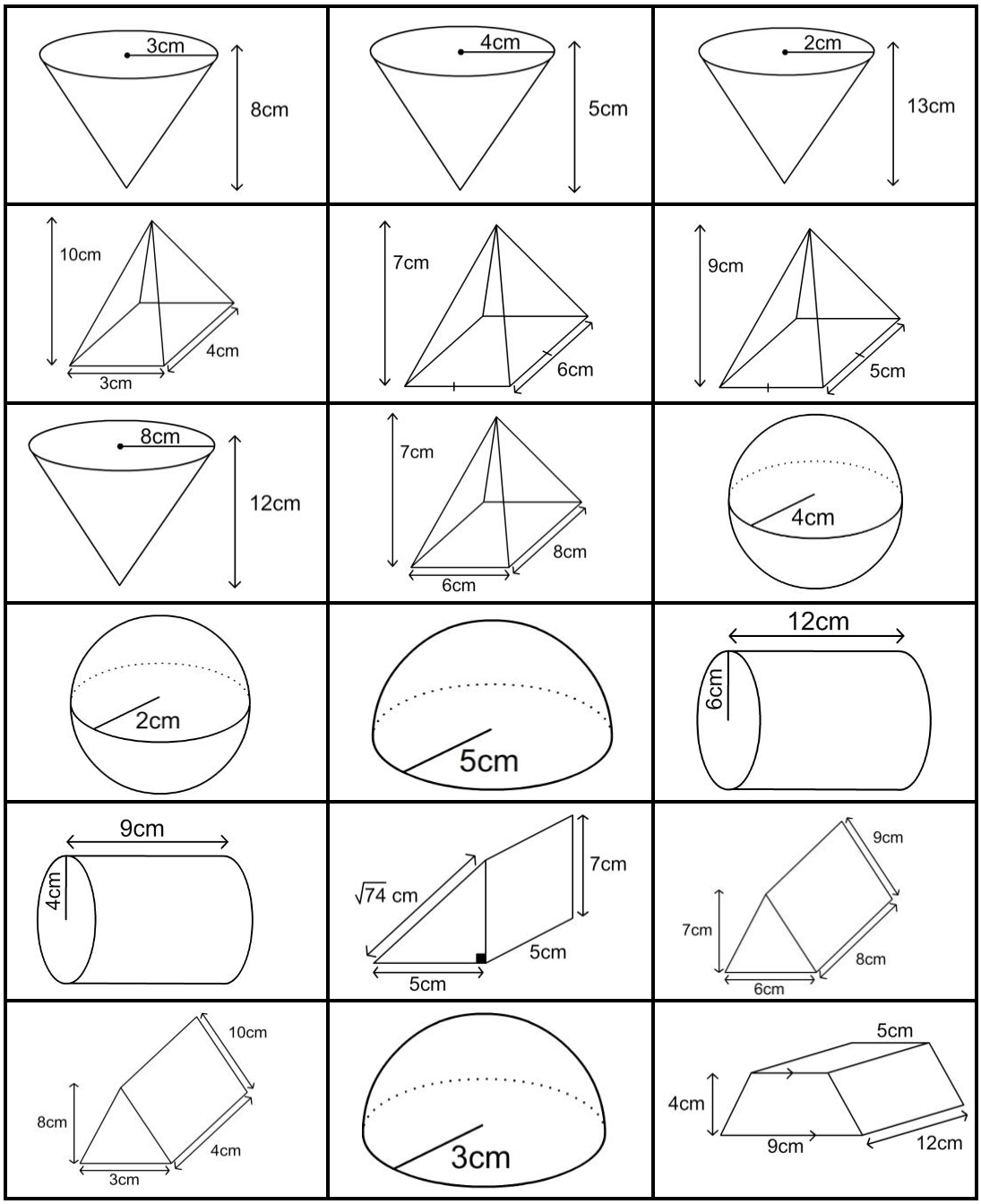


(k)

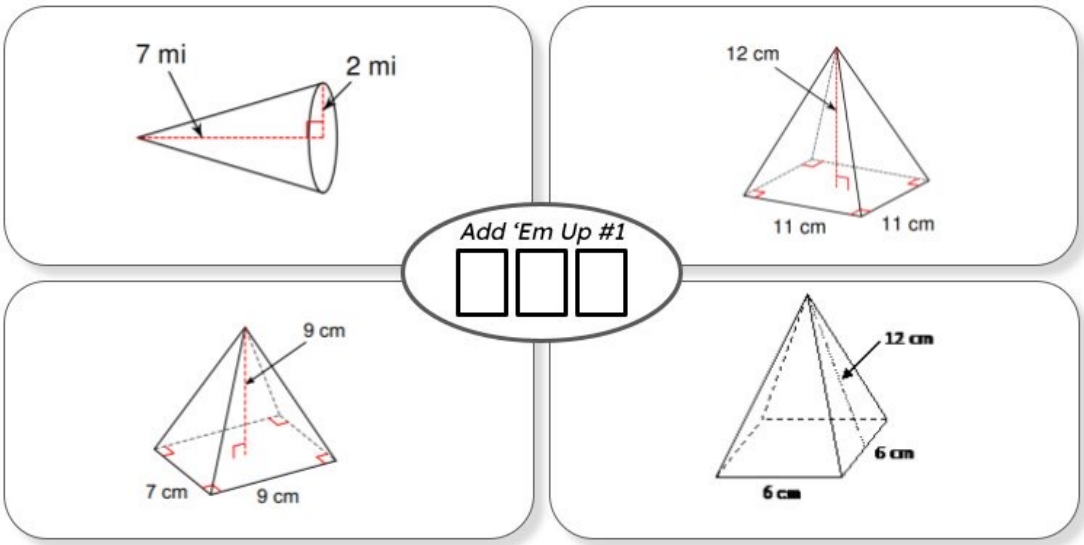
A shape is made by joining a cone to a hemisphere, where both shapes have the same radius. The total volume is $402\pi\ \text{cm}^3$. Find the height of the cone.

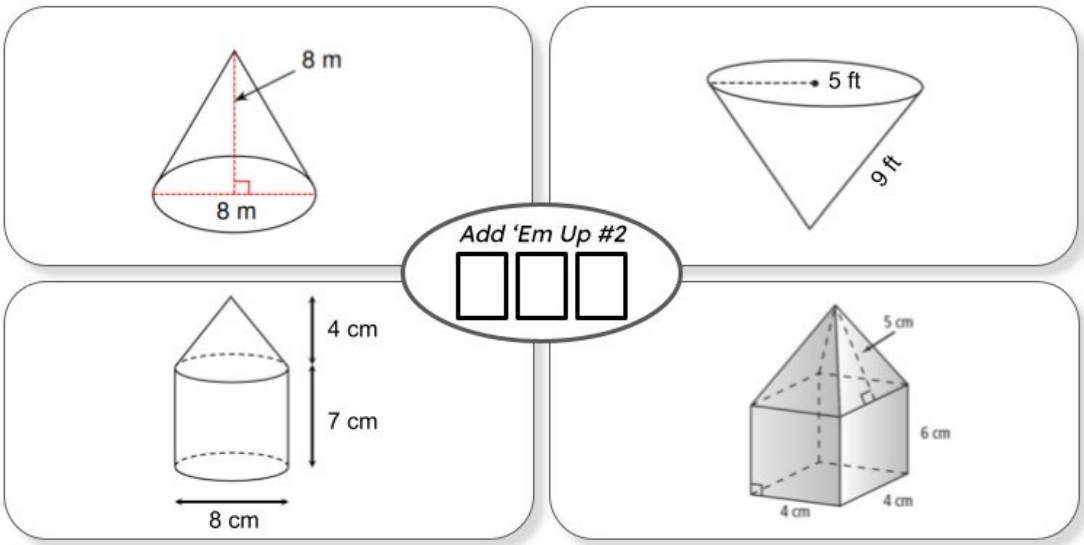


In each row, determine which shape has the largest volume.



Find the volume of each shape. Add the solutions to all four problems in each set together and record in the centre oval. Round all answers to the nearest whole unit.





13 The diagram shows a swimming pool in the shape of a prism.

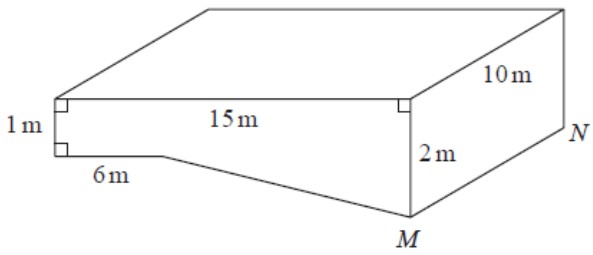


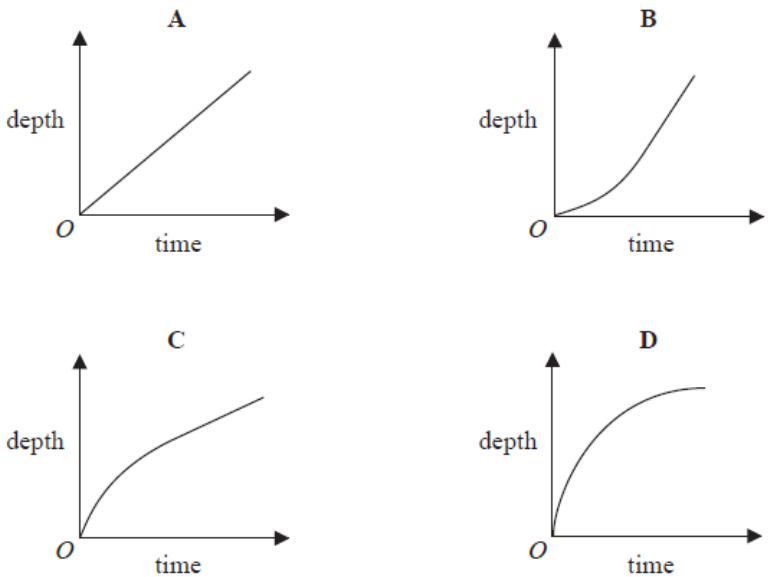
Diagram **NOT** accurately drawn

The swimming pool is empty.

The swimming pool is filled with water at a constant rate of 50 litres per minute.

(a) Work out how long it will take for the swimming pool to be completely full of water.
Give your answer in hours.
($1\,m^3 = 1000$ litres)

Here are four graphs.



(b) Write down the letter of the graph that best shows how the depth of the water in the pool above the line *MN* changes with time as the pool is filled.

.....

(1)

(Total for Question 13 is 6 marks)

17 Sumeet has a pond in the shape of a prism.

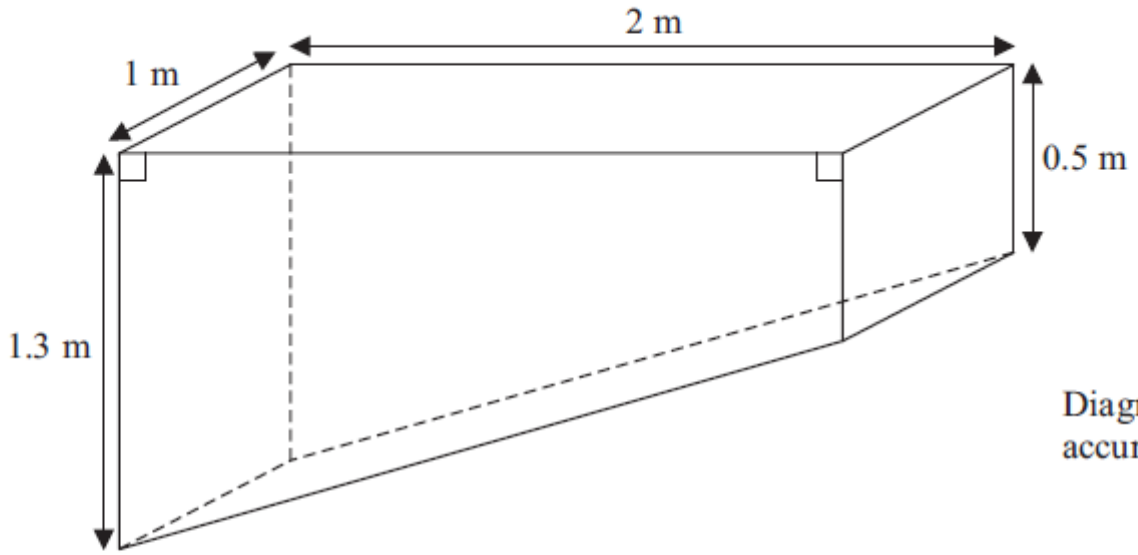


Diagram **NOT** accurately drawn

The pond is completely full of water.
Sumeet wants to empty the pond so he can clean it.
Sumeet uses a pump to empty the pond.

The volume of water in the pond decreases at a constant rate.
The level of the water in the pond goes down by 20 cm in the first 30 minutes.

Work out how much more time Sumeet has to wait for the pump to empty the pond completely.

23 The diagram shows a container for grain.

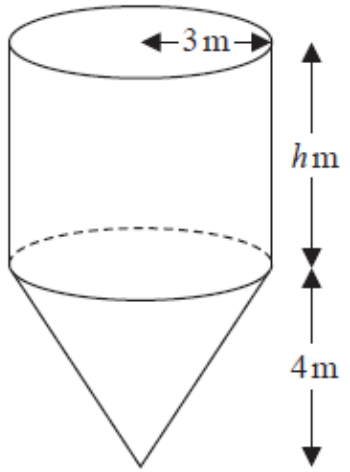


Diagram **NOT** accurately drawn

The container is a cylinder on top of a cone.
The cylinder has a radius of 3 m and a height of h m.
The cone has a base radius of 3 m and a vertical height of 4 m.

The container is empty.
The container is then filled with grain at a constant rate.

After 5 hours the depth of the grain is 6 metres above the vertex of the cone.
After 9 hours the container is full of grain.

Work out the value of h.
Give your answer as a fraction in its simplest form.
You must show all your working.

6.

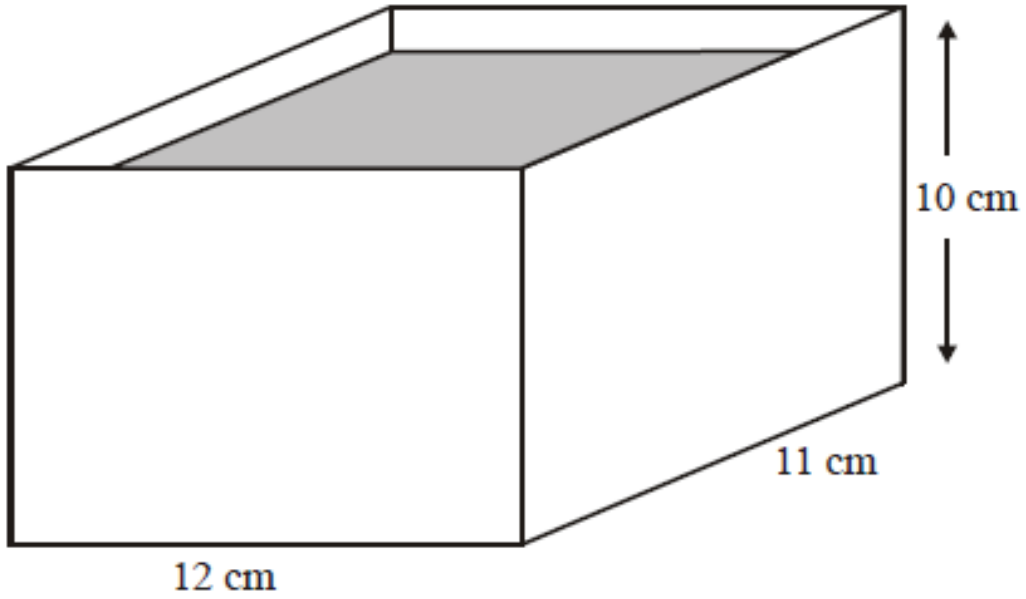


Diagram **NOT** accurately drawn

A rectangular container is 12 cm long, 11 cm wide and 10 cm high.
The container is filled with water to a depth of 8 cm.

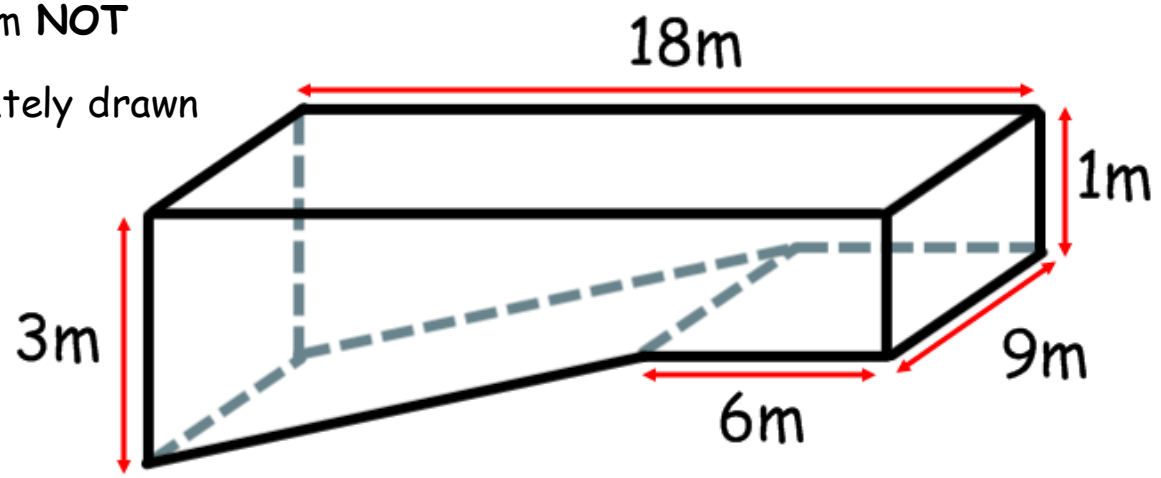
A metal sphere of radius 3.5 cm is placed in the water.
It sinks to the bottom.

Calculate the rise in the water level.
Give your answer correct to 3 significant figures.

1) James has a swimming pool in the shape of a prism.

Diagram **NOT**

Accurately drawn



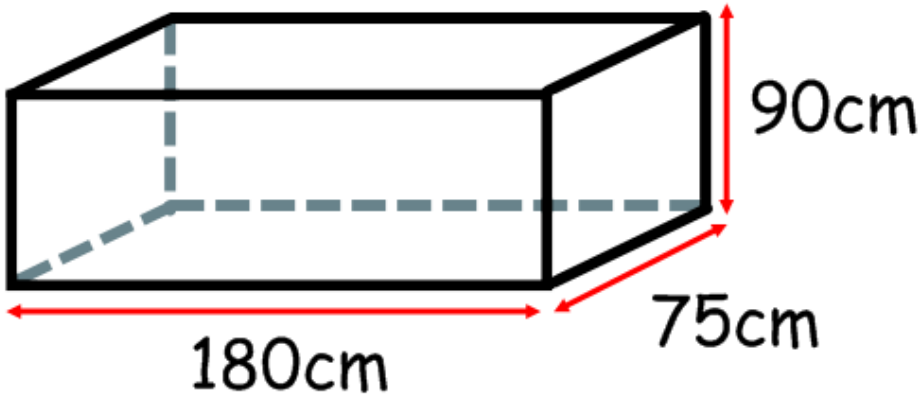
The swimming pool is empty.
It is filled with water at a constant rate.
It takes 4 hours for the water to be 2 meters deep from the deepest point.

a) How long will it take to completely fill the pool?
Give your answer in hours.
($1m^3$ = 1000litres)
You must show all your working.

2) Abigail has a fish tank in the shape of a cuboid.

Diagram **NOT**

Accurately drawn



The fish tank is filled with water at a constant rate.

It takes 6.35 seconds to fill 1 gallon.

The fish tank is empty.

a) How long will it take to fill the whole tank?

$1cm^3$ = 0.001 litres

1 gallon = 4.54609 litres

You must show all your working.

3) Sachyham has built a new structure to store grain.

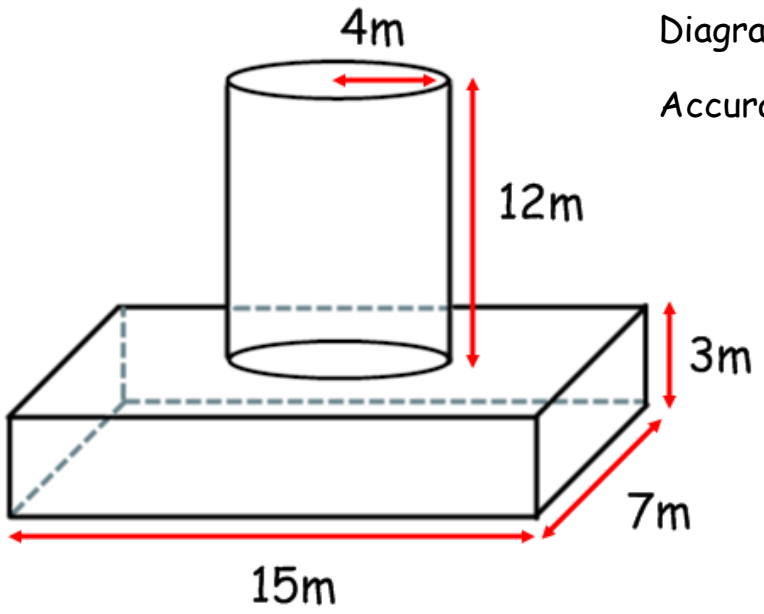


Diagram **NOT**

Accurately drawn

The structure is empty.
The structure fills with grain at a constant rate.
After 3 hours the structure is filled 5 meters above the centre of the base.

a) How long will it take to fill the structure?
Round your answer to the nearest minute.
You must show all your working.

4) The diagram shows a storage container for flour.

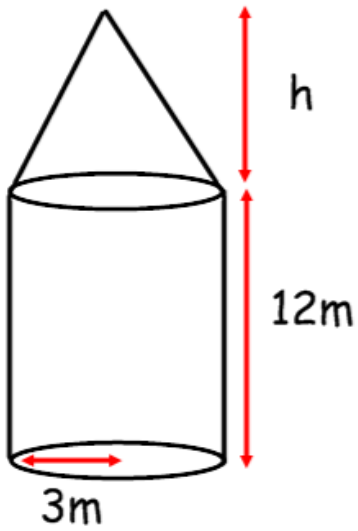


Diagram **NOT** Accurately drawn

The container is a cone on top of a cylinder.
The cylinder has a radius of 3m and a height of 12m.
The cone has a radius of 3m and a height of hm.

The container is empty.
The container is then filled with flour at a constant rate.

After 3 hours the depth of the flour is 5 meters high.
After 8 hours the container if full of flour.

Work out the value of h.
You must show all your working.

5) The diagram shows a glass water container.

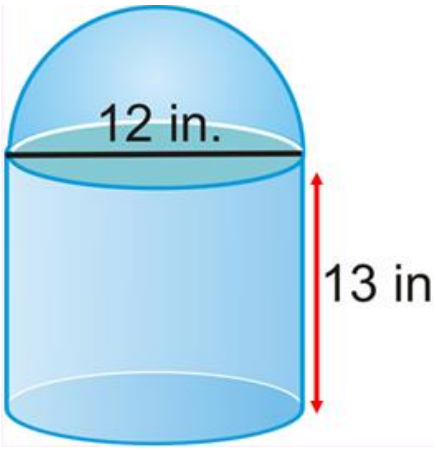


Diagram **NOT**

Accurately drawn

The container is a hemisphere on top of a cylinder.
The cylinder has a diameter of 12 inches and a height of 13 inches.
The hemisphere has a diameter of 12 inches.

The container is empty.
The container is then filled with water at a constant rate.

After 9 minutes the depth of the water is 7 inches high.
Will the container be full after 21 minutes?

Explain your answer.
You must show all your working.

6) The diagram shows a rocket.

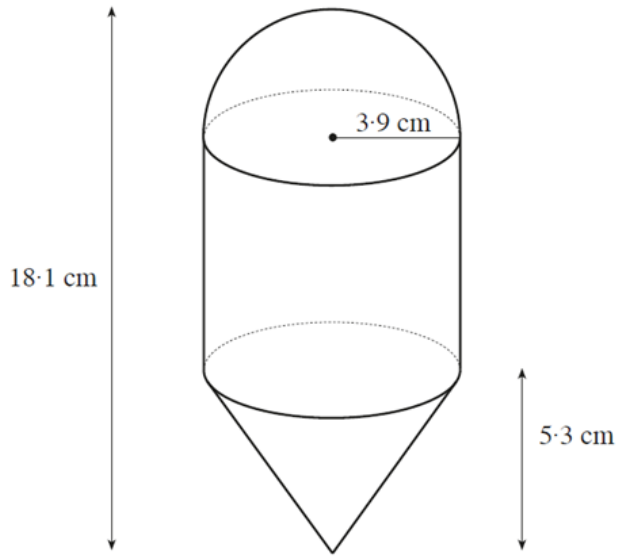


Diagram **NOT**

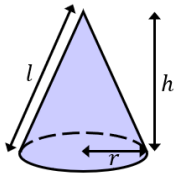
Accurately drawn

The rocket is a hemisphere on top of a cylinder on top of a cone.
The hemisphere has a radius of 3.9cm.
The cone has a height of 5.3cm.
The whole rocket has a height of 18.1cm.

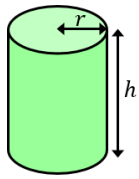
The container is empty.
The rocket is then filled to the top with fuel at a constant rate.

After 15 minutes the depth of the fuel will be 7.7cm above the vertex of the cone.
How long will it take to fill the whole rocket?
Round your answer to the nearest minute.
You must show all your working.

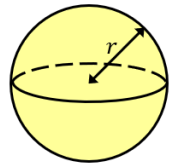
Surface Areas of Cylinders, Cones and Spheres



Curved Surface Area of Cone $= \pi rl$



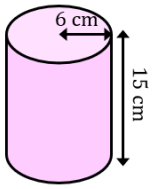
Curved Surface Area of Cylinder $= 2\pi rh$



Surface Area of Sphere $= 4\pi r^2$

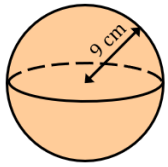
(a)

Find the curved surface area, giving your answer in terms of π



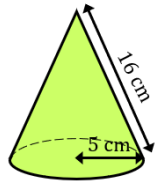
(b)

Find the surface area, giving your answer to 3 significant figures



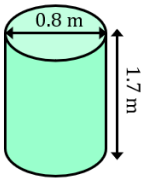
(c)

Find the curved surface area giving your answer to the nearest cm^2



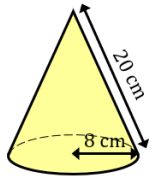
(d)

Find the **total** surface area, giving your answer to 2 decimal places



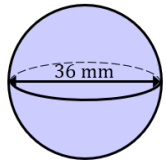
(e)

Find the **total** surface area, giving your answer to the nearest cm^2



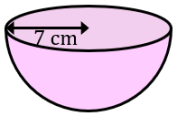
(f)

Find the surface area, leaving your answer in terms of π



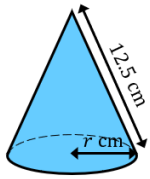
(g)

Find the total surface area of the hemisphere, leaving your answer in terms of π



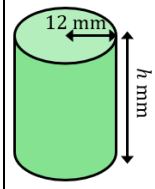
(h)

The cone has a curved surface area of $177\ \text{cm}^2$. Find the radius r to 1 decimal place.



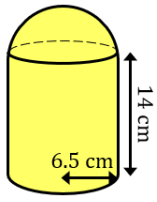
(i)

The total surface area is $744\pi\ \text{mm}^2$. Find the height of the cylinder.



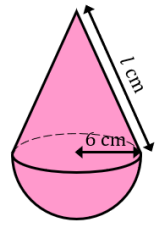
(j)

A shape is made by joining a hemisphere to a cylinder. Both have a radius of $6.5\ \text{cm}$. Find the surface area of the compound shape to the nearest cm^2.



(k)

A shape is made by joining a cone to a hemisphere, where both shapes have the same radius. The total surface area is $246\pi\ \text{cm}^2$. Find the slanted height l of the cone.

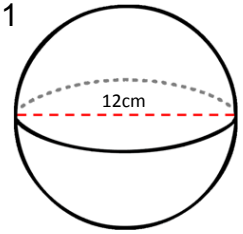


Fluency Practice

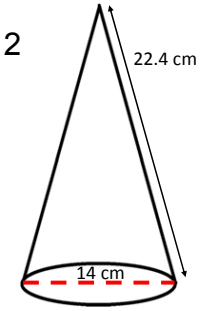
surface area

Calculate the surface area of these solids. Match your answers to the ones below. There is one missing answer. Complete this yourself.

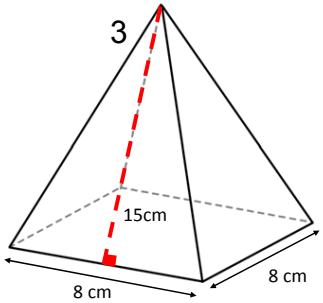
1



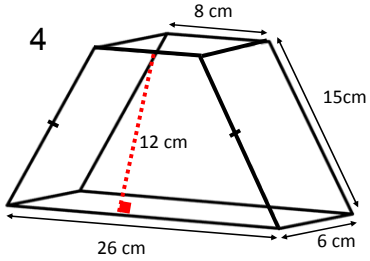
2



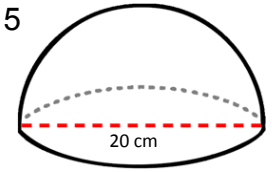
3



4

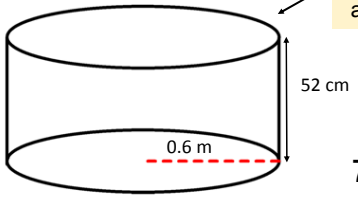


5

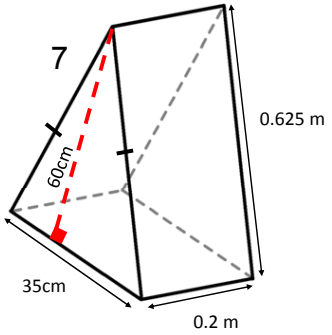


6

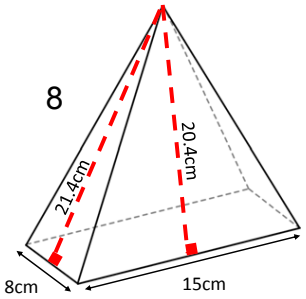
Careful! The units are not the same.



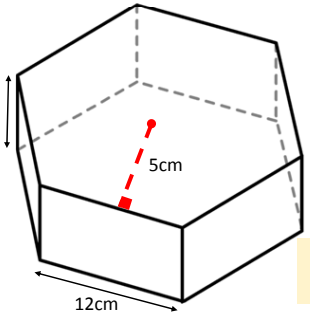
7



8



9



This is a regular hexagonal prism.

$942.5cm^2$

$646.5cm^2$

$5300cm^2$

$452.4cm^2$

$304cm^2$

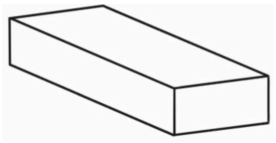
$42223cm^2$

$936cm^2$

$792cm^2$

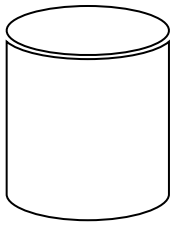
Find the missing value in each question.

1



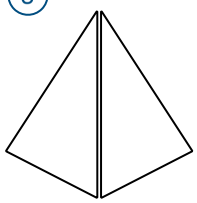
Volume = 800cm^3
Height = ?
Area of Base = 8cm

2



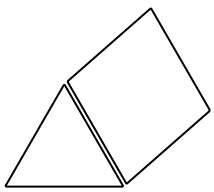
Volume = 1200cm^3
Height = ?
Radius = 8cm

3



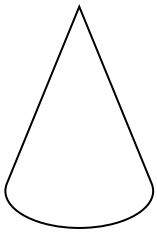
Volume = 1500cm^3
Height = ?
Area of base = 30cm^2

4



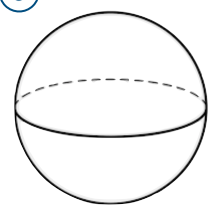
Volume = 2000cm^3
Height = ?
Area of cross-section = 15cm^2

5



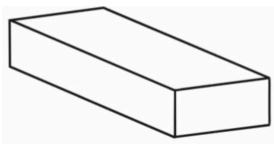
Volume = 300cm^3
Height = ?
Radius = 4cm

6



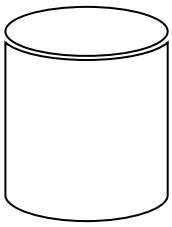
Volume = 800cm^3
Radius = ?

7



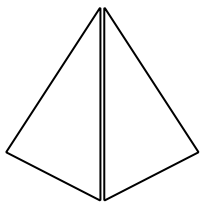
Volume = 1200cm^3
Height = 12cm
Area of Base =

8



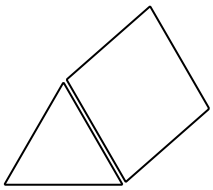
Volume = 900cm^3
Height = 15cm
Radius = ?

9



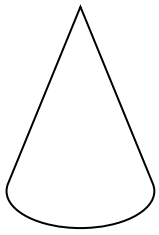
Volume = 600cm^3
Height = 12cm
Area of base = ?

10



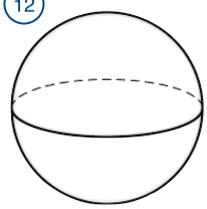
Volume = 3000cm^3
Height = 60cm
Area of cross-section = ?

11



Volume = 2000cm^3
Height = 5cm
Radius = ?

12

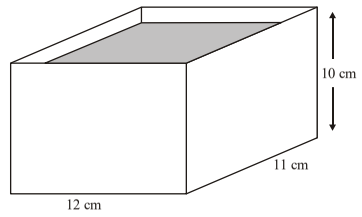


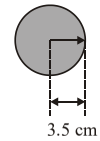
Volume = 9000cm^3
Diameter = ?

1. A rectangular container is 12 cm long, 11 cm wide and 10 cm high.
The container is filled with water to a depth of 8 cm.

A metal sphere of radius 3.5 cm is placed in the water.
It sinks to the bottom.

Calculate the rise in the water level.
Give your answer correct to 3 significant figures.

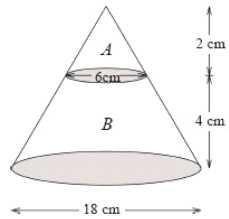


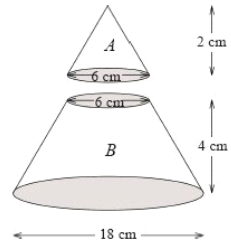


2. The diagram represents a large cone of height 6 cm and base diameter 18 cm.

The large cone is made by placing a small cone A of height 2 cm and base diameter 6 cm on top of a frustum B.

Calculate the volume of the frustum B.
Give your answer in terms of π.

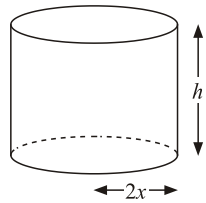


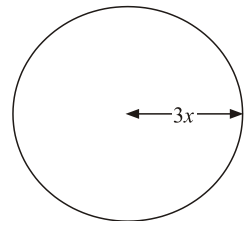


3. The diagram shows a cylinder and a sphere.

The radius of the base of the cylinder is $2x$ cm and the height of the cylinder is h cm.
The radius of the sphere is $3x$ cm.
The volume of the cylinder is equal to the volume of the sphere.

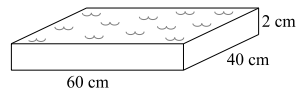
Express h in terms of x.
Give your answer in its simplest form.

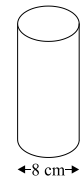




4. A rectangular tray has length 60 cm, width 40 cm and depth 2 cm.
It is full of water.
The water is poured into an empty cylinder of diameter 8 cm.

Calculate the depth, in cm, of water in the cylinder.
Give your answer correct to 3 significant figures.

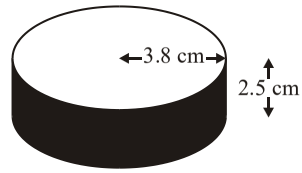




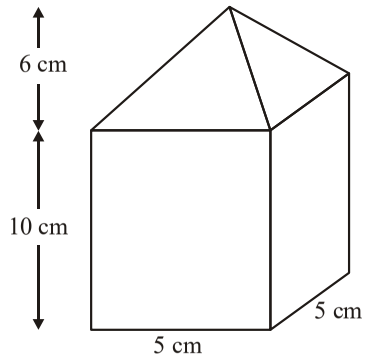
5. An ice hockey puck is in the shape of a cylinder with a radius of 3.8 cm, and a thickness of 2.5 cm.

It is made out of rubber with a density of 1.5 grams per cm^3.

Work out the mass of the ice hockey puck.
Give your answer correct to 3 significant figures.



6. The diagram shows a model.
The model is a cuboid with a pyramid on top.
The base of the model is a square with sides of length 5 cm.
The height of the cuboid in the model is 10 cm.
The height of the pyramid in the model is 6 cm.

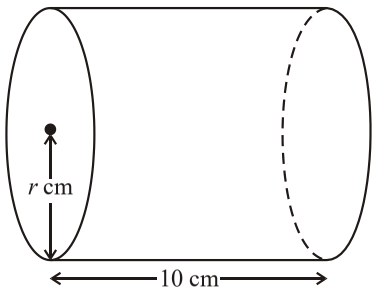


(a) Calculate the volume of the model.

The model represents a concrete post.
The model is built to a scale of 1:30
The surface area of the model is 290 cm^2.

(b) Calculate the surface area of the post.
Give your answer in square metres.

7. The diagram shows a cylinder.
The radius of the cylinder is r cm.
The length of the cylinder is 10 cm.



The volume of the cylinder is 140 cm^3.

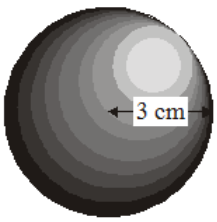
Work out the value of r.
Give your answer correct to 3 significant figures.

8. Ben fills a container with boxes.
Each box is a cube of side 0.5 m.

The container is a cuboid of
length 9 m,
width 4 m and
height 3 m.

Work out how many boxes will fit exactly into the container.

9.



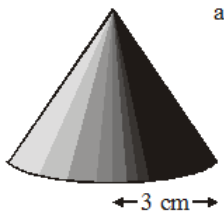


Diagram **NOT** accurately drawn

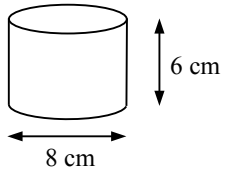
The radius of a sphere is 3 cm.
The radius of the base of a cone is also 3 cm.
The volume of the sphere is 3 times the volume of the cone.

Work out the curved surface area of the cone.
Give your answer as a multiple of π.

Fluency Practice

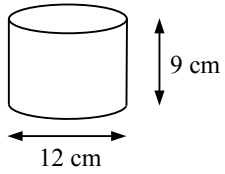
A1

Calculate the **curved** surface area.



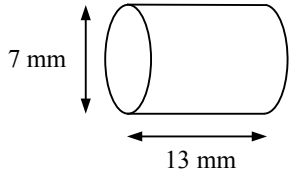
A2

Calculate the volume.



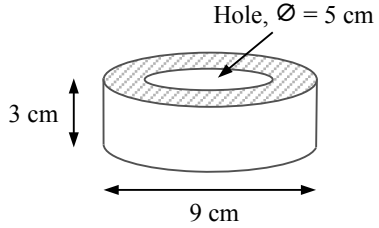
A3

Calculate the **total** surface area.



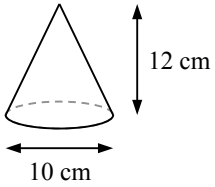
A4

Calculate the volume.



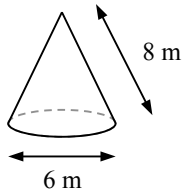
B1

Calculate the volume.



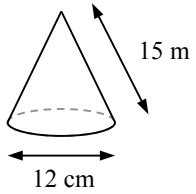
B2

Calculate the **curved** surface area.



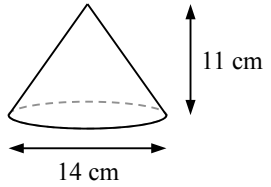
B3

Calculate the volume.



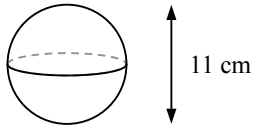
B4

Calculate the **total** surface area.



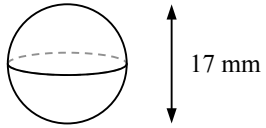
C1

Calculate the surface area.



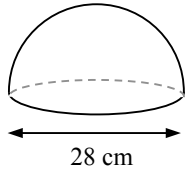
C2

Calculate the volume.



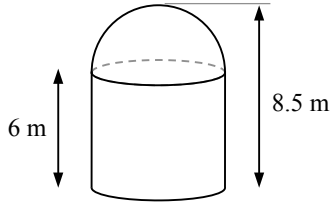
C3

Calculate the **total** surface area of the hemisphere.



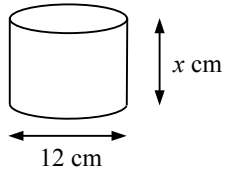
C4

Calculate the **total** surface area.



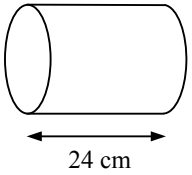
Fluency Practice

A1 Volume is 340 cm^3



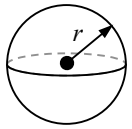
Calculate the height, x.

A2 Volume is 1500 cm^3



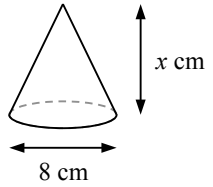
Calculate the radius.

A3 Surface area is 180 cm^2



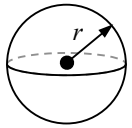
Calculate the radius, r.

A4 Volume is 140 cm^3



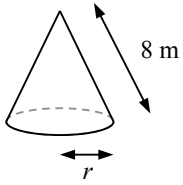
Calculate the height, x.

B1 Volume is 540 cm^3



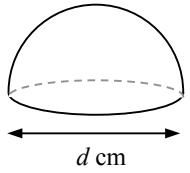
Calculate the radius, r.

B2 **Curved** surface area is 90 m^2



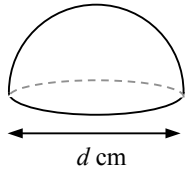
Calculate the radius, r.

B3 Volume is 300 cm^3



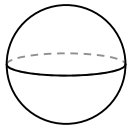
Calculate the diameter, d.

B4 **Total** surface area is 540 cm^2



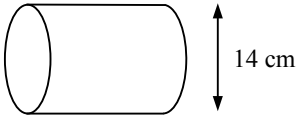
Calculate the diameter, d.

C1 Volume is 268 cm^3



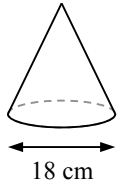
Find the surface area.

C2 Volume is 770 cm^3



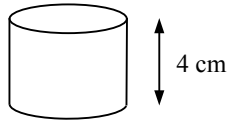
find the **total** surface area.

C3 Volume is 594 cm^3



Find the curved surface area.

C4 **Total** surface area is 100 cm^2



Find the volume.

Fluency Practice

Volume and Surface Area Revision

(a)

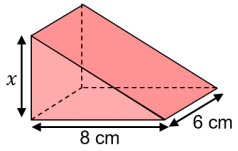
The volume of a cuboid is $320\ cm^3$. Its height is $16\ cm$ and the length is $5\ cm$. Find the width of the cuboid.

(b)

Find the surface area of a cube with side length $8.5\ cm$.

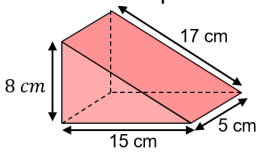
(c)

The volume of the prism is $264\ cm^3$. Find x.



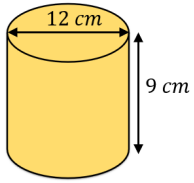
(d)

Find the surface area of this prism.



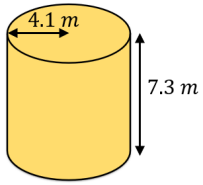
(e)

Find the volume of the cylinder, giving your answer to 3 significant figures.



(f)

Find the total surface area of the cylinder, leaving your answer in terms of π.

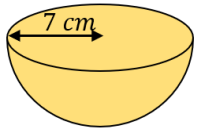


(g)

The volume of a sphere is $288\pi\ cm^3$. Find the radius of the sphere.

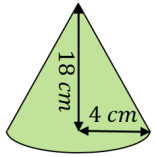
(h)

Find the total surface area of the hemisphere, giving your answer to 3 significant figures.



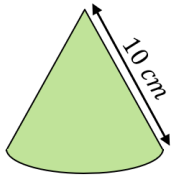
(i)

Find the volume of the cone, leaving your answer in terms of π.



(j)

A cone has a slanted height of $10\ cm$ and a curved surface area of $60\pi\ cm^2$. Find the volume of the cone, giving your answer to 3 significant figures.

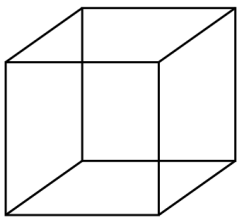


(k)

A cylinder has a height of $16\ cm$ and a radius of $x\ cm$. A sphere has a radius of $2x\ cm$. The volume of the cylinder and the sphere are equal. Find the value of x.

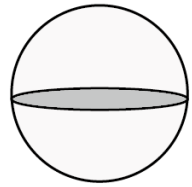
Fluency Practice

Mixed Volume and Surface Area Problems			
(a)	**(b)**	**(c)**	**(d)**
The surface area of a sphere with radius $10\ cm$ is equal to the curved surface area of a cylinder with the same radius as the sphere and height $h\ cm$. Find the height h.	A cylinder with height $h\ cm$ and radius $6\ cm$ has the same volume as a sphere with radius $9\ cm$. Find the value of h.	A metal cylinder is to be melted down and turned into spheres with radius $3\ cm$. The cylinder has a radius of $12\ cm$ and a height of $25\ cm$. How many whole spheres can be made?	A cone with slanted height $25\ cm$ and radius $8\ cm$ has the same curved surface area as a hemisphere. Find the radius r of the hemisphere.
(e)	**(f)**	**(g)**	**(h)**
A cylinder has a radius r and height $15r$. A sphere has radius $3r$. Find the ratio of the volume of the sphere to the volume of the cylinder in its simplest form.	A hemisphere with radius $2r$ has the same total surface area as a cylinder with radius r. Find the height of the cylinder in terms of r.	A cone has a radius of $\frac{3}{2}x$ and a height of $3x$. A sphere has a radius of kx. The ratio of the volume of the cone to the volume of the sphere is $4:1$. Find the value of k as a fraction in its simplest form.	A hemisphere of radius $(r+2)$ is attached to the base of a cone with radius $(r+2)$ and slant height $5r$. The total surface area of the compound shape is 273π. Find the volume of the compound shape.



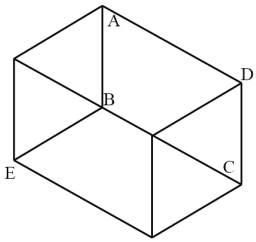
The cube has volume $125cm^3$

Find its surface area.



The shaded circle bisects the sphere.

If the circle has area $49\pi\ cm^2$ find the surface area of the sphere.

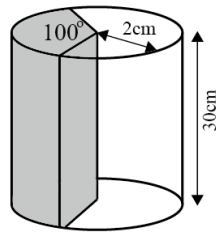


BC = 6cm

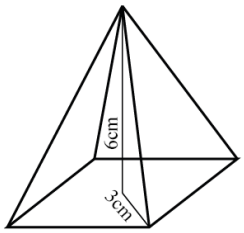
AC = $3\sqrt{5}$ cm

AE = $\sqrt{13}$ cm

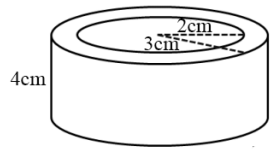
Find the surface area of the shape.



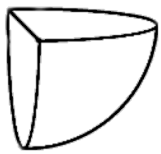
Find the surface area of the shaded shape.



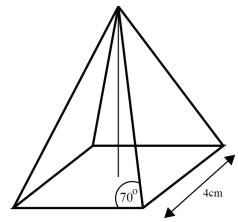
Find the surface area of the square based pyramid



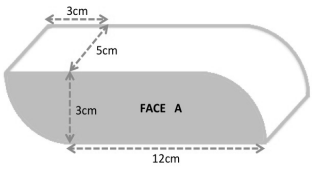
Find the surface area of the hollow tube.



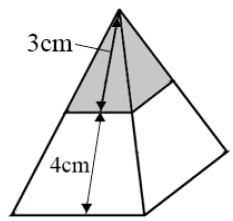
A quarter of a sphere is shown. If the volume of this shape is $9\pi\ cm^3$ what is its surface area?



Find the surface area of the square-based pyramid.

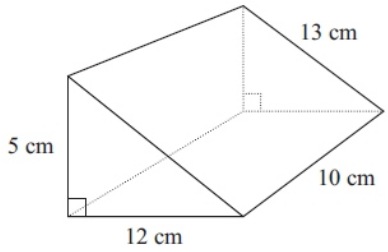


Assuming face A has rotational symmetry of order 2, calculate the surface area of the whole 3-D shape.



The tip of the square-based pyramid is painted as shown, using $30cm^2$ of paint. If $200cm^2$ more paint is needed to cover the entire shape, what is the width of the base?

Q1.



Work out the total surface area of this triangular prism.

Q2. Here is a vase in the shape of a cylinder.

The vase has a radius of 5 cm.

There are 1000 cm^3 of water in the vase.

Work out the depth of the water in the vase.
Give your answer correct to 1 decimal place.

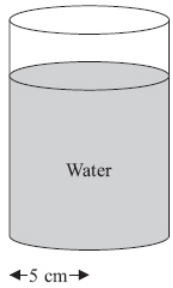


Diagram **NOT** accurately drawn

Q3. The diagram shows a circle drawn inside a square.

The circle has a radius of 6 cm. The square has a side of length 12 cm. Work out the shaded area. Give your answer in terms of π.

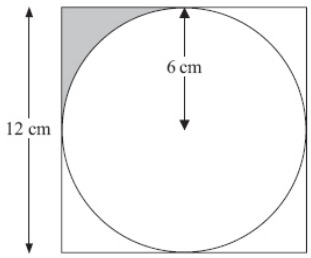


Diagram **NOT** accurately drawn

Q4.

The diagram shows a solid hemisphere of radius 5 cm.

Find the **total** surface area of the solid hemisphere.
Give your answer in terms of π.

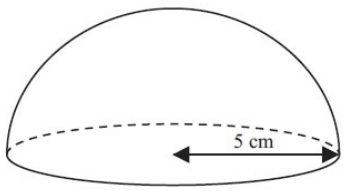


Diagram **NOT** accurately drawn

Q5.

A frustum is made by removing a small cone from a similar large cone.

The height of the small cone is 20 cm.
The height of the large cone is 40 cm.
The diameter of the base of the large cone is 30 cm.

Work out the volume of the frustum.
Give your answer correct to 3 significant figures.

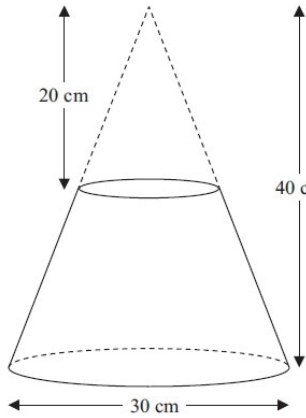


Diagram **NOT** accurately drawn

Q6. Sumeet has a pond in the shape of a prism.

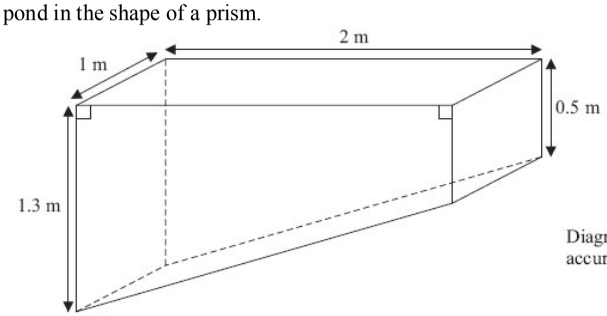


Diagram **NOT** accurately drawn

The pond is completely full of water.
Sumeet wants to empty the pond so he can clean it.
Sumeet uses a pump to empty the pond.

The volume of water in the pond decreases at a constant rate.
The level of the water in the pond goes down by 20 cm in the first 30 minutes.

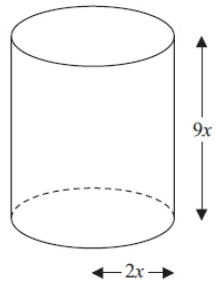
Work out how much more time Sumeet has to wait for the pump to empty the pond completely.

Q7. The diagram shows a solid metal cylinder.

The cylinder has base radius $2x$ and height $9x$.

The cylinder is melted down and made into a sphere of radius r.

Find an expression for r in terms of x.

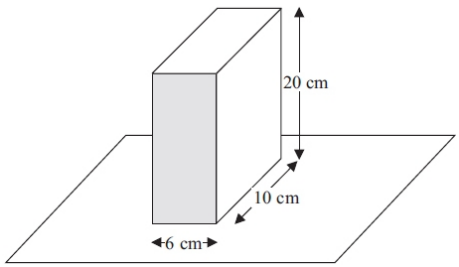


Q8. Jane has a carton of orange juice.
The carton is in the shape of a cuboid.

The depth of the orange juice in the carton is 8 cm.

Jane closes the carton.
Then she turns the carton over so that it stands on the shaded face.

Work out the depth, in cm, of the orange juice now.



Q9. Ella is designing a glass in the shape of a cylinder.

The glass must hold a minimum of ½ litre of liquid.

The glass must have a diameter of 8 cm.

Calculate the minimum height of the glass.

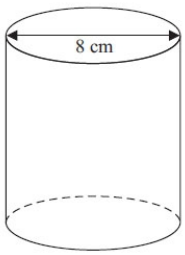


Diagram **NOT** accurately drawn

Q10. * The diagram shows the triangle PQR.

$PQ = x$ cm
$PR = 2x$ cm
Angle $QPR = 30°$

The area of triangle $PQR = A$ cm^2

Show that $x = \sqrt{2A}$

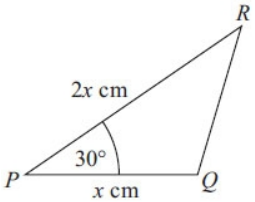


Diagram **NOT** accurately drawn

Q11.

The diagram shows a pyramid.

$BCDE$ is a square with sides of length 10 cm.
The other faces of the pyramid are equilateral triangles with sides of length 10 cm.

(a) Calculate the volume of the pyramid.
Give your answer correct to 3 significant figures.

(b) Find the size of angle DAB.

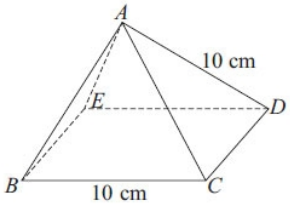


Diagram **NOT** accurately drawn

Q12.

ABCD is a square with a side length of $4x$
M is the midpoint of *DC*.
N is the point on *AD* where $ND = x$

BMN is a right-angled triangle.

Find an expression, in terms of x, for the area of triangle *BMN*.
Give your expression in its simplest form.

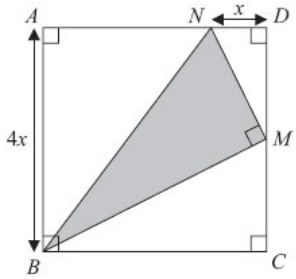
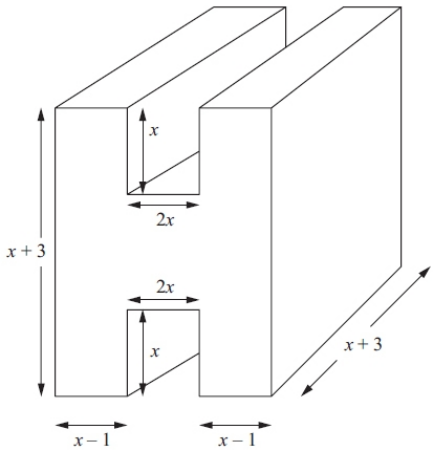


Diagram **NOT** accurately drawn



Q13.

The diagram shows a prism.
All measurements are in cm.
All corners are right angles.
The volume of the prism is V cm^3.

Find a formula for V.

Q14. A water trough is in the shape of a prism.

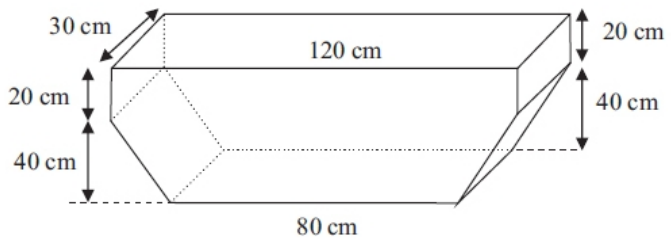


Diagram **NOT** accurately drawn

Hamish fills the trough completely.

Water leaks from the bottom of the trough at a constant rate.
2 hours later, the level of the water has fallen by 20 cm.

Water continues to leak from the trough at the same rate.

How many more minutes will it take for the trough to empty completely?

---

3 Arcs, Sectors and Segments

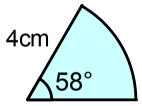
learn by heart

Length of an arc:

$\frac{\theta}{360} \times 2\pi r$

example

Calculate the arc length of the sector, correct to the nearest 0.01cm.



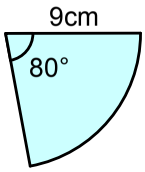
Arc length $= \frac{58}{360} \times 2 \times \pi \times 4$

$= 4.05\text{cm}$ (nearest 0.01cm)

exercise

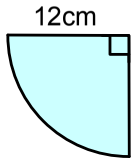
1. Calculate the arc length of each sector, correct to the nearest 0.01cm.

a)

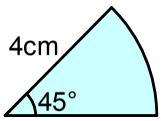


b)

c)

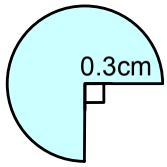


d)

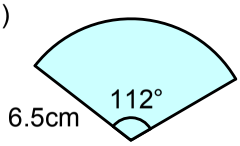


e)

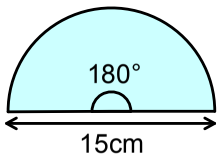
f)



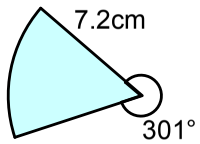
g)



h)

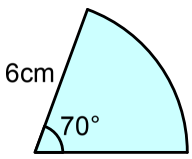


i)

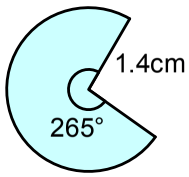


2. Calculate the **perimeter** of these shapes, correct to the nearest 0.01cm.

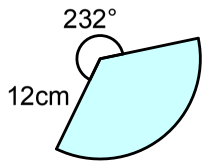
a)



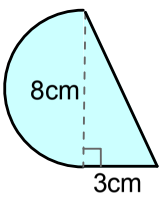
b)



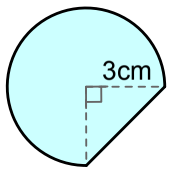
c)



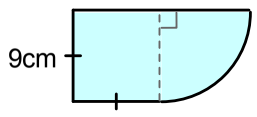
d)



e)



f)

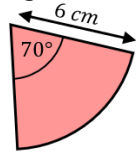


Fluency Practice

Arc Length and Perimeter of a Sector

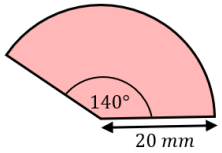
(a)

Find the arc length.



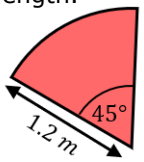
(b)

Find the arc length.



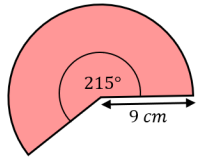
(c)

Find the arc length.



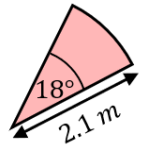
(d)

Find the arc length.



(e)

Find the arc length.

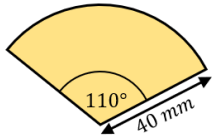


(f)

Find the perimeter.

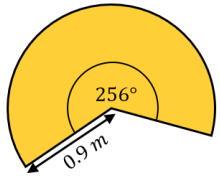
(g)

Find the perimeter.



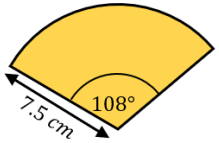
(h)

Find the perimeter.



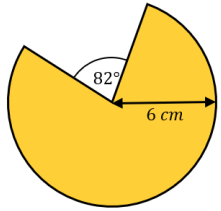
(i)

Find the perimeter.



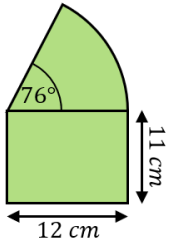
(j)

Find the perimeter.



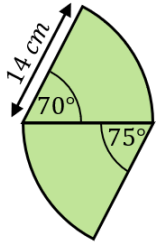
(k)

Find the perimeter.



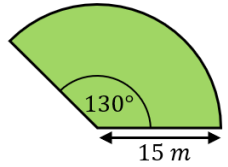
(l)

Find the perimeter.



(m)

Farmer Jo wants to put a fence around his sector-shaped field. Fencing costs £18.99 per metre. How much will it cost Farmer Jo?

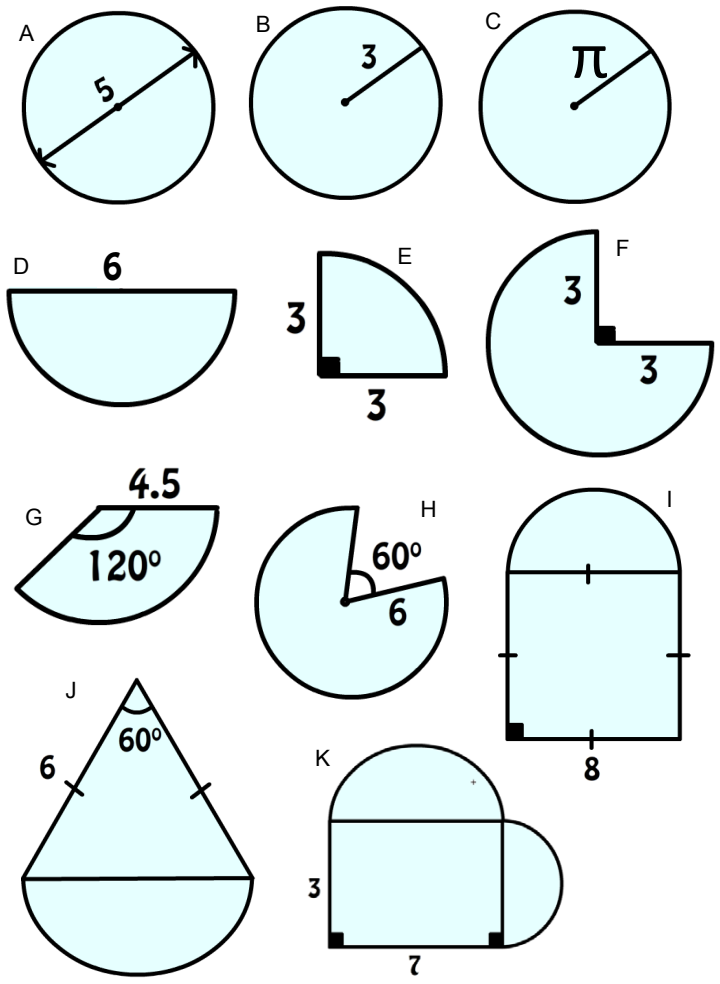


(n)

The pendulum of a clock is 48 cm long and swings through an angle of 15°. The pendulum swings back and forth every two seconds. How far does the end of the pendulum swing in a minute?

Circumference: Match Three!

Each of these pictures contains circles or parts of circles. Calculate the total perimeter of each shape. Find your answer in terms of Pi and as a decimal in the list below.

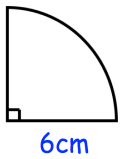


Perimeter in terms of π	Perimeter (2 d.p.)
$4.5\pi + 6$	18.42
$2\pi^2$	19.74
$10\pi + 12$	18.85
$3\pi + 12$	20.14
6π	36.57
5π	10.71
$3\pi + 9$	25.71
$5\pi + 10$	15.71
$1.5\pi + 6$	43.42
$4\pi + 24$	21.42
$3\pi + 6$	15.42

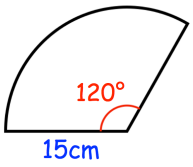
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		

Question 1: For each sector below, calculate the length of the arc.
Give your answers to one decimal place and include suitable units.

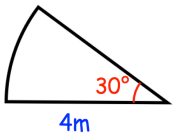
(a)



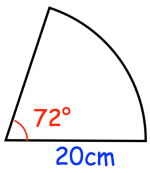
(b)



(c)

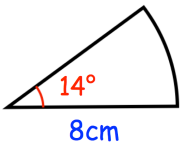


(d)

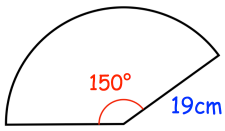


Question 2: For each sector below, calculate the length of the arc.
Give your answers to one decimal place and include suitable units.

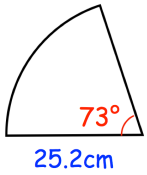
(a)



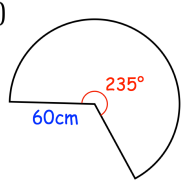
(b)



(c)

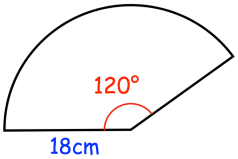


(d)

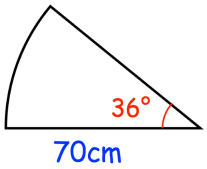


Question 3: For each sector below, calculate the length of the arc.
Leave your answer in terms of **π**

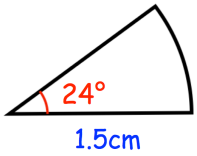
(a)



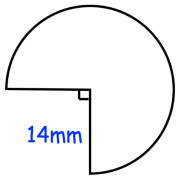
(b)



(c)

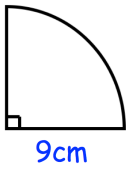


(d)

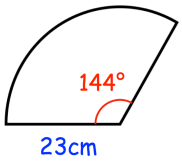


Question 4: Calculate the perimeter of each sector below
Give your answers to one decimal place and include suitable units.

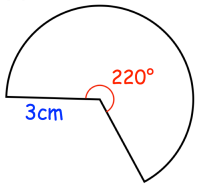
(a)



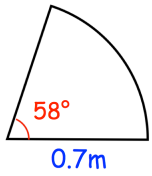
(b)



(c)

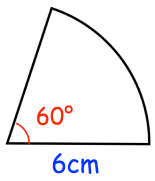


(d)

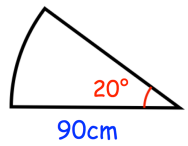


Question 5: Calculate the perimeter of each sector below
Leave your answer in terms of **π**

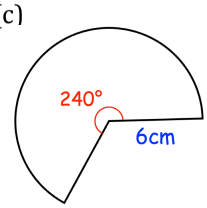
(a)



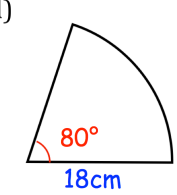
(b)



(c)



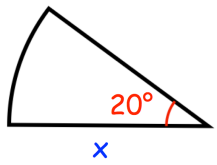
(d)



Question 6: The arc length of each sector has been given.
Calculate x
Give your answers to one decimal place and include suitable units.

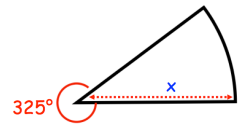
(a)

Arc Length = 50cm



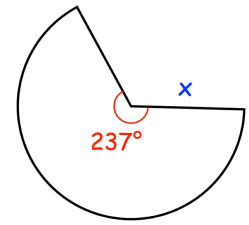
(b)

Arc Length = 85.2cm



(c)

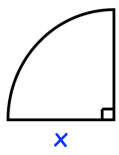
Arc Length = 9cm



Question 7: The arc length of each sector has been given.
Calculate x

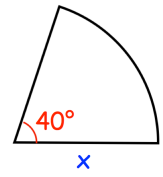
(a)

Arc Length = 15π cm



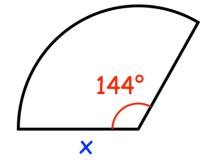
(b)

Arc Length = 6π cm



(c)

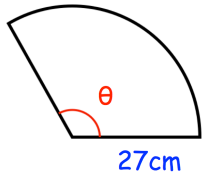
Arc Length = 10π cm



Question 8: The arc length of each sector has been given.
Calculate the size of the angle
Give your answers to one decimal place.

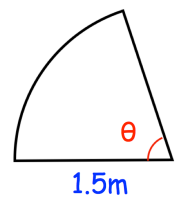
(a)

Arc Length = 56cm



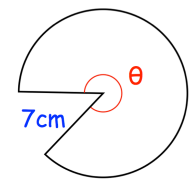
(b)

Arc Length = 1.8m



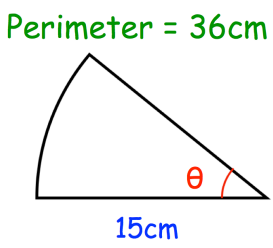
(c)

Arc Length = 41cm

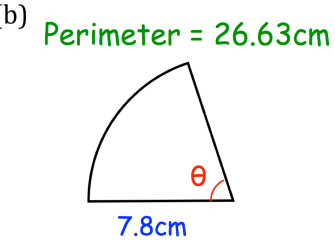


Question 9: The perimeter of each sector has been given.
Calculate the size of the angle
Give your answers to one decimal place.

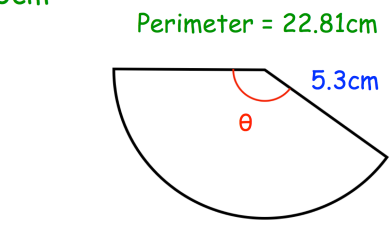
(a)



(b)

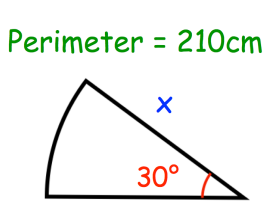


(c)

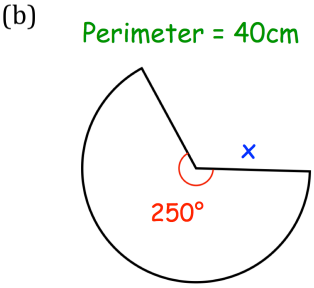


Question 10: The perimeter of each sector has been given.
Calculate x
Give your answers to one decimal place.

(a)

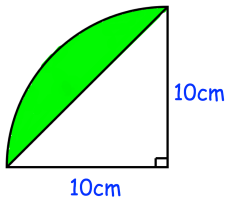


(b)

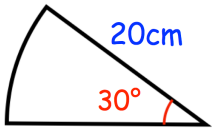


Apply

Question 1: Calculate the perimeter of the segment.



Question 2: James is calculating the perimeter of the sector.
Can you spot any mistakes?

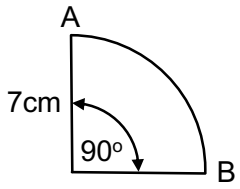


$$\begin{aligned} &Perimeter \\ &= \frac{30}{360} \times \pi \times 20 \\ &= 5.236cm \end{aligned}$$

Fluency Practice

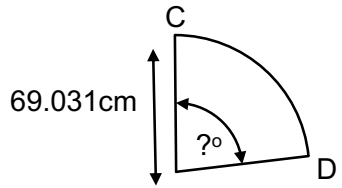
lengths of arcs (i)

(1)



what is the length of arc AB ?

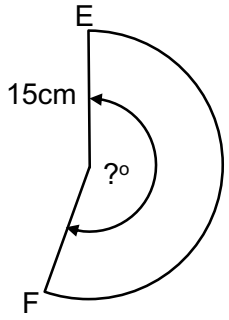
(2)



the length of the arc CD is 100cm

what is the angle?

(3)



the arc EF is to have an length of 50cm

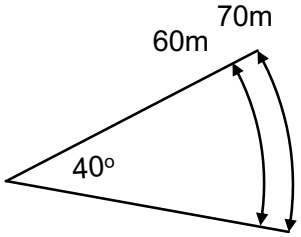
what is the angle?

(4) the radius of the London Eye is 60m

there are 32 equally spaced capsules

how far is it between two adjacent capsules?

(5)



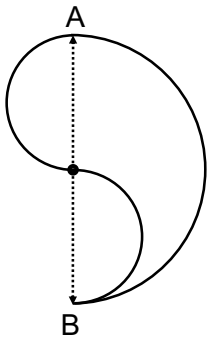
in a 40° (college) discus throwing area

what is the difference in arc lengths for the 60m and 70m arcs?

Fluency Practice

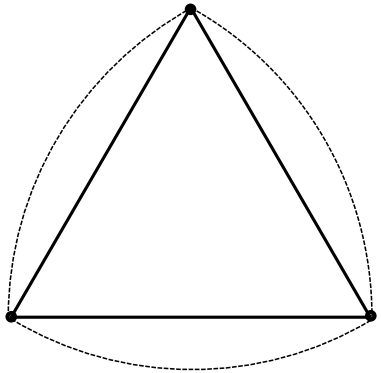
lengths of arcs (ii)

(1)



what is the perimeter of the shape if AB is a diameter of the larger circle, of length 20cm?

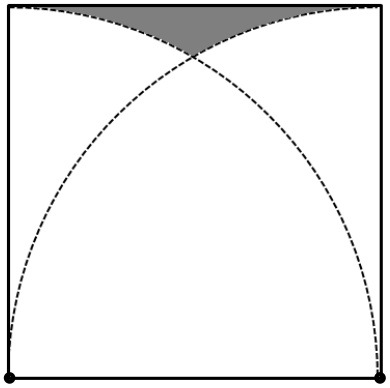
(2)



the length of a side of the equilateral triangle is 8cm

what is the total distance around the three arcs (drawn with centres on the corners of the triangle)?

(3)

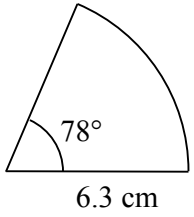


what is the perimeter of the shaded shape if a square has a length of 4cm?

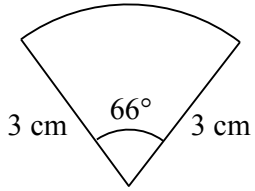
the arcs are drawn with centres on the corners of the square

Fluency Practice

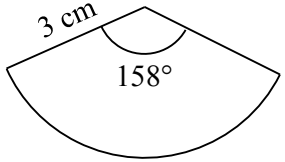
A1 Find the length of the arc



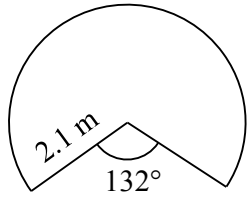
A2 Work out the perimeter



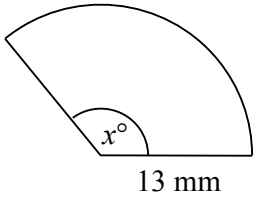
A3 Find the length of the arc



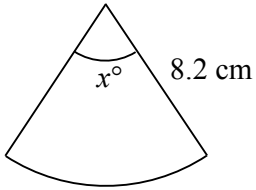
A4 Work out the perimeter



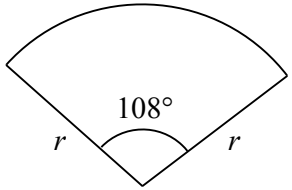
B1 Arc length is 32 mm, find x



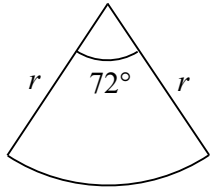
B2 Perimeter is 25 cm, find x



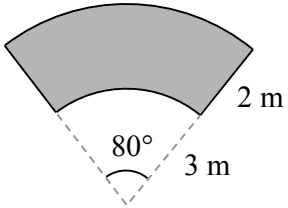
B3 Arc length is 9 cm, find r



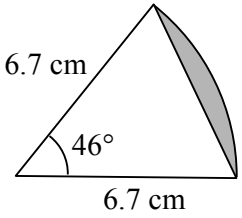
B4 Perimeter is 40 cm, find r



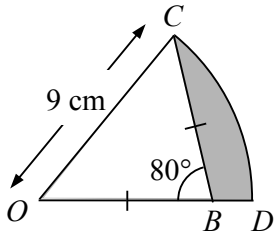
C1 Find the perimeter of the shaded shape.



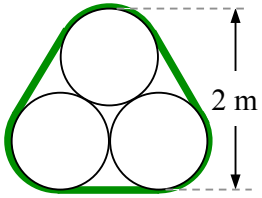
C2 Find the perimeter of the shaded segment.



C3 $OB = BC$. Find the perimeter of the shaded shape.



C4 Find length of the band that goes around the circles.



Fluency Practice

The radius is 5cm in every diagram. Find the area of each of the sectors.

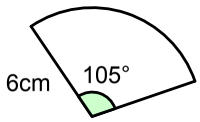
learn by heart

Area of a sector

$\frac{\theta}{360} \times \pi r^2$

example

Calculate the area of the sector, correct to the nearest $0.01cm^2$.



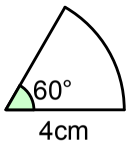
Area = $\frac{105}{360} \times \pi \times 6^2$

= $32.99cm^2$ (nearest $0.01cm^2$)

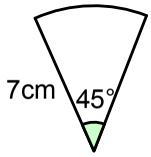
exercise

1. Calculate the area of these shapes, correct to the nearest $0.01cm^2$.

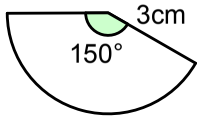
a)



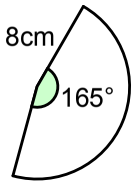
b)



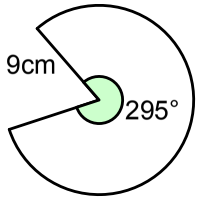
c)



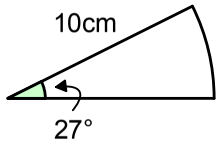
d)



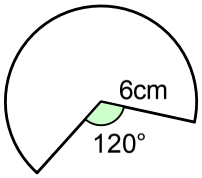
e)



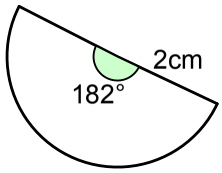
f)



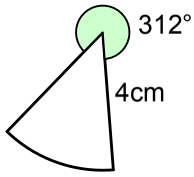
g)



h)

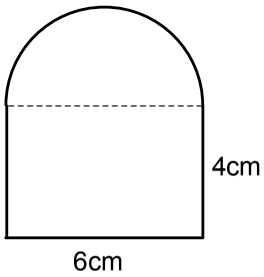


i)

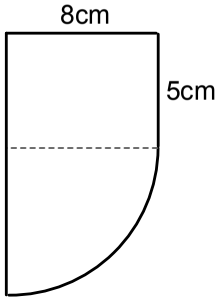


2. Calculate the area of these shapes, correct to the nearest $0.01cm^2$.

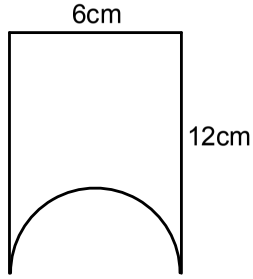
a)



b)

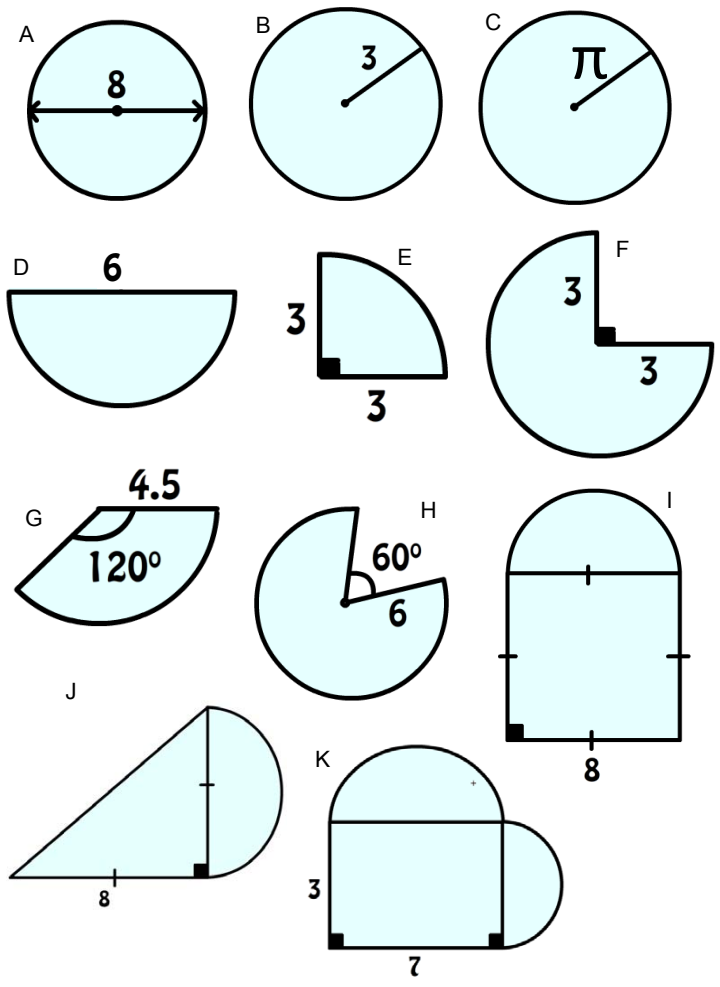


c)



area: match three!

Each of these pictures contains circles or parts of circles. Calculate the total area of each shape. Find your answer in terms of Pi and as a decimal in the list below.



Area in terms of π	Area (2 d.p.)
4.5π	14.14
6.75π	43.78
30π	94.25
π^3	
$32 + 8\pi$	28.27
9π	89.13
$64 + 8\pi$	21.21
16π	31.01
$21 + 7.25\pi$	57.13
6.75π	21.21
2.25π	7.07
	50.27

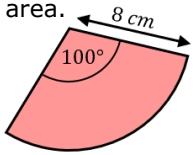
A		
B		
C		
D		
E		
F		
G		
H		
I		
J		
K		

Fluency Practice

Area of a Sector

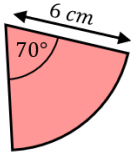
(a)

Find the area.



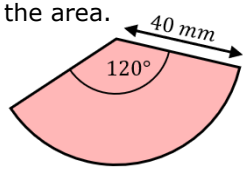
(b)

Find the area.



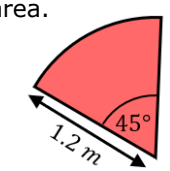
(c)

Find the area.



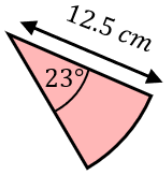
(d)

Find the area.



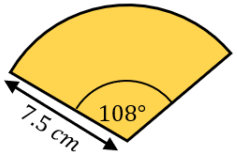
(e)

Find the area.



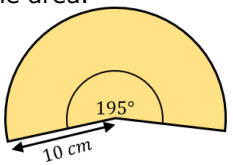
(f)

Find the area.



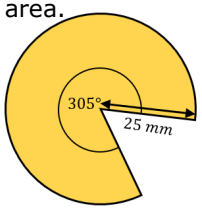
(g)

Find the area.



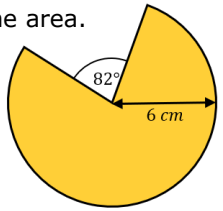
(h)

Find the area.



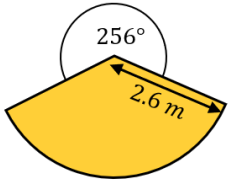
(i)

Find the area.



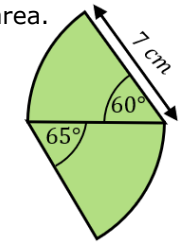
(j)

Find the area.



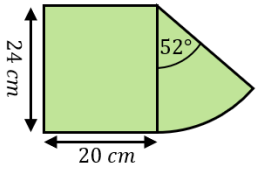
(k)

Find the area.



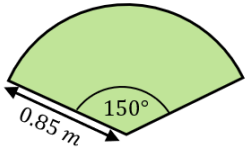
(l)

Find the area.



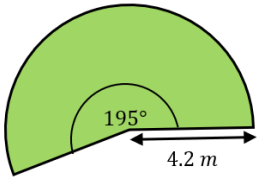
(m)

Tia wants a fan-shaped mirror, as shown in the diagram. Mirrored glass costs £82.50 per m^2. How much will Tia's mirror cost to make?



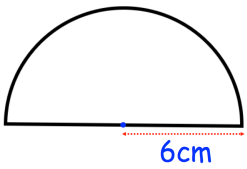
(n)

Pablo has a sector-shaped swimming pool with dimensions shown. He wants to fill his pool to a depth of 1.8 m. How many litres of water will he need?

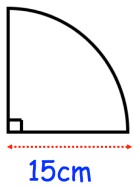


Question 1: Calculate the area of each of the following sectors.
Give each answer to one decimal place and include units.

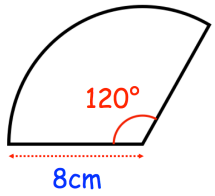
(a)



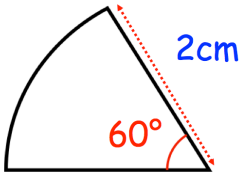
(b)



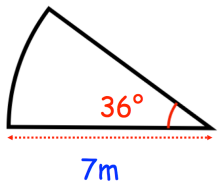
(c)



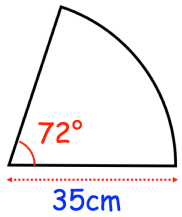
(d)



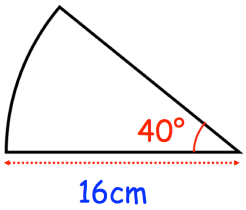
(e)



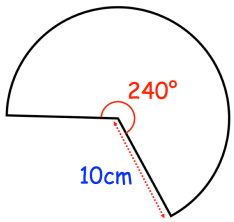
(f)



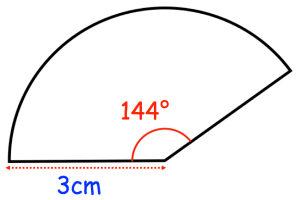
(g)



(h)

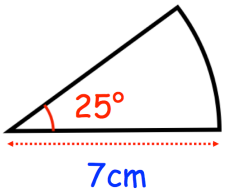


(i)

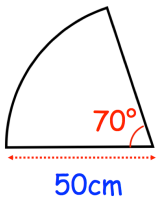


Question 2: Calculate the area of each of these sectors.
Give each answer to 2 decimal places and include suitable units.

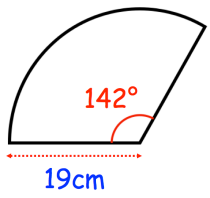
(a)



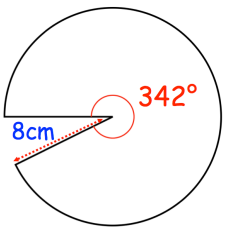
(b)



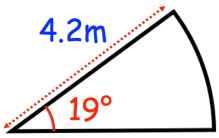
(c)



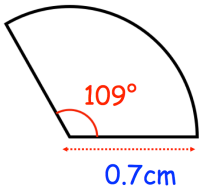
(d)



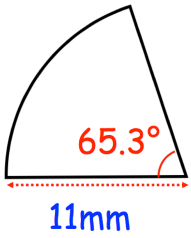
(e)



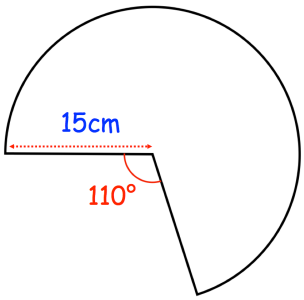
(f)



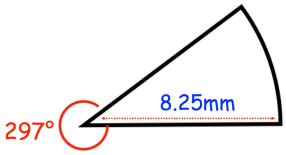
(g)



(h)

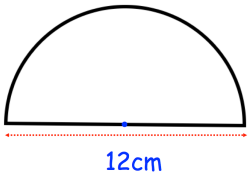


(i)

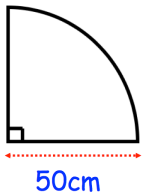


Question 3: Find the area of these sectors.
Leave your answer in terms of π

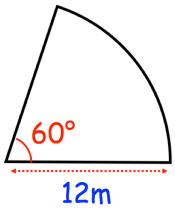
(a)



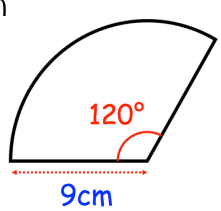
(b)



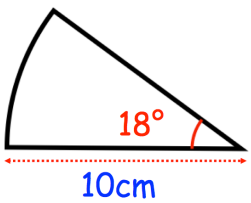
(c)



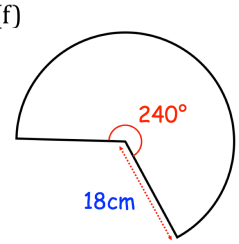
(d)



(e)

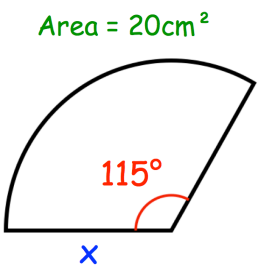


(f)

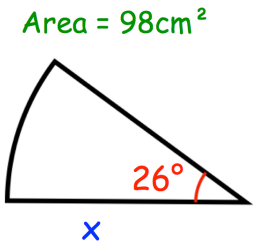


Question 4: The areas of these sectors have been given.
Calculate x.

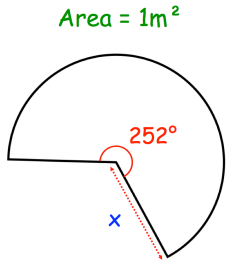
(a)



(b)

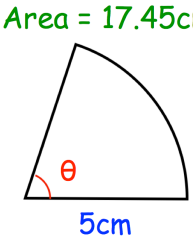


(c)

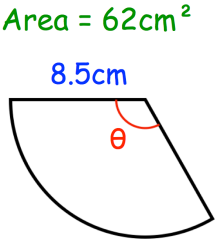


Question 5: The areas of these sectors have been given.
Calculate the missing angles.

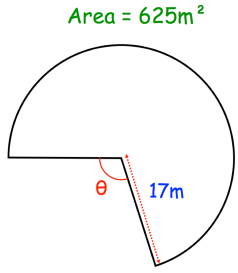
(a)



(b)

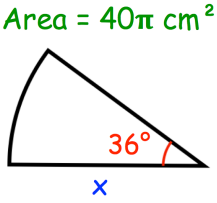


(c)

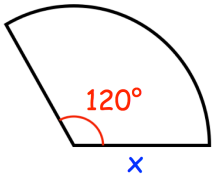


Question 6: The areas these sectors have been given in terms of **π**.
Work out x

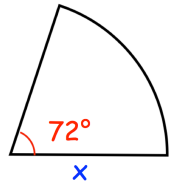
(a)



(b)

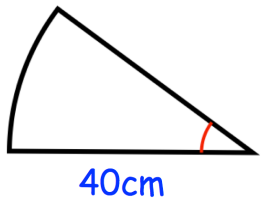


(c)

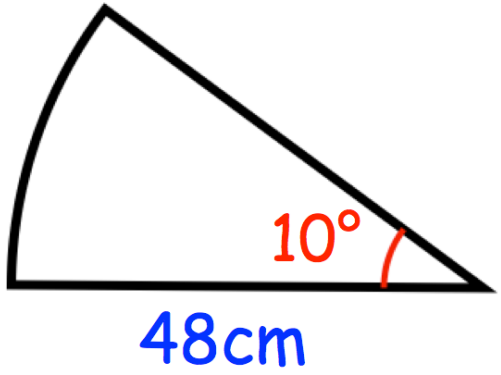


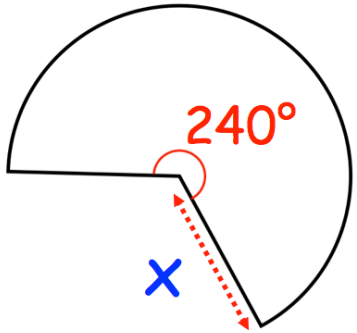
Apply

Question 1: The perimeter of the sector below is 103cm
Find the area of the sector



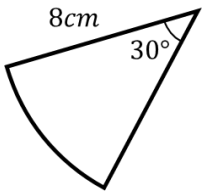
Question 2: These two sectors have the same area.
James says x is 2cm.
Is he correct?



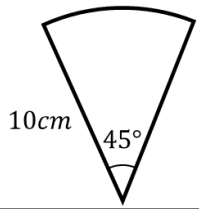


Find the area of the following shapes/shaded areas:

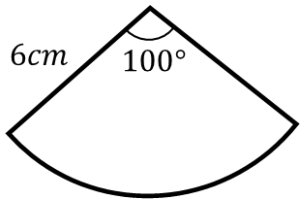
1 Leave to 2 d.p.



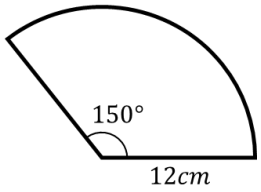
2 Leave in terms of π



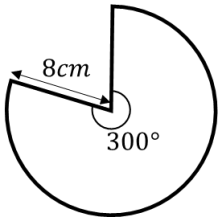
3 Leave to 3 s.f.



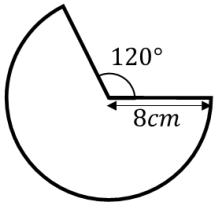
4 Leave in terms of π



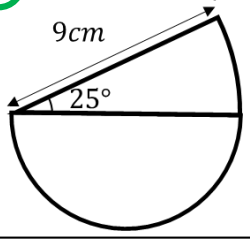
5 Leave to 1 d.p.



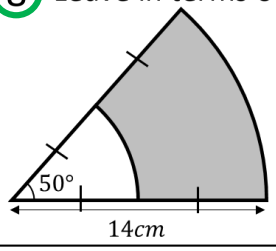
6 Leave in terms of π



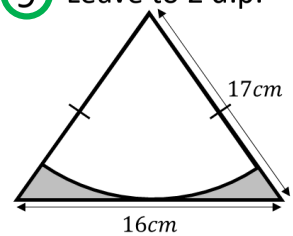
7 Leave to 2 d.p.



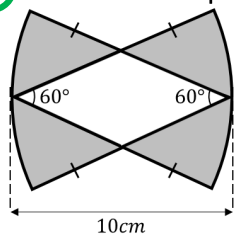
8 Leave in terms of π



9 Leave to 2 d.p.



10 Leave to 2 d.p.



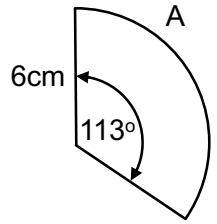
Fluency Practice

Find the area and the perimeter of the shaded section in each sector.

Fluency Practice

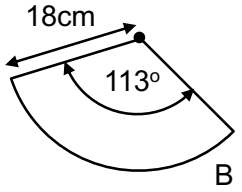
areas of sectors (i)

(1)



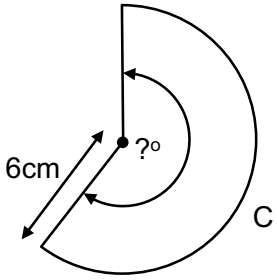
what is the area of sector A ?

(2)



what is the area of sector B ?

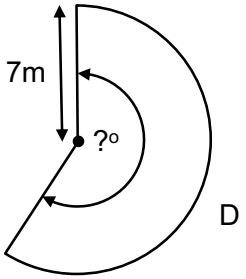
(3)



sector C is to have an area of $60.0044cm^2$

what angle must it have?

(4)

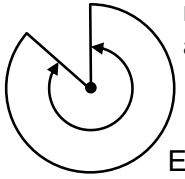


sector D is to have an area of $91.08m^2$

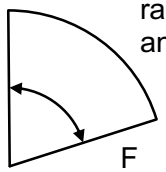
what angle must it have?

(5) which sector (E or F) has a bigger area?

explain your answer



radius = 23cm
angle = 305°

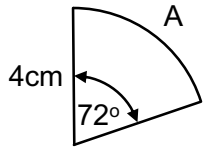


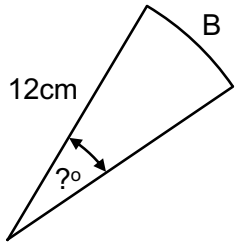
radius = 46cm
angle = 76.25°

Fluency Practice

areas of sectors (ii)

(1)

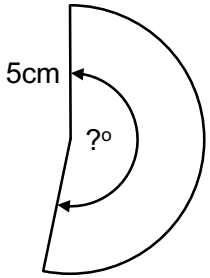




the two sectors A and B are to have exactly the same area

what angle must sector B have?

(2)



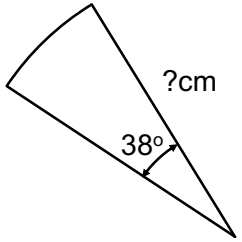
the sector (with a radius of 5cm) is to have an area of $40cm^2$

(a) what angle must the sector have?

what angle do you need for an area of $40cm^2$ for a sector with radius (b) 4cm (c) 6cm ?

(give your answers correct to the nearest 0.1^o)

(3)

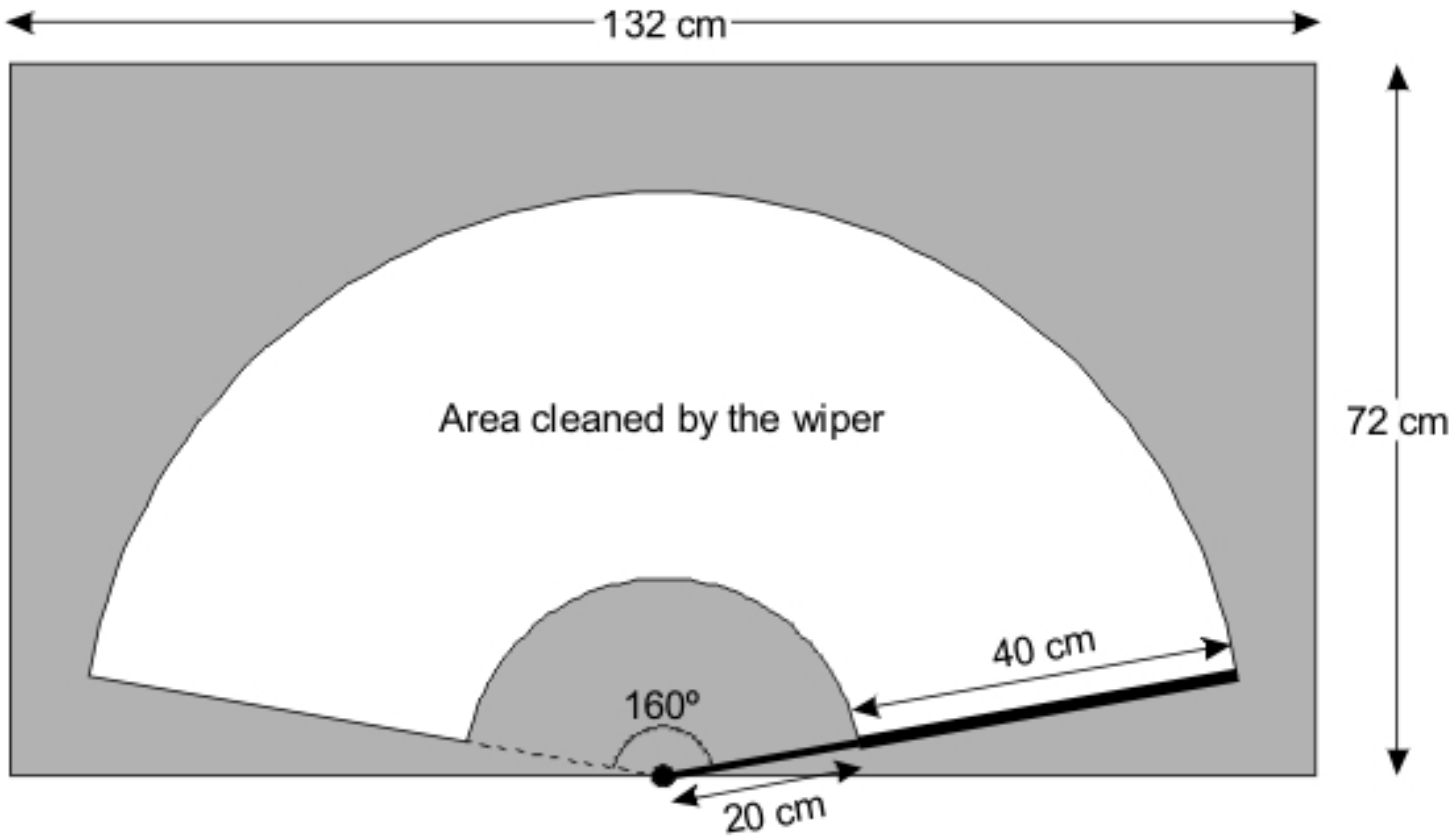


what radius must the sector have so that the area of the sector is $65cm^2$?

Exam Question

The diagram models a rectangular rear windscreen of a car.

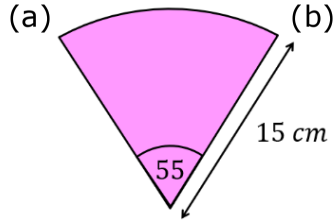
The windscreen wiper can rotate through 160°



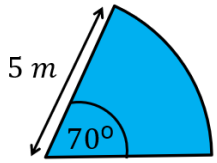
What percentage of the rear windscreen is cleaned by the wiper?

Find the area of each of these sectors.

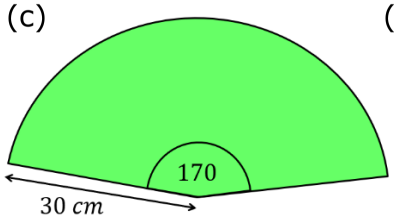
(a)



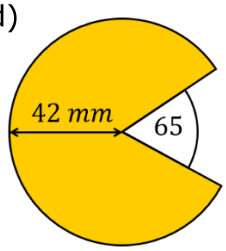
(b)



(c)

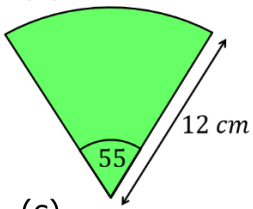


(d)

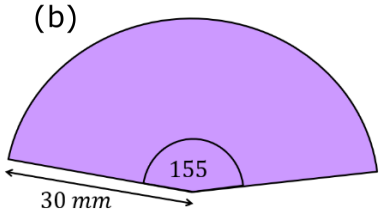


Find the arc length and perimeter of each of these sectors.

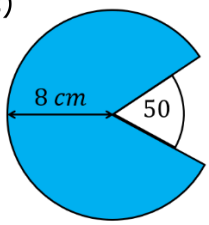
(a)



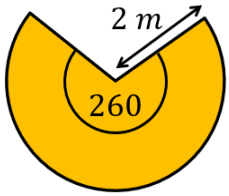
(b)



(c)

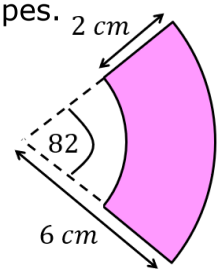


(d)

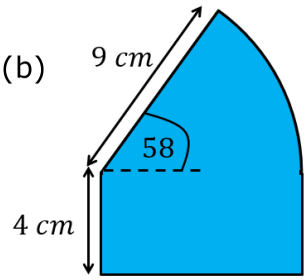


Find the area and perimeter of these shapes.

(a)



(b)



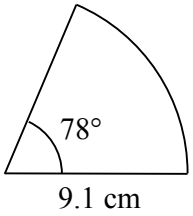
(a) A sector has an area of $70cm^2$ and a radius of $8\,cm$. Find its angle.

(b) A sector with angle $75°$ has an area of $30\,cm^2$. Find the radius of the sector.

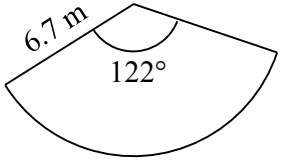
(c) A sector has a perimeter of $35\,cm$ and a radius of $7\,cm$. Find its angle.

Fluency Practice

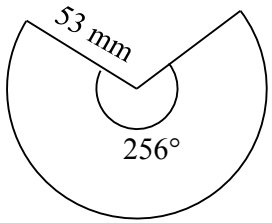
A1 Find the area.



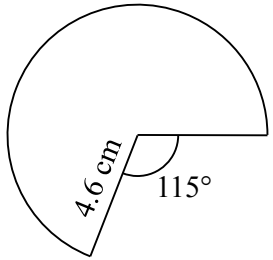
A2 Find the area.



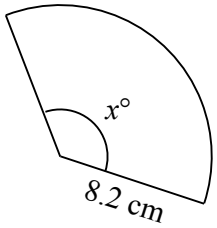
A3 Find the area.



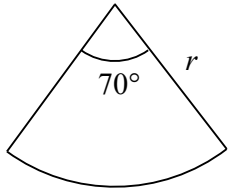
A4 Find the area.



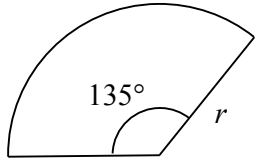
B1 The area is 75 cm^2, find x



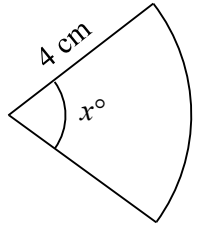
B2 The area is 20 cm^2, Find r



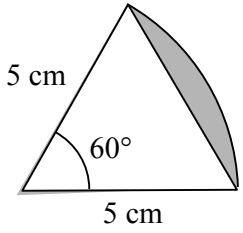
B3 The area is 65 cm^2, Find r



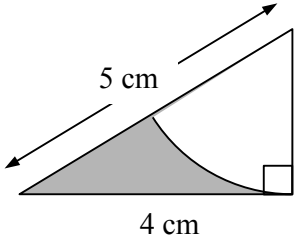
B4 The area is 8 cm^2, find x



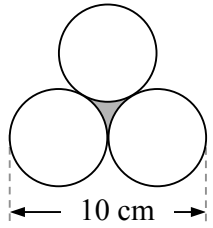
C1 Find the shaded area.



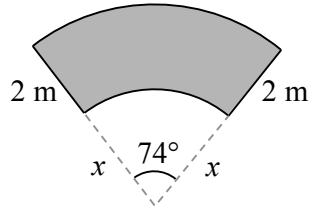
C2 Find the shaded area.



C3 Find the shaded area.

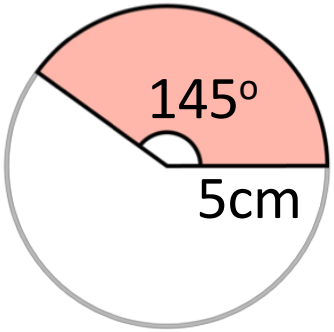


C4 The area of the shaded shape is 12 cm^2. Find the value of x.

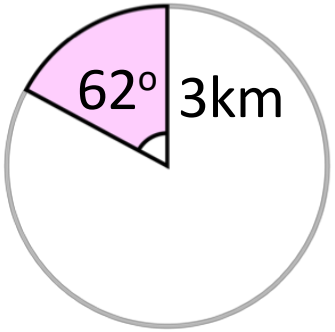


1 Work out the area and perimeter of each sector, correct to one decimal place.

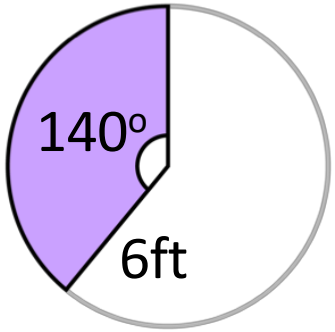
(a)



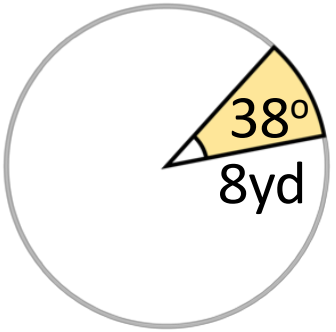
(b)



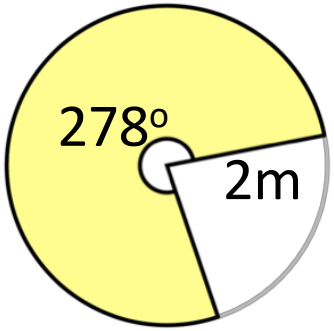
(c)



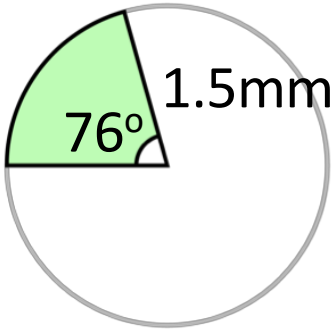
(d)



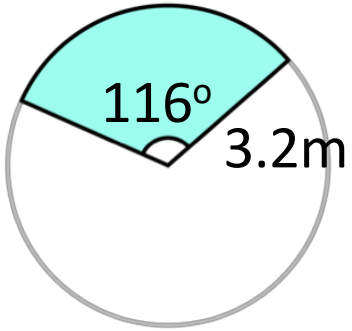
(e)



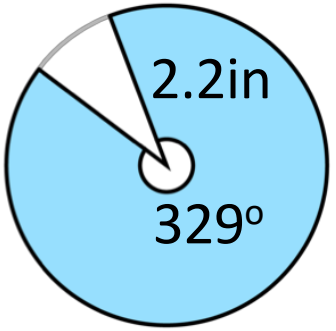
(f)



(g)



(h)



Fluency Practice

2 Use $\pi = 3.14$ to work out the **area** of a sector, to two decimal places, with radius, r and angle, θ given as:

(a) $r = 11\text{m}$, $\theta = 82^o$ **(b)** $r = 4\text{km}$, $\theta = 20^o$

(c) $r = 18\text{yd}$, $\theta = 18^o$ **(d)** $r = 9\text{in}$, $\theta = 30^o$

(e) $r = 10\text{cm}$, $\theta = 45^o$

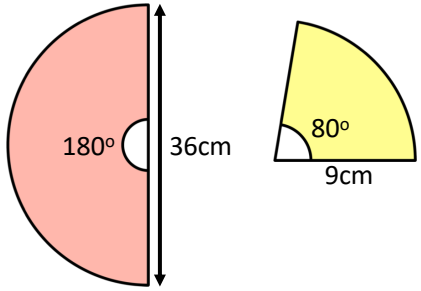
3 Use your calculator to work out the **perimeter** of a sector leaving your answer in terms of π, with radius, r and angle, θ given as:

(a) $r = 6\text{ft}$, $\theta = 60^o$ **(b)** $r = 9\text{km}$, $\theta = 160^o$

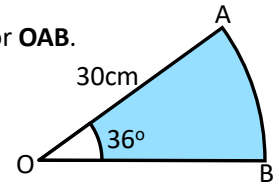
(c) $r = 15\text{m}$, $\theta = 45^o$ **(d)** $r = 18\text{mm}$, $\theta = 210^o$

(e) $r = 12\text{cm}$, $\theta = 330^o$

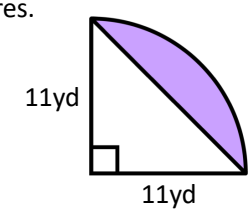
4 Express the **area** of the red sector to the area of the yellow sector as a ratio in its simplest form.



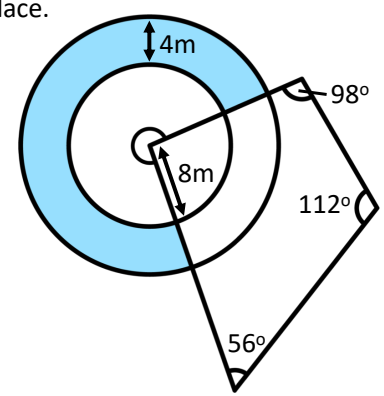
5 OAB is a sector of a circle, centre O.
The **radius** of the circle is **30cm**.
The **angle** of the sector is **36°**.
Calculate the **perimeter** of the sector **OAB**.
Leave your answer in terms of π.



6 Calculate the perimeter of the shaded region, correct to three significant figures.

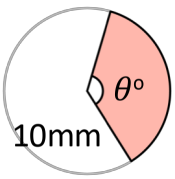


7 Find the area of the shaded region below, correct to one decimal place.



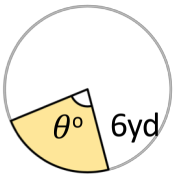
1 Work out the angle of each sector, correct to the nearest whole number, given its **area** and radius.

(a)



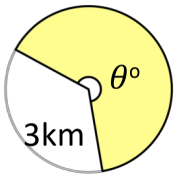
Area $= 115\text{mm}^2$

(b)



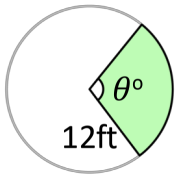
Area $= 8\pi\ \text{yd}^2$

(c)



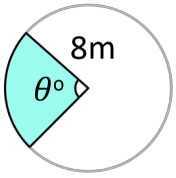
Area $= 18\text{km}^2$

(d)



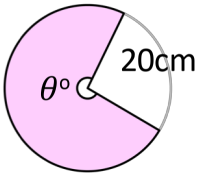
Area $= 42\pi\ \text{ft}^2$

(e)



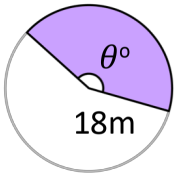
Area $= 49\text{m}^2$

(f)



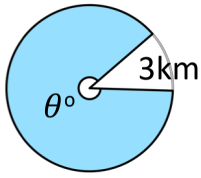
Area $= 930\text{cm}^2$

(g)



Area $= 135\pi\ \text{m}^2$

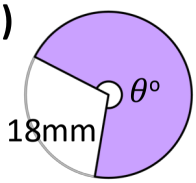
(h)



Area $= 8\pi\ \text{km}^2$

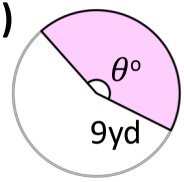
2 Work out the **angle** of each sector, correct to the nearest whole number, given its **arc length** and radius.

(a)



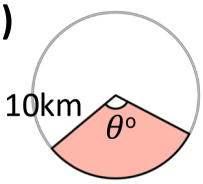
Arc length $= 25\pi$ mm

(b)



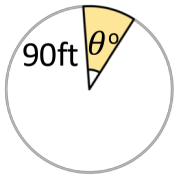
Arc length $= 8\pi$ yd

(c)



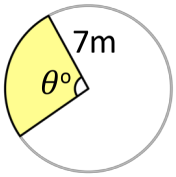
Arc length $= 20$ km

(d)



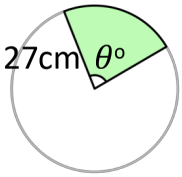
Arc length $= 19\pi$ ft

(e)



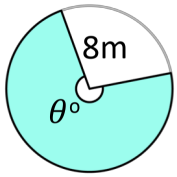
Arc length $= 12$ m

(f)



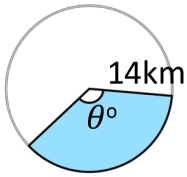
Arc length $= 12\pi$ cm

(g)



Arc length $= 36$ mm

(h)



Arc length $= 32$ km

Fluency Practice

3 Use $\pi = 3.14$ to work out the **angle** of a sector, correct to one decimal place, with **area**, A and **radius**, r given as:

(a) $A = 6\text{m}^2$, $r = 6\text{m}$ **(b)** $A = 150\text{km}^2$, $r = 12\text{km}$

(c) $A = 54\text{yd}^2$, $r = 8\text{yd}$ **(d)** $A = 85\text{in}^2$, $r = 10\text{in}$

(e) $A = 12\text{cm}^2$, $r = 2.5\text{cm}$

4 Use your calculator to work out the **angle** of a sector, correct to one decimal place, with **arc length**, L and **radius**, r given as:

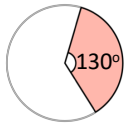
(a) $L = 6\pi\text{ft}$, $r = 12\text{m}$ **(b)** $L = 10\pi\text{km}$, $r = 6\text{m}$

(c) $L = 75\text{m}$, $r = 15\text{m}$ **(d)** $L = 12.5\text{mm}$, $r = 4.5\text{m}$

(e) $L = 31.2\text{cm}$, $r = 10\text{m}$

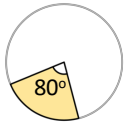
5 Work out the angle of each sector, correct to the nearest whole number, given its **area** and radius.

(a)



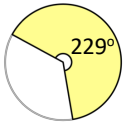
Area $= 52\pi$ mm^2

(b)



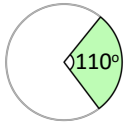
Area $= 18\pi$ yd^2

(c)



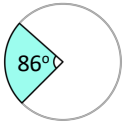
Area $= 450$ km^2

(d)



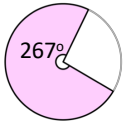
Area $= 44\pi$ ft^2

(e)



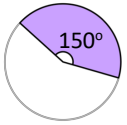
Area $= 27$ m^2

(f)



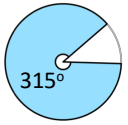
Area $= 760$ cm^2

(g)



Area $= 240\pi$ m^2

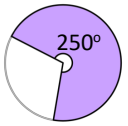
(h)



Area $= 1694\pi$ km^2

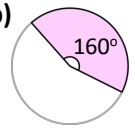
6 Work out the **radius** of each sector, correct to the nearest whole number, given its **arc length** and angle.

(a)



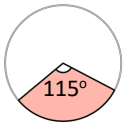
Arc length $= \frac{25}{2}\pi$ mm

(b)



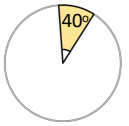
Arc length $= 80\pi$ yd

(c)



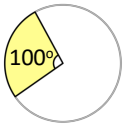
Arc length $= 44$ km

(d)



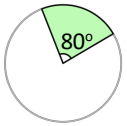
Arc length $= 2\pi$ ft

(e)



Arc length $= 15\pi$ m

(f)

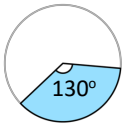


Arc length $= 40\pi$ cm

(g)

Arc length $= 68$ mm

(h)

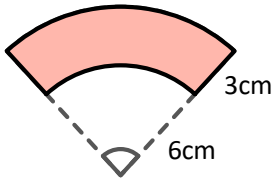


Arc length $= 200$ km

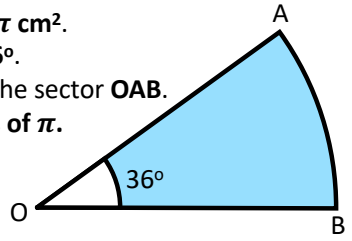
7 The perimeter of shaded section the sector below is,

$$6 + \frac{20}{3}\pi \text{ cm}$$

Find the angle, θ, in the sector.

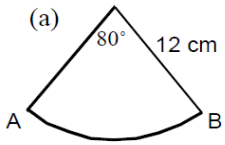


8 OAB is a sector of a circle, centre O.
The **area** of the sector is **10π cm^2**.
The **angle** of the sector is **36°**.
Calculate the **perimeter** of the sector **OAB**.
Leave your answer in terms of π.

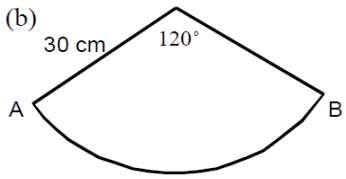


1. Calculate the length of arc AB in each question below:

(a)

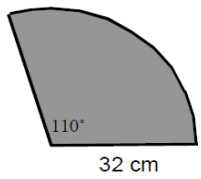


(b)

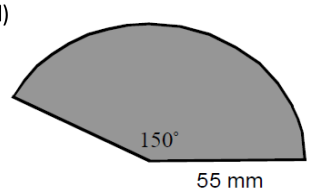


Calculate the area of each sector below:

(c)

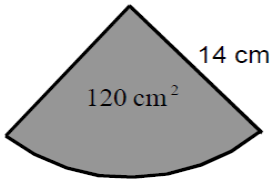


(d)

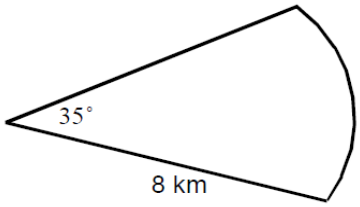


2. The length of arc AB is given. Calculate the size of the angle at the centre of the sector.

3. The area of the sector is given. Calculate the size of the angle at the centre of the sector.

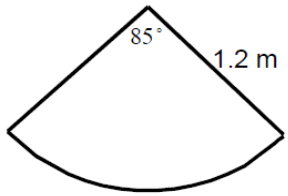


4. The beam from a lighthouse reaches a distance of 8 kilometres and spreads to an angle of 35°.



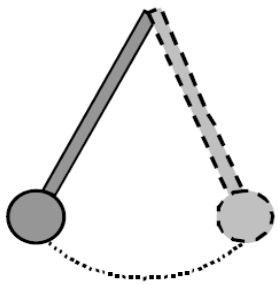
Calculate the area covered by the beam from the lighthouse.

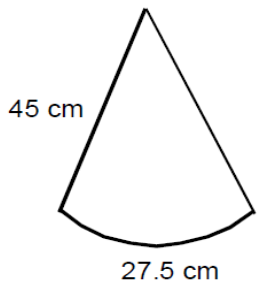
5. The curved part on an anchor is in the shape of an arc of a circle which has radius 1.2 metres.



Calculate the length of this arc.

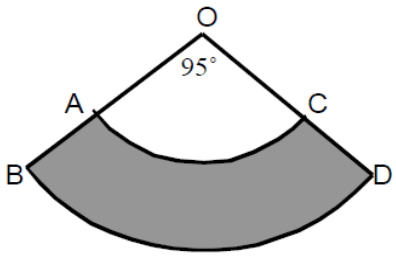
6. A pendulum is 45 centimetres long. When the pendulum swings it travels along the arc of a circle and covers a distance of 27.5 centimetres.





Calculate the size of the angle through which the pendulum travels.

7. In the diagram below AC and BD are arcs of circles with centres at O. The radius, OA, is 10 centimetres and the radius, OB, is 16 centimetres.



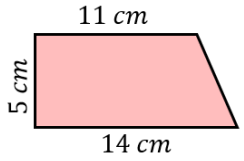
Find the shaded area.

Fluency Practice

Area and Perimeter Revision

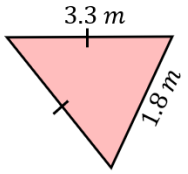
(a)

Find the area.



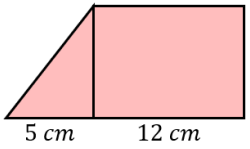
(b)

Find the perimeter of the triangle.



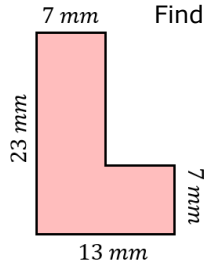
(c)

The area of the triangle is $20\ cm^2$. Find the area of the rectangle.



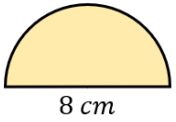
(d)

Find the perimeter.



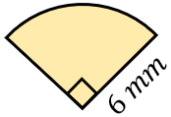
(e)

Find the area of the semi-circle to 1 decimal place.



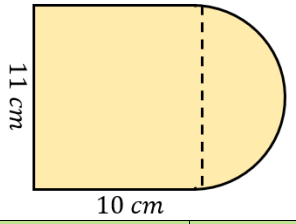
(f)

Find the perimeter to 3 significant figures.



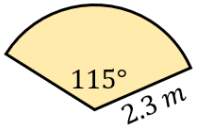
(g)

Find the perimeter to 1 decimal place.



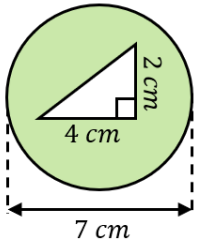
(h)

Find the area, giving your answer to 1 decimal place.



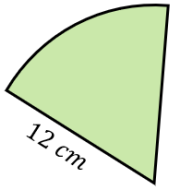
(i)

Work out the area of the shaded region.



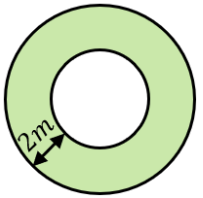
(j)

The area of the sector is $24\pi\ cm^2$. Find the perimeter of the sector in terms of π.



(k)

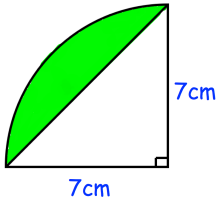
A circular lawn is surrounded by a path of constant width $2\ m$. The area of the path is $30\pi\ m^2$. Find the area of the lawn in terms of π.



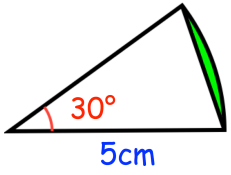
Fluency Practice

Question 1: Calculate the area of each segment.
Give each answer to 3 decimal places.

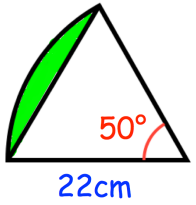
(a)



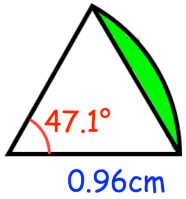
(b)



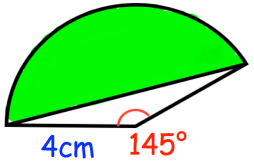
(c)



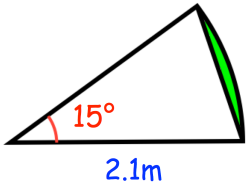
(d)



(e)

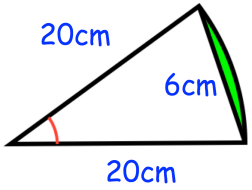


(f)

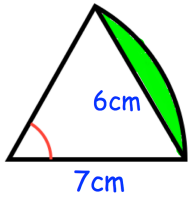


Question 2: Calculate the area of each segment.
Give each answer to 3 decimal places.

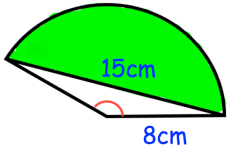
(a)



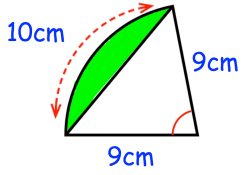
(b)



(c)

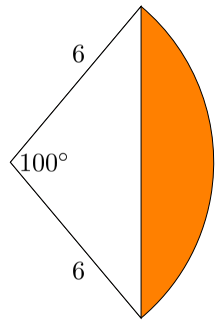


(d)

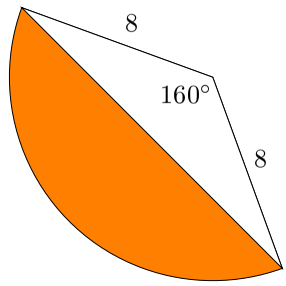


All units are in centimetres. Give all answers to three significant figures.

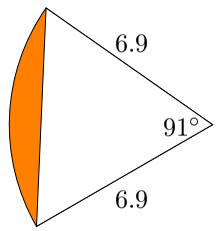
1. Find the area of the orange segment in the diagram below.



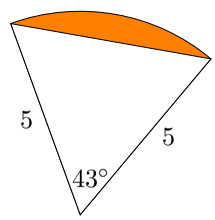
2. Find the area of the orange segment in the diagram below.



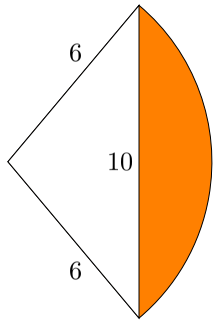
3. Find the area of the orange segment in the diagram below.



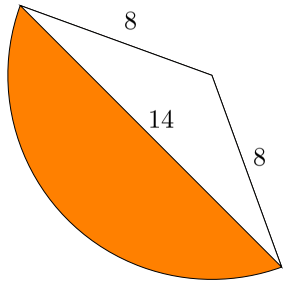
4. Find the area of the orange segment in the diagram below.



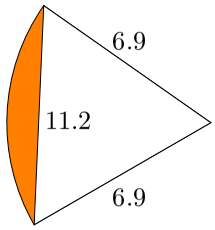
5. Find the area of the orange segment in the diagram below.



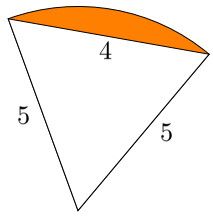
6. Find the area of the orange segment in the diagram below.



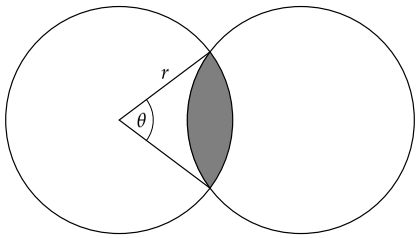
7. Find the area of the orange segment in the diagram below.



8. Find the area of the orange segment in the diagram below.

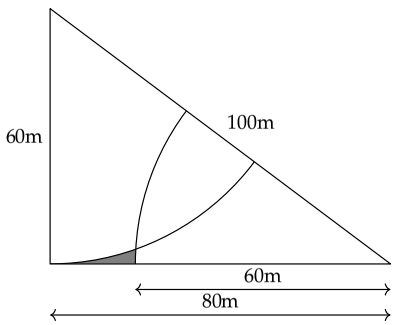


1. Shown are two circles, both with radius r. The angle subtended from the centre of each circle to the points of intersection of the circles is θ. Find the perimeter and area of region of intersection of the two circles, shown shaded in the diagram.



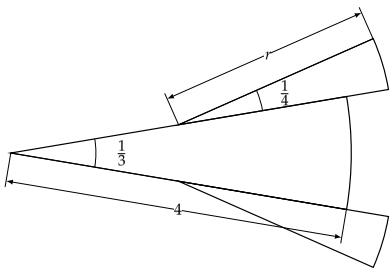
2. A sector of a circle has fixed area A, but the radius and angle may vary. Show that the minimum possible perimeter of the sector is $4\sqrt{A}$, and find the angle of the sector when this occurs.

3. A farmer has a field in the shape of a right angled triangle with sides 60m, 80m, and 100m. A goat is tethered to each of the two angles which are not the right angle by a rope of length 60m. Find, to three significant figures, the area of the field that cannot be reached by either goat.



4. The logo of a company is made up of three sectors, as shown in the diagram. The larger sector has radius 4 units and angle $\frac{1}{3}$ radians. The two smaller sectors both have radius r and angle $\frac{1}{4}$ radians.

The area of the logo is $\frac{203}{48}$ units^2. Find the perimeter of the logo.



5. Three circles are drawn such that the centre of each circle lies on the circumference of the other two. What fraction of the total area inside this shape lies inside all three of the circles?

4 Advanced Probability

Illustrate with a Venn diagram.

$$\xi = \{1 \text{ to } 10 \text{ inclusive}\}$$

$$A = \{1 \text{ to } 5 \text{ inclusive}\}$$

$$B = \{\text{even numbers}\}$$

$$C = \{3 \text{ to } 7 \text{ inclusive}\}$$

Illustrate with a Venn diagram.

$$\xi = \{1 \text{ to } 12 \text{ inclusive}\}$$

$$A = \{\text{prime numbers}\}$$

$$B = \{\text{multiples of } 6\}$$

$$C = \{\text{multiples of } 3\}$$

Illustrate with a Venn diagram.

$$\xi = \{a \text{ to } j \text{ inclusive}\}$$

$$A = \{h, i, j\}$$

$$B = \{a, c, e, g, i\}$$

$$C = \{e, f, g, h, i\}$$

Illustrate with a Venn diagram.

$$\xi = \{10 \text{ to } 20 \text{ inclusive}\}$$

$$A = \{\text{multiples of } 2\}$$

$$B = \{\text{multiples of } 3\}$$

$$C = \{\text{multiples of } 5\}$$

Illustrate with a Venn diagram.

$$\xi = \{1 \text{ to } 15 \text{ inclusive}\}$$

$$A = \{x : 3 \leq x \leq 9\}$$

$$B = \{\text{odd numbers}\}$$

$$C = \{7, 8, 9, 10, 11\}$$

Fluency Practice

Statement	True or false
$1 \in \{odd\ numbers\}$	True
$1 \in \{prime\ numbers\}$	
$2 \notin \{odd\ numbers\}$	
$Red \in \{colours\}$	
$Red \notin \{colours\}$	
$9 \in \{prime\ numbers\}$	
$Chocolate \notin \{fruits\}$	
$scalene \in \{types\ of\ triangle\}$	
$cylinder \in \{2D\ shapes\}$	

Fluency Practice

$A = \{1, 9, 15, 23, 100\}$

B =	$B \subset A$?
$\{1, 9\}$	Yes
$\{0, 1, 9\}$	
$\{23, 9\}$	
$\{100, 1\}$	
$\{9\}$	
$\{23, 9, 1, 100, 5\}$	
$\{0\}$	
$\{\square\}$ *also known as* $\emptyset$	
$\{100, 23, 15, 9, 1\}$	

Fluency Practice

For each of the following say if the statement is true or false.

$A = \{1, 4, 9, 16, 25\}$ $B = \{1, 4\}$ $C = \{1, 8, 27, 64, 125\}$ $D = \{1, 27\}$ $E = \{4, 1\}$

a) $B \subset A$	b) $A \subset B$	c) $6 \in A$
d) $B = E$	e) $B \subset C$	f) $D \subset C$
g) $27 \in C$	f) $E \subset D$	h) $25 \in A$

Fluency Practice

1. $\xi = \{4,5,6,7,8\}$,
   $A = \{5,6,7\}$
   $B = \{6,7,8\}$
   Construct a Venn Diagram to show these sets.

2. $\xi = \{1,2,3,4,5,6,7,8\}$
   $A = \{1,2,3,8\}$,
   $B = \{3,4,5,8\}$
   $C = \{1,5,6,8\}$
   Construct a Venn Diagram to show these sets.

3. $\xi = \{1,2,\ldots,10\}$
   $A =$ set of all primes
   $B =$ triangular numbers
   $C =$ 1 less than multiple of 4
   Construct a Venn diagram for these sets.

4. You have two sets A and B and $B \subset A$. Draw a Venn Diagram (without any numbers) that indicates the relationship between the sets.

5. You have three sets A, B and C and $A \subset C$. Draw a Venn Diagram (without any numbers) that indicates the relationship between the sets.

☠ The **power set** of a set is the set of all possible subsets, including the empty set and itself. E.g.

$$P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$$

   a. Determine $P(\{1,2,3\})$
   b. Determine how many members $P(A)$ has for a set of A of size n.

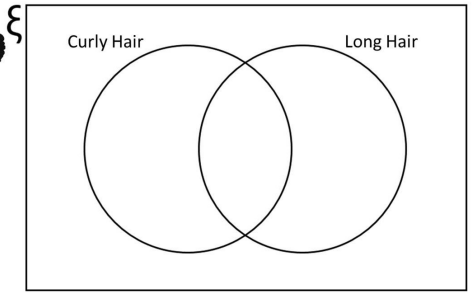
Fluency Practice

① 24 students were surveyed about their hair.

The results are below.

Curly	Long	Short
Chris	Chris	Edward
Fred	Diana	Anna
Anna	Kris	Viv
Tess	Fred	Lisa
Olly	Noel	Will
Mark	Robert	Tess
Iona	Yuna	Ben
Jessica	Zoe	Olly
Ben	Shay	Xena
	Peter	Jessica
	Mark	
	George	
	Iona	
	Hannah	

Complete the Venn diagram using letters for the student's names.



If you pick a student at random, what is the probability...

P(Curly Hair) =

P(NOT Curly Hair) =

P(Long AND Curly Hair) =

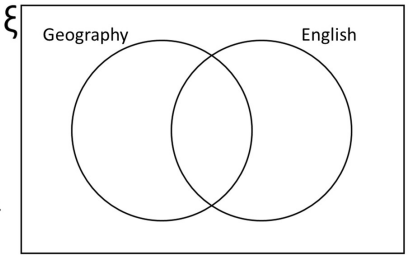
P(Long AND NOT Curly Hair) =

P(Curly OR Short Hair) =

②  

50 students were asked if they study Geography or English.

- 5 study Geography and <u>not</u> English.
- 25 study English and <u>not</u> Geography.
- 20 study English <u>and</u> Geography.



If I pick a student at random, what is the probability they…

P(study English) =

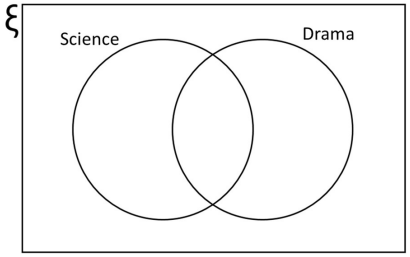
P(study Geography) =

P(study Geography AND English) =

③  

70 students were asked if they study Science or Drama.

- 13 study Science <u>and</u> Drama.
- 37 study Science and <u>not</u> Drama.
- 8 study Drama and <u>not</u> Science.



If I pick a student at random, what is…

P(study Drama) =

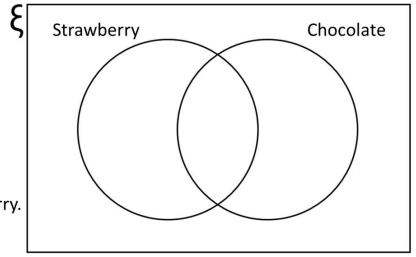
P(study Science) =

P(don't study Science OR Drama) =

④ 

45 people bought one or two scoops of ice cream.

- 20 bought strawberry.
- 10 didn't buy chocolate or strawberry.
- 30 bought chocolate.



How many people bought both flavours?

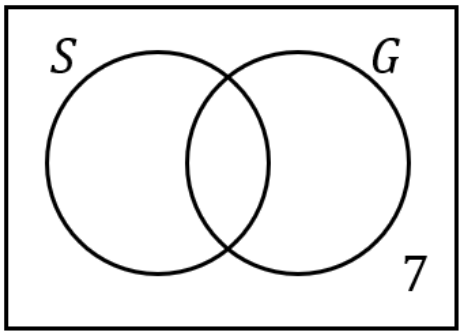
If I pick a person at random, what is…

P(strawberry) =

P(only chocolate) =

P(only one flavour) =

There are 30 students in a class. 18 study Spanish. 13 study German. 7 study neither Spanish nor German. Complete the Venn diagram to represent this information and find the number of students who study Spanish and German.



In a group of 25 people, 8 said they had been to the theatre recently. 15 said they had been to the library recently, and 10 said they had been to neither the theatre or the library recently. Draw a Venn diagram to represent this information and find the number of people who had only been to the library recently.

In a class of 30 students, 15 play football and 11 play tennis. Twice as many students play neither sport as play both sports. Draw a Venn diagram to represent this information and find the number of students who play both football and tennis.

There are 50 cars in a car park. 40% of the cars are Fords. Of the Fords, 6 were white. There were three times as many cars that were neither white nor Ford as there were white Fords. Draw a Venn diagram to represent this information.
Find:
(a) the number of cars that were white, but not Fords.
(b) the number of cars that were Fords, but not white.

In a group of 100 students, 42 study Statistics, 40 study Mathematics, and 50 study Physics. 21 study Mathematics and Physics, 19 study Statistics and Physics, 17 study Statistics and Mathematics and 5 study all three.

Draw a Venn diagram to represent this information.

(a) How many students study only **one** of these subjects?

(b) How many students study none of these subjects?

A group of 200 adults were asked which types of magazines they read. Their replies showed that 82 read Sports magazines, 80 read Garden magazines, and 84 read Fashion magazines. 36 read Sports magazines and Garden magazines. 31 read Sports magazines and Fashion magazines. 25 read Garden magazines and Fashion magazines. 14 read all three magazines.
(a) How many adults read Sports and Garden magazines, but not Fashion magazines?

(b) How many adults reads exactly two of these types of magazine?

There are 3 clubs - chess, drama and art. All the members of a group of 35 students belong to at least one club. 8 of the students belong to only art club. 6 of the students belong to all 3 clubs. 3 of the students belong to chess and art clubs but not to drama club. 18 of the students belong to art club. 3 of the students belong only to chess club. 4 of the students belong only to drama club.

(a) How many students belongs to chess club and to drama club but not to art club?

(b) How many students belong to chess club?

Fluency Practice

1. Draw a Venn diagram to represent each set of information

a.
There are 80 students.
9 students study French (F) and German (G).
35 students only study French.
2 students do not study French or German.

b.
50 children were eating lunch.
11 students only had chips (C).
20 students only had a burger (B).
13 students had both chips and a burger.

c.
In a group of 24 students,
13 like only oranges (O).
2 like only bananas (B).
4 don't like oranges or bananas.

d.
There are 32 students.
24 students study History (H), Geography (G) or both.
15 students only study History.
5 students only study Geography.

2. Draw a Venn diagram to represent each set of information

a.
40 people were asked if they had a cat (C) or a dog (D).
10 people owned both pets.
17 people owned a cat.
14 people didn't own either pet.

b.
110 workers were asked if they liked tea (T) or coffee (C).
90 workers liked tea.
41 worked liked coffee.
25 worked like tea and coffee.

c.
In a class of 24 students,
12 students play the piano (P).
13 students play the guitar (G).
4 students play neither instrument.

d.
100 people were asked if they had visited Rome (R) or Paris (P).
38 had been to Rome.
80 had been to Paris.
11 had not been to either city.

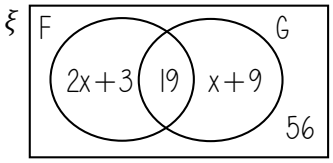
3. Draw a Venn diagram to represent each set of information

a.
32 students were asked which club they belong to.
6 students are only part of the art club (A).
2 students are only part of the chess club (C).
3 students are only part of the drama club (D).
5 students belong to all three clubs.
2 students are part of the art and chess clubs, but not the drama club.
15 student is part of the art and drama clubs, but not the chess club.
7 students are not part of any of the clubs.

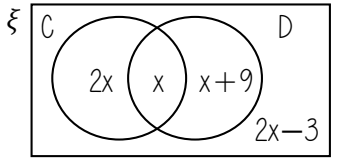
b.
60 people were asked which fruits they liked.
40 people said they liked melons (M).
42 people said they liked pears (P).
25 people said they liked strawberries (S).
6 people said they liked all three fruits.
20 people said they liked melons and pears, but not strawberries.
10 people said they liked pears and strawberries, but not melons.
6 people said they liked melons and strawberries, but not pears.

4. Find the value of x, and then find the frequency of each region.

a.
150 students were asked whether they studied French or German.



b.
30 people were asked whether they had a dog (D) or a cat (C).



Patrons are reminded to *define their variables*; i.e. if you introduce an x somewhere then you must define what it is (for example "Let x be the number of students in school who don't define their variables").

1. 100 people are surveyed as to whether they enjoy rugby or football.

   70 like football.
   60 like rugby.
   45 like both.

   (a) Draw a Venn diagram of the situation.

   A person from the survey is selected at random.

   (b) Find the probability they like neither rugby nor football.

   (c) Find the probability they like only one of the two sports.

2. Fifty people are surveyed as to whether they read the Guardian or the Telegraph. Forty read the Telegraph. Thirty read the Guardian. Five read neither.

   (a) Draw a Venn diagram of the situation.

   (b) Find the number of people who read both the Guardian and the Telegraph.

   A person from the survey is selected at random.

   (c) Find the probability they read the Telegraph but not the Guardian.

   (d) Find the probability they read only one paper.

3. Sixty A level students are surveyed as to whether they study maths or history. Fifty study maths. Twenty study history. Seven read neither.

   (a) Draw a Venn diagram of the situation.

   (b) Find the number of students who study both maths and history.

   A student from the survey is selected at random.

   (c) Find the probability they read history but not maths.

   (d) Find the probability they read neither subject.

4. A college has 80 students in Year 12.

   20 students study Biology.
   28 students study Chemistry.
   30 students study Physics.
   7 students study both Biology and Chemistry.
   11 students study both Chemistry and Physics.
   5 students study both Physics and Biology.
   3 students study all 3 of these subjects.

   (a) Draw a Venn diagram to represent this information.

   A Year 12 student at the college is selected at random.

   (b) Find the probability that the student studies Chemistry but not Biology or Physics.

   □

   (c) Find the probability that the student studies Chemistry or Physics or both.

5. A group of 100 students are asked if they like folk music, rock music or soul music.

   All students who like folk music also like rock music.
   No students like both rock music and soul music.
   75 students do not like soul music.
   12 students who like rock music do not like folk music.
   30 students like folk music.

   (a) Draw a Venn diagram to illustrate this information.

   (b) State two of these types of music that are mutually exclusive.

   Find the probability that a randomly chosen student

   (c) does not like folk music, rock music or soul music,

   (d) likes rock music,

   (e) likes folk music or soul music.

6. The following shows the results of a survey on the types of exercise taken by a group of 100 people.

   65 run
   48 swim
   60 cycle
   40 run and swim
   30 swim and cycle
   35 run and cycle
   25 do all three

   (a) Draw a Venn Diagram to represent these data.

   Find the probability that a randomly selected person from the survey

   (b) takes none of these types of exercise,

   (c) swims but does not run,

   (d) takes at least two of these types of exercise.

7. There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

   112 take systems support,
   70 take developing software,
   81 take networking,
   35 take developing software and systems support,
   28 take networking and developing software,
   40 take systems support and networking,
   4 take all three extra options.

   (a) Draw a Venn diagram to represent this information.

   A student from the course is chosen at random. Find the probability that the student takes

   (b) none of the three extra options,

   (c) networking only.

8. A person's blood group is determined by whether or not it contains any of 3 substances A, B and C. A doctor surveyed 300 patients' blood and produced the table below.

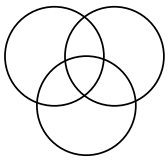
Blood contains	**No. of patients**
only C	100
A and C but not B	100
only A	30
B and C but not A	25
only B	12
A, B and C	10
A and B but not C	3

(a) Draw a Venn diagram to represent this information.

(b) Find the probability that a randomly chosen patient's blood contains substance C.

Patients whose blood contains none of these substances are called universal blood donors.

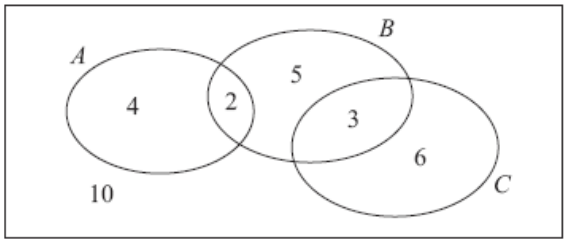
(c) Find the probability that a randomly chosen patient is a universal blood donor.

A group of friends are comparing whether they received homework from Maths, English and PE.	James had more homework than Miya.	Ellie had homework from all three subjects. Rory and Jack only had one piece each.
 You may find this useful.	Seven students in total have Maths homework but Miya isn't one of them.	Tom received homework from Maths and PE but not English.
Four students received two pieces of homework.	Two of the group just have PE homework and they are both male.	Katie is the only girl with no homework.
Rory has more homework than Aidan.	Maddie is one of two people who just have Maths homework.	Sophie and Rory do not have PE homework.
Wayne has less homework than Ashley.	Nobody in the group just has English homework.	None of the girls has exactly the same homework as another girl.
Five students must write an essay for English.	What is the probability that one of the students chosen at random has three pieces of homework?	There are 7 boys and 5 girls.

Fluency Practice

1. **[JMC 2002 Q11]** The Pythagoras School of Music has 100 students. Of these, 60 are in the band and 20 are in the orchestra. Given that 12 students are in both the band and the orchestra, how many are in neither the band nor the orchestra?

2. **[Edexcel S1 May 2010 Q4]** The Venn diagram in Figure 1 shows the number of students in a class who read any of 3 popular magazines A, B and C.



One of these students is selected at random.

(a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$. **(2)**

(b) Find the probability that the student reads A or B (or both). **(2)**

(c) Write down the probability that the student reads both A and C. **(1)**

Given that the student reads at least one of the magazines,

(d) find the probability that the student reads C. **(2)**

3. **[Edexcel S1 Jan 2010 Q4]** There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

112 take systems support,
70 take developing software,
81 take networking,
35 take developing software and systems support,
28 take networking and developing software,
40 take systems support and networking,
4 take all three extra options.

(*a*) Draw a Venn diagram to represent this information. **(5)**

A student from the course is chosen at random.
Find the probability that the student takes
(*b*) none of the three extra options, **(1)**
(*c*) networking only. **(1)**

Students who want to become technicians take systems support and networking. Given that a randomly chosen student wants to become a technician,
(*d*) find the probability that this student takes all three extra options. **(2)**

4. [Edexcel S1 May 2008 Q5] A person's blood group is determined by whether or not it contains any of 3 substances *A*, *B* and *C*.
A doctor surveyed 300 patients' blood and produced the table below.

Blood contains	No. of Patients
only *C*	100
A and *C* but not *B*	100
only *A*	30
B and *C* but not *A*	25
only *B*	12
A, *B* and *C*	10
A and *B* but not *C*	3

(*a*) Draw a Venn diagram to represent this information. **(4)**
(*b*) Find the probability that a randomly chosen patient's blood contains substance *C*. **(2)**
Harry is one of the patients. Given that his blood contains substance *A*,

(*c*) find the probability that his blood contains all 3 substances. **(2)**
Patients whose blood contains none of these substances are called universal blood donors.

(*d*) Find the probability that a randomly chosen patient is a universal blood donor. **(2)**

5. [Edexcel S1 Jan 2008 Q5] The following shows the results of a wine tasting survey of 100 people.
96 like wine *A*, 93 like wine *B*,
96 like wine *C*, 92 like *A* and *B*,
91 like *B* and *C*, 93 like *A* and *C*,
90 like all three wines.

(*a*) Draw a Venn Diagram to represent these data. **(6)**
Find the probability that a randomly selected person from the survey likes
(*b*) none of the three wines, **(1)**
(*c*) wine *A* but not wine *B*, **(2)**
(*d*) any wine in the survey except wine *C*, **(2)**
(*e*) exactly two of the three kinds of wine. **(2)**
Given that a person from the survey likes wine *A*,
(*f*) find the probability that the person likes wine *C*. **(3)**

6. [Edexcel S1 May 2006 Q6] A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left-handed and having dark hair.

Glasses were worn by 36 people, 28 were left-handed and 36 had dark hair. There were 17 who wore glasses and were left-handed, 19 who wore glasses and had dark hair and 15 who were left-handed and had dark hair. Only 10 people wore glasses, were left-handed and had dark hair.

(*a*) Represent this on a Venn diagram. **(6)**

A person was selected at random from this group.
Find the probability that this person
(*b*) wore glasses but was not left-handed and did not have dark hair, **(1)**
(*c*) did not wear glasses, was not left-handed and did not have dark hair, **(1)**
(*d*) had only two of the attributes, **(2)**
(*e*) wore glasses, given they were left-handed and had dark hair. **(3)**

7. [Edexcel S1 Jan 2005 Q5] Articles made on a lathe are subject to three kinds of defect, *A*, *B* or *C*. A sample of 1000 articles was inspected and the following results were obtained.

31 had a type *A* defect
37 had a type *B* defect
42 had a type *C* defect
11 had both type *A* and type *B* defects
13 had both type *B* and type *C* defects
10 had both type *A* and type *C* defects
6 had all three types of defect.

(*a*) Draw a Venn diagram to represent this. **(6)**
Find the probability that a randomly selected article from this sample had
(*b*) no defects, **(1)**
(*c*) no more than one of these defects. **(2)**

An article selected at random from this sample had only one defect.
(*d*) Find the probability that it was a type *B* defect. **(2)**

Two different articles were selected at random from this sample.
(*e*) Find the probability that both had type *B* defects. **(2)**

☠ *[SMC 2011 Q17]* Jamie conducted a survey on the food preferences of pupils at a school and discovered that 70% of the pupils like pears, 75% like oranges, 80% like bananas and 85% like apples. What is the smallest possible percentage of pupils who like all four of these fruits?

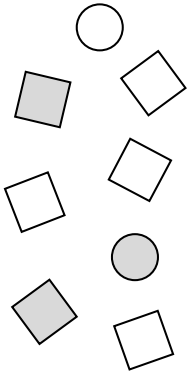
A. At least 10% B. At least 15%
C. At least 20% D. At least 25%
E. At least 70%

Fluency Practice

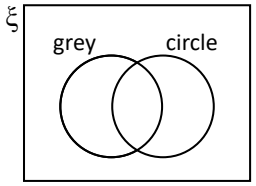
Creating Venns

For each *population* of shapes, complete the 4 Venn diagrams.

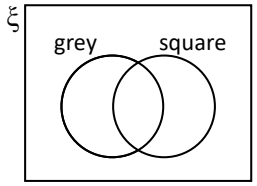
1



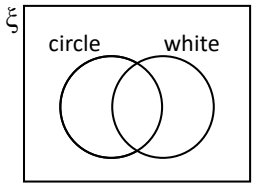
a



b



c



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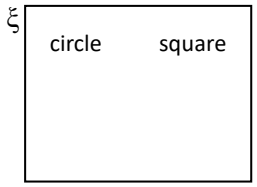

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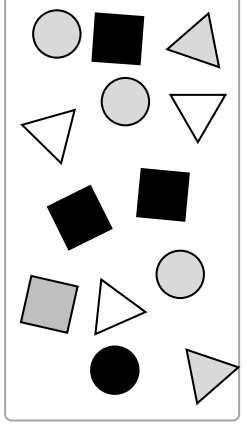
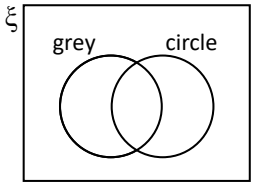
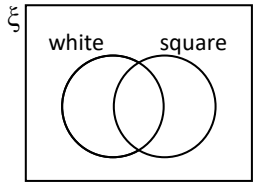


a

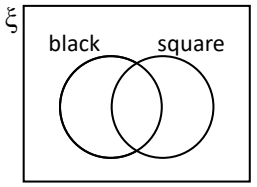


b



Is this a correct diagram?

c



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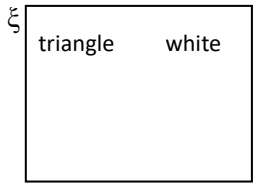

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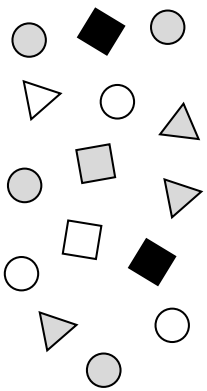
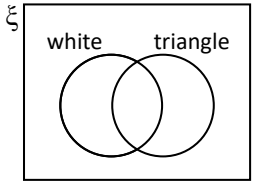


a



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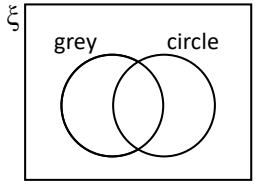
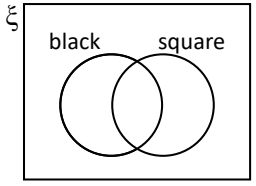


c



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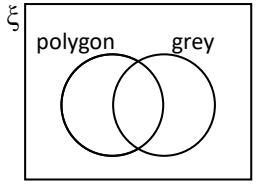

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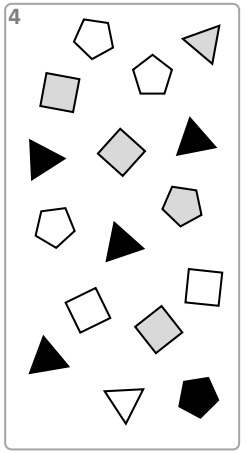
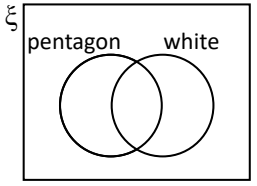


a



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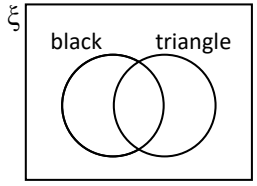
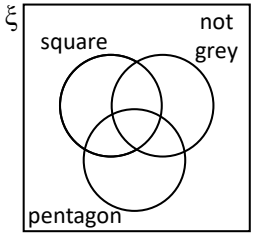


c



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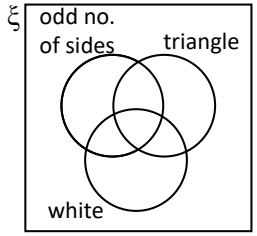


☆ How else could we draw these last two Venn diagrams? ☆

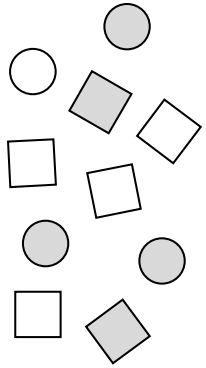
Fluency Practice

Creating Venns

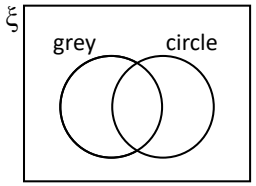
Each diagram shows a different bag of coloured shapes.
Complete the Venn diagrams & use them to calculate each probability.

If a shape is picked from this bag…

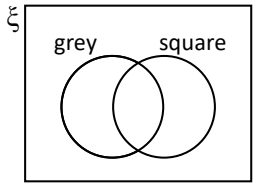
1



a



b



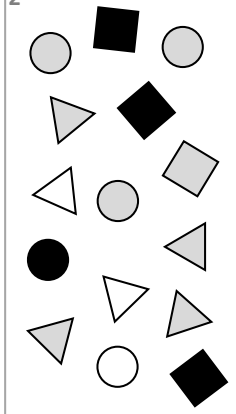
What is…

P(grey shape) =

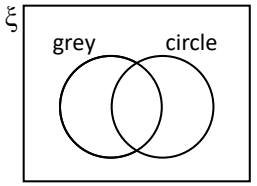
P(grey circle) =

P(grey square) =

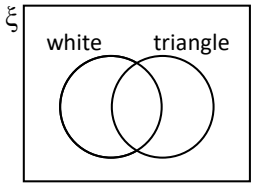
2



a



b



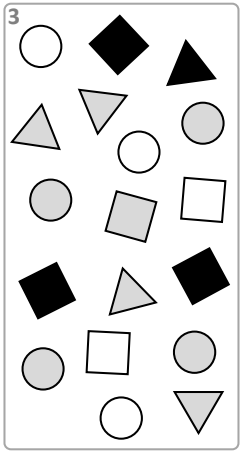
What is…

P(grey circle) =

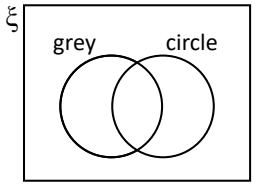
P(not a white triangle) =

P(not grey and not a circle) =

3



a



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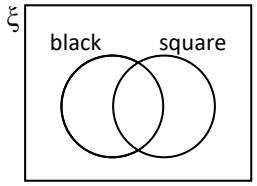


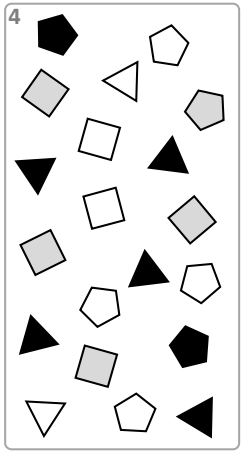
What is…

P(a grey circle) =

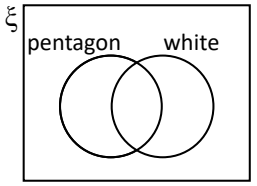
P(not a black square) =

A square has been picked from the bag.
What is the probability it is black?

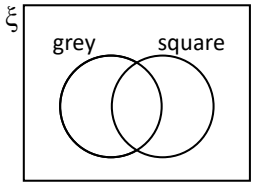
4



a



b



As a percentage, what is…

P(not a white pentagon) =

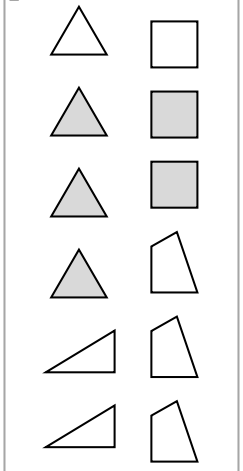
P(a grey shape is a square) =

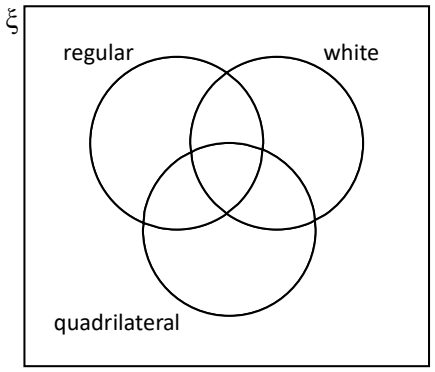
P(a square is not grey) =

Fluency Practice

Shape Venns

1



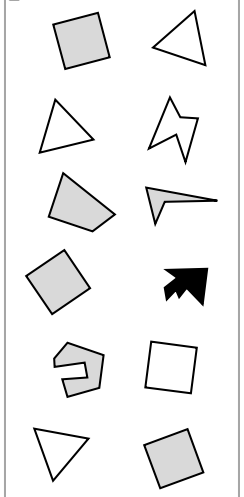


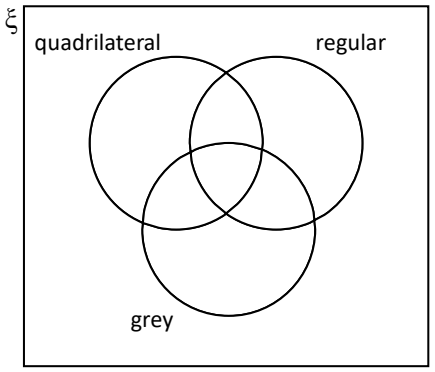
If a shape is picked randomly, what is:

P(a regular white quadrilateral)

P(a white quadrilateral)

2



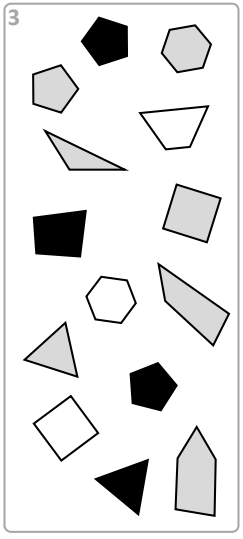


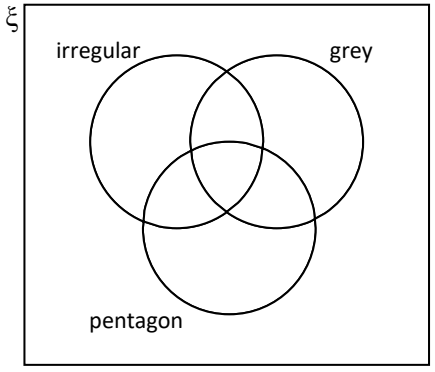
If a shape is picked randomly, what is:

P(a irregular grey quadrilateral)

P(a regular non-grey shape)

3

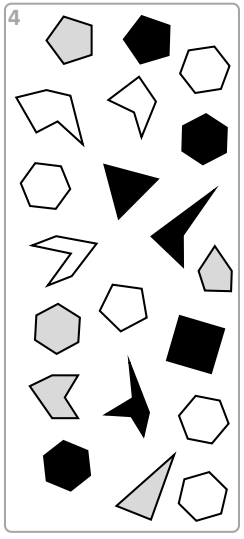


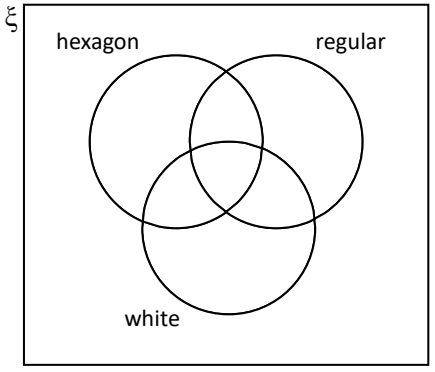


A pentagon has been picked,
what is P(it is grey)?

A grey shape has been picked,
what is P(it is an irregular pentagon)?

4



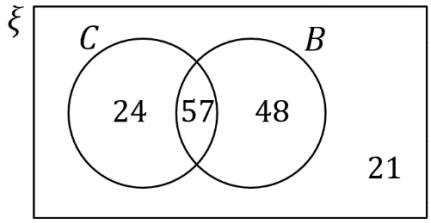


What is…

P(the shape is white, given that it is regular)?

P(an irregular shape is not white)

The Venn diagram shows information of 150 patients in a local surgery. They were asked if they took any medication for cholesterol (C) or blood pressure (B).



A patient is chosen at random.
(a) Work out the probability that a patient took neither medication.
(b) Work out the probability that a patient took cholesterol not but blood pressure medication.
(c) Given that the patient took blood pressure medication, what is the probability that they also took cholesterol medication?

90 people in a sports club were surveyed. 19 play tennis and squash. 50 play tennis. 32 play squash.
(a) Represent this with a Venn diagram.
One person is chosen at random.
(b) Work out the probability that the person chosen does not play tennis
(c) Work out the probability that the person chosen plays tennis or squash or both.
(d) Given that the person plays tennis, work out the probability that they also play squash.

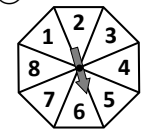
In a group of 40 children there are 19 who can swim and 16 who can ride a bike. There are 5 children who can swim and ride a bike.
(a) Draw a Venn diagram.
A child is selected at random.
(b) Find the probability that this child cannot swim or ride a bike.
Another child is selected at random.
(c) Given that this child can ride a bike, work out the probability that this child can swim.

90 children were asked they had in their packed lunch. Their replies are as follows: 38 had sandwich, crisps and cake. 60 had sandwich and a cake. 52 had sandwich and crisps. One student had crisps and cake only, and 5 students had a sandwich only. 10 students had none of these items in their packed lunch.
(a) Show this on a Venn diagram.
(b) Find the probability that a child chosen at random has both crisps and cake in their packed lunch.
(c) Given that a child had a sandwich, find the probability that this child also had crisps.

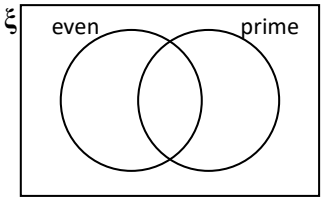
100 people were asked which sports they watched on television. Here are the results.
36 people watched cricket, 28 people watched rugby, 36 people watched football, 17 people watched both cricket and rugby, 19 people watched both cricket and football, 15 people watched both rugby and football, and 10 people watched all three sports.
(a) Draw a Venn diagram.
(b) One of the 100 people is selected at random. Find the probability that they watch none of these sports.
(c) Given that a person watches cricket, find the probability that this person also watches football.
(d) Given that a person watches at least one of the sports, find the probability that this person watches all three.

Fluency Practice

The General Addition Law

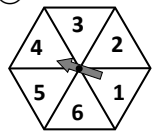
① 

A fair spinner.

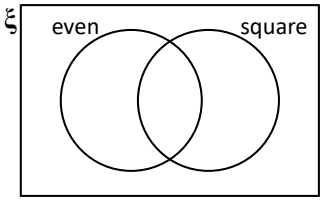


P(even) = P(prime) =

P(even or prime) =

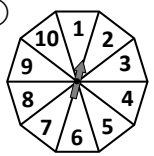
② 

A fair spinner.

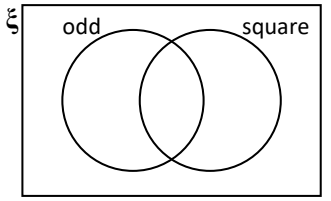


P(even) = P(square) =

P(even or square) =

③ 

A fair spinner.

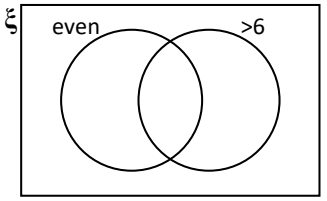


P(odd) = P(square) =

P(even or square) =

④ 

10 cards: 1 to 10

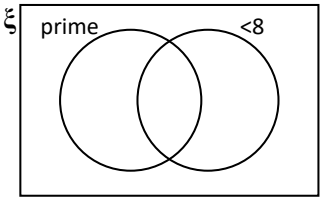


P(even) = P(greater than 6) =

P(even or greater than 6) =

⑤ 

15 cards: 1 to 15

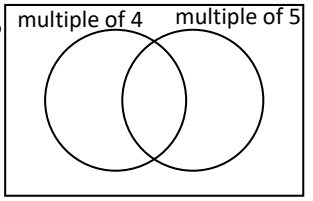


P(prime) = P(less than 8) =

P(prime or less than 8) =

⑥ 

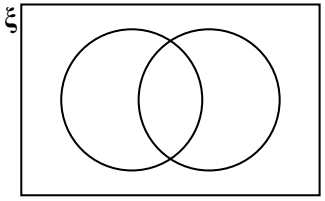
20 cards: 1 to 20
You can complete the Venn diagram with *totals*.



P(multiple of 4) = P(multiple of 5) =

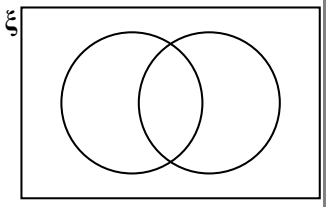
P(multiple of 4 or multiple of 5) =

⑦ A bag contains 50 spheres with the numbers 1 to 50.



P(multiple of 7 or even) =

⑧ A bag contains 100 spheres with the numbers 1 to 100.



P(multiple of 4 or a multiple of 10) =

⑨ 
30 students:
12 with blonde hair
13 with glasses
10 students without blonde hair or glasses

If we pick a student that is blonde, what is the probability they wear glasses?

Fluency Practice

conditional probability
and venn diagrams

(1)

30 students in a class
20 have a brother
16 have a sister
12 have a brother but not a sister

what is the probability of picking a student at random that has a brother, given that they have a sister?

(2)

a survey of 125 students
asked whether they have a cat or a dog in their home
61 had a cat
68 had a dog
13 had neither

what is the probability of picking a student at random that has a cat, given that they have a dog?

(3)

50 students
went to an activity camp
38 did climbing
13 did climbing and canoeing
4 did neither climbing or canoeing

what is the probability of picking a student at random that did canoeing, given that they did not do climbing?

(4)

a discrete survey of 90 students
asked if they have a laptop or a tablet, or both, or neither
40 had a laptop
57 had a tablet
23 had neither

what is the probability of picking a student at random that has a laptop, given that they own just one of these devices?

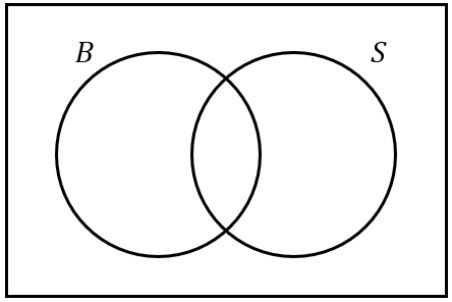
Fluency Practice

Venn Diagrams with Algebra

(a)

Some children are asked about whether they have any sisters or brothers. Six children said they had both sisters and brothers. Eight children said they had no sisters or brothers. Seven children said they had brothers but no sisters. x children said they had sisters.

(i) Complete the Venn diagram, giving numbers in terms of x where needed.

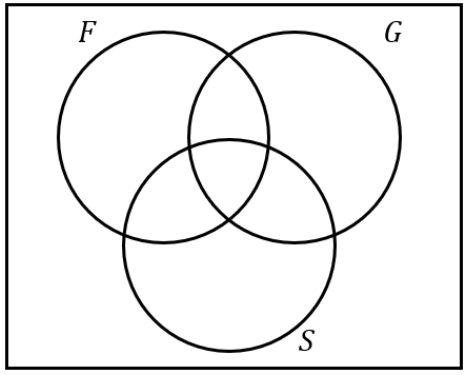


(ii) Given that there are 30 children in total, find the value of x using algebra.

(b)

Some students are asked whether they study French, German or Spanish. Four students study all three languages. Six study both French and German, and x students study French and Spanish. No students study both German and Spanish but not French. 21 students study French. 15 students study Spanish. 18 students study German. All students must study at least one language.

(i) Complete the Venn diagram, giving numbers in terms of x where needed.

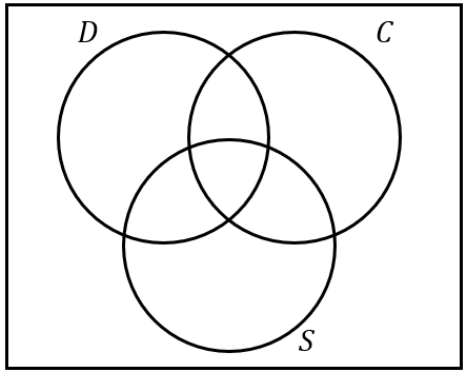


(ii) Given that there are 40 students in total, find the value of x using algebra.

(c)

Some TV viewers are asked whether they enjoy drama, comedy or sport programmes. Six viewers enjoy none of them. x viewers enjoy all three programmes. 15 viewers enjoy both drama and sport. 30 viewers enjoy both comedy and drama. 21 viewers enjoy both sport and comedy. 43 viewers enjoy comedy. 56 viewers enjoy sport. 38 viewers enjoy drama.

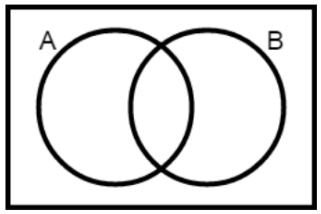
(i) Complete the Venn diagram, giving numbers in terms of x where needed.



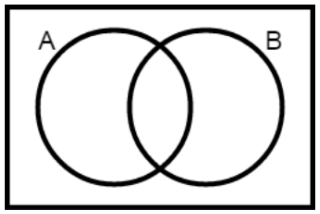
(ii) Given that there are 90 viewers in total, find the value of x using algebra.

Fluency Practice

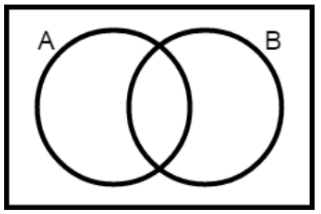
shade the regions corresponding to the sets



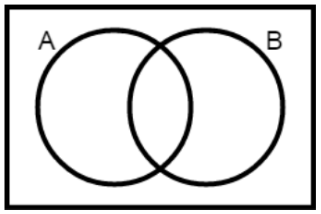
A



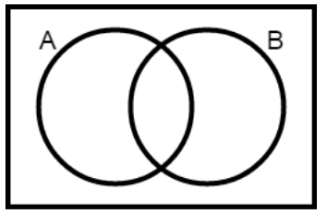
A ∪ B



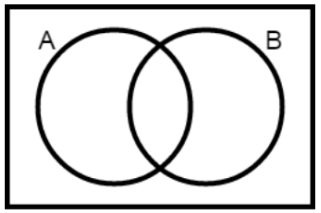
A ∩ B



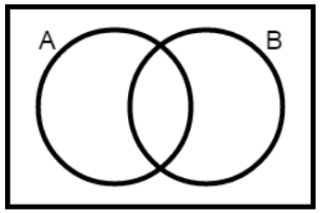
B'



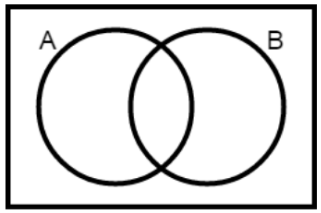
A ∪ B'



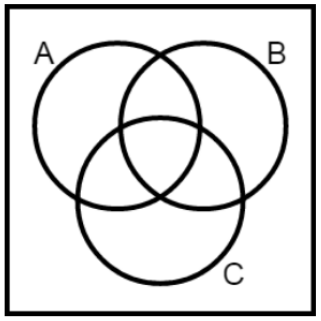
A' ∩ B



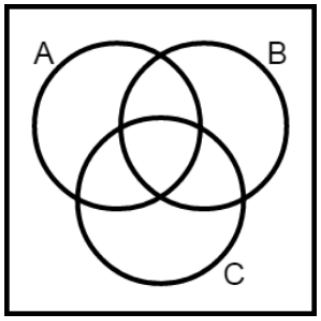
A' ∩ B'



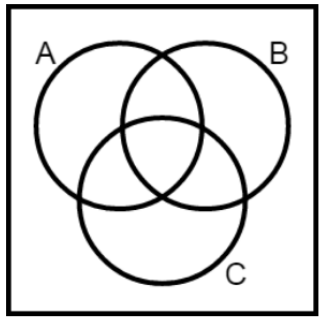
(A ∪ B)'



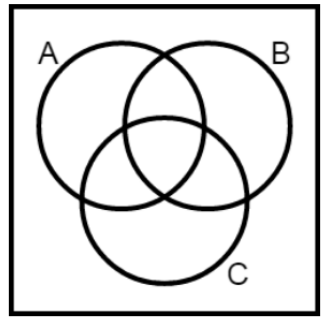
A ∩ C



C'



B ∪ C



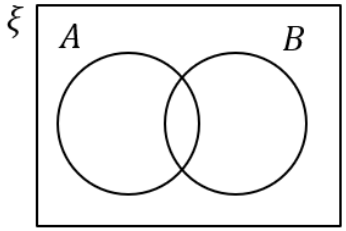
A ∩ C'

Fluency Practice

Shading Two Set Venn Diagrams

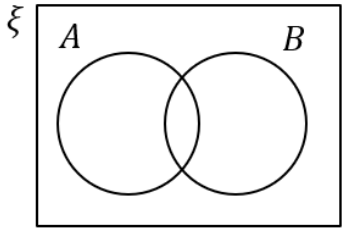
(a)

Shade A'



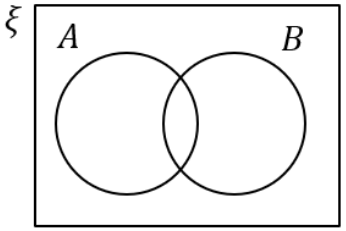
(b)

Shade $A \cap B$



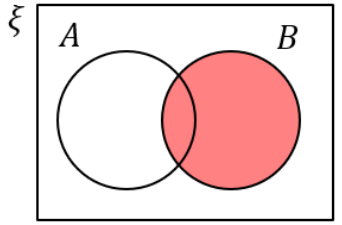
(c)

Shade $A \cup B$



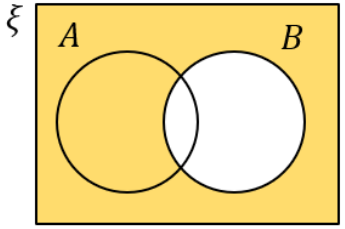
(d)

Describe the region shaded



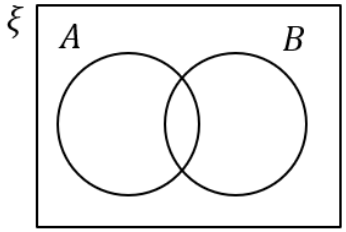
(e)

Describe the region shaded



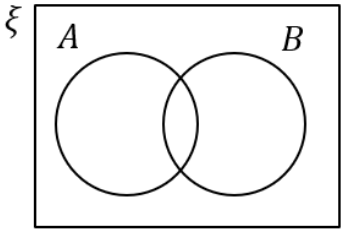
(f)

Shade $A \cap B'$



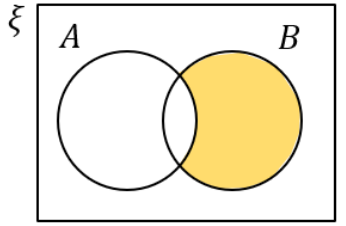
(g)

Shade $A' \cap B'$



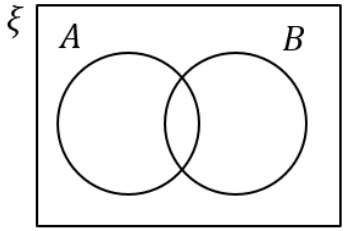
(h)

Describe the region shaded



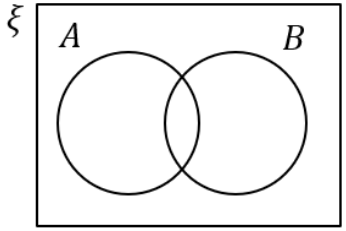
(i)

Shade $A' \cup B$



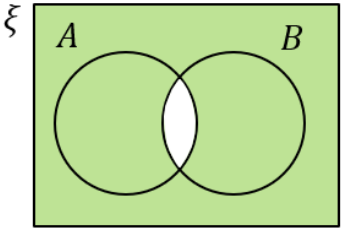
(j)

Shade $(A \cup B)'$



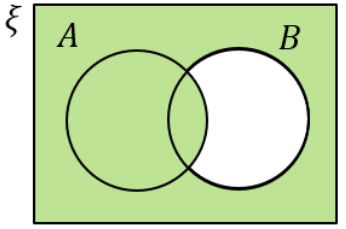
(k)

Describe the region shaded



(l)

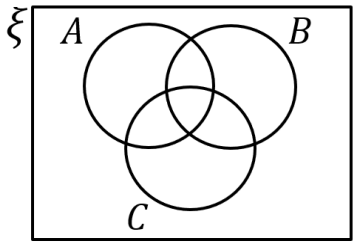
Describe the region shaded



Shading Three Set Venn Diagrams

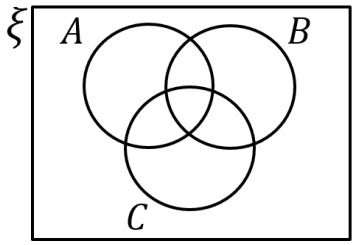
(a)

Shade the region corresponding to
$A \cap B$



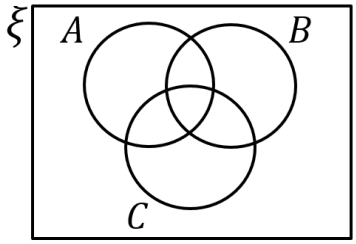
(b)

Shade the region corresponding to
$(B \cup C)'$



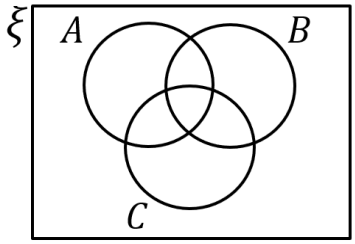
(c)

Shade the region corresponding to
$A' \cap B$



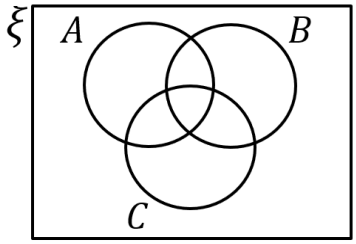
(d)

Shade the region corresponding to
$(A \cap B \cap C)'$



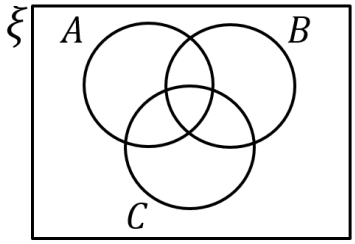
(e)

Shade the region corresponding to
$(A \cup B \cup C)'$



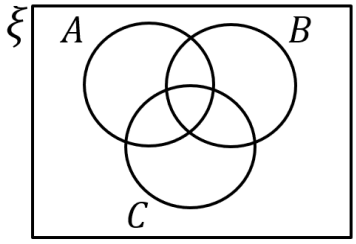
(f)

Shade the region corresponding to
$(A \cap C) \cap B'$



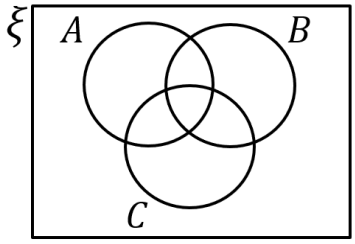
(g)

Shade the region corresponding to
$A' \cap (B \cup C)$



(h)

Shade the region corresponding to
$(B \cup C) \cup A'$



Describe these sets in words.

(a) {4, 8, 12, 16, 20, 24, 28}

(b) {1, 4, 9, 16, 25}

(c) {Europe, Asia, Africa,…}

(d) {1, 2, 3, 4, 6, 12}

List the elements of the sets:

(a) Multiples of 7 less than 30

(b) Months of the year

(c) Factors of 25

$A = \{factors\ of\ 6\}$

$B = \{prime\ numbers\ less\ than\ 20\}$

$C = \{integers\ from\ 1\ to\ 10\}$

(a) List the elements of A

(b) List the elements of B

(c) List the elements of C

(d) Write down $A \cup B$

(e) Write down $B \cup C$

(f) Write down $A \cap B$

(g) Write down $C \cap A$

$A = \{factors\ of\ 6\}$

$B = \{prime\ numbers\ less\ than\ 20\}$

$C = \{integers\ from\ 1\ to\ 10\}$

State whether the following are true or false?

(h) $9 \in C$

(i) $9 \in B$

(j) $9 \in A$

(k) $6 \notin C$

(l) $19 \in B$

(m) $1 \notin C$

Set Notation

A ∪ B	**Union**: the combination of **A** and **B**.
A ∩ B	**Intersection**: The overlap of **A** and **B**.
A′	**Complement**: Not in **A**.

ξ = { 1 to 25 inclusive }

A = { Factors of 12 } = { 1, 2, }

B = { Factors of 18 } =

C = { Factors of 32 } =

D = { Prime Numbers } =

E = { Multiples of 4 } =

F = { Square numbers } =

A ∪ B = { 1, 2, }

A ∪ F =

A ∩ B =

A ∩ D =

D′ =

A ∪ D′ =

B ∩ D′ =

B ∩ C ∩ F =

B ∪ C ∪ E =

A ∩ B ∩ C =

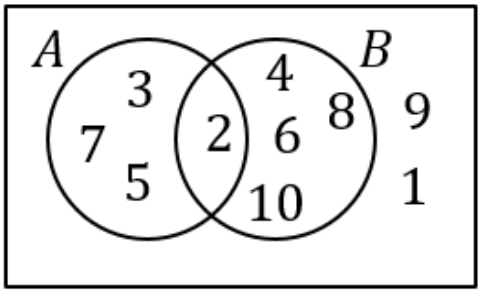
A ∪ B ∩ D′ =

$\xi = \{Integers\ from\ 1\ to\ 12\}$
$A = \{multiples\ of\ 2\}$
$B = \{multiples\ of\ 3\}$

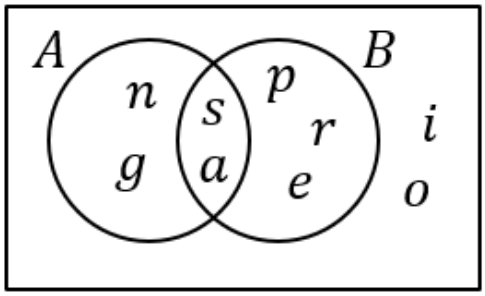
(a) Show these sets on a Venn diagram
(b) List the elements of $A \cap B$
(c) List the elements of A'
(d) List the elements of $A' \cup B$

$\xi = \{Integers\ from\ 1\ to\ 10\}$
$A = \{Integers\ five\ or\ less\}$
$B = \{factors\ of\ 12\}$

(a) Show these sets on a Venn diagram
(b) List the elements of $A \cup B$
(c) List the elements of B'
(d) List the elements of $A' \cap B'$



(a) List the elements of $A \cap B$
(b) List the elements of $(A \cap B)'$
(c) List the elements of B'



(a) List the elements of $A \cup B$
(b) List the elements of $(A \cup B)'$
(c) List the elements of $A' \cap B'$

Fluency Practice

Sets and Venns Revision

(a)

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 2, 3, 4, 5, 6\}$
$B = \{even\ numbers\}$
List the members of $A \cap B$

(b)

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{factors\ of\ 9\}$
$B = \{multiples\ of\ 4\}$
List the members of $A \cup B$

(c)

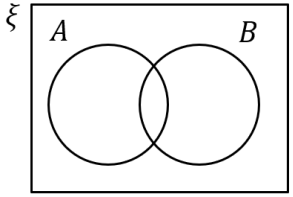
$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{factors\ of\ 9\}$
$B = \{multiples\ of\ 4\}$
Anna says that $A \cap B = \emptyset$. Is she correct?

(d)

$A = \{1, 3, 5, 7, 9\}$
$A \cap B = \{1, 3\}$
$A \cup B = \{0, 1, 2, 3, 4, 5, 7, 9\}$
List the members of B

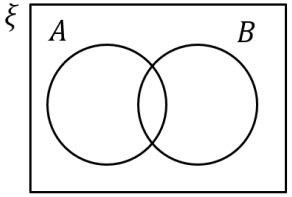
(e)

Shade the region which represents $A \cap B'$



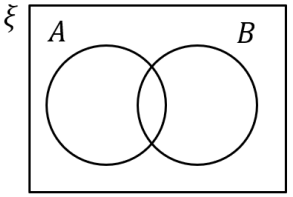
(f)

Shade the region which represents $A' \cup B$



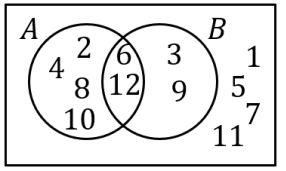
(g)

Show in a Venn diagram.
$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 4, 9, 10\}$
$B = \{2, 4, 6, 8, 10\}$



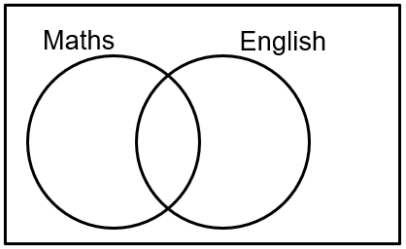
(h)

List the members of B' and $A' \cap B'$



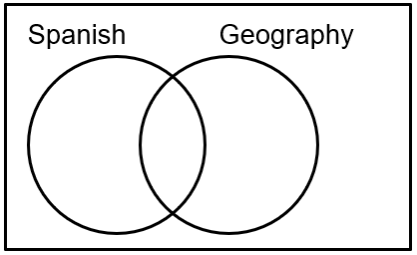
(i)

In a group of 20 students, 11 like Maths and 10 like English. 2 like neither subject.
(a) Complete the Venn diagram.
(b) How many students like Maths but not English?



(j)

There are 32 students in a class. 21 students like Spanish and 15 like Geography. There are twice as many students who like both subjects as like neither.
(a) Complete the Venn diagram.
(b) How many students like only Spanish?



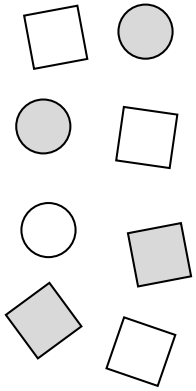
Fluency Practice

Creating Venns

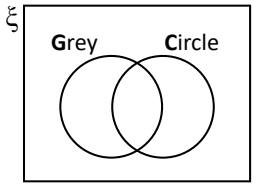
Each diagram shows a different bag of coloured shapes.
Complete the Venn diagrams & use them to calculate each probability.
Shade the diagrams to show the areas defined by the notation.

A shape is picked from this bag…

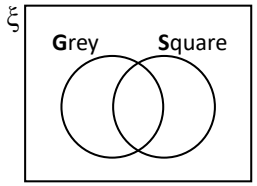
1



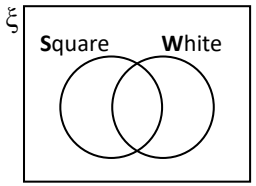
a



b



c

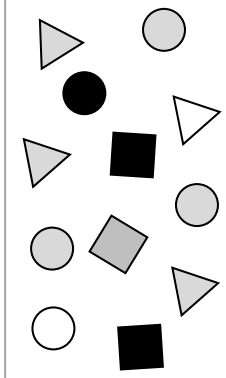


P(G ∪ C) =

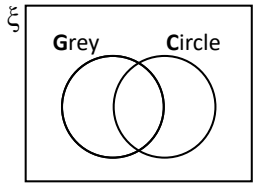
P(G ∪ S) =

P(S ∪ W) =

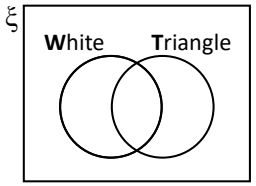
2



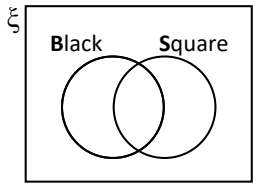
a



b



c

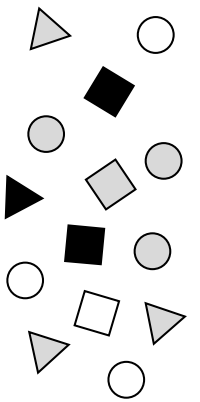


P(G ∩ C) =

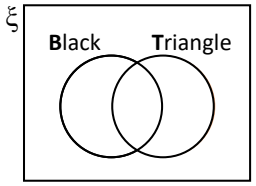
P(W ∩ T) =

P(B ∩ S) =

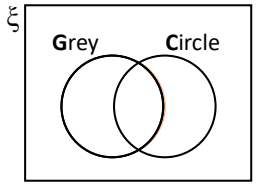
3



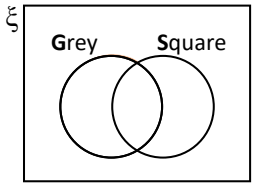
a



b



c

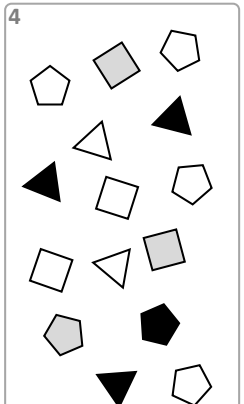


P(T′) =

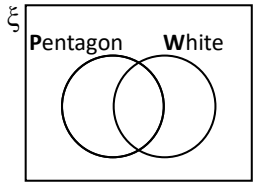
P(G′) =

P(S ∪ G′) =

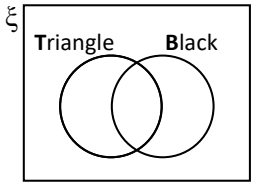
4



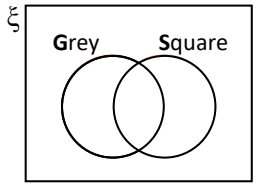
a



b



c



P(P ∪ W) =

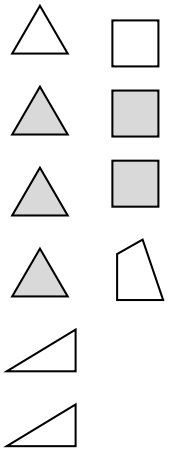
P(T ∩ B) =

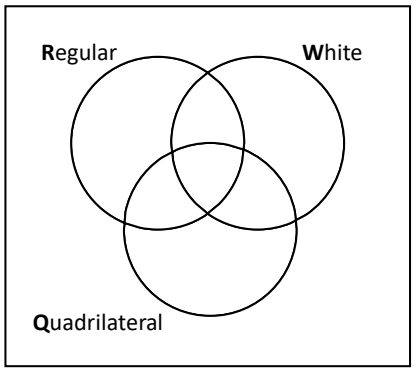
P(G′ ∩ S′) =

Fluency Practice

Shape Venns

1



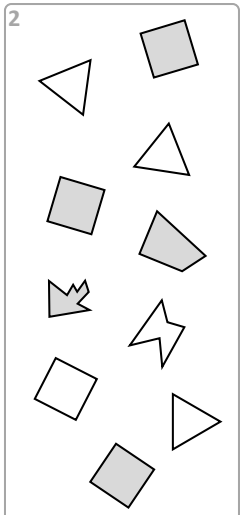


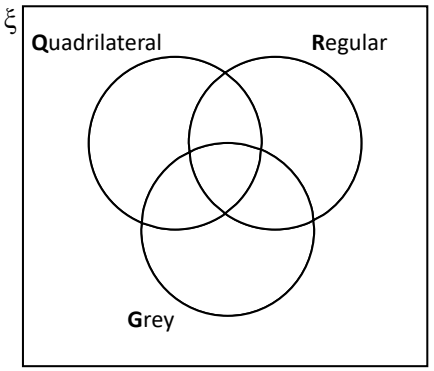
Shade **R**∩**W**∩**Q**

If a shape is picked randomly, what is:

P(**W**') =

2



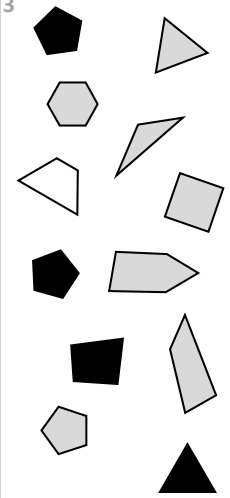


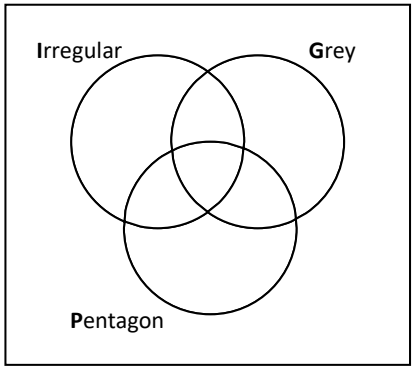
Shade **Q**∪**R**

If a shape is picked randomly, what is:

P(**Q**∪**R**∪**G**) =

3

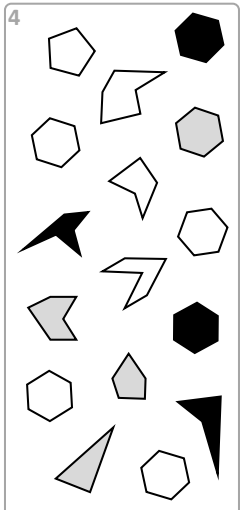


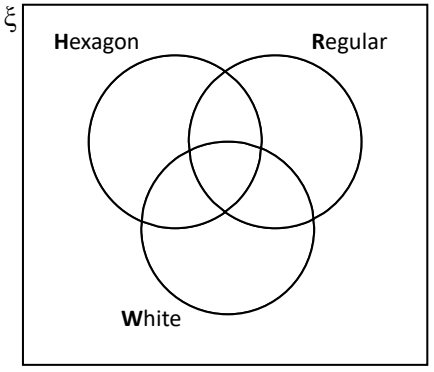


Shade **G**∩**P**

What is the probability of randomly picking a shape in the set (**G**∩**P**)' ?

4





Shade **R**∪**W**

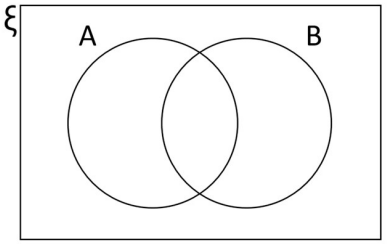
What is the probability of randomly picking a shape in the set (**H**∪**W**)' ?

Fluency Practice

①

ξ = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
A = {1, 2, 3, 4, 5, 9, 10}
B = {3, 5, 6, 7, 10}

Complete the Venn diagram.



If I pick a number at random what is…

P(A) =

P(A AND B) = P(A ∩ B) =

P(NOT A) = P(A') =

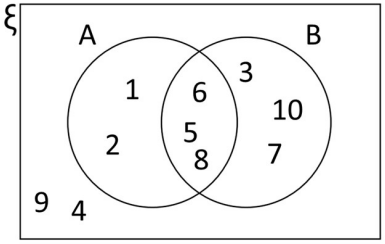
P(A OR B) = P(A ∪ B) =

P(B') =

P(A OR NOT B) = P(A ∪ B') =

②

Remember!
A' = **NOT** A
A ∩ B = A **AND** B
A ∪ B = A **OR** B



True or False?

$P(A) = \frac{1}{2}$ T / F

$P(A \cap B) = \frac{3}{10}$ T / F

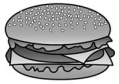
$P(B) = \frac{4}{5}$ T / F

$P(A \cup B) = \frac{7}{10}$ T / F

$P(A') = \frac{4}{10}$ T / F

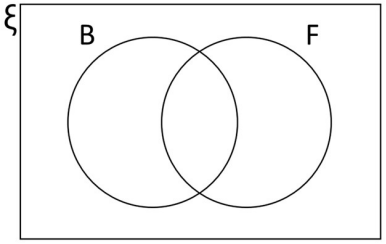
$P(A \cup B') = \frac{2}{5}$ T / F

③



50 students were asked about their lunch.

25 students had a burger (B)
35 students had fries (F)
15 students had both.



Complete the Venn diagram.
How many students had neither F or B?
If a student was chosen at random, what is…

P(B ∩ F) =

P(B) =

P(F ∪ B) =

P(F) =

P(B') =

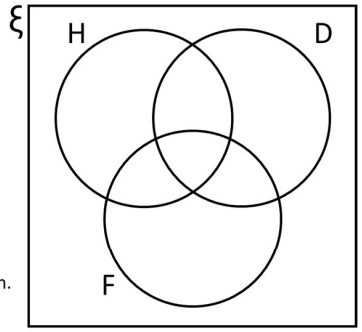
P(F' ∩ B') =

④

100 students were asked about their school subjects:

15 studied History (H), French (F) and Drama (D).
40 studied Drama and History.
25 studied History and French.
55 studied History.
22 studied French and Drama.
3 studied only Drama.
40 studied French.

Complete the Venn diagram with this information.



P(H) =

P(D') =

P(D ∩ F) =

P(H ∪ D) =

P(F') =

P(H ∩ F ∩ D) =

P(D ∪ F) =

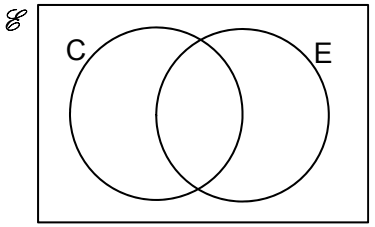
venn diagrams 5

1. 60 people were asked if they read either of two local newspapers, the Chronicle and the Echo.

12 people read both newspapers.
36 people read the Chronicle.
25 people read the Echo.

(a) Write the number of people who belong in each section of the Venn diagram.

(b) What proportion of people read neither newspaper?

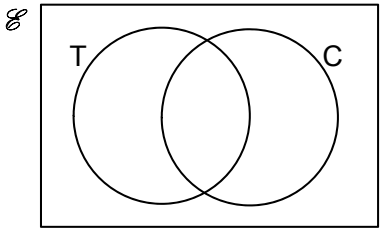


2. 85 people were asked about whether they like tea or coffee.

67 like at least one of the drinks.
29 like tea, but not coffee.
22 like coffee, but not tea.

(a) Write the number of people who belong in each section of the Venn diagram.

(b) One of the people is chosen at random. Work out P(T ∩ C) =



3. At a school, all pupils study French, Spanish or both.

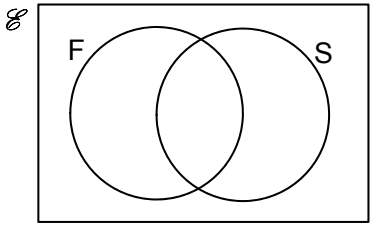
In a class of 30 pupils 21 pupils study French and 13 pupils study Spanish.

(a) Write the number of people who belong in each section of the Venn diagram.

(b) A pupil is picked at random from the class.
Work out:

P(F ∩ S) =

P(F') =

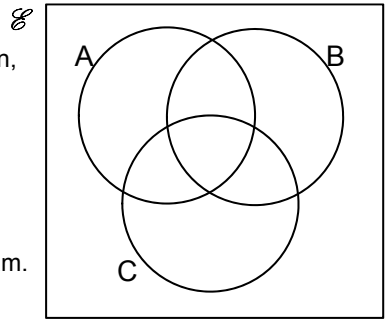


4. Students at a college can take part in different extra-curricular activities, including Aerobics classes, Badminton and Chess club.

In a group of 140 students,
7 do all three activities,
34 do aerobics and badminton,
22 do aerobics and chess,
19 do badminton and chess,
76 do aerobics,
62 do badminton,
46 do chess.

(a) Complete the Venn diagram.

(b) How many students do none of the three activities?



venn diagrams 6

1. The proportion of elements in three of the sections of a Venn diagram are shown.

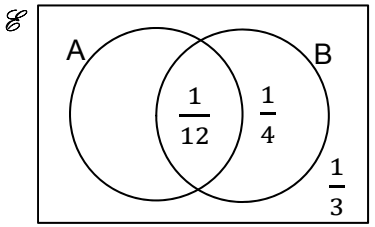
An element is chosen at random. Work out:

(a) P(A ∪ B) =

(b) P(B) =

(c) P(A') =

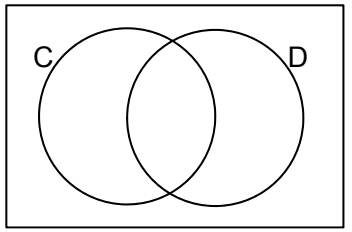
(d) P(B') =



2. In a survey, people were asked about their pets.

34% had a pet cat (C)
42% had a pet dog (D)
11% had both a cat and a dog.

(a) Write the proportion of people who belong in each section of the Venn diagram.



(b) One of the people is chosen at random. Work out the probability that they have a cat, given that they have a dog.

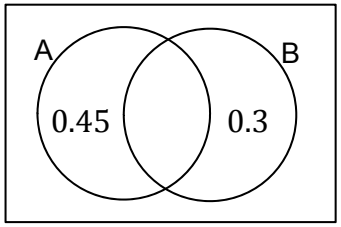
3. The proportion of elements in two of the sections of a Venn diagram are shown.

A fifth of the elements are in neither A nor B.

(a) Complete the Venn diagram.

(b) One of the elements is chosen at random.

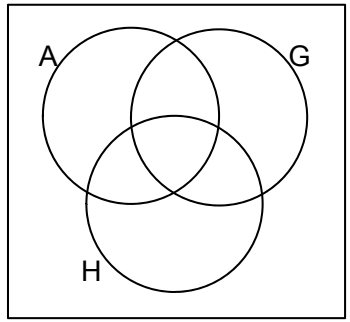
Work out the probability that the element is in A, given that it is not in B.



4. Students at a school can choose to study GCSEs in Art, Geography and History.

In a group of students,
12% study all of the subjects,
19% study Art and Geography,
23% study Art and History,
27% study Geography and History,
40% study Art,
54% study Geography,
2% study none of the subjects.

(a) Complete the Venn diagram.



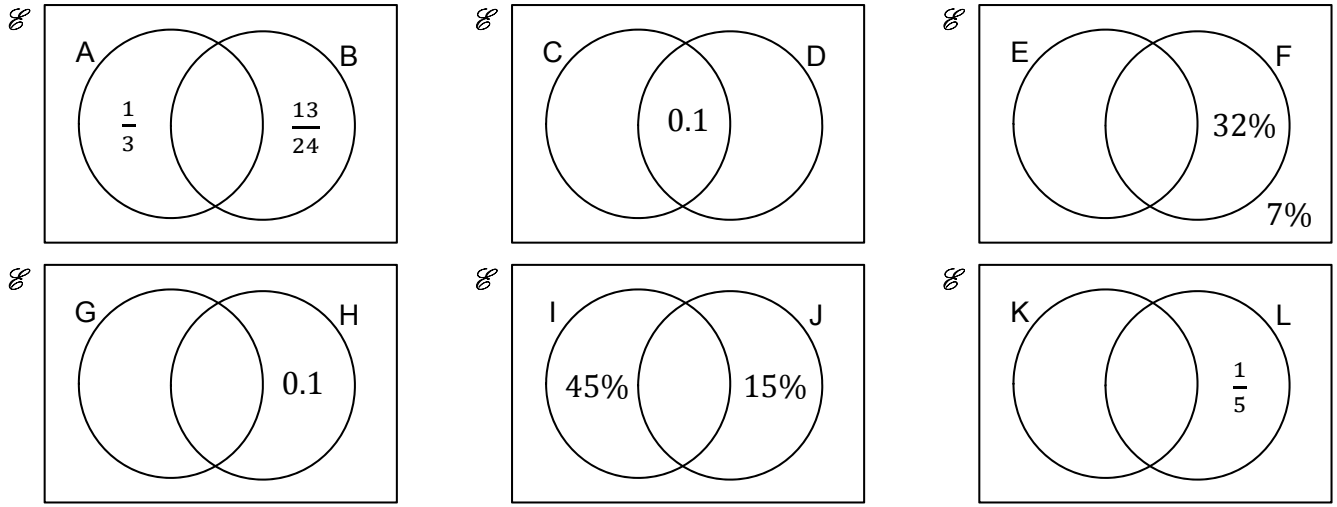
(b) What proportion of the Art students also study History?

venn diagrams 7

The numbers in the Venn diagrams show the proportions of elements that belong in each section. Use the clues to complete them.

$P(A \cap B) = \frac{1}{8}$ | $P(C) = 0.5$ | $P(C \cup D) = 0.6$ | $P(E') = P(F)$ | $P(G \cap H) = 0$

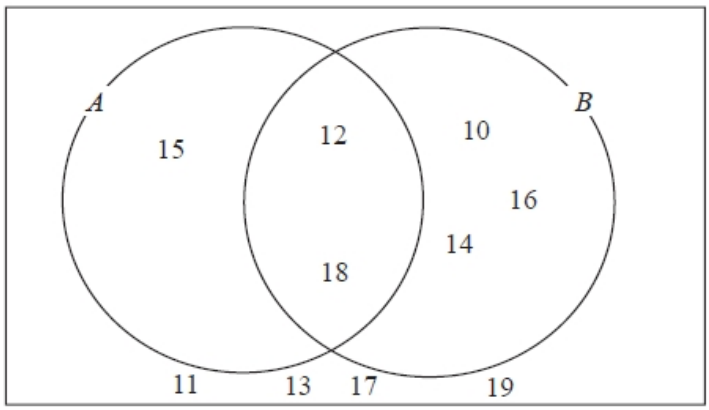
$P(G) = 0.3$ | $P(I \cup J) = P(I) + P(J)$ | $P(K) = P(K')$ | $P(L) : P(L') = 2 : 3$



Put a tick in the **5** correct boxes.

	A & B	C & D	E & F	G & H	I & J	K & L
Exhaustive						
Mutually Exclusive						
Independent						

Q1.
Here is a Venn diagram.



(a) Write down the numbers that are in set

(i) $A \cup B$

.....

(ii) $A \cap B$

.....

(2)

One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set A'

.....

(2)

(Total for question = 4 marks)

Q2.
Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 people like at least one of the drinks
19 people like all three drinks.
16 people like tea and coffee but do **not** like milk.
21 people like coffee and milk.
24 people like tea and milk.
40 people like coffee.
1 person likes only milk.

Sami selects at random one of the 50 people.

(a) Work out the probability that this person likes tea.

.....

(4)

(b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

.....

(2)

(Total for question = 6 marks)

Q3.
There are 80 students at a language school.

All 80 students speak at least one language from French, German and Spanish.

9 of the students speak French, German and Spanish.
19 of the students speak French and German.
28 of the students speak French and Spanish.
17 of the students speak Spanish and German.
45 students speak French.
50 students speak Spanish.

(a) Draw a Venn diagram to show this information.

(3)

One of the 80 students is selected at random.

(b) Find the probability that this student speaks German but not Spanish.

(1)

Given that the student speaks German,

(c) find the probability that this student also speaks French.

(2)

(Total for question = 6 marks)

Exam Questions

The following results were obtained from a survey concerning the reading habits of students.

60 % read magazine P
50 % read magazine Q
50 % read magazine R
30 % read magazines P and Q
20 % read magazines Q and R
30 % read magazines P and R
10 % read all three magazines

(a) Represent all of this information on a **Venn diagram**.

(b) What percentage of students read exactly two magazines?

(c) What percentage of students read at least two magazines?

(d) What percentage of students do not read any of the magazines?

A group of 30 children are surveyed to find out which of the three sports cricket (C), basketball (B) or volleyball (V) they play. The results are as follows:

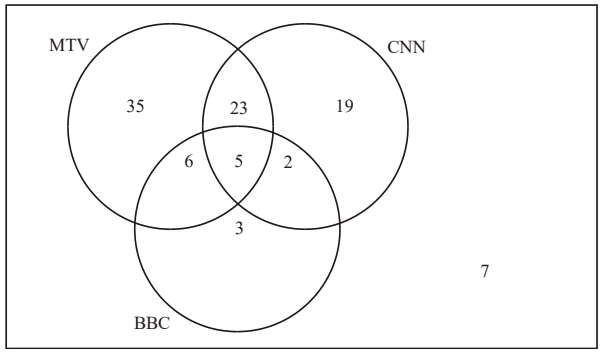
3 children do not play any of these sports
2 children play all three sports
6 play volleyball and basketball
3 play cricket and basketball
6 play cricket and volleyball
16 play basketball
12 play volleyball.

(a) Draw a **Venn diagram** to illustrate the relationship between the three sports played.

(b) On your Venn diagram indicate the number of children that belong to each region.

(c) How many children play only cricket?

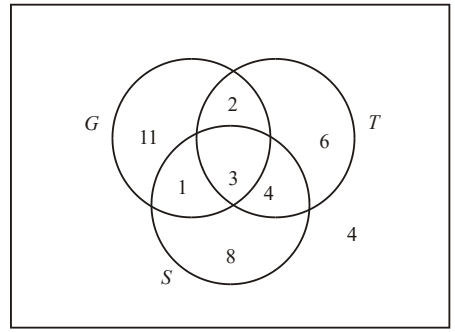
100 students were asked which television channel (MTV, CNN or BBC) they had watched the previous evening. The results are shown in the Venn diagram below.



From the information in the **Venn diagram**, write down the number of students who watched

(a) both MTV and BBC;
(b) MTV or BBC;
(c) CNN and BBC but not MTV;
(d) MTV or CNN but not BBC.

The sports offered at a retirement village are Golf (G), Tennis (T) and Swimming (S). The Venn diagram shows the numbers of people involved in each activity.



(a) How many people
  (i) only play golf?
  (ii) play both tennis and golf?
  (iii) do not play golf?

(b) Shade the part of the **Venn diagram** that represents the set G' ∩ S.

In a club with 60 members, everyone attends either on Tuesday for Drama (D) or on Thursday for Sports (S) or on both days for Drama and Sports.

One week it is found that 48 members attend for Drama and 44 members attend for Sports and x members attend for both Drama and Sports.

(a) (i) Draw and label fully a **Venn diagram** to illustrate this information.

(ii) Find the number of members who attend for both Drama and Sports.

(iii) Describe in words the set represented by $(D \cap S)'$.

(iv) What is the **probability** that a member selected at random attends for Drama only or Sports only?

The club has 28 female members, 8 of whom attend for both Drama and Sports.

(b) What is the **probability** that a member of the club selected at random

(i) is female and attends for Drama only or Sports only?

(ii) is male and attends for both Drama and Sports?

The sets A, B and C are subsets of U. They are defined as follows:

$\mathcal{E}$ = {positive integers less than 16}
A = {prime numbers}
B = {factors of 36}
C = {multiples of 4}

(a) List the elements (if any) of the following:
(i) A;
(ii) B;
(iii) C;
(iv) $A \cap B \cap C$.

(b) (i) Draw a **Venn diagram** showing the relationship between the sets U, A, B and C.
(ii) Write the elements of sets U, A, B and C in the appropriate places on the **Venn diagram**.

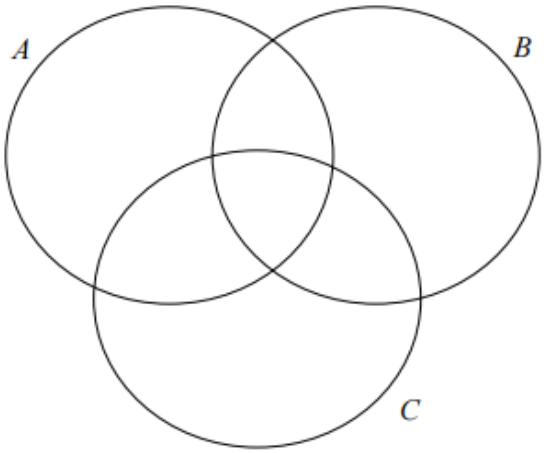
(c) From the Venn diagram, list the elements of each of the following
(i) $A \cap (B \cup C)$;
(ii) $(A \cap B)'$;
(iii) $(A \cap B)' \cap C$.

(d) Find the **probability** that a number chosen at random from the universal set $\mathcal{E}$ will be
(i) a prime number;
(ii) a prime number, but **not** a factor of 36;
(iii) a factor of 36 or a multiple of 4, but **not** a prime number;
(iv) a prime number, given that it is a factor of 36.

The sets A, B, and C are as follows:

$A = \{1, 2, 3, 5, 6, 7\}$ $B = \{2, 3, 4, 5, 8, 9\}$ $C = \{1, 4, 5, 10\}$.

(a) Complete the Venn diagram below.



(b) List the elements of each of the following sets.

$A \cup B =$ _____

$A \setminus C =$ _____

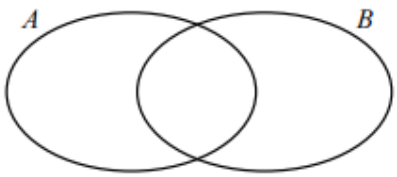
$A \cup (B \cap C) =$ _____

(c) Complete the following identity.

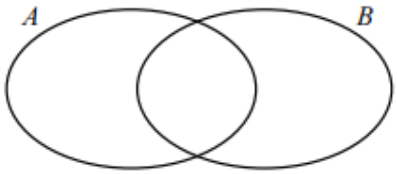
$A \cup (B \cap C) = (A \cup B) \cap$ (_____)

Question 2 **(Suggested maximum time: 5 minut**

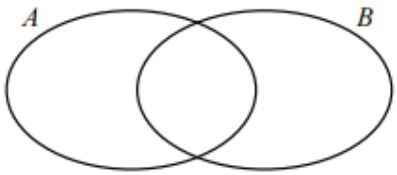
(a) On the Venn diagram below, **shade in** the region that represents $A \cap B$.



(b) On the Venn diagram below, **shade in** the region that represents $A \cup B$.



(c) On the Venn diagram below, **shade in** the region that represents $(A \cup B) \setminus (A \cap B)$.



(d) Put a tick (✓) in the correct box to show which of the following represents the elements that are **in *A* but not in *B***.

$B \setminus A$ ❒ $A + B$ ❒ $A \setminus B$ ❒

Question 5 **(Suggested maximum time: 5 minutes)**

(a) John was asked to give an example of a set.
He said: "The set of good websites."

Explain why this is **not** a set.

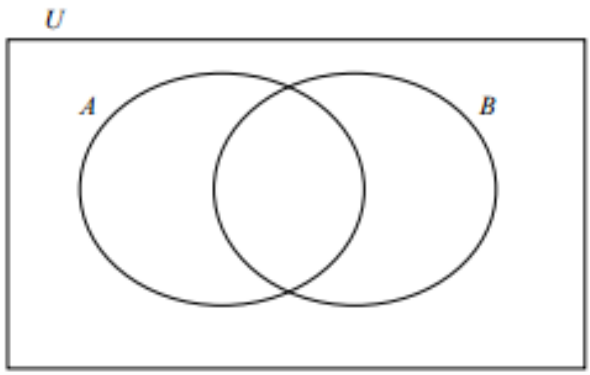
(b) The sets U, A, and B are defined as follows:

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

A is the set of multiples of 2, from 1 to 10 inclusive

B is the set of multiples of 3, from 1 to 10 inclusive.

(i) Use these sets to fill in the Venn diagram.



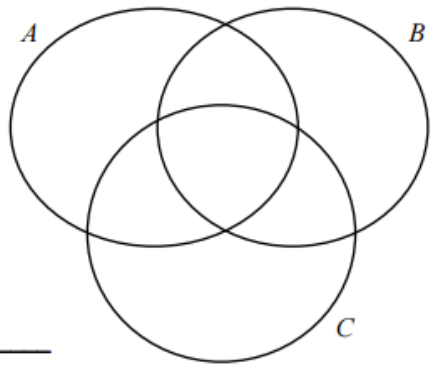
(ii) Using your Venn diagram, find the smallest number that is both a multiple of 2 **and** a multiple of 3 (the least common multiple).

Question 3 **(Suggested maximum time: 10 minutes)**

(a) The sets A, B, and C are as follows:

$A = \{2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8, 10\}$, and $C = \{1, 4, 8, 12, 14\}$.

(i) Complete the Venn diagram.



(ii) List the elements of each of the following sets:

$A \cap B =$ _____

$B \setminus (A \cap C) =$ _____

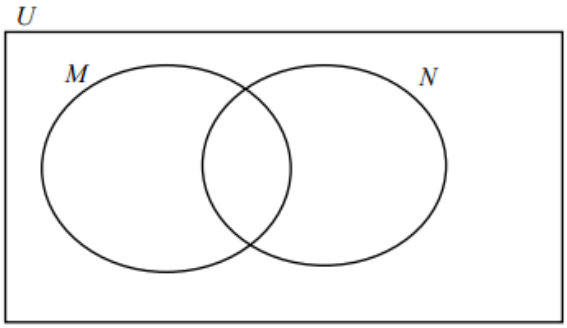
$(B \setminus A) \cup (B \setminus C) =$ _____

(iii) Write down a null set, in terms of A, B, and C. _____

(b) In a table quiz, 100 questions were asked. Team M answered 72 questions correctly. Team N answered 38 questions correctly.

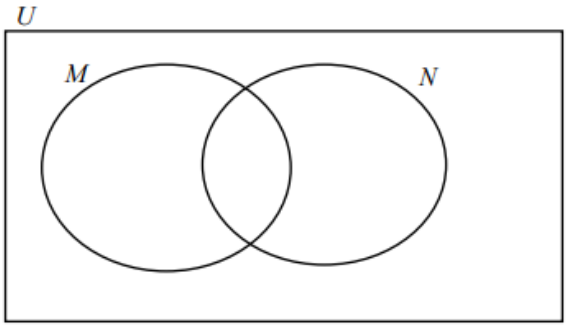
(i) Find, with the aid of the Venn diagram, the minimum number of questions answered correctly by both teams.

Minimum =



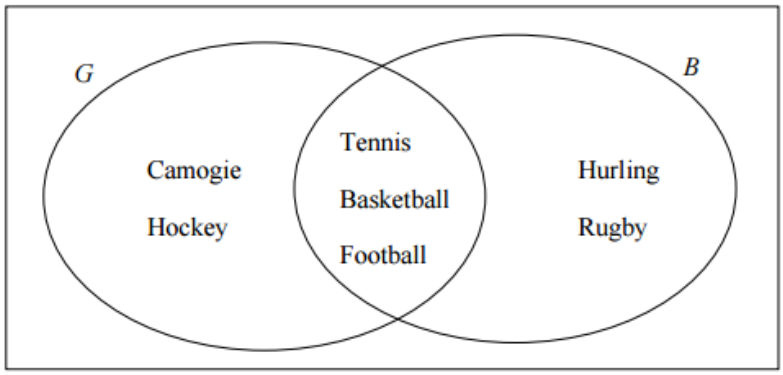
(ii) Find, with the aid of the Venn diagram, the maximum number of questions answered correctly by both teams.

Maximum =



Question 4 **(Suggested maximum time: 5 minutes)**

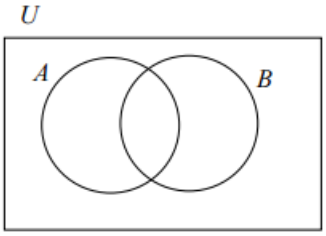
The sports played by a set of girls G and a set of boys B in a Limerick school are shown in the Venn diagram.



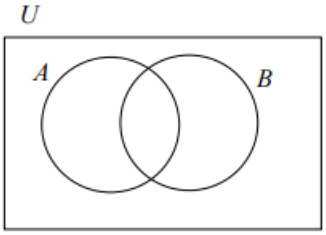
(a) Describe the region of the diagram where camogie and hockey are located.

(b) Describe the region of the diagram where tennis, basketball and football are located.

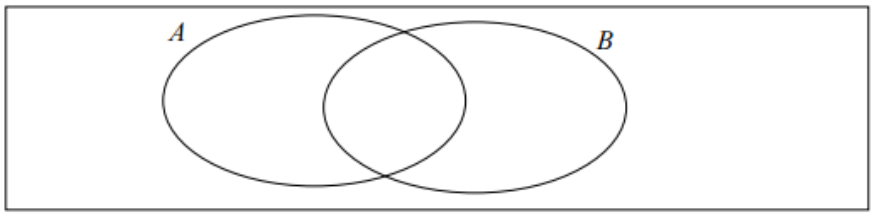
(c) **(i)** In the Venn diagram, shade the set $A \cup B$.



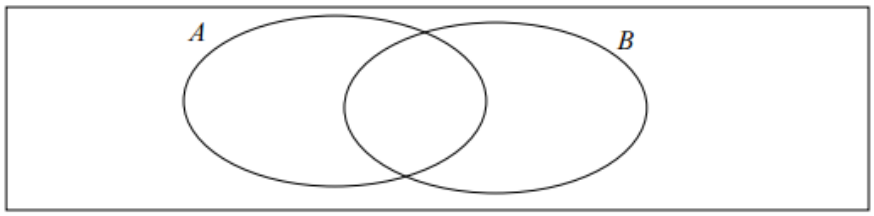
(ii) In the Venn diagram, shade the set $(A \cup B)'$, where $(A \cup B)'$ is the complement of $A \cup B$.



1. **(a)** **(i)** Using the Venn diagram below, shade in the region that represents $A \cap B$.



(ii) Using the Venn diagram below, shade in the region that represents $B \setminus A$.

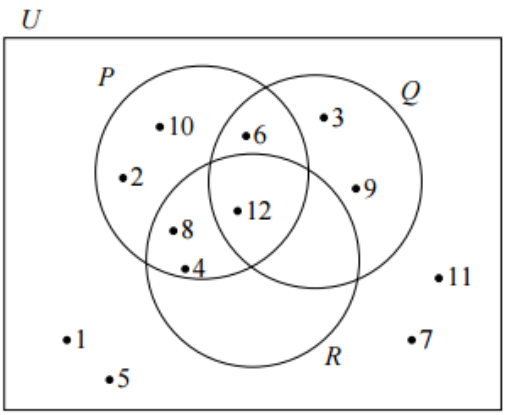


(b) U is the universal set.

$P = \{2, 4, 6, 8, 10, 12\}$ is the set of multiples of 2 less than 13.

$Q = \{3, 6, 9, 12\}$ is the set of multiples of 3 less than 13.

$R = \{4, 8, 12\}$ is the set of multiples of 4 less than 13.



(i) List the elements of $Q \cup R$.

(ii) List the elements of P', the complement of the set P.

(iii) Write down $\#R$.

(iv) What is the lowest common multiple of 2, 3 and 4?

(c) A survey was carried out in a class to find which of the films A, B or C the students had seen.

The following data was collected:

42% saw film A | 41% saw film B | 45% saw film C

12% saw both A and B | 18% saw both B and C | 15% saw both A and C

15% saw none of these films.

(i) ✍ Represent this information on a Venn diagram.

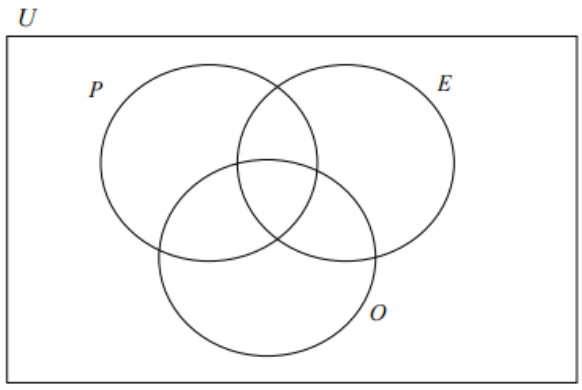
(ii) ✍ What percentage of the students in the class saw all three films?

(iii) ✍ What percentage of the students in the class saw two or more of the films?

Question 2 **(Suggested maximum time: 5 minutes)**

$U = \{1, 2, 3, ..., 12\}$. P is the set of prime numbers less than 12. E is the set of even numbers less than 12. O is the set of odd numbers less than 12.

(a) Represent these sets on the Venn diagram.



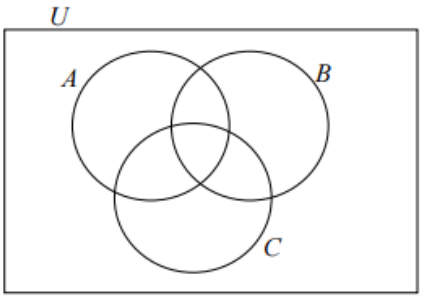
(b) Name any set on this diagram (after part **(a)** has been completed) that is a null set.

(c) If a number is drawn at random from **set *P***, what is the probability that it is even?

Question 3 **(Suggested maximum time: 15 minutes)**

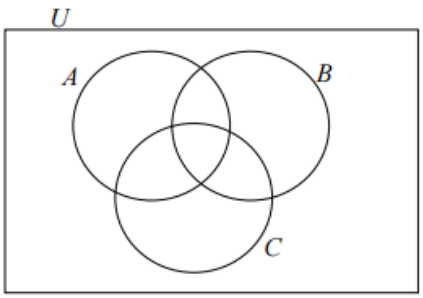
(a) For diagrams **(i)** and **(ii)** below, shade in the named region.

(i)



$A \cap B \cap C$

(ii)



$(A \cap B) \setminus C$

(b) The box on the right contains six statements, (**note**: P', is the complement of a set P).

A number of the statements are incorrect.

Write down one incorrect statement.

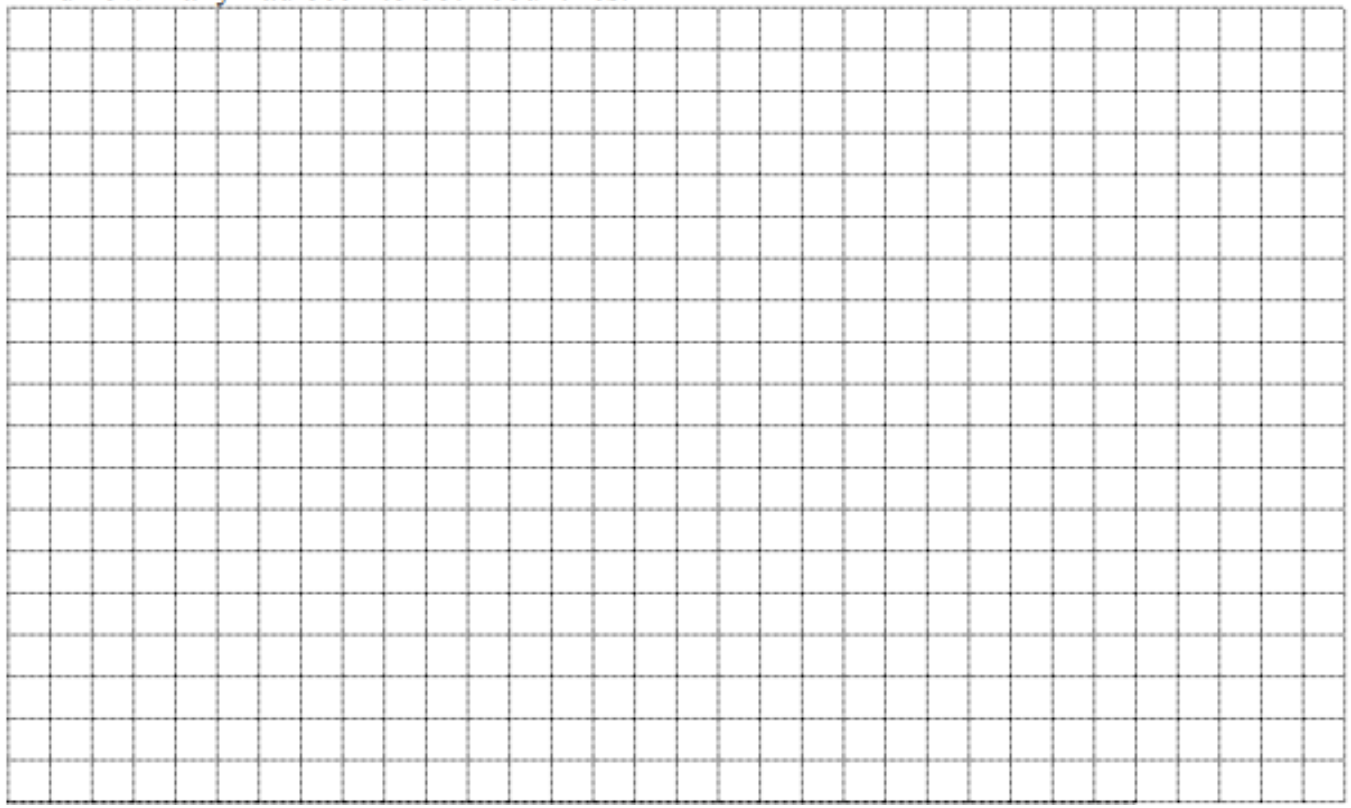
_____

Statements	
(i)	$A \cup B = B \cup A$
(ii)	$(A \cup B) \cup C = A \cup (B \cup C)$
(iii)	$(A \setminus B) \setminus C = A \setminus (B \setminus C)$
(iv)	$(A \cap B)' = U \setminus (A \cap B)$
(v)	$A \setminus B = B \setminus A$
(vi)	$B \setminus (A \cup C) = (B \cup C) \setminus A \setminus C$

Draw a diagram **or** give an example to explain your choice.

(c) A group of 38 students were asked if they had ever been to France or Spain.
The number who had been to Spain only was 3 more than the number who had been to both countries.
Twice as many had been to France as Spain.
4 students had not been to either country.

Find how many had been to both countries.



Question 6 **(Suggested maximum time: 5 minutes)**

The universal set, $U = \{ 1, 2, 3, 4, 5, 7, 10, 11, 13, 17, 19, 20 \}$.
A is the set of prime numbers between 1 and 20. B is the set of factors of 20.

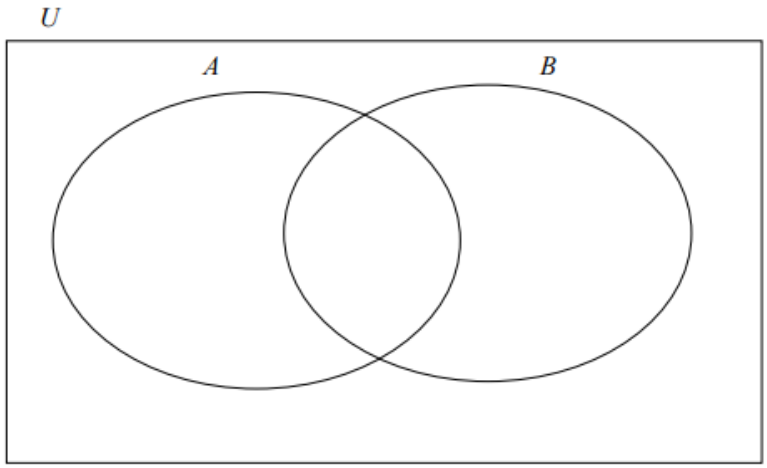
(a) List the elements of the set A.

$A = \{$, , , , , , , $\}$

(b) List the elements of the set B.

$B = \{$, , , , , $\}$

(c) Fill in the Venn diagram below placing all elements of U in the correct regions.



(d) List the elements of $A \cap B$.

$A \cap B = \{$ $\}$

(e) Complete the sentence below.

If an element is in the region $A \cap B$, it has two properties: it is a prime number and it is _____.

(f) The number 16 is added to the universal set. Place 16 in the correct region in the Venn diagram in part **(c)** and explain why you placed it there.

Reason:

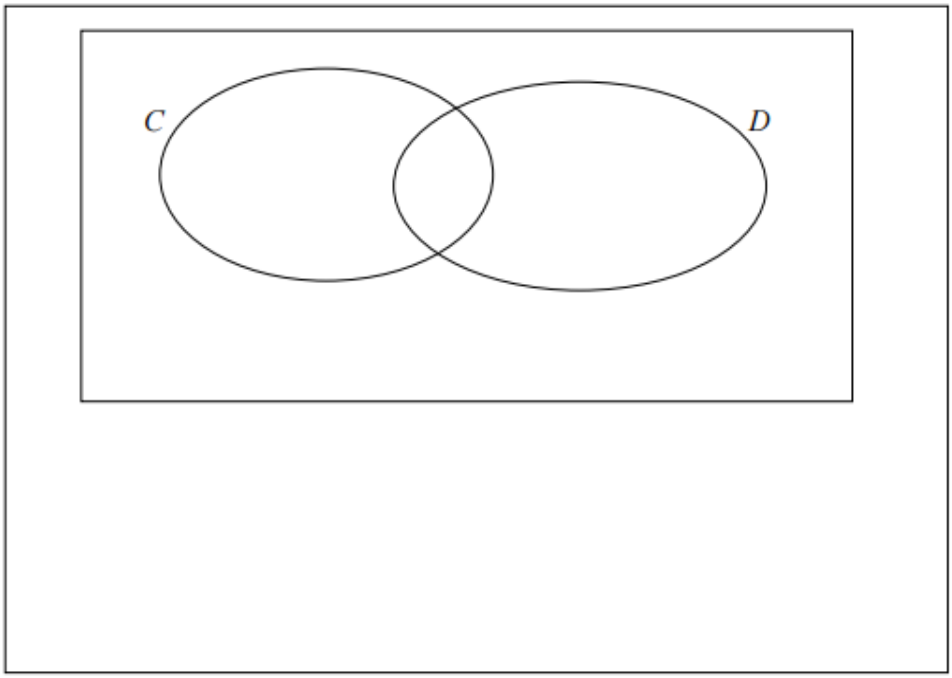
(c) In a survey, 60 households were asked if they had a cat (C) or a dog (D).

20 said they had a cat.

25 said they had a dog.

12 said they had both a cat and a dog.

(i) Represent this information in the Venn diagram below.



(ii) How many households had only a cat or a dog?

(iii) What percentage of households had neither a cat nor a dog?

(b) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ is the universal set.

$P = \{3, 5, 6, 8, 10\}$, $Q = \{2, 4, 6, 8, 10, 12\}$ and $R = \{2, 5, 6, 7, 9, 12\}$ are three subsets of U.

(i) Represent the above information on a Venn diagram.

Hence list the elements of:

(ii) $(P \cup Q \cup R)'$

(iii) $(P \cap Q) \setminus R$.

(c) U is the universal set and P and Q are two subsets of U.

$\# U = 30$, $\# P = 16$ and $\# Q = 6$.

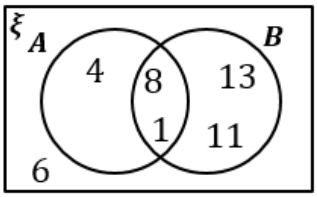
(i) ✍ Find with the aid of a Venn diagram the minimum value of $\# (P \cup Q)'$.

(ii) ✍ Find with the aid of a Venn diagram the maximum value of $\# (P \cup Q)'$.

$\# U = u$, $\# P = p$, $\# Q = q$ and $\# (P \cup Q)' = x$.

(iii) ✍ Show with the aid of a Venn diagram, that if $p > q$ and x is a maximum, then $u = p + x$.

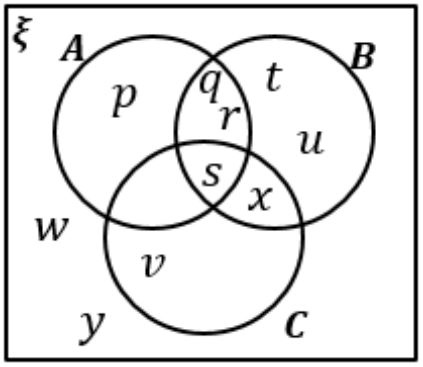
1.



List the numbers in:

a. $A \cap B$
b. $A \cup B$
c. A'
d. $A' \cap B$
e. Given that a number is chosen at random, find the probability it is in set B.

2.



Determine the sets:

a. $A \cap B$
b. $A \cup C$
c. $A' \cap C'$
d. $A \cap B \cap C$
e. $A \cap B' \cap C$
f. $(A \cup C) \cap B'$
g. Given that a number is chosen at random from the set $A \cup C$, find the probability it is in set B.

3. A and B are two sets such that:

$$n(\xi) = 20$$
$$n(A) = 14$$
$$n(A \cap B) = 3$$
$$n(B') = 12$$

Form a Venn diagram, where the number in each region is the **number** of elements in it.

4. $A = \{f, r, o, s, t\}$
$B = \{b, a, r, t, o, n\}$
Determine the sets:

a. $A \cap B$
b. $A \cup B$
c. $A' \cap B$

5. $\xi = \{1,2,3,4,5,6,7,8,9,10\}$
$A =$ { all prime numbers }
$B =$ { all multiples of 6 }
Determine:

a. $A \cup B$
b. $A \cap B$

6. $\xi = \{1,2,3,4,5,6,7,8,9,10\}$
$A =$ { all even numbers }
$B =$ { all factors of 8 }
Determine:

a. $A \cup B$
b. $A \cap B$
c. $A \cap B'$

7. Construct a Venn Diagram for sets A, B, C (without numbers) such that:

a. $A \subset B,\ \ B \cap C = \emptyset$
b. $A \subset B,\ \ A \cap C = \emptyset$

☠ A, B, C are sets such that:

$$n(A) = 20,$$
$$n(B) = 20,$$
$$n(C) = 20,$$
$$n(A \cap B) = 10$$
$$n(A \cap C) = 10$$
$$n(B \cap C) = 10$$
$$n(A \cap B \cap C) = 5$$

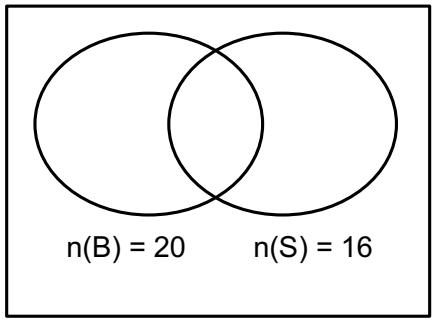
Determine $n(A \cup B \cup C)$.

Fluency Practice

conditional probability
and venn diagrams

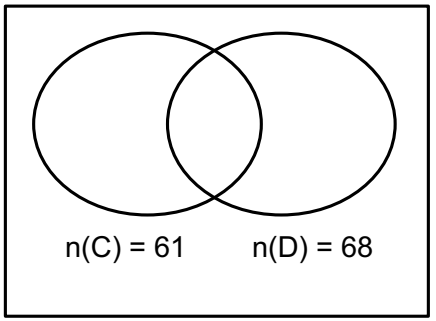
(1)

n(E) = 30



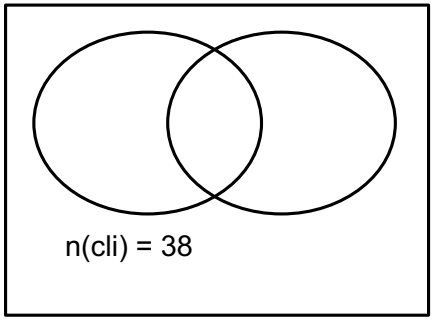
(2)

n(E) = 125



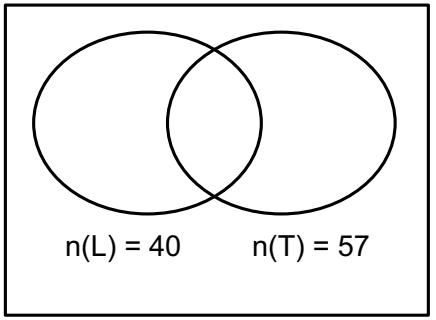
(3)

n(E) = 50



(4)

n(E) = 90



A garage has 50 cars for sale. 15 of the cars have both air conditioning and cruise control. 31 of the cars have air conditioning. 17 of the cars have cruise control. Draw a Venn diagram to represent this information.

(a) How many cars do not have air conditioning or cruise control?

(b) How many cars have air conditioning but not cruise control?

90 people in a sports club were surveyed. 19 play tennis and squash. 50 play tennis. 32 play squash. Draw a Venn diagram to represent this information.

(a) How many people do not play squash, but do play tennis?

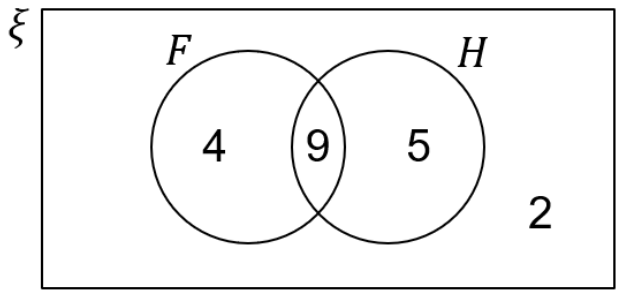
(b) How many people play neither squash nor tennis?

There are 16 people in a queue. Two are wearing both a hat and a scarf. Eight are wearing a hat. Seven are wearing neither a hat nor a scarf. Draw a Venn diagram to represent this information.

(a) How many people are wearing a scarf?

(b) How many people are wearing either a hat or a scarf or both?

The Venn shows how many students in a class play football (F) or hockey (H).



(a) How many students play both sports?

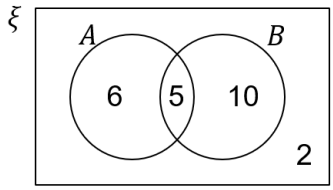
(b) Write down $n(H')$

(c) Write down $n(F' \cap H)$

Set Notation and Venn Diagrams

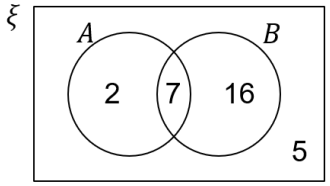
In each question, the values represent the number of elements in each set.

(a)



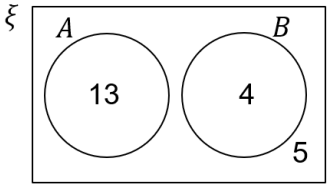
Write down:
(a) $n(A \cap B)$
(b) $n(A)$
(c) $n(B')$
(d) $n(\xi)$

(b)



Write down:
(a) $n(B)$
(b) $n(A')$
(c) $n(A \cup B)$
(d) $n(A \cap B')$

(c)



Write down:
(a) $n(A \cap B)$
(b) $n(A')$
(c) $n(\xi)$
(d) $n(A' \cap B')$

(d)

$n(A \cap B) = 5$
$n(A) = 13$
$n(A \cup B) = 16$
$n(\xi) = 20$
Draw a Venn diagram to represent this information.

(e)

$n(A \cap B) = 7$
$n(A) = 13$
$n(B) = 10$
$n(B') = 10$
Draw a Venn diagram to represent this information.

(f)

$n(A \cap B) = 10$
$n(A \cup B) = 18$
$n(A) = 13$
$n(\xi) = 20$
Draw a Venn diagram to represent this information.

(g)

$n(A \cup B) = 20$
$n(A') = 15$
$n(A \cap B) = 6$
$n(B) = 16$
Draw a Venn diagram to represent this information.

(h)

$n(B) = 7$
$n(A) = 10$
$n(A \cup B) = 12$
$n(\xi) = 20$
Draw a Venn diagram to represent this information.

(i)

$n(A' \cap B') = 10$
$n(\xi) = 25$
$n(B) = 11$
$n(A) = 8$
Draw a Venn diagram to represent this information.

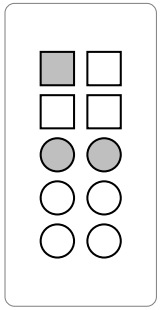
Fluency Practice

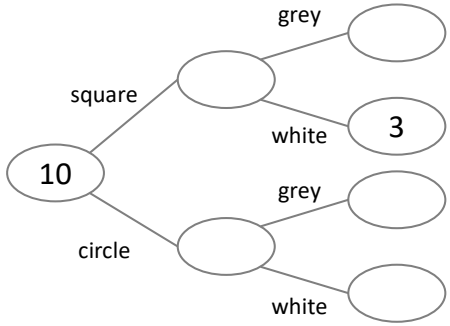
A1 $A = \{2, 3, 5, 7, 11\}$ $B = \{3, 5, 7, 9\}$ List $A \cup B$	**A2** $A = \{\text{factors of } 100\}$ $B = \{\text{multiples of } 5\}$ List $A \cap B$	**A3** $A = \{c, h, i, n, a\}$ $B = \{i, t, a, l, y\}$ List $A \cup B$	**A4** $A = \{s, u, p, e, r\}$ $B = \{c, o, m, p, u, t, e, r\}$ List $A \cap B$
B1 $A = \{1, 3, 6, 10, 15\}$ $B = \{3, 6, 9, 12\}$ Find $n(A \cup B)$	**B2** $S = \{s, q, u, a, r, e\}$ $V = \{a, e, i, o, u\}$ Find $n(S \cap V)$	**B3** $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{2, 3, 5\}$ Find $n(A')$	**B4** $M = \{2, 4, 6, 8, 10\}$ $N = \{1, 3, 5, 7, 9\}$ Find $n(M \cap N)$
C1 $A = \{2, 4, 6, 8, 10\}$ $B = \{1, 3, 5, 7, 9\}$ Explain why $A \cap B = \varnothing$	**C2** $A = \{\text{multiples of } 5\}$ $D = \{\text{prime numbers}\}$ Is it true that $A \cap D = \varnothing$?	**C3** $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{\text{even numbers}\}$ $B = \{\text{multiples of } 3\}$ $x \in B$ and $x \notin A$ What are the possible values of x?	**C4** $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{1, 3, 5, 7\}$ $B = \{2, 4, 6, 8\}$ $x \in \mathcal{E}$ and $x \notin A \cup B$ What is the value of x?
D1 $\mathcal{E} = \{\text{even numbers}\}$ $A = \{2, 4, 6, 8, 10\}$ B is such that $A \cap B = \{4, 8\}$ and $n(B) = 3$. List a possible set B.	**D2** $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7\}$ $A = \{2, 3, 4, 5\}$ C is such that $A \cap C = \{4, 5\}$ and $n(C) = 4$. List a possible set C.	**D3** $\mathcal{E} = \{2, 4, 6, 8, 10, 12, 14\}$ $A = \{2, 6, 8, 12\}$ C is such that $A \cap C = \varnothing$ and $n(C) = 3$. List set C.	**D4** $A = \{3, 4, 5\}$ $A \cup B = \{1, 2, 3, 4, 5\}$ B has as few members as possible. List set B.

Fluency Practice

FPR Frequency Trees

1) Complete the frequency tree for this group of shapes.

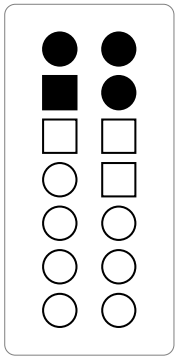


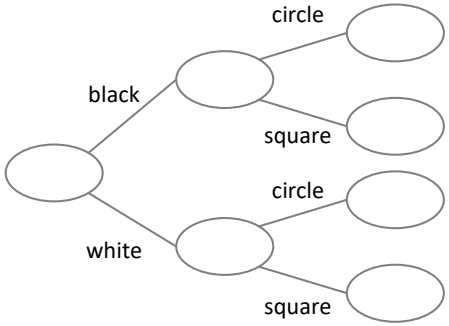


What fraction of the shapes are white squares?

Express the ratio grey circles : white circles

2) Complete the frequency tree.

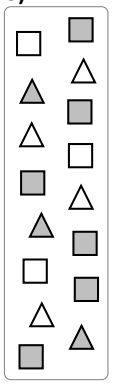


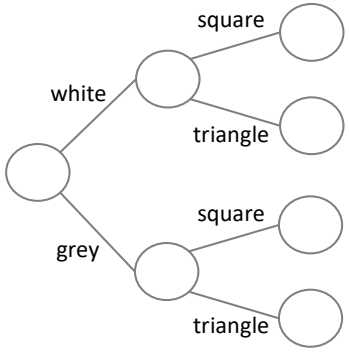


What fraction of the shapes are black?

What percentage of the white shapes are squares?

3)

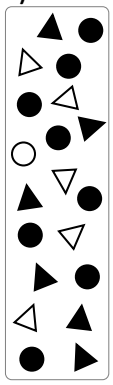


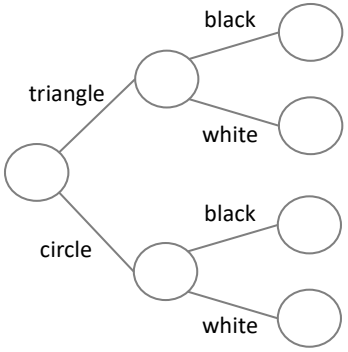


What percentage of the grey shapes are triangles?

Express the ratio white shapes : grey squares

4)

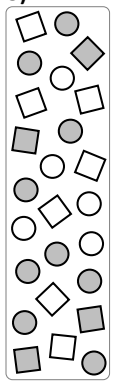


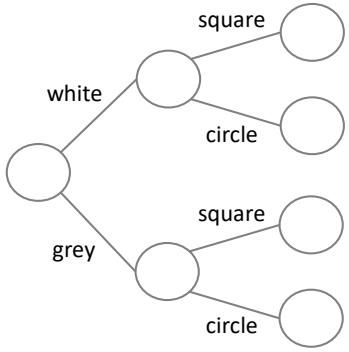


What percentage of the shapes are black?

Express the ratio circles : black triangles

5)





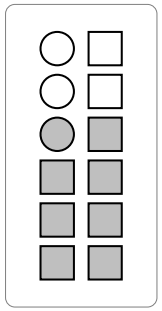
What fraction of the circles are white?

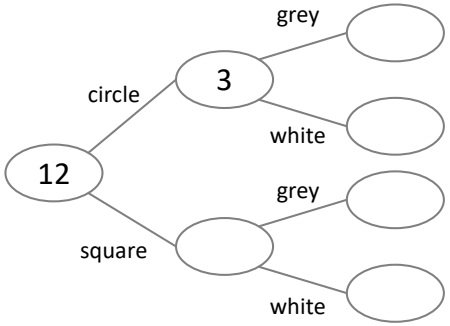
What % of the shapes are grey or circles?

Fluency Practice

Frequency Trees

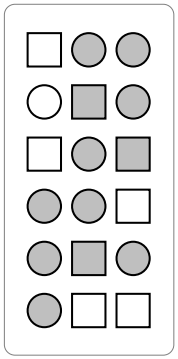
1) Complete the frequency tree for this group of shapes.

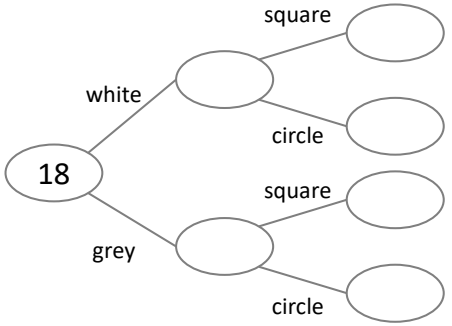




If a shape is picked randomly from the group,
what is the probability it is a grey square?

2) Complete the frequency tree.



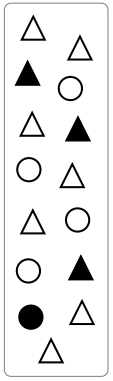


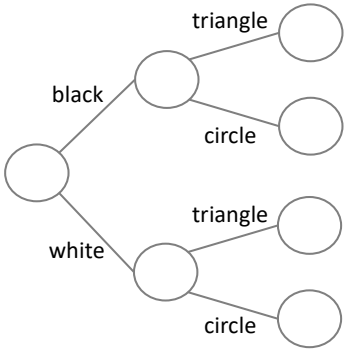
What is the probability of picking…

… a white square?

… a grey circle?

3) Complete the frequency tree.

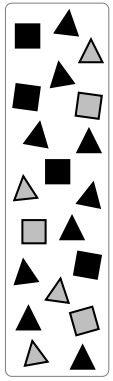


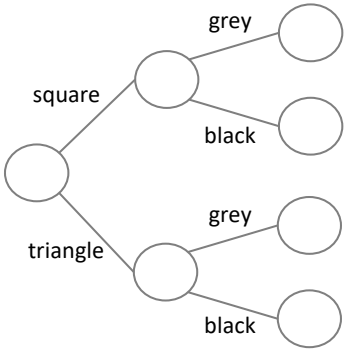


What is… Probability (picking a black shape)?

Probability (picking a circle)?

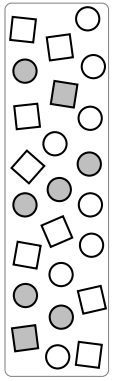
4) Complete the frequency tree.

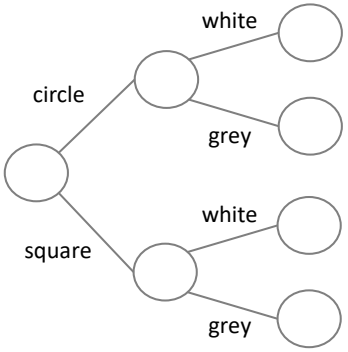




A square is picked from the group.
What is the probability it is grey?

5) Complete the frequency tree.



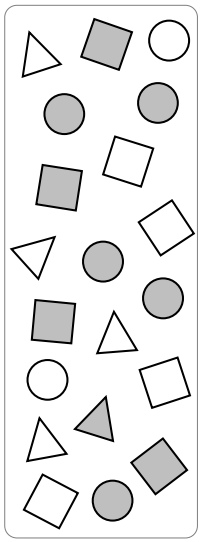


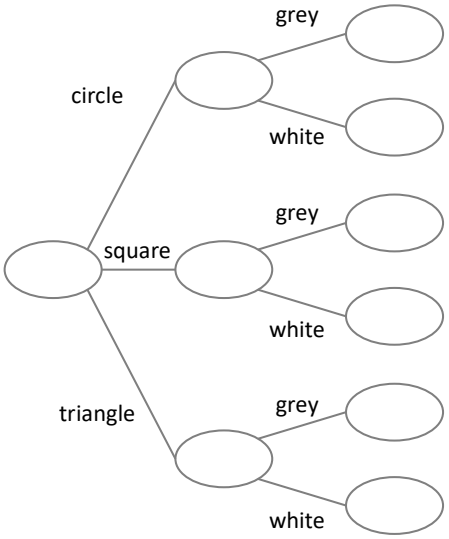
A grey shape is picked from the group.
What is the probability it is a square?

Fluency Practice

Frequency Trees

1) Complete the frequency tree for this group of shapes.





If we pick a shape at random, what is…

P(square)?

P(grey circle)?

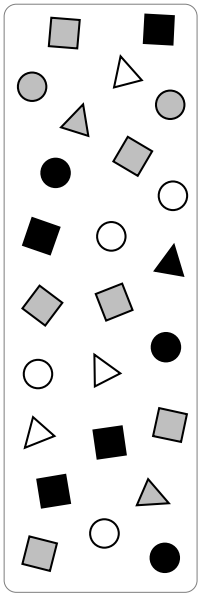
P(white shape)?

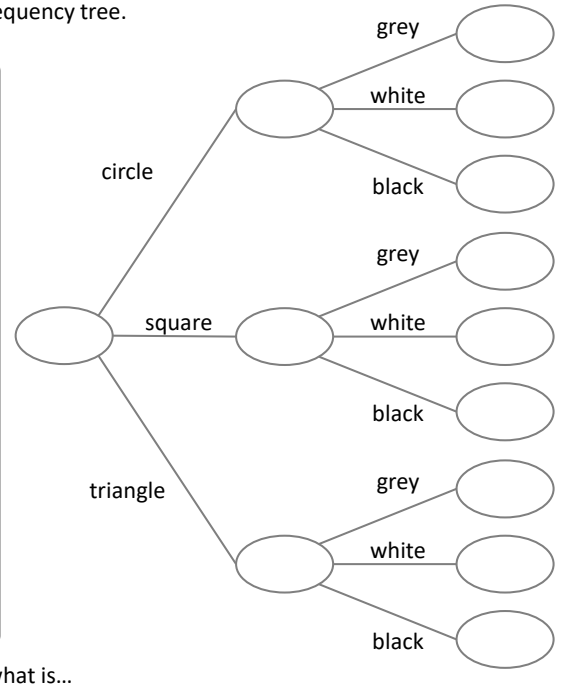
What is the probability a circle or a square is picked?

A triangle is picked, what is the probability it is grey?

A grey shape is picked, what is the probability it is a triangle?

2) Complete the frequency tree.





As a percentage, what is…

P(triangle)?

P(white circle)?

P(a grey circle or a black triangle)?

P(a black shape **given that** it is a square)?

P(a square **given that** it is black)?

What is the probability a white shape or a triangle is picked?

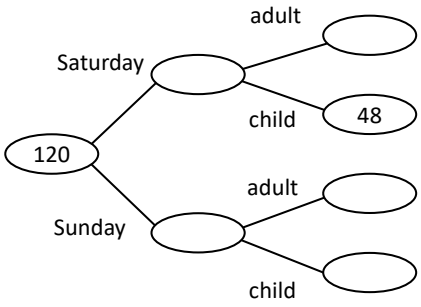
Fluency Practice

Frequency Trees: FPR

a) A shop recorded the customers it had over the weekend.

$\frac{2}{3}$ of the customers came on Saturday.

30% of Sunday customers were children.



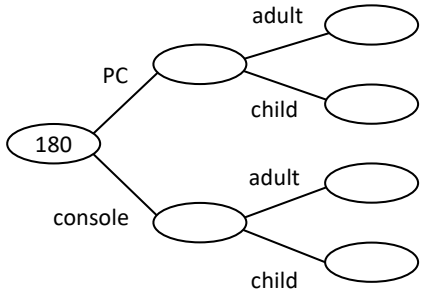
What fraction of the customers were adults?

b) A game company complied data on players.

The ratio of PC to console users was 4 : 5

$\frac{3}{5}$ of console users were children.

25% of PC users were adults.

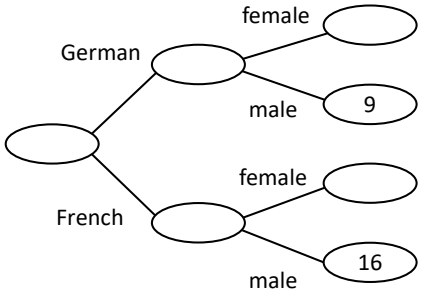


What fraction of the players were children?

c) Students were surveyed.

$\frac{3}{8}$ of the students that studied German were male.

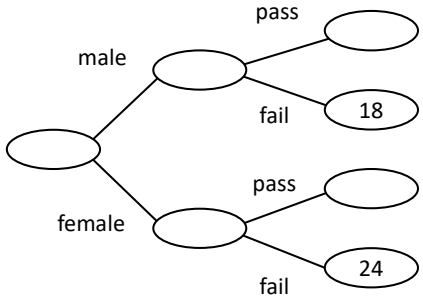
The ratio of male to female French students was 2 : 5



What percentage of the students studied French?

d) For a science test, the ratio of passing students to failing students was 5 : 3

$\frac{4}{9}$ of the female students failed.

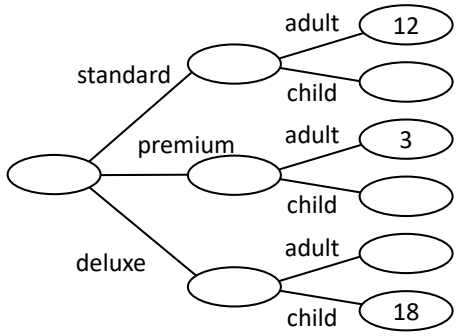


What fraction of the students were male?

e) $\frac{5}{7}$ of deluxe cinema tickets were sold to adults.

75% of standard tickets were sold to children.

The ratio of adult tickets to child tickets was 4 : 7



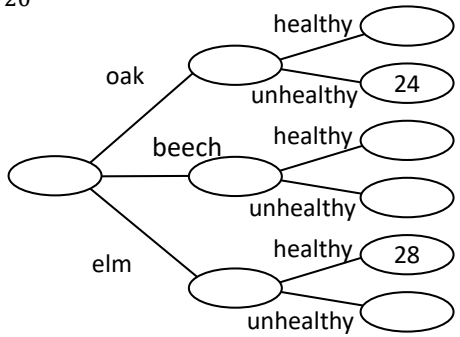
What fraction of the tickets were deluxe?

f) 15% of trees surveyed were unhealthy oaks.

The ratio of healthy elms to healthy oaks was 7 : 10

The ratio beech : elm was 5 : 7

$\frac{7}{20}$ of the beech trees were healthy.

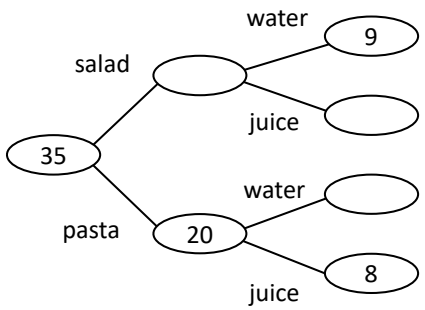


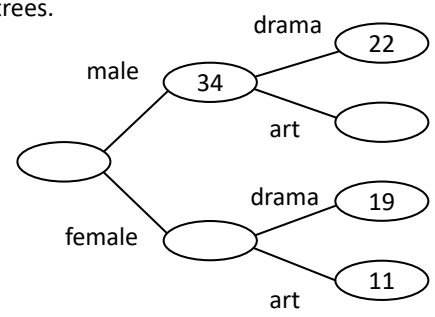
Which species was healthiest?

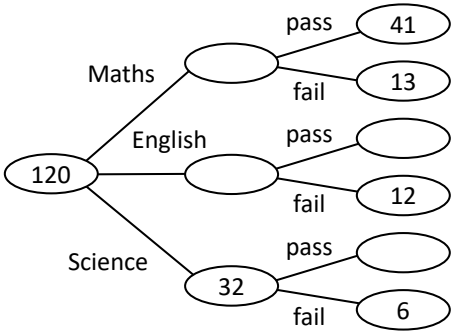
Fluency Practice

① **Frequency Trees**

1) Complete the frequency trees.

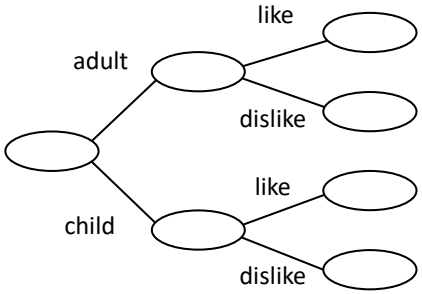






2) A computer game company surveyed 40 players about a game update.

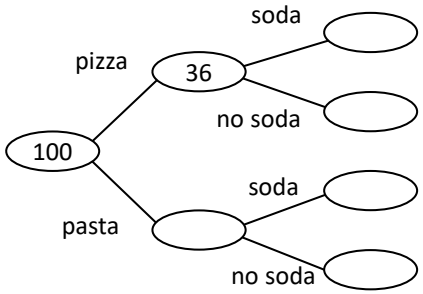
7 adults liked the update.
14 of the 22 children asked liked the update.



What fraction of the adults disliked the update?

3) A restaurant recorded if customers ordered a soda with their meal.

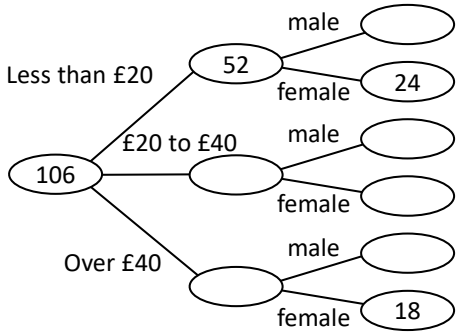
A third of those that had pizza had soda.
34 people ordered pasta with a soda.



What fraction of those surveyed had pasta but no soda?

4) A clothes shop recorded the gender & the money spent by customers spent in a day.

Twice as many customers spent less than £20, than spent over £40.
Of the customers that spent £20-£40, three-quarters were female.



What fraction of customers… …were male?
…spent over £20?

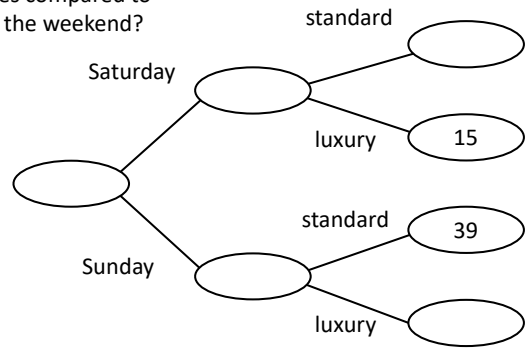
Fluency Practice

Frequency Trees ②

1) A cake shop sells 'standard' (£2) & 'luxury' cakes (£3).

On Sunday, a quarter of the customers ordered a luxury cake.
Over the weekend, a third of the customers ordered a luxury cake.

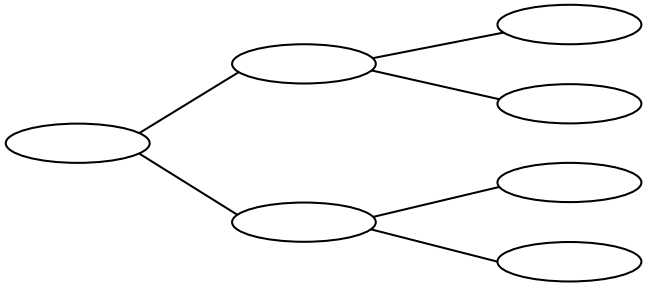
How much more revenue did the shop make from standard cakes compared to luxury cakes over the weekend?



2) Mike recorded the meals 120 students had at lunch.

14 students had a sandwich and no snack.
Two-thirds of the students had a sandwich.
Three-quarters of the students had a snack.
24 students had a snack but no sandwich.

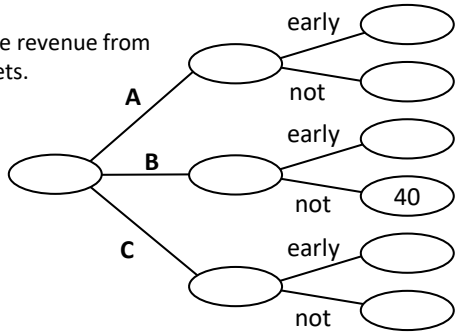
What fraction of students had no sandwich and no snack?
Complete a frequency tree to help.



3) There are 3 types of seating at a concert **A**: £30 **B**: £20 **C**: £15
Tickets bought early get a 20% discount.

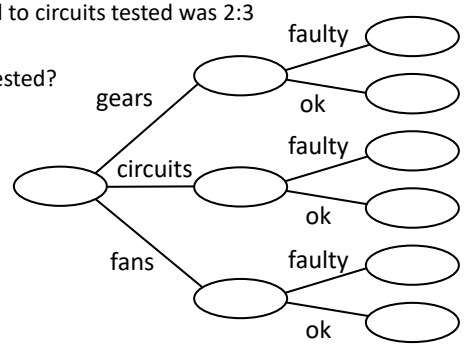
Three-fifths of the 80 **A** tickets were bought early.
12 more people bought **B** tickets early than **A** tickets early.
133 people bought tickets early.
For **C** tickets, the ratio of those bought early to on the door was 1 : 5

Show that the concert made more revenue from **C** tickets than **A** tickets.

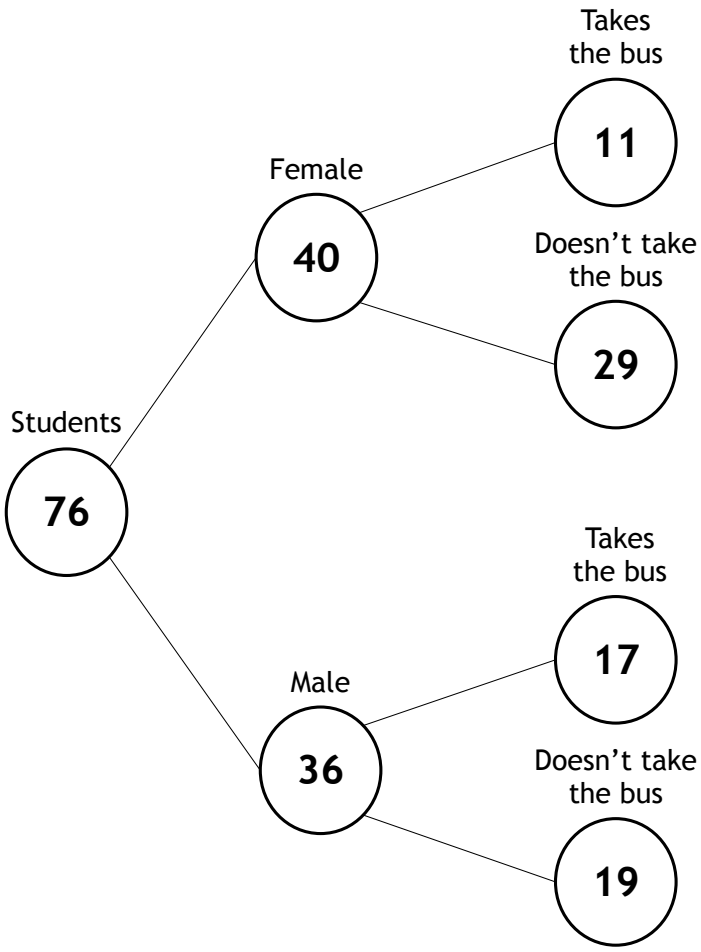


4) A robot factory makes gears, circuits & fans.
After testing, two-thirds of the gears were faulty
& one-fifth of the circuits were faulty.
60% of the items tested worked correctly.
56 fans that were tested worked correctly.
30% of the items tested were gears.
The ratio of gears tested to circuits tested was 2:3

How many items were tested?



Fluency Practice

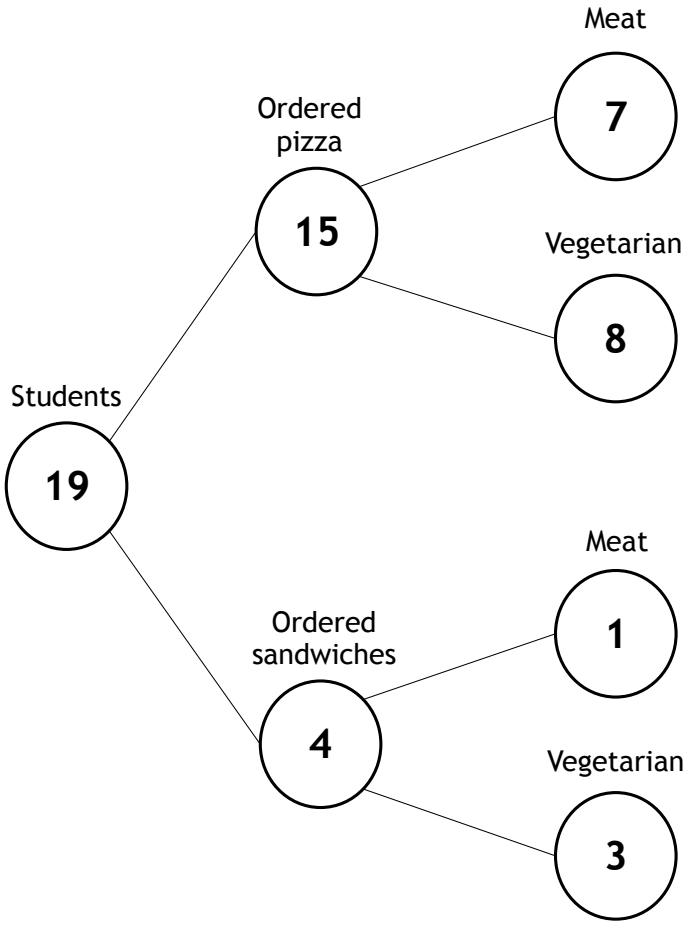


A frequency tree diagram shows the number of students that take the bus to school and don't.

Complete the table.

What fraction of students are male?	
What fraction of females take the bus?	
What fraction of students are females that take the bus?	
What fraction of males take the bus?	
What fraction of students are males who don't take the bus?	
What fraction of students don't take the bus?	
What fraction of males don't take the bus?	

Fluency Practice

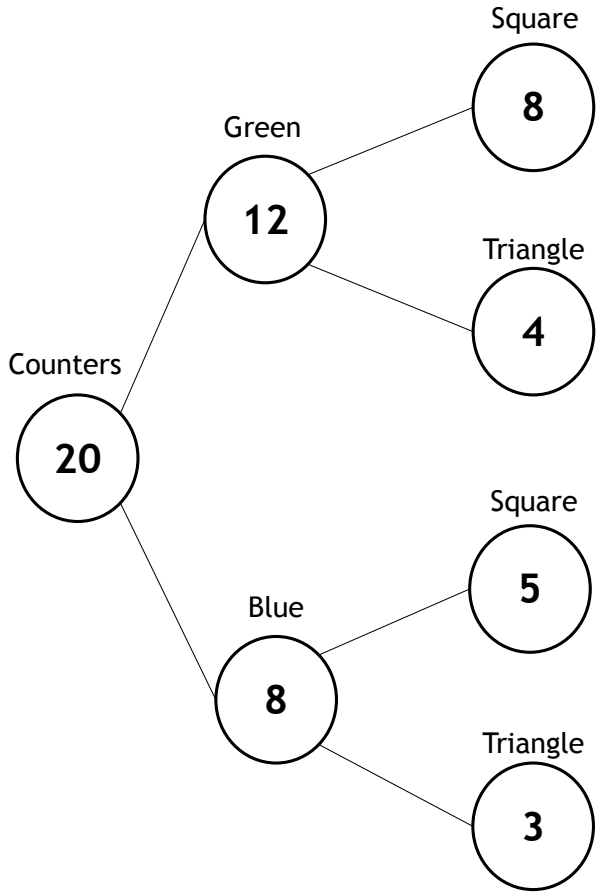


A frequency tree diagram shows the number of students that ordered meals and whether they are vegetarian or not.

Complete the table.

	$\frac{8}{15}$
What fraction of students ordered vegetarian pizza?	
	$\frac{1}{19}$
What fraction of those that ordered sandwiches are not vegetarian?	
	$\frac{15}{19}$
What fraction of students that ordered pizza are not vegetarian?	
	$\frac{3}{4}$

Fluency Practice

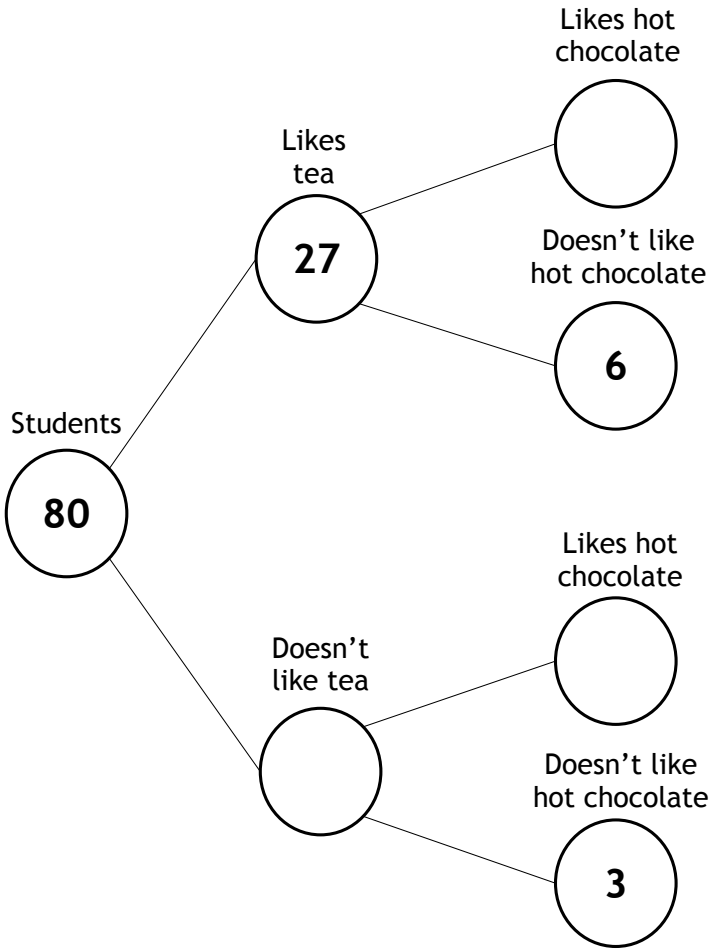


A frequency tree diagram shows the number of counters and their colour and shape.

Complete the table.

What fraction of blue counters are squares?	
	$\frac{3}{20}$
What fraction of blue counters are triangles?	
	$\frac{13}{20}$
What fraction of squares are green?	
What fraction of greens are squares?	
	$\frac{1}{3}$

Fluency Practice

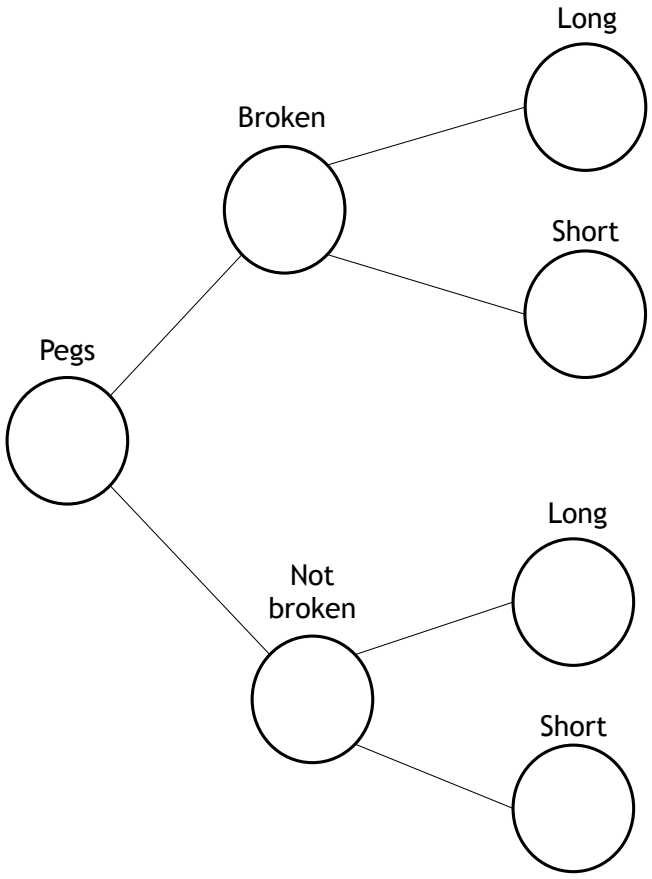


A frequency tree diagram shows the number of students that like hot chocolate and like tea.

Complete the frequency tree and the table

	$\frac{3}{80}$
	$\frac{53}{80}$
	$\frac{21}{80}$
	$\frac{3}{40}$
	$\frac{3}{53}$
	$\frac{5}{8}$
	$\frac{2}{3}$

Fluency Practice



The information below describes the number of short and long tent pegs that are broken or not broken in shop stockroom.

Complete the frequency tree

$\frac{53}{150}$ of the pegs are broken

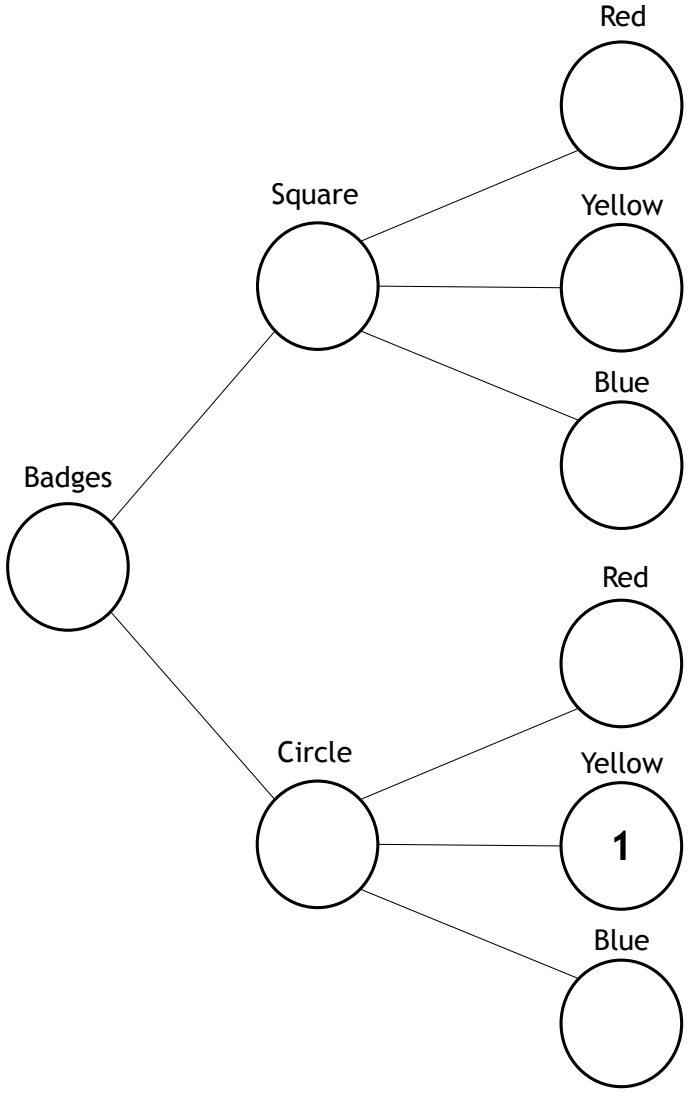
$\frac{51}{150}$ of the of the pegs are long and not broken

$\frac{3}{10}$ of the pegs are long and broken

There are 96 long pegs

$\frac{4}{26}$ of the short pegs are broken

Fluency Practice



A company has ordered badges with their logo on.
There is a choice of two shapes and three colours.

Use the information below to
complete the frequency tree

$\frac{1}{4}$ of the badges are circles

$\frac{1}{4}$ of the badges are red

$\frac{1}{4}$ of the of the red logos are circles

$\frac{1}{4}$ of the circles are red

$\frac{1}{4}$ of the squares are yellow

$\frac{1}{4}$ of the badges are yellow

$\frac{1}{4}$ of the circles are yellow

$\frac{1}{4}$ of the blue logos are circles

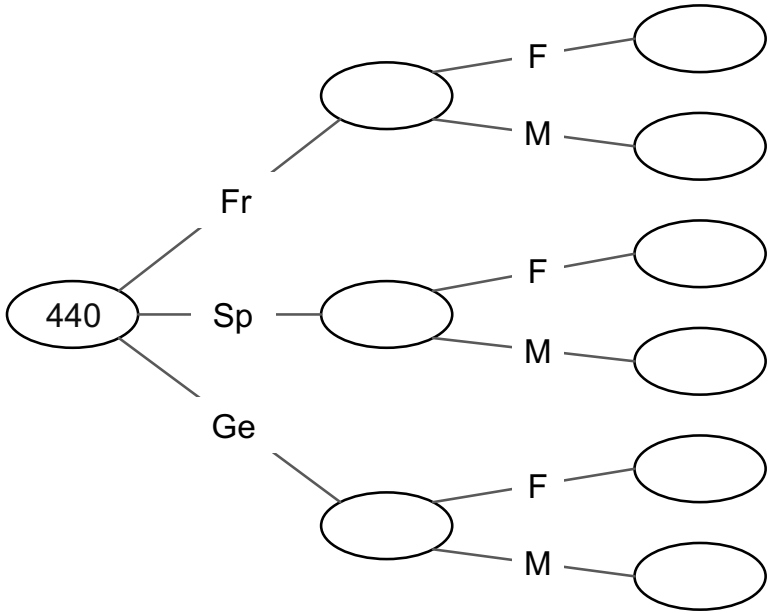
Fluency Practice

in a small school, all of the 440 students study just one modern foreign language

the ratio of numbers of students taking French, Spanish and German is 9 : 5 : 8 (respectively)

33⅓% of the students that study French are female
40% of the students that study Spanish are male
55% of the students that study German are female

complete the frequency tree:



what percentage of students in the whole school are male? [round your answer to the nearest whole number]

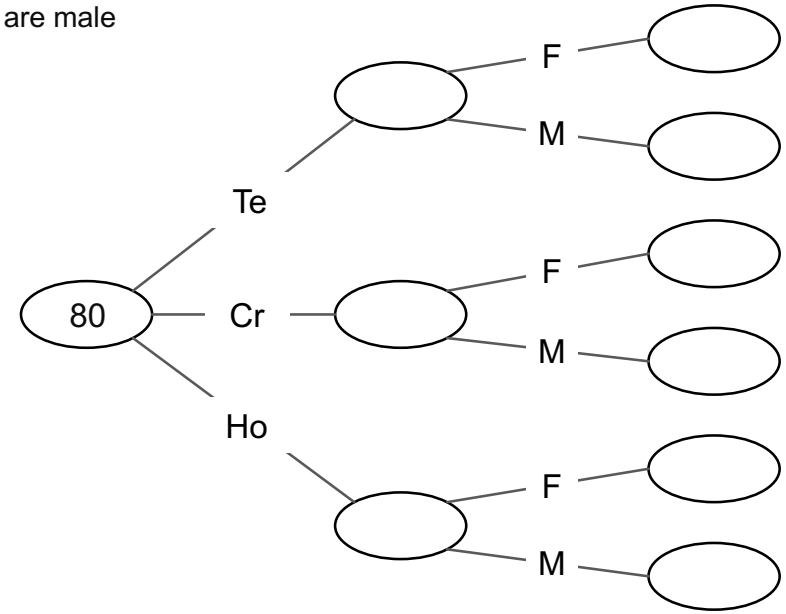
Fluency Practice

80 of the students in Y9 take part in an activity during their PE lesson

the same number of students play cricket as play hockey
twice the number of students play tennis than play cricket

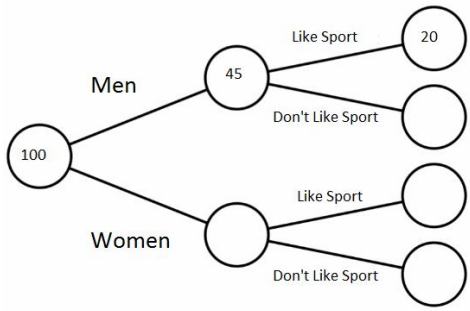
45% of the students that chose tennis are male
35% of the students that chose cricket are female
45% of the students that chose hockey are male

complete the frequency tree:



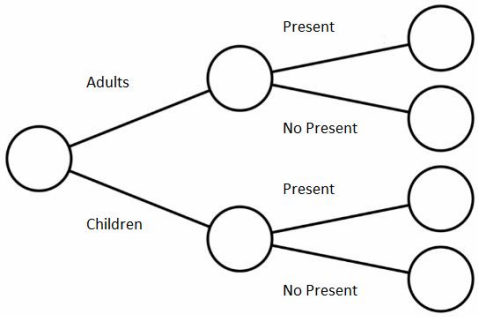
show that the numbers of girls and boys
in the school are equal

(1) There are 100 people in an office. 45 are men and the rest are women. 20 of the men like sport and the rest of the men don't. 28 of the women like sport and the rest don't.
The frequency tree shows some of this information below.



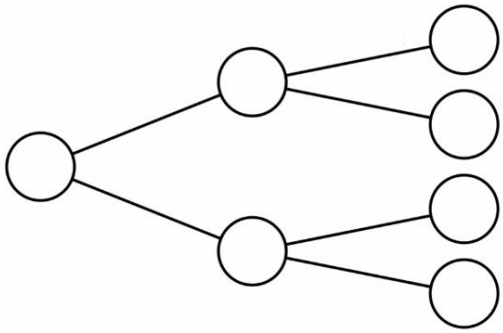
(a) Complete the frequency tree above.
(b) One person is chosen at random. Find the probability that the person is a woman who didn't like sport.
(c) A man is chosen at random. What is the probability that he liked sport?

(2) There are 80 people at a party. 20 of the people are adults and the rest are children. Of the children 45 take a present and the rest don't. Of the adults, all but one take a present.
(a) Complete the frequency tree below



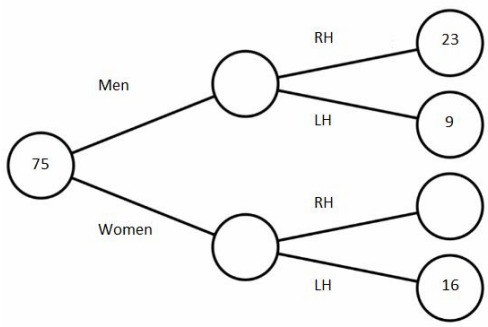
(b) What proportion of the people at the party are adults who have taken a present?
(c) One person is chosen at random. Find the probability that they were an adult who didn't take a present.

(3) In a box there are pink toys and black toys. There are 90 toys in total. Half of the toys are pink and half of the toys are black.
Of the pink toys 20 are plastic and the rest are metal. Of the black toys 10 are plastic and the rest are metal.
(a) Label and complete the frequency tree below.



(b) How many more metal toys were there than plastic toys?
(c) One toy is chosen at random. Find the probability that it's a black plastic toy.
(d) Write down the ratio of pink toys to black toys.

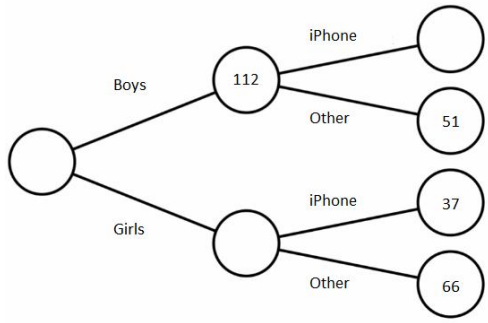
(4) Complete the frequency tree below



(5) There are 90 people at a party. 36 are adults and the rest are children. Of the children 10 take a drink and the rest don't. Of the adults 16 take a drink and the rest don't.
Draw a frequency tree to show this information.

(6) In Y10 there are 250 students. 130 are boys and the rest are girls. 50 boys take pack lunches and 80 girls take pack lunches. Those that don't take pack lunches eat in the canteen. Draw a frequency tree to represent the information.

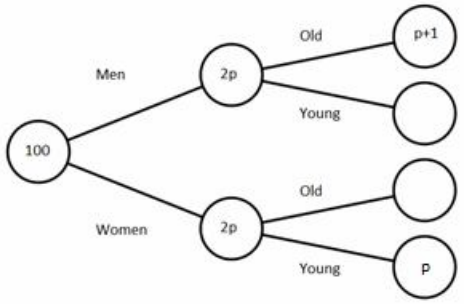
(7) The frequency tree below shows what phone some students at a school owned.
(a) Complete the frequency tree.



(b) One person is chosen at random. Find the probability of the person being a boy with an iPhone.
(c) A girl is chosen at random. Find the probability that she doesn't own an iPhone.
(d) How many more people own 'Other' phones than iPhones?

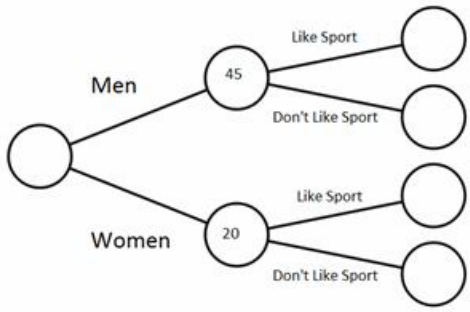
(8) There are 180 people at a party. 1/3 are men and the rest are women. Of the men half take food and the rest don't. Of the women 20% take food and the rest don't.
(a) Show this information in a frequency tree.
(b) One person is chosen at random. Find the probability that the person is a man who doesn't take food.
(c) A woman is chosen at random. Find the probability that she takes food.

(9) (a) Find the value of p in the frequency tree below.



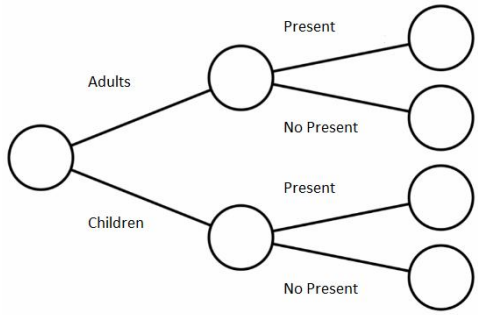
(b) Complete the frequency tree.

(1) There are a number of people in an office. 10 of the men like sport and the rest don't. 8 of the women like sport and the rest don't.
The frequency tree shows some of this information below.



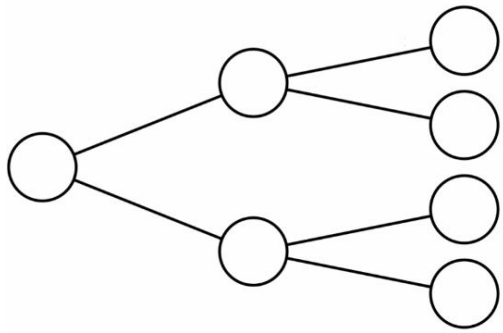
(a) Complete the frequency tree above.
(b) One person is chosen at random. Find the probability that the person is a woman who didn't like sport.
(c) A man is chosen at random. What is the probability that he liked sport?

(2) There are 92 people at a party. 33 of the people are adults and the rest are children. Of the children 12 take a present and the rest don't.
Of the adults, 9 don't take a present.
(a) Complete the frequency tree below



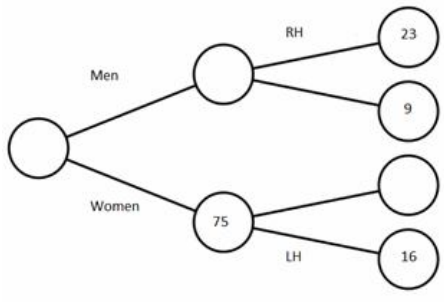
(b) What fraction of the people at the party are adults who have taken a present?
(c) One person is chosen at random. Find the probability that they were a child who didn't take a present.

(3) In a box there are blue toys and red toys. There are 112 toys in total. All but 30 of of the toys are red.
Of the red toys 20 are plastic and the rest are metal. Of the blue toys half are plastic and the rest are metal.
(a) Label and complete the frequency tree below.



(b) How many more metal toys were there than plastic toys?
(c) One toy is chosen at random. Find the probability that it's a red plastic toy.
(d) Write down the ratio of metal toys to plastic toys.

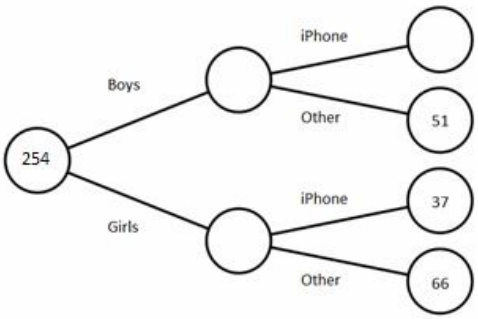
(4) Complete the frequency tree below



(5) There are 80 people at a party. 54 are adults and the rest are children. Of the children 8 take a drink and the rest don't. Of the adults 16 take a drink and the rest don't.
Draw a frequency tree to show this information.

(6) In Y10 there are 180 students. 130 are boys and the rest are girls. 40 boys take pack lunches and 20 girls take pack lunches. Those that don't take pack lunches eat in the canteen. Draw a frequency tree to represent the information.

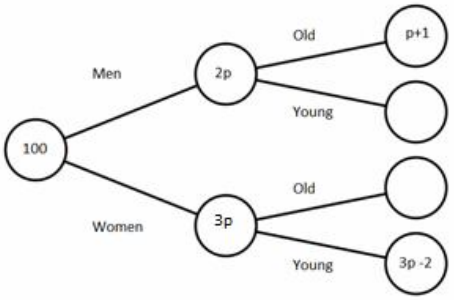
(7) The frequency tree below shows what phone some students at a school owned.
(a) Complete the frequency tree.



(b) One person is chosen at random. Find the probability of the person being a boy with an iPhone.
(c) A girl is chosen at random. Find the probability that she doesn't own an iPhone.
(d) How many more people own 'Other' phones than iPhones?

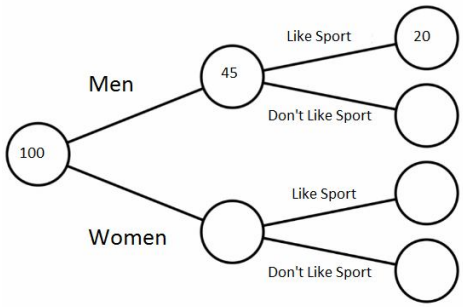
(8) There are 120 people at a party. 1/4 are men and the rest are women. Of the men half take food and the rest don't. Of the women 20% take food and the rest don't.
(a) Show this information in a frequency tree.
(b) One person is chosen at random. Find the probability that the person is a man who doesn't take food.
(c) A woman is chosen at random. Find the probability that she takes food.

(9) (a) Find the value of p in the frequency tree below.



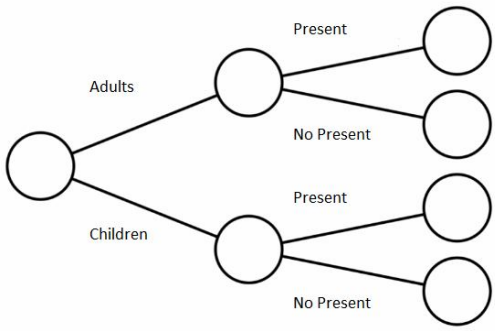
(b) Complete the frequency tree.

(1) There are 100 people in an office. 45 are men and the rest are women. 20 of the men like sport and the rest of the men don't. 28 of the women like sport and the rest don't.
The frequency tree shows some of this information below.



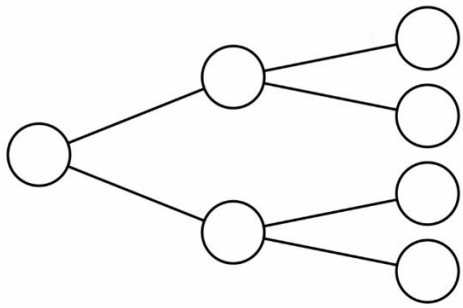
(a) Complete the frequency tree above.
(b) One person is chosen at random. Find the probability that the person is a woman who didn't like sport.
(c) A man is chosen at random. What is the probability that he liked sport?

(2) There are 80 people at a party. ¼ of the people are adults and the rest are children. Of the children 30 take a present and the rest don't. Of the adults, all but one take a present.
(a) Complete the frequency tree below



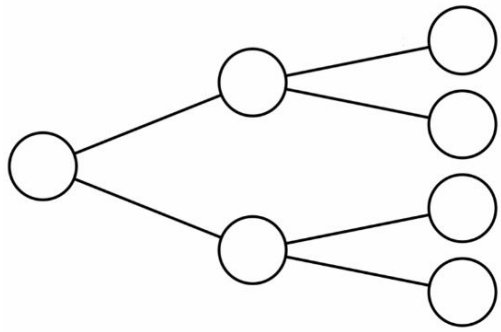
(b) What proportion of the people at the party are adults who have taken a present?
(c) What percentage of the children took a present?
(d) One person is chosen at random. Find the probability that they were an adult who didn't take a present.
(e) Fill in the blank: P(_____) = 19/80

(3) In a sports club there are 50 people. The ratio of males to females is 2:3. Of the males the ratio of those swimming to not swimming is 7:3. Of the females 60% swim and the rest don't
(a) Use the diagram below to show the information using a frequency tree



(b) What percentage of the people in the club were males who swam?
(c) What is the ratio of the women that swim to the men that swim?
(d) One person is chosen at random. Find the probability that the person is a female who doesn't swim.
(e) What percentage of the men didn't swim?

(4) There are men and women in an office.
The ratio of women to men is 4:1.
Of the women 25% bring their own lunch. The rest eat in the canteen.
Of the men, 6 bring their own lunch and 2 eat in the canteen.
(a) Show this information using the frequency tree below.

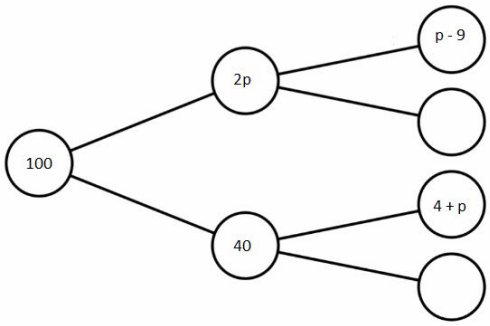


(b) One person is chosen at random. Find the probability that the person is a man who eats in the canteen.
(c) A woman is chosen at random. Find the probability that she brings he own lunch.
(d) What proportion of the people in the office brings their own lunch?

(5) There are 130 people at a party. 20% are adults and the rest are children. Half of the adults take food and 2/13 of the children take food.
(a) Explain why 15% of the people can't be adults.
(b) Draw a frequency tree in the space below to show the information given.

(c) Write the ratio of the number of adults bringing food to the number of children bringing food.
(d) What percentage of party goers were adults who took food?

(6) In a toy box there are pink toys and black toys. The toys are either electronic or they are manual. The frequency tree shows some information about the toys below.



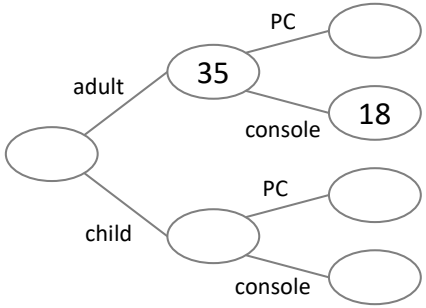
There are 40 black toys in the box.
Of the black toys 6 are manual.
(a) Complete the frequency tree above.
(b) One black electronic toy is taken from the box. What proportion of the toys left in the box are pink manual toys?
(c) Write the ratio of pink toys to black toys in its simplest form.
(d) n pink toys are removed from the box. What proportion of the toys in the box are now pink?
(e) K% of the items are electronic pink toys. Find the value of K.

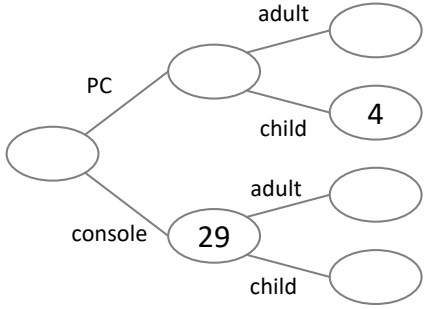
Fluency Practice

Two-Way Tables & Frequency Trees: Gap Fill

① A computer game company collects data about players: their age & what platform they prefer to use.

	Adult	Child	Total
PC	17		21
Console		11	
Total			50



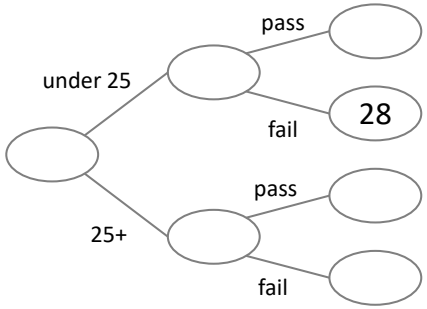


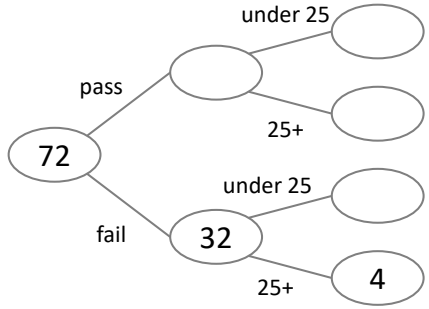
What fraction of the players played on console?

What fraction of the children played on console?

② A driving school records the age of students & their success when taking a driving test.

	Pass	Fail	Total
Under 25			
25+	27		
Total	40		



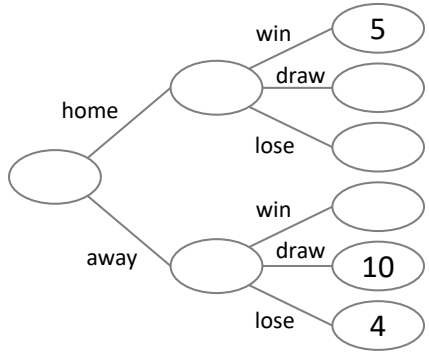


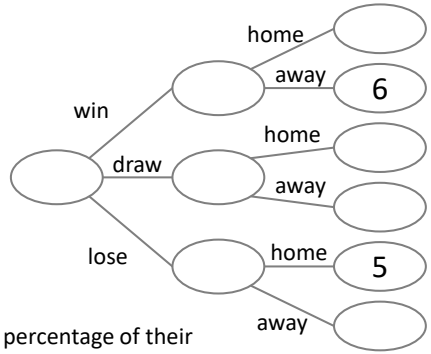
What fraction of the students were 25 or over and failed?

What fraction of those that failed were under 25?

③ Leesham United look at their results since new owners took over.

	Home	Away	Total
Win			
Draw	6		
Lose			
Total			





What percentage of their away games did they win?

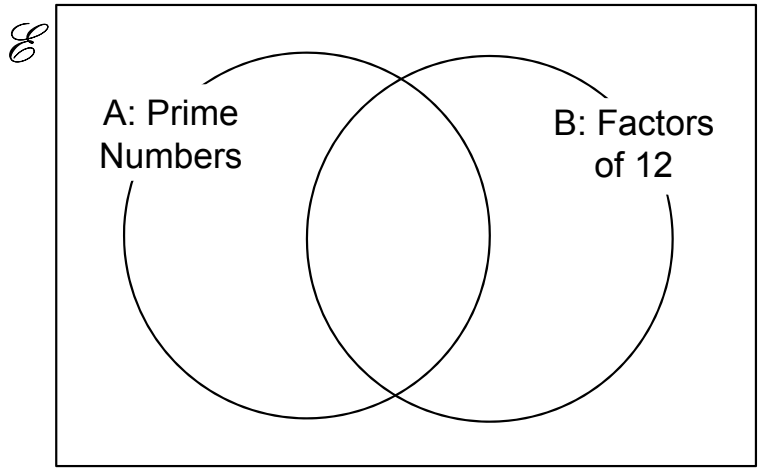
What fraction of the draws were away games?

Fluency Practice

① Place each element in the correct section of the Venn diagram.

1 2 3 4 5 6
7 8 9 10 11 12

Elements $\mathscr{E}$



② Shade the TRUE statements.

A has 5 elements	B has the same number of elements as B'
B has 4 elements	B has twice as many elements as A ∩ B
A ∩ B has 9 elements	There are 4 elements in A'
A ∪ B has 9 elements	¼ of the elements are outside both circles

③ One of the numbers is chosen at random. Choose the correct probabilities.

$P(A)$	
$P(B)$	
$P(A')$	
$P(B')$	
$P(A \cap B)$	
$P(A \cup B)$	

$\frac{1}{6}$ $\quad$ 75%

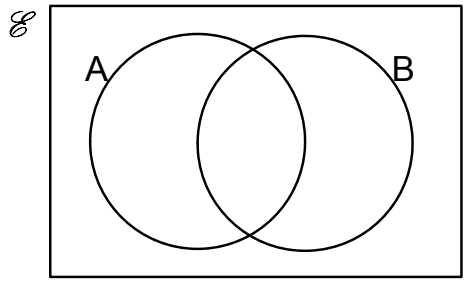
$\frac{7}{12}$

$\frac{5}{12}$ $\quad$ 0.5

$1 - P(B)$

Fluency Practice

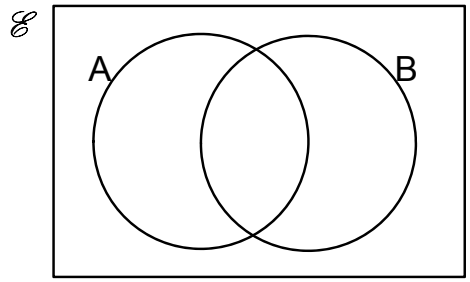
1. $\mathscr{E}$ = {1,2,3,4,5,6,7,8,9,10}
A = {2,3,5,6}
B = {1,3,5,7,9}
(a) Complete the Venn diagram.



(b) An element is picked at random. Shade the TRUE statements.

$P(B) = \frac{1}{2}$
$P(A \cap B) = 2$
$P(A \cup B) = 0.7$
$P(A') = \frac{7}{10}$

2. $\mathscr{E}$ = {8,9,10,11,12,13,14,15,16}
A = {even numbers}
B = {square numbers}
(a) Complete the Venn diagram.



(b) An element is picked at random. Work out:

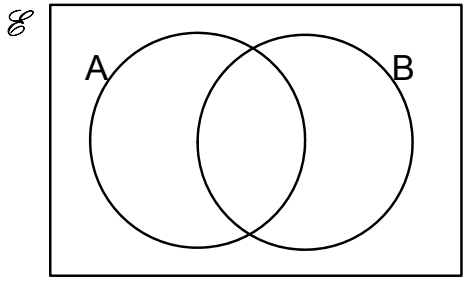
P(A') =

P(B') =

$P(A \cap B)$ =

$P(A \cup B)$ =

3. $\mathscr{E}$ = {5,6,7,8,9,10,11}
A' = {5,6,10}
B = {6,8,10,11}
$A \cap B$ = {8,11}
(a) Complete the Venn diagram.

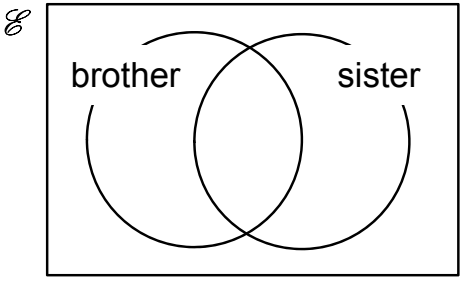


(b) An element is picked at random. Shade the TRUE statements.

P(A) < P(B)
$P(A \cap B) = P(B)$
$P(B) \geq P(B')$
$P(A \cup B) = P(A \cap B)$

4. In a class of 32 pupils,
23 pupils have a brother,
14 pupils have a brother and a sister,
6 pupils have no brothers or sisters.

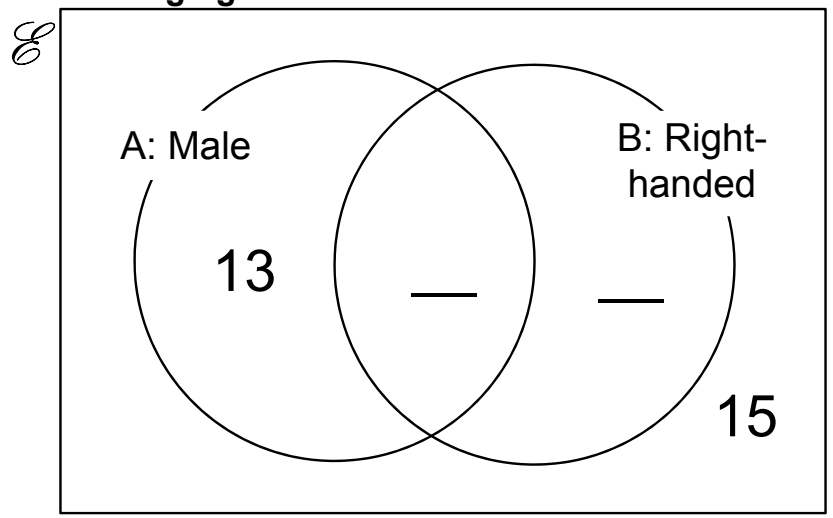
Write the number of pupils who belong in each section of the Venn diagram.



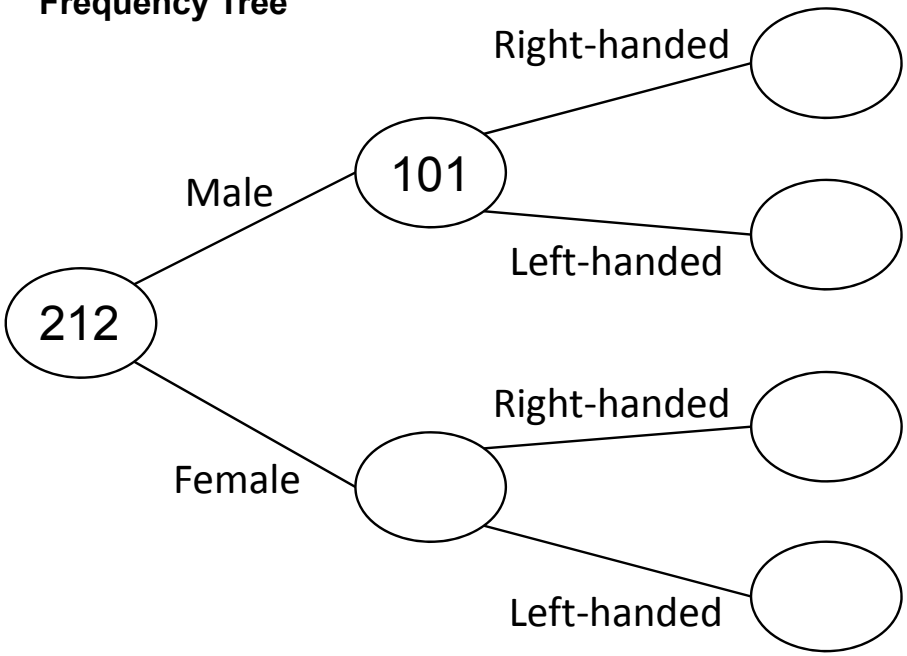
Fluency Practice

The Venn diagram, Frequency Tree and Two-Way Table all show the same information. Complete them all.

Venn diagram – the number of elements belonging to each section is labelled.



Frequency Tree

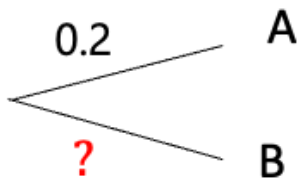


Two-Way Table

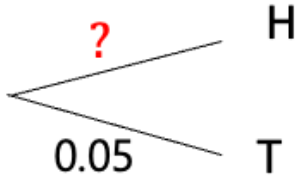
	Male	Female	Total
Right-handed			
Left-handed			
Total			

Fluency Practice

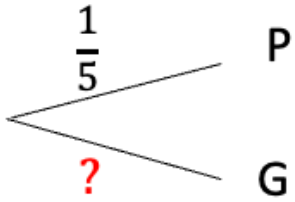
a.



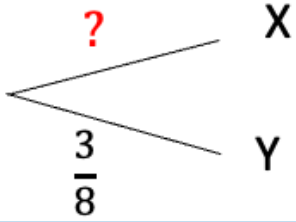
b.



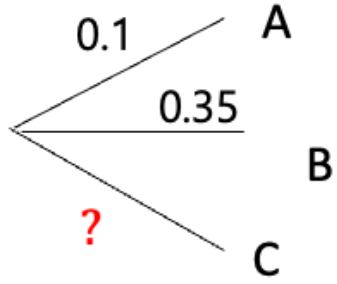
c.



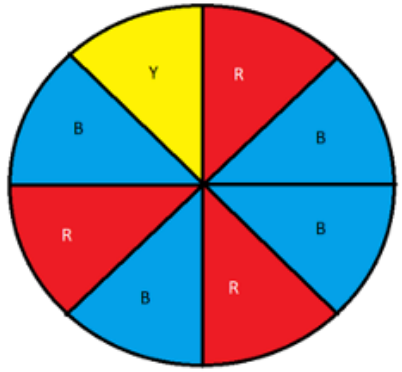
d.



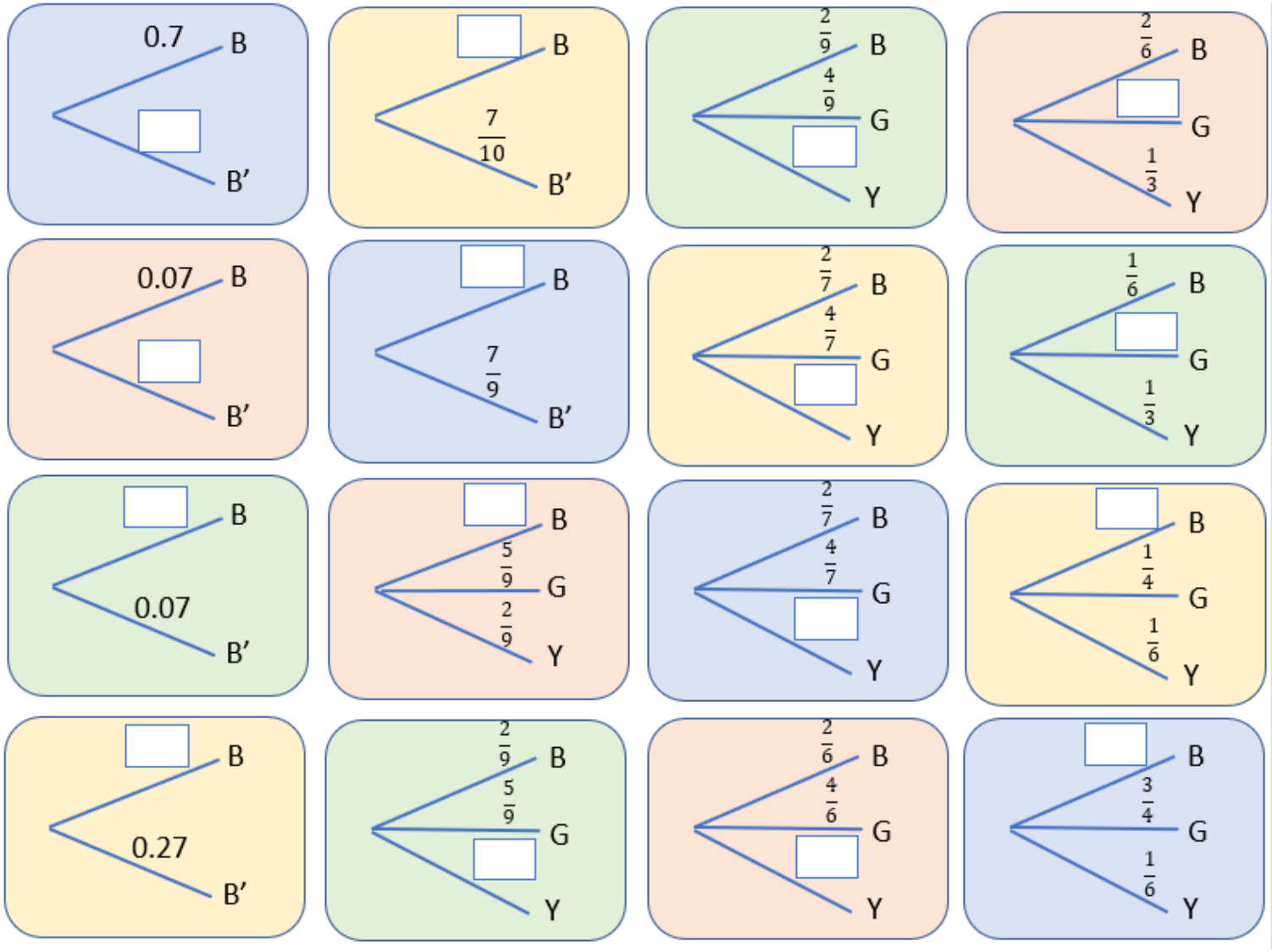
e.



f. Draw a probability tree for this spinner.



Fluency Practice



Fluency Practice

Add or multiply probabilities?

Getting a 6 on a die and a T on a coin.

+ ×

Hitting a bullseye or a triple 20.

+ ×

Getting a HHT or a THT after three throws of an unfair coin (presuming we've already worked out P(HHT) and P(THT).

+ ×

Getting 3 on the first throw of a die and a 4 on the second.

+ ×

Bart's favourite colour being red and Pablo's being blue.

+ ×

Shaan's favourite colour being red or blue.

+ ×

Fluency Practice

Independent?

Event 1	Event 2		
Throwing a heads on the first flip.	Throwing a heads on the second flip.	No	Yes
It rains tomorrow.	It rains the day after.	No	Yes
That I will choose maths at A Level.	That I will choose Physics at A Level.	No	Yes
Have a garden gnome.	Being called Bart.	No	Yes

Fluency Practice

Independent Events?

For Tim, what will happen tomorrow?

Which of these events are **independent** of each other?

How might one event affect the probability of a different event?

It will rain.

Tim's son will pass his maths test.

Tim will have pasta for lunch.

Tim will bump into Sarah.

Tim will be late to work.

Tim's car will break down.

Tim will feel unwell.

Tim will get a pay raise.

Tim will start looking for a new job.

Tim will go to bed early.

Tim will be early for work.

It will be sunny.

Tim will have dinner at a nice restaurant.

Tim will miss his evening running club.

It will be cloudy.

Tim's alarm clock will not work.

Stockley United will win their football match.

Tim's will choose coffee at the café.

Are any of the events **mutually exclusive**? (they cannot both happen)

In a group of 15 boys, 10 like apples, 8 like bananas and 6 like both apples and bananas.

Let A = a boy likes apples and B = a boy likes bananas.

(a) Show that A and B are not independent.

(b) Find the probability that a boy likes apples or bananas.

Of the 26 cars on a garage forecourt, 12 had ABS brakes, 16 had a CD-player. 6 had neither ABS brakes nor a CD-player.

Let A = has ABS and B = has CD-player.

(a) Show that A and B are not independent.

(b) Find the probability that a car has ABS or CD-player.

In a group of 20 boys, 8 have blue eyes, 10 have fair hair and 4 have both blue eyes and fair hair.

Let A = has blue eyes and B = has fair hair.

(a) Show that A and B are independent.

(b) Find the probability that a boy has either blue eyes or fair hair.

6/

There were 38 books on a shelf. 17 had large print, 12 were novels with large print. 10 were not novels and did not have large print.

Let A = is large print and B = is a novel.

(a) Show that A and B are not independent.

(b) Find the probability that a book is either large print or is a novel.

In a group of 24 girls, 18 like oranges, 8 like bananas and 4 like neither oranges nor bananas.

Let A = likes oranges and B = likes bananas.

(a) Show that A and B are independent.

(b) Find the probability that a girl either likes oranges or bananas.

In a group of 40 people, 24 played cricket and 30 played tennis.

The events 'played cricket' and 'played tennis' are independent.

Find the probability that a person plays either cricket or tennis.

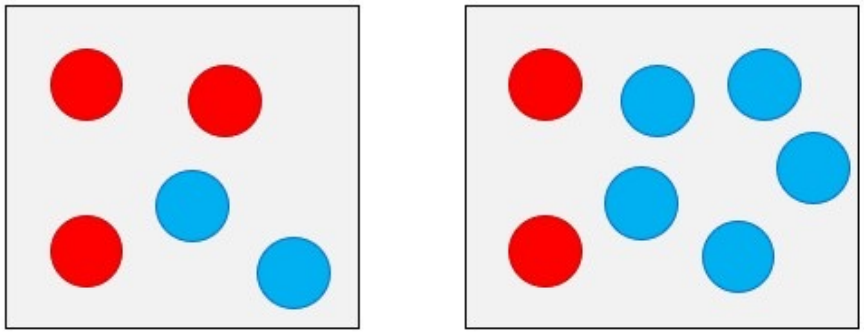
In a group of 30 people, 10 read books and 24 read newspapers.

The events 'read books' and 'read newspapers' are independent.

Find the probability that a person either reads books or newspapers.

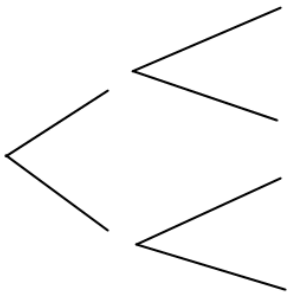
Problem Solving

Leyland picks a marble from each box.

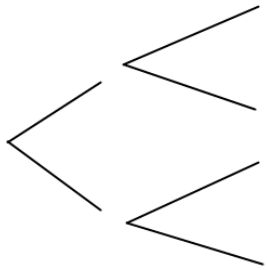


- Does it matter which box Leyland chooses from first?
- Will the order affect the probabilities of the combined outcomes?

1. A bag contains 6 blue counters and 4 green counters. One is picked and then replaced. A second is then picked. **Complete the tree diagram below.**



2. A box of chocolates contains three dark chocolates and two white chocolates. A chocolate is picked and then replaced. A second chocolate is then picked. **Complete the tree diagram below**



3. A wallet contains 2 gold coins and 8 silver coins. A coin is picked at random and then replaced. A second coin is then picked. **Draw a probability tree. What is the probability of getting two silver coins?**

4. Two suitcases each contain 3 hoodies and 9 t-shirts. An item of clothing is picked at random from both suitcases. **Draw a probability tree and calculate the probability of picking two t-shirts.**

5. A fair coin is thrown twice. **What is the probability of getting one head and one tail?**

6. A bag contains 5 red balls and 3 blue. A ball is picked at random and then replaced. A second ball is then picked. **Draw a probability tree. What is the probability that**

a) **Both balls are red?**
b) **Both balls are blue?**
c) **One of each?**

7. There are 4 banana smoothies and three apple smoothies in a box. Jennie takes a random smoothie from a box then replaces it. She then takes another. **What is the probability that the smoothies will be the same flavour?**

8. There are 10 books on a shelf. 7 are fiction and 3 are non-fiction. A member of the public takes a book at random, looks at it, then replaces it. They then take another book. **What is the probability that**

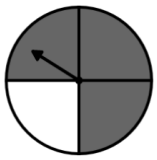
a) **Both books are non-fiction?**
b) **Both books are fiction?**
c) **one of each?**

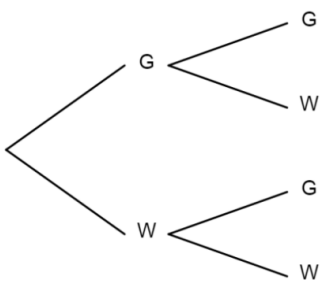
9. The probability that a biased coin will show heads is 0.4. It is flipped twice. **What is the probability that the coin will show heads on both flips?**

10. Suitcase A contains 3 hats and 7 t-shirts. Suitcase B contains 8 hats and 2 t-shirts. You randomly take an item of clothing from both suitcases. **What is the probability that you will have one hat and one t-shirt?**

Extension: A bag contains 5 red balls and 3 blue. A ball is picked at random and then **not** replaced. A second ball is then picked. **Draw a tree diagram. What is the probability of picking at least one red ball?**

1. A spinner has four equally sized sectors: three grey and one white. The spinner is to be spun twice.
(a) Complete the tree diagram.





P(GG) =

P(GW) =

(b) Work out the probability that the spinner will land on the same colour on both spins.

(c) Work out the probability that the spinner will land on grey on at least one of the spins.

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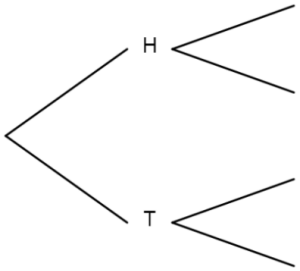
2. Simon has a biased coin, which has a probability of 0.68 of landing on heads.
He is going to flip the coin twice.
(a) Complete the tree diagram.



(b) Shade the true statements.

The probability of the coin landing on heads twice is greater than 50%.
The probability of two tails is a quarter of the probability of two heads.
P(HT) = P(TH).
The probability that the coin lands on heads at least once is 0.90 (correct to 2 d.p.)

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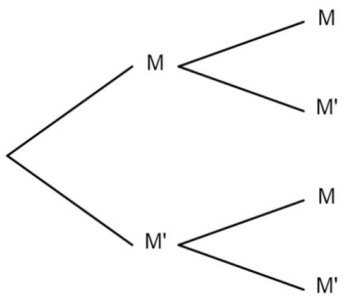
3. Angela is playing a game with two fair dice. She rolls both dice and wins a point for each die that lands on a multiple of 3.
(a) Complete the tree diagram, in which M stands for a multiple of 3.



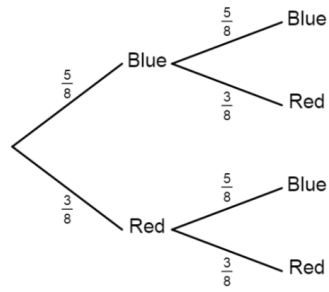
(b) Work out the probability that Angela scores 2 points.

(c) Work out the probability that Angela scores at least 1 point.

(d) Work out the probability that Angela scores no points.

4. A pack of cards contains red and blue cards only. Sally is going to pick a card at random, replace it and then pick another. A tree diagram for this situation is shown.

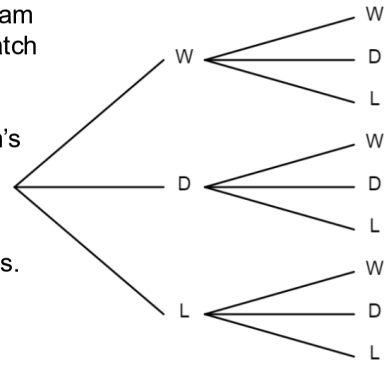
There are 18 red cards in the pack. How many blue cards are there?



5. Liam coaches a football team. Based on their past performance, he works out that the team have a 44% probability of winning each match and a 21% probability of drawing.

(a) Complete the tree diagram for the team's next two matches.

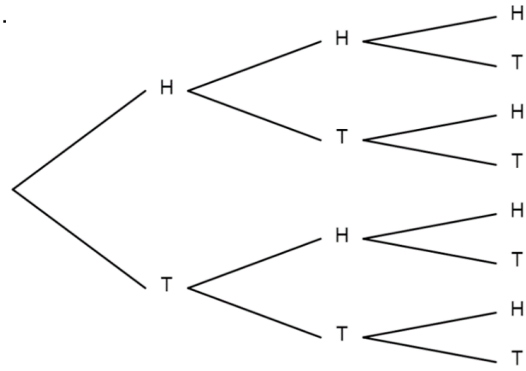
(b) Work out the probability that the team will not lose either of their next two matches.



6. A fair coin is to be flipped three times.
(a) Complete the tree diagram.

(b) Work out the probability that the coin lands on heads exactly twice.

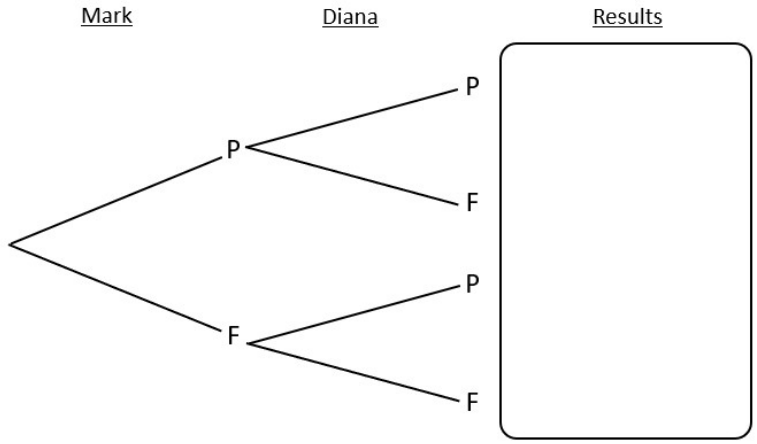
(c) Work out the probability that the coin lands on tails at least once.



7. John cycles to and from work. He passes a level crossing on his route, which has the gates closed 8% of the time. Work out the probability that on a given day the gates are closed on at least one of his two journeys.

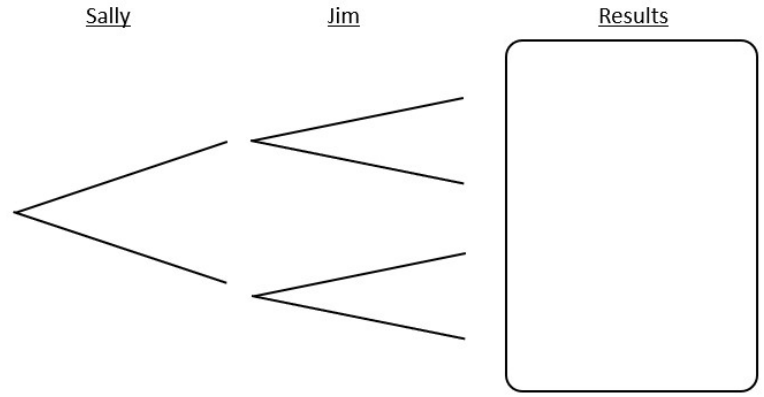
Fluency Practice

1. The probability Mark passes his driving test is $\frac{1}{3}$. The probability Diana passes is $\frac{5}{6}$.
Complete the Tree Diagram and find the probability they both pass their tests.



2. The probability Sally passes her history test is $\frac{4}{5}$. The probability Jim passes is $\frac{2}{3}$.

a) Complete the Tree Diagram and find the probability they both fail their tests.
b) What is the probability at least one of them passes?



3. The probability Jim passes his maths test is $\frac{7}{10}$. The probability he passes English is $\frac{4}{5}$.

a) Complete a Tree Diagram and find the probability he passes both tests.
b) What is the probability he fails both tests?
c) What is the probability he passes only one test?

4. The probability Helen passes her English test is $\frac{4}{9}$. The probability she passes History is $\frac{3}{7}$.

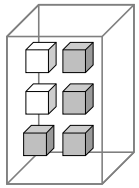
Complete a Tree Diagram and find the probability she passes at least one test.

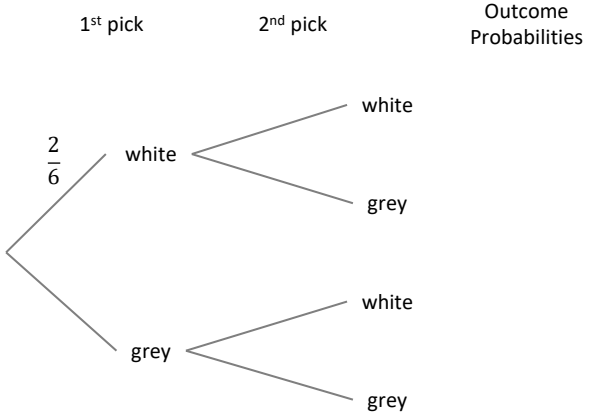
Fluency Practice

Probability Trees

A cube is picked at random from the box.
It is **replaced**, then another cube is picked.
Complete the probability tree & the probability of each outcome.

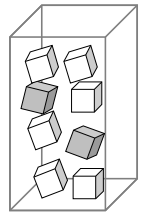
1)

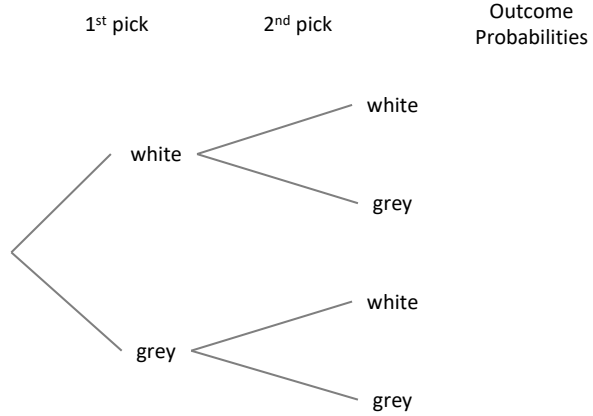




A cube is randomly picked from the box & replaced.
What is the probability 2 grey cubes are picked?
What is the probability 2 white cubes are picked?

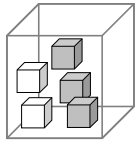
2)

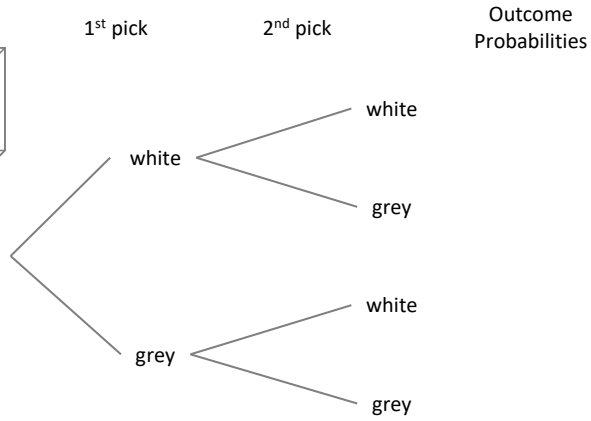




A cube is picked at random from the box.
It is **not replaced** before another cube is picked.
Complete the probability tree & the probability of each outcome.

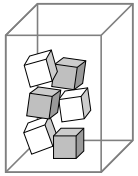
3)

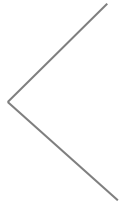




A cube is picked from the box, and **not replaced** before another is picked.
What is the probability 2 white cubes are picked?
What is the probability only 1 grey cube is picked?

4)



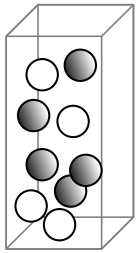


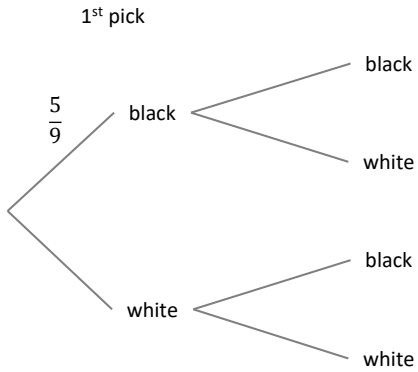
Fluency Practice

Probability Trees

A sphere is randomly picked from the box & replaced.
What is P(two black spheres)?
What is the probability 2 spheres of different colours are picked?

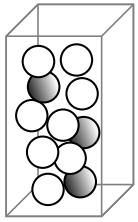
1)

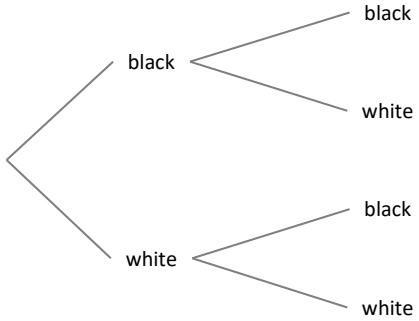




A sphere is randomly picked from the box & replaced.
What is P(two spheres of the same colour)?
What is P(at least one black sphere)?

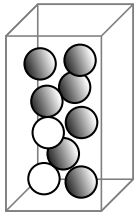
2)

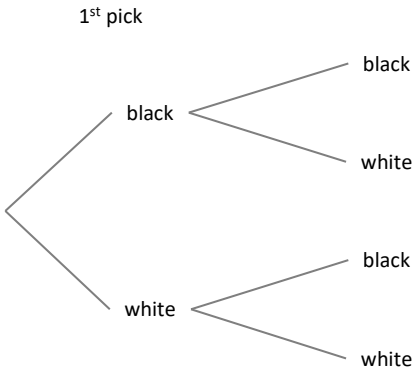




Two spheres are picked from the box (**without** the first being replaced).
What is P(2 white spheres)?
What is the probability at least one of the spheres is white?

3)

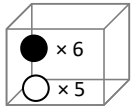


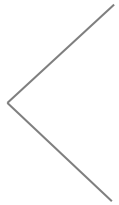


If two disks are picked from this box, what is P(different coloured disks)?

If **three disks** are picked from the box, what is P(3 white disks)?

4)

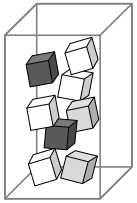




Fluency Practice

Probability Trees

1) A cube is picked at random from the box, then replaced.



1st pick

2nd pick

Outcome Probabilities

Calculate:

P(two white cubes)

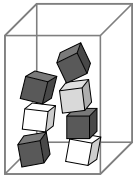
P(**not** two grey cubes)

P(two cubes of the same colour)

P(a white & a grey cube)

P(at least one white cube)

2) Two cubes are picked randomly and simultaneously from the box.



1st pick

2nd pick

Outcome Probabilities

black

white

grey

Calculate:

P(two black cubes)

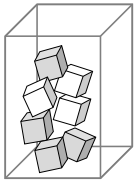
P(**not** two white cubes)

P(two cubes of different colours)

P(a grey cube & a black cube)

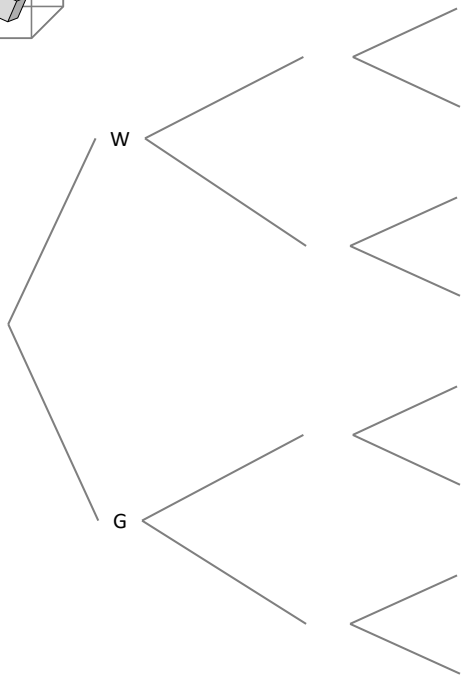
P(at least one black cube)

Fluency Practice



1) A cube is picked at random from the box, replaced, then another is picked & replaced, then another is picked.

(7×7×7 = 343)



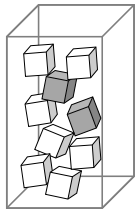
Calculate:

P(GGG)

P(WGW in order)

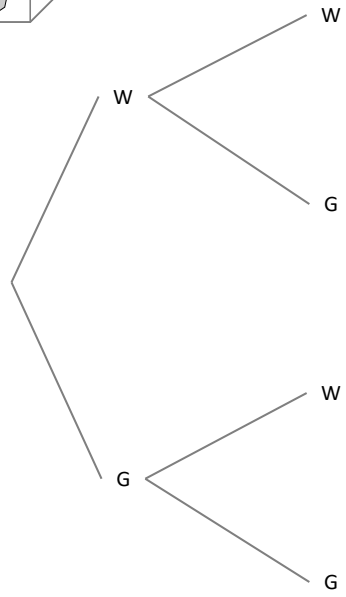
P(3 cubes of the same colour)

P(exactly 2 white cubes)



2) **Three cubes** are picked at random from the box.

(9×8×7 = 504)



Calculate:

P(WWW)

P(2 or more white cubes)

P(2 of the cubes are grey)

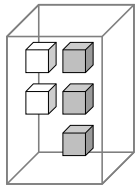
If 4 cubes are picked, what is P(WWWW)?

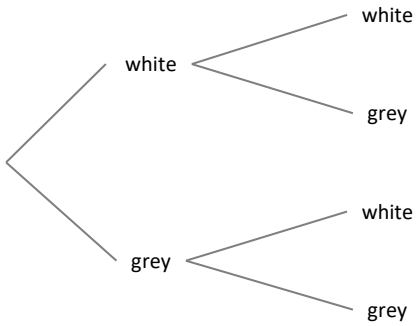
Fluency Practice

Probability Trees

A cube is picked at random from the box.
It is **replaced**, then another cube is picked.
Complete the probability tree & the probability of each outcome.

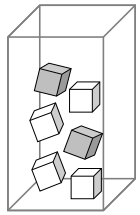
1)

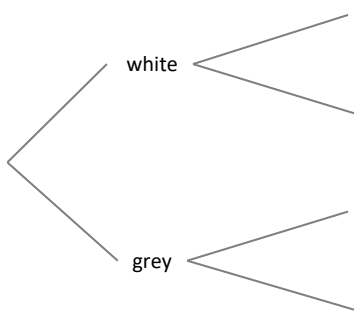




A cube is randomly picked from the box & replaced.
What is the probability 2 grey cubes are picked?
What is the probability 2 white cubes are picked?

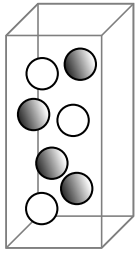
2)

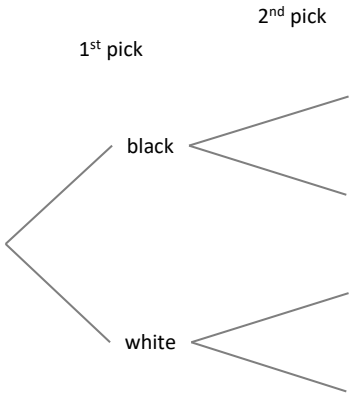




A sphere is randomly picked from the box & replaced.
What is P(two black spheres)?
What is the probability 2 spheres of different colours are picked?

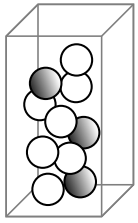
3)





A sphere is randomly picked from the box & replaced.
What is P(two spheres of the same colour)?
What is P(at least one black sphere)?

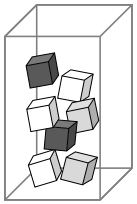
4)

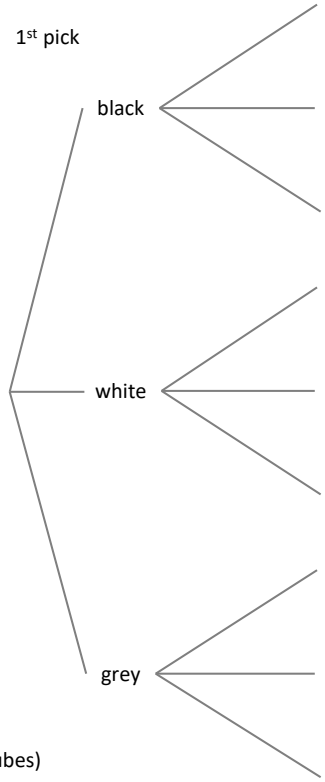


Fluency Practice

Probability Trees

1) A cube is picked at random from the box, then replaced.





Calculate:

P(two white cubes)

P(**not** two grey cubes)

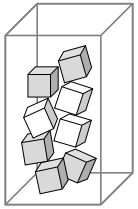
P(two cubes of the same colour)

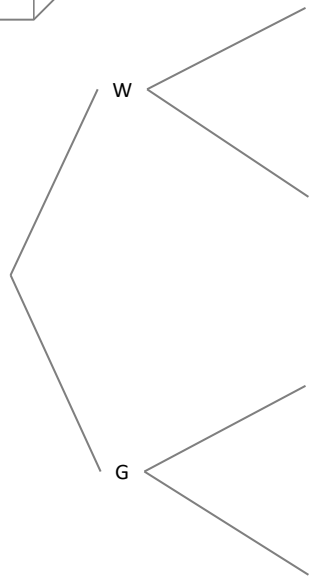
P(a white & a grey cube)

P(at least one white cube)

2) A cube is picked at random from the box, replaced, then another is picked, replaced, then another is picked.

(8×8×8 = 512)





Calculate:

P(GGG)

P(WGW in order)

P(3 cubes of the same colour)

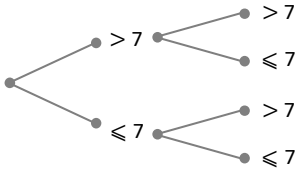
P(exactly 2 grey cubes)

Fluency Practice

Tree Diagrams: One Branch

10 cards are numbered 1 to 10.
A card is picked randomly & replaced, before another card is picked.

Complete the probabilities on each branch of this tree diagram.



What is the probability both cards have a value above 7?

Did we need to draw the entire tree diagram?

To find specific probabilities, we can draw only **one branch** of a tree diagram.

10 cards are numbered 1 to 10.
A card is picked randomly & replaced, before another card is picked.

What is the probability both cards have a value above 5?

What is the probability both cards have a value over 8?

12 cards are numbered 1 to 12.
A card is picked randomly & replaced, before another card is picked.

Complete single branches to calculate these probabilities:

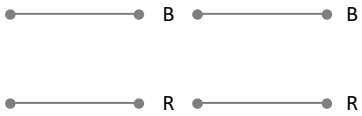
P(both cards are odd)

P(both cards have a value over 8)

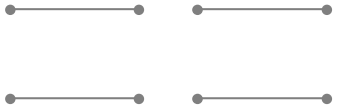
P(both cards are not greater than 5)

A bag contains 3 black & 7 red cubes.
Cubes are picked and then replaced.

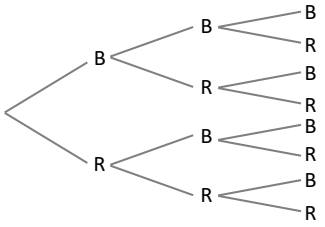
Use these two branches to calculate:
P(2 cubes of the same colour are picked)



Use two branches to calculate:
P(2 cubes of different colours are picked)



This tree diagram shows 3 picks from the same bag.



Calculate:
P(3 cubes of the same colour)

On the diagram highlight every branch where **only 1** of the 3 cubes picked is red.

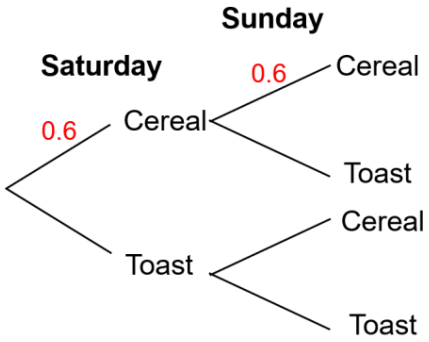
Find: P(only 1 of the 3 cubes is red)

If **4 picks** are made, calculate…

P(only 1 of the cubes is red)

P(exactly 2 of the cubes are red)

Ray has cereal or toast for his breakfast. The probability that he has cereal is 0.6. Ray has breakfast on Saturday and Sunday. Complete the tree diagram.



Find the probability that Ray has toast for breakfast on both days.

Zoya throws a biased coin twice. The probability that it lands on heads is 0.55. Represent this with a tree diagram. Find the probability that:
(a) Zoya gets two heads
(b) Zoya gets one of each - head and tail

Joy and Peter both sit a French test. The probability of passing the test is $\frac{2}{5}$. Represent this with a tree diagram. Find the probability that
(a) only one of them passes the test
(b) they both fail the test

The probability that Anna gets up late on a Monday is $\frac{1}{3}$. The probability that she gets up late on a Tuesday is $\frac{1}{4}$. Represent this with a tree diagram. Find the probability that Anna gets up late on at least one of the days.

Jamal plays three games of Monopoly against Iris. The probability that Jamal wins Monopoly is 0.4. Find the probability that Jamal wins at least two of the three games.

Almost all medical tests have a small chance of a 'false positive' or a 'false negative'.

- **False positive**: The test says you **do** have the condition when you really **don't**.
(you're fine, but the test thinks you're sick)
- **False negative**: The test says you **don't** have the condition when you really **do**.
(you're sick, but the test thinks you're fine)

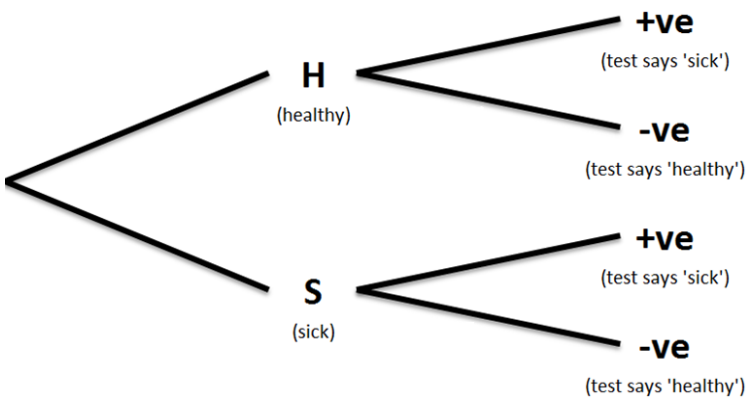
- The test for a particular disease has a false positive rate of 5%.

*This means that 5% of people who **don't** have the disease will be told that they **do**.*

- The test has a false negative rate of 1%.

*This means that 1% of people who **do** have the disease will be told that they **don't**.*

Assuming this particular disease affects 10% of people, complete the tree diagram below:



Use the tree diagram to calculate the probabilities for each of the four outcomes:

	Healthy	Sick
Tested Positive		
Tested Negative		

You're sick, and the test was correct

You're sick, but the test thinks you're fine
FALSE NEGATIVE:
You may not get the treatment you need.

You're fine, but the test thinks you're sick
FALSE POSITIVE:
You may get treatment you don't need, or be worried for no reason.

You're fine, and the test was correct

If the test says I'm sick, what is the chance that I really am?

Hint: Imagine 1000 people take the test. Work out how many would test positive, and out of those people, how many are really sick?

_____

WORDED: PROBABILITY

You may want to use a
probability tree diagram to help answer these questions.

1) Kelly goes ocean fishing. In that area there is a 30% chance that a caught fish is a barracuda. What is the probability Kelly catches two barracuda in a row?

2) James plays an online computer game. The probability of him winning any match is 0.6. What is the probability he wins his first match, then loses his second match?

3) In a factory, 5% of the finished TVs have a defect that needs to be fixed before they are shipped. As a fraction, what is the probability 3 TVs in a row have a defect?

4) Woodgreen United will play a home and an away match against Hughton FC. The probability of Woodgreen winning the home match is 0.9. The probability of them winning the away match is 0.6. What is the probability Woodgreen United win only one match?

5) A bag contains 12 red discs and 8 green discs. A disc is picked and replaced before another is picked. What is the probability at least one green disc is picked?

6) The probability of Shanice winning any darts match is $\frac{5}{7}$. If she plays twice, what is the probability the same person wins both matches?

7) In a box there are 3 triangles and 2 squares. Create a probability tree diagram to show the possible results when we randomly pick and replace a shape 3 times. What is the probability 2 of the 3 picks are squares?

Fluency Practice

Jim thinks about all the probabilities for the next day.
We can assume these events are **independent**.

P(Jim will be disappointed with his new haircut.) = $\frac{2}{7}$

P(Jim's bus to work will be late.) = $\frac{1}{5}$

P(Jim will bump into an old friend.) = $\frac{3}{20}$

P(Jim's favourite coffee shop will run out of Jim's favourite coffee.) = $\frac{1}{20}$

P(Jim will win the lottery.) = $\frac{1}{1{,}000{,}000}$

P(Jim's alarm clock will not work.) = $\frac{1}{10}$

P(Jim will get a rude email at work.) = $\frac{4}{5}$

P(It will be sunny after work.) = $\frac{3}{4}$

P(Jim will get a pay raise at work.) = $\frac{5}{12}$

P(Jim's football team will win their mid-week match.) = $\frac{1}{2}$

P(Jim will get rained on.) = $\frac{5}{8}$

P(Jim's partner will cook his favourite meal for dinner.) = $\frac{2}{3}$

P(Jim will find a £5 note on the street.) = $\frac{3}{100}$

Find the probabilities for these combined events (a tree-diagram can help):

a) Jim's bus is late but he enjoys some sun after work.

b) Jim's bumps into an old friend and his football team win their match.

c) Jim's gets a rude email at work, but he gets his favourite meal in the evening.

d) Jim's alarm clock will not work, he will get a rude email and he will be disappointed with his new haircut.

e) Jim's alarm clock works, but the bus to work is late.

f) Jim doesn't get a pay raise, but he does win the lottery.

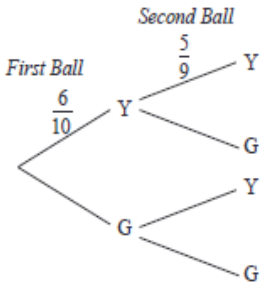
g) Jim's will be happy with his haircut and he doesn't get rained on.

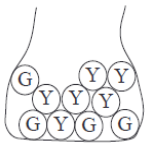
h) While enjoying some sun after work, Jim will bump into an old friend and then find £5 on the street.

Create possible good & bad days for Jim. How close to a 50% probability can you get?

Describe how tomorrow's events may not be independent.

1. A bag contains 6 yellow balls (Y) and 4 green balls (G). One ball is taken out at random and not put back. A second ball is then taken out. Copy and complete the tree diagram:





a. Calculate P(2 yellow balls)
b. Calculate P(2 green balls)
c. Calculate P(a green ball and a yellow ball in any order)

2. A drawer contains 8 green socks and 10 blue socks. One sock is taken out of the drawer at random and not replaced. A second sock is then taken out. Draw a probability tree and determine the probability that two socks of the same colour have been taken out of the drawer.

3. A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded. Draw a tree diagram to represent this information.

   Find the probability that
   (a) both balls selected are red
   (b) the second ball selected is red

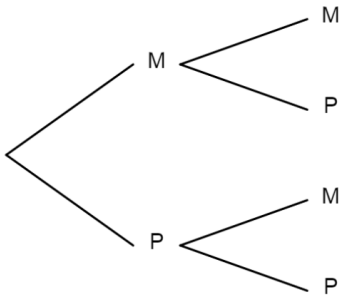
4. A packet contains stamps from three different countries. The packet contains 4 Spanish stamps, 10 French stamps and 6 German stamps. Two stamps are to be removed, without replacement. By drawing a probability tree, calculate the probability that both stamps will be from the same country.

5. In a village, 3/5 of the pensioners have had a flu jab. If a pensioner has had the flu jab, the probability of catching flu is 1/30. If a pensioner has **not** had the flu jab, the probability of catching flu is 7/10.
   a) Calculate the probability that a pensioner, picked at random from this village, catches flu.
   b) A statistician calculated that 120 pensioners from this village are expected to catch flu. Calculate how many pensioners live in the village.

6. After a flood, Debbie finds that all the labels have come off the tins in her cupboard. Debbie knows that she had 5 tins of tomatoes and 7 tins of baked beans. She opens 3 tins at random. What is the probability that she has opened 2 tins of baked beans and one tin of tomatoes?

7. A captain and five remaining passengers remain aboard a sinking ship. There are three lifejackets remaining. The captains knows that three of the passengers cannot swim. In his panic, the captain forgets who the non-swimmers are and hands out the lifejackets randomly to three of the five passengers. What is the probability that the gives the lifejackets to just two of the three non-swimmers?

1. A box contains 8 milk chocolates and 4 plain chocolates. Linda is going to choose a chocolate at random, eat it, and then choose a second.
(a) Complete the tree diagram.



(b) Work out the probability that Linda will choose two milk chocolates.

(c) Work out the probability that Linda will choose one of each type of chocolate.

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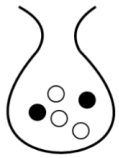
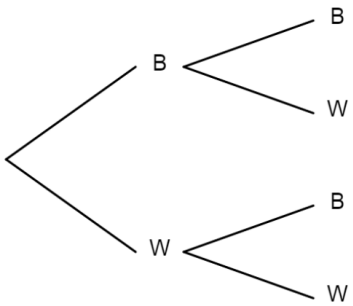
2. A bag contains five counters: three white and two black. Two counters are selected from the bag at random at the same time.
(a) Complete the tree diagram.





(b) Work out the probability that the counters will be the same colour.

(c) Work out the probability that the counters will be different colours.

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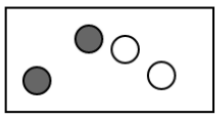
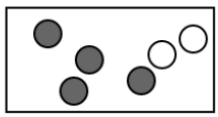
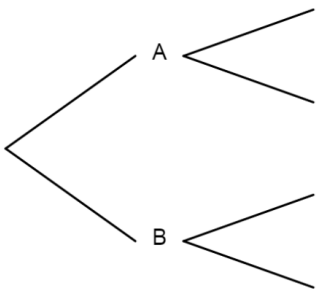
3. Two boxes contain grey and white marbles, as shown. A box is chosen at random and then a marble from the box is chosen.
(a) Complete the tree diagram.



Box A



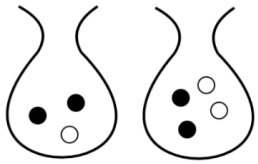
Box B



(b) Work out the probability that a grey marble is selected.

(c) Work out the probability that a white marble is selected.

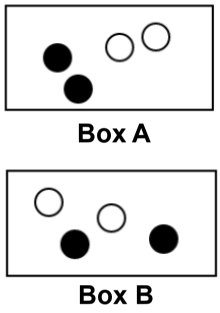
4. Two bags contain black and white counters as shown.



A counter is to be picked at random from Bag A and placed into Bag B.
A counter is then to be picked from Bag B.

Work out the probability that the counter picked from Bag B will be white.

5. Two boxes contain black and white counters as shown.



A counter will be picked from Box A and placed into Box B.
A counter will then be picked from Box B and placed into Box A.

(a) Work out the probability that Box A will end up containing 3 white counters.

(b) Work out the probability that Box A will end up containing exactly 2 white counters.

6. A jar contains eight 10p coins and six 5p coins. Three coins are picked at random from the jar, without replacement.

(a) Work out the probability that the total value of the three coins is 30p.

(b) Work out the probability that the total value of the three coins is 15p.

(c) Work out the probability that at least one of each type of coin is picked.

7. Each day, there is a 40% chance that Luke will cycle to work, otherwise he will take the bus.

If he cycles, there is a 5% chance he will arrive late. If he takes the bus, there is a 15% chance he will arrive late.

(a) Work out the probability that Luke will arrive late to work on any given day.

(b) Given that Luke was late one day, work out the probability that he had cycled on this day.

8. A particular disease affects 5% of the trees in a forest.

A test on a sample from the tree has a 90% chance of showing a positive result if the tree is diseased. The test has a 15% chance of showing a positive result if the tree is not diseased.

(a) Work out the probability that the test will show a positive result for a randomly selected tree.

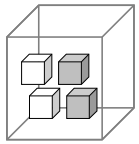
(b) Given that the test on a tree sample has returned a positive result, work out the probability that the tree is diseased.

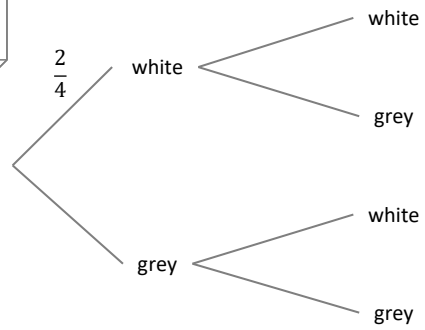
Fluency Practice

Probability Trees

A cube is picked at random from the box.
It is **not replaced** before another cube is picked.
Complete the probability tree & the probability of each outcome.

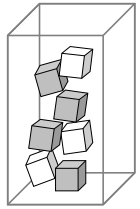
1)

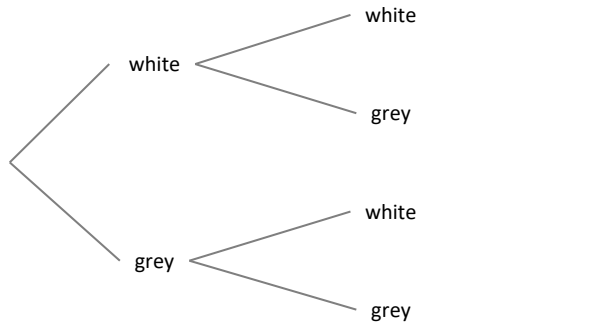




A cube is picked from the box, and **not replaced** before another is picked.
What is the probability 2 white cubes are picked?
What is the probability only 1 grey cube is picked?

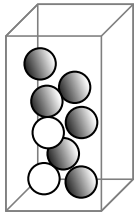
2)

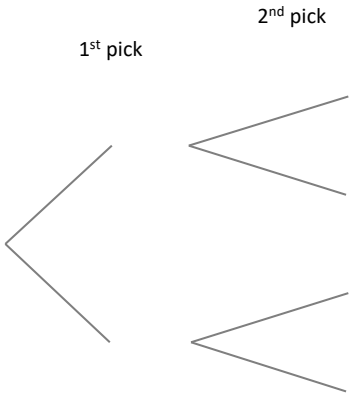




Two spheres are picked from the box.
What is P(2 white spheres)?
What is the probability at least one of the spheres is white?

3)

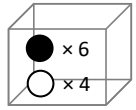




If two disks are picked from this box, what is P(different coloured disks)?

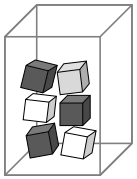
If three disks are picked from the box, what is P(3 white disks)?

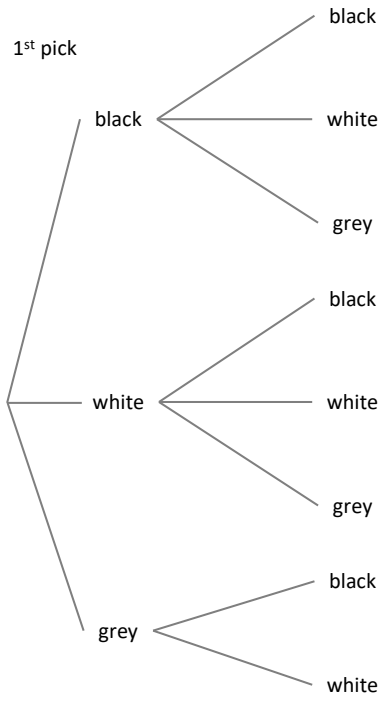
4)



Fluency Practice

1) Two cubes are picked at random from the box.





Calculate:

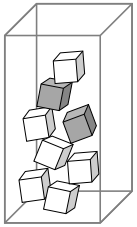
P(two black cubes)

P(**not** two white cubes)

P(two cubes of different colours)

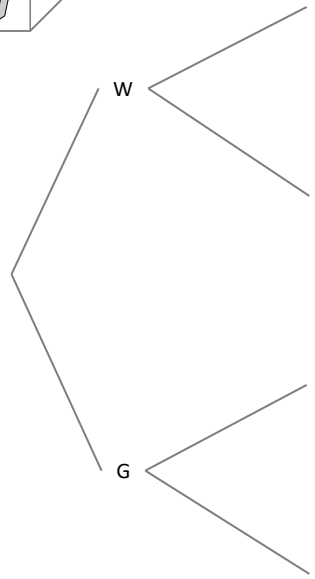
P(a grey cube & a black cube)

P(at least one black cube)



2) **Three cubes** are picked at random from the box.

(8×7×6 = 336)



Calculate:

P(WWW)

P(2 or more white cubes)

P(2 grey cubes)

If 4 cubes are picked, what is P(WWWW)?

1 A box contains 3 red discs, 5 blue discs and 6 green discs. I remove one disc at random, note its colour then return it to the box. I then remove a second disc at random.

(a) Find the probability that both discs are the same colour.

(b) Find the probability that exactly one disc is red.

(c) If I take a total of 5 discs, replacing the disc each time, find the probability that all 5 discs are red.

2 Another box contains 4 yellow discs and 7 black discs. I remove one at random, place it in my pocket then remove a second disc at random.

(a) Find the probability that the discs are different colours.

(b) If I take a total of three discs, find the probability that:

(i) all are yellow (ii) all three are different colours.

3 A bag contains some red balls, some blue balls and some green balls. When a ball is removed at random the probability that it is red is $\frac{1}{3}$ and the probability that it is blue is $\frac{2}{9}$. There are more than 30 balls in the box. What is the fewest number of green balls?

4 Each morning the probability that Anna gets up late is $\frac{1}{5}$. If she gets up late the probability that misses her bus is $\frac{5}{6}$. If she doesn't get up late the probability that she misses her bus is $\frac{1}{4}$.

(a) Find the probability that, on Monday morning, she misses her bus.

(b) Find the probability that she catches her bus every morning from Monday to Friday.

5 A box contains some red ball and some blue balls. There are four more blue balls than red balls. A ball is removed at random, replaced and a second ball randomly removed. The probability that the two balls are different colours is $\frac{21}{50}$. How many balls of each colour are in the box?

6 A box contains some white balls and some blue balls. There are 5 more blue balls than white balls. One ball is removed at random and not replaced. A second ball is then removed at random. The probability that the balls are different colours is $\frac{52}{105}$. Find the probability that both balls are white.

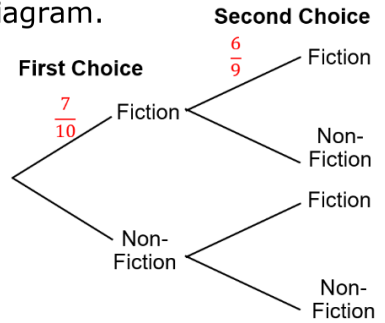
7 A prize must be randomly awarded to just one of a group of ten people. Which of the following methods is the fairest?

A: Fold 10 identical pieces of paper in half and put them in a bag. Nine have 'LOSE' written on them and one has 'WIN'. Line the people in alphabetical order. The first person randomly takes a piece of paper. If it says 'WIN' they are the winner and the game stops there. If it says 'LOSE' they do not replace the paper and the next person has a go.

B: Same as 'A' but this time the paper is put back in the bag before the next person has a go.

C: Same as 'B', that is each piece of paper is returned to the bag, but there are 999 'LOSE' papers and one 'WIN' paper.

A bookshelf contains three non-fiction books and seven fiction books. Bob chooses two books at random. Complete the tree diagram.



Find the probability that Bob chooses two non-fiction books.

A drawer contains five red socks and three black socks. A sock is taken out at random and not replaced. A second sock is then taken out. Draw a tree diagram and calculate the probability that either a pair of red socks or a pair of black socks is chosen.

A bag contains 4 yellow balls and 6 green balls. A ball is taken from the bag and not replaced. A second ball is then taken. Draw a tree diagram and find the probability that the two balls are different colours.

Amir has a biscuit tin containing 3 bourbons and 5 custard creams. He chooses a biscuit at random, eats it, then chooses another and eats that too. Draw a tree diagram and use it to find the probability that Amir has eaten at least one custard cream.

In a zoo, there are four elephants, three tigers and two giraffes. The zookeeper wants to choose two animals at random to take part in a promotional photo. Draw a tree diagram and use it to find the probability that the zookeeper chooses two of the same animal.

(a) A bag contains seven red balls and three black balls. Anton takes out a ball, notes the colour and replaces it. He does this three times. What is the probability that he takes out at least one red ball?

(b) Debbie has 5 tins of tomatoes and 7 tins of beans in the cupboard, but the labels have come off the tins. Debbie chooses two tins at random. Calculate the probability that Debbie chooses two tins of the same type.

(c) Ashok has six coins in his pocket. He has one 5 cent coin, two 10 cent coins and three 20 cent coins. He takes at random a coin from his pocket, records its value and puts the coin back into his pocket. He does this twice. Calculate the probability that the second coin he takes has a higher value than the first coin he takes.

(d) A bag contains 3 black beads, 5 red beads and 2 green beads. Gianna takes a bead at random from the bag, records its colour and replaces it. She does this two more times. Work out the probability that, of the three beads Gianna takes, exactly two are the same colour.

(e) Barney has a biased coin. When the coin is thrown once, the probability that the coin will land heads is 0.3. Barney throws the coin 4 times. Work out the probability that the coin will land heads at least once.

(f) Carolyn has 20 biscuits in a tin. She has 12 plain biscuits, 5 chocolate biscuits and 3 ginger biscuits. Carolyn takes at random two biscuits from the tin. Work out the probability that the two biscuits were **not** the same type.

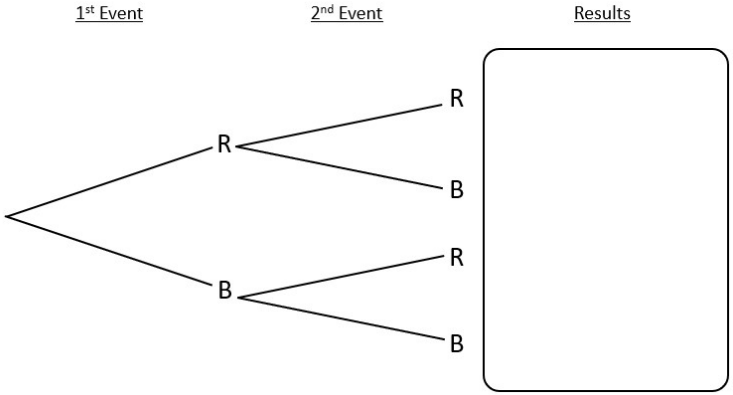
Fluency Practice

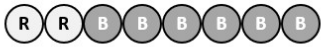
Conditional Tree Diagrams

a) 

There are 5 red and 3 blue buttons in a sack.

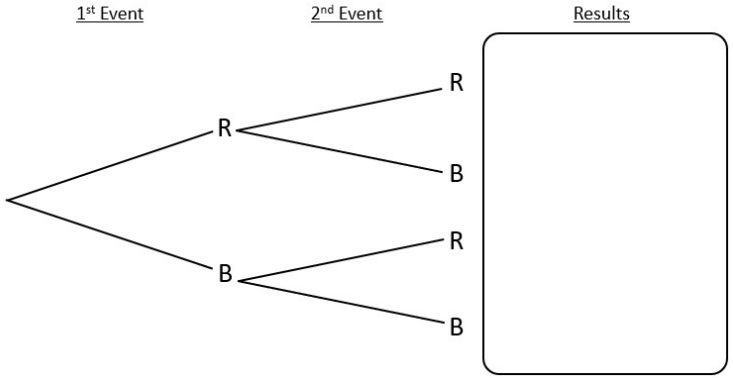
A button is taken out and not replaced. Then another button is taken out.
What is the probability both buttons are blue?



b) 

There are 2 red and 6 blue buttons in a sack.

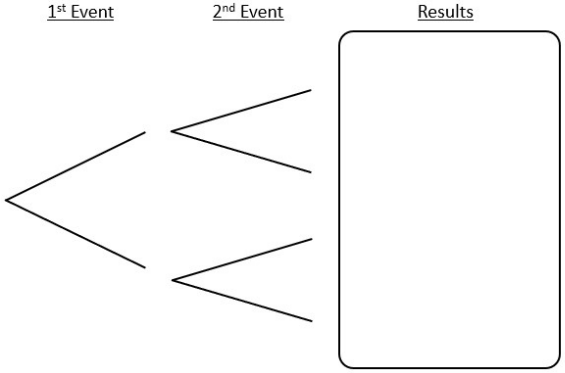
A button is taken out and not replaced. Then another button is taken out.
What is the probability both buttons are red?
What is the probability of choosing *at least one* red button?



c) 

There are 4 red and 5 green buttons in a sack.

A button is taken out and not replaced. Then another button is taken out.
What is the probability of choosing *at least one* red button?
What is the probability of choosing *only one* red button?



d) A science assessment has a practical and written test. Students have an 80% chance of passing the written test. If they pass the written test there is a 60% chance they pass the practical test. If they fail the written test, there is a 30% chance they will pass the practical test.

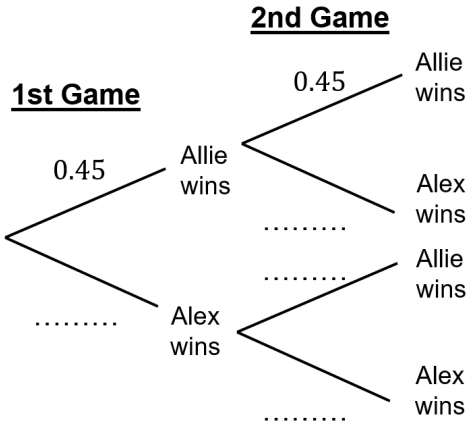
1) Draw a Probability Tree to show these events.
2) What is the probability a student passes both tests?
3) What is the probability a student passes only one of the tests?

Fluency Practice

Tree Diagrams Revision

(a)

Allie and Alex play two games of tennis. The probability of Allie winning the game is 0.45.
(a) Complete the tree diagram.



(b) Find the probability that Alex wins both matches.

(c) Find the probability that Allie and Alex win one match each.

(b)

Yusuf spins two fair spinners, A and B. Spinner A can land on a 1, 2 or 3. Spinner B can land on a 2, 3 or 4.
(a) Draw a tree diagram.

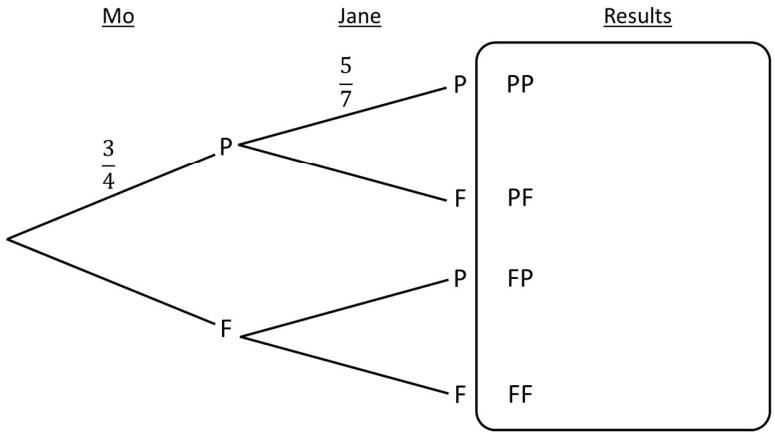
(b) Find the probability that the total on the spinners is an odd number.

(c)

There are 10 biscuits in a tin. 7 are digestives and 3 are bourbons. Temi takes a biscuit at random from the tin and eats it. She does this two more times. Calculate the probability that she has eaten at least two digestives.

Fluency Practice

① The probability Mo passes his driving test is $\frac{3}{4}$. The probability Jane passes is $\frac{5}{7}$.
Complete the Tree Diagram and find the probability they both fail their tests.



③ 

There are 2 red & 7 blue buttons in a sack.

A button is taken out and not replaced. Then another button is taken out.
a) What is the probability both buttons are red?
b) What is the probability of choosing two buttons of the same colour?

② The probability Latifa passes her Maths test is $\frac{7}{9}$.

The probability she passes her English test is $\frac{5}{6}$.

a) Complete a Tree Diagram and find the probability she passes both tests.
b) What is the probability she fails both tests?
c) What is the probability she passes only one test?

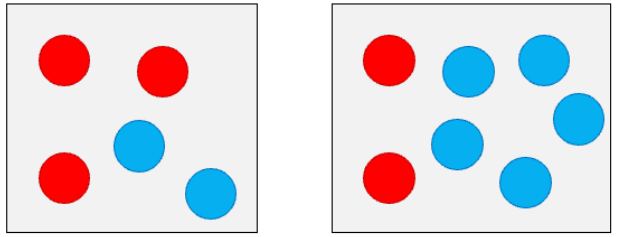
④ A science assessment has a practical & a written test. Kara has a 70% chance of passing the written test. If she passes the written test there is a 80% chance she passes the practical test.
If she fails the written test, there is a 30% chance she will pass the practical test.

a) Draw a Probability Tree to show these events.
b) What is the probability Kara fails both tests?
c) What is the probability Kara passes only one of the tests?

Problem Solving

Laura picks a marble from Box A and puts it into Box B.

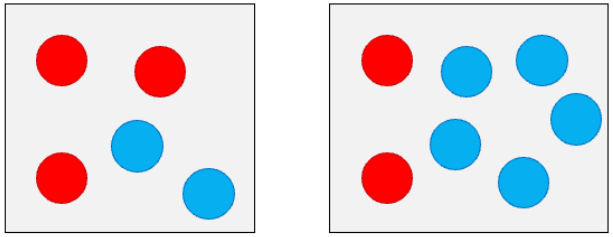
She then picks a marble from Box B.



vs

Laura picks a marble from Box B and puts it into Box A.

She then picks a marble from Box A.



- Does it matter which box Laura chooses from first?
- Will the order affect the probabilities of the combined outcomes?

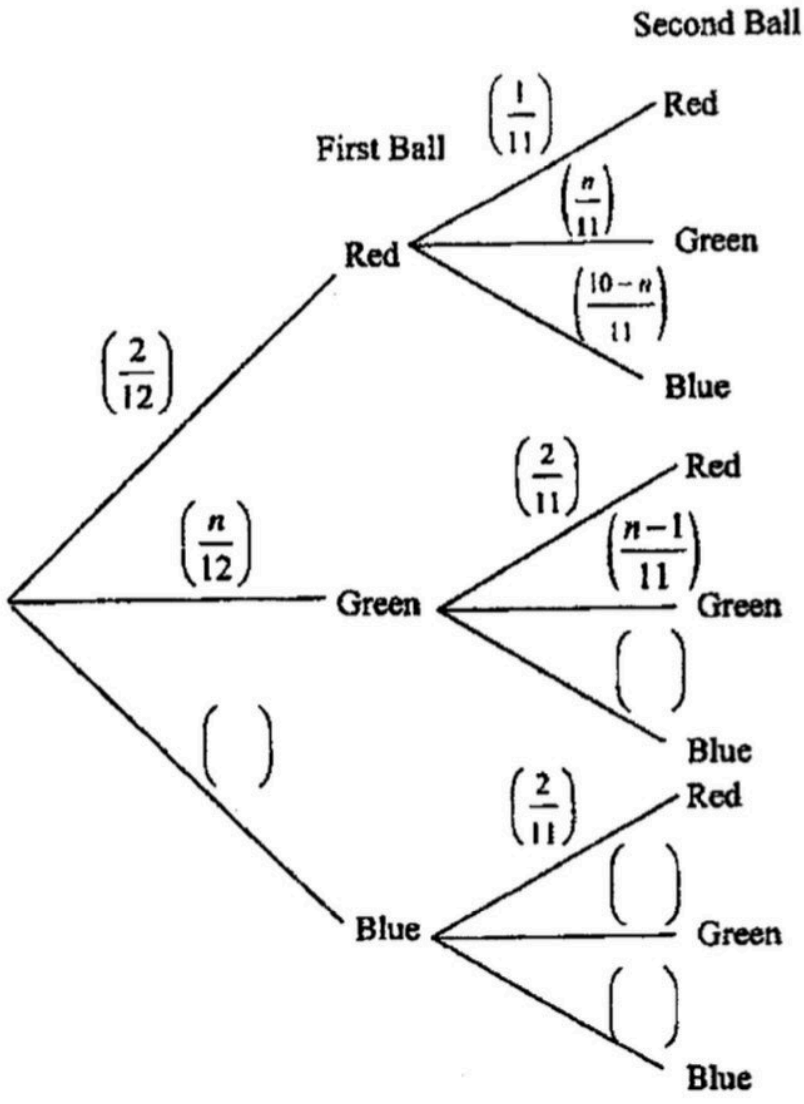
Problem Solving

Donna and James each arrange 36 square tiles into a quadrilateral. They then calculate the area and perimeter of their quadrilaterals.

What is the probability that they both make quadrilaterals where the value of the perimeter is greater than the value of the area?

A bag contains 12 balls.
2 of the balls are red, n are green balls and the rest are blue balls.
Eunice takes two balls from the bag, at random, without replacement.

(a) Complete the tree diagram.



(b) The probability that Eunice takes two blue balls is $\frac{5}{22}$.

Write down an equation to represent this information and show that it simplifies to

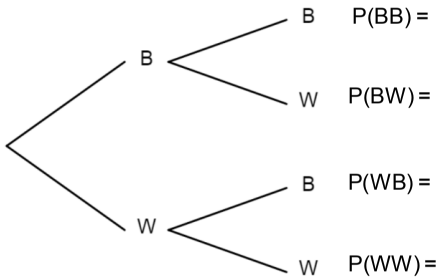
$$n^2 - 19n + 60 = 0 .$$

1. A fair spinner has ten equally likely sectors. n of the sectors are black and the rest are white.

(a) The spinner is to be spun once.
(b) Write expressions in terms of n for:

(i) P(black) (ii) P(white)

(b) The spinner is to be spun twice. Complete the tree diagram with expressions in terms of n for the probabilities.



(c) Given that when the spinner is spun twice, the probability that it lands on black both times is $\frac{16}{25}$, work out the value of n.

2. A fair spinner has eight equally sized sectors. n of the sectors are red and the rest are blue.

(a) The spinner is to be spun once. Write expressions in terms of n for:

(i) P(red) (ii) P(blue)

(b) If the spinner is spun twice, the probability of it landing on red both times is $\frac{9}{16}$. Work out the value of n.

3. A fair six-sided die has n green faces. The rest of the faces are black.

(a) The die is to be rolled once. Write expressions in terms of n for:

(i) P(green) (ii) P(black)

(b) If the die is rolled twice, the probability of it landing on two different colours is $\frac{1}{2}$. Work out the value of n.

4. There are 10 cards in a pack. x of the cards have a triangle and the rest have a star.

(a) A card is to be picked at random. Write expressions in terms of x for:

(i) P(triangle) (ii) P(star)

(b) If a card is picked at random, replaced, and then a second card picked, the probability that the two counters have the same shape is $\frac{29}{50}$. Work out the two possible values of x.

5. A bag contains 15 counters. x of the counters are yellow.

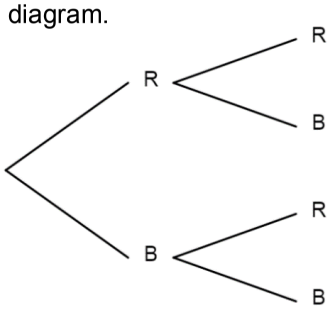
(a) A counter is to be picked at random. Write expressions in terms of x for:

(i) P(yellow) (ii) P(yellow')

(b) If a counter is picked from the bag at random, replaced, and then a second counter picked, the probability that at least one counter will be yellow is $\frac{24}{25}$. Work out the value of x.

6. A bag contains red and blue counters.
There are n red counters and twice as many blue counters.

(a) Two counters are picked at random without replacement.
Work out the probabilities (in terms of n) for the tree diagram.



The probability that the counters are both red is $\frac{1}{10}$.

(b) Work out the value of n.

7. A bag contains yellow and red sweets in the ratio 1 : 4.

If two counters are picked at random (without replacement), the probability that they will both be yellow is $\frac{1}{35}$.

Work out how many yellow sweets are in the bag.

8. A bag contains black and white tokens.
There are 3 times as many white tokens as back discs.

If two tokens are picked at random (without replacement), the probability that they will be the same colour is $\frac{3}{5}$.

Work out how many black tokens are in the bag.

9. A bag contains n counters. 5 of the counters are blue and the rest are pink.

If two counters are picked at random (without replacement), the probability that they will both be blue is $\frac{5}{18}$.

(a) Show that $n^2 - n - 72 = 0$.

(b) Work out the value of n.

10. A bag contains n counters. 8 of the counters are green and the rest are yellow.

If two counters are picked at random (without replacement), the probability that they will be different colours is $\frac{16}{45}$.

(a) Show that $n^2 - 46n + 360 = 0$.

(b) Work out the two possible values of n.

11. A bag contains n red counters and 3 yellow counters.

If two counters are picked at random (without replacement), the probability that they will both be red is $\frac{5}{12}$.

(a) Show that $7n^2 - 37n - 30 = 0$.

(b) Work out the value of n.

12. A bag contains 5 gold counters and at least one silver counter.

If two counters are picked at random (without replacement), the probability that they will be the same colour is $\frac{2}{3}$.

Work out the two possible numbers of silver counters.

Question 1: A drawer contains 6 red socks and 2 green socks.

A red sock is taken out of the drawer and not replaced.

Dexter is going to pick a sock at random from the remaining socks in the drawer.
(a) Write down the probability that Dexter picks a red sock?
(b) Write down the probability that Dexter picks a green sock?

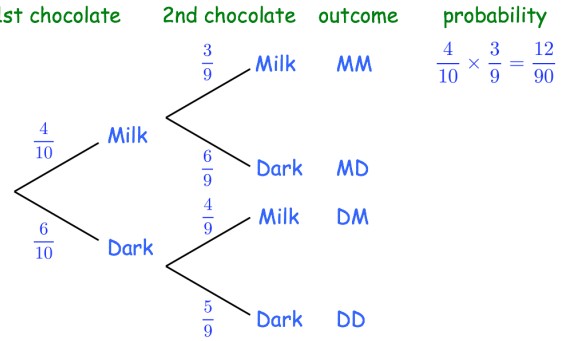
Question 2: There are 10 chocolates in a bag.
Four are milk chocolate and six are dark chocolate.
Two chocolates are taken out at random **without** replacement.

(a) Copy and complete the tree diagram

(b) Find the probability of getting two dark chocolates.

(c) Find the probability of getting one chocolate of each flavour.

(d) Find the probability of getting at least one milk chocolate.

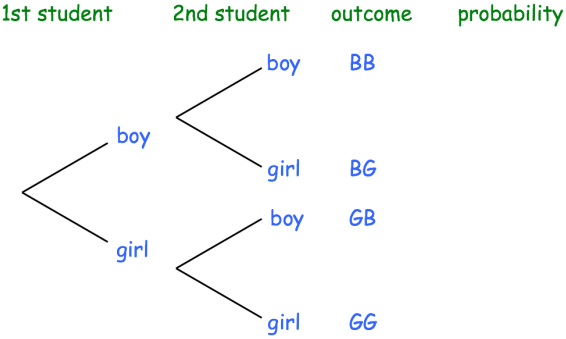


Question 3: 13 of the 20 students in Mr Davidson's class are girls.
Two students are chosen at random.

(a) Copy and complete the tree diagram

(b) Work out the probability of two boys being selected.

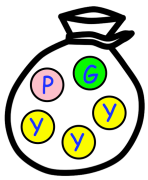
(c) Work out the probability of two girls being selected.



Question 4: There are 12 sweets in a bag
7 are lemon and 5 are orange.
Two sweets are taken out at random **without** replacement.

(a) Work out the probability that the two sweets are both lemon
(b) Work out the probability that the two sweets are both orange
(c) Work out the probability that the two sweets are the same flavour
(d) Work out the probability that the two sweets are different flavours

Question 5: There are five counters in a bag.
One counter is pink, one counter is green and three counters are yellow.
A counter is selected at random from the bag **without** replacement.
Then a second counter is taken at random.



(a) Find the probability that both counters are yellow.
(b) Find the probability of a pink counter then a yellow counter.

Question 6: In dry weather, the probability of a bus being late is $\frac{1}{10}$

In rainy weather, the probability of a bus being late is $\frac{1}{4}$

In snowy weather, the probability of a bus being late is $\frac{2}{3}$

The probability of dry weather is $\frac{3}{4}$

The probability of wet weather is $\frac{1}{5}$

The probability of snow is $\frac{1}{20}$

(a) Show this information on a tree diagram
(b) Calculate the probability that the weather is dry **and** the bus is on time.
(c) Calculate the probability that the bus is late

Question 7: Catherine has two bags of counters.
Bag A contains 6 red counters and 2 black counters.
Bag B contains 1 red counters and 4 black counters

Catherine rolls a fair ordinary six-sided dice.
If the dice lands on a 1, she takes a counter at random from Bag B.
If the dice lands on any other number, Catherine takes a counter randomly from Bag A.

Calculate the probability of Catherine getting a black counter.

Question 8: There are three flavours of crisps in a cafe.
There are 3 packets of salt and vinegar
5 packets of cheese and onion
1 packet of roast chicken
Bella takes two packets of crisps at random.

(a) Work out the probability that she takes 2 packets of crisps that are the **same** flavour.
(b) Work out the probability that she takes 2 packets of crisps that are **different** flavours.

Question 9: Toby has 20 counters in a bag.
11 counters are yellow.
6 counters are red.
3 counters are white.
Toby takes two counters from the bag at random.

Work out the probability that the two counters are **not** the same colour.

Question 10: There are 9 sweets in a bag.
Five sweets are purple, three sweets are white and one sweet is pink.
Three sweets are selected at random **without** replacement.
Calculate the probability that the sweets are **not** all the same colour.

Question 11: Florence has these letter tiles.

She picks three tiles at random without replacement.

(a) Calculate the probability that all three times are vowels.
(b) Calculate the probability that there are no vowels.
(c) Calculate the probability that exactly one T is taken from the bag.

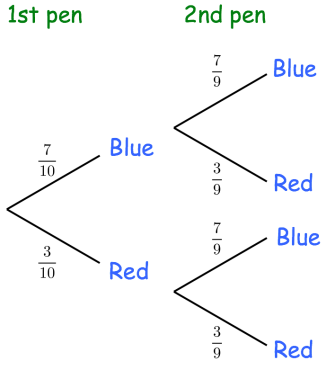
Question 12: There are 11 lego blocks in a bag, with each block the same size.
Five are red and six are yellow.
Four blocks are selected at random.

(a) Calculate the probability that all the blocks are yellow
(b) Calculate the probability that at least 3 of the blocks are red.

Apply

Question 1: Samantha has 10 pens.
7 of the socks are blue and the rest are red.
Samantha takes a pen at random without replacement.
She then takes a second pen at random.

Samantha drew this tree diagram.



(a) Write down what is wrong with the probabilities in the tree diagram.

(b) Calculate the probability that Samantha picks at least one red pen.

Question 2: Here are six number tiles.

3	3	4	5	5	5

Charlie takes a tile at random without replacement.
Charlie then takes a second tile at random.

(a) Work out the probability the both tiles have the number 5 on them

(b) Work out the probability that the number on the second is less than the number on the first tile.

Question 3: There are 20 passengers on a coach.
70% of the passengers are going to Bristol.
The rest are going to Bath.
Four passengers are chosen at random to complete a survey.

Calculate the probability that all four passengers are going to Bath.

Question 4: Ethan has 12 coins.
There are three 10p coins and nine 20p coins.
Ethan chooses 3 coins at random.
Work out the probability that he takes exactly 50p.

Question 5: There are 50 students in Year 11.
Each student studies one language.

	French	German
Female	11	12
Male	7	20

Two of these students are selected at random.

Calculate the probability that the chosen students will be a female studying French and any male.

Question 6: A box contains 40 pens.
30 of the pens are blue and the rest are green.
One pen is taken out at random and replaced by five of the other colour.
Another pen is taken out at random and replaced by five of the other colour.
A final pen is taken out at random.

(a) Work out the probability that all three pens are green.
(b) Work out the probability that all three pens are the same colour.
(c) Work out the probability that all three pens are **not** the same colour.

Question 7: Jamie has some coins.

Jamie has to pay 60p for a car park ticket.
He selects 3 coins at random, without replacement, from his pocket.

Work out the probability that he has chosen the exact price of the ticket.

Question 8: Rebecca has 10 cards, each with a number on it.

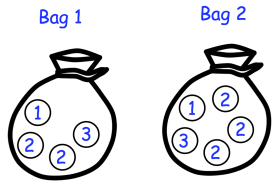
2	2	2	3	4	5	6	7	7	9

She picks three cards at random, without replacement.
Rebecca adds the three numbers together to get a score.
Calculate the probability that the score is an odd number.

Question 9: Harrison has two bags, each containing discs.

Bag 1 contains one disc labelled one, two discs labelled two and one disc labelled three.
Bag 2 contains one disc labelled one, three discs labelled two and one disc labelled three.

Harrison chooses a disc at random from Bag 1.
If the disc is labelled 1, he puts the disc in Bag 2.
If the disc is not labelled 1, he does **not** put the disc in Bag 2.
Harrison then chooses a disc at random from Bag 2.



Harrison adds together the numbers from the two discs he selected to give his score.

Find the probability of Harrison scoring 4.

Question 10: 45 students were asked if they have visited Canada, Mexico or the USA.

11 students had been to Canada
1 student had visited all three countries
2 students had visited Canada and Mexico but not the USA.
3 students had visited Mexico and the USA.
12 students had not visited any of the countries.
6 out the 19 students who had visited the USA, had been to at least one of the other countries.

Two of the 45 students are chosen at random.

Work out the probability that they both had only visited Mexico.

Question 11: There are x apples in a crate.
2 of the apples are bad.

Jesse chooses two apples from the crate, without replacement.
The probability that he selects two bad apples is $\frac{1}{28}$

(a) Prove $x^2 - x - 56 = 0$

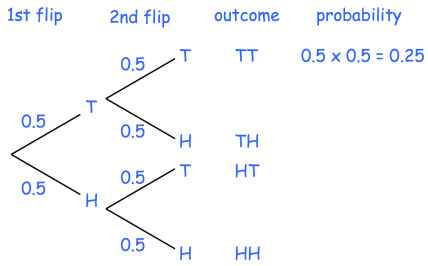
(b) Find x, the number of apples in the crate

Question 1: A fair coin is flipped twice.

(a) Find the probability of getting two heads

(b) Find the probability of getting a head and then a tail

(c) Find the probability of getting at least one head



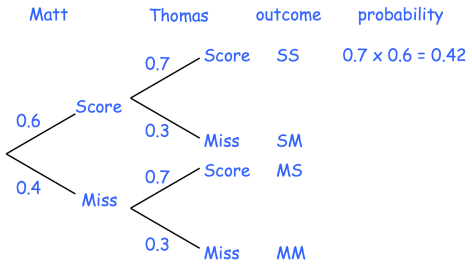
Question 2: Matt and Thomas each take a penalty.

The probability that Matt scores is 0.6
The probability that Thomas scores is 0.7

(a) Find the probability of both boys missing

(b) Find the probability of one boy scoring

(c) Find the probability of at least one boy missing



Question 3: Megan and Rosie sit their driving tests.

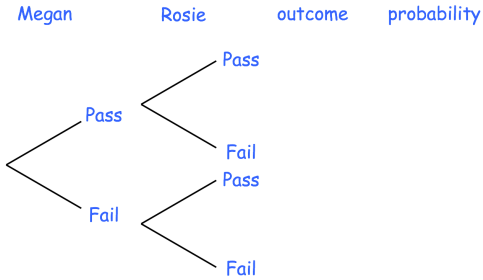
The probability that Megan passes the test is 0.8
The probability the Rosie fails the test is 0.3

(a) Copy and complete the tree diagram

(b) Find the probability that both women pass

(c) Find the probability that Megan fails and Rosie passes

(d) Find the probability that at least one woman passes



Question 4: Harry goes to an arcade. He has one go on the Teddy Grabber and one go on the Penny Drop.

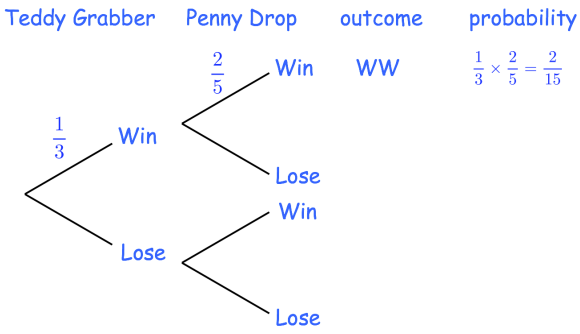
The probability that he wins on the Teddy Grabber is ⅓

The probability that he wins on the Penny Drop is ⅖

(a) Copy and complete the tree diagram

(b) Work out the probability that Harry loses on the Teddy Grabber and he also loses on the Penny Drop

(c) Work out the probability that Harry wins on exactly one machine



Question 5: There are 5 lemon and 4 strawberry sweets in a bag.

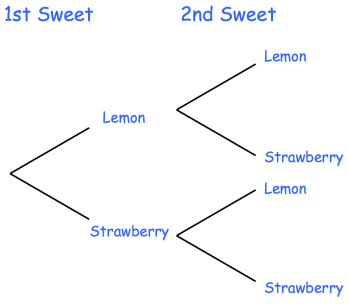
Hailey takes out a sweet at random, writes down its flavour and puts it back into the bag. Then Hailey takes out a second sweet, at random, and writes down its flavour.

(a) Copy and complete the tree diagram

(b) Find the probability of two lemon sweets

(c) Find the probability of one of each flavour

(d) Find the probability of no lemon sweets



Question 6: The probability that a bus arrives late is 0.1

Victor is travelling by bus on Monday and Tuesday.

(a) Show this information on a tree diagram

(b) Calculate the probability that the bus is on time both days.

Question 7: Each morning Martina attempts a crossword and a Sudoku

The probability that Martina successfully completes the crossword is 0.3
The probability that Martina successfully completes the Sudoku is 0.6

(a) Show this information on a tree diagram

(b) Work out the number of days that Martina successfully completes both the crossword and Sudoku over a period of 200 days.

Apply

Question 1: Timothy is taking part in an archery competition.

The probability of windy weather is 0.2

If it is windy, the probability of Timothy hitting the target is 0.35
If it is not windy, the probability of Timothy hitting the target is 0.8

(a) Draw a tree diagram to show this information

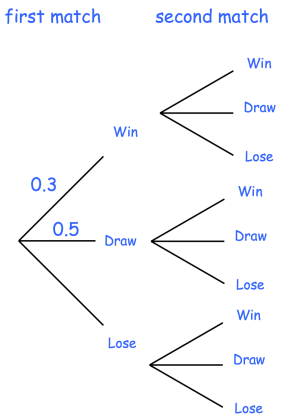
(b) Find the probability of Timothy hitting the target.

Question 2: A football team has two matches to play.

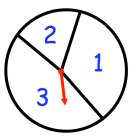
The probability that the team wins is 0.3
The probability that the team draws is 0.5

A win is worth 3 points, a draw 1 point and a loss 0 points.

Calculate the probability that the team will score at least 3 points over the two matches.



Question 3: Shown is a spinner.



The probability of a 1 is 3x
The probability of a 2 is x
The probability of a 3 is 4x

(a) Calculate the value of x

The spinner is spun twice and the scores are multiplied together.

(b) Work out the probability that the final score is odd.

Question 4: Freddie and Martha have dentist appointments.

The probability that Freddie is on time to his appointment is 0.9
The probability that both Freddie and Martha are on time to their appointments is 0.72

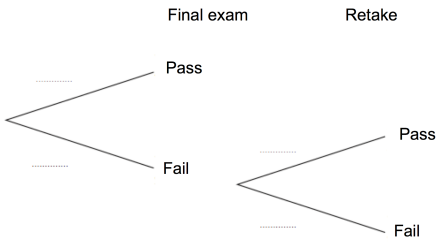
(a) Draw a tree diagram for this information

(b) Find the probability that both people are late for their appointments

Question 5: A college course consists of 8 weeks of teaching with a final exam at the end of the course

If a student fails the final exam, they have one opportunity to retake the exam.

The probability of a student passing the final exam is $\frac{7}{8}$
The probability of a student passing the retake is $\frac{2}{3}$



(a) Complete the tree diagram

If a student passes the final exam or retake, they receive a certificate.

(b) Work out the probability that a student receives a certificate.

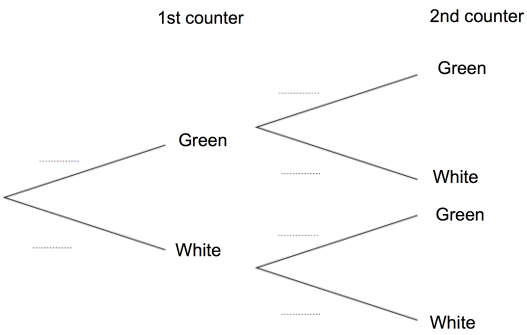
Question 6: There are 10 counters in a bag, 7 are green and the rest of white.

Erin takes out a counter at random and records its colour.
Without replacement, Erin takes out another counter, at random.

(a) Complete the tree diagram

(b) Find the probability that both counters are different colours

(c) Find the probability that both counters are the same colour



Question 7: Jenson is going to choose a ball at random from a bag and then flip a coin.

There are 5 balls in the bag, 2 white and 3 black.
A ball is picked at random from the bag and its colour is recorded.

If the ball is white, a fair coin is flipped.
If the ball is black, a biased coin is flipped, where heads has a probability of $\frac{7}{8}$

(a) Draw a tree diagram to show this information

Jenson selects a ball and flips the appropriate coin.

(b) Find the probability that he obtains a tail.

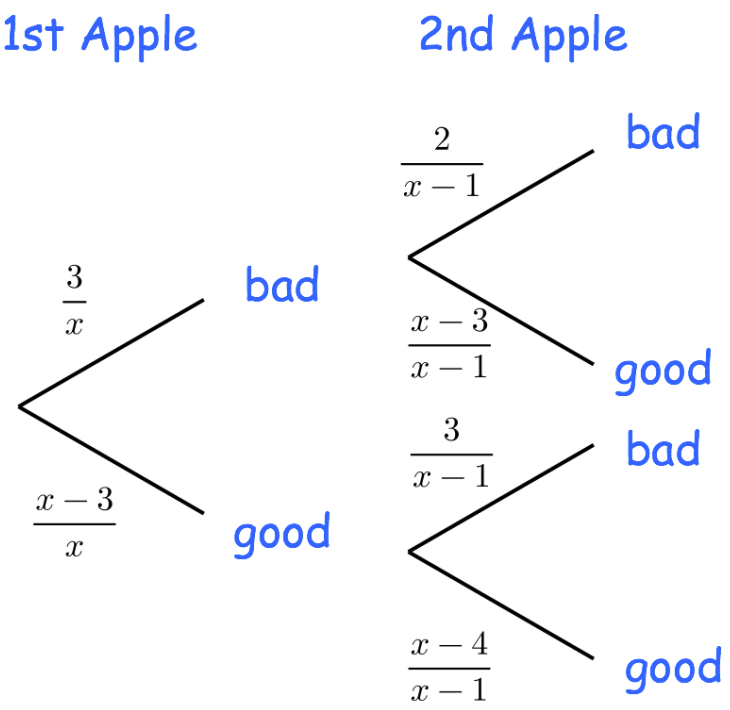
Question 8: There are x apples in a crate.
3 of the apples are bad.

Robert chooses two apples from the crate, without replacement
The probability that he selects two bad apples is $^1/_{12}$

(a) Using the tree diagram, prove $x^2 - x - 72 = 0$

(b) Find the number of apples in the crate, x.

(c) Find the probability that both apples are good



Question 1: A fair coin is flipped twice.

(a) Find the probability that the coin lands on heads twice.
(b) Find the probability that the coin lands on tails twice.
(c) Find the probability that the coin lands on heads exactly once.

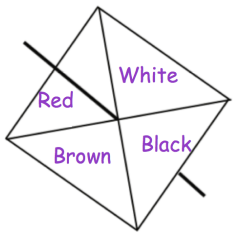
Question 2: Penelope is playing football.
When attempting to score a penalty, the probability she scores is $\frac{2}{3}$
During the game, Penelope takes two penalties.
Find the probability that Penelope scores both.

Question 3: Trevor is taking part in a quiz.
The probability that he answer a question correctly is $\frac{3}{5}$
Trevor is asked two questions.

(a) Calculate the probability that Trevor answers both questions correctly.
(b) Calculate the probability that Trevor answers both questions incorrectly.

Question 4: Daisy has a biased spinner.
The probability of each colour is:

Colour	Red	White	Black	Brown
Probability	0.1	0.4	0.3	0.2



Daisy spins the spinner twice.

(a) Find the probability of the spinner landing on white twice.
(b) Find the probability of the spinner landing on black and then brown.
(c) Find the probability of the spinner landing on the same colour in both spins.

Question 5: A fair six sided dice is rolled three times.

(a) Find the probability of getting a two all three times.
(b) Find the probability of getting no twos

Question 6: Mark is playing darts.
The probability he hits the bullseye is 0.4
Mark throws two darts

(a) Find the probability of Mark hitting the bullseye once.
(b) Find the probability of Mark hitting the bullseye at least once.

Question 7: A bag contains five yellow sweets, three green sweets and one purple sweet.
A sweet is taken out of the bag and replaced.
Another sweet is taken out.

(a) Find the probability that both sweets are yellow.
(b) Find the probability of neither sweet is green.
(c) Find the probability that the two sweets are different colours.

Question 8: The probability of a bus being on time is $\frac{3}{4}$

Archie catches the bus to work three times each week.

(a) Work out the probability that the bus is late every time.
(b) Work out the probability that the bus is on time every time.
(c) Work out the probability that the bus is late exactly once.

Question 9: Jackson, Frederick and Kelvin each sit a test.

The probability Jackson passes is $\frac{9}{10}$

The probability Frederick passes is $\frac{2}{3}$

The probability Kelvin passes is $\frac{1}{2}$

(a) Find the probability that Jackson and Kelvin pass, but Fredrick fails.

(b) Find the probability that Frederick passes, but Jackson and Kelvin fail.

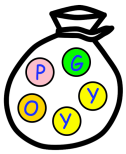
(c) Find the probability that at least two boys pass.

Question 10: The probability that Dylan reads at night is $\frac{4}{5}$

Calculate the probability that Dylan reads every night in one week.

Apply

Question 1: Amelia is organising a game for a charity fête.
She has put 1 orange, 1 pink, 1 green and 2 yellow counters into a bag.



To play, each person will pay £1 and take out a counter at random.
They will then replace the counter and then take a second counter at random.
The person will win £2.50 if both counters are the same colour.

Amelia expects 200 people to play the game.

How much money would Amelia expect to raise for charity?

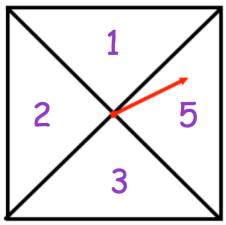
Question 2: There are12 tiles in a bag, each with a letter written on it.

C	O	R	B	E	T	T	M	A	T	H	S

A tile is selected at random and then replaced.
Another tile is then selected.

Find the probability that both tiles have different letters on them.

Question 3: A fair spinner has four sections.



The spinner is spun three times.
The three numbers are added together to give a score.

(a) Find the probability that the score is odd.
(b) Find the probability that the score is greater than 3.

Question 4: Tom and Ben sit their driving test.
The probability Tom passes is 0.4
The probability that only one man passes is 0.56
Find the probability they both fail.

(a) Arthur tosses a fair coin three times. Find the probability that the coin lands on tails exactly once.

(b) Iris and Usma play three games of tennis. The probability that Iris wins a game of tennis is $\frac{3}{8}$. Find the probability that Usma wins at least one game of tennis.

(a) Erik throws a biased coin twice. The probability of the coin landing on tails twice is $\frac{25}{81}$. Find the probability that the coin lands on heads twice.

(b) A spinner has two unequal sections coloured black and white. When the spinner is spun three times, the probability that it lands on black three times is 0.064 Find the probability that when the spinner is spun three times, it lands on white exactly twice.

(a) There are 20 counters in a bag – some are red and the rest are blue. A counter is chosen at random from the bag, it is replaced and then a second counter is chosen. The probability that two red counters are chosen is 0.09. Find the number of red counters, the number of blue counters and the probability of choosing one counter of each colour.

(b) Peta and Paul play one game of tennis and one game of pool against each other. The probability that Peta wins the game of pool is 0.66. The probability that Paul wins both games is 0.272. Find the probability that they win exactly one game each.

A spinner has three unequal sections – white, grey and black. The grey section is twice the size of the black section. When the spinner is spun four times, the probability of it landing on white four times in a row is 0.2401. Find the probability of the spinner landing on grey four times in a row.

Milo and Milly both sit their driving theory test. The probability of passing the test is higher than the probability of failing the test. The probability that Milo passes the test but Millie fails the test is $\frac{21}{100}$. Find the probability that both of them fail the test.

There are 9 yellow pencils and some green pencils in a pencil case. Two pencils are pulled out at random. The probability that both pencils are yellow is $\frac{6}{35}$. Find the number of green pencils in the pencil case.

There are x balls in a bag. 11 of the balls are white and the rest are orange. Two balls are chosen at random from the bag. The probability that the first ball chosen is white and the second ball chosen is orange is $\frac{77}{300}$. Find the value of x.

A jar contains only orange sweets and lemon sweets. There is one more orange sweet than there are lemon sweets. Two sweets are chosen at random from the jar. The probability of choosing two lemon sweets is $\frac{11}{50}$. Find the number of orange sweets and the number of lemon sweets in the jar.

There are black and white socks in a drawer. There are four more black socks than white socks. Finn chooses two socks at random from the drawer. The probability of him choosing one of each colour sock is $\frac{48}{95}$. Find the number of white socks and the number of black socks in the drawer.

1. Mr Stone gets breakfast in the morning with probability $\frac{1}{3}$, else he has nothing. If he has breakfast there is a $\frac{3}{4}$ chance he will be in a good mood. If he has not had breakfast there is a $\frac{1}{5}$ chance he will be in a good mood. On a randomly chosen day what is the probability Mr Stone will be in a good mood?

2. A darts player is aiming for trebles. If his previous throw is a treble then the probability his next throw is a treble is $\frac{2}{3}$. If his previous throw is not a treble then the probability his next throw is not a treble is $\frac{3}{4}$. On his first throw of three he has a $\frac{1}{2}$ probability of hitting a treble. On a round of three throws, find the probability he hits exactly two trebles.

3. There are 4 red, 2 green and 5 black counters in a bag. Two are taken from the bag at once. Find the probability they are

   (a) the same colour.

   (b) different colours.

   What do you notice about your answers to parts (a) and (b)? Explain.

4. There are 3 red, 4 yellow and 5 green balls in a bag. Three balls are taken *without replacement*. Find the probability that the three balls contain exactly two of one colour and one of another.

5. There are 3 red, 4 yellow and 5 green balls in a bag. Three balls are taken one at a time *with replacement*. Find the probability that the three balls contain exactly two of one colour and one of another.

6. There are r red beads and 3 blue beads in a bag. Two beads are taken at once from the bag. The probability they are both blue is $\frac{3}{28}$. Find r.

7. I have 44 socks in my drawer, each sock either red or black. In the dark I randomly pick two socks. The probability they do not match is $\frac{192}{473}$. How many of the 44 socks are red? [Maclaurin 2011]

8. A bag contains 3 red, 4 blue and g green beads. Two balls are taken from the bag at once. The probability they are different colours is $\frac{13}{18}$. Find g.

9. There are n sweets in a bag. 6 of the sweets are orange. The rest are yellow. Hannah takes a sweet at random from the bag and eats it. Hannah then takes another sweet at random from the bag and eats it. The probability that she eats two orange sweets is $\frac{1}{3}$.

   (a) Show that $n^2 - n - 90 = 0$.

   (b) Find the number of yellow sweets in the bag.

10. A prisoner is brought in front of the Emperor. The Emperor tells the prisoner he must distribute 100 white beads and 100 black beads into two urns as he wishes (there must be at least one bead in each urn). He is told that he will then be blindfolded and he will then pick one of the urns at random and pick a bead at random from the selected urn. If the bead is white he will be set free. Otherwise he will have to listen to Justin Bieber songs for all eternity (i.e. a fate worse than death). Find the maximum probability that he is set free (if he is smart and comes up with the best possible solution).

Fluency Practice

11. A pack of seeds tells me that each seed has a $\frac{2}{3}$ chance of flowering (independent of each other).

    (a) If I plant three seeds, what is that probability that two of them flower?

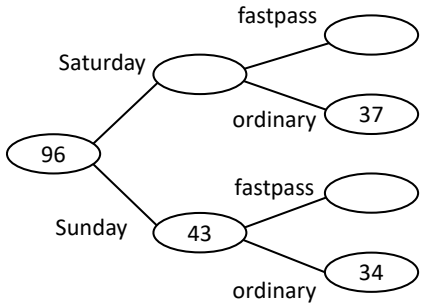
    (b) If I plant six seeds, what is the probability that four of them flower? [Here you don't want to draw the tree diagram, but visualise it and try to count the number of routes through the diagram with the desired properties.]

12. There are r red beads and b blue beads in a bag. Two beads are taken from the bag *with replacement*. The probability they are both different colours is $\frac{12}{25}$. Find $\frac{b}{r}$.

Fluency Practice

Frequency Trees

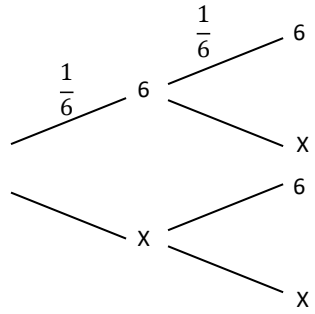
1A) A theme park recorded ticket sales on the weekend & whether it was upgraded to a fastpass.



P(A ticket was bought on Saturday) =
P(A ticket on Sunday was a fastpass) =

Probability Trees: Unconditional

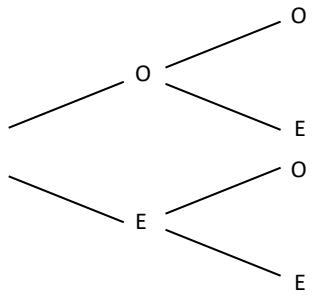
1B) A dice is rolled twice.



P(2 sixes) =
P(Exactly one six) =
P(1 or more sixes) =

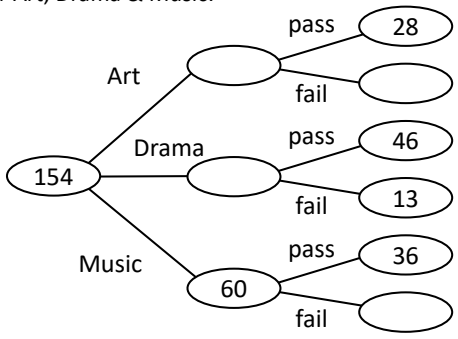
Probability Trees: Conditional

1C) 10 cards are numbered 1-10.
Two cards are picked simultaneously.



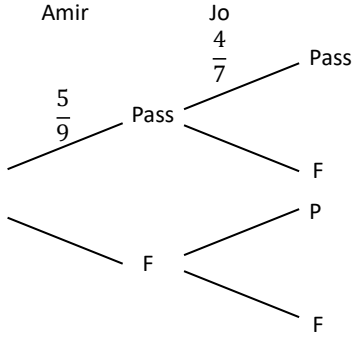
P(Both odd numbers) =
P(1 odd number & 1 even number) =
P(Both odd, or both even) =

2A) Ms Doolittle recorded end-of-year results for Art, Drama & Music.



Did a greater proportion of Art students pass compared to Music students?
P(A student failed) =
P(A passing student took Drama) =

2B) Amir & Jo will both take a driving test.



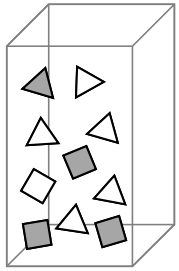
P(They both pass) =
P(Only one passes) =
P(They get the same result) =

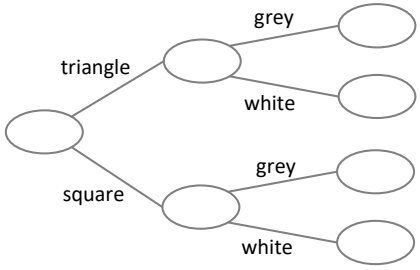
2C) P(Sophie wins her first tennis match) = 0.7
If she wins that match, the probability she wins her next match is 0.8 If she loses, that probability changes to 0.4

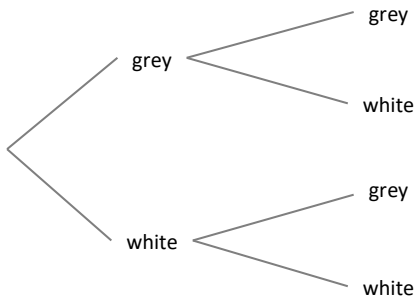
P(Wins then loses) =
P(Wins or loses both) =
P(Wins at least 1) =

Fluency Practice

① Complete the frequency & tree diagrams (2 shapes are picked **with replacement**).







P(grey shape then grey shape) =

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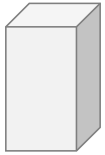
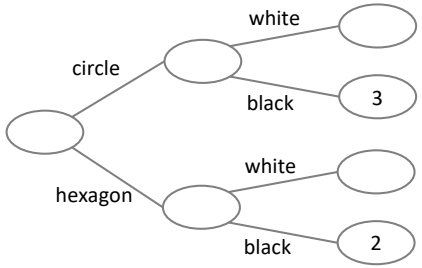
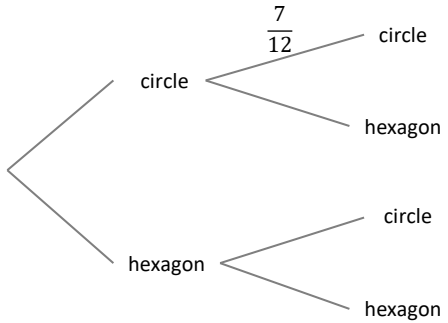
②







P(two different shapes) =

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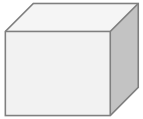
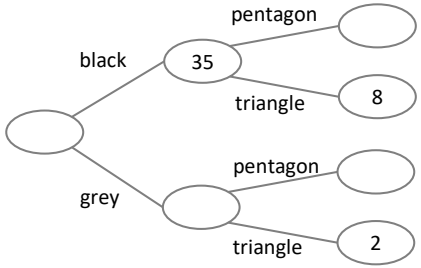
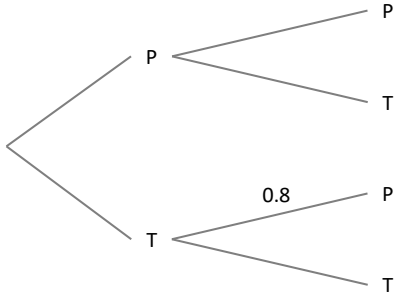
③



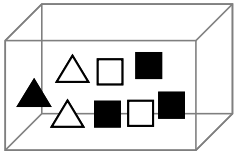




P(at least one pentagon) =

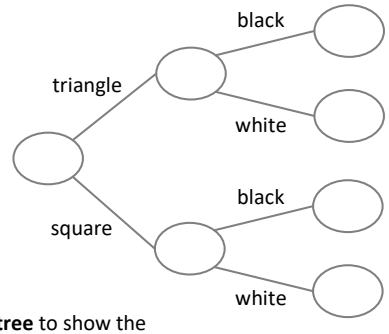
Fluency Practice

① Here is a box of shapes.

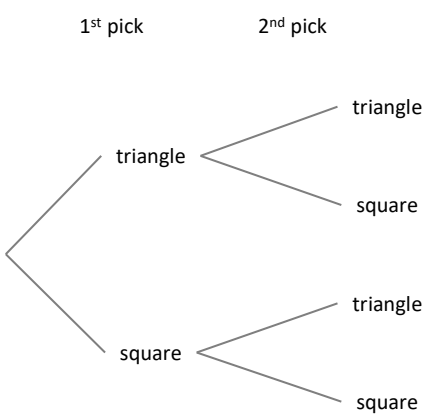


Mary picks a shape at random, records it & replaces it.

a) Complete the **frequency tree** for the group of shapes.



b) Complete the **probability tree** to show the probabilities if Mary picks a shape, replaces it, then picks another.

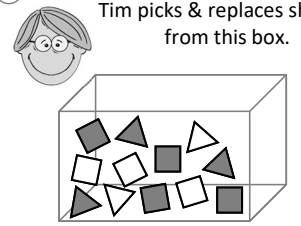


Using the diagrams…

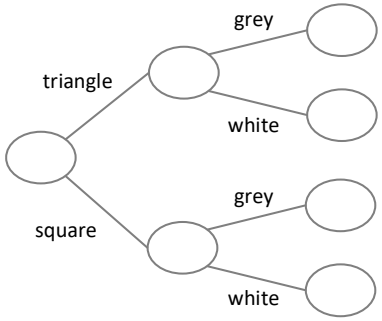
On her first pick, what is the probability Mary picks a white triangle?

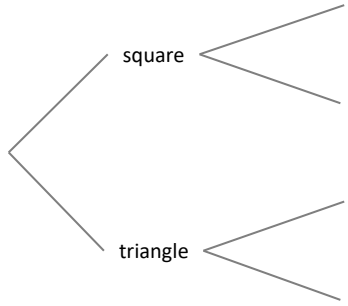
What is the probability Mary picks a square, then picks another square?

② Tim picks & replaces shapes from this box.



Complete the frequency tree & probability tree diagram.





What is the probability Tim picks…

…a grey square?

…a white shape?

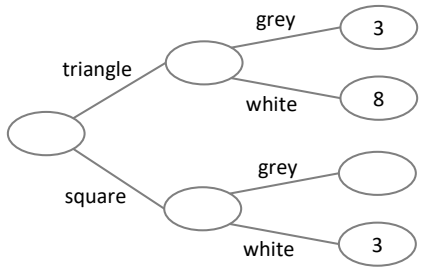
…two triangles in a row?

…two squares in a row?

…two of the same shape?

Fluency Practice

③ Complete this frequency tree for a group of 20 shapes.

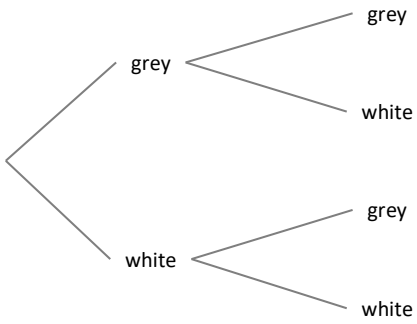


What is the % probability a grey square is picked randomly from the group?

What is P(a grey shape is picked)?

A square has been picked, what is the probability it is grey?

A shape is picked from the group and replaced before another is picked. Complete this tree diagram.

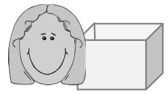


What is the probability a grey shape is picked before a white shape is picked?

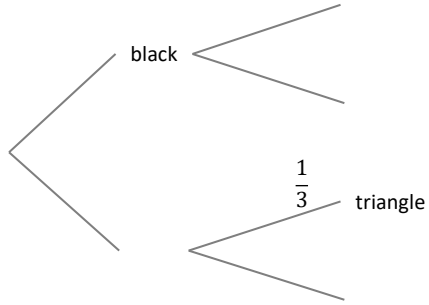
What is P(2 white shapes are picked)?

What is P(2 shapes of the same colour)?

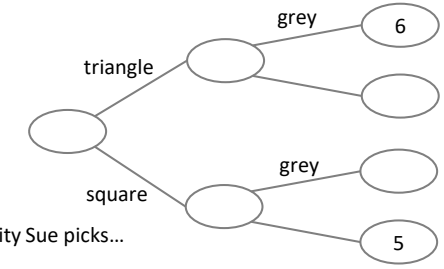
What is the probability a grey square is picked, then a white triangle is picked?

④ 

Sue randomly picks & replaces from a group of 30 shapes.
For her **first pick** she records the **colour**.
For her **second pick** she records the **type**.



Complete both tree diagrams.



What is the probability Sue picks…

…a black triangle?

…a black shape?

…a black shape then a square?

…a grey shape then a square?

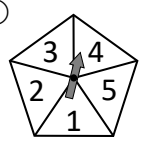
…a square, given the shape picked is grey?

…a triangle given the shape picked before was black?

Fluency Practice

Multiple Event Probabilities: A

①

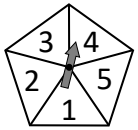


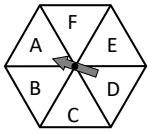
This is a fair spinner.

What is the percentage probability the arrow lands on **1** or on **4**?

②

These are fair spinners.

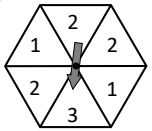


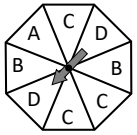


If both spinners are spun, what is the probability the results are **2** & **E** ?

③

These are fair spinners.





If both spinners are spun, what is…

P(**1** & **A**)?

P(**2** & **C**)?

④

On a fair dice, what is the probability two **sixes** are rolled in a row?

What is the probability two **ones** are rolled in a row?

What is the probability someone rolls two **sixes** or two **ones** in a row?

⑤

For a £4.49 'Meal Deal' you can choose 1 drink, 1 main & 1 desert.

Drink	Main	Desert
Soda	Pie	Ice Cream
Juice	Roll	Cake
	Curry	

A customer is chosen at random. By listing all the different possible 'Meal Deals' someone can order, what is the probability the customer has chosen 'Juice-Pie-Cake'?

⑥

A phone's security code is 4 digits long.

X	X	X	X

By choosing 4 digits randomly. what is the probability someone can guess the code in one try?

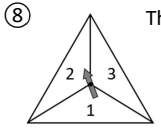
⑦

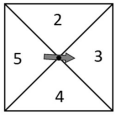
A car company gives each car an individual code using a letter and 2 digits.

For example, D-78

How many cars can be produced before the code needs to be expanded?

⑧





These two fair spinners are spun and their scores are **multiplied**.

What is the probability the final score is 6?

1st Spinner

2nd Spinner	1	2	3
2			6
3			
4			
5			

What is…

P(even)?

P(above 7)?

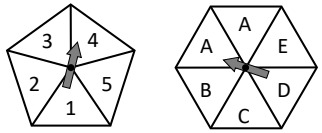
⑨

Kim has 2 sets of cards. The first set has the first 4 prime numbers on them.
The second set has the first 3 positive square numbers on them.
Kim takes one card from each set and sums the two numbers.
What is the probability the total is prime?
What is the probability the total is even or less than 6?

Fluency Practice

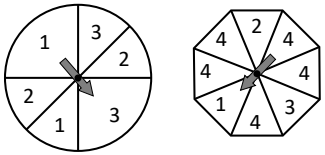
Multiple Event Probabilities: B

① These are fair spinners.



If both spinners are spun, what is the probability the results are **1** & **A** ?

② These are fair spinners.



Both spinners are spun & their scores are multiplied. What is the probability the total is 12?

③ A shoe company offers:
5 fabric colours, 4 sole colours &
7 colours for the laces.

How many different shoe colour combinations are there?

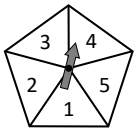
④ A credit card PIN is 4 digits long.

X	X	X	X

Bo knows the PIN is odd,
he knows it starts with
a positive even digit,
he knows it does not contain any zeros.

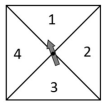
What is the probability Bo guesses the PIN correctly?

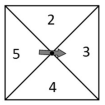
⑤ This fair spinner is spun twice & the two scores are added.



What is the probability the score is exactly 8?

⑥





These two fair spinners are spun and their scores are **added**.

1st Spinner

2nd Spinner	1	2	3	4
2				
3				
4				
5				

Complete the sample space.

What is the probability the final score is 7?

What is…

P(prime)?

P(less than 6)?

⑦ A bag contains 4 red disks,
3 black disks & 3 white disks.

Three times a disk is chosen at random & then replaced.
What is the probability…

….3 red disks are chosen?

….3 disks of the same colour are chosen?

⑧

A	B	C	D	E	F

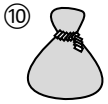
How many ways can we arrange these 6 cards?

If we pick 3 cards at random, what is the probability we pick the cards showing **A** & **B** & **C**?

⑨ 8 players are on a basketball squad.
Only 5 players play at once.

How many different options does the manager have when choosing the first 5 players to play?

⑩



A bag contains 6 green disks, 5 blue disks & 3 pink disks.
2 disks are taken at random.
What is the probability at least one blue disk is picked?

Fluency Practice

1 A fair coin is flipped, and a fair 6-sided die is rolled. Calculate:

a) $P(H \cap 6)$

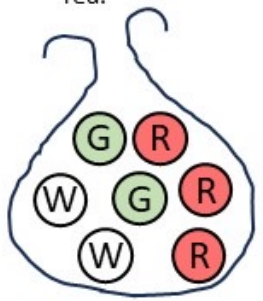
b) $P(T \cap Even\ Number)$

c) $P(T \cap 1')$

2 Calculate the probability of rolling three 1s on three fair 8-sided dice.

3 Charlie plays 2 games of chance, the probability he wins both games is 0.21. The probability of winning game A is 0.25. Calculate the probability he wins game B.

4 Chloe picks out two balls from the bag. Calculate the probability that both balls are red.



5 A box contains pink, green and purple counters in the ratio 2:4:7.

A counter is taken out of the box and replaced, a second is then taken out. Calculate the probability at least one of the counters is purple.

6 The following is a set meal at a restaurant.

Starter:
Tomato Soup
Mozzarella Sticks
Beef Nachos

Main:
Roast Chicken
Salad
Steak
Pesto Pasta

Dessert
Brownie
Lava Cake
Ice Cream

a) Calculate the number of combinations of the set menu.

b) Assuming the items of the set menu are chosen randomly, what is the probability a vegetarian meal set is chosen.

7 There are n balls in a bag of colours blue, red and green. A ball is picked randomly, the colour is recorded and returned to the bad. This is repeated a second time.

$$P(B \cap R) = 0.07$$
$$P(R \cap G) = 0.09$$
$$P(G \cap G) = 0.2025$$

Calculate the least number of balls the bag could contain.

8 A set of 10 counters labelled 1-10 are in a bag. A counter is taken out of the bag, recorded and returned. Another counter is then taken.

Ali wins if the sum of the numbers is even.

Grace wins if the product of the numbers is odd.

Calculate the probabilities of each player winning and determine whether this is a fair game.

5 Capture-Recapture

CHECK IT

Estimating population size using the capture- recapture method

1) In a wildlife park, 9 deer are caught, marked and released. A week later 10 deer are caught and 1 is found to be marked. Estimate the number of deer in the park

2) In a wildlife park, 28 fish are caught, marked and released. A week later 50 fish are caught and 2 are found to be marked. Estimate the number of fish in the park

3) In a wildlife park, 15 fish are caught, marked and released. A week later 30 fish are caught and 3 are found to be marked. Estimate the number of fish in the park

4) In a wildlife park, 16 deer are caught, marked and released. A week later 25 deer are caught and 2 are found to be marked. Estimate the number of deer in the park

5) In a wildlife park, 120 badgers are caught, marked and released. A week later 60 badgers are caught and 8 are found to be marked. Estimate the number of badgers in the park

6) In a wildlife park, 10 deer are caught, marked and released. A week later 20 deer are caught and 5 are found to be marked. Estimate the number of deer in the park

7) In a wildlife park, 25 fish are caught, marked and released. A week later 40 fish are caught and 1 is found to be marked. Estimate the number of fish in the park

8) In a wildlife park, 32 badgers are caught, marked and released. A week later 25 badgers are caught and 2 are found to be marked. Estimate the number of badgers in the park

9) In a wildlife park, 27 pheasants are caught, marked and released. A week later 50 pheasants are caught and 3 are found to be marked. Estimate the number of pheasants in the park

10) In a wildlife park, 14 pheasants are caught, marked and released. A week later 20 pheasants are caught and 2 are found to be marked. Estimate the number of pheasants in the park

11) In a wildlife park, 31 rabbits are caught, marked and released. A week later 30 rabbits are caught and 1 is found to be marked. Estimate the number of rabbits in the park

12) In a wildlife park, 13 pheasants are caught, marked and released. A week later 20 pheasants are caught and 1 is found to be marked. Estimate the number of pheasants in the park

13) In a wildlife park, 6 fish are caught, marked and released. A week later 10 fish are caught and 2 are found to be marked. Estimate the number of fish in the park

14) In a wildlife park, 20 badgers are caught, marked and released. A week later 20 badgers are caught and 4 are found to be marked. Estimate the number of badgers in the park

15) In a wildlife park, 100 deer are caught, marked and released. A week later 40 deer are caught and 5 are found to be marked. Estimate the number of deer in the park

16) In a wildlife park, 20 fish are caught, marked and released. A week later 20 fish are caught and 5 are found to be marked. Estimate the number of fish in the park

17) In a wildlife park, 60 deer are caught, marked and released. A week later 50 deer are caught and 4 are found to be marked. Estimate the number of deer in the park

18) In a wildlife park, 63 badgers are caught, marked and released. A week later 40 badgers are caught and 3 are found to be marked. Estimate the number of badgers in the park

19) In a wildlife park, 12 rabbits are caught, marked and released. A week later 30 rabbits are caught and 3 are found to be marked. Estimate the number of rabbits in the park

20) In a wildlife park, 11 badgers are caught, marked and released. A week later 20 badgers are caught and 1 is found to be marked. Estimate the number of badgers in the park

21) In a wildlife park, 15 badgers are caught, marked and released. A week later 10 badgers are caught and 3 are found to be marked. Estimate the number of badgers in the park

22) In a wildlife park, 30 deer are caught, marked and released. A week later 30 deer are caught and 3 are found to be marked. Estimate the number of deer in the park

23) In a wildlife park, 15 rabbits are caught, marked and released. A week later 20 rabbits are caught and 5 are found to be marked. Estimate the number of rabbits in the park

24) In a wildlife park, 50 rabbits are caught, marked and released. A week later 20 rabbits are caught and 2 are found to be marked. Estimate the number of rabbits in the park

25) In a wildlife park, 30 pheasants are caught, marked and released. A week later 40 pheasants are caught and 2 are found to be marked. Estimate the number of pheasants in the park

26) In a wildlife park, 7 pheasants are caught, marked and released. A week later 10 pheasants are caught and 1 is found to be marked. Estimate the number of pheasants in the park

27) In a wildlife park, 7 fish are caught, marked and released. A week later 40 fish are caught and 1 is found to be marked. Estimate the number of fish in the park

28) In a wildlife park, 33 rabbits are caught, marked and released. A week later 50 rabbits are caught and 3 are found to be marked. Estimate the number of rabbits in the park

29) In a wildlife park, 21 fish are caught, marked and released. A week later 50 fish are caught and 3 are found to be marked. Estimate the number of fish in the park

30) In a wildlife park, 12 rabbits are caught, marked and released. A week later 20 rabbits are caught and 1 is found to be marked. Estimate the number of rabbits in the park

1. John is trying to work out the total number of fish in a lake.
   One day he captures 10 fish, marks them, and then returns them.
   On the next day, he captures 24 fish and finds that 3 of them are marked.
   Work out an estimate for the total number of fish in the lake.

2. Sophie is trying to work out the total number of fish in a lake.
   One day she captures 30 fish, marks them, and then returns them.
   On the next day, she captures 40 fish and finds that 8 of them are marked.
   Work out an estimate for the total number of fish in the lake.

3. Preet is trying to work out the total number of fish in a lake.
   One day she captures 12 fish, marks them, and then returns them.
   On the next day, she captures 24 fish and finds that 6 of them are marked.
   Work out an estimate for the total number of fish in the lake.

4. Graham is trying to work out the total number of fish in a lake.
   One day he captures 18 fish, marks them, and then returns them.
   On the next day, he captures another sample of fish and finds that 40% of them are marked.
   Work out an estimate for the total number of fish in the lake.

5. A park has a population of ducks. 20 of the ducks are marked with tags.
   A week later, 24 ducks are caught. 5 of these ducks have tags on.

   a) Work out an estimate for the number of ducks in the park.

   b) Some of the tags may have fallen off during the week.
      Explain the effect this would this have on your estimate.

Capture & Recapture

A) A fisherman catches, tags 30 fish & releases them back into a pond.
The next day, the fisherman catches 20 fish & 6 of them are tagged.
Estimate the fish population in the lake.

B) Ralph waits 2 months between tagging some fish in a pond and then taking a recapture sample. Why might this affect the accuracy of any population estimate?

C) In a national park, Dr Howe catches, tags & releases 40 eagles.
The next day, Dr Howe captures 20 eagles. 4 of those eagles are tagged.

1) Estimate the hawk population in the park.
2) What assumptions do we need to make to estimate the population?

D) A scientist tags & releases 50 salamander into a river.
In a recapture sample, the scientist finds 15 out of 40 salamanders are tagged.
Estimate the salamander population in the river.

E) A marine biologist tags fish using a shiny metal tag on one fin.
Why might this affect the accuracy of a capture-recapture population estimate?

F) A scientist tags & releases 200 bees back into a hive.
In a recapture sample, the scientist finds 7% of the bees are tagged.
Estimate the bee population in the hive.

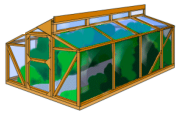
G) In a recapture sample, Dr Williams finds 3 out of 90 deer are tagged.
She uses this to estimate a deer population of 300.
How many deer were originally tagged?

H) If we tag 20 individuals from a population of 180, how many individuals do we expect to be tagged in a recapture sample of 50?

Question 1: Hannah wants to estimate the number of eels in a lake.
She catches and rings 50 eels.
She returns the 50 eels to the lake.
The next day Hannah catches 400 eels.
Of these 400 eels, 10 are ringed.

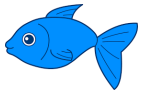
Work out an estimate for the total number of eels in the lake.

Question 2: Tom wants to estimate the number of woodlice in a greenhouse.
He catches and marks 32 woodlice.
The woodlice are then returned to the greenhouse.
The next day Tom catches 20 woodlice.
8 of these are marked.



Work out an estimate for the total number of woodlice in the greenhouse.

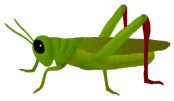
Question 3: A scientist wants to estimate the total number of fish in a pond.
On Thursday, she catches 180 fish.
These fish are marked and returned to the pond.
On Friday, the scientist catches 305 fish.
45 of these fish are marked.



(a) Work out an estimate for the total number of fish in the pond

(b) What assumptions have you made?

Question 4: Darren wants to estimate how many grasshoppers live in a field.
He catches and marks 24 grasshoppers.
He then releases the grasshoppers.
The next day, Darren returns to the same field and captures 51 grasshoppers.
7 of these have been marked.



Work out an estimate for the total number of grasshoppers in the field.

Question 5: Heather has a large jar of jelly beans.

Heather wants to find an estimate for the total number of jelly beans in the jar.
She takes out all the jelly beans and marks 200 of them.
Heather mixes the jelly beans and puts them back into the jar.

Heather then takes 140 jelly beans from the jar.
3 of the jelly beans are marked.
Heather then puts all the jelly beans back into the jar.

(a) Work out an estimate for the number of jelly beans in the jar.

(b) What assumptions have you made?

Apply

Question 1: Charlotte wants to work out an estimate of the number of fish living in a pond.
She captures X fish and tags them.
Charlotte returns the fish to the pond.
The next day Charlotte catches 50 fish.
Of these 50 fish, 32 are tagged.

Charlotte's estimate of the number of fish in the pond is 125.

Work out how many fish Charlotte tagged, X.

Question 2: Ronan wants to estimate the number of honey bees in a beehive.

On Wednesday, Ronan catches 660 honey bees from the beehive.
He marks the honey bees and then releases them.

On Thursday, Ronan catches 400 honey bees and notes how many were marked.

Ronan then calculates his estimate as 22,000 honey bees in the beehive.

How many of the 400 honey bees caught on Thursday were marked?

Question 3: Rhys has a large tub of yellow counters.
Alex has a large tub of blue counters.

40 yellow counters are taken from Rhys' tub and placed into Alex's tub.
40 blue counters are taken from Alex's tub and placed into Rhys' tub.

Rhys randomly selects 100 counters from his tub.
8 of the 100 counters are blue.

Alex randomly selects 50 counters from his tub.
48 of the 50 counters are blue.

All the counters are then placed into one tub.

Work out an estimate for the ratio of yellow to blue counters in the tub.

Question 4: A scientist wants to estimate the number of lions living in a region.

On Thursday, he locates and tags some lions.
On Friday he returns and locates 10 less lions than he had on Thursday.
He notices that 4 of the lions are tagged.

The scientist works out an estimate for the total number of lions living in the region.

He notices that the number of lions that he had caught on Thursday, was a fifth of the total number of lions.

How many lions live in the region?