



Year 11 2024 Mathematics 2025 Unit 26 Booklet





Tasks



Dr Frost Course



Name:

Class:

Contents

- 1 <u>Differentiation (L2FM Only)</u>
- 2 Matrices (L2FM Only)

1 Differentiation (L2FM Only)

Basic Differentiation

Worked Example	Your Turn
Differentiate with respect to x: a) x^4 b) $5x^4$ c) $4x - 5$ d) $\frac{1}{x^3}$ e) $\frac{6}{5x^3}$	Differentiate with respect to x: a) x^5 b) $-3x^5$ c) $6x - 7$ d) $\frac{1}{x^4}$ e) $\frac{7}{8x^4}$

Your Turn
Your Turn Differentiate with respect to x: a) $y = 5x^4 - 2x^7 + 12345 - x^5$ b) $y = (3x - 5)^2$ c) $f(x) = x^2(x - 3)$

Worked Example	Your Turn
Worked Example Differentiate with respect to x: a) $f(x) = \frac{(3x-2)^2}{5x}$ b) $y = \frac{3x^4(2x^5-11x)}{x^2}$	Your Turn Differentiate with respect to x: a) $f(x) = \frac{(2x+3)^2}{5x}$ b) $f(x) = \frac{2x^2(3x^3-7x)}{x}$

Worked Example	Your Turn
Find an expression for the rate of change of y with respect to x: $y = (x - 3)(x - 4)^2$	Find an expression for the rate of change of y with respect to x: $y = (x + 1)(x + 2)^2$

Worked Example	Your Turn
a) Find the gradient of the curve: $y = 2x^3 - x + 5$ at $(-1, 4)$ b) Find the coordinates of the point(s) where the gradient is 4: $y = 5x^2 - x + 7$	a) Find the gradient of the curve: $y = 3x^2 - 2x + 1$ at $(-2, 17)$ b) Find the coordinates of the point(s) where the gradient is 3: $y = 3x^2 - 9x + 7$

Worked Example

Let $f(x) = 4x^2 - 8x + 3$

- a) Find the gradient of y = f(x) at the point $\left(\frac{1}{2}, 0\right)$
- b) Find the coordinates of the point on the graph of y = f(x) where the gradient is 8.
- c) Find the gradient of y = f(x) at the points where the curve meets the line y = 4x 5.

Your Turn

Let $f(x) = x^2 - 4x + 2$

- a) Find the gradient of y = f(x) at the point (1, -1)
- b) Find the coordinates of the point on the graph of y = f(x) where the gradient is 5.
- c) Find the gradient of y = f(x) at the points where the curve meets the line y = 2 x.

Worked Example	Your Turn
Worked Example Find the coordinates of the point(s) where the gradient is 10: $y = x^3 + 6x^2 - 11x + 7$	Your Turn Find the coordinates of the point(s) where the gradient is 2: $y = x^3 - 3x^2 - 7x + 8$

A curve has gradient function $\frac{dy}{dx} = 3x^2 + 7$ Work out the values of x for which the rate of change of y with respect to x is 55 A curve has gradient function $\frac{dy}{dx} = 5x^2 - 7$ Work out the values of x for which the rate of change of y with respect to x is 38	Worked Example	Your Turn
	A curve has gradient function $\frac{dy}{dx} = 3x^2 + 7$ Work out the values of x for which the rate of change of y with respect to x is 55	A curve has gradient function $\frac{dy}{dx} = 5x^2 - 7$ Work out the values of <i>x</i> for which the rate of change of <i>y</i> with respect to <i>x</i> is 38

Worked Example	Your Turn
Worked Example $y = 3x^2 + bx$ The rate of change of y with respect to x when $x = 4$ is triplethe rate of change of y with respect to x when $x = -2$.Work out the value of b	Your Turn $y = 2x^3 + ax$ The rate of change of y with respect to x when $x = 2$ is twicethe rate of change of y with respect to x when $x = -1$.Work out the value of a

Fill in the Gaps

Tangents and Normals

	Worked Example		Your Turn
a) b)	Worked Example Find the equation of the tangent to the curve $y = x^4$ when $x = 2$ Find the equation of the normal to the curve $y = x^4$ when $x = 2$	a) b)	Your Turn Find the equation of the tangent to the curve $y = x^3$ when $x = 2$ Find the equation of the normal to the curve $y = x^3$ when $x = 2$

Worked Example	Your Turn
Worked Example Find the equation of the tangent to the curve with equation $y = x^3 - 5x^2 - 3x + 2$ at the point $(5, -13)$	Your TurnFind the equation of the tangent to the curve with equation $y = x^3 - 3x^2 + 2x - 1$ at the point (3, 5)

Worked Example	Your Turn
Worked Example The tangent to the curve $y = \frac{1}{4}x^3 + x^2 - 3x$ at the point P is parallel to the line with equation $y = -\frac{17}{4}x + 2$. Find the two possible values for the <i>x</i> -coordinate of the point P.	Your Turn The tangent to the curve $y = \frac{1}{5}x^3 - x^2 - x$ at the point P is parallel to the line with equation $y = 4x - 9$. Find the two possible values for the <i>x</i> -coordinate of the point P.

Worked Example	Your Turn
Worked ExampleThe point P with x-coordinate $\frac{1}{4}$ lies on the curve with equation $y = 2x^2$. The normal to the curve at P intersects the curve at points P and Q. Find the coordinates of Q	Your TurnThe point P with x-coordinate $\frac{1}{2}$ lies on the curve with equation $y = 4x^2$. The normal to the curve at P intersects the curve at points P and Q. Find the coordinates of Q

Worked Example	Your Turn
Worked Example The curve $y = 4x^3 - 7$ intersects the y-axis at C. The tangent to the curve at P(3, 101) intersects the y-axis at D. Work out the length of CD.	Your Turn The curve $y = 2x^3 - 5$ intersects the y-axis at C. The tangent to the curve at P(2, 11) intersects the y-axis at D. Work out the length of CD.

Worked Example	Your Turn
Worked Example Point B lies on the curve $y = x^3 - 5x + 4$ The x-coordinate of B is -8 Show that the equation of the normal to the curve at B is $187y + x = -87524$	Your TurnPoint A lies on the curve $y = x^2 + 5x + 8$ The x-coordinate of A is -4 Show that the equation of the normal to the curve at A is $3y - x = 16$

Increasing and Decreasing Functions

Worked Example	Your Turn
Find the values of x for which the function $f(x) = x^2 - 5x + 8$ is increasing	Find the values of x for which the function $f(x) = x^2 + 8x - 5$ is decreasing

Worked Example	Your Turn
Find the interval(s) on which the function $f(x) = x^3 - 6x^2 - 135x + 1$ is increasing	Find the interval(s) on which the function $f(x) = x^3 + 6x^2 - 135x - 2$ is increasing

Worked Example	Your Turn
Find the interval on which the function $f(x) = x^3 - 3x^2 - 9x - 10$ is decreasing	Find the interval on which the function $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing

Worked Example	Your Turn
Show that the function $f(x) = x^3 + 26x - 1$ is increasing for all real values of x	Show that the function $f(x) = x^3 + 16x - 2$ is increasing for all real values of x

Worked Example	Your Turn
Worked Example Show that the function $f(x) = x^3 - 3x^2 + 8x - 5$ is increasing for all real values of x	Your Turn Show that the function $f(x) = x^3 + 6x^2 + 21x + 2$ is increasing for all real values of x

Worked Example	Your Turn
Worked Example Show that the function $5 - x(4x^2 + 3)$ is decreasing for all $x \in \mathbb{R}$	Your Turn Show that the function $3 + 4x(-x^2 - 5)$ is decreasing for all $x \in \mathbb{R}$

Stationary Points

Worked Example	Your Turn
Find the least value of $f(x) = x^2 + 6x - 9$	Find the least value of $f(x) = x^2 - 4x + 9$

Worked Example	Your Turn
Worked ExampleFind the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = 2x^3 + 6x^2 - 4$	Your TurnFind the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = x^3 + 3x^2 - 4$

Worked Example	Your Turn
Find the stationary points on the curve $y = \frac{5}{3}x^3 - 80x$	Find the stationary points on the curve $y = x^3 - 12x$

Worked Example	Your Turn
Worked Example Find the coordinates of the turning points of $y = x^3 - 6x^2 - 15x$	Your Turn Find the coordinates of the turning points of $y = x^3 + 6x^2 - 135x$

Worked Example	Your Turn
Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = \frac{2}{3}x^3 - 3.5x^2 + 3x + 5$	Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = x^3 + \frac{1}{2}x^2 - 2x + 4$

Worked Example	Your Turn
$y = 4ax^3 + \frac{3}{2}$	$y = 8ax^3 + \frac{6}{-}$
x y has a minimum when $x = \frac{1}{2}$	x y has a minimum when $x = \frac{1}{2}$
Work out the value of <i>a</i>	Work out the value of a
Second Derivative and Maxima/Minima

Worked Example	Your Turn
Worked Example Show that one of the stationary points of the curve with equation $y = x^3 - 3x^2 + 45x$ is $(-3, -159)$, and by testing the gradient of the curve either side of the stationary point, determine whether it is a maximum or a minimum.	Your Turn Show that one of the stationary points of the curve with equation $y = x^3 + 3x^2 - 45x$ is $(3, -81)$, and by testing the gradient of the curve either side of the stationary point, determine whether it is a maximum or a minimum.

Worked Example	Your Turn
$y = 5x^3 - 7x^2 - x + 4$	$y = 2x^3 - 3x^2 + x + 9$
Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$	Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$

Worked Example	Your Turn
Worked Example Find the coordinates of the stationary points on the curve with equation $y = 4x^3 + 30x^2 + 48x - 3$ and use the second derivative to determine their nature	Your Turn Find the coordinates of the stationary points on the curve with equation $y = 2x^3 - 15x^2 + 24x + 6$ and use the second derivative to determine their nature

Fill in the Blanks

Maximum Points

Equation of Curve	$\frac{dy}{dx}$	$\frac{dy}{dx} = 0$	<i>x</i> -coordinate	y-coordinate	Maximum or Minimum Point
$y = x^2 - 10x + 2$				<i>y</i> = -23	Minimum
$y = 3x^2 + 12x + 20$					
$y = 15 - 2x - x^2$					
$y = 3 + 8x - 2x^2$					
$y = x^2 + 12x + \bigcirc$				y = -6	Minimum
$y = x^2 - 9x + \square$				$y = -\frac{21}{4}$	
$y = x^2 - \bigcup x + 15$			x = 4		
$y = \square + \square x - x^2$			<i>x</i> = 2	<i>y</i> = 10	

Fill in the Gaps					
Function	Derivative	Derivative at $x = 2$	2 nd Derivative	Function at $x = 1$	Stationary Point(s)
$y = x^2 - 2x + 10$					
	6 <i>x</i> ²			2	
	5 <i>x</i>			$-\frac{3}{2}$	
		8	6 <i>x</i> – 2	-10	
	$x^2 - 3x + 2$			1	
			-2	7	<i>x</i> = 3
	3			5	

Optimisation

Worked Example	Your Turn
$U = 81y + \frac{49}{y}, y > 0$	$V = 49x + \frac{81}{x}, x > 0$
Use calculus to show that V has a minimum value and work out the minimum value of V	Use calculus to show that V has a minimum value and work out the minimum value of V

Worked Example

An F shape is made from rectangles.

All lengths are in centimetres.

The perimeter of the shape is $128\ cm$

The area of the shape is $A cm^2$

- a) Find an expression for *y*
- b) Hence find an expression for *A*.
- c) Use calculus to derive an expression for the rate of change of *A* as *x* varies.
- d) Hence work out the maximum area





Worked Example

A cuboid is to be made with volume $81 \ cm^3$

The cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cmThe volume of the cuboid is 81 cm^3

a) Show that the total length, L, of the twelve edges of the cuboid is given by $L = 12x + \frac{162}{r^2}$

b) Given that *x* can vary, use differentiation to find the maximum or minimum value of *L*

c) Justify that the value of L you have found is a minimum

Your Turn

A cuboid is to be made from $54m^2$ of sheet metal.

The cuboid has a horizontal base and no top.

The height of the cuboid is x metres.

Two of the opposite vertical faces are squares.

a) Show that the volume, V m³, of the tank is given by $V = 18x - \frac{2}{3}x^3$

b) Given that x can vary, use differentiation to find the maximum or minimum value of V

c) Justify that the value of V you have found is a maximum

Graph Sketching

Worked Example	Your Turn
Worked Example Sketch the following graph, labelling all intercept(s) and any turning point(s): $y = 2x^3 - 3x^2 - 11x + 6$	Your Turn Sketch the following graph, labelling all intercept(s) and any turning point(s): $y = 2x^3 - 3x^2 - 11x + 6$

Worked Example	Your Turn
Sketch the graph of $y = \frac{1}{x} + \frac{256}{3}x^3$ labelling the stationary points.	Sketch the graph of $y = \frac{1}{x} + 27x^3$ labelling the stationary points.

Extra Notes

2 Matrices (L2FM Only)

Multiplication of Matrices

Worked Example	Your Turn
Find: $5\begin{pmatrix} -1 & 2\\ 3 & -4 \end{pmatrix}$	Find: $4\begin{pmatrix} 3 & 4\\ -2 & -5 \end{pmatrix}$

Worked Example	Your Turn
Find: $ \begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix} \begin{pmatrix} 5 \\ -6 \end{pmatrix} $	Find: $ \begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix} \begin{pmatrix} 5 \\ -6 \end{pmatrix} $

Worked Example	Your Turn
Find: $\begin{pmatrix} 2 & 3 \\ -4 & -5 \end{pmatrix} \begin{pmatrix} -1 & 7 \\ 8 & -6 \end{pmatrix}$	Find: $ \begin{pmatrix} 5 & -3 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -7 & 6 \end{pmatrix} $

Worked Example	Your Turn
Find:	Find:
$\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}^4$	$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^4$

Worked Example	Your Turn
Find the value of <i>t</i> , given that $\begin{pmatrix} -5 & 3 \\ 7 & -4 \end{pmatrix} \begin{pmatrix} 4 & t \\ 7 & 5 \end{pmatrix} = I$	Find the value of <i>t</i> , given that $\begin{pmatrix} -7 & 4 \\ 5 & -3 \end{pmatrix} \begin{pmatrix} -3 & -4 \\ -5 & t \end{pmatrix} = I$

Worked Example	Your Turn
$M = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ Show that $M^3 = I$	$M = \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$ Show that $M^3 = I$

Worked Example	Your Turn
$2\begin{pmatrix} 5 & 0 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ Find the values of a and b	$3\begin{pmatrix} 2 & 1 \\ -5 & 0 \end{pmatrix} \begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ Find the values of <i>a</i> and <i>b</i>

Worked Example	Your Turn
Worked Example Work out all solutions for x and y, given that $\begin{pmatrix} x & 1 \\ -4 & y \end{pmatrix} \begin{pmatrix} x \\ -3 \end{pmatrix} = \begin{pmatrix} 2x \\ 9 \end{pmatrix}$	Your Turn Work out all solutions for x and y, given that $\begin{pmatrix} x & 3 \\ 1 & y \end{pmatrix} \begin{pmatrix} x \\ -4 \end{pmatrix} = \begin{pmatrix} 4x \\ 8 \end{pmatrix}$

Worked Example	Your Turn
$ \begin{pmatrix} 2 & 1 \\ 8x & 5x \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} $ Work out the possible values for x and y	$ \begin{pmatrix} 1 & 2 \\ x & 3x \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix} $ Work out the possible values for x and y



Transformations

Worked Example	Your Turn
Worked ExampleFind a 2×2 matrix that represents:a) A reflection in the y -axisb) A reflection in the line $y = x$	Your TurnFind a 2×2 matrix that represents:a) A reflection in the x-axisb) A reflection in the line $y = -x$

Worked Example	Your Turn
Find a 2×2 matrix that represents: Rotation 90° anticlockwise about the origin	Find a 2×2 matrix that represents: Rotation 270° anticlockwise about the origin

Worked Example	Your Turn
Describe geometrically the effect of the following matrices: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Describe geometrically the effect of the following matrices: $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

Worked Example	Your Turn
The transformation matrix $\begin{pmatrix} a & -2 \\ 1 & 1 \end{pmatrix}$ maps the point (2, 5) onto the point (3, b). Work out the values of a and b .	The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point (3, 4) onto the point (2, b). Work out the values of a and b .

Worked Example	Your Turn
The matrix $\begin{pmatrix} -a & 3b \\ 2a & b \end{pmatrix}$ maps the point (2, 3) onto the point (-31, -1). Work out the values of a and b.	The matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point (5, 4) onto the point (1, 17). Work out the values of a and b.



Worked Example	Your Turn
A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$	A triangle T has vertices (1, 1), (1, 2) and (2, 2)
 a) Find the vertices of the image of T under the transformation given by the matrix M = (2 0) (0 3) b) Sketch T and its image, T' on a coordinate grid. c) Describe the geometric transformation. 	 a) Find the vertices of the image of T under the transformation given by the matrix M = (3 0) (0 2) b) Sketch T and its image, T' on a coordinate grid. c) Describe the geometric transformation.
Combinations of Transformations

Worked Example	Your Turn
A triangle T has vertices (1, 1), (1, 2) and (2, 2).	A triangle T has vertices (1, 1), (1, 2) and (2, 2).
Find the vertices of the image of T under the transformation given by the matrix $M = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.	Find the vertices of the image of T under the transformation given by the matrix $M = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

Worked Example	Your Turn
A, B and C are transformations in the $x - y$ plane.	A, B and C are transformations in the $x - y$ plane.
A: Rotation through 270° anticlockwise about the origin. B: Reflection in the <i>y</i> -axis C: Transformation B followed by transformation A.	A: Rotation through 90° anticlockwise about the origin. B: Reflection in the <i>x</i> -axis C: Transformation A followed by transformation B.
Use matrix multiplication to show that C is equivalent to a single reflection.	Use matrix multiplication to show that C is equivalent to a single reflection.

Worked Example	Your Turn
Worked Example Use matrix multiplication to show that, in the $x - y$ plane, a reflection in the line $y = x$, followed by a rotation, 90° clockwise about the origin, followed by a reflection in the x -axis is equivalent to a transformation by the identity matrix.	Your Turn Use matrix multiplication to show that, in the $x - y$ plane, a reflection in the line $y = -x$, followed by a rotation, 90° anticlockwise about the origin, followed by a reflection in the <i>x</i> -axis is equivalent to a transformation by the identity matrix.

Extra Notes	