



KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS



KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM

# Year 11

## 2024 Mathematics 2025

### Unit 23 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_



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# Year 11

## 2024 Mathematics 2025

### Unit 23 Booklet – Part 2

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_

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## 1 Completing the Square



**Worked Example**

- a) Express  $x^2 + 12x + 36$  in the form  $(x + a)^2$   
b) Express  $4x^2 + 12x + 9$  in the form  $(ax + b)^2$

**Your Turn**

- a) Express  $x^2 - 14x + 49$  in the form  $(x + a)^2$   
b) Express  $9x^2 - 12x + 4$  in the form  $(ax + b)^2$

### Worked Example

Complete the square on the following expressions:

a)  $x^2 + 10x$

b)  $x^2 - 10x$

### Your Turn

Complete the square on the following expressions:

a)  $x^2 + 6x$

b)  $x^2 - 6x$

### Worked Example

Write  $x^2 + 4x + 6$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are integers.

### Your Turn

Write  $x^2 - 2x + 5$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are integers.

### Worked Example

Write  $x^2 + 5x - 7$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are constants to be found.

### Your Turn

Write  $x^2 - 11x - 4$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are constants to be found.

### Worked Example

Write  $x^2 + 2px + 4$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are in terms of  $p$ .

### Your Turn

Write  $x^2 + 8kx + 5$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are in terms of  $k$ .

# Fill in the Blanks Completing the Square $(x^2 + bx + c)$

| Quadratic Expression | $\left(x + \frac{b}{2}\right)^2$ | $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ | $\pm c$                                             | Completed Square                                |
|----------------------|----------------------------------|---------------------------------------------------------------|-----------------------------------------------------|-------------------------------------------------|
| $x^2 + 10x$          | $(x + 5)^2$                      | $(x + 5)^2 - 25$                                              | $(x + 5)^2 - 25$                                    | $(x + 5)^2 - 25$                                |
| $x^2 + 8x - 2$       | $(x + 4)^2$                      | $(x + 4)^2 - 16$                                              | $(x + 4)^2 - 16 - 2$                                |                                                 |
| $x^2 + 2x + 5$       | $(x + 1)^2$                      | $(x + 1)^2 - 1$                                               |                                                     |                                                 |
| $x^2 + 4x + 7$       | $(x + 2)^2$                      |                                                               |                                                     |                                                 |
| $x^2 + 6x - 11$      |                                  |                                                               |                                                     |                                                 |
| $x^2 + 18x + 50$     |                                  |                                                               |                                                     |                                                 |
| $x^2 - 12x$          | $(x - 6)^2$                      | $(x - 6)^2 - 36$                                              | $(x - 6)^2 - 36$                                    | $(x - 6)^2 - 36$                                |
| $x^2 - 2x + 5$       | $(x - 1)^2$                      | $(x - 1)^2 - 1$                                               | $(x - 1)^2 - 1 + 5$                                 |                                                 |
| $x^2 - 6x - 1$       | $(x - 3)^2$                      | $(x - 3)^2 - 9$                                               |                                                     |                                                 |
| $x^2 - 8x + 16$      | $(x - 4)^2$                      |                                                               |                                                     |                                                 |
| $x^2 - 4x + 5$       |                                  |                                                               |                                                     |                                                 |
| $x^2 + 7x + 1$       | $\left(x + \frac{7}{2}\right)^2$ | $\left(x + \frac{7}{2}\right)^2 - \frac{49}{4}$               | $\left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + 1$ | $\left(x + \frac{7}{2}\right)^2 - \frac{45}{4}$ |
| $x^2 - 5x - 3$       |                                  |                                                               |                                                     |                                                 |
| $x^2 + x + 6$        |                                  |                                                               |                                                     |                                                 |
| $x^2 - 3x + 2$       |                                  |                                                               |                                                     |                                                 |
|                      |                                  |                                                               |                                                     | $(x + 5)^2 - 7$                                 |
|                      |                                  |                                                               |                                                     | $\left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$  |

### Worked Example

Write the expression  $4x^2 - 8x - 9$  in the form  $r(x + p)^2 + q$  where  $r, p$ , and  $q$  are integers.

### Your Turn

Write the expression  $3x^2 + 12x + 19$  in the form  $r(x + p)^2 + q$  where  $r, p$ , and  $q$  are integers.

| Quadratic Expression | Take out Common Factor | Complete the Square                                               | Multiply by Common Factor                            | Completed Square                                 |
|----------------------|------------------------|-------------------------------------------------------------------|------------------------------------------------------|--------------------------------------------------|
| $2x^2 + 16x$         | $2[x^2 + 8x]$          | $2[(x + 4)^2 - 16]$                                               | $2(x + 4)^2 - 32$                                    | $2(x + 4)^2 - 32$                                |
| $3x^2 - 18x$         | $3[x^2 - 6x]$          |                                                                   |                                                      |                                                  |
| $2x^2 + 12x + 1$     | $2[x^2 + 6x] + 1$      | $2[(x + 3)^2 - 9] + 1$                                            | $2(x + 3)^2 - 18 + 1$                                | $2(x + 3)^2 - 17$                                |
| $2x^2 - 20x - 7$     | $2[x^2 - 10x] - 7$     | $2[(x - 5)^2 - 25] - 7$                                           |                                                      |                                                  |
| $3x^2 + 6x - 5$      | $3[x^2 + 2x] - 5$      |                                                                   |                                                      |                                                  |
| $4x^2 + 16x - 1$     |                        |                                                                   |                                                      |                                                  |
| $5x^2 - 30x + 11$    |                        |                                                                   |                                                      |                                                  |
| $2x^2 - 10x + 3$     | $2[x^2 - 5x] + 3$      | $2\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 3$ | $2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2} + 3$ | $2\left(x - \frac{5}{2}\right)^2 - \frac{19}{2}$ |
| $2x^2 + 6x - 1$      |                        |                                                                   |                                                      |                                                  |
| $3x^2 - 9x + 2$      |                        |                                                                   |                                                      |                                                  |



### Worked Example

Write  $-3x^2 + 2x + 6$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are rational numbers.

### Your Turn

Write  $-3x^2 + 8x + 5$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are rational numbers.

**Worked Example**

Solve the equation:  
 $(x + 1)^2 = 4$

**Your Turn**

Solve the equation:  
 $(x - 4)^2 = 9$

**Worked Example**

Solve the equation:  
 $(2x + 3)^2 = 4$

**Your Turn**

Solve the equation:  
 $(3x - 2)^2 = 9$

**Worked Example**

Solve the following quadratic equation, leaving your answer in exact form:  $x^2 - 6x + 2 = 0$

**Your Turn**

Solve the following quadratic equation, leaving your answer in exact form:  $x^2 + 4x - 11 = 0$

**Worked Example**

Solve the following quadratic equation, leaving your answer in exact form:  $3x^2 - 7x + 1 = 0$

**Your Turn**

Solve the following quadratic equation, leaving your answer in exact form:  $4x^2 - 7x + 3 = 0$

### Worked Example

Make  $x$  the subject:

a)  $y = x^2 + 6x - 2$

b)  $y = 2x^2 + 16x - 2$

### Your Turn

Make  $x$  the subject:

a)  $y = x^2 + 8x - 3$

b)  $y = 2x^2 + 12x - 3$

## Extra Notes

## 2 Advanced Functions



**Worked Example**

$$\text{If } f(x) = 3x + 4,$$

$$g(x) = 2x - 5$$

$$\text{a) } fg(6) =$$

$$\text{b) } gf(6) =$$

**Your Turn**

$$\text{If } f(x) = 4x - 3,$$

$$g(x) = 5x + 2$$

$$\text{a) } fg(8) =$$

$$\text{b) } gf(8) =$$

**Worked Example**

If  $f(x) = 5x^2$ ,  
 $g(x) = 2x + 3$

a)  $fg(2) =$

b)  $gf(2) =$

**Your Turn**

If  $f(x) = 4x^2$ ,  
 $g(x) = 3x + 2$

a)  $fg(4) =$

b)  $gf(4) =$

**Worked Example**

$$\text{If } f(x) = x + 3,$$

$$g(x) = \frac{1}{x-2}$$

$$\text{a) } fg(5) =$$

$$\text{b) } gf(5) =$$

**Your Turn**

$$\text{If } f(x) = x - 5,$$

$$g(x) = \frac{1}{x+4}$$

$$\text{a) } fg(8) =$$

$$\text{b) } gf(8) =$$

| Question                                            | Input          | 1 <sup>st</sup> Function | 2 <sup>nd</sup> Function | Output       | Answer    |
|-----------------------------------------------------|----------------|--------------------------|--------------------------|--------------|-----------|
| $f(x) = 3x$ $g(x) = x - 1$<br>Find $fg(2)$          | <div>2</div> → | <div>-1</div> →          | <div>× 3</div> →         | <div>3</div> | $fg(2) =$ |
| $f(x) = 5x$ $g(x) = x + 3$<br>Find $gf(6)$          | <div>6</div> → | <div>× 5</div> →         | <div></div> →            | <div></div>  |           |
| $f(x) = x - 1$ $g(x) = x^2$<br>Find $fg(3)$         | <div></div> →  | <div></div> →            | <div></div> →            | <div></div>  |           |
| $f(x) = x + 9$ $g(x) = \sqrt{x}$<br>Find $gf(-5)$   | <div></div> →  | <div></div> →            | <div></div> →            | <div></div>  |           |
| $f(x) = \frac{x}{2}$ $g(x) = x + 7$<br>Find $fg(4)$ | <div></div> →  | <div></div> →            | <div></div> →            | <div></div>  |           |
| $g(x) = \sqrt{x}$ $h(x) = x - 3$<br>Find $gh(3.25)$ | <div></div> →  | <div></div> →            | <div></div> →            | <div></div>  |           |
| $f(x) = \frac{1}{x}$ $g(x) = x^2$<br>Find $gf(0.4)$ | <div></div> →  | <div></div> →            | <div></div> →            | <div></div>  |           |

# Fill in the Blanks

## Evaluating Composite Two-Step Functions

| Question                                                     | Input | 1 <sup>st</sup> Function                                     | 2 <sup>nd</sup> Function                                | Output | Answer     |
|--------------------------------------------------------------|-------|--------------------------------------------------------------|---------------------------------------------------------|--------|------------|
| $f(x) = x^2 + 2$<br>$g(x) = 3x - 1$<br>Find $fg(4)$          | 4 →   | <div>× 3</div> <div>→</div> <div>-1</div> <div>→</div>       | <div>square</div> <div>→</div> <div></div> <div>→</div> |        | $fg(4) =$  |
| $f(x) = 3\sqrt{x}$<br>$g(x) = 2x + 5$<br>Find $gf(9)$        | →     | <div>square root</div> <div>→</div> <div></div> <div>→</div> | <div></div> <div>→</div> <div></div> <div>→</div>       |        | $gf(9) =$  |
| $f(x) = \frac{1}{x} - 3$<br>$g(x) = 2x + 4$<br>Find $fg(-1)$ | →     | <div></div> <div>→</div> <div></div> <div>→</div>            | <div></div> <div>→</div> <div></div> <div>→</div>       |        | $fg(-1) =$ |
| $g(x) = \frac{x}{2} + 1$<br>$h(x) = 4x^2$<br>Find $hg(0.5)$  | →     | <div></div> <div>→</div> <div></div> <div>→</div>            | <div></div> <div>→</div> <div></div> <div>→</div>       |        |            |
| $f(x) = x^2 + 3$<br>$g(x) = 2x - 7$<br>Find $fg(5)$          | →     | <div></div> <div>→</div> <div></div> <div>→</div>            | <div></div> <div>→</div> <div></div> <div>→</div>       |        |            |

**Worked Example**

$$\text{If } f(x) = 3x + 4,$$

$$g(x) = 2x - 5$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

**Your Turn**

$$\text{If } f(x) = 4x - 3,$$

$$g(x) = 5x + 2$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

**Worked Example**

$$\text{If } f(x) = 2x - 3$$
$$ff(x) =$$

**Your Turn**

$$\text{If } g(x) = 3x - 2$$
$$gg(x) =$$

**Worked Example**

$$\text{If } f(x) = 5x^2,$$
$$g(x) = 2x + 3$$

a)  $fg(x) =$

b)  $gf(x) =$

**Your Turn**

$$\text{If } f(x) = 4x^2,$$
$$g(x) = 3x + 2$$

a)  $fg(x) =$

b)  $gf(x) =$



**Worked Example**

$$\text{If } f(x) = 4x^2 - 3$$
$$ff(x) =$$

**Your Turn**

$$\text{If } g(x) = 5x^2 - 2$$
$$gg(x) =$$

**Worked Example**

$$\text{If } f(x) = x + 3,$$

$$g(x) = \frac{1}{x-2}$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

**Your Turn**

$$\text{If } f(x) = x - 5,$$

$$g(x) = \frac{1}{x+4}$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

**Worked Example**

$$\text{If } f(x) = \frac{x+2}{3}$$
$$ff(x) =$$

**Your Turn**

$$\text{If } g(x) = \frac{x-3}{2}$$
$$gg(x) =$$

# Fill in the Blanks

## Composite One-Step Functions

| Question                                            | Input           | 1 <sup>st</sup> Function    | 2 <sup>nd</sup> Function | Output  | Answer             |
|-----------------------------------------------------|-----------------|-----------------------------|--------------------------|---------|--------------------|
| $f(x) = 5x$ $g(x) = x + 2$<br>Find $fg(x)$          | $x \rightarrow$ | $+2 \rightarrow$            | $\times 5 \rightarrow$   | $fg(x)$ | $fg(x) = 5(x + 2)$ |
| $f(x) = 5x$ $g(x) = x + 2$<br>Find $gf(x)$          | $x \rightarrow$ | $\times 5 \rightarrow$      | $+2 \rightarrow$         |         |                    |
| $f(x) = x - 1$ $g(x) = x^2$<br>Find $fg(x)$         | $x \rightarrow$ | $\text{square} \rightarrow$ |                          |         |                    |
| $f(x) = x + 3$ $g(x) = \sqrt{x}$<br>Find $gf(x)$    | $x \rightarrow$ |                             |                          |         |                    |
| $f(x) = \frac{x}{2}$ $g(x) = x + 7$<br>Find $fg(x)$ | $x \rightarrow$ |                             |                          |         |                    |
| $g(x) = x - 4$ $h(x) = \sqrt{x}$<br>Find $gh(x)$    | $x \rightarrow$ |                             |                          |         |                    |
| $f(x) = \frac{1}{x}$ $g(x) = x^2$<br>Find $gf(x)$   | $x \rightarrow$ |                             |                          |         |                    |

# Fill in the Blanks

# Fill in the Blanks Two-Step Functions

| Question                                                     | Input | 1 <sup>st</sup> Function | 2 <sup>nd</sup> Function | Output  | Answer                                         |
|--------------------------------------------------------------|-------|--------------------------|--------------------------|---------|------------------------------------------------|
| $f(x) = 3x - 1$<br>$g(x) = x^2 + 5$<br>Find $fg(x)$          | $x$   | square → +5              | $\times 3$ →             | $fg(x)$ | $fg(x)$<br>$= 3(x^2 + 5) - 1$<br>$= 3x^2 + 14$ |
| $f(x) = 2\sqrt{x}$<br>$g(x) = 4x - 3$<br>Find $gf(x)$        | $x$   | square root →            | →                        | $gf(x)$ | $gf(x)$<br>$=$                                 |
| $f(x) = \frac{x}{2} + 1$<br>$g(x) = 3x^2$<br>Find $fg(x)$    | $x$   | →                        | →                        | $fg(x)$ | $fg(x)$<br>$=$                                 |
| $g(x) = \frac{1}{x-2}$<br>$h(x) = 4\sqrt{x}$<br>Find $hg(x)$ | $x$   | →                        | →                        | $hg(x)$ |                                                |
| $f(x) = 5x^2$<br>$g(x) = 2x - 1$<br>Find $fg(x)$             | $x$   | →                        | →                        | $fg(x)$ |                                                |

## Fill in the Gaps

$$f(x) = 3x + 2 \quad g(x) = 5x - 4 \quad h(x) = x^2 + 1$$

| Composite function | Description                      | Unsimplified     | Simplified |
|--------------------|----------------------------------|------------------|------------|
| $fg(x)$            | into $g$ first,<br>then into $f$ | $3(5x - 4) + 2$  | $15x - 10$ |
|                    | into $f$ first,<br>then into $g$ |                  |            |
|                    |                                  | $(3x + 2)^2 + 1$ |            |
|                    |                                  |                  | $3x^2 + 5$ |
|                    |                                  | $3(3x + 2) + 2$  |            |
|                    |                                  |                  | $25x - 24$ |

## Fill in the Blanks

## Composite Functions

| $f(x)$                | $g(x)$                 | $fg(x)$                     | $gf(x)$                     |
|-----------------------|------------------------|-----------------------------|-----------------------------|
| $f(x) = x - 3$        | $g(x) = x^2$           | $fg(x) = x^2 - 3$           | $gf(x) = (x - 3)^2$         |
| $f(x) = \frac{x}{5}$  | $g(x) = x + 1$         |                             |                             |
| $f(x) = 3x$           | $g(x) = 7 - x$         |                             |                             |
| $f(x) = \sqrt{x}$     | $g(x) = \frac{x}{4}$   |                             |                             |
| $f(x) = 2x + 9$       | $g(x) = x - 3$         | $fg(x) = 2x + 3$            |                             |
| $f(x) = x^2 - 1$      | $g(x) = \frac{x}{3}$   |                             |                             |
| $f(x) = \sqrt{x}$     | $g(x) = 4 - 3x$        |                             |                             |
| $f(x) = \frac{2x}{5}$ | $g(x) = x^2$           |                             | $gf(x) = \frac{4x^2}{25}$   |
| $f(x) = \frac{1}{x}$  | $g(x) = 2x - 3$        |                             |                             |
| $f(x) = 9 - x$        | $g(x) = \sqrt{2x}$     |                             |                             |
| $f(x) = 3x - 1$       | $g(x) = \frac{2}{x+1}$ |                             |                             |
|                       | $g(x) = x - 3$         | $fg(x) = \frac{x-3}{10}$    |                             |
| $f(x) = 2x + 1$       |                        | $fg(x) = 2x^3 + 1$          |                             |
|                       |                        | $fg(x) = \frac{1}{x^2 + 2}$ | $gf(x) = \frac{1}{x^2} + 2$ |

**Worked Example**

Given that  $f(x) = x^2 - 6x - 4$ , find the possible values of  $x$  when  $f(x) = 3$ .

**Your Turn**

Given that  $g(x) = x^2 - 3x - 12$ , find the possible values of  $x$  when  $g(x) = -8$ .



**Worked Example**

Given  $f(x) = \sqrt{-2x + 17}$

Solve  $f(x^2) = 3$

**Your Turn**

Given  $g(x) = \sqrt{-3x + 12}$

Solve  $g(x^2) = 3$

**Worked Example**

Given

$$f(x) = 2x^2 + 3x + 1$$

$$g(x) = 6x^2 + 11x - 11$$

Solve  $f(x) = g(x)$ **Your Turn**

Given

$$g(x) = x^2 + 2x - 1$$

$$h(x) = 5x^2 + 18x - 1$$

Solve  $g(x) = h(x)$

### Worked Example

Find the inverse function:

$$f(x) = x^2 + 4x - 5$$

### Your Turn

Find the inverse function:

$$g(x) = x^2 + 8x - 5$$

### Worked Example

Find the inverse function:

$$f(x) = 2x^2 - 10x + 9$$

### Your Turn

Find the inverse function:

$$g(x) = 2x^2 - 12x + 3$$

## Extra Notes

### 3 Quadratic Graphs

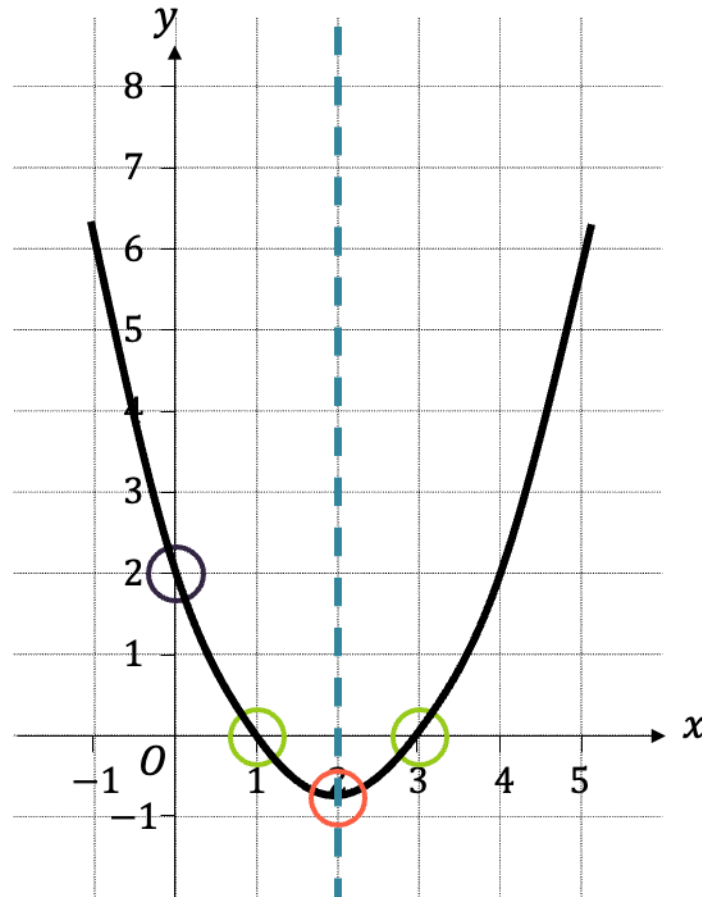
## Key Features of Quadratic Graphs

There are some key features of quadratic graphs that we can use if we want to **sketch** a quadratic rather than plot it.

1 The shape of the graph - U or  $\cap$

2 Where the graph crosses the  $y$ -axis

3 Where the graph crosses the  $x$ -axis



4 The line of symmetry of the graph

5 The turning point of the graph

### Worked Example

A quadratic graph has equation  $y = (8 - 2x)(3x + 7)$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.

### Your Turn

A quadratic graph has equation  $y = (2x - 5)(8 + 3x)$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.



### Worked Example

A quadratic graph has equation  $y = 5x^2 + 4x - 1$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.

### Your Turn

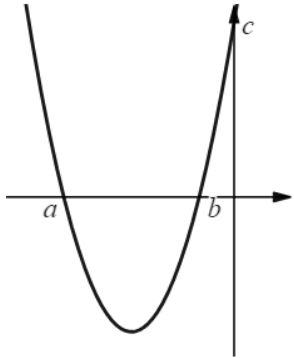
A quadratic graph has equation  $y = 5x^2 - 24x - 5$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.

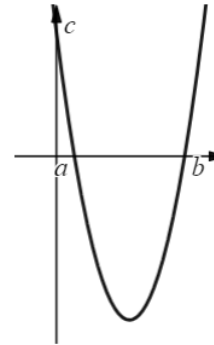
### Worked Example

A quadratic graph has equation  $y = (x + 1)(x + 5)$   
Find the values of  $a$ ,  $b$  and  $c$ .



### Your Turn

A quadratic graph has equation  $y = (x - 7)(x - 1)$   
Find the values of  $a$ ,  $b$  and  $c$ .



### Worked Example

Sketch the following graph:

$$y = (x - 7)(x + 10)$$

### Your Turn

Sketch the following graph:

$$y = (x + 3)(x + 8)$$

### Worked Example

Sketch the following graph:

$$y = x(x + 2)$$

### Your Turn

Sketch the following graph:

$$y = x(x - 3)$$

### Worked Example

Sketch the following graph:

$$y = x^2 + 5x - 36$$

### Your Turn

Sketch the following graph:

$$y = x^2 + 6x + 8$$

### Worked Example

Sketch the following graph:

$$y = (3x + 7)(2x - 3)$$

### Your Turn

Sketch the following graph:

$$y = (2x + 3)(3x - 5)$$

### Worked Example

Sketch the following graph:

$$y = 2x^2 - 13x + 15$$

### Your Turn

Sketch the following graph:

$$y = 5x^2 + 13x + 6$$

### Worked Example

Sketch the following graph:

$$y = (2x - 1)(7 - 2x)$$

### Your Turn

Sketch the following graph:

$$y = (1 - 2x)(2x + 5)$$



### Worked Example

Sketch the following graph:

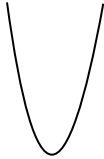
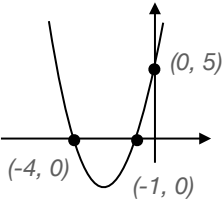
$$y = -3x^2 - 2x + 8$$

### Your Turn

Sketch the following graph:

$$y = -2x^2 + 5x + 7$$

## Fill in the Gaps

| Function           | Roots                                                                                      | Y intercept                                   | Shape                                                                                                                   | Graph                                                                               |
|--------------------|--------------------------------------------------------------------------------------------|-----------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|
| $y = x^2 + 5x + 4$ | $x^2 + 5x + 4 = 0$<br>$(x + 4)(x + 1) = 0$<br>$x = -4, = x - 1$<br>$(-1, 0)$ and $(-4, 0)$ | $y = (0)^2 + 5(0) + 4$<br>$y = 4$<br>$(0, 5)$ | <i>Positive <math>x^2</math></i><br> |  |
| $y = x^2 + 7x + 6$ |                                                                                            |                                               |                                                                                                                         |                                                                                     |
| $y = x^2 - 6x + 5$ |                                                                                            |                                               |                                                                                                                         |                                                                                     |
| $y = 6 - x - x^2$  |                                                                                            |                                               |                                                                                                                         |                                                                                     |

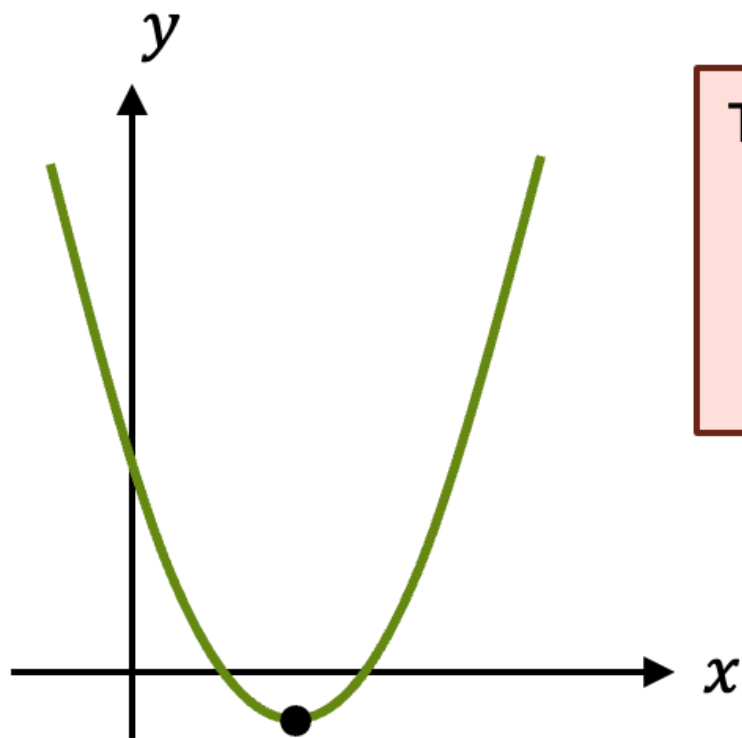
## Fill in the Gaps

| Function            | Roots | Y intercept | Shape | Graph |
|---------------------|-------|-------------|-------|-------|
| $y = x^2 + 8x$      |       |             |       |       |
| $y = x^2 - 25$      |       |             |       |       |
| $y = 9 - x^2$       |       |             |       |       |
| $y = 35 - 2x - x^2$ |       |             |       |       |

## Fill in the Gaps

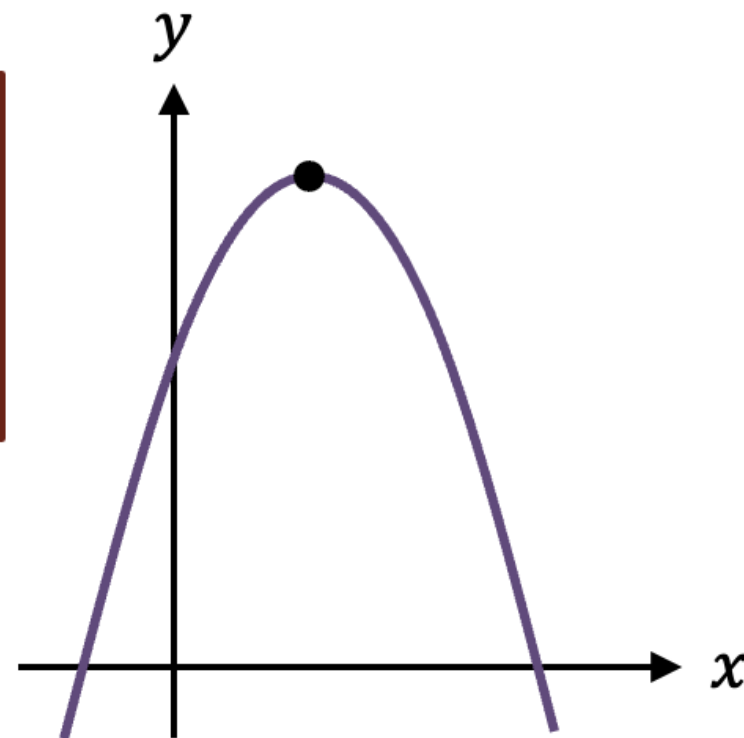
| Function              | Roots | Y intercept | Shape | Graph |
|-----------------------|-------|-------------|-------|-------|
| $y = (x - 5)(x - 2)$  |       |             |       |       |
| $y = (x - 3)(x + 3)$  |       |             |       |       |
| $y = -(x + 4)(x - 1)$ |       |             |       |       |
| $y = (2 - x)(3 + x)$  |       |             |       |       |

## Turning Point



For a **positive** quadratic, the turning point is a **minimum**

The turning point of a quadratic graph is its maximum or minimum point



For a **negative** quadratic, the turning point is a **maximum**

**Worked Example**

$x^2 + 8x + 11$  can be expressed as  $(x + 4)^2 - 5$ .

Hence, or otherwise, state the coordinates of the minimum point of the graph  $y = x^2 + 8x + 11$ .

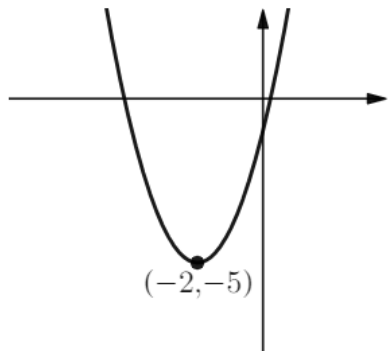
**Your Turn**

$x^2 + 6x + 15$  can be expressed as  $(x + 3)^2 + 6$ .

Hence, or otherwise, state the coordinates of the minimum point of the graph  $y = x^2 + 6x + 15$

### Worked Example

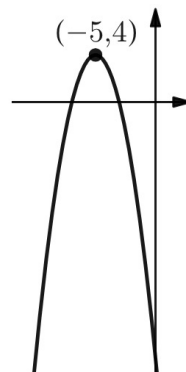
The graph below shows part of the parabola with equation of the form  $y = (x + a)^2 + b$



The minimum turning point  $(-2, -5)$  is shown on the diagram. State the values of  $a$  and  $b$ .

### Your Turn

The graph below shows part of the parabola with equation of the form  $y = -(x + a)^2 + b$



The maximum turning point  $(-5, 4)$  is shown on the diagram. State the values of  $a$  and  $b$ .

### Worked Example

$-x^2 - 6x - 10$  can be expressed as  $-(x + 3)^2 - 1$ .

Hence, or otherwise, state the coordinates of the maximum point of the graph  $y = -x^2 - 6x - 10$ .

### Your Turn

$-x^2 + 8x - 19$  can be expressed as  $-(x - 4)^2 - 3$ .

Hence, or otherwise, state the coordinates of the maximum point of the graph  $y = -x^2 + 8x - 19$ .



### Worked Example

Find the minimum point of the graph with equation  
 $y = x^2 - 2x - 3$

### Your Turn

Find the minimum point of the graph with equation  
 $y = x^2 + 6x + 8$

### Worked Example

Find the maximum point of the graph with equation  
 $y = -x^2 + 2x - 4$

### Your Turn

Find the maximum point of the graph with equation  
 $y = -x^2 - 4x + 1$

### Worked Example

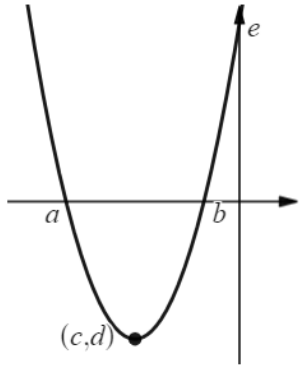
A graph has equation  $y = x^2 + 10x - 9$   
Find the equation of its line of symmetry.

### Your Turn

A graph has equation  $y = x^2 - 9x - 16$   
Find the equation of its line of symmetry.

### Worked Example

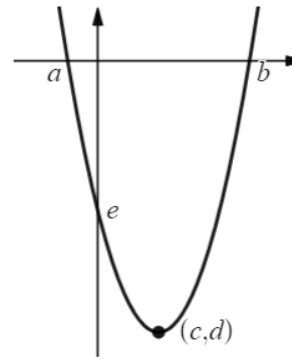
A quadratic graph has equation  $y = x^2 + 6x + 5$



Find the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$

### Your Turn

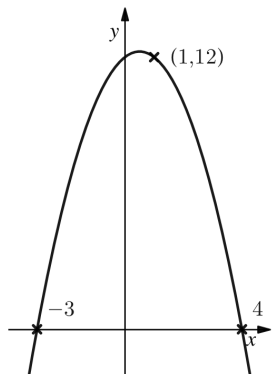
A quadratic graph has equation  $y = x^2 - 4x - 5$



Find the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$

### Worked Example

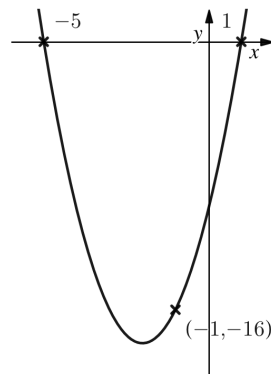
The graph below shows a quadratic curve.



Find the equation of the curve, giving your answer in the form  $y = ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are integers.

### Your Turn

The graph below shows a quadratic curve.



Find the equation of the curve, giving your answer in the form  $y = ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are integers.

### Worked Example

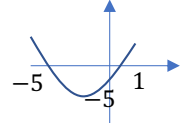
By completing the square, explain why the curve  $y = 2x^2 - 8x + 9$  does not intersect the  $x$ -axis

### Your Turn

By completing the square, explain why the curve  $y = 2x^2 - 20x + 51$  does not intersect the  $x$ -axis

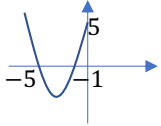
| Worked Example                                                                                                                                                                                        | Your Turn                                                                                                                                                                                             |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $y = x^2 + 2x - 15$ <p>Factorise:</p><br><br><br><br><br><p>Complete the square:</p><br><br><br><br><br><p>Intercepts:</p><br><br><br><br><br><p>Turning point:</p><br><br><br><br><br><p>Sketch:</p> | $y = x^2 + 2x - 24$ <p>Factorise:</p><br><br><br><br><br><p>Complete the square:</p><br><br><br><br><br><p>Intercepts:</p><br><br><br><br><br><p>Turning point:</p><br><br><br><br><br><p>Sketch:</p> |

## Fill in the Gaps

|    | $x^2 + bx + c$ | $(x + d)(x + e)$ | $(x + p)^2 + q$ | $x$<br>intercepts      | $y$<br>intercept | Turning point | Sketch                                                                              |
|----|----------------|------------------|-----------------|------------------------|------------------|---------------|-------------------------------------------------------------------------------------|
| 1. | $x^2 - 2x - 3$ |                  |                 |                        |                  |               |                                                                                     |
| 2. |                | $(x - 1)(x + 3)$ |                 |                        |                  |               |                                                                                     |
| 3. |                |                  |                 |                        |                  |               |  |
| 4. |                |                  | $(x + 1)^2 - 9$ |                        |                  |               |                                                                                     |
| 5. |                |                  |                 | $(-2, 0)$<br>$(-4, 0)$ |                  |               |                                                                                     |
| 6. |                | $(x - 4)(x + 4)$ |                 |                        |                  |               |                                                                                     |
| 7. |                |                  |                 | $(4, 0)$<br>$(0, 0)$   |                  |               |                                                                                     |

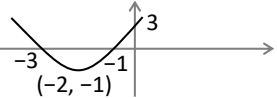
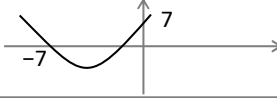
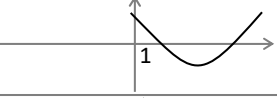
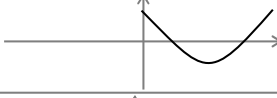
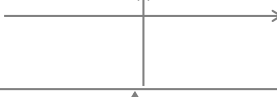
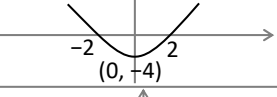
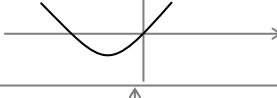
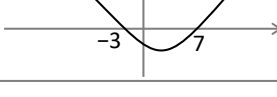


## Fill in the Gaps

|     | $x^2 + bx + c$ | $(x + d)(x + e)$  | $(x + p)^2 + q$ | $x$<br>intercepts | $y$<br>intercept | Turning point | Sketch                                                                              |
|-----|----------------|-------------------|-----------------|-------------------|------------------|---------------|-------------------------------------------------------------------------------------|
| 8.  | $x^2 + 4x$     |                   |                 |                   |                  |               |                                                                                     |
| 9.  |                |                   |                 |                   |                  | $(-2, 0)$     |                                                                                     |
| 10. |                | $(x + 3)(x + 3)$  |                 |                   |                  |               |                                                                                     |
| 11. |                |                   |                 |                   |                  |               |  |
| 12. |                |                   | $(x + 3)^2 + 4$ |                   |                  |               |                                                                                     |
| 13. |                | $-(x + 1)(x + 5)$ |                 |                   |                  |               |                                                                                     |
| 14. |                |                   |                 |                   | $(0, 5)$         |               |                                                                                     |

# Fill in the Gaps

## ① Significant Points of a Quadratic Curve Using Factorisation & Symmetry

| Equation            | Factorisation        | 1 <sup>st</sup> Root | Turning Point | 2 <sup>nd</sup> Root | y-intercept | Sketch                                                                                |
|---------------------|----------------------|----------------------|---------------|----------------------|-------------|---------------------------------------------------------------------------------------|
| $y = x^2 + 4x + 3$  | $(\quad)(x + 1) = 0$ |                      |               | $(-1, 0)$            | $+3$        |    |
| $y = x^2 + 8x + 7$  | $(x + 1) = 0$        | $(-7, 0)$            | $(x, -9)$     |                      |             |    |
| $y = x^2 - 6x + 5$  | $= 0$                |                      | $(3, y)$      |                      |             |    |
| $y = x^2 - 8x + 12$ |                      |                      |               |                      |             |    |
|                     | $(x + 3)(x - 5) = 0$ |                      |               |                      | $-15$       |    |
| $y = x^2 + 2x - 24$ |                      |                      |               |                      |             |    |
|                     |                      | $(-2, 0)$            |               | $(2, 0)$             | $-4$        |    |
| $y = x^2 + 5x$      |                      |                      | $(x, -6.25)$  |                      |             |   |
| $y = x^2 + 8x + 16$ |                      |                      |               |                      |             |  |
|                     |                      |                      |               |                      |             |  |

Fill in the Gaps

| Equation             | y-intercept | Shape<br>U or ∩? | Discriminant | Factorisation         | Roots              | Completing<br>the<br>square | Maximum<br>or minimum<br>point | Sketch |
|----------------------|-------------|------------------|--------------|-----------------------|--------------------|-----------------------------|--------------------------------|--------|
| $y = x^2 - 4x - 5$   |             |                  |              |                       |                    |                             |                                |        |
| $y = x^2 + 2x - 3$   |             |                  |              |                       |                    |                             |                                |        |
|                      |             |                  |              | $(x - 2)(x - 10)$     |                    |                             |                                |        |
|                      |             |                  |              |                       |                    | $(x + 4)^2 - 9$             |                                |        |
|                      |             |                  |              |                       | $x = 1$<br>$x = 1$ |                             |                                |        |
|                      | (0,9)       |                  |              |                       |                    |                             |                                |        |
|                      |             |                  |              | Does not<br>factorise |                    |                             | (-2,5)                         |        |
| $y = -x^2 + 6x - 16$ |             |                  |              |                       |                    |                             |                                |        |
|                      |             |                  |              | $(1 - x)(x + 7)$      |                    |                             |                                |        |
| $y = -x^2 + 4x + 5$  |             |                  |              |                       |                    |                             |                                |        |
|                      |             |                  |              |                       | $x = 0$<br>$x = 4$ |                             | (2, -4)                        |        |
|                      |             |                  | 36           |                       |                    |                             |                                |        |

Can you draw any general conclusions from the table?

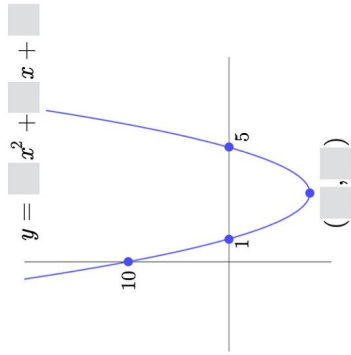
## Fill in the Gaps

| Forms            |                    |                                                     | Intercepts |     |     | Values     |            |            | Vertex          |                 |
|------------------|--------------------|-----------------------------------------------------|------------|-----|-----|------------|------------|------------|-----------------|-----------------|
| Standard         | Factorised         | Vertex                                              | $x$        | $x$ | $y$ | at $x = 1$ | at $x = 2$ | at $x = 3$ | $x$ -coordinate | $y$ -coordinate |
| $x^2 - 8x + 15$  | $(x - 3)(x - 5)$   | $(x - 4)^2 - 1$                                     | 3          | 5   | 15  | 8          | 3          | 0          | 4               | -1              |
| $x^2 + 6x + 5$   | $(x - 2)(x - 3)$   | $(x + 2)^2 - 25$                                    |            |     |     |            |            |            |                 |                 |
|                  |                    |                                                     | -4         | 3   | -12 | -4         | -3         | 0          |                 |                 |
| $6x^2 - 13x + 6$ | $(2x - 3)(4x - 5)$ |                                                     |            |     |     |            |            |            |                 |                 |
|                  |                    | $35\left(x - \frac{1}{35}\right)^2 - \frac{36}{35}$ |            |     |     |            |            |            |                 |                 |

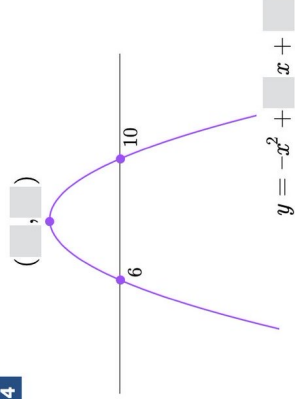
# Fill in the Gaps

Fill in the blanks:

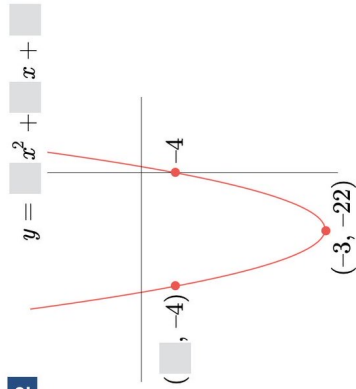
1



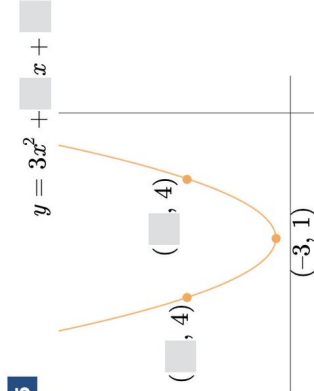
4



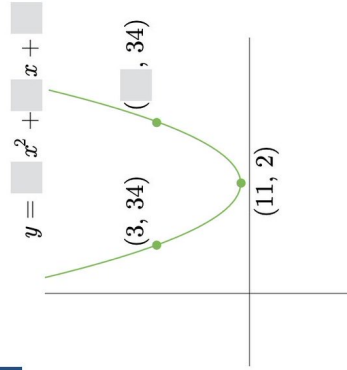
2



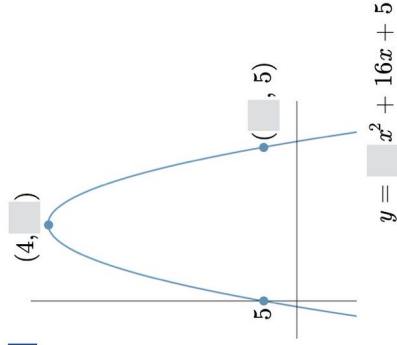
5



3

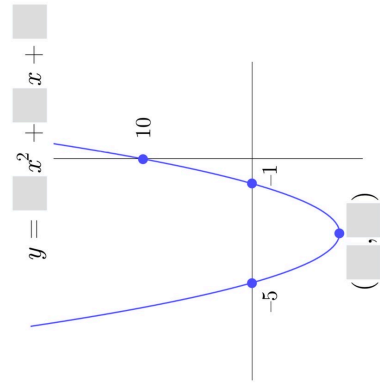


6

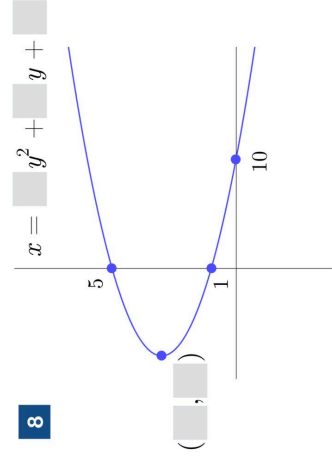


Fill in the blanks. It may help to compare questions 7 and 8 to question 1.

7



8

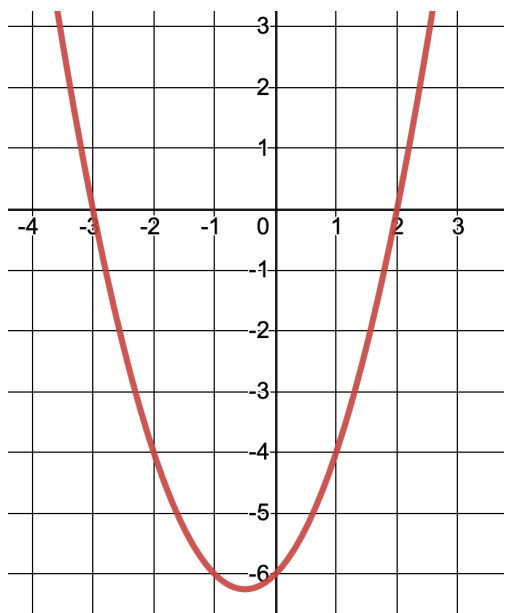


## Extra Notes

## 4 Quadratic Inequalities

### Worked Example

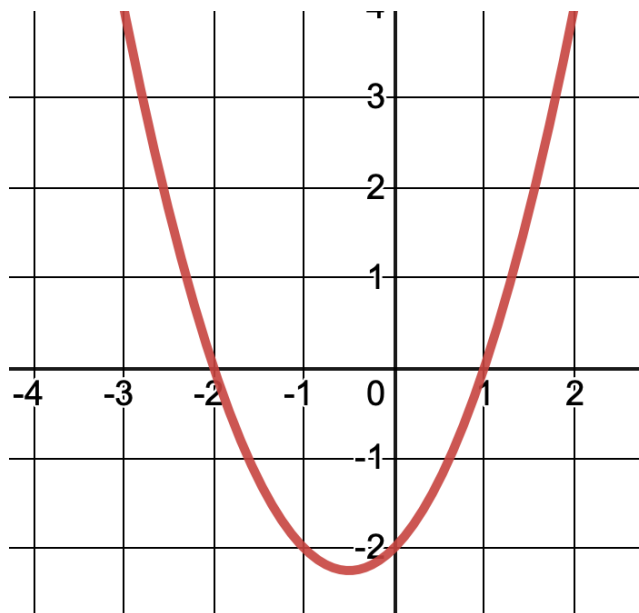
Here is the graph of  
 $y = (x + 3)(x - 2)$



- a) Show which part of the graph that represents  $(x + 3)(x - 2) < 0$
- b) State the solution set of  $(x + 3)(x - 2) < 0$

### Your Turn

Here is the graph of  
 $y = (x + 2)(x - 1)$

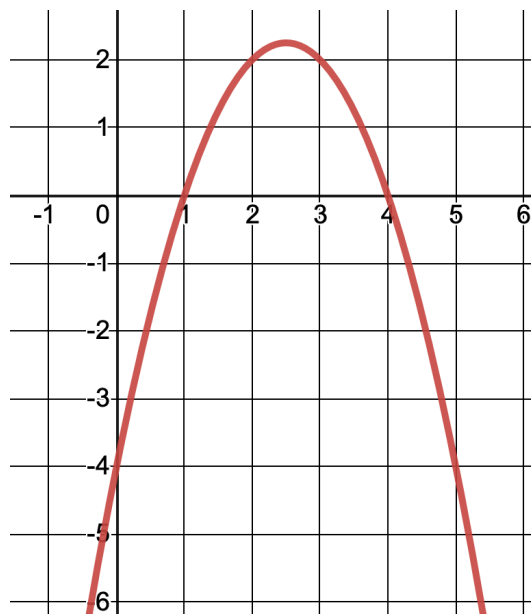


- a) Show which part of the graph that represents  $(x + 2)(x - 1) < 0$
- b) State the solution set of  $(x + 2)(x - 1) < 0$



## Worked Example

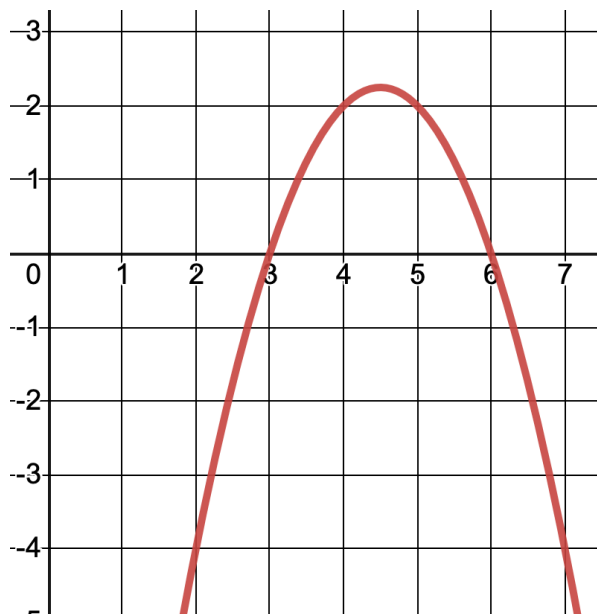
Here is the graph of  
 $y = (4 - x)(x - 1)$



- a) Show which part of the graph that represents  $(4 - x)(x - 1) < 0$
- b) State the solution set of  $(4 - x)(x - 1) < 0$

## Your Turn

Here is the graph of  
 $y = (3 - x)(x - 6)$



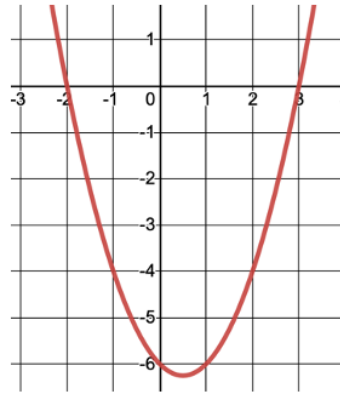
- a) Show which part of the graph that represents  $(3 - x)(x - 6) < 0$
- b) State the solution set of  $(3 - x)(x - 6) < 0$

## Fluency Practice

1) Here is the graph of  
 $y = (x + 2)(x - 3)$

Show which part of the graph  
 that represents  
 $(x + 2)(x - 3) < 0$

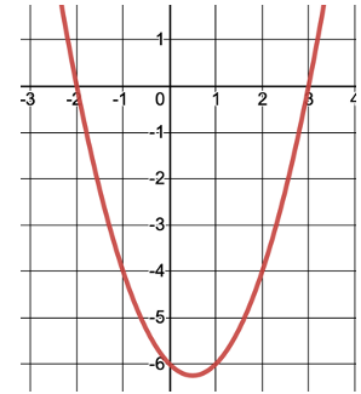
State the solution set of  
 $(x + 2)(x - 3) < 0$



2) Here is the graph of  
 $y = (x + 2)(x - 3)$

Show which part of the graph  
 that represents  
 $(x + 2)(x - 3) > 0$

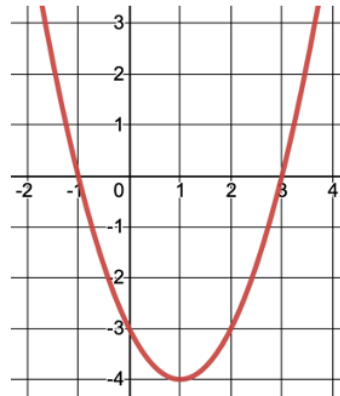
State the solution set of  
 $(x + 2)(x - 3) > 0$



3) Here is the graph of  
 $y = (x + 1)(x - 3)$

Show which part of the graph  
 that represents  
 $(x + 1)(x - 3) < 0$

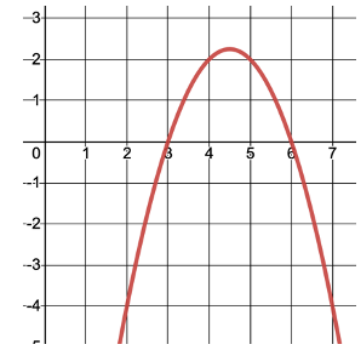
State the solution set of  
 $(x + 1)(x - 3) < 0$



4) Here is the graph of  
 $y = (-x + 6)(x - 3)$

Show which part of the graph  
 that represents  
 $(-x + 6)(x - 3) > 0$

State the solution set of  
 $(-x + 6)(x - 3) > 0$

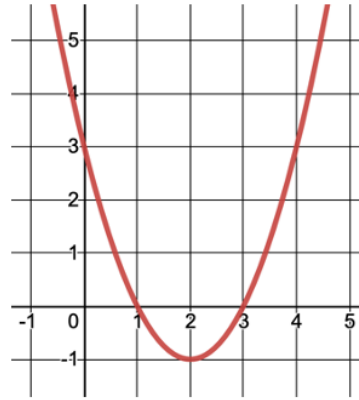


## Fluency Practice

5) Here is the graph of  
 $y = (x - 1)(x - 3)$

Show which part of the graph  
 that represents  
 $(x - 1)(x - 3) < 0$

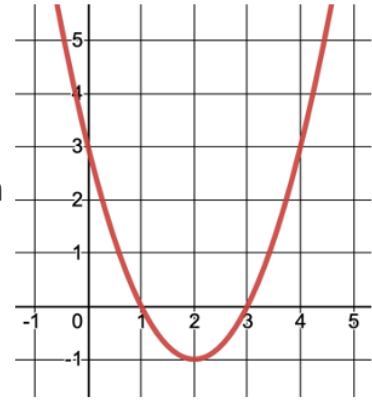
State the solution set of  
 $(x - 1)(x - 3) < 0$



6) Here is the graph of  
 $y = (x - 1)(x - 3)$

Show which part of the graph  
 that represents  
 $(x - 1)(x - 3) > 0$

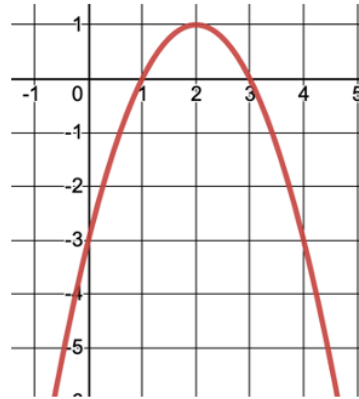
State the solution set of  
 $(x - 1)(x - 3) > 0$



7) Here is the graph of  
 $y = (-x + 1)(x - 3)$

Show which part of the graph  
 that represents  
 $(-x + 1)(x - 3) < 0$

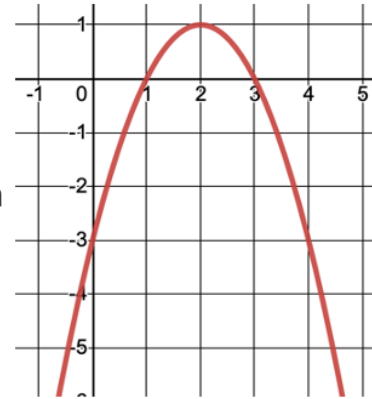
State the solution set of  
 $(-x + 1)(x - 3) < 0$



8) Here is the graph of  
 $y = (-x + 1)(x - 3)$

Show which part of the graph  
 that represents  
 $(-x + 1)(x - 3) > 0$

State the solution set of  
 $(-x + 1)(x - 3) > 0$



**Worked Example**

Solve  $x^2 + 3x - 10 < 0$

**Your Turn**

Solve  $x^2 - 7x + 12 \geq 0$

**Worked Example**

Solve the inequality  $x(x - 5) > 2(x - 6)$

**Your Turn**

Solve the inequality  $x(x - 3) < 2(x - 2)$

**Worked Example**

Solve the inequality  $9x^2 \geq 4$

**Your Turn**

Solve the inequality  $3x^2 > 27$

**Worked Example**

Solve the inequality  $(5x + 4)(x + 4) \leq 21$

**Your Turn**

Solve the inequality  $(5x + 2)(x - 2) > -7$

**Worked Example**

Solve  $5 - 4x - x^2 > 0$

**Your Turn**

Solve  $3x - 2 - x^2 < 0$



**Worked Example**

Solve  $2 \leq \frac{5x}{x^2+1}$

**Your Turn**

Solve  $1 > \frac{7x}{x^2+6}$

| Worked Example                                                                                                                                                            | Your Turn                                                                                                                                                             |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><math>n</math> is an integer such that <math>7n + 4 &lt; 18</math> and <math>\frac{5n}{n^2+1} \geq 2</math></p> <p>Find all the possible values of <math>n</math>.</p> | <p><math>n</math> is an integer such that <math>7n + 1 &gt; 8</math> and <math>2n^2 - 7n + 5 \leq 0</math></p> <p>Find all the possible values of <math>n</math>.</p> |

**Worked Example**

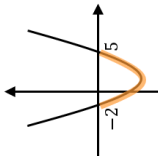
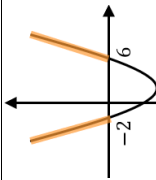
Solve  $44 < 5x^2 - 1 < 79$

**Your Turn**

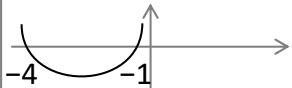
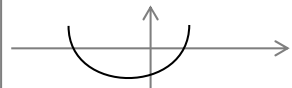
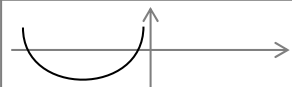
Solve  $76 < 3x^2 + 1 < 148$

## Fill in the Gaps



| Quadratic Inequality  | Factorised Quadratic | Critical Values      | Sketch and Shade                                                                    | Solution             |
|-----------------------|----------------------|----------------------|-------------------------------------------------------------------------------------|----------------------|
| $x^2 - 3x - 10 < 0$   | $(x - 5)(x + 2) < 0$ | $x = 5,$<br>$x = -2$ |    | $-2 < x < 5$         |
| $x^2 + 4x - 12 < 0$   | $(x + 6)(x - 2) < 0$ |                      |                                                                                     |                      |
| $x^2 - 6x + 5 < 0$    |                      |                      |                                                                                     |                      |
| $x^2 + 5x + 6 \leq 0$ |                      |                      |                                                                                     |                      |
| $x^2 - 4x - 12 > 0$   | $(x - 6)(x + 2) > 0$ | $x = 6,$<br>$x = -2$ |  | $x < -2,$<br>$x > 6$ |
| $x^2 + 4x + 3 \geq 0$ |                      |                      |                                                                                     |                      |
| $x^2 - x - 6 \geq 0$  |                      |                      |                                                                                     |                      |
|                       |                      |                      |                                                                                     | $-4 < x < 1$         |

## Fill in the Gaps

| Quadratic Inequalities  |                      |                 |  |                                                                                       |                |                     |
|-------------------------|----------------------|-----------------|--|---------------------------------------------------------------------------------------|----------------|---------------------|
| Inequality              | Factorisation        | Critical Values |  | Sketch                                                                                | Solution(s)    | Integer Answers (?) |
| $x^2 + 5x + 4 > 0$      | $(x + 1)(x + 4) = 0$ | -1              |  |    | $x >$<br>$x <$ |                     |
| $x^2 + 7x + 10 > 0$     |                      | -2              |  |    |                |                     |
| $x^2 + 2x - 15 > 0$     |                      |                 |  |    | $x > 3$        |                     |
|                         | $(x + 3)(x - 4) = 0$ |                 |  |    |                |                     |
| $x^2 + 7x + 6 < 0$      |                      |                 |  |    |                |                     |
|                         | $(x - 2)(x + 4) = 0$ |                 |  |    |                |                     |
| $x^2 + x - 20 \leq 0$   |                      |                 |  |    |                |                     |
|                         |                      |                 |  |   | $3 < x < 4$    |                     |
| $x^2 - 8x + 16 \leq 0$  |                      |                 |  |  |                |                     |
| $x^2 - 12x + 35 \geq 0$ |                      |                 |  |  |                |                     |



## Fill in the Gaps

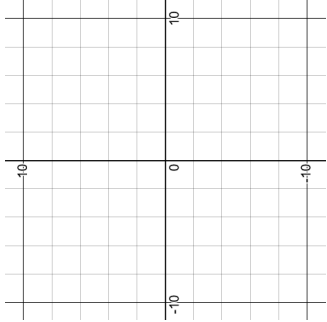
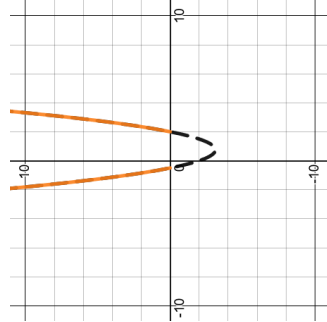
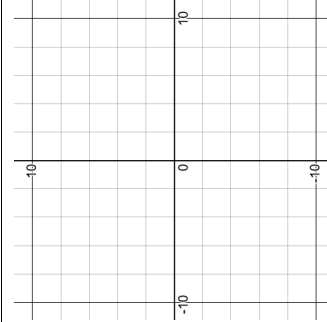
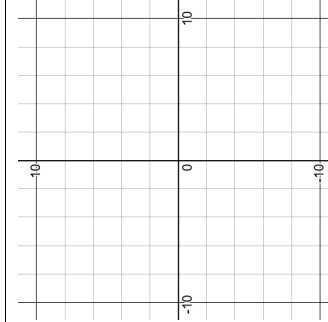
| Quadratic Inequality         | Rearrange           | Factorise | Critical Values | Sketch and Shade | Solution |
|------------------------------|---------------------|-----------|-----------------|------------------|----------|
| $x^2 < 7x - 12$              | $x^2 - 7x + 12 < 0$ |           |                 |                  |          |
| $3x^2 + 5 \geq 16x$          |                     |           |                 |                  |          |
| $x^2 > 3x$                   |                     |           |                 |                  |          |
| $2x^2 < 6 + 11x$             |                     |           |                 |                  |          |
| $\frac{4x^2 + 5x}{3} \leq 2$ |                     |           |                 |                  |          |

Fill in the Gaps

Fill in the blanks in the table below:

| Question                       | Factorise            | Sketch | Answer             |
|--------------------------------|----------------------|--------|--------------------|
| Solve<br>$x^2 + 4x + 3 < 0$    | $(x + 3)(x + 1) < 0$ |        |                    |
| Solve<br>$x^2 - 2x - 8 \geq 0$ |                      |        |                    |
| Solve<br>$x^2 + x > 2$         |                      |        |                    |
| Solve                          |                      |        | $-1 \leq x \leq 3$ |

# Fill in the Gaps

|                                |                                                                                    |                                                                                     |                             |
|--------------------------------|------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------|-----------------------------|
| Solve<br>$5 + 4x - x^2 > 0$    |                                                                                    |    |                             |
| Solve<br>$(2x + 1)(x - 2) > 0$ |  |                                                                                     |                             |
| Solve                          |                                                                                    |  | $\frac{2}{3} \leq x \leq 2$ |
| Solve<br>$5x + 3 - 2x^2 < 0$   |                                                                                    |  |                             |



## Extra Notes

## 5 Real-Life Graphs

## Conversion Graphs

## Worked Example

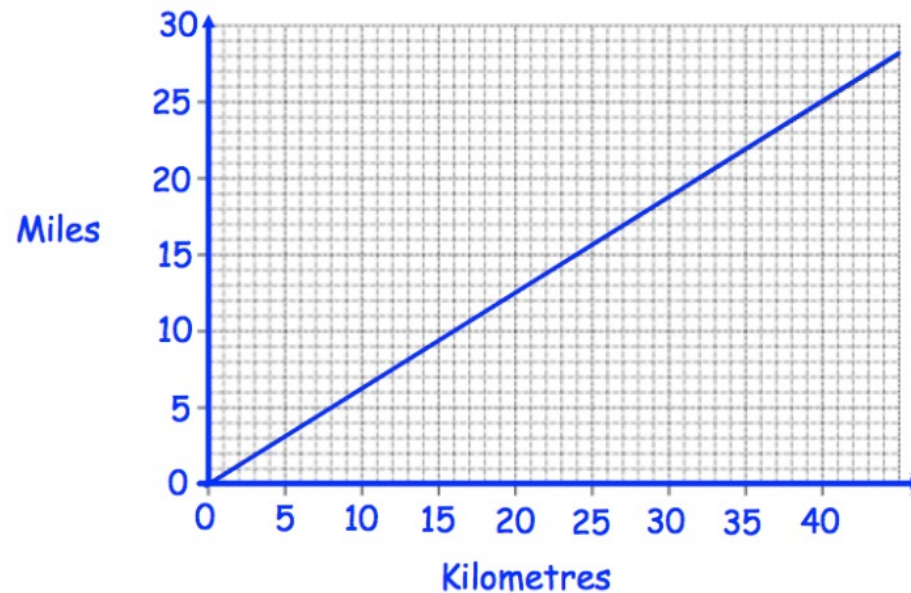
Use the graph to convert:



- a) 5 kilograms to pounds
- b) 8 pounds to kilograms
- c) 150 kilograms to pounds

## Your Turn

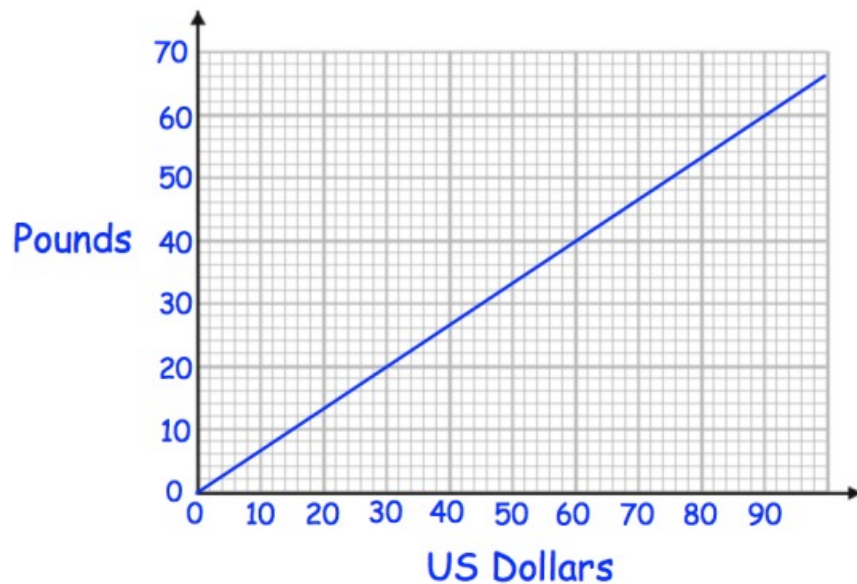
Use the graph to convert:



- a) 40 kilometres to miles
- b) 10 miles to kilometres
- c) 800 kilometres to miles

## Worked Example

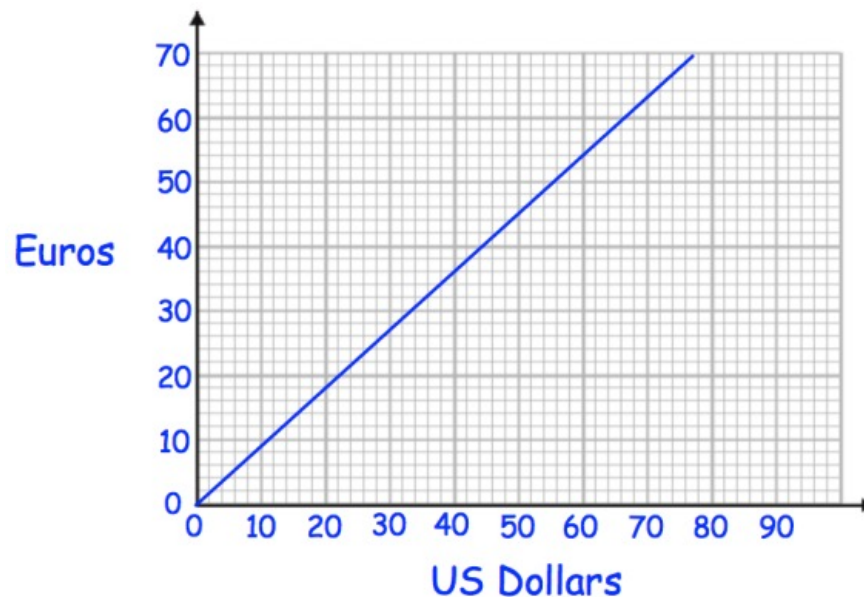
Use the graph to convert:



- a) 60 US Dollars to Pounds
- b) 20 Pounds to US Dollars
- c) 1800 US dollars to Pounds

## Your Turn

Use the graph to convert:



- a) 60 US Dollars to Euros
- b) 20 Euros to US Dollars
- c) 1800 US dollars to Euros

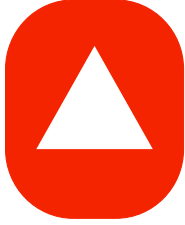


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## Conversion Graphs

Videos 151 and 152 on [www.corbettmaths.com](http://www.corbettmaths.com)

Examples



Click here

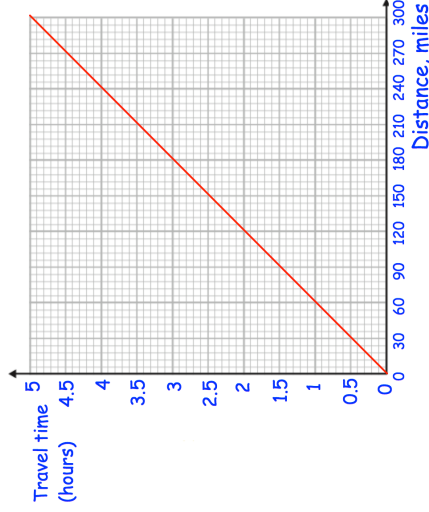


Scan here

Workout

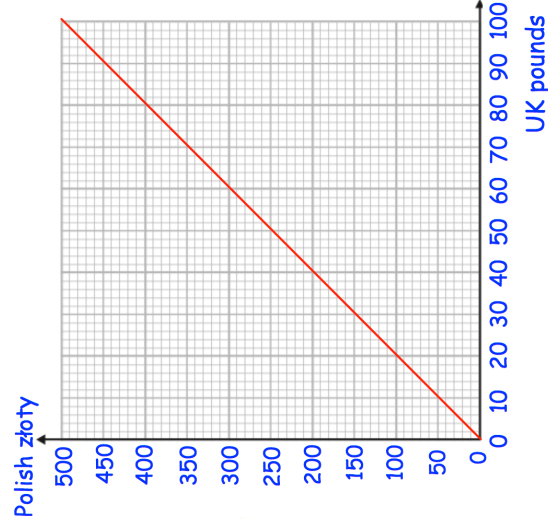
Question 1:

- How long should a 120 mile journey take?
- How long should a 270 mile journey take?
- Carlos has spent 1 hour travelling.  
What distance is he expected to have travelled?
- Rosie has spent 3.5 hours travelling.  
What distance is she expected to have travelled?



Question 2:

- Change £20 into Polish zloty
- Change £90 into Polish zloty
- Change 300zł into UK pounds
- Change 450zł into UK pounds
- Change £50 into Polish zloty
- Change £200 into Polish zloty
- Change 800zł into UK pounds



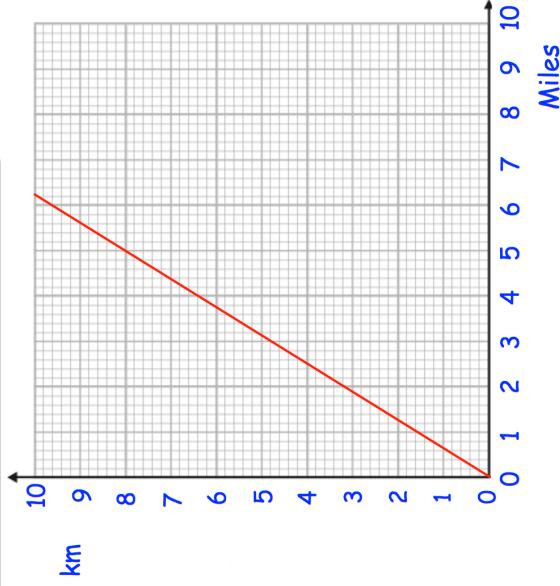
© CORBETTMATHS 2019



## Conversion Graphs

Videos 151 and 152 on [www.corbettmaths.com](http://www.corbettmaths.com)

Question 3: This conversion graph can be used to change between miles and kilometres.



- (a) Change 5 miles into kilometres
- (b) Change 1 mile into kilometres
- (c) Change 6km into miles
- (d) Change 4.8km into miles
- (e) Change 20 miles into kilometres
- (f) Change 16km into miles

Question 4: Draw conversion graphs for the following sets of information.

- (a) A conversion graph to convert between UK pound and South African Rand.

|         |   |    |    |    |
|---------|---|----|----|----|
| Pounds  | 0 | 1  | 2  | 3  |
| SA Rand | 0 | 20 | 40 | 60 |

Horizontal axis: UK pound from £0 to 10  
Vertical axis: South African Rand from 0 to 200 Rand

- (b) A conversion graph for hours and cost of driving lessons.

|       |     |     |
|-------|-----|-----|
| Hours | 1   | 4   |
| Cost  | £15 | £60 |

Horizontal axis: Hours from 0 to 10  
Vertical axis: Cost (decide scale yourself)

- (c) A conversion graph to show how to change from kilograms to pounds.

|               |    |
|---------------|----|
| Kilograms, kg | 5  |
| Pounds, lb    | 11 |

Horizontal axis: Kilograms from 0kg to 10kg  
Vertical axis: Pounds (decide scale yourself)

- (d) A conversion graph to convert between Euros and US Dollars.

|            |       |
|------------|-------|
| US Dollars | \$ 77 |
| Euros      | € 70  |

Horizontal axis: Euros from 0 to €100  
Vertical axis: US Dollar (decide scale yourself)



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## Conversion Graphs

Videos 151 and 152 on [www.corbettmaths.com](http://www.corbettmaths.com)

Apply

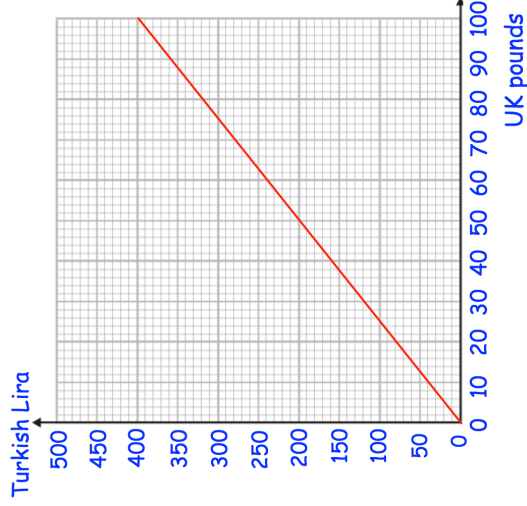
Question 1:

Richard has ₺300 and £800.

He buys a flight that costs ₺900

He pays use the ₺300 and some of the pounds.

Work out how many pounds he has left.

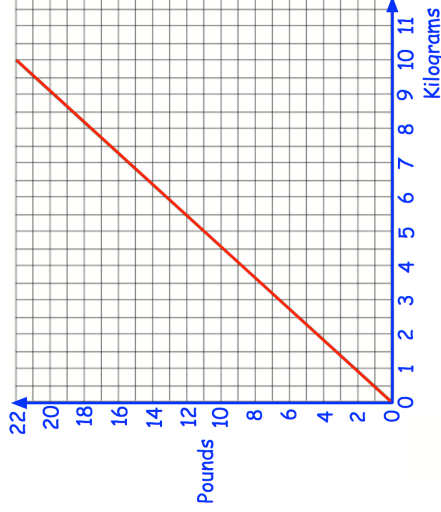


Question 2:

Jenny's weight is 65kg.

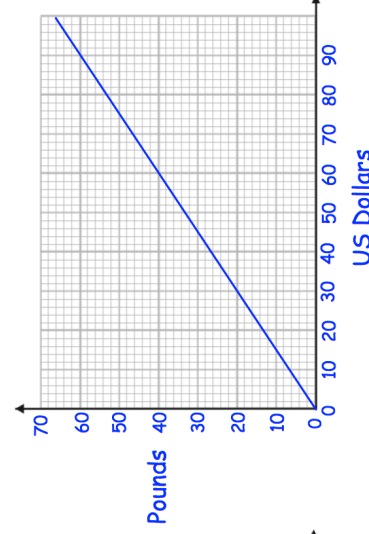
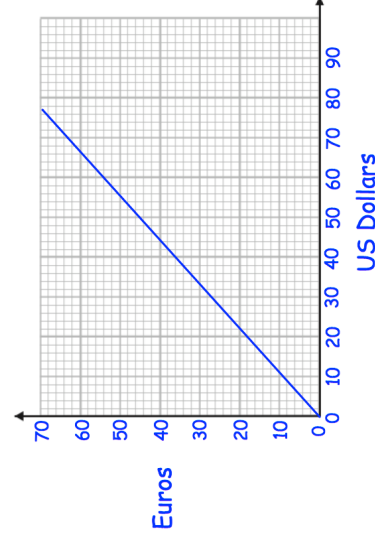
1 stone = 14 pounds.

What is Jenny's weight in stones and pounds?



Question 3: The two conversion graphs below show how to change between some currencies.

Convert £800 into Euros.



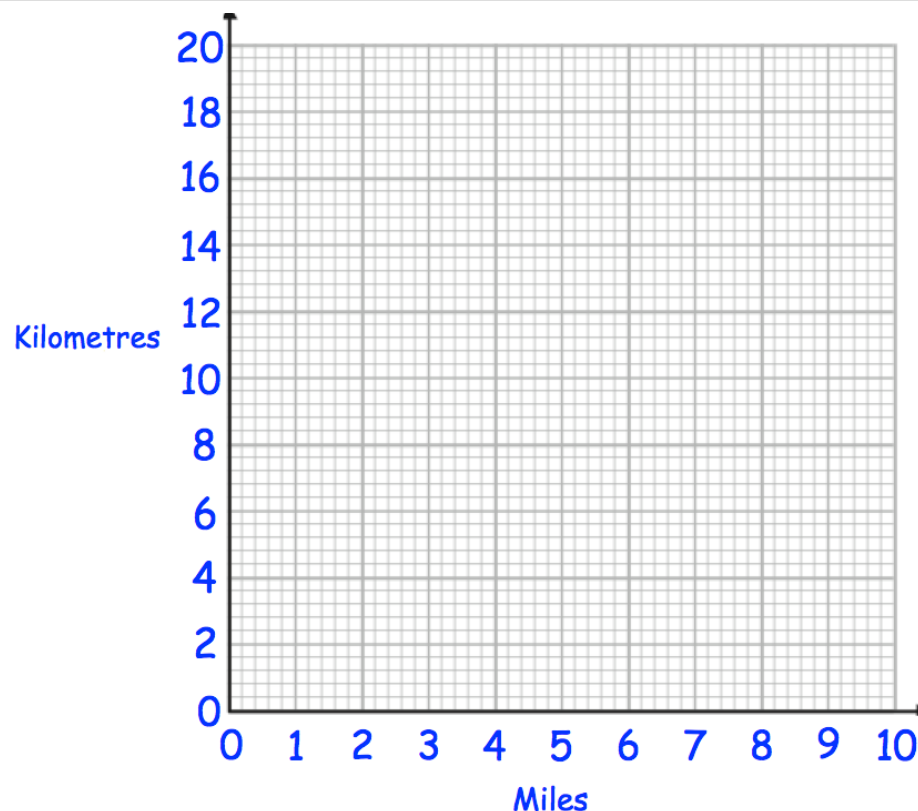


Question 4: 5 miles  $\approx$  8 kilometres

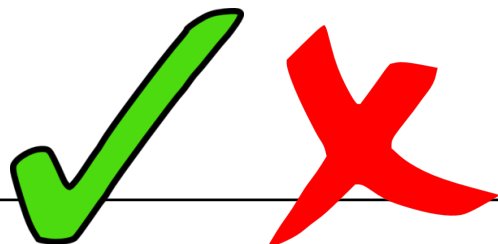
(a) Use this information to draw a conversion graph.

A car is travelling 60km/h.

(b) Use the graph to convert this into mph.



Answers



Click here <sup>Page 152</sup>



Scan here

### Worked Example

The A Level Maths mark,  $y$  %, and GCSE Maths mark,  $x$  %, is recorded for several students.

Assume the line goes through  $(0, 40)$  and  $(60, 80)$ .

- a) Write a linear model.
- b) Interpret the gradient and  $y$ -intercept in this context.

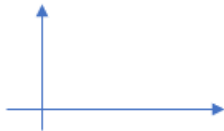
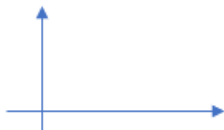
### Your Turn

The temperature  $y$  at different points on a mountain is recorded at different altitudes  $x$ .

Assume the line goes through  $(0, 70)$  and  $(250, 20)$ .

- a) Write a linear model.
- b) Interpret the gradient and  $y$ -intercept in this context.

## Fill in the Gaps

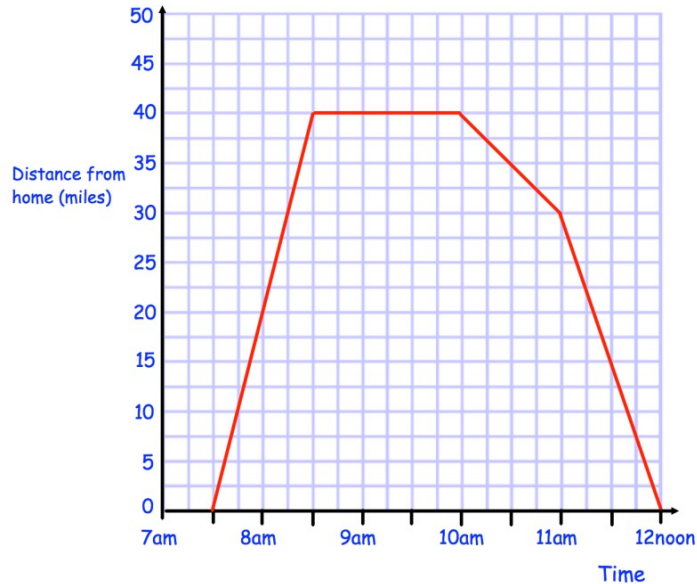
| Independent variable                     | Response variable                     | Gradient & intercept      | Equation of line     | Sketch of graph (with labelled axes)                                                 | Interpretation of the gradient                                         | Interpretation of the intercept                       |
|------------------------------------------|---------------------------------------|---------------------------|----------------------|--------------------------------------------------------------------------------------|------------------------------------------------------------------------|-------------------------------------------------------|
| $t$ : time (s)                           | $M$ : mass (kg)                       | $m = 3.5$<br>$c = -0.04$  |                      |    |                                                                        |                                                       |
| $V$ : Volume of water in kettle (litres) | $t$ : time taken to boil (s)          |                           | $t = 167V + 12.6$    |    |                                                                        |                                                       |
| $t$ :                                    | $h$ :                                 |                           |                      |    | Each day the shoot grows by 0.0206mm.                                  | Initially the shoot is 0.312mm tall.                  |
| $x$ :                                    | $v$ :                                 | $m =$<br>$c = 2.73$       |                      |    | Walking speed decreases by $0.011\text{ms}^{-1}$ for each year of age. |                                                       |
| $r$ : altitude of runway (m)             |                                       |                           | $t = 0.258r + \dots$ |   |                                                                        | Take off distance at sea level (0m altitude) is 623m. |
| $x$ : amount of fertilizer (kg/hectare)  | $y$ : yield of maize (tonnes/hectare) | $m = 0.76$<br>$c = 0.058$ |                      |  |                                                                        |                                                       |

## Distance-Time Graphs

## Worked Example

Study the distance-time graph below.

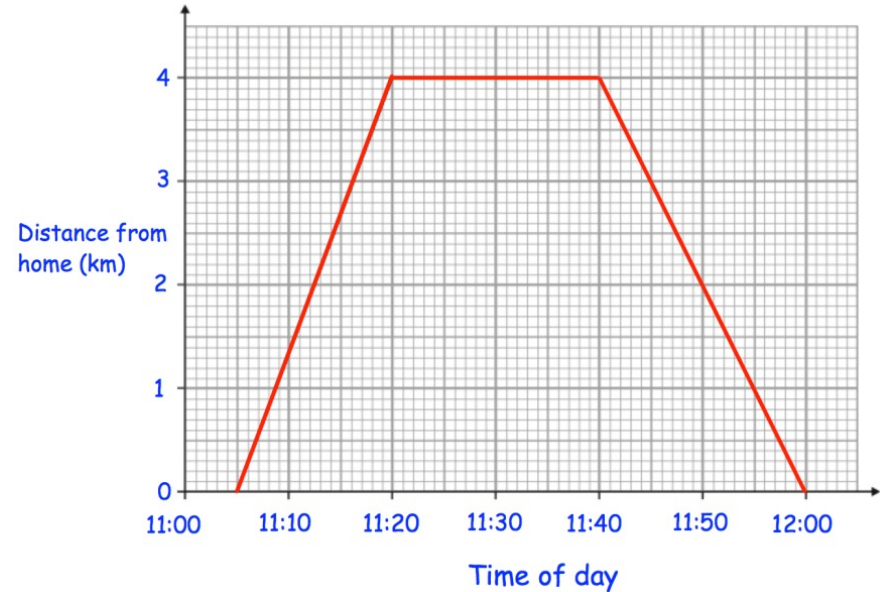
- a) How far was the person from home at 8.30am?
- b) How long did the person stop for?
- c) How far did the person travel in total?



## Your Turn

Study the distance-time graph below

- a) How far was the person from home at 11.20?
- b) How long did the person stop for?
- c) How far did the person travel in total?

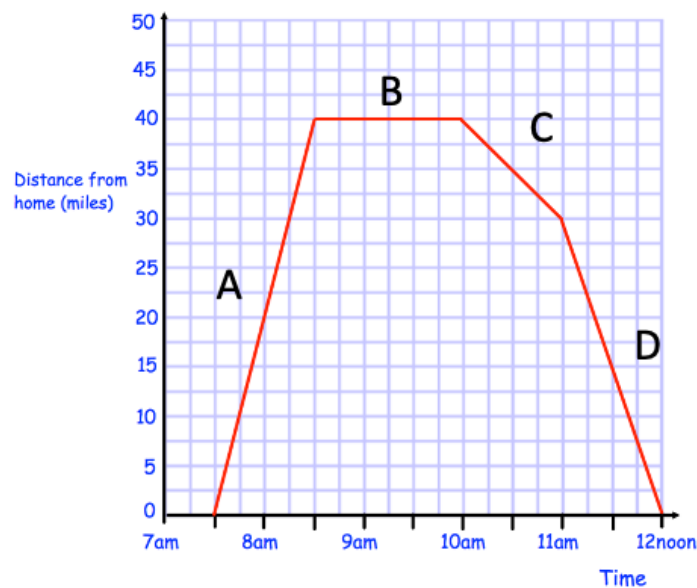


## Worked Example

Study the distance-time graph below.

What was the person's speed in each section of the journey:

*A*  
*B*  
*C*  
*D*

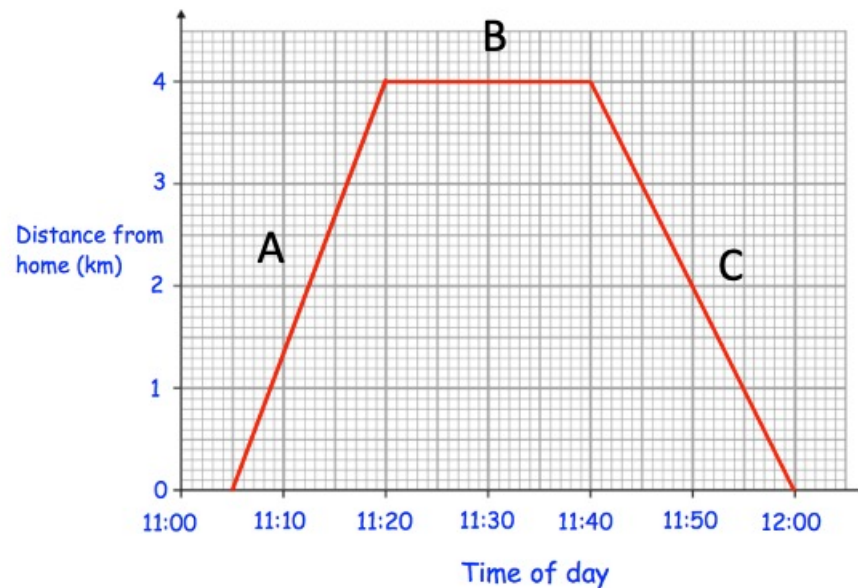


## Your Turn

Study the distance-time graph below.

What was the person's speed in each section of the journey:

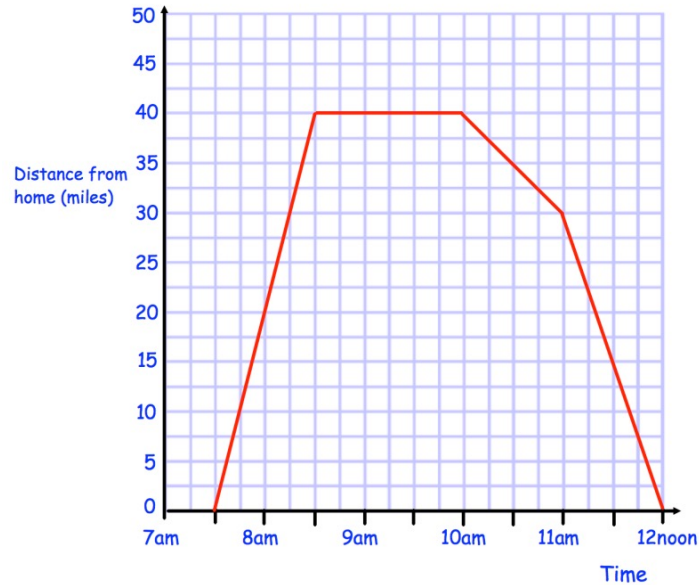
*A*  
*B*  
*C*



## Worked Example

Study the distance-time graph below.

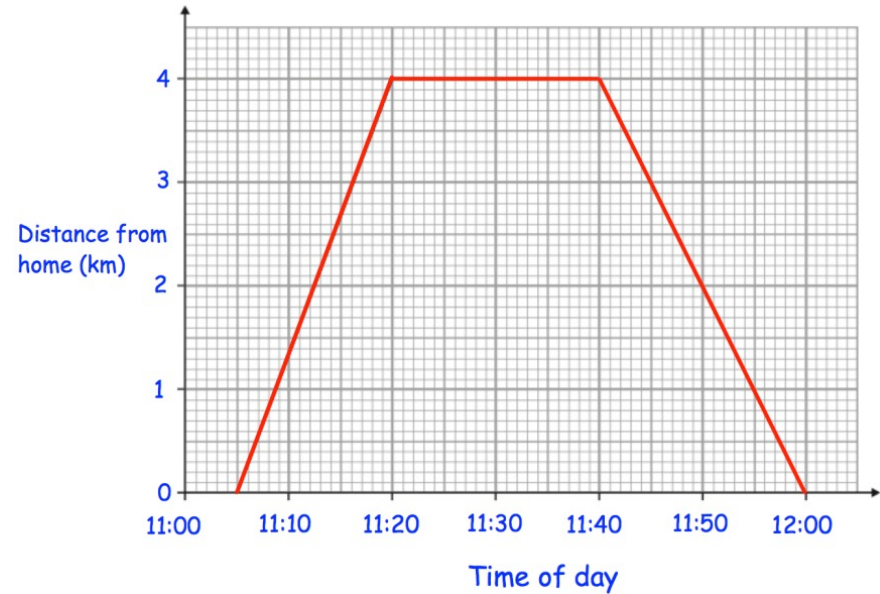
What was the person's average speed for the whole journey?



## Your Turn

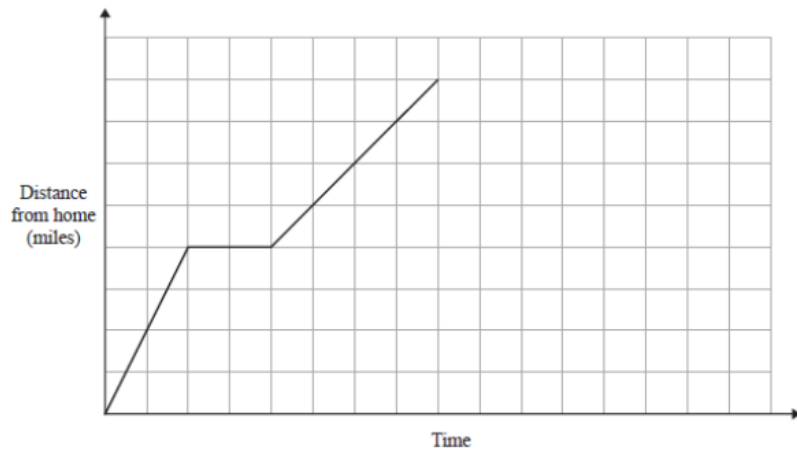
Study the distance-time graph below.

What was the person's average speed for the whole journey?



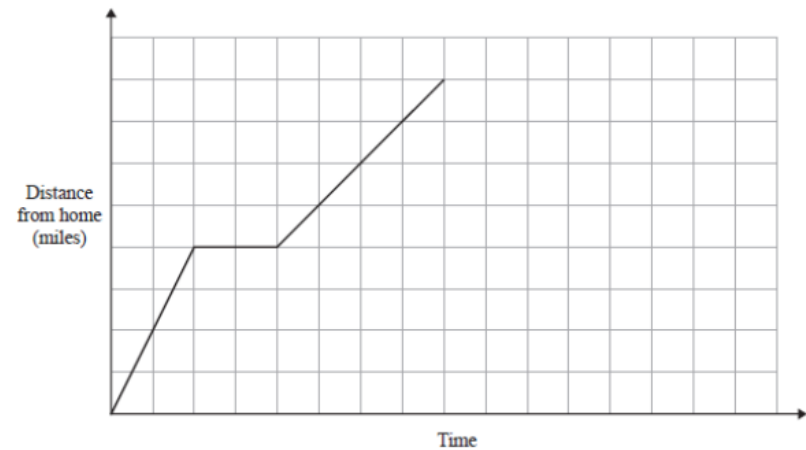
## Worked Example

A person leaves home at 13 : 00.  
She drives for 1 hour at a constant speed of 20 mph.  
She then stops for a break.  
She then drives to the hospital at a constant speed.  
She then is at the hospital for 90 minutes.  
She then drives home at a constant speed of 48 mph.  
What time does she arrive home?

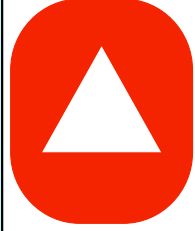


## Your Turn

A person leaves home at 13 : 00.  
She drives for 30 minutes at a constant speed of 40 mph.  
She then stops for a break.  
She then drives to the hospital at a constant speed.  
She then is at the hospital for 60 minutes.  
She then drives home at a constant speed of 32 mph.  
What time does she arrive home?







Examples

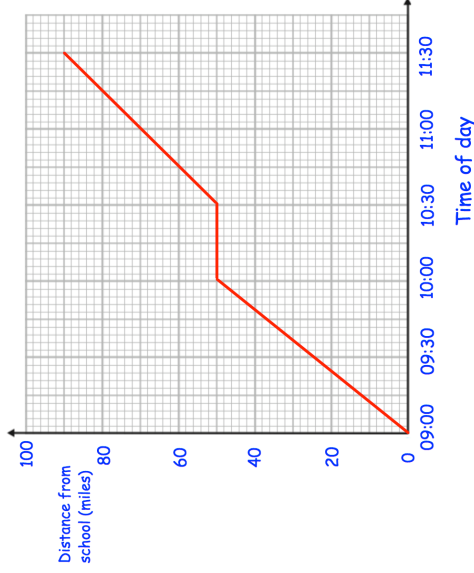


Click here

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Workout

Question 1: The distance-time graph shows class 8A's journey to the zoo. They stopped for a picnic on the way to the zoo.

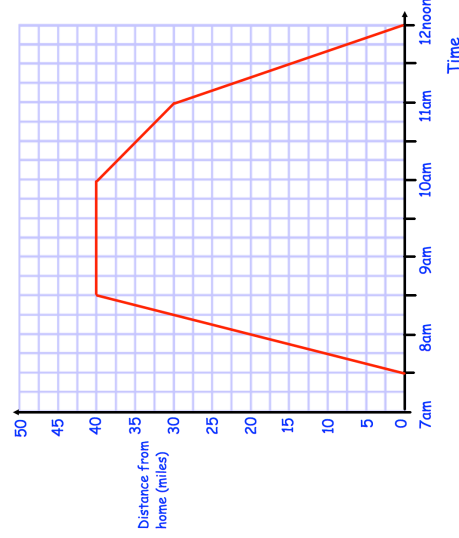


- (a) What time did the bus leave school?
- (b) What time did they stop for a picnic?
- (c) How far had they travelled when they stopped for a picnic?
- (d) How long did they stop for?
- (e) What time did they arrive at the zoo?
- (f) How far is the zoo from school?

Question 2: Emma travelled to her Grandmother's house and back.

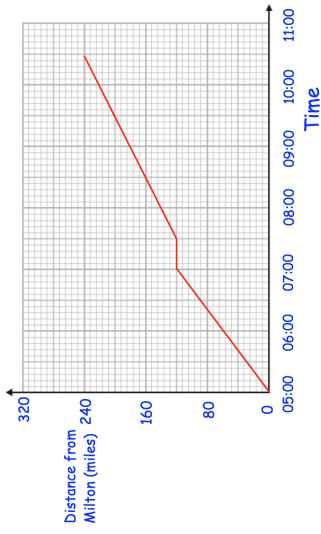
The distance-time graph shows information about her journey.

- (a) What time did Emma begin her journey?
- (b) How far was Emma from home at 8am?
- (c) How long did Emma stay at her Grandmother's house?
- (d) What time did Emma leave her Grandmother's house?
- (e) How far was Emma from home at 11:45?
- (f) How far did Emma travel in total?



Question 3: A train travels from Milton to Redville, stops for 30 minutes, then travels to Leek.

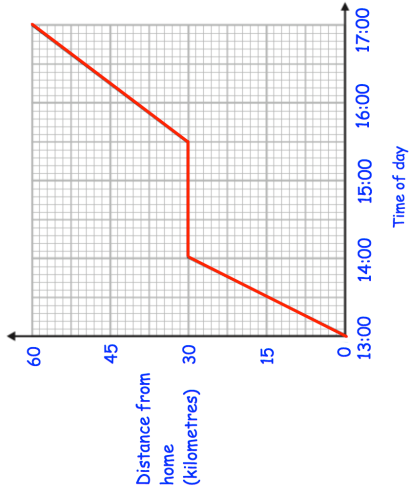
- How long did it take the train to travel from Milton to Redville?
- How far is Redville from Milton?
- Work out the speed of the train for the journey from Milton to Redville.
- How long did it take the train to travel from Redville to Leek?
- How far is Leek from Redville?



(f) Work out the speed of the train for the journey from Redville to Leek.

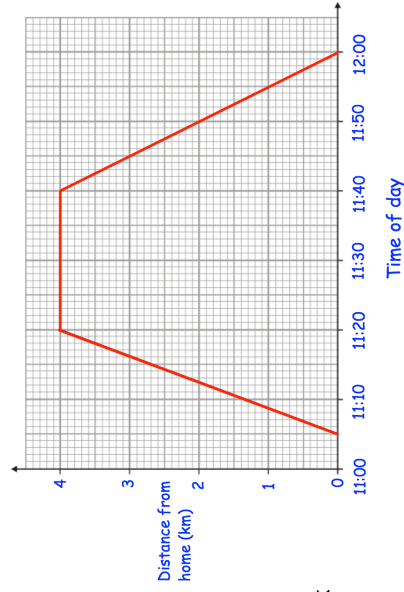
Question 4: Ben drove 60 kilometres, from his home to Liverpool. He stopped and visited his friend Tim on the way.

- Work out Ben's speed for the first part of his journey.
- How long did Ben spend visiting Tim?
- Work out Ben's speed for the last part of his journey.



Question 5: Laura goes for a cycle from her house to the post office, 4km away.

- How long did it take Laura to cycle to the post office?
- Work out Laura's speed cycling to the post office.
- How long did Laura spend at the post office?
- Work out Laura's speed cycling back home.





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## Distance-Time Graphs

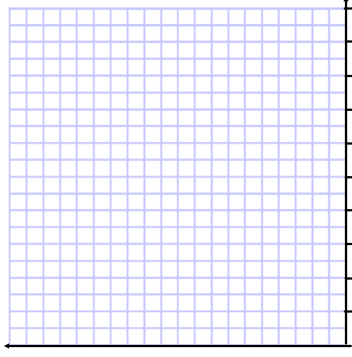
Video 171 on [www.corbettmaths.com](http://www.corbettmaths.com)

### Apply

#### Question 1:

Erin leaves home at 11am.  
She cycles at a speed of 16 miles per hour for 90 minutes.  
She stops for half an hour.  
Erin then cycles home and arrives at 3pm.

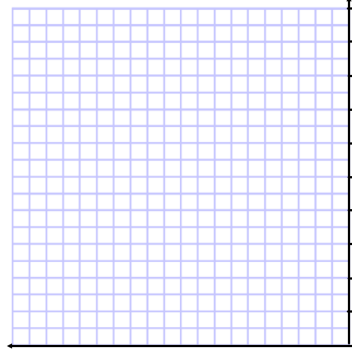
- (a) Draw a distance-time graph to show Erin's journey.  
(b) What is Erin's average speed on the return part of her cycle?



#### Question 2:

Thomas leaves home at 14:00  
He drives at an average speed of 40mph for  $3\frac{1}{2}$  hours  
Thomas stops for 30 minutes.  
He then drives home at an average speed of 70mph.

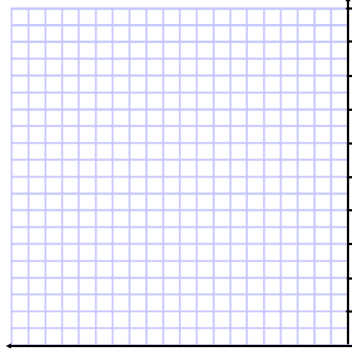
Draw a distance-time graph to show Thomas's journey



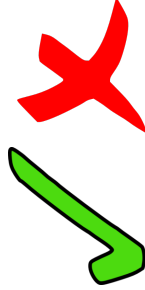
#### Question 3:

A helicopter leaves Bristol at 10:00.  
It flies for 45 minutes at 80km/h.  
It lands for 30 minutes and then flies a further 65 kilometres in 30 minutes.  
The helicopter then immediately returns to its base in Bristol, flying at 100km/h.

Draw a distance-time graph to show the journey.



### Answers



Click here



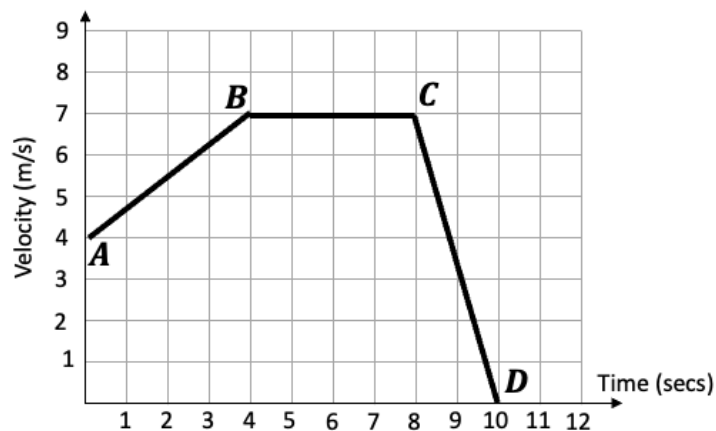
Scan here

## Speed-Time Graphs

## Worked Example

State and describe the acceleration between:

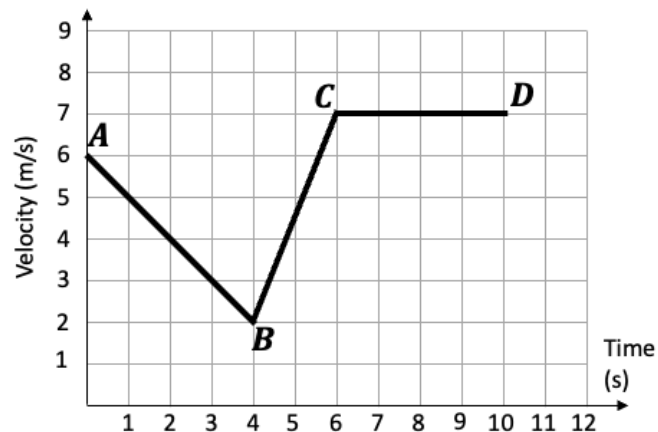
- a)  $A$  and  $B$
- b)  $B$  and  $C$
- c)  $C$  and  $D$



## Your Turn

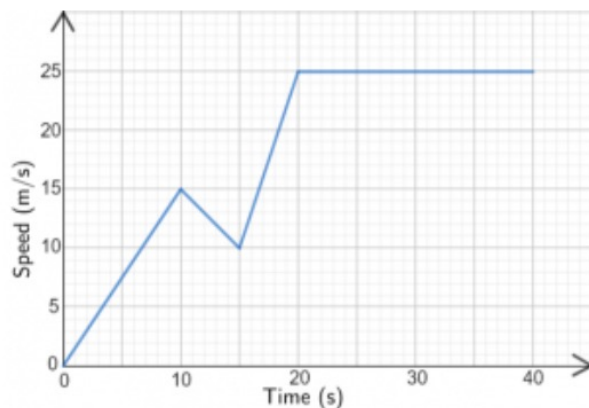
Determine the acceleration from:

- a)  $A$  to  $B$
- b)  $B$  to  $C$
- c)  $C$  to  $D$



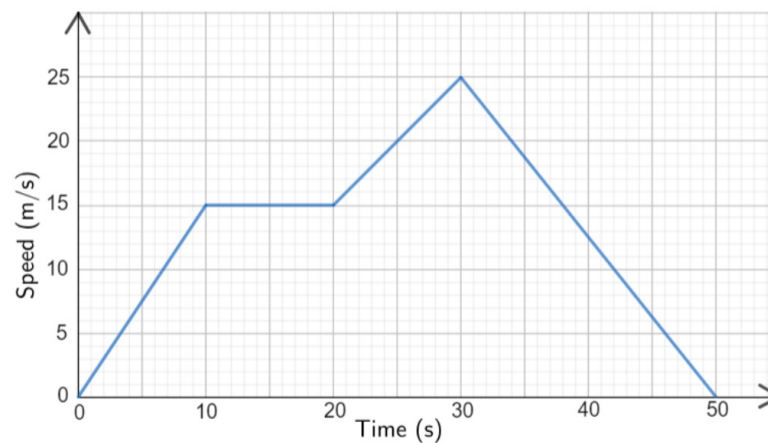
## Worked Example

Below is the speed-time graph of 40-second cycle journey. Work out the total distance travelled by the cyclist over the journey.



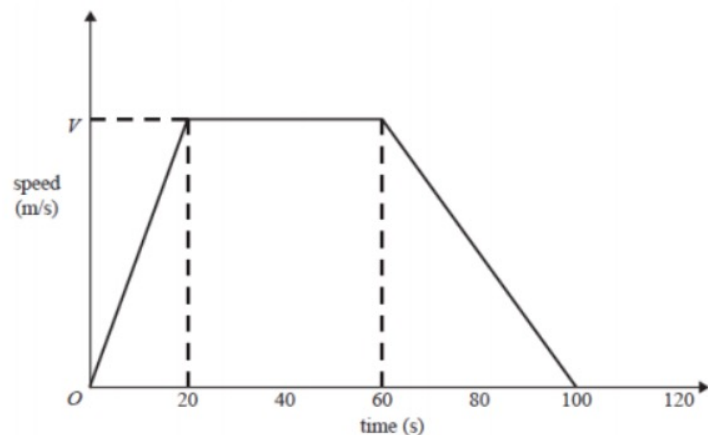
## Your Turn

Below is the speed-time graph of a 50-second car journey. Work out the total distance travelled by the car over the journey.



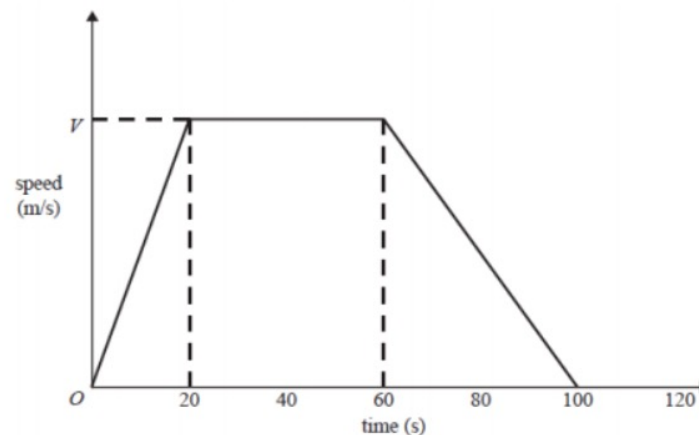
### Worked Example

Below is a speed-time graph for a car journey.  
The car travelled 3500 m in 100 seconds.  
Calculate the value of  $V$ .



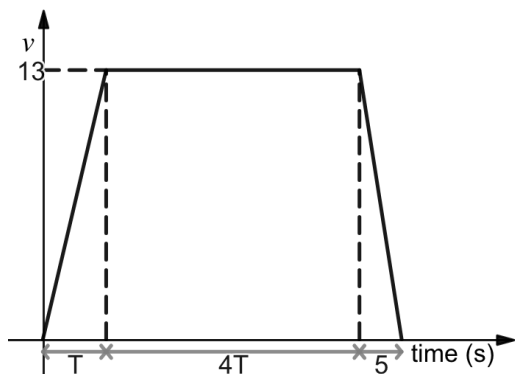
### Your Turn

Below is a speed-time graph for a car journey.  
The car travelled 1750 m in 100 seconds.  
Calculate the value of  $V$ .



### Worked Example

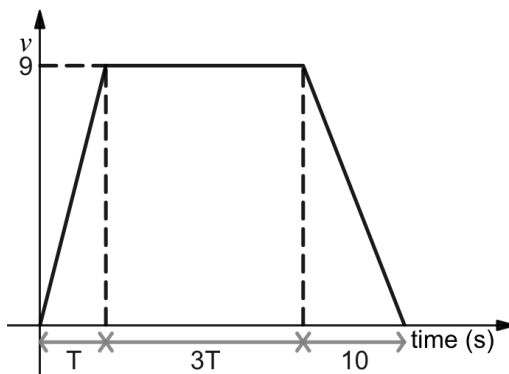
The velocity-time graph illustrate the motion of a vehicle moving in a straight line.



Given that the total displacement of the vehicle is 471.25 m, find the value of  $T$ .

### Your Turn

The velocity-time graph illustrate the motion of a particle moving in a straight line.



Given that the total displacement of the particle is 249.75 m, find the value of  $T$ .



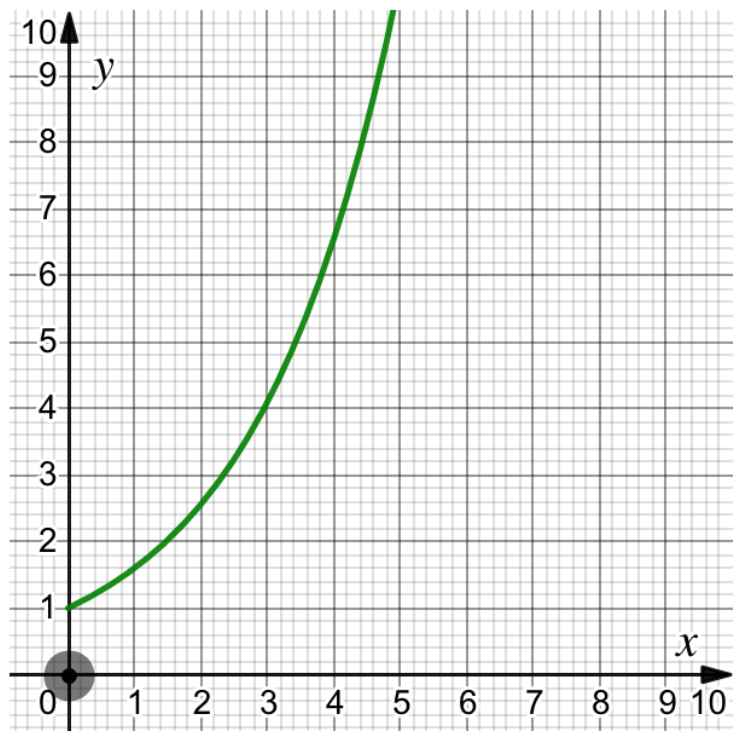
## Extra Notes

## 6 Gradients and Areas of Graphs

## Gradients of Curves

## Worked Example

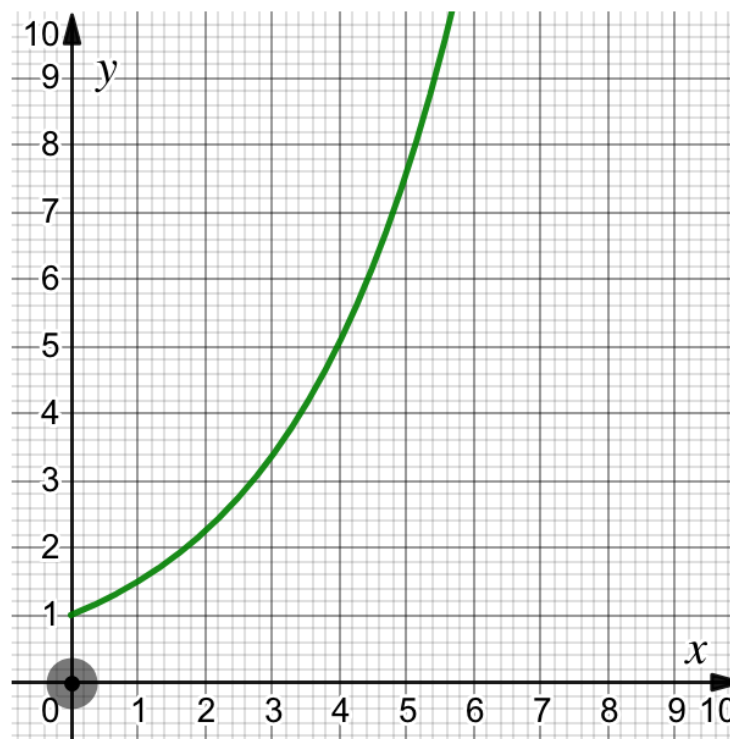
The diagram below shows part of the graph of  $y = f(x)$ .



$P$  is the point on the graph of  $y = f(x)$  where  $x = 3$ .  
Work out an estimate for the gradient of the graph at the point  $P$ .

## Your Turn

The diagram below shows part of the graph of  $y = f(x)$ .

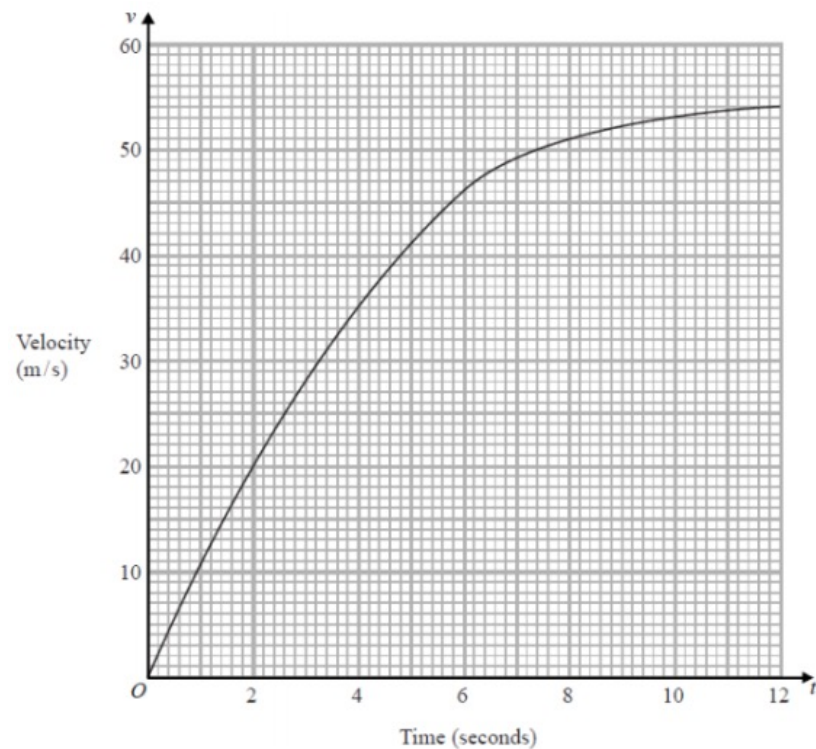


$P$  is the point on the graph of  $y = f(x)$  where  $x = 4$ .  
Calculate an estimate for the gradient of the graph at the point  $P$ .

## Worked Example

The graph shows the velocity,  $v$  m/s, of a parachutist  $t$  seconds after leaving a plane.

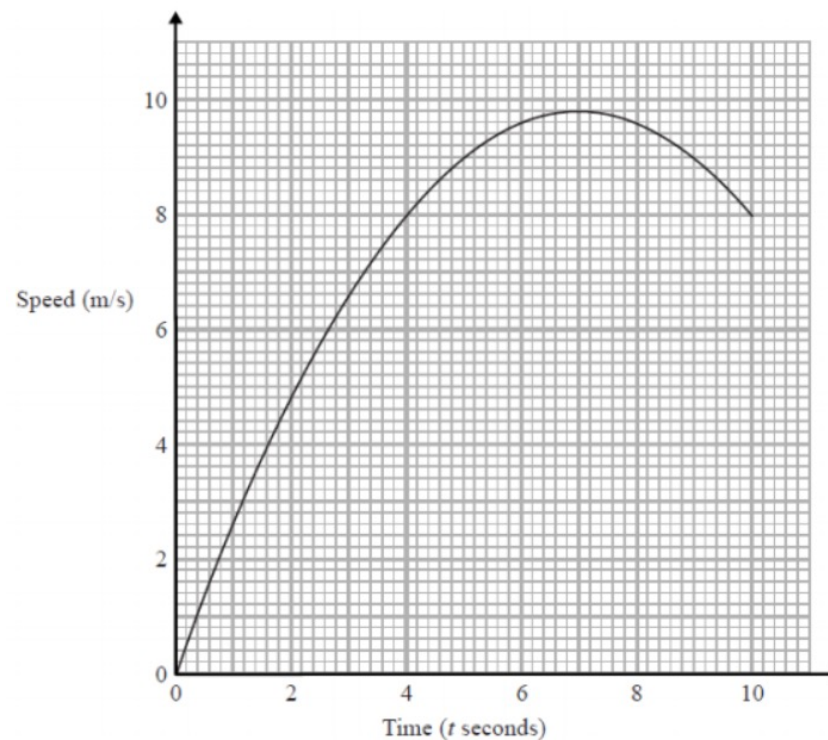
Calculate an estimate for the acceleration of the parachutist at  $t = 6$ .



## Your Turn

The graph shows the speed, m/s, of a runner,  $t$  seconds after the start of a race.

Calculate an estimate for the acceleration of the runner at  $t = 4$ .





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## Rates of Change

Videos 390a and 390b on [www.corbettmaths.com](http://www.corbettmaths.com)

Examples

Instantaneous



Click here

Average



Click here

Instantaneous



Scan here

Average

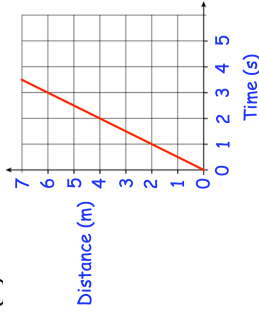


Scan here

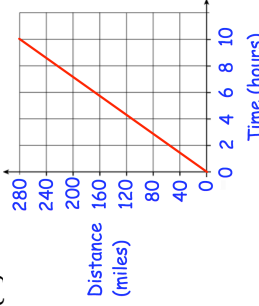
Workout

Question 1: For each graph below, work out the speed.

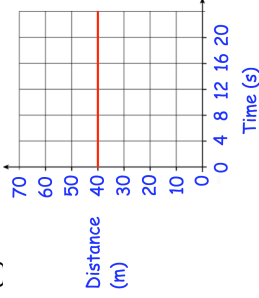
(a)



(b)



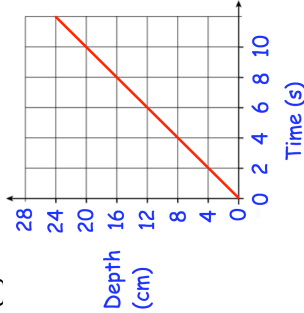
(c)



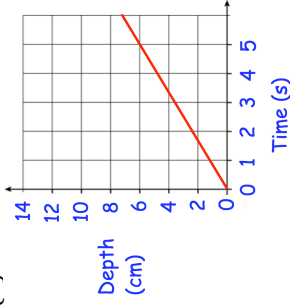
Question 2: For each graph below, work out the rate of change of depth.

Give each answer in  $cm/s$

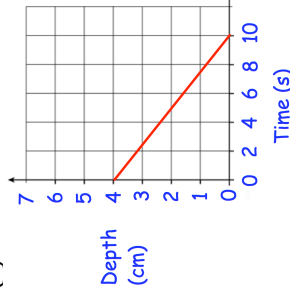
(a)



(b)



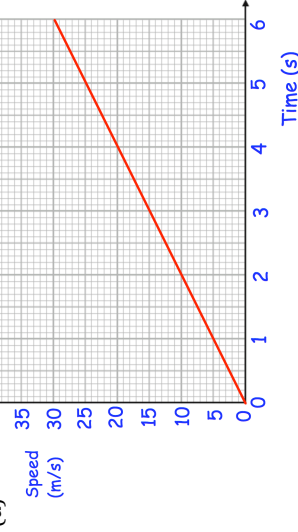
(c)



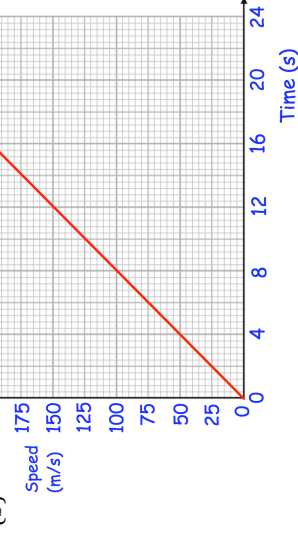
Question 3: For each graph below, work out the acceleration.

Give each answer in  $m/s^2$

(a)



(b)



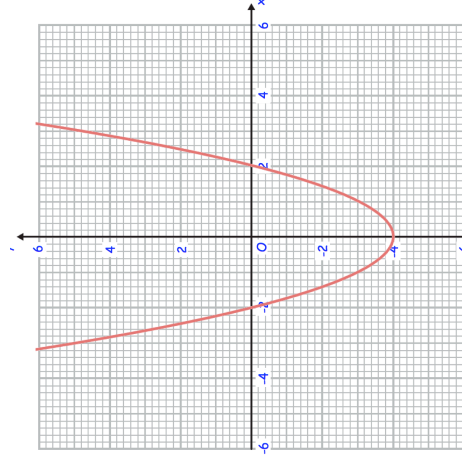


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## Rates of Change

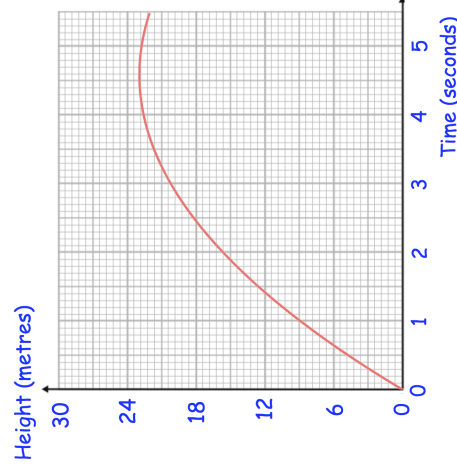
Videos 390a and 390b on [www.corbettmaths.com](http://www.corbettmaths.com)

Question 4: Here is part of a quadratic graph



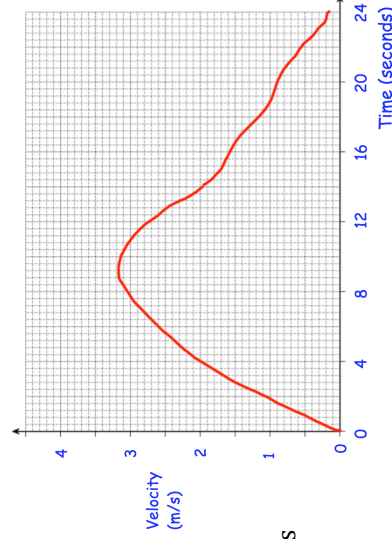
- (a) What is the gradient of the graph at the point  $(0, -4)$ ?
- (b) Calculate an estimate of the gradient of the graph at the point  $(2, 0)$
- (c) Calculate an estimate of the gradient of the graph at the point  $(-1, -3)$

Question 5: The graph shows the height of a ball above the ground.



- (a) Use the graph to work out an estimate of the speed of the ball at 1 second.
- (b) When was the speed  $0 \text{ m/s}$ ?
- (c) Use the graph to work out an estimate of the speed of the ball at 4 seconds.

Question 6: The graph shows the velocity of object.



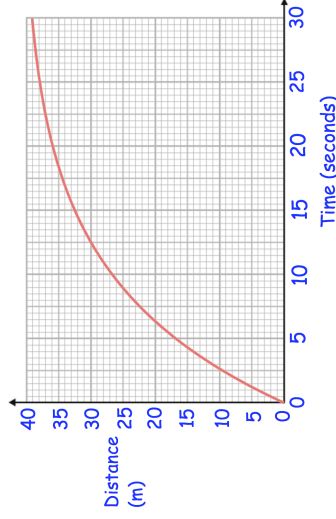
- (a) Use the graph to work out an estimate of the acceleration of the object at 2 seconds.
- (b) Use the graph to work out an estimate of the acceleration of the object at 8 seconds.
- (c) Use the graph to work out an estimate of the deceleration of the object at 16 seconds

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Question 7: Shown is a distance-time graph of part of a journey.

(a) Work out the average speed over the first 5 seconds of the journey.

(b) Work out the average speed between 15 and 30 seconds.



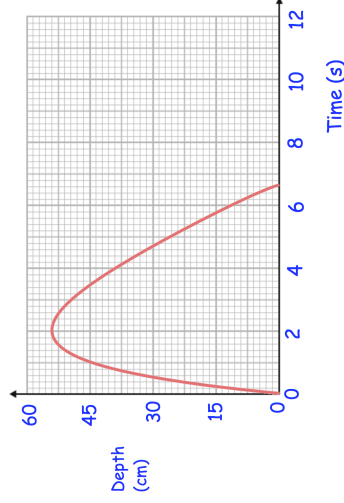
Question 8: Hugh has a bucket with holes in it.

Hugh fills the bucket with water and records the depth of water

The graph shows the depth of water in the bucket.

(a) Work out the average rate of change of depth of water between 0 and 2 seconds.

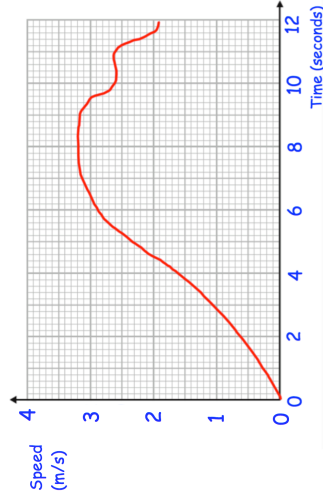
(b) Work out the average rate of change of depth of water between 2 and 6 seconds.



Question 9: Here is the speed of a toy car during 12 seconds.

(a) Work out the average acceleration of the toy car between 1 and 5 seconds.

(b) Work out the average acceleration of the toy car between 8 and 12 seconds.

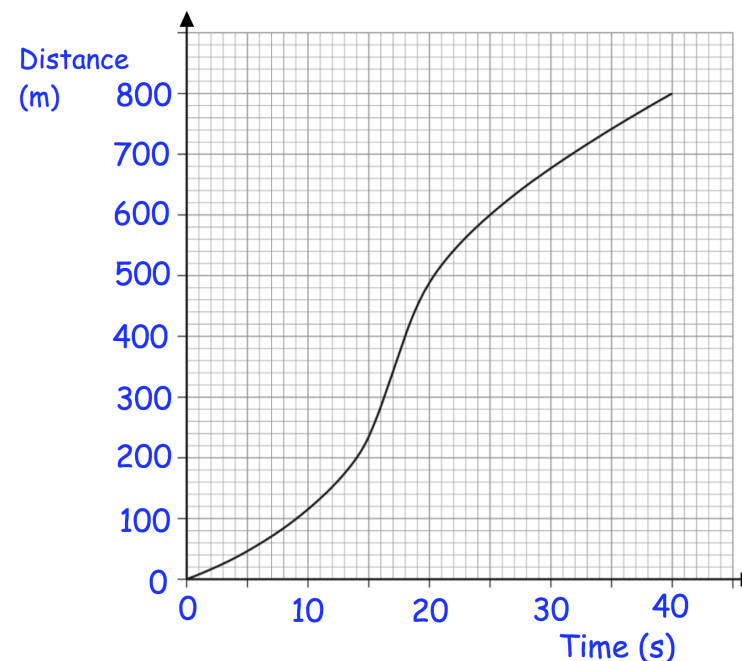




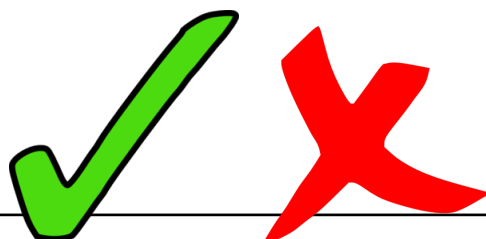
## Apply

Question 1: Below is the distance-time graph for the first 40 seconds of a train journey.

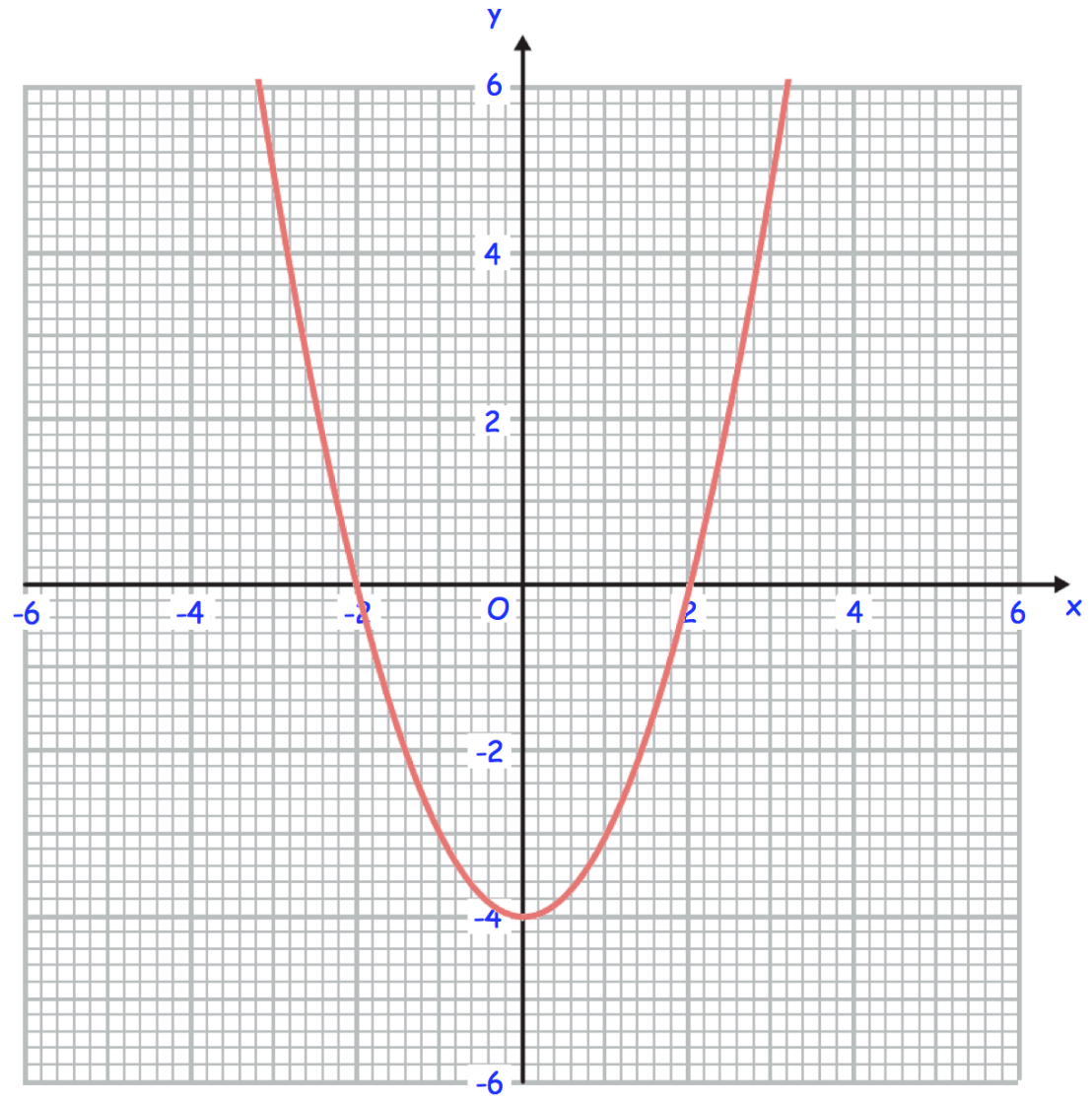
- (a) Use the graph to calculate an estimate for the speed of the train at 30 seconds.
- (b) Explain why your answer to (a) is only an estimate.
- (c) Estimate the highest speed reached by the train on the journey.



## Answers



Question 4:



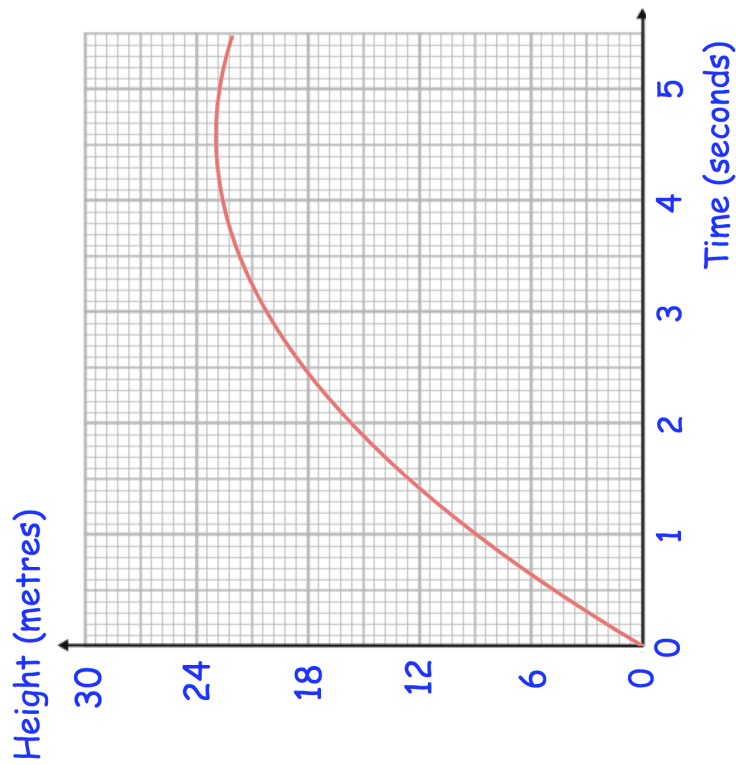


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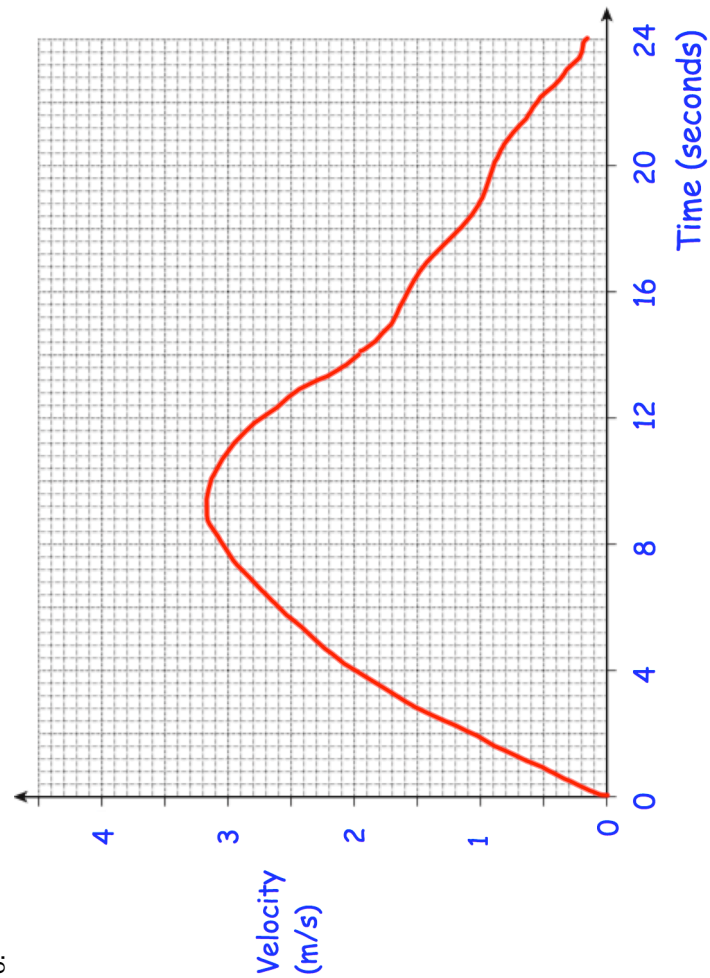
## Rates of Change

Videos 390a and 390b on [www.corbettmaths.com](http://www.corbettmaths.com)

Question 5:



Question 6:



## Templates

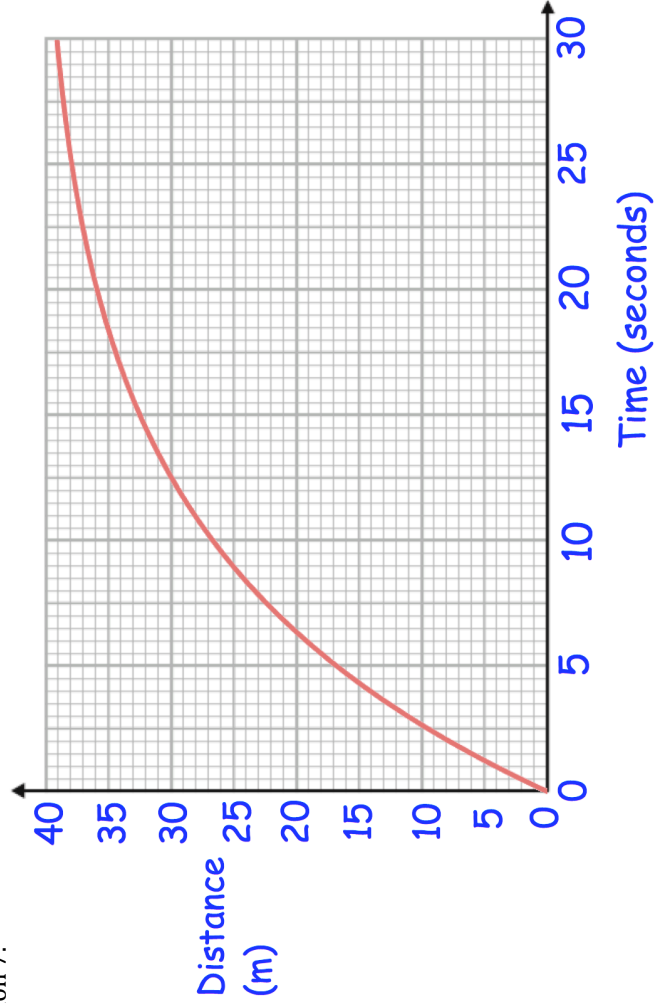


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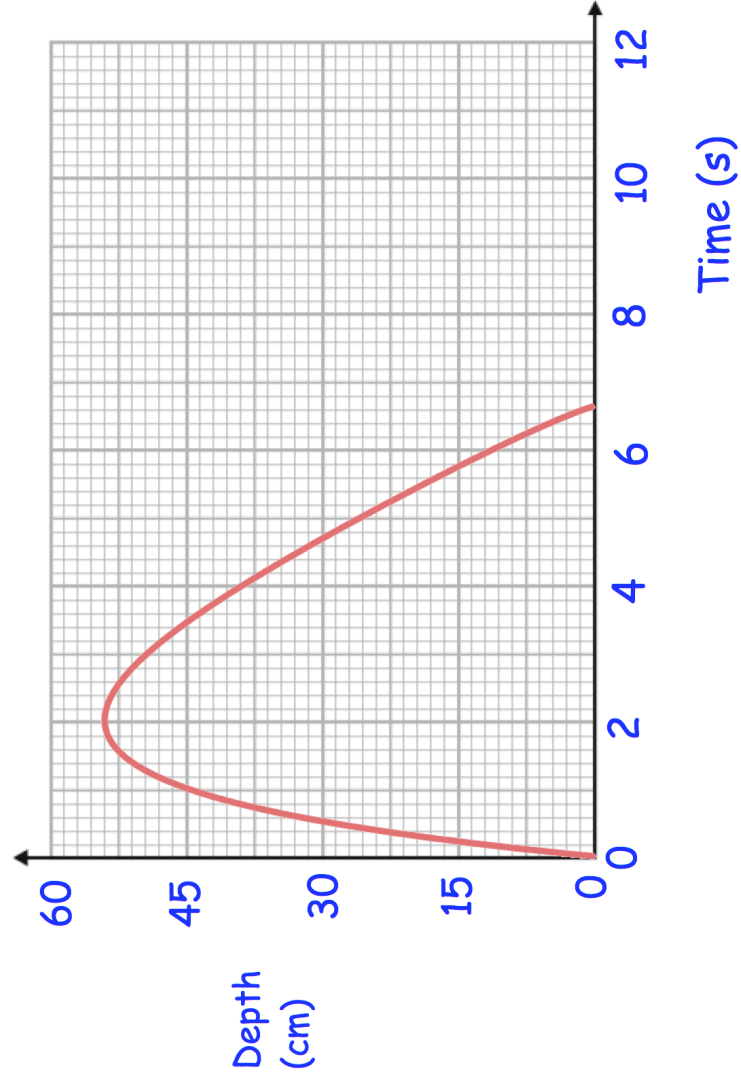
## Rates of Change

Videos 390a and 390b on [www.corbettmaths.com](http://www.corbettmaths.com)

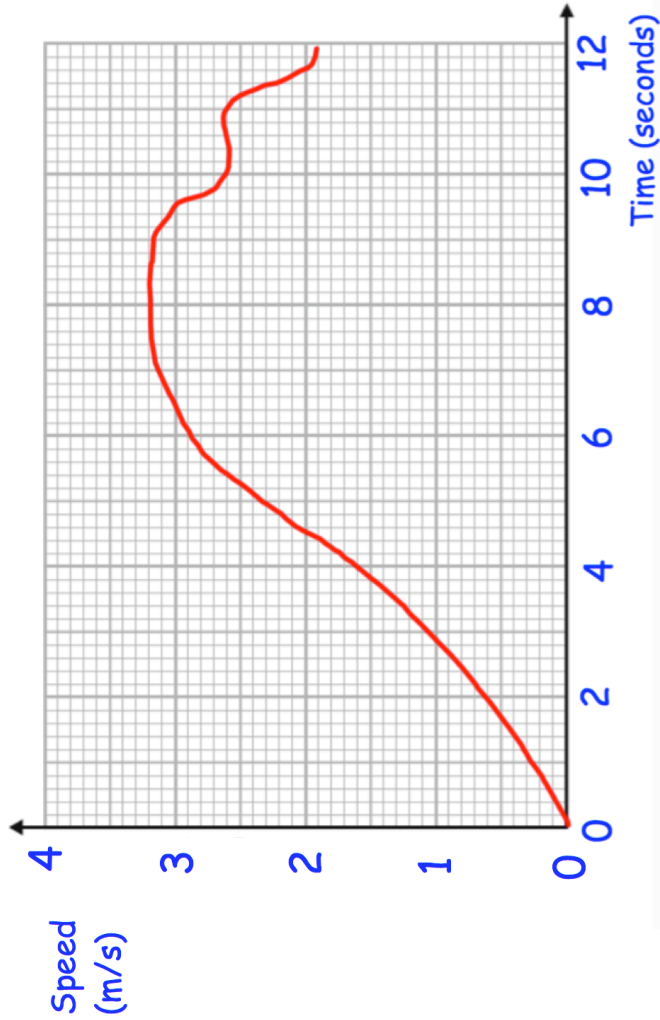
Question 7:



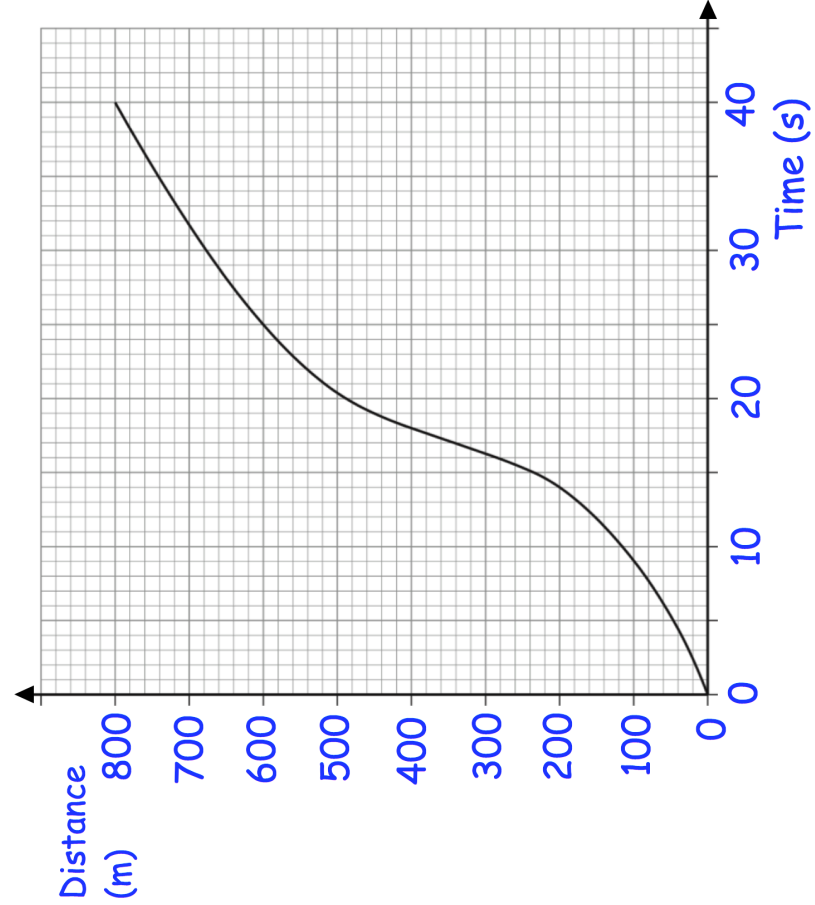
Question 8:



Question 9:



## Templates



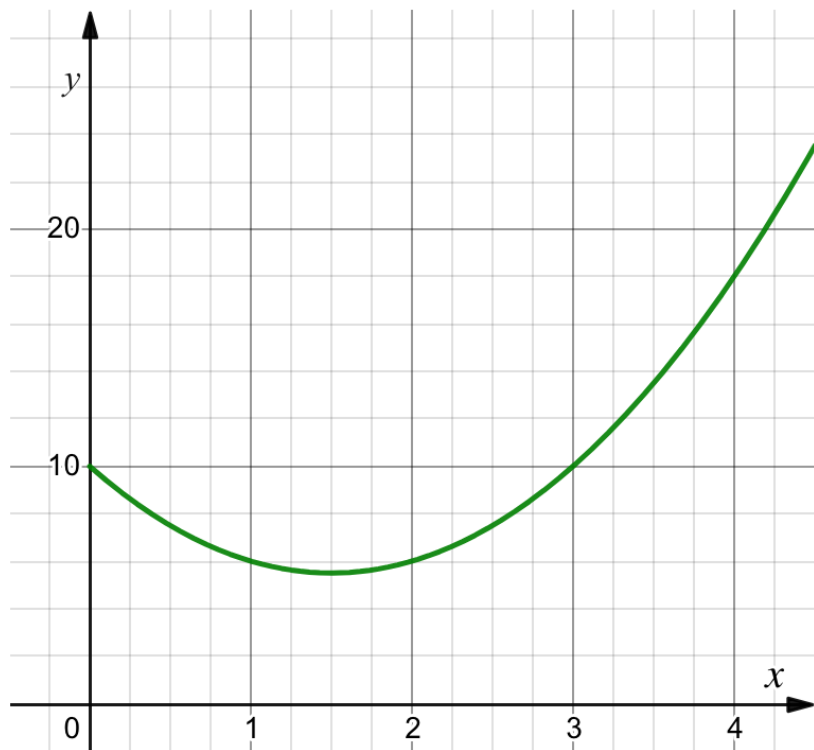
Apply 1:

## Area under Curves

### Worked Example

Work out an estimate of the area between the curve, the axes and  $x = 4$ .

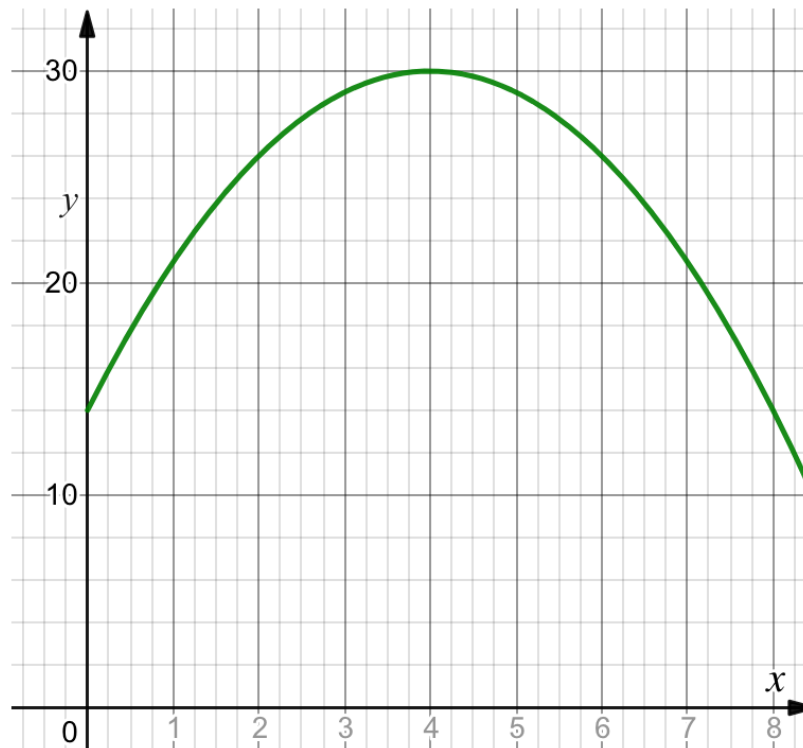
Use 4 strips of equal width.



### Your Turn

Work out an estimate of the area between the curve, the axes and  $x = 8$ .

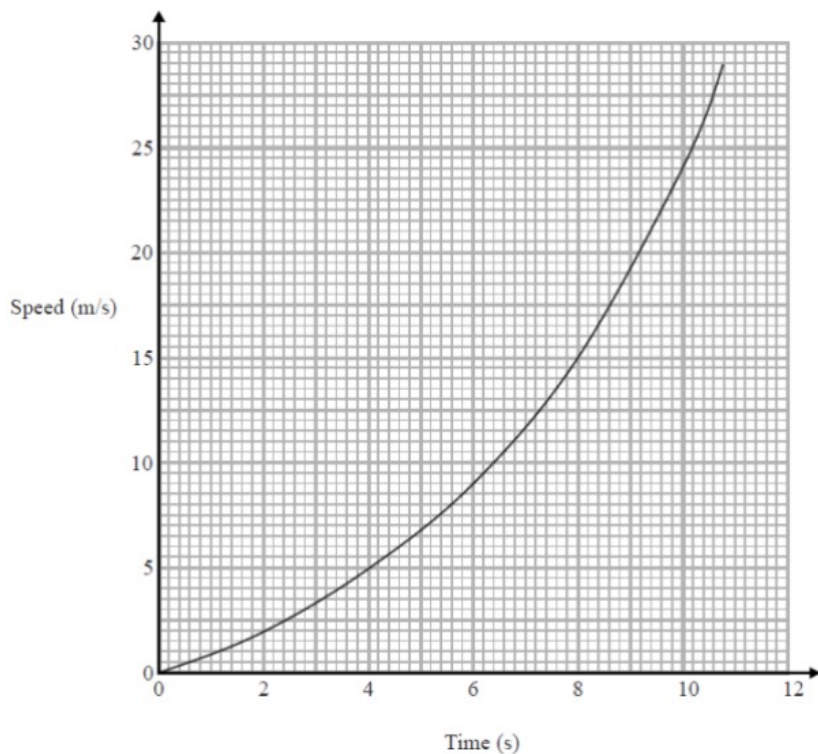
Use 4 strips of equal width.



## Worked Example

Below is a speed-time graph showing the speed, in metres per second, of an object  $t$  seconds after it started to move.

- a) Use 5 strips of equal width to find an estimate for the distance travelled in the first 10 seconds.

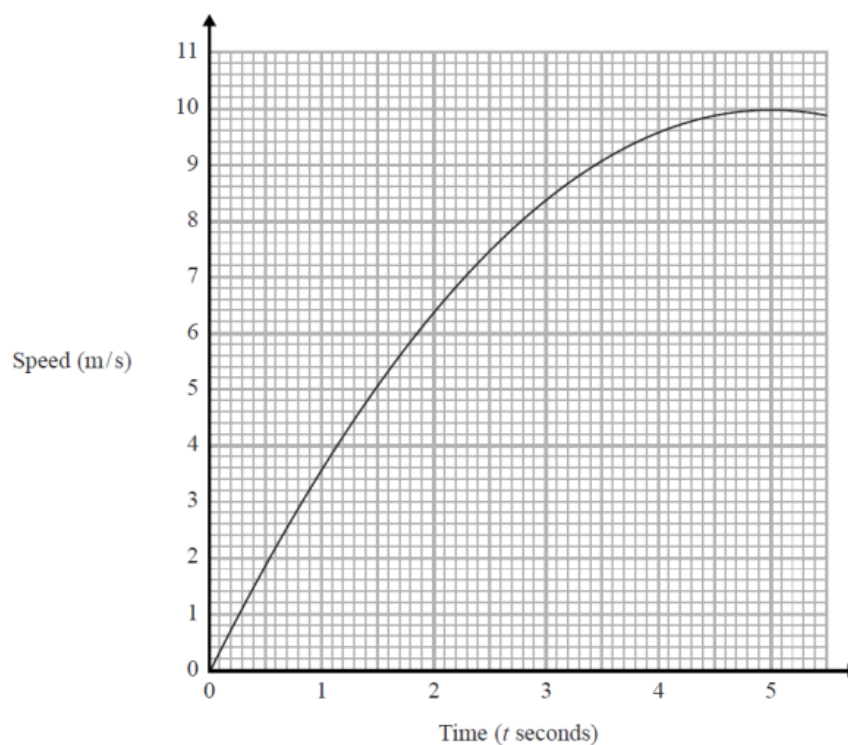


- b) Is your estimate an overestimate or underestimate?

## Your Turn

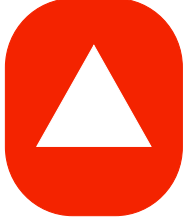
Below is a speed-time graph showing the speed, in metres per second, of an object  $t$  seconds after it started to move.

- a) Use 3 strips of equal width to find an estimate for the distance travelled between  $t = 1$  and  $t = 4$ .



- b) Is your estimate an overestimate or underestimate?

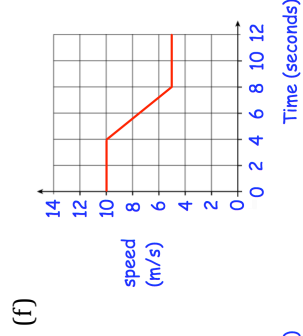
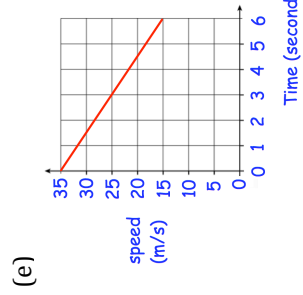
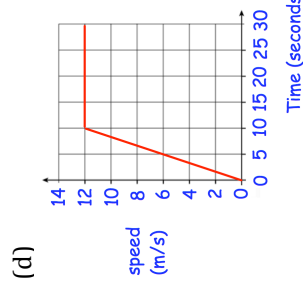
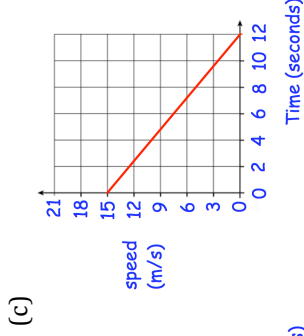
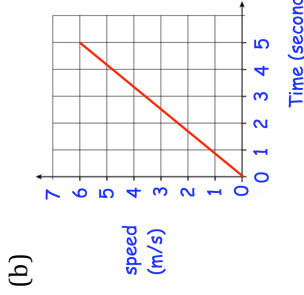
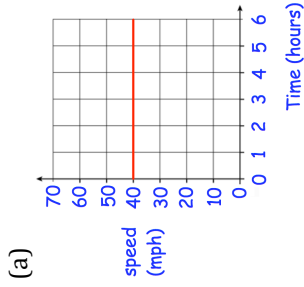




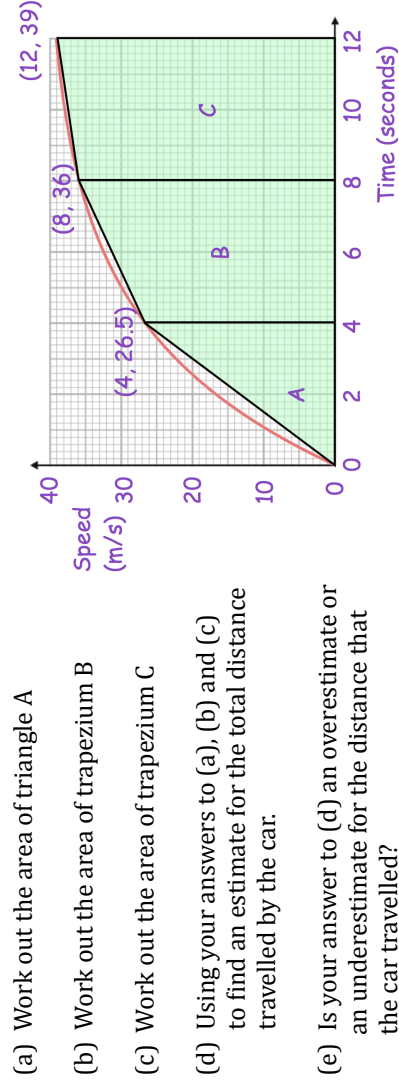
Click here

Scan here

Question 1: Shown below are speed-time graphs for some journeys.  
For each journey, calculate the total distance travelled.



Question 2: Here is the speed-time graph for a car's journey.



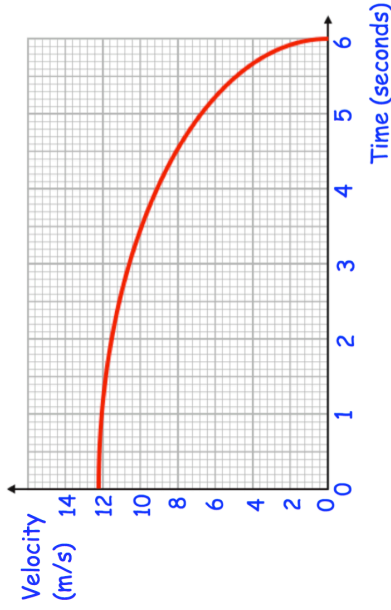


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## Area under a Graph

Video 389 on [www.corbettmaths.com](http://www.corbettmaths.com)

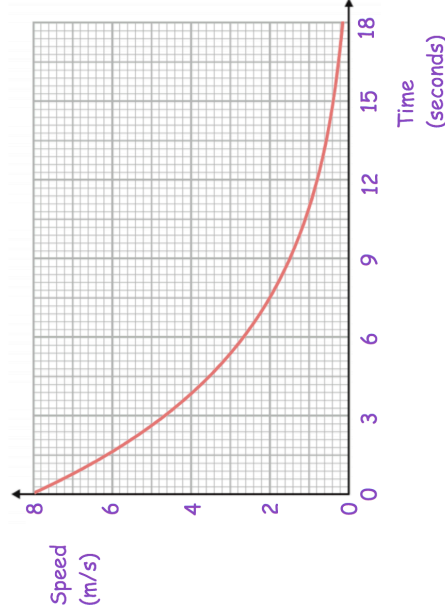
Question 3: Here is a velocity-time graph for 6 seconds of a journey.



(a) Work out an estimate for the distance travelled over 6 seconds. Use 3 strips of equal width.

(b) Is your answer to (a) an overestimate or an underestimate of the actual distance travelled?

Question 4: Here is a speed-time graph for a remote-controlled car



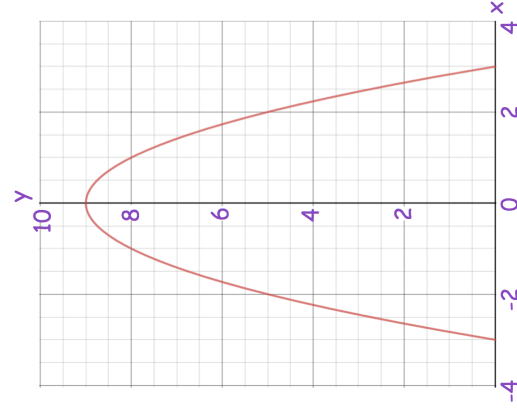
(a) Work out an estimate for the distance travelled over the first 12 seconds of the journey. Use 4 strips of equal width.

(b) Is your answer to (a) an overestimate or an underestimate of the actual distance travelled?

Question 5: Here is a sketch of  $y = 9 - x^2$

The graph is used to model the cross section of a tunnel.

Calculate an estimate of the area under the graph.





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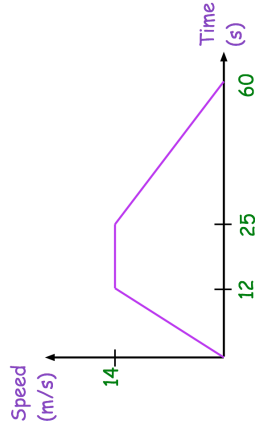
## Area under a Graph

Video 389 on [www.corbettmaths.com](http://www.corbettmaths.com)

Apply

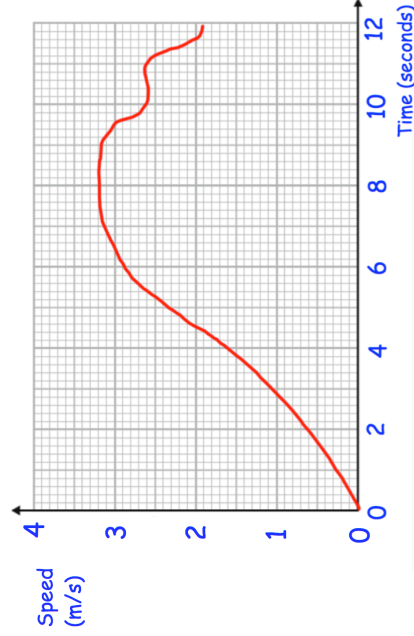
# Fluency Practice

Question 1: The graph shows the speed of a bicycle between two houses.



Calculate the distance between the houses.

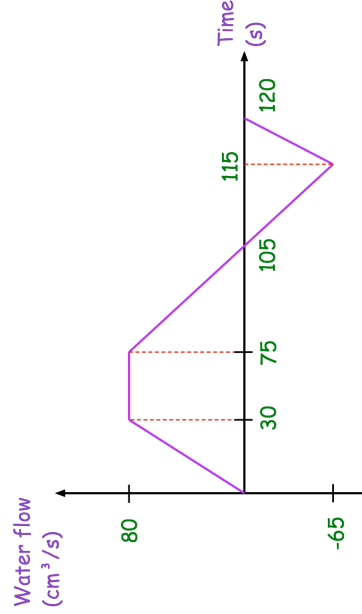
Question 2: Here is a speed-time graph for the first 12 seconds of a journey.



Calculate an estimate for the total distance travelled over the 12 seconds.

Question 3: The graph below shows information on how an empty container is being filled with water.

How much water is in the container after 120 seconds?

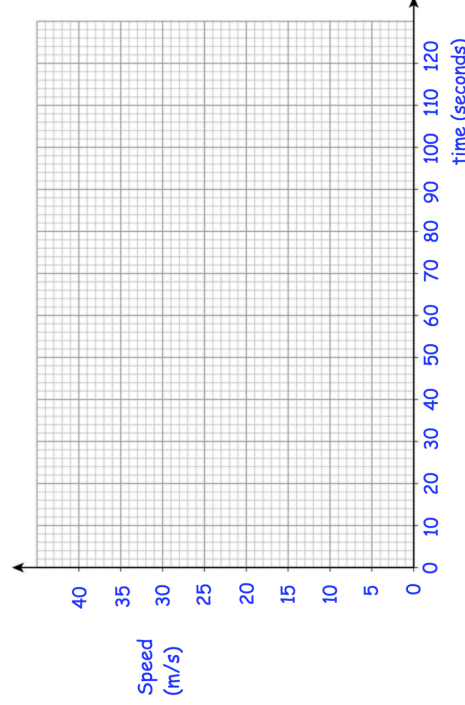


## Fluency Practice

Question 4: Finn is driving his car in a straight line.

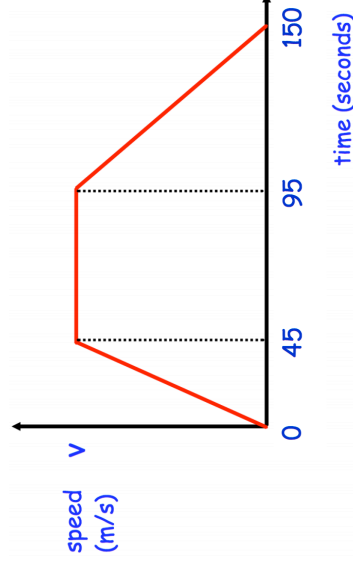
The car begins at rest.  
He accelerates uniformly at  $2\text{m/s}^2$  for 15 seconds.  
Finn drives at the same speed for the next 25 seconds.  
He then accelerates uniformly to a speed of  $40\text{m/s}$  by 80 seconds.  
The remainder of the 2 minute journey is spent decelerating to rest.

(a) Draw a speed-time graph for his journey.



(b) Write down the average speed for the total journey.

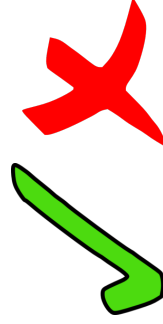
Question 5: Here is a speed-time graph for a train journey.



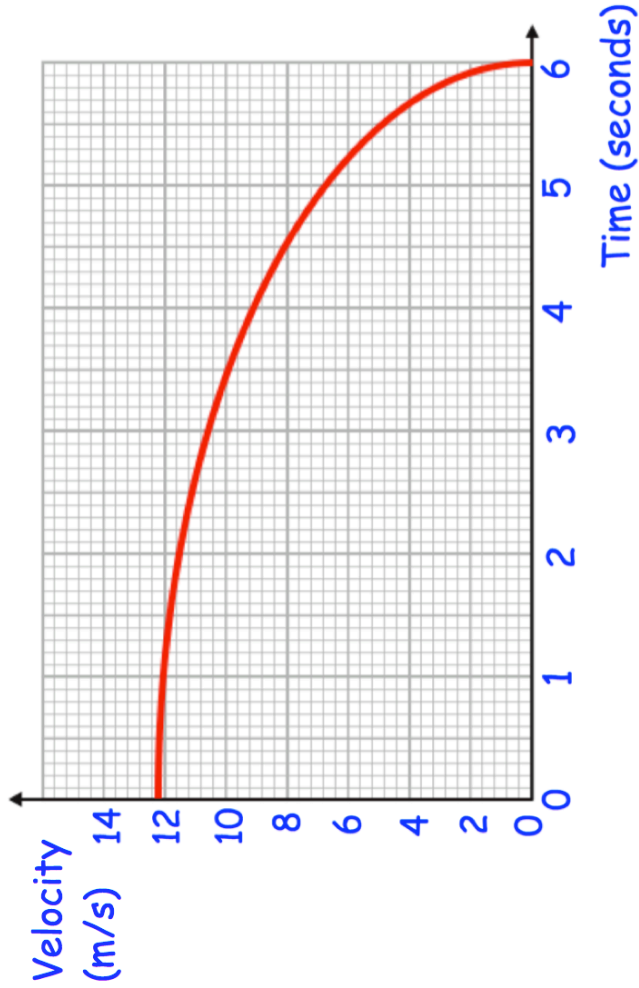
The journey took 150 seconds.  
The train travelled 1.53km in the 150 seconds.

Work out the value of  $v$ .

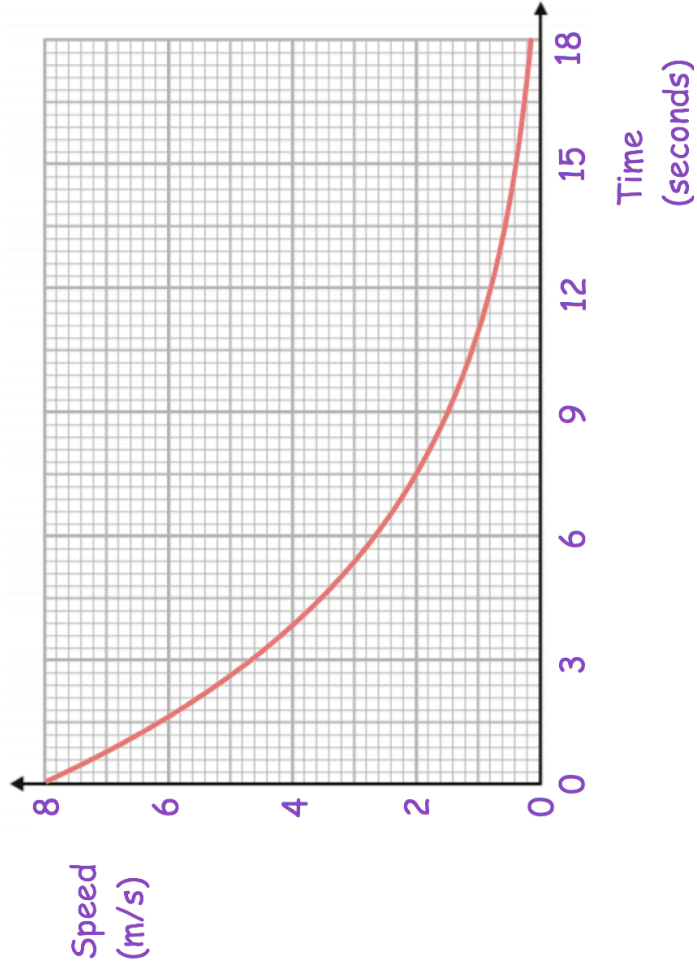
Answers



Question 3:



Question 4:

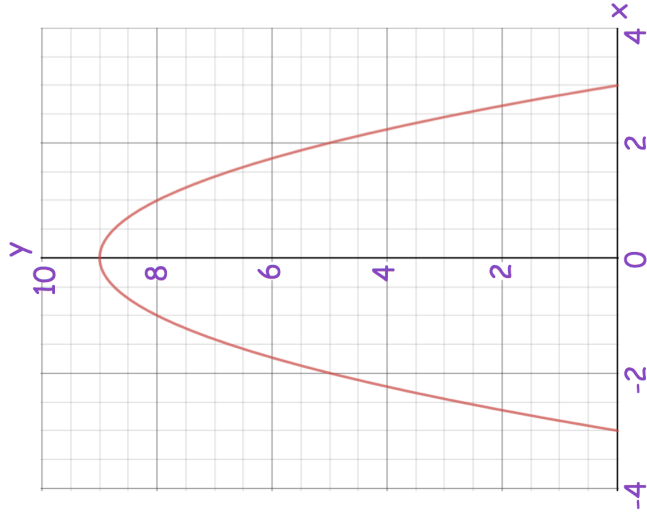




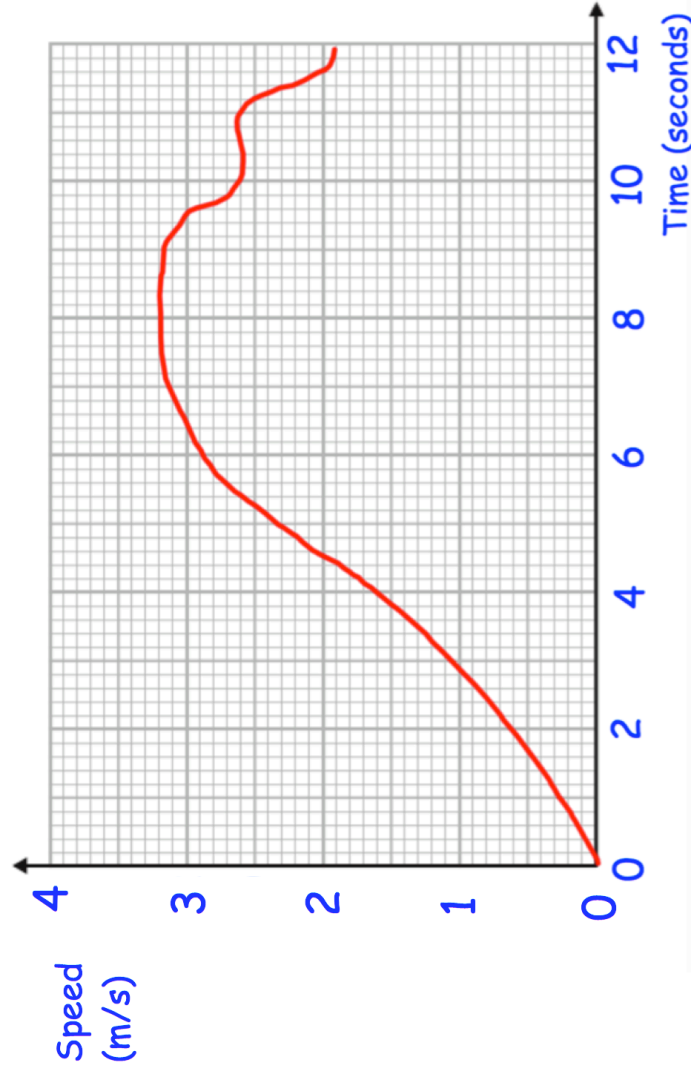
## Area under a Graph

Video 389 on [www.corbettmaths.com](http://www.corbettmaths.com)

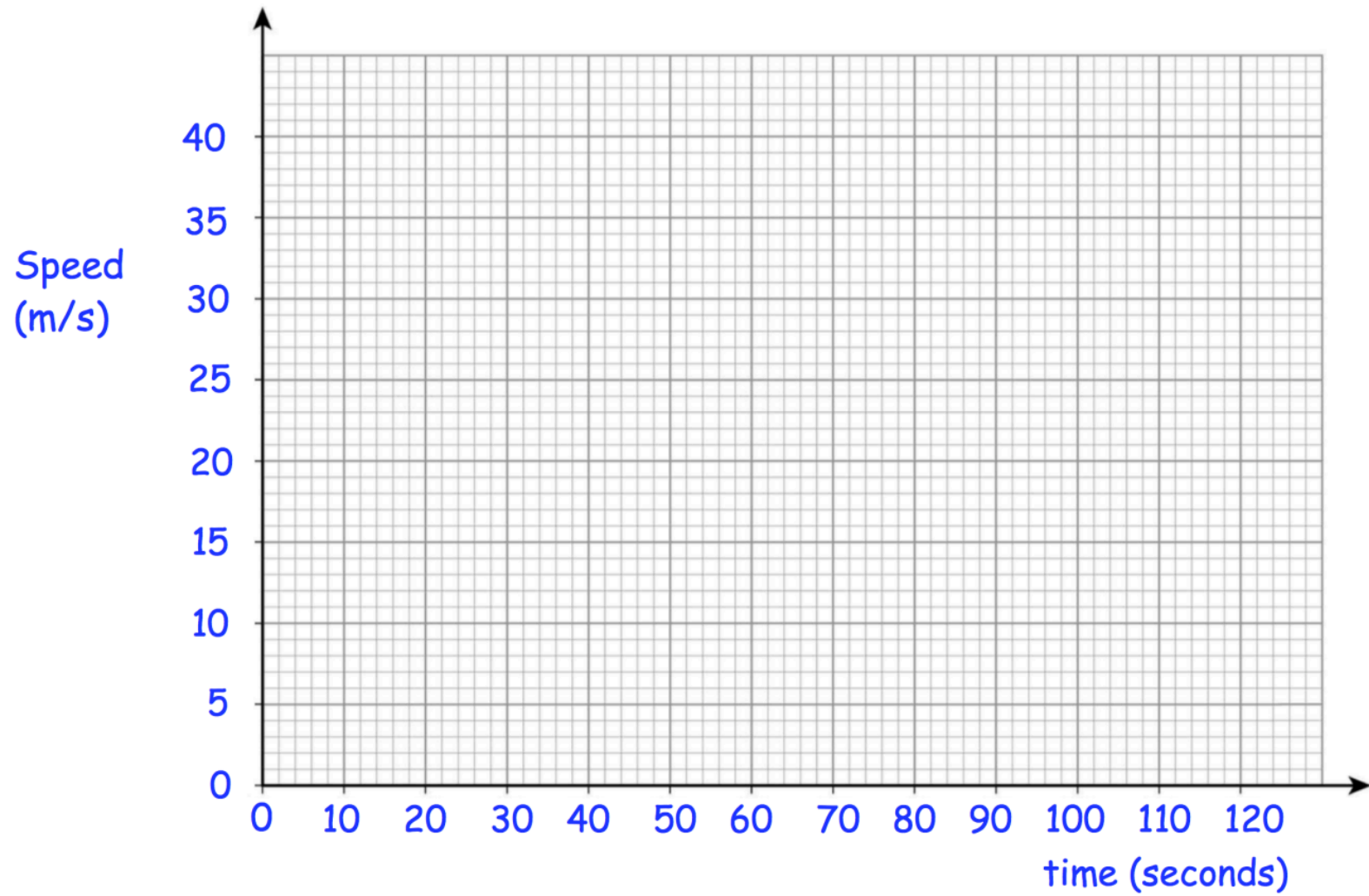
Question 5:



Apply Question 2:



## Templates



## Extra Notes



## 7 Iterations

**Worked Example**

Show that the equation  $3 + 5x - 2x^2 - x^3 = 0$  has a solution between  $-4$  and  $-3$ .

**Your Turn**

Show that the equation  $x^3 + 4x^2 - x - 5 = 0$  has a solution between 1 and 2.

### Worked Example

Show that the equation  $\sqrt{2x+4} - 3 = 0$  has a solution between 2 and 3.

### Your Turn

Show that the equation  $\frac{3}{\sqrt{2+x}} - 4 = 0$  has a solution between  $-1.5$  and  $-1$ .

**Worked Example**

Find the solution to the equation  $x^3 + x^2 - 0.5 = 0$  between 0 and 1, to 1 decimal place.

**Your Turn**

Find the solution to the equation  $x^3 - 5x - 1 = 0$  between 2 and 3, to 1 decimal place.

| Worked Example                                                                                                                                                                                                                                                         | Your Turn                                                                                                                                                                                                                                                                                         |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>An approximate solution to the equation <math>x^3 - 2x^2 - x - 1 = 0</math> can be found using the iteration formula <math>x_{n+1} = \sqrt[3]{2x_n^2 + x_n + 1}</math></p> <p>Find the value of <math>x_6</math>. Give your answer correct to 3 decimal places.</p> | <p>The equation <math>3x^3 + 8x^2 + 7x + 2 = 0</math> can be rearranged to give the iterative formula <math>x_{n+1} = \frac{-3(x_n)^3 - 8(x_n)^2 - 2}{7}</math></p> <p>Starting with <math>x_0 = -2</math>, find the value of <math>x_5</math>. Give your answer correct to 3 decimal places.</p> |

### Worked Example

Use the iterative formula  $x_{n+1} = \sqrt{3 - x_n}$  to find a solution to the equation  $x^2 + x - 3 = 0$  to 3 decimal places. Starting with  $x_0 = 1$ .

### Your Turn

Use the iterative formula  $x_{n+1} = \frac{1-4x_n^3}{3}$  to find a solution to the equation  $4x^3 + 3x - 1 = 0$  to 2 decimal places. Starting with  $x_0 = 0$ .

**Worked Example**

Show that  $x^3 - 2x^2 - x - 1 = 0$  can be rearranged to  
 $x = \sqrt[3]{2x^2 + x + 1}$

**Your Turn**

Show that  $x^3 - 2x + 5 = 0$  can be rearranged to give  
 $x = \sqrt[3]{2x - 5}$

## Extra Notes