KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 26 Tasks

DO NOT WRITE INSIDE

Contents

1 Differentiation (L2FM Only)

Find the gradient function $\frac{dy}{dx}$ when:

(a) $y = x^4$ (b) $y = x^9$

(c) $y = x^7$ (d) $y = x^6$

(e) $y = x$ (f) $y = x^{10}$

Find the gradient function $\frac{dy}{dx}$ when:

(a) $y = 7x^2$

(b) $y = 3x^5$

(c) $y = 10x^6$

(d) $y = 2x^9$

(e) $y = \frac{1}{2}x^8$

(f) $y = \frac{1}{5}x^4$

(g) $y = 0.3x^5$

(h) $y = -6x^3$

Find the gradient function $\frac{dy}{dx}$ when:

(a) $y = x^2 + x^5$

(b) $y = 3x^2 + 7x^5$

(c) $y = 5x^4 - x^3$

(d) $y = 2x^3 - x^2 + 5x$

(e) $y = 3x + 6x^4$

(f) $y = 0.5x^7 + 3$

(g) $y = \frac{1}{4}x^5 - x^3 + 7x$

(h) $y = x^3 + 2x^2 - 7x + 10$

(a) Expand and simplify $(x + 3)(x^2 - 5)$

(b) Hence find the gradient function $\frac{dy}{dx}$ when $y = (x + 3)(x^2 - 5)$

Find $\frac{dy}{dx}$ when:

(a) $y = x^3(x + 2)$

(b) $y = 2x(x^5 - 4x^3)$

(c) $y = (x + 7)(x - 3)$

(d) $y = (3x - 5)(2x + 1)$

(e) $y = (x^2 + 3)(x - 5)$

(f) $y = x(x + 4)(x - 4)$

Find $\frac{dy}{dx}$ when:

(a) $y = \frac{8x^5+6x^2}{2}$

(b) $y = \frac{x^4- 2x^3}{x}$

(c) $y = \frac{10x^4- 5x^3}{2x}$

(d) $y = \frac{9x^7+ 2x^3}{3x^2}$

(e) $y = \frac{4x^2(x-7)}{2x}$

Find $\frac{dy}{dx}$ when:

(a) $y = \frac{7}{x}$ (b) $y = -\frac{3}{x^2}$

(c) $y = \frac{5}{2x}$ (d) $y = \frac{4}{5x^3}$

(e) $y = 2x^5 + x^3 - \frac{3}{x}$

(f) $y = 7x^2 + 4x + \frac{5}{2x}$

(g) $y = 6x^3 + \frac{1}{x} - \frac{5}{x^2}$

(h) $y = (x + 3)\left(x + \frac{1}{x}\right)$

(i) $y = \frac{10x^4+4x^2+2}{2x}$

Fluency Practice

1 For each of the following, find the gradient function $\frac{dy}{dx}$, and hence find the gradient of the tangent to the curve when $x = 2$

a $y = 2x^5 \rightarrow$?

b $y = 7x^3 \rightarrow$?

c $y = 3x + 4 \rightarrow$?

d $y = 2x + x^2 \rightarrow$?

e $y = 2x^3 + x^{-1} \rightarrow$?

f $y = \frac{1}{2}x^{-2} \rightarrow$?

g $y = 2\pi x^2 - 1 \rightarrow$?

2 The tangent to the curve $y = x^3 + 2x^2 - x$ has gradient 6. Determine the possible values of x.

?

3 For the curve $y = x^2 + x - 2$, determine:

(a) The gradient of the tangent to the curve at the point $(1,0)$

?

(b) The point on the curve where the gradient is 5.

?

4 Find the points on the curve $y = \frac{1}{3}x^3 + x^2 - 4x$ where the gradient is 11.

?

(a) Find the equation of the tangent to the curve $y = x^2 + 2x - 3$ at the point $(1, 0)$.

(b) Find the equation of the tangent to the curve $y = x^2 + 4x - 5$ at the point $(-1, -7)$.

(c) Find the equation of the tangent to the curve $y = x^3 + x$ at the point $(2, 10)$.

(a) Find the equation of the normal to the curve $y = x^2 - 4$ at the point $(1, -3)$.

(b) Find the equation of the normal to the curve $y = x^2 - 5x - 6$ at the point $(3, -12)$.

(c) Find the equation of the normal to the curve $y = 2x^3 - 3x + 1$ at the point $(1, 0)$.

(a) Find the equation of the tangent to the curve $y = x^2 + \frac{1}{x}$ at the point where $x = 1$.

(b) Find the equation of the normal to the curve $y = x(x + 2)(x - 1)$ at the point where $x = -2$.

(a) Find the equation of the tangent to the curve $y = 3x - x^2$ at the point $x = 2$.

(b) The tangent crosses the x-axis and y-axis at A and B respectively. Find the area of the triangle AOB.

Fluency Practice

1 [IGCSEFM June 2012 Paper 1 Q8] A curve has equation $y = x^3 + 5x^2 + 1$

(a) When $x = -1$, show that the value of $\frac{dy}{dx}$ is -7.

?

(b) Work out the equation of the tangent to the curve $y = x^3 + 5x^2 + 1$ at the point where $x = -1$.

?

2 [IGCSEFM June 2013 Paper Q8] A curve has equation $y = x^4 - 5x^2 + 9$

(a) Work out $\frac{dy}{dx}$. $\frac{dy}{dx} =$?

(b) Work out the equation of the tangent to the curve at the point where $x = 2$

Give your answer in the form $y = mx + c$

?

3 [IGCSEFM Set Paper 1 Q11] Show that the tangents to the curve $y = x^3 + 3x^2 + 3x + 1$ at $x = 1$ and $x = -3$ are parallel.

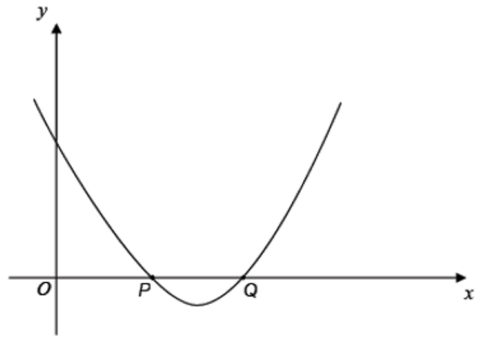
?

4 [IGCSEFM Set 1 Paper 2 Q17] Work out the equation of the normal to the curve $y = 2x^3 - x^2 + 1$ at the point (1, 2). Give your answer in the form $y = mx + c$.

?

Fluency Practice

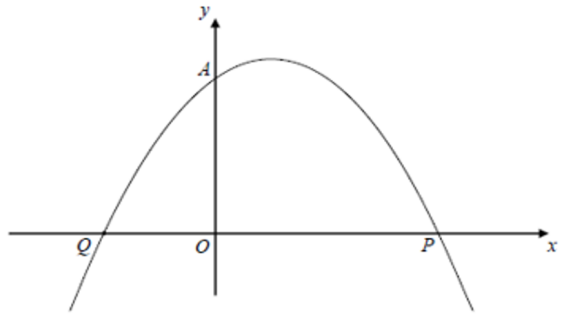
5 [IGCSEFM Set 2 Paper 1 Q15] The graph shows a sketch of $y = (x-2)(x-3)$. The curve intersects the x-axis at P and Q.



Show that the tangents at P and Q are perpendicular.

?

6 [IGCSEFM Set 4 Paper 2 Q20] A sketch of the curve $y = (x+1)(2-x)$ is shown. $A(0,2), P(2,0)$ and Q are points on the curve.

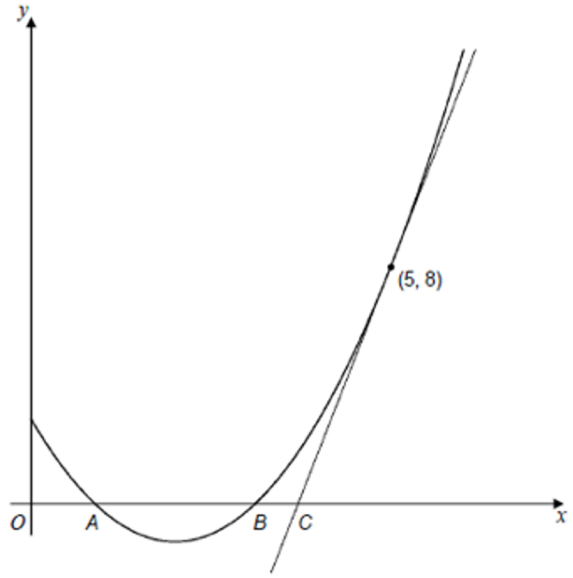


(a) Write down the coordinates of point Q.
(b) Show that the normal to the curve at A intersects the curve again at P.

?

Fluency Practice

7



[IGCSEFM Specimen Paper 2 Q22] The diagram shows the graph of $y = x^2 - 4x + 3$
The curve cuts the x-axis at the points A and B.
The tangent to the curve at the point (5,8) cuts the x-axis at the point C.
Show that $AB = 3BC$.

?

Fluency Practice

1 $y = \frac{1}{3}x^3 + 3x^2 + 10x + 1$

a) Find $\frac{dy}{dx}$?

b) By completing the square, hence show that this function is increasing for all x.

?

2 $f(x) = x^2 + 3x + 5$

a) Find $f'(x)$?

b) Hence or otherwise, determine the values of x for which $f(x)$ is increasing.

?

3 $y = x^2 - x + 2$

Determine the values of x for which y is decreasing.

?

4 $y = x^3 - 7x^2 - 5x$

Determine the values of x for which y is increasing.

?

5 Show that $f(x) = x^3 - 9x^2 + 39x - 4$ is an increasing function for all values of x.

?

6 [AQA IGCSEFM June 2012 Paper 2 Q20] For what values of x is $y = 150x - 2x^3$ an increasing function?

?

7 [Set 4 Paper 1 Q10] $y = 10 - 8x - x^3$ for all values of x. Show that y is a decreasing function for all values x.

?

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ when:

(a) $y = x^2 + 4x - 3$

(b) $y = 5x^3 + x^2 + 8x - 3$

(c) $y = x^4 - 7x^2$

(d) $y = x^2 - \frac{2}{x}$

Find the coordinates of the stationary points on each of these curves. By differentiating for a second time, establish whether these points are maximums or minimums.

(a) $y = 4x^2 - 8x$

(b) $y = 5 + 2x - x^2$

(c) $y = (8 + x)(2 - x)$

(d) $y = x^4 - 8x^2$

(e) $y = 2x^3 - 3x^2 - 12x + 5$

(f) $y = x + \frac{1}{x}$

(a) Find the coordinates of the stationary point on the curve $y = x^3 + 3x^2 + 3x + 1$.

(b) By considering the gradient either side of the stationary point, show that the stationary point is a point of inflection.

(a) Find the coordinates of the stationary point on the curve $y = (2 - x)^3$.

(b) By considering the gradient either side of the stationary point, show that the stationary point is a point of inflection.

(a) Find the gradient of the curve $y = x^2 - 3x + 7$ at the point $(3, 7)$

(b) Find the gradient of the curve $y = x^3 + 4x^2 - 9x$ at the point $(2, 6)$

(c) Find the gradient of the curve $y = x + \frac{9}{x}$ at the point $(3, 6)$

(a) Find the coordinates of the minimum point on the curve $y = x^2 - 4$

(b) Find the coordinates of the minimum point on the curve $y = x^2 + 8x + 15$

(c) Find the coordinates of the maximum point on the curve $y = 7 - 6x - x^2$

(d) Find the coordinates of the maximum point on the curve $y = 2 + 5x - x^2$

(a) Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. By sketching the graph, determine whether each point is a minimum point or a maximum point.

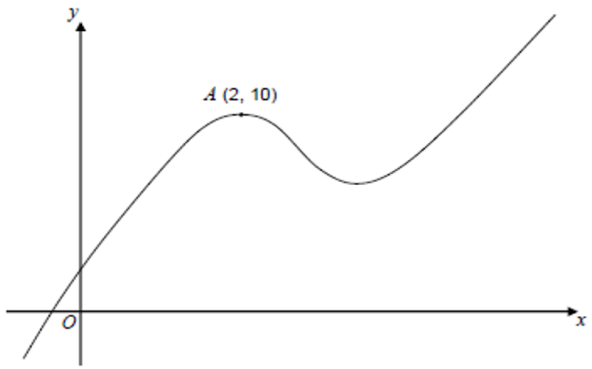
(b) Find the coordinates of the stationary point on the curve $y = 3x + \frac{12}{x^2}$. Is this point a minimum point or a maximum point?

(a) The curve with equation $y = x^2 + ax + b$ has a stationary point at $(-4, -11)$. Find the values of a and b.

(b) The curve with equation $y = c + dx - x^2$ has a stationary point at $(3, 10)$. Find the values of c and d.

Fluency Practice

1 [Set 4 Paper 2 Q22] A sketch of $y = f(x)$, where $f(x)$ is a cubic function, is shown.



There is a maximum point at $A(2,10)$.

(a) Write down the equation of the tangent to the curve at A. ?

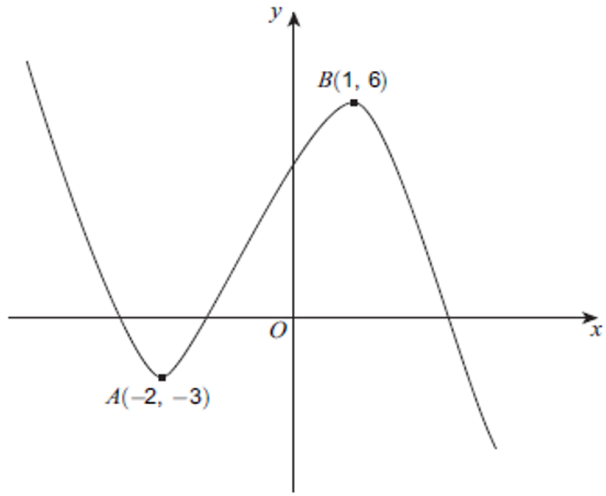
(b) Write down the equation of the normal to the curve at A. ?

(c) Circle the word that describes the cubic function when $x < 2$.

positive negative increasing decreasing

?

2 [June 2013 Paper 2 Q8] A sketch of $y = f(x)$ is shown. There are stationary points at A and B.



(a) Write down the equation of the tangent to the curve at A. ?

(b) Write down the equation of the normal to the curve at B. ?

(c) Circle the range of values of x for which $f(x)$ is an increasing function.

$x < -2$, $-2 < x < 1$, $-3 < x < 6$

$x > 1$

?

Fluency Practice

3 [Set 2 Paper 2 Q12] A curve has equation

$$y = x^3 - 9x^2 + 24x - 16$$

(a) Work out $\frac{dy}{dx}$?

(b) Work out the coordinates of the two stationary points on the curve.

?

4 [Set 1 Paper 2 Q14] (a) Work out the stationary points on the curve $y = x^3 - 12x$.

(b) Sketch the curve $y = x^3 - 12x$

?

6 [Specimen Paper 1 Q13] (a) Work out the coordinates of the stationary point for the curve $y = x^2 + 3x + 4$.

(b) Explain why $x^2 + 3x + 4 = 0$ has no real solutions.

?

(a) A rectangle has a width x cm and a length $(30 - 2x)$ cm. Using calculus, find the maximum area of the rectangle.
(b) A car sales company sells x cars per week. Its revenue R per week is given by the equation $R = 0.2x^2 - 10x + 1750$. Using differentiation, find the number of cars which generates the maximum revenue, and the value of this revenue.

(a) The cost C of a car journey when driving at a speed of x mph is given by $C = \frac{720}{x} + 0.2x + 6$. Using differentiation, find the value of x that minimises the cost, and the minimum value of C.
(b) The volume of a box is given by $V = x(5 - x)^2$. Use calculus to find the maximum volume of the box, and the value of x for which this occurs.

(a) A picture frame has a perimeter of 120 cm. If the width of the frame is x cm, then show that the height of the frame is $(60 - x)$ cm. Hence use calculus to find the value of x that gives a maximum area for the frame. Calculate this maximum area.
(b) A farmer has enough stone for 80 m of dry stone walling. He wants to create a field with the largest area possible. Find the dimensions of the field that gives this maximum area.

Optimisation

One of the most useful applications of differentiation is optimisation.

It allows us to exactly calculate the ideal size for a tin or a box to minimise surface area, and therefore minimise the cost of production.

Dimensions

Baked Beans:
$Radius = 3.8cm$
$Height = 11cm$

Tuna:
$Radius = 4.3cm$
$Height = 3.5cm$

1. Calculate the volume and surface area of the baked beans tin, to 2 significant figures.
Hint: consider the net of a cylinder.

2. Find an expression for the surface area of a cylinder of radius r and volume $500cm^3$.
Hint: start by writing the height in terms of r.

3. Use differentiation to find the optimal radius and the resulting minimum surface area.
Hint: stationary points occur when the derivative equals 0.

4. Calculate the potential saving in steel if tuna were to be sold in optimal cylindrical tins (of the same volume) rather than using current dimensions.
Hint: use scale factors to generalise the result you've already found.

Extension:
The majority of tins are cylindrical in shape – an optimal circular prism (cylinder with equal diameter and height) has a surface area almost 8% smaller than that of an optimal rectangular-based prism (cube). However, even among cylindrical packaging, some are far from optimal (eg Pringles tubes, spice jars, olive oil bottles). Why is this?

2 Matrices (L2FM Only)

Write down the order of these matrices.

(a) $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 3 \\ 2 & -2 \\ 4 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 2 & 6 \end{pmatrix}$ (d) $\begin{pmatrix} 0.5 & 0 \\ 1.5 & 1 \end{pmatrix}$

Work out:

(a) $\begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 0 \end{pmatrix}$ (b) $\begin{pmatrix} -2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

(c) $\begin{pmatrix} -1 & 0 \\ 4 & 7 \end{pmatrix} + \begin{pmatrix} 2 & -3 \\ 0 & -2 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & 0.5 \\ -0.5 & 3 \end{pmatrix} - \begin{pmatrix} 2 & 1 \\ -4 & 0.5 \end{pmatrix}$

Work out:

(a) $2 \times \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$ (b) $4 \times \begin{pmatrix} 3 & -1 \\ 0.5 & 6 \end{pmatrix}$

(c) $-3 \times \begin{pmatrix} 11 \\ -8 \end{pmatrix}$ (d) $\frac{1}{2} \times \begin{pmatrix} 4 & -6 \\ 2 & 0 \\ 3 & 8 \end{pmatrix}$

Given that

$A = \begin{pmatrix} -2 & 3 \\ 0 & 5 \end{pmatrix}$ $B = \begin{pmatrix} 4 & -1 \\ -3 & 7 \end{pmatrix}$ $C = \begin{pmatrix} -2 & 0 \\ 8 & -3 \end{pmatrix}$

Find:

(a) $A + B - C$

(b) $C - B$

(c) $2A$

(d) $A + 2B$

(e) $3C + B$

(f) $4B - A$

(g) $2A + 3C$

(h) $-4B + A + 2B$

Is it possible to multiply the matrices shown?

(a) $\begin{pmatrix}2\\0\\4\end{pmatrix} \times \begin{pmatrix}5 & -1\\4 & 0\end{pmatrix}$

(b) $\begin{pmatrix}-7 & 4\end{pmatrix} \times \begin{pmatrix}2\\6\end{pmatrix}$

(c) $\begin{pmatrix}1 & 0\\4 & -3\end{pmatrix} \times \begin{pmatrix}3 & 2 & 5\\6 & 0 & -1\end{pmatrix}$

Work out:

(a) $\begin{pmatrix}4\\2\end{pmatrix} \times \begin{pmatrix}-2 & 5\end{pmatrix}$

(b) $\begin{pmatrix}0 & 3\\2 & 5\end{pmatrix} \times \begin{pmatrix}-1 & 3\\0 & 6\end{pmatrix}$

(c) $\begin{pmatrix}4 & 7 & -2\end{pmatrix} \times \begin{pmatrix}0\\1\\5\end{pmatrix}$

(d) $\begin{pmatrix}1 & -2\\3 & 7\end{pmatrix} \times \begin{pmatrix}-1 & 4\\0 & -2\end{pmatrix}$

(e) $\begin{pmatrix}0 & 2\\-5 & 3\end{pmatrix} \times \begin{pmatrix}1 & 6\\-3 & 0\end{pmatrix}$

(f) $\begin{pmatrix}-2 & 1\\8 & 0\end{pmatrix} \times \begin{pmatrix}-3 & 5\\1 & 2\end{pmatrix}$

(a) Given that

$\begin{pmatrix}-2 & a\\-4 & 3\end{pmatrix}\begin{pmatrix}3\\7\end{pmatrix} = \begin{pmatrix}22\\9\end{pmatrix}$

work out the value of a.

(b) Matrix **P** = $\begin{pmatrix}2 & 3\\a & b\end{pmatrix}$

Matrix **Q** = $\begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$

You are given that **PQ** = **QP**. Work out the values of a and b.

$A = \begin{pmatrix} -1 & 3 \\ 2 & -2 \end{pmatrix}$ $B = \begin{pmatrix} -2 & 0 \\ 4 & 5 \end{pmatrix}$

(a) Given that $B + C = I$, find C

(b) Given that $D - A = I$, find D

(c) Given that $B + 2I = E$, find E

(a) Given that

$$\begin{pmatrix} x & -2 \\ -7 & y \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix} = I$$

Find the values of x and y.

(b) Given that

$$\begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}\begin{pmatrix} 2 & p \\ q & 4 \end{pmatrix} = I$$

Find the values of p and q.

(a) Find I^2

(b) Given that $2A + I^2 = \begin{pmatrix} 6 & -4 \\ -1 & 5 \end{pmatrix}$

find A.

(a) Given that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 3 & -2 \\ 4 & -3 \end{pmatrix} = I$

find the values of a, b, c and d.

(b) Given that $\begin{pmatrix} -5 & 3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = I$

find the values of a, b, c and d.

(c) Given that

$$\begin{pmatrix} x & \frac{1}{2} \\ -2 & y \end{pmatrix}\begin{pmatrix} 1 & z \\ -\frac{2}{3} & -\frac{4}{9} \end{pmatrix} = I^2$$

find the values of x, y, and z.

Fluency Practice

1 Work out

(a) $\begin{pmatrix} 4 & 2 \\ -3 & 5 \end{pmatrix}\begin{pmatrix} 7 \\ 1 \end{pmatrix}$?

(b) $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} -3 \\ -4 \end{pmatrix}$?

(c) $2\begin{pmatrix} 5 & -2 \\ 6 & -3 \end{pmatrix}$?

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$?

(e) $6\begin{pmatrix} -4 & 7 \\ -1 & -3 \end{pmatrix}$?

(f) $\begin{pmatrix} 8 & 4 \\ 4 & 2 \end{pmatrix}\begin{pmatrix} -3 \\ 6 \end{pmatrix}$?

2 $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 7 & 4 \\ 5 & 3 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} -2 & 3 \\ 1 & -1 \end{pmatrix}$

Work out

(a) **AB** ?

(b) **BC** ?

(c) **3A** ?

(d) **BA** ?

(e) **–C** ?

(f) $\mathbf{B}\begin{pmatrix} 1 & -4 \\ -5 & 7 \end{pmatrix}$?

3 $\mathbf{P} = \begin{pmatrix} -2 & 0 \\ 5 & 1 \end{pmatrix}$ $\mathbf{Q} = \begin{pmatrix} -4 & 1 \\ 3 & -2 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$

Work out

(a) $\mathbf{P}^2$?

(b) **QP** ?

(c) **5Q** ?

(d) **PC** ?

(e) **IQ** ?

(f) **3I** ?

Fluency Practice

4

(a) $\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 0 & 3 \\ 1 & -4 \end{pmatrix}$?

(b) $\begin{pmatrix} -3 & -2 \\ -1 & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ 3 & 4 \end{pmatrix}$?

(c) $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}\begin{pmatrix} 5 & -2 \\ -7 & 3 \end{pmatrix}$?

(d) $\begin{pmatrix} 10 & -7 \\ 9 & 8 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ -2 & 3 \end{pmatrix}$?

(e) $\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$?

(f) $\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}$?

5 Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} \sqrt{2} & 1 \\ -1 & 3\sqrt{2} \end{pmatrix}\begin{pmatrix} \sqrt{2} & 0 \\ -3 & -2\sqrt{2} \end{pmatrix}$?

(b) $\begin{pmatrix} -\frac{1}{2} & -1 \\ \frac{3}{2} & 5 \end{pmatrix}\begin{pmatrix} -2 & 4 \\ -\frac{1}{2} & 3 \end{pmatrix}$?

(c) $\begin{pmatrix} 3 & 2 \\ 7 & 5 \end{pmatrix}^2$?

(d) $\begin{pmatrix} 3\sqrt{3} & -4 \\ 2 & 3\sqrt{3} \end{pmatrix}\begin{pmatrix} \sqrt{3} & 1 \\ -4 & 0 \end{pmatrix}$?

(e) $\begin{pmatrix} \frac{1}{3} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$?

(f) $\begin{pmatrix} \sqrt{2} & 2 \\ 7 & \sqrt{3} \end{pmatrix}^2$?

6 Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} p \\ p+1 \end{pmatrix}$?

(b) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$?

(c) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} m \\ 2m \end{pmatrix}$?

(d) $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -a & 0 \\ 0 & a \end{pmatrix}$?

(e) $\begin{pmatrix} 4t & 0 \\ 0 & 4t \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$?

(f) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -2 \end{pmatrix}$?

Fluency Practice

7

Work out, giving your answers as simply as possible.

(a) $\begin{pmatrix} 2x & -3 \\ -5 & 4x \end{pmatrix}\begin{pmatrix} x & 3x \\ -3 & 0 \end{pmatrix}$?

(b) $\begin{pmatrix} a & 3a \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 7 & 8 \\ -10 & 11 \end{pmatrix}$?

(c) $\begin{pmatrix} x & 0 \\ 1 & x \end{pmatrix}^2$?

(d) $\begin{pmatrix} y & y \\ -3 & x \end{pmatrix}\begin{pmatrix} 2 & 3y \\ 0 & 1 \end{pmatrix}$?

(e) $\begin{pmatrix} a+1 & a \\ a+2 & a+1 \end{pmatrix}\begin{pmatrix} a+1 & -a \\ -a-2 & a+1 \end{pmatrix}$?

(f) $\begin{pmatrix} 3x & -3 \\ -9 & x+1 \end{pmatrix}^2$?

Fluency Practice

1 $\begin{pmatrix} -2 & a \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 3 \\ 7 \end{pmatrix} = \begin{pmatrix} 22 \\ 9 \end{pmatrix}$

?

Work out the value of a.

2 June 2013 Paper 2 Q11

(a) Work out $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}\begin{pmatrix} 0 & b \\ a & c \end{pmatrix}$

?

Give your answer in terms of a, b and c.

(b) You are given that $\begin{pmatrix} 2 & -1 \\ \frac{1}{3} & 0 \end{pmatrix}\begin{pmatrix} 0 & b \\ a & c \end{pmatrix} = \mathbf{I}$ where **I** is the identity matrix.

Work out the values of a, b and c.

?

3 Set 2 Paper 2 Q16

Matrix $\mathbf{P} = \begin{pmatrix} 2 & 3 \\ a & b \end{pmatrix}$ Matrix $\mathbf{Q} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

You are given that $\mathbf{PQ} = \mathbf{QP}$

Work out the values of a and b.

?

4 Set 4 Paper 1 Q17

$\begin{pmatrix} 2 & a \\ 1 & -3 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$

Work out **all** possible pairs of values of a and b.

?

$$PQ = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$$

$$QP = \begin{pmatrix} e & f \\ g & h \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} ae+cf & be+df \\ ag+ch & bg+dh \end{pmatrix}$$

2×2 matrix multiplication is not necessarily commutative

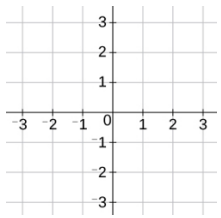
BUT, can you find 2×2 matrices A and B such that AB = BA? (it is possible!)

Fluency Practice

Rotations Using Matrices

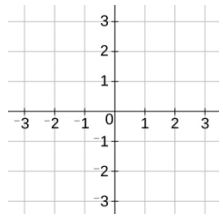
(a)

By considering the unit square, determine the matrix which describes a rotation $90°$ clockwise about the origin.



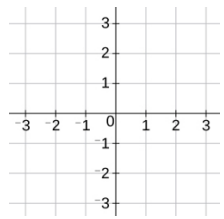
(b)

Describe fully the single transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$



(c)

By considering the unit square, determine the matrix which describes a rotation $180°$ about the origin.



(d)

The point $(1, -6)$ is mapped onto the point (a, b) when rotated $90°$ anti-clockwise about the origin. Using matrix algebra, find the values of a and b.

(e)

The point (c, d) is mapped onto the point $(2, 4)$ when rotated $270°$ anti-clockwise about the origin. Using matrix algebra, find the values of c and d.

(f)

A triangle with vertices at $(1, 1)$, $(5, 2)$ and $(4, -1)$ is rotated $180°$ about the origin. Use matrix algebra to find the coordinates of the vertices of the rotated triangle.

(g)

Use matrix algebra to show that a rotation of $90°$ clockwise about the origin, followed by a rotation of $180°$ is equivalent to a rotation of $90°$ anti-clockwise about the origin.

(h)

The point $(a, 6)$ is mapped onto the point $(b, -4)$ following a rotation of $90°$ anti-clockwise about the origin. Use matrix algebra to find the values of a and b.

(i)

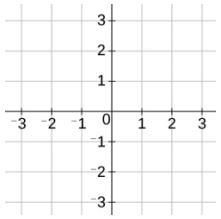
The point $(x, 2y + 6)$ is mapped onto the point $(2x, y - 7)$ following a rotation of $90°$ clockwise about $(0, 0)$. Use matrix algebra to find the values of x and y.

Fluency Practice

Reflections Using Matrices

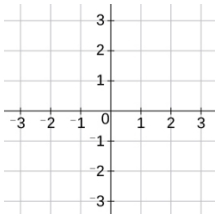
(a)

By considering the unit square, determine the matrix which describes a reflection in the x-axis.



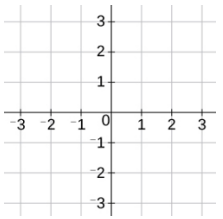
(b)

Describe fully the single transformation represented by the matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$



(c)

By considering the unit square, determine the matrix which describes a reflection in the line $y = -x$.



(d)

The point $(-4, 2)$ is mapped onto the point (a, b) when reflected in the x-axis. Using matrix algebra, find the coordinates (a, b).

(e)

The point (c, d) is mapped onto the point $(7, -5)$ when reflected in the line $y = -x$. Using matrix algebra, find the coordinates (c, d).

(f)

A triangle with vertices at $(0, 5)$, $(4, 3)$ and $(1, -1)$ is reflected in the line $y = x$. Use matrix algebra to find the coordinates of the vertices of the reflected triangle.

(g)

A triangle with vertices at $(0, 1)$, $(1, 0)$ and $(3, 2)$ is reflected so its vertices map to $(0, -1)$, $(-1, 0)$ and $(-2, -3)$. Find the transformation matrix and the line of reflection.

(h)

The point $(-2, a)$ is mapped onto the point $(b, 3)$ following a reflection in the line $x = 0$. Use matrix algebra to find the values of a and b.

(i)

The point $(x, 3x - 7)$ is mapped onto the point $(y + 3, y)$ following a reflection in the line y-axis. Use matrix algebra to find the values of x and y.

A triangle with coordinates (3,2), (5,2) and (3,6) is transformed by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. By pre-multiplying, find the coordinates of the transformed triangle. Draw this transformation on a grid and hence describe it fully.

A triangle with coordinates (-3,2), (-5,2) and (-3,5) is transformed by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. By pre-multiplying, find the coordinates of the transformed triangle. Draw this transformation on a grid and hence describe it fully.

A triangle with coordinates (2,3), (4,3) and (4,7) is transformed by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$. By pre-multiplying, find the coordinates of the transformed triangle. Draw this transformation on a grid and hence describe it fully.

A triangle with coordinates (3,1), (5,1) and (3,5) is transformed by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. By pre-multiplying, find the coordinates of the transformed triangle. Draw this transformation on a grid and hence describe it fully.

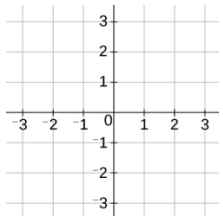
The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point $(3,\ 4)$ onto the point $(2,\ b)$. Work out the values of a and b.

Fluency Practice

Enlargements Using Matrices

(a)

By considering the unit square, determine the matrix which describes an enlargement about the origin with scale factor 3.



(b)

Describe fully the single transformation represented by the matrix $\begin{pmatrix} \frac{5}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix}$

(c)

Use matrix algebra to show that an enlargement of scale factor 2 about $(0, 0)$, followed by an enlargement of scale factor 1.5 about $(0, 0)$ is equivalent to an enlargement of scale factor 3 about $(0, 0)$.

(d)

The point $(-5, 3)$ is mapped onto the point (a, b) when enlarged by a scale factor 2 about the origin. Using matrix algebra, find the values of a and b.

(e)

The unit square OABC with coordinates O(0, 0), A(0, 1), B(1, 1) and C(1, 0) is mapped to OA'B'C' under matrix $\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$. Use matrix algebra to find the coordinates of A', B' and C'.

(f)

The point (c, d) is mapped onto the point $(-1, -4)$ when enlarged by a scale factor 0.5 about the origin. Using matrix algebra, find the values of c and d.

(g)

Use matrix algebra to show that an enlargement of scale factor 2 about $(0, 0)$, followed by an enlargement of scale factor -0.5 about $(0, 0)$ is the same as a rotation of $180°$ about the origin.

(h)

The point $(a, 3)$ is mapped to the point $(6, 2a)$ when enlarged with scale factor b about the origin. Use matrix algebra to find the possible values of a and b.

(i)

The point $(x - 4, y)$ is mapped to the point $(2y, 2x - 18.5)$ when transformed under the matrix $\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$. Find the values of x and y.

Fluency Practice

1 [Jan 2013 Paper 2 Q15] Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

?

2 [Set 2 Paper 1 Q4] The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point $(3,4)$ onto the point $(2, b)$. Work out the values of a and b.

?

3 [Set 3 Paper 1 Q6] The matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point $(5,4)$ onto the point $(1,17)$. Work out the values of a and b.

?

4 [Worksheet 2 Q5] Work out the image of the point *D* (–1, 2) after transformation by the matrix $\begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$

?

5 [Worksheet 2 Q6] The point *A*(*m*, *n*) is transformed to the point *A*′ (–2, 0) by the matrix $\begin{pmatrix} 2 & 3 \\ 1 & 1 \end{pmatrix}$

Work out the values of *m* and *n*.

?

6 [Worksheet 2 Q8] Describe fully the transformation given by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

?

7 [Worksheet 2 Q9] The unit square *OABC* is transformed by the matrix $\begin{pmatrix} h & 0 \\ 0 & h \end{pmatrix}$ to the square *OA*′*B*′*C*′.

The area of *OA*′*B*′*C*′ is 27. Work out the exact value of *h*.

?

Fluency Practice

Harder Transformations Using Matrices		
(a)	**(b)**	**(c)**
Find the single matrix that represents an enlargement about the origin with scale factor 3, followed by a rotation of $90°$ clockwise about the origin.	Find the single matrix that represents a reflection in the y-axis, followed by a rotation of $180°$ about the origin.	$P = \begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix}$ $Q = \begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix}$ Matrices P and Q represent different transformations. Find the single matrix that represents transformation P followed by transformation Q.
(d)	**(e)**	**(f)**
The point P $(4, -2)$ is mapped to the point Q following a reflection in the line $y = x$, then an enlargement with scale factor 2 about the origin. Use matrix algebra to find the coordinates of point Q.	The point (a, b) is mapped to the point $(-5, 1)$ following a rotation of $180°$ about the origin, then a reflection in the x-axis. Using matrix algebra, find the coordinates (a, b).	The matrix $\begin{pmatrix} 0 & b \\ -2 & 4 \end{pmatrix}$ maps the point $(a, -3)$ onto the point $(-9, 5)$. Use matrix algebra to find the values of a and b.
(g)	**(h)**	**(i)**
The transformation matrix $\begin{pmatrix} a & 2b \\ -a & 3 \end{pmatrix}$ maps the point $(2, -1)$ to the point $(6, 7)$. Find the values of a and b.	The transformation matrix $\begin{pmatrix} b & 2a \\ a & -b \end{pmatrix}$ maps the point $(6, 3)$ to the point $(24, b)$. Find the values of a and b.	Point $(c, 4)$ is mapped to the point $(-2, d)$ by the transformation matrix $\begin{pmatrix} c & -3 \\ 2 & -1 \end{pmatrix}$. Use matrix algebra to find the two possible values of c and d.

Find the matrices that represent the following transformations:
(a) A reflection in the x-axis, followed by a rotation through $180°$ centre the origin.
(b) An enlargement with centre the origin and scale factor 2, followed by a reflection in the line $y = x$.
(c) A reflection in the y-axis followed by a reflection in the line $y = x$.
(d) A reflection in the line $y = x$ followed by enlargement about the origin with scale factor 3.

Point $(3, -2)$ is transformed by the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ followed by a further transformation by the matrix $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$.
(i) Work out the matrix for the combined transformation.
(ii) Work out the co-ordinates of the image point of P.

Point $(-1,4)$ is transformed by the matrix $\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$ followed by a further transformation by the matrix $\begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}$.
(i) Work out the matrix for the combined transformation.
(ii) Work out the co-ordinates of the image point of W.

The transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps a point P to Q. The transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ maps point Q to point R. Point R is $(-4,3)$. Work out the coordinates of point P.

Fluency Practice

1 Point $(3, -2)$ is transformed by the matrix $\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$ followed by a further transformation by the matrix $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$.

(i) Work out the matrix for the combined transformation. ?

(ii) Work out the co-ordinates of the image point of P. ?

2 Point $(-1,4)$ is transformed by the matrix $\begin{pmatrix} 3 & -1 \\ -2 & 2 \end{pmatrix}$ followed by a further transformation by the matrix $\begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}$.

(i) Work out the matrix for the combined transformation. ?

(ii) Work out the co-ordinates of the image point of W. ?

3 The unit square is reflected in the x-axis followed by a rotation through $180°$ centre the origin. Work out the matrix for the combined transformation.

?

4 The unit square is enlarged, centre the origin, scale factor 2 followed by a reflection in the line $y = x$. Work out the matrix for the combined transformation.

?

Fluency Practice

5 [Jan 2013 Paper 2 Q17] $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a reflection in the y-axis. $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ represents a reflection in the line $y = x$.
Work out the matrix that represents a reflection in the y-axis followed by a reflection in the line $y = x$.

?

6 [June 2012 Paper Q22] The transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps a point P to Q. The transformation matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ maps point Q to point R.
Point R is $(-4,3)$. Work out the coordinates of point P.

?

7 [Set 1 Paper Q14b] The unit square OABC is transformed by reflection in the line $y = x$ followed by enlargement about the origin with scale factor 2. What is the matrix of the combined transformation?

?

$A = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$.
The point $P(2,7)$ is transformed by matrix BA to P'. Show that P' lies on the line
$7x + 2y = 0$.

?