KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 24 Tasks – Part 1

DO NOT WRITE INSIDE

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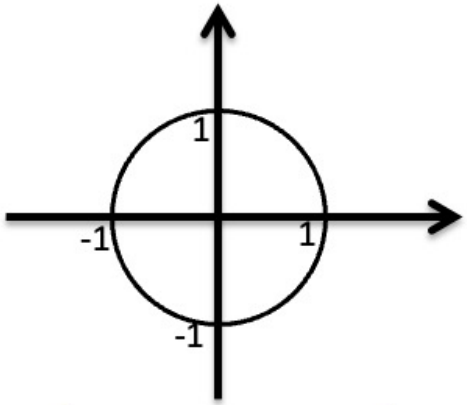
Unit 24 Tasks – Part 2

DO NOT WRITE INSIDE

Contents

1 Equations of Circles and Tangents

Fluency Practice



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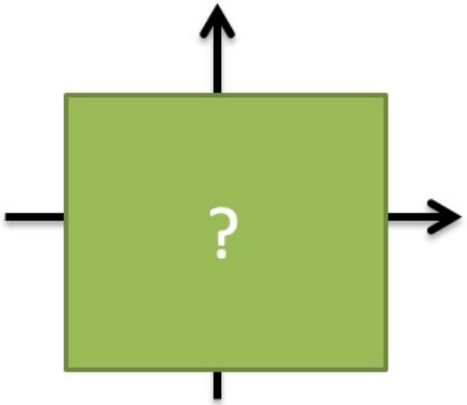
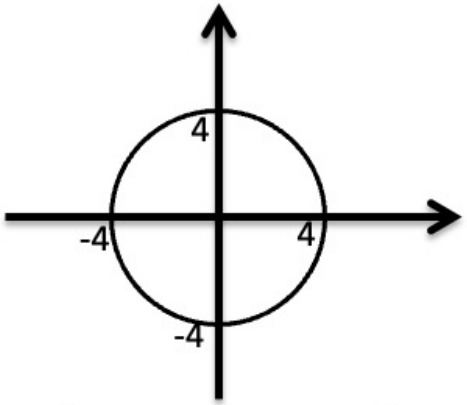


$x^2 + y^2 = 9$



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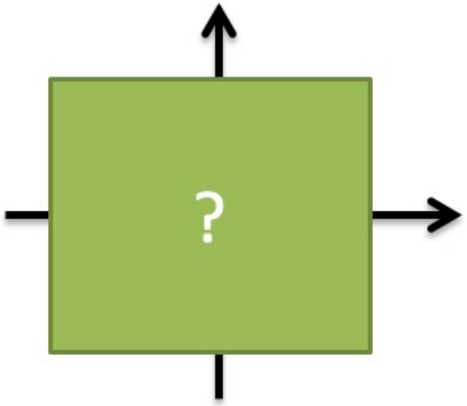
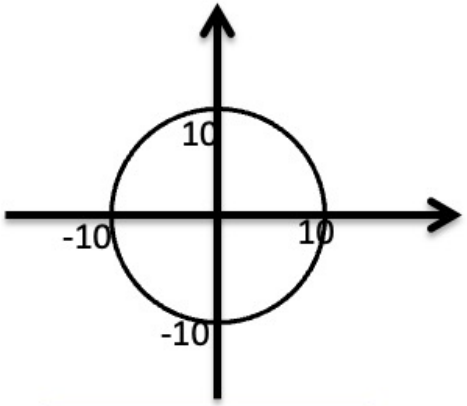


$x^2 + y^2 = 64$



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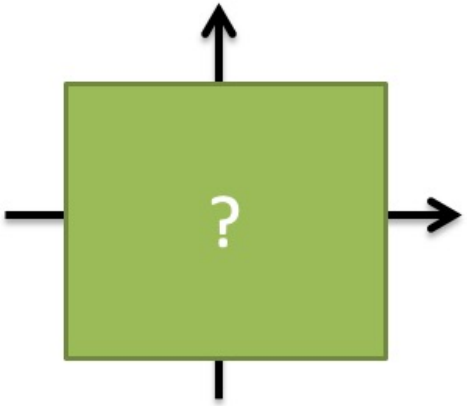


$x^2 + y^2 = 36$

Fluency Practice

Find the radius of the circle: $x^2 + y^2 = 36$	Find the radius of the circle: $x^2 + y^2 = 45$
Find the diameter of the circle: $x^2 + y^2 = 100$	Find the area of the circle: $x^2 + y^2 = 144$
Find the circumference of this circle: $x^2 + y^2 = 9$	Find the area of the circle: $x^2 + y^2 = 48$
Write down the equation of the circle with centre $(0, 0)$ and diameter 800.	Write down the equation of the circle with centre $(0, 0)$ and area 201.
Write the equation of a circle with centre $(0, 0)$ and radius $\sqrt{11}$.	Write the equation of a circle with centre $(0, 0)$ and radius $2\sqrt{3}$.
The point $(3, a)$ is on the circle $x^2 + y^2 = 25$ Find the possible values of a.	The point $(b, 12)$ is on the circle $x^2 + y^2 = 169$ Find the possible values of b.

Plotting Different Graphs

① 

On the grid, plot the graph

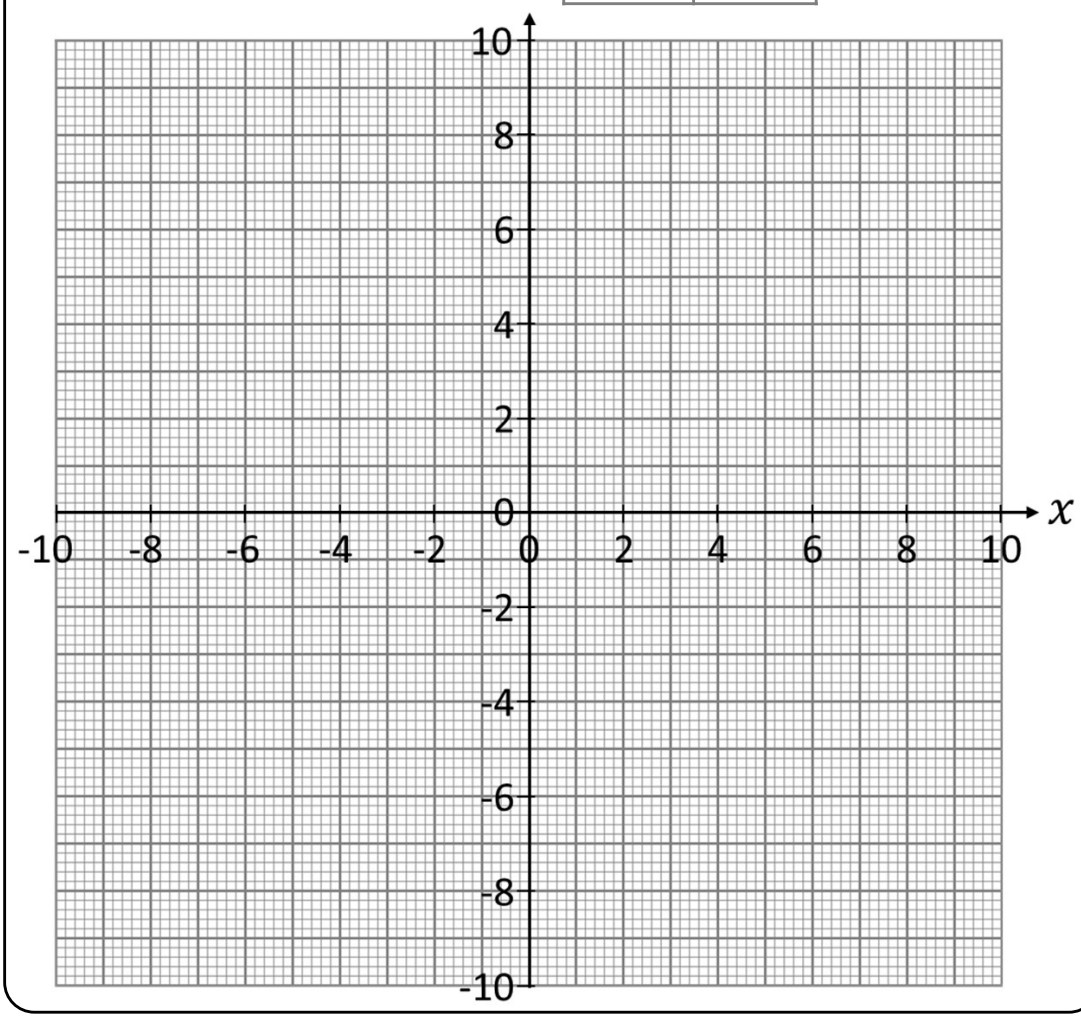
$x^2 + y^2 = 100$

If $x = 0$,
what values will y be?

Form equations to help find coordinates.

x	y
−10	
−8	
−6	
−4	
−2	
0	

x	y
10	
8	
6	
4	
2	



Plotting Different Graphs

② 

On the grid, plot the graph

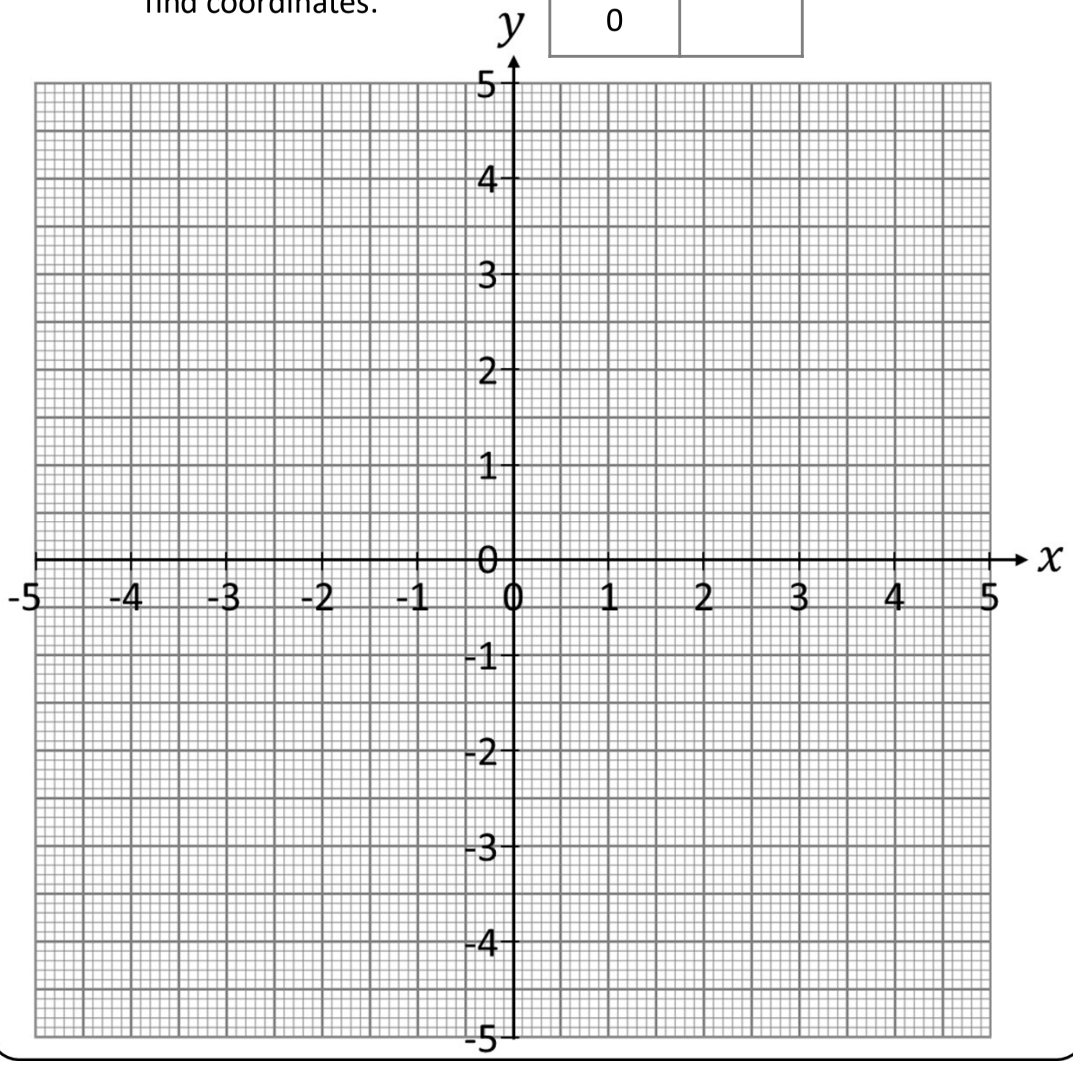
$x^2 + y^2 = 16$

If x = 0,
what values will y be?

Form equations to help find coordinates.

x	y
−4	
−3	
−2	
−1	
0	

x	y
4	
3	
2	
1	



Plotting Different Graphs

③

On the grid, plot the graph

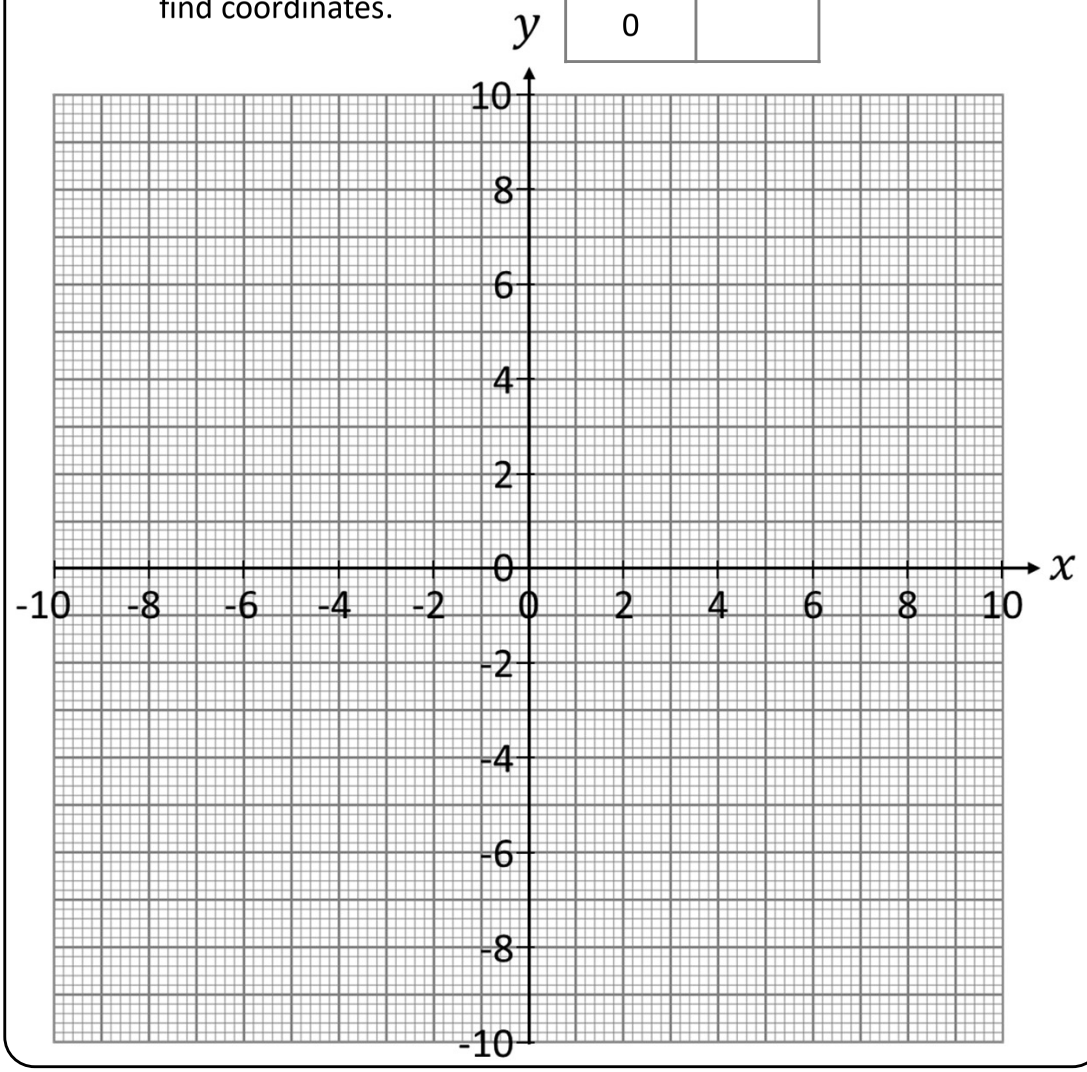
$x^2 + y^2 = 64$

If $x = 0$,
what values will y be?

Form equations to help find coordinates.

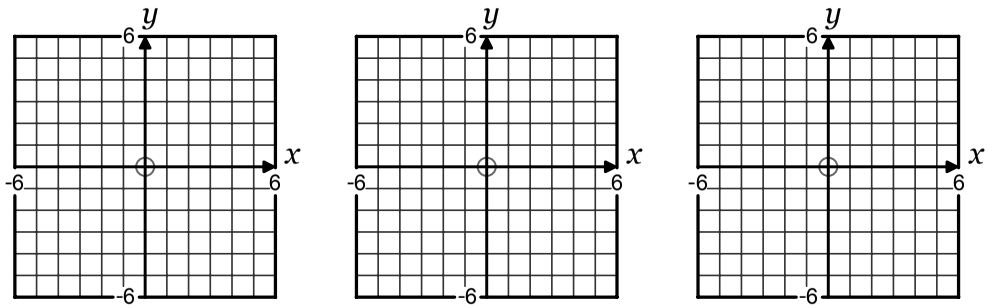
x	y
−8	
−6	
−4	
−2	
0	

x	y
8	
6	
4	
2	



1. Draw the graph of each function:

   a) $x^2 + y^2 = 4$ b) $x^2 + y^2 = 25$ c) $x^2 + y^2 = 4^2$



2. A circle has equation $x^2 + y^2 = 64$. Work out the diameter of this circle.

3. Write down the equation of a circle centred on the origin with:

   a) a radius of 7 b) a diameter of 20 c) an area of 36π

4. Which of the following points lie on the circle with equation $x^2 + y^2 = 100$? Select all that apply.

   a) (10, 1) b) (-8, 6) c) (7, -7) d) $(5, 5\sqrt{3})$

5. Which of the following points lie on the circle with equation $x^2 + y^2 = 80$? Select all that apply.

   a) $(0, 4\sqrt{5})$ b) $(3\sqrt{10}, -\sqrt{10})$ c) $(-7, -\sqrt{31})$

6. Ravi draws the graphs with equations $x^2 + y^2 = 169$ and $y = -5$.
   Work out the coordinates of the points where Ravi's two graphs intersect.

7. Circle **C** has equation $x^2 + y^2 = 81$.
   Write down the equations of the two horizontal lines that are tangent to **C**.

8. Which of the following are equations of circles? Select all that apply.

   a) $x^2 + y = 121$ b) $(x + y)^2 = 9$ c) $x^2 + y^2 - 24 = 0$

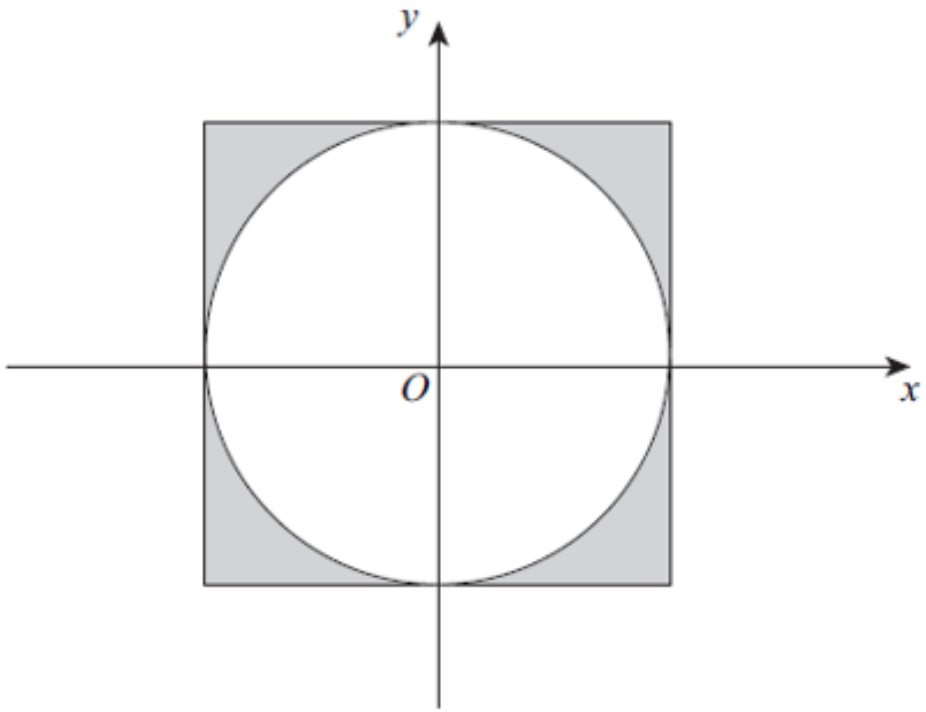
   d) $x = 16 - y^2$ e) $20 = y^2 + x^2$ f) $x^2 + y^2 = \sqrt{5}$

Fluency Practice

1) Write down the centre and radius of the circle with equation $x^2 + y^2 = 36$

2) A circle has the equation $x^2 + y^2 = 36$. Work out its circumference.

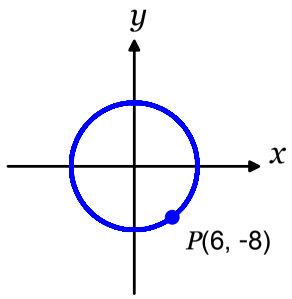
3) The circle $x^2 + y^2 = 25$ touches each side of the square as shown. Work out the total shaded area.



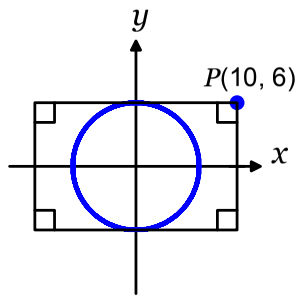
Circle graphs: find the radius

Work out the radius of each circle graph centred on the origin. Lines that look like tangents *are* tangents.

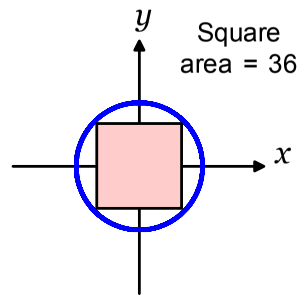
A



B



C



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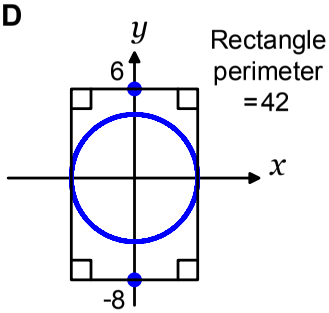

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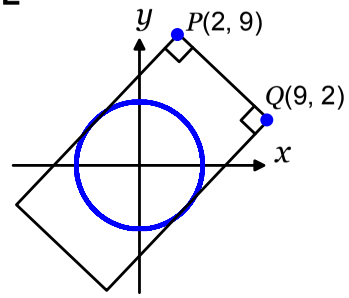

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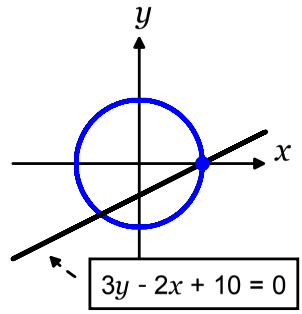
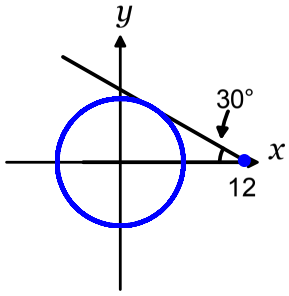
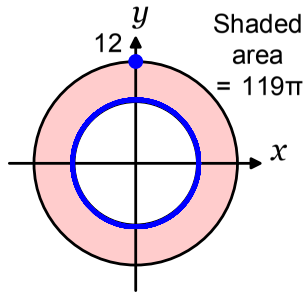


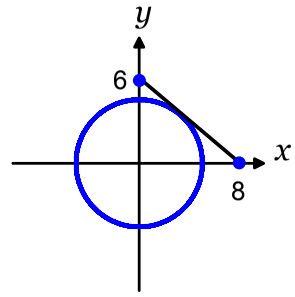
G



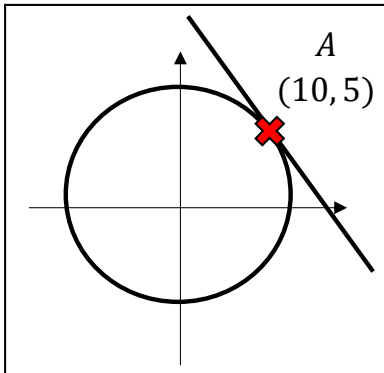
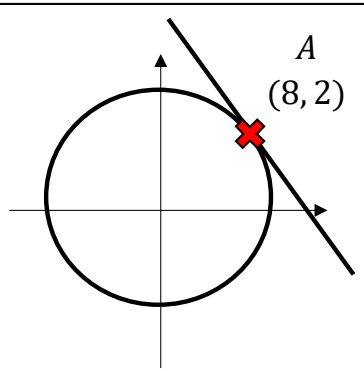
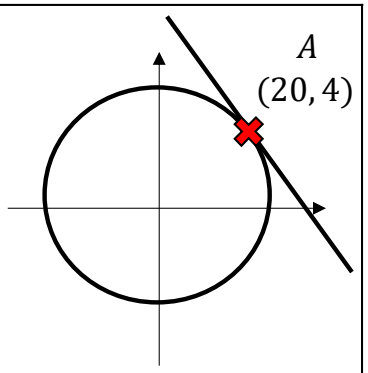
H *Radius of inner circle?*



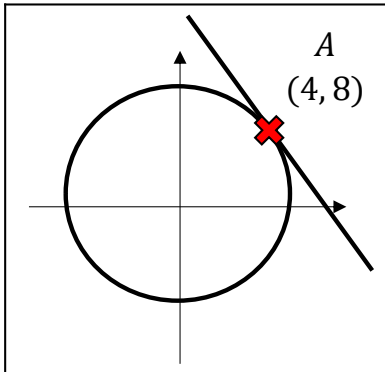
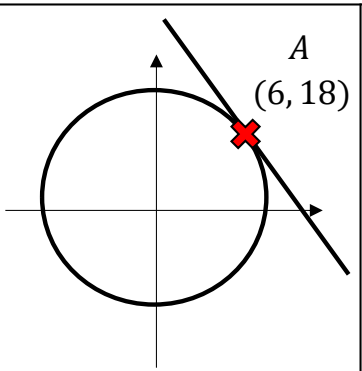
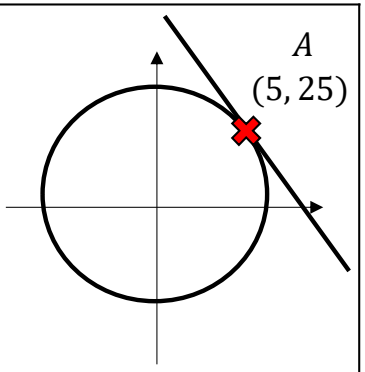
I *Challenge*



Fluency Practice

			
What is the gradient between A and the origin?			
What is the gradient of the tangent?			
What is the equation of the tangent (so far)?			
Substitute the co-ordinate to find the value of c			
Final Answer: The equation of the tangent is…			

Fluency Practice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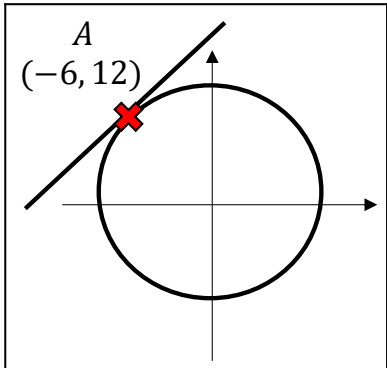
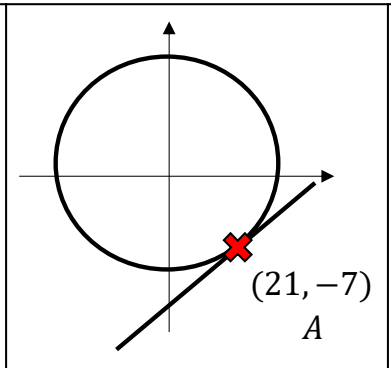
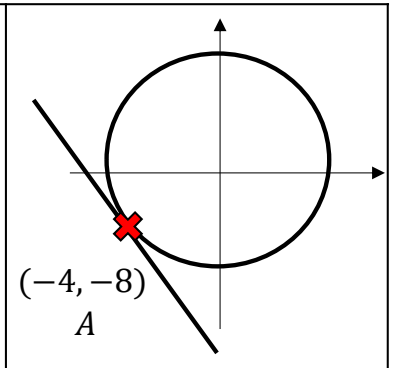
Fluency Practice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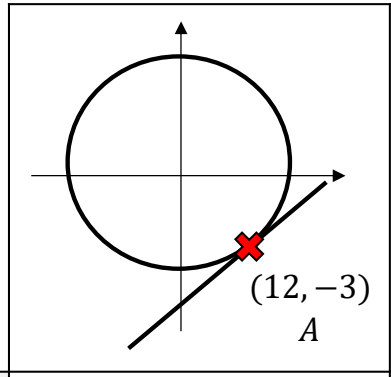
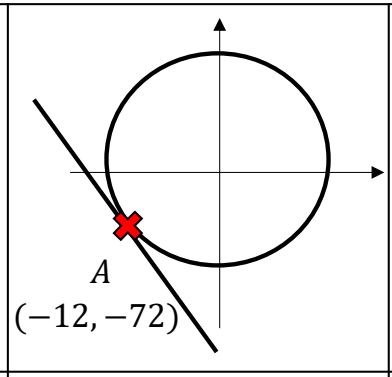
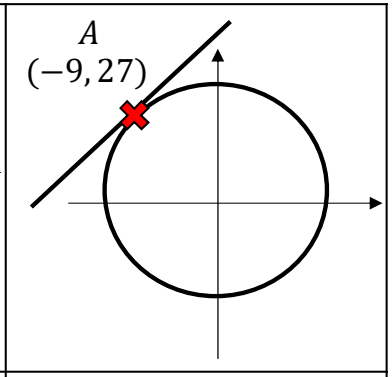
Fluency Practice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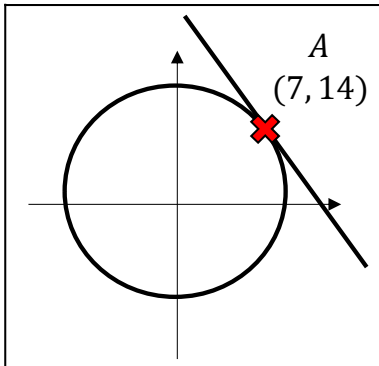
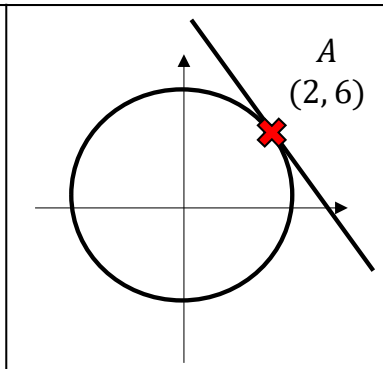
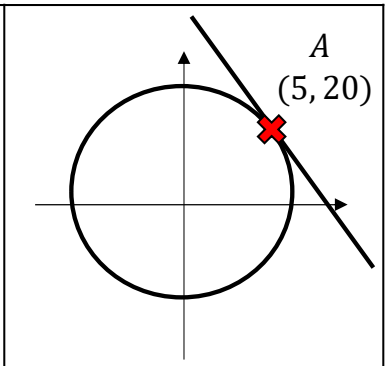
What is the gradient of the tangent?			
What is the equation of the tangent (so far)?			
Substitute the co-ordinate to find the value of c			
Final Answer: The equation of the tangent is…			

Fluency Practice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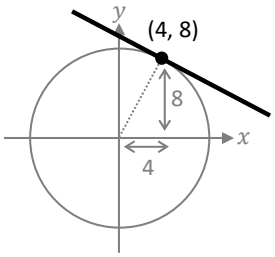
What is the gradient of the tangent?			
What is the equation of the tangent (so far)?			
Substitute the co-ordinate to find the value of c			
Final Answer: The equation of the tangent is…			

Fluency Practice

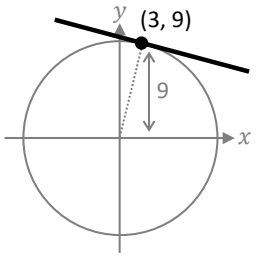
Rad. Grad. = 2 Tan. Grad. = $-\frac{1}{2}$



$y = -\frac{1}{2}x + 8$

$y = -\frac{1}{2}x + 10$

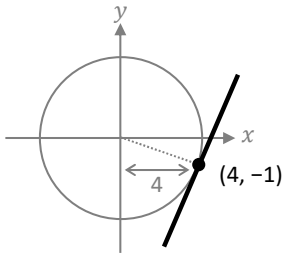
Rad. Grad. = Tan. Grad. =



$y = -\frac{1}{3}x + 10$

$y = -\frac{1}{3}x + 11$

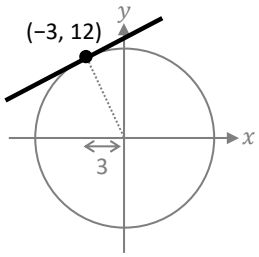
Rad. Grad. = $-\frac{1}{4}$ Tan. Grad. =



$y = 4x - 22$

$y = 4x - 17$

Rad. Grad. = −4 Tan. Grad. =

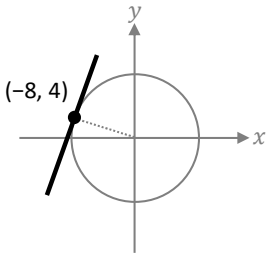


$y = \frac{1}{4}x + 13$

$y = \frac{1}{4}x + 12\frac{3}{4}$

Tangent Match

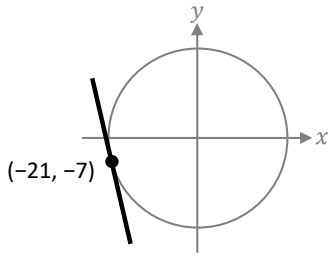
Rad. Grad. = Tan. Grad. =



$y = \quad x + 20$

$y = \quad x + 24$

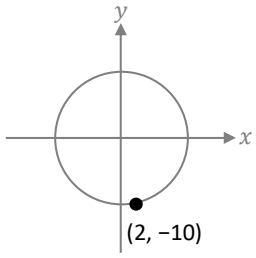
Rad. Grad. = Tan. Grad. =



$y = \quad x - 56$

$y = \quad x - 54$

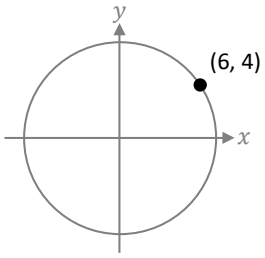
Rad. Grad. = Tan. Grad. =



$y = \quad x - 10.2$

$y = \quad x - 10.4$

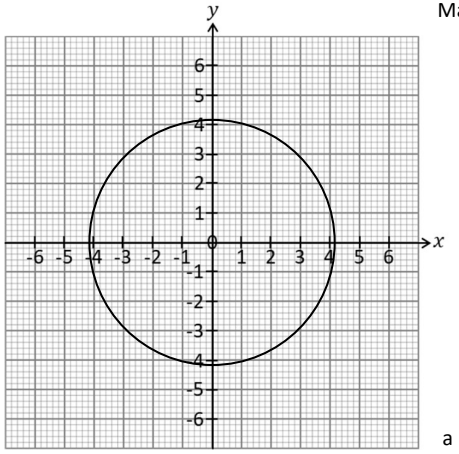
Rad. Grad. = Tan. Grad. =



Fluency Practice

Lines around Circles

① The graph $x^2 + y^2 = 18$ is on the grid.



Mark point **A** on the circle at (3, 3), then draw line **OA** from the origin (**O**) to point **A**.

What is the equation of line **OA**?

_____

Plot the line $y = 6 - x$

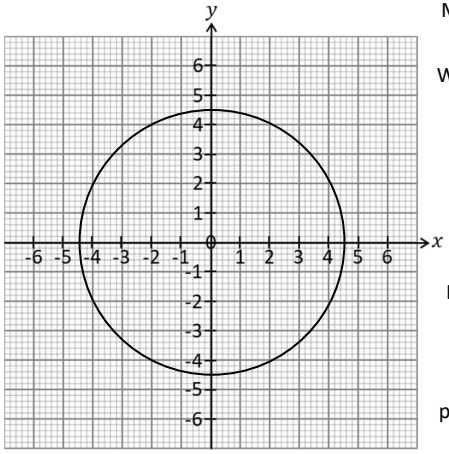
How are the gradients of these graphs related?

_____

$y = 6 - x$ is a _____ to the circle.

Draw & label the line that is parallel to $y = 6 - x$ & touches the circle at one point.

② The graph $x^2 + y^2 = 20$ is on the grid.



Mark point **B** on the circle at (2, 4), & draw line **OB** from the origin.

What is the gradient of the line **OB**?

_____

What is the gradient of a line perpendicular to **OB**?

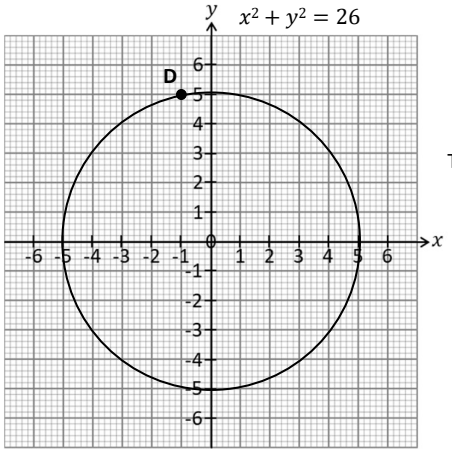
_____

Plot & label line **CD** - the line that is perpendicular to **OB** & passes through **B**.

Draw & label the line that is parallel to **CD** & touches the circle at one point.

Where on the circle will a tangent have a gradient of 2? _____

③



Sketch the line **OD**.

Find the gradient of this radius, is it positive or negative?.

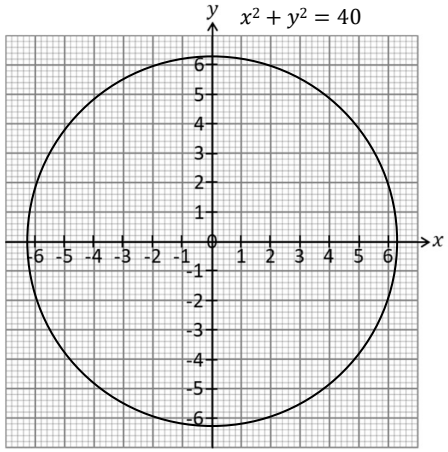
The tangent to the circle at **D** has a gradient of $\frac{1}{5}$ and passes through (−1, 5).

We can express this as:

$$y = \frac{1}{5}x + c$$

Plot the tangent & calculate its y-intercept.

④



Find the equation of the tangent at (6, −2)

_____

Find the equation of the tangent at (−2, 6)

_____

(a) The point P $(6, -2)$ lies on the circle with equation $x^2 + y^2 = 40$. Find the gradient of the tangent to the circle at point P.

(b) The point Q $(2, 1)$ lies on the circle with equation $x^2 + y^2 = 5$. Find the gradient of the tangent to the circle at point Q.

(a) The point A $(2, -5)$ lies on the circle with equation $x^2 + y^2 = 29$. Find the equation of the tangent to the circle at point A.

(b) The point B $(-3, -2)$ lies on the circle with equation $x^2 + y^2 = 13$. Find the equation of the tangent to the circle at point A.

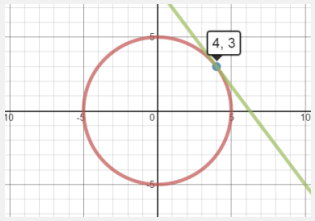
(a) The point R $(-2, -3)$ lies on the circle with centre $(1, 2)$ and radius $\sqrt{34}$. Find the equation of the tangent to the circle at point R.

(b) The point S $(7, 1)$ lies on the circle with centre $(x - 3)^2 + (y + 2)^2 = 25$. Find the equation of the tangent to the circle at point S.

(a) Point P $(-2, 8)$ lies on a circle with centre $(-1, 6)$. Point Q with coordinates $(a, 5)$ lies on the tangent to the circle at P. Find the value of a.

(b) Point A $(-1, -3)$ lies on the circle with equation $(x - 3)^2 + (y + 2)^2 = 17$. The line L is the tangent to the circle at point A. Find the coordinates of the point where line L crosses the x-axis.

Fluency Practice

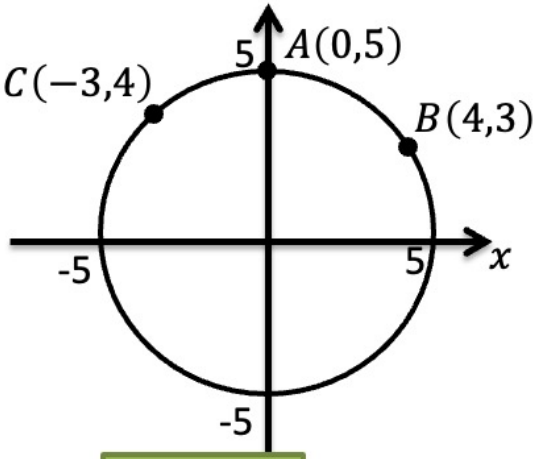
Question	Radius	Sketch	Gradient of radius at point	Gradient of tangent at point	Equation of tangent at point
Example: Find the equation of the tangent to $x^2 + y^2 = 25$ at $(4, 3)$	5		$\frac{3}{4}$	$-\frac{4}{3}$	$y = -\frac{4}{3}x + \frac{25}{3}$
1. Find the equation of the tangent to $x^2 + y^2 = 5$ at $(2, 1)$					
2. Find the equation of the tangent to $x^2 + y^2 = 100$ at the point on the circumference with x-coordinate 6 and a positive y-coordinate					
3. Find the equation of the tangent to $x^2 + y^2 = 45$ at $(-6, 3)$					

Fluency Practice

4. Find the equation of the tangent to $x^2 + y^2 - 20 = 0$ at $(-4, -2)$					
5. Find the equation of the tangent to $x^2 + y^2 = 13$ at the point on the circumference with x-coordinate 3 and a negative y-coordinate					
6. Find the equation of the tangent to the circle with centre $(0, 0)$ and diameter $\sqrt{32}$ at the point $(2, 2)$					
7. Find the equation of the tangent to $x^2 + y^2 = 25$ at the point $(5, 0)$					

Fluency Practice

1 The diagram shows the circle with equation $x^2 + y^2 = 25$. What is the equation of the tangent at the following points?

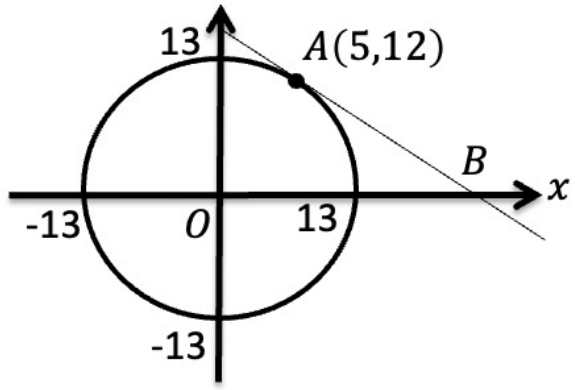


a $A(0,5)$: ?

b $B(4,3)$: ?

c $C(-3,4)$: ?

2



The tangent to the above circle at the point $A(5,12)$ intersects the x axis at the point B.

a Find the equation of the tangent to the circle at the point A.

?

b) Find the area of triangle OAB.

b ?

The line l is tangent at the point $P(x,y)$ to the circle with equation $x^2 + y^2 = 1$. The gradient of l is $-\frac{1}{2}$. Determine the point $P(x,y)$.

?

WARM-UP

1. Work out the gradient of the line from the origin to each of these points:

   a) (5, 8) b) (-3, 7) c) $(-2, \sqrt{3})$ d) $(-2\sqrt{2}, 4)$

2. In each case, the point P lies on the circle C. Work out a, b and c.

   a) C: $x^2 + y^2 = 100$
   P: $(-6, a)$
   $a > 0$

   b) C: $x^2 + y^2 = 4^2$
   P: $(2, b)$
   $b > 0$

   c) C: $x^2 + y^2 - 25 = 0$
   P: $(-2, c)$
   $c < 0$

3. Work out the gradient of a line **perpendicular** to a line with gradient:

   a) -4 b) $\frac{2}{5}$ c) $-\frac{3}{4}$ d) $\frac{1}{5}$

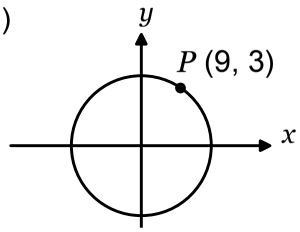
4. In each case, the point P lies on a line with gradient m. Work out the equation of the line in the form $y = mx + c$.

   a) P(2, 5), $m = 3$ b) P(-3, 4), $m = -1$ c) P(-3, -2), $m = \frac{1}{4}$

QUESTIONS

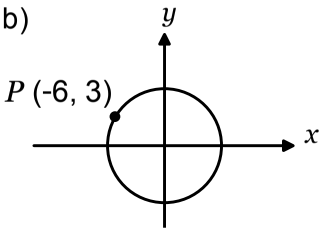
1. Work out the equation of the tangents to these circles at the given point P.
   Give your answers in the form $y = mx + c$.

   a)



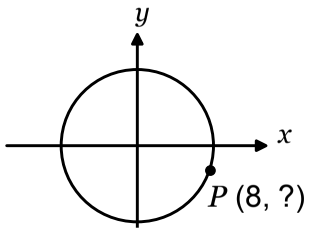
   $x^2 + y^2 = 90$

   b)



   $x^2 + y^2 = 45$

   c)



   $x^2 + y^2 = 80$

2. Circle C has equation $x^2 + y^2 = 400$.
   Point P(a, b) lies on C such that $a = -12$ and $b < 0$.

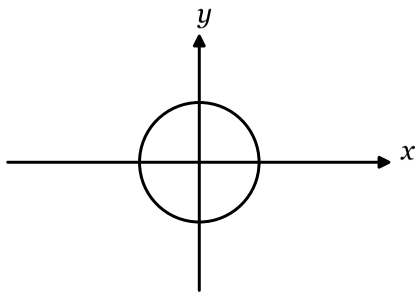
   a) Work out the value of b.

   b) Work out the equation of the tangent to C at P in the form $y = mx + c$.

3. Circle C has equation $x^2 + y^2 = 72$.
Two points on C have a tangent with a gradient of 0.

Work out the coordinates of these two points.

4. Circle C is centred on the origin and has a radius of 15 units.
Point P(a, b) lines on C, such that $a = 12$ and $b > 0$.
Line T is the tangent to C at P.



a) Work out the equation of T.

b) Work out the coordinates of the point at which T meets the y-axis.

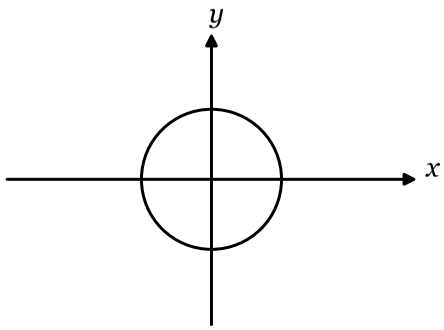
c) Work out the coordinates of the point at which T meets the x-axis.

d) Work out the area of the triangle formed by the x-axis, the y-axis and T.

5. Circle C has equation $x^2 + y^2 = 32$.
Two different lines are both tangents to C and have a gradient of 1.
Work out the coordinates of the points where the tangents meet C.

6. A circle is centred on the origin.
The points P(-15, -20) and Q(-24, c) lie on the circle, where $c > 0$.
Work out the equation of the tangent to the circle at Q.

7. Circle C has equation $x^2 + y^2 = 50$.
O is the origin (0, 0).
P(5, s) is a point on the circle.
Q(1, t) is a point on the circle.
$s > 0$ and $t < 0$.



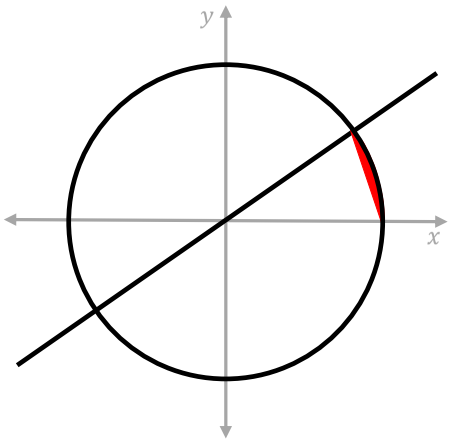
a) Work out the values of s and t.

b) Work out the equations of the tangents to C at P and Q.

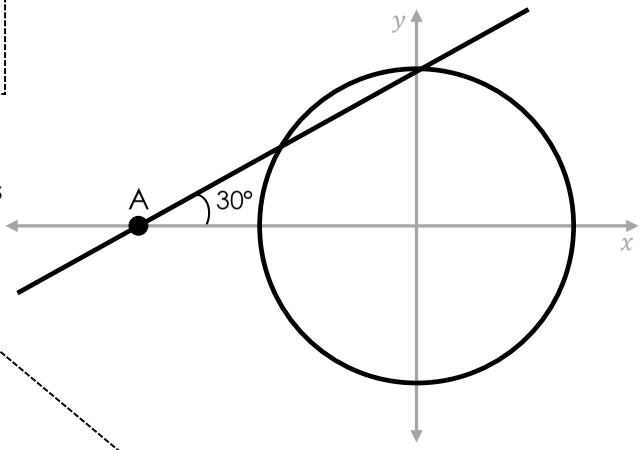
c) Work out the coordinates of the point R, where the two tangents meet.

d) Given that $P\hat{O}Q = 127°$, work out $P\hat{R}Q$.

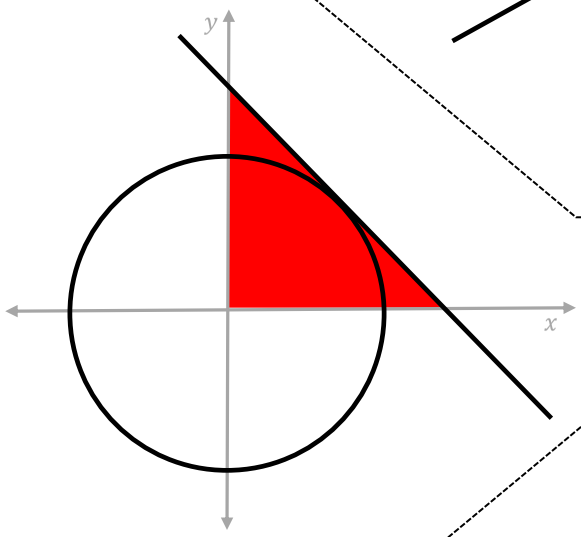
8 fiendishly difficult circle questions for GCSE



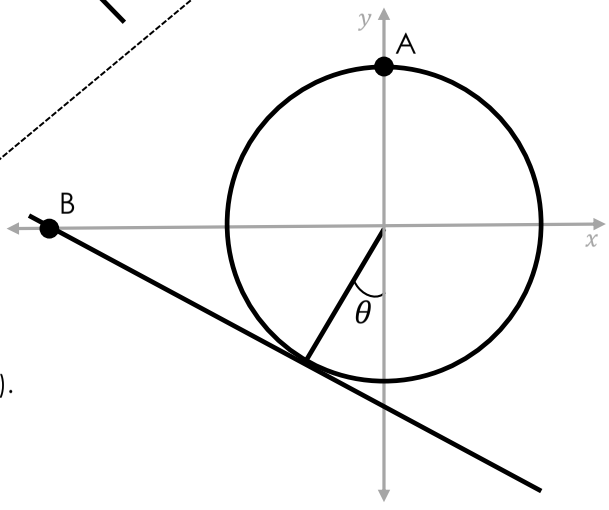
The equation of the circle is $x^2 + y^2 = 36$. The equation of the straight line is $y = \frac{\sqrt{3}}{3}x$.What is the area of the shaded segment?



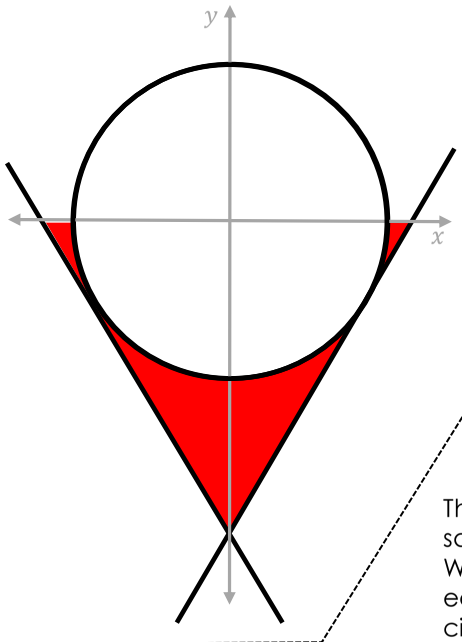
The point A has coordinates (-6,0). The straight line shown makes an angle of 30° with the x-axis. What is the equation of the circle?



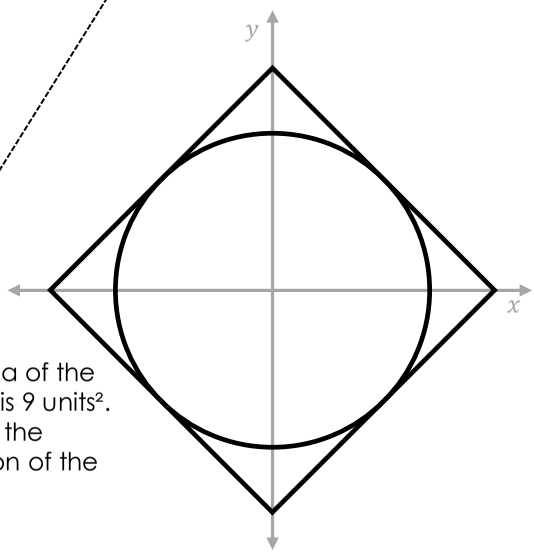
A tangent meets the circle at (5,12) What is the area of the shaded triangle?



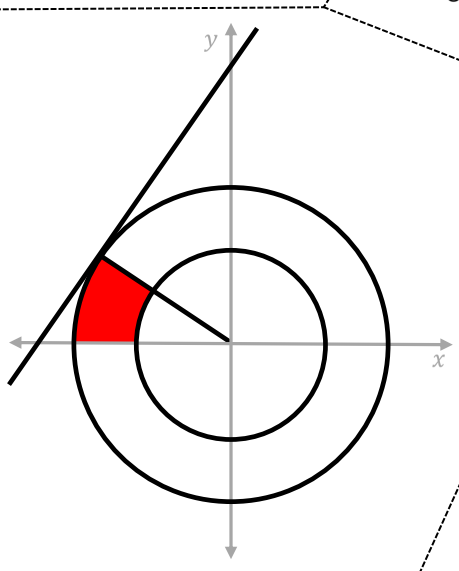
The point A has coordinates $(0, \frac{5}{3})$.
The point B has coordinates $(-\frac{8}{3}, 0)$.
Calculate the angle labelled θ.



One tangent meets the circle at (6,-8), and another meets the circle at (-6,-8) What is the total area of the shaded sections?



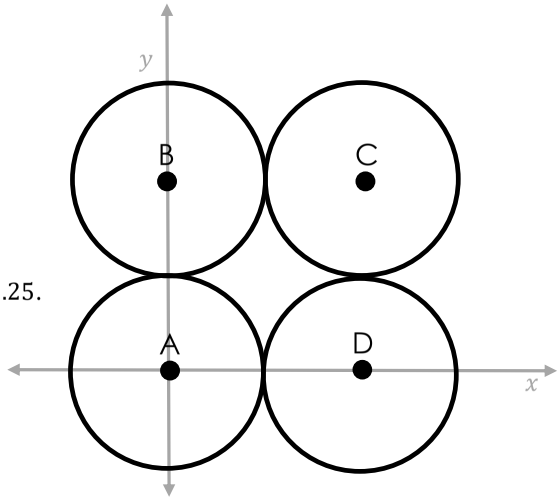
The area of the square is 9 units². What is the equation of the circle?



Two concentric circles are shown, one is an enlargement of the other by a scale factor 3 and has the equation $x^2 + y^2 = 144$.

The tangent shown has the equation $y = \sqrt{3}\,x + 24$.

Calculate the are of the shaded section.



Four congruent circles are shown, with centres A,B,C, and D marked.

One circle has the equation $y^2 + x^2 = 2.25$.

Calculate the distance A to C

3 Advanced Simultaneous Equations

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $xy = 12$
$y = x + 1$

2. $xy = 12$
$y = x - 1$

3. $xy = 12$
$y = x - 4$

4. $xy = 12$
$y = x + 4$

5. $xy = 12$
$y = 2x + 5$

6. $xy = 12$
$y = 2x - 5$

7. $xy = 12$
$2y = x + 5$

8. $xy = 12$
$2y = x - 5$

9. $4xy = 12$
$y = x - 2$

10. $4xy = 12$
$y = x + 2$

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $x^2 + y^2 = 25$
$y = x + 5$

2. $x^2 + y^2 = 25$
$y = x - 5$

3. $x^2 + y^2 = 25$
$y = 2x + 5$

4. $x^2 + y^2 = 25$
$y = 2x - 5$

5. $x^2 + y^2 = 17$
$y = x + 5$

6. $x^2 + y^2 = 17$
$y = x - 5$

7. $2x^2 + y^2 = 17$
$y = x + 5$

8. $2x^2 + y^2 = 17$
$y = x - 5$

Intelligent Practice

Solve the following pairs of simultaneous equations:

1. $y = x^2 + 5x - 2$
$y = x + 3$

2. $y = x^2 - 3x - 2$
$y = x + 3$

3. $y = x^2 - 5x - 2$
$y = -x + 3$

4. $y = x^2 + 3x - 2$
$y = -x + 3$

5. $y = x^2 - 3x - 2$
$y = 2x - 8$

6. $y = x^2 - 3$
$y = x - 3$

7. $y = x^2 - 3$
$y = x + 3$

Fluency Practice

① Solve:

$$y = x^2 + 3x - 1$$
$$y = 2x + 1$$

② Solve:

$$y = x^2 + 5x - 7$$
$$y = 3x + 8$$

③ Solve:

$$xy = 3$$
$$y = x - 2$$

④ Solve:

$$xy = -8$$
$$y = x + 6$$

⑤ Solve:

$$x^2 + y^2 = 13$$
$$y = x + 5$$

⑥ Solve:

$$x^2 + y^2 = 17$$
$$y = x - 3$$

⑦ Solve:

$$x^2 + y^2 = 45$$
$$y = x - 9$$

⑧ Solve:

$$x^2 + y^2 = 5$$
$$y = 2x + 3$$

Solve these simultaneous equations.

(a) $y = x^2 - 4$

$y = 3x$

(b) $y = x^2 + 5x$

$y = 2x + 10$

(c) $y = 2x^2 + x - 3$

$y = 3x + 1$

Solve these simultaneous equations.

(a) $x^2 + 8y = 13$

$x + 2y = 2$

(b) $y = 2x^2$

$3x + y = 20$

(c) $y = 3x^2 - 4$

$y = 2x - 3$

Solve these simultaneous equations.

(a) $x^2 + y^2 = 25$

$x + y = 7$

(b) $x^2 + y^2 = 9$

$y = x + 3$

(c) $x^2 + y^2 = 5$

$y = 3x + 5$

A netball court has an area of 224 m^2. If the length were decreased by 1 m and the width increased by 1 m, the area would be increased by 1 m^2. Find the dimensions of the court.

(a) Solve algebraically $xy = 4$
$y = x + 3$

(b) Solve algebraically $xy = -8$
$x + y = 2$

(c) Solve algebraically $xy + 12 = 0$
$x = 2 - 2y$

(a) Solve algebraically $y = \frac{3}{x}$
$2x = y - 1$

(b) Solve algebraically $y = \frac{4}{x} - 2$
$x = 4y + 8$

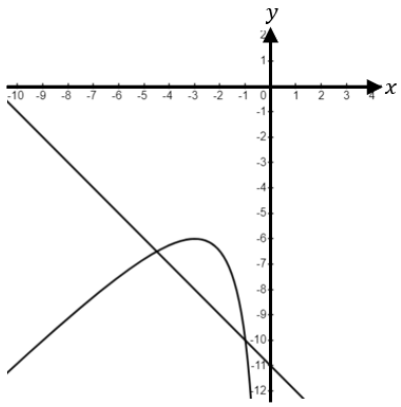
(c) Solve algebraically $y = \frac{4}{x} + x$
$x + y = 6$

(a) Solve algebraically $x^2 + xy = 20$
$x = 4y$

(b) Solve $x^2 + y^2 - xy = 16$
$x + y = 4$

(c) Solve $x^2 - y^2 = 4 + xy$
$y = 2x - 6$

The diagram shows the graphs of $y = x + \frac{9}{x}$ and $x + y + 11 = 0$. Find the coordinates of the points of intersection.



Fluency Practice

Solve the following simultaneous equations.

1. $\begin{matrix} x+y=3 \\ x^2+y^2=5 \end{matrix}$.
2. $\begin{matrix} x-4y=2 \\ 3=5x-2y \end{matrix}$.
3. $\begin{matrix} 2x+y=1 \\ x^2+y^2=2 \end{matrix}$.
4. $\begin{matrix} x+2y=1 \\ x^2+2y^2=3 \end{matrix}$.
5. $\begin{matrix} x-3y=2 \\ 4x-y=1 \end{matrix}$.
6. $\begin{matrix} x-3y=2 \\ y^2-x^2=-4 \end{matrix}$.
7. $\begin{matrix} 2x+y=3 \\ 4x^2+y^2=5 \end{matrix}$.
8. $\begin{matrix} 2x-3y=2 \\ 3x+y=1 \end{matrix}$.
9. $\begin{matrix} x+2y=3 \\ 3x^2+4y=6 \end{matrix}$.
10. $\begin{matrix} 2x-y=4 \\ x^2+2y^2=17 \end{matrix}$.
11. $\begin{matrix} x+3y=7 \\ x^2+x+y=21 \end{matrix}$.

Fluency Practice

A1 Solve $y = x^2$ $y = 11x - 10$	**A2** Solve $y = x^2$ $y + 3x = 4$	**A3** Solve $y = 3x^2$ $y = 5x + 8$	**A4** Solve $y + 7x = 15$ $y = 2x^2$
B1 Solve $x^2 + y^2 = 29$ $y = 3x + 1$	**B2** Solve $x^2 + y^2 = 41$ $y = 2x - 3$	**B3** Solve $x^2 + y^2 = 17$ $y + 2x = 7$	**B4** Solve $3x = 4 - y$ $x^2 + y^2 = 8$
C1 Solve $y = (x+1)^2$ $y = 5x - 1$	**C2** Solve $y = x^2 + 7x + 3$ $y = 9x + 2$	**C3** Solve $x + y = 11$ $xy = 30$	**C4** Solve $xy = 6$ $2x + y = 7$
D1 Solve $3x - y = 4$ $x^2 + y^2 = 34$	**D2** Solve $x^2 - y^2 = 8$ $3x = y + 8$	**D3** Solve $2x^2 - xy = 12$ $x = 4 + y$	**D4** Solve $3y^2 + 2x^2 = 21$ $y - 2x = -7$

Question 1: Solve the following simultaneous equations

(a) $y = x + 3$
$y = x^2 + 5x - 2$

(b) $y = x^2 + x - 14$
$y = x - 5$

(c) $y = 2x - 1$
$y = x^2 - 2x + 2$

(d) $y = 2x^2 + 9x + 1$
$y = 3x + 9$

(e) $y = 2x^2 + x + 1$
$y = x^2 - 5x - 7$

(f) $y = -x^2 + 5x + 2$
$y = 3x^2 - x - 2$

Question 2: Solve the following simultaneous equations

(a) $x + y = 4$
$y = x^2 + 3x - 1$

(b) $x + y = 7$
$xy = 10$

(c) $x^2 + y^2 = 13$
$x + y = 5$

(d) $2x - y + 4 = 0$
$y = x^2 + x - 2$

(e) $x^2 + y^2 = 29$
$7 + x + y = 0$

(f) $xy = -6$
$x + 2y = -4$

(g) $y = x^2 - 3x + 3$
$y = 10x - 39$

(h) $x^2 + y^2 = 1$
$x + 2y = 1$

(i) $5x + y = 5$
$2x^2 - 9x - y = 11$

(j) $y = 9x^2 + 11x + 3$
$5x - y + 2 = 0$

(k) $2x + y = 5$
$2x^2 + y^2 = 11$

(l) $x - 4y + 1 = 0$
$x^2 - 4xy + y^2 = 13$

Question 3: Solve the following simultaneous equations

(a) $2x + y = 7$
$x^2 - y^2 = 8$

(b) $x^2 + y^2 = 20$
$y = x + 3$

(c) $y = x^2 - 9x - 3$
$y = x$

(d) $2x^2 + y^2 = 10$
$2x - y = 5$

(e) $y = x^2 + x - 7$
$4x + 2y + 1 = 0$

(f) $y = x - 2$
$2x^2 - xy = 11$

Apply

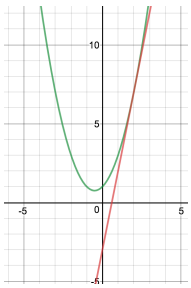
Question 1: Find the coordinates where the line $x + y = 3$ and the curve $x^2 + 3y = 27$ intersect

Question 2: How many points of intersection does the curve $y = (x - 3)(x + 4)$ have with the line $y = x - 8$?

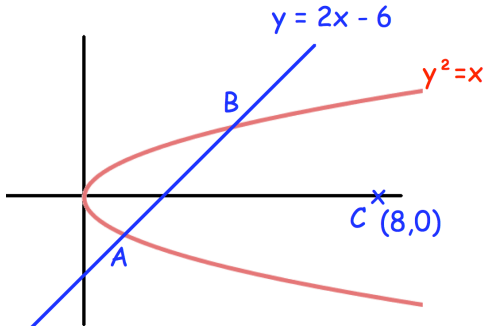
Question 3: The line $y = x + 4$ and the curve $y = x^2 + 3x + 4$ intersect at the points A and B. Find the distance between the points A and B.

Question 4: Find the coordinates of the points where the line $x + 5y = 37$ and the curve $y = x^2 + x + 2$ meet.

Question 5: Show, by using simultaneous equations, that the line $y = 5x - 3$ is a tangent to the curve $y = x^2 + x + 1$

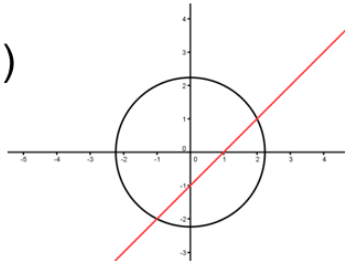


Question 6: Shown below is the curve $y^2 = x$ and the line $y = 2x - 6$
The curve and the line meet at the points A and B
The point C is (8, 0)
Show ABC is a right angled triangle.



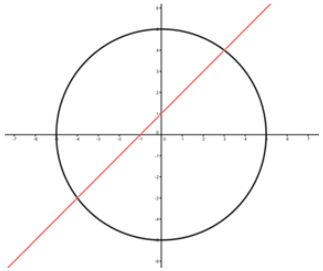
Fluency Practice

solving simultaneous equations: one linear, one quadratic (i)
find the coordinates of the points of intersection

(1) 

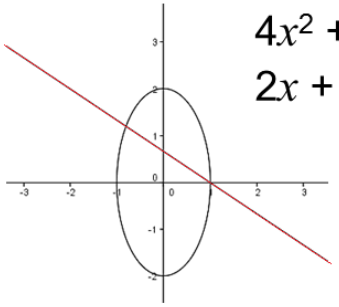
$x^2 + y^2 = 5$
$x - y = 1$

(2) $x^2 + y^2 = 58$
$y - x = 4$

(3) 

$x^2 + y^2 = 25$
$-x + y = 1$

(4) $x^2 + y^2 = 2$
$x + 3y = 2$

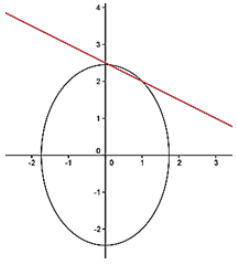
(5) 

$4x^2 + y^2 = 4$
$2x + 3y = 2$

(6) $2x^2 + xy + 1 = 0$
$y - x = 4$

(7) $4x^2 + y^2 = 37$
$2y + x = 1$

(8) $2x^2 + y^2 = 6$
$x + 2y = 5$



factorise any quadratic equations …

Fluency Practice

solving simultaneous equations: one linear, one quadratic (ii)

find the coordinates of the points of intersection

(1) $y = 3x^2 - 14x - 5$
$y = 4x - 32$

(2) $x^2 + 8y = 13$
$2y + x = 2$

(3) $4x^2 + y^2 = 10$
$2x - y = 4$

(4) $2x^2 + y^2 = 57$
$2y + x = 6$

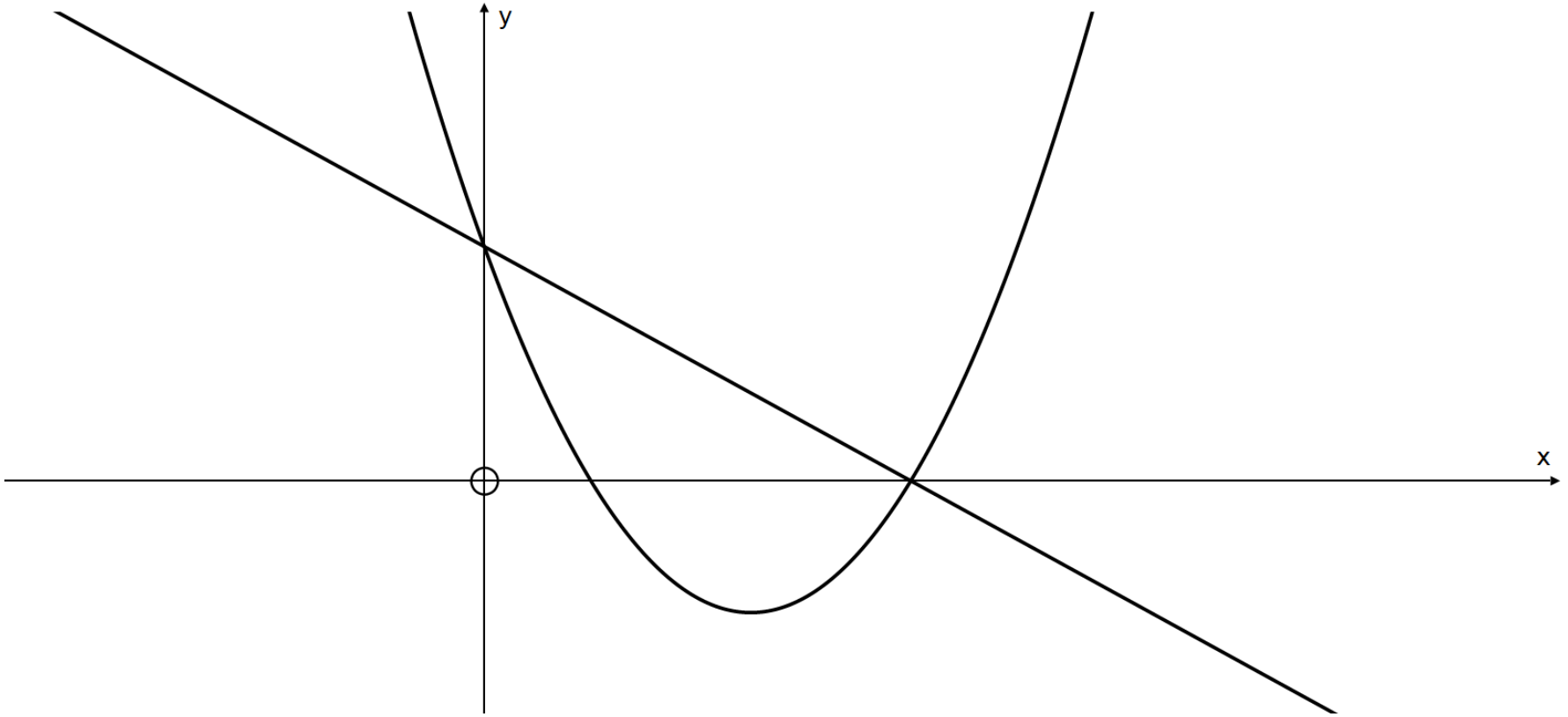
(5) $2y^2 + x^2 = 48$
$y + x = 8$

(6) $x^2 + y^2 + 2x - 4y = 8$
$x + y = 6$

(7) $x^2 + y^2 + 6x - 4y = 4$
$y = 3x + 4$

(8) $2x^2 - y^2 + xy = 14$
$4x + 5y = 0$

Problem Solving



Suggest possible equations for the line and the parabola.

Justify your answer.

Generalise your answer.

Fluency Practice

Solving Linear & Non-Linear Simultaneous Equations Graphically

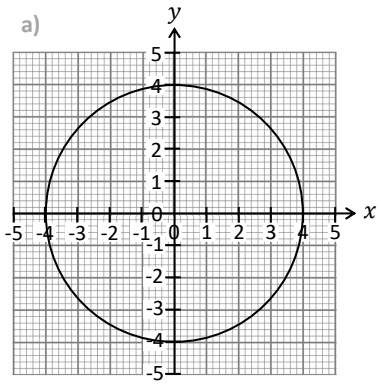
Estimate solutions to each pair of simultaneous equations.

Substitute the linear equation into the non-linear equation to express a quadratic equation in the general form: $ax^2 + bx + c = 0$

a)

$x^2 + y^2 = 16$

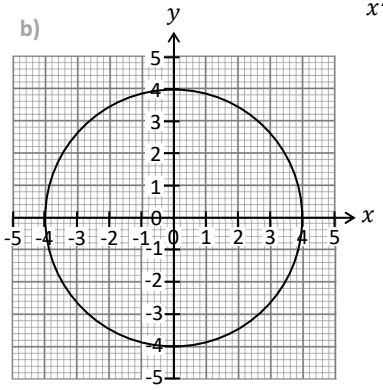
$y = 2x$



b)

$x^2 + y^2 = 16$

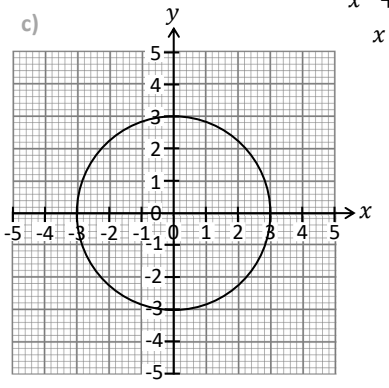
$y = x + 1$



c)

$x^2 + y^2 = 9$

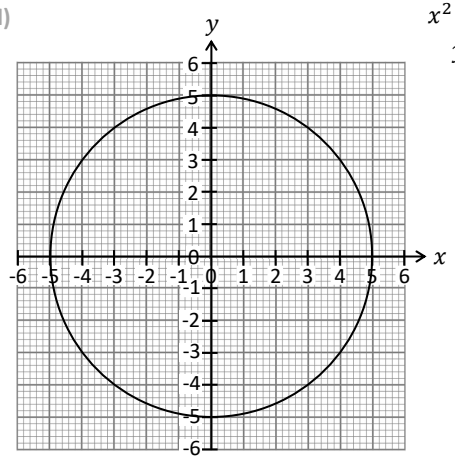
$x + y = 1$



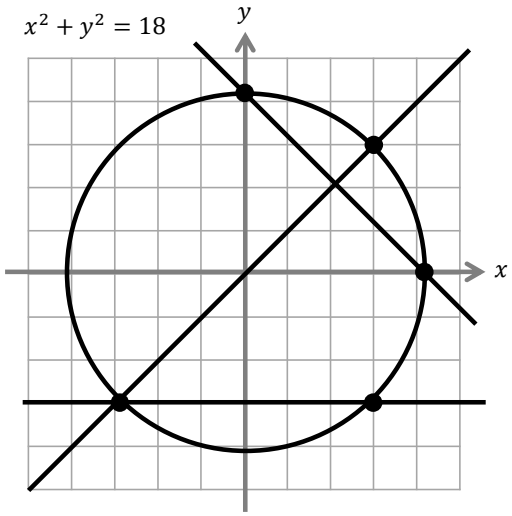
d)

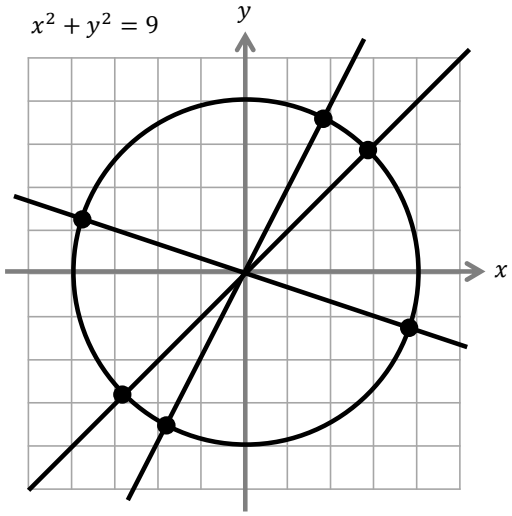
$x^2 + y^2 = 25$

$y = 3x - 2$



Fluency Practice

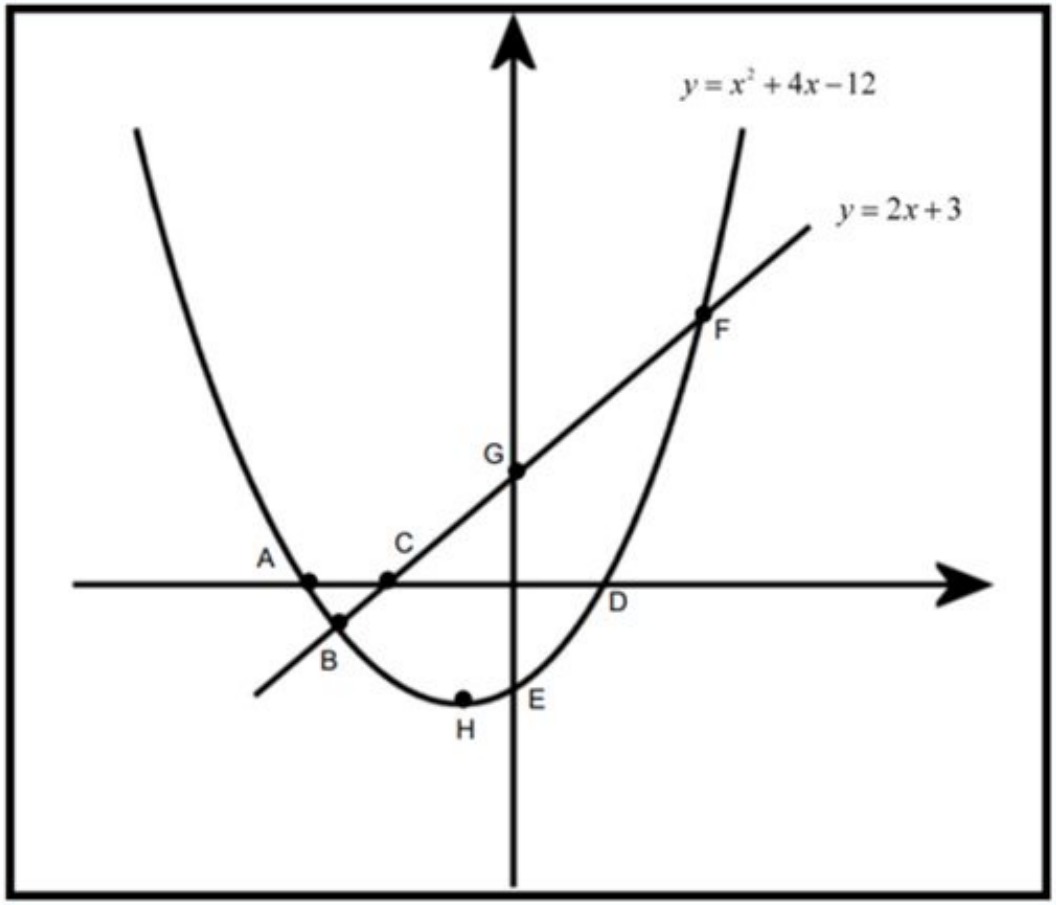




Each grid shows **3 simultaneous equations** involving **1 linear graph & 1 non-linear graph**.
Calculate **exact solutions** for each. Can you substitute & simplify the equations?

Problem Solving

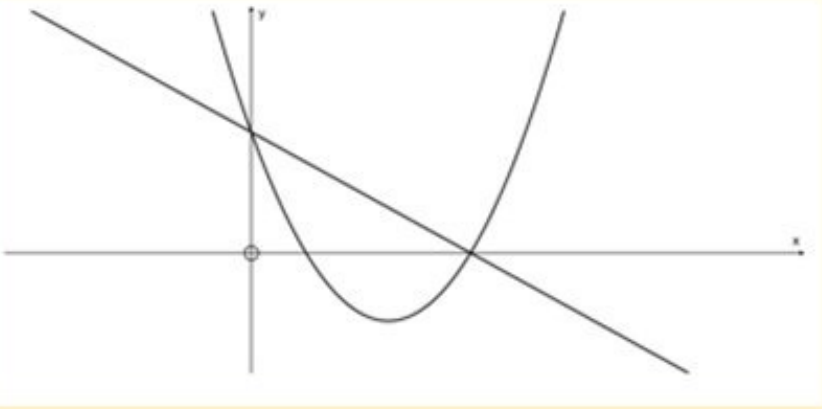
Find the coordinates of the points A, B, C, D, E, F, G and H



TASK

CHALLENGE

When you finish the task on the left, look at the diagram below. Suggest possible equations for the line and parabola. Justify your answer. Generalise your answer.



Fluency Practice

① Max wants to spend £100 on plants.
But each plant costs £5 more than expected, so
Max has to buy 10 less than planned.

How many plants did Max plan to buy?

② **Changing Prices**

Hae wants to spend £90 on plant pots.
But each pot costs £3 more than expected, so
Hae has to buy 5 less than planned.

How many pots did Hae plan to buy?

③ Jia wants to spend £160 on calculators.
But each calculator costs £2 more than expected, so
Jia can afford 4 less than they planned.

How many calculators can Jia now buy?

④ A rectangle has an area of 84 m^2.
The length is increased by 1 metre.
The width is increased by 1 metre.
The new area is 104 m^2.

What was the width of the original rectangle?

⑤ **Changing Values**

A rectangle has an area of 60 m^2.
The length is decreased by 2 metres.
The width is increased by 3 metres.
The new area is 72 m^2.

What was the length of the original rectangle?

⑥ With £360 a shop can buy q t-shirts.
With a price decrease of £8, the shop can
now buy 12 more t-shirts.

How many t-shirts can the shop now buy?

Fluency Practice

Speed Change

Tim runs 300 metres in t seconds.
By increasing their speed (s) by 1 m/s,
Tim runs the 300 metres 10 seconds faster.

What was Tim's original 300 m time?

$$s = \frac{d}{t} \qquad st = d$$

Original: $st = 300$

New: $(s + 1)(t - 10) = 300$

Jan runs 200 metres in t seconds.
By increasing their speed by 3 m/s,
Jan runs the 200 metres 15 seconds faster.

What was Jan's original 200 m time?

A cyclist travels 1 km in t seconds.
Their speed decreases by 5 m/s and they
travel the next kilometre 10 seconds slower.

How long did it take the cyclist to cover the 2 km?

Fluency Practice

Harder Simultaneous Equations			
(a)	**(b)**	**(c)**	**(d)**
Solve $2x + 3y - 18 = 0$ $x = y + 4$	Solve $3x + 4y = 8$ $6 - x = 2y$	Solve $y = x^2 - 2x + 6$ $y = x + 4$	Solve $x^2 + y^2 = 50$ $y = x - 8$
(e)	**(f)**	**(g)**	**(h)**
Solve $x^2 - 5x = y - 5$ $2x + y = 5$	Solve $x^2 + 2y^2 = 22$ $3x = 2y$	Solve $x^2 + y^2 + xy = 12$ $x = 6 - 2y$	Solve $y = x^2 + 3x - 5$ $x - y = 4$
(i)	**(j)**	**(k)**	**(l)**
Find the coordinates of the points where the curve $y = 2x^2 - 3x - 4$ intersects with the line $y = 2x - 1$	Solve $xy = 16$ $x + y = 10$	Solve $x + 2y = 5$ $(x - 1)^2 + (y - 2)^2 = 20$	Find the length of the line joining the points of intersection of $y = \frac{x}{2} + 1$ and $x^2 + y^2 = xy + 4$

Fluency Practice

More Quadratic Equations and Inequalities Revision			
(a)	**(b)**	**(c)**	**(d)**
Solve $x^2 + x - 20 = 0$	Solve, by completing the square, $x^2 + 6x + 3 = 0$ giving answers in surd form	Solve $(x + 3)(x - 5) > 0$	Solve $3x^2 - 11x - 7$ giving your answers to 3 significant figures
(e)	**(f)**	**(g)**	**(h)**
Solve $x^2 \leq 25$	Solve $5x^2 + 18x = 8$	Solve $y = x + 1$ $y = x^2 + 5x - 11$	Solve $2x^2 - 13x + 10 < 0$

(i)	**(j)**	**(k)**
The area of a triangle with base $(x + 9)$ cm and height $(2x + 1)$ cm is $21\ cm^2$. Find the value of x.	A rectangle has a length of $(x - 6)$ cm and a width of $4x$ cm. The area of the rectangle is less than $13\ cm^2$. Find the range of possible values of x.	Solve $x + 2y = 7$ $x^2 + y^2 = 10$

4 Advanced Sequences

irrational sequences: what comes next?

Look at these sequences, can you work out the term-to-term rule?
What will be the next number in the sequence?

a $\sqrt{2}$, 2, $2\sqrt{2}$, 4, ?...

b $\sqrt{3}$, 6, $12\sqrt{3}$, 72, ?...

c $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$, ?...

d $\sqrt{75}, \sqrt{48}, \sqrt{27}, \sqrt{12}$, ?...

e 9, $3\sqrt{3}$, 3, $\sqrt{3}$, ?...

f $2\sqrt{5}$, -10, $10\sqrt{5}$, -50, ?...

g $\sqrt{3}-2$, $3-2\sqrt{3}$, $3\sqrt{3}-6$, $9-6\sqrt{3}$, ?...

h $\sqrt{2}+1$, $2\sqrt{2}+3$, $5\sqrt{2}+7$, $12\sqrt{2}+17$, ?...

extension:

Make up your own sequence. Can you partner figure out your term-to-term rule?

Fluency Practice

geometric series: what are the constant ratios?

(1) 9, 576, 294,912, …

(2) 21, 189, 5,103, …

(3) 80, 1280, …

(4) 324, 26,244, 2,125,764, …

(5) 48, 10,368, …

(6) 4, 5,324, …

(7) 27, 729, …

(8) 3000, 81, …

(9) 4,782,969, 177,147, …

(10) 12, 192, …

what are the first terms (a) and the ratios (r) of these geometric series?

(10) 35, 21,875, …

(11) 441, 1,058,841, …

(12) 1221, 1,625,151, …

(13) 200, 0.00125, …

(14) 111, 26,973, …

Fluency Practice

geometric series: what are the missing terms?

(1) 24
—
54
—
…

(2) —
30
—
480
…

(3) —
—
72
—
32
…

(4) —
—
1200
—
1875
…

(5) —
8
—
—
1
…

(6) 25
—
—
102.4
…

(7) —
9.9
—
11.979
—
…

(8) —
150
—
216
—
…

(9) 192
—
—
81
…

(10) 250
—
—
54
…

what are ‘a’ and ‘r’ for these geometric series?

(10) $ar = 4$
$ar^4 = 4000$

(11) $ar^2 = 144$
$ar^6 = 729$

(12) $ar^3 = 1792$
$ar^6 = 917504$

(13) $ar^2 = 1728$
$ar^6 = 729$

(14) $ar = 1.6$
$ar^7 = 0.0001024$

Fluency Practice

geometric series: find possible values of **n** and the corresponding constant ratios

(1)
$n - 1$
$2n$
$3n + 9$
—
—
…

(2)
$n - 2$
$2n$
$3n + 18$
—
—
…

(3)
$2n - 6$
n
$n - 4$
—
—
…

(4)
$n - 2$
$2n + 2$
$3n + 88$
—
—
…

(5)
n
$10 + n$
$35 + 2n$
—
—
…

(6)
$n - 2$
$2n - 2$
$3n + 6$
—
—
…

(7)
$n - 1$
$n + 1$
$3n - 1$
—
—
…

(8)
$n + 1$
$2n - 1$
$5n - 7$
—
—
…

(9)
$2n + 2$
$3n - 1$
$4n - 3$
—
—
…

(10)
$n - 2$
$2n$
$3n + 20$
—
—
…

(11)
$n + 1$
$2n + 7$
$3n - 3$
—
—
…

(12)
$2n - 3$
$2 - 3n$
$3n + 8$
—
—
…

(13)
$3n - 2$
$4n - 2$
$5n - 1$
—
—
…

(14)
$n - 4$
$3n - 4$
$10n - 4$
—
—
…

(15)
$3n - 2$
$3n + 3$
$4n$
—
—
…

1. Below is the first number in a geometric sequence. Write the first 4 terms of 2 possible sequences that could start with that number:

a. 2
b. 4
c. 8
d. 10
e. 7

f. 0.4
g. -4
h. ¼
i. -0.3
j. $\sqrt{2}$

k. ab
l. $\frac{a^2b}{2}$
m. $\sqrt{a}$
n. ab^2c^3
o. $\frac{\pi}{2}$


Easy


Medium


Hard

2. Calculate the first 5 terms of these sequences:

a. 2^n
b. 2^{n+1}
c. $(-2)^n$
d. $-(2)^n$
e. $0.2^{(2n)}$

f. 3×3^n
g. $0.4 \times 3^{(n-1)}$
h. $\frac{3}{5} \times 1.1^n$
i. $9.1 \times 10^{(\frac{1}{n})}$
j. $0.03 \times 0.9^{(3n)}$

k. $a \times 3^{bn}$
l. $\frac{h}{3^n}$
m. $n \times a^n p^2$
n. $m \times 0.03^{(b+n)}$
o. $\frac{1}{f} \times ad^{(n-3)}$


Easy


Medium


Hard

3. Work out T(n) for the following geometric series:

a. 4, 8, 16, 32
b. 4, 2, 1, 0.5
c. 5, 15, 45, 135
d. 3, 18, 108, 648
e. -5, -10, -20, -40

f. -2, 4, -8, 16
g. 4, -4, 4, -4
h. $\frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}$
i. $\sqrt{2}, 2, 2\sqrt{2}, 4$
j. $5, 5\sqrt{5}, 25, 25\sqrt{5}$

k. ab, ab^2, ab^3, ab^4
l. $\frac{a}{b^2}, \frac{a^2}{b^3}, \frac{a^3}{b^4}, \frac{a^4}{b^5}$
m. $-a^2, a^2, -a^2, a^2$
n. $-ab, ab^2, -ab^3, ab^4$
o. $a^2b^2, a^2b, a^2, \frac{a^2}{b}$


Easy


Medium


Hard

Fluency Practice

Geometric Sequences			
(a)	**(b)**	**(c)**	**(d)**
Find the next two terms in the sequence $7, 14, 28, 56, \ldots$	Find the next two terms in the sequence $40, 20, 10, 5, \ldots.$	Find the first four terms of the sequence with first term 2 and common ratio 3	Find the first term and common ratio for the sequence: $3, 15, 75, 375, \ldots$
(e)	**(f)**	**(g)**	**(h)**
Find the first term and common ratio for the sequence: $160, 80, 40, 20, \ldots$	Find the next two terms in the sequence $2, -4, 8, -16, \ldots.$	Find the first four terms of the sequence with first term 120 and common ratio 0.5	Find the first term and common ratio for the sequence: $4, -8, 16, -32, \ldots$
(i)	**(j)**	**(k)**	**(l)**
Find the first four terms of the sequence with first term 5 and common ratio -2	Find the first four terms of the sequence with nth term $6 \times 3^{n-1}$	Find the nth term of the sequence with first term 10 and common ratio 4	Find the nth term of the sequence with first term 250 and common ratio 0.2
(m)	**(n)**	**(p)**	
Find the first four terms of the sequence with nth term $400 \times \left(\frac{1}{2}\right)^{n-1}$	Find the nth term of the sequence with first term 8 and common ratio -5	A tree starts with four branches. Every month each branch splits into two. How many branches will the tree have after 5 months? Find a formula for the number of branches b after n months.	

By finding the first and second differences, decide whether each of these sequences is quadratic.

(a) 1, 5, 11, 19, 29, 41

(b) 2, 5, 8, 11, 14, 17

(c) 0, 8, 22, 41, 68, 98

(d) 2, 9, 20, 35, 54, 77

(e) 4, 1, 0, 1, 4, 9

(f) 6, 17, 36, 65, 98, 141

(g) 18, 37, 62, 93, 130, 173

(h) 3, 9, 23, 43, 75, 113

(i) $-10, -4, 12, 38, 74, 120$

(j) 17, 39, 69, 107, 153, 207

Generate the first five terms of each of these quadratic sequences.

(a) $n^2 + 10$ (b) $n^2 - 1$

(c) $n^2 + n$ (d) $n^2 + 2n + 1$

(e) $n^2 - 3n$ (f) $n^2 - n - 2$

(g) $2n^2 + 5$ (h) $3n^2 - 7$

(i) $2n^2 + n - 5$ (j) $4n^2 + 3n - 1$

Find the 10^{th} and 50^{th} term of the following quadratic sequences.

(a) $n^2 + 5$ (b) $n^2 - 2$

(c) $n^2 - n$ (d) $n^2 + 2n$

(e) $n^2 - 3n + 1$ (f) $n^2 - n - 2$

(g) $4n^2 + 1$ (h) $3n^2$

(i) $2n^2 + n - 1$ (j) $5n^2 + 3n$

Quadratic Sequences

Substitute to complete each term.

	1st	2nd	3rd	4th	5th	10th
n^2				16		
$n^2 - 5$				11	20	
$n^2 + n$		6	12			
$n^2 + 2n$	3				35	
$n^2 - n$				12	20	
$n^2 + n + 3$			15	23		
$n^2 + 4n$	5				45	
$n^2 - 3n$				4		
$n^2 + 2n + 6$		14	21			
$n^2 - n + 5$		7			25	
$n^2 + 3n - 4$			14	24		
$n^2 - 2n + 5$	4	5				
$2n^2 + 3n$				44	65	
$2n^2 - 4n$				16		
$2n^2 + 2n + 3$			27		63	
$4n^2 - 3n - 5$		5		47		

Find the nth term of these sequences:

(a) $8, 11, 16, 23, \ldots$

(b) $-4, -1, 4, 11, \ldots$

(c) $6, 12, 22, 36, \ldots$

(d) $-1, 5, 15, 29, ..$

(e) $4, 13, 28, 49, \ldots$

(f) $2, 14, 34, 62, \ldots$

Find the nth term of these sequences:

(a) $3, 7, 13, 21, \ldots$

(b) $2, 7, 14, 23, \ldots$

(c) $9, 18, 31, 48, \ldots$

(d) $1, 5, 13, 25, \ldots$

(e) $7, 23, 49, 85, \ldots$

(f) $7, 15, 29, 49, \ldots$

Find the nth term of these sequences:

(a) $10.5, 12, 14.5, 18, \ldots$

(b) $5.5, 9, 13.5, 19, \ldots$

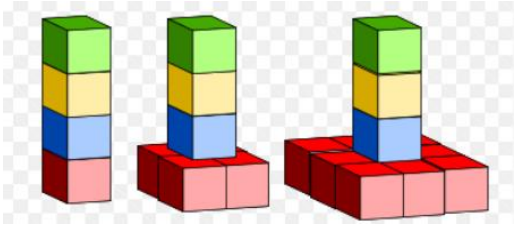
(c) $1.5, 5, 11.5, 21, \ldots$

(d) $3.5, 10, 19.5, 32, \ldots$

(e) $19, 16, 11, 4, -5, \ldots$

(f) $5, 6, 5, 2, -3, \ldots$

Here are some patterns made from cubes. Find an expression for the nth term of this sequence.



Quadratic Sequences

Quadratic sequences take the form

$$an^2 + bn + c$$

Section 1

For each of the following quadratic sequences, identify the values of a, b and c:

1. $2n^2 + 3n + 4$
2. $4n^2 + 8n + 5$
3. $5n^2 - 11n$
4. $n^2 + n - 2$
5. $\frac{n^2}{2} - 4$
6. $-n^2$

Section 2

Find the first 3 terms for these quadratic sequences:

7. $n^2 + 3$
8. $2n^2 + 2$
9. $3n^2$
10. $4n^2 - 30$
11. $n^2 + n - 9$
12. $2n^2 - 2n - 4$
13. $\frac{n^2}{2} + 3n + 9$
14. $n(n + 3)$
15. $(n + 3)(2n + 6)$

Find the 10th and 100th term of these sequences:

16. $n^2 + 3$
17. $2n^2 - 4n$
18. $-8n^2$
19. $\frac{n^2}{3} + 20$

Section 3

Which of these terms are **not** in the sequences provided in **bold:**:

20. $\{12, 39, 55, 103\}$ $(\boldsymbol{n^2 + 3})$
21. $\{15, 35, 63, 82\}$ $(\boldsymbol{n^2 - 1})$
22. $\{108, 116, 128, 162\}$ $(\boldsymbol{2n^2})$
23. $\{1, 38, 56, 119\}$ $(\boldsymbol{n^2 + 10n})$

Match the sequences to the descriptions below (assume n is always a whole number):

24. $n^2 + 1$
25. $n^2 + n$
26. $n^2 \div 3$
27. $2n^2$
28. $n^2 + 3n + \frac{3}{5}$

A: Always even
B: Always odd
C: Sometimes even, sometimes odd
D: Sometimes an integer
E: Never an integer

Section 4

Find the nth term of these quadratic sequences:

29. 1, 4, 9, 16
30. -3, 0, 5, 12
31. 3, 12, 27, 48
32. 2, 5, 10, 17
33. 8, 11, 16, 23
34. 0, 6, 16, 30
35. 8, 14, 24, 38
36. 11, 20, 35, 56
37. 0.5, 2, 4.5, 8
38. $9.\dot{3}$, $10.\dot{3}$, 12, $14.\dot{3}$
39. 3, 7, 13, 21
40. 5, 13, 25, 41
41. 7, 16, 27, 40
42. 0, 1, 4, 9
43. - 5, 8, 27, 52

Easiest | Medium | Hardest

Find the nth term for each sequence and the specified term.

-1 , 5 , 15 , 29 , 47

10^{th} term

5 , 14 , 29 , 50 , 77

20^{th} term

3 , 6 , 11 , 18 , 27

50^{th} term

5 , 11 , 21 , 35 , 53

25^{th} term

6 , 17 , 34 , 57 , 86

10^{th} term

3 , 18 , 41 , 72 , 111

20^{th} term

Fluency Practice

find the next two terms in these quadratic sequences

(1) 6 , 7 , 10 , 15 , 22 , 31 , ? , ?

(2) 11 , 15 , 21 , 29 , 39 , 51 , ? , ?

(3) – 18 , – 14 , – 8 , 0 , 10 , 22 , ? , ?

(4) – 7 , – 8 , – 7 , – 4 , 1 , 8 , ? , ?

(5) – 10 , – 13 , – 14 , – 13 , – 10 , – 5 , ? , ?

Fluency Practice

two of the terms in these quadratic sequences are wrong

(1) 5 , 0 , – 1 , – 4 , – 1 , 0 , 5 , 12 , 21 , 40

(2) 10 , 6 , 4 , 4 , 6 , 12 , 16 , 24 , 32 , 46

(3) 5 , 1 , – 1 , – 3 , – 1 , 5 , 11 , 19 , 29 , 41

(4) 17 , 11 , 3 , – 1 , – 3 , – 3 , – 1 , 3 , 11 , 17

(5) – 26 , – 22 , – 20 , – 14 , – 6 , 4 , 14 , 30 , 46

Fluency Practice

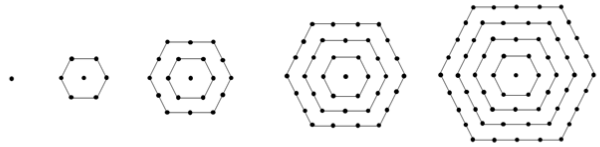
fit a quadratic rule to these sequences

(1)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*	(6)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*
	2	3	6	11	18	27	…….			9	18	31	48	69	94	…….	
(2)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*	(7)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*
	3	6	11	18	27	38	…….			31	48	69	94	123	156	…….	
(3)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*	(8)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*
	3	2	3	6	11	18	…….			4	9	18	31	48	69	…….	
(4)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*	(9)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*
	6	11	18	27	38	51	…….			6	3	4	9	18	31	…….	
(5)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*	(10)	*1*	*2*	*3*	*4*	*5*	*6*	…….	*n*
	11	18	27	38	51	66	…….			24	13	6	3	4	9	…….	

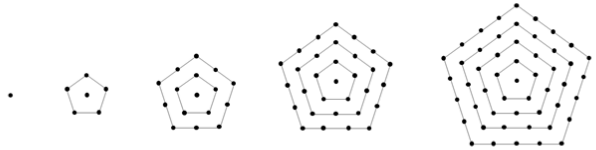
Fluency Practice

find a rule for the numbers of dots/circles in these quadratic sequences

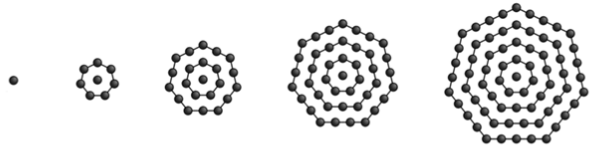
1) centred hexagonal numbers



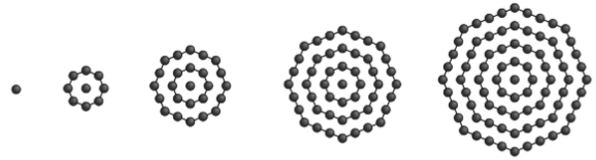
2) centred pentagonal numbers



3) centred heptagonal numbers



4) centred octagonal numbers



5) corner hexagonal numbers

6) corner pentagonal numbers

7) corner heptagonal numbers

8) corner octagonal numbers

Fluency Practice

The nth term of a sequence is in the form $an^2 + bn$
The 2nd term in the sequence is 14.
The 4th term in the sequence is 44.
Find a & b.

1) Form two equations in a & b.

$a(2)^2 + b(2) = 14$
$4a + 2b = 14$ ×2 → $16a + 4b = 44$

$a(4)^2 + b(4) = 44$
$16a + 4b = 44$ → $8a + 4b = 28$

2) Solve the simultaneous equations.

$16a + 4b = 44$
$8a + 4b = 28$
$8a = 16$
$a = 2$

$4(2) + 2b = 14$
$2b = 6$
$b = 3$

nth term = $2n^2 + 3n$

Finding the nth Term of a Quadratic Sequence

① The nth term of a sequence is in the form $an^2 + bn$
The 2nd term in the sequence is 18.
The 4th term in the sequence is 52.
Find a & b.

How can we check our answer?

② A sequence is in the form $an^2 + bn$
The 3rd term in the sequence is 30.
The 6th term in the sequence is 114.
Find the nth term

③ A sequence is in the form $an^2 + bn$
The 2nd term in the sequence is 22.
The 3rd term in the sequence is 45.
Find the nth term

④ The 4th term in a sequence is 64.
The 5th term in a sequence is 105.
Find the 3rd term in the sequence.

Fluency Practice

(a) A quadratic sequence begins 3, 15, _ , 51, 75,...
What is the the third term?

(b) The quadratic sequence beginning –3, 3, 11, 21, 33...
has nth term $n^2 + 3n - 7$.
Find the nth term of the quadratic sequence that begins
3, 11, 21, 33...

(c) The first term of a quadratic sequence is 10, the fifth term is 25, and the ninth term 45. Find the thirteenth term of the sequence.

Question 1: Find the next two terms for each quadratic sequence

(a) 4, 6, 10, 16, 24 … …

(b) 1, 2, 4, 7, 11 … …

(c) 2, 5, 10, 17, 26 … …

(d) 3, 9, 19, 33, 51 … …

(e) 50, 48, 44, 38, 30 … …

(f) 3, 14, 29, 48, 71 … …

Question 2: List the first 5 terms of the sequences with n^{th} term:

(a) n^2

(b) $n^2 + 1$

(c) $n^2 + 4$

(d) $n^2 - 2$

(e) $2n^2$

(f) $5n^2$

(g) $\frac{1}{2}n^2$

(h) $\frac{1}{4}n^2$

(i) $3n^2 + 10$

(j) $\frac{3}{5}n^2$

Question 3: The quadratic n^{th} term of the sequence below is n^2

1, 4, 9, 16, 25, 36, 49 …

Find the n^{th} term of each of these sequences

(a) 4, 7, 12, 19, 28, 39, 52 …

(b) 51, 54, 59, 66, 75, 86, 99 …

(c) −5, −2, 3, 10, 19, 30…

(d) 3, 12, 27, 48, 75, 108 …

(e) 20, 80, 180, 320, 500, 720 …

(f) 0.2, 0.8, 1.8, 3.2, 5 …

(g) 3, 9, 19, 33, 51, 73, 99 …

(h) 2.5, 4, 6.5, 10, 14.5, 20 …

Question 4: For each n^{th} term, work out the first five terms of the sequence.

(a) $n^2 + n$

(b) $n^2 + 2n$

(c) $n^2 - n$

(d) $n^2 - 3n$

(e) $n^2 + n + 2$

(f) $n^2 - 2n + 5$

(g) $n^2 + 4n - 10$

(h) $2n^2 + n$

(i) $3n^2 - n + 6$

(j) $10n^2 + 5n - 7$

Question 5: For each n^{th} term, work out the first five terms of the sequence.

(a) $-n^2$

(b) $-2n^2$

(c) $-4n^2 + 2$

(d) $-n^2 + 3n$

(e) $50 - n^2$

(f) $6n - n^2$

(g) $-n^2 - 7n - 2$

Question 6: For each n^{th} term, work out the first five terms of the sequence.

(a) $n(n + 1)$ (b) $n(n + 3)$ (c) $(n + 1)(n + 5)$ (d) $n(n - 2)$

(e) $(n - 3)(n + 1)$ (f) $(n - 8)(n - 3)$

Question 7: Work out the n^{th} term for each quadratic sequence

(a) 7, 12, 19, 28, 39 … (b) 7, 16, 31, 52, 79 … (c) 6, 13, 24, 39, 58 …

(d) 3, 13, 27, 45, 67 … (e) 9, 20, 35, 54, 77 … (f) 9, 24, 45, 72, 105 …

(g) −6, −1, 6, 15, 26 … (h) −5, −4, −1, 4, 11 … (i) 7, 10, 17, 28, 43 …

(j) 2.5, 5, 8.5, 13, 18.5 … (k) −0.5, 1, 4.5, 10, 17.5 …

Question 8: Calculate the 10^{th} term of each sequence in question 7

Question 9: Work out the n^{th} term for each quadratic sequence

(a) 3, 1, −3, −9, −17… (b) −4, −12, −24, −40, −60 … (c) 6, 5, 2, −3, −10 …

(d) 100, 96, 90, 82, 72 … (e) −17, −30, −49, −74, −105 … (f) 6, 5.5, 4.5, 3, 1 …

Question 10: Calculate the 10^{th} term of each sequence in question 9

Question 11: A sequence has an n^{th} term of $n^2 + n - 20$
Work out which term in the sequence has a value of 52.

Question 12: A sequence has an n^{th} term of $n^2 + 2n - 5$
Work out which term in the sequence has a value of 58.

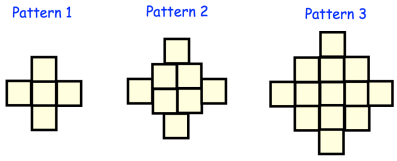
Question 13: A sequence has an n^{th} term of $n^2 - 6n + 7$
Work out which term in the sequence has a value of 23.

Apply

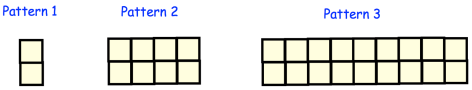
Question 1: The first 5 terms of a quadratic sequence are: 4, 10, 18, 28, 40
Work out the difference between the 10^{th} and 20^{th} terms.

Question 2: Below are patterns of tiles.
The number of tiles in each form quadratic sequences.
Find the number of tiles in pattern n for each.

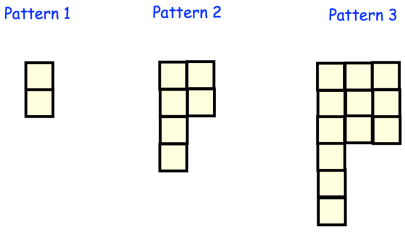
(a)



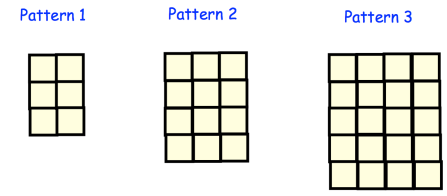
(b)



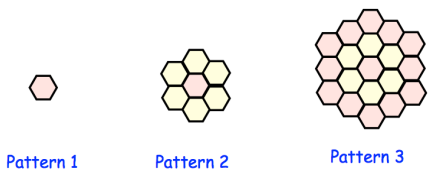
(c)



(d)



Question 3: Here is a pattern made from tiles.
How many tiles are needed to make Pattern 20?



Question 4: The first 4 terms of a sequence are: 400, 390, 375, 355 ...
Which term is the first to be negative?

Question 5: The n^{th} term of a quadratic sequence is $n^2 + 4n$
Two consecutive terms have a difference of 25.
Work out the two terms.

Question 6: Prove every term in the sequence $n^2 - 8n + 21$ is positive

Fluency Practice

Using the Nth Term of Quadratic Sequences			
$u_n = n^2 + 3n - 5$	$u_n = 3n^2 - n + 1$	$u_n = n^2 - 2n$	$u_n = n^2 + an - b$
(a)	**(a)**	**(a)**	**(a)**
Find the value of u_4	Find the value of u_6.	Find the 9^{th} term of the sequence.	Find the value of u_5 in terms of a and b.
(b)	**(b)**	**(b)**	**(b)**
Find the difference between the 6^{th} term and the 7^{th} term.	Find the sum of the 9^{th} term and the 10^{th} term.	Find an expression for the $(n+1)^{th}$ term.	Find the value of u_7 in terms of a and b.
(c)	**(c)**	**(c)**	**(c)**
A term of the sequence is 65 Find the value of n.	A term of the sequence is 103 Find the value of n.	Find an expression for the difference between the n^{th} and the $(n+1)^{th}$ term.	Given that $u_5 = 25$ and $u_7 = 70$, find the values of a and b.

Fluency Practice

Using the Nth Term of Sequences			
$u_n = \frac{6n}{n+7}$	$u_n = \frac{4n+3}{n+1}$	$u_n = \frac{10-3n}{2+n}$	$u_n = \frac{4n^2}{n^2+8}$
(a)	**(a)**	**(a)**	**(a)**
Find the value of u_8 as a fraction in its simplest form.	Find the value of u_9.	Find the 6^{th} term.	Find the value of u_5 as a mixed number.
(b)	**(b)**	**(b)**	**(b)**
A term of the sequence is $\frac{11}{3}$ Find the value of n.	A term of the sequence is $\frac{15}{4}$ Find the value of n.	A term of the sequence is $-\frac{7}{5}$ Find the value of n.	Find the term in the sequence closest to 3.8
(c)	**(c)**	**(c)**	**(c)**
Find the difference between the 5^{th} term and the 9^{th} term.	Find the sum of the 4^{th} term and the 14^{th} term.	Find $2u_{10} - u_{16}$	Find the difference between the 8^{th} term and the 10^{th} term.
(d)	**(d)**	**(d)**	**(d)**
Find the first term in the sequence that is greater than 4	Find the first term in the sequence that is greater than 3.9	Find the first term in the sequence that is negative.	Find the smallest value of n for which $u_n > \frac{7}{2}$

6 Algebraic Proof

Fluency Practice

Prove It!

a) $2n + 4(n + 1) + 4$
is always even

b) $2(n + 1) + 4(n + 2)$
is always even

c) $5(n + 3) - 3(n - 1)$
is always even

d) $4(2n - 3) - 3(2n - 1)$
is always odd

A number (n)
An even number $2(n)$
An odd number $2(n) + 1$

e) $(n + 1)^2 - n^2 + 1$
is always even

f) $(n + 2)^2 - n^2$
is always even

g) $n^2 + (n + 1)^2$
is always odd

h) $n^2 + (n - 3)^2$
is always odd

i) $(2n + 1)^2 + 3$
is divisible by 4

j) $(n + 1)^2 + (n + 2)^2$
is always odd

k) $(n + 3)^2 - (n + 1)^2$
is divisible by 4

l) $(n - 4)^2 - (n - 2)^2$
is even

m) $(2n + 1)^3$
is always odd

n) $(4n + 1)^2 - (4n - 1)^2$
is divisible by 8

o) $(2n - 1)^2 - 3(n - 2)^2 - 2(n - 10)$
is a square number

1. n is a positive integer. Write each expression in the correct box.

$6n$ | $n + 2$ | $2n - 3$ | $5n$ | $4n + 8$ | $2n + 1$ | $(2n)^2$ | $4(n + 1)$ | $2n^2 + 4n - 3$ | $n^2 + 1$

Always an even number	**Sometimes** an even number	**Never** an even number

	Example	*Your turn*
2.	n is a positive integer. Prove algebraically that $6(n + 3) - n + 2$ is **always** a **multiple of 5**.	n is a positive integer. Prove algebraically that $n(2n + 1) + n^2 + 2n$ is **always** a **multiple of 3**.
3.	n is a positive integer. Prove algebraically that $(n + 2)^2 - n^2$ is **always** a **multiple of 4**.	n is a positive integer. Prove algebraically that $(4n + 1)^2 - 1$ is **always** a **multiple of 8**.
4.	n is a positive integer. Prove algebraically that $(4n + 3)^2 + 2(n + 1)$ is **always** an **odd number**.	n is a positive integer. Prove algebraically that $(n + 1)(n + 2) - n^2$ is **never** a **multiple of 3**.

Fluency Practice

expanding quadratic brackets (n is a positive integer)

proofs

(1) prove that $(n + 6)(n + 3)$ is always 10 greater than $(n + 8)(n + 1)$

(2) prove that $(n + 1)(n + 2) – n(n + 3)$ is always 2

(3) prove that $n(n + 1) + (n + 1)(n + 2)$ is double a square number

(4) prove that $(n – 3)(n – 2)$ is always 2 greater than $(n – 4)(n – 1)$

(5) prove that $(3n – 1)(3n + 1) + 1$ is $9 \times$ a square number

(6) prove that $(2n + 1)(2n + 3) + 1$ is an even square number

(7) prove that $(3n + 2)^2$ is always 1 more than $(3n + 1)(3n + 3)$

(8) prove that $(3n + 1)(3n + 2) – 3n(3n + 3)$ is always 2

(9) prove that $(2n + 1)^2 + (n + 3)^2$ is always a multiple of 5

1. n is a positive integer. Find a counter-example to show that these statements are **false**:

   a) $3n + 1$ is always even.

   b) $6n - 1$ is always prime.

	Example	*Your turn*
2.	Prove algebraically that for any three consecutive integers, the difference between the square of the middle number and the product of the two outer numbers is always 1.	Prove algebraically that the sum of any pair of consecutive odd numbers is always a multiple of 4.
3.	Prove algebraically that the sum of any two odd numbers is an even number.	Prove algebraically that the difference between any two odd numbers is an even number.

4. n is a positive integer. Prove algebraically that $(n + 3)^2 - (n + 1)^2$ is a multiple of 4.

5. Prove that the difference between the squares of two consecutive odd numbers is always a multiple of 8.

(a) Show that $4(n+3)-n$ is a multiple of 3 for all integer values of n

(b) Show that $(n+2)^2+3n^2$ is a multiple of 4 for all integer values of n

(c) Show that $(3n-1)^2-(2n+1)^2$ is a multiple of 5 for all integer values of n

(d) Show that $(2n+1)(4n-3)-(n+2)^2-n$ is a multiple of 7 for all integer values of n

(a) Show that the sum of three consecutive integers is always a multiple of 3

(b) Show that the sum of three consecutive even numbers is always a multiple of 6

(c) Show that the product of two consecutive even numbers is always a multiple of 4

(a) Prove algebraically that the sum of three consecutive square numbers is never a multiple of 3

(b) Prove algebraically that the sum of the squares of any two odd numbers is never a multiple of 4

(c) Prove algebraically that the product of two consecutive odd numbers is never a multiple of 4

(a) Prove algebraically that the product of three consecutive even numbers is always a multiple of 8

(b) Prove algebraically that the sum of the cubes of two consecutive even numbers is always a multiple of 8

(c) Prove algebraically that the product of the squares of two odd numbers is always one more than a multiple of 4

Write down algebraic expressions for:

(a) two different odd numbers

(b) two consecutive numbers

(c) two consecutive even numbers

(d) two different odd numbers squared

(a) Prove algebraically that the sum of any two odd numbers is always even.

(b) Prove algebraically that the product of an odd number and an even number is always even

(c) Prove algebraically that the sum of two consecutive numbers is always odd

(d) Prove algebraically that the product of two consecutive even numbers is always even

(a) Prove algebraically that the mean of two consecutive odd numbers is always even

(b) Prove algebraically that the difference between an odd number and an even number is always odd

(c) Prove algebraically that the mean of three consecutive odd numbers is always equal to the middle number

(a) Prove algebraically that the sum of the squares of two even numbers is always a multiple of 4

(b) Prove algebraically that the difference between the squares of two odd numbers is always a multiple of 4

(c) Prove algebraically that the sum of the squares of two consecutive numbers is always odd

(a) Show for all integers values of n that $(n+3)^2 - n(n-6) + 2$ is always odd

(b) Show for all integer values of n that $(3n+5)^2 + (3-n)^2$ is always even

(a) Prove that the difference of two odd numbers is even.

(b) Prove that $n^2 - 2 - (n-2)^2$ is always an even number.

(c) Prove that the product of two odd numbers is always odd.

(d) Prove that the square of an even number is always even.

(e) Prove that the difference between any two consecutive odd numbers is always two.

(f) Prove that the mean of three consecutive integers is always the middle number.

(g) Prove that the difference between the squares of any two consecutive numbers is always odd.

(h) Prove that $7(n+8) + 5(n-4)$ is always a multiple of 12.

(i) Prove that $(m+2)^2 - m^2 - 12$ is always a multiple of 4.

(j) Prove that the sum of three consecutive odd numbers is always a multiple of three.

(k) Prove that the sum of the squares of any two positive odd integers is always even.

(l) Prove that $(3n+1)^2 - (3n-1)^2$ is always a multiple of 12 for all positive integer values of n.

(m) Prove that the sum of four consecutive integers is not divisible by four.

Fluency Practice

triangular number proofs

(1)
$1 + 3 = 4$
$3 + 6 = 9$
$6 + 10 = 16$
…
general rule? proof?

(2)
$3 - 1 = 2$
$6 - 3 = 3$
$10 - 6 = 4$
…
general rule? proof?

(3)
$1 \times 3 = 3 \times 1$
$2 \times 6 = 4 \times 3$
$3 \times 10 = 5 \times 6$
…
general rule? proof?

(4)
$8 \times 1 + 1 = 3 \times 3$
$8 \times 3 + 1 = 5 \times 5$
$8 \times 6 + 1 = 7 \times 7$
…
general rule? proof?

(5)
$3 - 2 \times 1 = 1$
$10 - 2 \times 3 = 4$
$21 - 2 \times 6 = 9$
…
general rule? proof?

(6)
$3 \times 3 + 1 = 10$
$3 \times 6 + 3 = 21$
$3 \times 10 + 6 = 36$
…
general rule? proof?

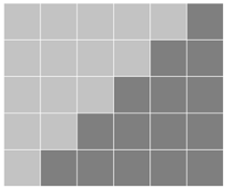
(7)
$3 \times 3 + 6 = 15$
$3 \times 6 + 10 = 28$
$3 \times 10 + 15 = 45$
…
general rule? proof?

(8)
$9 \times 1 + 1 = 10$
$9 \times 3 + 1 = 28$
$9 \times 6 + 1 = 55$
…
general rule? proof?

(9)
$1^2 + 3^2 = 10$
$3^2 + 6^2 = 45$
$6^2 + 10^2 = 136$
…
general rule? proof?

(10)
$3^2 - 1^2 = 8$
$6^2 - 3^2 = 27$
$10^2 - 6^2 = 64$
…
general rule? proof?

$T_n = \frac{1}{2} n (n + 1)$



n	T_n
1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36
9	45
10	55
11	66
12	78
13	91
14	105
15	120
16	136
17	153
18	171
19	190
20	210
21	231
22	253
23	276
24	300
25	325
26	351
27	378
28	406
29	435

Fluency Practice

consecutive numbers – setting up and using an equations to solve a problem

find three consecutive numbers that add up to 45

find four consecutive numbers that add up to 38

find five consecutive numbers that add up to 60

any number is **n**; the next number is **n + 1**

any even number is **2n**; the next even number is **2n + 2** etc

find three consecutive even numbers that add up to 42

find four consecutive even numbers that add up to 60

find five consecutive even numbers that add up to 60

find three consecutive odd numbers that add up to 57

find four consecutive odd numbers that add up to 48

find even numbers of consecutive odd numbers that add up to 72

any odd number is **2n + 1**; the next odd number is **2n + 3** etc

any multiple of 4 is **4n**; the next multiple of 4 is **4n + 4** etc

find three consecutive multiples of 3 that add up to 63

find four consecutive multiples of 5 that add up to 150

find five consecutive multiples of 2 that add up to 94

1 Write down algebraic expressions for

(a) an even number

(b) an odd number

(c) a multiple of four

(d) a positive number which leaves a remainder of 1 upon division by 5

(e) the sum of two consecutive even numbers

(f) the sum of two even numbers

(g) the product of two odd numbers

(e) the cube of a multiple of 6

(h) the sum of the squares of two consecutive multiples of 3

2 Prove algebraically that

(a) the sum of two consecutive numbers is odd

(b) the product of two even numbers is even

(c) the sum of any three consecutive numbers is a multiple of 3

3 (a) Prove algebraically that the sum of the squares of two consecutive odd numbers is **not** a multiple of 4.

(b) What is the remainder when the sum of the squares of two consecutive odd numbers is divided by 4? Is the number odd or even?

4 Johnny has an even number. He adds 3 to this number and then squares it.
The result is 81.
What even number did Johnny start with?

5 Prove algebraically that $(5n + 1)^2 - (5n - 1)^2$ is both a multiple of 4 and a multiple of 5 for positive integers n.

6 Anne has a number which is not a multiple of 3.

(a) Explain why it must be of the form $(3n + 1)$ or $(3n + 2)$, where n is an integer.

(b) Hence, by considering these two different cases, show that the square of Anne's number leaves a remainder of 1 when divided by 3.

7 (a) Prove that if n is an even integer, then $3n^2 + n + 14$ is even.

(b) Prove that if n is an odd integer, then $3n^2 + n + 14$ is even.

(c) What can you deduce about the number $3n^2 + n + 14$ for all integers n?

Negative…. when? x = any number

Can these expressions have a negative value?

For each expression, test values & *limits* for x.

(hint: factorising the expression, or a part of it may help)

x^2

$x^2 - 4$

$x^2 + x$

$x^2 + 2x$

$x^2 - 3x$

$x^2 + 2x + 1$

$x^2 + 2x + 1$

Question 1: Prove the following

(a) $(n+7)^2-(n+1)^2=12(n+4)$ (b) $(n+1)^2-(n-3)^2=8(n-1)$

(c) $(n+1)^2+(n+5)^2-(n+9)^2=(n+5)(n-11)$

Question 2: Prove the following

(a) $(n+4)^2-(n+2)^2$ is always a multiple of 4 for all positive integer values of n.

(b) $(n+10)^2-(n+2)^2$ is always a multiple of 16 for all positive integer values of n.

(c) $(2n+3)^3-(2n+1)$ is always even for all positive integer values of n.

(d) $(5n+2)^2-(5n-1)^2$ is always a multiple of 3 for all positive integer values of n.

(e) $(2n+9)^2-(2n+5)^2$ is always a multiple of 4 for all positive integer values of n.

(f) $(n+2)^2-(n-2)^2+3$ is always odd for all positive integer values of n.

Question 3: n is a positive integer.

(a) Write an expression for an even number.
(b) Write an expression of an odd number.

Question 4: Prove the following

(a) The sum of any three consecutive integers is divisible by 3.
(b) The sum of any three consecutive even numbers is always a multiple of 6.
(c) The sum of two consecutive odd numbers is even.
(d) The sum of three consecutive odd numbers is always a multiple of 3.
(e) The sum of four consecutive odd numbers is always a multiple of 8.
(f) The sum of two consecutive integers is always odd.
(g) The sum of four consecutive integers is **not** a multiple of 4

Question 5: Prove the following

(a) Prove the product of two odd numbers is always odd.

(b) Prove the product of two even consecutive numbers is always a multiple of 4.

(c) The difference between the squares of any two consecutive integers is equal to the sum of the two integers.

(d) Prove the sum of the squares of any two consecutive even numbers is always a multiple of 4.

(e) Prove that when any odd integer is squared, the result is always one more than a multiple of 8.

Apply

Question 1: The first five terms of a linear sequence are 2, 7, 12, 17, 22

(a) Find the nth term of the sequence

A new sequence is generated by squaring each term of the linear sequence and then adding 1.

(b) Prove that all terms in the new sequence are divisible by 5.

Question 2: The first two terms of a fibonacci sequence are ***a*** and ***b***.

(a) Show the 4th term of the sequence is ***a+2b***
(b) Prove that the sum of the first 10 terms is equal to 11 times the 7th term.

Question 3: Cara writes down a 3-digit number where the first digit is greater than the last. e.g. 681
She then reverses the number to give 186.
Cara then subtracts this number from her starting number. 681 – 186 = 495
She then reverses her answer to give 594.
Cara then adds these number 495 + 594 = 1089.

$$\begin{array}{r} 681 \\ -186 \\ \hline 495 \end{array} \qquad \begin{array}{r} 594 \\ +495 \\ \hline 1089 \end{array}$$

Cara repeats this several times and always gets 1089 as her answer.
Prove algebraically that the answer is always 1089.

Fluency Practice

$8(ax + 2) \equiv 24x + 16$	$a =$	
$5(2x + a) \equiv 10x + 45$	$a =$	
$3(9x + 1) \equiv 27x + a$	$a =$	
$6(5 - x) \equiv 30 - ax$	$a =$	
$4(7x + 2) + a \equiv 28x + 10$	$a =$	
$5(3x + 4) + ax \equiv 19x + 20$	$a =$	
$2x + 9 + ax - b \equiv 10x + 6$	$a =$	$b =$
$a(7x - 3) \equiv 35x - 15$	$a =$	
$2(ax + 5) + 3(2x + b) \equiv 18x + 13$	$a =$	$b =$
$9(ax + b) + 4(x - 2) \equiv 13x + 37$	$a =$	$b =$

1.	$2(3x+8)+6(x-5) \equiv ax+b$
2.	$10x(x-3)-3x(2x-2) \equiv cx^2+dx$
3.	$6(2x+1)+2(4x-1)+10 \equiv ex+f$
4.	$6(x+3)+g(2x+10) \equiv 14x+h$
5.	$(x+7)(x-6)+3 \equiv x^2+jx+k$
6.	$(x+3)(x+n)+mx^2+5x \equiv 4x^2+16x+p$
7.	$(x+5)(x-2)+qx(2x+8) \equiv 9x^2+35x-10$
8.	$(x+r)(x-s)+2x+15 \equiv x^2+8x-25$

Answers

a	b	c	d	e	f	g	h

j	k	m	n	p	q	r	s

Fluency Practice

1	$(2x+3)(x-5)+7 \equiv ax^2+bx+c$	$a =$ $b =$ $c =$
2	$(x+1)(x+4)(x-1) \equiv x^3+ax^2+bx+c$	$a =$ $b =$ $c =$
3	$2x^2+12x-9 \equiv a(x+b)^2+c$	$a =$ $b =$ $c =$
4	$(ax+1)(x+c)+b \equiv 5x^2+11x+8$	$a =$ $b =$ $c =$
5	$3(ax+1)-(6x+b) \equiv 21x-8$	$a =$ $b =$
6	$(ax+5)(x-b)+4x \equiv 2x^2+cx-10$	$a =$ $b =$ $c =$
7	$4(3x+a)+5(ax+b) \equiv 22x+28$	$a =$ $b =$
8	$4x^2+ax+3 \equiv b(x+1)^2+c$	$a =$ $b =$ $c =$
9	$\dfrac{ax^2+x-b}{x+2} \equiv x-1$	$a =$ $b =$
10	$2(5x+1)(3x+b)+ax(2x+3) \equiv 44x^2+37x+c$	$a =$ $b =$ $c =$

Fluency Practice

Find the value of each letter in the identities below. Circle the answers at the bottom as you find them.

$8(ax+2) \equiv 24x+16$ $a =$ ☐	$5(2x+b) \equiv 10x+45$ $b =$ ☐	$c(9x+d) \equiv 18x+26$ $c =$ ☐, $d =$ ☐	$e(2+fx) \equiv 24x-8$ $e =$ ☐, $f =$ ☐
$6(3x+5)+g \equiv 18x+44$ $g =$ ☐	$4(5x-3)+hx \equiv 13x-12$ $h =$ ☐	$i(8x-3) \equiv 4x+j$ $i =$ ☐, $j =$ ☐	$12(6x+k)-lx \equiv 71x+4$ $k =$ ☐, $l =$ ☐
$(x+m)(x-2) \equiv x^2+5x-14$ $m =$ ☐	$(x+4)(x-7) \equiv x^2+nx-28$ $n =$ ☐	$(x+\sigma)(x+20) \equiv x^2+px-160$ $\sigma =$ ☐, $p =$ ☐	$(2x+1)(x-5) \equiv 2x^2+rx+q$ $q =$ ☐, $r =$ ☐
$(x-s)^2+10 \equiv x^2-10x+35$ $s =$ ☐	$(x+3)^2-3 \equiv x^2+tx+t$ $t =$ ☐	$(x+u)^2-v \equiv x^2+22x+71$ $u =$ ☐, $v =$ ☐	$x^2-5x+6 \equiv (x+w)^2-y$ $w =$ ☐, $y =$ ☐
$\frac{\alpha x-8}{4x+\beta} \equiv 4$ $\alpha =$ ☐, $\beta =$ ☐	$\frac{x^2+\gamma x+44}{x+11} \equiv x+\delta$ $\gamma =$ ☐, $\delta =$ ☐	$(x+5)^2(x-2) \equiv x^3+\epsilon x^2+5x+\zeta$ $\epsilon =$ ☐, $\zeta =$ ☐	$(x-\eta)(x+8) \equiv (x+\theta)^2-81$ $\eta =$ ☐, $\theta =$ ☐

Answers (One of the questions has multiple solutions. Which one? What are the other possible solutions?)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
-1	-2	-3	-4	-5	-6	-7	-8	-9	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$-1\frac{1}{2}$	$-2\frac{1}{2}$	-50	50

Fluency Practice

Find the value of each letter in the identities below. Circle the answers at the bottom as you find them.

$\frac{ax+12}{4x+6} \equiv 2$ $a =$ ☐	$\frac{7x+b}{21x-12} \equiv \frac{3x+5}{9x+15}$ $b =$ ☐	$\frac{cx-10}{18x-12} \equiv \frac{35x-5}{42x+d}$ $c =$ ☐, $d =$ ☐	$\frac{12-9x}{6x+e} \equiv f$ $e =$ ☐, $f =$ ☐
$\frac{x}{3} - \frac{x}{g} \equiv \frac{x}{12}$ $g =$ ☐	$\frac{x+1}{7} - \frac{x}{2} \equiv \frac{hx+20}{140}$ $h =$ ☐	$\frac{2x+i}{5} + \frac{x+5}{3} \equiv \frac{jx+22}{15}$ $i =$ ☐, $j =$ ☐	$\frac{kx-8}{6} - \frac{4x-11}{8} \equiv \frac{l}{12}$ $k =$ ☐, $l =$ ☐
$\frac{x^2+mx}{x^2+7x+10} \equiv \frac{x}{x+2}$ $m =$ ☐	$\frac{x^2-x-12}{x^2-9x+20} \equiv \frac{x+3}{x+n}$ $n =$ ☐	$\frac{x+4}{x^2-\sigma} \equiv \frac{p}{x-4}$ $\sigma =$ ☐, $p =$ ☐	$\frac{x^2+qx+r}{x^2+x-20} \equiv \frac{x^2+3x-18}{x^2+11x+30}$ $q =$ ☐, $r =$ ☐
$\frac{3x^2-sx-5}{x-5} \equiv 3x+1$ $s =$ ☐	$\frac{3x^2+tx}{2x^2-18} \equiv \frac{3x}{2x-6}$ $t =$ ☐	$\frac{x^2+ux-18}{2x^2-x-6} \equiv \frac{x+9}{vx+3}$ $u =$ ☐, $v =$ ☐	$\frac{wx^2+12x-1}{26x^2+11x-1} \equiv \frac{11x^2+yx-1}{22x^2+9x-1}$ $w =$ ☐, $y =$ ☐
$\frac{5}{x+3} - \frac{4}{x+\alpha} = \frac{x-27}{x^2+\beta}$ $\alpha =$ ☐, $\beta =$ ☐	$\frac{\gamma}{x+4} - \frac{x-44}{x^2-16} \equiv \frac{\delta}{10x-40}$ $\gamma =$ ☐, $\delta =$ ☐	$\frac{6\zeta}{5-x} + \frac{3}{x+7} \equiv \frac{29-x}{14\epsilon-2x-x^2}$ $\epsilon =$ ☐, $\zeta =$ ☐	$\frac{1}{x} + \frac{1}{x+1} + \frac{\eta}{x+2} \equiv \frac{12\theta x+2}{x^3+3x^2+2x}$ $\eta =$ ☐, $\theta =$ ☐

Answers

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
-1	-2	-3	-4	-5	-6	-7	-8	-9	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$-1\frac{1}{2}$	$-2\frac{1}{2}$	-50	50

Fluency Practice

versions

find the values of the letters in these (equivalent) 'families' of quadratic expressions

(1) $(x + 2)(x + 6) + 3$

$(x + 7)(x + a) + b$

$(x + c)^2 - d$

$(x + 1)(x + 15) - ex$

$(x + f)(x + g)$

(2) $(x - 8)(x + 1) + 14$

$(x - 2)(x - a) - b$

$(x - 4)(x - c) - d$

$(x - 3)(x - 2) - ex$

$(x - f)(x - g)$

(3) $(x - 2)(x - 10) + 15$

$(x + 3)(x - a) + b$

$(x - c)^2 - d$

$(x + 5)(x + 7) - ex$

$(x - f)(x - g)$

(4) $(x - 3)(x - 5) - 35$

$(x - 11)(x + a) + b$

$(x - c)^2 - d$

$x^2 - 4(2x + e)$

$(x + f)(x - g)$

(5) $(2x + 3)(x + 8) + 11$

$(2x + 7)(x + a) - b$

$(x + 9)(2x + c) + d$

$(x - 5)(2x - 7) + ex$

$(2x + f)(x + g)$

(6) $(2x + 1)(x + 2) - 14$

$(2x - 1)(x + a) - b$

$(x + 6)(2x - c) + d$

$2(x - 3)(x + 2) + ex$

$(2x - f)(x + g)$

Prove it!

Prove that $(a + b)^2$ is not equal to $a^2 + b^2$

Show that $5a + 2a + 4 + 3$ is always a multiple of 7

$n - 1 + n + 1$ will be even. Why?

The sum of three consecutive numbers is 3 times the middle number

(hint – call your first number a, your second number a + 1)

The sum of two consecutive odd numbers is always even

For any three consecutive numbers, the difference between the square of the middle number and the product of the largest and smallest is always 1

Show that $5(2x + 5) + 11(x - 2) = 3(7x + 1)$

Fluency Practice

from exercises in algebra part three by HF Browne (1936)

solve the equations

$$\frac{2}{x} + \frac{3}{y} = 4$$

$$\frac{5}{x} + \frac{6}{y} = 7$$

find two numbers such that
the sum of their squares is
five times their sum

and

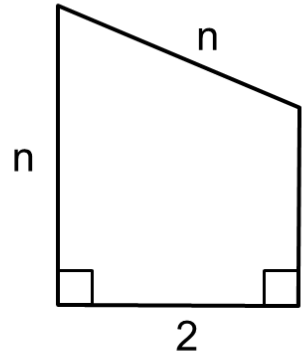
the difference of their squares
is nine times their difference

the perimeter of a
right-angled triangle is 40 cm
and the area is 60 cm^2

find the lengths of the sides

the area of the
right trapezium
is 4

find n



Fluency Practice

Number and Algebra Proof Revision			
(a)	**(b)**	**(c)**	**(d)**
Show that $8\frac{1}{2} - 3\frac{2}{3} = 4\frac{5}{6}$	Show that $\sqrt{80}$ can be written in the form $k\sqrt{5}$ where k is an integer to be found	Show that $0.\dot{7}\dot{5} = \frac{25}{33}$	Show that $3\frac{5}{8} \div 1\frac{5}{6} = 1\frac{43}{44}$
(e)	**(f)**	**(g)**	**(h)**
Show that $0.3\dot{1} = \frac{14}{45}$	Show that $0.4\dot{2}\dot{7} = \frac{47}{110}$	Show that $(7 - 5\sqrt{3})^2 = a + b\sqrt{3}$ where a and b are integers to be found	Show that the product of an even number and an odd number is always even.

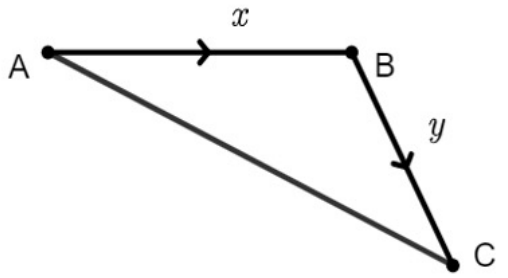
(i)	**(j)**	**(k)**
Show that the sum of three consecutive odd numbers is always a multiple of 3	Show that $\frac{3\sqrt{12}}{2-\sqrt{3}}$ can be written in the form $c + d\sqrt{3}$, where c and d are integers to be found.	Show that $(3n + 4)(n - 3) + n(n - 3)$ is a multiple of 4 for all integer values of n

7 Advanced Vectors

Fluency Practice

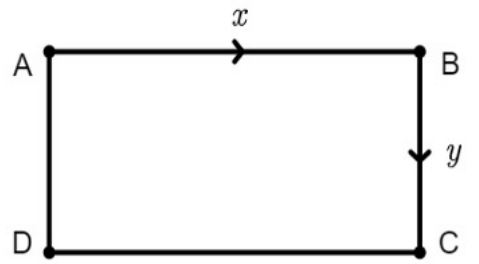
1. ABC is a triangle. Find, in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$:

$(a)\ \overrightarrow{BA}\ \ (b)\ \overrightarrow{CB}$
$(c)\ \overrightarrow{AC}\ (d)\ \overrightarrow{CA}$



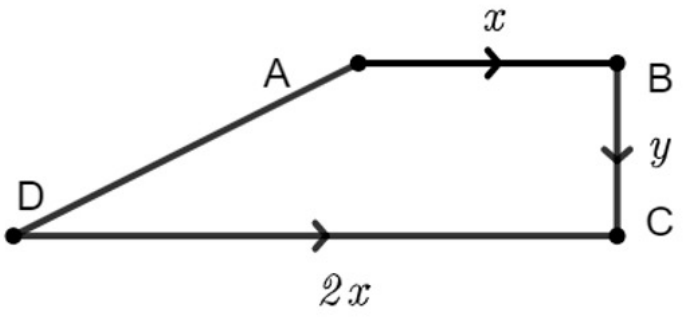
2. ABCD is a rectangle. Find, in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$:

$(a)\ \overrightarrow{DA}\ \ (b)\ \overrightarrow{AC}$
$(c)\ \overrightarrow{CA}\ (d)\ \overrightarrow{BD}$



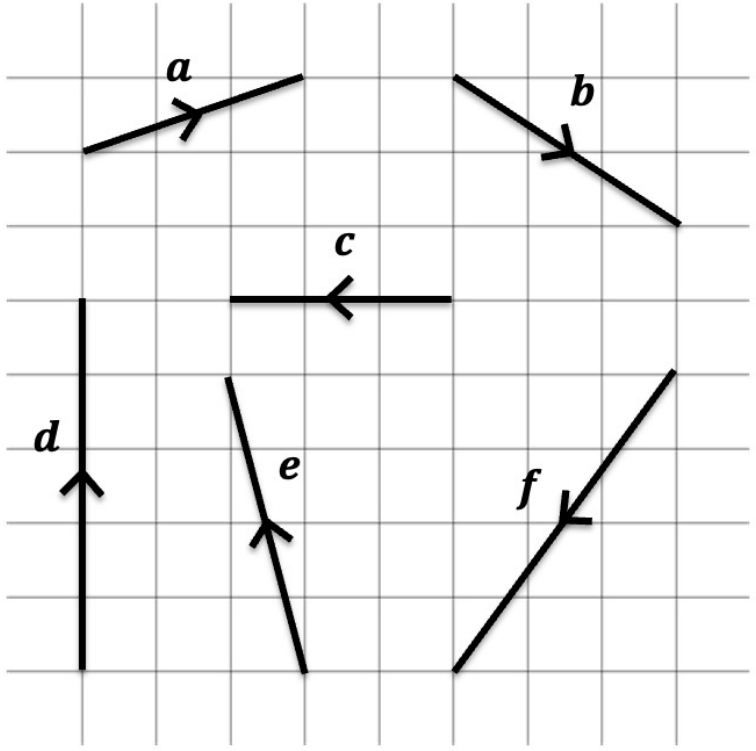
3. ABCD is a trapezium. Find, in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$:

$(a)\ \overrightarrow{AC}\ \ (b)\ \overrightarrow{DB}$
$(c)\ \overrightarrow{AD}\ (d)\ \overrightarrow{DA}$



Fluency Practice

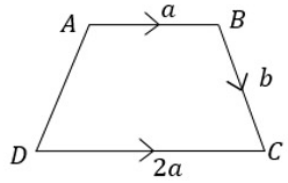
1 State the value of each vector.



$a =$? $b =$? $c =$?

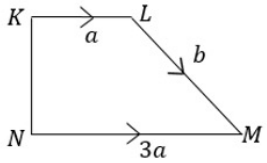
$d =$? $e =$? $f =$?

2



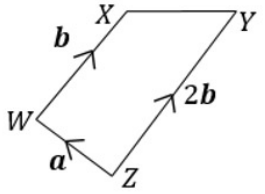
a. $\overrightarrow{BA} =$?
b. $\overrightarrow{AC} =$?
c. $\overrightarrow{DB} =$?
d. $\overrightarrow{AD} =$?

3



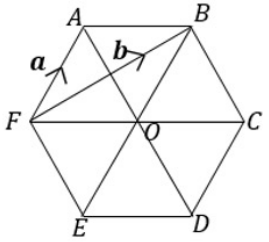
a. $\overrightarrow{MK} =$?
b. $\overrightarrow{NL} =$?
c. $\overrightarrow{NK} =$?
d. $\overrightarrow{KN} =$?

4



a. $\overrightarrow{ZX} =$?
b. $\overrightarrow{YW} =$?
c. $\overrightarrow{XY} =$?
d. $\overrightarrow{XZ} =$?

5



a. $\overrightarrow{AB} =$?
b. $\overrightarrow{FO} =$?
c. $\overrightarrow{AO} =$?
d. $\overrightarrow{FD} =$?

Fluency Practice

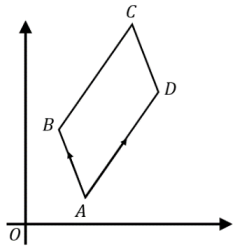
Vectors and Coordinates

(a)

$ABCD$ is a parallelogram.

$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$

Given that the coordinates of A are $(3, 1)$, find the coordinates of points B, C and D.

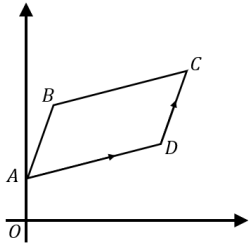


(b)

$ABCD$ is a parallelogram.

$\overrightarrow{AD} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$ and $\overrightarrow{DC} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

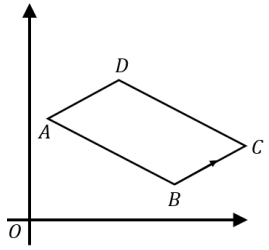
Given that the coordinates of A are $(0, 1)$, find the coordinates of points B, C and D.



(c)

$ABCD$ is a parallelogram. $\overrightarrow{BC} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

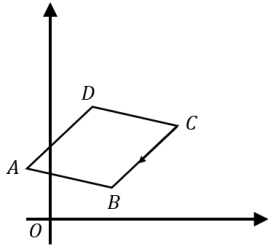
The coordinates of A are $(2, 7)$ and of B are $(8, 3)$. Find the coordinates of C and D, and the vector $\overrightarrow{DC}$.



(d)

$ABCD$ is a rhombus. $\overrightarrow{CB} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

The coordinates of A are $(-1, 4)$ and of B are $(3, 1)$. Find the coordinates of C and D, and the vector $\overrightarrow{DC}$.

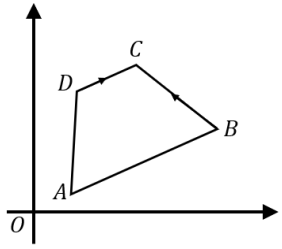


(e)

$ABCD$ is a trapezium. $\overrightarrow{AB} = 2\overrightarrow{DC}$.

$\overrightarrow{DC} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

The coordinates of D are $(2, 8)$. Find the coordinates of A, and the vector $\overrightarrow{AD}$.

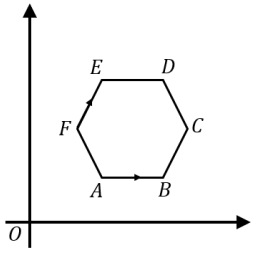


(f)

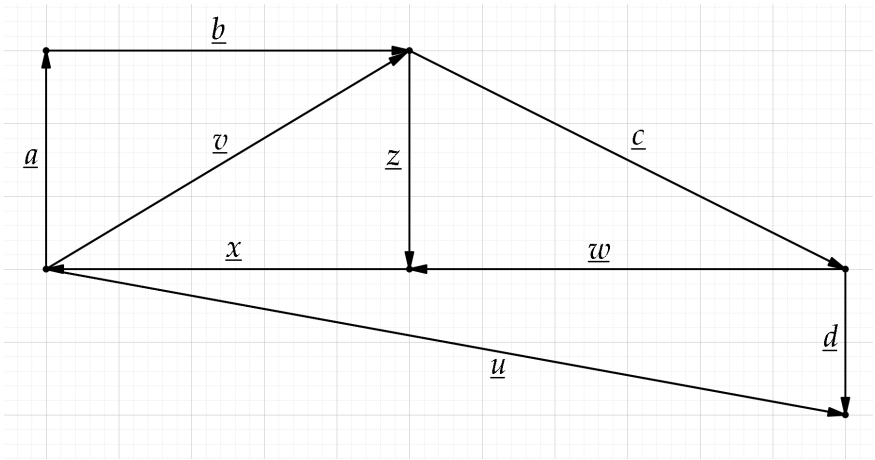
$ABCDEF$ is a regular hexagon.

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ and $\overrightarrow{FE} = \begin{pmatrix} 2 \\ 2\sqrt{3} \end{pmatrix}$

$\overrightarrow{FC} = 2\overrightarrow{AB}$. The coordinates of A are $(5, 2)$. Find the coordinates of B, C and D.



1. Based on the picture, determine whether each of the given statements is true or false.



a) $\underline{v} = \underline{a} + \underline{b}$

b) $\underline{v} = \underline{z} + \underline{x}$

c) $\underline{v} = \underline{x} - \underline{z}$

d) $\underline{c} = \underline{d} - \underline{u} + \underline{v}$

e) $\underline{a} = \underline{z}$

f) $\underline{w} + \underline{x} + \underline{u} = \underline{d}$

g) $\underline{d} + \underline{u} = \underline{x} + \underline{w}$

h) $\underline{w} - \underline{c} = \underline{x} - \underline{v}$

i) $\underline{a} + \underline{b} + \underline{z} + \underline{x} = \underline{0}$

j) $\underline{a} + \underline{b} + \underline{c} + \underline{w} + \underline{x} = \underline{0}$

k) $\underline{x} + \underline{w} + \underline{v} = -\underline{c}$

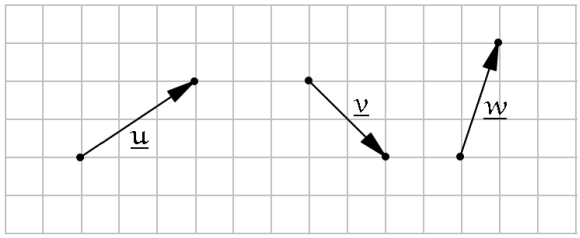
l) $\underline{v} + \underline{c} + \underline{d} = -\underline{u}$

m) $\underline{a} + \underline{z} = \underline{0}$

n) $\underline{a} - \underline{b} = -\underline{v}$

o) $\underline{z} - \underline{w} = \underline{c}$

2. Given the vectors $\underline{u}$, $\underline{v}$ and $\underline{w}$, draw each of the following vectors.

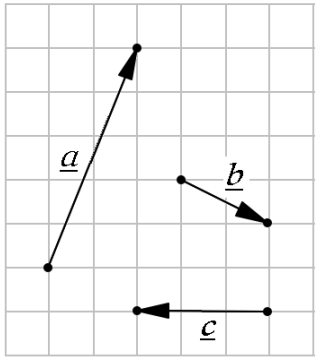


a) $\underline{u} + \underline{v}$

b) $\underline{v} - \underline{w}$

c) $2\underline{u} - \underline{v} + \underline{w}$

3. Given the vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$, draw each of the following vectors.



a) $\underline{a} + \underline{b}$

b) $3\underline{b}$

c) $\underline{b} - \underline{a}$

d) $\underline{a} + \underline{b} + \underline{c}$

e) $\underline{a} - 2\underline{b} + \underline{c}$

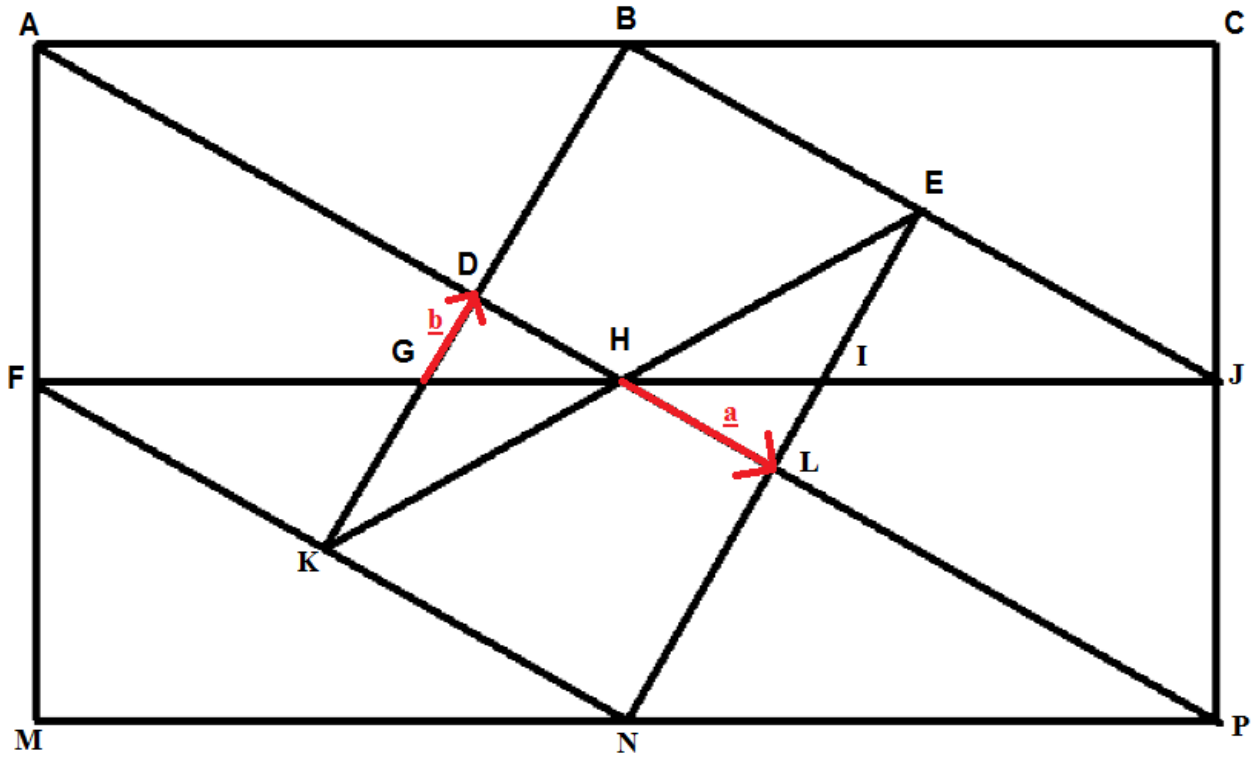
f) $-2\underline{c}$

g) $-2\underline{a} + 3\underline{b} - \underline{c}$

Fluency Practice

4. Suppose that M is the midpoint of line segment AB. Let O be a point in the plane of AB. Let $\underline{a}$ denote the vector pointing from O to A, let $\underline{b}$ denote the vector pointing from O to B, and $\underline{m}$ denote the vector pointing from O to M. Prove that $\underline{m} = \dfrac{\underline{a} + \underline{b}}{2}$. (Hint: Let $\underline{x}$ be the vector pointing from A to M. Express first $\underline{x}$ in tems of $\underline{a}$ and $\underline{b}$. $\underline{m} = \underline{a} + \underline{x}$).

Purposeful Practice

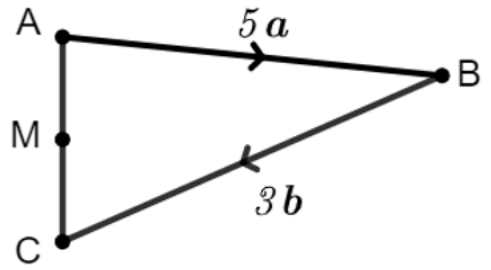


If HL = **a** and GD = **b** how many other vectors can you define in terms of **a** and **b**?

Fluency Practice

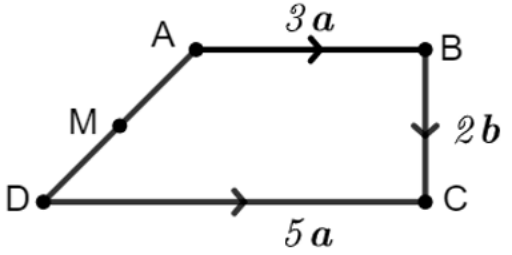
1. ABC is a triangle. M is the midpoint of AC. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

   $(a)\ \overrightarrow{AC}\ \ (b)\ \overrightarrow{CA}$
   $(c)\ \overrightarrow{AM}\ (d)\ \overrightarrow{BM}$



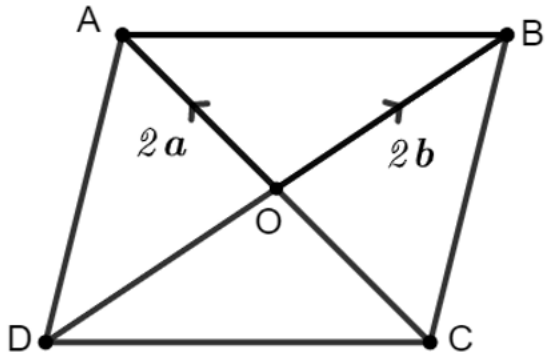
2. ABCD is a trapezium. M is the midpoint of AD. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

   $(a)\ \overrightarrow{DA}\ \ (b)\ \overrightarrow{DM}$
   $(c)\ \overrightarrow{AD}\ (d)\ \overrightarrow{MC}$



3. ABCD is a parallelogram. Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

   $(a)\ \overrightarrow{OC}\ \ (b)\ \overrightarrow{AC}$
   $(c)\ \overrightarrow{AB}\ (d)\ \overrightarrow{AD}$



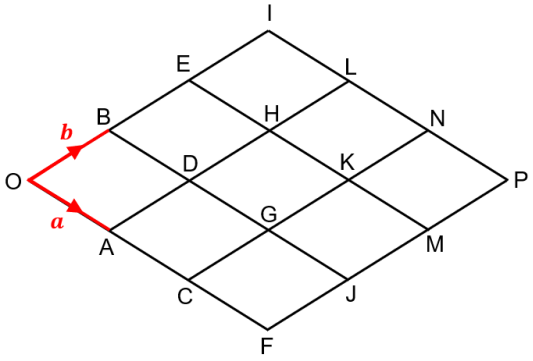
Fluency Practice

Defining Vectors

(a)

The diagram is made up of nine congruent rhombuses.

$\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$.



Define the following vectors in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $\overrightarrow{OE}$

(b) $\overrightarrow{OF}$

(c) $\overrightarrow{GJ}$

(d) $\overrightarrow{MP}$

(e) $\overrightarrow{AO}$

(f) $\overrightarrow{LD}$

(g) $\overrightarrow{OH}$

(h) $\overrightarrow{DP}$

(i) $\overrightarrow{IM}$

(j) $\overrightarrow{MD}$

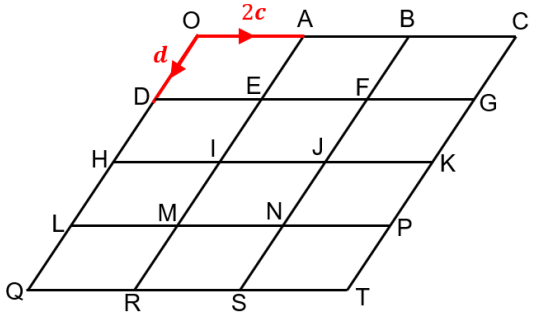
(k) $\overrightarrow{CB}$

(l) $\overrightarrow{NO}$

(b)

The diagram is made up of twelve congruent parallelograms.

$\overrightarrow{OA} = 2\boldsymbol{c}$ and $\overrightarrow{OD} = \boldsymbol{d}$.



Define the following vectors in terms of $\boldsymbol{c}$ and $\boldsymbol{d}$.

(a) $\overrightarrow{EG}$

(b) $\overrightarrow{BS}$

(c) $\overrightarrow{KJ}$

(d) $\overrightarrow{RE}$

(e) $\overrightarrow{OF}$

(f) $\overrightarrow{JT}$

(g) $\overrightarrow{MJ}$

(h) $\overrightarrow{QF}$

(i) $\overrightarrow{IL}$

(j) $\overrightarrow{HB}$

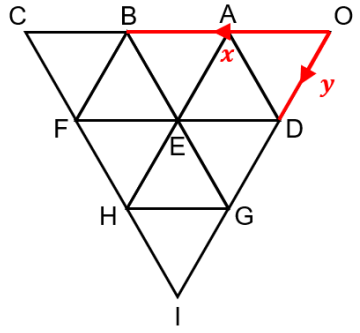
(k) $\overrightarrow{TE}$

(l) $\overrightarrow{KD}$

(c)

The diagram is made up of nine congruent equilateral triangles.

$\overrightarrow{OB} = \boldsymbol{x}$ and $\overrightarrow{OD} = \boldsymbol{y}$.



Define the following vectors in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

(a) $\overrightarrow{FD}$

(b) $\overrightarrow{HA}$

(c) $\overrightarrow{GH}$

(d) $\overrightarrow{OC}$

(e) $\overrightarrow{DB}$

(f) $\overrightarrow{OE}$

(g) $\overrightarrow{FO}$

(h) $\overrightarrow{CE}$

(i) $\overrightarrow{HI}$

(j) $\overrightarrow{BG}$

(k) $\overrightarrow{HD}$

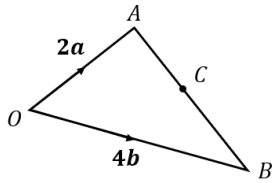
(l) $\overrightarrow{IF}$

Fluency Practice

Vectors and Midpoints

(a)

In the triangle OAB, $\overrightarrow{OA} = 2\boldsymbol{a}$ and $\overrightarrow{OB} = 4\boldsymbol{b}$. C is the midpoint of the line AB.



Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

(a) $\overrightarrow{AB}$

(b) $\overrightarrow{BA}$

(c) $\overrightarrow{AC}$

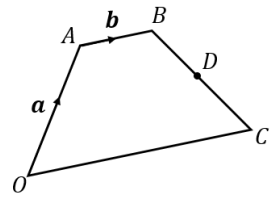
(d) $\overrightarrow{BC}$

(e) $\overrightarrow{OC}$

(f) $\overrightarrow{CO}$

(b)

$OABC$ is a trapezium, where $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{AB} = \boldsymbol{b}$. D is the midpoint of BC and $\overrightarrow{OC} = 2\overrightarrow{AB}$.



Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

(a) $\overrightarrow{OC}$

(b) $\overrightarrow{CB}$

(c) $\overrightarrow{BC}$

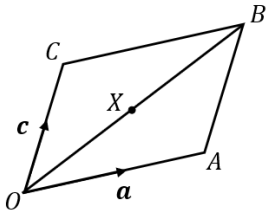
(d) $\overrightarrow{BD}$

(e) $\overrightarrow{AD}$

(f) $\overrightarrow{DO}$

(c)

In the parallelogram $OABC$, $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{c}$.
X is the midpoint of the line OB.



Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$:

(a) $\overrightarrow{CB}$

(b) $\overrightarrow{BA}$

(c) $\overrightarrow{OB}$

(d) $\overrightarrow{XB}$

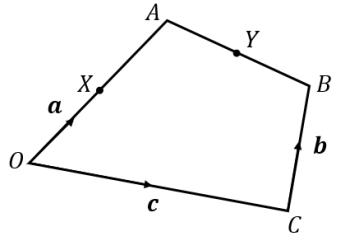
(e) $\overrightarrow{XC}$

(f) $\overrightarrow{AX}$

What do the answers to (e) and (f) tell us about the points C, X and A?

(d)

$OABC$ is a quadrilateral.
$\overrightarrow{OX} = \boldsymbol{a}$, $\overrightarrow{OC} = \boldsymbol{c}$ and $\overrightarrow{CB} = \boldsymbol{b}$.
X is the midpoint of OA and Y is the midpoint of AB.



Express the following in terms of $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$:

(a) $\overrightarrow{OA}$

(b) $\overrightarrow{OB}$

(c) $\overrightarrow{CA}$

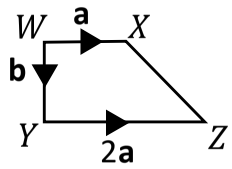
(d) $\overrightarrow{AB}$

(e) $\overrightarrow{AY}$

(f) $\overrightarrow{XY}$

What do the answers to (b) and (f) tell us about vectors $\overrightarrow{OB}$ and $\overrightarrow{XY}$?

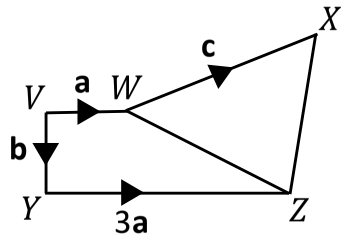
Vectors Around Shapes



1) $\overrightarrow{WX}$ = $\overrightarrow{WY}$ =

$\overrightarrow{YZ}$ = $\overrightarrow{WZ}$ =

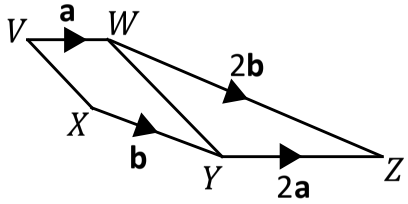
$\overrightarrow{XY}$ = $\overrightarrow{XZ}$ =



2) $\overrightarrow{VX}$ = $\overrightarrow{WV}$ =

$\overrightarrow{XV}$ = $\overrightarrow{VZ}$ =

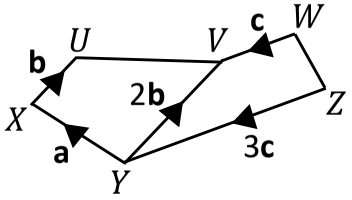
$\overrightarrow{WZ}$ = $\overrightarrow{XZ}$ =



3) $\overrightarrow{VZ}$ = $\overrightarrow{VY}$ =

$\overrightarrow{XZ}$ = $\overrightarrow{WY}$ =

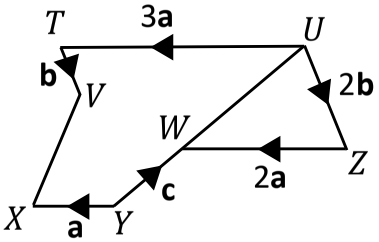
$\overrightarrow{YW}$ = $\overrightarrow{XV}$ =



4) $\overrightarrow{YZ}$ = $\overrightarrow{VX}$ =

$\overrightarrow{ZW}$ = $\overrightarrow{UY}$ =

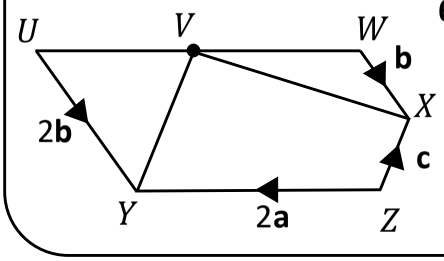
$\overrightarrow{UZ}$ = $\overrightarrow{UW}$ =



5) $\overrightarrow{VZ}$ = $\overrightarrow{WT}$ =

$\overrightarrow{ZX}$ = $\overrightarrow{XZ}$ =

$\overrightarrow{YT}$ = $\overrightarrow{XV}$ =



6) (V is the midpoint of UW)

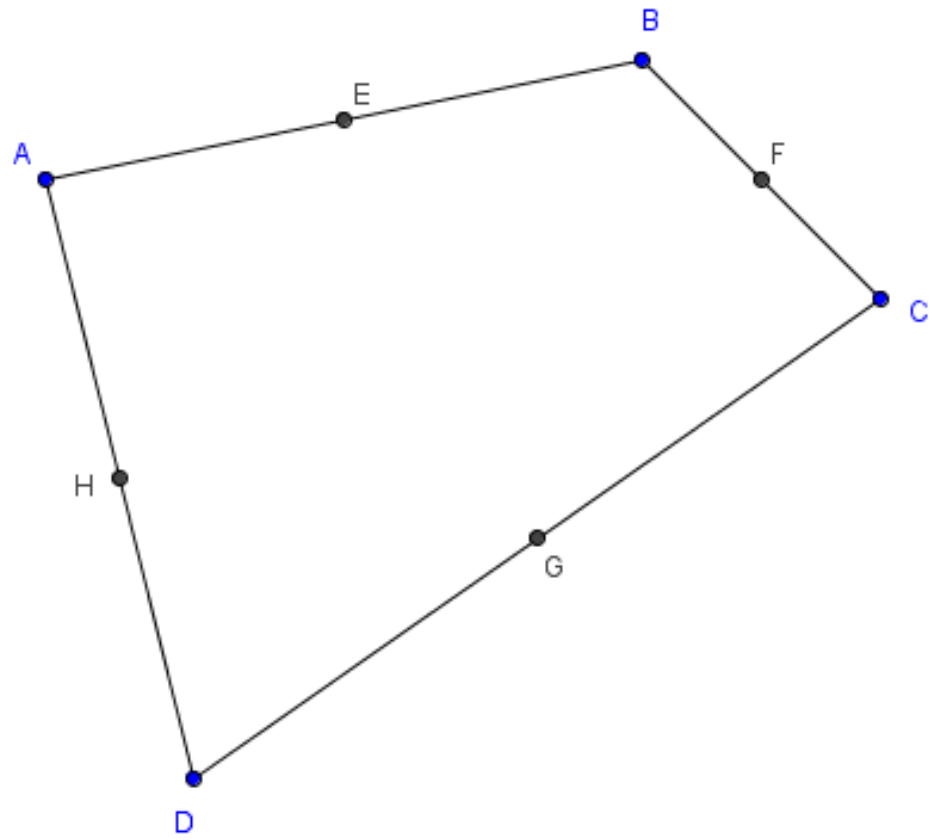
$\overrightarrow{WU}$ = $\overrightarrow{WV}$ =

$\overrightarrow{UW}$ = $\overrightarrow{UV}$ =

$\overrightarrow{XV}$ = $\overrightarrow{YV}$ =

Fluency Practice

The diagram shows the quadrilateral ABCD.
The point E is the midpoint of A and B. The point F is the midpoint of B and C.
The point G is the midpoint of C and D. The point H is the midpoint of D and A.



$\overrightarrow{AB} = \boldsymbol{a}$ $\overrightarrow{BC} = \boldsymbol{b}$ $\overrightarrow{CD} = \boldsymbol{c}$

1. Express in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and/or $\boldsymbol{c}$:

$\overrightarrow{DC} =$

$\overrightarrow{AD} =$

2. Express in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and/or $\boldsymbol{c}$:

$\overrightarrow{HE} =$

$\overrightarrow{GF} =$

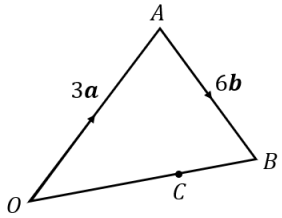
What do you notice?

Fluency Practice

Vectors and Ratio

(a)

In the triangle OAB, $\overrightarrow{OA} = 3\boldsymbol{a}$ and $\overrightarrow{AB} = 6\boldsymbol{b}$. The point C divides the line OB in the ratio $2:1$.



Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

(a) $\overrightarrow{OB}$

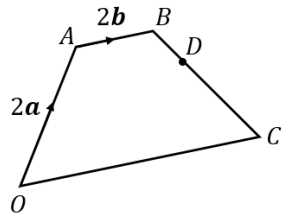
(b) $\overrightarrow{OC}$

(c) $\overrightarrow{BC}$

(d) $\overrightarrow{AC}$

(b)

$OABC$ is a trapezium, where $\overrightarrow{OA} = 2\boldsymbol{a}$ and $\overrightarrow{AB} = 2\boldsymbol{b}$. $\overrightarrow{OC} = 2\overrightarrow{AB}$ and D splits the line BC in the ratio $1:3$.



Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

(a) $\overrightarrow{BO}$

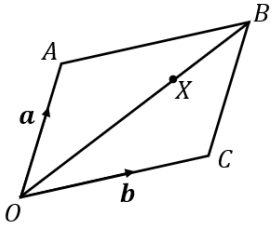
(b) $\overrightarrow{BC}$

(c) $\overrightarrow{BD}$

(d) $\overrightarrow{DO}$

(c)

In the parallelogram $OABC$, $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OC} = \boldsymbol{b}$. The point X divides the line OB in the ratio $3:2$.



Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$:

(a) $\overrightarrow{AB}$

(b) $\overrightarrow{OB}$

(c) $\overrightarrow{OX}$

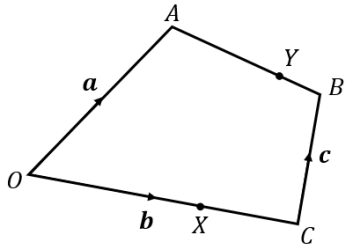
(d) $\overrightarrow{BX}$

(e) $\overrightarrow{AX}$

(f) $\overrightarrow{XC}$

(d)

$OABC$ is a quadrilateral. $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OC} = \boldsymbol{b}$ and $\overrightarrow{CB} = \boldsymbol{c}$. The point X divides the line OC in the ratio $2:1$. The point Y divides the line AB in the ratio $3:1$.



Express the following in terms of $\boldsymbol{a}, \boldsymbol{b}$ and $\boldsymbol{c}$:

(a) $\overrightarrow{OX}$

(b) $\overrightarrow{XC}$

(c) $\overrightarrow{AB}$

(d) $\overrightarrow{AY}$

(e) $\overrightarrow{AX}$

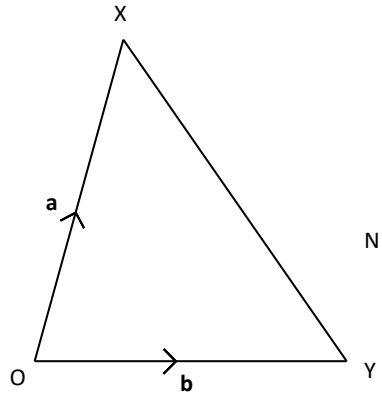
(f) $\overrightarrow{XY}$

Fluency Practice

Vectors Around Shapes

1)

M is the midpoint of line XY.



$\overrightarrow{YX}$ =

$\overrightarrow{YM}$ =

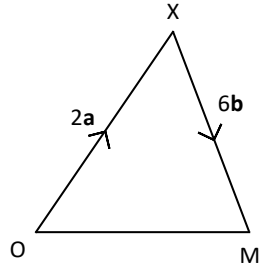
$\overrightarrow{OM}$ =

N is the midpoint of line OM.

$\overrightarrow{ON}$ =

2)

M divides the line OY in the ratio 2:3



$\overrightarrow{OM}$ =

$\overrightarrow{OY}$ =

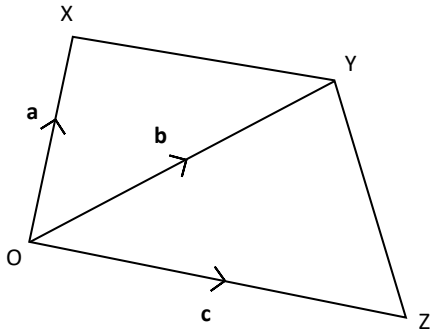
$\overrightarrow{MY}$ =

$\overrightarrow{YM}$ =

$\overrightarrow{YX}$ =

3)

M is the midpoint of line XY.
N divides the line ZY in the ratio 1:2



$\overrightarrow{XM}$ =

$\overrightarrow{ZN}$ =

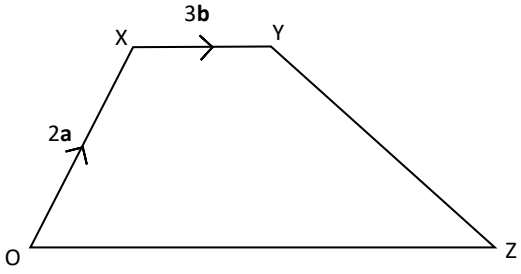
Express using **a**, **b** & **c**.

$\overrightarrow{ZM}$ =

$\overrightarrow{MN}$ =

4)

OZ = 2XY
M divides the line XZ in the ratio 1:2
N divides the line YZ in the ratio 1:2



$\overrightarrow{XZ}$ =

$\overrightarrow{XM}$ =

$\overrightarrow{YZ}$ =

$\overrightarrow{YN}$ =

$\overrightarrow{MN}$ =

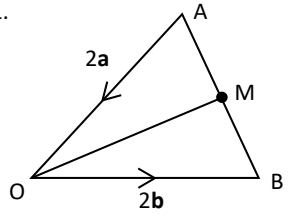
What statements can you make about
the magnitude & direction of lines XY & MN?

Fluency Practice

Vector Proofs

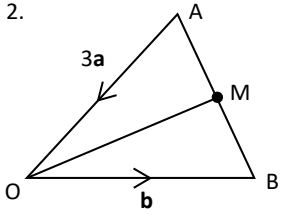
M is the midpoint of the line.

1.



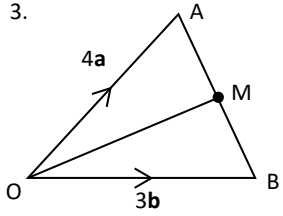
Prove $\overrightarrow{AM} = \boldsymbol{a} + \boldsymbol{b}$

2.



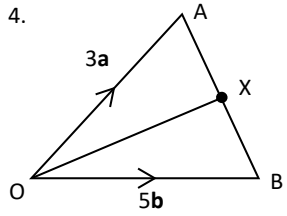
Prove $\overrightarrow{AM} = \frac{3}{2}\boldsymbol{a} + \frac{1}{2}\boldsymbol{b}$

3.



Prove $\overrightarrow{AM} = \frac{3}{2}\boldsymbol{b} - 2\boldsymbol{a}$

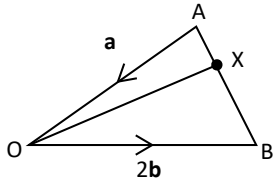
4.



$AX : AB = 1 : 2$

Prove $\overrightarrow{AX} = 2.5\boldsymbol{b} - 1.5\boldsymbol{a}$

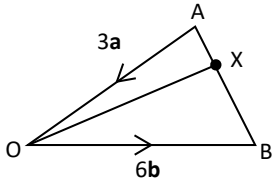
5.



$AX : AB = 1 : 3$

Prove $\overrightarrow{AX} = \frac{1}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b}$

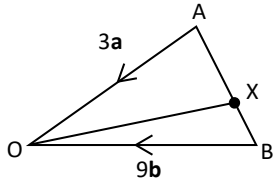
6.



$AX : XB = 1 : 2$

Prove $\overrightarrow{AX} = \boldsymbol{a} + 2\boldsymbol{b}$

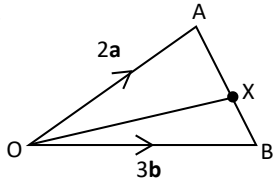
7.



$AX : XB = 2 : 1$

Prove $\overrightarrow{OX} = -\boldsymbol{a} - 6\boldsymbol{b}$

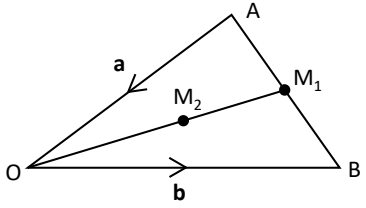
8.



$AX : XB = 3 : 2$

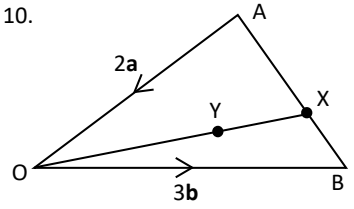
Prove $\overrightarrow{AX} = 1.8\boldsymbol{b} - 1.2\boldsymbol{a}$

9.



Prove $\overrightarrow{AM_2} = \frac{1}{4}\boldsymbol{b} - \frac{1}{4}\boldsymbol{a}$

10.



$AX : XB = 2 : 1$

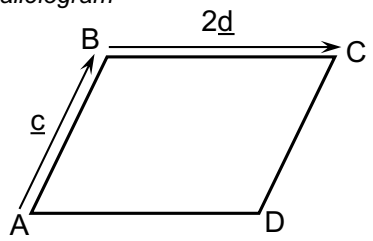
$OY : YX = 2 : 1$

Prove $\overrightarrow{OY} = \frac{4}{3}\boldsymbol{b} - \frac{4}{9}\boldsymbol{a}$

Fluency Practice

vector questions (i)

a parallelogram



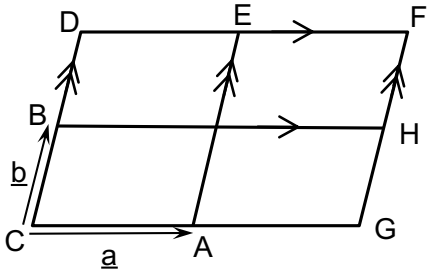
$\overrightarrow{CD}$ =

$\overrightarrow{BD}$ =

$\overrightarrow{DB}$ =

$\overrightarrow{CA}$ =

a parallelogram



$\overrightarrow{BA}$ =

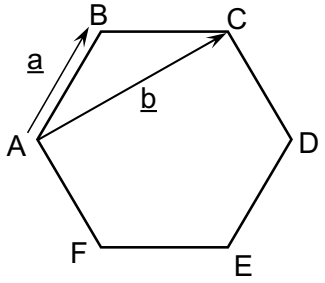
$\overrightarrow{CF}$ =

$\overrightarrow{HC}$ =

$\overrightarrow{?}$ = 2a – b

A is the mid-point of CG B is the mid-point of CD

a regular hexagon



$\overrightarrow{BC}$ =

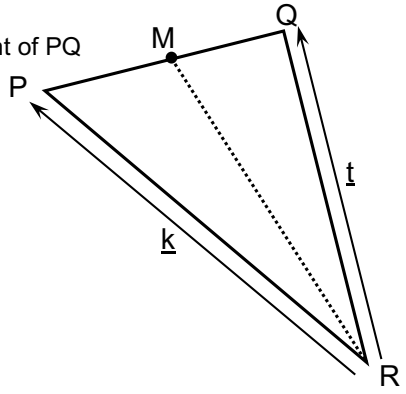
$\overrightarrow{AD}$ = 2$\overrightarrow{BC}$ =

$\overrightarrow{CD}$ =

$\overrightarrow{AE}$ =

a triangle

M is the midpoint of PQ



$\overrightarrow{PQ}$ =

$\overrightarrow{PM}$ =

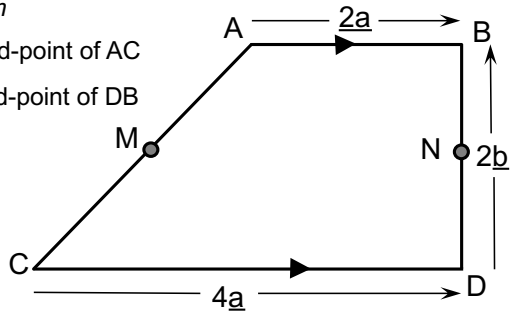
$\overrightarrow{QM}$ =

$\overrightarrow{RM}$ =

a trapezium

M is the mid-point of AC

N is the mid-point of DB



$\overrightarrow{AD}$ =

$\overrightarrow{AC}$ =

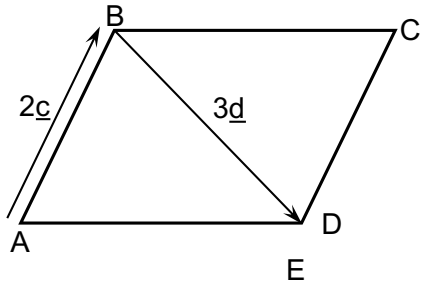
$\overrightarrow{MC}$ =

$\overrightarrow{MN}$ =

Fluency Practice

vector questions (ii)

a parallelogram



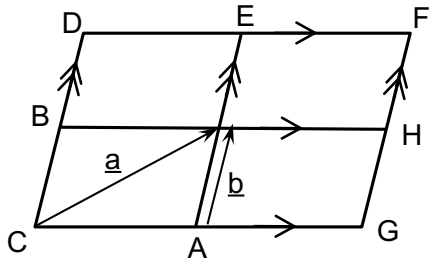
$\overrightarrow{DC}$ =

$\overrightarrow{BC}$ =

$\overrightarrow{AC}$ =

$\overrightarrow{CA}$ =

a parallelogram



A is the mid-point of CG
B is the mid-point of CD

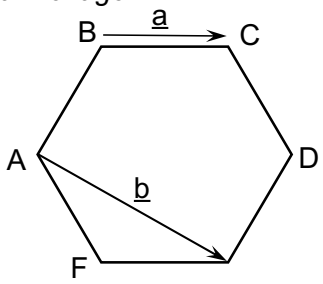
$\overrightarrow{CA}$ =

$\overrightarrow{DF}$ =

$\overrightarrow{CH}$ =

$\overrightarrow{?}$ = 2b – a

a regular hexagon



$\overrightarrow{BD}$ =

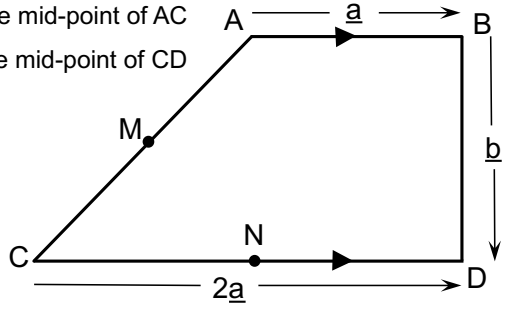
$\overrightarrow{AD}$ = $2\overrightarrow{BC}$ =

$\overrightarrow{CD}$ =

$\overrightarrow{BA}$ =

a trapezium

M is the mid-point of AC
N is the mid-point of CD



$\overrightarrow{AD}$ =

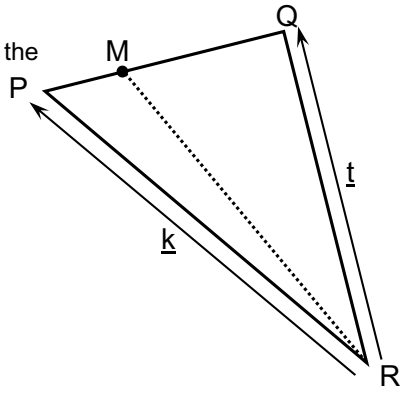
$\overrightarrow{CA}$ =

$\overrightarrow{CM}$ =

$\overrightarrow{MN}$ =

a triangle

M divides PQ in the ratio 1 : 2



$\overrightarrow{PQ}$ =

$\overrightarrow{PM}$ =

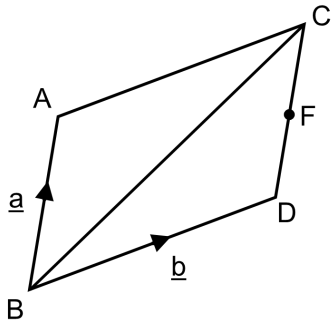
$\overrightarrow{QM}$ =

$\overrightarrow{RM}$ =

Fluency Practice

vector questions (iii)

ACDB is a parallelogram



F is the midpoint of CD.

work out the vectors:

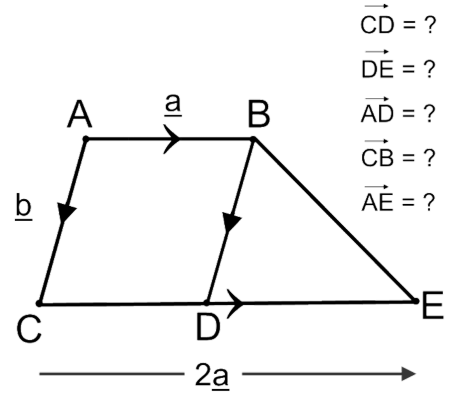
$\overrightarrow{DC}$ = ?

$\overrightarrow{DF}$ = ?

$\overrightarrow{AD}$ = ?

$\overrightarrow{CB}$ = ?

$\overrightarrow{BF}$ = ?



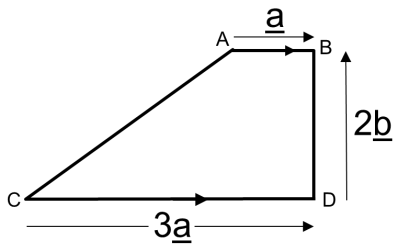
$\overrightarrow{CD}$ = ?

$\overrightarrow{DE}$ = ?

$\overrightarrow{AD}$ = ?

$\overrightarrow{CB}$ = ?

$\overrightarrow{AE}$ = ?

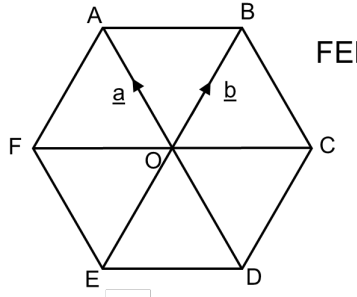


find, in terms of a and b, the vectors:

1) $\overrightarrow{CA}$

2) $\overrightarrow{CB}$

3) $\overrightarrow{DA}$



FEDCBA is a regular hexagon

OA = a

OB = b

write down, in terms of **a** and **b**, the vectors

(i) $\overrightarrow{AB}$ (ii) $\overrightarrow{FC}$

what fact can you deduce about $\overrightarrow{AB}$ and $\overrightarrow{FC}$?

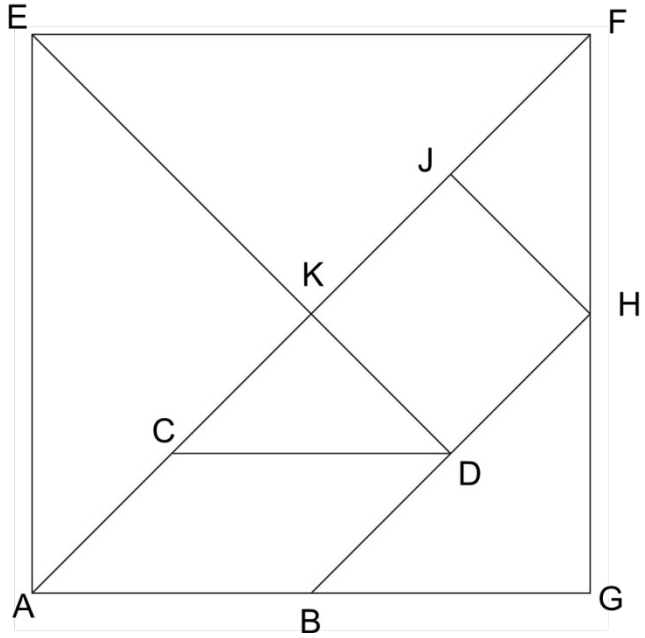
Fluency Practice

vector questions (iv)

a tangram puzzle

AB = $\underline{b}$

AC = $\underline{c}$



$\overrightarrow{AF}$ =

$\overrightarrow{BH}$ =

$\overrightarrow{DK}$ =

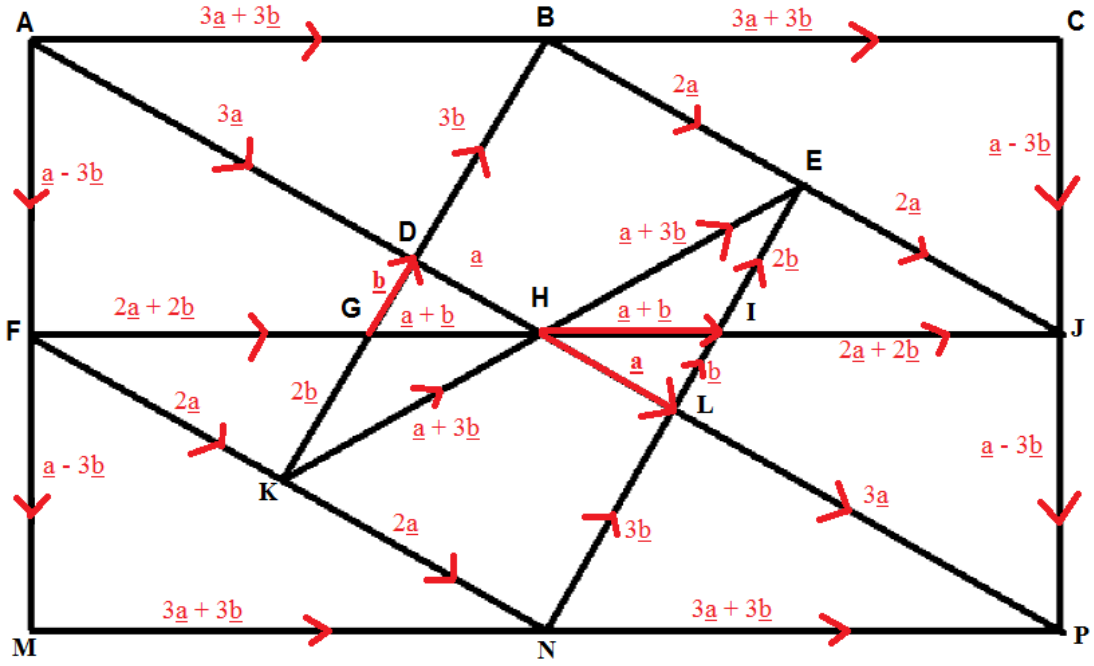
$\overrightarrow{GH}$ =

$\overrightarrow{JC}$ =

$\overrightarrow{BH}$ =

$\overrightarrow{HC}$ =

$\overrightarrow{DE}$ =



Which of these points lie on straight lines? Prove your answers.

(a) HEC (b) FDE (c) MKI (d) BIP

(e) BHN (f) AGN

Work out the ratio of the lengths of the line segments

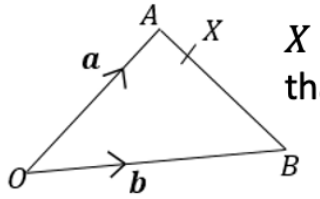
(a) AL:LP (b) KG:GB (c) FI:IJ (d) KL:LJ

(e) KE:EC (f) MK:KC

The point X is between A and B so that XDN is a straight line. Find the ratio AX:XB

Fluency Practice

1



X is a point such that $AX : XB = 1 : 4$

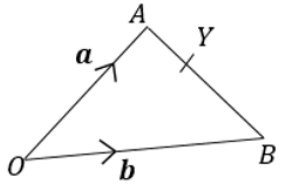
a. $\overrightarrow{AB} =$?

b. $\overrightarrow{AX} =$?

c. $\overrightarrow{OX} =$?

d. $\overrightarrow{BX} =$?

2



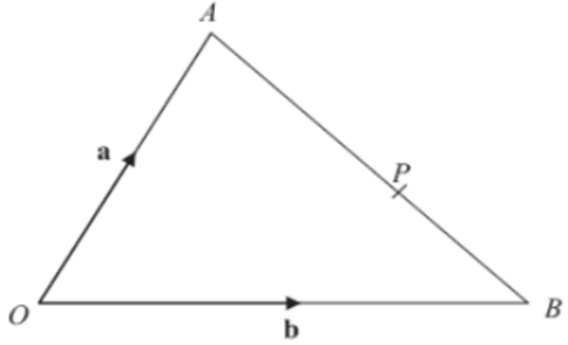
Y is a point such that $YB = 2AY$

a. $\overrightarrow{AY} =$?

b. $\overrightarrow{OY} =$?

c. $\overrightarrow{YO} =$?

3



[June 2009 2H Q23]

a) Find $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

?

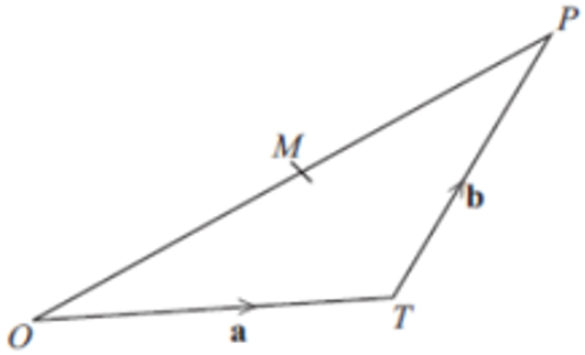
b) P is on AB such that $AP : PB = 3 : 2$.

Show that $\overrightarrow{OP} = \frac{1}{5}(2\boldsymbol{a} + 3\boldsymbol{b})$

?

Fluency Practice

4 [Nov 2010 1H Q27] M is the midpoint of OP.



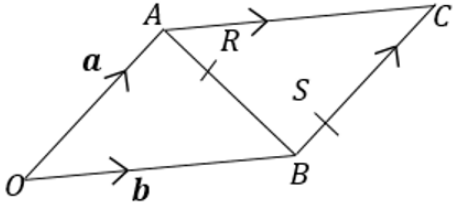
a) Express $\overrightarrow{OM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

?

b) Express $\overrightarrow{TM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ giving your answer in its simplest form.

?

5



$OACB$ is a parallelogram. R is a point such that $AR:RB = 2:3$. S is a point such that $BS:SC = 1:3$.

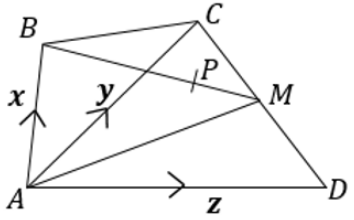
a. $\overrightarrow{OR} =$?

b. $\overrightarrow{BS} =$?

c. $\overrightarrow{OS} =$?

d. $\overrightarrow{RS} =$?

6



M is the midpoint of CD, $BP:PM = 2:1$

a. $\overrightarrow{DC} =$?

b. $\overrightarrow{DM} =$?

c. $\overrightarrow{AM} =$?

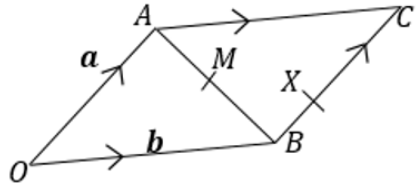
d. $\overrightarrow{BM} =$?

e. $\overrightarrow{BP} =$?

f. $\overrightarrow{AP} =$?

Fluency Practice

7



a. $\overrightarrow{AB} =$?

b. $\overrightarrow{BC} =$?

c. $\overrightarrow{MB} =$?

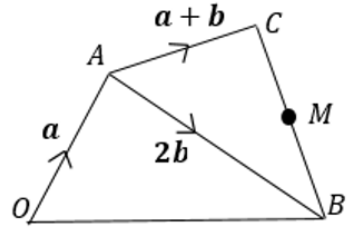
d. $\overrightarrow{MX} =$?

e. $\overrightarrow{XA} =$?

f. $\overrightarrow{CM} =$?

g. $\overrightarrow{XO} =$?

8



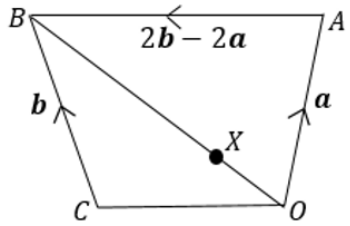
a. $\overrightarrow{OB} =$?

b. $\overrightarrow{BC} =$?

c. $\overrightarrow{AM} =$?

d. $\overrightarrow{OM} =$?

9



a. $\overrightarrow{OC} =$?

b. $\overrightarrow{XC} =$?

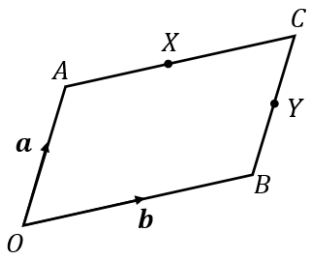
c. $\overrightarrow{AX} =$?

Fluency Practice

Vector Proof – Parallel Lines

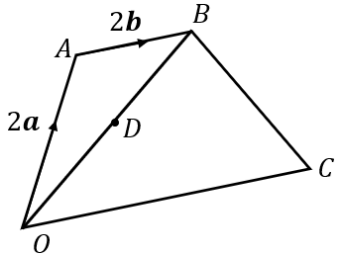
(a)

$OACB$ is a parallelogram. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$. X is the midpoint of AC and Y is the midpoint of BC. Show that XY and AB are parallel.



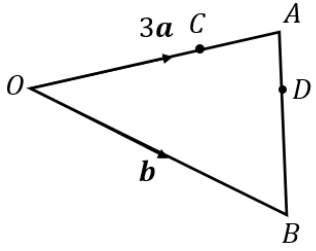
(b)

$OACB$ is a trapezium. $\overrightarrow{OA} = 2\boldsymbol{a}$ and $\overrightarrow{AB} = 2\boldsymbol{b}$. $\overrightarrow{OC} = 2\overrightarrow{AB}$ and D is the midpoint of OB. Show that AD is parallel to BC.



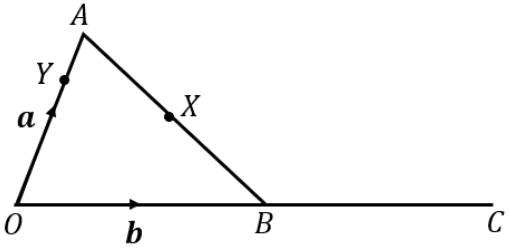
(c)

In the triangle OAB, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OA} = 3\boldsymbol{a}$. The point C divides the line OA in the ratio $2:1$ and the point D divides the line AB in the ratio $1:2$. Show that CD is parallel to OB.

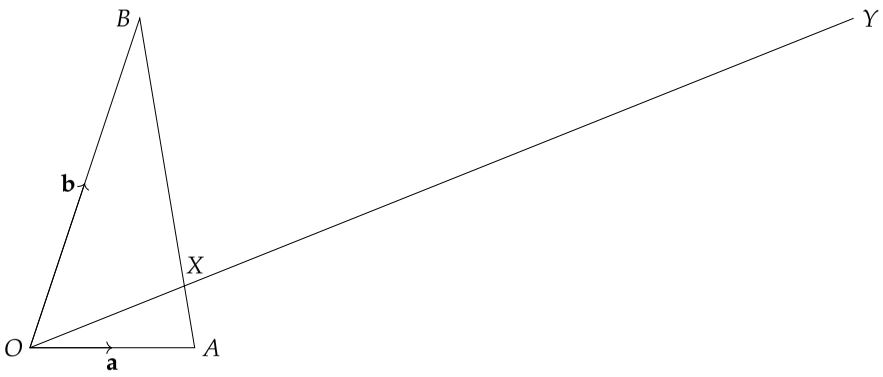


(d)

In the triangle OAB, $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{OA} = \boldsymbol{a}$. Point B is the midpoint of the line OC and X is the midpoint of AB. The point Y divides the line OA in the ratio $3:1$. Show that YX is parallel to AC.



1.

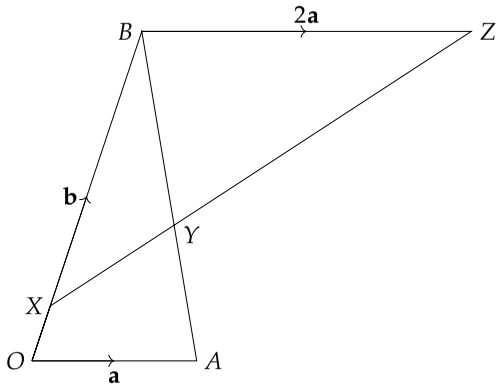


In the diagram,

- $\overrightarrow{OA} = \mathbf{a}$
- $\overrightarrow{OB} = \mathbf{b}$
- $AX : XB = 1 : 4$
- $\overrightarrow{OY} = 5 \times \overrightarrow{OX}$

Show that $\overrightarrow{OA}$ and $\overrightarrow{BY}$ are parallel.

2.

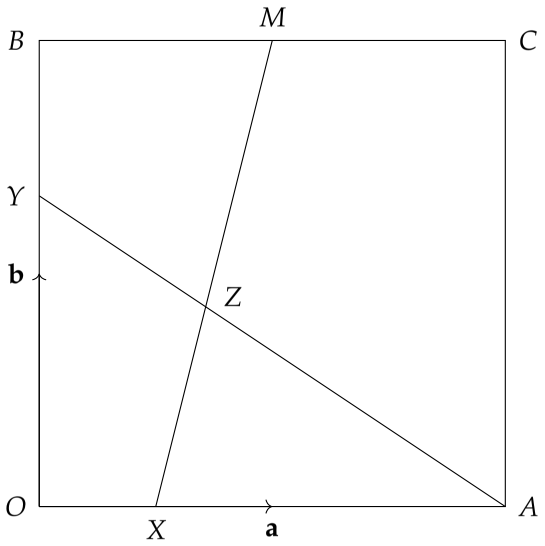


In the diagram,

- $\overrightarrow{OA} = \mathbf{a}$
- $\overrightarrow{OB} = \mathbf{b}$
- $\overrightarrow{BY} = 2\mathbf{b}$
- $OX : XB = 1 : 5$

Find the ratio $BY : BA$.

3.

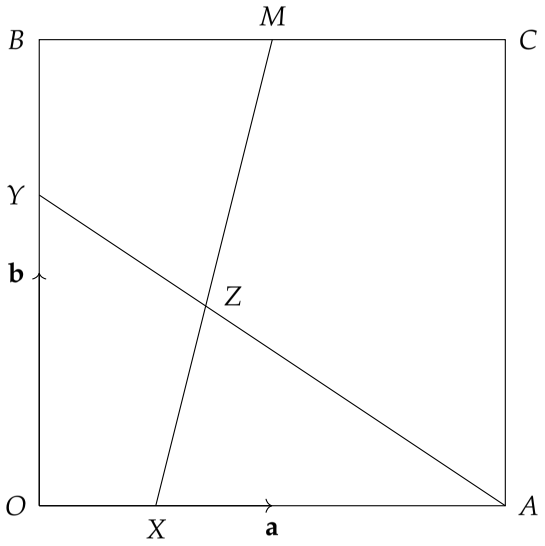


In the diagram,

- $ABCD$ is a square.
- $\overrightarrow{OA} = \mathbf{a}$
- $\overrightarrow{OB} = \mathbf{b}$
- M is the midpoint of BC
- $OY : YB = 2 : 1$
- $OX : XA = 1 : 3$

Find the vector $\overrightarrow{OZ}$.

4.



In the diagram,

- $ABCD$ is a square.

- $\overrightarrow{OA} = \mathbf{a}$
- $\overrightarrow{OB} = \mathbf{b}$
- M is the midpoint of BC
- $\overrightarrow{OX} = m \times \overrightarrow{OA}$
- $\overrightarrow{OY} = n \times \overrightarrow{OB}$

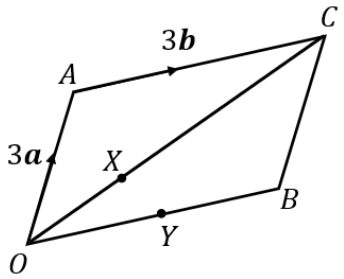
Find the vector $\overrightarrow{OZ}$, giving your answer in terms of m and n.

Fluency Practice

Vector Proof – Collinear Points

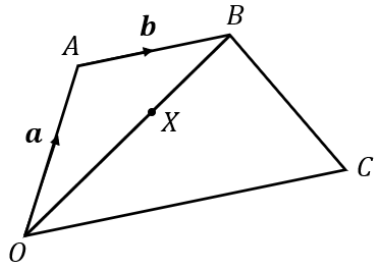
(a)

$OACB$ is a parallelogram. $\overrightarrow{OA} = 3\boldsymbol{a}$ and $\overrightarrow{AC} = 3\boldsymbol{b}$. Y is the midpoint of OB and X divides the line OC in the ratio $1 : 2$. Show that the points A, X and Y are collinear.



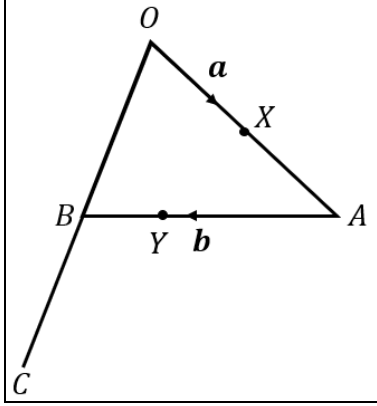
(b)

$OACB$ is a trapezium. $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{AB} = \boldsymbol{b}$. $\overrightarrow{OC} = 3\overrightarrow{AB}$ and X divides the line OB in the ratio $3 : 1$. Show that the points A, X and C are collinear.



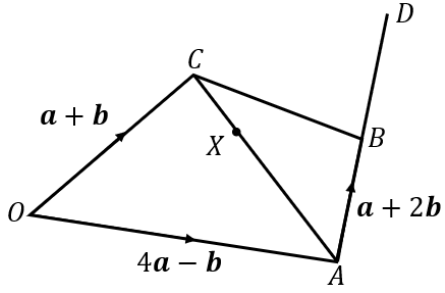
(c)

In the triangle OAB, $\overrightarrow{OX} = \boldsymbol{a}$ and $\overrightarrow{AB} = \boldsymbol{b}$. X is the midpoint of OA and the point Y divides the line AB in the ratio $2 : 1$. $\overrightarrow{OB} = \overrightarrow{BC}$. Show that the points X, Y and C are collinear.



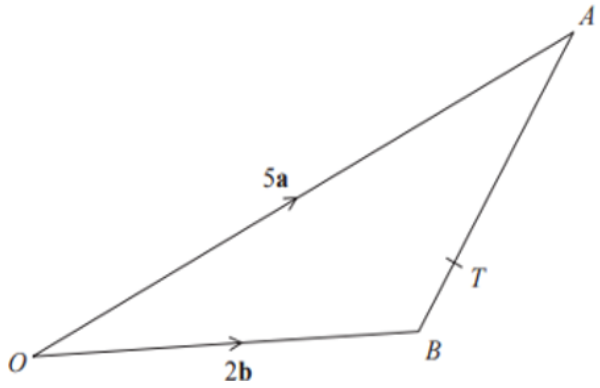
(d)

$\overrightarrow{OA} = 4\boldsymbol{a} - \boldsymbol{b}$, $\overrightarrow{AB} = \boldsymbol{a} + 2\boldsymbol{b}$ and $\overrightarrow{OC} = \boldsymbol{a} + \boldsymbol{b}$. $\overrightarrow{AB} = \overrightarrow{BD}$. The point X divides the line AC in the ratio $6 : 1$. Show that O, X and D are collinear.



Fluency Practice

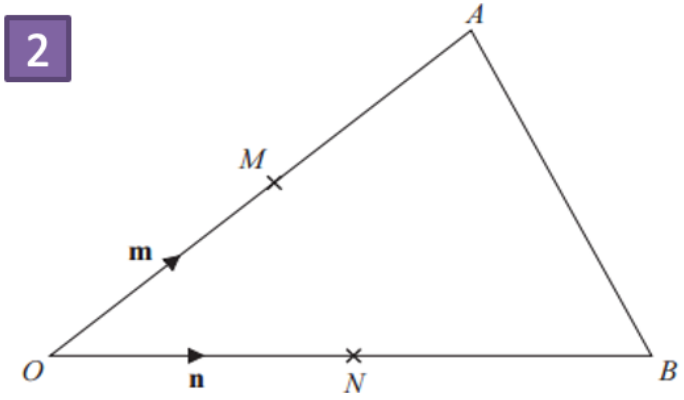
1



T is the point on AB such that $AT:TB = 5:1$. Show that OT is parallel to the vector $\boldsymbol{a} + 2\boldsymbol{b}$.

?

2

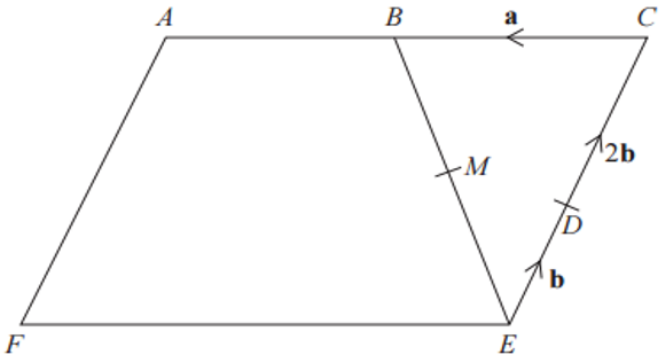


M is the midpoint of OA. N is the midpoint of OB. Prove that AB is parallel to MN.

?

Fluency Practice

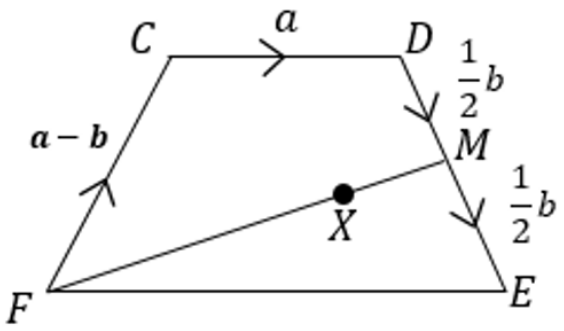
3



$ACEF$ is a parallelogram. B is the midpoint of AC. M is the midpoint of BE. Show that AMD is a straight line.

?

4



$\overrightarrow{CD} = \boldsymbol{a}$, $\overrightarrow{DE} = \boldsymbol{b}$ and $\overrightarrow{FC} = \boldsymbol{a} - \boldsymbol{b}$

i) Express $\overrightarrow{CE}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

?

ii) Prove that $\overrightarrow{FE}$ is parallel to $\overrightarrow{CD}$.

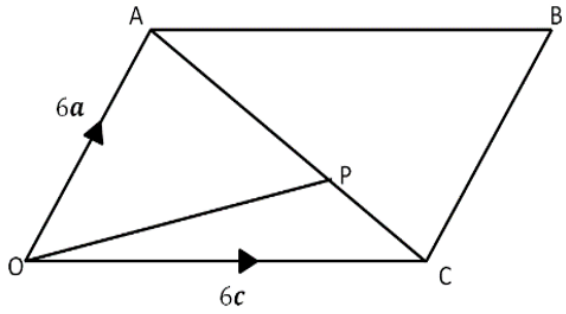
?

iii) X is the point on FM such that such that $FX:XM = 4:1$. Prove that C, X and E lie on the same straight line.

?

Fluency Practice

5



$OABC$ is a parallelogram. P is the point on AC such that $AP = \frac{2}{3}AC$.

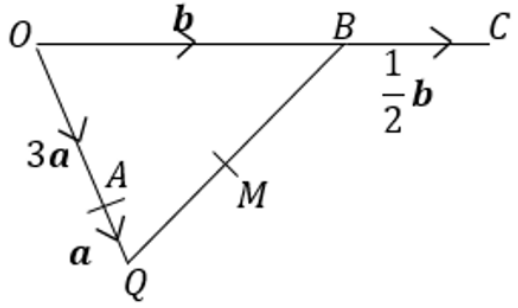
i) Find the vector $\overrightarrow{OP}$. Give your answer in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

?

ii) Given that the midpoint of CB is M, prove that OPM is a straight line.

?

6



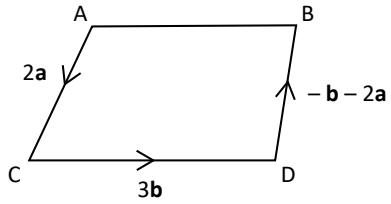
$\overrightarrow{OA} = 3\boldsymbol{a}$ and $\overrightarrow{AQ} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$ and $\overrightarrow{BC} = \frac{1}{2}\boldsymbol{b}$. $M$is the midpoint of QB. Prove that AMC is a straight line.

?

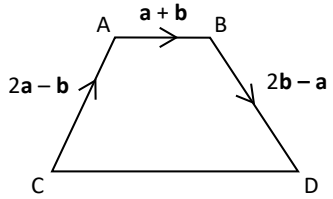
Fluency Practice

Vectors: Proving Parallel & Straight Lines

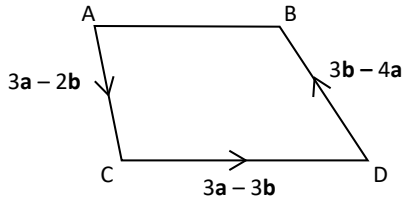
1) Prove $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$



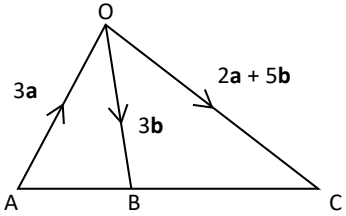
2) Prove $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$



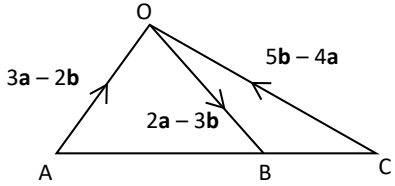
3) Prove $\overrightarrow{AB}$ is parallel to $\overrightarrow{CD}$



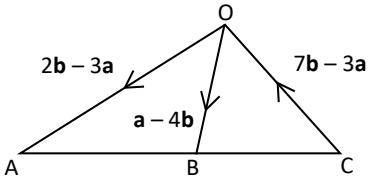
4) Prove $\overrightarrow{ABC}$ is a straight line.



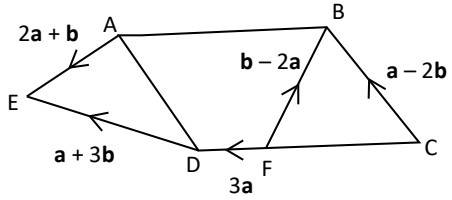
5) Prove $\overrightarrow{ABC}$ is a straight line.



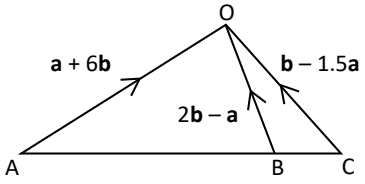
6) Prove $\overrightarrow{ABC}$ is a straight line.



7) Prove ABCD is a parallelogram.



8) Prove $\overrightarrow{ABC}$ is a straight line.



Find the values of λ and μ by equating coefficients of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(a) $4\boldsymbol{a} + \lambda\boldsymbol{b} = \mu\boldsymbol{a} + 6\boldsymbol{b}$

(b) $\lambda\boldsymbol{a} - 5\boldsymbol{b} = -2\boldsymbol{a} + \mu\boldsymbol{b}$

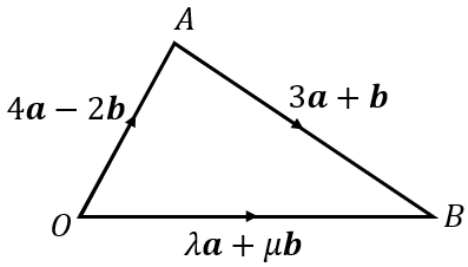
(c) $\lambda\boldsymbol{a} + \mu\boldsymbol{b} = 2\boldsymbol{a} + 6\boldsymbol{b} + 3\boldsymbol{a} - 3\boldsymbol{b}$

(d) $-3\boldsymbol{a} + \lambda\boldsymbol{b} = 2\boldsymbol{a} - \boldsymbol{b} + \mu\boldsymbol{a} + 5\boldsymbol{b}$

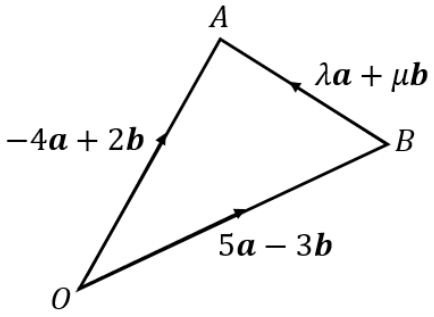
(e) $2(2\boldsymbol{a} - \mu\boldsymbol{b}) + 3\boldsymbol{a} + 10\boldsymbol{b} = \lambda\boldsymbol{a} + 6\boldsymbol{b}$

In the vector diagrams shown, find the values of λ and μ.

(a)

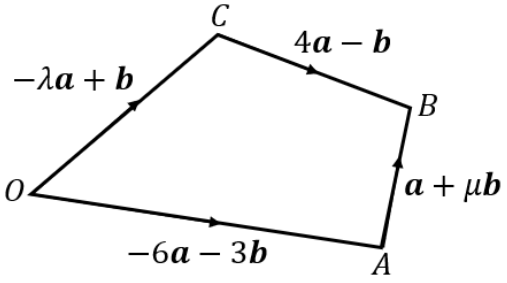


(b)

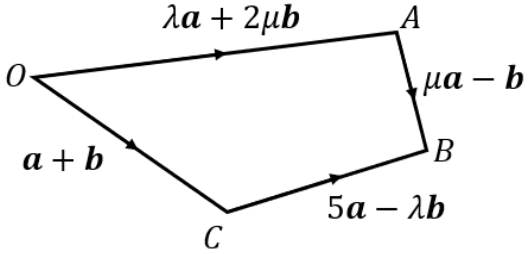


In the vector diagrams shown, find the values of λ and μ.

(a)



(b)

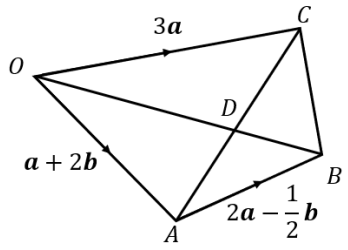


Fluency Practice

Vector Proof – Equating Coefficients

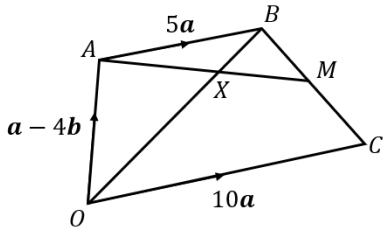
(a)

$OABC$ is a quadrilateral, where $\overrightarrow{OC} = 3\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} + 2\boldsymbol{b}$ and $\overrightarrow{AB} = 2\boldsymbol{a} - \frac{1}{2}\boldsymbol{b}$. The point D is on OB and AC such that $OD : OB = \lambda : 1$ and $AD : AC = \mu : 1$. By finding two ways to express the vector $\overrightarrow{OD}$, find the values of λ and μ.



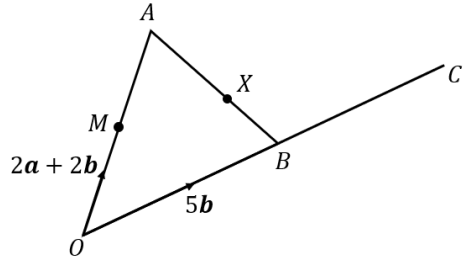
(b)

$OABC$ is a trapezium, where $\overrightarrow{OC} = 10\boldsymbol{a}$, $\overrightarrow{OA} = \boldsymbol{a} - 4\boldsymbol{b}$ and $\overrightarrow{AB} = 5\boldsymbol{a}$. M is the midpoint of the line BC. The point X is on OB and AM such that $OX : OB = \lambda : 1$ and $AX : AM = \mu : 1$. Find the values of λ and μ and the vector $\overrightarrow{OX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



(c)

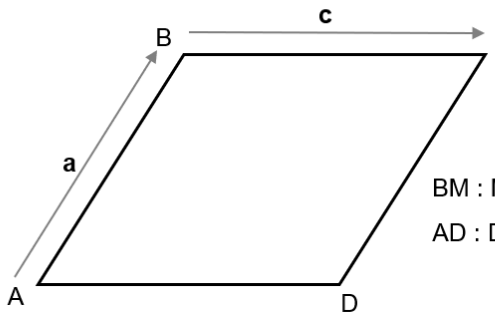
In the triangle OAB, $\overrightarrow{OB} = 5\boldsymbol{b}$ and $\overrightarrow{OM} = 2\boldsymbol{a} + 2\boldsymbol{b}$, where M is the midpoint of OA. OC is the line OB produced and $\overrightarrow{OB} = \overrightarrow{BC}$. The point X is on the line AB such that $AX : AB = \lambda : 1$. Given that MXC is a straight line, find the value of λ and the vector $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



Fluency Practice

vector proofs

(1) a **parallelogram** (ABCD)

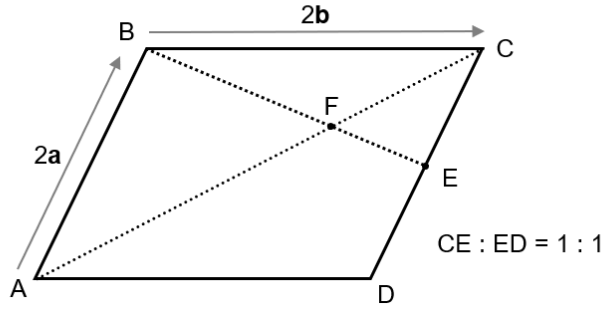


BM : MD = 1 : 1

AD : DE = k : 1

$\overrightarrow{ME} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$ find the value of k

(2) a **parallelogram** (ABCD)



CE : ED = 1 : 1

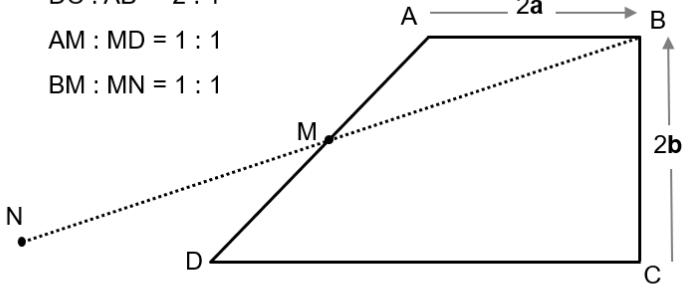
show that AF : FC = 2 : 1

(3) a **trapezium** (ABCD)

DC : AB = 2 : 1

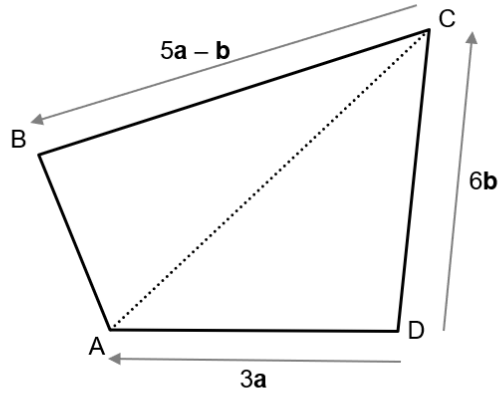
AM : MD = 1 : 1

BM : MN = 1 : 1



show that CDN is a straight line

(4)



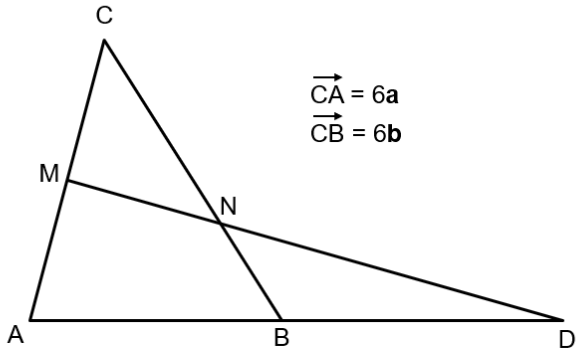
AX : XC = 1 : 2

DX : DB = k : 1 find the value of k

Fluency Practice

vector proofs

(1)

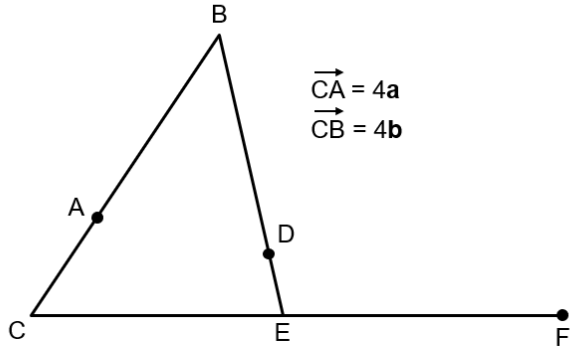


CM : MA = 1 : 1

AB : BD = 1 : 1

$\overrightarrow{CN} = k\mathbf{b}$ find the value of k

(2)



CA : AB = 1 : 2

ED : DB = 1 : 3

CE : EF = 1 : 2

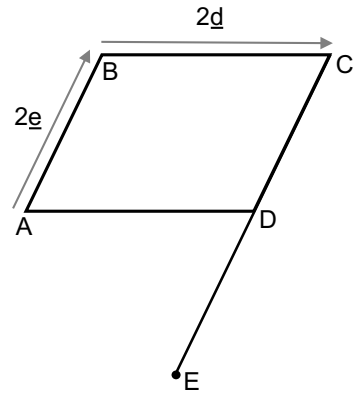
show that ADF is a straight line

(3) choose some other ratios to explore

Fluency Practice

vector proofs (i)

(1) a **parallelogram** (ABCD)



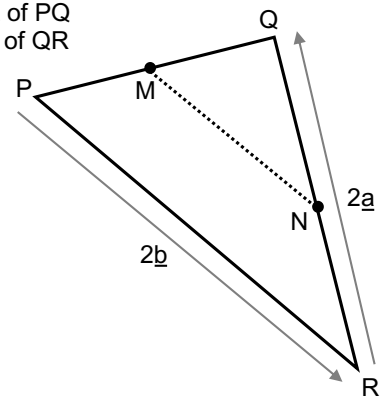
CE is twice CD

establish that ABDE is a parallelogram

(2) a **triangle** (PQR)

M is the midpoint of PQ

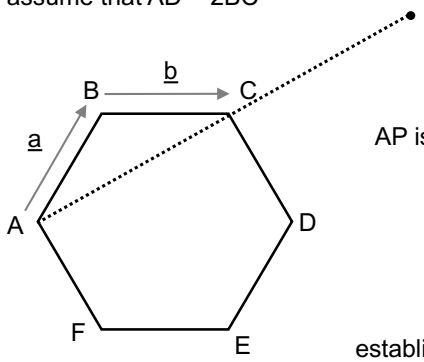
N is the midpoint of QR



prove that MN is parallel to PR and half the length of it

(3) a **regular hexagon**

you can assume that AD = 2BC



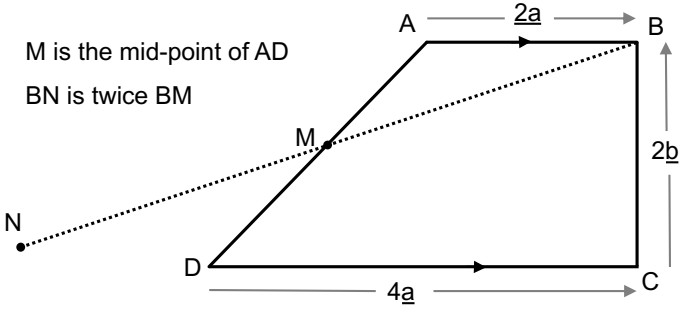
AP is twice AC

establish that EDP is a straight line

(4) a **trapezium** (ABCD)

M is the mid-point of AD

BN is twice BM

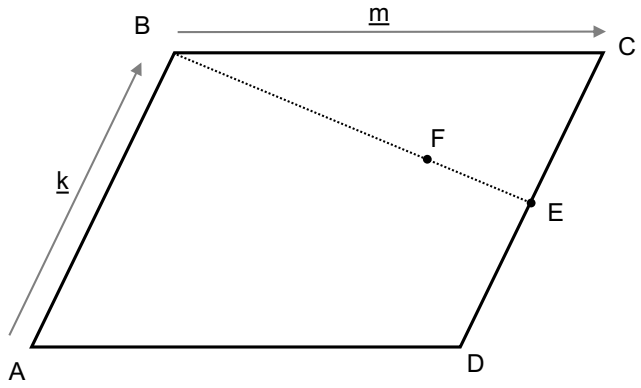


prove that CDN is a straight line

Fluency Practice

vector proofs (ii)

(4) a **parallelogram** (ABCD)

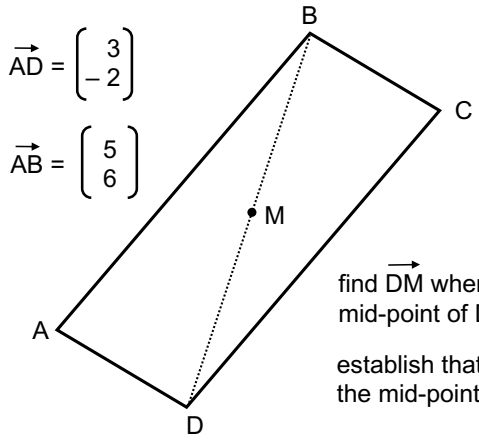


E is the mid-point of CD

BF : FE = 2 : 1

establish that F is ⅔ of the way along AC

(5) a **parallelogram** (ABCD)



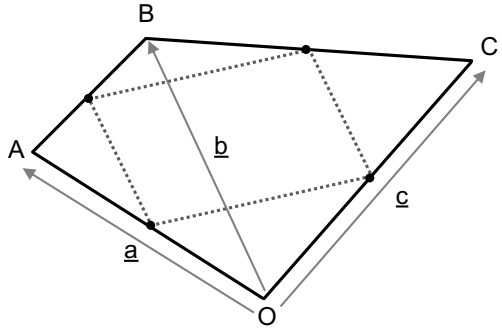
find $\overrightarrow{DM}$ where M is the mid-point of DB

establish that M is also the mid-point of AC

(6) any **quadrilateral** (OABC)

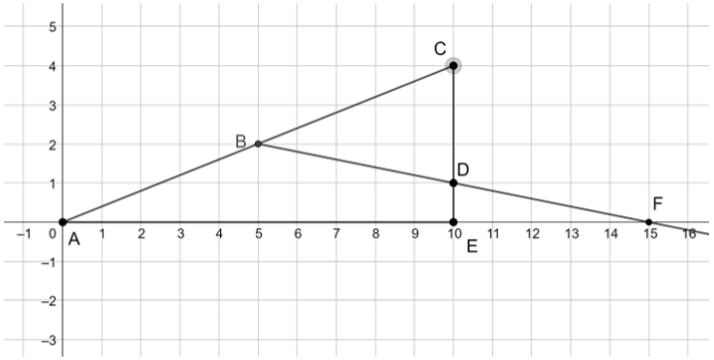
any quadrilateral (OABC) is drawn and the mid-points of the four sides are joined

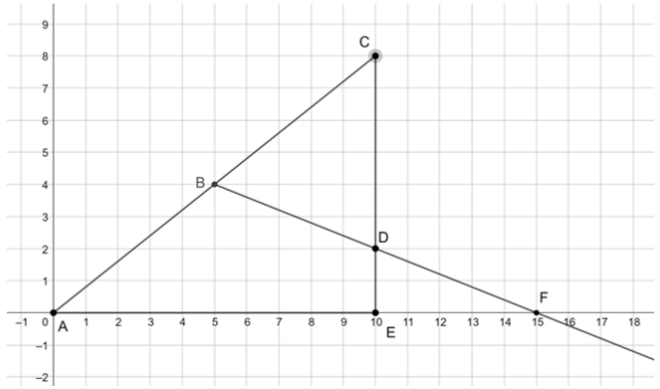
establish that the resulting shape is a parallelogram



Fluency Practice

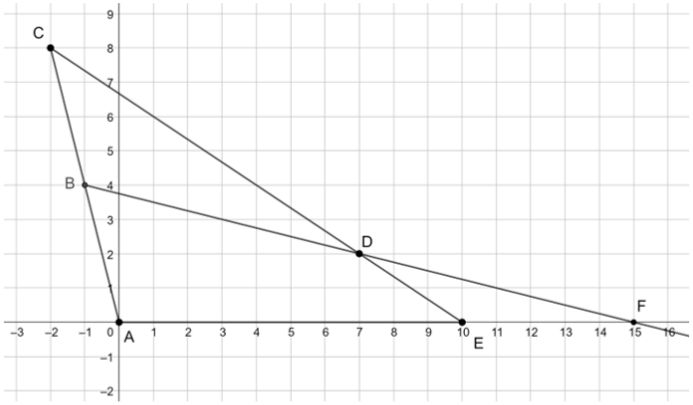
prove the general result

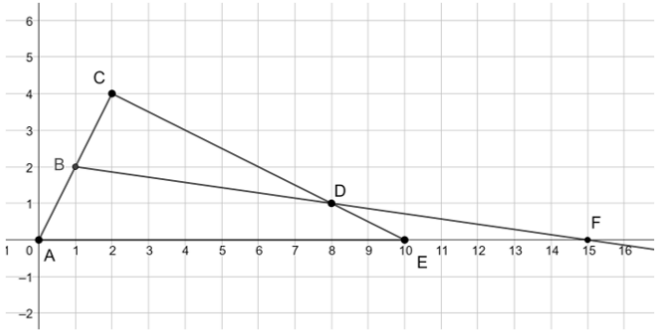




AB : BC = 1 : 1
ED : DC = 1 : 3
AE : EF = 2 : 1

prove
BD : DF = 1 : 1





Q1.

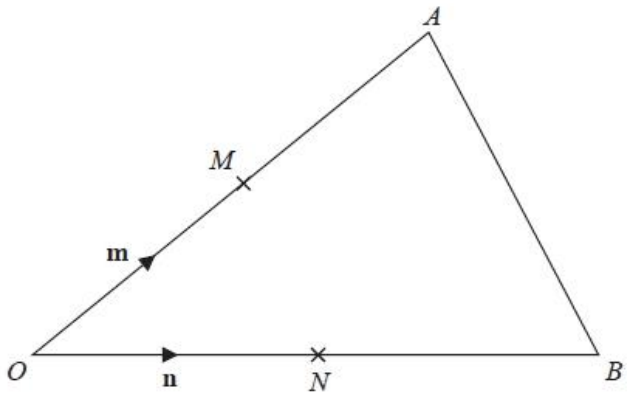


Diagram **NOT** accurately drawn

OAB is a triangle.
M is the midpoint of *OA*.
N is the midpoint of *OB*.

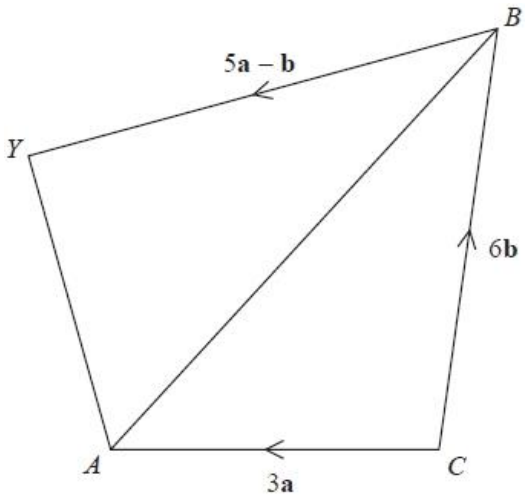
$\overrightarrow{OM}$ = **m**

$\overrightarrow{ON}$ = **n**

Show that *AB* is parallel to *MN*.

(Total for Question is 3 marks)

Q2.



CAYB is a quadrilateral.

$\overrightarrow{CA} = 3\mathbf{a}$
$\overrightarrow{CB} = 6\mathbf{b}$
$\overrightarrow{BY} = 5\mathbf{a} - \mathbf{b}$

X is the point on *AB* such that *AX* : *XB* = 1 : 2

Prove that $\overrightarrow{CX} = \frac{2}{5}\overrightarrow{CY}$

(Total for question is 5 marks)

Q3.

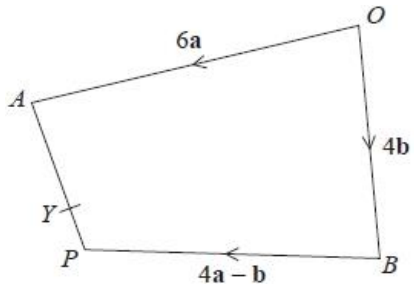


Diagram **NOT** accurately drawn

OBPA is a quadrilateral.

$\overrightarrow{OA} = 6\mathbf{a}$

$\overrightarrow{OB} = 4\mathbf{b}$

$\overrightarrow{BP} = 4\mathbf{a} - \mathbf{b}$

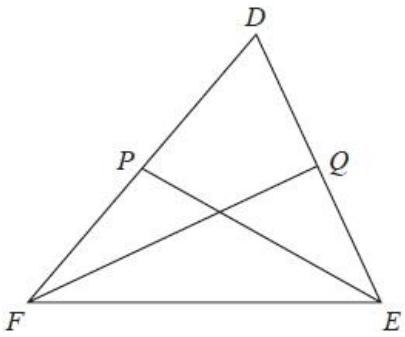
Y is the point on *AP* such that *AY* : *YP* = 2 : 1

Show that $\overrightarrow{OY}$ is parallel to the vector 7**a** + 3**b**

(Total for question = 4 marks)

Q4.

DEF is a triangle.



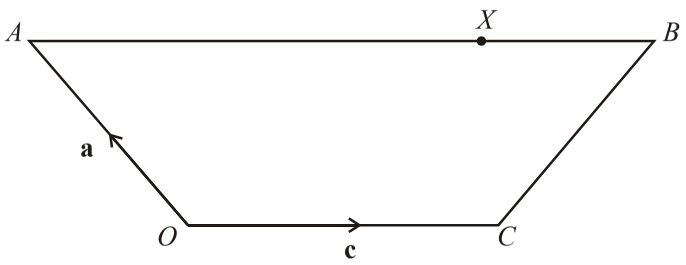
P is the midpoint of *FD*.
Q is the midpoint of *DE*.

$\overrightarrow{FD} = \mathbf{a}$ and $\overrightarrow{FE} = \mathbf{b}$

Use a vector method to prove that *PQ* is parallel to *FE*.

(Total for question = 4 marks)

1.



$OABC$ is a trapezium.

OC is parallel to AB.

$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$

$AB = 2OC$.

X is the point on AB such that $AX:XB = 3:1$.

Express $\overrightarrow{XC}$ in terms of **a** and **c**.

2.

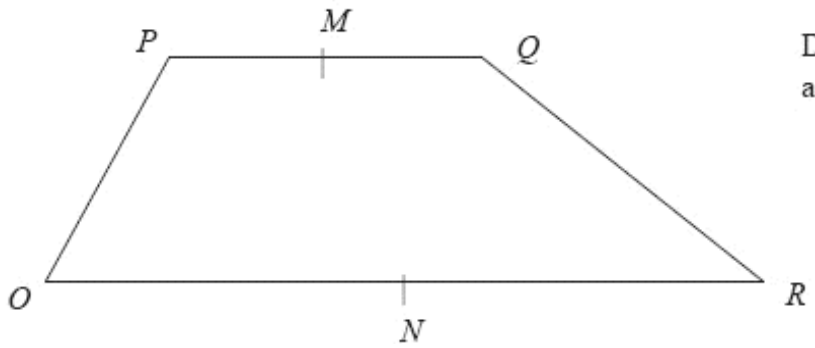


Diagram **NOT** accurately drawn

$OPQR$ is a trapezium with PQ parallel to OR.

$\overrightarrow{OP} = 2\mathbf{b}$ $\overrightarrow{PQ} = 2\mathbf{a}$ $\overrightarrow{OR} = 6\mathbf{a}$

M is the midpoint of PQ and N is the midpoint of OR.

(a) Find the vector $\overrightarrow{MN}$ in terms of **a** and **b**.

X is the midpoint of MN and Y is the midpoint of QR.

(b) Prove that XY is parallel to OR.

3.

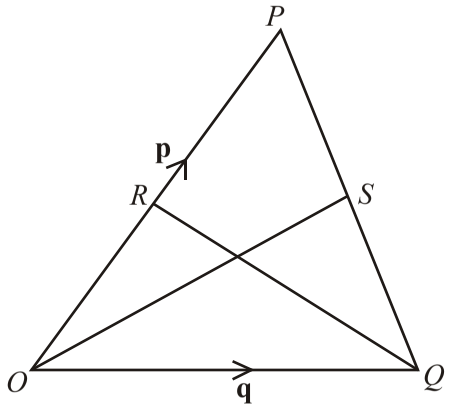


Diagram **NOT** accurately drawn

OPQ is a triangle.
R is the midpoint of OP.
S is the midpoint of PQ.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$

(i) Find $\overrightarrow{OS}$ in terms of **p** and **q**. (ii) Show that RS is parallel to OQ.

4.

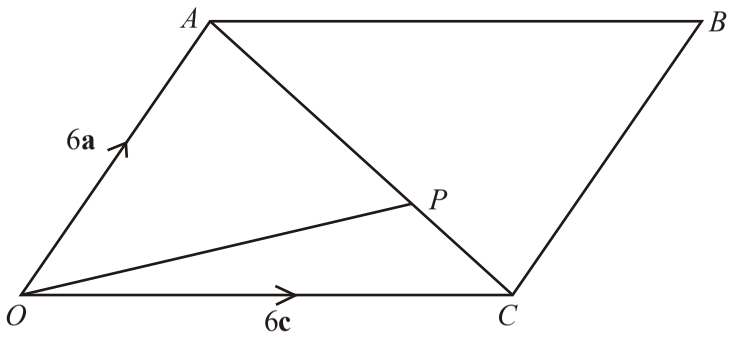


Diagram **NOT** accurately drawn

$OABC$ is a parallelogram.

P is the point on AC such that $AP = \frac{2}{3}AC$.

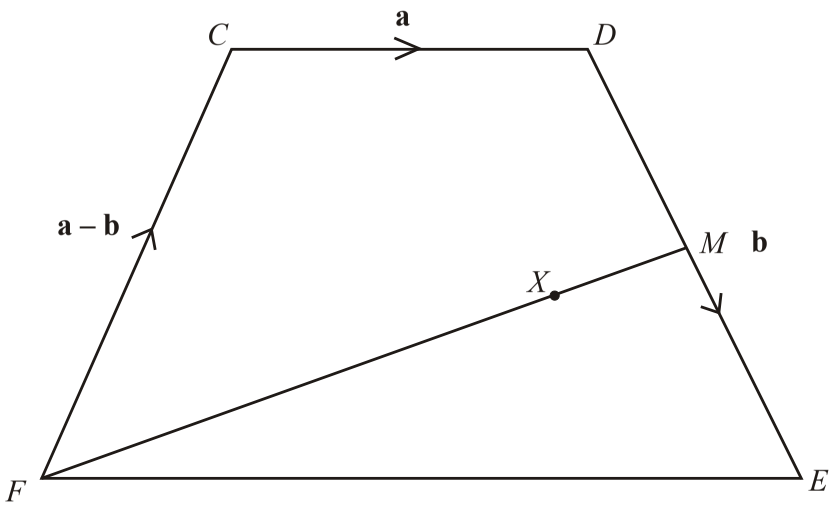
$\overrightarrow{OA} = 6\mathbf{a}$. $\overrightarrow{OC} = 6\mathbf{c}$.

(a) Find the vector $\overrightarrow{OP}$. Give your answer in terms of **a** and **c**.

The midpoint of CB is M.

(b) Prove that OPM is a straight line.

5.



CDEF is a quadrilateral with $\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

(a) Express $\overrightarrow{CE}$ in terms of **a** and **b**.

(b) Prove that *FE* is parallel to *CD*.

M is the midpoint of *DE*.

(c) Express $\overrightarrow{FM}$ in terms of **a** and **b**.

X is the point on *FM* such that *FX* : *XM* = 4 : 1.

(d) Prove that *C*, *X* and *E* lie on the same straight line.

6.

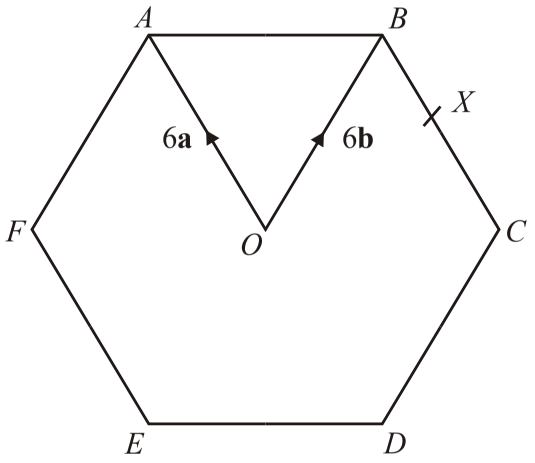


Diagram **NOT** accurately drawn

The diagram shows a regular hexagon *ABCDEF* with centre *O*.

$\overrightarrow{OA} = 6\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$

(a) Express in terms of **a** and/or **b**

(i) $\overrightarrow{AB}$,

(ii) $\overrightarrow{EF}$.

X is the midpoint of BC.

(b) Express $\overrightarrow{EX}$ in terms of **a** and/or **b**

Y is the point on AB extended, such that $AB : BY = 3:2$

(c) Prove that E, X and Y lie on the same straight line.

7.

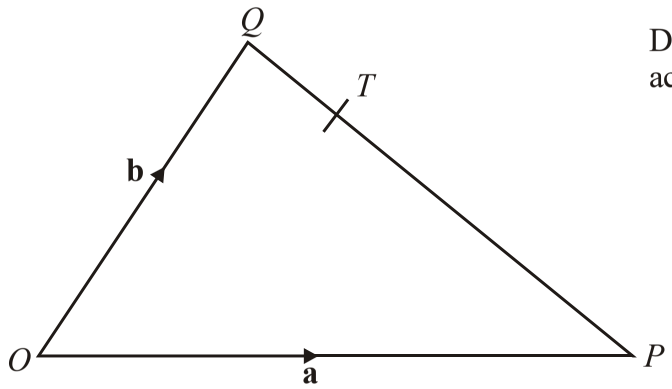


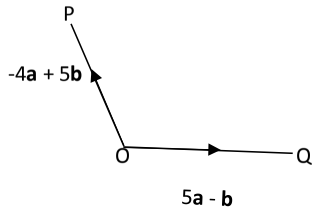
Diagram **NOT** accurately drawn

OPQ is a triangle.
T is the point on PQ for which $PT : TQ = 2 : 1$.
$\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OQ} = \mathbf{b}$.

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{PQ}$.

(b) Express $\overrightarrow{OT}$ in terms of **a** and **b**.
Give your answer in its simplest form.

1. $\overrightarrow{OP} = -4\mathbf{a} + 5\mathbf{b}$ and $\overrightarrow{OQ} = 5\mathbf{a} - \mathbf{b}$.



(a) Express $\overrightarrow{PQ}$ in terms of **a** and **b**.

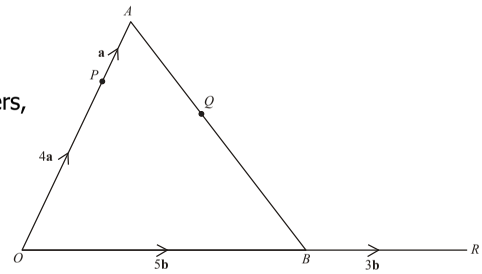
R is a point on $\overrightarrow{PQ}$ such that *PR* : *RQ* = 1 : 2.

(b) Express $\overrightarrow{OR}$ in terms of **a** and **b**.

(Total 3 marks)

2. In the diagram $\overrightarrow{OP} = 4\mathbf{a}$, $\overrightarrow{PA} = \mathbf{a}$,

$\overrightarrow{OB} = 5\mathbf{b}$, $\overrightarrow{BR} = 3\mathbf{b}$ and $\overrightarrow{AQ} = \frac{2}{5}\overrightarrow{AB}$



(a) Find, in terms of **a** and **b**, simplifying your answers,

(i) $\overrightarrow{AB}$

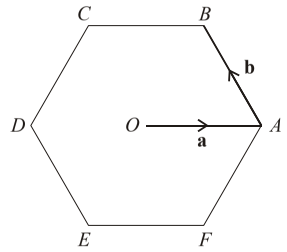
(ii) $\overrightarrow{PQ}$

(b) Show clearly that points *P, Q* and *R* lie on a straight line.

(Total 6 marks)

3. *ABCDEF* is a regular hexagon with centre *O*.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$

Diagram drawn accurately



(a) Find expressions, in terms of **a** and **b**, for

(i) $\overrightarrow{OB}$

(ii) $\overrightarrow{AC}$

(iii) $\overrightarrow{EC}$

(Total 3 marks)

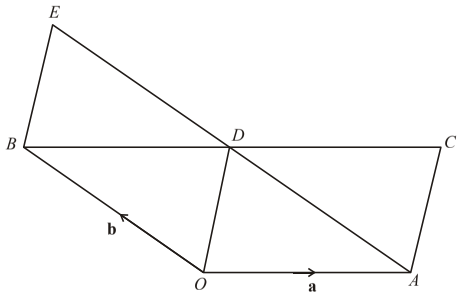
(b) The positions of points *P* and *Q* are given by the vectors
$\overrightarrow{OP} = \mathbf{a} - \mathbf{b}$ $\overrightarrow{OQ} = \mathbf{a} + 2\mathbf{b}$

(i) Draw and label the positions of points *P* and *Q* on the diagram.

(ii) Hence, or otherwise, deduce an expression for $\overrightarrow{PQ}$.

(Total 6 marks)

4. In the diagram *OACD, OADB* and *ODEB* are parallelograms.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$



(a) Express, in terms of **a** and **b**, the following vectors.
Give your answers in their simplest form.

(i) $\overrightarrow{OD}$

(ii) $\overrightarrow{OC}$

(iii) $\overrightarrow{AB}$

(b) The point *F* is such that *OCFE* is a parallelogram.
Write the vector $\overrightarrow{CF}$ in terms of **a** and **b**.

(c) What geometrical relationship is there between the points *O, D* and *F*?
Justify your answer.

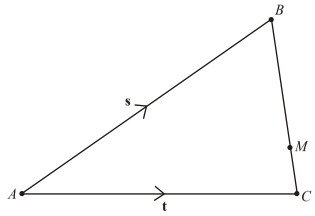
(Total 7 marks)

5. In triangle *ABC*, *M* lies on *BC* such that $BM = \frac{3}{4} BC$.

$\overrightarrow{AB} = \mathbf{s}$ and $\overrightarrow{AC} = \mathbf{t}$

Find $\overrightarrow{AB}$ in terms of **s** and **t**.
Give your answer in its simplest form.

Not drawn accurately



(Total 3 marks)

6. In the diagram $\overrightarrow{OP} = 4\mathbf{a}$, $\overrightarrow{PA} = \mathbf{a}$,
$\overrightarrow{OB} = 5\mathbf{b}$, $\overrightarrow{BR} = 3\mathbf{b}$ and $\overrightarrow{AQ} = \frac{2}{5} \overrightarrow{AB}$

Not drawn accurately

(a) Find, in terms of **a** and **b**, simplifying your answers,

(i) $\overrightarrow{AB}$

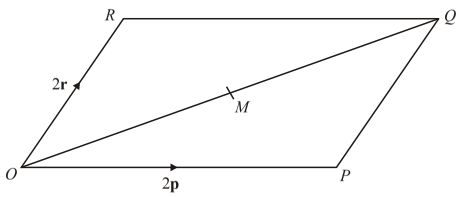
(ii) $\overrightarrow{PQ}$

(b) Show clearly that points *P, Q* and *R* lie on a straight line.

(Total 6 marks)

7. *OPQR* is a parallelogram.
M is the mid-point of the diagonal *OQ.*

$\overrightarrow{OP} = 2\mathbf{p}$ and $\overrightarrow{OR} = 2\mathbf{r}$



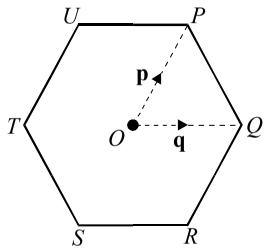
(a) Express $\overrightarrow{OM}$ in terms of **p** and **r**.

(b) Use vectors to prove that *M* is also the mid-point of *PR.*

(Total 4 marks)

8. *PQRSTU* is a regular hexagon and *O* is the centre of the hexagon.

$\overrightarrow{OP}$ = **p** and $\overrightarrow{OQ}$ = **q**



Express each of the following vectors in terms of **p** and **q**

(a) $\overrightarrow{PQ}$

(b) $\overrightarrow{SP}$

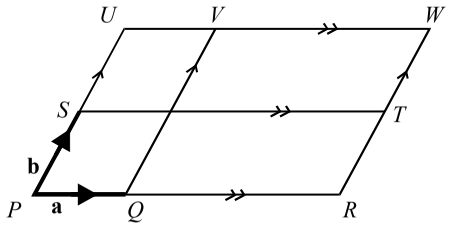
(c) $\overrightarrow{SQ}$

(Total 4 marks)

9. The diagram shows two sets of parallel lines.

Vector $\overrightarrow{PQ}$ = **a** and vector $\overrightarrow{PS}$ = **b**

$\overrightarrow{PR} = 3\overrightarrow{PQ}$ and $\overrightarrow{PU} = 2\overrightarrow{PS}$



(a) Write the vector $\overrightarrow{PV}$ in terms of **a** and **b**

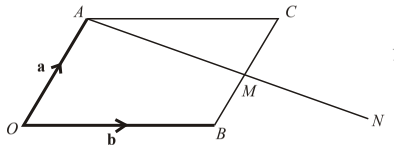
(b) Write the vector $\overrightarrow{RU}$ in terms of **a** and **b**

(c) Find **two** vectors that can be written as 3**a** – **b**

(Total 4 marks)

10. *OACB* is a parallelogram and *M* is the mid-point of *BC*.

$\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**



(a) Express the following vectors in terms of **a** and **b**

(i) $\overrightarrow{BA}$

(iii) $\overrightarrow{AM}$

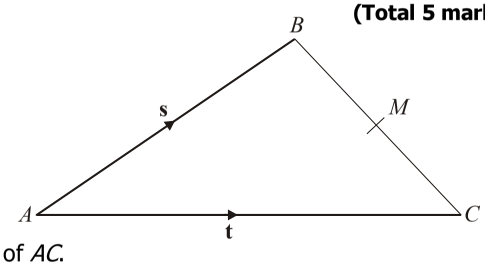
(b) *AM* is extended to *N*, where $\overrightarrow{AN} = 2\overrightarrow{AM}$.
Show that $\overrightarrow{BN}$ = **b**

(c) What does this tell you about the position of *N*?

(Total 5 marks)

11. In triangle *ABC*, *M* is the mid-point of *BC*.

$\overrightarrow{AB}$ = **s** and $\overrightarrow{AC}$ = **t**



(a) Find $\overrightarrow{AM}$ in terms of **s** and **t.**
Give your answer in its simplest form.

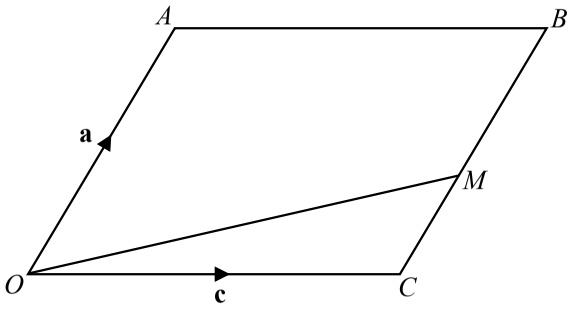
(b) $\overrightarrow{AD}$ = **s** + **t**
The length of *AB* is **not** equal to the length of *AC*.

(i) Write down the name of the shape *ABDC*.

(ii) Write down one fact about the points *A*, *M* and *D*.
Explain your answer.

(Total 6 marks)

1 $OABC$ is a parallelogram.



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OC} = \mathbf{c}$

M is the point on BC such that $BM : MC = 3 : 2$

The line AB is extended to the point N.
OMN is a straight line.

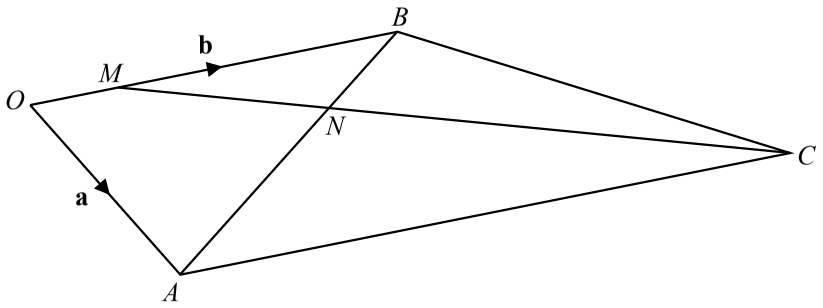
Find the ratio $AB : BN$.

Give your answer in the form $k : 1$

$AB : BN =$ .....

(Total for Question 1 is 4 marks)

2 $OABC$ is a quadrilateral.



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

MNC and ANB are straight lines.
M is the point on the line OB such that $OM : MB = 1 : 5$
N is the point such that $AN : NB = 3 : 2$
$\overrightarrow{OB}$ is parallel to $\overrightarrow{AC}$.

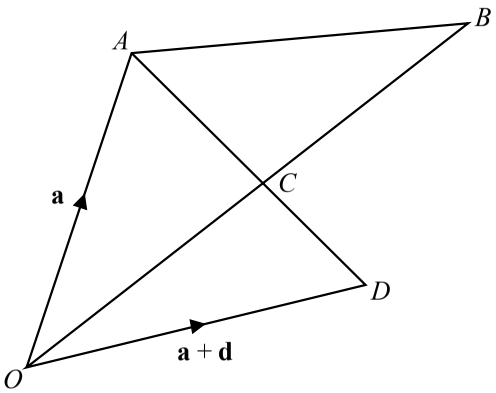
Work out the ratio $OB : AC$.

Give your answer in the form $k : 1$

$OB : AC =$ .....

(Total for Question 2 is 5 marks)

3 *OAD* is a triangle.



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OD} = \mathbf{a} + \mathbf{d}$

C is the point on the line *AD* such that *AC* : *AD* = 2 : 3
OCB is a straight line such that *OC* : *CB* = 3 : 5

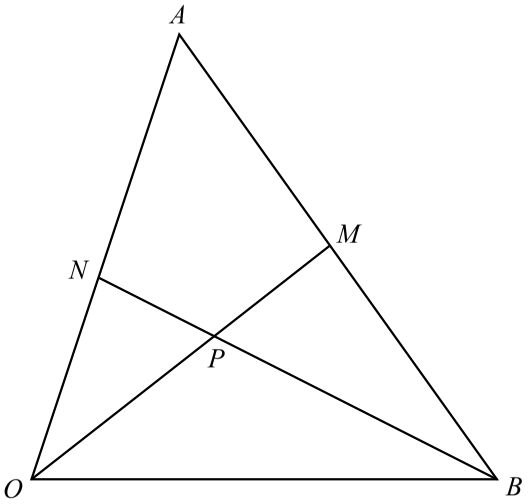
The vector $\overrightarrow{AB} = \lambda\mathbf{a} + \mu\mathbf{d}$

Work out the ratio $\lambda : \mu$
Give your answer as a ratio in its simplest form.

$\lambda : \mu =$ .....

(Total for Question 3 is 4 marks)

4 OAB is a triangle.



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

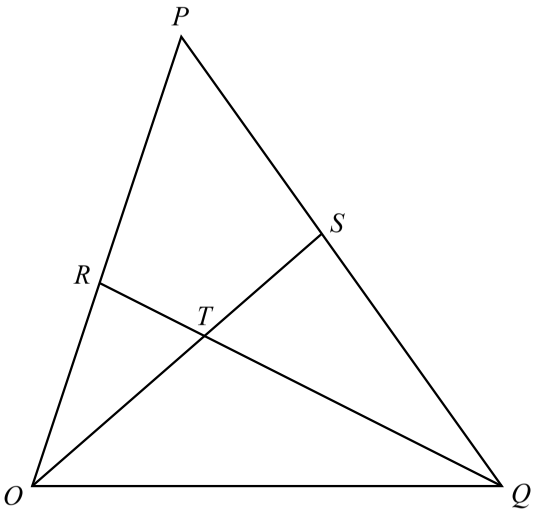
M is the midpoint of AB.
OPM and BPN are straight lines.
P is the point such that $OP : PM = 4 : 3$

Find the ratio $ON : NA$

$ON : NA =$ .....

(Total for Question 4 is 5 marks)

5 OPQ is a triangle.



$\overrightarrow{OP} = \mathbf{p}$ $\overrightarrow{OQ} = \mathbf{q}$

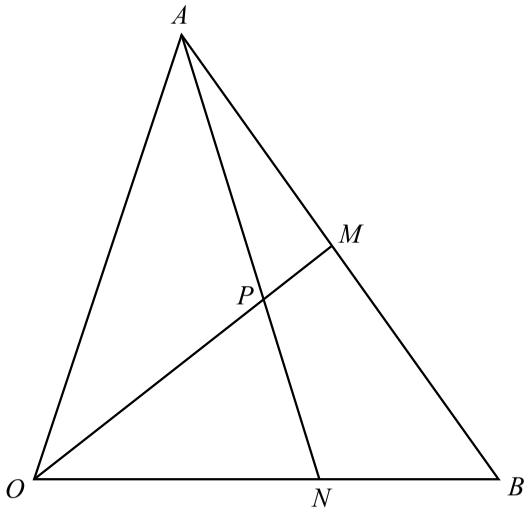
OTS and RTQ are straight lines.
R is the point on OP such that $OR : RP = 3 : 4$
T is the point such that $TQ : RQ = 3 : 4$

Find the ratio $PS : SQ$
Give your answer in the form $a : b$ where a and b are integers.

$PS : SQ =$ .....

(Total for Question 5 is 5 marks)

6 OAB is a triangle.



OPM and APN are straight lines.
M is the midpoint of AB.

$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

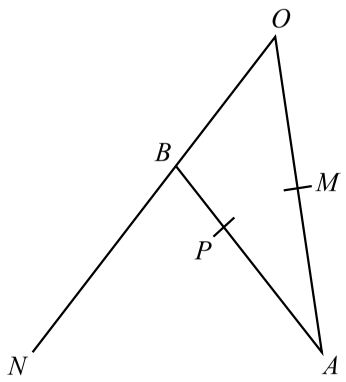
$OP : PM = 5 : 2$

Work out the ratio $ON : NB$

.....

(Total for Question 6 is 5 marks)

7



OBN, *OMA* and *APB* are straight lines.
$BN = 2OB$.
M is the midpoint of *OA*.

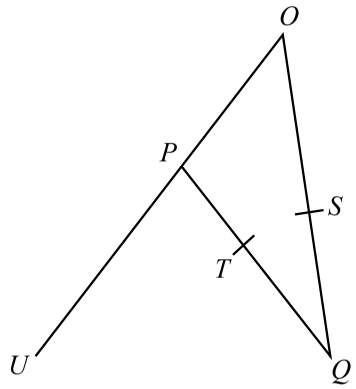
$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

Given that *MPN* is a straight line, work out the ratio *AP* : *PB*

.....

(Total for Question 7 is 5 marks)

8



OPU, *OSQ* and *PTQ* are straight lines.

$OP : PU = 3 : 4$
$OS : OQ = 5 : 8$

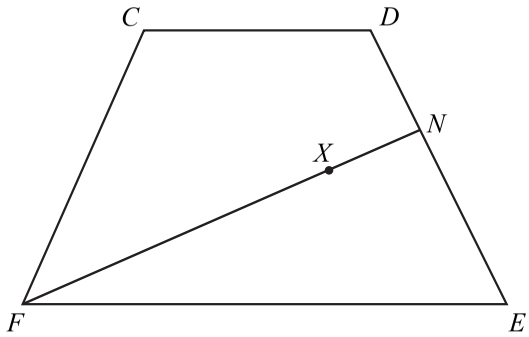
$\overrightarrow{OP} = \mathbf{p}$ $\overrightarrow{OQ} = \mathbf{q}$

Given that *STU* is a straight line, work out the ratio $PT : TQ$

.....

(Total for Question 8 is 5 marks)

9 *CDEF* is a quadrilateral.



$\overrightarrow{CD} = \mathbf{a}$, $\overrightarrow{DE} = \mathbf{b}$ and $\overrightarrow{FC} = \mathbf{a} - \mathbf{b}$.

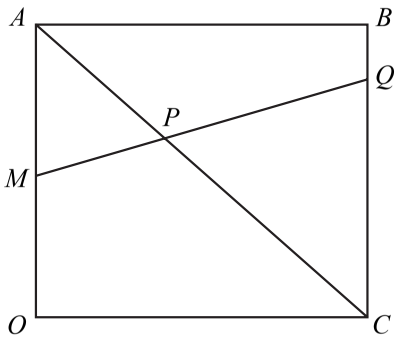
N is the point on DE such that $DN : NE = 2 : 3$
X is the point on FN such that $FX : XN = k : 1$
CXE is a straight line.

Work out the value of k.

$k =$ .....

(Total for Question 9 is 5 marks)

10 $OABC$ is a square.



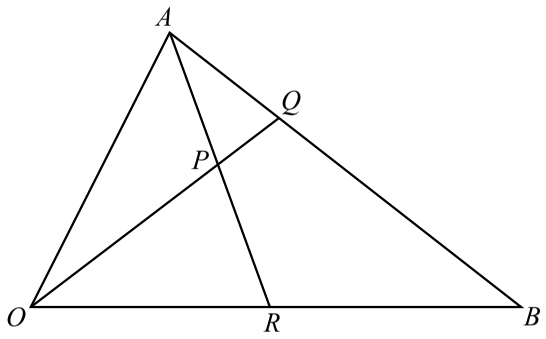
$\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = \mathbf{c}$.

M is the midpoint of OA.
Q divides BC in the ratio $BQ : QC = 1 : 3$
APC and MPQ are straight lines.

Show that P divides AC in the ratio 2 : 3

(Total for Question 10 is 5 marks)

11 OAB is a triangle.



$\overrightarrow{OA} = \mathbf{a}$ $\overrightarrow{OB} = \mathbf{b}$

OPQ and APR are straight lines.

The point P divides APR in the ratio $AP : PR = 3 : 4$
The point Q divides AQB in the ratio $AQ : QB = 2 : 5$

Find the ratio $OR : RB$
Give your answer in its simplest form.

.....

(Total for Question 11 is 6 marks)