KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 25 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Contents

1 Exponential and Trigonometric Graphs

Exponential Graphs

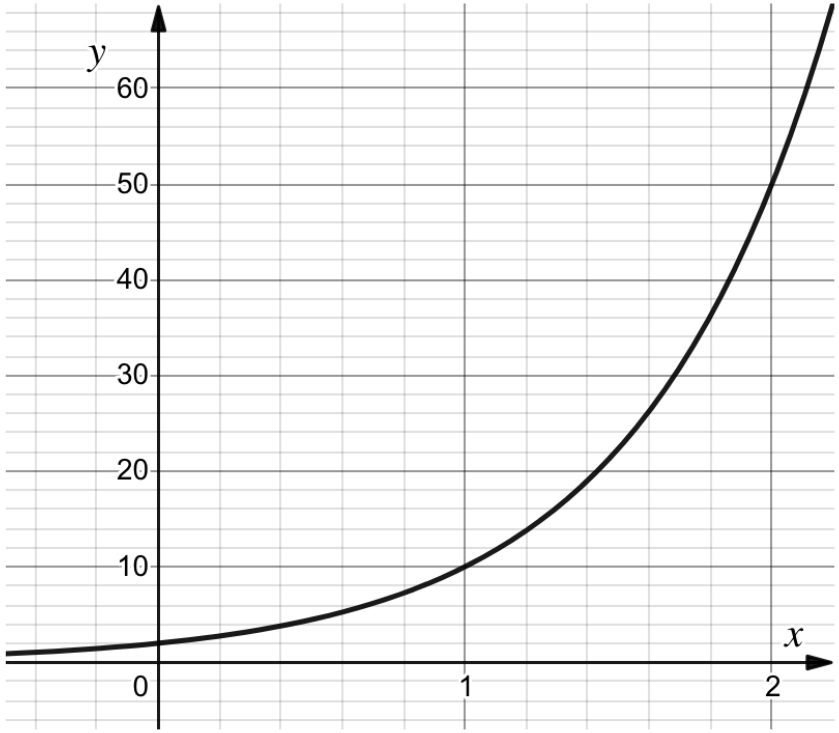
Worked Example	Your Turn
Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 1750\times(1.02)^t$ State the initial amount of money invested.	Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 3000\times(1.005)^t$ State the initial amount of money invested.

Worked Example	Your Turn
Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 500\times(1.04)^t$ Determine the interest rate offered by the bank.	Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 1250\times(1.025)^t$ Determine the interest rate offered by the bank.

Worked Example	Your Turn
The sketch graph shows a curve with equation $y = ab^x$ The curve passes through the points $(0, 3.25)$ and $(3, 87.75)$. Calculate the value of a and the value of b.	The sketch graph shows a curve with equation $y = ab^x$ The curve passes through the points $(0, 2.75)$ and $(2, 68.75)$. Calculate the value of a and the value of b.

Worked Example

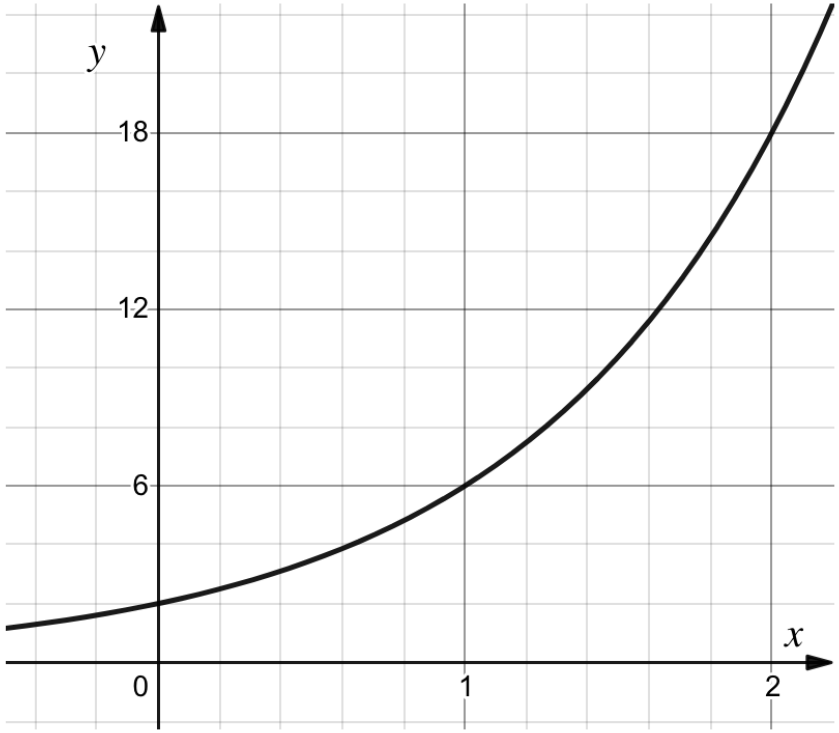
This graph shows a curve with equation $y = pq^x$



Calculate the value of p and q.

Your Turn

This graph shows a curve with equation $y = pq^x$



Calculate the value of p and q.

Worked Example	Your Turn
At the start of an experiment, a petri dish contained $4{,}000{,}000$ bacteria. After 4 days, there were $6{,}000{,}000$ bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 7 days after the start of the experiment, giving your answer to 3 significant figures.	At the start of an experiment, a petri dish contained $4{,}000{,}000$ bacteria. After 5 days, there were $13{,}000{,}000$ bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 11 days after the start of the experiment, giving your answer to 3 significant figures.

Trigonometric Graphs

Angle (θ Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
$\sin(\theta)$								
$\cos(\theta)$								
$\tan(\theta)$								

Worked Example

Sketch the graph $y = \sin(x)$ for $-360° \leq x \leq 360°$

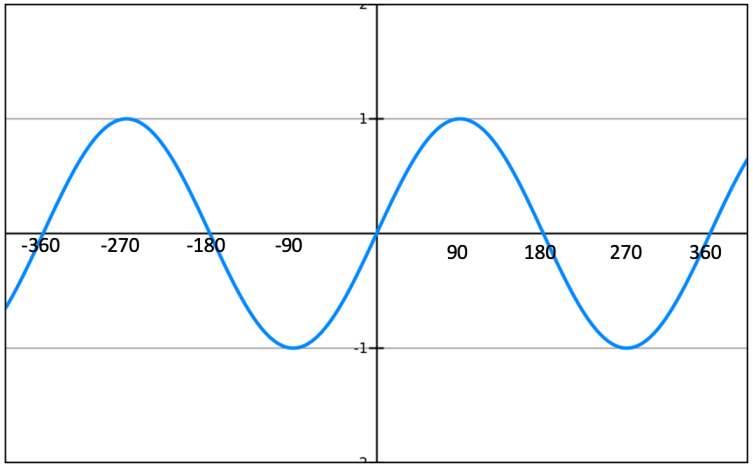
Worked Example

Sketch the graph $y = \cos(x)$ for $-360° \leq x \leq 360°$

Worked Example

Sketch the graph $y = \tan(x)$ for $-360° \leq x \leq 360°$

Worked Example



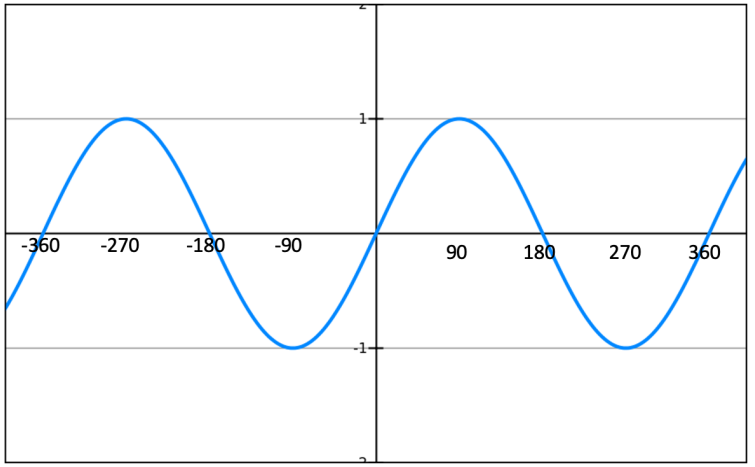
Suppose we know that $\sin(30) = 0.5$. By thinking about symmetry in the graph, work out:

a) $\sin(150) =$

b) $\sin(-30) =$

c) $\sin(210) =$

Your Turn



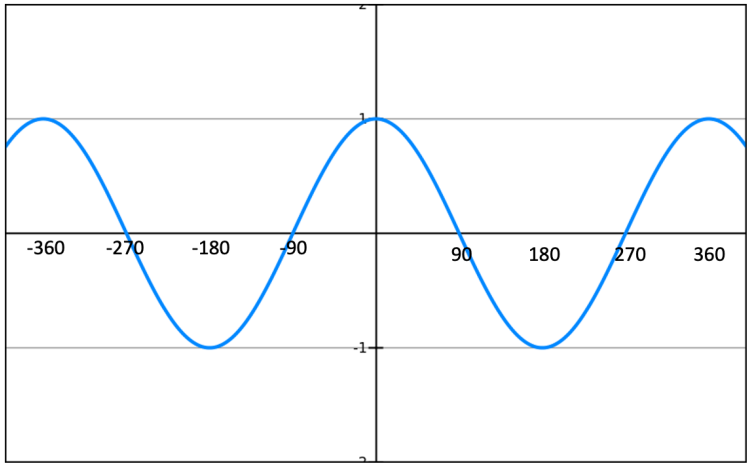
Suppose we know that $\sin(60) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out:

a) $\sin(240) =$

b) $\sin(120) =$

c) $\sin(-60) =$

Worked Example



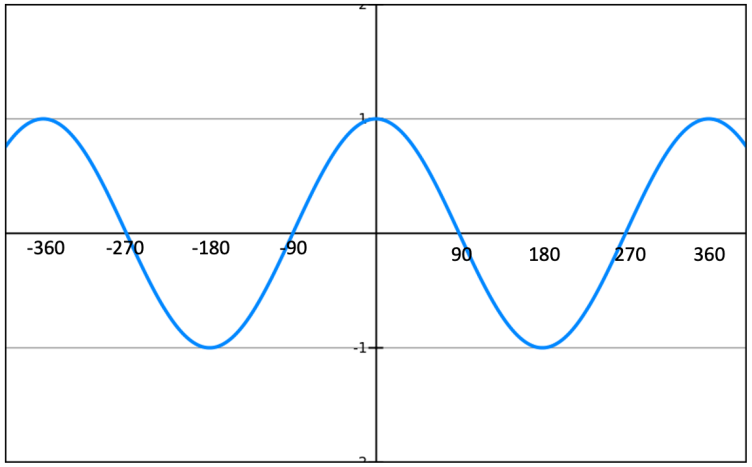
Suppose we know that $\cos(60) = 0.5$. By thinking about symmetry in the graph, work out:

a) $\cos(120) =$

b) $\cos(-60) =$

c) $\cos(240) =$

Your Turn



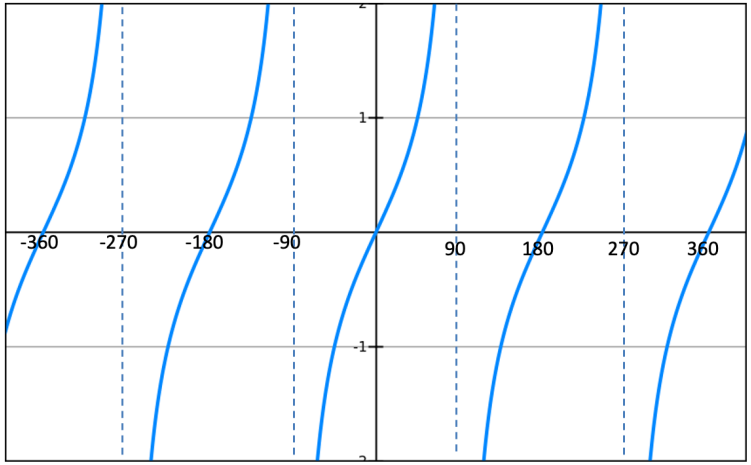
Suppose we know that $\cos(30) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out:

a) $\cos(-30) =$

b) $\cos(210) =$

c) $\cos(150) =$

Worked Example

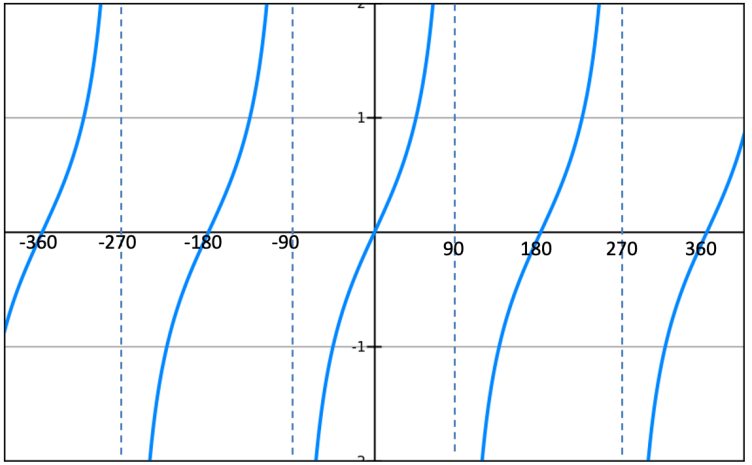


Suppose we know that $\tan(30) = \frac{1}{\sqrt{3}}$. By thinking about symmetry in the graph, work out:

a) $\tan(-30) =$

b) $\tan(150) =$

Your Turn



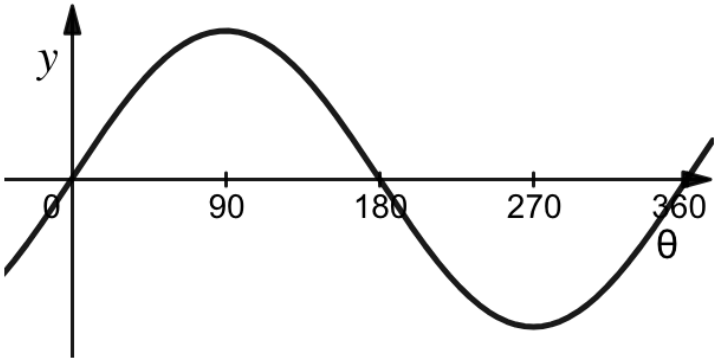
Suppose we know that $\tan(60) = \sqrt{3}$. By thinking about symmetry in the graph, work out:

a) $\tan(120) =$

b) $\tan(-60) =$

Worked Example

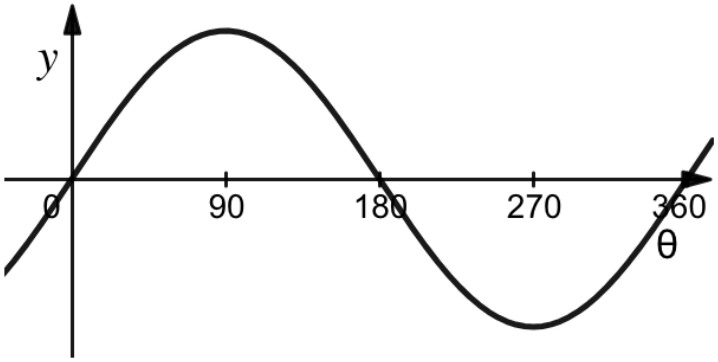
Here is a graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$.



Solve $\sin\theta = -0.8$ for $0° \leq \theta \leq 360°$.
Give your solutions correct to 2 decimal places.

Your Turn

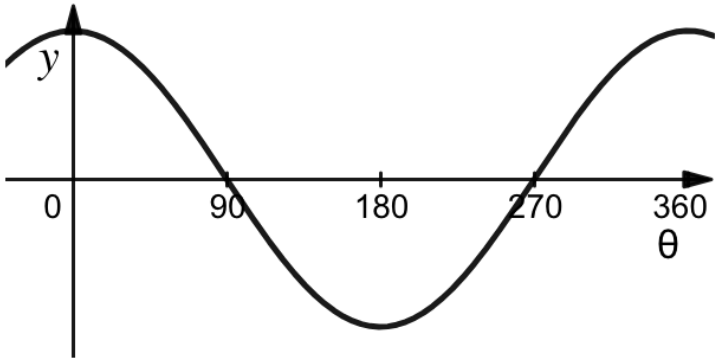
Here is a graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$.



Solve $\sin\theta = 0.3$ for $0° \leq \theta \leq 360°$.
Give your solutions correct to 2 decimal places.

Worked Example

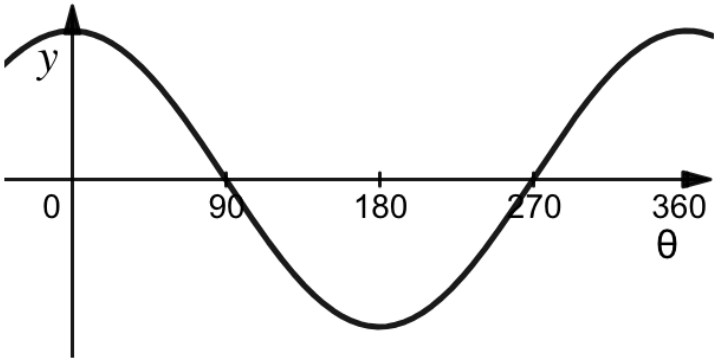
Here is a graph of $y = \cos\theta$ for $0° \leq \theta \leq 360°$.



Solve $\cos\theta = 0.7$ for $0° \leq \theta \leq 360°$.
Give your solutions correct to 2 decimal places.

Your Turn

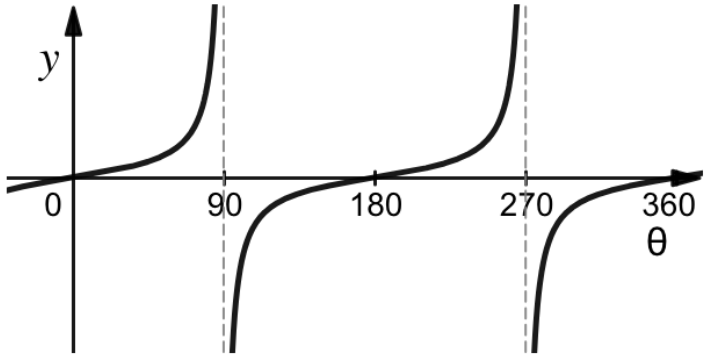
Here is a graph of $y = \cos\theta$ for $0° \leq \theta \leq 360°$.



Solve $\cos\theta = -0.2$ for $0° \leq \theta \leq 360°$.
Give your solutions correct to 2 decimal places.

Worked Example

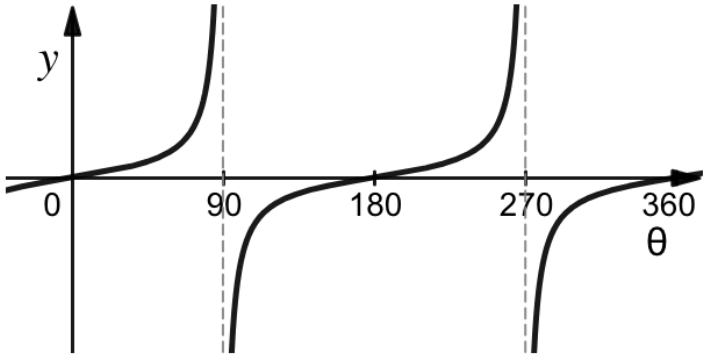
Here is a graph of $y = \tan\theta$ for $0° \leq \theta \leq 360°$.



Solve $\tan\theta = -7$ for $0° \leq \theta \leq 360°$.
Give your solutions correct to 2 decimal places.

Your Turn

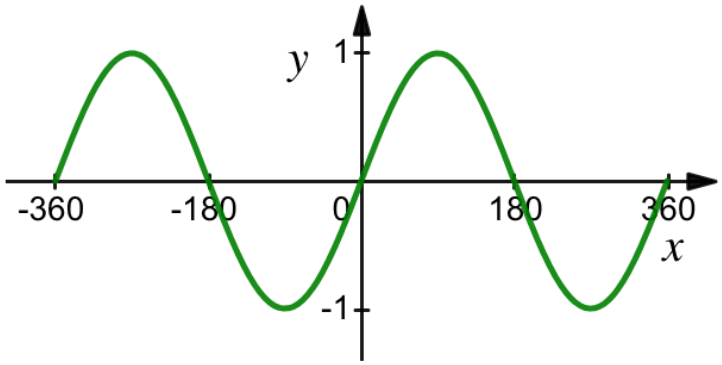
Here is a graph of $y = \tan\theta$ for $0° \leq \theta \leq 360°$.



Solve $\tan\theta = 6$ for $0° \leq \theta \leq 360°$.
Give your solutions correct to 2 decimal places.

Worked Example

Here is the graph of $y = \sin x$ for the interval $-360 \leq x \leq 360°$.

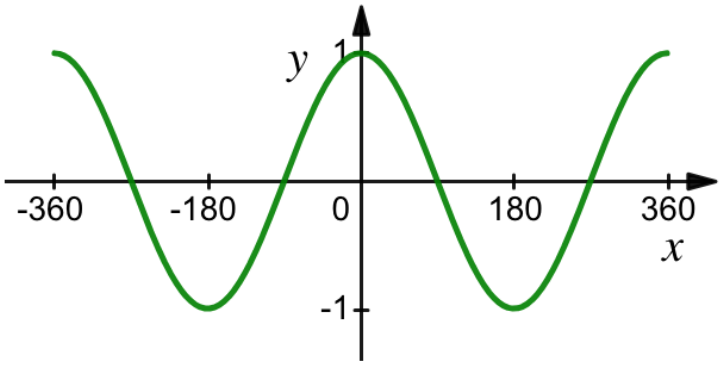


$x = 203.6$ is a solution to the equation $\sin x = -0.4$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$.
Give your solutions to one decimal place.

Your Turn

Here is the graph of $y = \cos x$ for the interval $-360 \leq x \leq 360°$.

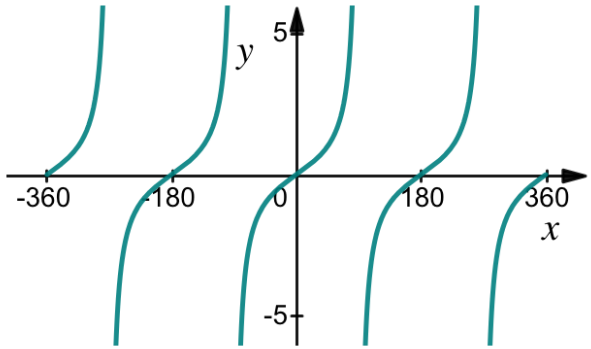


$x = 107.5$ is a solution to the equation $\cos x = -0.3$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$.
Give your solutions to one decimal place.

Worked Example

Here is the graph of $y = tan\, x$ for the interval $-360 \leq x \leq 360°$.

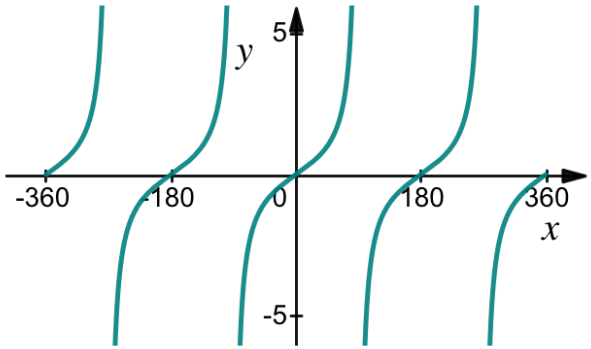


$x = 105.95$ is a solution to the equation $\tan x = -3.5$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$.
Give your solutions to two decimal places.

Your Turn

Here is the graph of $y = tan\, x$ for the interval $-360 \leq x \leq 360°$.

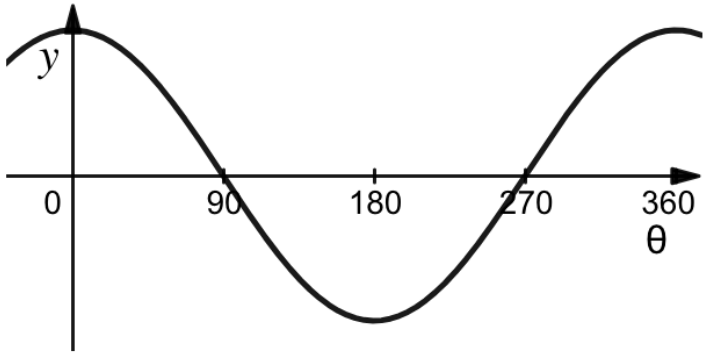


$x = 56.31$ is a solution to the equation $\tan x = 1.5$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$.
Give your solutions to two decimal places.

Worked Example

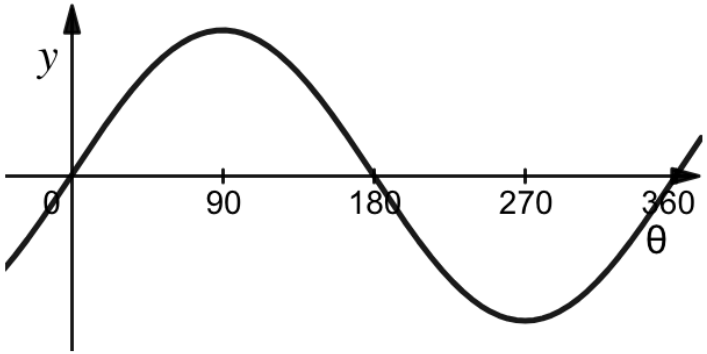
Here is a graph of $y = \cos\theta$ for $0° \leq \theta \leq 360°$.



Solve $\cos\theta = -0.4$ for $0° \leq \theta \leq 720°$.
Give your solutions correct to 2 decimal places.

Your Turn

Here is a graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$.



Solve $\sin\theta = -0.7$ for $0° \leq \theta \leq 720°$.
Give your solutions correct to 2 decimal places.

Extra Notes

5 Graph Transformations

Worked Example	Your Turn
The point $A(2, 5)$ is on the graph of $y = f(x)$. Write the new coordinates of A after the transformation: a) $y = f(x) + 3$ b) $y = f(x + 3)$ c) $y = -f(x)$ d) $y = f(-x)$ e) $y = -f(x) + 3$ f) $y = f(-x) + 3$	The point $A(3, 4)$ is on the graph of $y = f(x)$. Write the new coordinates of A after the transformation: a) $y = f(x) - 4$ b) $y = f(x - 4)$ c) $y = f(-x)$ d) $y = -f(x)$ e) $y = -f(x) - 6$ f) $y = -f(-x) - 6$

Dr Frost 448d, 448c, 449c and 449d

Worked Example	Your Turn
Sketch $y = \cos x + 1, 0 \leq x \leq 360°$	Sketch $y = \sin x - 2, 0 \leq x \leq 360°$

Worked Example	Your Turn
Sketch $y = \sin(x - 45°)$, $0 \leq x \leq 360°$	Sketch $y = \cos(x + 45°)$, $0 \leq x \leq 360°$

Worked Example	Your Turn
Sketch $y = -\sin x, 0 \leq x \leq 360°$	Sketch $y = -\tan x, 0 \leq x \leq 360°$

Worked Example	Your Turn
Sketch $y = \cos(-x)$, $0 \leq x \leq 360°$	Sketch $y = \tan(-x)$, $0 \leq x \leq 360°$

Worked Example

The graph of $y = f(x)$ is shown in Figure 1.

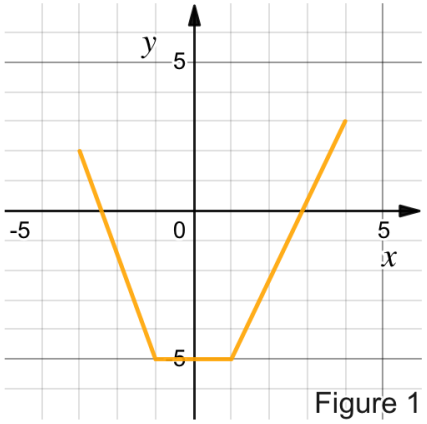


Figure 1

The graph of $y = f(x) + a$ is shown in Figure 2.

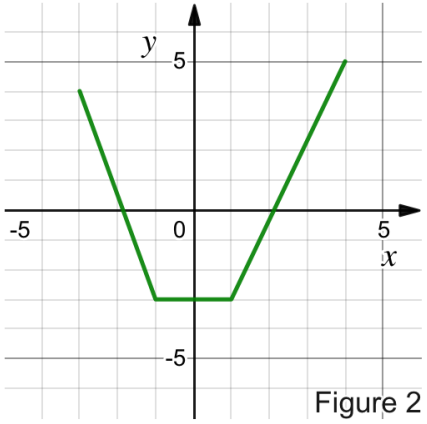


Figure 2

Determine the value of a.

Your Turn

The graph of $y = f(x)$ is shown in Figure 1.

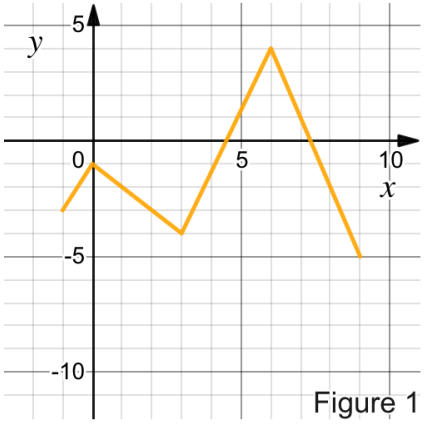


Figure 1

The graph of $y = f(x) + a$ is shown in Figure 2.

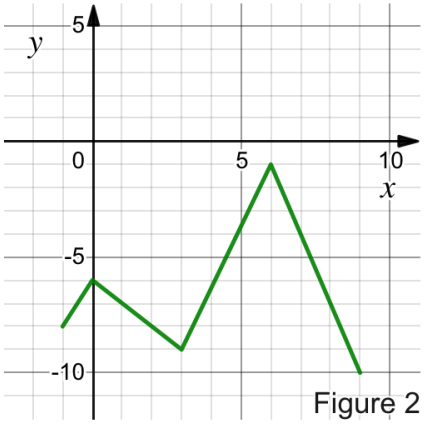


Figure 2

Determine the value of a.

Worked Example

The graph of $y = f(x)$ is shown in Figure 1.

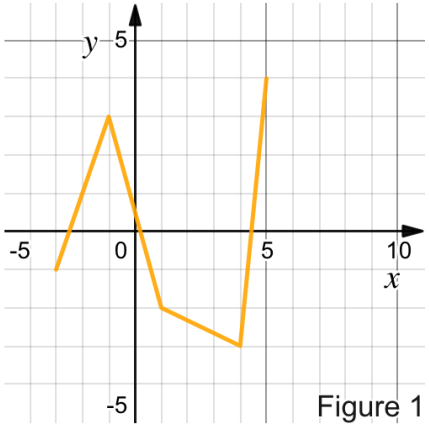


Figure 1

The graph of $y = f(x + a)$ is shown in Figure 2.

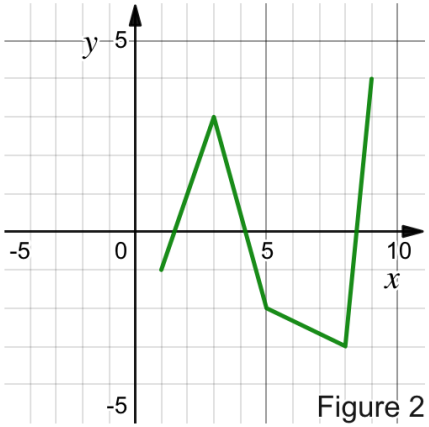


Figure 2

Determine the value of a.

Your Turn

The graph of $y = f(x)$ is shown in Figure 1.

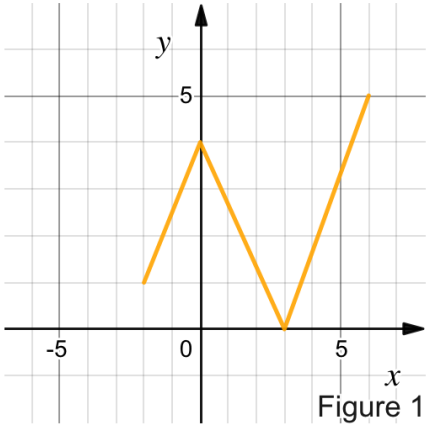


Figure 1

The graph of $y = f(x + a)$ is shown in Figure 2.

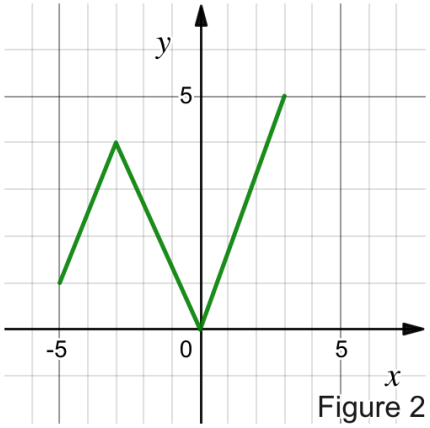


Figure 2

Determine the value of a.

Worked Example	Your Turn
a) The curve $y = \cos(4x + 90)$ is translated by $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$ State the equation of the new curve after this transformation.	a) The curve $y = \tan(3x - 30)$ is translated by $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ State the equation of the new curve after this transformation.
b) The curve $y = \frac{4}{2x-5}$ is translated by $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ State the equation of the new curve after this transformation.	b) The curve $y = \frac{1}{3x-3}$ is translated by $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$ State the equation of the new curve after this transformation.

Worked Example	Your Turn
The curve $y = 2x^2 + 3x$ is translated by $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$ State the equation of the new curve after this transformation.	The curve $y = 2x^3 - x^2$ is translated by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ State the equation of the new curve after this transformation.

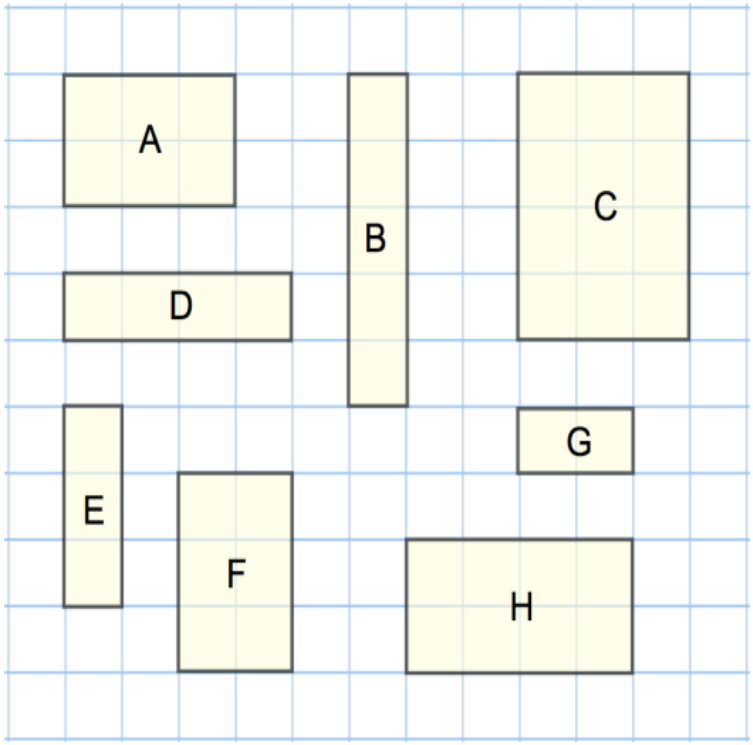
Fill in the Gaps

$f(x)$	Function notation	Description of translation	Vector of translation	New function
$2x+1$	$f(x-3)$	3 places right	$\begin{pmatrix}3\\0\end{pmatrix}$	$f(x-3) = 2(x-3)+1$ $= 2x-6+1$ $= 2x-5$
$3x+1$	$f(x-2)$			
x^2	$f(x-1)$			
x^2		2 places left		
x^2+5			$\begin{pmatrix}-3\\0\end{pmatrix}$	
				$4(x+5)+2$
x^2+2x-1		1 place left		
	$f(x-4)$			

Extra Notes

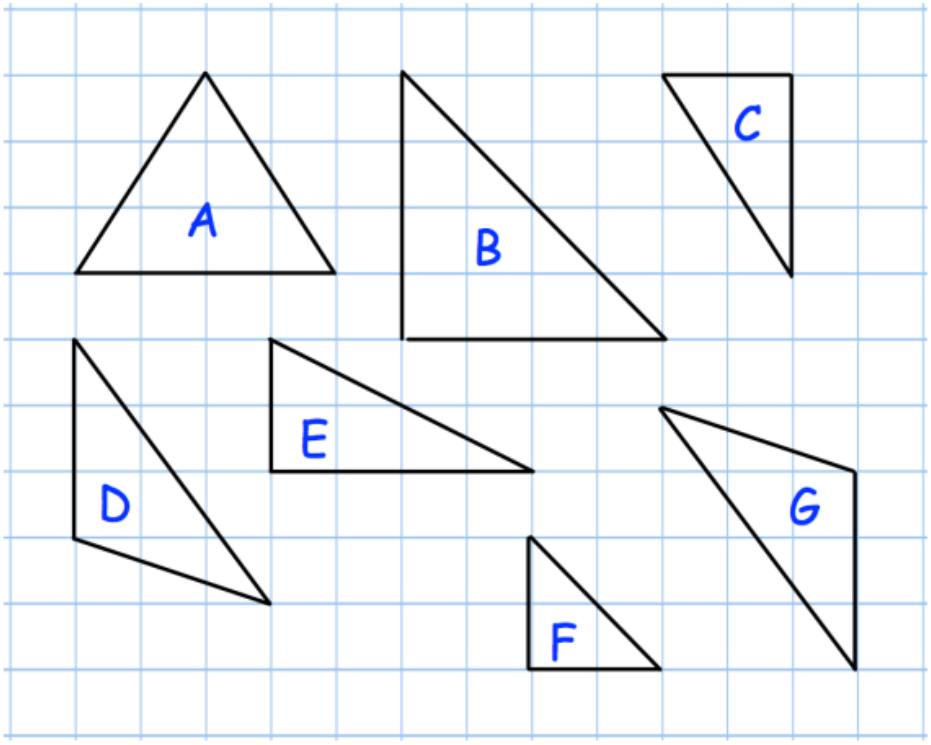
6 Congruency

Worked Example



a) Which two shapes are congruent?
b) Which two shapes are similar?

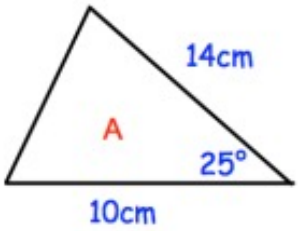
Your Turn

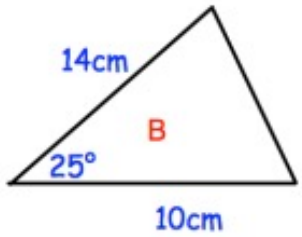


a) Which two shapes are congruent?
b) Which two shapes are similar?

Worked Example

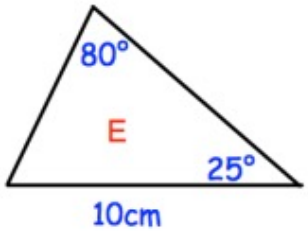
State the condition why these two triangles are congruent.

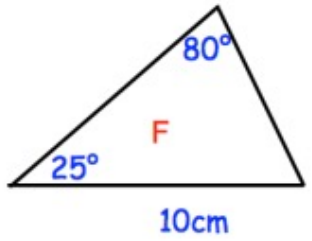




Your Turn

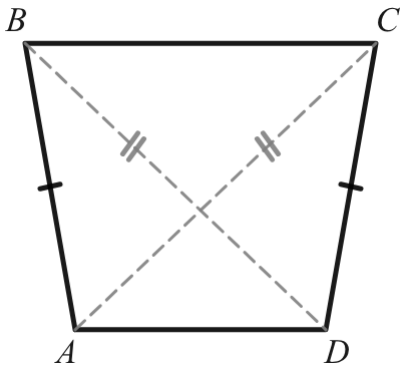
State the condition why these two triangles are congruent.





Worked Example

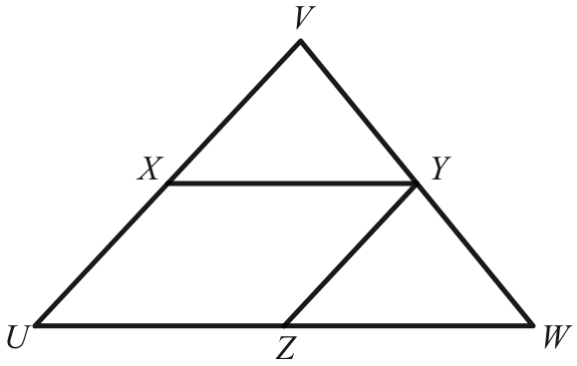
Below is a quadrilateral $ABCD$.



Prove that triangle ABD is congruent to triangle ACD.

Your Turn

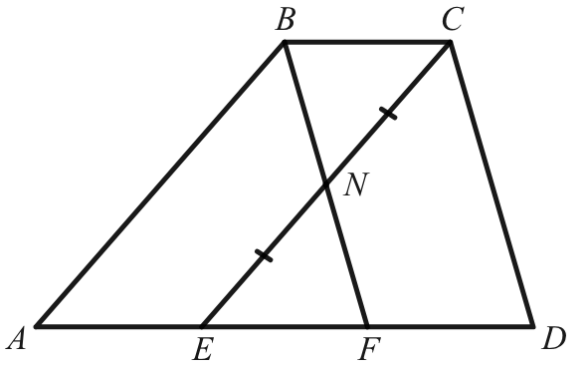
The diagram shows triangle UVW.



$UXYZ$ is a parallelogram where
X is the midpoint of UV,
Y is the midpoint of VW,
and Z is the midpoint of UW.
Prove that triangle XVY and triangle ZYW are congruent.

Worked Example

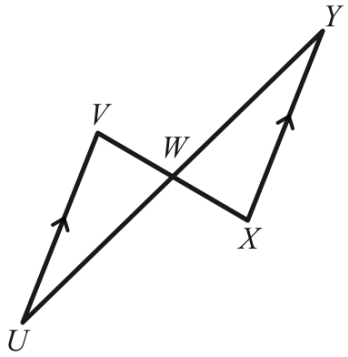
In the diagram below, $ABCE$ and $BCDF$ are parallelograms and N is the midpoint of CE.



Prove that triangle BCN and triangle EFN are congruent.

Your Turn

In the diagram below, UWY and VWX are straight lines.



UV and XY are parallel.
W is the midpoint of UWY.
Prove that triangle UVW and triangle WXY are congruent.

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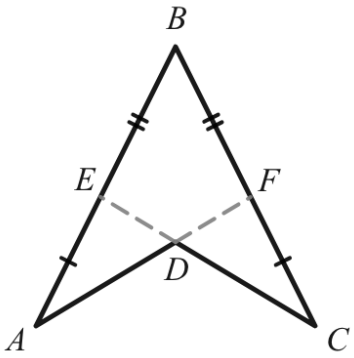
Worked Example

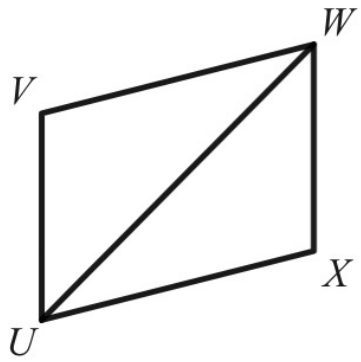
Below is a quadrilateral $ABCD$.



Prove that triangle ABF and triangle BCE are congruent.

Your Turn

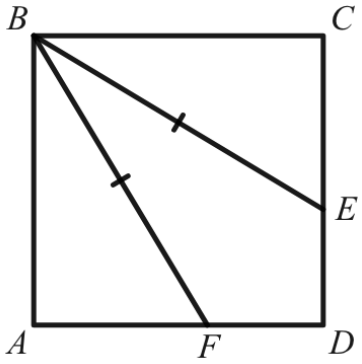
The diagram below shows a parallelogram $UVWX$.



Prove that triangle UVW and triangle UWX are congruent.

Worked Example

Below is a square $ABCD$.

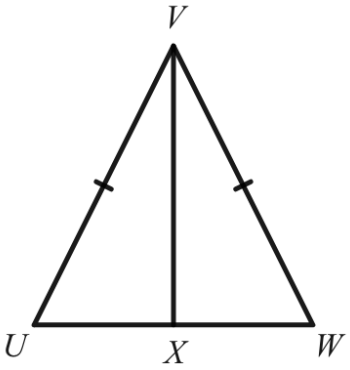


$BE = BF$

Prove that triangle ABF and triangle BCE are congruent.

Your Turn

Below is an isosceles triangle UVW.



VX is the perpendicular bisector of UW.

Prove that triangle UVX and triangle VWX are congruent.

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Extra Notes

7 Circle Theorem Proofs

Worked Example

Prove angles in a semicircle are 90°.

Worked Example

Prove the angle at the centre of a circle is twice the angle at the circumference.

Worked Example

Prove angles in the same segment are equal.

Worked Example

Prove opposite angles of a cyclic quadrilateral add to 180°.

Worked Example

Prove the alternate segment theorem.

Extra Notes