KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Mathematics 2026

Unit 21 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Mathematics 2026

Unit 21 Booklet – Part 2

HGS Maths

Tasks

Dr Frost Course

Name: _____

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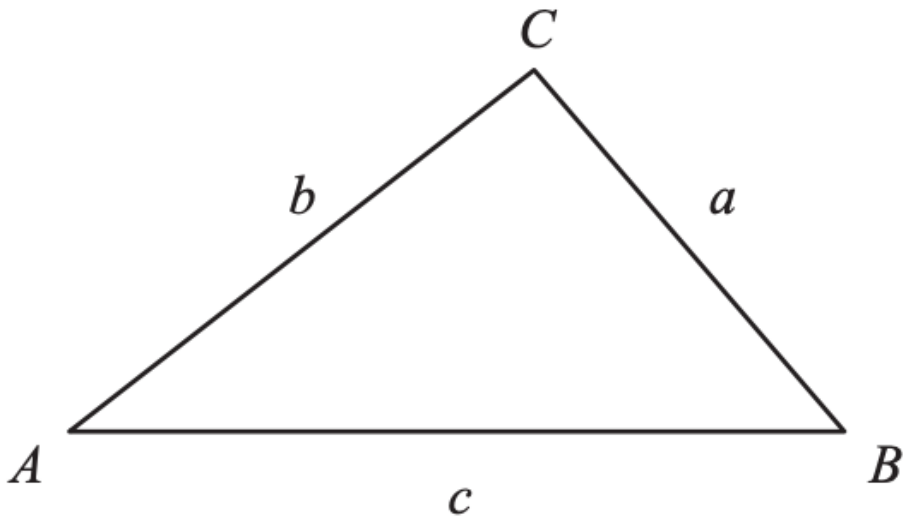
Contents

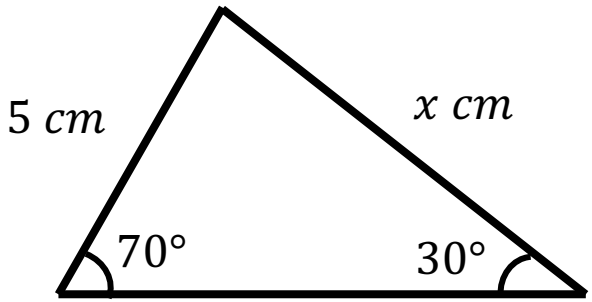
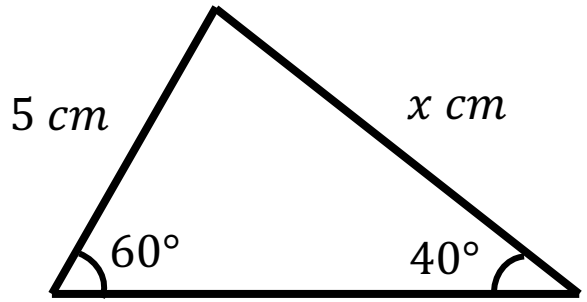
1 Advanced Trigonometry

Sine Rule



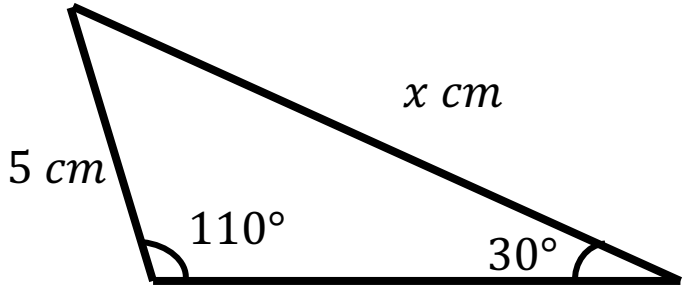
In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Worked Example	Your Turn
Find the value of x. Give your answer to 2 decimal places.  	Find the value of x. Give your answer to 2 decimal places. 

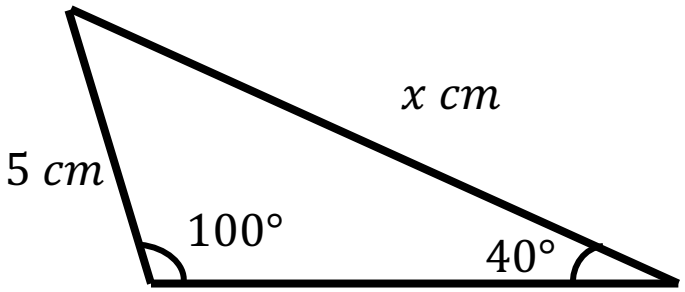
Worked Example

Find the value of x. Give your answer to 2 decimal places.

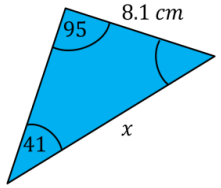
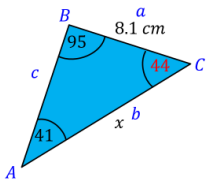
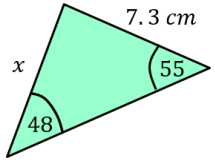
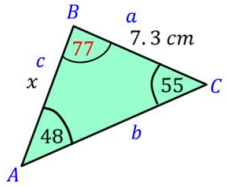
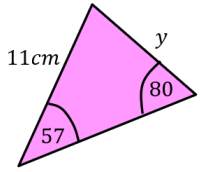
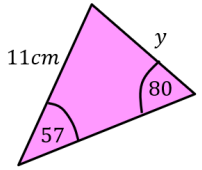
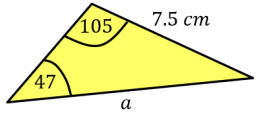
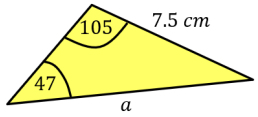
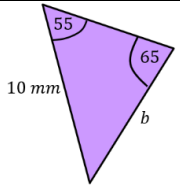
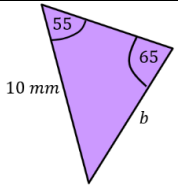


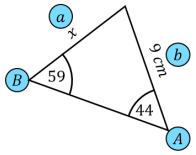
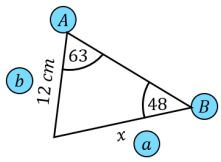
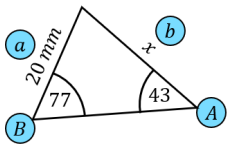
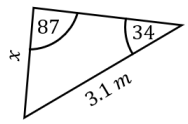
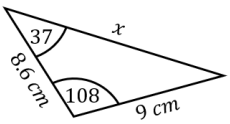
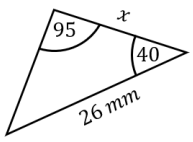
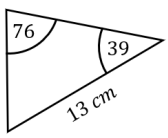
Your Turn

Find the value of x. Give your answer to 2 decimal places.



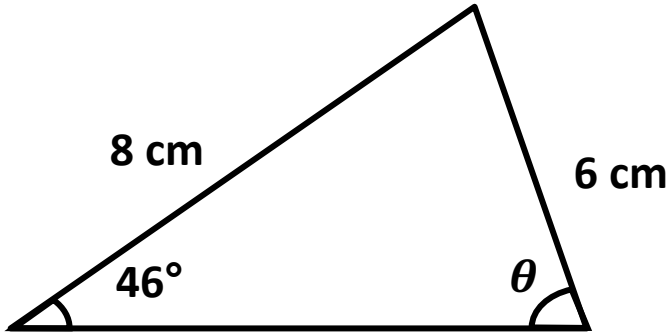
Fill in the Gaps

Question	Label the triangle and calculate any angles	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing length.
		$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\frac{8.1}{\sin 41} = \frac{x}{\sin 95} = \frac{c}{\sin 44}$	$x = \sin 95 \times \frac{8.1}{\sin 41}$	$x = 12.3\ cm$
		$\frac{7.3}{\sin 48} = \frac{b}{\sin 77} = \frac{x}{\sin 55}$		
				
				
				

Labelled diagram	Substitute into formula	Rearrange formula	Length (1dp)
	$\frac{x}{\sin 44} = \frac{9}{\sin 59}$	$x = \frac{9 \times \sin 44}{\sin 59}$	
	$\frac{x}{\sin 63} = \frac{12}{\sin 48}$		
			
			
			
			
	$\frac{x}{\sin 65} = \frac{13}{\sin 76}$		
		$x = \frac{3.5 \times \sin 36}{\sin 68}$	

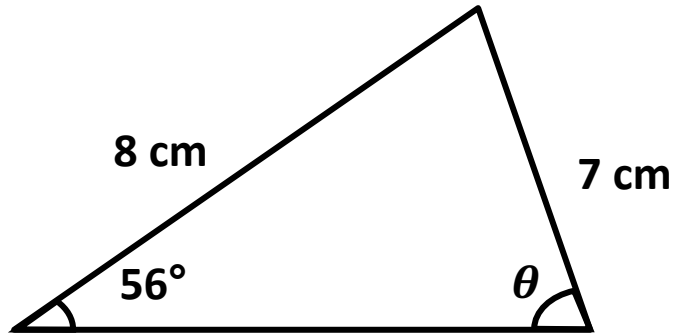
Worked Example

Find the value of θ. Give your answer to 2 decimal places.

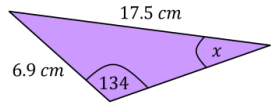
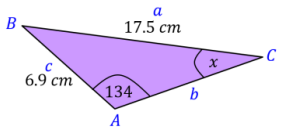
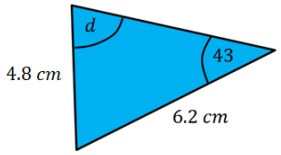
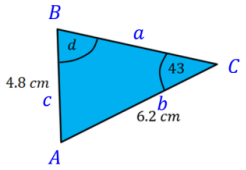
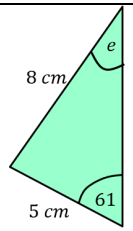
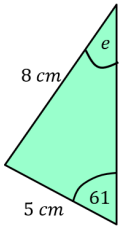
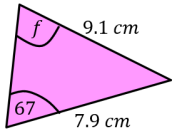
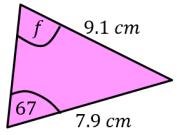
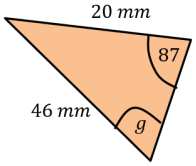
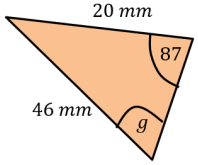


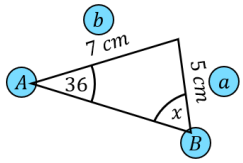
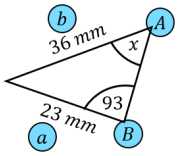
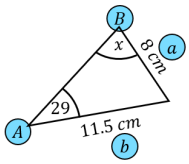
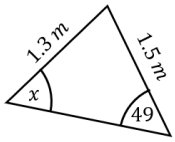
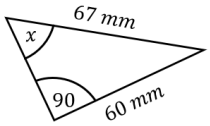
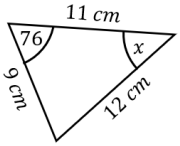
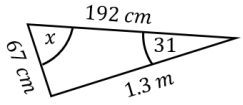
Your Turn

Find the value of θ. Give your answer to 2 decimal places.



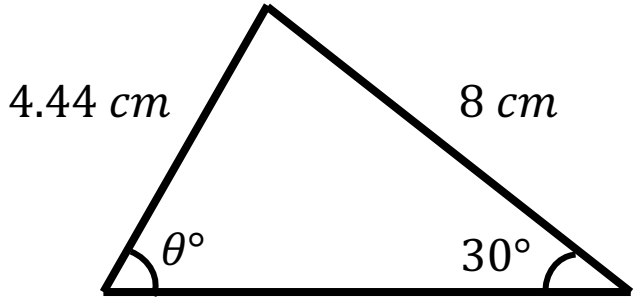
Fill in the Gaps

Question	Label the triangle	Fill into the formula and cross out the part not needed	Rearrange the formula	Use calculator to find missing angle.
		$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\frac{\sin 134}{17.5} = \frac{\sin B}{b} = \frac{\sin x}{6.9}$	$\sin x = 6.9 \times \frac{\sin 134}{17.5}$	$x = sin^{-1}(0.2836)$ $x = 16.5°$
		$\frac{\sin A}{a} = \frac{\sin d}{6.2} = \frac{\sin 43}{4.8}$		
				
				
				

Labelled diagram	Substitute into formula	Rearrange formula	Acute Angle (1dp)
	$\frac{\sin 36}{5} = \frac{\sin x}{7}$	$\sin x = \frac{7 \times \sin 36}{5}$	$x = 55.4°$
	$\frac{\sin x}{23} = \frac{\sin 93}{36}$		
			
			
			
			
			
		$\sin x = \frac{5 \times \sin 47}{10}$	

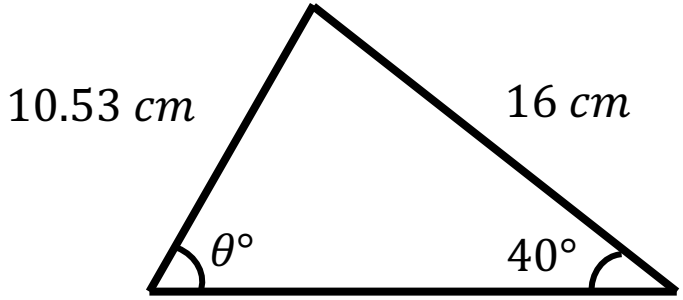
Worked Example

Find the possible values of θ. Give your answer to 2 decimal places.

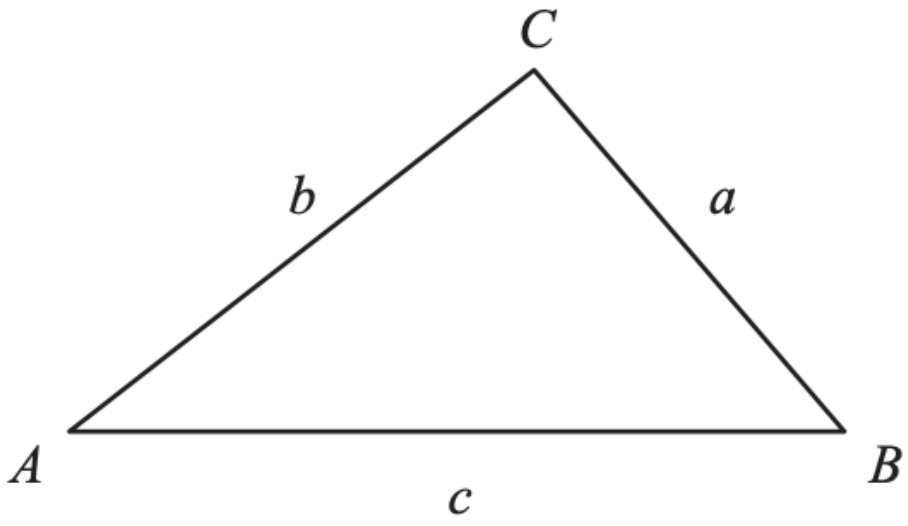


Your Turn

Find the possible values of θ. Give your answer to 2 decimal places.



Cosine Rule

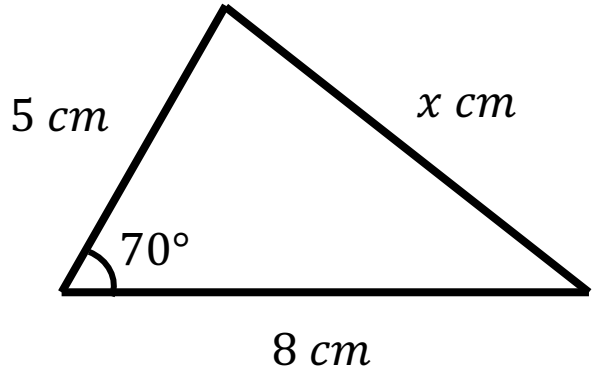


In any triangle ABC where a, b and c are the length of the sides:

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

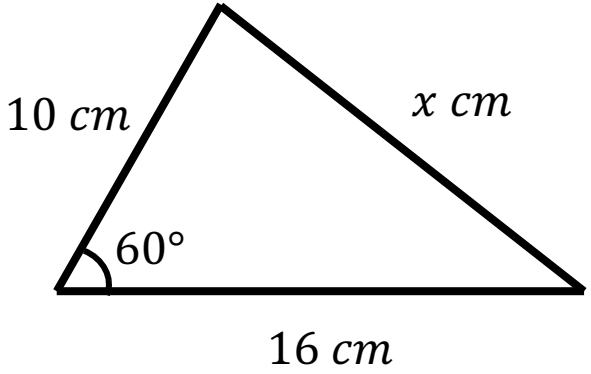
Worked Example

Find the value of x. Give your answer to 2 decimal places.

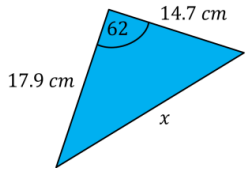
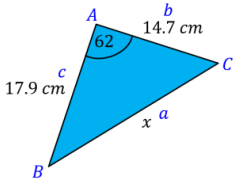
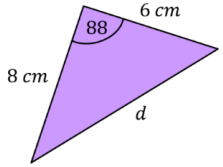
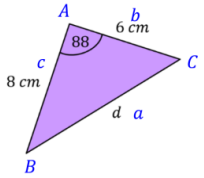
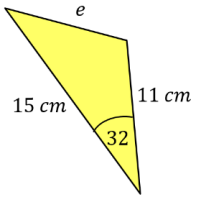
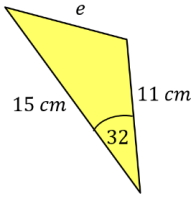
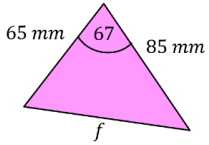
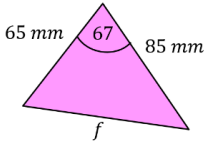
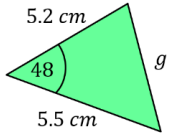
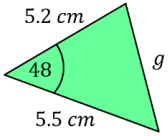


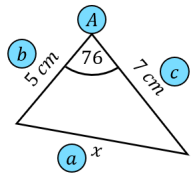
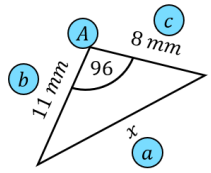
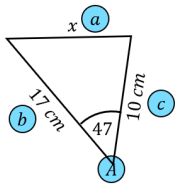
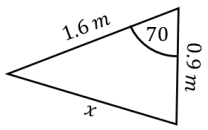
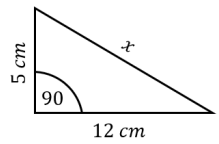
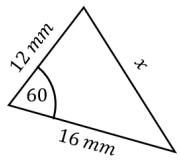
Your Turn

Find the value of x. Give your answer to 2 decimal places.



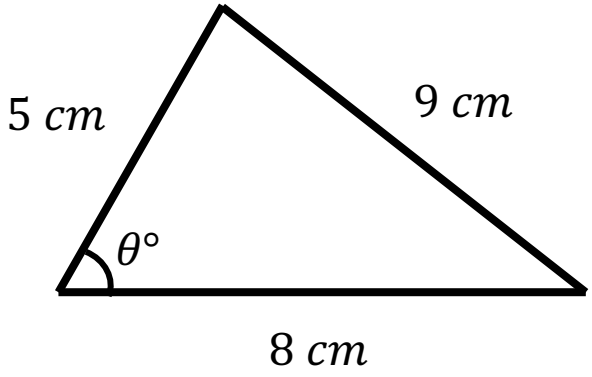
Fill in the Gaps

Question	Label the triangle with the angle being used as A	Fill into the formula	Use calculator to find missing length.
		$a^2 = b^2 + c^2 - 2bc\cos A$ $x^2 = 14.7^2 + 17.9^2 - 2 \times 14.7 \times 17.9 \cos 62$	$x^2 = 289.436$ $x = 17.0\ cm\ (1\ dp)$
		$a^2 = b^2 + c^2 - 2bc\cos A$ $x^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 88$	
			
			
			

Labelled diagram	Substitute into formula	x^2	x to 1dp
	$x^2 = 7^2 + 5^2$ $-2 \times 7 \times 5 \times \cos 76$	$x^2 = 57.065..$	
	$x^2 = 11^2 + 8^2$ $-2 \times 11 \times 8 \times \cos 96$		
			
			
			
			
	$x^2 = 32^2 + 14^2$ $-2 \times 32 \times 14 \times \cos 53$		

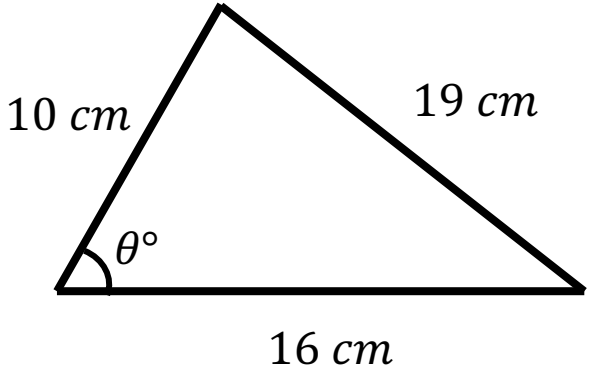
Worked Example

Find the value of θ. Give your answer to 2 decimal places.

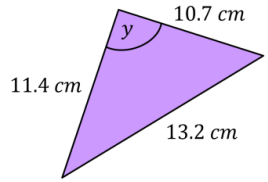
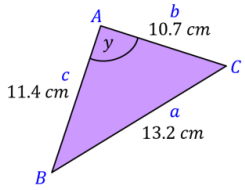
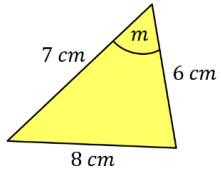
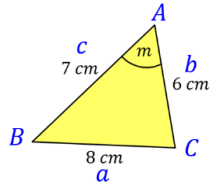
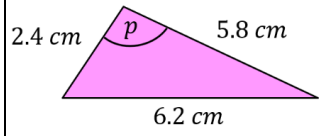
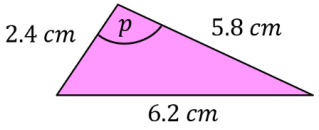
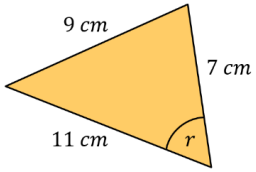
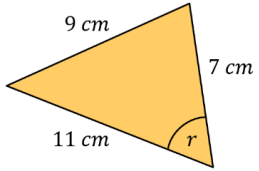
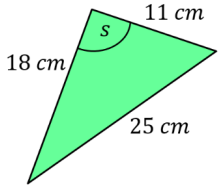
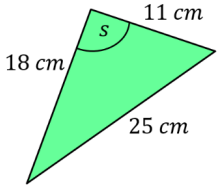


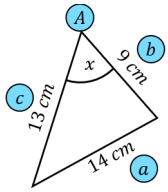
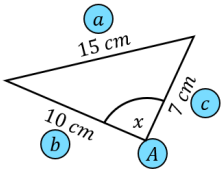
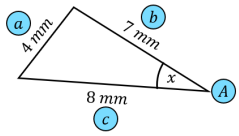
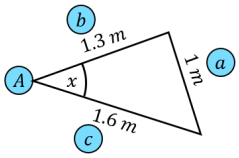
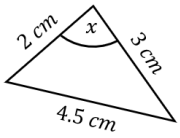
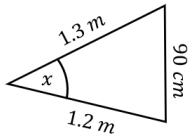
Your Turn

Find the value of θ. Give your answer to 2 decimal places.



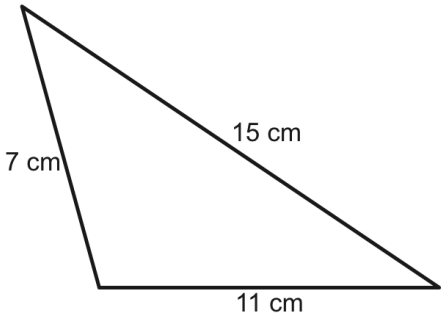
Fill in the Gaps

Question	Label the triangle with the angle being found as A	Fill into the formula	Use calculator to find missing angle
		$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos A = \frac{10.7^2 + 11.4^2 - 13.2^2}{2 \times 10.7 \times 11.4}$	$\cos A = 0.2878$ $A = \cos^{-1}(0.2878)$ $A = 73.3°$
		$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos m = \frac{6^2 + 7^2 - 8^2}{2 \times 6 \times 7}$	
			
			
			

Labelled diagram	Substitute into formula	Rearrange formula	Angle (1dp)
	$14^2 = 9^2 + 13^2$ $-2 \times 9 \times 13 \times \cos x$	$\cos x = \frac{9^2 + 13^2 - 14^2}{2 \times 9 \times 13}$	$x = 76.7°$
	$15^2 = 10^2 + 7^2$ $-2 \times 10 \times 7 \times \cos x$	$\cos x = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	
	$4^2 = 7^2 + 8^2$ $-2 \times 7 \times 8 \times \cos x$		
			
			
			
		$\cos x = \frac{6^2 + 5^2 - 3^2}{2 \times 6 \times 5}$	

Worked Example

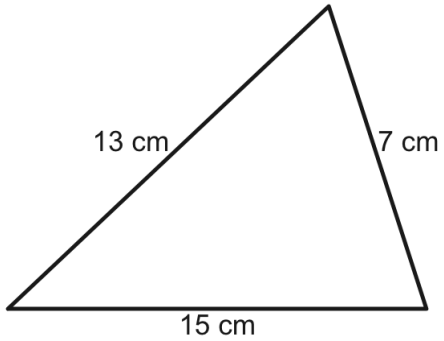
Find the value of the largest angle.



Give your answer correct to 1 decimal place.

Your Turn

Find the value of the largest angle.



Give your answer correct to 1 decimal place.

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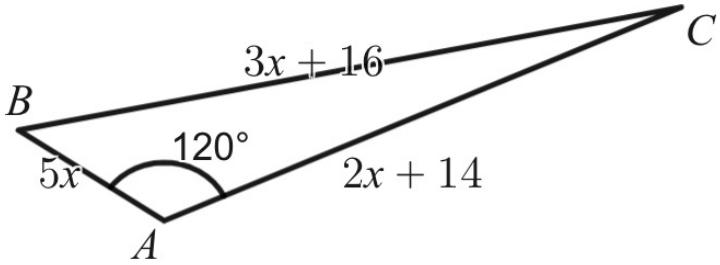
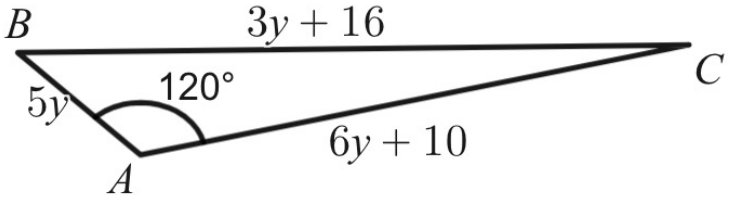
Worked Example



Determine the value of x

Your Turn

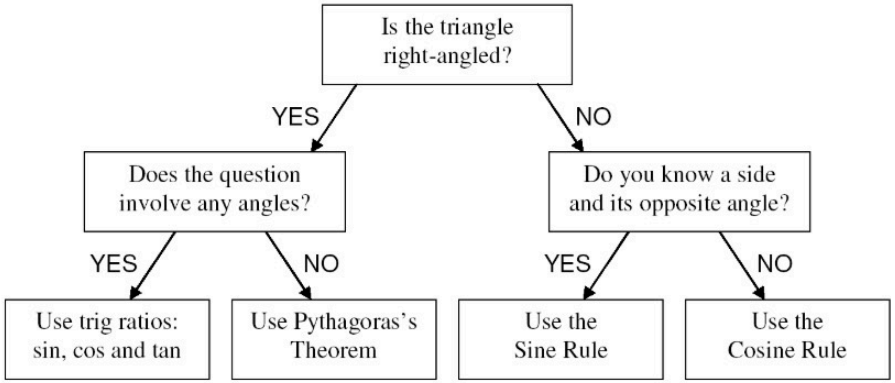


Determine the value of y

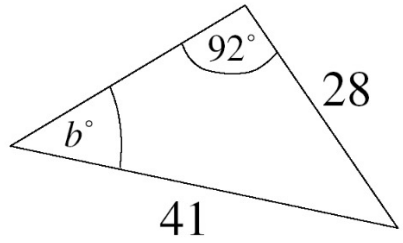
Review

Choosing The Appropriate Technique

Sometimes more than one technique from the formula table at the top of this page can be used to solve a trig problem, but you will want to choose the most efficient and easiest method to use:

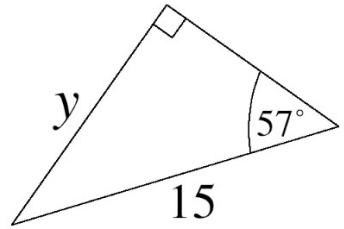


e.g. 1



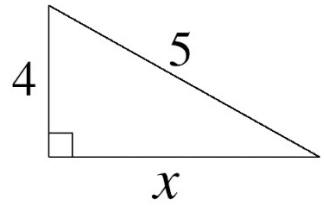
1. The triangle is not right-angled.
2. We do know a side and its opposite angle.
3. Therefore we use the Sine Rule.

e.g. 2



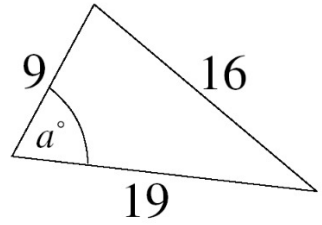
1. The triangle is right-angled.
2. The question involves angles.
3. Therefore we use trig ratios - sin, cos and tan.

e.g. 3



1. The triangle is right-angled.
2. The question does not involve angles.
3. Therefore we use Pythagoras's Theorem.

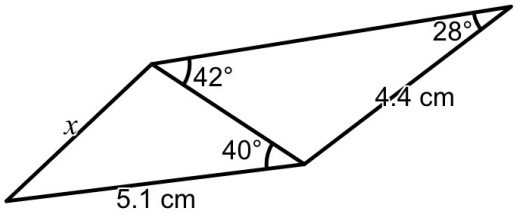
e.g. 4



1. The triangle is not right-angled.
2. We do not know a side and its opposite angle.
3. Therefore we use the Cosine Rule.

Worked Example

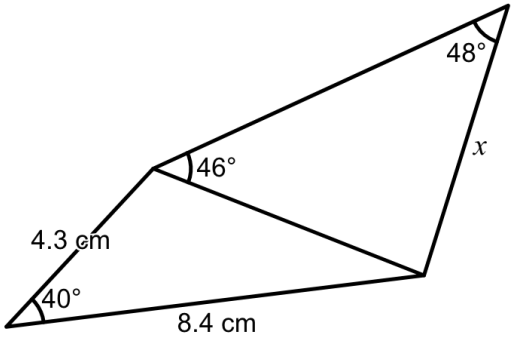
Find the value of x



Give your answer correct to 2 decimal places.

Your Turn

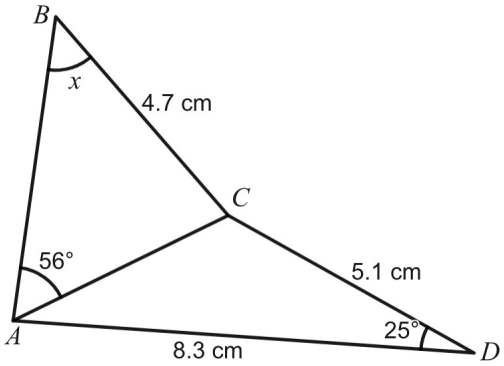
Find the value of x



Give your answer correct to 2 decimal places.

Worked Example

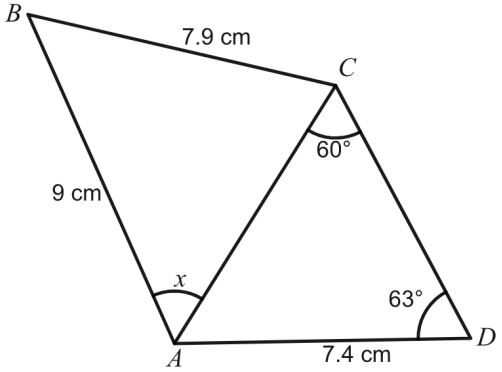
ABC and ADC are adjoined triangles.



Work out the value of x. Give your answer correct to the nearest whole degree.

Your Turn

ABC and ADC are adjoined triangles.



Work out the value of x. Give your answer correct to the nearest whole degree.

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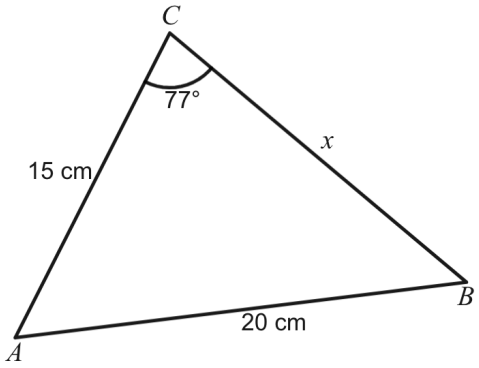
Worked Example

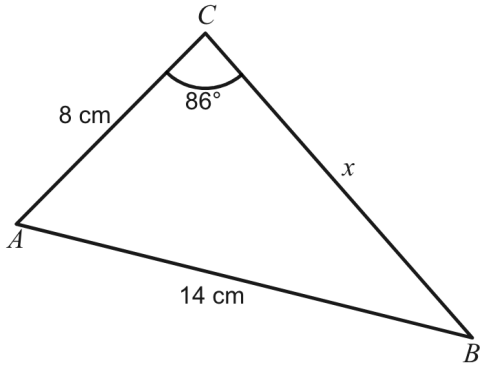
Find the value of x



Give your answer correct to 1 decimal place.

Your Turn

Find the value of x



Give your answer correct to 1 decimal place.

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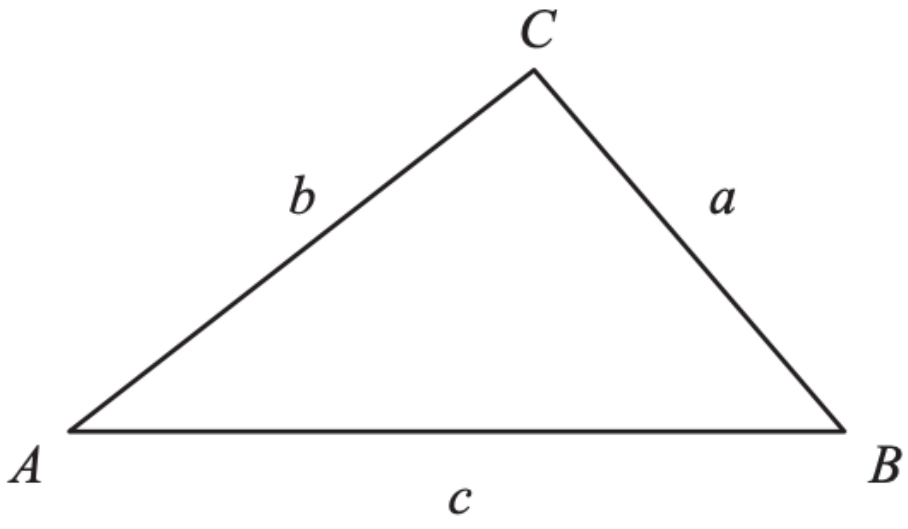
Area of Triangles

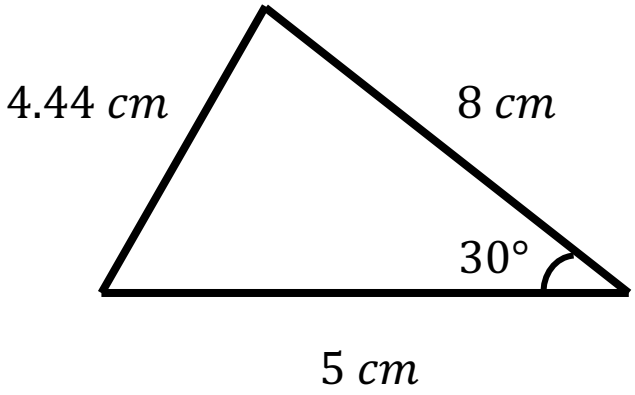


In any triangle ABC where a, b and c are the length of the sides:

$$\text{Area of triangle} = \frac{1}{2}\, a\, b \sin C$$

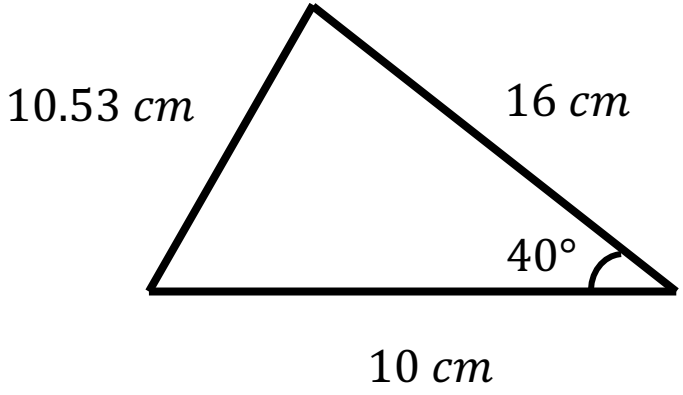
Worked Example

Calculate the area of the triangle. Give your answer to 2 decimal places.



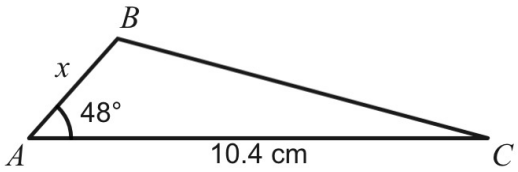
Your Turn

Calculate the area of the triangle. Give your answer to 2 decimal places.



Worked Example

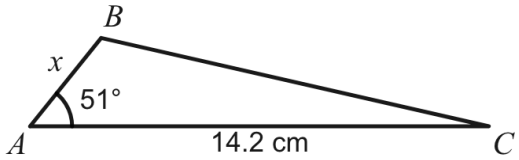
The triangle ABC has an area of 11.6 cm^2



Work out the length of AB
Give your answer correct to 1 decimal place.

Your Turn

The triangle ABC has an area of 19.3 cm^2

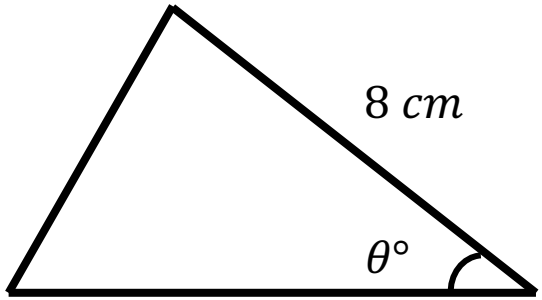


Work out the length of AB
Give your answer correct to 1 decimal place.

Worked Example

The area is $10\ cm^2$

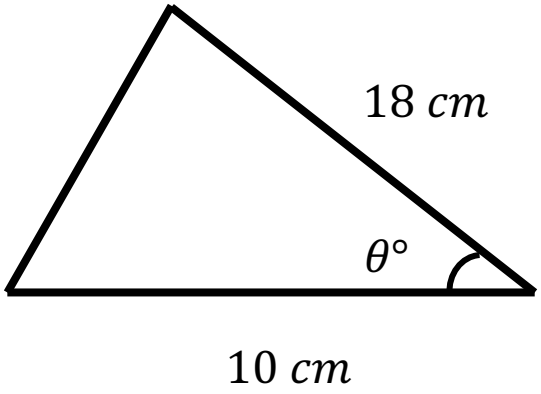
Calculate θ. Give your answer to 2 decimal places.



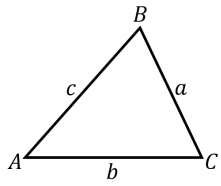
Your Turn

The area is $51.42\ cm^2$

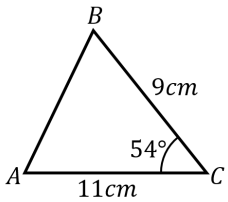
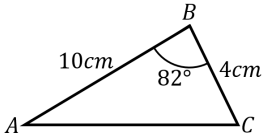
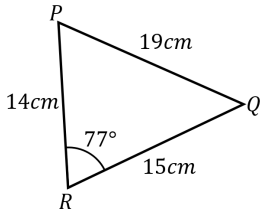
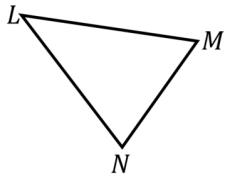
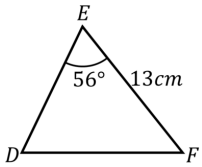
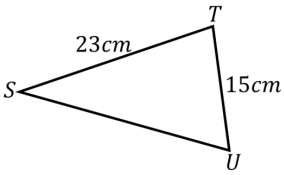
Calculate θ. Give your answer to 2 decimal places.



Fill in the blanks for each triangle and calculation (to 1dp) below using the area formula:

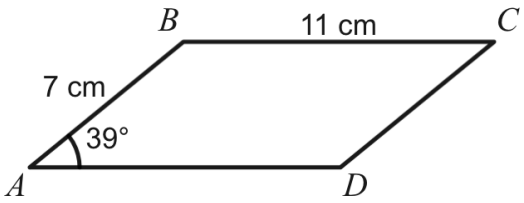


$$Area = \frac{1}{2}ab\sin C$$

Shape	Calculation	Answer
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad °$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad °$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times \quad \times \quad \sin \quad °$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times 8 \times 5 \sin 63°$	$Area = \quad cm^2$
	$A = \frac{1}{2} \times 13 \times \quad \sin 56°$	$Area = 38.8cm^2$
	$A = \frac{1}{2} \times 23 \times 15 \sin \quad °$	$Area = 172.3cm^2$

Worked Example

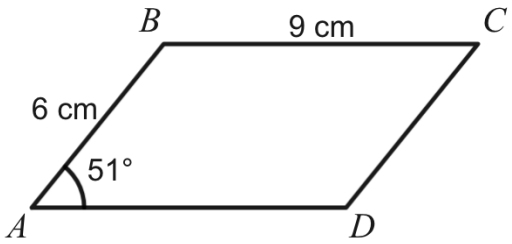
In the parallelogram $ABCD$



Find the area of the parallelogram $ABCD$
Give your answer correct to 3 significant figures.

Your Turn

In the parallelogram $ABCD$

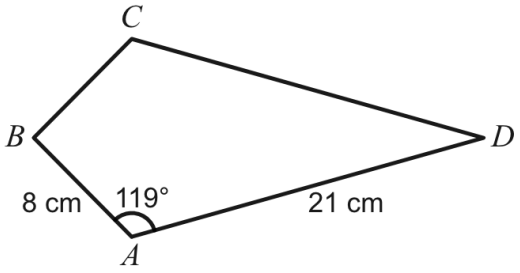


Find the area of the parallelogram $ABCD$
Give your answer correct to 3 significant figures.

Dr Frost 467b

Worked Example

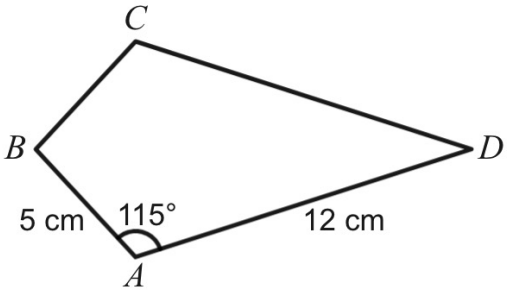
In the kite $ABCD$



Find the area of the kite $ABCD$
Give your answer correct to 3 significant figures.

Your Turn

In the kite $ABCD$

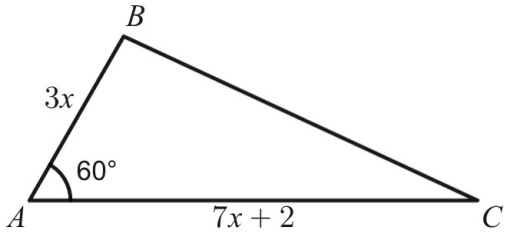


Find the area of the kite $ABCD$
Give your answer correct to 3 significant figures.

Dr Frost 467c

Worked Example

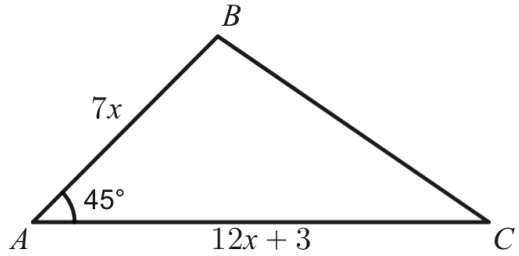
In the triangle ABC



Write an expression for the area of the triangle ABC. Give your answer in the form $Dx^2 + Ex$, where D and E are surds.

Your Turn

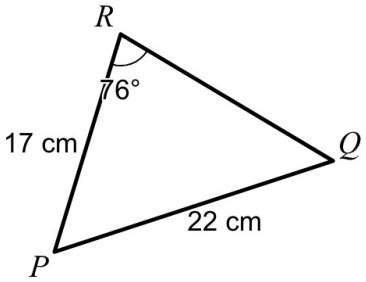
In the triangle ABC



Write an expression for the area of the triangle ABC. Give your answer in the form $Dx^2 + Ex$, where D and E are surds.

Worked Example

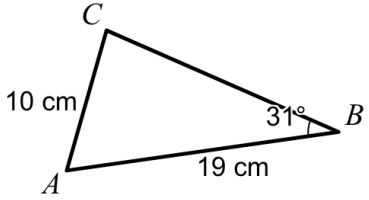
Find the area of the triangle PQR



Give your answer correct to 1 decimal place.

Your Turn

Find the area of the triangle PQR

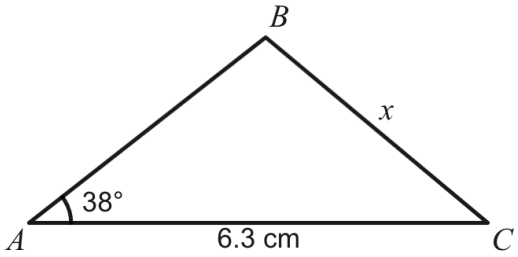


Give your answer correct to 1 decimal place.

Dr Frost 467d

Worked Example

The triangle ABC has an area of 8 cm^2

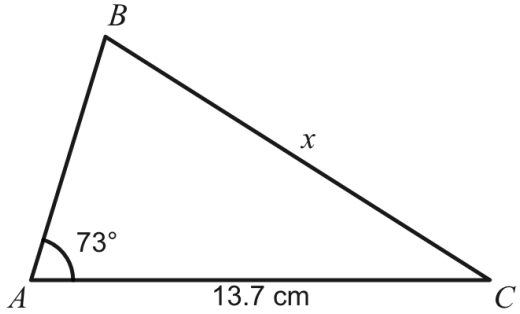


Work out the length of BC

Give your answer correct to 1 decimal place.

Your Turn

The triangle ABC has an area of 49.8 cm^2

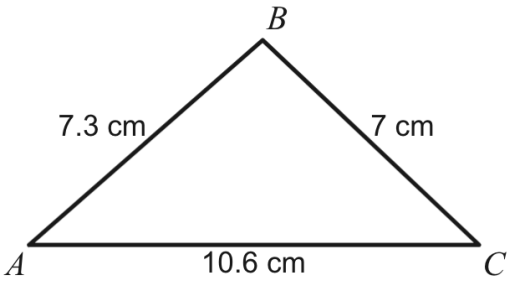


Work out the length of BC

Give your answer correct to 1 decimal place.

Worked Example

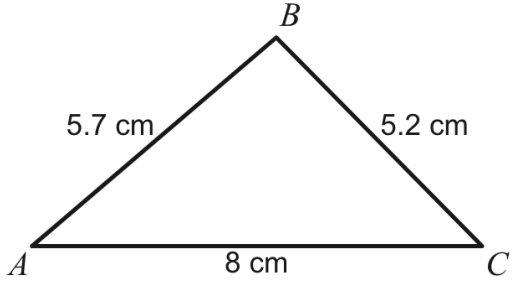
In the triangle ABC



Work out the area of triangle ABC.
Give your answer correct to 1 decimal place.

Your Turn

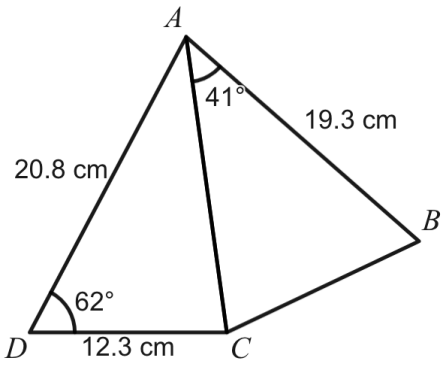
In the triangle ABC



Work out the area of triangle ABC.
Give your answer correct to 1 decimal place.

Worked Example

A shape is made from two triangles ACD and ABC

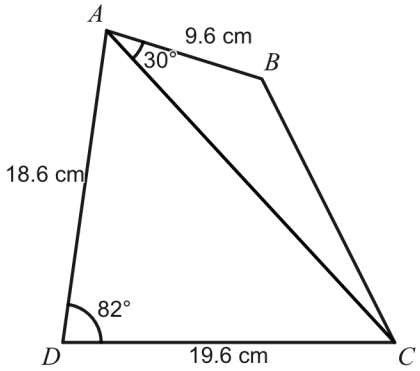


Work out the area of triangle ABC

Give your answer correct to 3 significant figures.

Your Turn

A shape is made from two triangles ACD and ABC

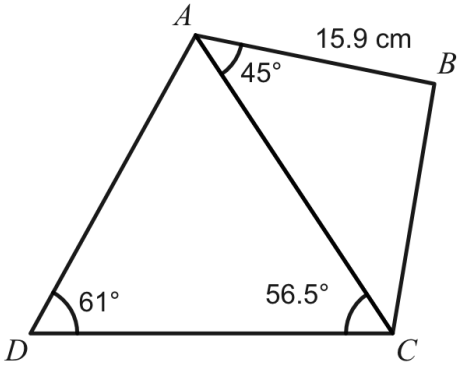


Work out the area of triangle ABC

Give your answer correct to 3 significant figures.

Worked Example

$ABCD$ is a quadrilateral.

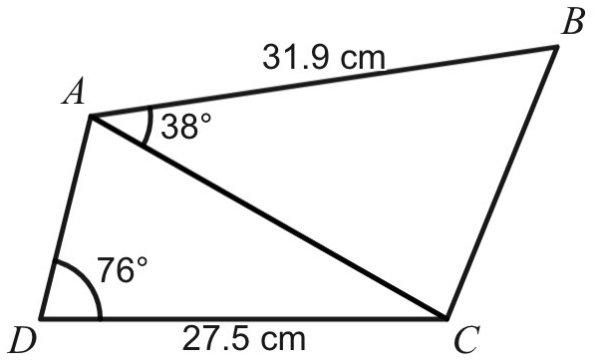


The area of triangle $ABC = 131$ cm^2

Work out the area of the quadrilateral $ABCD$
Give your answer correct to 3 significant figures.

Your Turn

$ABCD$ is a quadrilateral.



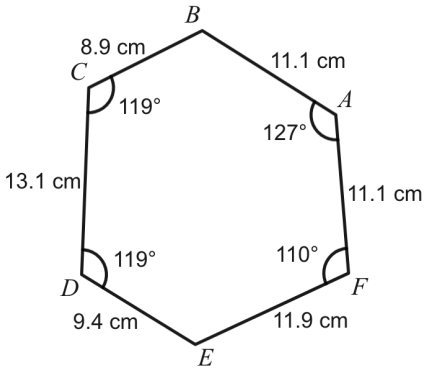
The area of triangle $ABC = 272$ cm^2

Work out the area of the quadrilateral $ABCD$
Give your answer correct to 3 significant figures.

Dr Frost 467o

Worked Example

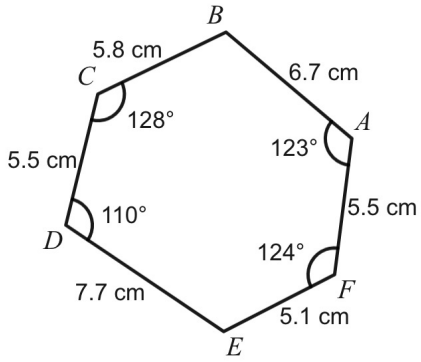
$ABCDEF$ is a hexagon.



Work out the area of hexagon $ABCDEF$
Give your answer correct to 3 significant figures.

Your Turn

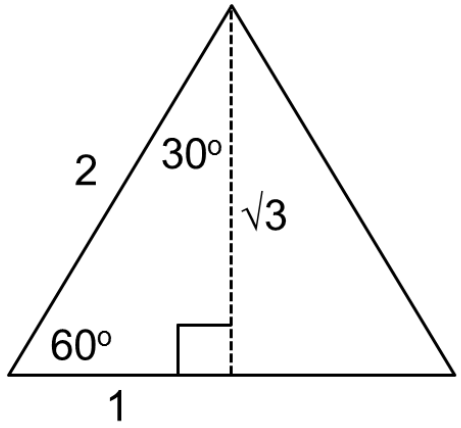
$ABCDEF$ is a hexagon.

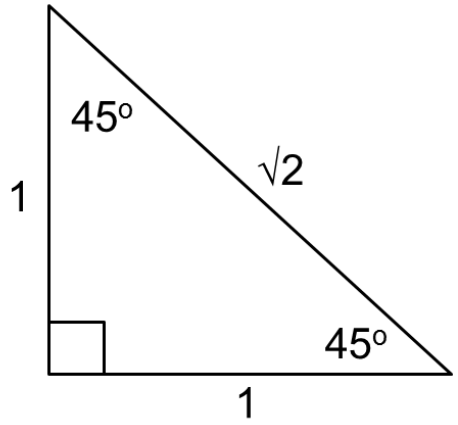


Work out the area of hexagon $ABCDEF$
Give your answer correct to 3 significant figures.

Exact Trigonometric Values

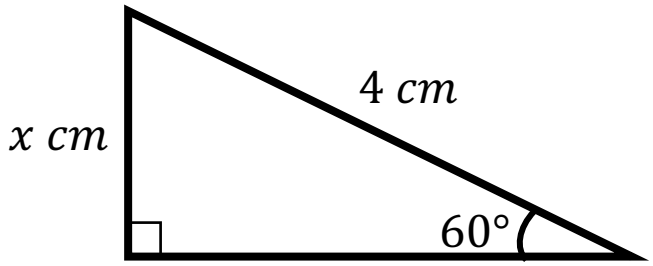
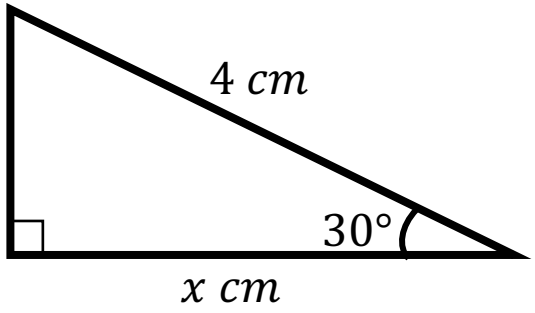
exact values in trigonometry





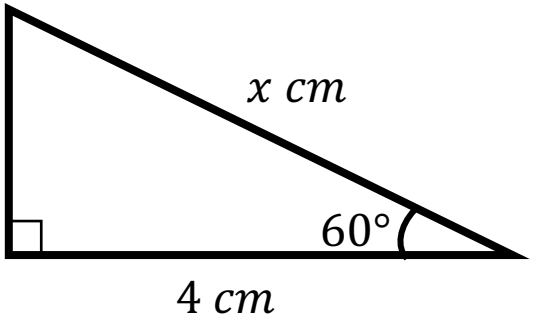
angle	sin	cos	tan
0°			
30°			
45°			
60°			
90°			

Worked Example	Your Turn
Show that $5 \sin 30° \times \cos 30° \times 8 \tan 30°$ is an integer	Show that $2 \sin 60° \times 5 \cos 60° \times 6 \tan 60°$ is an integer

Worked Example	Your Turn
Without a calculator, calculate x:  	Without a calculator, calculate x: 

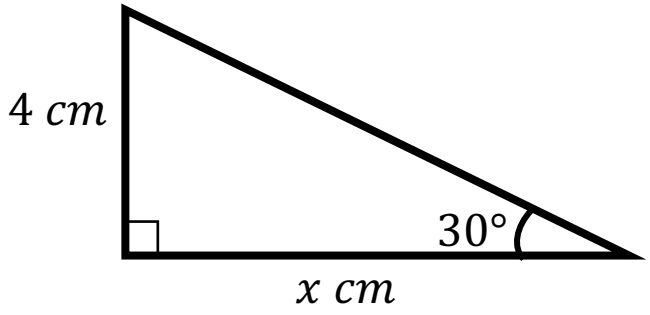
Worked Example

Without a calculator, calculate x:



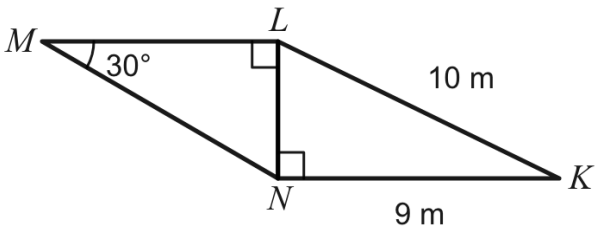
Your Turn

Without a calculator, calculate x:



Worked Example

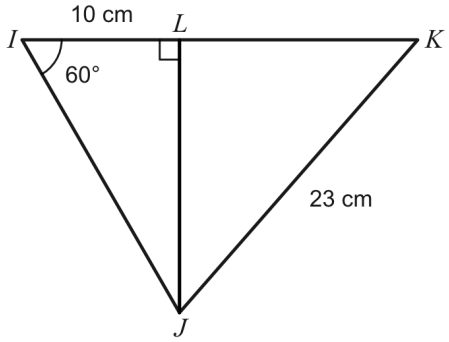
The diagram shows two right-angled triangles KLN and LMN.
$KL = 10$ m, $KN = 9$ m and $\angle LMN = 30°$.



Without using a calculator, work out the length of MN.

Your Turn

The diagram shows two right-angled triangles IJL and JKL.
$\angle JIL = 60°$, $IL = 10$ cm and $JK = 23$ cm.



Without using a calculator, work out the length of KL.

Dr Frost 412d

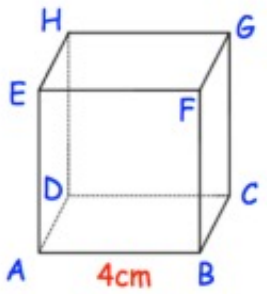
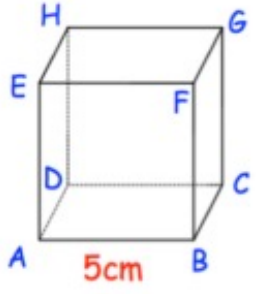
Extra Notes

2 3D Pythagoras' Theorem and Trigonometry

3D Pythagoras' Theorem

Pythag Triangles in a Box

How many ways can you join 3 vertices of a cuboid to make a right-angled triangle? Mark the right-angle.

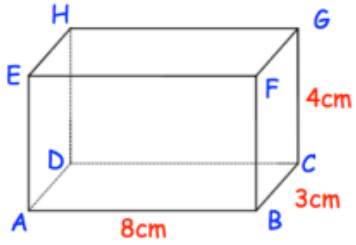
Worked Example	Your Turn
Shown below is a cube. a) Calculate the length AC. b) Calculate the length AG. 	Shown below is a cube. a) Calculate the length BD. b) Calculate the length BH. 

Worked Example

Shown below is a cuboid.

a) Calculate the length AC.

b) Calculate the length AG.

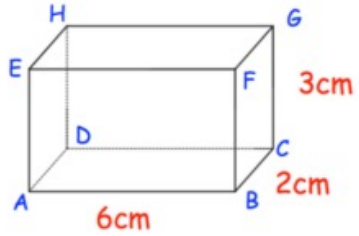


Your Turn

Shown below is a cuboid.

a) Calculate the length AC.

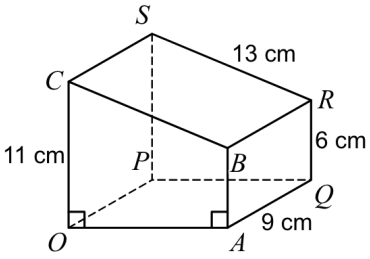
b) Calculate the length AG.



Dr Frost 410b and 410a

Worked Example

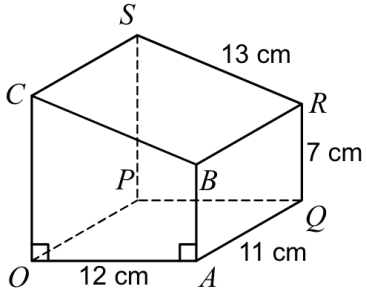
The diagram below shows a prism with a trapezium cross-section.



Find the length of OA

Your Turn

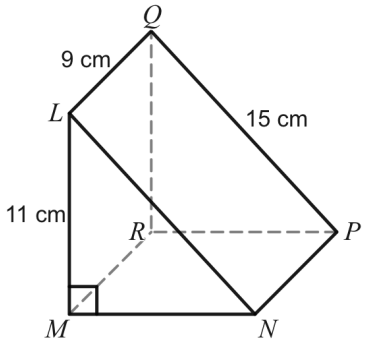
The diagram below shows a prism with a trapezium cross-section.



Find the length of OC

Worked Example

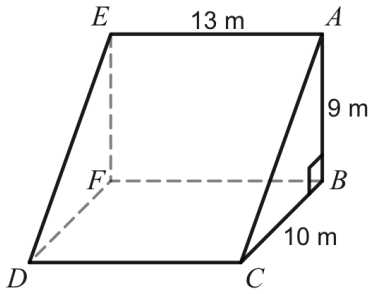
$LMNPQR$ is a triangular prism.
$LM = 11$ cm, $QP = 15$ cm and $LQ = 9$ cm.
Angle $LMN = 90°$



Find the length of the line MP.
Give your answer correct to 1 decimal place.

Your Turn

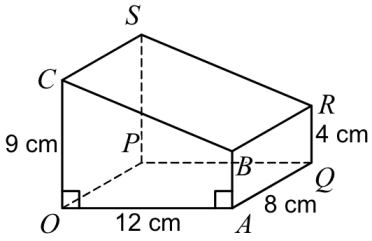
$ABCDEF$ is a triangular prism.
$AB = 9$ m, $BC = 10$ m and $AE = 13$ m.
Angle $ABC = 90°$.



Find the length of the line CE.
Give your answer correct to 1 decimal place.

Worked Example

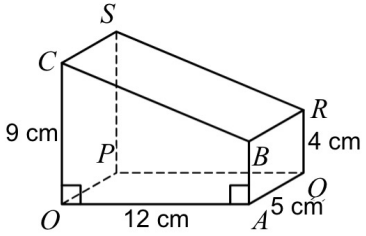
The diagram below shows a prism with a trapezium cross-section.



Find the length of OR
Give your answer to 1 decimal place.

Your Turn

The diagram below shows a prism with a trapezium cross-section.



Find the length of QC
Give your answer to 1 decimal place.

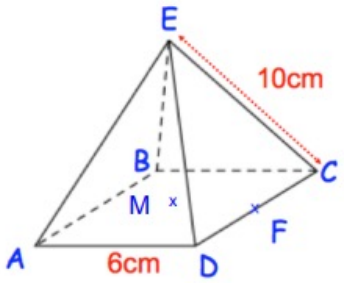
Worked Example

Shown below is a square based pyramid.

a) Find the length BD.

b) Find the length EM.

c) Find the length EF.



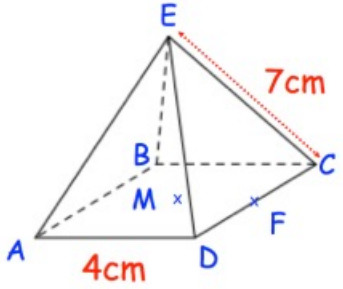
Your Turn

Shown below is a square based pyramid.

a) Find the length BD.

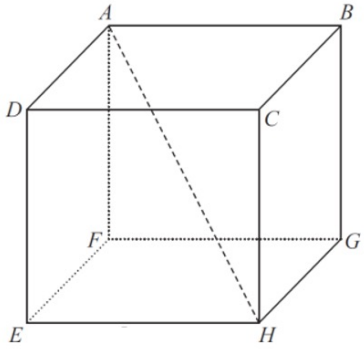
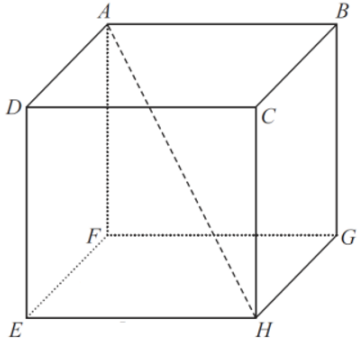
b) Find the length EM.

c) Find the length EF.



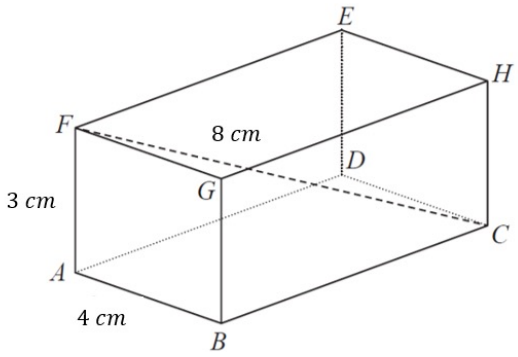
Dr Frost 410f and 410g

3D Trigonometry

Worked Example	Your Turn
A cube $ABCDEFGH$ has side lengths of 10 cm. Find the angle between the diagonal AH and the base $EFGH$.  	A cube $ABCDEFGH$ has side lengths of 6 cm. Find the angle between the diagonal AH and the base $EFGH$. 

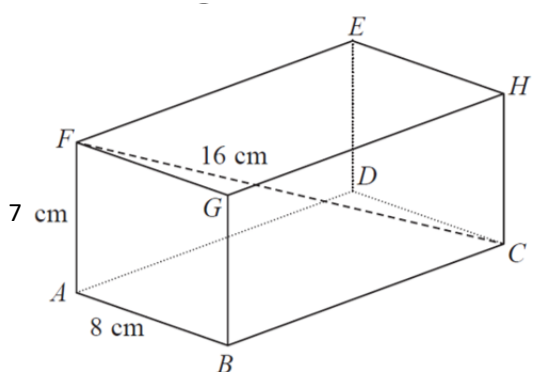
Worked Example

Calculate the angle between the line FC and the plane $ABGF$.



Your Turn

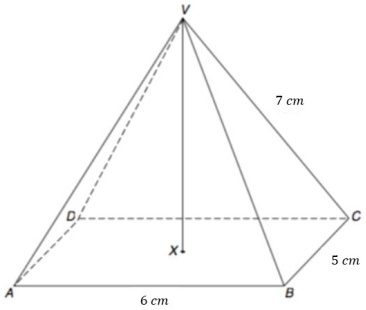
Calculate the angle between the line FC and the plane $ABGF$.



Dr Frost 411a

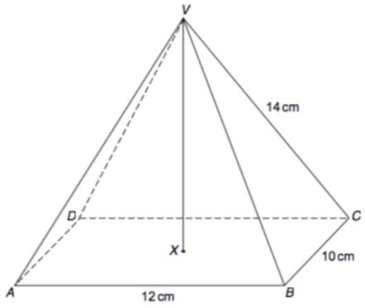
Worked Example

$VABCD$ is a rectangular based pyramid.
Calculate the angle between VC and the plane $ABCD$.



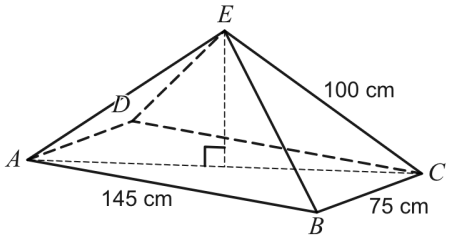
Your Turn

$VABCD$ is a rectangular based pyramid.
Calculate the angle between VC and the plane $ABCD$.



Worked Example

$ABCDE$ is a rectangle-based pyramid.

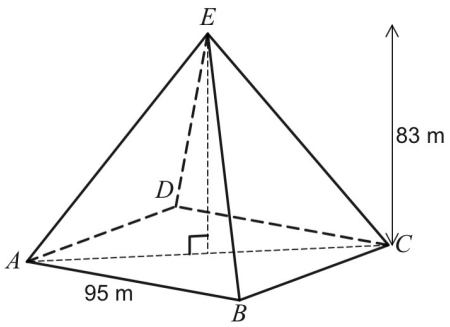


$AB = 145$ cm and $BC = 75$ cm
$CE = 100$ cm

Find the size of angle CEA
Give your answer correct to one decimal place.

Your Turn

$ABCDE$ is a square-based pyramid.

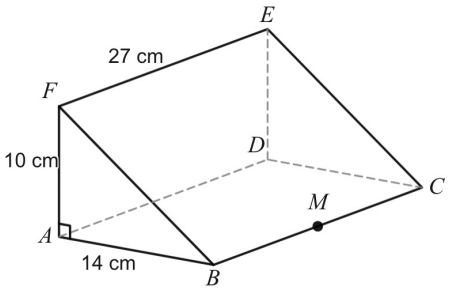


$AB = BC = 95$ m
The perpendicular height of $ABCDE$ is 83 m

Find the size of angle CEA
Give your answer correct to one decimal place.

Worked Example

The diagram shows a triangular prism $ABCDEF$



Angle $BAF = 90°$

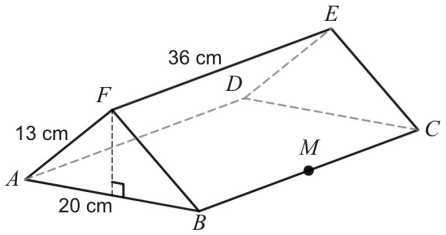
$AB = 14$ cm, $FE = 27$ cm and $AF = 10$ cm

M is the midpoint of BC

Calculate the size of angle between FM and the base $ABCD$
Give your answer correct to one decimal place.

Your Turn

The diagram shows a triangular prism $ABCDEF$



F is vertically above the midpoint of AB

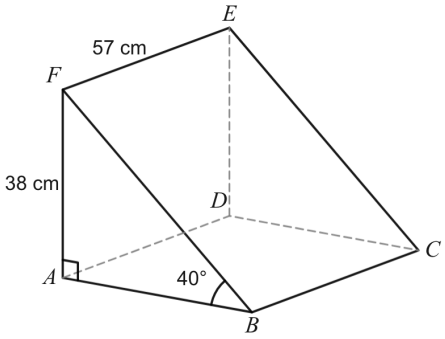
$AB = 20$ cm, $FE = 36$ cm and $AF = 13$ cm

M is the midpoint of BC

Calculate the size of angle between FM and the base $ABCD$
Give your answer correct to one decimal place.

Worked Example

The diagram shows a triangular prism $ABCDEF$

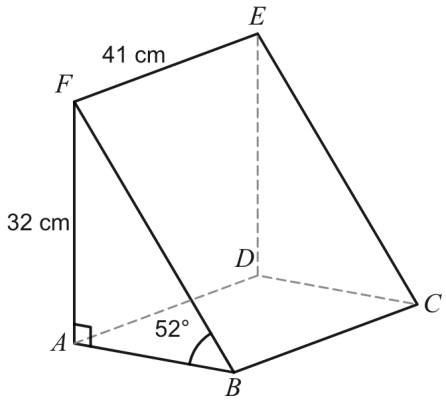


$AF = 38$ cm and $FE = 57$ cm
Angle $FBA = 40°$

Calculate the size of angle between FC and the base $ABCD$
Give your answer correct to one decimal place.

Your Turn

The diagram shows a triangular prism $ABCDEF$

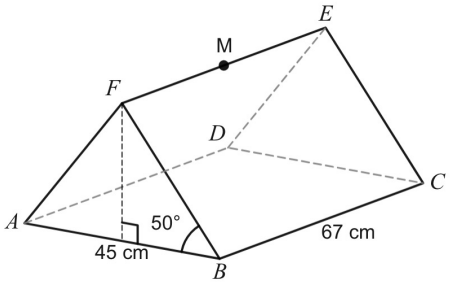


$AF = 32$ cm and $FE = 41$ cm
Angle $FBA = 52°$

Calculate the size of angle between FC and the base $ABCD$
Give your answer correct to one decimal place.

Worked Example

The diagram shows a triangular prism $ABCDEF$



F is vertically above the midpoint of AB
$AB = 45$ cm and $BC = 67$ cm
Angle $FBA = 50°$

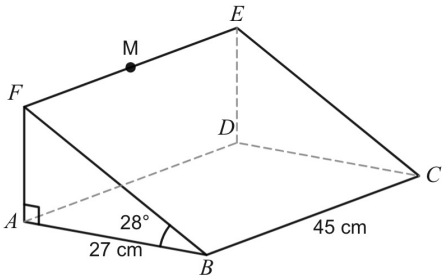
M is the midpoint of EF

Calculate the length of MB

Give your answer correct to one decimal place.

Your Turn

The diagram shows a triangular prism $ABCDEF$



Angle $BAF = 90°$
$AB = 27$ cm and $BC = 45$ cm
Angle $FBA = 28°$

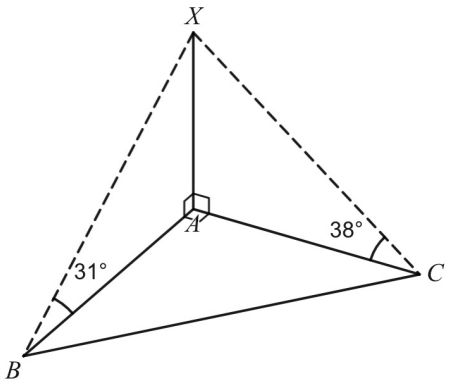
M is the midpoint of EF

Calculate the length of MB

Give your answer correct to one decimal place.

Worked Example

The diagram shows a vertical tower AX and two points B and C
Points A, B and C are on a horizontal plane.
B is due south of A and C is due east of A

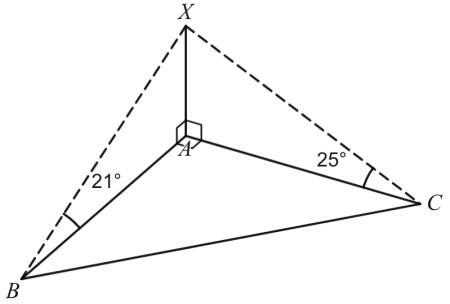


The tower, AX, is 22 m high.
The angle of elevation of X from B is $31°$
The angle of elevation of X from C is $38°$

Calculate the distance BC
Give your answer correct to one decimal place.

Your Turn

The diagram shows a vertical tower AX and two points B and C
Points A, B and C are on a horizontal plane.
B is due south of A and C is due east of A

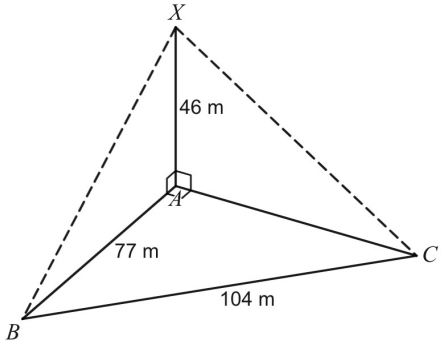


The tower, AX, is 13 m high.
The angle of elevation of X from B is $21°$
The angle of elevation of X from C is $25°$

Calculate the distance BC
Give your answer correct to one decimal place.

Worked Example

The diagram shows a vertical tower AX and two points B and C
Points A, B and C are on a horizontal plane.
Angle $BAC = 90°$



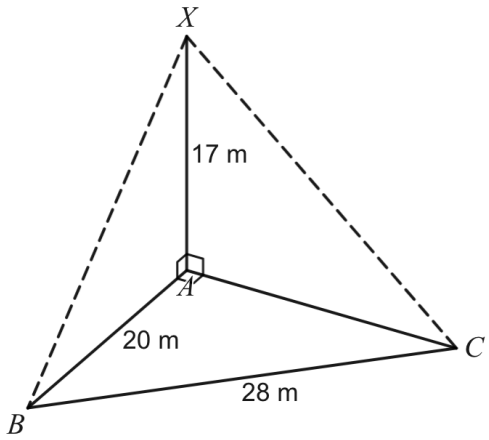
The tower, AX, is 46 m high.
$AB = 77$ m
$BC = 104$ m

D is the point on BC which is nearest to A

Find the angle of elevation from D to the top of the tower.
Give your answer correct to one decimal place.

Your Turn

The diagram shows a vertical tower AX and two points B and C
Points A, B and C are on a horizontal plane.
Angle $BAC = 90°$



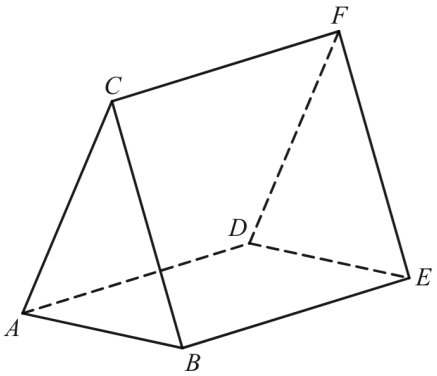
The tower, AX, is 17 m high.
$AB = 20$ m
$BC = 28$ m

D is the point on BC which is nearest to A

Find the angle of elevation from D to the top of the tower.
Give your answer correct to one decimal place.

Worked Example

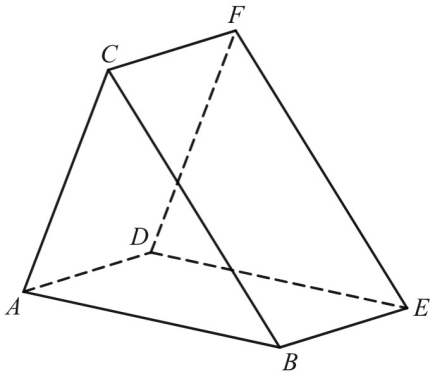
The diagram shows the prism $ABCDEF$ with cross section triangle ABC.



Angle $BEC = 36°$
$BE = 137$ m
Angle $CAB = 64°$
Angle $ACB = 47°$
Find the length of AB
Give your answer correct to 1 decimal place.

Your Turn

The diagram shows the prism $ABCDEF$ with cross section triangle ABC.

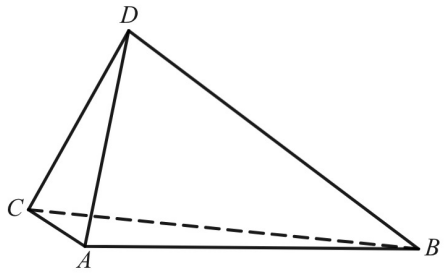


$\mathrm{CE} = 102$ cm
$BE = 52$ cm
Angle $CAB = 66°$
Angle $ACB = 67°$
Find the length of AB
Give your answer correct to 1 decimal place.

Dr Frost 470a

Worked Example

The diagram shows the triangular based pyramid $ABCD$

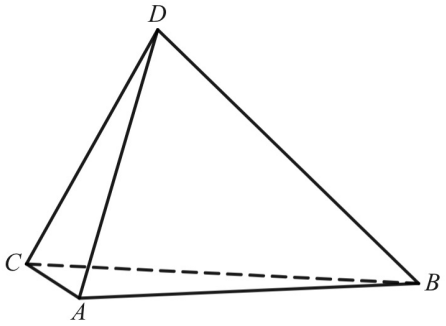


Angle $ACB = 45°$
$AC = 15$ mm
$CB = 31$ mm
$BD = 25$ mm
Angle $DBA = 37°$

Find the length of AD

Your Turn

The diagram shows the triangular based pyramid $ABCD$

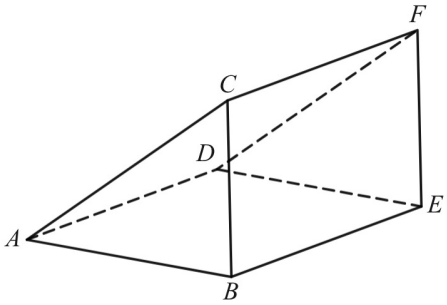


Angle $ACB = 59°$
$AC = 22$ mm
$CB = 44$ mm
$BD = 41$ mm
Angle $DBA = 45°$

Find the length of AD

Worked Example

The diagram below shows the prism $ABCDEF$ with cross-section triangle ABC

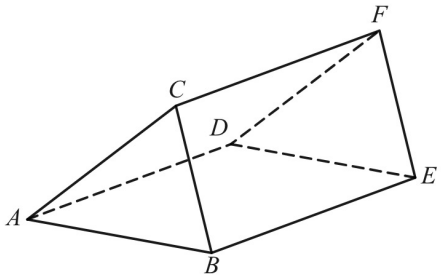


$AB = 147$ m and $BE = 195$ m
Angle $CEB = 36°$ and angle $ACB = 55°$

Find the size of angle CAB
Give your answer correct to 1 decimal place.

Your Turn

The diagram below shows the prism $ABCDEF$ with cross-section triangle ABC

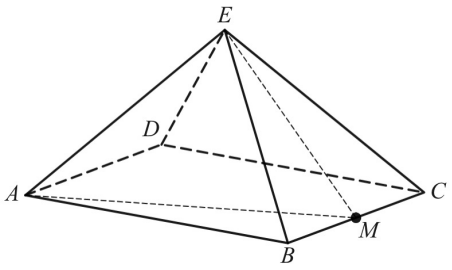


$AB = 26$ m and $BE = 41$ m
Angle $CEB = 34°$ and angle $ACB = 68°$

Find the size of angle CAB
Give your answer correct to 1 decimal place.

Worked Example

The diagram shows a rectangle-based pyramid $ABCDE$

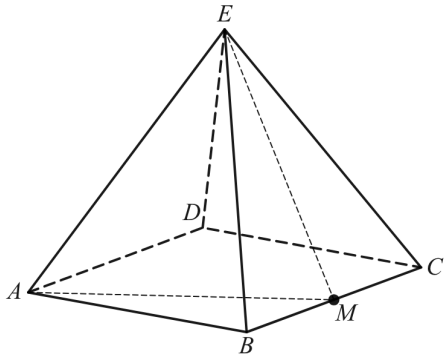


$AE = BE = CE = DE = 88$ cm
$AB = 108$ cm and $BC = 80$ cm
M is the midpoint of BC

Find the size of angle EMA
Give your answer correct to 1 decimal place.

Your Turn

The diagram shows a rectangle-based pyramid $ABCDE$



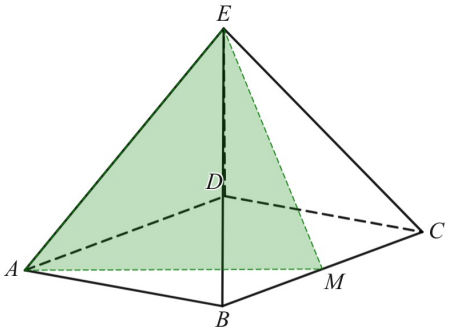
$AE = BE = CE = DE = 245$ mm
$AB = 199$ mm and $BC = 227$ mm
M is the midpoint of BC

Find the size of angle EMA
Give your answer correct to 1 decimal place.

Dr Frost 470d

Worked Example

The diagrams shows the rectangle-based pyramid $ABCDE$

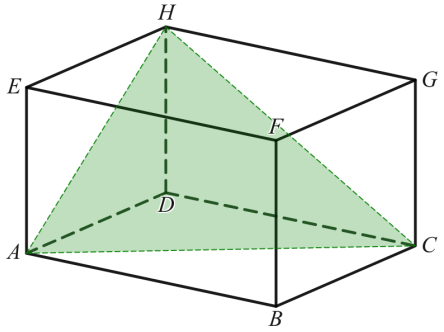


$AB = BE = CE = DE = 61$ cm
$AB = 47$ cm and $BC = 68$ cm
M is the midpoint of BC

Find the area of triangle AME
Give your answer correct to 1 decimal place.

Your Turn

The diagrams shows the cuboid $ABCDEFGH$



$AB = 148$ mm, $BC = 118$ mm and $AE = 71$ mm

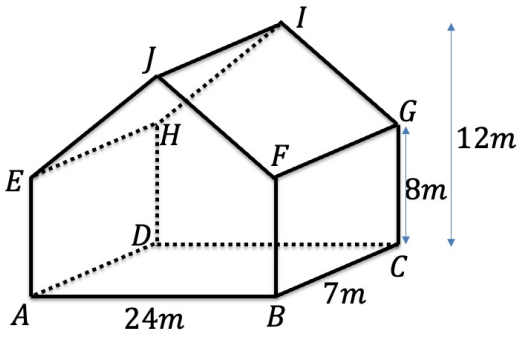
Find the area of triangle ACH
Give your answer correct to 1 decimal place.

Worked Example

Frost Manor is as pictured, with $EFGH$ horizontally level.

a) Find the angle between the line AG and the plane $ABCD$.

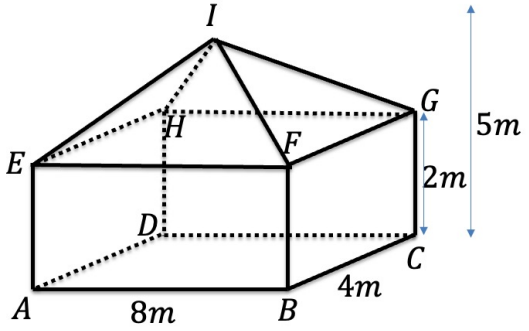
b) Find the angle between the planes $FGIJ$ and $EFGH$.



Your Turn

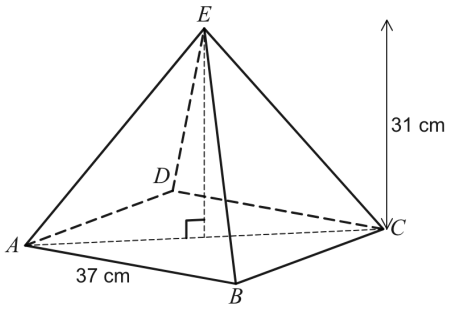
a) Determine the angle between the line AI and the plane $ABCD$.

b) Determine the angle between the planes FGI and $EFGH$.



Worked Example

The diagram shows a pyramid $ABCDE$ with a square base $ABCD$



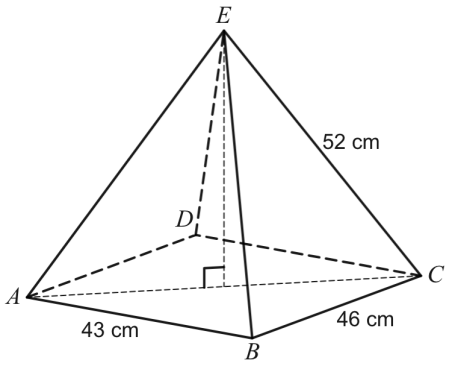
The vertex E of the pyramid is vertically above the centre of the base.

$AB = BC = CD = AD = 37$ cm
The perpendicular height of the pyramid is 31 cm

Calculate the angle between the plane BCE and the plane $ABCD$
Give your answer correct to one decimal place.

Your Turn

The diagram shows a pyramid $ABCDE$ with a rectangular base $ABCD$



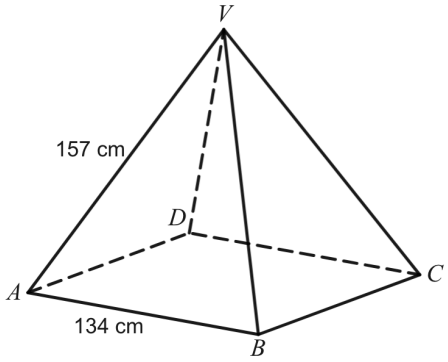
The vertex E of the pyramid is vertically above the centre of the base.

$AB = CD = 43$ cm and $BC = AD = 46$ cm
$AE = BE = CE = DE = 52$ cm

Calculate the angle between the plane BCE and the plane $ABCD$
Give your answer correct to one decimal place.

Worked Example

The diagram below shows the pyramid $ABCDV$ with rectangular base $ABCD$

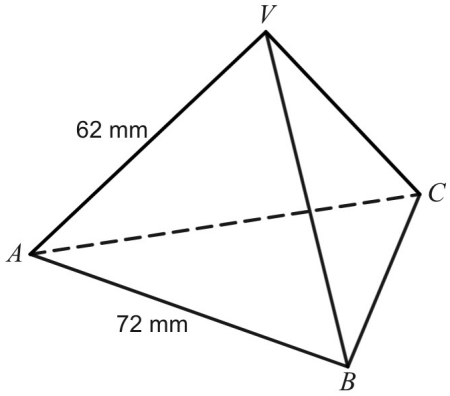


$AB = BC = CD = DA = 134$ cm
$AV = BV = CV = DV = 157$ cm

Find the non-reflex angle between the plane VAB and the plane VBC
Give your answer correct to 1 decimal place.

Your Turn

The diagram below shows the pyramid $ABCV$ with equilateral triangle base ABC



$AB = BC = CD = 72$ mm
$AV = BV = CV = 62$ mm

Find the non-reflex angle between the plane VAB and the plane VBC
Give your answer correct to 1 decimal place.

Extra Notes

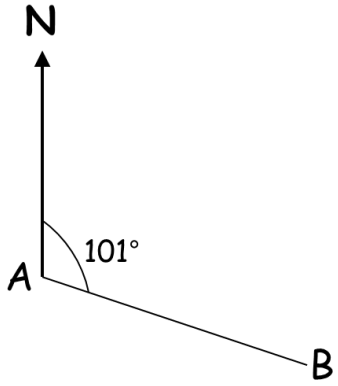
3 Bearings

Can this be a Bearing?	
040	Yes / No
90	Yes / No
90.5	Yes / No
158.50	Yes / No
58.5	Yes / No

Intelligent Practice

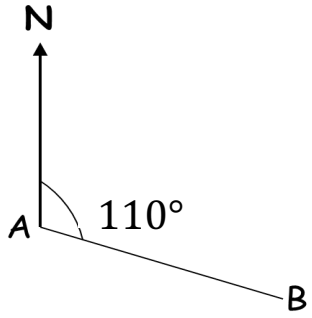
1) 045 Yes / No
2) 090 Yes / No
3) 45 Yes / No
4) 360 Yes / No
5) 361 Yes / No
6) 450 Yes / No
7) 30 Yes / No
8) 030 Yes / No
9) -145 Yes / No
10) 260 Yes / No
11) 365 Yes / No
12) 180 Yes / No
13) 27 Yes / No
14) -049 Yes / No
15) 049.5 Yes / No
16) 0180 Yes / No
17) 045 Yes / No
18) 145 Yes / No
19) -260 Yes / No
20) 0100 Yes / No
21) 80 Yes / No
22) 080 Yes / No
23) 0005 Yes / No
24) 000.5 Yes / No
25) 100.005 Yes / No

Worked Example



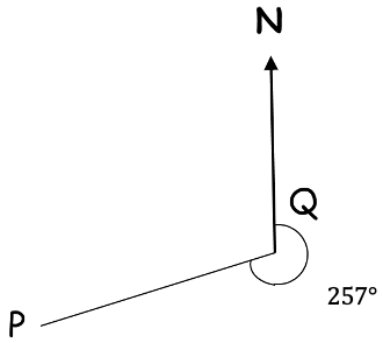
a) Find the bearing of B from A
b) Find the bearing of A from B

Your Turn



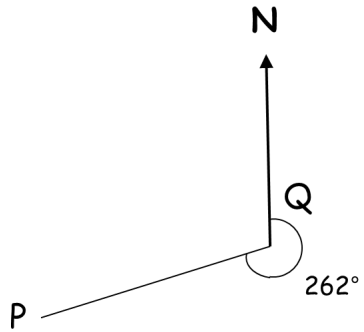
a) Find the bearing of B from A
b) Find the bearing of A from B

Worked Example



a) Find the bearing of P from Q
b) Find the bearing of Q from P

Your Turn

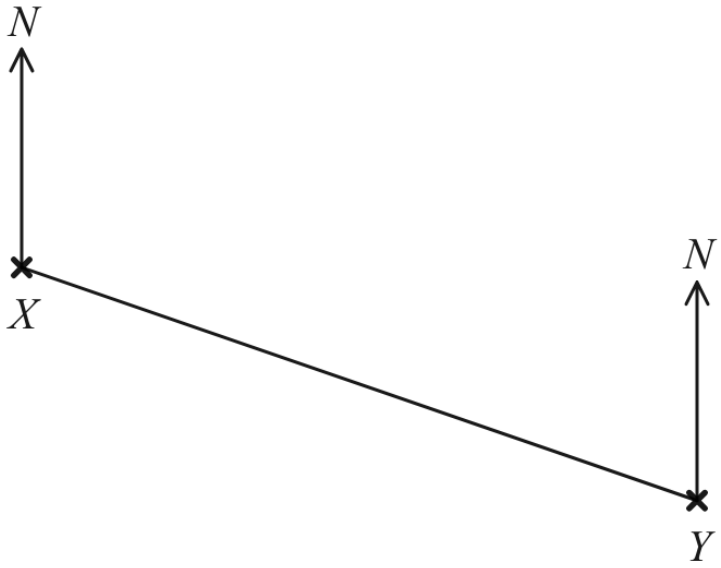


a) Find the bearing of P from Q
b) Find the bearing of Q from P

Dr Frost 264b and 264d

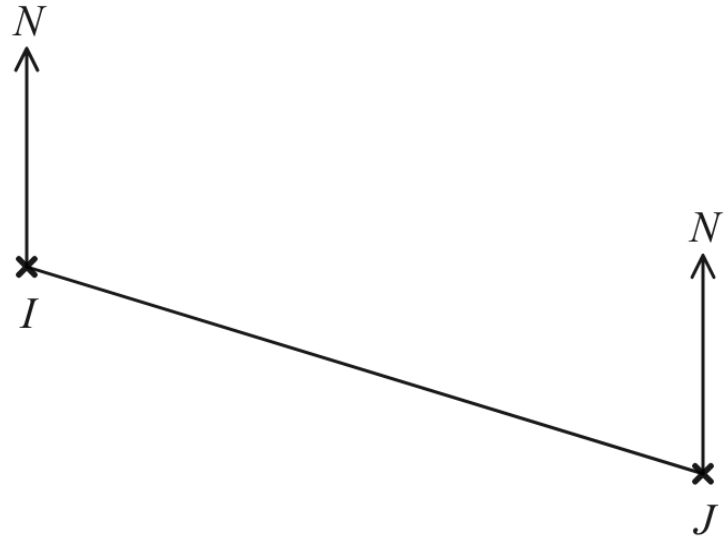
Worked Example

Measure the bearing of X from Y.



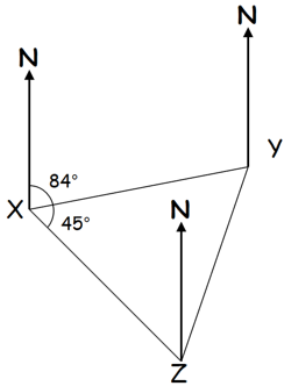
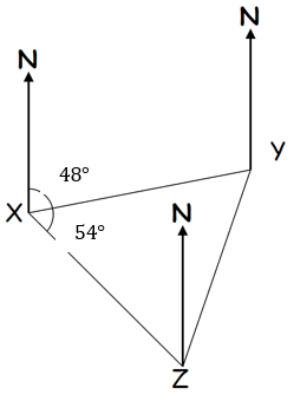
Your Turn

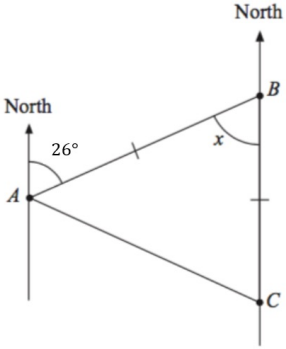
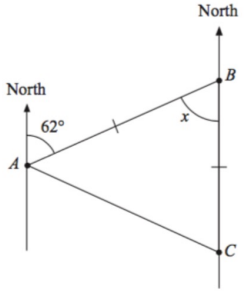
Measure the bearing of I from J.



Dr Frost 264c

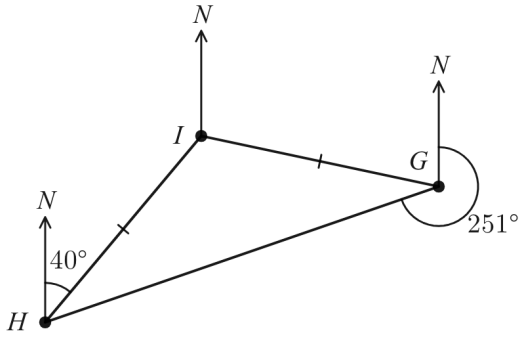
Worked Example	Your Turn
a) The bearing of B from A is $030°$. What is the bearing of A from B? b) The bearing of B from A is $130°$. What is the bearing of A from B? c) The bearing of B from A is $230°$. What is the bearing of A from B?	a) The bearing of B from A is $250°$. What is the bearing of A from B? b) The bearing of B from A is $050°$. What is the bearing of A from B? c) The bearing of B from A is $150°$. What is the bearing of A from B?

Worked Example	Your Turn
Find the bearing of:  Y from X X from Y Z from X X from Z	Find the bearing of:  Y from X X from Y Z from X X from Z

Worked Example	Your Turn
Calculate the bearing of C from A  	Calculate the bearing of C from A 

Worked Example

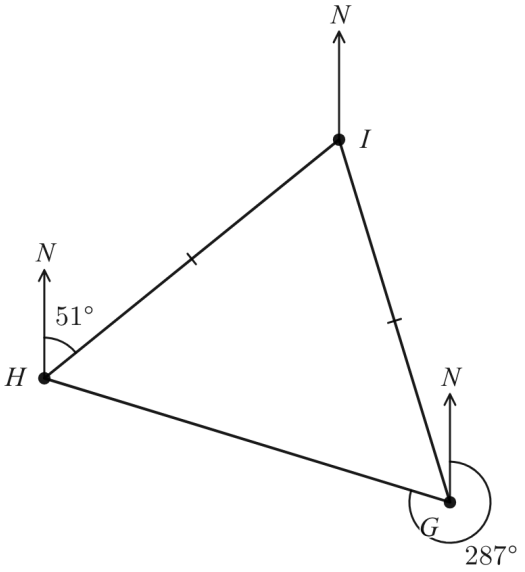
The diagram shows three points, G, H and I.
The bearing of H from G is $251°$.
The bearing of I from H is $040°$.
$HI = IG$



Find the bearing of I from G.

Your Turn

The diagram shows three points, G, H and I.
The bearing of I from H is $051°$.
The bearing of H from G is $287°$.
$HI = IG$



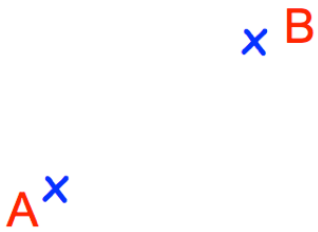
Find the bearing of I from G.

Question 1: Write down the bearing of B from A in each of the following.
Give each answer as a three figure bearing.

(a)

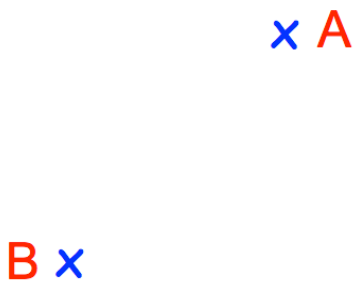
(b)

(c)



(d)

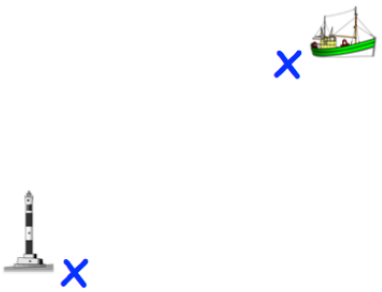
(e)



(f)

Question 2: Write down the bearing of the boat from the lighthouse in each of the following. Give each answer as a three figure bearing.

(a)

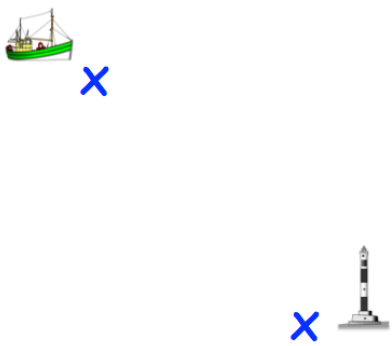


(b)

(c)

(d)

(e)



(f)

Question 3: There are eight airplanes in the airspace above a radar.
Put an x in the middle of your page to represent the radar.
Letting 1cm = 1mile, mark the position of each airplane.

(a) Airplane 1 is 6 miles from the radar on a bearing of 025°
(b) Airplane 2 is 4 miles from the radar on a bearing of 075°
(c) Airplane 3 is 5 miles from the radar on a bearing of 125°
(d) Airplane 4 is 8 miles from the radar on a bearing of 150°
(e) Airplane 5 is 4 miles from the radar on a bearing of 190°
(f) Airplane 6 is 3 miles from the radar on a bearing of 250°
(g) Airplane 7 is 6.5 miles from the radar on a bearing of 310°
(h) Airplane 8 is 9 miles from the radar on a bearing of 351°

Question 4: There are eight boats in the sea around an island.
Put an x in the middle of your page to represent the island.
Letting 1cm = 1km, mark the position of each boat.

(a) Boat 1 is 4 km from the island on a bearing of 080°
(b) Boat 2 is 3 km from the island on a bearing of 016°
(c) Boat 3 is 5 km from the island on a bearing of 111°
(d) Boat 4 is 5.5 km from the island on a bearing of 308°
(e) Boat 5 is 3.5 km from the island on a bearing of 055°
(f) Boat 6 is 6 km from the island on a bearing of 214°
(g) Boat 7 is 6 km from the island on a bearing of 199°
(h) Boat 8 is 5 km from the island on a bearing of 154°

Question 5: Give these directions of travel as three figure bearings

(a) North (b) South-east (c) West (d) North-east

(e) East (f) South-west (g) South (h) North-west

Question 6: A dolphin is on a bearing of 100° from the island.
The same dolphin is on a bearing of 015° from the lighthouse.
On a sketch of the diagram below, mark the location of the dolphin.

Question 7: A hot-air balloon is on a bearing of 140° from the radar A.
The same hot-air balloon is on a bearing of 065° from the radar B.
On a sketch of the diagram below, mark the location of the hot-air balloon.

Question 8: A UFO is on a bearing of 015° from the radar A.
The same UFO is on a bearing of 315° from the radar B.
On a sketch of the diagram below, mark the location of the UFO.

Question 9:

(a) The bearing of A from B is 025°, find the bearing of B from A.
(b) The bearing of A from B is 061°, find the bearing of B from A.
(c) The bearing of A from B is 098°, find the bearing of B from A.
(d) The bearing of A from B is 102°, find the bearing of B from A.
(e) The bearing of A from B is 193°, find the bearing of B from A.
(f) The bearing of A from B is 222°, find the bearing of B from A.
(g) The bearing of A from B is 315°, find the bearing of B from A.

Question 10: Make a copy of the diagram below into your book.

(a) Find the bearing of B from A.
(b) Find the bearing of A from B.

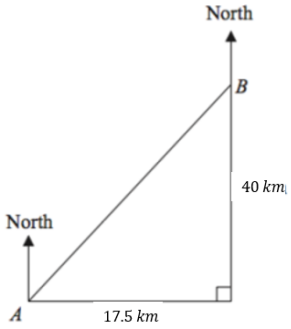
Use the scale 1cm represents 20miles.

(c) From your diagram, work out the real distance between A and B.

C is 140 miles from B on a bearing of 110°.

(d) On your diagram, mark C with a cross.

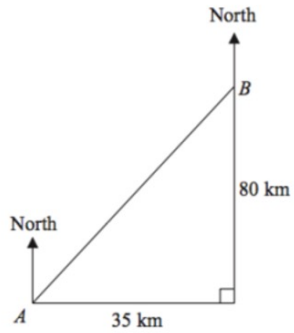
Worked Example



a) Work out the bearing of town A from town B

b) Work out the bearing of town B from town A

Your Turn

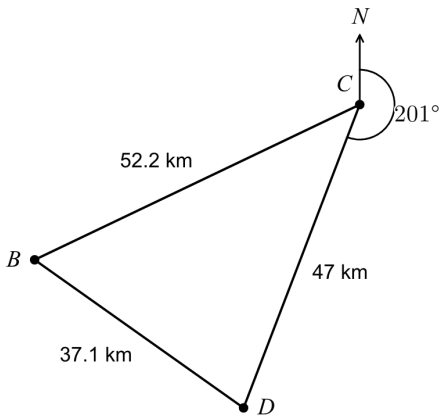


a) Work out the bearing of town A from town B

b) Work out the bearing of town B from town A

Worked Example

The diagram shows the position of three towns B, C and D



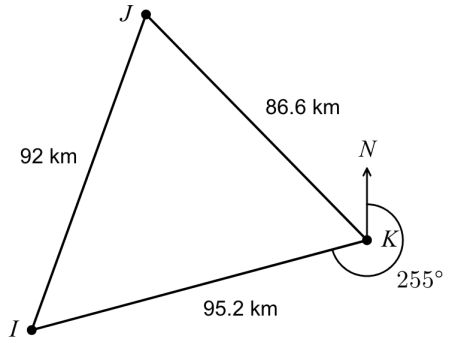
The distance between C and D is 47 km
The distance between C and B is 52.2 km
The distance between D and B is 37.1 km

The bearing of D from C is $201°$

Find the bearing of B from C
Give your answer correct to the nearest degree.

Your Turn

The diagram shows the position of three towns I, J and K



The distance between K and I is 95.2 km
The distance between K and J is 86.6 km
The distance between I and J is 92 km

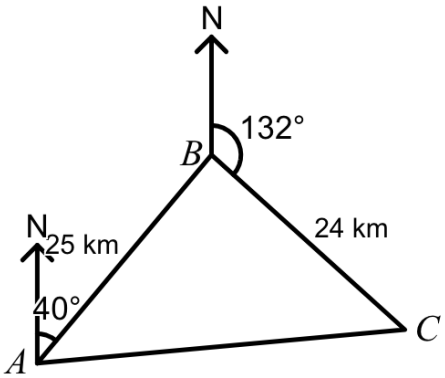
The bearing of I from K is $255°$

Find the bearing of J from K
Give your answer correct to the nearest degree.

Dr Frost 469a

Worked Example

In the diagram, the three towns Abbotsley, Bowden and Cowbridge are represented by the letters A, B and C

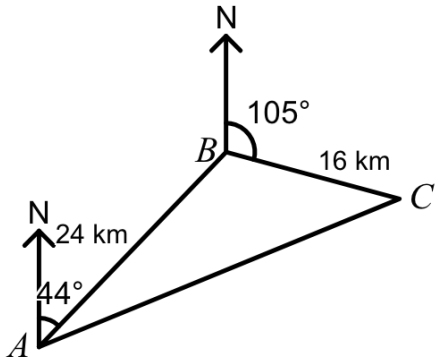


A plane flies from Abbotsley for 25 km on a bearing of $040°$ to Bowden.
It then flies from Bowden for 24 km on a bearing of $132°$ to Cowbridge.
The plane then returns directly to Abbotsley.

Find the total distance travelled by the plane.

Your Turn

In the diagram, the three towns Abercarn, Bagham and Caldmore are represented by the letters A, B and C



A helicopter flies from Abercarn for 24 km on a bearing of $044°$ to Bagham.
It then flies from Bagham for 16 km on a bearing of $105°$ to Caldmore.
The helicopter then returns directly to Abercarn.

Find the total distance travelled by the helicopter.

Dr Frost 469b

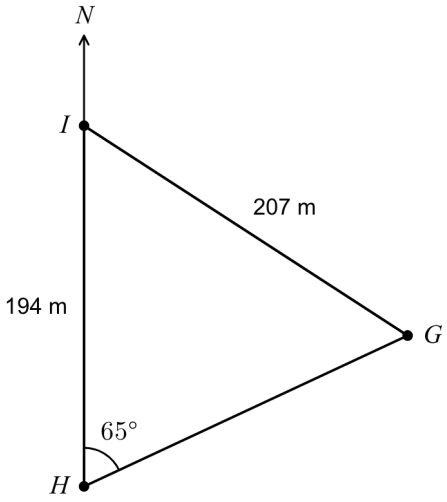
Worked Example

The diagram shows the position of three people G, H and I

I is due north of H

The bearing of G from H is $065°$
The distance between I and G is 207 m.
The distance between H and I is 194 m.

Calculate the bearing of G from I
Give your answer to the nearest degree.



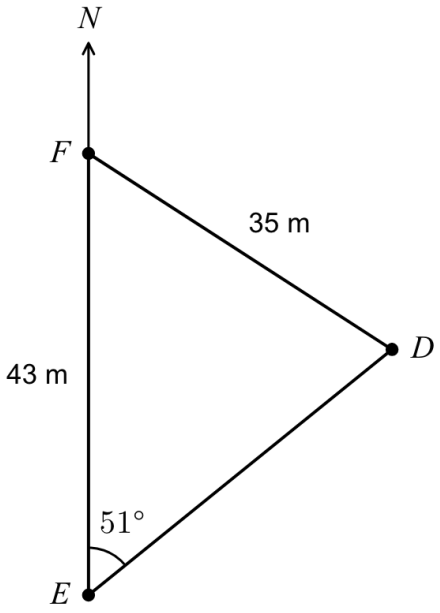
Your Turn

The diagram shows the position of three people D, E and F

F is due north of E

The bearing of D from E is $051°$
The distance between F and D is 35 m.
The distance between E and F is 43 m.

Calculate the bearing of D from F
Give your answer to the nearest degree.



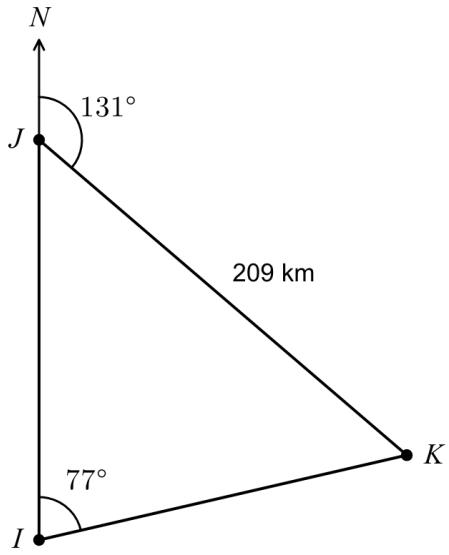
Dr Frost 469c

Worked Example

The diagram shows the position of three radio towers I, J and K

J is due north of I

The bearing of K from I is $77°$
The bearing of K from J is $131°$
The distance between J and K is 209 km.



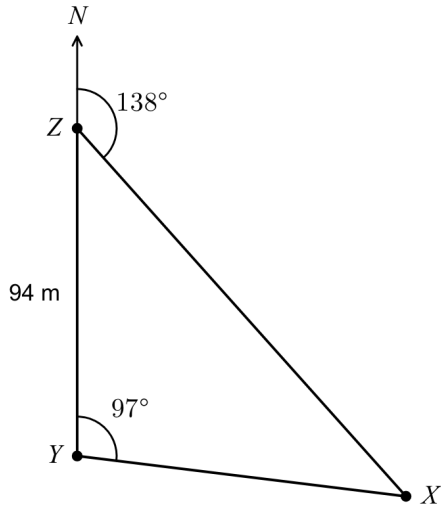
Find the distance between K and I
Give your answer correct to one decimal place.

Your Turn

The diagram shows the position of three radio towers X, Y and Z

Z is due north of Y

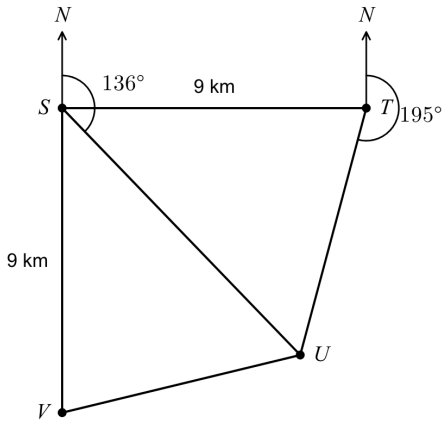
The bearing of X from Y is $97°$
The bearing of X from Z is $138°$
The distance between Y and Z is 94 m.



Find the distance between X and Y
Give your answer correct to one decimal place.

Dr Frost 469d

Worked Example



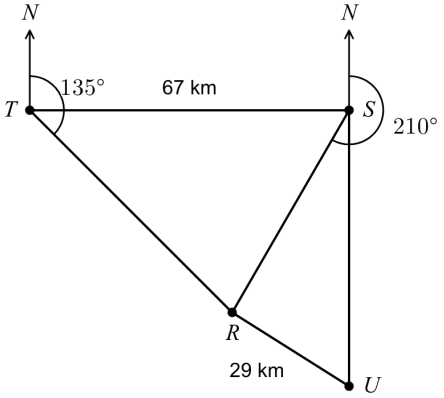
There is a lifeguard station at point S and at point T
T is 9 km due east of S

There is a lifeboat at point U
U is on a bearing of $136°$ from S
U is on a bearing of $195°$ from T

There is a yacht at point V
V is 9 km due south of S

Find the bearing of U from V
Give your answer correct to the nearest degree.

Your Turn

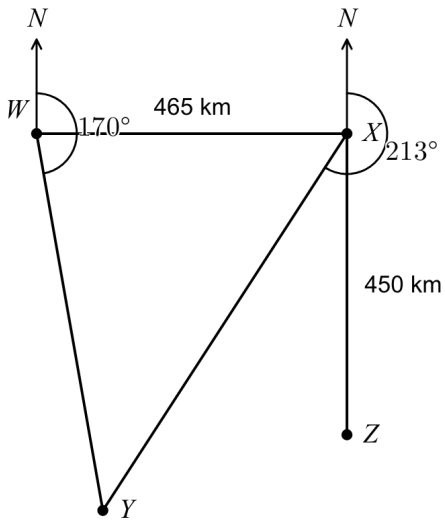


There are three towns at points S, T and R
T is 67 km due west of S
R is on a bearing of $210°$ from S
R is on a bearing of $135°$ from T

A hiker walks due south from S to point U, which is 29 km from R

Find the bearing of R from U
Give your answer correct to the nearest degree.

Worked Example

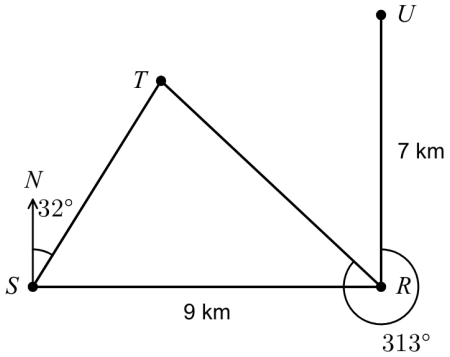


There are three airports at points X, W and Y
W is 465 km due west of X
Y is on a bearing of $213°$ from X
Y is on a bearing of $170°$ from W

A plane flies 450 km due south from X to point Z

Find the distance between Y and Z
Give your answer correct to one decimal place.

Your Turn



There is a coastguard station at point R and at point S
S is 9 km due west of R

There is a rowing boat at point T
T is on a bearing of $313°$ from R
T is on a bearing of $32°$ from S

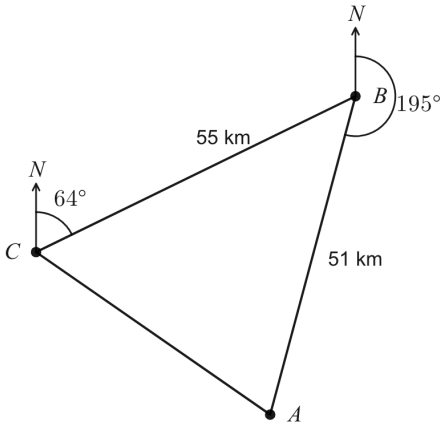
There is a speedboat at point U
U is 7 km due north of R

Find the distance between T and U
Give your answer correct to one decimal place.

Dr Frost 469f

Worked Example

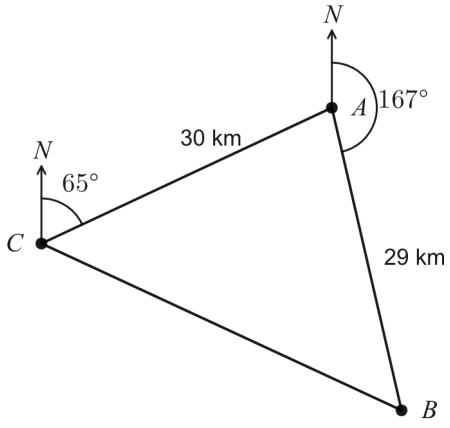
The diagram shows the position of three radio towers, A, B and C.
The bearing of A from B is $195°$.
The bearing of B from C is $64°$.
The distance between B and A is 51 km.
The distance between B and C is 55 km.



Calculate the area of triangle ABC.
Give your answer correct to 1 decimal place.

Your Turn

The diagram shows the position of three radio towers, A, B and C.
The bearing of B from A is $167°$.
The bearing of A from C is $65°$.
The distance between A and B is 29 km.
The distance between A and C is 30 km.



Calculate the area of triangle ABC.
Give your answer correct to 1 decimal place.

Extra Notes

4 Advanced Ratio

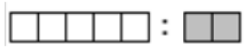
Worked Example

a) Find the ratio of Green to Blue
b) Write green as a fraction of the whole bar
c) Write blue as a fraction of the whole bar
d) Write green as a fraction of blue
e) Write blue as a fraction of green
f) Form a linear equation linking green and blue
Green =
Blue =

Your Turn

a) Find the ratio of Green to Blue
b) Write green as a fraction of the whole bar
c) Write blue as a fraction of the whole bar
d) Write green as a fraction of blue
e) Write blue as a fraction of green
f) Form a linear equation linking green and blue
Green =
Blue =

Fill in the Gaps

x : y	Visual representation	x as a fraction of whole	y as a fraction of a whole	x as a fraction of y	y as a fraction of x	Linear equation	x = ...	y = ...
								
								
1 : 7								
		$\frac{3}{8}$						
				$\frac{3}{2}$				
						$3x = 7y$		
							$x = \frac{10}{3}$	
								$y = \frac{2}{9}$

Worked Example	Your Turn
Given that $3y = 7x$, work out the ratio $x : y$	Given that $9q = 4p$, work out the ratio $p : q$

Worked Example	Your Turn
The ratio of x to y is $4 : 5$ Write a linear function involving x and y	The ratio of p to q is $5 : 3$ Write a linear function involving p and q

Worked Example	Your Turn
The ratio $5x + 3 : 2y - 1$ is equal to $5 : 4$ Express x in terms of y	The ratio $a + 1 : 2b + 5$ is equal to $5 : 7$ Express a in terms of b

Worked Example	Your Turn
Given that $3x - 10 : 9x - 51 = 2 : 3$ Find the value of x	Given that $9a - 4 : 7a + 21 = 7 : 2$ Find the value of a

Worked Example	Your Turn
Given that $7x - 6 : 4x + 12 = 5x - 2 : 5x + 10$ Find the possible values of x.	Given that $6a + 11 : 3a + 3 = 5a + 8 : 2a + 4$ Find the possible values of a.

Dr Frost 464c

Worked Example	Your Turn
$2a^2 - 5ab = 12b^2$, where $a > 0$ and $b > 0$ Find the ratio $a : b$ in its simplest form.	$5q^2 - 16pq = 16p^2$, where $p > 0$ and $q > 0$ Find the ratio $p : q$ in its simplest form.

Worked Example	Your Turn
There are blue counters and white counters in a bag in the ratio $4:3$ 10 blue counters are added, and the ratio becomes $2:1$ Work out how many white counters there are in the bag.	There are black counters and red counters in a bag in the ratio $3:4$ 20 black counters are removed, and the ratio becomes $1:3$ Work out how many red counters there are in the bag.

Worked Example	Your Turn
There are black counters and red counters in a bag in the ratio $3:7$ 5 black counters are removed, and 10 red counters are added to the bag, and the ratio becomes $2:5$ Work out the original number of red counters in the bag.	There are white counters and red counters in a bag in the ratio $3:4$ 10 white counters are removed, and 1 red counter is added to the bag, and the ratio becomes $2:3$ Work out the original number of red counters in the bag.

Worked Example	Your Turn
a, b, c and d are integers with no common factors. $a : b = 4 : 3$ $c : d = 1 : 6$ $2a = 3d$ Find $a : b : c : d$	a, b, c and d are integers with no common factors. $4a = 7b$ $c : d = 3 : 2$ $a : d = 4 : 7$ Find $a : b : c : d$

Worked Example	Your Turn
Green shapes and purple shapes are used in a game. Some of the shapes are triangles. All the other shapes are hexagons. The ratio of triangles to hexagons is $5:2$ The ratio of green triangles to purple triangles is $3:5$ Work out the fraction of shapes that are green triangles.	Blue shapes and red shapes are used in a game. Some of the shapes are circles. All the other shapes are squares. The ratio of circles to squares is $4:5$ The ratio of blue circles to red circles is $3:2$ Work out the fraction of shapes that are red circles.

Worked Example	Your Turn
White shapes and black shapes are used in a game. Some of the shapes are circles. All of the other shapes are squares. The ratio of the number of white shapes to the number of black shapes is $4:5$ The ratio of the number of white circles to the number of white squares is $3:4$ The ratio of the number of black circles to the number of black squares is $2:1$ Work out what fraction of all the shapes are circles.	Blue shapes and red shapes are used in a game. Some of the shapes are circles. All of the other shapes are squares. The ratio of the number of blue shapes to the number of red shapes is $4:1$ The ratio of the number of blue circles to the number of blue squares is $3:4$ The ratio of the number of red circles to the number of red squares is $3:2$ Work out what fraction of all the shapes are circles.

Worked Example	Your Turn
A drink seller has two types of drink mix in stock. Drink A is made from apple juice and orange juice in the ratio $8:3$ Drink B is made from apple juice and orange juice in the ratio $2:9$ Both drinks come in the same size cartons. Drink A and Drink B can be combined to produce Drink C. The vendor wants Drink C to contain apple juice and orange juice in the ratio $7:6$ If the seller is to use whole cartons only, find the least number of cartons that she needs of each type.	A drink seller has two types of drink mix in stock. Drink A is made from apple juice and orange juice in the ratio $9:5$ Drink B is made from apple juice and orange juice in the ratio $6:1$ Both drinks come in the same size cartons. Drink A and Drink B can be combined to produce Drink C. The vendor wants Drink C to contain apple juice and orange juice in the ratio $7:2$ If the seller is to use whole cartons only, find the least number of cartons that she needs of each type.

Extra Notes