KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 23 Tasks – Part 1

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Unit 23 Tasks – Part 2

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Unit 23 Tasks – Part 3

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1 Completing the Square

Fluency Practice

Express each of the following expressions in the form $(x+a)^2$.

1. x^2+6x+9
2. a^2+4a+4
3. $p^2-10p+25$
4. $s^2-12s+36$
5. $x^2-5x+\frac{25}{4}$
6. $b^2+3b+\frac{9}{4}$
7. $x^2+9x+\frac{81}{4}$
8. $a^2-a+\frac{1}{4}$
9. $x^2-\frac{1}{2}x+\frac{1}{16}$
10. $x^2+8x+16$
11. $x^2+x+\frac{1}{4}$
12. $x^2+\frac{2}{3}x+\frac{1}{9}$
13. $p^2+18p+81$
14. $a^2-\frac{4}{5}a+\frac{4}{25}$
15. $t^2-\frac{3}{2}t+\frac{9}{16}$
16. $x^2+2bx+b^2$
17. $x^2-2cx+c^2$
18. $x^2+\frac{b}{a}x+\frac{b^2}{4a^2}$

Express the following expressions in the form $(ax+b)^2$.

19. $9x^2+6x+1$
20. $4x^2-12x+9$
21. $100x^2-60x+9$
22. $9x^2-24x+16$
23. $4x^2-4x+1$
24. $25x^2+20x+4$
25. $9x^2-6x+1$
26. $4x^2+2x+\frac{1}{4}$
27. $\frac{9x^2}{4}+2x+\frac{4}{9}$

Intelligent Practice

Complete the square on the following expressions:

1) $x^2 + 4x$

2) $x^2 + 8x$

3) $x^2 + 12x$

4) $x^2 - 4x$

5) $x^2 - 8x$

6) $x^2 - 12x$

7) $x^2 + 3x$

8) $x^2 + 5x$

9) $x^2 - 7x$

The missing values a, b, c, d, e and f are the digits 1, 2, 3, 4, 5 and 6 in some order (no repeats!)

$$x^2 + \square^{a}\, x + \square^{b}\square^{c}$$

$$= (x + \square^{d})^{\square^{e}} + \square^{f}$$

Complete the square.

(a) $x^2 + 10x$

(b) $x^2 + 8x$

(c) $x^2 - 8x$

(d) $x^2 - 6x + 3$

(e) $x^2 - 6x - 3$

(f) $x^2 + 4x - 3$

Write in completed square format.

(a) $x^2 - 5x$

(b) $x^2 + 7x$

(c) $x^2 + 7x - 5$

(d) $x^2 - 9x - 5$

(e) $x^2 - 9x + 5$

(f) $x^2 - 9x + 13$

Complete the square.

(a) $2x^2 + 12x$

(b) $2x^2 + 8x$

(c) $2x^2 - 8x$

(d) $2x^2 - 8x + 3$

(e) $2x^2 - 6x + 3$

(f) $3x^2 - 6x + 3$

Write in completed square format.

(a) $3x^2 - 6x + 5$

(b) $3x^2 + 6x + 5$

(c) $4x^2 + 16x + 5$

(d) $4x^2 - 16x - 5$

(e) $5x^2 - 30x + 9$

(f) $5x^2 + 30x - 9$

Fluency Practice

A1 Express $x^2 + 8x$ in the form $(x + a)^2 + b$	**A2** Express $x^2 + 6x$ in the form $(x + a)^2 + b$	**A3** Express $x^2 - 4x$ in the form $(x + a)^2 + b$	**A4** Express $x^2 - 12x$ in the form $(x + a)^2 + b$
B1 Express $x^2 + 3x$ in the form $(x + p)^2 + q$	**B2** Express $x^2 - 9x$ in the form $(x + p)^2 + q$	**B3** Express $x^2 - 7x$ in the form $(x + p)^2 + q$	**B4** Express $x^2 + 11x$ in the form $(x + p)^2 + q$
C1 Express $x^2 + 6x + 5$ in the form $(x + m)^2 + n$	**C2** Express $x^2 - 6x + 5$ in the form $(x + m)^2 + n$	**C3** Express $x^2 + 10x - 11$ in the form $(x + m)^2 + n$	**C4** Express $x^2 - 4x + 20$ in the form $(x + m)^2 + n$
D1 Express $2x^2 + 8x$ in the form $a(x + b)^2 + c$	**D2** Express $3x^2 - 9x$ in the form $a(x + b)^2 + c$	**D3** Express $3x^2 + 12x - 6$ in the form $a(x + b)^2 + c$	**D4** Express $2x^2 - 6x + 11$ in the form $a(x + b)^2 + c$

Fluency Practice

A1 Express $x^2 - 2x$ in the form $(x + a)^2 + b$	**A2** Express $x^2 + 14x$ in the form $(x + h)^2 + k$	**A3** Express $x^2 + 5x$ in the form $(x + m)^2 + n$	**A4** Express $x^2 - 13x$ in the form $(x + p)^2 + q$
B1 Express $x^2 + 18x$ in the form $(x + p)^2 + q$	**B2** Express $x^2 + 21x$ in the form $(x + m)^2 + n$	**B3** Express $x^2 + 10x$ in the form $(x + h)^2 + k$	**B4** Express $x^2 - x$ in the form $(x + a)^2 + b$
C1 Express $x^2 - 4x + 20$ in the form $(x + h)^2 + k$	**C2** Express $x^2 - 3x - 10$ in the form $(x + p)^2 + q$	**C3** Express $x^2 + 14x + 6$ in the form $(x + a)^2 + b$	**C4** Express $x^2 + 9x + 8$ in the form $(x + m)^2 + n$
D1 Express $3x^2 + 12x$ in the form $a(x + b)^2 + c$	**D2** Express $5x^2 + 10x + 20$ in the form $a(x + b)^2 + c$	**D3** Express $2x^2 - 6x + 5$ in the form $a(x + b)^2 + c$	**D4** Express $2x^2 + 9x - 3$ in the form $a(x + b)^2 + c$

Fluency Practice

Solve the equations:

1. $(x+1)^2 = 9$
2. $(x-2)^2 = 16$
3. $(x-3)^2 = 25$
4. $(x+6)^2 = 100$
5. $(x+7)^2 = 1$
6. $(x-1)^2 = 25$
7. $(x+2)^2 = 49$
8. $(x-5)^2 = 16$
9. $(x-7)^2 = 4$
10. $(x+4)^2 = 16$
11. $(x+3)^2 = 25$
12. $(x-9)^2 = 36$
13. $(x+1)^2 = \frac{1}{4}$
14. $(x-2)^2 = \frac{9}{4}$
15. $(x-\frac{1}{2})^2 = \frac{25}{4}$

Solve the equations:

16. $(2x-1)^2 = 16$
17. $(3x+2)^2 = 25$
18. $(5x-1)^2 = 36$
19. $(3x-4)^2 = 1$
20. $(7x+2)^2 = 100$
21. $(2x+1)^2 = 36$
22. $(3x-4)^2 = 49$
23. $(5x+2)^2 = 25$
24. $(4x-3)^2 = 1$
25. $(9x-5)^2 = 4$
26. $(5x+3)^2 = 16$
27. $(7x-5)^2 = 81$

Fluency Practice

Solve the following equations by completing the square.

1. $x^2+4x=5$
2. $x^2-6x=7$
3. $x^2+10x=11$
4. $x^2+8x+3=0$
5. $x^2-4x+1=0$
6. $x^2+8x=3$
7. $x^2-4x=9$
8. $x^2+9x+4=0$
9. $x^2-7x+5=0$
10. $x^2-x-4=0$
11. $x^2+9x-3=0$
12. $x^2+8x+4=0$

Solve the following equations by completing the square.

13. $2x^2+6x=9$
14. $6x^2-12x=5$
15. $4x^2+8x=3$
16. $2x^2-3x-4=0$
17. $3x^2+12x-8=0$
18. $3x^2-5x=1$
19. $5x^2-5x=4$
20. $5x^2+8x+2=0$
21. $4x^2-7x-3=0$
22. $6x^2-5x-1=0$
23. $7x^2+7x-4=0$
24. $3x^2-9x=2$

Question 1: Write the following expressions in the form $(x + a)^2 + b$

(a) $x^2 + 8x + 1$ (b) $x^2 + 10x + 3$ (c) $x^2 + 2x - 1$

(d) $x^2 - 6x - 10$ (e) $x^2 - 4x - 13$ (f) $x^2 - 12x + 3$

(g) $x^2 + 14x + 3$ (h) $x^2 - 2x - 15$ (i) $x^2 + 4x - 11$

(j) $x^2 + x - 8$ (k) $x^2 + 3x + 1$ (l) $x^2 - 7x - 2$

(m) $x^2 - 9x - 1$ (n) $x^2 + 11x + 3$ (o) $x^2 - 100x - 25$

Question 2: Solve the following equations (use completing the square).

(a) $x^2 + 4x + 1 = 0$ (b) $x^2 + 8x - 10 = 0$ (c) $x^2 + 14x - 4 = 0$

(d) $x^2 - 8x - 2 = 0$ (e) $x^2 - 10x + 10 = 0$ (f) $x^2 + 18x + 7 = 0$

(g) $x^2 + 12x + 3 = 19$ (h) $x^2 = 2x + 10$ (i) $x^2 - 7x - 3 = 0$

(j) $x^2 + x - 7 = 0$ (k) $x^2 + 3x + 8 = 0$ (l) $2x^2 - 10x - 30 = x^2 - 4x$

Question 3: Write the following expressions in the form $a(x + b)^2 + c$

(a) $2x^2 + 8x + 2$ (b) $2x^2 + 12x - 3$ (c) $3x^2 - 12x + 2$

(d) $4x^2 + 12x - 5$ (e) $2x^2 - 3x - 5$ (f) $5x^2 - 20x + 30$

Question 4: Solve the following equations (use completing the square).

(a) $3x^2 + 12x + 3 = 0$ (b) $2x^2 + 16x - 20 = 0$ (c) $3x^2 - 6x + 1 = 0$

(d) $5x^2 + 10x - 9 = 0$ (e) $2x^2 - 5x - 3 = 0$ (f) $2x^2 - 7x + 1 = 0$

Question 1: Solve each of the equations below using completing the square

(a) $x^2 + 6x + 8 = 0$ (b) $x^2 + 10x + 24 = 0$ (c) $x^2 + 14x + 40 = 0$

(d) $x^2 - 4x - 45 = 0$ (e) $x^2 - 12x + 35 = 0$ (f) $x^2 - 2x - 3 = 0$

(g) $x^2 + 14x - 51 = 0$ (h) $x^2 - 6x - 16 = 0$ (i) $x^2 - 2x + 1 = 0$

Question 2: Solve each of the following equations using completing the square

(a) $x^2 + 5x + 4 = 0$ (b) $x^2 - 3x - 18 = 0$ (c) $x^2 + x - 12 = 0$

(d) $x^2 - 7x + 12 = 0$ (e) $x^2 - 11x + 24 = 0$ (f) $x^2 - 7x - 30 = 0$

Question 3: Solve each of the following equations using completing the square
Write each answer in simplified surd form

(a) $x^2 + 4x - 3 = 0$ (b) $x^2 + 6x - 10 = 0$ (c) $x^2 - 2x - 5 = 0$

(d) $x^2 - 10x + 1 = 0$ (e) $x^2 + 8x + 3 = 0$ (f) $x^2 - 8x - 22 = 0$

(g) $x^2 + 20x + 7 = 0$ (h) $x^2 - 12x + 1 = 0$ (i) $x^2 - 30x - 100 = 0$

Question 4: Solve each of the following equations using completing the square
Write each answer in simplified surd form

(a) $x^2 + x - 7 = 0$ (b) $x^2 - 3x + 1 = 0$ (c) $x^2 + 11x - 5 = 0$

(d) $x^2 - 7x + 9 = 0$ (e) $x^2 - x - 50 = 0$ (f) $x^2 + 13x + 1 = 0$

Question 5: Solve each of the following equations using completing the square
Write each answer in simplified surd form

(a) $x^2 - 4x + 1 = 10$ (b) $x^2 + 2x + 5 = 20 - 8x$ (c) $3x^2 = 2x^2 + x + 7$

(d) $x^2 + 6x + 4 = 2x^2 + 8x - 1$ (e) $\frac{12}{x} = 10 + x$ (f) $\frac{x-1}{2} = \frac{5}{x+3}$

Question 6: Solve each of the following equations using completing the square

(a) $5x^2 + 30x - 10 = 0$ (b) $2x^2 + 7x + 3 = 0$ (c) $3x^2 + 12x - 2 = 0$

(d) $2x^2 - 3x - 7 = 0$ (e) $5x^2 + 2x - 8 = 0$ (f) $10x^2 - 2x - 1 = 0$

Apply

Question 1: Find the points where the curve $y = x^2 + 10x + 3$ meets:
(a) the y-axis
(b) the x-axis

Question 2: The length of a rectangle is 20cm longer than its width.
The area of the rectangle is $1000cm^2$
Find the width and length of the rectangle.
Give your answers in surd form.

Question 3: Abby is trying to solve $x^2 + 4x + 15 = 0$
By using completing the square, explain why there are no (real) solutions

Question 4: The curve $y = x^2 + 8x - 1$ meets the x-axis at the points A and B
The point C is (2, 5)
Find the area of triangle ABC

Question 5: James has solved the equation $x^2 + ax + b = 0$
His solutions are $x = -3 + \sqrt{17}$ and $x = -3 - \sqrt{17}$
Find a and b

Question 6: By using completing the square on $ax^2 + bx + c = 0$ to establish the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve these quadratic equations by completing the square.

(a) $x^2 - 2x - 1 = 0$

(b) $x^2 - 4x - 1 = 0$

(c) $x^2 + 4x + 2 = 0$

(d) $x^2 + 4x - 2 = 0$

(e) $x^2 + 10x + 2 = 0$

(f) $x^2 + 12x - 5 = 0$

Solve these quadratic equations by completing the square.

(a) $x^2 + 11x - 1 = 0$

(b) $x^2 + 11x - 3 = 0$

(c) $x^2 + 5x - 3 = 0$

(d) $x^2 - 5x - 3 = 0$

(e) $x^2 - x - 3 = 0$

Solve these quadratic equations by completing the square.

(a) $2x^2 - 8x + 3 = 0$

(b) $2x^2 - 4x - 3 = 0$

(c) $3x^2 + 12x + 3 = 0$

(d) $3x^2 - 18x - 2 = 0$

(e) $4x^2 + 16x - 2 = 0$

(f) $5x^2 + 20x - 5 = 0$

Solve these quadratic equations by completing the square.

(a) $2x^2 + 3x - 1 = 0$

(b) $2x^2 + 5x - 1 = 0$

(c) $2x^2 - 5x - 3 = 0$

(d) $2x^2 - 11x - 3 = 0$

Fluency Practice

Create 14 groups of 3 by matching up one card from each table

Quadratic Equations	
$x^2 + 6x + 7 = 0$	$x^2 + 14x + 21 = 0$
$x^2 - 10x + 19 = 0$	$x^2 - 10x - 3 = 0$
$x^2 - 4x + 2 = 0$	$x^2 - 2x - 5 = 0$
$x^2 - 3x + 1 = 0$	$x^2 + 4x - 6 = 0$
$x^2 + 5x - 8 = 0$	$x^2 + 8x + 6 = 0$
$x^2 - 20x + 53 = 0$	$x^2 - 2x - 9 = 0$
$x^2 + 6x + 3 = 0$	$x^2 + 18x - 2 = 0$

Exact Answers	
$-4 \pm \sqrt{10}$	$\frac{3}{2} \pm \frac{\sqrt{5}}{2}$
$2 \pm \sqrt{2}$	$10 \pm \sqrt{47}$
$-9 \pm \sqrt{83}$	$-7 \pm 2\sqrt{7}$
$-\frac{5}{2} \pm \frac{\sqrt{57}}{2}$	$-2 \pm \sqrt{10}$
$-3 \pm \sqrt{6}$	$-3 \pm \sqrt{2}$
$1 \pm \sqrt{10}$	$5 \pm 2\sqrt{7}$
$5 \pm \sqrt{6}$	$1 \pm \sqrt{6}$

Rounded Answers (2 d.p.)	
1.27 −6.27	16.86 3.14
10.29 −0.29	0.11 −18.11
−1.71 −12.29	7.45 2.55
−0.84 −7.16	1.16 −5.16
2.62 0.38	−0.55 −5.45
3.45 −1.45	3.41 0.59
4.16 −2.16	−1.59 −4.41

Fluency Practice

(a)	(b)	(c)	(d)
Solve $x^2 = 9$	Solve $x^2 - 7x + 10 = 0$	Solve $x^2 - 5x + 6 = 0$	Solve $x^2 + 8x + 12 = 0$
(e)	**(f)**	**(g)**	**(h)**
Solve $x^2 + 2x - 8 = 0$	Solve $x^2 + 10x + 21 = 0$	Solve $x^2 - 3x - 18 = 0$	Solve $x^2 - 1 = x + 5$
(i)	**(j)**	**(k)**	**(l)**
Solve $3x^2 - 7x + 2 = 0$	Solve $x^2 + 5x + 2 = 0$, giving your solutions to 3 significant figures.	Solve $x^2 + 3x - 8 = 0$, giving your solutions to 3 significant figures.	Solve $3x^2 + 2x - 9 = 0$, giving your solutions in surd form.

Fluency Practice

There are three algebraic methods for solving quadratic equations: (a) By factorising (b) Using the quadratic formula (c) By completing the square

Solve each of the following quadratic equations using each of the three methods, remembering that sometimes it is not possible to solve by factorising. When using the quadratic formula, give your answers to 2 decimal places.

Equation	By Factorising	By Formula	By Completing the Square	Equation	By Factorising	By Formula	By Completing the Square
$x^2 + 4x + 3 = 0$	$(x+3)(x+1) = 0$ $x = -3$ $or\ x = -1$	$a = 1, b = 4, c = 3$ $x = \frac{-4 \pm \sqrt{16 - 12}}{2}$ $x = -3\ or\ x = -1$	$(x+2)^2 - 4 + 3 = 0$ $(x+2)^2 = 1$ $x = -2 \pm \sqrt{1}$ $x = -3\ or\ x = -1$	$3x^2 - 12x = 0$			
$x^2 - 6x + 8 = 0$				$2x^2 - 9x - 5 = 0$			
$x^2 - x - 12 = 0$				$2x^2 + 8x - 1 = 0$			
$x^2 + 4x - 2 = 0$				$x^2 + 3x = 18$			
$x^2 + 2x - 5 = 0$				$2x^2 = 4x + 1$			

Fluency Practice

Complete the square then solve…

1. $x^2 + 6x + 7 = 0$
2. $x^2 + 10x + 25 = 0$
3. $2x^2 + 8x + 5 = 0$
4. $2x^2 + 6x + 6 = 5$
5. $x(2x + 5) + 10 = 3$

Factorise and solve…

1. $x^2 + 7x + 12 = 0$
2. $3x^2 + 10x + 7 = 0$
3. $4x^2 - 17x - 15 = 0$
4. $100x^2 + 60x + 10 = 1$
5. $x(15x - 4) = 4$

Quadratic formula to solve..

1. $x^2 + 6x - 5 = 0$
2. $4x^2 - 12x + 9 = 0$
3. $3x^2 + 9x + 6 = 0$
4. $x(x - 2) = 2$
5. $2x^2 - 4x + 5 = 2$

Problem Solving

In each row, explain which equation is the odd one out.

$(x+5)^2 + 10 = 0$	$x^2 + 10x + 35 = 0$	$x^2 + 10x + 25 = 10$	$(x+5)^2 - 25 + 35 = 0$
$5x^2 + 10x = 10$	$(x^2 + 2x) = 2$	$(x+1)^2 = 3$	$(x+1)^2 - 2 = 2$
$2x^2 + 12x - 20 = 0$	$2(x^2 + 6x) = 20$	$(x+3)^2 = 19$	$x = 3 \pm \sqrt{19}$
$6x^2 + 36x = 18$	$x^2 + 6x = 3$	$(x+3)^2 + 9 = 3$	$(x+3)^2 = 12$

*** Challenge ***

How many equations can you write that are equivalent to : $(\boldsymbol{x+3})^{\boldsymbol{2}} = \boldsymbol{30}$

Can you write some equations that are equivalent to $\boldsymbol{x^2 + 20x = 26}$?

2 Advanced Functions

Intelligent Practice

$f(x) = 2x \quad g(x) = 3x$ Evaluate: $fg(1)$ $fg(2)$ $fg(3)$ $fg(4)$ $fg(5)$	$f(x) = 3x \quad g(x) = x + 1$ Evaluate: $fg(1)$ $fg(2)$ $fg(3)$ $fg(4)$ $fg(5)$	$f(x) = 2x - 1 \;\; g(x) = 3x + 2$ Evaluate: $fg(1)$ $fg(2)$ $fg(3)$ $fg(4)$ $fg(5)$
$f(x) = 2x \quad g(x) = 3x$ Evaluate: $gf(1)$ $gf(2)$ $gf(3)$ $gf(4)$ $gf(5)$	$f(x) = 3x \quad g(x) = x + 1$ Evaluate: $gf(1)$ $gf(2)$ $gf(3)$ $gf(4)$ $gf(5)$	$f(x) = 2x - 1 \;\; g(x) = 3x + 2$ Evaluate: $gf(1)$ $gf(2)$ $gf(3)$ $gf(4)$ $gf(5)$

Describe any patterns you find in your answers.
Can you explain the patterns?

Fluency Practice

$a(x) = 3x + 1$	$b(x) = 2x - 3$	$c(x) = \frac{x}{3} + 2$	$d(x) = \frac{x}{2} - 4$	$e(x) = \frac{3x + 5}{3}$	$f(x) = \frac{2x - 3}{2}$
$g(x) = \frac{5x}{4} + 3$	$h(x) = \frac{x + 6}{3} - 1$	$m(x) = 3(x - 4)$	$n(x) = 3(2x + 3)$	$o(x) = \frac{4x - 5}{3} + 4$	$p(x) = \frac{3x + 2}{4} - 3$

Inverse Functions

$a^{-1}(10) =$

$b^{-1}(5) =$

$c^{-1}(11) =$

$d^{-1}(3) =$

$e^{-1}(4) =$

$f^{-1}(5) =$

$g^{-1}(-2) =$

$h^{-1}(0) =$

$m^{-1}(5) =$

$n^{-1}(-9) =$

$o^{-1}(7) =$

$p^{-1}(3) =$

You can check your answers by reversing the input & output.

Composite Functions

$ab(2) =$

$aa(2) =$

$ba(3) =$

$bb(4) =$

$ca(1) =$

$db(0) =$

$ac(9) =$

$bd(-2) =$

$ea(3) =$

$fd(8) =$

$ah(6) =$

$hm(-2) =$

$eo(2) =$

$op(10) =$

$f(x) = 2x + 5$ $g(x) = x^2 - 1$

Evaluate:

(a) $fg(3)$ (b) $gf(-1)$

(c) $ff(0.6)$ (d) $gg(-4)$

$f(x) = 4 - x$ $g(x) = \frac{8}{x+1}$

Evaluate:

(a) $fg(-5)$ (b) $gf(1)$

(c) $ff\left(\frac{2}{3}\right)$ (d) $gg(-3)$

$f(x) = 3x^2$ $g(x) = \frac{x}{4} - 5$

Evaluate:

(a) $fg(16)$ (b) $gf\left(\sqrt{2}\right)$

(c) $ff\left(-\frac{1}{2}\right)$ (d) $gg(0)$

$g(x) = \sqrt{x-1}$ $h(x) = 5x + 2$

Evaluate:

(a) $gh(3)$ (b) $hg(10)$

(c) $hh(-2.8)$ (d) $gg(17)$

$f(x) = x^2 + 2x + 1$

$g(x) = 20 - 3x$

$h(x) = \frac{2}{x+3}$

Evaluate:

(a) $fg(5)$ (b) $hf(-4)$

(c) $fhg(6)$ (d) $ghf(-1)$

(e) $hgh(1)$ (f) $fff(-2)$

Fluency Practice

$a(x) = 2x + 3$	$b(x) = 3x - 4$	$c(x) = \frac{x}{2} + 1$	$d(x) = \frac{x}{3} - 5$	$e(x) = \frac{2x+3}{3}$	$f(x) = \frac{3x-1}{2}$
$g(x) = \frac{2x}{3} + 2$	$h(x) = \frac{x+3}{2} - 4$	$m(x) = 2(x - 3)$	$n(x) = 2(3x + 5)$	$o(x) = \frac{3x-1}{2} + 2$	$p(x) = \frac{4x+5}{3} - 1$

Inverse Functions

$a^{-1}(x) =$ $b^{-1}(x) =$

$c^{-1}(x) =$ $d^{-1}(x) =$

$e^{-1}(x) =$ $f^{-1}(x) =$

$g^{-1}(x) =$ $h^{-1}(x) =$

$m^{-1}(x) =$ $n^{-1}(x) =$

$o^{-1}(x) =$ $p^{-1}(x) =$

You can check your answers by substituting inputs & outputs.

Composite Functions

$ab(x) =$ $aa(x) =$

$ba(x) =$ $bb(x) =$

$ca(x) =$ $db(x) =$

$ac(x) =$ $bd(x) =$

$ea(x) =$ $fd(x) =$

$cd(x) =$ $fe(x) =$

$eb(x) =$ $og(x) =$

Intelligent Practice

For each pair of functions, find:

a) $fg(2)$ b) $gf(3)$ c) $fg(x)$ d) $gf(x)$

Use your answers to parts a and b to check your answers to parts c and d.

1) $f(x) = 4x \quad g(x) = 5x$	5) $f(x) = \frac{x}{2} \quad g(x) = 4x$
2) $f(x) = x + 3 \quad g(x) = x - 4$	6) $f(x) = x^2 \quad g(x) = x + 2$
3) $f(x) = 2x \quad g(x) = x - 3$	7) $f(x) = x^2 - 1 \quad g(x) = x - 3$
4) $f(x) = 3x - 4 \quad g(x) = 2x + 1$	8) $f(x) = x^2 \quad g(x) = x^3$

For some of these pairs of functions, $fg(x) = gf(x)$

Explain why this happens for these pairs of functions.

Find an expression for $fg(x)$ for each of these functions:

(a) $f(x) = x - 1$ and $g(x) = 5 - 2x$

(b) $f(x) = 2x^2$ and $g(x) = x + 3$

Find an expression for $gf(x)$ for each of these functions:

(a) $f(x) = 2x + 1$ and $g(x) = 4x + 3$

(b) $f(x) = \frac{3}{x}$ and $g(x) = 2x - 1$

The function f is such that $f(x) = 2x - 3$

(i) Find $ff(2)$

(ii) Solve the equation $ff(x) = x$

Functions f and g are such that

$f(x) = x^2$ and $g(x) = 5 + x$

(a) Find (i) $fg(x)$ (ii) $gf(x)$

(b) Solve $fg(x) = gf(x)$

The function g is such that

$g(x) = \frac{1}{1-x}$ for $x \neq 1$

(a) Prove that $gg(x) = \frac{x-1}{x}$

(b) Find $ggg(3)$

Functions f, g and h are such that

$f(x) = 3 - x$

$g(x) = x^2 - 14$ and

$h(x) = x - 2$

Given that $f(x) = gfh(x)$, find the values of x.

Using $f(x) = 4x + 3$ and $g(x) = x - 2$, find:

1. $f(g(5))$
2. $g(f(-6))$
3. $f(f(7))$
4. $g(f(x))$

Using $f(x) = 6x^2$ and $g(x) = 14x + 4$ find:

5. $(f \circ g)(x)$
6. $(g \circ f)(x)$
7. Are these two answers the same? What does this information tell you about composition?

The notation $[x]$ means the greatest integer not exceeding the value of x.
Given $f(x) = [x]$, $g(x) = 12x$ and $h(x) = 6/x$ find:

8. $(f \circ g)(5)$
9. $(f \circ h)(x)$
10. $(h \circ f)(3)$

Using $f(x) = 5x + 4$ and $g(x) = x - 3$, find:

1. $f(g(6))$
2. $g(f(-7))$
3. $f(f(8))$
4. $g(f(x))$

Using $f(x) = 8x^2$ and $g(x) = 2x + 8$ find:

5. $(f \circ g)(x)$
6. $(f \circ g)(x)$
7. Are these two answers the same? What does this information tell you about composition?

The notation $[x]$ means the greatest integer not exceeding the value of x. Given $f(x) = [x]$, $g(x) = 15x$ and $h(x) = 8/x$ find:

8. $(f \circ g)(6)$
9. $(f \circ h)(4)$
10. $(h \circ f)(4)$

Using $f(x)=6x+2$ and $g(x)= x-5$, find:

1. $f(g(7))$
2. $g(f(3))$
3. $f(f(2))$
4. $g(g(x))$

Using $f(x)=2x^2$ and $g(x) =3x+4$ find:

5. $(g \circ f)(5)$
6. $(f \circ g)(5)$
7. Are these two answers the same? What does this information tell you about composition?

The notation $[x]$ means the greatest integer not exceeding the value of x.
Given $f(x) = [x]$, $g(x) =8x$ and $h(x) =5/x$ find:

8. $(f \circ g)\ (4)$
9. $(f \circ h)\ (2)$
10. $(h \circ f)\ (x)$

Using f(x) = 7x + 4 and g(x) = 2x - 4,find:

1. f(g(3))
2. g(f(4))
3. f(f(3))
4. g(g(5))

Using f(x) = 8x and g(x) = 4x + 2 find:

5. (g ∘ g)(x)
6. (f ∘ f)(x)
7. Are these two answers the same? What does this information tell you about composition?

The notation [x]means the greatest integer not exceeding the value of x.
Given f(x) = [x],g(x) = 4x and h(x) = 4/x find:

8. (f∘ g) (x)
9. (f∘ h) (4)
10. (h ∘f) (2)

Using $f(x) = 8x + 5$ and $g(x) = 7x - 2$, find:

1. $f(g(4))$
2. $g(f(6))$
3. $f(f(3))$
4. $g(g(2))$

Using $f(x) = 7x^2$ and $g(x) = 5x + 1$ find:

5. $(g \circ g)(2)$
6. $(f \circ f)(2)$
7. Are these two answers the same? What does this information tell you about composition?

The notation $[x]$ means the greatest integer not exceeding the value of x.
Given $f(x) = [x]$, $g(x) = 6x^2$ and $h(x) = 6/2x$ find:

8. $(f \circ g)(3)$
9. $(f \circ h)(5)$
10. $(h \circ f)(3)$

Fluency Practice

functions into functions

Work out the value of each of these functions.

Match them to the jumbled answers at the bottom.

$G(x)=\frac{x}{3}$	$y(x) = 2x - 3$	$p(x) = 10 - x$	$f(x) = \frac{1}{2}x^2$
$G(3)$	$y(3)$	$p(4)$	$f(4)$
$G(-9)$	$y(-3)$	$p(\frac{1}{4})$	$f(12)$
$G(\frac{1}{3})$	$y(?) = -3$	$p(-2)$	$f(-6)$
$G(x+4)$	$y(x+3)$	$p(-9)$	$f(2x)$
$G(6x + 6)$	$y(x-3)$	$p(x+2)$	$f(4x)$
$G(3x)$	$y(4x)$	$p(x-3)$	$f(\frac{x}{2})$
$G(9x^3)$	$y(\frac{x}{2})$	$p(x-7)$	$f(x^2)$

jumbled answers

$8x^2$ | -3 | 6 | $13 - x$ | 3 | $2x^2$ | $8x - 3$ | $\frac{x^4}{2}$

0 | $2x - 9$ | $10 - (x+2)$ | $\frac{x+4}{3}$ | $2x + 3$ | -9 | 72 | 9.75 | $\frac{1}{9}$ | $x - 3$

12 | $17 - x$ | 1 | $2x+2$ | 18 | x | $3x^3$ | 8 | 19 | $\frac{x^2}{8}$

composite functions:

Using the five functions at the top, work out the composite functions.
Match them to the answers at the bottom.

$f(x) = x+2$	$G(x) = 10-x$	$p(x) = \frac{x}{3}$	$t(x) = \sqrt{x}$	$y(x) = 3x^2$

fG(3)	py(2)	tG(1)	GG(6)
Gf(3)	py(5)	tG(6)	GG(8)
tf(−1)	pt(81)	pG(−11)	tp(300)
yt(25)	yt(100)	yy(2)	ff(10)
		fG(12)	fG(-12)

jumbled answers

0	1	2	3	3	4	5	6	7	8	9	10	14	24	25	75	300	432

composite functions:

Using the four functions at the top, work out the composite functions.
Match them to the jumbled expressions on the side.

$f(x) = 2x + 6$ | $G(x) = x^2$ | $H(x) = (x + 3)^2$ | $K(x) = \sqrt{4x}$

$fG(x)$	$GK(x)$
$Gf(x)$	$KG(x)$
$GH(x)$	$HK(x)$
$HG(x)$	$KH(x)$
$fK(x)$	$Hf(x)$
$Kf(x)$	$fH(x)$
$ff(x)$	$HH(x)$

jumbled answers

$4x + 18$

$4x + 12\sqrt{x} + 9$

$2x^2 + 6$

$(x + 3)^4$

$\sqrt{8x + 24}$

$4x$

$4x^2 + 24x + 36$

$4x^2 + 36x + 81$

$2x^2 + 12x + 24$

$2x$

$2(\sqrt{4x}) + 6$

$(x^2 + 6x + 12)^2$

$2(x + 3)$

$x^4 + 6x^2 + 9$

Problem Solving

Match the cards into groups of three:

$f(x) = x^2$ $g(x) = 2x + 2$	$f(x) = \frac{x-3}{2}$ $g(x) = 2x + 3$	$f(x) = 4x + 6$ $g(x) = 3x^2$	$f(x) = x^2 + 4$ $g(x) = 2x$
$(g * f)(3) = 26$	$(f * g)(3) = 114$	$(f * g)(x) = (g * f)(x)$	$(f * g)(x) = 4x^2 + 8x + 4$
$(g * g)(x) = 4x + 9$	$(g * f)(x) = 2x^2 + 8$	$(g * f)(x) = 2x^2 + 2$	$(g * f)(3) = 972$

Challenge

1. If $f\big(g(x)\big) = 4x^2 - 8x$ and $f(x) = x^2 - 4$, then what is $g(x)$?
2. If $g(x) = 3x + 2\ and\ g\big(f(x)\big) = x$, what is $f(2)$?
3. If $q(x) = 4x - 6$ and $q(a) = 0$, then what is a?
4. Can you create two functions such that$(f * g)(x) \equiv (g * f)(x)$?

Game

	$b(x) = x + 1$ $c(x) = 2x + 1$ $cb(x) = ?$		$i(x) = 5x - 10$ $w(x) = x - 2$ $iw(x) = ?$		$p(x) = 3x - 4$ $d(x) = 0.5x$ $pd(x) = ?$		LAP FINISH
$h(x) = 4x$ $n(x) = x - 13$ $nh(x) = ?$		$r(x) = 2x$ $q(x) = x - 1$ $rq(x) = ?$		$j(x) = 2x + 2$ $k(x) = 5 - x$ $kj(x) = ?$		$s(x) = 4 - 0.5x$ $d(x) = x + 3$ $ds(x) = ?$	
	$v(x) = 2x$ $c(x) = x - 1$ $vc(x) = ?$		$d(x) = 5 - x$ $w(x) = x + 2$ $wd(x) = ?$		$z(x) = 2x - 3$ $zz(x) = ?$		$t(x) = 7 - x$ $y(x) = 2x$ $ty(x) = ?$
$f(x) = 2x - 7$ $ff(x) = ?$		$i(x) = x + 4$ $b(x) = 2x - 1$ $ib(x) = ?$		$t(x) = -x$ $u(x) = x + 5$ $ut(x) = ?$		$y(x) = -2x + 1$ $f(x) = x + 1$ $yf(x) = ?$	
	$f(x) = x - 3$ $d(x) = 3x$ $df(x) = ?$		$q(x) = 2x + 1$ $qq(x) = ?$		$l(x) = x - 2$ $t(x) = 0.5x$ $tl(x) = ?$		$o(x) = 3x - 15$ $p(x) = 2x$ $op(x) = ?$
$d(x) = -x + 3$ $r(x) = 2x$ $rd(x) = ?$		$v(x) = x - 9$ $b(x) = 4x - 2$ $vb(x) = ?$		$d(x) = 0.5x$ $q(x) = 2x + 1$ $qd(x) = ?$		$n(x) = x + 1$ $m(x) = 3x$ $mn(x) = ?$	
	$t(x) = 0.5x + 2$ $e(x) = x - 3$ $et(x) = ?$		$k(x) = 2x - 5$ $l(x) = 3x$ $lk(x) = ?$		$y(x) = x - 1$ $z(x) = 3x$ $zy(x) = ?$		$d(x) = 2x + 3$ $h(x) = x - 2$ $dh(x) = ?$
$f(x) = x - 4$ $d(x) = x + 1$ $df(x) = ?$		$t(x) = x + 2$ $w(x) = 2x - 1$ $tw(x) = ?$		$c(x) = x + 1$ $a(x) = 3x - 6$ $ca(x) = ?$		$h(x) = 0.5x + 1$ $g(x) = 2x$ $hg(x) = ?$	
START	$f(x) = x + 1$ $g(x) = 2x$ $fg(x) = ?$		$t(x) = x - 1$ $r(x) = 3x$ $rt(x) = ?$		$y(x) = 2x - 3$ $k(x) = x - 2$ $yk(x) = ?$		$a(x) = x + 1$ $b(x) = 3x$ $ba(x) = ?$

FUNCTION BOOST

Youngest goes first. Move the amount of spaces on the dice. If you land on a function, calculate your **FUNCTION BOOST**...

Rolled a **1 or 2**: $x = 2$
Rolled a **3 or 4**: $x = 4$
Rolled a **5 or 6**: $x = 6$

Use x again if you get a double-boost!
First to complete 1, 2, 3 or 4 laps wins!

Intelligent Practice

1) $f(x) = 10 - 3x$
$g(x) = x^2$
Solve: $f(x) = g(x)$

2) $f(x) = 10 - 3x$
$g(x) = x^2$
Solve: $f(2x) = g(2x)$

3) $f(x) = 10 - 3x$
$g(x) = x^2$
Solve: $f(3x) = g(3x)$

4) $f(x) = 10 - 3x$
$g(x) = x^2 + 10$
Solve: $f(3x) = g(3x)$

5) $f(x) = 10 - 3x$
$g(x) = x^2 + 10$
Solve: $f(x) = g(x)$

6) $f(x) = 3x + 10$
$g(x) = x^2 + 10$
Solve: $f(x) = g(x)$

7) $f(x) = 3x + 10$
$g(x) = x^2$
Solve: $f(x) = g(x)$

8) $f(x) = 3x + 10$
$g(x) = x^2$
Solve: $fg(x) = f(x)$

9) $f(x) = 3x + 10$
$g(x) = 2x^2$
Solve: $fg(x) = f(x)$

10) $f(x) = 3x + 10$
$g(x) = 2x^2$
Solve: $gf(x) = g(x)$

11) $f(x) = 6x + 10$
$g(x) = x^2$
Solve: $gf(x) = fg(x)$

12) $f(x) = 6x + 10$
$g(x) = x^2$
$h(x) = 2x - 3$
Solve: $gh(x) = gf(x)$

Fluency Practice

Linear	Quadratic factorise	Quadratic formula	Quadratic complete the square
$f(x) = 4x - 3$, Solve $f(x) = 7$	$f(x) = x^2 + 3x$ Solve $f(x) = 18$	$f(x) = x^2 - 2$, $g(x) = 4x + 10$ Solve $f(x) = g(x)$	$f(x) = x^2 + 8x$ Solve $f(x) = 30$
$f(x) = 5x + 1$, $g(x) = 2x - 10$ Solve $f(x) = g(x)$	$f(x) = x^2 - 1$, $g(x) = 2x + 23$ Solve $f(x) = g(x)$	$f(x) = x^2 + 6x$, $g(x) = 4x + 35$ Solve $f(x) = g(x)$	$f(x) = x^2 + 6x$ Solve $f(x) = -4$
$f(x) = 5x + 1$, $g(x) = 2x - 1$ $3f(x) = 2g(x)$	$f(x) = 6x^2 + 8x$, $g(x) = x + 3$ Solve $f(x) = g(x)$	$f(x) = 2x^2 + x + 7$, $g(x) = 7x + 4$ Solve $f(x) = g(x)$	$f(x) = 2x^2 - 7x$, $g(x) = x - 3$ Solve $f(x) = g(x)$

1	For all values of x $f(x) = 2x - 3$ and $g(x) = x^2 + 2$ Find $g(-4)$
2	Given that $f(x) = 6x + 1$ Find the value of x when $f(x) = 7$
3	$f(x) = 3x^2 - 2x - 8$ Express $f(x + 2)$ in the form $ax^2 + bx$
4	Let $f(x) = x^2 + x$ Solve $f(x - 1) + f(x + 1) = 6$
5	f is the function such that $f(x) = 2x - 5$ g is the function such that $g(x) = x^2 - 10$ Find $fg(-4)$
6	f is the function such that $f(x) = 2x - 5$ g is the function such that $g(x) = x^2 - 10$ Solve $gf(x) = -1$
7	f is the function such that $f(x) = \frac{x}{3x + 1}$ Find $ff(-1)$
8	$f(x) = 2x - 3$ and $g(x) = x^2 + 2$ (b) Determine $gf(x)$, giving your expression in its simplest form.
9	$f(x) = 2x - 3$ and $g(x) = x^2 + 2$ (c) Solve $fg(x) = gf(x)$

10	The function f is such that $f(x) = 4x - 1$ The function g is such that $g(x) = kx^2$ where k is a constant. Given that $fg(2) = 12$, work out the value of k
11	The functions f and g are such that $f(x) = 3(x - 4)$ and $g(x) = \frac{x}{5} + 1$ Find $ff(x)$, simplifying your expression.
12	The functions f and g are such that $f(x) = 3(x - 4)$ and $g(x) = \frac{x}{5} + 1$ Find $g^{-1}(x)$
13	$f(x) = \frac{2x}{x-1}$ Find $f^{-1}(x)$
14	$f(x) = \frac{1}{x+2} + \frac{1}{x-3}$ Write down a value of x for which $f(x)$ is not defined.
15	$f(x) = \frac{3}{x+1} + \frac{1}{x-2}$ Find the value of x for which $f(x) = 0$ Show clear algebraic working.
16	$f(x) = \frac{1}{x+2} + \frac{1}{x-3}$ Given that $f(x) = 4$ find the possible values of x . Give your answer in the form $\frac{p \pm \sqrt{q}}{r}$ where p, q and r are positive integers.

Find the value of the constants a and b in each of the function problems.

The function of f and g are such that $f(x)=5x+3$ and $g(x)=ax+b$ where a and b are constants.

Given that $g(3) = 20$ and $f^{-1}(33)=g(1)$

The function of f and g are such that $f(x)=3x+5$ and $g(x)=ax+b$ where a and b are constants.

Given that $g(5) = 43$ and $f^{-1}(26)=g(1)$

The function of f and g are such that $f(x)=2x-1$ and $g(x)=ax+b$ where a and b are constants.

Given that $g(3) = 21$ and $f^{-1}(25)=g(2)$

The function of f and g are such that $f(x)=6x+2$ and $g(x)=ax+b$ where a and b are constants.

Given that $g(4) = 27$ and $f^{-1}(134)=g(3)$

The function of f and g are such that $f(x)=6x+7$ and $g(x)=ax+b$ where a and b are constants.

Given that $g(3) = 22$ and $f^{-1}(85)=g(2)$

The function of f and g are such that $f(x)=3x+11$ and $g(x)=ax+b$ where a and b are constants.

Given that $g(9) = 61$ and $f^{-1}(107)=g(5)$

Find the value of the constant k in each of the function problems.

The function of f is such that $f(x) = 4x-1$

The function g is such that

$g(x) = kx^2$ where k is a constant.

Given that $fg(2) = 12$

(a) Work out the value of k

The function of f is such that $f(x) = 5x-3$

The function g is such that

$g(x) = kx^2$ where k is a constant.

Given that $fg(3) = 17$

(a) Work out the value of k

The function of f is such that $f(x) = 4x+3$

The function g is such that

$g(x) = kx^2$ where k is a constant.

Given that $fg(2) = 9$

(a) Work out the value of k

The function of f is such that $f(x) = 6x-14$

The function g is such that

$g(x) = kx^2$ where k is a constant.

Given that $fg(3) = 27$

(a) Work out the value of k

The function of f is such that $f(x) = 9x+2$

The function g is such that

$g(x) = kx^2$ where k is a constant.

Given that $fg(4) = 15$

(a) Work out the value of k

The function of f is such that $f(x) = 4x+7$

The function g is such that

$g(x) = kx^2-3$ where k is a constant.

Given that $fg(3) = 25$

(a) Work out the value of k

Fluency Practice

Functions Revision

(a)

$f(x) = x^2 + 6$

Find $f(4)$

(b)

$g(x) = \frac{x}{x+5}$

Find $g(-1)$

(c)

$f(x) = 2(x-1)^2$

Find $f(1.5)$

(d)

$f(x) = 3x - 1$

Given $f(a) = 11$, find the value of a

(e)

$f(x) = \frac{3}{2x-4}$

Solve $f(x) = 1$

(f)

$f(x) = x^2 \quad g(x) = x + 6$

Solve $f(x) = g(x)$

(g)

$g(x) = \frac{3x}{x-4}$

Find the value of x that cannot be included in any domain of g.

(h)

$f(x) = 2x^2 \quad g(x) = x - 5$

Find $fg(8)$

(i)

$f(x) = 4 - 3x \quad g(x) = \frac{1}{2x+1}$

Find $gf(x)$, simplifying your answer.

(j)

$g(x) = 4x - 7$

Find the inverse function $g^{-1}(x)$

(k)

$f(x) = \frac{3x}{2x-1}$

Find the inverse function $f^{-1}(x)$

(l)

$f(x) = \frac{3}{2x+1} \quad g(x) = 5 - x$

Solve $fg(x) = -1$

(m)

$f(x) = \frac{2x}{1-3x}$

Solve $f(x) = f^{-1}(x)$

(n)

$g(x) = \frac{2x}{x+1}$

Find $gg(x)$

Question 1: Given $f(x) = 3x + 5$

Work out the values of

(a) $f(2)$ (b) $f(8)$ (c) $f(0)$ (d) $f(-2)$

Question 2: Given $g(x) = \frac{2x + 9}{4}$

Work out the values of

(a) $g(6)$ (b) $g(-1)$ (c) $g(0)$ (d) $g(-10)$

Question 3: Given $h(x) = x^2 - 5$

Work out the values of

(a) $h(7)$ (b) $h(-1)$ (c) $h(-3)$ (d) $h(15)$

Question 4: The function f is such that $f(x) = 3x - 8$

Solve $f(x) = 7$

Question 5: The function g is such that $g(x) = 19 - 4x$

Solve $g(x) = 31$

Question 6: The function h is such that $h(x) = \frac{5x - 1}{2}$

Solve $h(x) = 32$

Question 7: The function f is such that $f(x) = x^2 - 2x + 3$

Solve $f(x) = 27$

Question 8: The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = x + 5$$
$$g(x) = 3x - 1$$

Calculate the value of:

(a) $fg(1)$ (b) $fg(-5)$ (c) $gf(4)$ (d) $gf(0)$

(e) $ff(2)$ (f) $ff(-4)$ (g) $gg(10)$ (h) $gg(-2)$

Question 9: The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = x^2 + 7$$
$$g(x) = 3x - 8$$
$$h(x) = \frac{x}{4}$$

Calculate the value of:

(a) $fg(3)$ (b) $hf(5)$ (c) $gh(20)$ (d) $gf(-2)$

(e) $fh(12)$ (f) $ff(1)$ (g) $gg(4)$ (h) $hh(40)$

Question 10: The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = \frac{32}{x^2} \qquad g(x) = 2x^3 \qquad h(x) = \frac{12 - 2x}{5}$$

Calculate the value of:

(a) $fg(1)$ (b) $gf(4)$ (c) $gh(-19)$ (d) $hf(2)$

(e) $ff(2)$ (f) $ggg(1)$ (g) $hgf(8)$ (h) $hgh(6)$

Question 11: The functions $f(x)$ and $g(x)$ are given by the following:

$$f(x) = 2x + 1$$
$$g(x) = x - 5$$

Find:

(a) $fg(x)$ (b) $gf(x)$ (c) $ff(x)$ (d) $gg(x)$

Question 12: The functions $f(x)$, $g(x)$ and $h(x)$ are given by the following:

$$f(x) = 4x - 3 \qquad g(x) = 2x + 6 \qquad h(x) = x^2$$

Find

(a) $fg(x)$ (b) $gf(x)$ (c) $hf(x)$ (d) $fh(x)$

(e) $hg(x)$ (f) $gh(x)$ (g) $fgh(x)$ (h) $hgf(x)$

Question 13: Find $f^{-1}(x)$ for each of the following:

(a) $f(x) = 2x$ (b) $f(x) = x - 6$ (c) $f(x) = \frac{x}{3}$

(d) $f(x) = 5x + 1$ (e) $f(x) = \frac{2x}{7}$ (f) $f(x) = \frac{x-2}{6}$

Question 14: Given $h(x) = \frac{x}{4}$

(a) Find $h^{-1}(x)$

(b) Calculate the value of $h^{-1}(1.5)$

Question 15: Given $f(x) = 2x - 3$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(7)$

Question 16: Given $g(x) = \dfrac{3x+1}{2}$

(a) Find $g^{-1}(x)$

(b) Calculate the value of $g^{-1}(11)$

Question 17: Given $f(x) = \dfrac{4x}{9} - 8$

(a) Find $f^{-1}(x)$

(b) Calculate the value of $f^{-1}(-10)$

Apply

Question 1: Given $f(x) = 5x + 7$ and $g(x) = 3x - 18$

Find the value of a such that $f(a) = g(a)$

Question 2: Given $f(x) = x^2 + 9$ and $g(x) = x + 21$

Find the values of a such that $f(a) = g(a)$

Question 3: Given $f(x) = \dfrac{x+1}{3}$ and $g(x) = \dfrac{2}{x+2}$

Find the values of a such that $f(a) = g(a)$

Question 4: Given $f(x) = x^2 + 4x - 1$

Express the following in the form $ax^2 + bx + c$

(a) $f(x+2)$ (b) $f(x-1)$ (c) $f(2x)$

(d) $f(3x)$ (e) $f(2x-1)$ (f) $f(4x+3)$

Question 5: The function f is such that $f(x) = kx + 7$

The function g is such that $g(x) = 3x - 2$

Given that $gf(1) = 34$

Work out the value of k

Question 6: The function g is such that $f(x) = \frac{kx + 2}{4}$

The function h is such that $g(x) = 2x + 5$

Given that $fg(4) = -9.25$

Work out the value of k

Question 7: For all values of x

$$f(x) = x^2 + 5$$

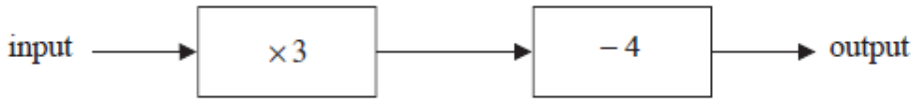
$$g(x) = x - 4$$

Solve $fg(x) = gf(x)$

Question 8: $f(x) = x^2 + 3x + 8$

Show that $f(x + 1) - f(x) = 2x + 4$

1. Here is a number machine.



(a) Work out the **output** when the input is 4

(b) Work out the **input** when the output is 11

(c) Show that there is a value of the input for which the input and the output have the same value.

2. Functions f and g is such that $f(x) = 2x - 1$ and $g(x) = \frac{3}{x}$

(a) Find the value of

(i) f(3)

(ii) fg(6)

(b) Express the inverse function in the form $f^{-1}(x)) = ...$

(c) Express the composite function gf in the form $gf(x) = ...$

3. The function f is such that $f(x) = 4x - 1$

(a) Find $f^{-1}(x)$

The function g is such that $g(x) = kx^2$ where k is a constant.

(b) Given that $fg(2) = 12$, work out the value of k

4. Functions f and g are such that $f(x) = 3(x - 4)$ and $g(x) = \frac{x}{5} + 1$

(a) Find the value of f(10)

(b) Find $g^{-1}(x)$

(c) Show that $ff(x) = 9x - 48$

5. Given that $f(x) = x^2$ and $g(x) = x - 6$, solve the equation $fg(x) = g^{-1}(x)$

6. f and g are functions such that $f(x) = 2x - 3$ and $g(x) = 1 + \sqrt{x}$

(a) Calculate f(−4)

(b) Given that $f(a) = 5$, find the value of a.

(c) Calculate gf(6).

(d) Find the inverse function $g^{-1}(x)$.

7. Functions f and g are such that

$f(x) = \dfrac{1}{x+2}$ and $g(x) = \sqrt{x-1}$

(a) Calculate fg(10)

(b) Find the inverse function $g^{-1}(x)$.

8. Functions f and g are such that

$f(x) = 3x + 2$ and $g(x) = 2x - 5$

(a) Find the composite function fg.
Give your answer as simply as possible.

(b) Find the inverse function $f^{-1}(x)$.

(c) Hence, or otherwise, solve $f^{-1}(x) = g^{-1}(x)$.

9. The function f is such that $f(x) = \dfrac{1}{x+3}$

(a) Find the value of f(2)

(b) Given that $f(a) = \dfrac{1}{10}$, find the value of a.

The function g is such that $g(x) = x + 2$

(c) Find the function gf.

Give your answer as a single algebraic fraction in its simplest form.

10. Functions f and g are such that $f(x) = x^2$ and $g(x) = x - 3$

(a) Find $gf(x)$.

(b) Find the inverse function $g^{-1}(x)$.

(c) Solve the equation $gf(x) = g^{-1}(x)$.

11. The function f is such that $f(x) = (x - 1)^2$

(a) Find f(8)

The function g is such that $g(x) = \dfrac{x}{x-1}$

(b) Solve the equation $g(x) = 1.2$

(c) (i) Express the inverse function g^{-1} in the form $g^{-1}(x) = ...$

(ii) Hence write down $gg(x)$ in terms of x.

12. f is a function such that $f(x) = \dfrac{1}{x^2+1}$

(a) Find $f(\frac{1}{2})$

g is a function such that $g(x) = \sqrt{x-1}, x \geq 1$

(b) Find $fg(x)$
Give your answer as simply as possible.

13. The function f is such that $f(x) = \dfrac{x-6}{2}$

(a) Find $f(8)$

(b) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

The function g is such that $g(x) = \sqrt{x-4}$

(c) Express the function gf in the form $gf(x) = \ldots$
Give your answer as simply as possible.

14. Functions f and g are such that $f(x) = 3x - 2$ and $g(x) = \dfrac{10}{x+2}$

(a) Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$

(b) Find $gf(x)$
Simplify your answer.

15. Functions f and g are such that $f(x) = \dfrac{2}{x}$ and $g(x) = \dfrac{x+1}{x}$

(a) Solve $gf(x) = 3$

(b) Express the inverse function g^{-1} in the form $g^{-1}(x) = \ldots$

16. Functions g and h are such that $g(x) = \dfrac{x}{2x-5}$ and $h(x) = x + 4$

(a) Find the value of $g(1)$

(b) Find $gh(x)$
Simplify your answer.

(c) Express the inverse function g^{-1} in the form $g^{-1}(x) = \ldots$

1. Find an expression for $fg(x)$ for each of these functions:

   (a) $f(x) = x - 1$ and $g(x) = 5 - 2x$

   (b) $f(x) = 2x + 1$ and $g(x) = 4x + 3$

   (c) $f(x) = \frac{3}{x}$ and $g(x) = 2x - 1$

   (d) $f(x) = 2x^2$ and $g(x) = x + 3$

2. Find an expression for $gf(x)$ for each of these functions:

   (a) $f(x) = x - 1$ and $g(x) = 5 - 2x$

   (b) $f(x) = 2x + 1$ and $g(x) = 4x + 3$

   (c) $f(x) = \frac{3}{x}$ and $g(x) = 2x - 1$

   (d) $f(x) = 2x^2$ and $g(x) = x + 3$

3. The function f is such that $f(x) = 2x - 3$

   Find (i) $ff(2)$ (ii) Solve the equation $ff(a) = a$

4. Functions f and g are such that

   $f(x) = x^2$ and $g(x) = 5 + x$

   Find (a)(i) $fg(x)$ (ii) $gf(x)$

   (b) Show that there is a single value of x for which $fg(x) = gf(x)$ and find this value of x.

5. Given that $f(x) = 3x - 1$, $g(x) = x^2 + 4$ and $fg(x) = gf(x)$, show that $x^2 - x - 1 = 0$

6. The function f is defined by $f(x) = \frac{x-1}{x}$, $x \neq 0$

   Solve $ff(x) = -2$

7. The function g is such that $g(x) = \frac{1}{1-x}$ for $x \neq 1$

   (a) Prove that $gg(x) = \frac{x-1}{x}$

   (b) Find $ggg(3)$

1. If f$(x) = 2x + 3$ find (fully simplified):

   (a) $f(1)$,
   (b) $f(\frac{1}{2})$,
   (c) $f(x + 1)$,
   (d) $f(1 - 3x)$.

2. If $g(x) = 2x^2 - x + 1$ find (fully simplified):

   (a) $g(5)$,
   (b) $g(-2)$,
   (c) $g(\frac{2}{3})$,
   (d) $g(x + 1)$,
   (e) $g(x^2 - 1)$,
   (f) $g(\sqrt{x})$.

3. If $h(x) = \frac{2}{3+x}$ find (fully simplified):

   (a) $h(7)$,
   (b) $h(-\frac{1}{2})$,
   (c) $h(x - 3)$,
   (d) $h(\frac{1}{x})$,
   (e) $h(\frac{7}{x-8})$.
   (f) $h(\frac{ax}{4-bx})$.

4. If $l(x) = \frac{1+2x}{2-3x}$ find (fully simplified):

   (a) $l(-3)$,
   (b) $l(\frac{1}{4})$,
   (c) $l(x + 1)$,
   (d) $l(\frac{1}{x})$,
   (e) $l(\frac{x}{x+1})$,
   (f) $l(\frac{x-1}{x-2})$.

5. If $f(x) = 2x - 3$, $g(x) = x^2 + x$ and $h(x) = \frac{1}{x+1}$ solve the following:

   (a) $f(x) = 3$,
   (b) $f(x + 3) = 8$,
   (c) $fff(x) = 2x - 1$,
   (d) $hf(x) = 7$,
   (e) $gf(x) = 0$,
   (f) $fh(x) = -1$,
   (g) $fg(x) = -3$,

6. If $f(x) = x + 1$, $g(x) = x^2$ and $h(x) = \frac{1}{x}$ solve the following:

   (a) $f(x) = 2$,
   (b) $f(x + 3) = -\frac{1}{2}$,
   (c) $ff(x) = 2x - 1$,
   (d) $fff(x) = 2x + 6$,
   (e) $hf(x) = x$,
   (f) $gf(x) = 1$,
   (g) $fh(x) = 0$,
   (h) $fg(x) = 2$,

7. If $f(x) = 2x + 1$, $g(x) = x^2 + x$ and $h(x) = \frac{1}{x}$, find fully simplified expressions for:

   (a) $fg(x)$.
   (b) $gf(x)$.
   (c) $hf(x)$.
   (d) $hfg(x)$.
   (e) $ff(x)$.
   (f) $fff(x)$.
   (g) $gg(x)$.
   (h) $hh(x)$.

. Find the natural domain of the following functions:

(a) $f(x) = -x + 7$.
(b) $f(x) = x^2 - 2x + 1$.
(c) $f(x) = \frac{1}{x}$.
(d) $f(x) = \sqrt{x - 6}$.
(e) $f(x) = \sqrt{2x + 1}$.
(f) $f(x) = \sqrt{1 - 4x}$.
(g) $f(x) = \frac{x+1}{x-1}$.
(h) $f(x) = \frac{ax+b}{cx+d}$.
(i) $f(x) = \frac{\sqrt{7-3x}}{x+3}$.

. Find the inverses of the following functions:

(a) $f(x) = 2x - 1$.
(b) $f(x) = \frac{1}{x}$.
(c) $f(x) = ax + b$.
(d) $f(x) = \sqrt{2 - x}$.
(e) $f(x) = \frac{1}{1+x}$.
(f) $f(x) = \frac{1+x}{1-x}$.
(g) $f(x) = \frac{2-3x}{1+5x}$.
(h) $f(x) = 1 + \frac{3x}{1-x}$.
(i) $f(x) = 5 - \frac{3}{x-2}$.
(j) $f(x) = a + \frac{1+x}{1-ax}$.
(k) $f(x) = \frac{ax+b}{cx+d}$.
(l) $f(x) = \sqrt{\frac{ax+x+c}{x-c}}$.

. The following functions are self-inverse (i.e. $f(x) \equiv f^{-1}(x)$ and $ff(x) = x$). Find conditions on the constants for this to be true:

(a) $f(x) = x + a$.
(b) $f(x) = \frac{x+a}{x-3}$.

. Given that

$$f(x) = \frac{2x}{1-x} \text{ and } g(x) = \frac{3+x}{2x-1},$$

find the value(s) of a such that

$$fg(1) = gf(a - 1).$$

Fluency Practice

A1 $f(x) = 2x - 3$ $g(x) = x^2 - 4$ Solve $gf(x) = 21$	**A2** $f(x) = 2x - 3$ $g(x) = x^2 - 4$ Solve $fg(x) = 21$	**A3** $f(x) = 2x - 9$ $g(x) = x^2 - 3$ Solve $fg(x) = x$	**A4** $f(x) = 2x + 3$ $g(x) = x^2$ Solve $gf(x) = x^2$
B1 $f(x) = x + 2$ $g(x) = x^2 + 3$ Solve $gf(x) = g(x)$	**B2** $f(x) = 2x + 1$ $g(x) = x^2 - 2$ Solve $fg(x) = f(x)$	**B3** $f(x) = 2x + 1$ $g(x) = x^2 - 2$ Solve $gf(x) = g(x)$	**B4** $f(x) = x^2$ $g(x) = 3x - 2$ Solve $fg(x) = gf(x)$
C1 $f(x) = x^2$ $g(x) = x - 3$ Solve $fg(x) = g^{-1}(x)$	**C2** $f(x) = x^2$ $g(x) = x - 3$ Solve $gf(x) = g^{-1}(x)$	**C3** $f(x) = x - 1$ $g(x) = x^2$ Solve $ff(x) = g^{-1}(x)$	**C4** $f(x) = 3x - 2$ $g(x) = \sqrt{2 - x}$ Solve $ff(x) = g^{-1}(x)$
D1 $f(x) = \frac{x}{2} + 3,\ g(x) = \frac{14}{2x - 3}$ Solve $gf(x) = f^{-1}(x)$	**D2** $f(x) = \frac{x + 1}{x},\ g(x) = x - 5$ Solve $fg(x) = f^{-1}(x)$	**D3** $f(x) = \frac{x + 1}{3x - 2},\ g(x) = x - 5$ Solve $fg(x) = f^{-1}(x)$	**D4** $f(x) = \frac{10 - x}{x - 3},\ g(x) = x - 5$ Solve $fg(x) = f^{-1}(x)$

Fluency Practice

A1

$f(x) = 2x - 5$, $g(x) = x^2 - 10$

Find fg(4)

A2

$f(x) = x + 2$, $g(x) = \dfrac{1}{x-3}$

Find fg(x)

A3

$f(x) = 2x + 5$, $g(x) = x^2 - 25$

Solve gf(x) = 0

B1

$f(x) = \dfrac{1}{2}x + 4$, $g(x) = \dfrac{2x}{x+1}$

work out fg(–3)

B2

$f(x) = x + 4$, $g(x) = \dfrac{x}{2x-5}$

Find gf(x)

B3

$f(x) = \dfrac{2}{x}$, $g(x) = \dfrac{x+1}{x}$

Solve gf(a) = 3

C1

$f(x) = \sqrt{x-1}$, $g(x) = \dfrac{1}{x+2}$

Calculate gf(10)

C2

$f{:}x \mapsto 2x^2 + 1$, $g{:}x \mapsto \dfrac{2x}{x-1}$

Express the composite function gf in the form $gf{:}x \mapsto \ldots$

C3

$f(x) = x^2$, $g(x) = 2 + x$

Solve the equation fg(x) = g(x)

D1

$f{:}x \mapsto 2x - 3$, $g{:}x \mapsto 1 + \sqrt{x}$

Calculate fg(6)

D2

$f(x) = \dfrac{x-6}{2}$, $g(x) = \sqrt{x-4}$

Express the function gf in the form gf(x) = …
Give your answer as simply as possible.

D3

$f(x) = x^2$, $g(x) = x - 3$

Solve the equation gf(x) = $g^{-1}(x)$

Fluency Practice

A1 $f(x) = x^2 + 4x$ Solve $f(x) = 12$	**A2** $f(x) = x^2 - 12$ Solve $f(x) = x$	**A3** $f(x) = 7x - 10$ Solve $f(x) = x^2$	**A4** $f(x) = (x + 4)(x + 2)$ Solve $f(x) = 3$
B1 $f(x) = x^2 - 6$ Solve $f^{-1}(x) = x$	**B2** $f(x) = \sqrt{x+3}$ Solve $f^{-1}(x) = 6$	**B3** $f(x) = 2x - 15$ Solve $f^{-1}(x) = x^2$	**B4** $f(x) = \dfrac{4x}{x-1}$ Solve $f^{-1}(x) = x + 1$
C1 $f(x) = x^2 - 6$ $g(x) = 3x + 4$ Solve $f(x) = g(x)$	**C2** $f(x) = x^2 - 6$ $g(x) = x^2 + 6$ Solve $f(x) = g(2)$	**C3** $f(x) = x^2 - 4$ $g(x) = 4x - 1$ Solve $f(x) = g(x) + 2$	**C4** $f(x) = 9x - 2$ $g(x) = x^2$ Solve $f(x) = g(x + 2)$
D1 $f(x) = x - 3$ $g(x) = x^2 - 3$ Solve $f(x) = g^{-1}(x)$	**D2** $f(x) = \sqrt{x+4}$ $g(x) = x - 8$ Solve $f^{-1}(x) = g^{-1}(x)$	**D3** $f(x) = \sqrt{3x+4}$ $g(x) = 2(x + 2)$ Solve $f^{-1}(x) = g(x)$	**D4** $f(x) = 3x^2 + 1$ $g(x) = x + 4$ Solve $f^{-1}(x) = g^{-1}(x + 3)$

Fluency Practice

A1

$f(x) = 2x - 5$, $g(x) = x^2 - 10$

Find fg(4)

A2

$f(x) = x + 2$, $g(x) = \dfrac{1}{x-3}$

Find fg(x)

A3

$f(x) = 2x + 5$, $g(x) = x^2 - 25$

Solve gf(x) = 0

B1

$f(x) = \dfrac{1}{2}x + 4$, $g(x) = \dfrac{2x}{x+1}$

work out fg(–3)

B2

$f(x) = x + 4$, $g(x) = \dfrac{x}{2x-5}$

Find gf(x)

B3

$f(x) = \dfrac{2}{x}$, $g(x) = \dfrac{x+1}{x}$

Solve gf(a) = 3

C1

$f(x) = \sqrt{x-1}$, $g(x) = \dfrac{1}{x+2}$

Calculate gf(10)

C2

$f: x \mapsto 2x^2 + 1$, $g: x \mapsto \dfrac{2x}{x-1}$

Express the composite function gf in the form $gf: x \mapsto \ldots$

C3

$f(x) = x^2$, $g(x) = 2 + x$

Solve the equation fg(x) = g(x)

D1

$f: x \mapsto 2x - 3$, $g: x \mapsto 1 + \sqrt{x}$

Calculate fg(6)

D2

$f(x) = \dfrac{x-6}{2}$, $g(x) = \sqrt{x-4}$

Express the function gf in the form gf(x) = …
Give your answer as simply as possible.

D3

$f(x) = x^2$, $g(x) = x - 3$

Solve the equation gf(x) = $g^{-1}(x)$

Fluency Practice

Composite Functions (1)

1. If $f(x) = 4x - 5$ and $g(x) = 3x + 10$, evaluate

a) $fg(2) =$ b) $gf(2) =$ c) $ff(2) =$

2. If $f(x) = 3x^2 - 4x$ and $h(x) = 4(x - 2)$, evaluate

a) $fh(5) =$ b) $hf(-5) =$ c) $f^2(1) =$

3. If $f(x) = \frac{x+5}{x-3}$ and $g(x) = 10 - 2x$, evaluate

a) $fg(4) =$ b) $gf(-2) =$ c) $g^2(3) =$

4. Given that $f(x) = 5x + 2x^2$ and $g(x) = 3x - 4$ evaluate,

a) $fg(1) =$ b) $gf(1) =$ c) $f^2(2) =$

5. Given that $f(x) = 3x + 7$ and $g(x) = 2x - 5$, write in simplest form

a) $gf(x) =$ b) $fg(x) =$

c) $gf(x) - fg(x) =$

6. If $f(x) = 10 - 4x$ and $g(x) = 3x + 5$, write in simplest form

a) $gf(x) =$ b) $fg(x) =$

c) $gf(x) - fg(x) =$

7. If, $f(x) = 5x - 4$ and $g(x) = 3 - 2x$, write in simplest form

a) $fg(x) =$ b) $gf(x) =$

c) $fg(x) - gf(x) =$

8. If, $f(x) = 3x$ and $g(x) = x^2 + 5x$, write in simplest form

a) $fg(x) =$ b) $gf(x) =$

c) $gf(x) - fg(x) =$

9. If, $f(x) = x - 2$ and $g(x) = x^2 - 4x$, write in simplest form

a) $fg(x) =$ b) $gf(x) =$

c) $fg(x) - gf(x) =$

Extension

10. If $f(x) = 3x - 4$

a) $f^2(x) =$ b) $f^3(x) =$

Fluency Practice

Composite Functions (2)

1. If $f(x) = 4x - 5$ and $g(x) = 3x + 10$,

a) Write as a simplified expression, $fg(x)$

b) Write as a simplified expression $gf(x)$

c) Solve $fg(x) = 10$

d) Show that $fg(x) - gf(x) = 40$

2. If $f(x) = 5x + 2$ and $g(x) = 10 - 3x$

a) Write as a simplified expression, $fg(x)$

b) Write as a simplified expression $gf(x)$

c) Solve $gf(x) = 1$

d) Show that $fg(x) - gf(x) = 48$

3. If $f(x) = x^2 - 4$ and $g(x) = x + 5$

a) Show that $fg(x) = x^2 + 10x + 21$

b) Show that $gf(x) = x^2 + 1$

c) For what values does $gf(x) = 10$

d) Solve $fg(x) - gf(x) = 0$

4. If $f(x) = x^2 + 3x$ and $g(x) = x - 8$,

a) Show that $fg(x) = x^2 - 13x + 40$

b) Write $fg(x) - gf(x)$ as an expression in simplest form

c) Solve $fg(x) - gf(x) = 0$

d) Solve $fg(x) = 10$

5. It is given that, $f(x) = 3kx - 10$ and $g(x) = 5x + 9$.

a) Find the value of k if $fg(-1) = 50$

b) Find the value of k if $gf(2) = 64$

6. It is given that, $f(x) = ax + b$ and $g(x) = 3ax - 2b$.

Given that $f(4) = 10$ and $g(4) = 40$, find the value of a and b

7. If $f(x) = \frac{1}{x+6}$ and $g(x) = 4x + 5$

a) Write in simplified form, $fg(x)$

b) Show that $gf(x) = \frac{5x+34}{x+6}$

c) Solve $gf(x) = 10$

Fluency Practice

combined functions show that $fg(x) = gf(x)$ for each of these

(1)

$f(x) = 3x + 2$
$g(x) = 6x + 5$

(2)

$f(x) = 4x + 1$
$g(x) = 13x + 4$

(3)

$f(x) = 7x + 3$
$g(x) = 9x + 4$

(4)

$f(x) = 7x - 3$
$g(x) = 5x - 2$

(5)

$f(x) = 1\frac{1}{2}\,x + 6$
$g(x) = 1\frac{1}{4}\,x + 3$

(6)

$f(x) = 7x - 2$
$g(x) = -8x + 3$

in general,

$f(x) = ax + b$
$g(x) = cx + d$

where 'a', 'b', 'c' and 'd' are constants

what relationship is there between the four constants if $fg(x) = gf(x)$?

make up some further examples and test that your rule works

combined linear functions

find the difference between fg(x)) and gf(x):

(1) f(x) = 3x + 5
g(x) = 2x + 3

(2) f(x) = 4x + 5
g(x) = 3x + 4

(3) f(x) = 4x + 9
g(x) = 3x + 7

(4) f(x) = 3x + 2
g(x) = 2x + 3

(5) f(x) = 5x + 1½
g(x) = 3x – 2

(6) f(x) = 2x + 3
g(x) = 3x + 12

Fluency Practice

combined linear functions

choose any four of these and find the difference between fg(x) and gf(x):

(1) f(x) = 3x + 1
g(x) = 4x + 2

(2) f(x) = 4x + 2
g(x) = 5x + 3

(3) f(x) = 7x + 1
g(x) = 6x + 1

(4) f(x) = 8x + 5
g(x) = 5x + 3

(5) f(x) = 9x + 5
g(x) = 4x + 2

(6) f(x) = 2x – 1
g(x) = 4x – 2

(7) f(x) = 4x – 2
g(x) = 3x – 1

(8) f(x) = 8x – 2
g(x) = 5x – 1

(9) f(x) = – 3x – 1
g(x) = 6x + 1

(10) f(x) = – 2x – 8
g(x) = 3x + 5

Fluency Practice

combined functions

(1) $f(x) = 2x + 3$
$g(x) = 3x - 2$

show that $gf(x) - fg(x) = 8$

(2) $f(x) = 9 - 4x$
$g(x) = 2x - 7$

show that $fg(x) - gf(x) = 26$

(3) $f(x) = 3x + k$
$g(x) = \frac{1}{3}(x - 4)$

for what value of k is $fg(x) - gf(x) = 0$?

(4) $f(x) = x^2 - 2x + 1$
$g(x) = 1 - 3x$

(a) solve $fg(x) = 1$

(b) solve $gf(x) = x^2$

(5) $f(x) = 2x - 3$

(a) solve $ff(x) = x$
(b) solve $f^3(x) = x$
(c) solve $f^4(x) = x$

(6) $f(x) = \frac{2x - 4}{x}$
$g(x) = \frac{4}{2 - x}$

(a) work out $fg(x)$
(b) work out $gf(x)$

(7) $g(x) = \frac{x + 2}{x - 2}$
$f(x) = \frac{2x + 2}{x - 1}$

(a) work out $fg(x)$
(b) work out $gf(x)$

(8) $f(x) = \frac{1}{x + 1}$

find $f^2(x)$
$f^3(x)$
$f^4(x)$
etc.
what do you notice?

Fluency Practice

combined functions

(1)

$f(x) = 2x + a$
$g(x) = ax + 3$

$fg(x) = 10x + b$

where 'a' and 'b' are constants

work out the values of 'a' and 'b'

(2)

$f(x) = dx - e$
$g(x) = ex - 1$

$fg(x) = 18x - 11$

where 'd' and 'e' are constants

work out the values of 'd' and 'e'

(3)

$f(x) = hx + k$
$g(x) = kx + h$

$fg(x) = 20x + 14$

where 'k' and 'h' are constants

work out the values of 'k' and 'h'

(4)

$f(x) = mx - n$
$g(x) = nx + m$

$gf(x) = 12x - 13$

where 'm' and 'n' are constants

work out the values of 'm' and ''

(5)

$f(x) = vx + w$
$g(x) = 2vx + 2w$

$gf(x) = 18x + 16$

work out two values for each of 'v' and 'w'

(6)

$f(x) = 2x + p$
$g(x) = px + 3$

$gf(x) = 2px + 5p - 1$

work out two values for 'p'

Fluency Practice

rational functions

f(7) means substitute 7 in place of x
$f^2(7)$ means work out f(7) and then use this value as a new input value
(i.e. do 'f' then do 'f' again)

(1) $f(x) = \dfrac{2x + 1}{x - 2}$

(i) work out:
(a) f(7) (d) f(3)
(b) f(1) (e) f(– 3)
(c) f(12) (f) f(2.5)

(ii) work out and simplify:
(a) f(27) (d) f(2.2)
(b) f(52) (e) f(2.1)
(c) f(17) (f) f(2⅓)

(2) $f(x) = \dfrac{3x + 1}{x - 3}$

(i) work out:
(a) f(4) (d) f(13)
(b) f(5) (e) f(8)
(c) f(1) (f) f(– 2)

(ii) work out and simplify:
(a) f(23) (d) f(103)
(b) f(28) (e) f(3.5)
(c) f(53) (f) f(3.4)

(3) $f(x) = \dfrac{2x + 2}{x - 2}$

(i) work out:
(a) f(3) (d) f(8)
(b) f(4) (e) f(5)
(c) f(1) (f) f(– 4)

(ii) work out and simplify:
(a) f(6) (e) f(17)
(b) f(7) (f) f(22)
(c) f(12) (g) $f^2(32)$
(d) f(14) (h) $f^2(62)$

(4) $f(x) = \dfrac{4x + 2}{x - 4}$

(i) work out:
(a) f(5) (d) f(10)
(b) f(6) (e) f(13)
(c) f(7) (f) f(22)

(ii) work out and simplify:
(a) f(14) (e) f(34)
(b) f(16) (f) f(40)
(c) f(19) (g) $f^2(49)$
(d) f(24) (h) $f^2(64)$

(i) f(?) = 4.2
(j) f(?) = 4.1
(k) f(?) = 4

Find the inverse function $f^{-1}(x)$ for each of the following functions:

(a) $f(x) = (x-2)^2 + 5$ for $x \geq 2$

(b) $f(x) = (x+3)^2 - 1$ for $x \leq -3$

(c) $f(x) = 2(x-1)^2 + 3$ for $x \leq 1$

(d) $f(x) = 4(x+2)^2 - 7$ for $x \geq -2$

Find the inverse function $f^{-1}(x)$ for each of the following functions:

(a) $f(x) = x^2 + 6x$ for $x \geq -3$

(b) $f(x) = x^2 - 10x + 3$ for $x \leq 5$

(c) $f(x) = x^2 + 2x - 5$ for $x \leq -1$

(d) $f(x) = x^2 - 8x + 1$ for $x \geq 4$

Find the inverse function $f^{-1}(x)$ for each of the following functions:

(a) $f(x) = 2x^2 - 8x$ for $x \geq 2$

(b) $f(x) = 3x^2 - 6x + 1$ for $x \leq 1$

(c) $f(x) = 4x^2 + 24x - 3$ for $x \geq -3$

(d) $f(x) = 2x^2 - 10x + 3$ for $x \leq 2.5$

The function $g(x)$ is defined as $g(x) = x^2 - 6x + 3$ where $x \geq 3$

(a) Find the inverse function $g^{-1}(x)$ in the form $g^{-1}(x) =$..

(b) Solve $g^{-1}(x) = 5$

The function $f(x)$ is defined as $f(x) = 3x^2 + 12x - 2$ where $x \leq -2$

(a) Find the inverse function $f^{-1}(x)$ in the form $f^{-1}(x) =$..

(b) Solve $f^{-1}(x) = -3$

3 Quadratic Graphs

For each of these, sketch a graph marking clearly any points where it crosses the axes.

1. $y = (x-4)(x-1)$
2. $y = (x-2)(x-1)$
3. $x(x-2)$
4. $y = (x+1)(x-1)$
5. $x(x-5)$
6. $y = (x-10)(x+2)$
7. $y = (-x+10)(x+2)$
8. $y = (x+3)(x-4)$
9. $y = (x-3)(x-2)$
10. $y = (x+15)(x-4)$
11. $y = x^2 + 2x + 1$
12. $y = x^2 - 2x + 1$
13. $y = x^2 + 5x + 4$
14. $y = x^2 + 7x + 12$
15. $y = x^2 - 3x - 10$
16. $y = x^2 + 3x - 10$
17. $y = x^2 + 5x - 14$
18. $y = x^2 - 7x - 30$
19. $y = x^2 - 17x - 30$
20. $y = x^2 - 6x - 8$

Don't forget that a line crosses the y-axis when $x = 0$ and crosses the x-axis when $y = 0$.

If $x = 0$ then put a zero in where there is an x and then find out what y equals.

If $y = 0$ then put a zero in where there is a y and rearrange the equation to find out what y equals.

For example, to find out where $y = (x+7)(x+2)$ crosses the y-axis, put in 0 instead of x and get $y = (0+7)(0+2) = 7 \times 2 = 14$. To find where it crosses the x-axis, put in 0 instead of y and get $(x+7)(x+2) = 0$. If two things multiplied make zero, then at least one of them must be zero, so either $x + 7 = 0 \Rightarrow x = -7$ or $x + 2 = 0 \Rightarrow x = -2$.

For each of these, sketch a graph marking clearly any points where it crosses the axes.

1. $y = 3x^2 + 4x + 1$
2. $y = 2x^2 + 7x + 6$
3. $y = 2x^2 - 5x - 3$
4. $y = 6x^2 - x - 2$
5. $y = 2x^2 - x - 15$
6. $y = 4x^2 - 31x + 21$
7. $y = -2x^2 - 7x + 4$
8. $y = 14x^2 - 3x - 2$
9. $y = -12x^2 - x + 1$
10. $y = 4x^2 - 1$

Don't forget that a line crosses the y-axis when $x = 0$ and crosses the x-axis when $y = 0$.

If $x = 0$ then put a zero in where there is an x and then find out what y equals.

If $y = 0$ then put a zero in where there is a y and rearrange the equation to find out what y equals.

For example, to find out where $y = (x+7)(x+2)$ crosses the y-axis, put in 0 instead of x and get $y = (0+7)(0+2) = 7 \times 2 = 14$. To find where it crosses the x-axis, put in 0 instead of y and get $(x+7)(x+2) = 0$. If two things multiplied make zero, then at least one of them must be zero, so either $x + 7 = 0 \Rightarrow x = -7$ or $x + 2 = 0 \Rightarrow x = -2$.

$$2x^2 - x - 6$$

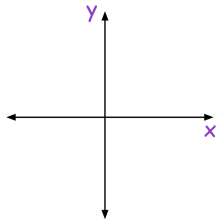
Look for factors of -12 (2 times -6)

$$2x^2 - 4x + 3x - 6$$

$$2x(x - 2) + 3(x - 2)$$

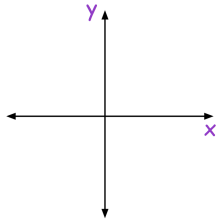
$$(2x + 3)(x - 2)$$

Question 1: Emilio wants to sketch the graph of $y = x^2 - 7x + 10$



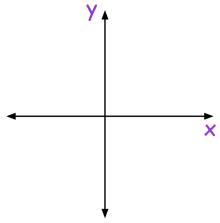
(a) Find the value of y when $x = 0$
(b) Use your answer to (a) to plot where the graph crosses the y–axis.
(c) Solve the equation $x^2 - 7x + 10 = 0$
(d) Use your answers to (c) to help you plot where the graph crosses the x–axis.
(e) Sketch the graph of $y = x^2 - 7x + 10$

Question 2: Rebecca wants to sketch the graph of $y = x^2 + 7x - 8$



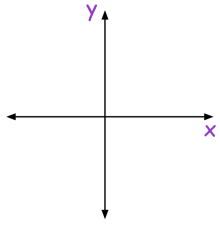
(a) Find the value of y when $x = 0$
(b) Use your answer to (a) to plot where the graph crosses the y–axis.
(c) Solve the equation $x^2 + 7x - 8 = 0$
(d) Use your answers to (c) to help you plot where the graph crosses the x–axis.
(e) Sketch the graph of $y = x^2 + 7x - 8$

Question 3: Michael wants to sketch the graph of $y = x^2 + 16x + 64$



(a) Find the value of y when $x = 0$
(b) Use your answer to (a) to plot where the graph crosses the y–axis.
(c) Solve the equation $x^2 + 16x + 64 = 0$
(d) Use your answers to (c) to help you plot where the graph meets the x–axis.
(e) Sketch the graph of $y = x^2 + 16x + 64$

Question 4: James wants to sketch the graph of $y = x^2 + 4x + 10$



(a) Find the value of y when $x = 0$
(b) Use your answer to (a) to plot where the graph crosses the y–axis.
(c) Show that the equation $x^2 + 4x + 10 = 0$ has no real roots.
(d) Explain why your answer to (c) means that the graph does not cross the x–axis.
(e) Find the value of y when $x = 1$
(e) Sketch the graph of $y = x^2 + 4x + 10$

Question 5: Sketch the following graphs.

(a) $y = x^2 + 6x + 8$ (b) $y = x^2 - x - 6$ (c) $y = x^2 + 6x + 9$

(d) $y = x^2 - 13x + 42$ (e) $y = x^2 + 5x - 36$ (f) $y = x^2 - 2x + 1$

(g) $y = x^2 + 5x + 11$ (h) $y = x^2 - 4x + 7$

Question 6: Sketch the following graphs.

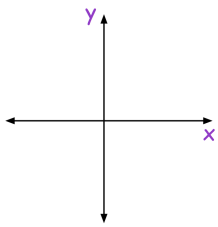
(a) $y = (x - 7)(x + 10)$ (b) $y = (x + 3)(x + 8)$ (c) $y = (x - 2)^2$

Question 7: Sketch the following graphs.

(a) $y = x^2 - 49$ (b) $y = x^2 - 1$ (c) $y = x^2 - 196$

Question 8: Michael wants to sketch the graph of $y = -x^2 + 5x + 14$

(a) Find the value of y when $x = 0$
(b) Use your answer to (a) to plot where the graph crosses the y–axis.
(c) Solve the equation $-x^2 + 5x + 14 = 0$
(d) Use your answers to (c) to help you plot where the graph crosses the x–axis.
(e) Sketch the graph of $y = -x^2 + 5x + 14$



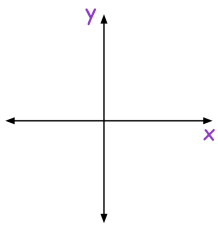
Question 9: Sketch the following graphs.

(a) $y = -x^2 - 5x - 4$ (b) $y = -x^2 + 9x - 18$ (c) $y = 84 - 5x - x^2$

(d) $y = (3 - x)(x + 8)$ (e) $y = -x^2 - 8x - 16$ (f) $y = 144 - x^2$

Question 10: Robyn wants to sketch the graph of $y = 2x^2 + 9x + 4$

(a) Find the value of y when $x = 0$
(b) Use your answer to (a) to plot where the graph crosses the y–axis.
(c) Solve the equation $2x^2 + 9x + 4 = 0$
(d) Use your answers to (c) to help you plot where the graph crosses the x–axis.
(e) Sketch the graph of $y = 2x^2 + 9x + 4$



Question 10: Sketch the following graphs

(a) $y = 5x^2 + 13x + 6$ (b) $y = 3x^2 - 16x - 12$ (c) $y = 2x^2 - 13x + 15$

(d) $y = (2x + 5)(2x - 1)$ (e) $y = 3 - 20x - 7x^2$ (f) $y = 4x^2 - 28x + 49$

Question 11: Sketch the following graphs.
Label exactly where each graph crosses the coordinates axes.

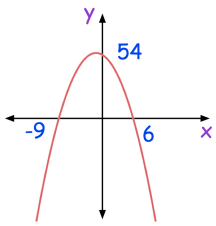
(a) $y = x^2 + 4x + 1$ (b) $y = x^2 - 10x + 10$ (c) $y = x^2 - 8x - 2$

(d) $y = x^2 + 18x + 7$ (e) $y = x^2 - 7x - 3$ (f) $y = 2x^2 + 8x + 2$

Apply

Question 1: Dominic sketches the graph of $y = x^2 + 3x - 54$

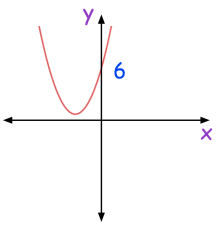
Can you spot any mistakes?



Question 2: Hannah sketches the graph of $y = x^2 - 2x + 6$

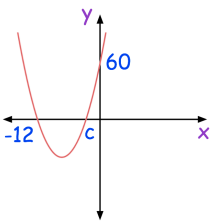
(a) Can you spot any mistakes?

(b) Use the discriminant, $b^2 - 4ac$, to explain why the graph of $y = x^2 - 2x + 6$ does not cross the x-axis.

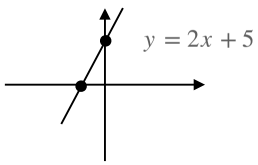


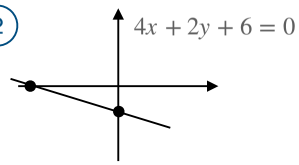
Question 3: Shown is the graph of $y = x^2 + ax + b$

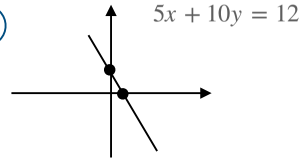
Find the values of a, b and c.

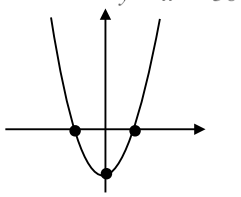


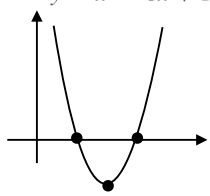
Find the coordinates of where each of the following cut the x-axis and y -axis.

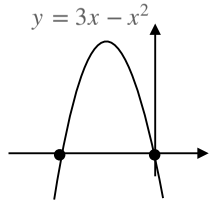
1 

2 

3 

4 

5 

6 

Find the coordinates of where each of the following cut the x-axis and y -axis.
State if the equation represents a line or parabola.

7 $y = 12x - 24$

8 $y = x^2 + 10x$

9 $y = 5 - x$

10 $y = x^2 + 5x + 4$

11 $3y = 6x + 12$

12 $y = x^2 + 7x + 6$

13 $y = (x - 5)(x - 2)$

14 $4y + 8x - 12 = 0$

15 $y = 35 - 2x - x^2$

16 $y = 5x^2 + 2x - 7$

17 $y = x^2 - 49$

18 $8y + 4x - 12 = 0$

19 $y = 9 - x^2$

20 $y = 21x^2 - 35x - 14$

21 $y = 6 - x - x^2$

22 $2x + 3y - 7 = 0$

23 $y = (x - 3)(x + 3)$

24 $y = 12x - 6x^2$

Intelligent Practice

Find the coordinates of the turning point of the following graphs:

1) $y = x^2 + 10x + 4$

2) $y = x^2 + 10x + 3$

3) $y = x^2 + 10x + 2$

4) $y = x^2 + 10x + 1$

5) $y = x^2 + 10x - 1$

6) $y = x^2 + 10x - 2$

7) $y = x^2 + 10x - 3$

8) $y = x^2 + 10x - 4$

1) $y = 2x^2 + 4x + 4$

2) $y = 2x^2 + 4x + 3$

3) $y = 2x^2 + 4x + 2$

4) $y = 2x^2 + 4x + 1$

5) $y = 2x^2 + 4x - 1$

6) $y = 2x^2 + 4x - 2$

7) $y = 2x^2 + 4x - 3$

8) $y = 2x^2 + 4x - 4$

Intelligent Practice

Quadratic	Completed the Square	Min/Max Point
$y = x^2$		
$y = x^2 - 11$		
$y = x^2 + 6$		
$y = x^2 + 6x$		
$y = x^2 - 6x$		
$y = x^2 - 6x - 11$		
$y = x^2 + 6x + 11$		
$y = x^2 + 7x + 11$		
$y = x^2 + 8x + 11$		

Intelligent Practice

Quadratic	Completed the Square	Min/Max Point
$y = x^2 + 8x + 11$		
$y = x^2 + 8x + 16$		
$y = x^2 + 10x + 25$		
$y = -x^2 - 10x - 25$		
$y = -x^2 + 10x - 25$		
$y = -x^2 + 10x$		
$y = x^2 - 10x$		
$y = x^2 - 2x$		
$y = x^2 - \frac{2}{3}x$		

Apply

Question 1: Write $(x + 3)^2 - 4$ in the form $x^2 + bx + c$

Question 2: Write $(x - 2)^2 - 9$ in the form $x^2 + bx + c$

Question 3: Write $(x - 7)^2 + 11$ in the form $x^2 + bx + c$

Question 4: Use completing the square to find the minimum point for each graph below

(a) $y = x^2 + 10x + 12$ (b) $y = x^2 + 4x + 1$ (c) $y = x^2 + 6x + 8$

(d) $y = x^2 - 2x + 3$ (e) $y = x^2 - 6x - 3$ (f) $y = x^2 - x - 4$

(g) $y = x^2 + 9x + 1$ (h) $y = x^2 - 6x - 2$ (i) $y = x^2 + 22x + 100$

Question 5: By using completing the square to solve $ax^2 + bx + c = 0$, prove the quadratic formula.

Question 6: Can you spot any mistakes?

Solve $x^2 + 10x + 2 = 0$

$(x + 5)^2 - 10 + 2 = 0$

$(x + 5)^2 - 12 = 0$

$(x + 5)^2 = 12$

$x + 5 = \sqrt{12}$

$x = -5 + \sqrt{12}$

1. By writing the following in the form $y = (x + a)^2 + b$, where a and b are integers, write down the coordinates of the turning point of the curve. Hence sketch the curve.

(a) $y = x^2 - 8x + 20$
(b) $y = x^2 - 10x - 1$
(c) $y = x^2 + 4x - 6$
(d) $y = 2x^2 - 12x + 8$
(e) $y = -x^2 + 6x + 10$
(f) $y = 5 - 2x - x^2$

2. Given the following minimum turning points of quadratic curves, find an equation of the curve in the form $y = x^2 + ax + b$. Hence sketch each curve.

(a) $(2, -3)$
(b) $(-4, 1)$
(c) $(-1, 5)$
(d) $(3, -12)$
(e) $(1, -7)$
(f) $(-4, -1)$

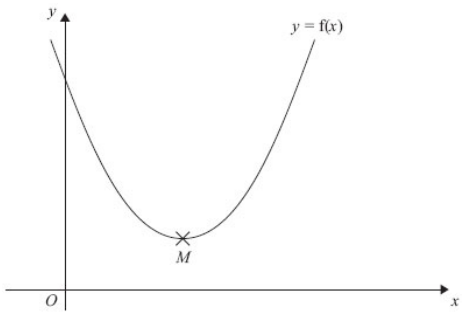
3. Find the maximum or minimum value of the following curves and sketch each curve.

(a) $y = x^2 + 4x + 2$
(b) $y = 1 - 6x - x^2$
(c) $y = -x^2 + 2x - 3$
(d) $y = x^2 - 8x + 8$
(e) $y = x^2 - 3x - 1$
(f) $y = -3x^2 + 12x - 9$

4. The expression $x^2 - 3x + 8$ can be written in the form $(x - a)^2 + b$ for all values of x.

(i) Find the value of a and the value of b.

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 3x + 8$
The diagram shows part of a sketch of the graph of $y = f(x)$.



The minimum point of the curve is M.

(ii) Write down the coordinates of M.

Fluency Practice

5. (i) Sketch the graph of $f(x) = x^2 - 6x + 10$, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

(ii) Hence, or otherwise, determine whether $f(x) - 3 = 0$ has any real roots.
Give reasons for your answer.

***6.** The minimum point of a quadratic curve is $(1, -4)$. The curve cuts the y-axis at -1.
Show that the equation of the curve is $y = 3x^2 - 6x - 1$

***7.** The maximum point of a quadratic curve is $(-2, -5)$. The curve cuts the y-axis at -13.
Find the equation of the curve. Give your answer in the form $ax^2 + bx + c$.

* = extension

By completing the square, find the coordinates of the turning points of these quadratic graphs:

(a) $x^2 + 6x + 1$

(b) $x^2 - 10x - 3$

(c) $x^2 + 8x + 4$

(d) $x^2 - 3x - 5$

(e) $x^2 + x + 9$

By completing the square, find the coordinates of the turning points of these quadratic graphs:

(a) $10 - 2x - x^2$

(b) $6 + 4x - x^2$

(c) $2x^2 + 8x - 1$

(d) $3x^2 - 18x - 4$

(e) $13 - 4x - 2x^2$

(a) A quadratic graph with equation $y = x^2 + 6x + b$ has a turning point at $(a, -13)$. Find the values of a and b.

(b) A quadratic graph with equation $y = x^2 + ax - \frac{7}{4}$ has a turning point at $(-\frac{3}{2}, b)$. Find the values of a and b.

(a) A quadratic graph has a turning point at $(2, 3)$ and passes through $(0, 7)$. Find the equation of the quadratic, giving your answer in the form $y = ax^2 + bx + c$.

(b) A quadratic graph has a turning point at $(-1, 6)$ and passes through $(0, 8)$. Find the equation of the quadratic, giving your answer in the form $y = ax^2 + bx + c$.

(c) A quadratic graph has a turning point at $(1, 11)$ and passes through $(0, 9)$. Find the equation of the quadratic, giving your answer in the form $y = ax^2 + bx + c$.

Fluency Practice

1 Express each of the following expressions in the form $(x-a)^2-b$, where a and b are constants to be found.

(a) x^2+2x+5 (b) x^2-4x+1 (c) x^2+6x-1 (d) $x^2-12x+8$

(e) $x^2-4x-60$ (f) $x^2+12x+32$ (g) x^2+3x+1 (h) x^2+8x-2

(i) $x^2+22x+57$ (j) x^2+10x (k) $x(x-2)+1$ (l) $(x+2)(x-3)$

2 Solve each of the quadratic equations below using completing the square.

(a) $x^2+5x+6=0$ (b) $x^2+9x-10=0$ (c) $x^2-8x+12=0$ (d) $x^2+5x=14$

(e) $x^2+6x-59=0$ (f) $x^2-12x+23=0$ (g) $2x+3=x^2$ (h) $x^2-10x+26=8$

3 The quadratic curve C has the equation $y=(x-a)^2+b$.

(a) Explain why the curve has a minimum point when $x = a$.

(b) Write down the y coordinate of the minimum point on C.

(c) Find the coordinates of the point where C crosses the y axis.

The curve C meets the x axis provided $-b \geq 0$.

(d) Explain the restriction $-b \geq 0$.

4 Find the real solutions to the quadratic equations below or prove that no real solutions exist.

(a) $x^2+8x-10=0$ (b) $x^2-5x-10=0$ (c) $x^2-5x+20=0$ (d) $x^2+1=0$

(e) $x^2+11x-2=0$ (f) $x^2+3x+4=0$ (g) $x^2-11x-60=0$ (h) $x^2+10x-13=0$

5 The curve is defined such that $y = f(x)$, where

$$f(x)=x^2+ax+5$$

and a is a constant.

(a) Given that the curve passes through the point (1, 12), find the value of a.

(b) Find the coordinates of the point where the curve crosses the y axis.

(c) Find the coordinates of the point where the curve crosses the x axis.

(d) (i) Find the coordinates of the turning point on the curve.

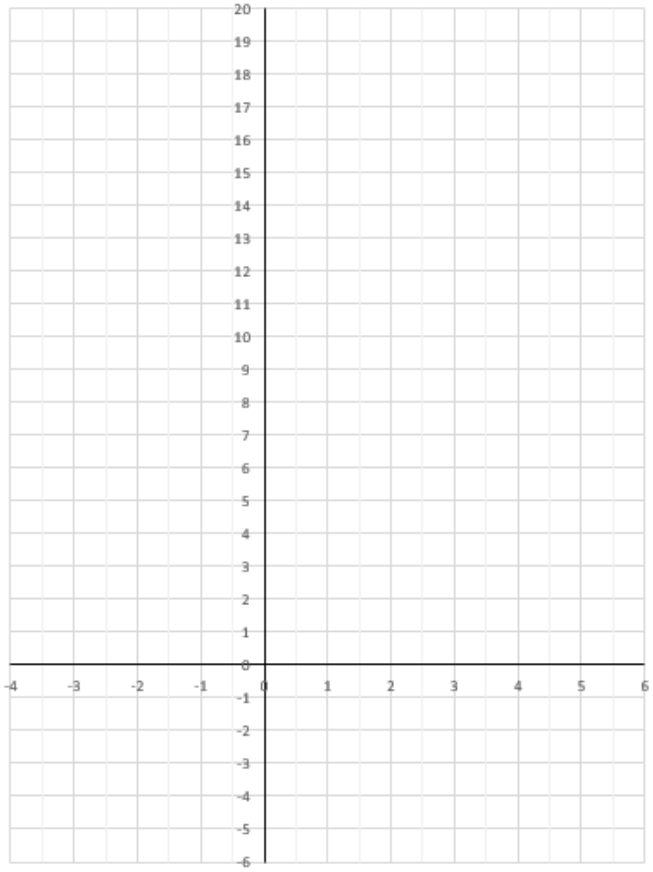
(ii) Is the turning point a minimum point or a maximum point? Justify your answer.

Fluency Practice

(a) Sketch the graph of $y = x^2 - 2x - 3$

Shape – Is it ∪-shaped or ∩-shaped?

Turning Point– Complete the square into the format $(x - p)^2 + q$ where the turning point is (p, q)



Y-Axis – Find out where it crosses the y-axis by putting $x = 0$.

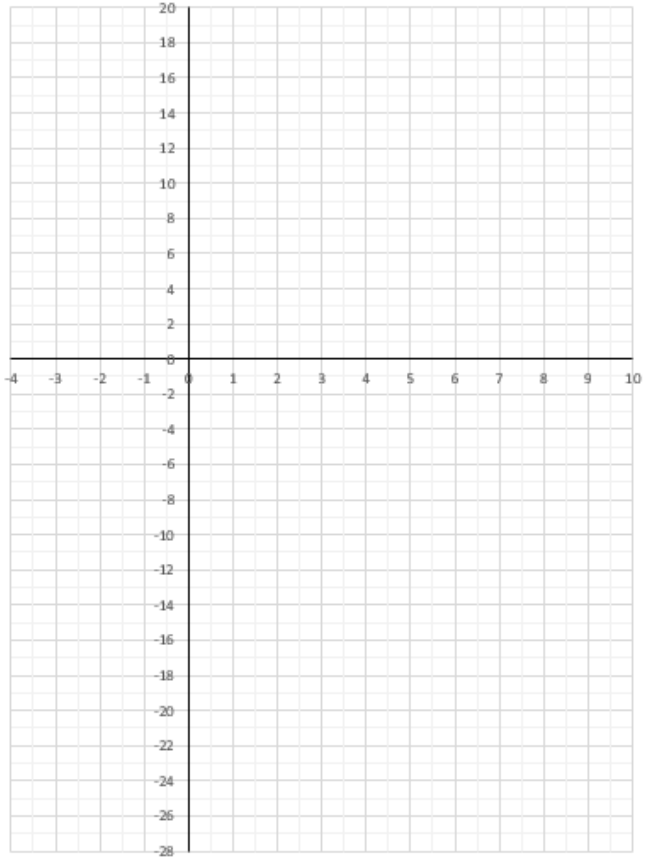
X-Axis – Find out where it crosses the x-axis by putting $y = 0$, then solving the quadratic equation.

Fluency Practice

(b) Sketch the graph of $y = x^2 - 6x - 16$

Shape – Is it ∪-shaped or ∩-shaped?

Turning Point– Complete the square into the format $(x - p)^2 + q$ where the turning point is (p, q)



Y-Axis – Find out where it crosses the y-axis by putting $x = 0$.

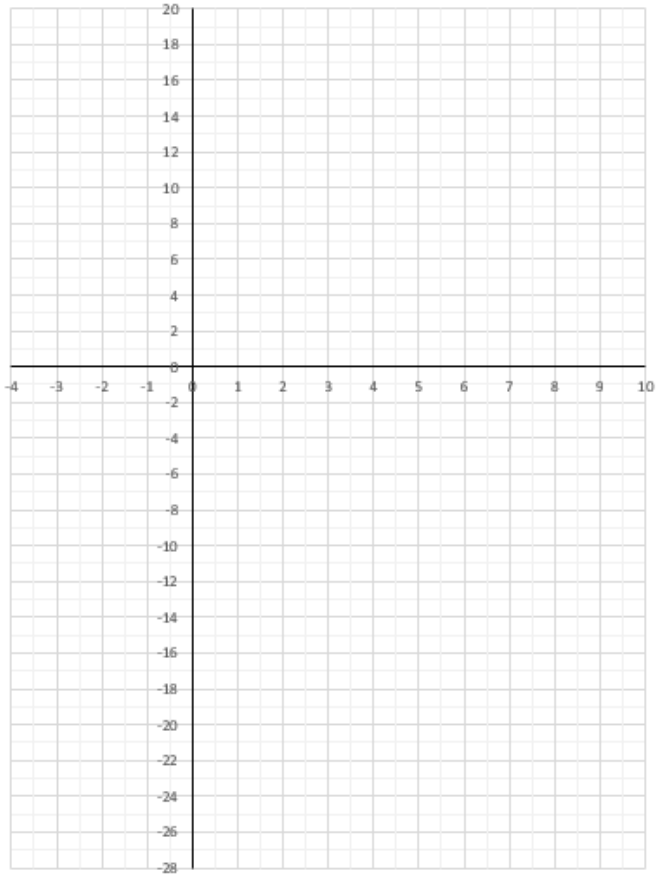
X-Axis – Find out where it crosses the x-axis by putting $y = 0$, then solving the quadratic equation.

Fluency Practice

(c) Sketch the graph of $y = 12 + 4x - x^2$

Shape – Is it ∪-shaped or ∩-shaped?

Turning Point– Complete the square into the format $q - (x - p)^2$ where the turning point is (p, q)



Y-Axis – Find out where it crosses the y-axis by putting $x = 0$.

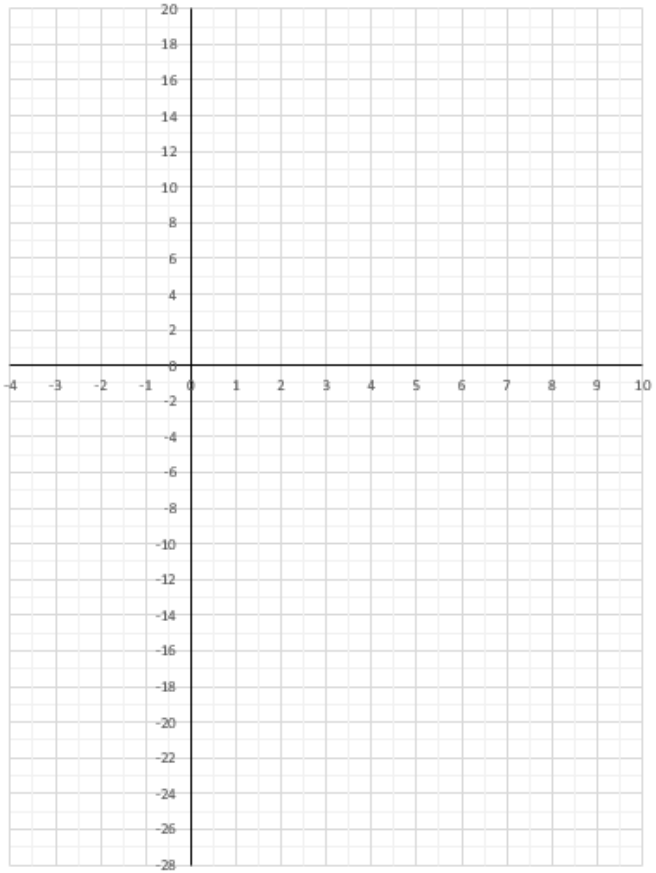
X-Axis – Find out where it crosses the x-axis by putting $y = 0$, then solving the quadratic equation.

Fluency Practice

(d) Sketch the graph of $y = 2x^2 - 4x + 7$

Shape – Is it ∪-shaped or ∩-shaped?

Turning Point– Complete the square into the format $a(x - p)^2 + q$ where the turning point is (p, q)

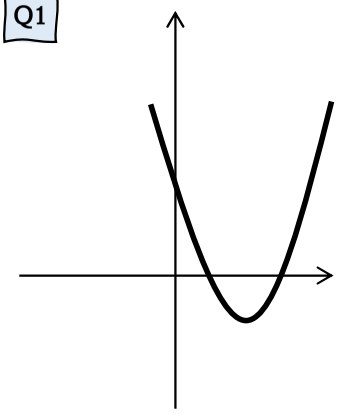


Y-Axis – Find out where it crosses the y-axis by putting $x = 0$.

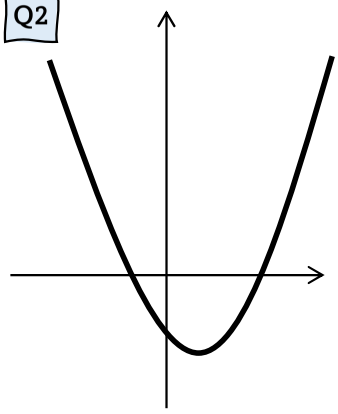
X-Axis – Find out where it crosses the x-axis by putting $y = 0$, then solving the quadratic equation.

In each question, find the roots, the y-intercept and the turning point of the graph.

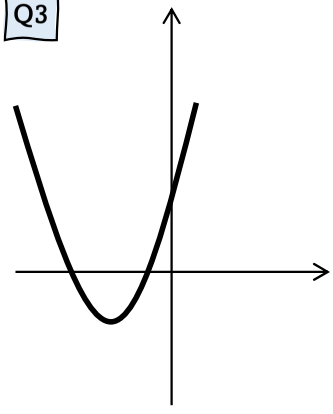
Q1 $y = (x - 2)(x - 4)$



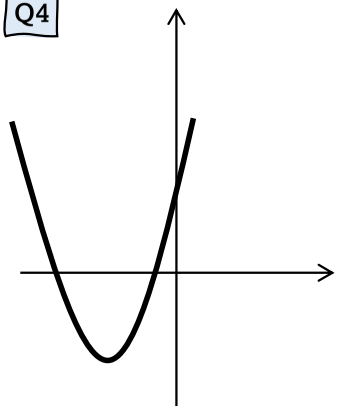
Q2 $y = (x + 2)(x - 4)$



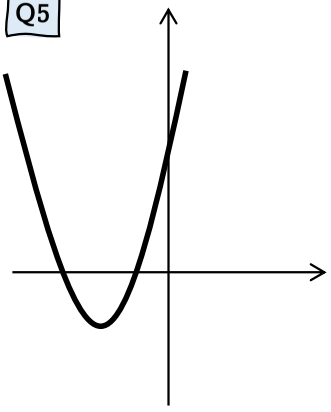
Q3 $y = x^2 + 4x + 3$



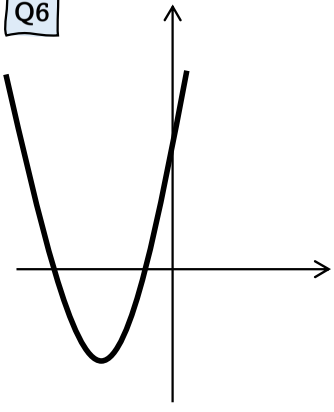
Q4 $y = x^2 + 8x + 12$



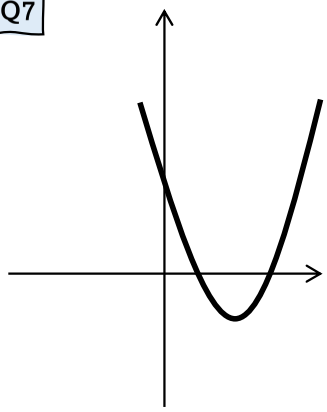
Q5 $y = x^2 + 12x + 35$



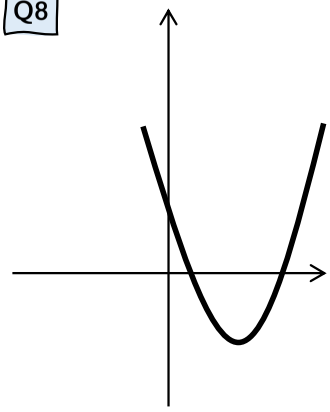
Q6 $y = x^2 + 10x + 21$



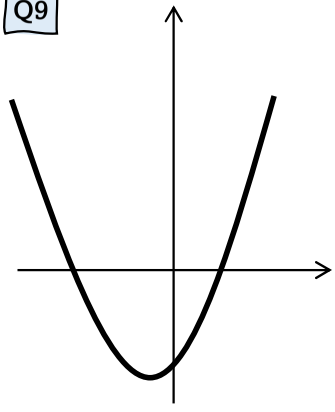
Q7 $y = x^2 - 8x + 15$



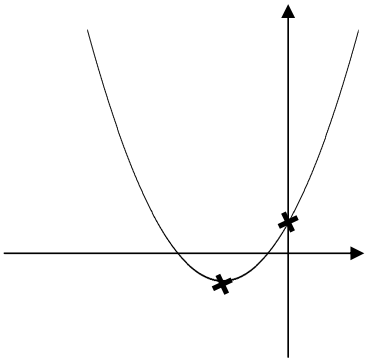
Q8 $y = x^2 - 6x + 5$



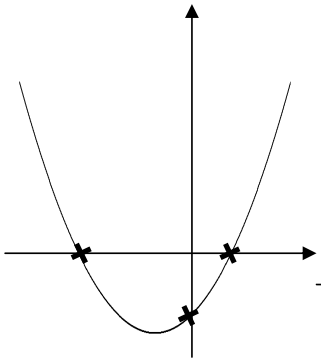
Q9 $y = x^2 + 2x - 24$



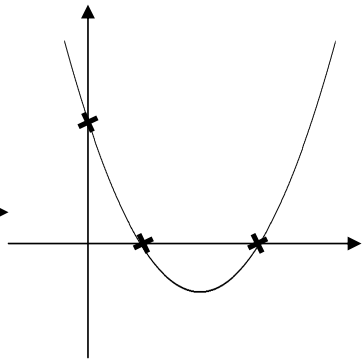
Label the coordinates of all the points marked x.



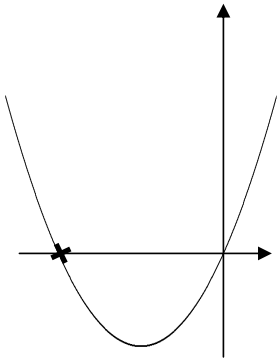
$y = (x + 2)^2 - 3$



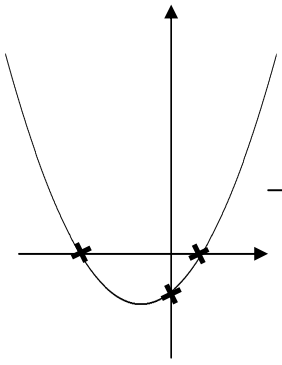
$y = (x + 4)(x - 1)$



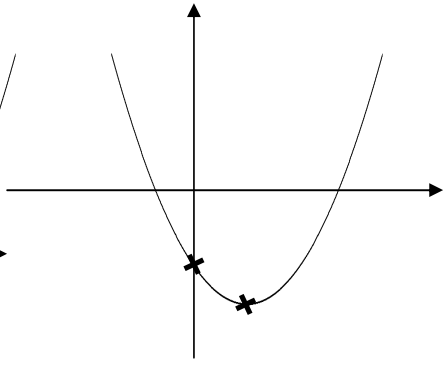
$y = (2x - 1)(x - 3)$



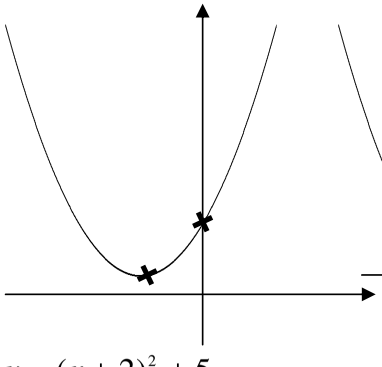
$y = x(x + 4)$



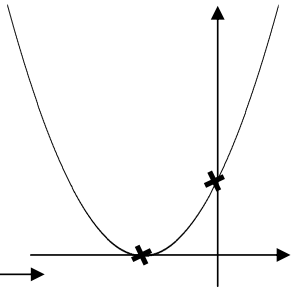
$y = (x - 2)(x + 4)$



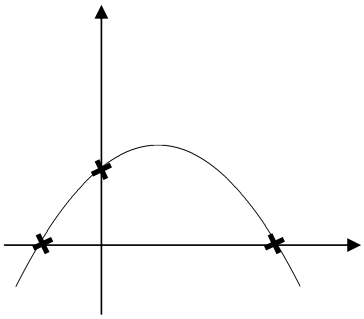
$y = (x - 1)^2 - 3$



$y = (x + 2)^2 + 5$



$y = (x + 3)^2$



$y = (6 - x)(x + 1)$

Fluency Practice

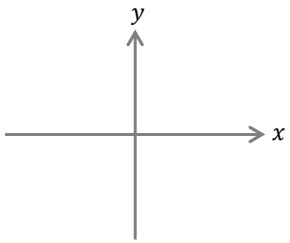
Plotting Quadratics

Factorise each quadratic expression. Use this factorisation to sketch the ***x*-intercepts**.

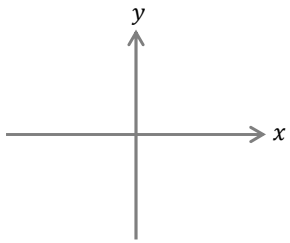
Every quadratic graph is **symmetrical**. Use this fact to find the **turning point** of each graph & its coordinates.

How can we calculate the y-intercept?

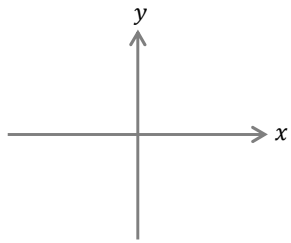
A) $y = x^2 - 4x$



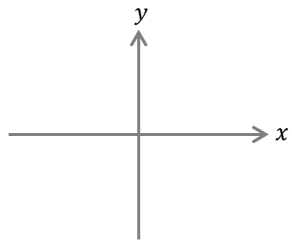
B) $y = x^2 + 8x$



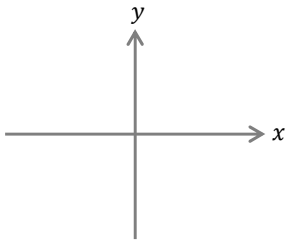
C) $y = x^2 + 6x + 8$



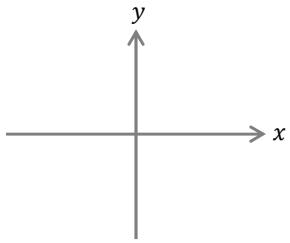
D) $y = x^2 - 8x + 12$



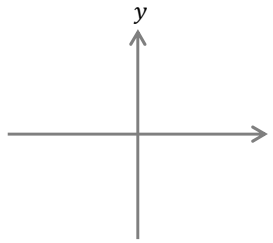
E) $y = x^2 - 2x - 3$



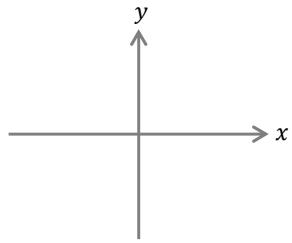
F) $y = x^2 - 9$



G) $y = x^2 - 5x + 6$



H) $y = x^2 + x - 20$

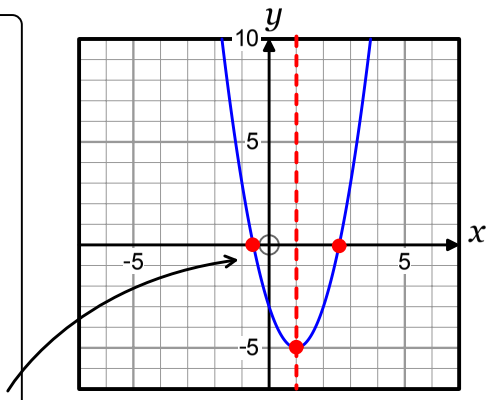


A Quadratic Graph has:

- a **vertex or turning point**
  (1,-5) on the graph shown,
  where the graph 'turns'

- a **line of symmetry**
  $x = 1$ on the graph
  it passes through the vertex

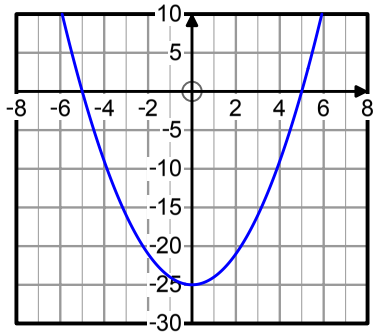
- 0, 1 or 2 **roots** (x intercepts)
  $x \approx -0.7$ and 2.7 for this graph
  this is where the graph crosses the x axis



exercise

1. Write down estimations or exact values of the y-intercept, roots, vertex and line of symmetry for each of these graphs:

a)



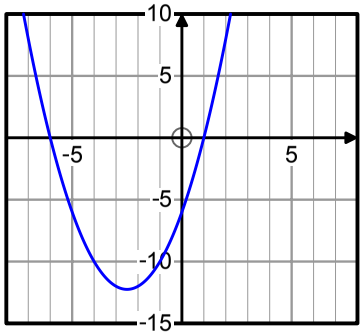
y - intercept:

turning point:

line of symmetry:

roots:

b)



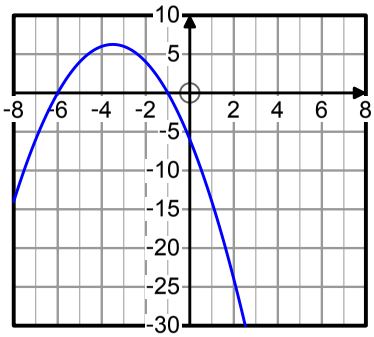
y - intercept:

turning point:

line of symmetry:

roots:

c)



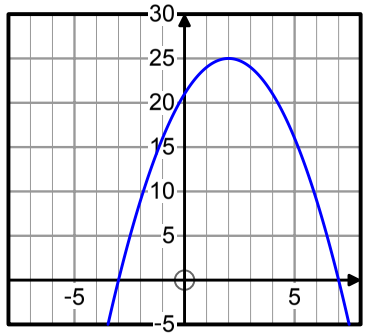
y - intercept:

turning point:

line of symmetry:

roots:

d)



y - intercept:

turning point:

line of symmetry:

roots:

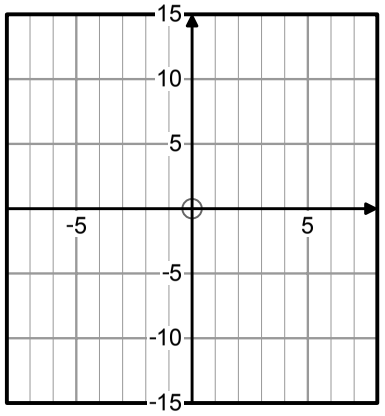
2. Consider the graph $y = (x + 4)(x - 2)$.

a) What are the roots of the function?

b) Can you work out the line of symmetry without drawing the graph?

c) Can you work out the y-intercept without drawing the graph?

d) Sketch the graph of $y = (x + 4)(x - 2)$.



3. Write down the roots, y-intercept and line of symmetry for each of these functions:

a) $y = (x + 4)(x - 4)$

y - intercept:

line of symmetry:

roots:

b) $y = (x + 1)(x - 4)$

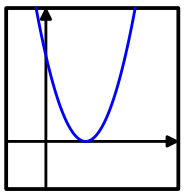
y - intercept:

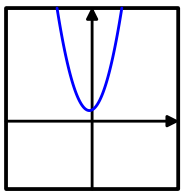
line of symmetry:

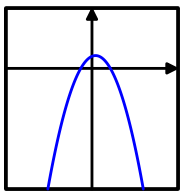
roots:

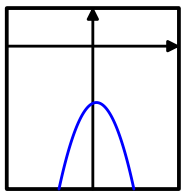
4. By first factorising, write down the roots of $y = x^2 + 8x + 12$

5. Which of these graphs show functions with no roots? Circle all that apply.

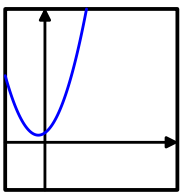
a) 

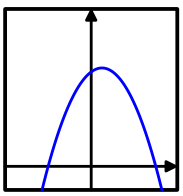
b) 

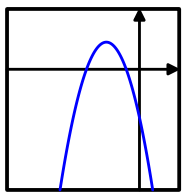
c) 

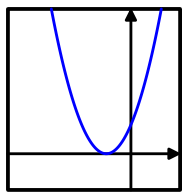
d) 

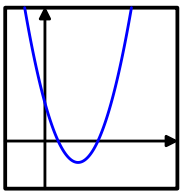
6. Three of the following are graphs of functions with roots at $x = 3$, $x = -2$.
Which three?

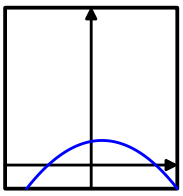
a) 

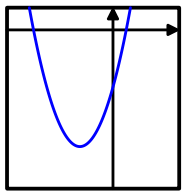
b) 

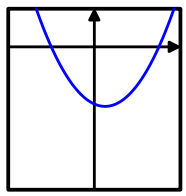
c) 

d) 

e) 

f) 

g) 

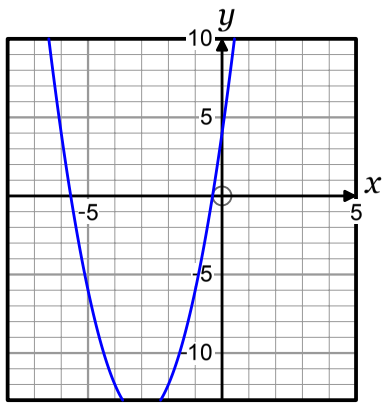
h) 

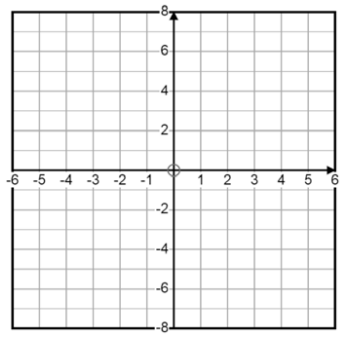
7. *Challenge!*

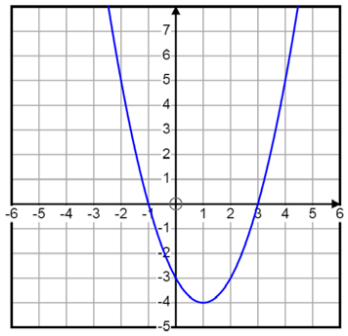
The graph shows the quadratic function $y = 2x^2 + 12x + 4$.

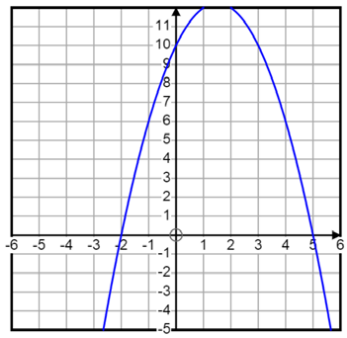
a) Can you work out the values of the roots of the function, correct to 2 decimal places?

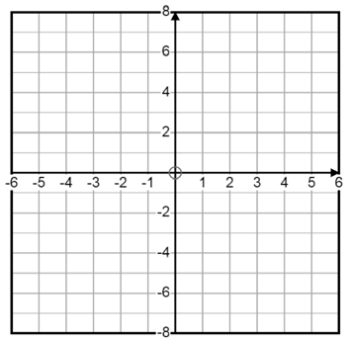
b) The line of symmetry is at $x = -3$. Can you work out the coordinates of the turning point?



A graph of this quadratic would look like: 	The factors of this quadratic are:	The roots of this quadratic are:
	The quadratic is: $x^2 - 5x + 4$	In completed square form this quadratic would be:
The y-intercept of the graph is:	This quadratic crosses the x axis at:	The minimum point of this quadratic is:

A graph of this quadratic would look like: 	The factors of this quadratic are:	The roots of this quadratic are:
	The quadratic is:	In completed square form this quadratic would be:
The y-intercept of the graph is:	This quadratic crosses the x axis at:	The minimum point of this quadratic is:

A graph of this quadratic would look like: 	The factors of this quadratic are:	The roots of this quadratic are:
	The quadratic is:	In completed square form this quadratic would be:
The y-intercept of the graph is:	This quadratic crosses the x axis at:	The maximum point of this quadratic is:

A graph of this quadratic would look like: 	The factors of this quadratic are: **2( )( )**	The roots of this quadratic are:
	The quadratic is:	In completed square form this quadratic would be:
The y-intercept of the graph is: **(0,8)**	This quadratic crosses the x axis at: **x =1, x = 4**	The minimum point of this quadratic is:

More-Same-Less

Instructions: Calculate the turning point in the middle box. Then complete the remaining boxes trying to make the minimal change possible.

Value of the y coordinate of the turning point

Value of the x coordinate of the turning point	Less	Same	More
More			
Same		$y = x^2 + 8x + 5$	
Less			

4 Quadratic Inequalities

Question 1: Solve the following inequalities

(a) $(x - 4)(x - 1) < 0$ (b) $(x - 2)(x + 1) < 0$ (c) $(x + 7)(x + 3) \leq 0$

(d) $(x - 5)(x + 4) \leq 0$ (e) $x(x - 9) > 0$ (f) $(x + 6)(x - 5) > 0$

(g) $(x + 10)(x + 1) \geq 0$ (h) $(x - 7)(x + 7) \geq 0$ (i) $(x + 8)(x + 2) < 0$

(j) $(x - 4)(x + 7) \geq 0$ (k) $(x + 1)(x - 5) \leq 0$ (l) $(x - 12)(x - 11) > 0$

Question 2: Solve the following inequalities

(a) $x^2 + 5x + 6 > 0$ (b) $x^2 + 7x + 10 < 0$ (c) $x^2 - 4x - 5 \leq 0$

(d) $x^2 + 2x - 24 > 0$ (e) $x^2 - 6x + 8 \geq 0$ (f) $x^2 + 3x - 4 < 0$

(g) $x^2 - x - 56 > 0$ (h) $x^2 + 9x + 18 < 0$ (i) $x^2 - 13x + 22 \leq 0$

(j) $x^2 - 4x - 32 < 0$ (k) $x^2 - 64 \geq 0$ (l) $x^2 - 14x + 48 > 0$

Question 3: Solve the following inequalities

(a) $x^2 - 2x < 15$ (b) $x^2 + 6x > x - 4$ (c) $x^2 < 36$

(d) $x^2 > 121$ (e) $2x^2 - x - 12 \leq x^2 - 2x$ (f) $6x > x^2 - 8x + 40$

(g) $x^2 + 6x < 36 - 10x$ (h) $x^2 + 5x + 1 \geq 7x + 25$

Question 4: Solve the following inequalities

(a) $-x^2 + 8x - 15 < 0$ (b) $-x^2 + 3x + 10 \geq 0$ (c) $-x^2 - 6x - 5 \leq 0$

(d) $18x - x^2 - 32 > 0$ (e) $7x + 44 - x^2 < 0$ (f) $-3x^2 + 4x > -4x^2 + 3x$

Question 5: Solve the following inequalities

(a) $(2x - 1)(x - 4) < 0$ (b) $(x + 4)(5x + 1) > 0$ (c) $(2x + 7)(x - 8) \leq 0$

(d) $(3x - 2)(x + 1) \leq 0$ (e) $(4x - 3)(2x - 9) > 0$ (f) $(3x + 2)(3x - 5) \geq 0$

Question 6: Solve the following inequalities

(a) $5x^2 + 7x + 2 > 0$ (b) $3x^2 + 8x - 3 < 0$ (c) $2x^2 - 9x + 4 > 0$

(d) $4x^2 - 3x - 1 \geq 0$ (e) $6x^2 - 13x + 7 < 0$ (f) $2x^2 + x - 6 \leq 0$

(g) $4x^2 - 11x + 6 > 0$ (h) $4x^2 - 27x + 18 > 0$ (i) $15x^2 + 4x - 35 < 0$

Apply

Question 1: Tia has attempted her maths homework.
Can you spot any mistakes?

Solve $x^2 - 8x - 33 > 0$

$(x - 11)(x + 3) = 0$
$x = 11$ or $x = -3$

$x > -3$
$x > 11$

Question 2: (a) Solve $x^2 + 6x + 3 = 0$ giving your answers in surd form.

(b) Solve the inequality $x^2 + 6x + 3 < 0$

Question 3: The set of values for x that satisfies a quadratic inequality is $-5 < x < -2$
Write down a possible quadratic inequality.

Question 4: The set of values for x that satisfies a quadratic inequality is $x < -3$ or $x > 6$
Write down a possible quadratic inequality.

Fluency Practice

Question 5: Find the set of values of x that satisfy both

$2x - 6 > 6 - 6x$ and $x^2 - 6x + 2 < 42$

Question 6: The set of values for x that satisfies a quadratic inequality is $x < -0.5$ or $x > 1.5$
Write down a possible quadratic inequality.

Question 7: The width of a rectangular field is x metres.
The length of the field is 30m longer than the width.
The perimeter of the field is less than 500m.
The area of the field is greater than $4000m^2$.

By writing suitable inequalities, find the possible values of x.

Solve these inequalities.

(a) $(x-3)(x-6) > 0$

(b) $(x-3)(x-6) < 0$

(c) $(x+3)(x-6) < 0$

(d) $(x+3)(x+4) \geq 0$

(e) $x(x+3) > 0$

(f) $(2x-1)(x+3) < 0$

Solve these inequalities.

(a) $x^2 - 6x + 16 > 0$

(b) $x^2 + 7x + 12 \geq 0$

(c) $x^2 - 7x + 12 < 0$

(d) $x^2 - 9x + 20 > 0$

(e) $x^2 - 16 < 0$

(f) $x^2 - 9x < 0$

Solve these inequalities.

(a) $x^2 - 2x > 35$

(b) $x^2 + 2x < 48$

(c) $2x^2 > 11x - 12$

(d) $16x - x^2 \leq 6x$

Find the solution sets for these inequalities.

(a) $\frac{x^2+12}{2} > 4x$

(b) $(x-3)(2x+3) < 2x(1-2x) - 5$

(c) $(x+5)^2 \geq 1$

(d) $(5-x)(x+3) \leq 1$

Fluency Practice

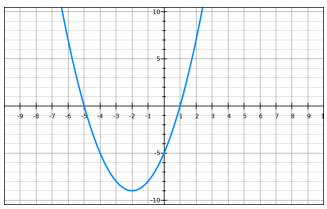
Instructions	1. $x^2 + x - 12 > 0$	2. $x^2 - x - 12 < 0$	3. $12 - x - x^2 > 0$
Factorise to find the roots			
Sketch to find the section or sections needed			
State the range of solutions			
4. $12 - 4x - x^2 < 0$	5. $x^2 + 3x - 10 > 0$	6. $8 + 2x - x^2 < 0$	7. $2x^2 - 6x - 8 > 0$

All but one of these answers are wrong! Can you work out which answer is correct?

A

Solve $x^2 + 4x - 5 > 0$

$(x + 5)(x - 1) > 0$

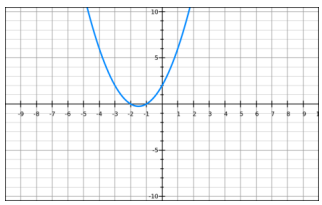


$x \leq -5 \quad x \geq 1$

B

Solve $x^2 + 3x + 2 \leq 0$

$(x + 2)(x + 1) \leq 0$



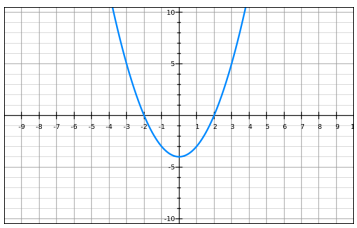
$x \leq -2 \quad x \geq -1$

C

Solve $x^2 < 4$

$x^2 - 4 < 0$

$(x + 2)(x - 2) < 0$



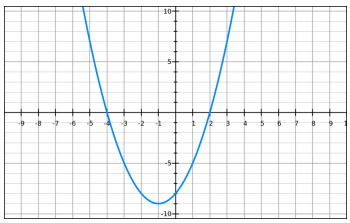
$-2 < x < 2$

D

Solve $x^2 + 2x < 8$

$x^2 + 2x - 8 > 0$

$(x + 4)(x - 2) > 0$

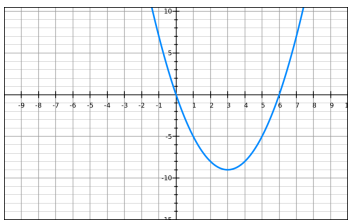


$x < -4 \quad x > 2$

E

Solve $6x - x^2 > 0$

$x(6 - x) > 0$

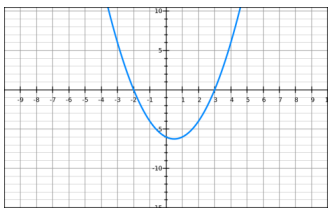


$x < 0 \quad x > 6$

F

Solve $x^2 + 5x - 6 \geq 0$

$(x + 2)(x - 3) \geq 0$



$x \leq -2 \quad x \geq 3$

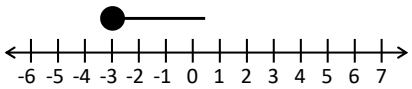
Fluency Practice

Quadratic Inequalities

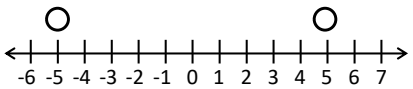
We can form & solve an **equation** to find the two **bounds**.

Solve each inequality and complete the answer on each number line.
How can you **check** you have marked the **correct region** of the number line?

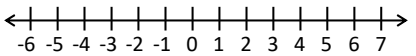
A) $x^2 \leqslant 9 \qquad x^2 = 9$



B) $x^2 > 25$



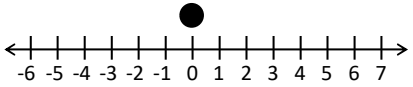
C) $x^2 < 25$



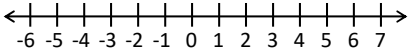
We need to **factorise** to find the two bounds.

D) $x^2 + 3x \leqslant 0$

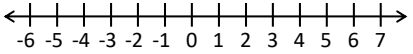
$x(x + 3) = 0$



E) $x^2 + 5x < 0$



F) $x^2 - 4x \geqslant 0$

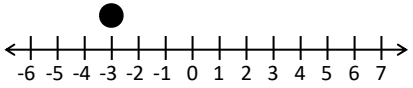


We will factorise into two brackets.

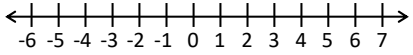
G) $x^2 + 4x + 3 \geqslant 0$

$(x + 3)(x + 1) = 0$

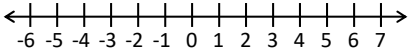
$x + 3 = 0 \qquad x + 1 = 0$



H) $x^2 + 6x + 5 > 0$



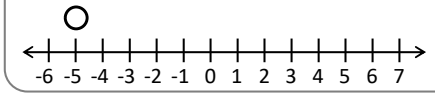
I) $x^2 - 2x - 24 \leqslant 0$



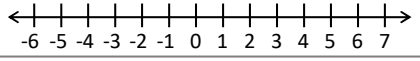
We need to **rearrange** before factorising.

J) $x^2 + 2x < 15$

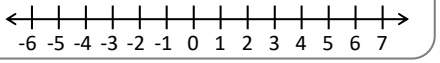
$x^2 + 2x - 15 = 0$



K) $x^2 - 6x \geqslant -8$



L) $x^2 > 3x + 28$



Fluency Practice

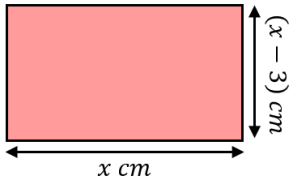
A1 Solve $x + 5 > 11$	**A2** Solve $4x + 11 \leq 29$	**A3** Solve $7 - x \geq 15$	**A4** Work out the integer values of x that satisfy both the inequalities $3x - 4 \leq 11$ and $2x + 3 > 9$
B1 Solve $\frac{2x+5}{3} > 7$	**B2** Solve $6x + 3 \leq 2x + 19$	**B3** Solve $3x + 9 < 4x + 5$	**B4** Work out the lowest integer which satisfies the inequality $5x - 2 \geq 3x + 7$
C1 List the integer values for x if: $-3 \leq x < 4$	**C2** List the integer values for x if: $-5 \leq 5x \leq 15$	**C3** List the integer values for x if: $4 \leq 3x + 1 < 12$	**C4** List the integer values for x if: $2x < 3x + 1 < 13$
D1 Solve $x^2 - 7 < 42$	**D2** Solve $3x^2 - 17 < 31$	**D3** Solve $5x^2 - 13 \leq 32$	**D4** List the integer values for x if: $2x + 3 < 4x + 5 \leq 3x + 7$

Fluency Practice

Solving Quadratic Inequalities in Context

(a)

A rectangle has sides of length $x\ cm$ and width $(x-3)\ cm$, as shown. If the area of the rectangle is greater than $10\ cm^2$:

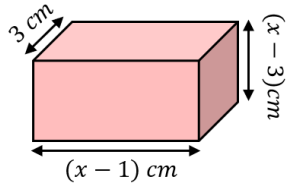


(i) Show that $x^2 - 3x - 10 > 0$

(ii) Find the range of possible values of x.

(b)

A cuboid has dimensions of $3\ cm$, $(x-1)\ cm$ and $(x-3)\ cm$, as shown. If the volume of the cuboid is greater than $45\ cm^3$:

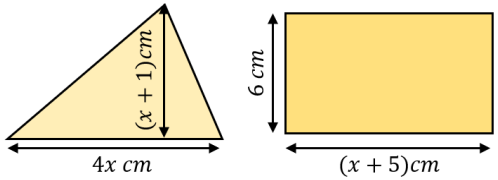


(i) Show that $x^2 - 4x - 12 > 0$

(ii) Find the range of possible values of x.

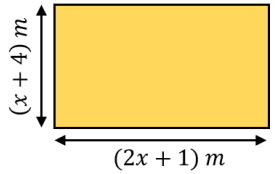
(c)

Given that the area of the rectangle is greater than the area of the triangle, find the range of possible values of x.



(d)

A rectangular lawn has a length of $(2x+1)\ m$ and a width of $(x+4)\ m$, as shown. Given that the area of the lawn is less than $49\ m^2$, find the range of possible values of x.

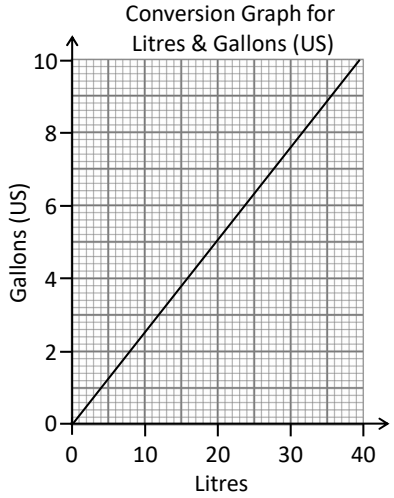


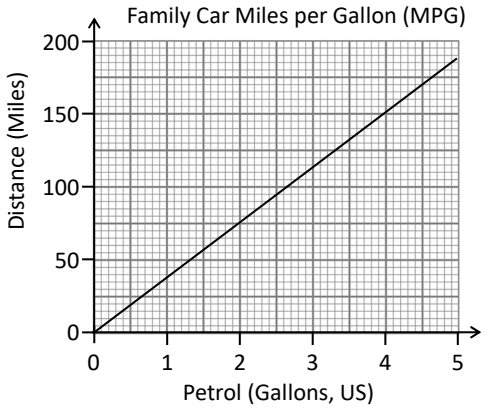
5 Real Life Graphs

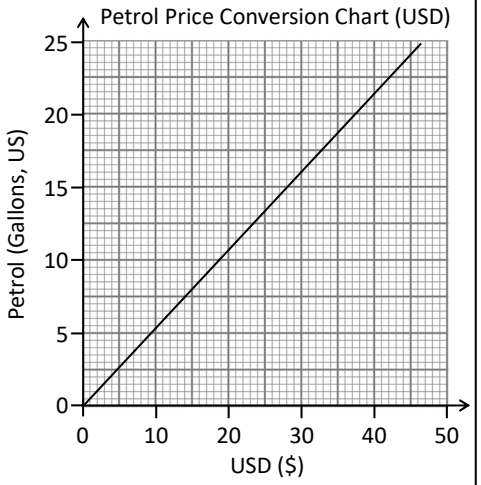
Fluency Practice

Reading Conversion Graphs

Use the conversion graphs to complete the tables.
Remember! Readings from conversion graphs are **estimates**.





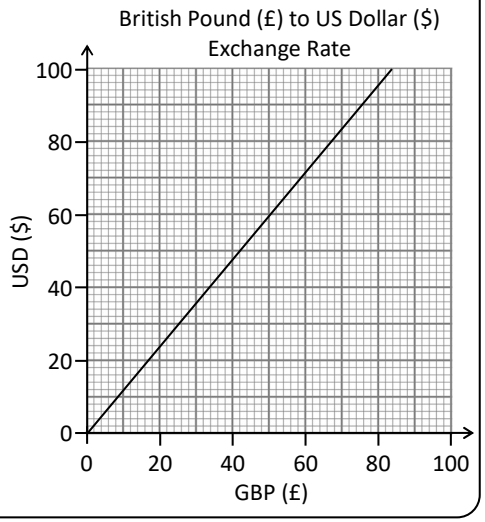


Litres	Gallons
20	
35	
	8
	12

Petrol (Gallons)	Distance (Miles)
3	
4.5	
	150
	300

USD ($)	Petrol (Gallons)
30	
42	
	11
	55

GBP £)	USD ($)
80	
250	
	70
	96
	250



a) In the USA, a family drives 140 miles. How many gallons of petrol did they use?
How many litres is this? How much did this cost?

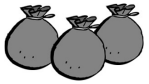
b) A family from the UK goes on holiday to the US. How much petrol can they buy with £35?
(They must exchange pounds for dollars first.)

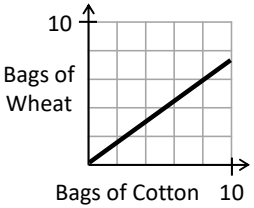
c) In the UK you can buy 25 litres of petrol for £14. Is this cheaper or more expensive than in the USA?

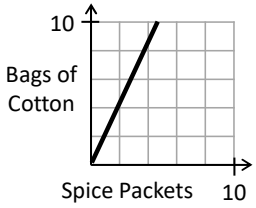
d) From San Francisco to Yellowstone is 550 miles. How much would this cost a British holidaymaker in GBP?

Fluency Practice

Ancient TRADERS

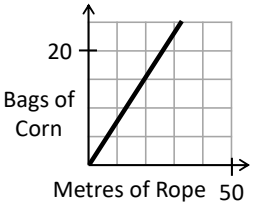


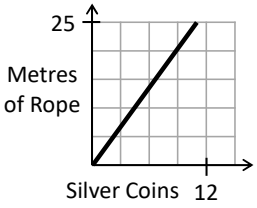




Trex has
6 bags of wheat.

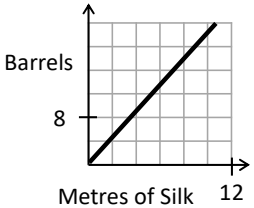
How many spice packets can they buy?

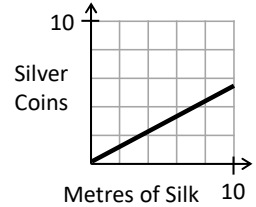




Ati has
15 bags of corn.

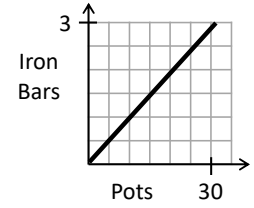
How many silver coins can they buy?

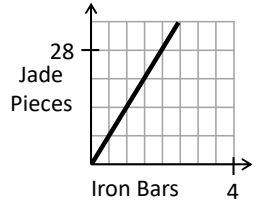




Mitith has 20 barrels.

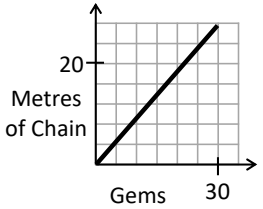
How many silver coins can they buy?

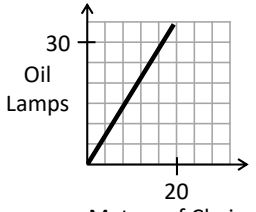




Dares has 15 pots.

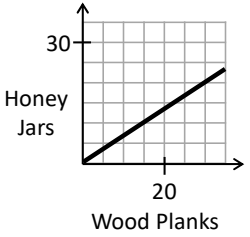
How many jade pieces can they buy?

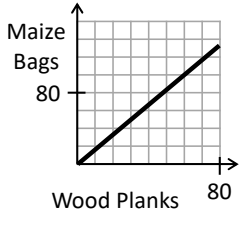




Arish has 25 oil lamps.

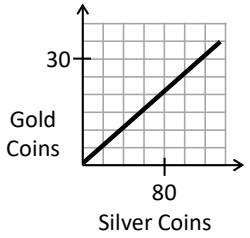
How many gems can they buy?

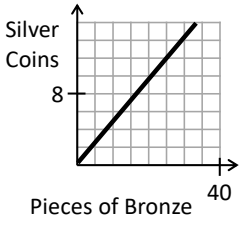




Oliz has 40 jars of honey.

How many bags of maize can they buy?





Poli has 25 pieces of bronze.

How many gold coins can they buy?

Approximately…

How many metres of rope can you buy with 15 gold coins?

How many pieces of bronze can you buy with 40 metres of silk?

Hegarth has 240 barrels. They will convert half to pieces of bronze and half into bags of corn. How much will they get of each?

Fluency Practice

My ship's come in!

Ships enter port and unload their goods to the market.
For each ship, **approximately** how many **gold coins** will they get for their **cargo**?

THE ST. JULES

60 bags of wheat
22 kg of red spice

MARISS HADDOCK

500 bags of wheat
500 kg of salt
36 kg of red spice

BADGER IV

110 oranges
6 Tonnes of salt
40 kg of red spice

CORNWALL TRADERS III

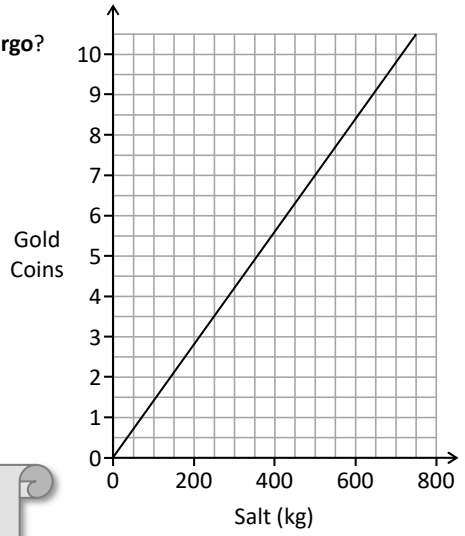
250 bags of wheat
65 oranges

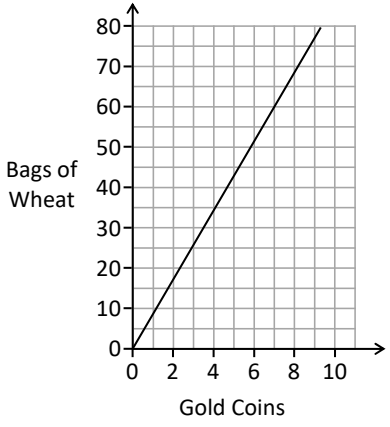
THE VICTORY

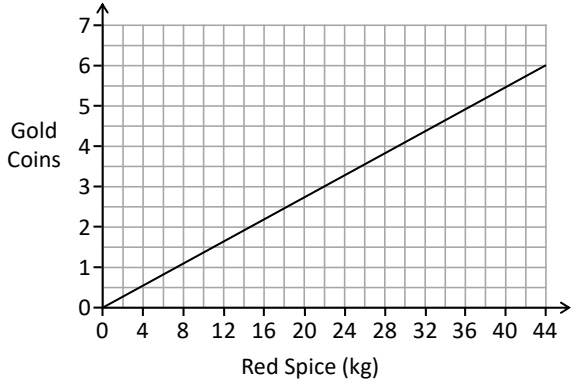
280 oranges
250 kg of salt
15 kg of red spice

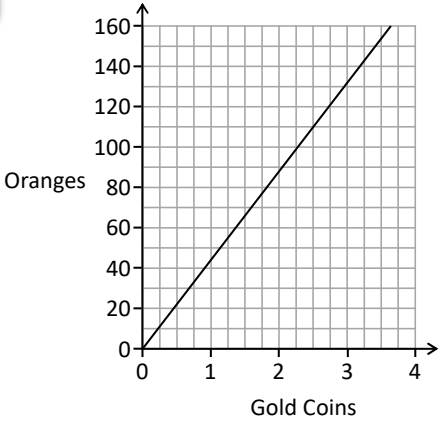
BETSY ROSS

650 bags of wheat
420 kg of salt
900 kg of red spice
1000 oranges



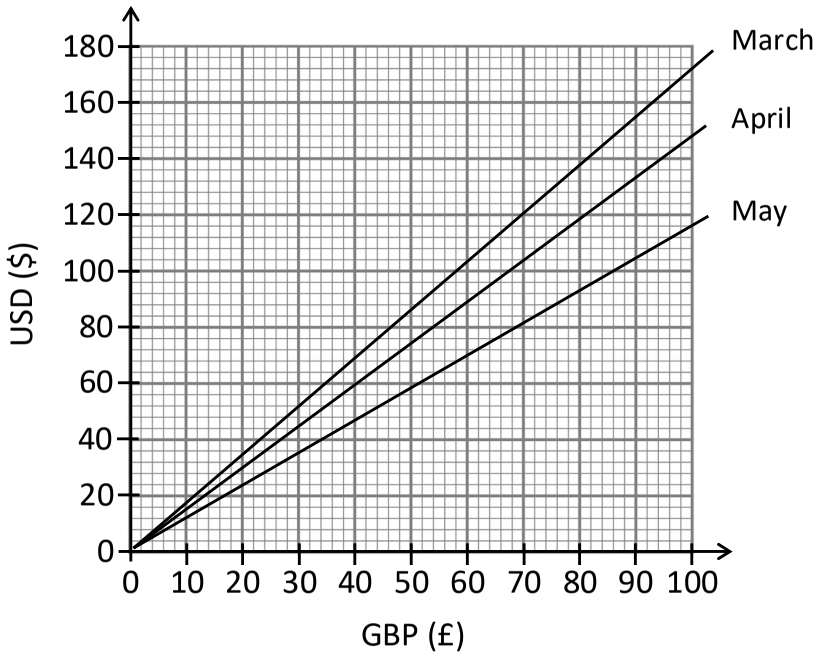






Time to Travel

This conversion graph shows the exchange rate between British Pounds and United States Dollars for 3 different months.



Reading means estimating...

How can we be accurate?

a) In March Hugh went to America and spent $300.
Estimate how much they spent in Pounds.

b) When Kim travelled to America in May, how much did it cost to exchange $300 to GBP?

c) Jack travelled to Dallas in April and exchanged £400 to USD.
How much did it cost them?

d) If Jack went in March, how many more Dollars would they have received?

e) Sam works in Milwaukee but is from Birmingham. If they are paid $1160 every week, how did their pay in GBP change from March to May?

f) The conversion line for June has a higher gradient than the conversion line for May. What will this mean for someone exchanging Pounds to Dollars?

g) Complete these statements:

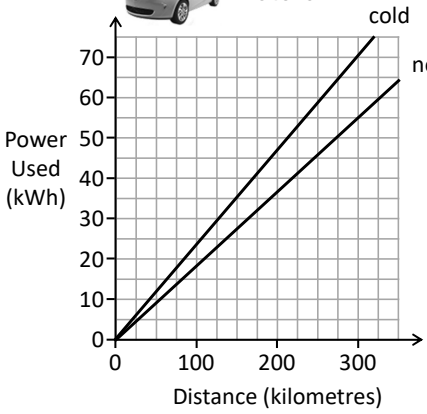
In March £1 = In April £1 = In May £1 =

h) If the conversion line for September has a gradient of 1.5, describe its position on the conversion graph.

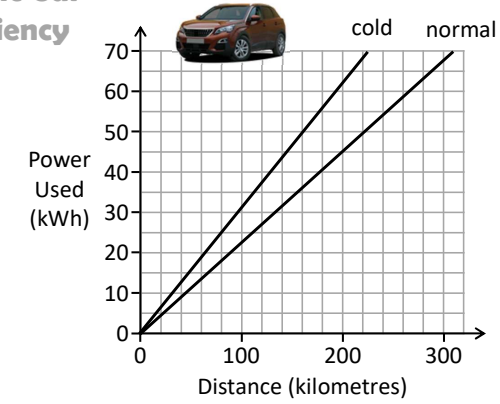
Fluency Practice

Electric Car Efficiency

Xeta 5

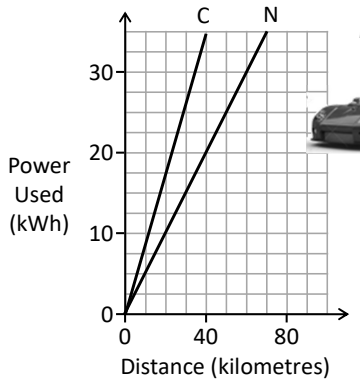


Colorado XT

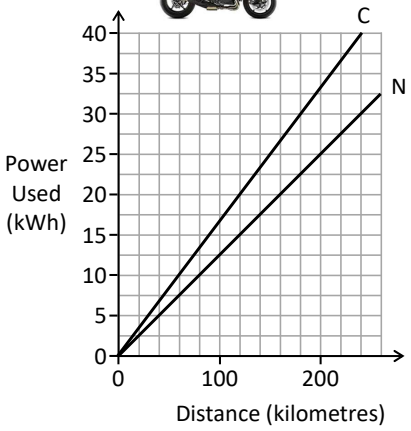


1 kWh = 1 kilowatt hour = 1000 watts of electricity for an hour

Migani



Wasp



Automax Magazine tests four electric vehicles by driving them and recording how much battery power has been used. Cold temperatures affect the rate the battery is used. How? Why?

a) How much battery power does the Xeta 5 use to travel 300 km in normal conditions?

b) How much power does the Xeta 5 use to travel 300 km in cold conditions?

c) How far can the Colorado XT travel using 50 kWh in normal conditions?

d) How much less is this distance in cold conditions?

e) The Migani has a 90 kWh battery. What is the furthest it can travel on a full charge?

f) In the same conditions, how much power would the Wasp use to travel the same distance?

g) The Xeta 5 has a battery with a capacity of 40 kWh.

i) Estimate how far it can travel in cold conditions on two charges of the battery.

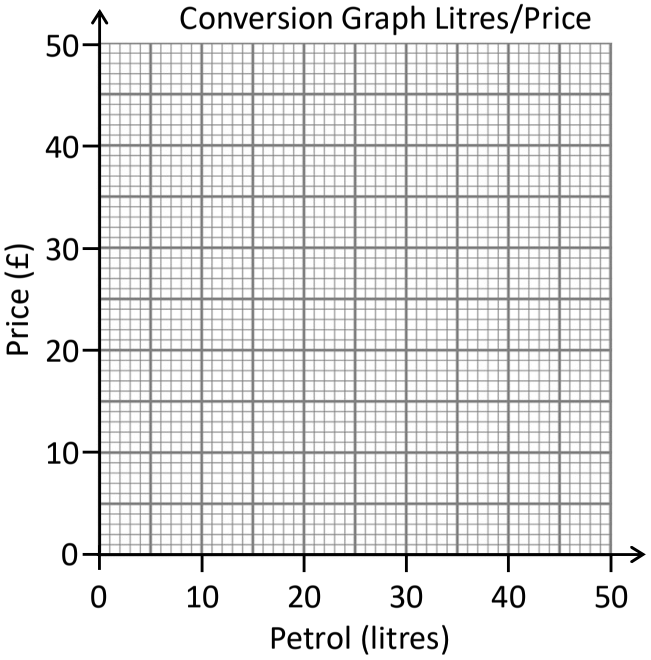
ii) With a battery capacity of 60 kWh, how much further can the Colorado XT travel?

iii) In the same test the Wasp travels 320 km. Estimate the size of its battery.

h) With a full charge, estimate how many km are lost in cold conditions for each vehicle.

i) By comparing percentage losses, which vehicle does best in cold conditions?

Creating Conversion Graphs



Mary drives for work.
She bought fuel 6 times over 2 weeks & recorded the quantity (litres) and cost (£).

40 L	£45	30 L	£35
15 L	£17	25 L	£30
8 L	£10	19 L	£21

Plot this data on the grid.

How can you draw a conversion line for the price of fuel?

Could you use a mean?

Mary will buy 35 litres of fuel, how much should she expect to pay?

Plastex LTD records the price ($) it pays for barrels (B) crude oil.

25 B	$1300	23 B	$1000
44 B	$2200	45 B	$950
31 B	$1700	15 B	$600
20 B	$1000	35 B	$2050

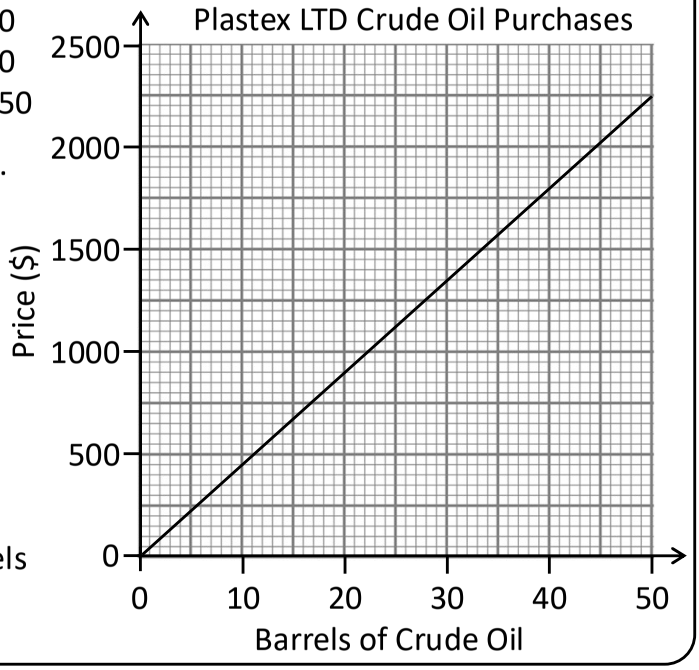
Plot this data on the grid.

Find the mean quantity, & the mean price.

Plot this & use it to create a conversion line.

Comment on the **anomaly** & its effect.

If Plastex bought 120 barrels how much would they expect to pay?



Sugar SHOCK!

Complete the conversion graph to show the sugar content of these foods.

Cookies have 15 grams of sugar in every 100 grams.

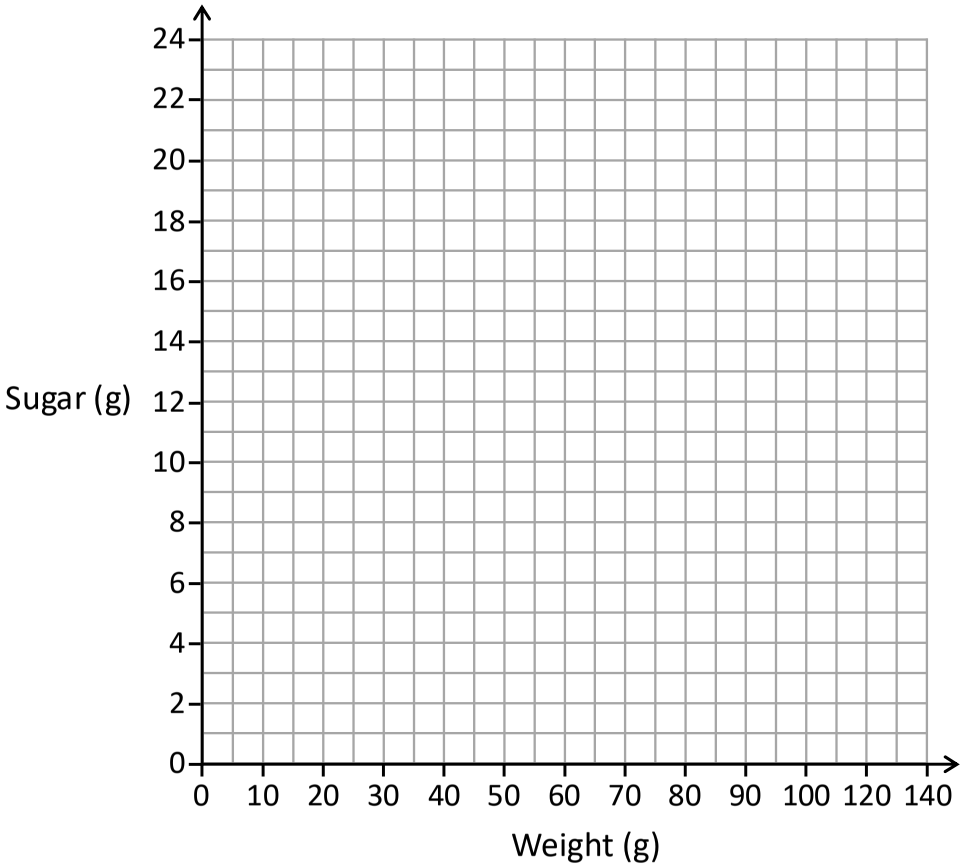
These is 96 grams of sugar in 200 grams of **chocolate**.

Shop-bought **tea** has about 0.7 grams of sugar in 10 grams

In a **doughnut** there is 11.5 grams of sugar in every 50 grams.

An **ice cream** contains 3.6 grams of sugar per 20 grams.

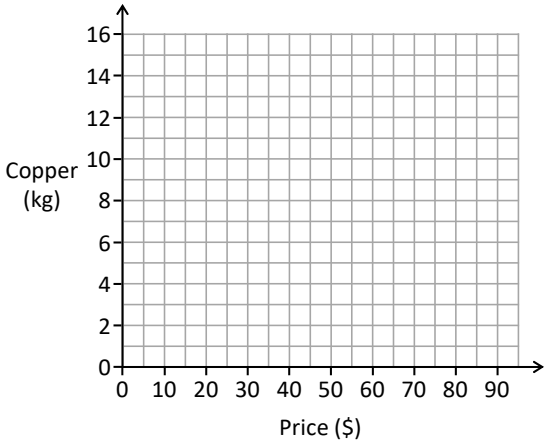
A 250 gram **soda** contains 30 grams of sugar.



Tay wants to reduce their sugar intake. If they want to only eat 20 grams of sugar, how much of each food can they eat?

Fluency Practice

Mineral Market



In February, Jax Jewels Ltd bought 13 kg of copper for $80.
In March they bought 9 kg for the same price.

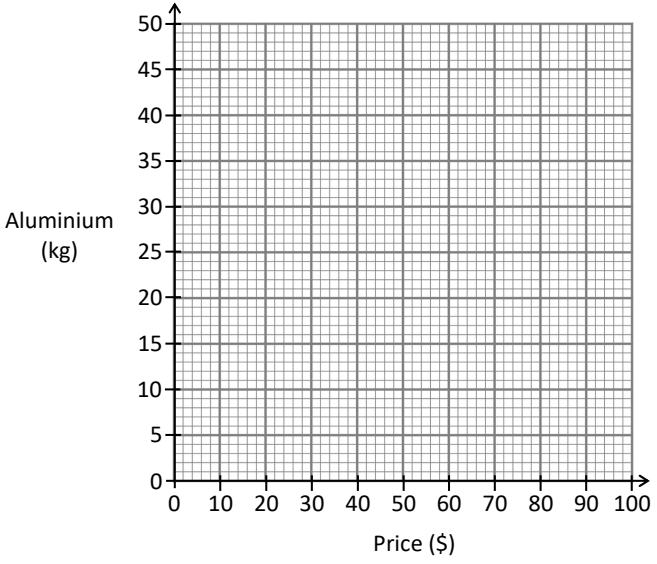
Plot 2 conversion graphs for the 2 different prices.

The company will buy another 20 kg of copper in March.
Approximately how much will it cost?

Krystal Designs Ltd buy $50 of copper every month.
How much less copper did they receive in March compared to February?

In April the price of copper doubles.
Draw a third conversion graph for this new price.

How much would 30 kg of copper cost in April?



In 2022 Maltrex Industries buys 300 kg of aluminium for $900.
In 2024 they bought 240 kg of aluminium for $420.

Plot these conversion graphs.

Tech-Nox buy 150 kg of aluminium every year.
Approximately how much did the price decrease between 2022 and 2024?

Industri-Corp bought $40,000 of aluminium in 2022.

How much is the aluminium worth now?

'Low-Grade Aluminium' is not as pure, but it is 20% cheaper.
Plot a conversion graph for the price of
low-grade aluminium in 2024.

How much does 120 kg of low-grade aluminium cost in 2024?

Fluency Practice

Taxi Travel

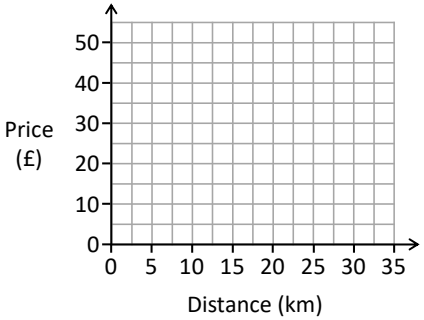
Taxis charge a **price per mile**. But they also charge a **fixed-charge** to collect you.

On-Time Taxis

Francis travelled 20 km for £35.

Jay travelled 30 km for £45.

a



Use these two data points to plot a conversion graph for On-Time Taxis.

How much is the fixed-charge?

How much would it cost to travel 15 km?

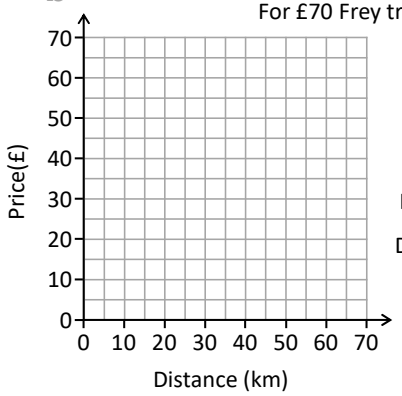
After the fixed-charge, what is the price per kilometre?

Luxury Point-to-Point

Meena travelled 10 km for £40.

For £70 Frey travelled 25 km.

b



Plot a conversion graph for this taxi company.

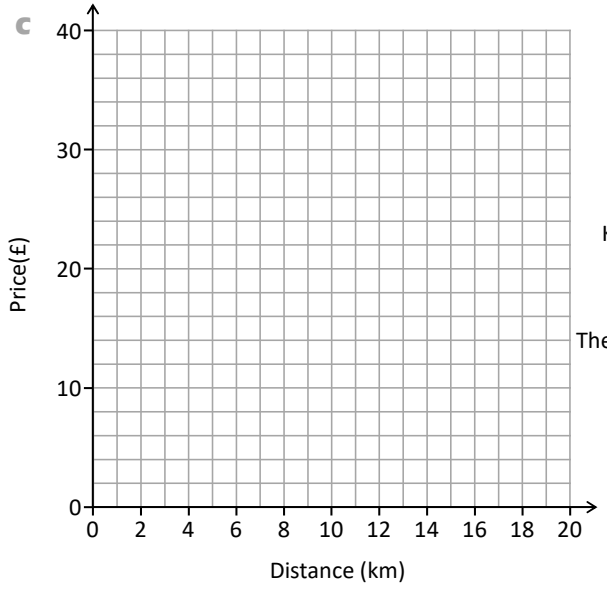
How much is the fixed-charge?

How far can you travel for £50?

Describe the gradient of the line as the price-per-kilometre.

---

c



Use this information to plot a conversion line for each company.

Ted's Taxis

Kay travelled 8 km for £18.

Kim spent £26 to travel 16 km.

Clarice's Cabs

The company has a £4 fixed-charge.

Dan travelled 17 km for £38.

Distance Drivers LTD

50p per kilometre

A £16 fixed-charge.

Hei wants to travel 4 km.

For each company, how much would it cost Hei?

For each company, how far can you travel for £24?

For what distance is Ted's Taxi's the cheapest option?

Describe the gradient of each conversion line.

Liam's Limos prices can be described with an equation:

$y = 2x + 20$ x is the distance in km

y is the total price

How much does it cost to travel 10 km?

Plot a graph for Liam's Limos on the grid.

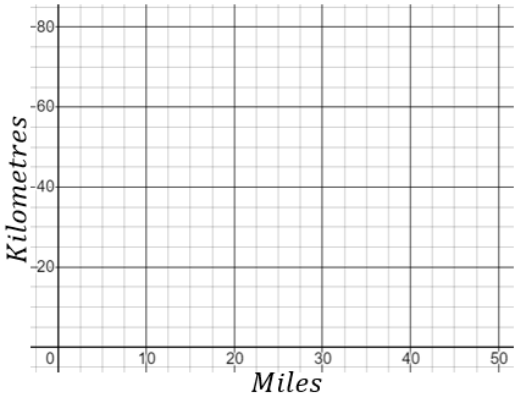
How can you describe the other companies using x and y?

Conversion Graphs

(a)

(i) Complete the table to convert miles to kilometres, then draw a conversion graph.

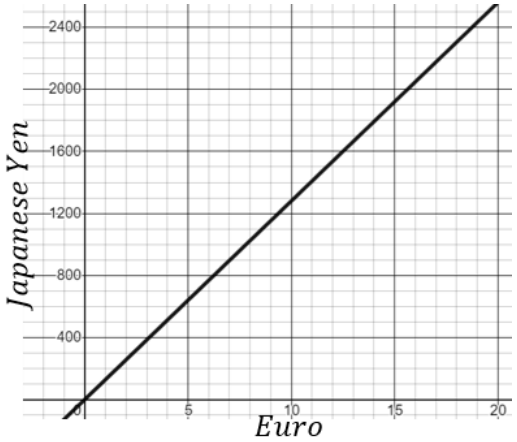
Miles	10	20	30	40	50
Kilometres	16			64	



(ii) Use the graph to convert 55 km into miles.

(b)

The graph shown converts euros to Japanese yen.

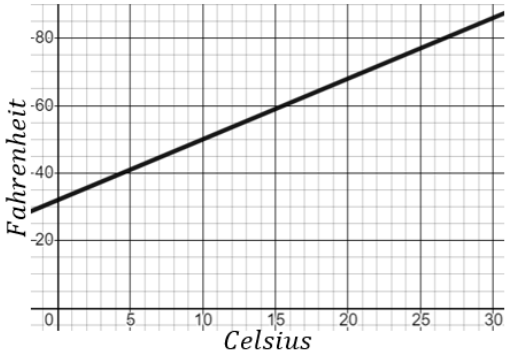


(a) Convert 10 euros to Japanese yen.

(b) Convert 400 Japanese yen into euros.

(c)

The graph shown converts temperatures from Fahrenheit (℉) to Celsius (℃).

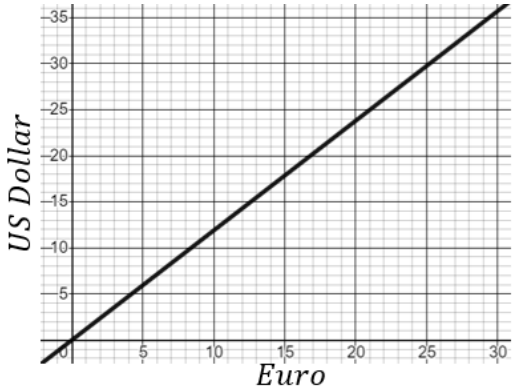


(a) Convert $10℃$ into ℉.

(b) The temperature of a room increases by $20℉$. What is the equivalent increase in ℃ ?

(d)

The graph shown converts Euros to US dollars.

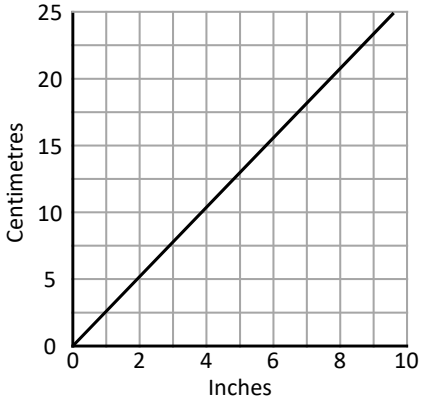


Given that $1\ Euro\ =\ 10.2\ Swedish\ krona$, convert 250 US dollars into Swedish krona.

Fluency Practice

Conversion Graphs

A) This graph shows the conversion rate from inches to centimetres.



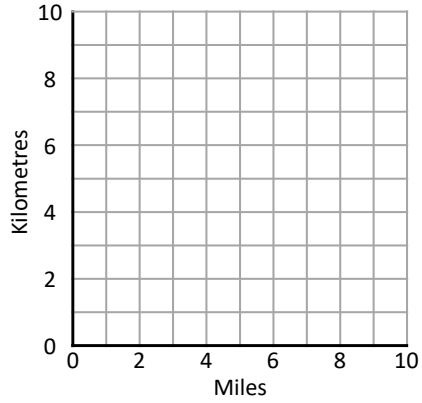
Use the graph to complete this table.

Remember, we **estimate** when reading from a graph.

Inches	Centimetres
6	
4	
9	
	5
	13
	19

Describe how we can use the graph to convert 80 inches to centimetres.

B)



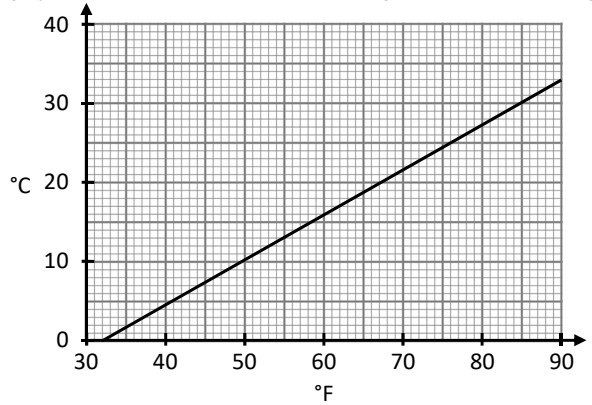
5 miles ≈ 8 kilometres
0 miles = ? kilometres

Use these data points to plot a conversion graph.

Use this line to complete the table.

Miles	Kilometres
2	
3	
12	
	10
	7
	70

C) This graph shows the conversion rate from degrees Fahrenheit to degrees Celsius.



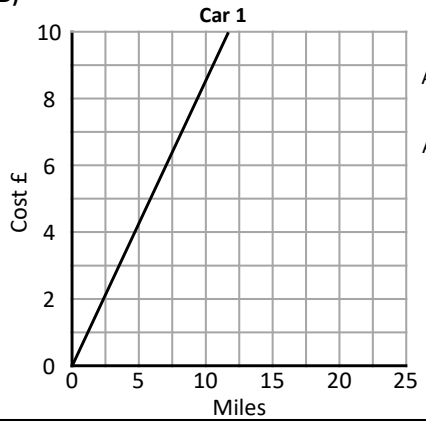
What is different about this conversion graph?

1) 70°F ≈ °C 2) 47°F ≈ °C 3) 24°C ≈ °F

4) When water is frozen, what is approximate temperature in °F?

5) Use this fact to convert 60°C to °F.

D)



This graph shows the cost of running a car (petrol + maintenance).

A different car costs £5 to go 13 miles. Draw the conversion line for this car.

Approximately how much does it cost each car to travel 40 miles?

Car 1

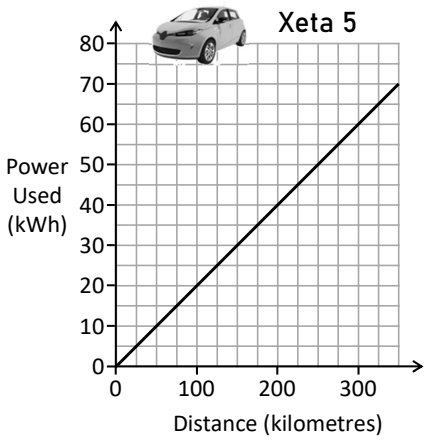
Car 2

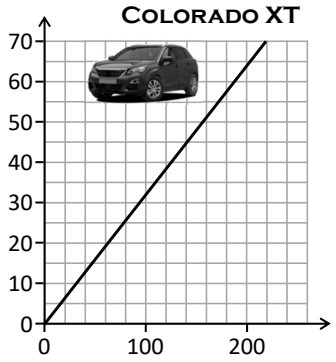
How much cheaper is it to travel 50 miles in Car 1 compared to Car 2?

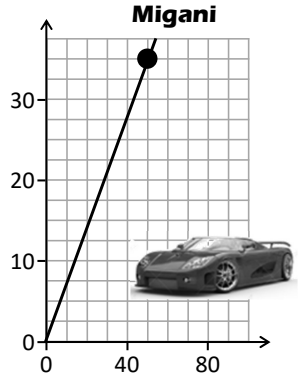
Fluency Practice

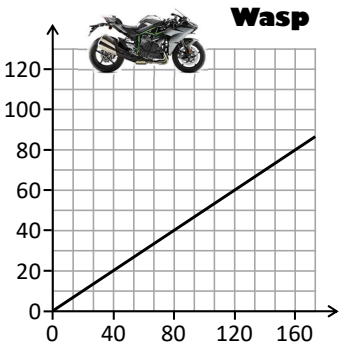
Combining Conversion Graphs

Researchers tested 4 electric vehicles by recording the **distance travelled** and the **battery power used**.









Use the research data to complete a conversion graph that shows all four vehicles.
How can you make the graph as accurate as possible?

Which vehicle is most efficient?

How far does each vehicle travel using 25 kWh of power?
How much battery would each vehicle use to travel 300 km?

Complete the table below using approximations.

	Xeta 5	Colorado XT	Migani	Wasp
25 kWh				
300 km				

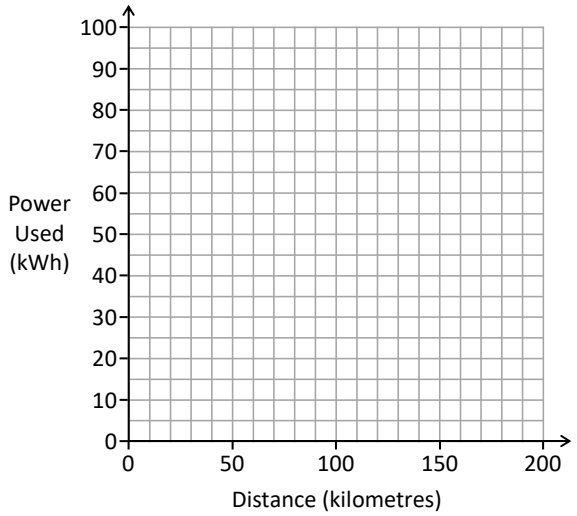
As a percentage, approximately how much further will the Colorado XT travel, compared to the Wasp with the same charge?

The E-Scoot electric scooter travels 15 km per kWh.
The Land-Racer XR uses 0.9 kWh per kilometre.

Add these vehicles to the conversion graph, how do they compare to the other vehicles?

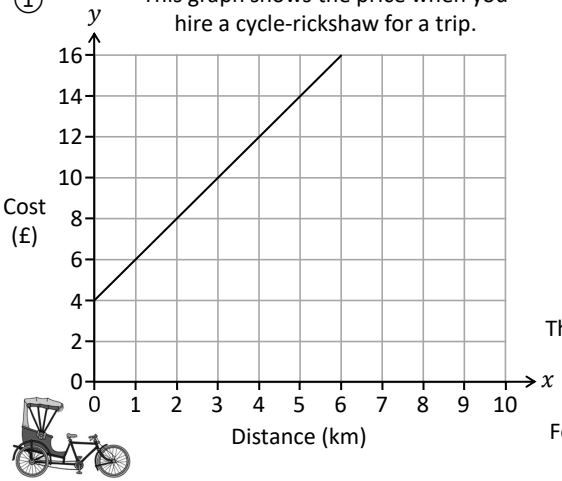
At what **rate** does the Xeta 5 use power? Express this as **kWh per kilometre**.

How does the rate of power consumption of the Migani compare to the Xeta 5?



Fluency Practice

① This graph shows the price when you hire a cycle-rickshaw for a trip.



How much is the fixed charge?
(the cost before you travel any distance).

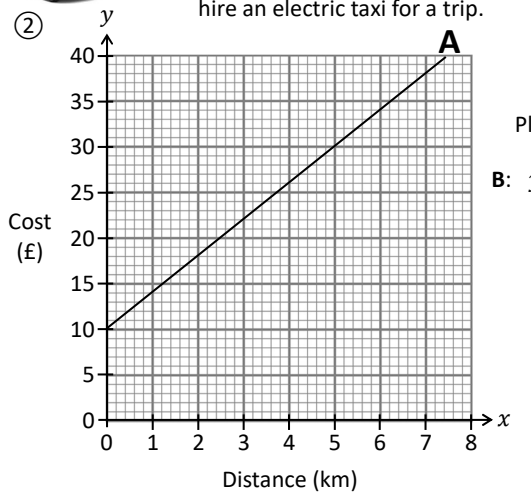
How much does it cost per km?

Use the gradient (cost per km)
& y-intercept (fixed charge)
to form a cost equation in
the form $y = mx + c$

The formula for a different company's cost is
$y = x + 8$
Plot this graph on the grid.

For what distance is this company cheaper?

② This graph shows the price when you hire an electric taxi for a trip.



How much does the taxi cost per km?
Describe the graph as a cost-equation.

A: _____

Plot these 3 different companies on the same grid.

B: $y = 5x$ **C**: $y = 2.5x + 15$ **D**: $y = x + 16$

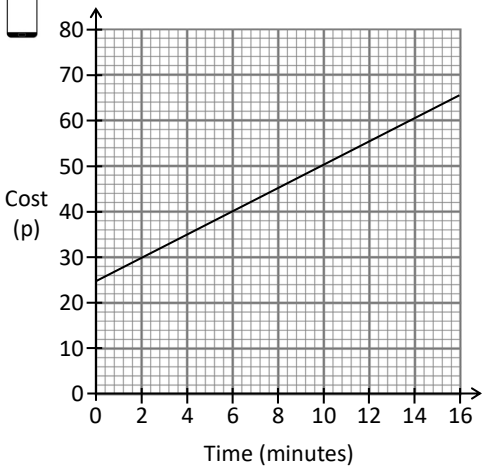
For what distance are
companies **B** & **C** the same price?

For what distance are
companies **B** & **D** the same price?

John travels 7 km. How much cheaper is
the cheapest company compared to
the most expensive?

Cost Relationships & Equations

③ This graph shows the price for long-distance satellite telephone calls.



Describe the graph using an equation.

With Inta-Fone™ Jake makes a 12 minute call.
He pays 58p which includes a
one-time 40p connection cost.

Plot & label a cost graph for this company.

Globe Net: $y = 0.5x + 60$

Light-Call: $y = 3.5x + 15$

Plot & label the cost graphs for these companies.

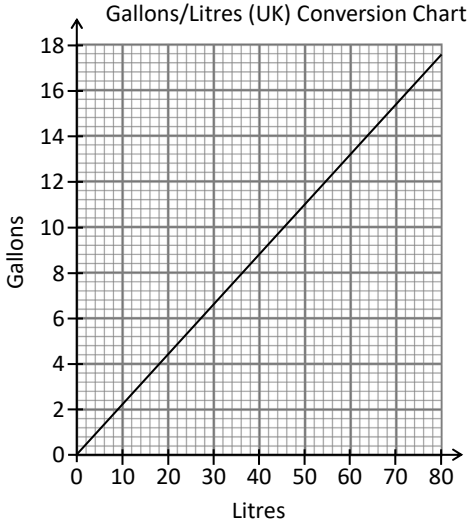
Plot a cost-graph for your
own company with a 30p connection cost.

Fluency Practice

Real-Life Graphs

① ②

Gallons/Litres (UK) Conversion Chart



Use the chart to complete these conversions:

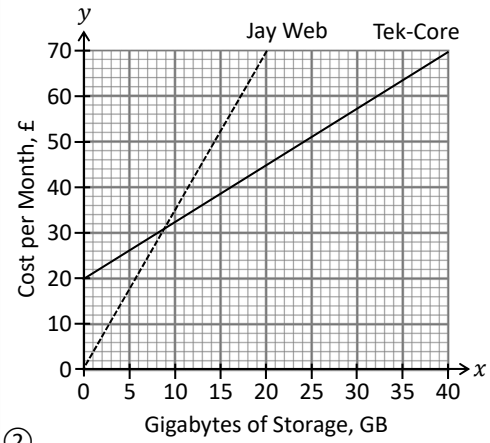
40 Litres = _____ Gallons

12 Gallons =

144 Litres =

40 Gallons =

This graph compares the website hosting fees for two companies.



How much does Jay Web charge per GB?

For what level of storage is Jay-Web cheaper?

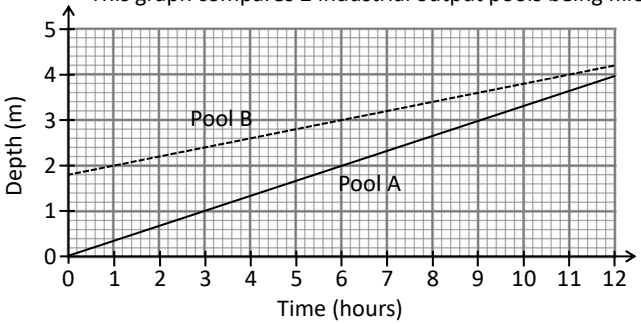
Why does Tek-Core's graph intercept the axis at (0,20)?

How much does Tek-Core charge per GB?

Form an equation in terms of x & y for Tek-Core's cost graph.

③ ④

This graph compares 2 industrial output pools being filled.

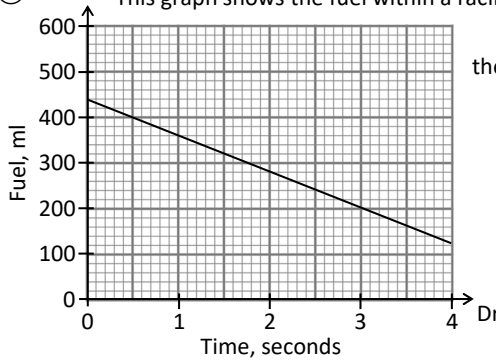


Estimate the depth difference after 8 hours and 10 minutes.

At what rate is Pool B being filled?

Pool A is 9 metres deep.
It started being filled at 8 am. When should it be full?

This graph shows the fuel within a racing drag car.



How much fuel does the car burn over 2 seconds?

Another tank design burns 60 ml of fuel per second & has a maximum capacity of 380 ml.

Draw this graph on the grid.

Form an equation for each graph.

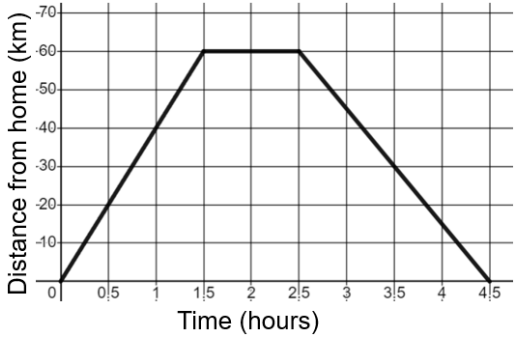
Use algebra to show when the 2 designs have used the same amount of fuel.

Fluency Practice

Reading Distance-Time Graphs

(a)

The distance-time graph shows Jamil's journey as he goes to visit a friend.



(a) How long after Jamil has set off from home does he stop to visit his friend?

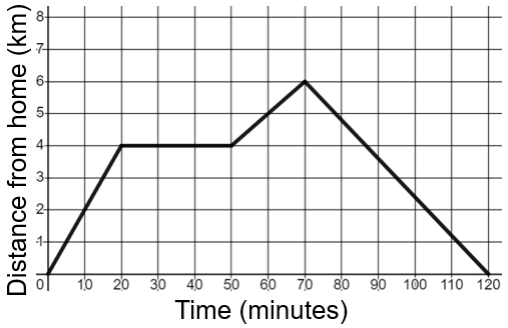
(b) Calculate Jamil's speed as he travels to his friend's house.

(c) How long does Jamil stay at his friend's house?

(d) Calculate the speed Jamil travels at as he returns home.

(b)

The travel graph shows Natalie's journey as she goes for a walk.



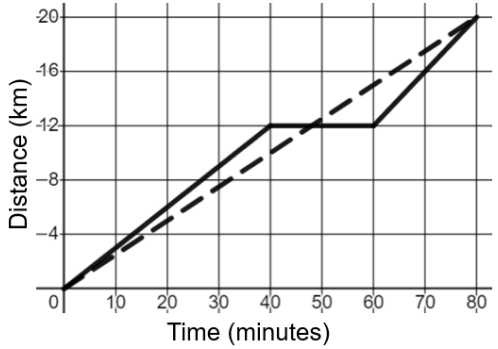
(a) Natalie sets off from home and arrives at her friend's house 20 minutes later. How long does Natalie stay at her friend's house?

(b) Natalie then walks for a further 20 minutes to the post box, before returning home. How far does she walk in total?

(c) Calculate Natalie's speed in km/h as she walks home from the post box.

(c)

The graph shows the journey of two runners, Pol and Pat, during a 20 km race.



(a) Pol runs the race at a constant speed over 80 minutes. Calculate Pol's speed in km/h.

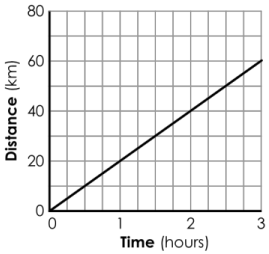
(b) Describe Pat's run, calculating any speeds in km/h.

(c) Pol runs past Pat 12 km into the race. At what time does this happen?

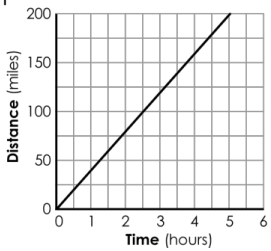
Fluency Practice

1. Calculate the average speed

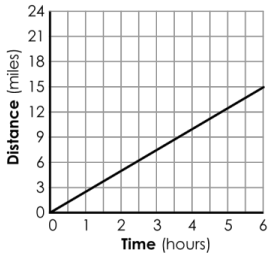
a.



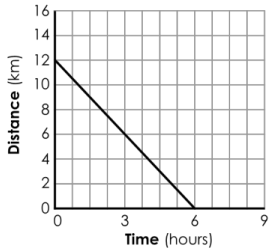
b.



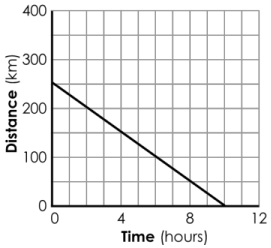
c.



d.

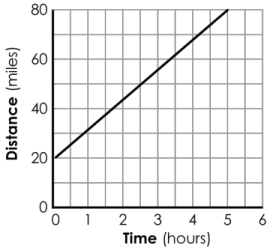


e.

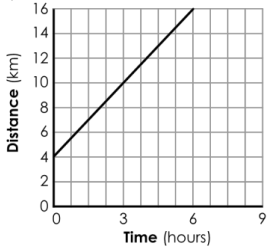


2. Calculate the average speed

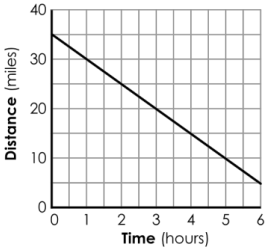
a.



b.



c.



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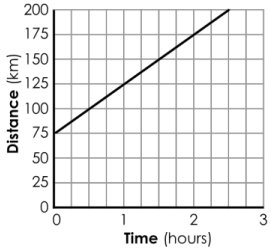
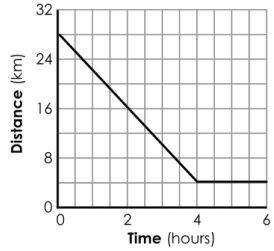
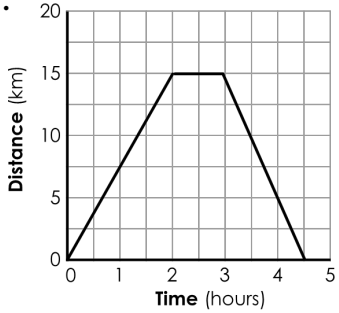


e.

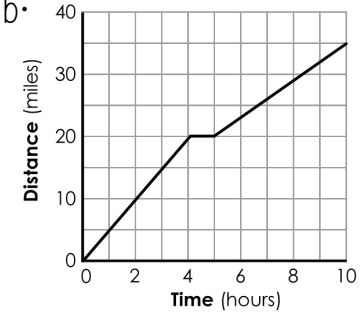


3. Calculate the average speed for each part of the journey that is spent travelling

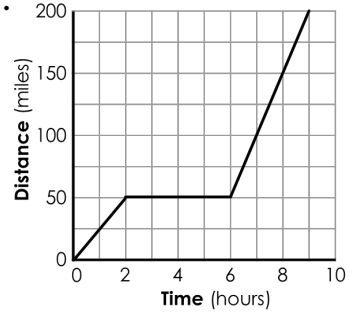
a.



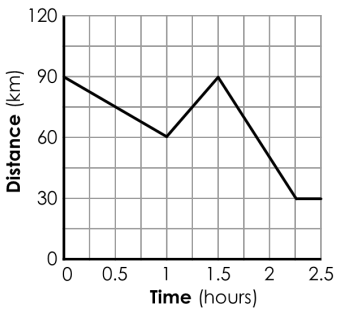
b.



c.



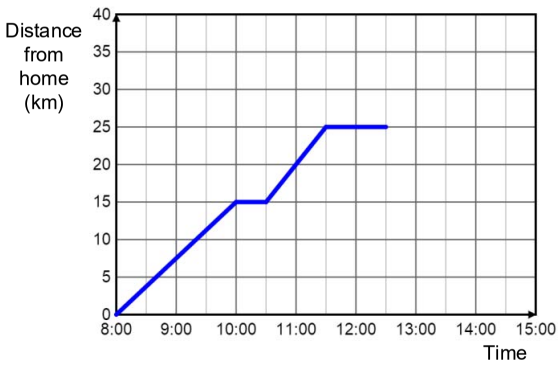
d.



distance-time graphs

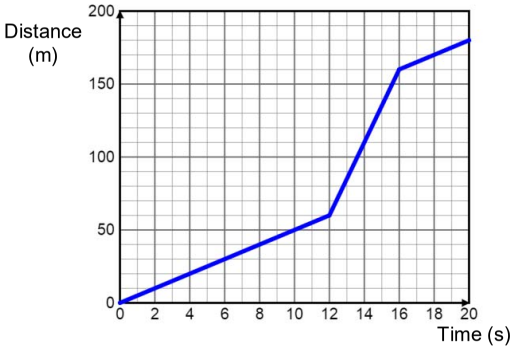
1. Sarah spent a day on a cycling trip.
The distance-time graph shows her journey.

(a) At what time did Sarah first stop for a break?
(b) What was Sarah's average speed between 8:00 and 10:00?
(c) What was Sarah's average speed between 10:30 and 11:30?
(d) At 12:30, Sarah started cycling home. She did not take another break and arrived home 2 hours later. Use this information to complete the graph of Sarah's trip.



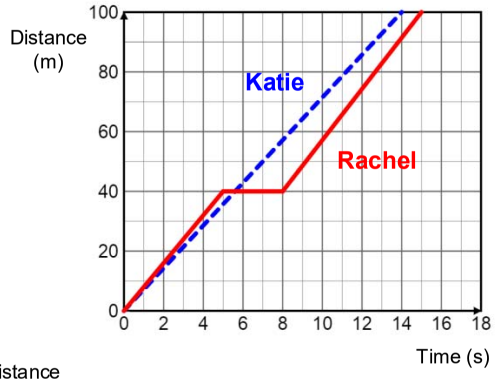
2. The distance-time graph shows the motion of a go-kart moving along a track.

(a) What was the average speed of the go-kart over the entire 20 seconds?
(b) True or false? The go-kart was travelling at the same speed for the fist twelve seconds as it was for the last 4 seconds.
(c) State the time interval during which the go-kart was travelling fastest.



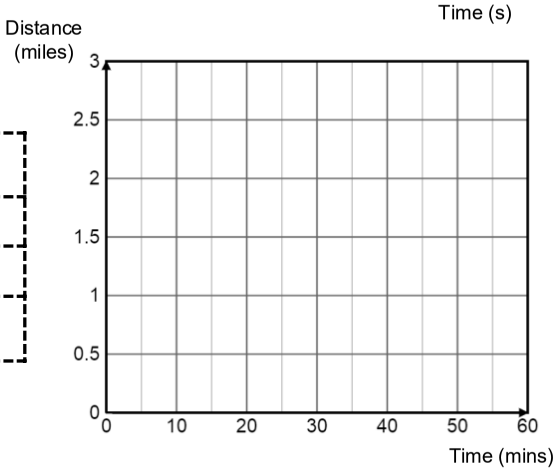
3. Katie and Rachel race each other over 100m.
The graph illustrates the race.

(a) Katie maintained a steady pace. What was her average speed?
(b) What was Rachel's speed between the 5th and 8th second of the race?
(c) Who won the race, and by how many seconds?



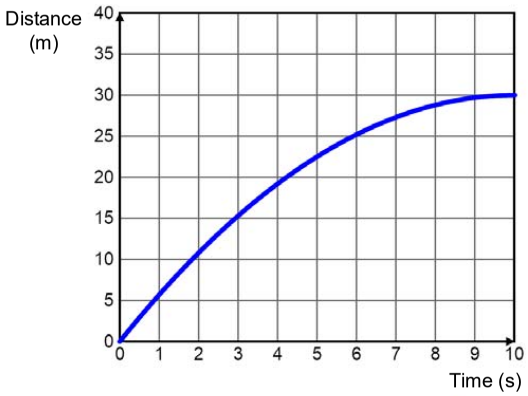
4. John drives from his home to a shop and back. Use the clues to complete the distance-time graph for his journey.

John left home and travelled to the shop at a constant speed, arriving after 20 minutes.
John spent 20 minutes at the shop.
The shop is two and a half miles from John's home.
John travelled home at twice the speed that he travelled to the shop.



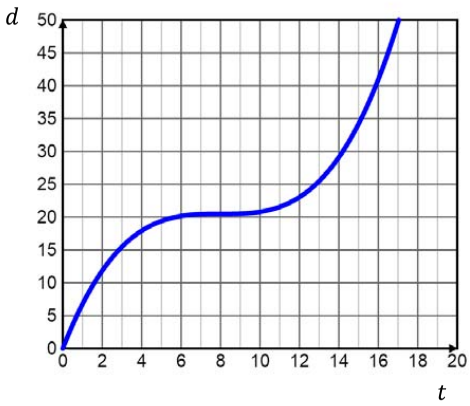
5. The graph shows the motion of a car.

(a) Work out the average speed of the car over the 10 second interval.
(b) Work out the instantaneous speed of the car at 6 seconds.
(c) Work out the time at which the instantaneous speed is equal to the average speed over the 10 second interval.



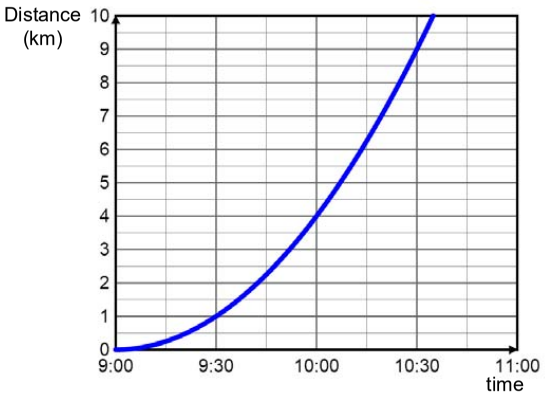
6. The graph shows the distance, d metres, travelled by a canoeist, t seconds after passing a marker point.

(a) Work out the canoeist's instantaneous speed when $t = 4$.
(b) Work out the canoeist's average speed over the interval $10 \leq t \leq 15$.
(c) Explain, without calculating, how you can tell that the canoeist is moving faster when $t = 16$ than at $t = 12$.

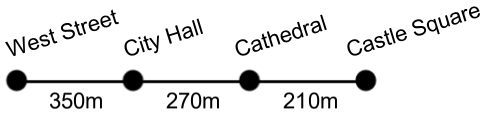


7. Mark and Sophie take part in a 10km run. Mark's progress is shown on the graph.

(a) Work out Mark's instantaneous speed at 9:30.
(b) Sophie started the run at the same time as Mark. She maintained a steady speed of 6km/h throughout her run. Draw Sophie's progress on the graph.
(c) At one point one of the two runners overtook the other. At what time did this happen, and who overtook who?

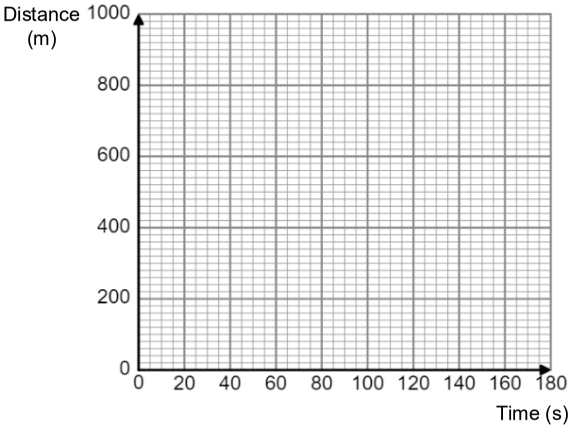


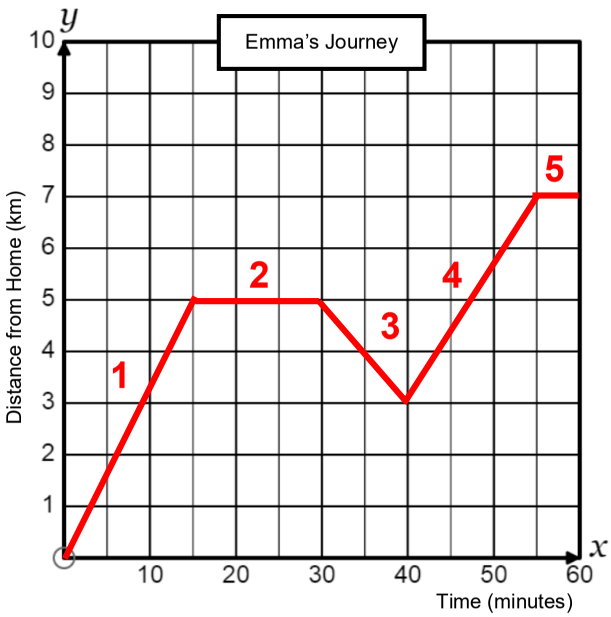
8. Four stops on a tramline in Sheffield are illustrated, with the distance between stops shown in metres.



A tram spends 30 seconds at each stop, and when it is moving has an average speed of 8 m/s.

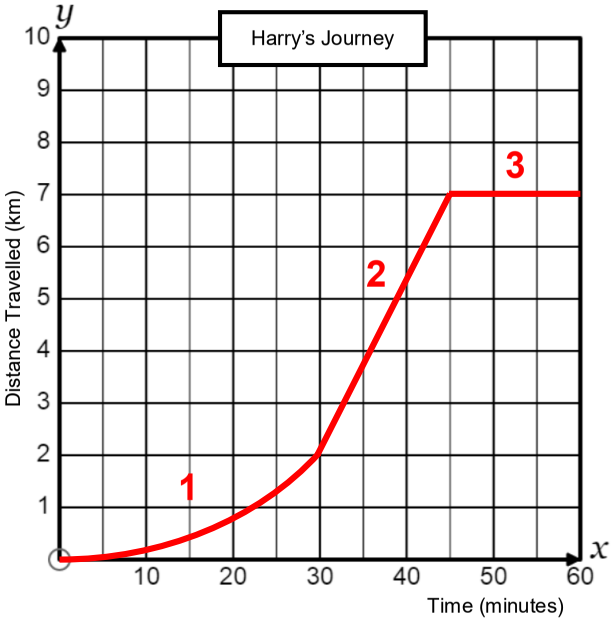
Complete the graph to show the journey of a tram from the moment it leaves West Street to the moment it arrives at Castle Square.





A.	During stage 1, Emma travelled at 3 km/minute.
B.	Emma was stationary during stage 2.
C.	The graph shows that Emma reached the top of two hills.
D.	Emma travelled 4000m during Stage 4.
E.	The slowest part of Emma's journey was during stage 3.
F.	Emma's average speed over the whole journey was 7 km/h.
G.	Emma was travelling faster during stage 4 than stage 1.
H.	When she was 5 minutes into her journey, Emma had travelled 3 km.

True Statements: _____



A.	It took Harry half an hour to travel the first 2km of his journey.
B.	Harry travelled at a constant speed during section 1.
C.	Harry's speed 35 minutes into his journey was 1⅔ km/h.
D.	During section 3 of his journey, Harry's speed was 0 mph.
E.	Harry was accelerating during section 1 of the journey
F.	After 45 minutes, Harry had travelled 7 km.
G.	During section 1, Harry travelled at an average speed of 15 km/h.
H.	Harry's speed was fastest during section 1 of the journey.

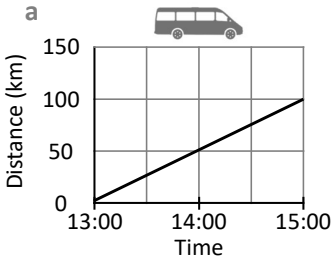
True Statements: _____

Fluency Practice

Distance-Time Graphs

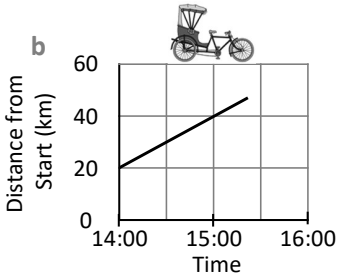
What **methods** can you use to calculate the speed of each object?

a



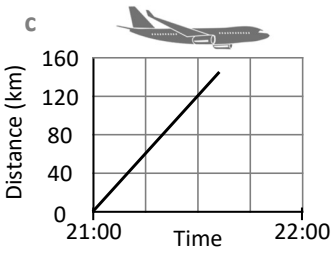
Speed:

b



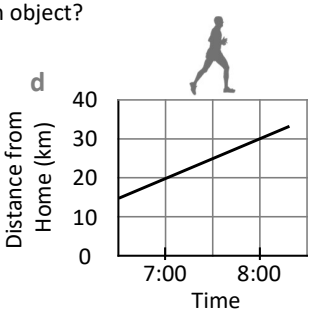
Speed:

c



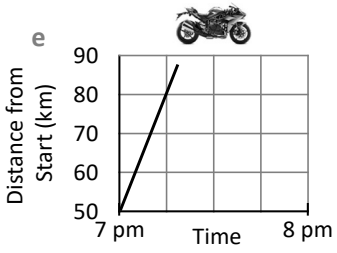
Speed:

d



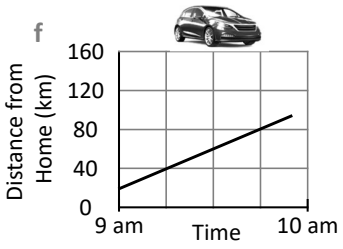
Speed:

e



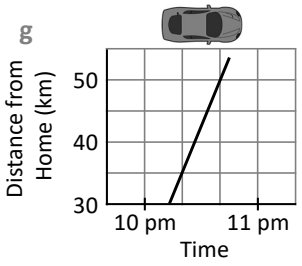
Speed:

f



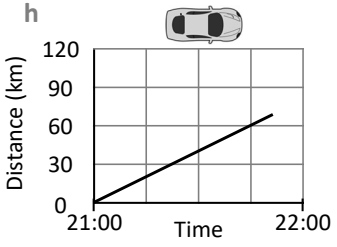
Speed:

g



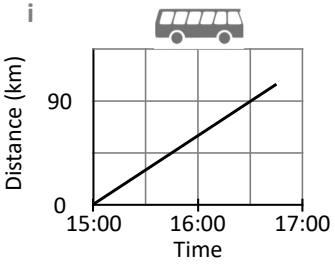
Speed:

h



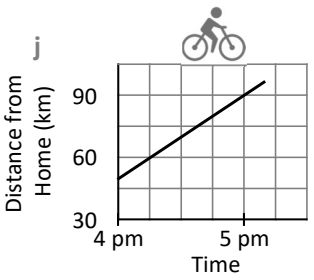
Speed:

i



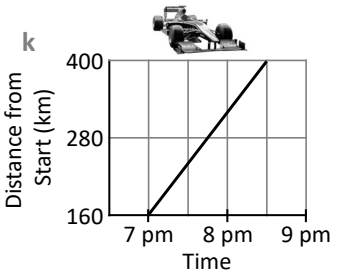
Speed:

j



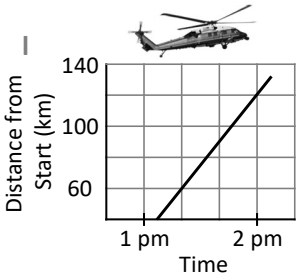
Speed:

k



Speed:

l

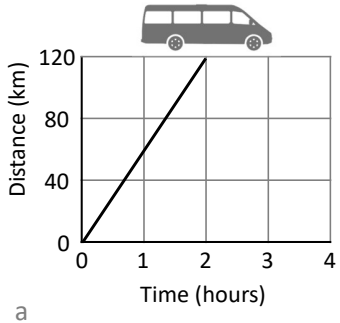


Speed:

Fluency Practice

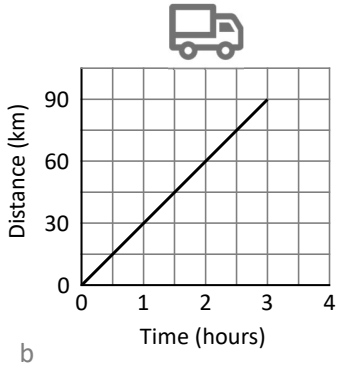
1 What units will we use for the speed?

Distance-Time Graphs



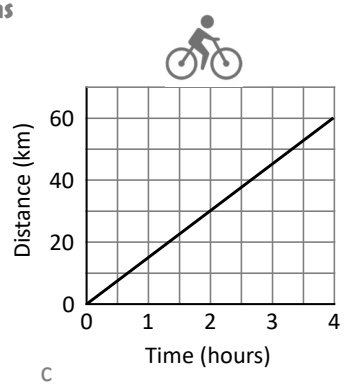
a

Speed:



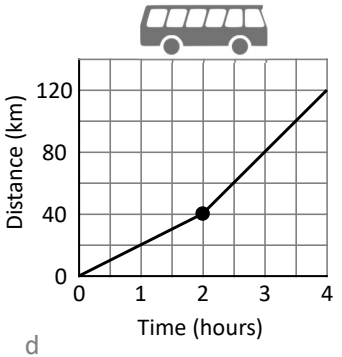
b

Speed:



c

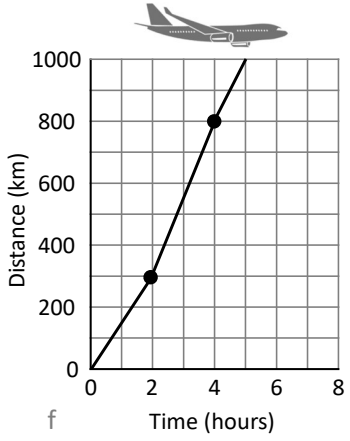
Speed:



d

Min. Speed:

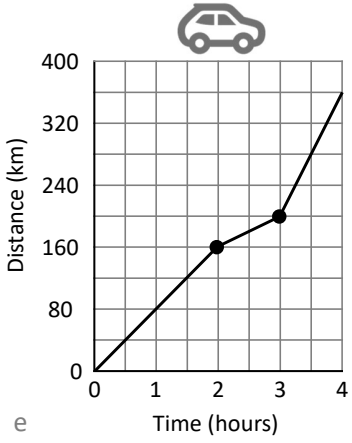
Max. Speed:



f

Max. Speed:

Average Speed for the whole journey:

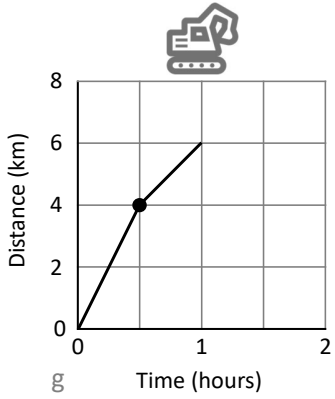


e

Max. Speed:

Min. Speed:

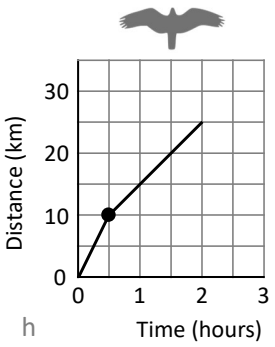
Average Speed:



g

Max. Speed:

Min. Speed:



h

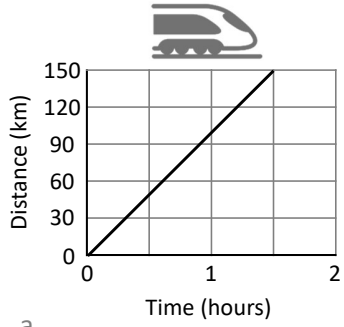
Max. Speed:

Min. Speed:

Average Speed:

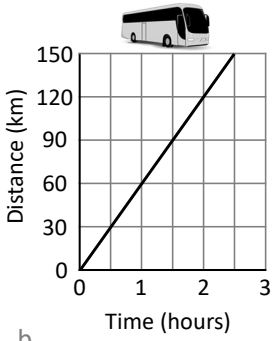
Fluency Practice

2 **Distance-Time Graphs**



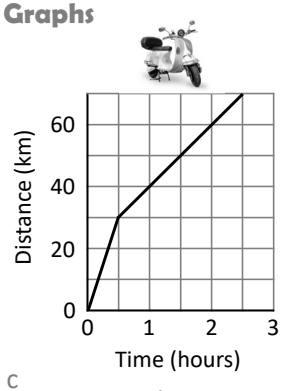
a

Speed:



b

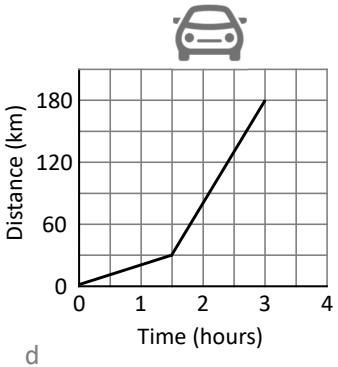
Speed:



c

Max. Speed:

Min. Speed:

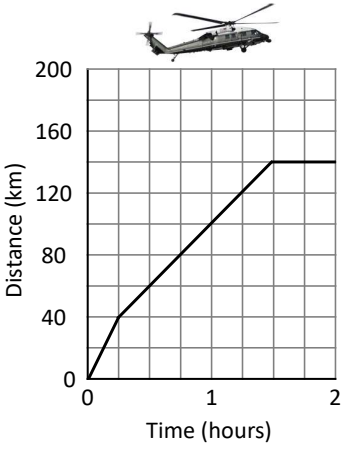


d

Max. Speed:

Min. Speed:

Average Speed:

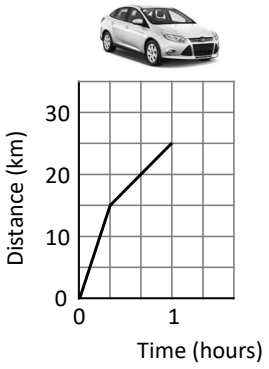


e

Max. Speed:

Min. Speed:

Average Speed:

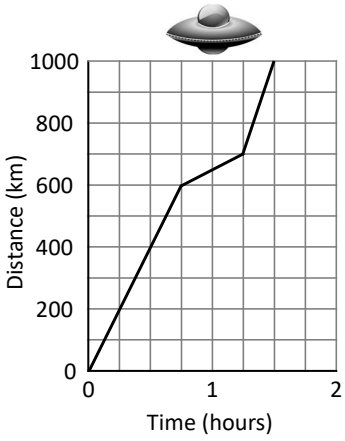


f

Max. Speed:

Min. Speed:

Average Speed:

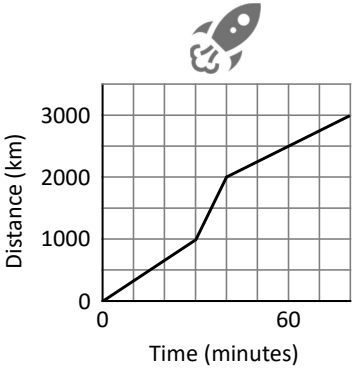


g

Max. Speed:

Min. Speed:

Average Speed:



h

Max. Speed:

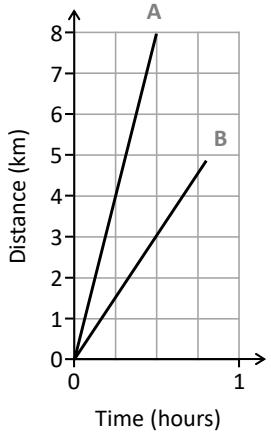
Min. Speed:

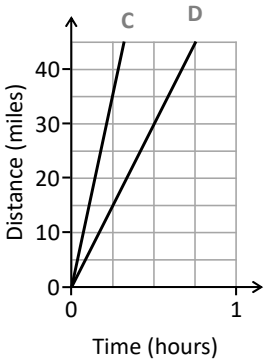
Average Speed:

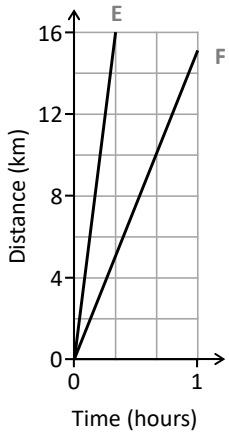
Fluency Practice

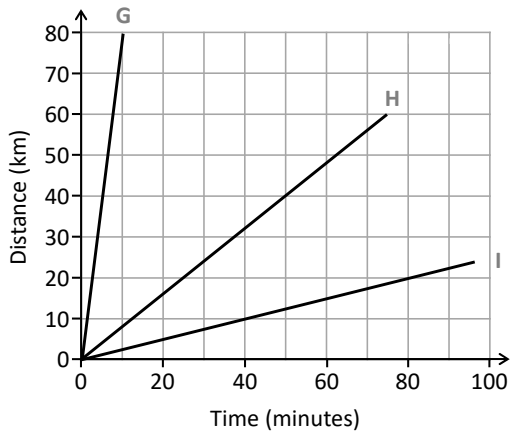
Fractional Times

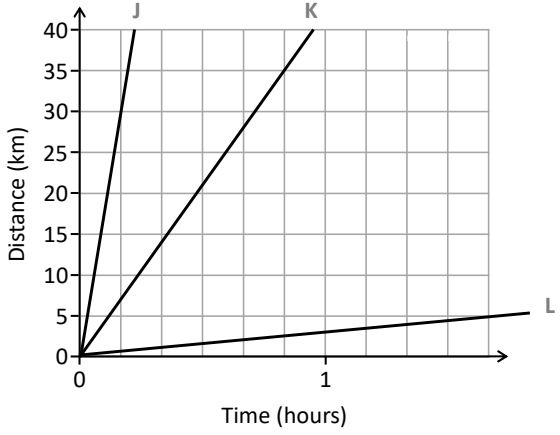
What method can we use to find the speed of each journey (A to S)?

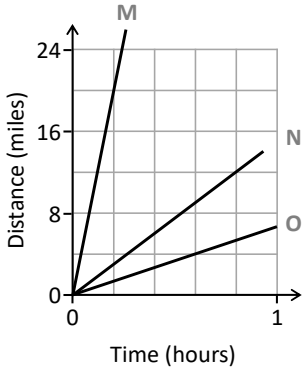


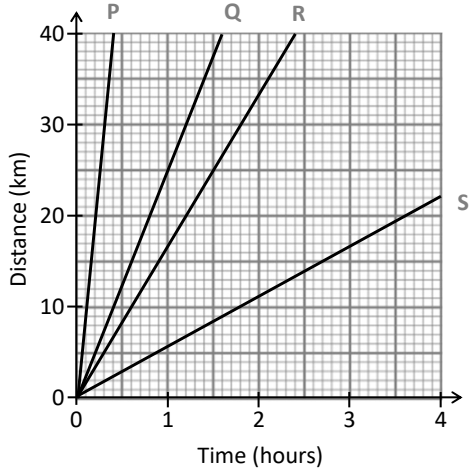






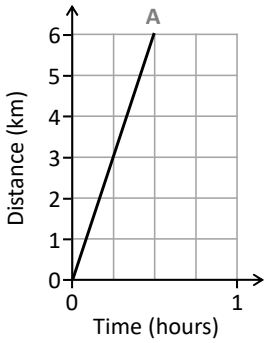






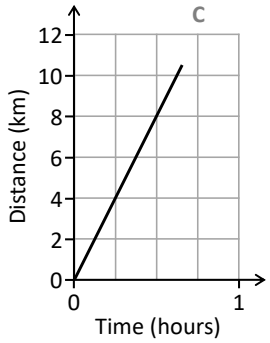
If we read a speed from a graph, is it always an estimate?

Fluency Practice



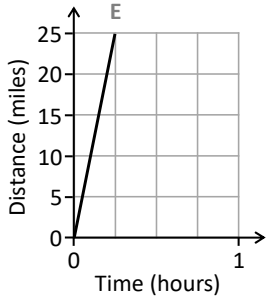
Journey A = _____ kmph

Journey B = 8 kmph



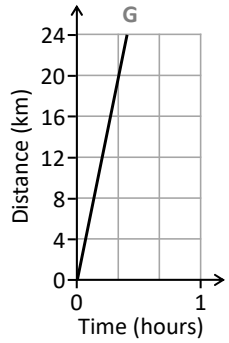
Journey C = _____ kmph

Journey D = 24 kmph



Journey E = _____ mph

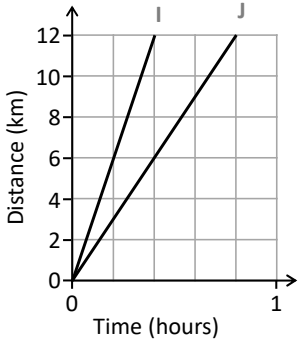
Journey F = 60 mph



Journey G = _____ kmph

Journey H = 48 kmph

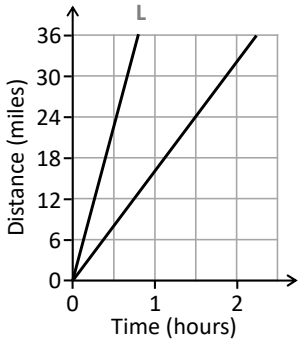
Fractional Times On each grid, how can we draw each journey accurately (using *exact* coordinates).



Journey I = _____ kmph

Journey J = _____ kmph

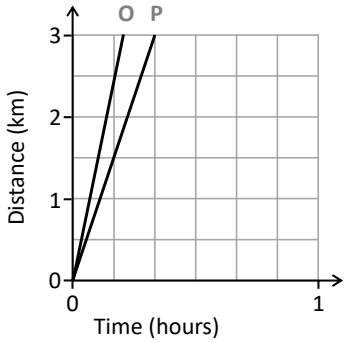
Journey K = 40 kmph



Journey L = _____ mph

Journey M = _____ mph

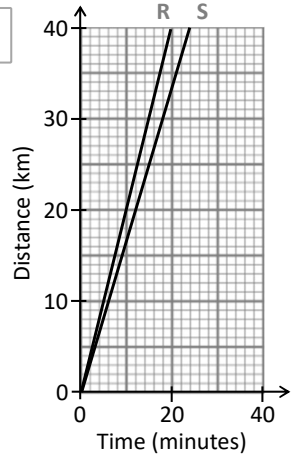
Journey N = 4 mph



Journey O = _____ kmph

Journey P = _____ kmph

Journey Q = 5 kmph

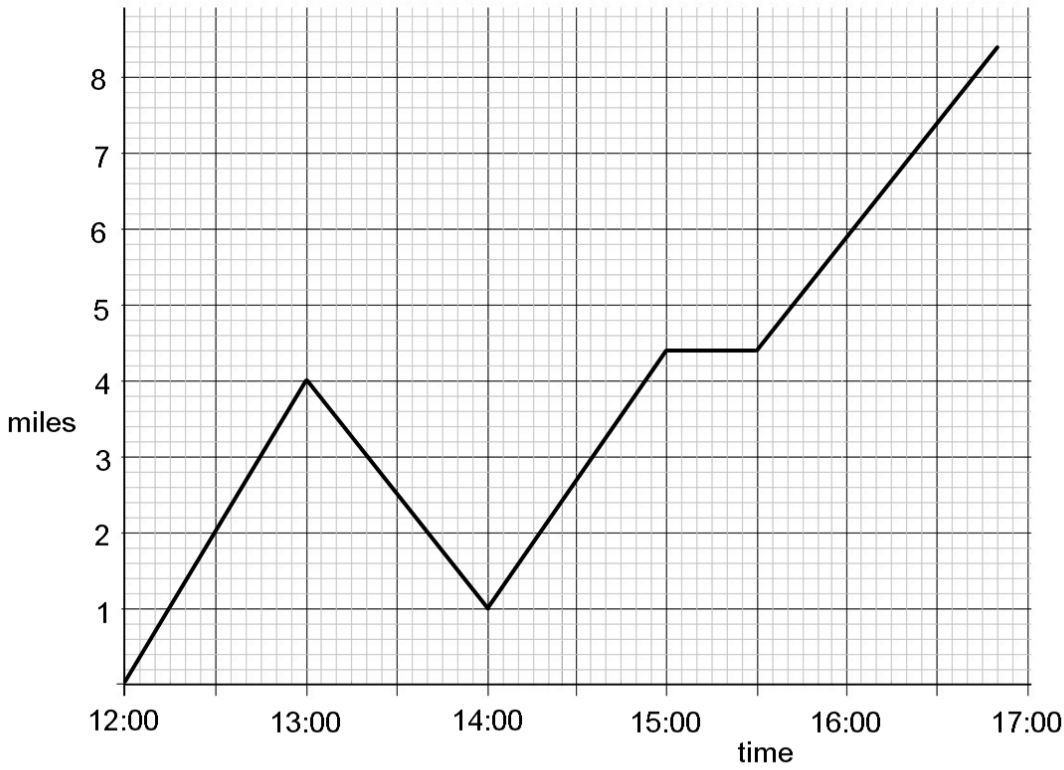


Journey R = _____ kmph

Journey S = _____ kmph

Journey T = 600 kmph

(1) **quite a long walk**



someone goes on an 8 mile walk

they go back to a shop

and, later, stop for half an hour

what is the average speed in mph for:

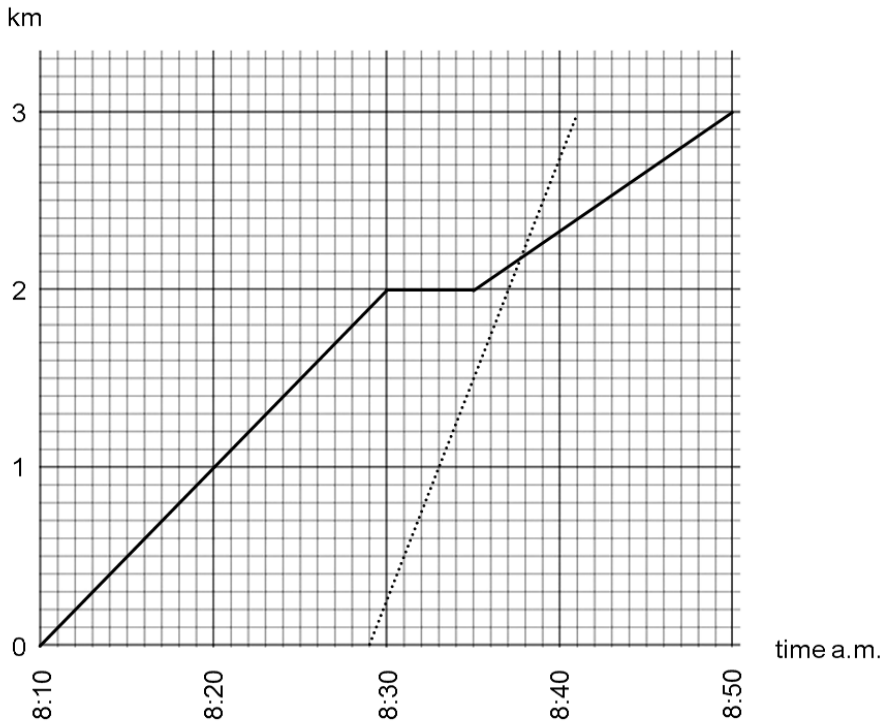
the first hour of the journey?

the journey back to the shop?

the journey from the shop to where they stop?

the journey from where they stop to the destination?

(2) **two students going to college**



starting from home, a student walked for ……. km

at a steady speed of ……. km per hour

after …… minutes from home the student

stopped to chat for ……. minutes

and then continued at steady speed of ……. km per hour

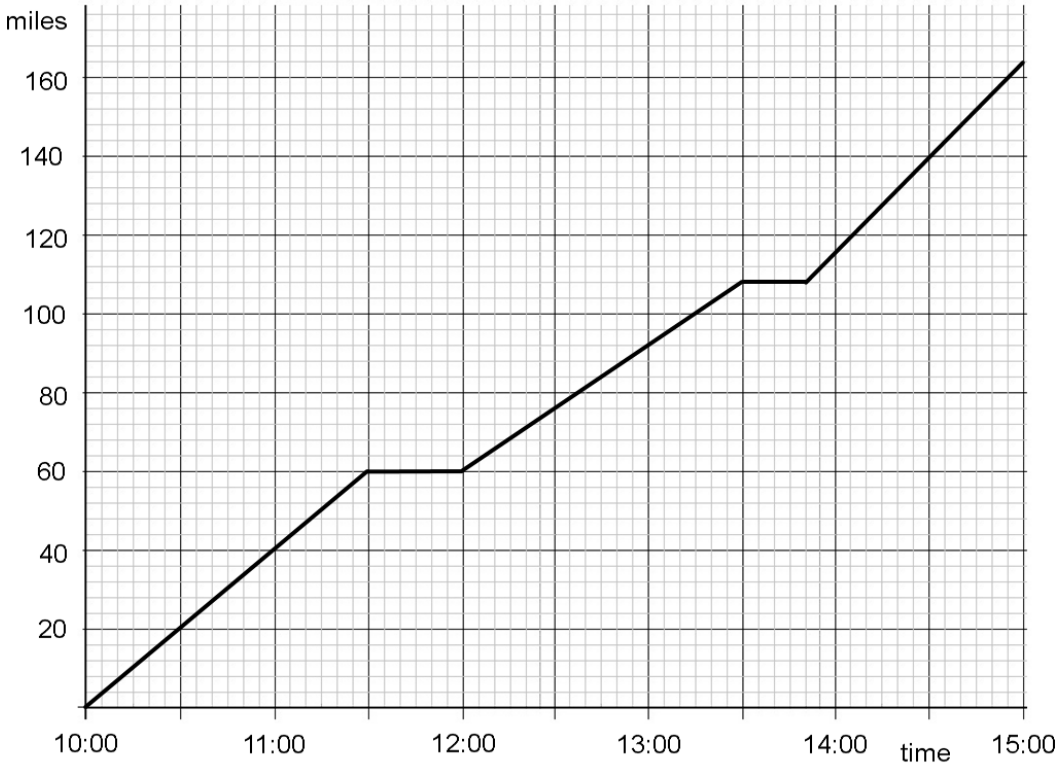
until they reached college at ……. …

another family member cycled to college

starting out at …….

and cycling at steady speed of ……. km/h to arrive at college at …….

(3) **transporter on the motorway**



a (fairly slow) transporter goes from Shrewsbury to London

it stops twice

what is the average speed in mph for:

the first leg of the journey?

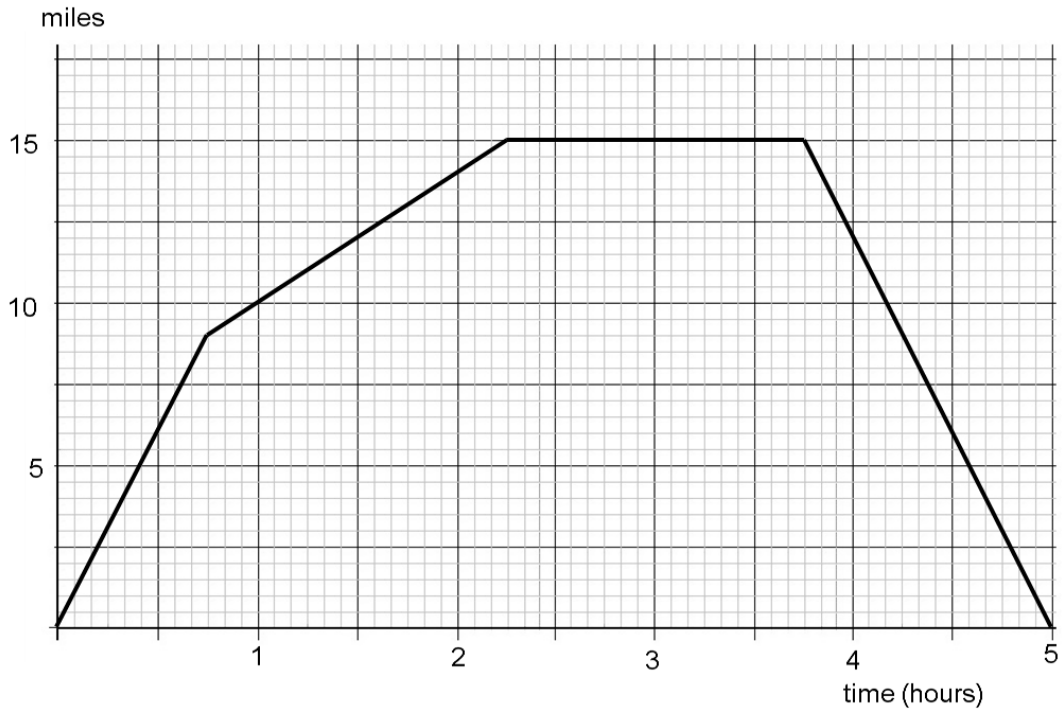
the second leg of the journey?

the last leg of the journey?

how far is it to London from Shrewsbury?

what is the average speed for the whole journey?

(4) **cyclist on a journey**



starting from home, a cyclist went for ……. miles

at a steady speed of ……. mph

after …… minutes from home the cyclist got a puncture

and so walked at a steady speed of ……. mph

for …… hours

when they were ……. miles from home

they stopped for ……. hours and fixed the puncture

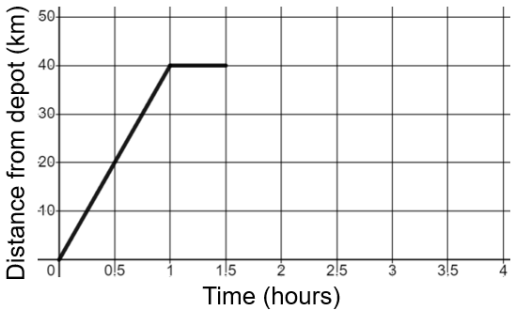
then they cycled home at a steady speed of ……. mph

Fluency Practice

Completing Distance-Time Graphs

(a)

A delivery driver sets off from the depot to deliver a parcel.



(a) Calculate the speed the driver travels at over the first part of the journey.

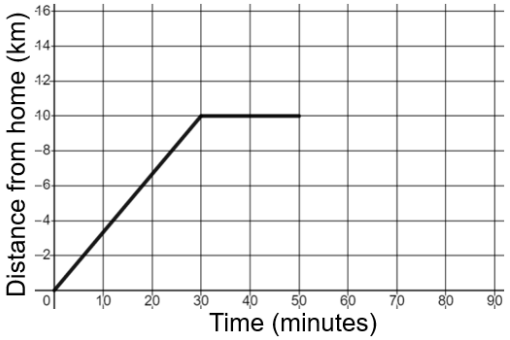
(b) How long does the driver stop for while delivering the parcel?

(c) The driver returns to the depot at a constant speed of 20 km/h. Complete the graph.

(d) How far has the delivery driver travelled in total?

(b)

Karen sets of on her bike to visit a friend. A graph showing her journey is shown.



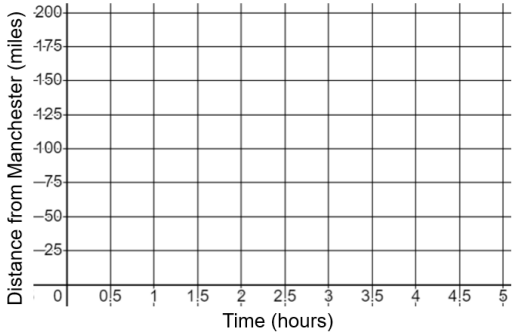
(a) At what speed does Karen ride during the first stage of her journey?

(b) How far away from home is Karen when she stops for a rest?

(c) After a rest, Karen continues on to her friend's house, which is 16 km away from her home. She travels at 12 km/h. Complete the graph.

(c)

A bus travels from Manchester to London, a distance of 200 miles.



(a) The bus sets off and travels at 50 mph for 90 minutes. It then stops at the services for 30 minutes, before setting off again. The bus continues its journey, again at 50 mph, for the next two hours. It then gets stuck in slow-moving traffic, travelling the last 25 miles in one hour. Draw a distance-time graph to represent this journey.

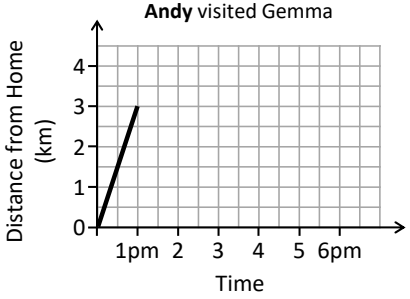
(b) Calculate the bus's average speed across the whole journey.

Fluency Practice

Visiting Friends

6 people visit their friends, by walking or on bicycle. For each journey, complete the graph & the 4 pieces of information.

Andy visited Gemma



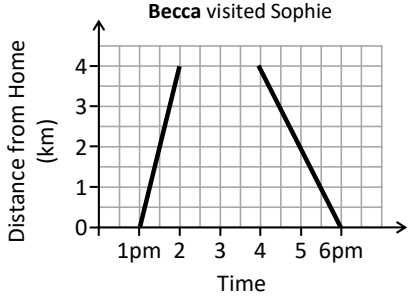
Speed to friend's house:

Time at friend's house: 3 hours

Speed home: 3 km/h

Total distance travelled:

Becca visited Sophie



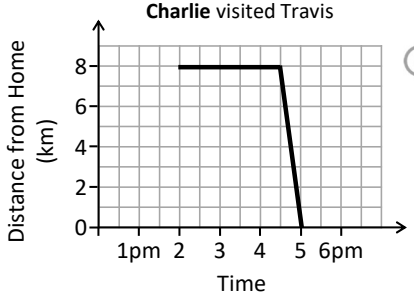
Speed to friend's house:

Time at friend's house: 2 hours

Speed home:

Total distance travelled:

Charlie visited Travis



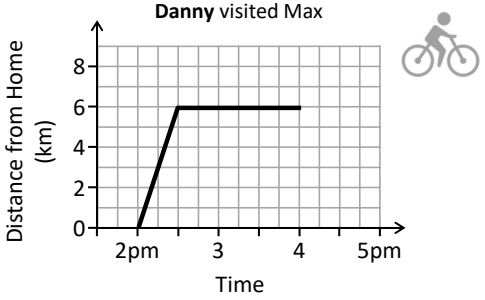
Speed to friend's house: 8 km/h

Time at friend's house:

Speed home:

Total distance travelled:

Danny visited Max



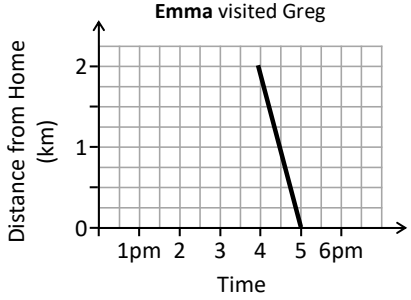
Speed to friend's house:

Time at friend's house:

Speed home: 6 km/h

Total distance travelled:

Emma visited Greg



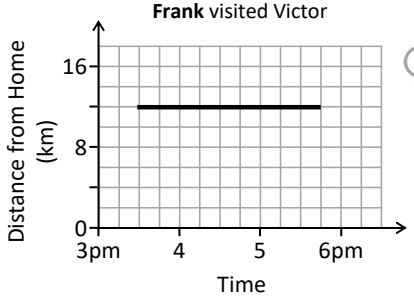
Speed to friend's house: 4 km/h

Time at friend's house: 3 hours 30 minutes

Speed home:

Total distance travelled:

Frank visited Victor



Speed to friend's house: 24 km/h

Time at friend's house:

Speed home: 16 km/h

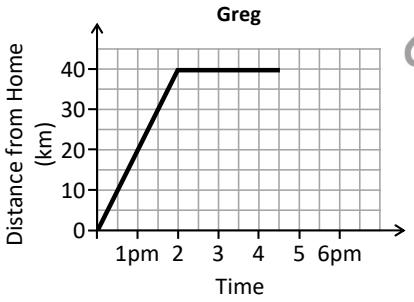
Total distance travelled:

Fluency Practice

Going Shopping

6 people travel to the shops and then back home. For each journey, complete the graph & the 4 pieces of information.

Greg



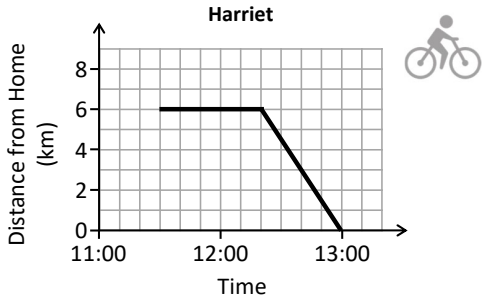
Speed to shops:

Time shopping:

Speed home: 40 km/h

Total distance travelled:

Harriet



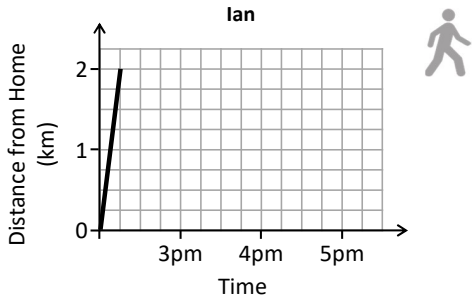
Speed to shops: 12 km/h

Time shopping:

Speed home:

Total distance travelled:

Ian



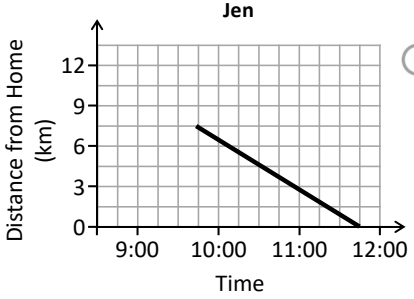
Speed to shops:

Time shopping: 2 hours

Speed home: 4 km/h

Total distance travelled:

Jen



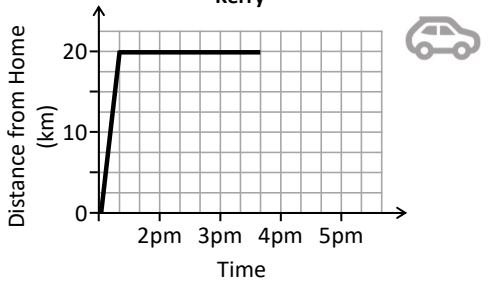
Speed to shops: 15 km/h

Time shopping: 45 minutes

Speed home:

Total distance travelled:

Kerry



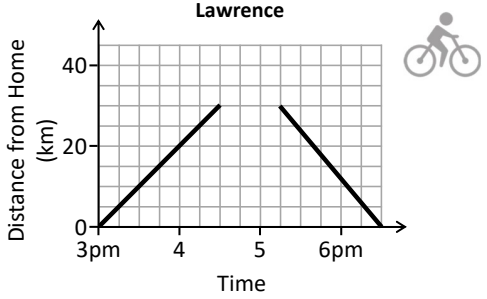
Speed to shops:

Time shopping:

Speed home: 30 km/h

Total distance travelled:

Lawrence



Speed to shops:

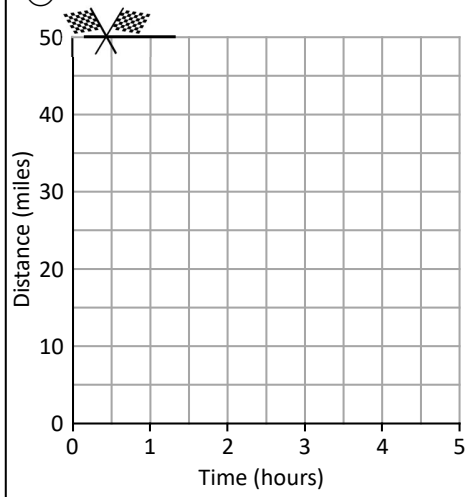
Time shopping: 45 minutes

Speed home:

Total distance travelled:

Fluency Practice

① **Race!** Plot the graphs for each 4 races. Who won? By how much time did they win? Who got the top speed?



Amir

20 miles in one hour
then 3 hours at 10 mph

Ash

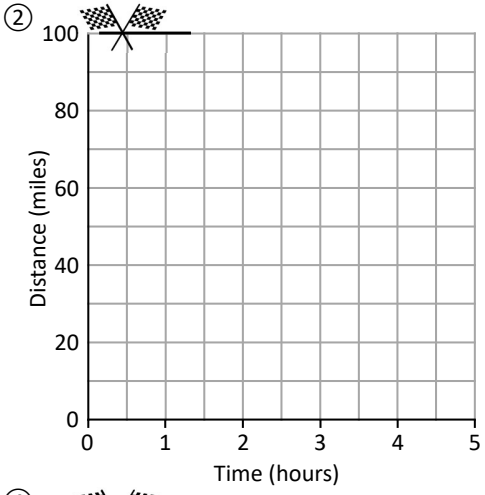
10 mph for 2.5 hours
then 25 mph

Winner:

Winning Margin (time):

Top Speed:

②



Brian

40 mph for 90 minutes
then a 1 hour breakdown
then 80 mph

Beth

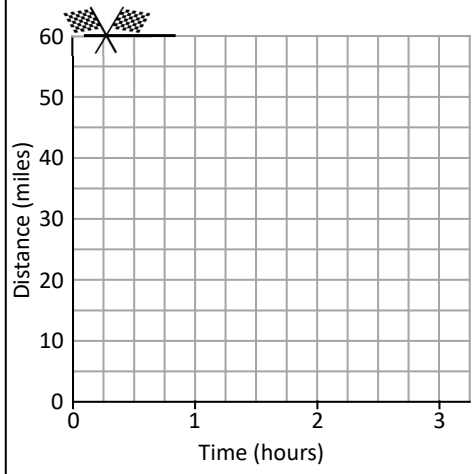
20 mph for 2 hours
then a 30 minute tire-change
then 30 mph

Winner:

Winning Margin:

Top Speed:

③



Christy

40 mph for 45 minutes
then 10 miles in 45 minutes
then 20 mph

Collin

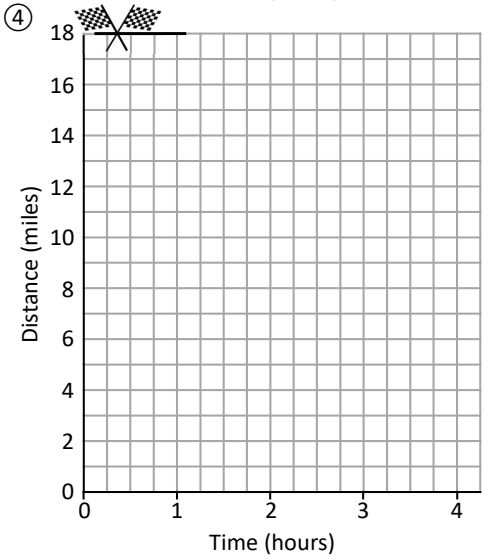
25 miles in 105 minutes
then 60 mph for 30 minutes
then 10 mph

Winner:

Winning Margin: es

Top Speed:

④



Donna

8 mph for 75 minutes
then 6 mph for 3 miles
then 2.5 mph

David

10 miles at 20 mph
then 45 minutes at 4 mph
then 16 mph

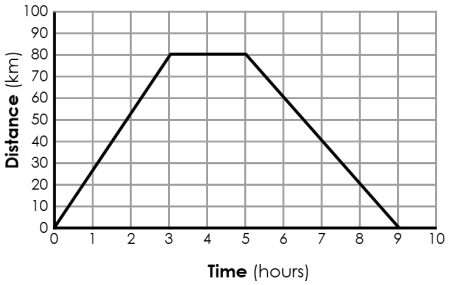
Daz

15 minutes late starting
90 minutes at 6 mph
then 28 mph for 0.25 hours
then 1 mph

Winner:

1. Use the distance time graphs to answer the questions

a. Delilah driving to and from work

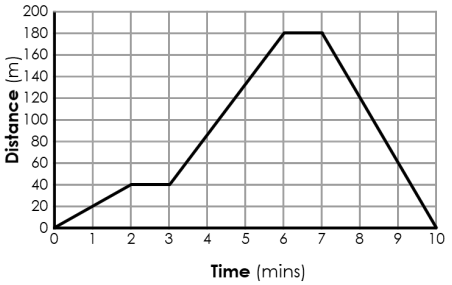


i. How far does she travel in total?

ii. How long does she spend at work?

iii. How long does it take her to get to work?

b. Jedidiah walking his dog

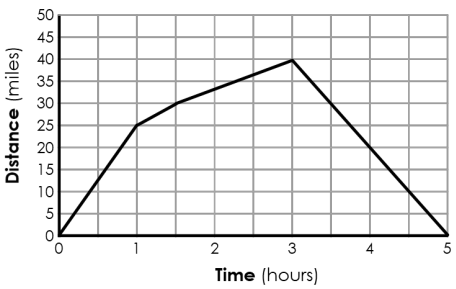


i. How far has he walked after 5 minutes?

ii. How long does he walk his dog for?

iii. How many times does he stop?

c. Sam riding his bike

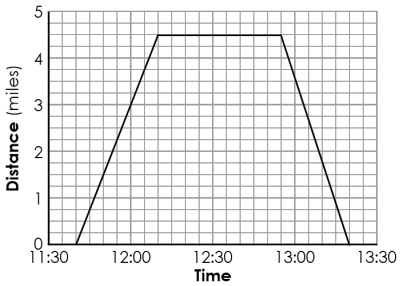


i. How far does he travel in total?

ii. How long did it take Sam to cycle 50 miles?

iii. How far had Sam travelled in 2 hours?

d. Penelope walking to the shops



i. What time does she leave home?

ii. How long was she in the shops?

iii. How long did it take to get home?

2. Use the information to draw the distance time graphs

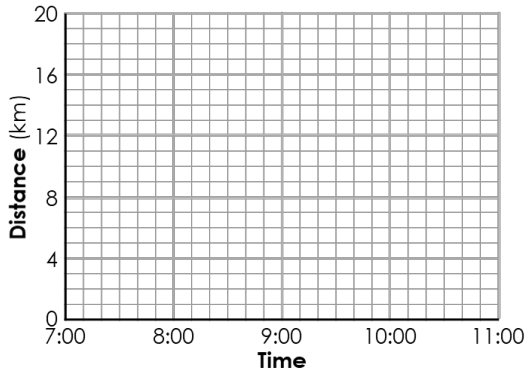
a.

Sally leaves the house at 7:00

She drives 12 km in 30 mins

She goes shopping for an hour

She gets back home at 10:20

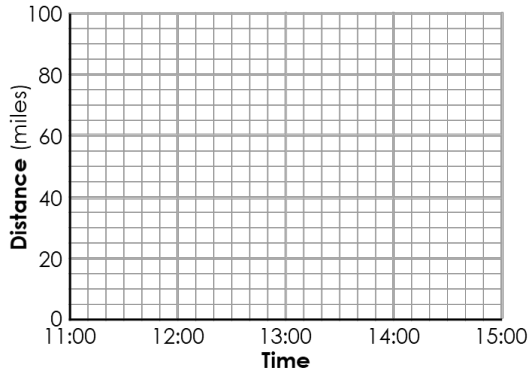


b.

Johnny leaves home at 11:40

He arrives at the restaurant, which is 50 miles away, at 12:30. He has lunch with his friends for 40 mins

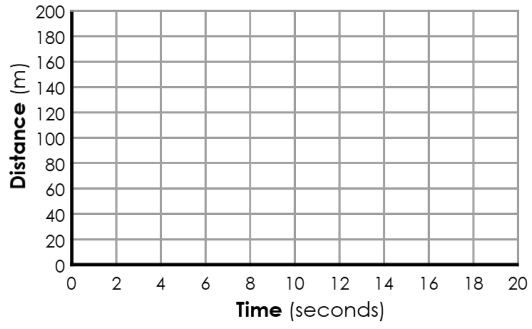
It takes him 30 mins to get home



c.

Tallulah drives 40 metres in 6 seconds until she arrives at a red light.

She waits for 4 seconds, before driving another 120 metres in 9 seconds.

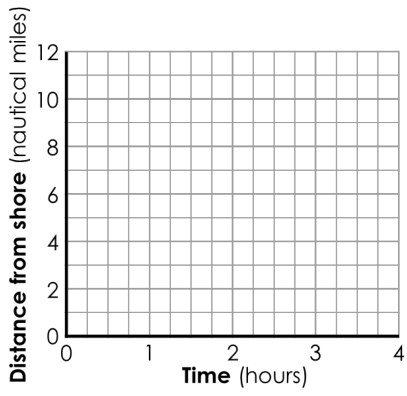


d.

A fishing boat travels 6 nautical miles in 45 mins

It anchors for one hour, before travelling another 5 nautical miles in 1 hour

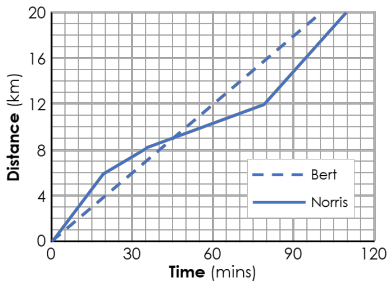
It anchors for 30 mins before sailing for 45 mins back to shore



3. Use the distance time graphs to answer the questions

a.

Bert and Norris are taking part in a race.



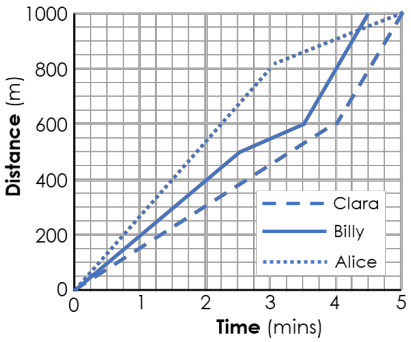
i. Who was in the lead after 30 mins?

ii. Who won the race, and by how much?

iii. What happened 45 mins into the race?

b.

Alice, Billy and Clara ran a 1000m race



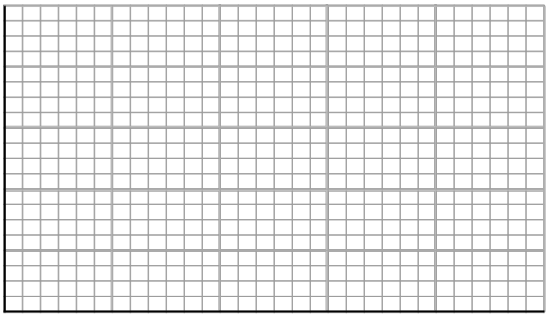
i. Who won the race?

ii. Describe Clara's race, giving as much detail as possible

c.

Teddy leaves home at 13:00
He drives 150 miles in 2 ½ hours
Teddy stops for 30 minutes
He then drives home in 1 hour 40 mins.

i. Show this information on the distance time graph

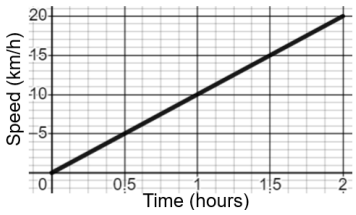


ii. A film starts at 5:45
Does Teddy get home in time for the start?

Speed-Time Graphs

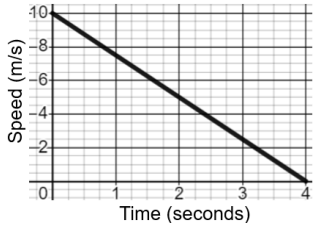
(a)

Find the acceleration.



(b)

Find the deceleration.

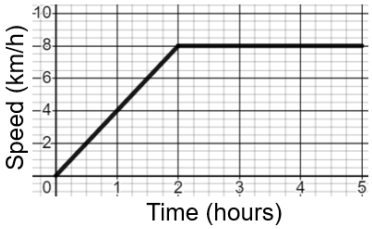


(c)

Find the acceleration.

(d)

Describe each step of the journey, stating the values of any acceleration or deceleration.

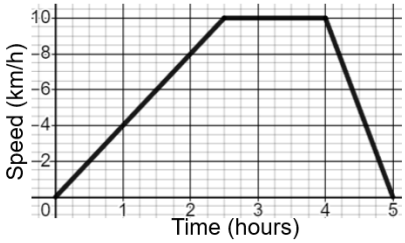


(e)

Describe each step of the journey, stating the values of any acceleration or deceleration.

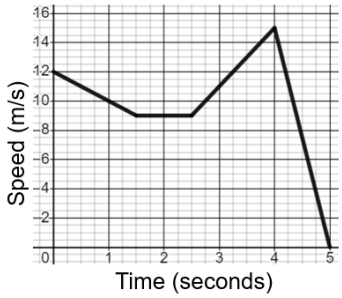
(f)

Describe each stage of the speed-time graph, calculating the values of any acceleration and deceleration.



(g)

Describe each stage of the speed-time graph, calculating the values of any acceleration and deceleration.



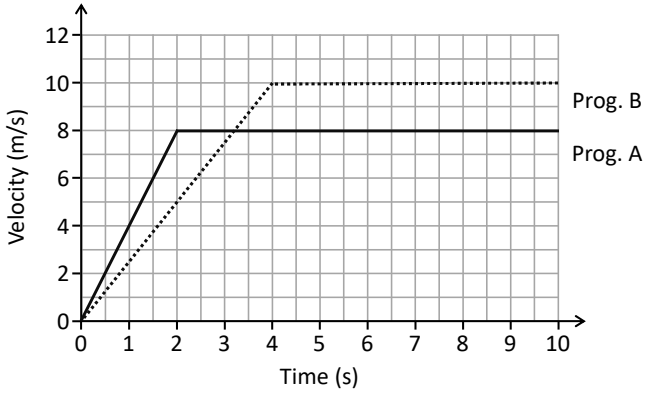
Fluency Practice

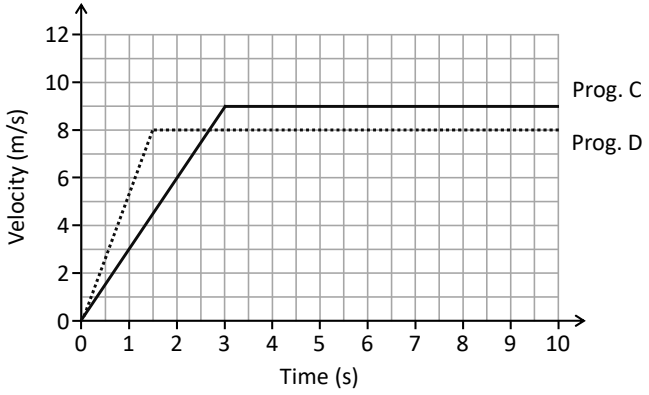
Sprint Training

To train to accelerate in the 100 m sprint,
athletes use a special pre-programmed treadmill.

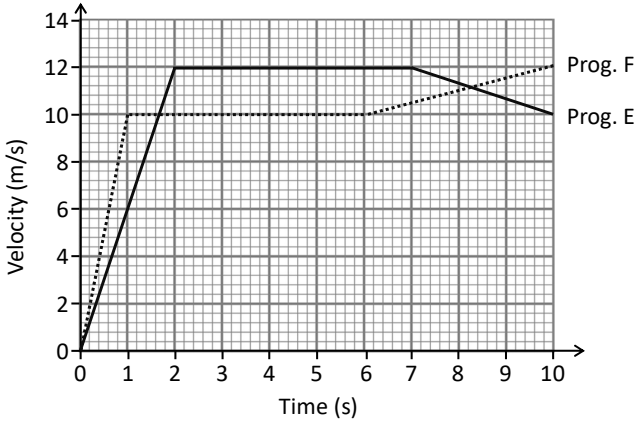
The athlete accelerates (sometimes in stages) then
tries to maintain a constant speed until 10 seconds have passed.

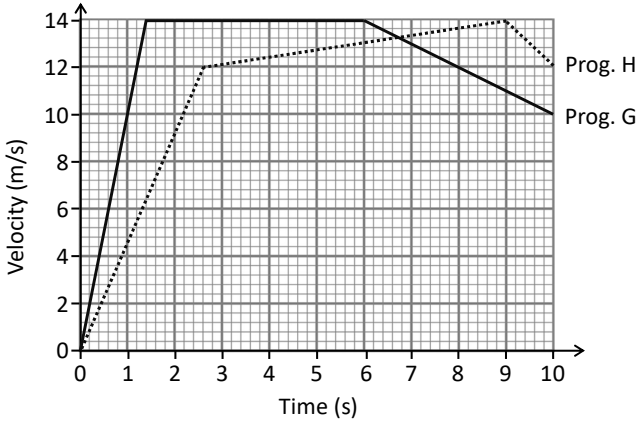
For each program,
calculate the rate of **acceleration** & the **average speed** for the 10 seconds.





These programs account for speed changes towards the end of the race.
Calculate total average speed & acceleration/deceleration rates.





Fluency Practice

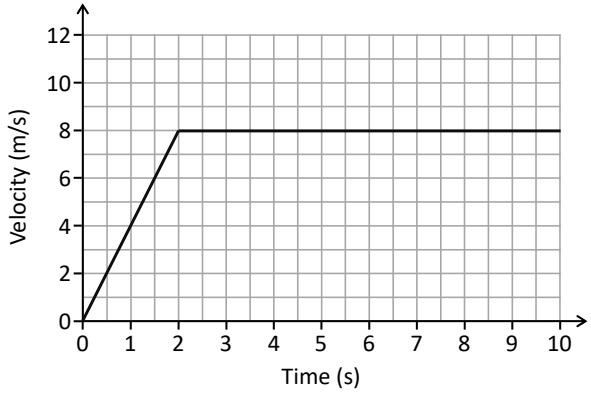
Sprint Training

To train for the 100 m sprint,
athletes use a special pre-programmed treadmill.

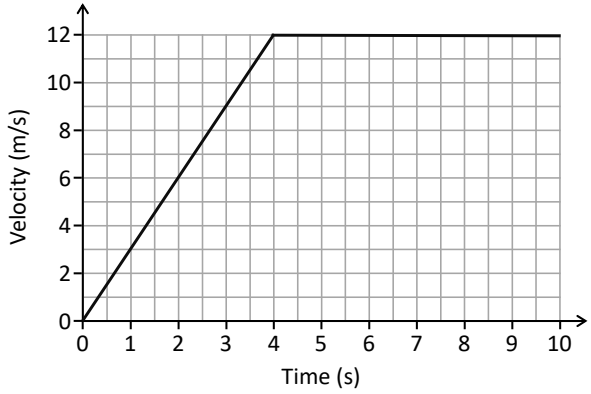
The athlete must accelerate (sometimes in stages) then maintain a constant speed for 10 seconds in total.

Calculate the **total distance travelled** &
the **average speed** for the 10 seconds.

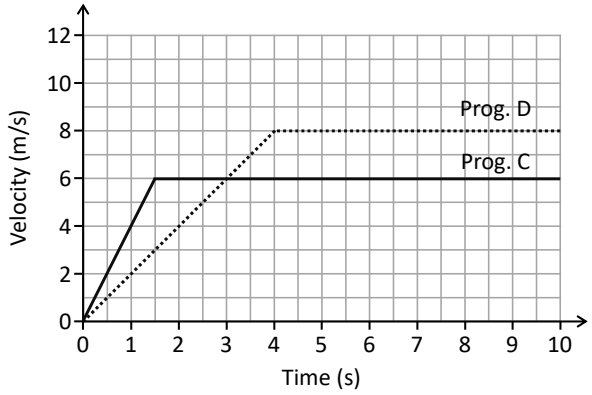
Program A

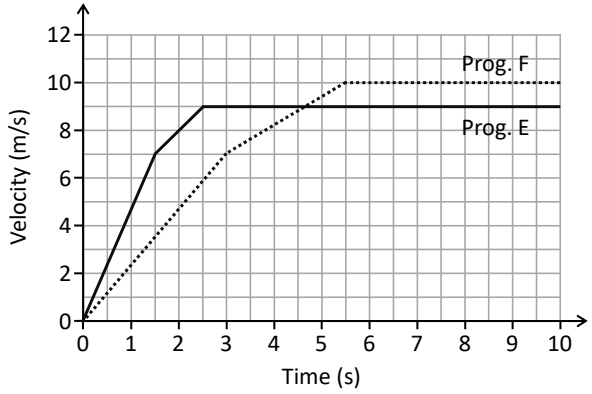


Prog. B



The same sprinter tries different programs.
These have been plotted on the same grid.
Which program (journey) has a better average speed?





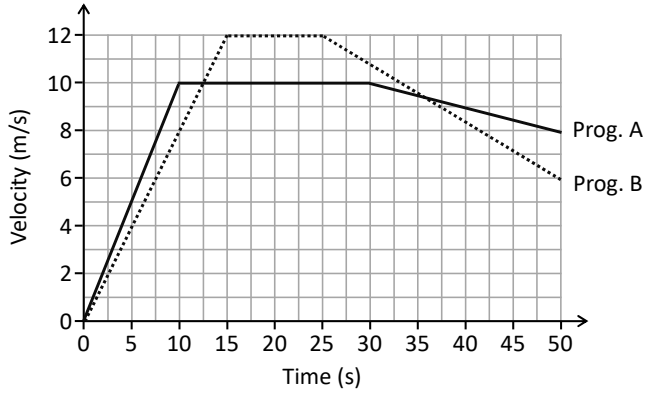
Fluency Practice

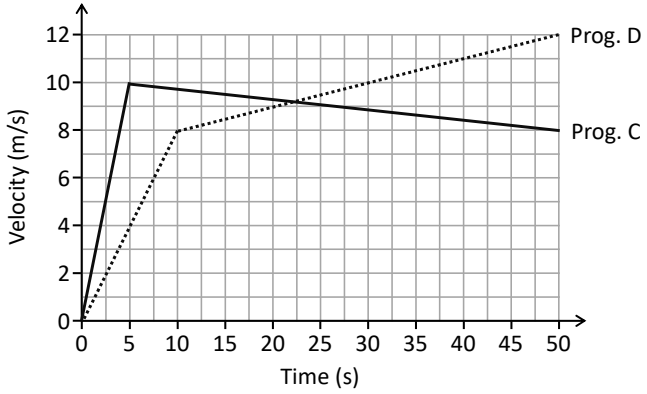
Distance Training

To train for the 400 m run,
athletes use a special pre-programmed treadmill.

Programs accelerate/decelerate to try & find a perfect strategy

Calculate the **average speed** for each program.

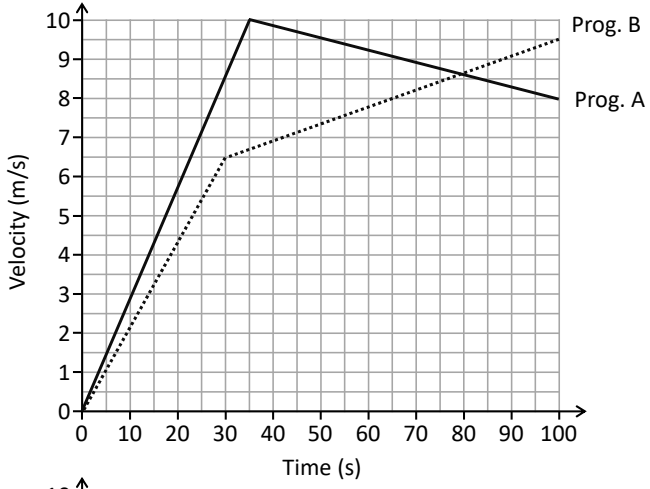


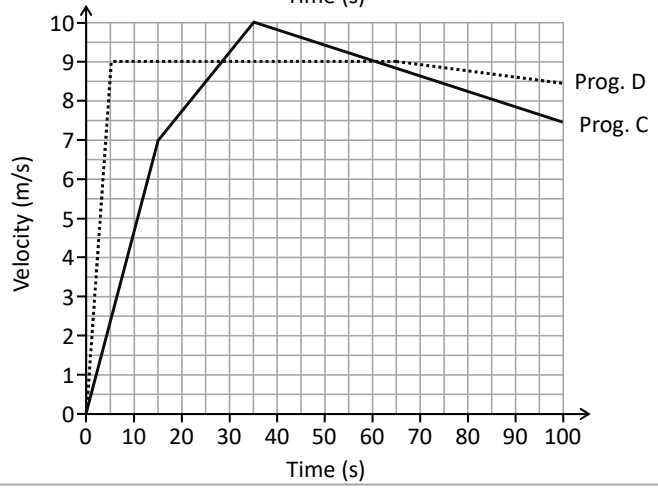


Which strategy do you prefer?

These programs are for the 800 m run.

Calculate **average speed** for each program.





Fluency Practice

Find the acceleration and distance travelled for these velocity time graphs

1)

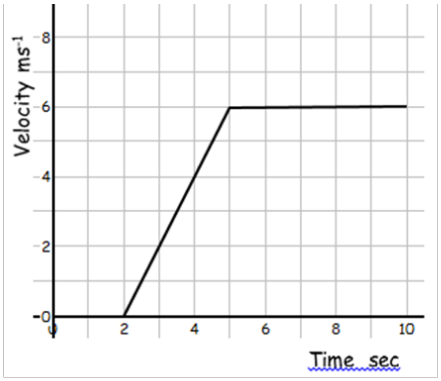
2)

3)

4)

5)

6)



7.

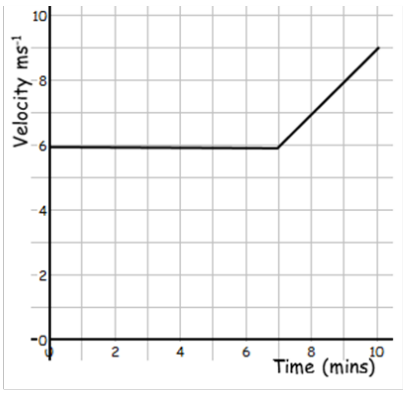
a) Calculate the acceleration between 2 and 5 seconds

b) Calculate the distance travelled

8.

a) Calculate the rate of deceleration between 7 and 10 seconds

b) Calculate the distance travelled



9.

a) Calculate the rate of acceleration between 7 and 10 minutes

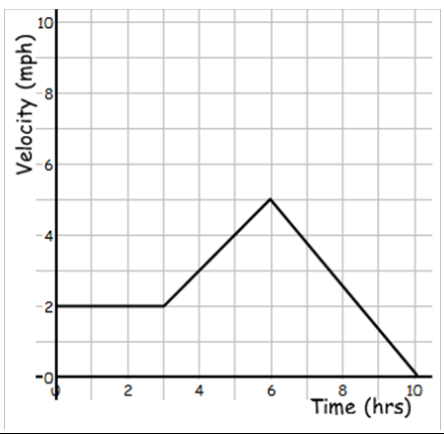
b) Calculate the distance travelled

10.

a) Calculate the rate of acceleration between 3 and 6 hours

b) Calculate the rate of deceleration between 6 and 10 hours

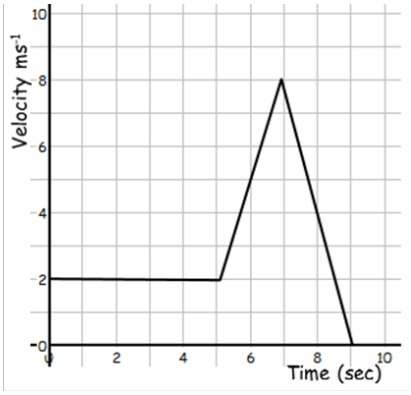
c) Calculate the distance travelled

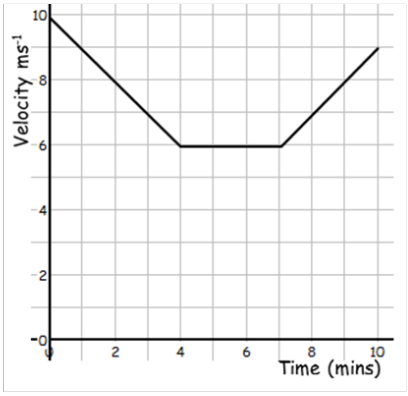


11. The graph on the right shows the motion of a marble travelling along a path.

a) Calculate the acceleration of the marble between 5 and 7 seconds

b) Calculate the distance travelled by the marble





12. The graph on the left shows part of the journey made by a train.

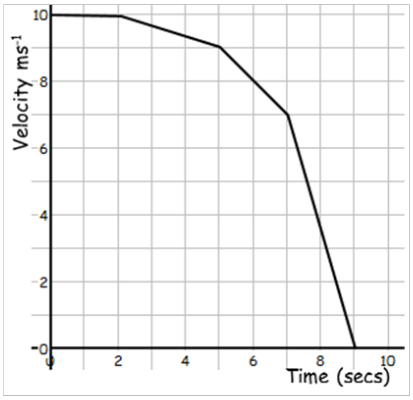
a) Calculate the deceleration of the train between 0 and 4 minutes

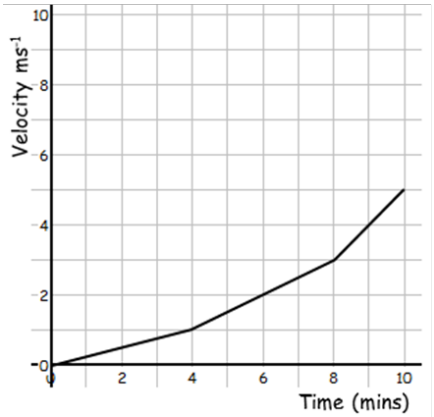
b) Calculate the distance travelled by the train during this period

13. The graph on the right shows the last 9 seconds of Dylan's bike ride.

a) Calculate Dylan's deceleration during the final 2 seconds

b) Calculate the distance cycled by Dylan during these 9 seconds.





14. The graph on the left shows the first 10 minutes of Kirsty's run.

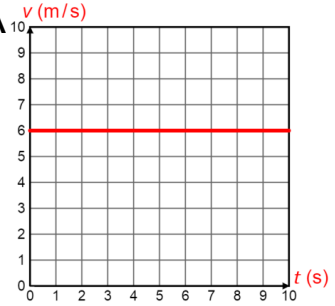
a) How much greater was Kirsty's acceleration between 8 and 10 minutes compared to the first 4 minutes?

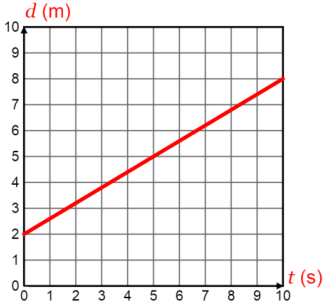
b) Calculate the distance ran by Kirsty over this 10 minute period

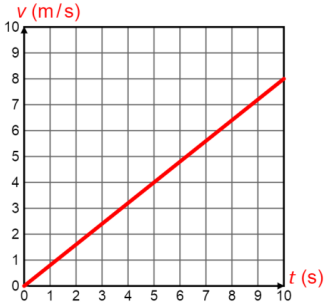
Fluency Practice

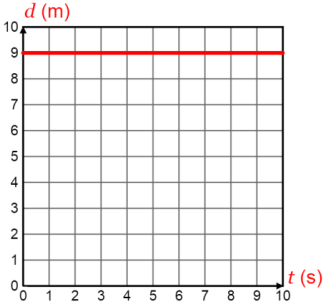
what distance has been travelled?

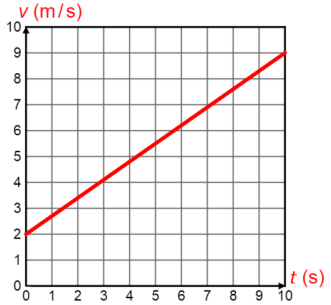
For each graph, work out the total distance travelled during the 10 seconds. Watch out – some are **distance-time** graphs and some are **velocity-time** graphs.

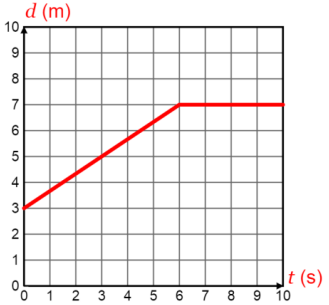
A


B


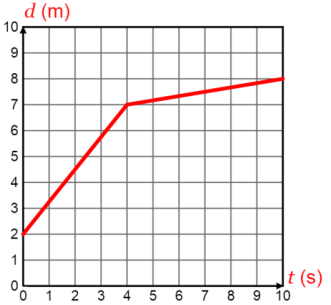
C


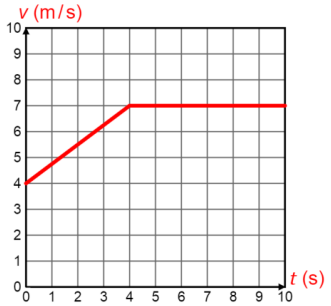
D


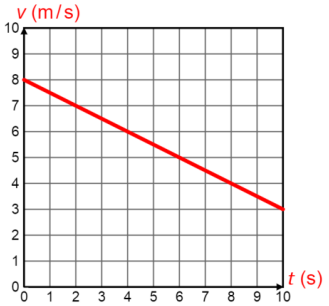
E


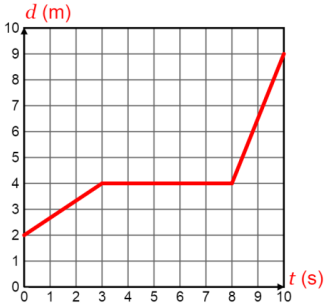
F


G


H


I


J


K


L


6 Gradients and Areas of Graphs

Fluency Practice

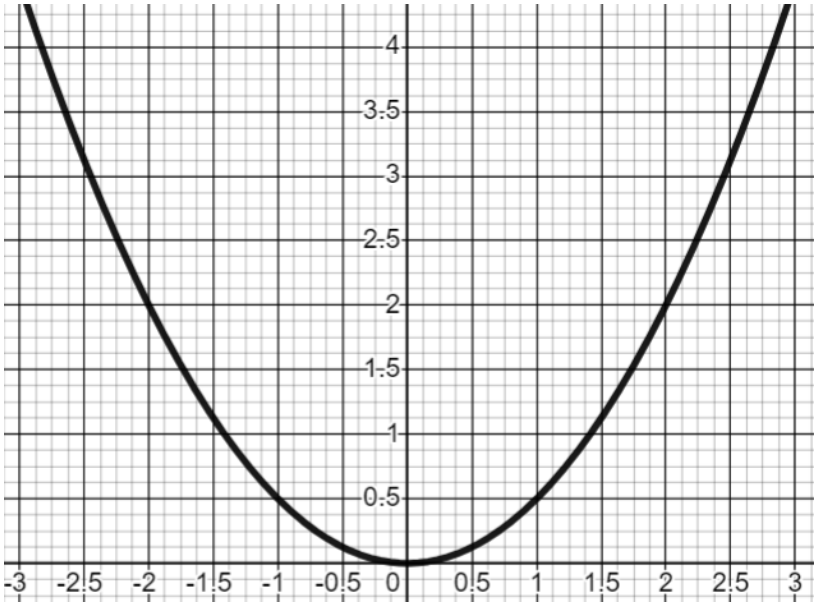
Estimating the Gradient of a Curve

(a)

Estimate the gradient of the curve at

(a) $x = 2$

(b) $x = -1$

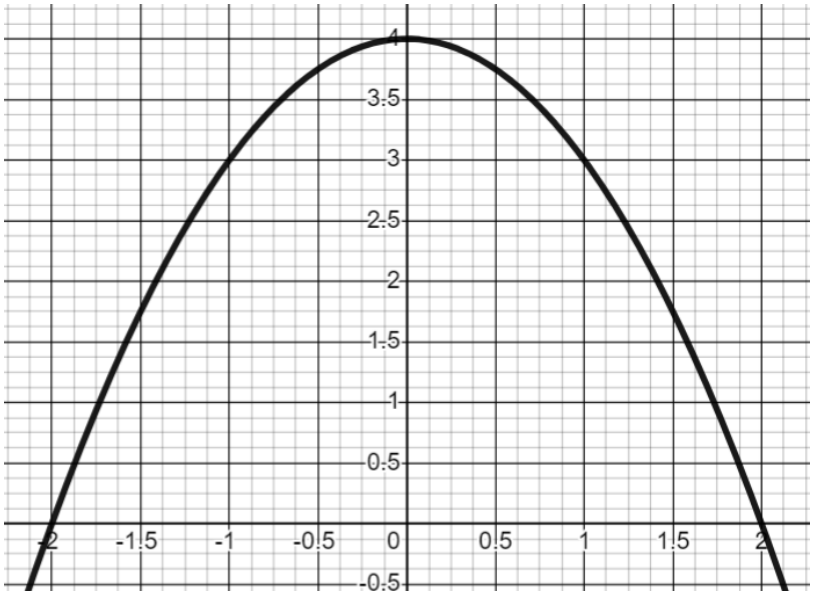


(b)

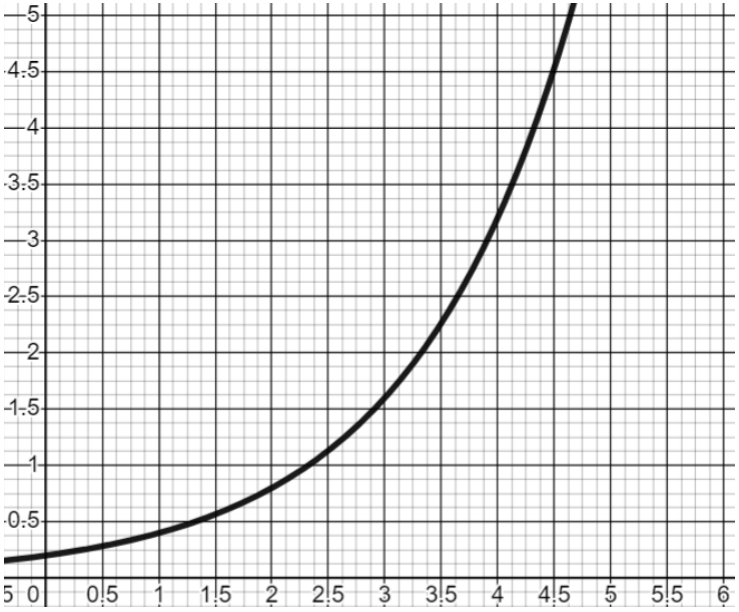
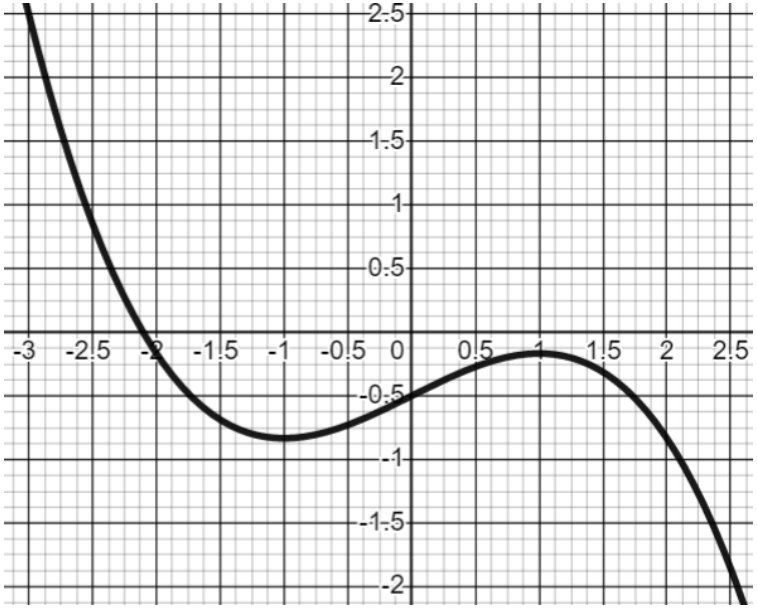
Estimate the gradient of the curve at

(a) $x = -1.5$

(b) $x = 1$



Fluency Practice

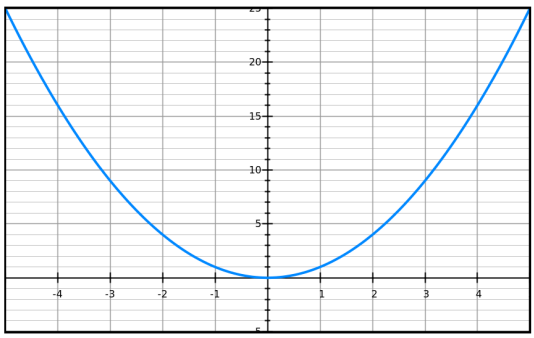
(c)	(d)
Estimate the gradient of the curve at (a) $x = 2$ (b) $x = 4$ 	Estimate the gradient of the curve at (a) $x = 2$ (b) $x = -1$ 

By drawing a suitable tangent, estimate the gradient of each curve at the points stated.

1. Estimate the gradient of $y = x^2$

a) at $(2, 4)$

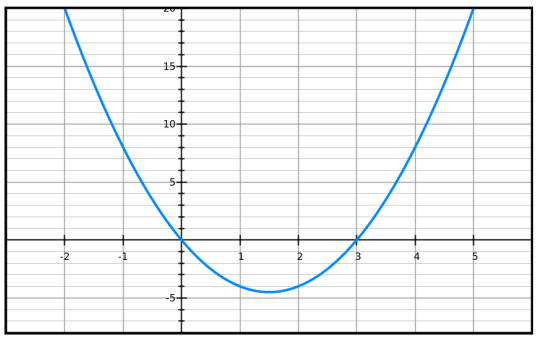
b) at $(-4, 16)$



2. Estimate the gradient of $y = 2x^2 - 6x$

a) at $(4, 8)$

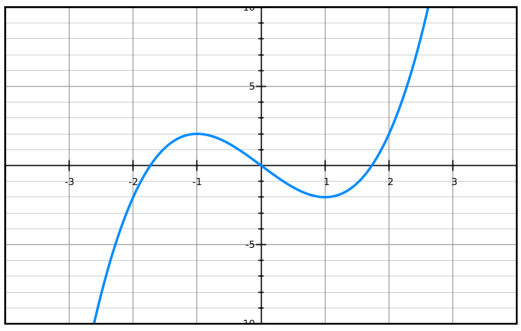
b) at $(0,0)$



3. Estimate the gradient of $y = x^3 - 3x$

a) at $(0,0)$

b) at $(-1,2)$

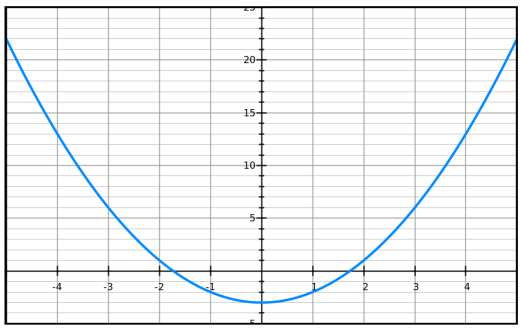


c) Which other point on the graph has the same gradient as your answer to part b?

4. Estimate the gradient of $y = x^2 - 3$

a) at $(2, 1)$

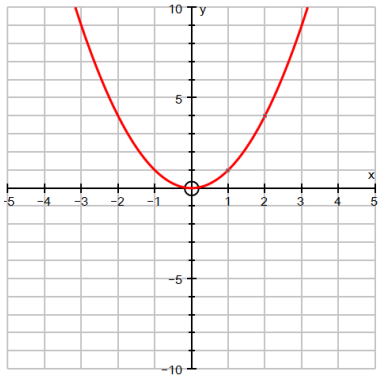
b) at $(-4, 13)$



c) Compare these answers to your answers to question 1. What do you notice? Why?

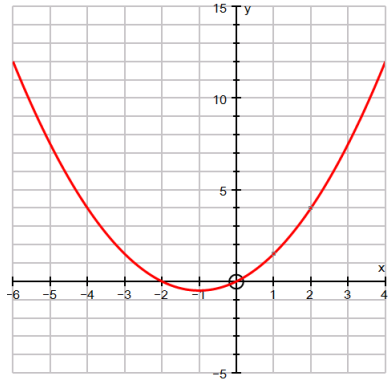
1. The graph below shows the curve of $y = x^2$

   Find the gradient at a) $x = 1$ b) $x = -2$



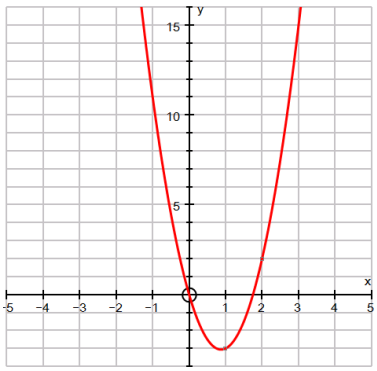
2. The graph below shows the curve of $y = \frac{1}{2}x^2 + x$

   Find the gradient at a) $x = 0$ b) $x = -4$



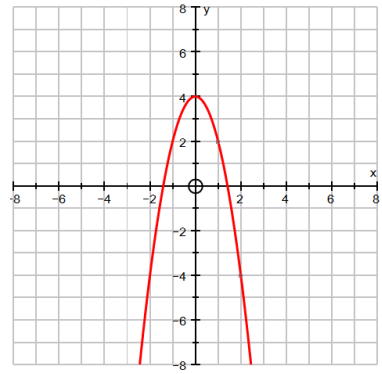
3. The graph below shows the curve of $y = 4x^2 - 7x$

   Find the gradient at a) $x = 2$ b) $x = 0$



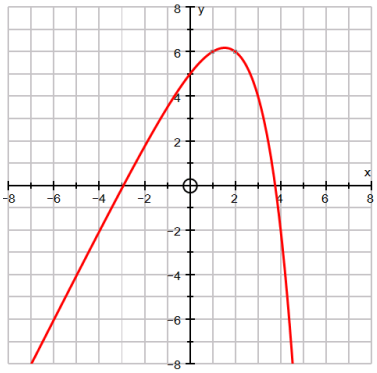
4. The graph below shows the curve of $y = -2x^2 + 4$

   Find the gradient at a) $x = -2$ b) $x = 1$



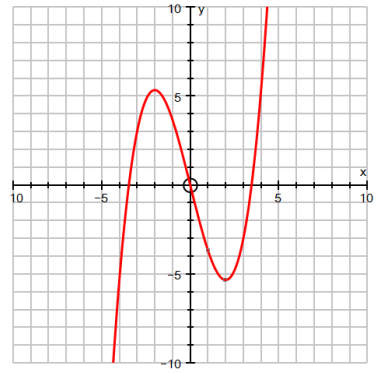
5. The graph below shows the curve of $y = 2^x + 2x + 6$

   Find the gradient at a) $x = 3$ b) $x = -5$



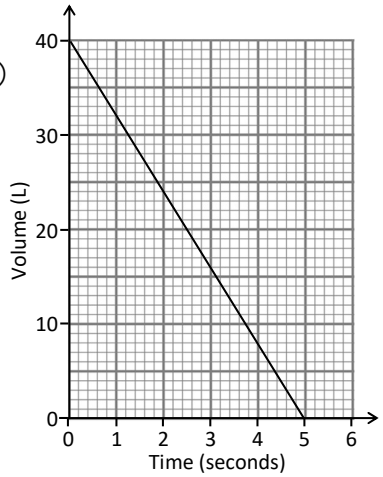
6. The graph below shows the curve of $y = \frac{1}{3}x^3 - 4x$

   Find the gradient at a) $x = 4$ b) $x = -3$

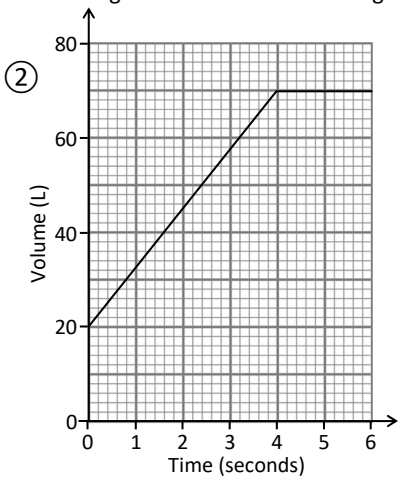


Fluency Practice

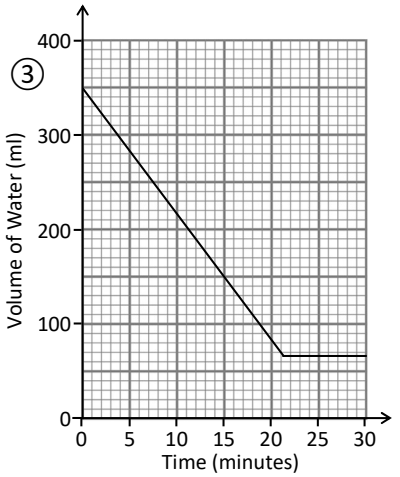
Finding the Rate of Volume Change

① 

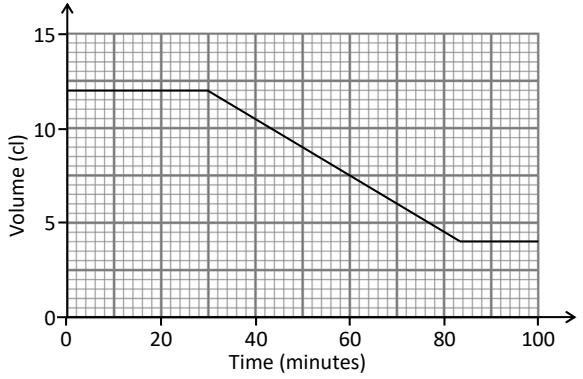
The container is emptied by a pump at _____ litres per second

② 

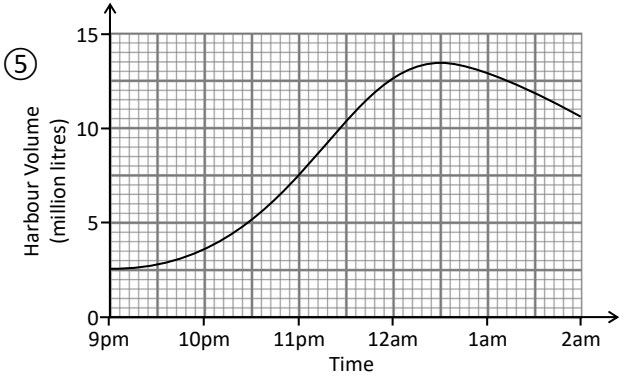
The container is filled at a rate of _____

③ 

Water leaves the container at a rate of _____

④ 

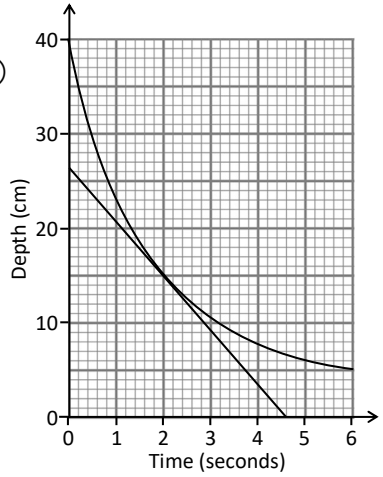
8 cl is pumped from the container at a rate of _____

⑤ 

The harbour fills at an **average rate** of _____

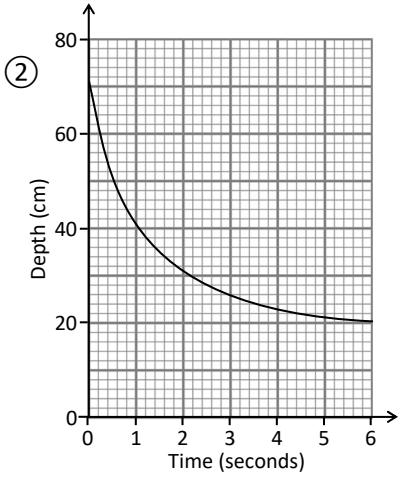
Fluency Practice

Estimating the Rate of Depth Change Using the Gradient of a Tangent

① 

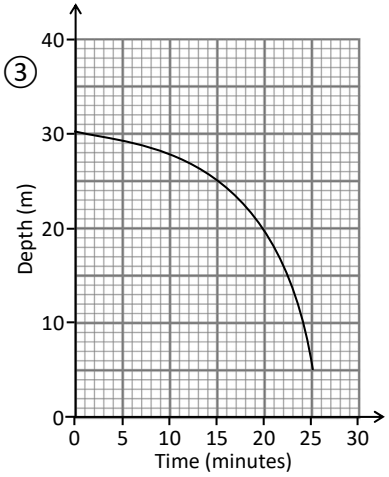
At 2 seconds the rate of depth change =

_____ cm per second

② 

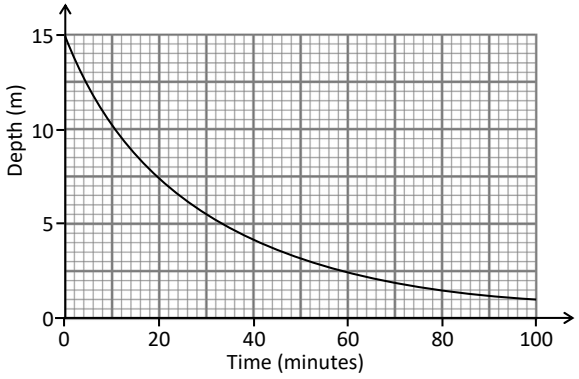
After 1 second the rate of depth change =

_____

③ 

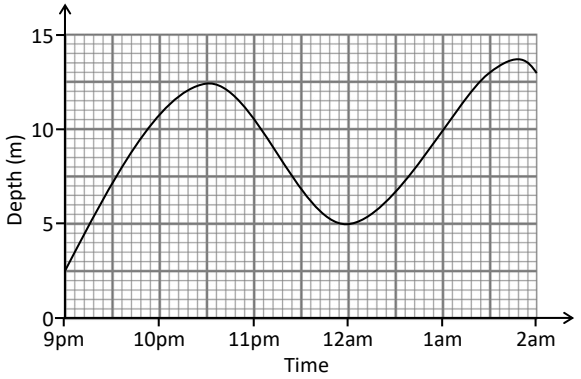
When T = 20 the rate of depth change =

_____

④ 

At 40 minutes the rate of depth change =

_____

⑤ 

At 23:30 the rate of depth change =

_____

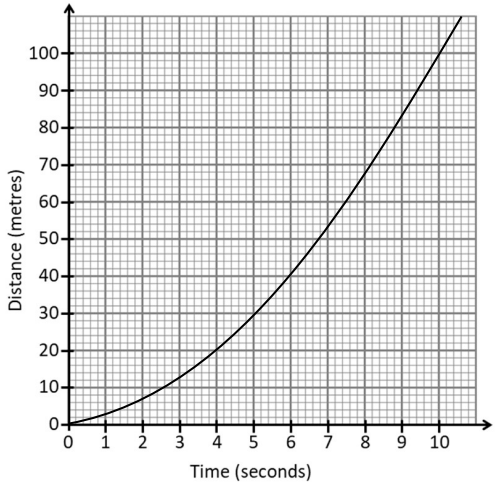
Fluency Practice

Distance-Time Graphs

① The graph shows a distance-time graph for a sprinter running a 100m race.

What time did they get for the race?

What was their average speed over the 100 metres?



How far did they go in the **first** 4 seconds of the race?

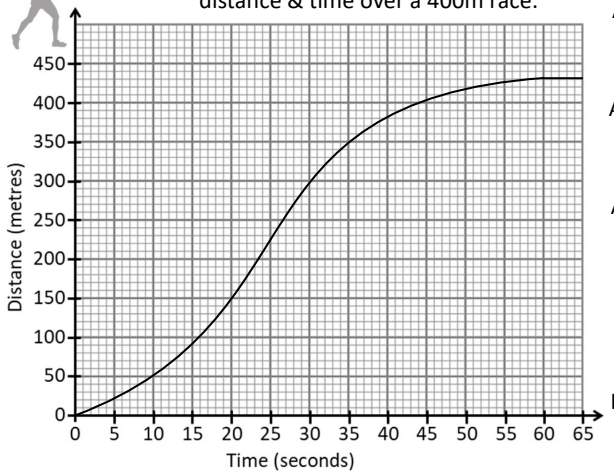
What was their average speed for those 4 seconds?

How far did they go in the **last** 4 seconds of the race?

What was their average speed for those 4 seconds?

Why is the sprinter slower at the beginning of the race?
How can you see this in a distance-time graph?

② The graph shows a runner's distance & time over a 400m race.



Average speed over the race = _____

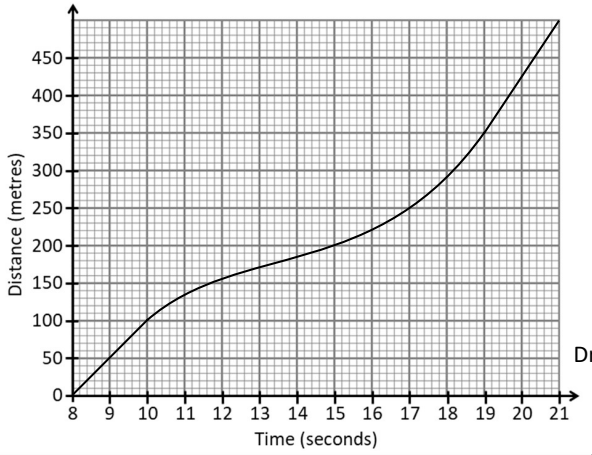
Av. speed for first 20 seconds = _____

Av. speed for last 10 seconds = _____

When do they stop running? _____

Estimate when the runner was going the fastest. _____

③ The graph shows the distance travelled by a car during 13 seconds of a race.



Why is the car changing speed? _____

What is the car's av. speed? _____

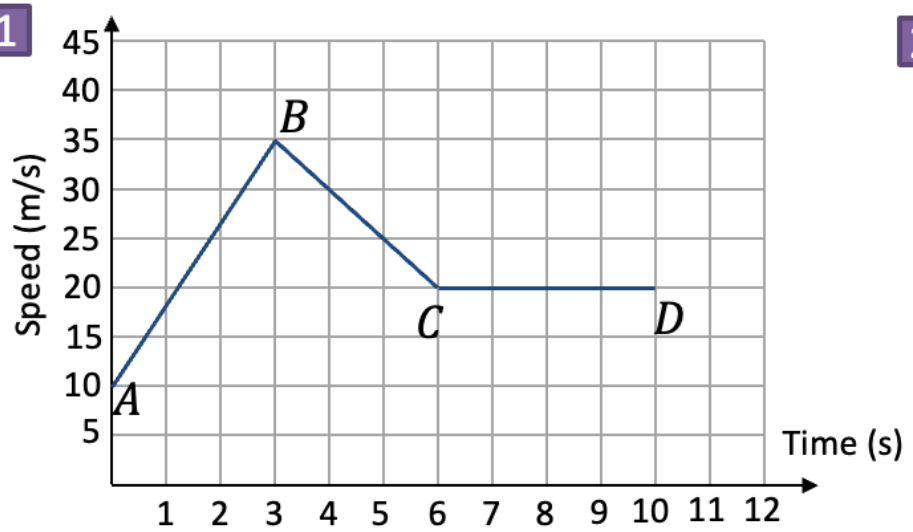
What is the car's top speed? _____

Estimate when the car is travelling the slowest _____

Draw a straight line to estimate the car's speed at 15 seconds. _____

Fluency Practice

1



The speed-time graph shows the motion of a toboggan as it descends a hill. Determine the acceleration of the toboggan between:

a) A and B = ?

b) B and C = ?

c) C and D = ?

2

The graph shows the speed of a car over time.
Estimate the acceleration of the car when:

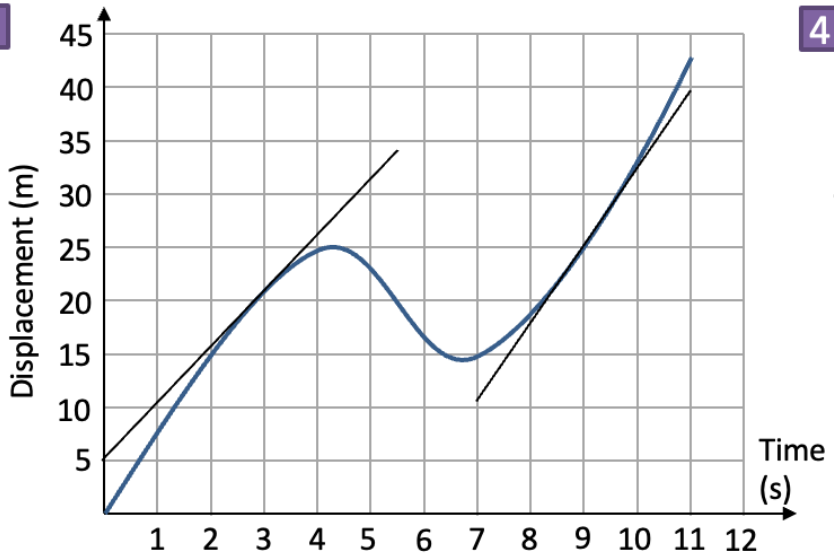
a) $t = 0$?

b) $t = 6$?

c) $t = 10$?

Fluency Practice

3



The graph shows Sheila's displacement from a wall during a game of dodgeball.
Estimate her velocity when:

a) $t = 3$

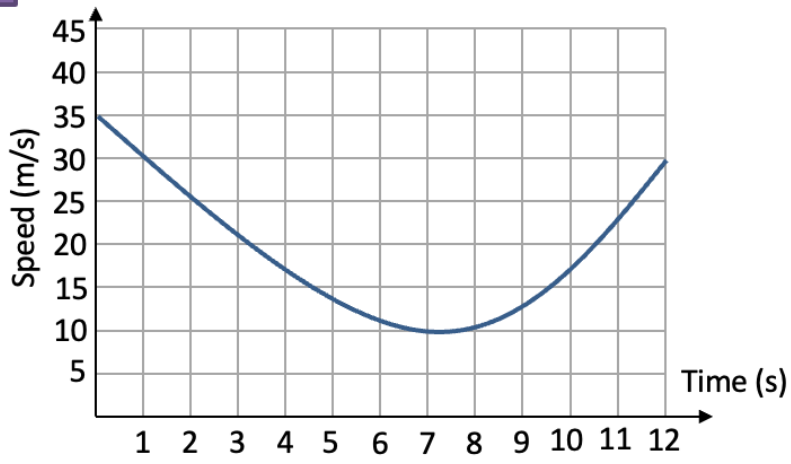
b) $t = 10$

?

?

Note: The difference between **'distance'** and **'displacement'** is that displacement can be negative, and can decrease, whereas distance travelled always increases.

4



The graph shows the motion of a car before and after the driver sees a speed camera.
Estimate the acceleration of the car when:

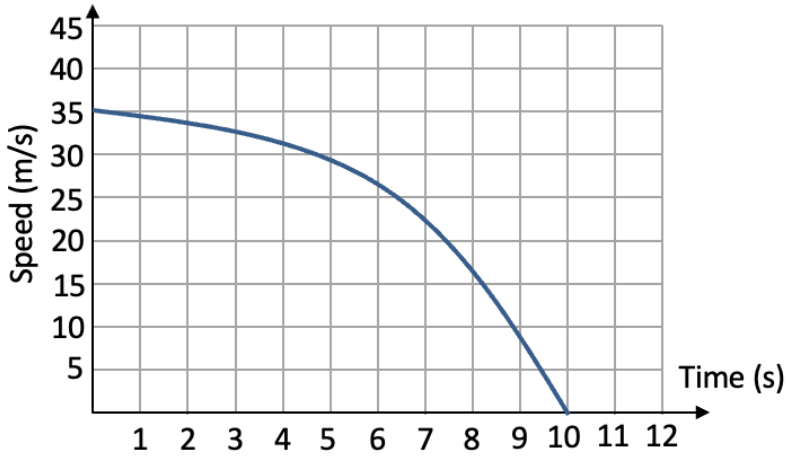
a) $t = 6$

b) $t = 12$

?

?

5



The graph shows the speed of a train after applying its breaks.

a) Determine the average acceleration of the train.

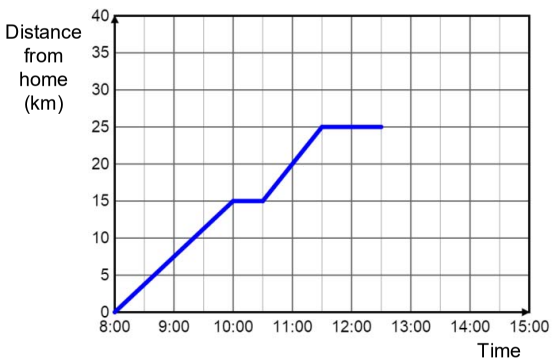
?

b) Estimate the time at which the acceleration of the train was this average.

?

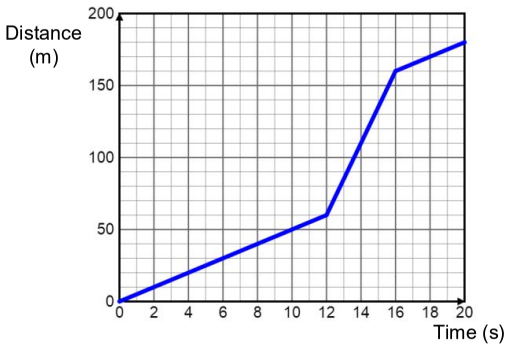
1. Sarah spent a day on a cycling trip.
The distance-time graph shows her journey.

(a) At what time did Sarah first stop for a break?
(b) What was Sarah's average speed between 8:00 and 10:00?
(c) What was Sarah's average speed between 10:30 and 11:30?
(d) At 12:30, Sarah started cycling home. She did not take another break and arrived home 2 hours later. Use this information to complete the graph of Sarah's trip.



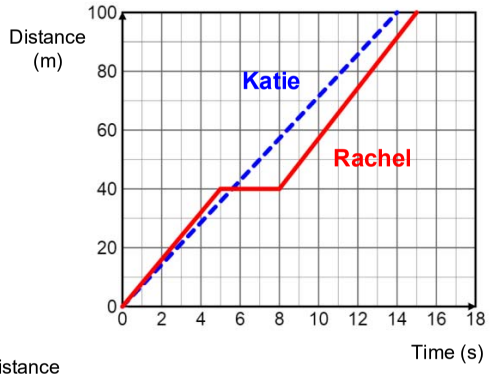
2. The distance-time graph shows the motion of a go-kart moving along a track.

(a) What was the average speed of the go-kart over the entire 20 seconds?
(b) True or false? The go-kart was travelling at the same speed for the fist twelve seconds as it was for the last 4 seconds.
(c) State the time interval during which the go-kart was travelling fastest.



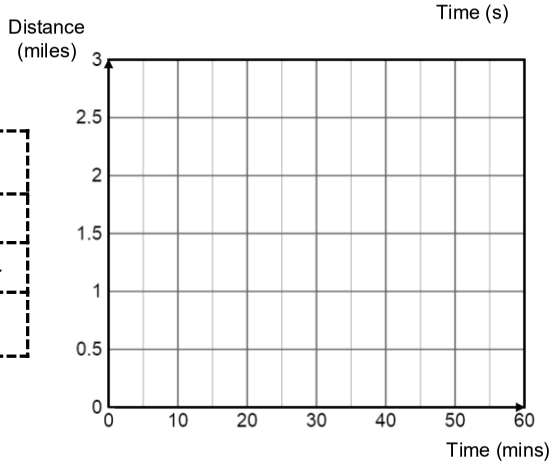
3. Katie and Rachel race each other over 100m.
The graph illustrates the race.

(a) Katie maintained a steady pace. What was her average speed?
(b) What was Rachel's speed between the 5th and 8th second of the race?
(c) Who won the race, and by how many seconds?



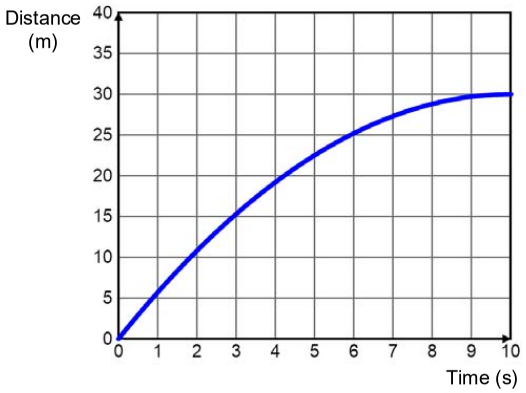
4. John drives from his home to a shop and back. Use the clues to complete the distance-time graph for his journey.

John left home and travelled to the shop at a constant speed, arriving after 20 minutes.
John spent 20 minutes at the shop.
The shop is two and a half miles from John's home.
John travelled home at twice the speed that he travelled to the shop.



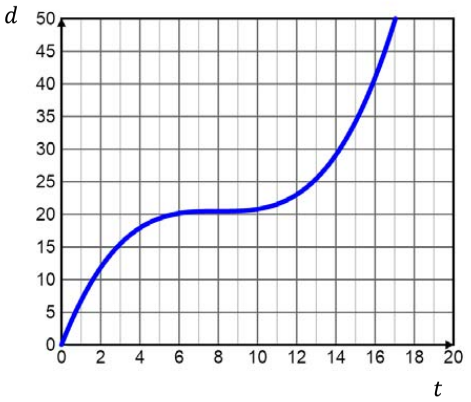
5. The graph shows the motion of a car.

(a) Work out the average speed of the car over the 10 second interval.
(b) Work out the instantaneous speed of the car at 6 seconds.
(c) Work out the time at which the instantaneous speed is equal to the average speed over the 10 second interval.



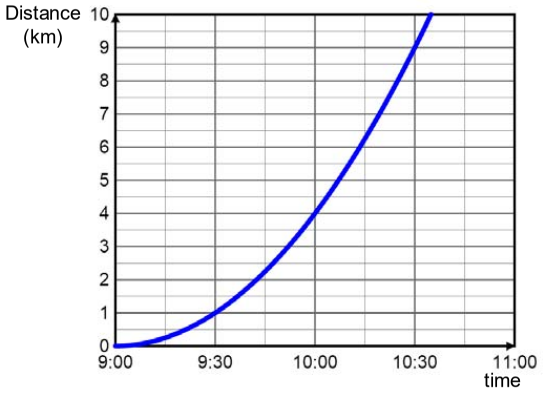
6. The graph shows the distance, d metres, travelled by a canoeist, t seconds after passing a marker point.

(a) Work out the canoeist's instantaneous speed when $t = 4$.
(b) Work out the canoeist's average speed over the interval $10 \leq t \leq 15$.
(c) Explain, without calculating, how you can tell that the canoeist is moving faster when $t = 16$ than at $t = 12$.

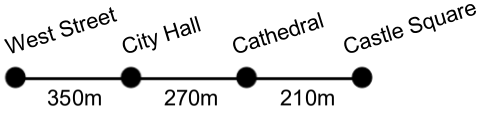


7. Mark and Sophie take part in a 10km run. Mark's progress is shown on the graph.

(a) Work out Mark's instantaneous speed at 9:30.
(b) Sophie started the run at the same time as Mark. She maintained a steady speed of 6km/h throughout her run. Draw Sophie's progress on the graph.
(c) At one point one of the two runners overtook the other. At what time did this happen, and who overtook who?

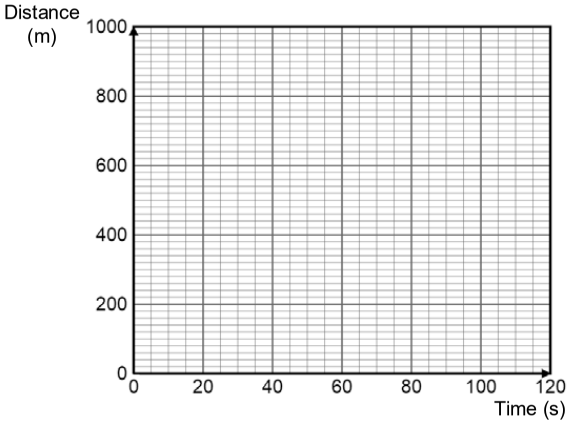


8. Four stops on a tramline in Sheffield are illustrated, with the distance between stops shown in metres.



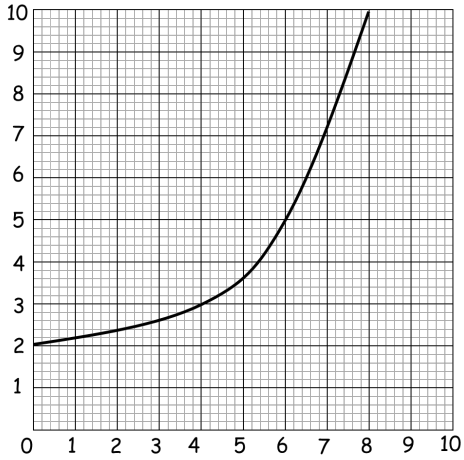
A tram spends 30 seconds at each stop, and when it is moving has an average speed of 8 m/s.

Complete the graph to show the journey of a tram from the moment it leaves West Street to the moment it arrives at Castle Square.

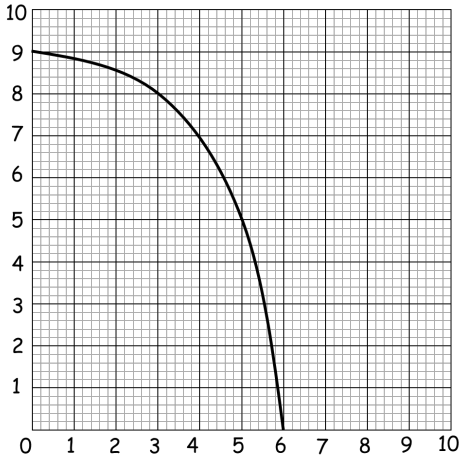


In each of the following graphs, use suitable shapes and an appropriate number is strips to estimate the area under each graph. Additionally, state whether your calculation is an overestimate or underestimate for the area.

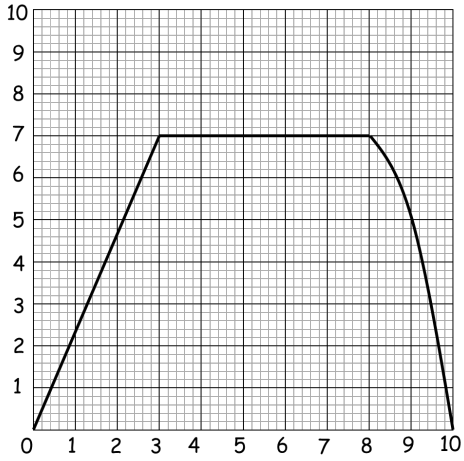
1)



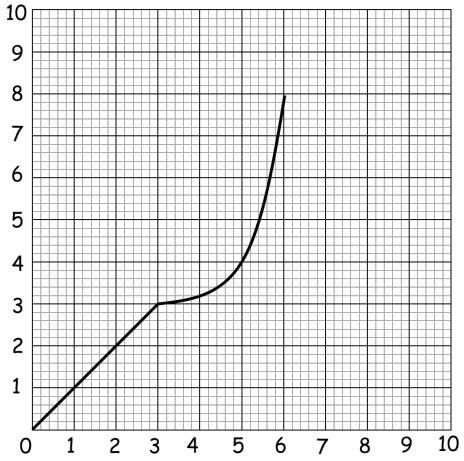
2)



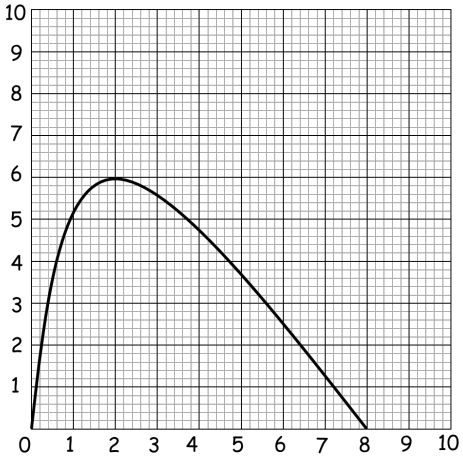
3)



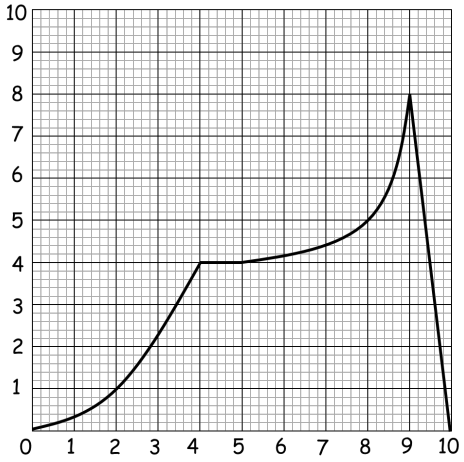
4)



5)

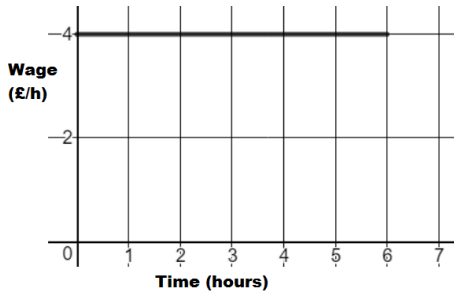


6)

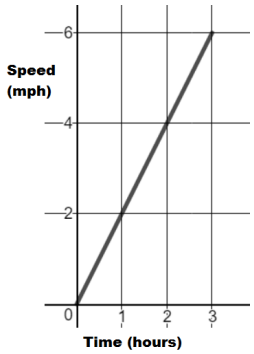


1. Find the area under each graph. Include units in your answer.

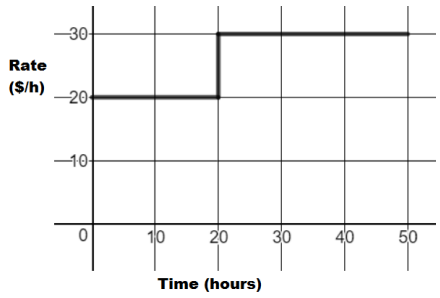
A



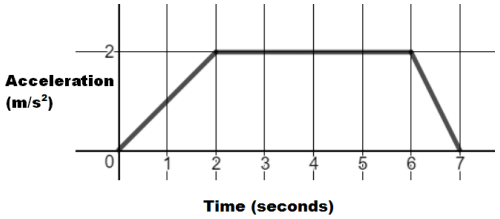
B



C

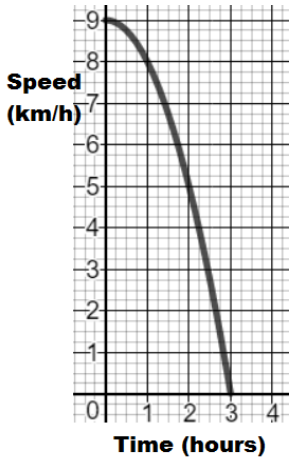


D



2. Estimate the area under each graph. Include units in your answer.

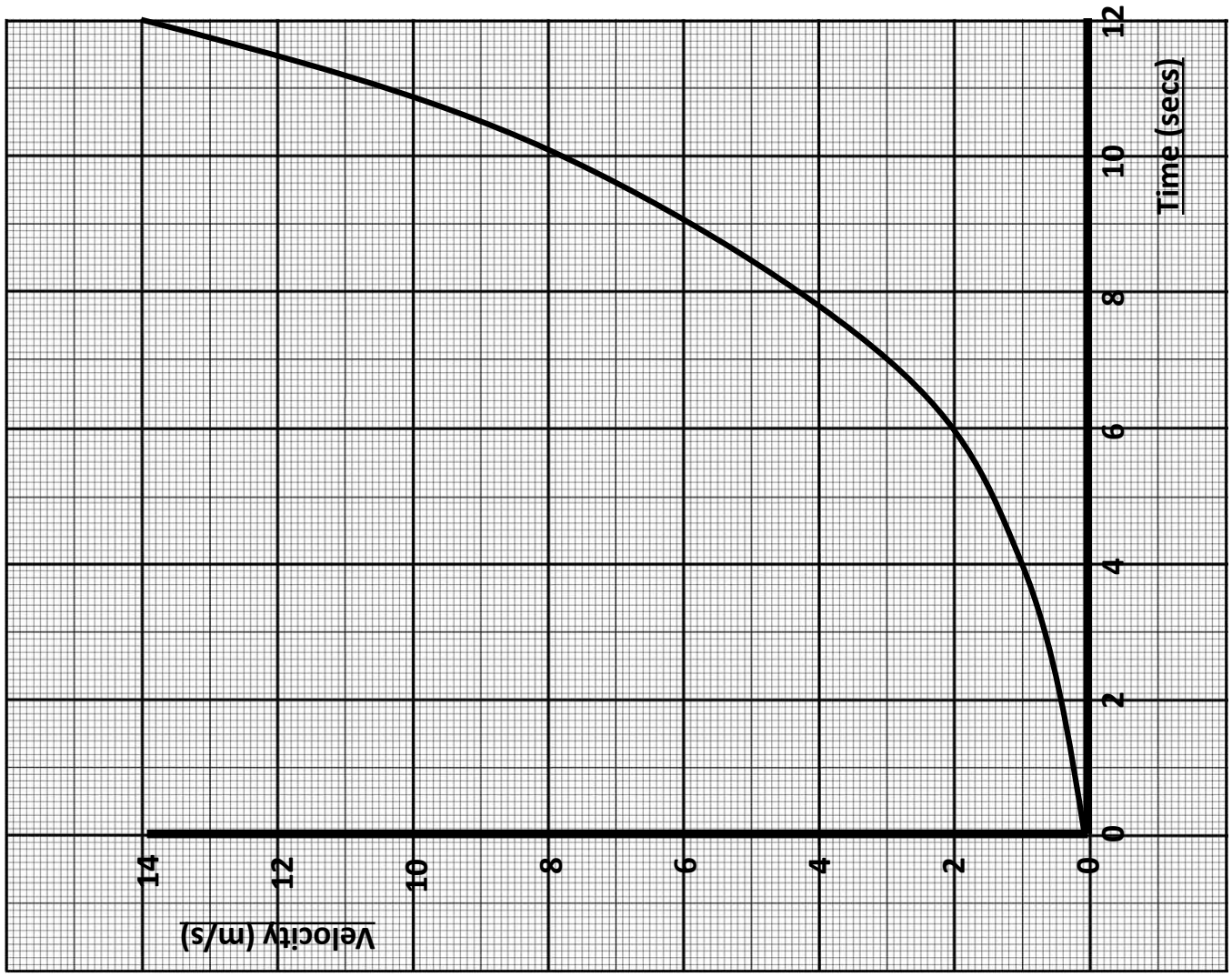
E



F

Exam Questions

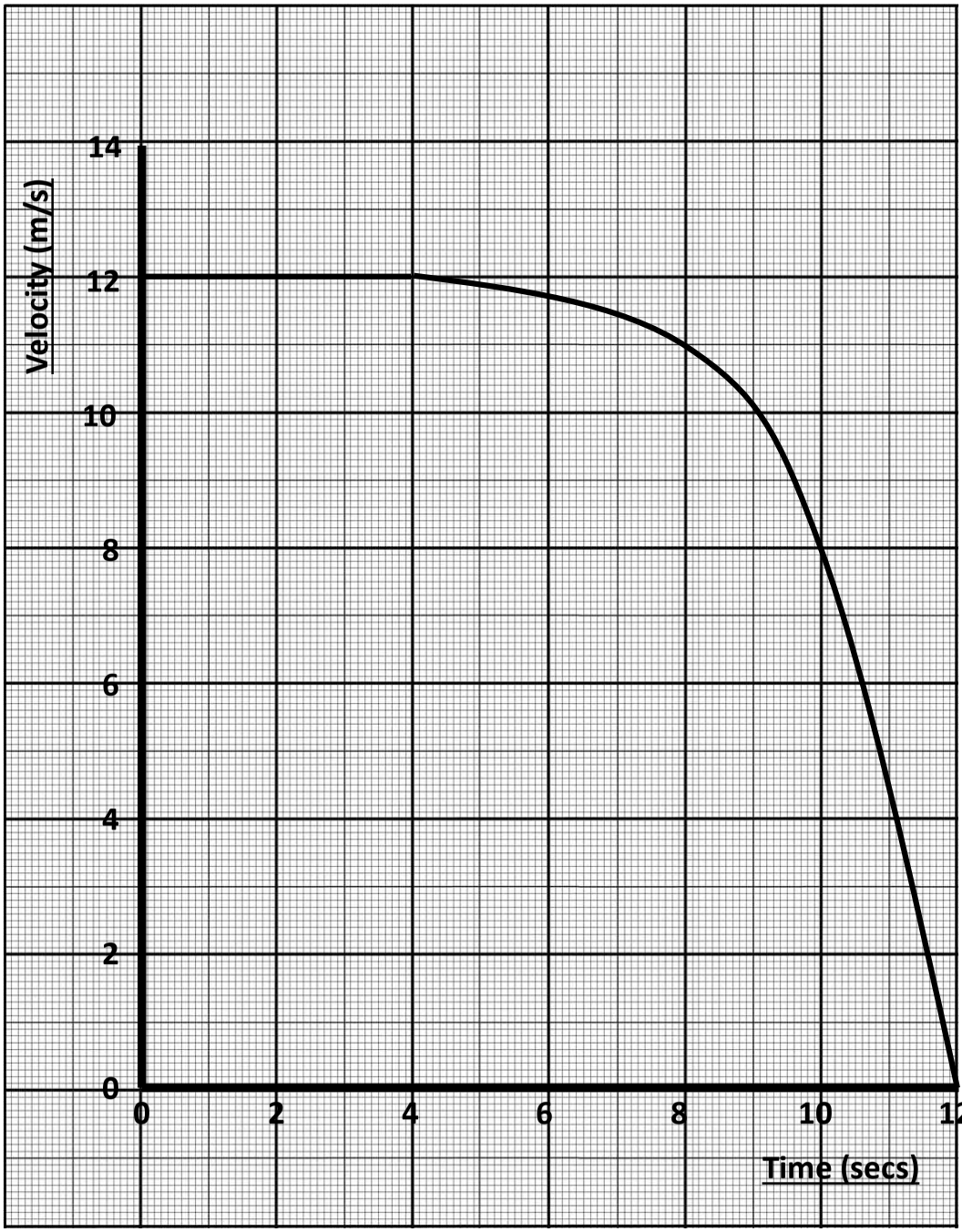
1. The velocity time graph below shows the first part of a train's journey.



a) Calculate the acceleration of the train at time t = 6

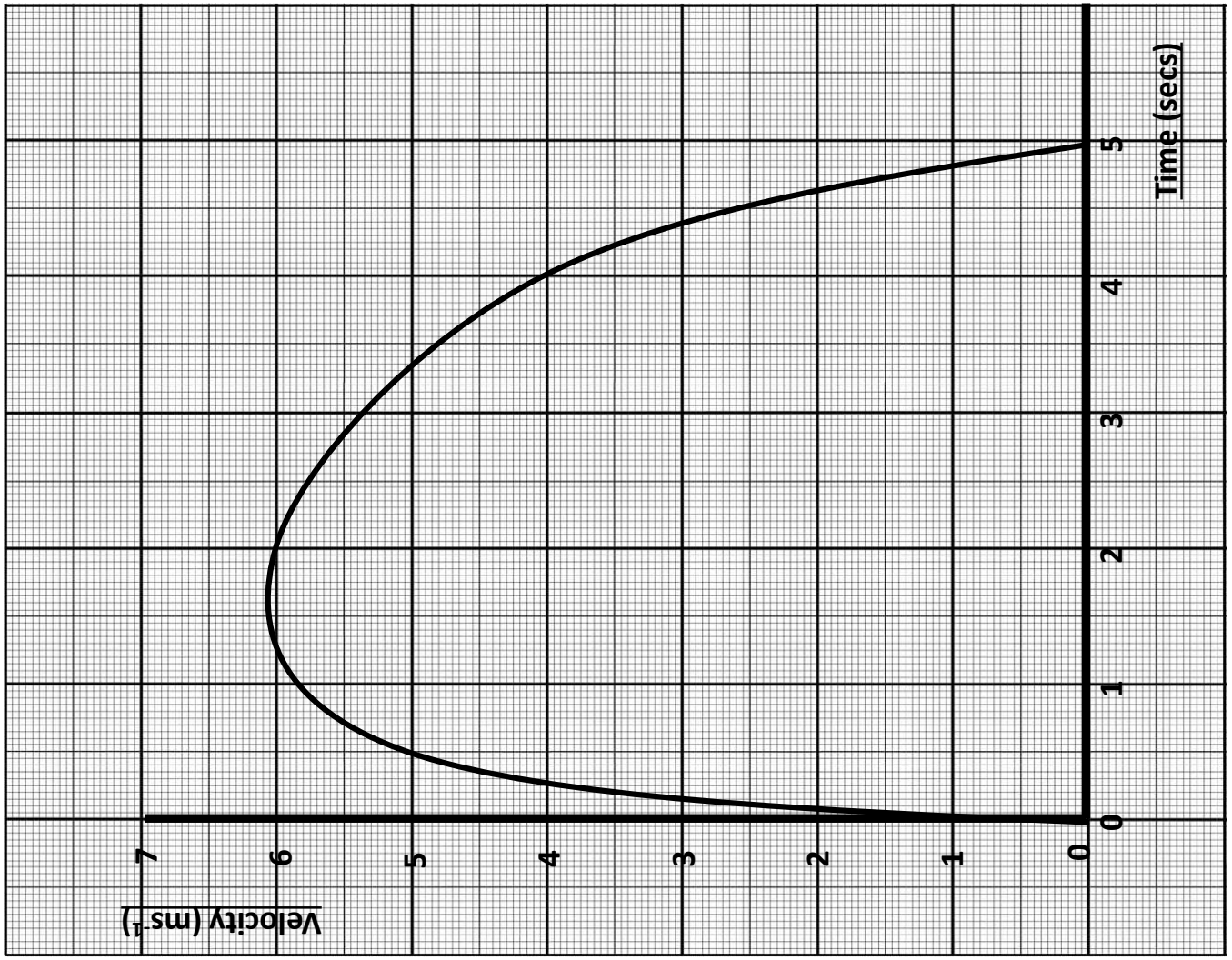
b) Calculate the distance travelled by the train between 2 and 10 seconds. Use 4 strips of equal width.

2. The graph below shows the speed of a car as it approaches a junction.



a) Describe what is happening between 0 and 4 seconds
b) Calculate the car's deceleration at time t = 8.
c) Calculate the distance travelled by the car between 8 and 12 seconds. Use 4 strips of equal width.

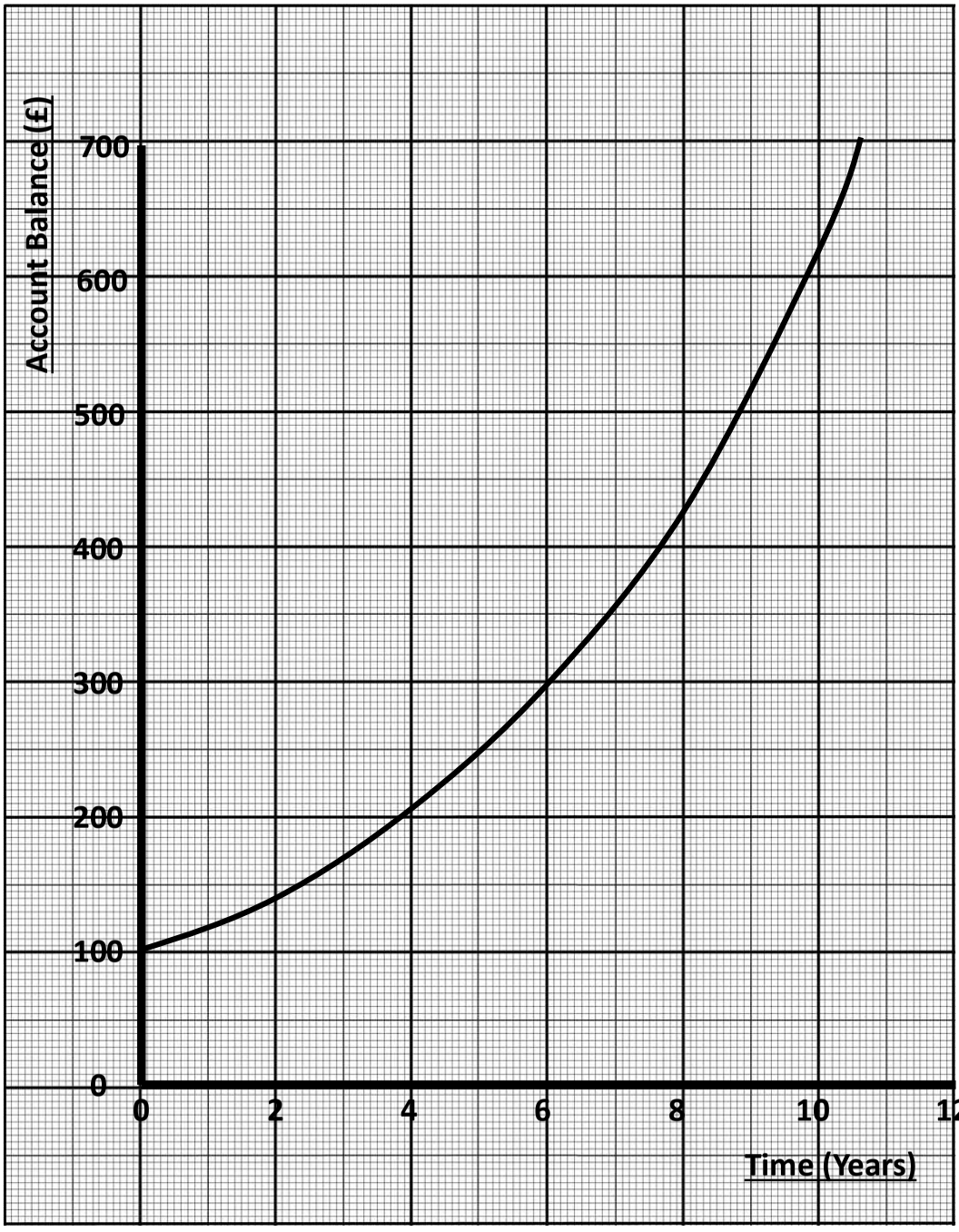
3. The graph below shows the journey of a javelin when thrown.



a) Calculate the acceleration of the javelin at time t = 4.

b) Calculate the distance travelled by the javelin between 2 and 5 seconds.

4. Will gets £100 which he puts in his bank. Each year he puts more money into his account to ensure the total increases by 20% each year. The graph below shows Will's account balance over a 10 year period.



a) How much money is in Will's account after 7 years?

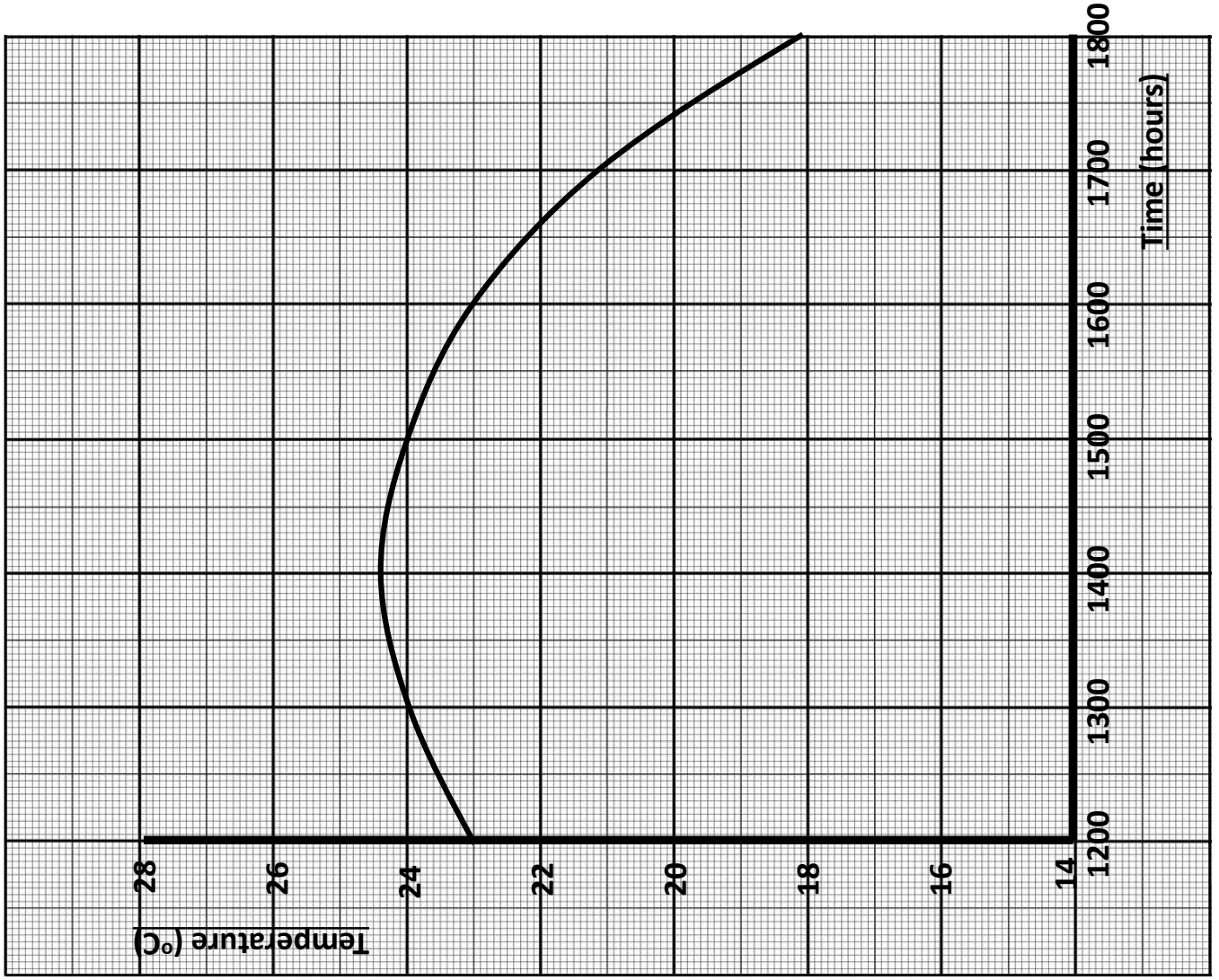
b) Calculate the rate of change of Will's balance in year 6. State the units in your answer.

5. The graph below shows the electricity usage by a church hall.

a) Calculate the rate of change of the electricity usage at 11:00.

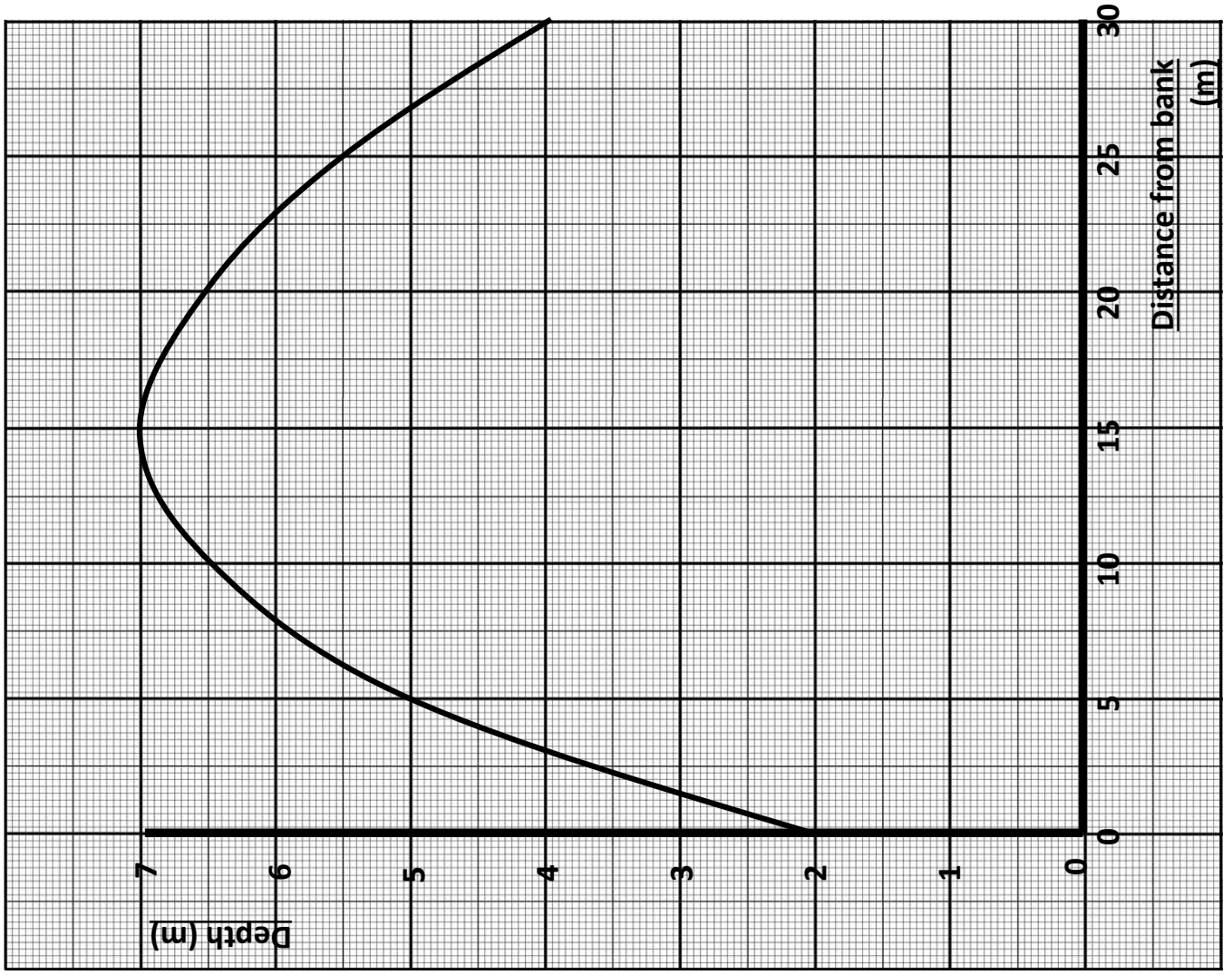
b) Calculate the amount of electricity used by the church hall between 10:00 and 12:00. State the units in your answer.

6. The graph below shows the temperature during part of one day in June.



a) What is the temperature at 16:00?
b) Calculate the gradient at 17:00
c) Describe what you have calculated in part b), stating any units.

7. The graph below shows the cross-section of a river.



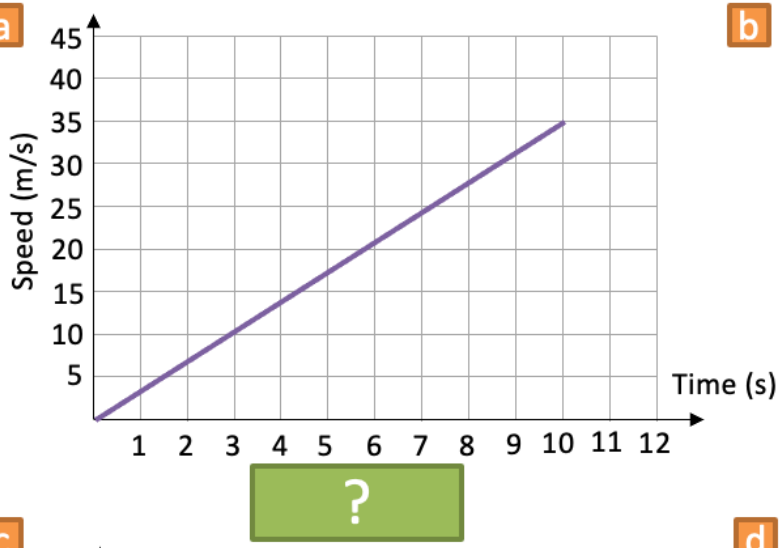
a) Calculate the rate of change in depth of the river 10m from the bank.

b) Calculate the area of the cross-section. Use 6 strips of equal widths

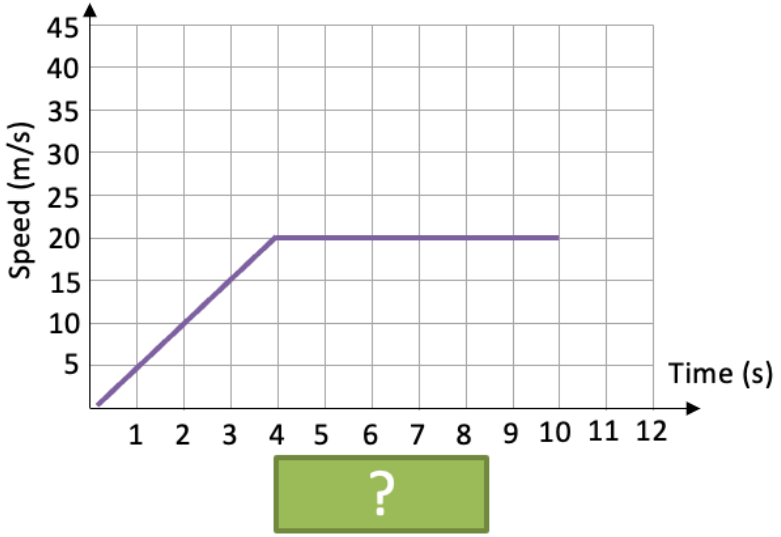
Fluency Practice

1 Calculate the distance travelled by these cars in their first 10 seconds.

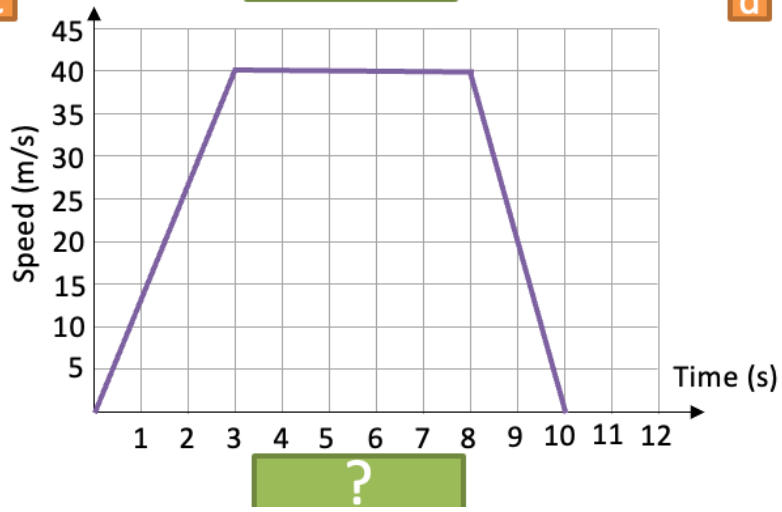
a



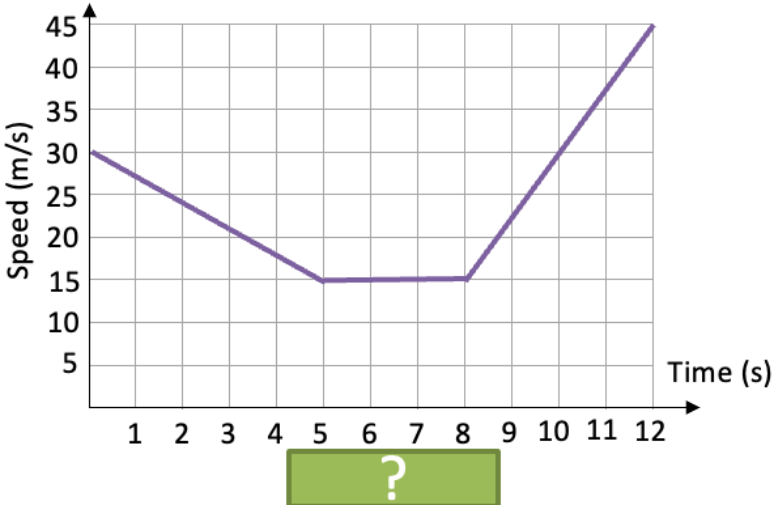
b



c

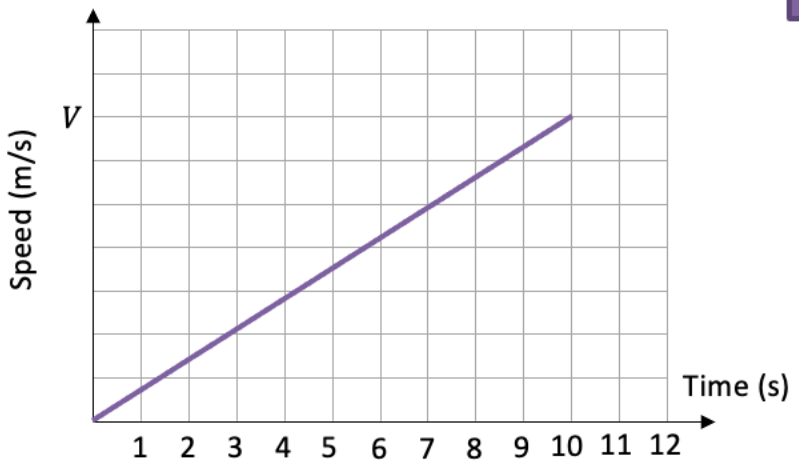


d



Fluency Practice

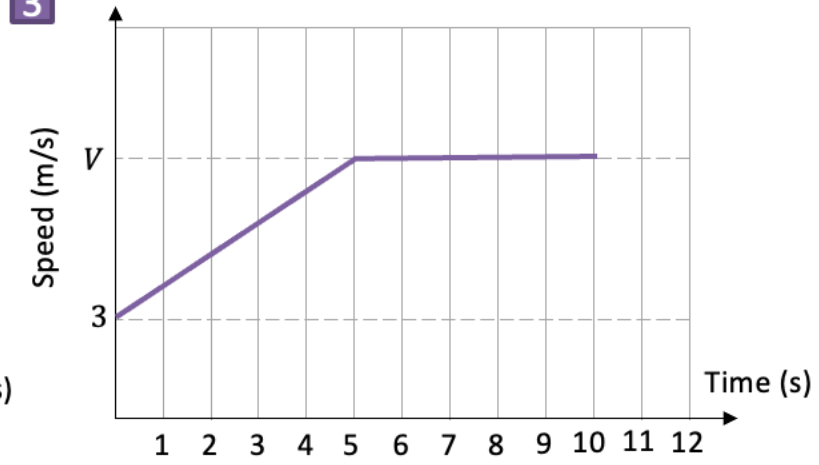
2



A car accelerates from rest with constant acceleration, until it reaches a speed of V m/s after 10 seconds. The distance travelled is 150m.
Determine the value of V.

?

3

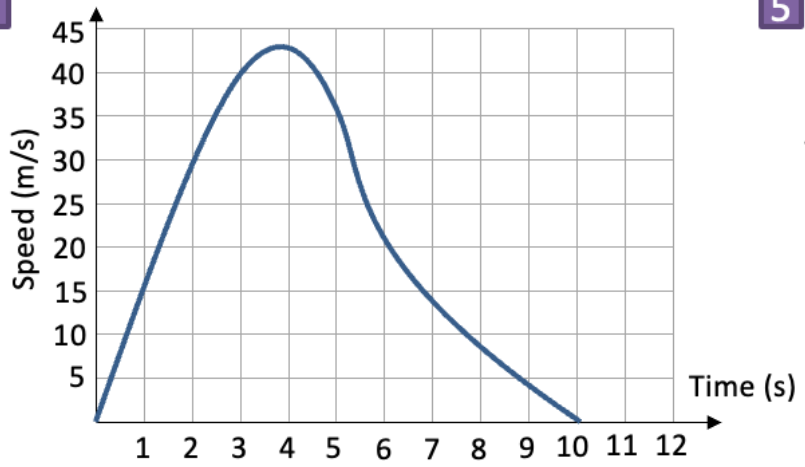


A tortoise starts running at a speed of 3 m/s, increasing to V m/s after 5 seconds. He maintains this speed for the next 5 seconds.
Over the 10 seconds he has travelled 155m. Determine V.

?

Fluency Practice

4



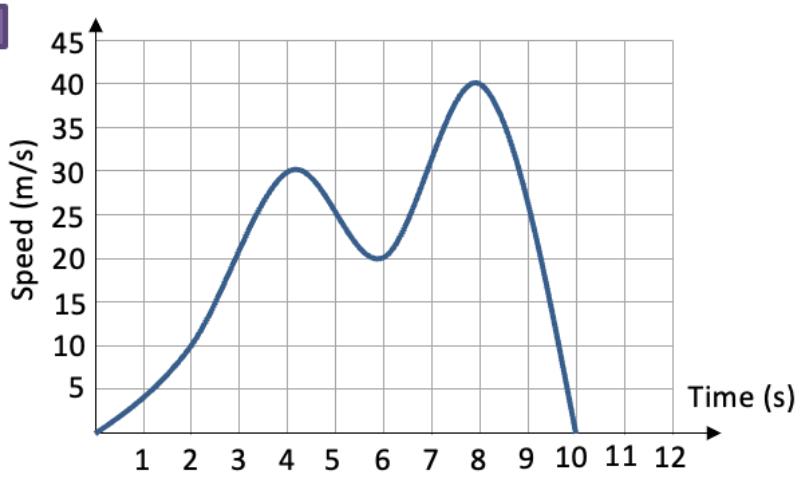
a) Using 2 strips, estimate the distance the car has travelled in the first 10 seconds.

?

b) Does this underestimate or overestimate the true area?

?

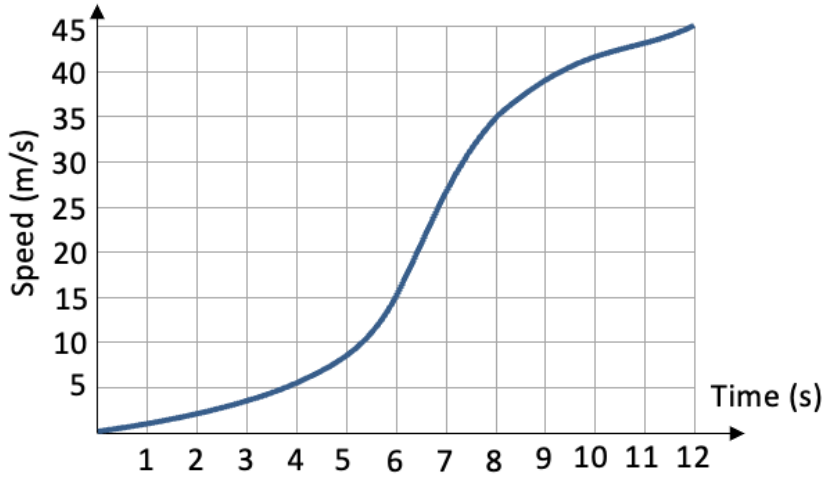
5



Using 5 strips, estimate the distance the car has travelled in the first 10 seconds.

?

Fluency Practice



Estimate the distance travelled in the first 12 seconds when using:

a) 1 strip (i.e. a single triangle)

?

b) 2 strips

?

c) 3 strips.

?

Fluency Practice

velocity-time graphs: tangents & areas

Work out the best estimates you can.

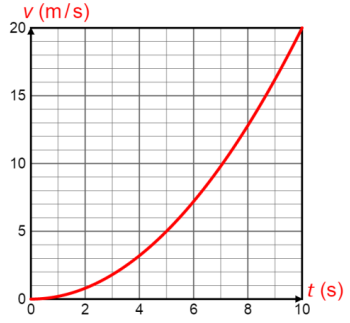
1.

(a) Maximum speed.

(b) Acceleration at $t = 4$.

(c) Total distance travelled.

2.

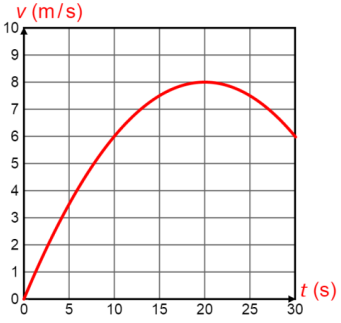


(a) Average acceleration.

(b) Acceleration at $t = 4$.

(c) Total distance travelled.

3.

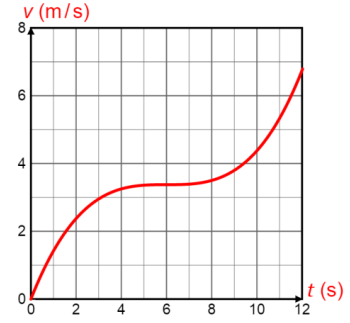


(a) Time at which acceleration is zero.

(b) Acceleration at $t = 10$.

(c) Total distance travelled.

4.

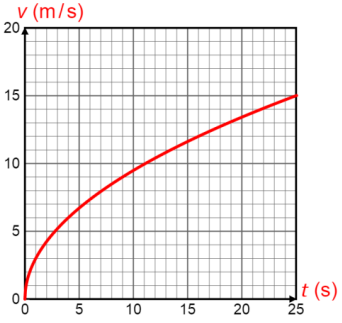


(a) Time at which speed is 2m/s.

(b) Acceleration at $t = 4$.

(c) Distance travelled in first 6 seconds.

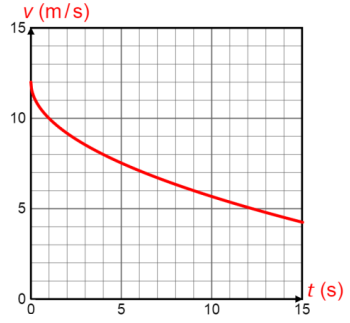
5.



(a) Average acceleration.

(b) Time at which:
instantaneous acceleration
= average acceleration

6.

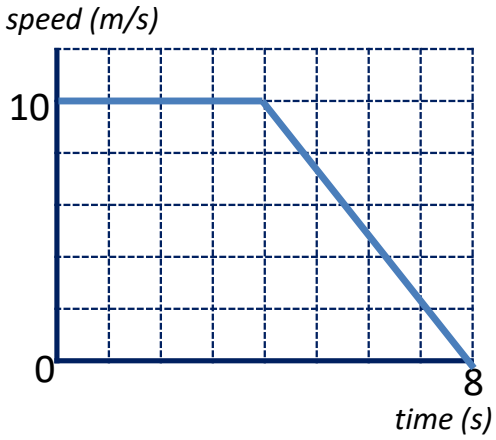


(a) Average acceleration.

(b) Total distance travelled.

(c) Is your distance an under-estimate or an over estimate?

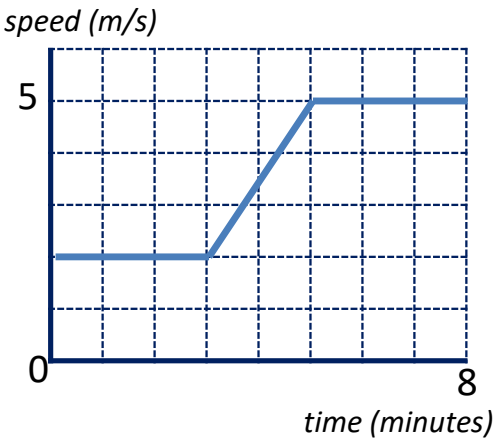
Fluency Practice



1. The graph shows the speed of a car approaching traffic lights.

a) What is the rate of deceleration between 4 and 8 seconds?

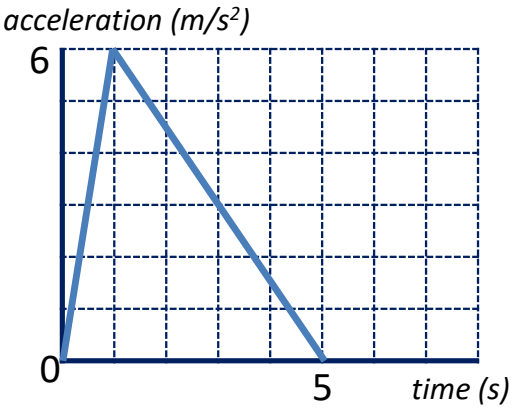
b) By calculating the area under the graph, find the distance travelled.



2. The graph shows the speed of a train for part of its journey.

a) What is the rate of acceleration between 3 and 5 seconds?

b) By calculating the area under the graph, find the distance travelled.

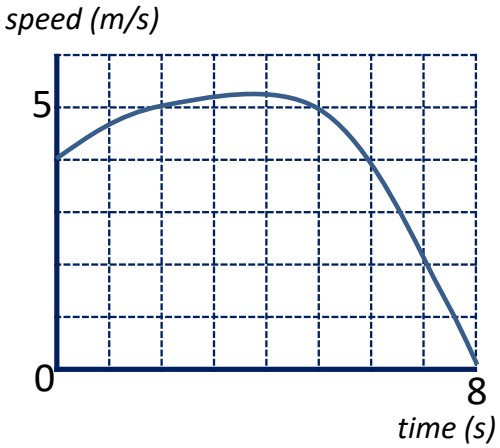


3. The graph shows the acceleration of a child going down a waterslide.

a) Describe what is happening after the first second.

b) By calculating the area under the graph, find the child's final speed (assume the child starts from a resting position).

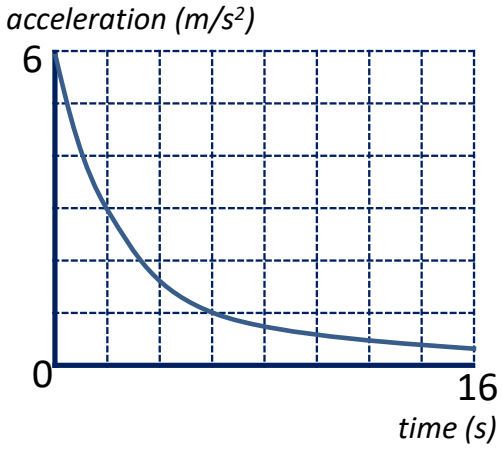
Fluency Practice



4. The graph shows the speed of a ball rolling down a hill.

a) By drawing a tangent, estimate the rate of deceleration at 6 seconds.

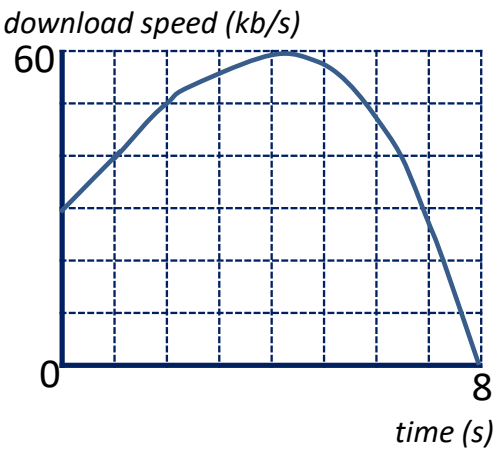
b) By considering the area under the graph, estimate the distance travelled by the ball.



5. The graph shows the acceleration of a box-car in a downhill race.

a) Use the graph to estimate the rate of change in acceleration at 3 seconds.

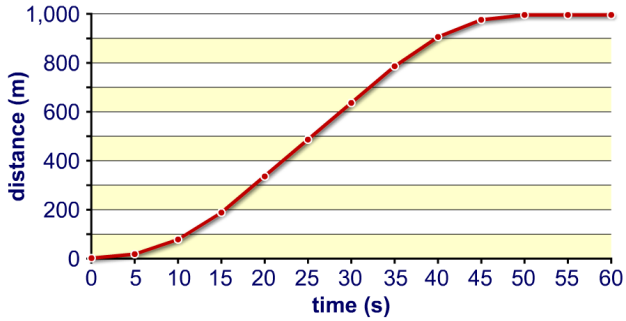
b) By considering the area under the graph, estimate the average speed in the first 10 seconds.



6. The graph shows the speed, in kilobits per second, of a download.

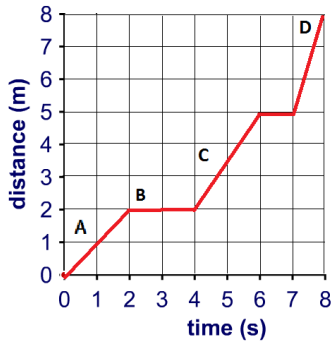
a) At what time is the download speed not changing?

b) By considering the area under the graph, estimate the number of kilobits downloaded.



1. What does the slope of the graph represent?

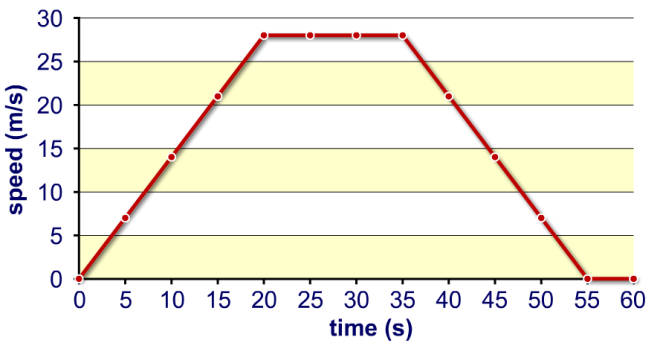
2. What does this graph tell you about how the speed of the object changes?



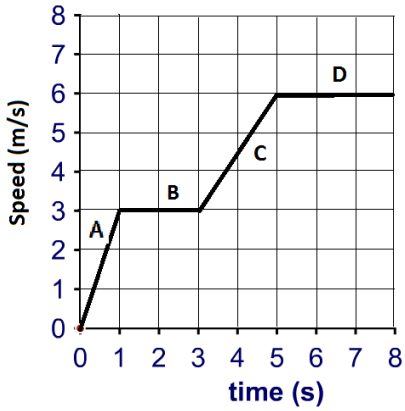
3. What is the speed at A, B, C and D?

4. What is the average speed of the whole journey?

5. What is the formula for calculating acceleration?



6. What does the slope of the graph on the right tell you about how acceleration is changing?



7. Calculate the acceleration at A, B, C and D on the graph to the left.

8. Calculate distance travelled for the whole journey on the graph to the left.

9. If you walk for 1 $^{1}/_{2}$ hours at an average speed of 4 miles/h. How far will you have walked?

10. A bird flies 10m in 2 seconds. What is its speed?

11. A plane takes 8 hours to fly 5000km. What is its speed?

12. A cheetah accelerates at 10m/s^2 from rest until it reaches 20m/s. How long does it take?

7 Iterations

Fluency Practice

1 Show that the equation $x^3 + 2.5x^2 - 1 = 0$ has a solution between -1 and 0.

?

?

2 Show that the equation $2x^3 + 5x^2 - 5x - 1 = 0$ has a solution between -3.5 and -3.

?

?

Fluency Practice

3 Show that the equation $x - \sqrt{x} - 1 = 0$ has a solution between 2.6 and 2.7.

?

?

4 Show that the equation $2x^3 - 7x^2 + 5x = 8$ has a solution between 3 and 3.5.

?

?

5 *[CCEA GCSE Summer 2021 M8.2 Q2 Edited]*

A solution to the equation $4x - x^3 - 2 = 0$ lies between $x = 0$ and $x = 1$

Find this solution correct to 1 decimal place.

x	$4x - x^3 - 2$
0	$4\times0 - 0^3 - 2 = -2$
0.1	$4\times0.1 - 0.1^3 - 2 = -1.601$
0.2	?
0.3	?
0.4	?
0.5	?
0.6	?
0.7	?
0.8	$4\times0.8 - 0.8^3 - 2 = 0.688$
0.9	$4\times0.9 - 0.9^3 - 2 = 0.871$
1	$4\times1 - 1^3 - 2 = 1$

?

?

6 *[OCR GCSE Nov 2021 6H Q15]*

a Show that the equation $x^3 - 5x - 1 = 0$ has a solution between $x = 2$ and $x = 3$.

?

b Find this solution correct to **1** decimal place.

?

?

?

Fluency Practice

1 Show that the equation

$$x^2 - 9x + 4 = 0$$

can be rearranged to give

$$x = \sqrt{9x - 4}.$$

?

2 Show that the equation

$$2x^3 - 8x^2 + 10x - 4 = 0$$

can be rearranged to give

$$x = \frac{4 + 8x^2 - 2x^3}{10}.$$

?

3 Show that the equation

$$2x^3 + 4x^2 - 3x - 2 = 0$$

can be rearranged to give

$$x = \sqrt[3]{\frac{2 + 3x - 4x^2}{2}}.$$

?

Fluency Practice

4 We wish to solve the equation $3x^3 - 10x^2 + 9x - 2 = 0$

This can be rearranged to

$$x = \frac{-3x^3 + 10x^2 + 2}{9}$$

Starting with $x_0 = 1$, use the iteration formula

$$x_{n+1} = \frac{-3(x_n)^3 + 10(x_n)^2 + 2}{9}$$

to find the value of x_2.

Give your answer correct to 3 decimal places.

?

5 We wish to solve the equation $x^3 - 5x^2 + 3x + 9 = 0$

This can be rearranged to

$$x = \sqrt[3]{5x^2 - 3x - 9}$$

Starting with $x_0 = 2$, use the iteration formula

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 3x_n - 9}$$

to find the value of x_2.

Give your answer correct to 3 decimal places.

?

Fluency Practice

6 We wish to solve the equation

$$3x^3 + 10x^2 + 9x + 2 = 0$$

a Show that this can be rearranged to

$$x = \frac{-3x^3 - 10x^2 - 2}{9}$$

?

b Starting with $x_0 = -2$, use the iteration formula

$$x_{n+1} = \frac{-3(x_n)^3 - 10(x_n)^2 - 2}{9}$$

to find the value of x_1.
Give your answer correct to 3 decimal places.

?

Fluency Practice

7 We wish to solve the equation

$$3x^3 - 10x^2 + 9x - 2 = 0$$

a Show that this can be rearranged to

$$x = \sqrt[3]{\frac{10x^2 - 9x + 2}{3}}.$$

?

b Starting with $x_0 = 0$, use the iteration formula

$$x_{n+1} = \sqrt[3]{\frac{10(x_n)^2 - 9x_n + 2}{3}}$$

to find the value of x_1.
Give your answer correct to 3 decimal places.

?

8 *[Edexcel GCSE Nov 2017 H3 Q15]*

a Show that the equation $x^3 + 7x - 5 = 0$ has a solution between $x = 0$ and $x = 1$.

(2 marks)

?

b Show that the equation $x^3 + 7x - 5 = 0$ can be arranged to give $x = \frac{5}{x^2+7}$.

(2 marks)

?

8

c Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \frac{5}{x_n^2+7}$ three times to find an estimate for the solution of $x^3 + 7x - 5 = 0$.

(3 marks)

?

d By substituting your answer to part (c) into $x^3 + 7x - 5$,
Comment on the accuracy of your estimate for the solution to $x^3 + 7x - 5 = 0$.

(2 marks)

?

[Edexcel GCSE Nov 2021 H2 Q16]

a Use the iteration formula $x_{n+1} = \sqrt[3]{10 - 2x_n}$ to find the values of x_1, x_2 and x_3. Start with $x_0 = 2$

?

[3 marks]

b The values of x_1, x_2 and x_3 found in part (a) are estimate of the solution of an equation of the form $x^3 + ax + b = 0$ where a and b are integers.

Find the value of a and the value of b.

?

[1 mark]

Fluency Practice

Solving Quadratic Equations using Iteration

Quadratic Equation: $x^2 - 2x - 1 = 0$

(rearrange)

Iterative Formula: $x_{n+1} = \sqrt{2x_n + 1}$

Initial Value: $x_1 = 3$

x_2	2.64575…
x_3	2.50828…
x_4	2.45287…
x_5	2.43017…
x_6	2.42081…
x_7	2.41694…
x_8	2.41534…
x_9	2.41468…
x_{10}	2.41440…
x_{11}	2.41429…
x_{12}	2.41424…

$x = 2.4$ Both solutions are the same (to 1 dp).

$x = 2.41$ Both solutions are the same (to 2 dp).

A For this quadratic equation find the solutions, to 1 dp & to 2 decimal places.

$x^2 - 3x - 1 = 0$

Iterative. Formula: $x_{n+1} = \sqrt{3x_n + 1}$

Initial Value: $x_1 = 4$

x_2	
x_3	3.437…
x_4	
x_5	
x_6	
x_7	
x_8	
x_9	3.303…
x_{10}	
x_{11}	
x_{12}	

B Find a solution for this equation to 2 dp & to 3 dp.

$x^2 - 5x + 2 = 0$

$x_{n+1} = \sqrt{5x_n - 2}$

Initial Value: $x_1 = 5$

x_2	
x_3	
x_4	
x_5	4.5991…
x_6	
x_7	
x_8	
x_9	
x_{10}	
x_{11}	
x_{12}	
x_{13}	

C Find a solution for this equation to 3 dp.

Choose your own initial value.

$x^2 - 4x - 7 = 0$

D Find a solution for this equation to 1 dp.

$x^2 + 4x - 13 = 0$

Why do only some initial values work?

E Find a solution for this cubic equation to 3 dp.

$x^3 - 5x + 3 = 0$

(Rearrange so x^3 is by itself, then take the cube root of both sides).

1 The sequence a_n is defined such that $a_{n+1} = a_n + 3$ and $a_0 = 4$.

(a) Find the values of a_1, a_2 and a_3.

(b) What type of sequence is a_n?

2 The sequence u_n is defined such that $u_{n+1} = 2u_n - 3$ and $u_1 = -2$.

(a) Find the values of u_2, u_3 and u_4.

(b) Find the value of u_0.

3 Given that $x_{n+1} = 3 - \frac{x_n}{4}$ and $x_1 = 1$, find the values of

(a) x_2, x_3 and x_4,

(b) x_0.

Where appropriate, give each value to three decimal places.

4 Given that $x_{n+1} = x_n^{\,2} - \frac{7}{8}$ and $x_0 = 0$,

(a) write down the smallest term in the sequence.

(b) Find the values of x_1, x_2 and x_3, giving each value to four significant figures.

5 A deadly disease infects a population of plants.

The number of plants infected with the disease n days after the first infection, p_n, is given by

$$p_{n+1} = 4p_n + 6$$

and $p_0 = 3$.

(a) Interpret the meaning of p_0 in this context.

(b) Find the number of plants infected with the disease 2 days after the first infection.

There are 20000 plants in the population.

(c) Show that all the plants in the population will be infected with the disease after 6 days.

6 Markov invests £P into a bank account.

He is paid r% per annum compound interest and takes out £D a year from this account.

The amount of money in the account after n years is A_n.

(a) Show that

$$A_{n+1} = A_n\left(1 + \frac{r}{100}\right) - D$$

Markov invests £1000 into the account and the interest rate is 2%.

After 3 years, Markov has £143 in the savings account.

(b) How much money does Markov take out of the account each year? Give your answer to the nearest pound.

7 Here is a table of values for the curve with equation $y = f(x)$.

x	-3	-2	-1	0	1
$f(x)$	-10	-3	2	15	82

The equation $f(x) = 0$ has at least one solution in the interval $x = a$ and $x = b$.
Using the table, write down the value of a and b. Explain your reasoning.

8 Show that the equation $x^3 - 2x^2 - 7x + 1 = 0$ has a solution between $x = 3$ and $x = 4$.

9 Show that the equation $5x^3 + x^2 - 8x + 4 = 0$ has a solution between $x = -2$ and $x = -1$.

10 (a) Show that the equation $-3x^3 + 2x^2 + 4x + 5 = 0$ has a solution α between $x = 1.5$ and $x = 2$.

(b) Show that the equation $-3x^3 + 4x + 5 = 0$ can be re-arranged to give $x = \left(\frac{4x+5}{3}\right)^{\frac{1}{3}}$.

(c) Starting with $x_0 = 1.5$, use the iterative formula

$$x_{n+1} = \left(\frac{4x_n + 5}{3}\right)^{\frac{1}{3}}$$

three times to find an estimate for α.
Give your answer to three decimal places.

(d) The actual value of $\alpha = 1.5516$ to four decimal places.

(i) Find the percentage error in your estimate in (c).

(ii) Explain how you could make your approximation in (c) more accurate.

iteration 1: roots & intervals

1. Show that there is a root of the equation $x^3 + x - 3 = 0$ in the interval $(1,\ 2)$.

2. Show that there is a root of the equation $x^2 - 5x + 2 = 0$ in the interval $(0.4,\ 0.5)$.

3. Here is a table of values for the continuous function $f(x)$:

x	1	2	3	4	5	6
$f(x)$	1.9	1.6	1.1	0.4	-0.5	-1.6

State two consecutive values of x between which there is a solution to the equation $f(x) = 0$.

4. Here is a table of values for the continuous function $g(x)$:

x	-3	-2	-1	0	1	2	3
$g(x)$	2.33	3.52	2.16	-0.24	-2.20	-2.20	1.24

State ***two pairs*** of consecutive values of x between which there are solutions to the equation $g(x) = 0$.

5. $f(x)$ is a continuous function. Choose the correct statement for each of the following:

(a) $f(3) = 2.34$
$f(4) = -0.01$

(b) $f(6) = -4$
$f(7) = -1.52$

(c) $f(-1) = 0.62$
$f(0.5) = 0.74$

(d) $f(7) < 0$
$f(8) > 0$

Statements

① There is a root of $f(x) = 0$ in the interval.

② There is **not** a root of $f(x) = 0$ in the interval.

③ It is not possible to tell if there is a root of $f(x) = 0$ in the interval.

6. $f(x) = x^2 - 4x - 3$.
(a) Show that there is a root of the equation $f(x) = 0$ in the interval $(4,\ 5)$.
(b) Work out $f(4.5)$.
(c) State an interval that is smaller than $(4,\ 5)$ and contains the root.

7. (a) $a = 2.2$ correct to 1 decimal place. State the lower and upper bounds of a.
(b) Use the bounds as an interval to prove that a is root of the equation $x^3 - x - 9 = 0$, correct to 1 decimal place.

iteration 2: recurrence relations

1. Given the recurrence relation $x_{n+1} = 3x_n - 2$ and $x_0 = 2$, work out x_1, x_2, x_3 and x_4.

2. Given the recurrence relation $x_{n+1} = (x_n)^2 - 5$ and $x_0 = 3$, work out x_1, x_2, x_3 and x_4.

3. Given that $P_{n+1} = 1.05P_n$ and $P_0 = 2000$, work out P_4 correct to 3 significant figures.

4. Given that $U_{n+1} = 1.02(U_n - 50)$ and $U_0 = 1500$, work out U_3 correct to 3 significant figures.

5. Match the recurrence relation rules to the correct sequence of values (x_0, x_1, x_2, x_3):

A: $x_{n+1} = 3x_n - 1$	P: 3, 5, 7, 9
B: $x_{n+1} = x_n + 2$	Q: 2, 5, 14, 41
C: $x_{n+1} = (x_n)^2 + 1$	R: 2, 5, 8, 11
D: $x_{n+1} = x_n + 3$	S: 3, 6, 15, 42
E: $x_{n+1} = 3(x_n - 1)$	T: 2, 5, 26, 677

6. An approximate solution to the equation $x^3 - 8x + 4 = 0$ can be found using the iterative formula:

$$x_{n+1} = \frac{{x_n}^3 + 4}{8}, \quad x_0 = 0$$

Work out x_1, x_2, x_3 and x_4. Round each value to 4 decimal places where necessary.

7. (a) Show that the equation $x^3 - 5x - 2 = 0$ has a root between $x = -1$ and $x = 0$.

(b) Starting with $x_0 = 0$, use the iterative formula:

$$x_{n+1} = \frac{{x_n}^3 - 2}{5}$$

three times to find an estimate for the root of $x^3 - 5x - 2 = 0$. Round your answer to 2 decimal places.

8. (a) Show that the equation $x^3 + x^2 = 10x - 6$ has a root in the interval $(0, 1)$.

(b) Starting with $x_0 = 1$, use the iterative formula

$$x_{n+1} = \frac{{x_n}^3 + {x_n}^2 + 6}{10}$$

three times to find an estimate for the root of $x^3 + x^2 = 10x - 6$. Round your answer to 3 significant figures.

iteration 3: rearrangement

1. Show that the equation $x^3 - 8x - 5 = 0$ can be rearranged to each of these forms:

   (a) $x = \sqrt[3]{8x + 5}$ (b) $x = \frac{x^3 - 5}{8}$

   (c) $x = \frac{5}{x^2 - 8}$ (d) $x = \pm\sqrt{\frac{8x+5}{x}}$

2. (a) Show that the equation $x^3 - 4x - 5 = 0$ has a root in the interval $(2,\ 3)$.

   (b) Show that the equation $x^3 - 4x - 5 = 0$ can be rearranged to give $x = \sqrt[3]{4x + 5}$.

   (c) Starting with $x_0 = 2$, use the iteration formula

   $$x_{n+1} = \sqrt[3]{4x_n + 5}$$

   three times to find an estimate for the root of $x^3 - 4x - 5 = 0$. Round your estimate to 3 significant figures.

3. (a) Show that the equation $x^3 + 3x - 2 = 0$ has a root between $x = 0$ and $x = 1$.

   (b) Show that the equation $x^3 + 3x - 2 = 0$ can be rearranged to give $x = \frac{2}{x^2+3}$.

   (c) Starting with $x_0 = 0.5$, use the iteration formula

   $$x_{n+1} = \frac{2}{{x_n}^2+3}$$

   twice to find an estimate for the root of $x^3 + 3x - 2 = 0$. Round your answer to 2 decimal places.

4. (a) Show that the equation $x^3 - 4x - 2 = 0$ has a root in the interval $(-1,\ 0)$.

   (b) Show that the equation $x^3 - 4x - 2 = 0$ can be rearranged to give $x = \frac{x^3-2}{4}$.

   (c) Starting with $x_0 = -0.5$, use the iteration formula

   $$x_{n+1} = \frac{{x_n}^3-2}{4}$$

   three times to find an estimate for the root of $x^3 - 4x - 2 = 0$. Round your answer to 3 significant figures.

iteration 4: review

1. The expected number of birds in a colony t years from now is P_t, where

$$P_0 = 600$$
$$P_{t+1} = 1.025P_t$$

Work out the expected number of birds in the colony 3 years from now.

2. The value of an investment t years from now is £V_t, where

$$V_0 = 20{,}000$$
$$V_{t+1} = 1.02V_t + 2000$$

Work out the value of the investment 2 years from now.

3. The expected value of a car n years from now is £c_n, where

$$c_0 = 18{,}000$$
$$c_{n+1} = 0.85c_n - 500$$

Work out the expected value of the car 2 years from now.

4. (a) Show that the equation $x^3 - 6x - 6 = 0$ has a root in the interval $(2,\ 3)$.

(b) Show that the equation $x^3 - 6x - 6 = 0$ can be rearranged to give $x = \sqrt{\frac{6x+6}{x}}$.

(c) Starting with $x_0 = 3$, use the iteration formula

$$x_{n+1} = \sqrt{\frac{6x_n+6}{x_n}}$$

three times to find an estimate for the root of $x^3 - 6x - 6 = 0$. Round your answer to 2 decimal places.

5. (a) Show that the equation $x^3 + 2x + 8 = 0$ has a root between $x = -2$ and $x = -1$.

(b) Show that the equation $x^3 + 2x + 8 = 0$ can be rearranged to give $x = \sqrt[3]{-2x - 8}$.

(c) Starting with $x_0 = -1.5$, use the iteration formula

$$x_{n+1} = \sqrt[3]{-2x_n - 8}$$

three times to find an estimate for the root of $x^3 + 2x + 8 = 0$. Round your answer to 2 decimal places.

Fluency Practice

solving cubic equations by iteration

example

find a solution to $x^3 - 10x = 30$ between 4 and 5
give your answer correct to 3 decimal places

$x^3 - 10x = 30$

rearrange this equation

$x = \sqrt[3]{(30 + 10x)}$

$x_{n+1} = \sqrt[3]{(30 + 10x_n)}$ the iteration equation

do $\sqrt[3]{(30 + 10\text{Ans})}$ enter the smaller number, 4 , then =

press the = button

carries forward the previous answer, x_n , to obtain the next one, x_{n+1}

repeatedly….

until you have a good reason to stop

the initial value, x_1, is 4

round this number to 3 decimal places

4.150638865
takes 12 steps on a calculator

4.151

use an iteration method
to try to find a solution

(1) $x^3 + 2x = 40$ between 3 and 4

(2) $x^3 + 10x = 51$ between 2 and 3

(3) $x^3 + 2x = 1$ between 0 and 1

(4) $x^3 + 4x = 10$ between 1 and 2

(5) $x^3 + 10x = 25$ between 1 and 2

what happens?

explore further…

Fluency Practice

solving cubic equations by iteration

example: another rearrangement

find a solution to $x^3 - 4x = 10$ between 2 and 3
give your answer correct to 3 decimal places

$x^3 - 4x = 10$

$x^3 = 4x + 10$

$x^2 = 4 + \frac{10}{x}$

$x_{n+1} = \sqrt{(4 + 10/x_n)}$ the iteration equation

enter the 'seed' value of $x_1 = 2$ [2 =]

do $\sqrt{(4 + 10/\text{Ans})}$

(15 steps on a calculator)

2.760817834 2.761

what happens if you enter a 'seed' value of
$x_1 = -1$ or
$x_1 = -2$? why?
what happens for seed values less than -2 ?

(6) a race: use two rearrangements to try to find a solution to:

$x^3 - 3x = 2$ between 1 and 3
(although the solution is obvious)

(i) $x_{n+1} = \sqrt[3]{(2 + 3x_n)}$

(ii) $x_{n+1} = \sqrt{(3 + 2/x_n)}$

which iteration equation is faster - needs fewer steps (iterations)?

(7) find a solution to:

$x^3 - 5x = 15$ between 3 and 4 using:

(i) $x_{n+1} = \sqrt[3]{(15 + 5x_n)}$

(ii) $x_{n+1} = \sqrt{(5 + 15/x_n)}$

which iteration equation is faster?

(8) find a solution to:

$x^3 - 5x = 2$ between 2 and 3 using two different iteration equations

Fluency Practice

a different type of cubic equation

(9) find a solution to $x^3 + 3x^2 = 4$ with $x_1 = 3$

$$x^3 + 3x^2 = 4$$

$$x^2 (x + 3) = 4$$

$$x^2 = \frac{4}{x + 3}$$

$$x_{n+1} = \sqrt{\frac{4}{x_n + 3}}$$

what happens for $x_1 = -4$?

try to find a solution to $x^3 + 3x^2 = 4$ with $x_1 = 3$
using:

$$x_{n+1} = \sqrt[3]{(4 - 3x_n^2)}$$

what happens ?

using different iteration equations

(10) find a solution to $x^3 - 8x = 3$ with $x_1 = 2$

$$x^3 - 8x = 3$$

$$x^3 - 3 = 8x$$

$$8x = x^3 - 3$$

$$x_{n+1} = \frac{x_n^3 - 3}{8}$$

try other x_1 values
find the (small) range of x_1 values for which the sequence converges (tends to a limit)

find a solution to $x^3 - 8x = 3$ with $x_1 = 2$
using

$$x_{n+1} = \sqrt[3]{(3 + 8x_n)}$$

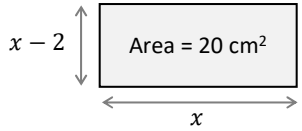
$$x_{n+1} = \sqrt{(8 + 3/x_n)}$$

$$x_{n+1} = -\sqrt{(8 + 3/x_n)}$$

Fluency Practice

Forming & Solving Equations using Iteration

①



1) Form a quadratic equation $(= 0)$.

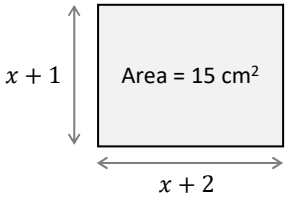
2) Rearrange to form an iterative function.

(Use a square root to form $x_{n+1} =$)

3) Use the function to find the value of x (to 2 dp)

How can we check our answer?

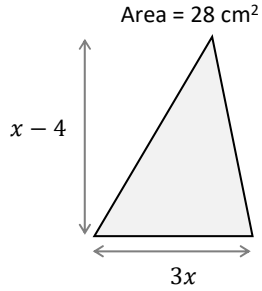
②



Use iteration to find the value of x to 2 dp

Why do we need to be careful choosing an initial value?

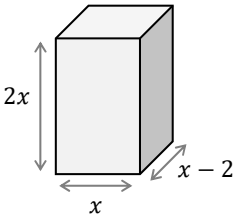
③



Use iteration to find the value of x to 3 dp

④

Volume = 132 cm³



Use iteration to find the value of x to 3 dp