Year 11

2025   Mathematics   2026

Unit 21 Tasks – Part 1

DO NOT WRITE INSIDE

Year 11

2025   Mathematics   2026

Unit 21 Tasks – Part 2

DO NOT WRITE INSIDE

Year 11

2025   Mathematics   2026

Unit 21 Tasks – Part 3

DO NOT WRITE INSIDE

Contents

1 Advanced Trigonometry

Fluency Practice

Sine Rule ×2

To find each missing length in these triangles, we will need to use the sin rule twice.
How can we maintain accuracy?
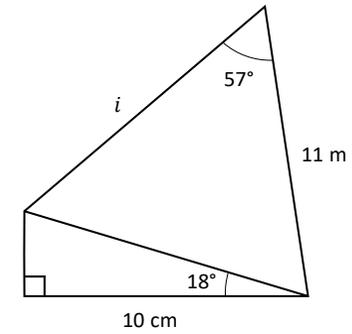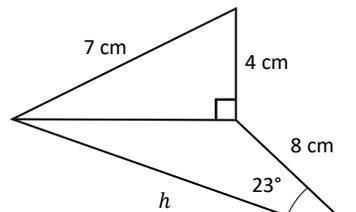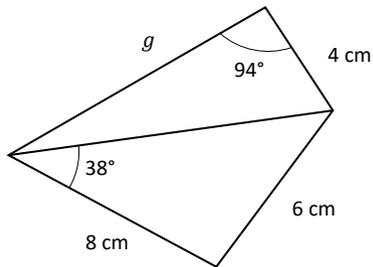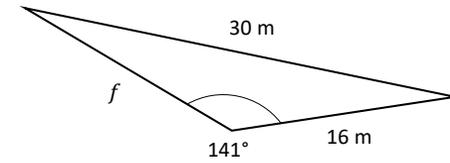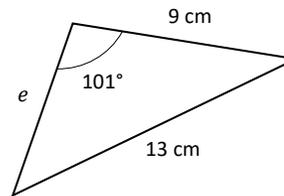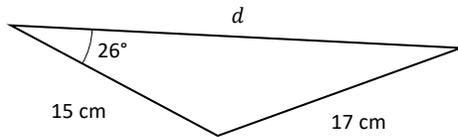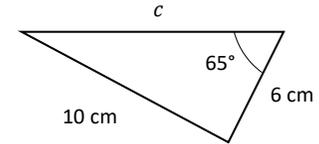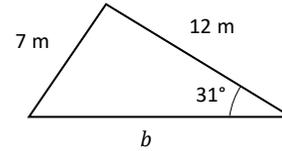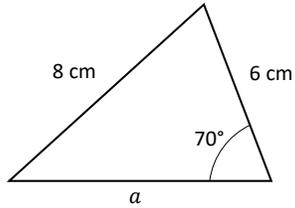
8 cm   6 cm   70°   a

7 m   12 m   31°   b

c   65°   6 cm   10 cm

d   26°   15 cm   17 cm

9 cm   101°   e   13 cm

30 m   f   141°   16 m

g   94°   4 cm   38°   6 cm   8 cm

7 cm   4 cm   8 cm   23°   h

i   57°   11 m   18°   10 cm

Fluency Practice

Find the missing length.

(a)

7.3 cm, 55, x, 48

(b)

y, 80, 57, 11cm

(c)

7.5 cm, 105, 47, a

(e)

4.5 cm, 96, 47, c

(d)

65, 55, b, 10 mm

Find the missing angle.

(b)

4.8 cm, e, 8 cm, 61, 5 cm

(a)

43, d, 6.2 cm, 4.8 cm

(c)

9.1 cm, f, 67, 7.9 cm

(e)

90, h, 4.1 cm, 5.7 cm

(d)

20 mm, 87, g, 46 mm
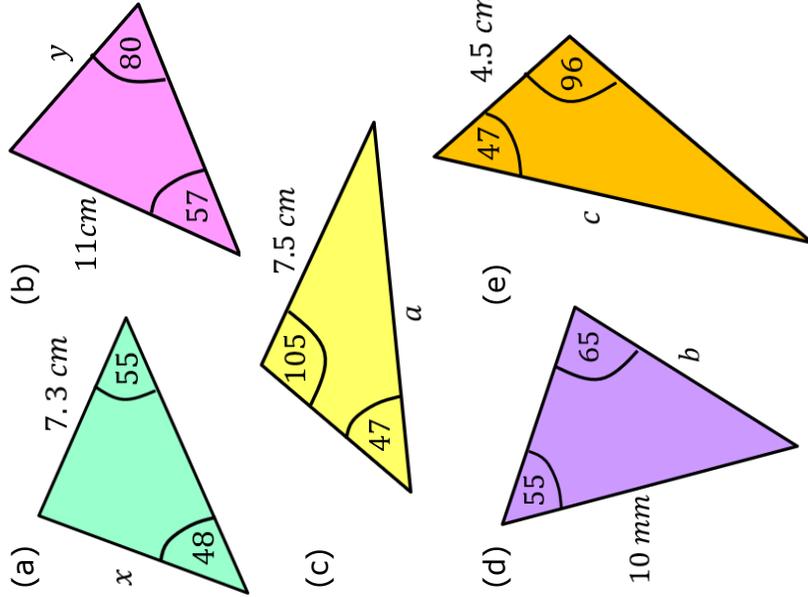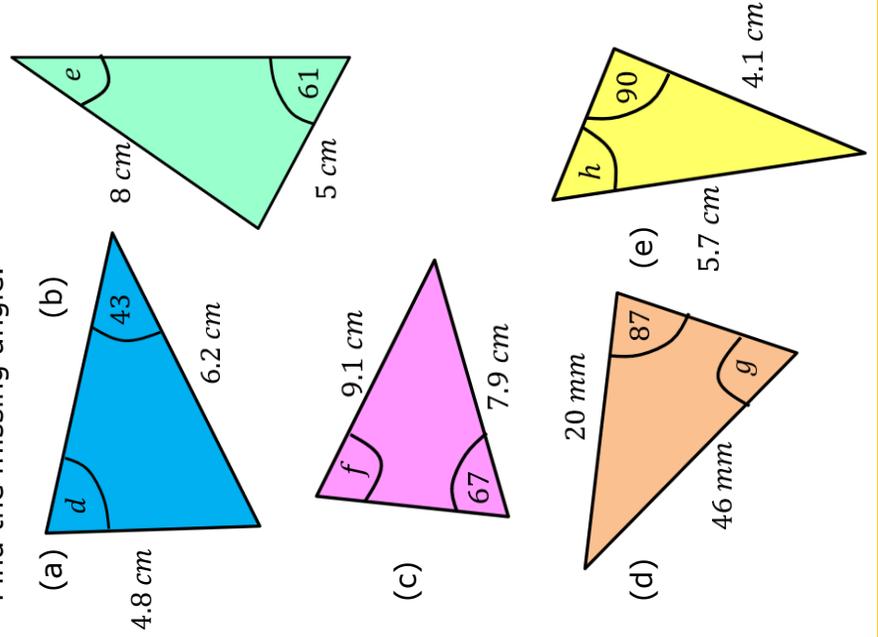
Purposeful Practice

The measurements for these different scalene triangles are correct, but they are **not to scale** and their **shapes are not drawn accurately**!

Which triangles can have two values for x... which are **ambiguous**?

Row 1:

Triangle 1: 10 cm, 6 cm, 30°, $x°$

Triangle 2: 10 cm, 9 cm, 30°, $x°$

Triangle 3: 6 cm, 9 cm, 30°, $x°$

Triangle 4: 10 cm, 9 cm, 30°, $x°$

Row 2:

Triangle 5: 10 cm, 6 cm, 85°, $x°$

Triangle 6: 6 cm, 9 cm, 85°, $x°$

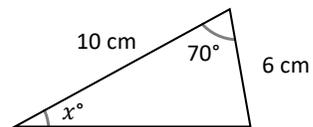Triangle 7: 10 cm, 9 cm, 85°, $x°$

Triangle 8: 8 cm, 9 cm, 60°, $x°$

Row 3:

Triangle 9: 10 cm, 6 cm, 70°, $x°$

Triangle 10: 10 cm, 9.5 cm, 70°, $x°$

Triangle 11: 6 cm, 9 cm, 70°, $x°$

Triangle 12: 10 cm, 9 cm, 45°, $x°$
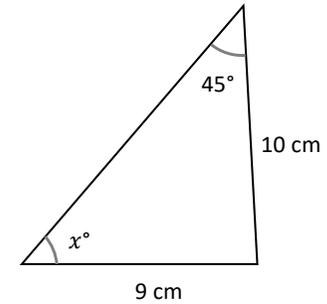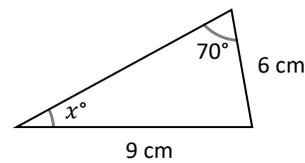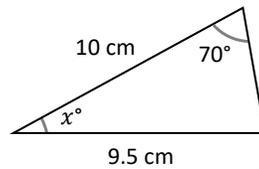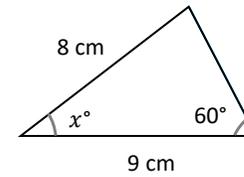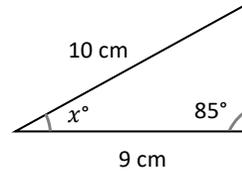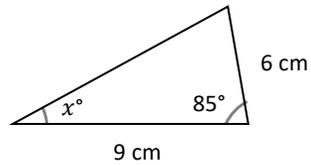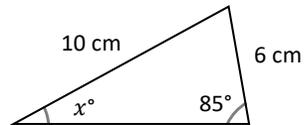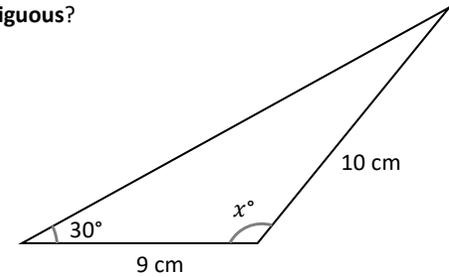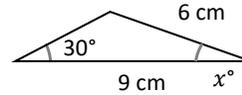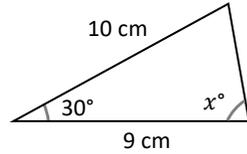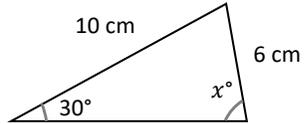
Can you draw an ambiguous triangle with a known obtuse angle?

What are the **conditions** for ambiguity?   Sketch 3 triangles where x is ambiguous.

Fluency Practice

1. In triangle ABC, $AB = 8cm$, $BC = 6cm$ and angle $CAB = 40°$ (see diagram)

   Find the two possible sizes of angle BCA.
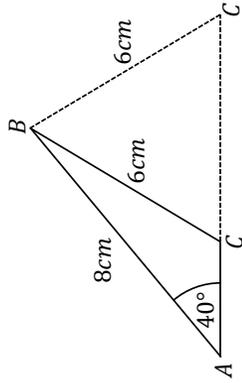
2. In triangle ABC, $AB = 10cm$, $BC = 7cm$ and angle $CAB = 30°$ (see diagram)

   Find the two possible sizes of angle BCA.
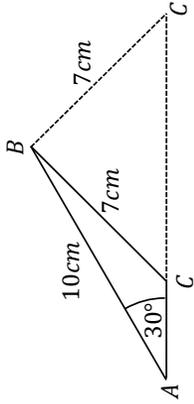
3. In triangle ABC, $AB = 12cm$, $BC = 9cm$ and angle $CAB = 47°$

   Find the two possible sizes of angle BCA.

4. In triangle ABC, $AB = 13cm$, $BC = 11cm$ and angle $CAB = 53°$

    Find the two possible sizes of angle BCA.

5. In triangle ABC, $AB = 5.1cm$, $BC = 3.8cm$ and angle $CAB = 42°$ (see diagram)

    (i)   Find the two possible sizes of angle BCA.
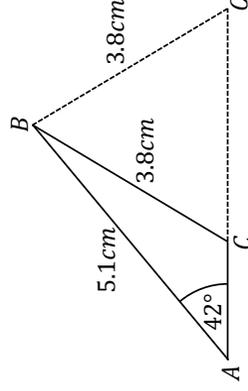
    (ii)   Find the two possible lengths of AC.

6. In triangle ABC, $AB = 17.4cm$, $BC = 10.7cm$ and angle $CAB = 23°$

    (i)   Find the two possible sizes of angle BCA.

    (ii)   Find the two possible lengths of AC.

Fluency Practice

Q	Work out the missing length	Answers	Q	Work out the missing angle	Answers
1a	5 cm, x cm, 60°, 40°		**1b**	*x* acute 7 cm, 5 cm, x°, 40°	
2a	x cm, 5 cm, 60°, 40°		**2b**	*x* obtuse 5 cm, 7 cm, x°, 40°	
3a	x cm, 60°, 40°, 5 cm		**3b**	*x* acute 5 cm, 7 cm, x°, 40°	
4a	5 cm, 60°, 40°, x cm		**4b**	*x* obtuse 7 cm, 5 cm, x°, 40°	

Fluency Practice

Triangle A: 85°, 10, 35°, A

Triangle B: 4, 40°, 110°, B

Triangle C: 3, 5, 75°, C°

Triangle D: 8, D°, 60°, 7

Triangle E: 20°, 9, 15°, E

Triangle F: 11, 100°, 15, F°

Triangle G: 12, 7, G°, 25°

Triangle H: 15°, 50, 18, H°

Triangle I: 62°, I, 50°, 5, 13

Triangle J: 19, 15, J°, 40°, 20

Triangle K: 26°, 18, K°, 28°, 10

Triangle L: 11, 6, 104°, L

10 Answers	46.2	49.3	17.4	133.6	35.4
	2.7	12.1	26.6	11.9	134.0

Fluency Practice

1. Work out the lengths of the sides marked x. Round your answers to the nearest 0.1cm.

a)
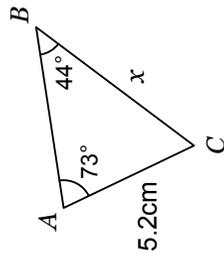
b)
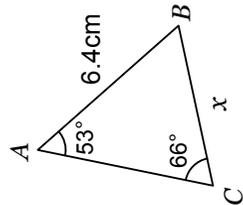
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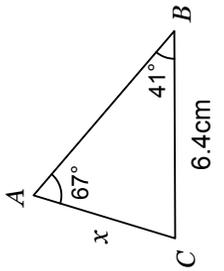
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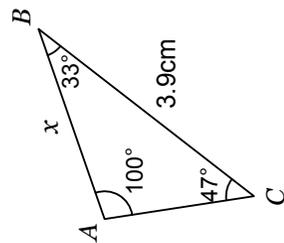
e)
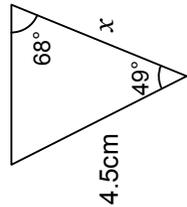
f)
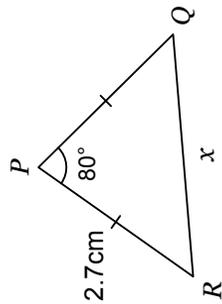
2. Work out the acute angles marked θ. Round your answers to the nearest degree.

a)
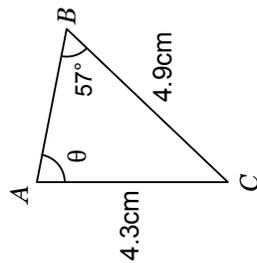
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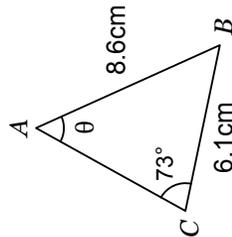
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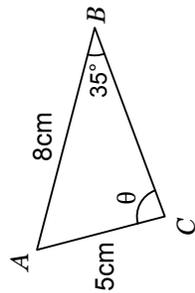
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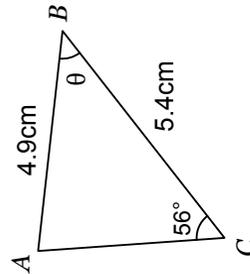
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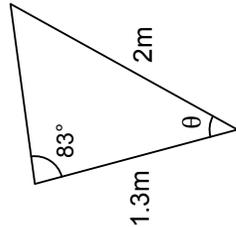
f)
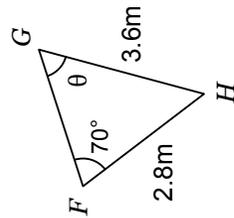
3. Triangle ABC is such that $AB = 4.5$cm, $A\hat{B}C = 37°$ and $B\hat{C}A = 78°$.
   Work out the length AC, correct to the nearest millimetre.

4. Work out the length marked x.
Give your answer correct to the nearest 0.1cm.
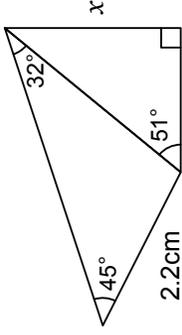
5. Work out the acute angle marked θ.
Give your answer correct to the nearest degree.
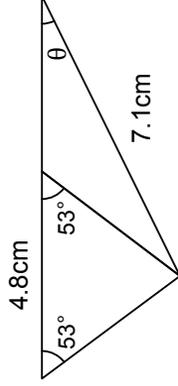
6. Hardeep is trying to work out the height of a building.

He measures the angle of elevation to the top of the building to be 42°. He walks 15m towards the building and the angle of elevation increases to 65°.
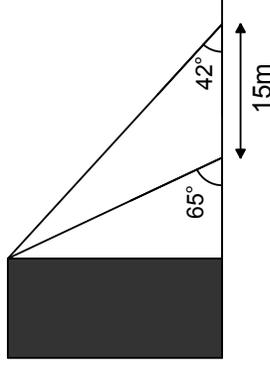
Work out the height of the building correct to the nearest metre.

7. Triangles ABC and DEF each have side lengths of 4cm and 7cm, and an angle of 33°, but they are not congruent. Angle x is **acute** and angle y is **obtuse**.

Work out the values of x and y, correct to the nearest degree.
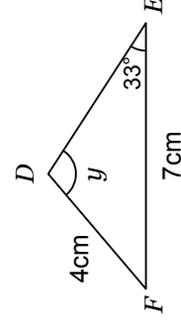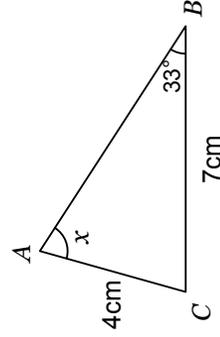
8. a) Use the sine rule to work out the value of $\sin\theta$.
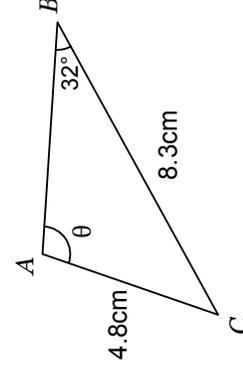
b) Given that θ is obtuse, work out the value of θ correct to the nearest degree.

Fluency Practice

Sine Rule

Videos 333, 334 on www.corbettmaths.com

Scan here

Click here

Examples

Workout

Question 1: Find x for each of the triangles below.

(a)

9cm
45°
85°
x

(b)

3.4cm
78°
30°
x

(c)

25mm
115°
22°
x
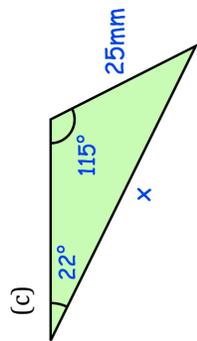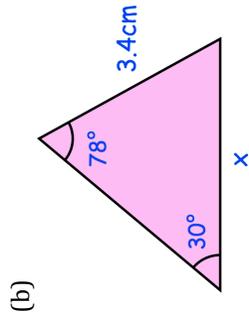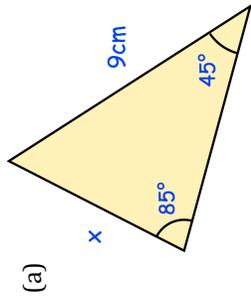
Question 2: Find x for each of the triangles below.
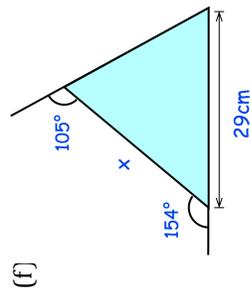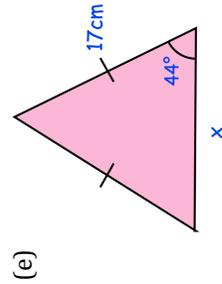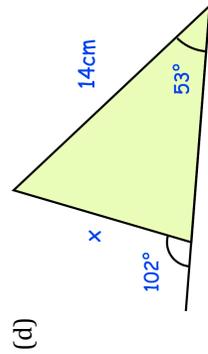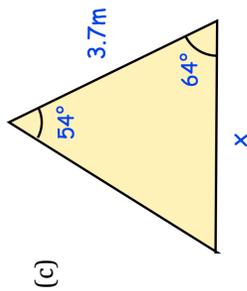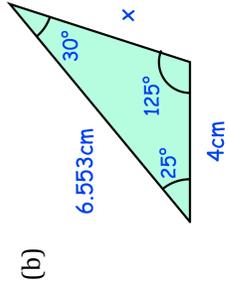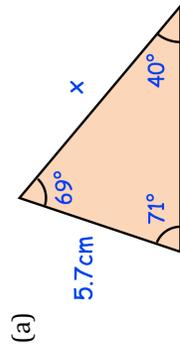
(a)

5.7cm
69°
71°
x

(b)

6.553cm
30°
125°
25°
4cm
x

(c)

3.7m
54°
64°
x

(d)

14cm
53°
102°
x

(e)

17cm
44°
x

(f)

105°
154°
29cm
x

Question 3: Find the size of θ for each of these triangles.

(a)

8cm
θ
75°
5cm

(b)

35cm
θ
80°
16cm

(c)

40cm
120°
θ
30cm

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Fluency Practice

Sine Rule

Videos 333, 334 on www.corbettmaths.com

Question 4: Find the size of θ for each of these triangles.

(a)

66°
31.865cm
θ
38cm
37.386cm

(b)

7cm
10.652cm
78°
θ
9.615cm

(c)

2m
θ
33°
57.4cm
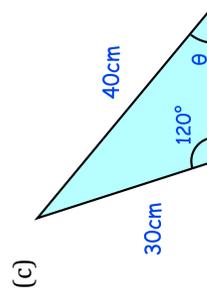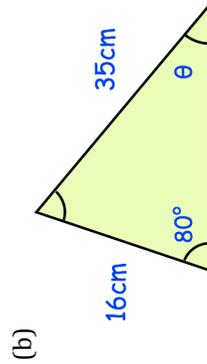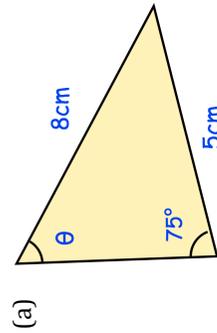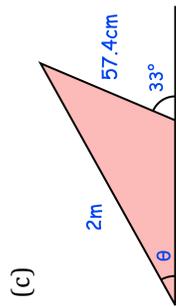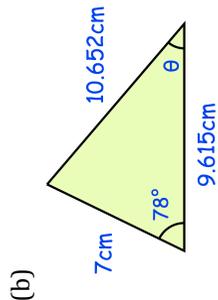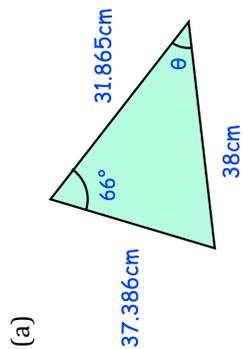
Question 5: Find the missing angle θ or side, x, for each of the triangles below.

(a)

A
52°
10cm
73°
C
x
B

(b)

4.9cm
B
θ
2cm
53.2°
A
C

(c)

E
49°
91°
40°
D
6mm
x
F

(d)

B
θ
20cm
120°
8cm
A
C

(e)

G
0.9m
135°
θ
0.5m
H
I

(f)

B
77°
87cm
29°
x
A
C
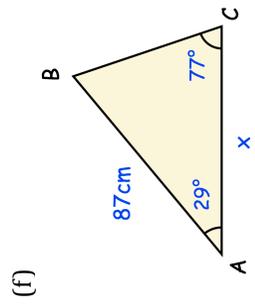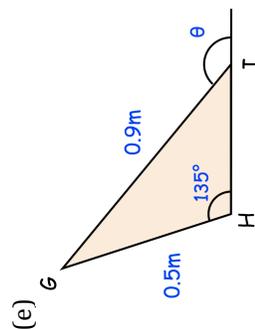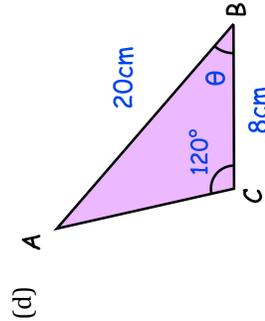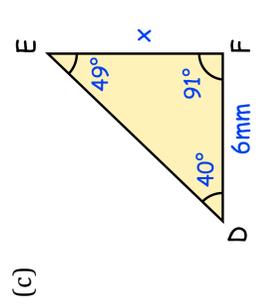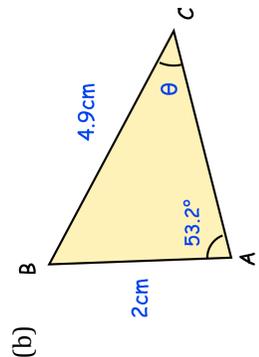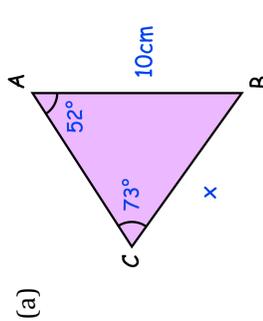
Question 6: The triangles below have two possible values for the angle θ.
Find the size of these possible angles.

(a)

B
15°
9cm
θ
2.5cm
A
C

(b)

A
34°
14cm
θ
B
8cm
C

(c)

θ
6cm
40°
7cm

Sine Rule

Videos 333, 334 on www.corbettmaths.com

Apply

Corbett mɑths
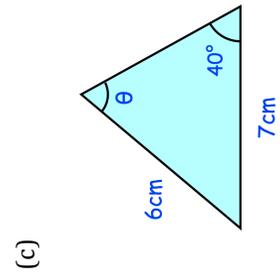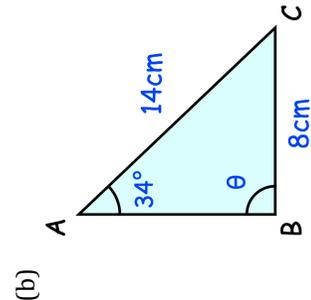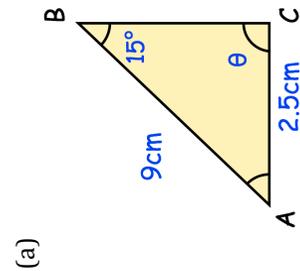
Question 1: ACD is a triangle and B is a point on AC.

BC = 10cm

Angle ACD = 77°   Angle BDC = 26°

Angle DAC = 40°
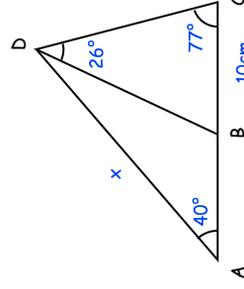
(a) Find the length of BD

(b) Find the size of angle ABD

(c) Find the length of AD

Question 2: Calculate the perimeter of triangle ABC
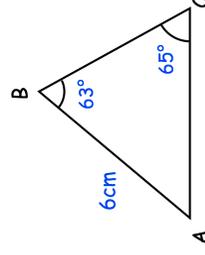
Question 3: Donhampton is 48km from Castletown on a bearing of 050°.
Eastville is on a bearing of 075° from Castletown and on a bearing of 150° from Donhampton.

Calculate the distance of Eastville from Castletown.
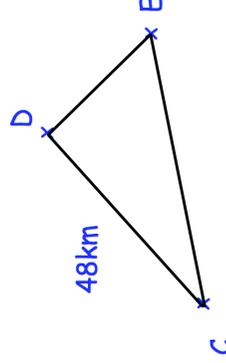
Question 4: The diagram shows 3 helicopters D, E and F.
Helicopter D is due South of helicopter E.
The bearing of helicopter F from helicopter E is 060°
The bearing of helicopter F from helicopter D is 012°
The distance of helicopter F from helicopter D is 900m
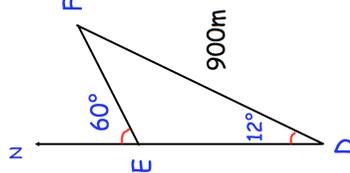
(a) Find the distance of helicopter F from helicopter E

(b) Find the distance of helicopter D from helicopter E.

Sine Rule

Videos 333, 334 on www.corbettmaths.com

Corbett mαths

Question 5: Two small boats are 28m apart.
The angle of elevation of the boats to the top of the lighthouses are 24° and 40° respectively.

Calculate the height of the lighthouse.
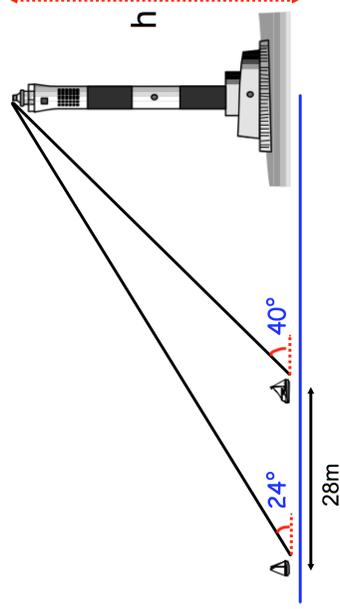
Question 6: There are two possible triangles DEF, where DE = 6.2cm, EF = 7.9cm and ∠DFE = 30°

Sketch both possible triangles with all sides and angles found.

Question 7: A UFO is flying above two people, standing on the ground at points A and B.
A and B are 300m apart.
The angle of elevation of the UFO from A is 30°
The angle of elevation of the UFO from B is 23°
Find the height of the UFO above the ground.
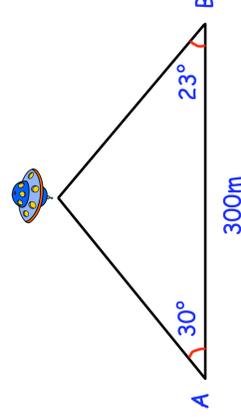
Question 8: Two ships, A and B, leave a port at midday.
A travels on a bearing of 085° at a speed of 18km/h.
B travels on a bearing of 113° at a speed of y km/h.
At 14:00 the distance between A and B is 30km.
Boat B was travelling at a slower speed than boat A
Work out y, the speed of boat B.

Question 9:   Shown below is triangle ABC.
              All values are correct to 2 significant figures.
              Calculate the upper bound for the length of AC.
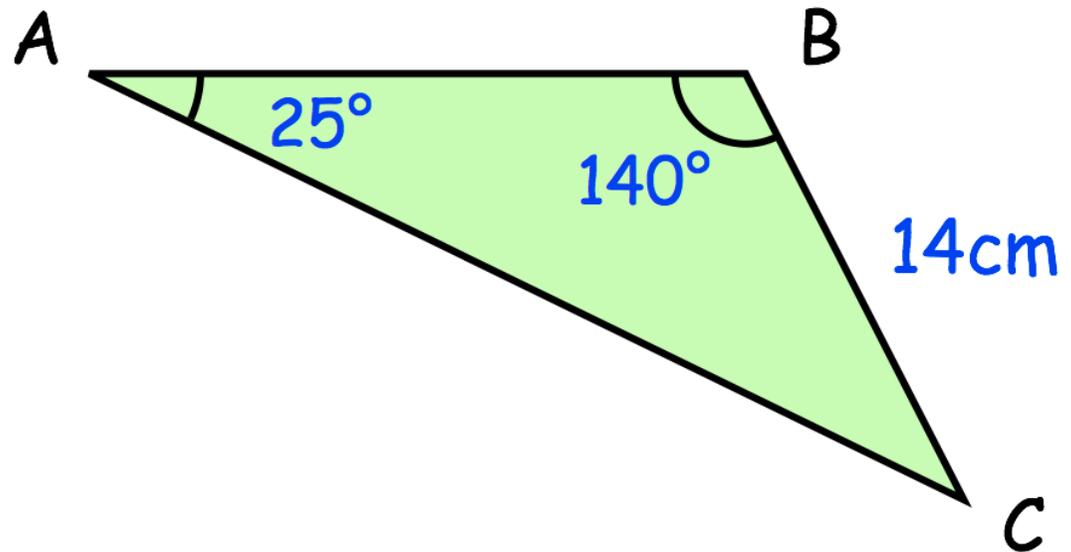
Question 10: Prove the Sine Rule.

Fluency Practice
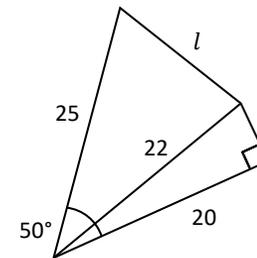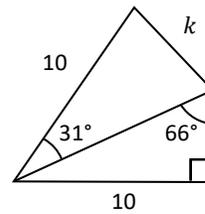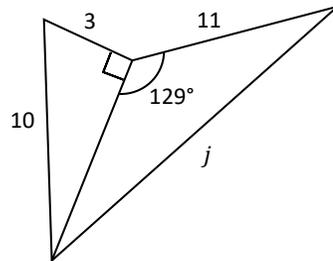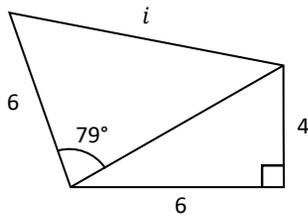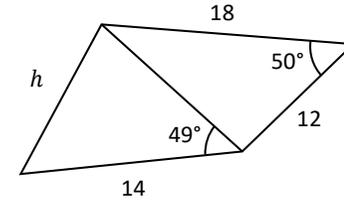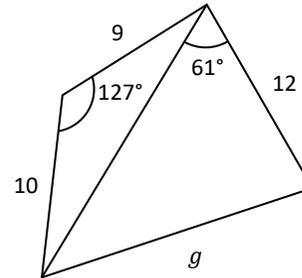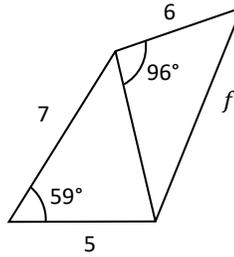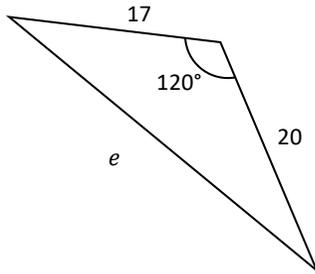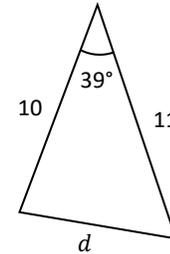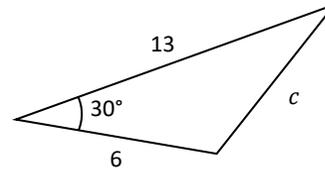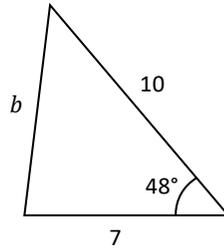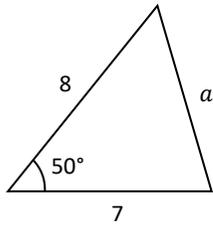
10 Answers	7.4	8.5	6.4	32.1	11.5
	15.3	7.1	9.0	18.5	8.4

Fluency Practice

Cosine Rule: Angles　　　　　Lengths are metres.　　　　　not to scale
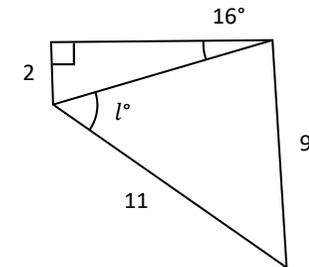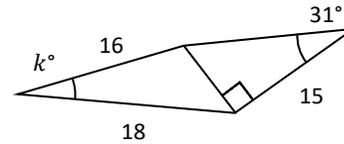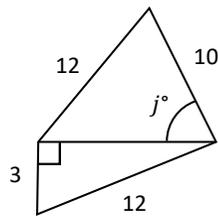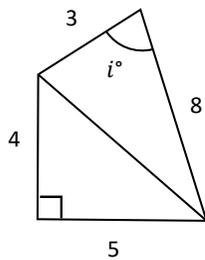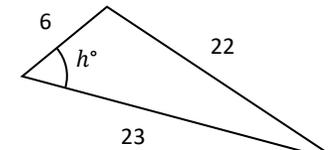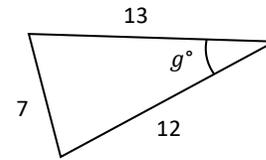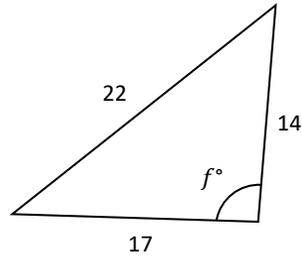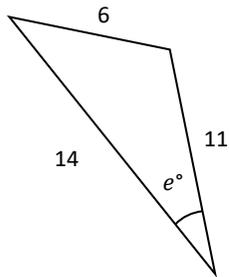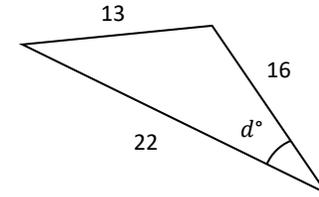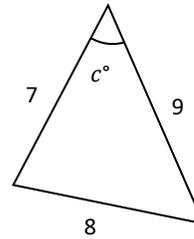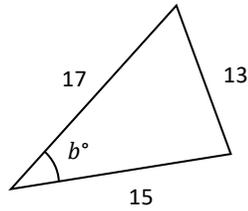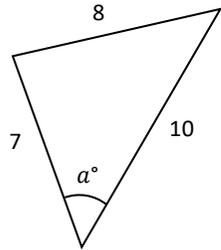
10 Answers	32.2	35.8	66.9	72.9	58.4
	47.4	90.0	52.6	24.2	48.2

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Fluency Practice

Find the missing length.

(a)

88°
6 cm
8 cm
d

(b)

e
11 cm
32°
15 cm

(c)

67°
85 mm
65 mm
f

(d)

g
48°
5.2 cm
5.5 cm

(e)

h
7 cm
78°
3 cm

Find the missing angle.

(a)

m
7 cm
6 cm
8 cm

(b)

7 mm
12 mm
n
16 mm

(c)

p
5.8 cm
2.4 cm
6.2 cm

(d)

9 cm
r
11 cm
7 cm

(e)

s
11 cm
18 cm
25 cm
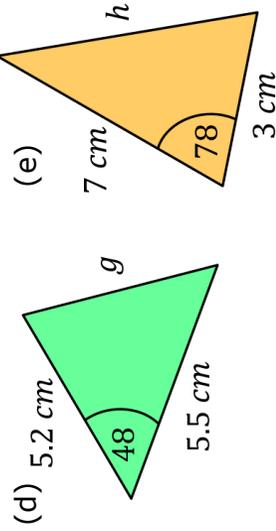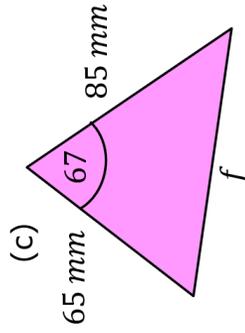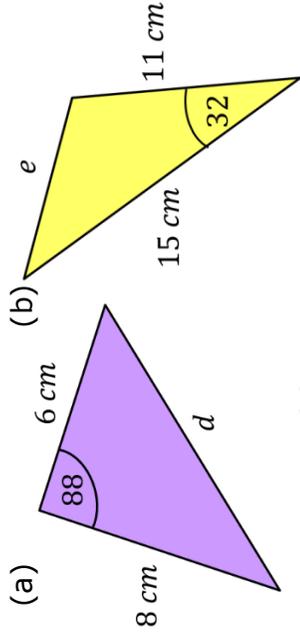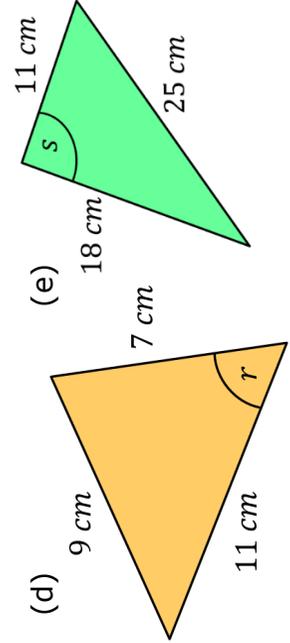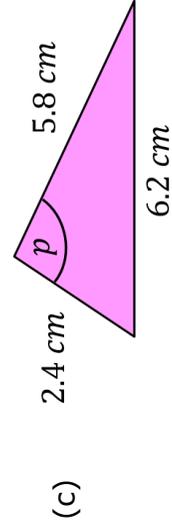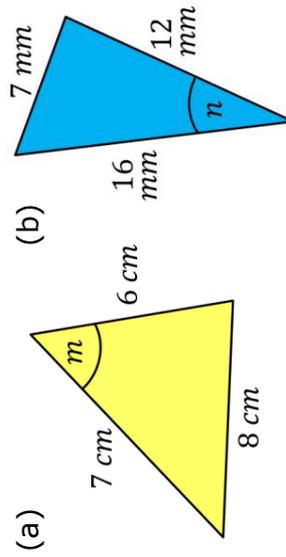
Fluency Practice

① **Cosine Rule** Lengths are centimetres. not to scale
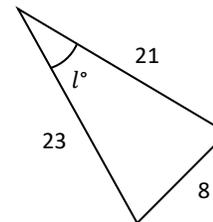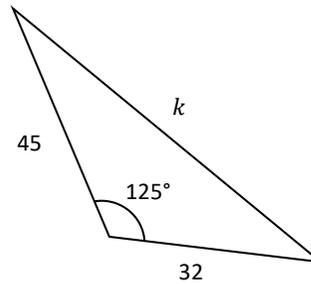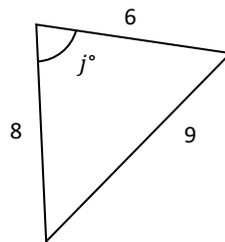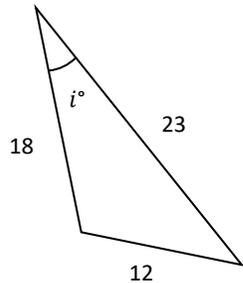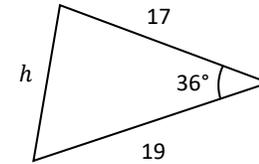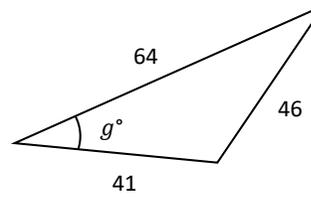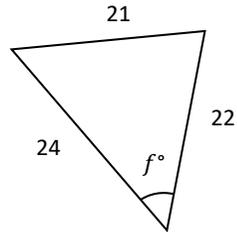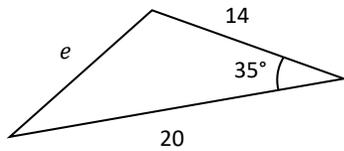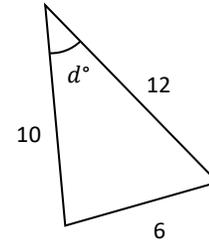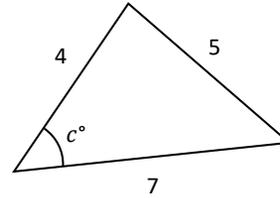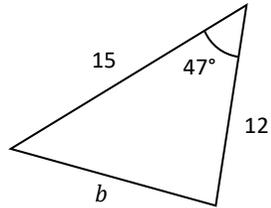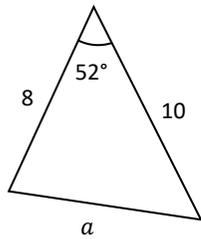
| 10 Answers | 29.9 | 45.8 | 8.1 | 11.7 | 44.4 |
| | 31.1 | 11.1 | 54.1 | 78.6 | 11.3 |

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Fluency Practice

Cosine Rule   Lengths are centimetres.   not to scale

44, $m°$, 14, 40	12, 57°, 20, n, 98°, 10	19, 63°, o, 16, 122°, 14	15, 53°, 47°, 20, 23, p
17, 18, 13, $q°$, 24	21, r, 31, 83°, 27	13, 11, $s°$, 4, 13	t, 131°, 21, 20, 7
28°, 18, 19, $u°$, 23	71°, 37°, v, 27	53°, 21, 17, w, 4	3, 20, $x°$, 18, 19°
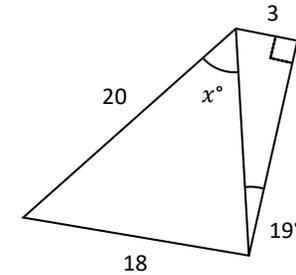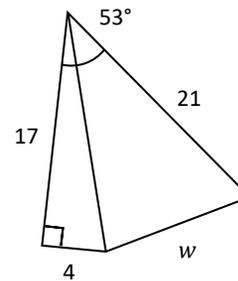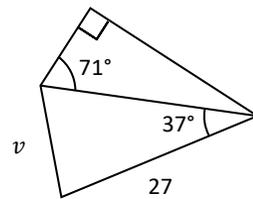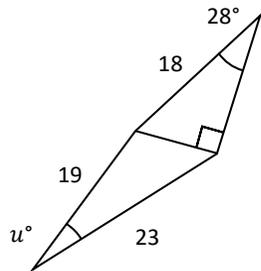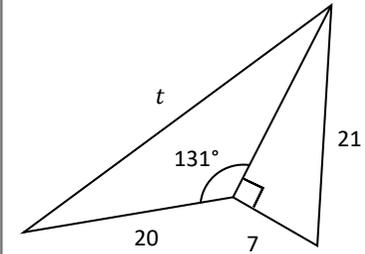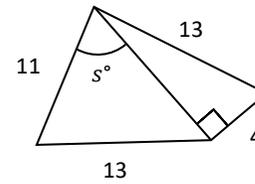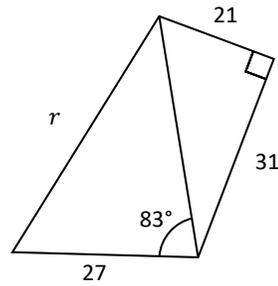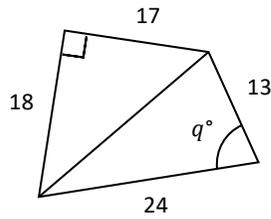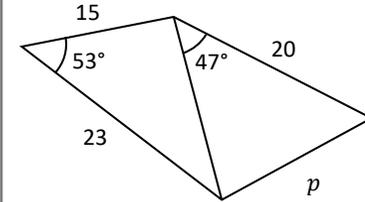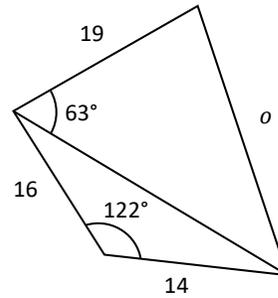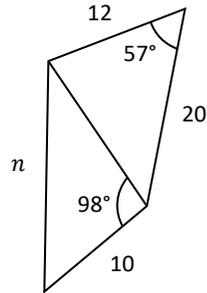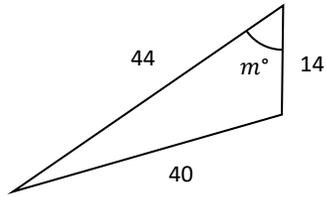
10 Answers	36.2	77.8	64.4	16.5	24.4
	20.7	20.5	43.4	15.4	67.3

Fluency Practice

Q	Work out the missing length	Answers	Q	Work out the missing angle	Answers
1a	5 cm, x cm, 40°, 7 cm		**1b**	5 cm, 9 cm, x°, 8 cm	
2a	x cm, 5 cm, 40°, 60°, 7 cm		**2b**	5 cm, 9 cm, x°, 8 cm	
3a	7 cm, 5 cm, 40°, 60°, x cm		**3b**	5 cm, x°, 9 cm, 8 cm	
4a	p cm, x cm, r°, q cm		**4b**	p cm, x°, q cm, r cm	
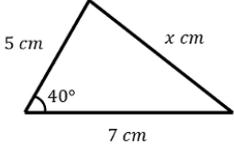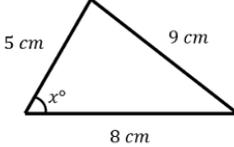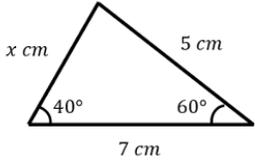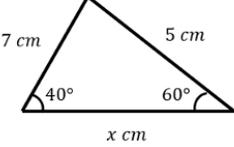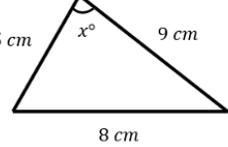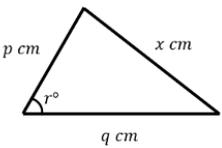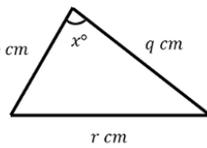
Fluency Practice

A1 Find the length of BC

B

5 cm

$43°$

A 3 cm C
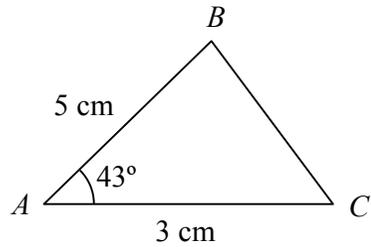
A2 Find the length of DF

E

7 cm

$46°$

D

4 cm

F
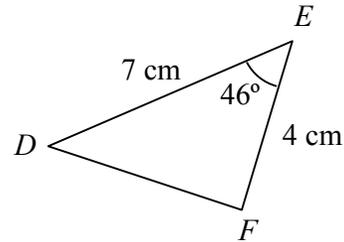
A3 Find the length of HI
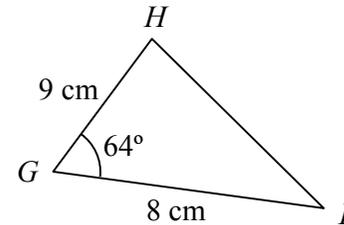
H

9 cm

$64°$

G

8 cm I

A4 Find the distance AB
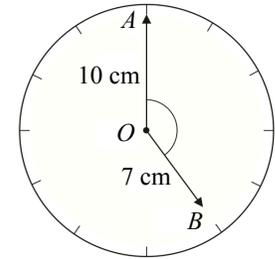
A

10 cm

O

7 cm

B

B1 Find angle MPN

P

11 cm

$x°$

7 cm

M

5 cm N

B2 Find angle QSR

R

5 cm

6 cm

Q

$x°$

S

7 cm

B3 Find angle TUV

U

3 cm

$x°$

6 cm

T

7 cm

V

B4 Find angle JKL

L

6 cm

$81°$ 3 cm

J

K

C1 Fi

V X

$2\sqrt{7}$
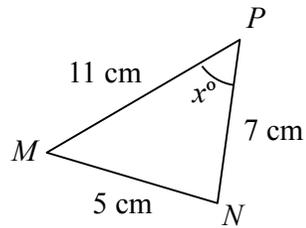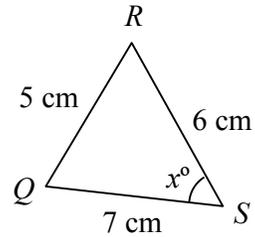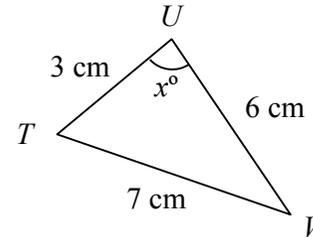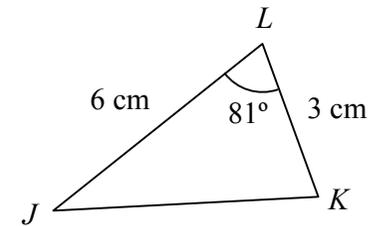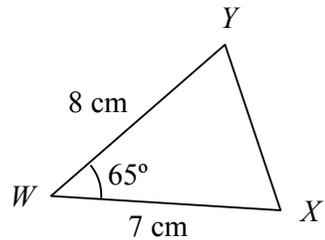
C2 Find the perimeter of XYZ

Z

Y

7 cm

$120°$

X 7 cm O

C3 Find x

$60°$

$x + 6$

x

$x + 4$

C4 Find x

B

$2x - 1$ $2\sqrt{7}$

$60°$

A $2x + 1$ C

$2\sqrt{7}$

Q1 Work out the missing side, x, of each of the following triangles to one decimal place.

[a]

11cm, x, 37°, 12cm

[b]

5yd, 80°, 8yd, x

[c]

18in, 114°, x, 23in

[d]

6.8ft, x, 44°, 7.6ft

[e]

x, 104°, 15.2m, 12.3m

[f]

9.2km, 20°, x, 14.1km

[g]

7.2mm, 55°, x, 4mm

[h]

16.5cm, x, 32°, 20cm
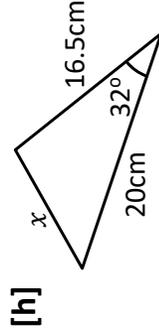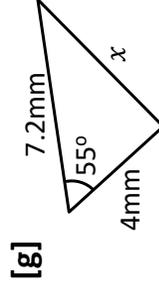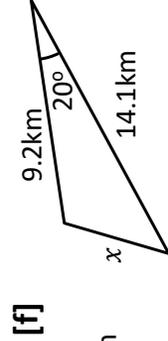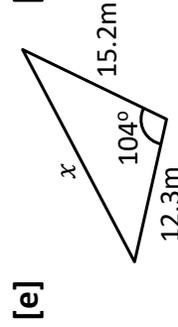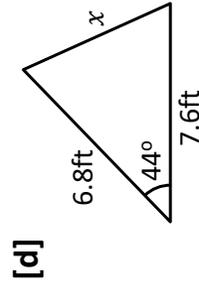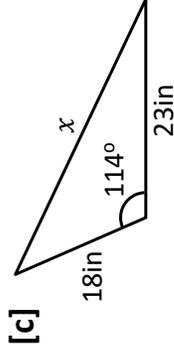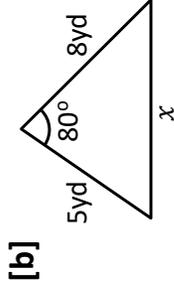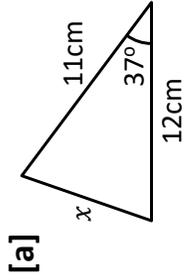
Q2 A ship sails from port A on a bearing of 060° for 100 miles and then on a bearing of 140° for 145 miles.
Calculate the distance of the ship from port A to the nearest mile.

N, 140°, 145 miles, N, 60°, 100 miles
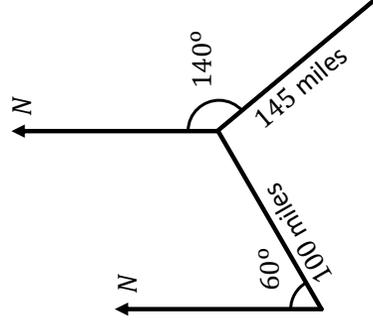
Q3 Find the perimeter of the following triangle.

24m, 71°, 18.5m
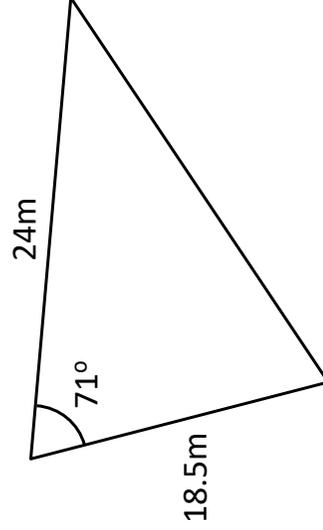
Purposeful Practice

Q1 Work out the missing angle, $\theta°$, in each of the following triangles to one decimal place.

[a]

6cm, 8cm, 9cm, $\theta°$

[b]

10.4m, 10m, 12m, $\theta°$

[c]

10cm, 8cm, 16cm, $\theta°$

[d]

8.6m, 11.4m, 13.1m, $\theta°$

[e]

12in, 8.1in, 10.2in, $\theta°$

[f]

12.2ft, 5ft, 11ft, $\theta°$

[d]

6.8km, 14.2km, 18.1km, $\theta°$

[e]

25yd, 6.2yd, 21yd, $\theta°$
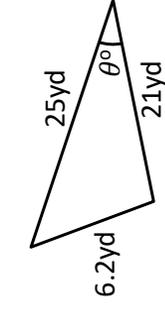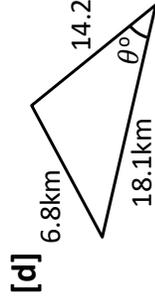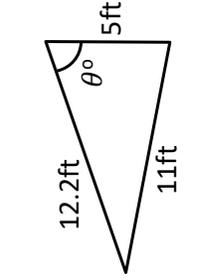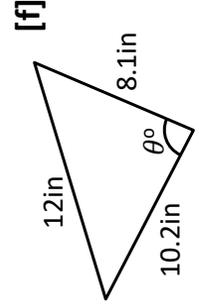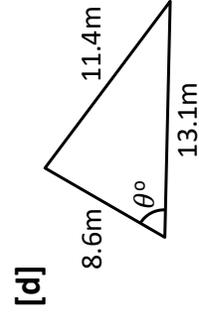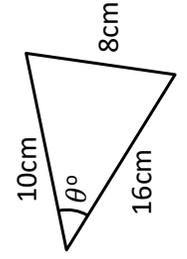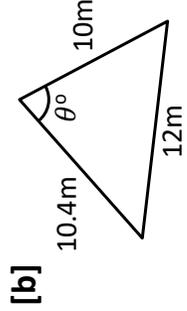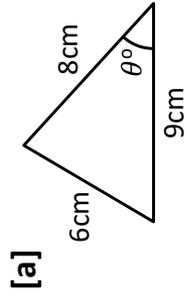
Q2 The diagram shows a kite.
Find the size of the missing angle, $\theta°$, correct to the nearest one decimal place.

7cm, 114°, 15cm, $\theta°$
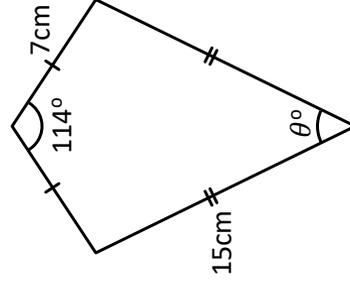
Q3 Find the size of the smallest angle in the following triangle.

28.4m, 19.6m, 27.5cm
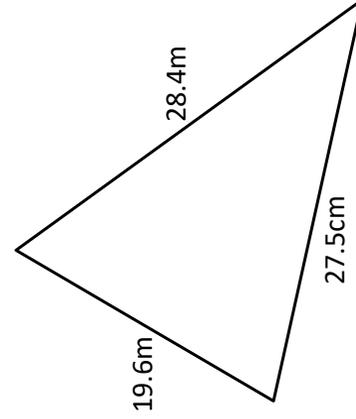
Fluency Practice

1. Work out the missing side lengths in each triangle, correct to the nearest 0.1cm:

a)

A, 5.1cm, B, 70°, 2.4cm, C

b)

C, 8.2cm, A, 37°, 6.8cm, B

c)

48°, 3.4cm, 3.7cm

d)

A, 8.8cm, B, 54°, 7.9cm, 64°, C

e)

C, 46°, 5.9cm, B, A

f)

R, 3.4cm, Q, 49°, P
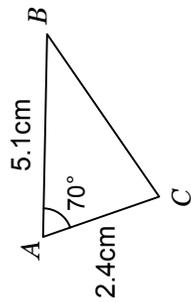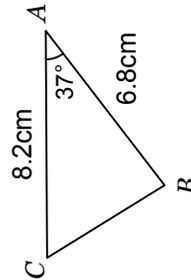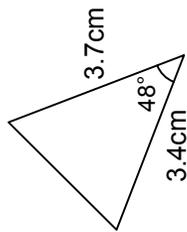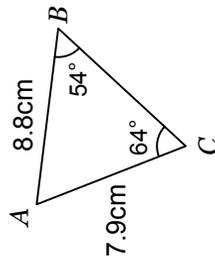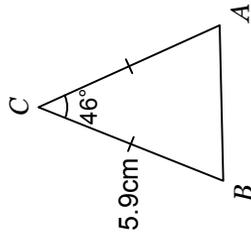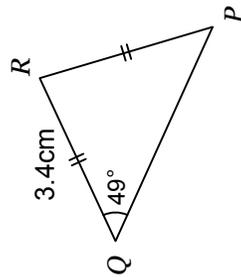
2. Work out the size of the angle marked θ in each triangle, correct to the nearest degree:

a)

B, 10cm, 9cm, A, θ, 6cm, C

b)

8cm, 5.8cm, θ, 5.7cm

c)

B, 9cm, A, θ, 8.5cm, 5.5cm, C

d)

F, 0.5m, 1.3m, θ, H, 1.2m, G

e)

Y, 11mm, X, θ, 9mm, Z

f)

A, 66mm, 59mm, θ, B, 10cm, C
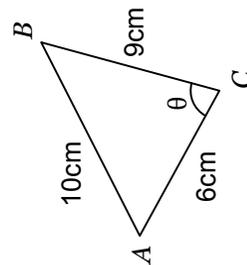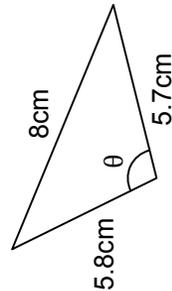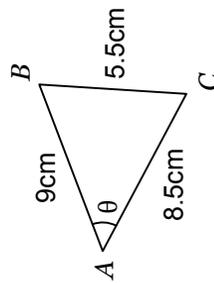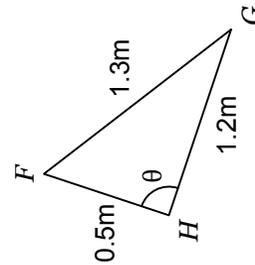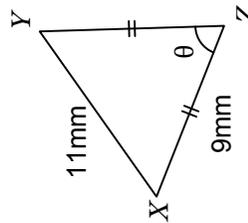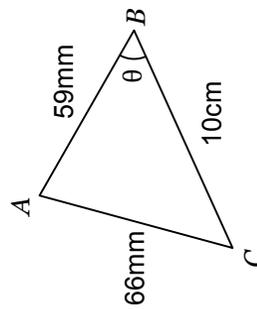
3. Triangle ABC is such that $AB = 6.1$cm, $AC = 7$cm and $B\hat{A}C = 65°$.
Work out the length BC to the nearest 0.1cm.

4. Triangle ABC is such that $AB = 6$m, $BC = 5$m and $AC = 8$m.
Work out the size of $B\hat{A}C$ to the nearest degree.

5.

a) Work out the length QS, correct to the nearest 0.1cm.

b) Work out the angle $Q\hat{R}S$.
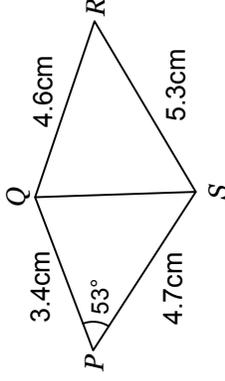
3.4cm
4.6cm
4.7cm
5.3cm
53°
P
Q
R
S

6. The diagram shows a kite.
Work out the angle marked θ, correct to the nearest degree.

74°
θ
6cm
11cm
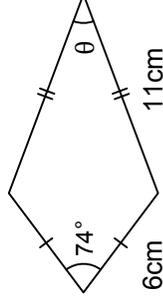
7. Two ships are being tracked from a lighthouse.
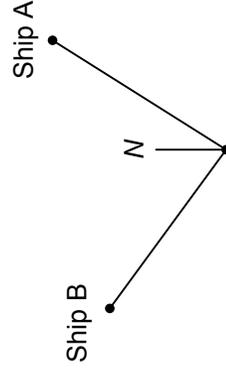
Ship A is 2.3km from the lighthouse on a bearing of 029°.

Ship B is 2.5km from the lighthouse on a bearing of 306°.

Work out the straight-line distance between the ships.

Ship A
N
Ship B
Lighthouse

8. Two aircraft, flying at the same height, are being tracked from a control centre.
Aircraft A is 4.1km from the control centre on a bearing of 078°.
Aircraft B is 1.6km from the control centre on a bearing of 163°.

Work out the straight-line distance between the aircraft.

9. Justin is going to buy lots of identical tiles in the shape of an isosceles triangle, as shown in *figure 1*.
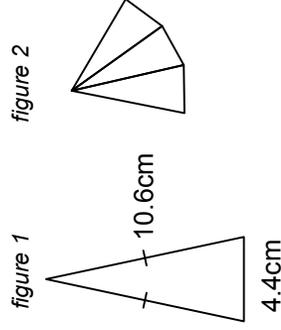
He wants to arrange them in a pattern around a point, as shown in *figure 2*.

Work out how many of the tiles are needed to fit completely around a point.

figure 1
10.6cm
4.4cm

figure 2

10. In the diagram, ADC is a straight line.
Work out the length BC, correct to the nearest 0.1m.

A
B
C
D
8.6m
3.5m
5.9m
3.7m

Fluency Practice

Cosine Rule

Videos 335, 336 on www.corbettmaths.com

Scan here

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Examples

Workout

Corbett maths

Question 1: Find x for each of the triangles below.

(a)
8cm, x, 60°, 10cm

(b)
x, 20cm, 120°, 15cm

(c)
42mm, 56°, 35mm, x

(d)
x, 17cm, 70°, 17cm

(e)
6.1cm, 148°, 5.5cm, x

(f)
x, 6cm, 19°, 7cm

(c)
22cm, 29cm, θ, 20cm

(f)
50cm, 1.2m, θ, 1m
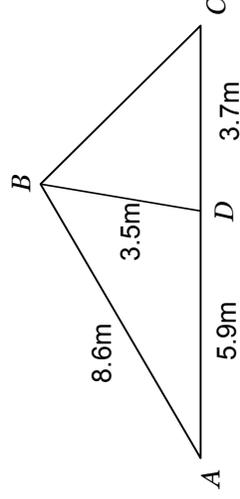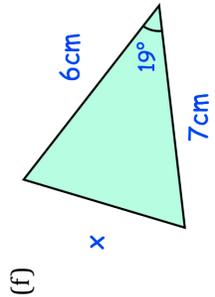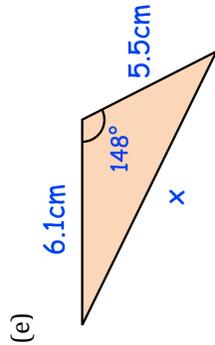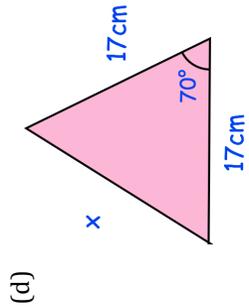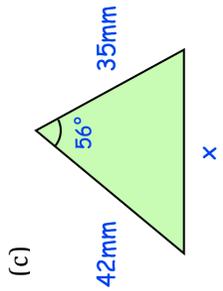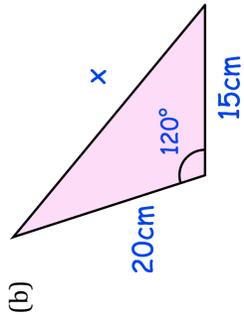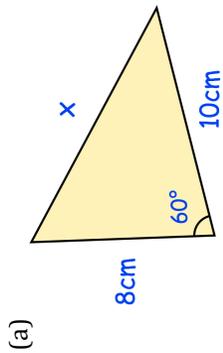
Question 2: Find the size of θ for each of these triangles.

(a)
8cm, θ, 9cm, 10cm

(b)
8m, θ, 5m, 7m

(d)
14.3cm, θ, 7.5cm, 8.1cm

(e)
4.9cm, 7.4cm, θ, 7.4cm

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Page 48

Purposeful Practice

Cosine Rule
Videos 335, 336 on www.corbettmaths.com
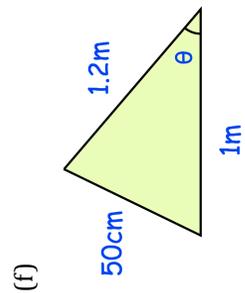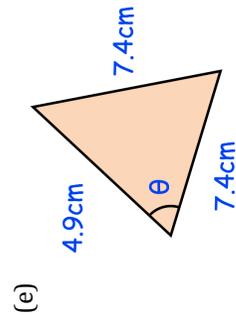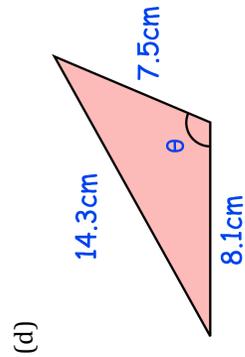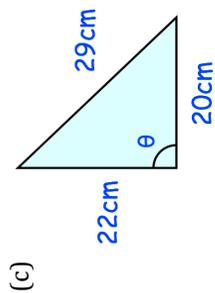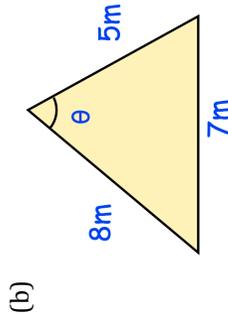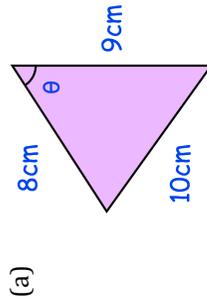
Question 3: In triangle ABC, the side AB = 6cm, the side BC = 8cm and angle ABC = 100°
Find the length of side AC.
Give your answer to 1 decimal place.

Question 4: In triangle DEF, the side DE = 30cm, the side DF = 40cm and the side EF = 45cm.
Find the size of angle DFE.
Give your answer to 3 significant figures.

Question 5: In triangle GHI, the side GH = 3cm, the side HI = 18cm and the side GI = 20.5cm.
Find the size of angle HGI.
Give your answer to 3 significant figures.

Apply

Question 1: Calculate the perimeter of triangle ABC
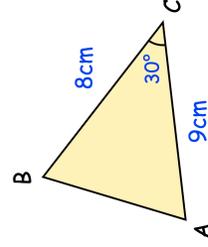
Question 2: Boat A is 16km from a lighthouse on a bearing of 055°
Boat B is 11km from the same lighthouse on a bearing of 152°
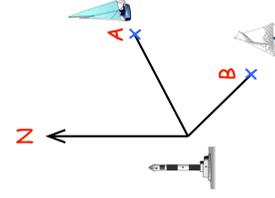
Calculate the distance between the two boats.

Question 3: Find the size of the smallest angle in this triangle.

Purposeful Practice

Cosine Rule

Videos 335, 336 on www.corbettmaths.com

Corbett mαths
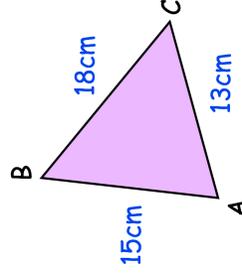
Question 4: A hot air balloon is flying above two point, standing on the ground at points A and B, 600m apart.

The hot air balloon is 300m from A and 500m from B.

(a) Work out the angle of elevation from point B

(b) How high is the hot air balloon from the ground?

300m   500m

A   600m   B
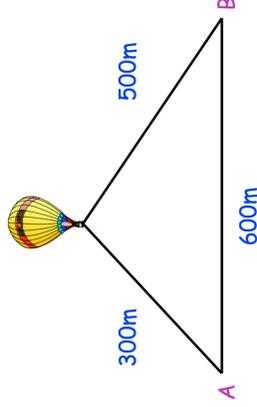
Question 5: Shown is sector OAB.
O is the centre of the circle with radius 9cm
A and B are points on the circle.
The length of the chord AB is 10.35cm

Find the area of sector OAB

B

10.35cm

A   9cm   O
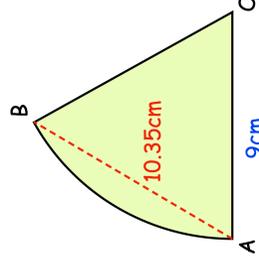
Question 6: ABCD is a quadrilateral.
AB = 7cm   BC = 5cm   BC = 11cm   AD = 10cm   Angle BAD = 130°

Work out the size of angle BCD.

5cm   C

B   11cm

7cm   130°

A   10cm   D

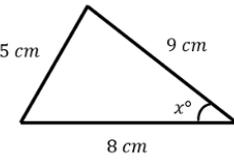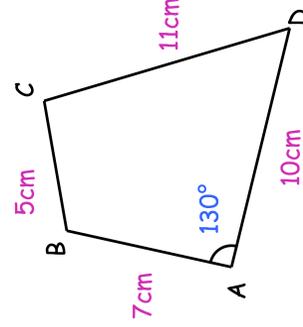# Purposeful Practice

Cosine Rule

Question 7: A is a vertex of a regular pentagon.
B is a vertex of a regular octagon.
C and D are vertices of both polygons.
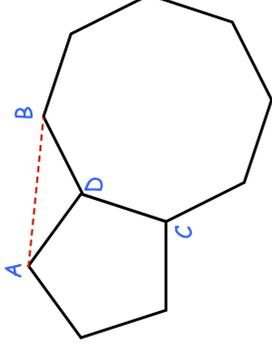
The perimeter of the octagon is 40cm.

Work out the length AB

Question 8: A clock has two hands.
A minute hand which is 5.5cm long and an hour hand which is 4cm long.

Find the distance between the tips of the two hands at 7:15am

Question 9: Two ships, A and B, leave a port at 10:30

Ship A travels on a bearing of 196° at a speed of 30km/h.
Ship B travels on a bearing of 244° at a speed of 24km/h.

(a) Work out the distance between A and B at 14:00

(b) Work out the bearing of B from A at 14:00

Question 10: In the diagram:

ABD is a straight line.
AC = 6cm   CD = 7.4cm   DE = 25cm
Angle BAC = 36°   Angle BDE = 110°

Calculate the length of BE
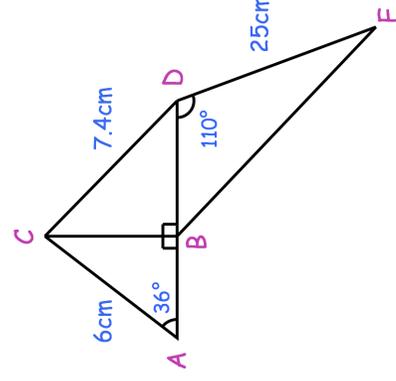
Question 11: The Cosine Rule is   a² = b² + c² − 2bcCosA

Make CosA the subject.

Question 12: Shown is kite ABCD

Prove $CosBAD = 1 - \dfrac{x^2}{50}$
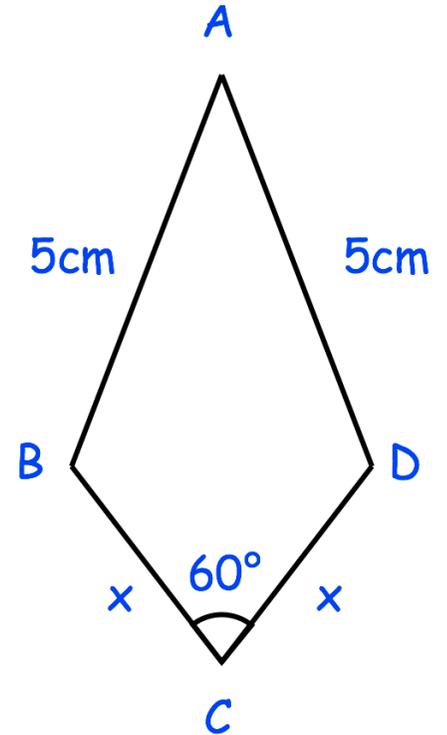
A

5cm   5cm

B   D

x   60°   x

C

Answers

Fluency Practice

Find the area of the triangle.

(a)

9 cm · 33° · 12 cm

(b)

98° · 2.5 cm · 1.3 cm

(c)

5 m · 3 m (right angle)

(d)

7.8 cm · 62° · 58° · 5.9 cm

(e)

17 mm · 75° · 41° · 29 mm

Find the area of the triangle.

(a)

11 cm · 63° · 71°

(b)

88° · 25° · 8.7 cm
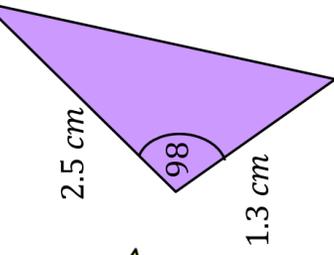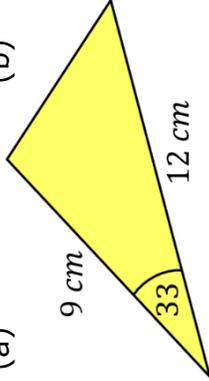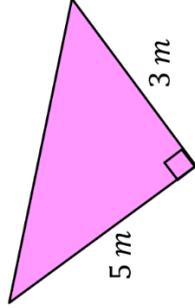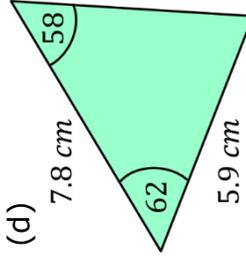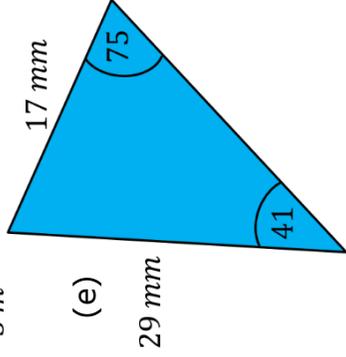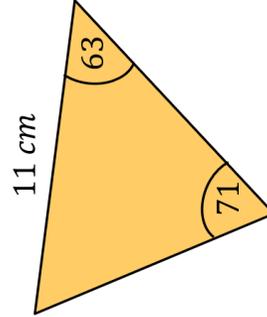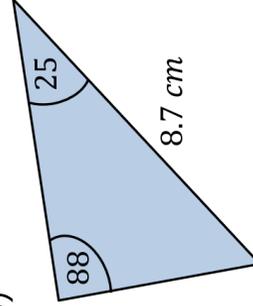
Find the missing length or angle.

(a) Area = 32 cm²

71° · 7 cm · x cm

(b) Area = 300 mm²

20 mm · x · 35 mm
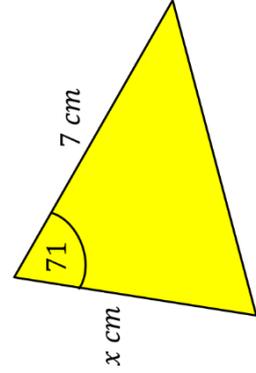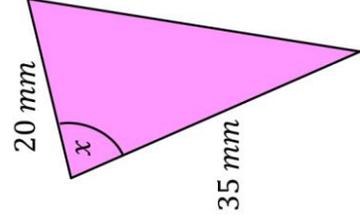
Fluency Practice

Finding the Area of a Triangle using the Area Rule   (lengths are cm)   not drawn accurately

A

10, 75°, 12

B

4, 71°, 6

C

21, 42°, 20

D

13, 63°, 16

E

18, 66°, 17

F

5, 132°, 8

G

19°, 21, 105°, 7

H

9, 12, 76°, 47°

I

17, 57°

J

13, 12, 51°, 70°, 14, 11

K

11, 8, 97°, 7

L

13, 54°, 15, 7, 121°

M

31°, 8, 278°, 9, 3, 5

N

14, 39°, x

Area = 57 cm²

O

15, x, 17
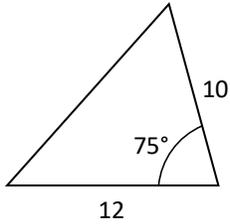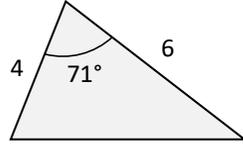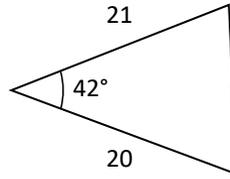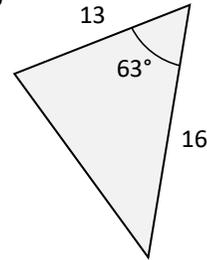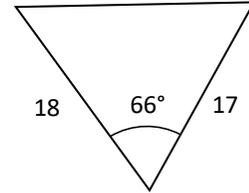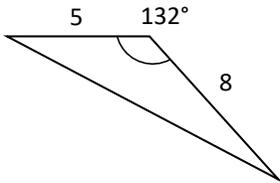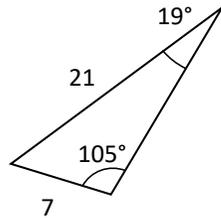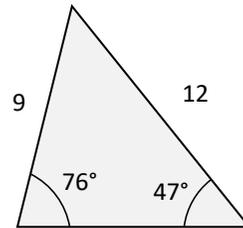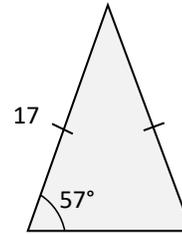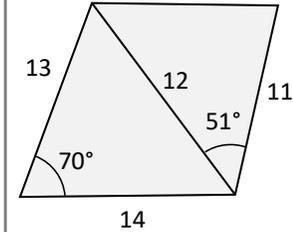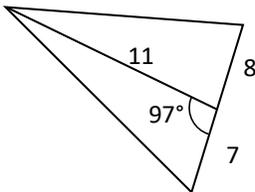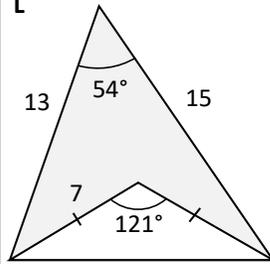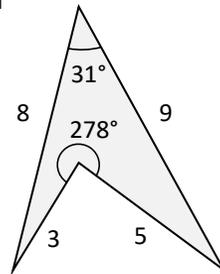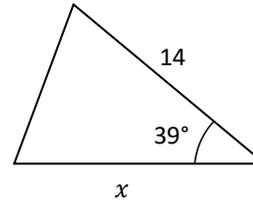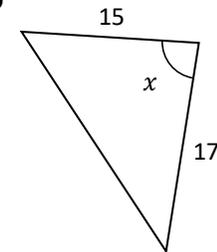
Area = 125 cm²

14 Answers	14.9	136.8	139.8	58.0	140.5	45.3	57.9
	11.1	11.3	132.0	60.9	81.9	12.9	92.7

Fluency Practice

Examples

Workout

Trigonometry: Area of any Triangle
Video 337 on Corbettmaths

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Question 1: Find the area of each of these triangles.

(a)

12cm

30°

15cm

(b)

19cm

120°

19cm

(c)

14cm

70°

8cm

(d)

20cm

60°

8cm

(e)

8m

100°

7m

(f)

12 miles

88°

7 miles

Question 2: Find the area of each of these triangles.

(a)

14cm

68°

13cm

(b)

9cm

42°

9.5cm

6.6cm

(c)

13cm

100°

10.5cm

6.4cm
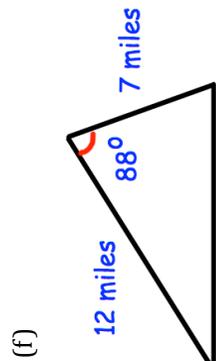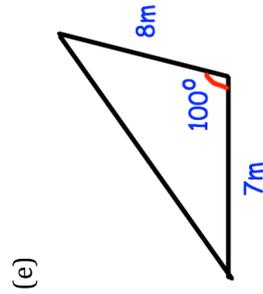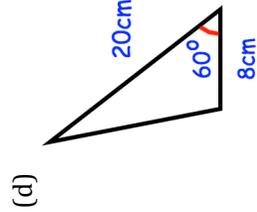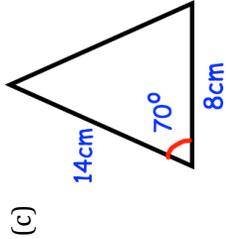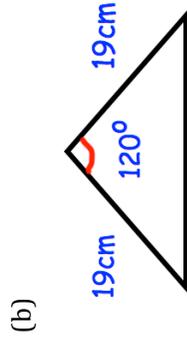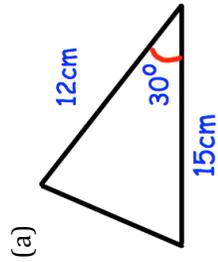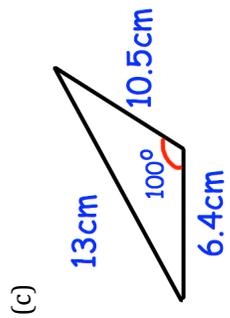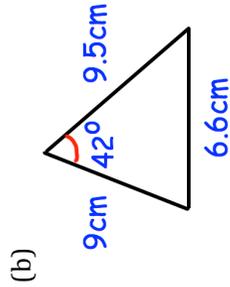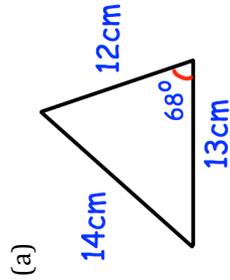
Question 3: Find the area of each of these triangles

(a) ABC with AB = 10cm, BC = 9cm and angle ABC = 44°.

(b) DEF with EF = 28cm, DF = 34cm and angle DFE = 81°.

(c) XYZ with YZ = 9mm, XY = 13mm and angle XYZ = 121°.

Trigonometry: Area of any Triangle
Video 337 on Corbettmaths

Corbettmaths

Question 4: Find the length of the missing side in each of these triangles.
Give each answer to one decimal place.

(a)

6cm
110°
Y
area = 24cm^2

(b)

5cm
32°
Y
area = 9.22cm^2

(c)

16cm
87°
Y
area = 63.5cm^2
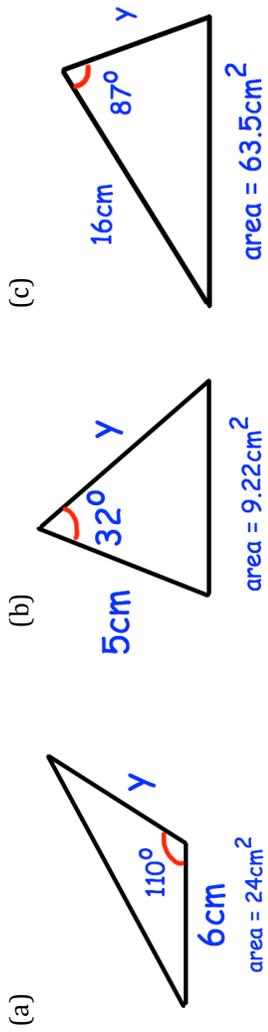
Question 5: Find the size of the missing acute angles below.
Give each answer to one decimal place.

(a)

16cm
x
10cm
area = 78.5cm^2

(b)

6cm
x
7cm
area = 11cm^2

(c)

8cm
x
area = 16cm^2
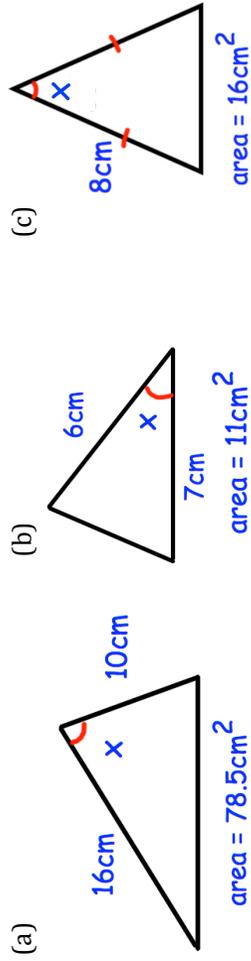
Apply

Question 1: Find the area of these triangles.

(a)

52°
60°
5cm

(b)

12cm
55°
45.1°

(c)

88°
32°
60°
40cm

Corbett mᾳths

Trigonometry: Area of any Triangle
Video 337 on Corbettmaths
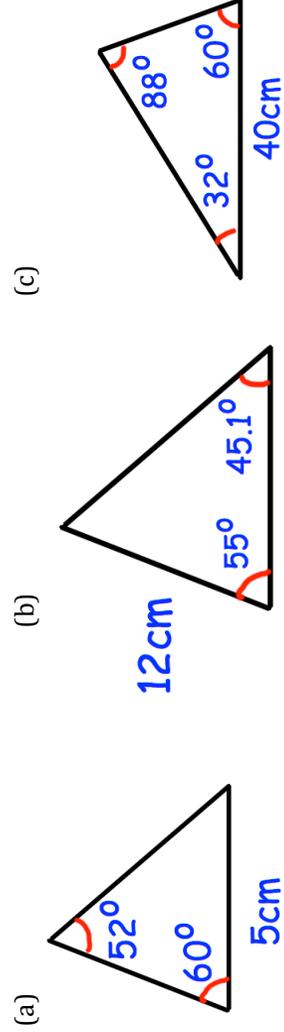
Question 2: Find the area of this triangle.

7cm

12cm

8cm
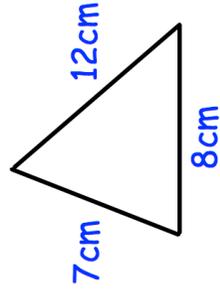
Question 3: A triangular field has three fences. One is 40m long, another is 50m and the other is 60m.
Find the area of the field.

Question 4: Find the area of quadrilateral DEFG.

D

E

70° 50°

7cm

45°

G

8cm

F
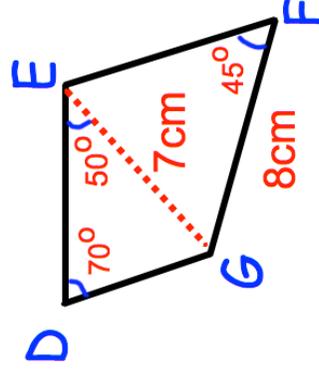
Question 5: Parallelogram ABCD has side lengths AB = 5cm and BC = 9cm.
Angle BCD = 55°.
Find the area of the parallelogram.

Question 6: Prove the area of any triangle is given by the formula $A = \frac{1}{2}abSinC$.

Answers

Click here

Scan here

Fluency Practice

A1 Find the area of triangle *ABC*

5.2 cm 83° 7.1 cm

B, *A*, *C*
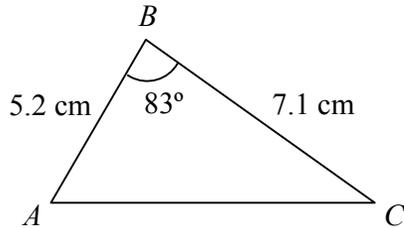
A2 Find the area of triangle *DEF*

E

8.5 cm 88°

32°

D 9.8 cm *F*
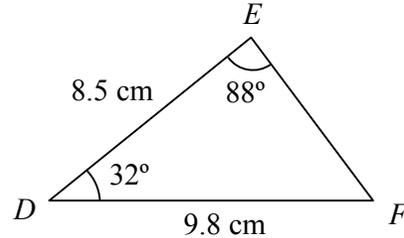
A3 Find the area of triangle *GHI*

H 74° 28° *I*

7.3 cm 11.4 cm

G
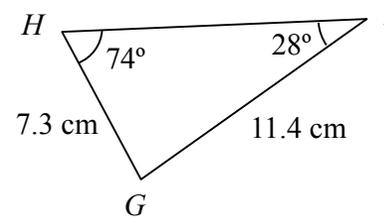
A4 Find the area of the kite

B

A 105° *C*

3.5 cm 5.5 cm

D
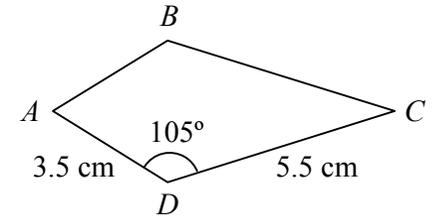
B1 Find the area of triangle *JKL*

K

9.6 cm 11.2 cm

83°

J *L*
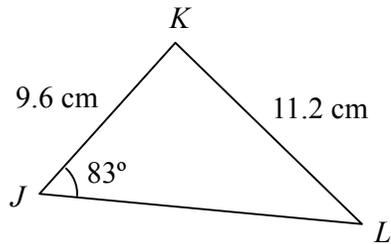
B2 Find the area of triangle *MNO*

M *N*

67°

15.7 cm 85°

O

B3 Find the area of triangle *PQR*

Q

Diagram **NOT** accurately drawn

C 7 cm

7 cm 9 cm

150°

A *B*

7.8 cm
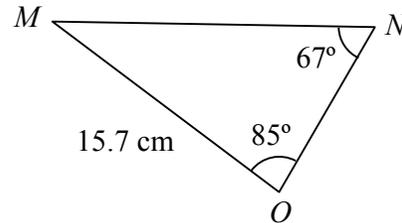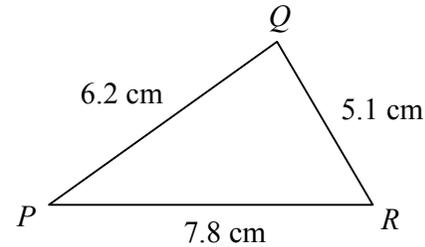
B4 Find the area of the shape

O

5 cm

6 cm 5 cm

140°

A *B*
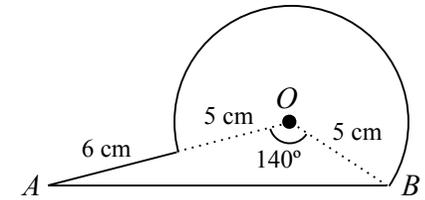
C1 The area of the triangle is 16.5 cm².

7 cm *x*° 5 cm
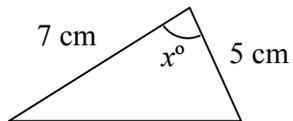
The angle *x*° is **acute**.
Find the value of *x*.

C2 The area of the triangle is 20 cm².

7 cm *x*° 9 cm
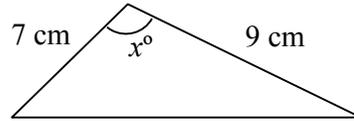
The angle *x*° is **obtuse**.
Find the value of *x*.

C3
ABC is a triangle.
AB = 11 cm
AC = 7 cm

The area of triangle *ABC* is 32 cm²

Find, in degrees, the **two** possible sizes of angle *BAC*.

C4 Find the **shaded** area

Z

Y

5.2 cm

132°

X 5.2 cm *O*
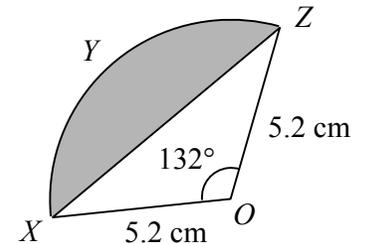
Fluency Practice

NOT TO SCALE

Section A
Questions 1 - 5 find the area of the triangles, for question 6 find the missing side x.

1)

7 cm

6 cm

100°

Area =

2)

3.2 cm

84°

4.5 cm

Area =

3)

8.1 cm

62°

12.3 cm

Area =

4)

13 cm

9 cm

27°

5 cm

Area =

5)

39°

1.4 cm

112°

2.7 cm

Area =

6)

x

Area = 31.7 cm²

44°
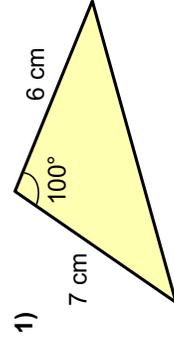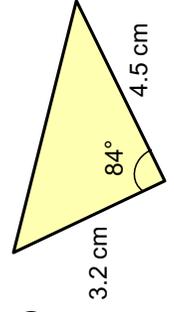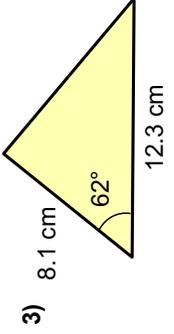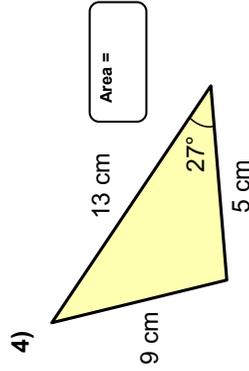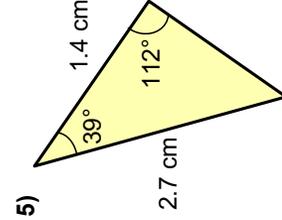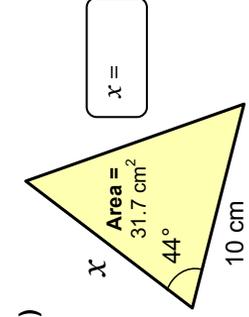
10 cm

$x =$

Section B
Work out the area of the parallelogram and irregular quadrilaterals.

1)

19.5 cm

15 cm

71°

Area =

2)

O

6 cm

47°

128°

2 cm

Area =

3)

2.2 cm

51°

1.9 cm

3.6 cm
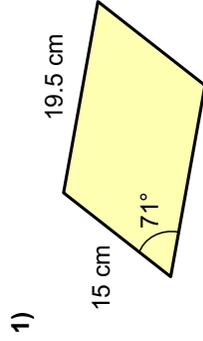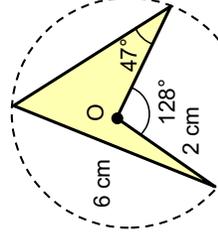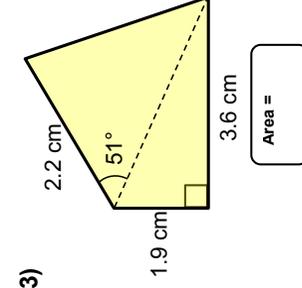
Area =

Section C
1) A regular hexagon has sides lengths 10 cm.
Calculate the area of the hexagon.

2) The area of triangle ABC is 19.6 cm².
AB = 5.9 cm, AC = 8.7 cm.
Calculate the two possible sizes of angle A.

Extension
The points X, Y and Z are are on the circumference of a circle, with centre O and radius 5 cm.
XY = 7 cm and YZ = 4.5 cm.
Calculate the area of quadrilateral OXYZ.

Fluency Practice

① Calculate x

8cm, 15cm

② Calculate x to 2 d.p.

9cm, 4cm, x
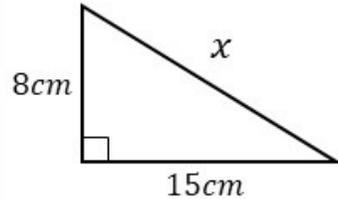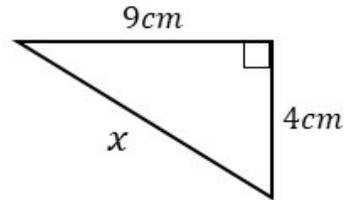
③ Calculate x to 1 d.p.

7.4cm, 8.7cm, x
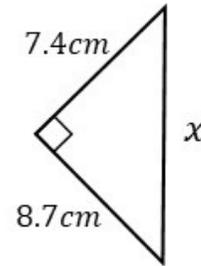
④ Calculate x, leave as a fraction

x, $\frac{4}{5}m$, $\frac{1}{3}m$
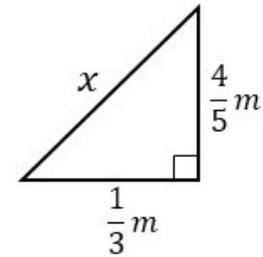
⑤ Find x leave in exact form

6cm, x
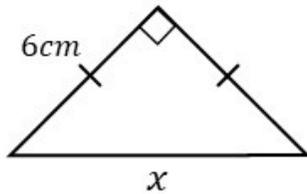
⑥ Calculate the perimeter to 1 d.p.

5cm, 7cm, 3cm
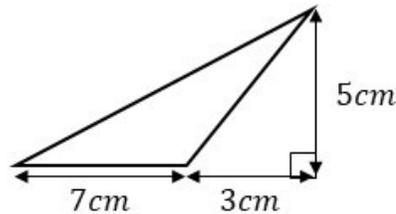
⑦ Calculate the length BD of the rectangle to 3 s.f.

A, B, 4cm, D, 11cm, C
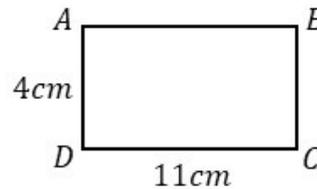
⑧ The area of the triangle is $63cm^2$, calculate x to 2 d.p.

x, 7cm
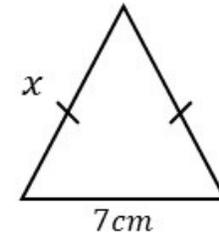
⑨ Calculate the perimeter to 1 d.p.

7cm, 5cm, 10cm
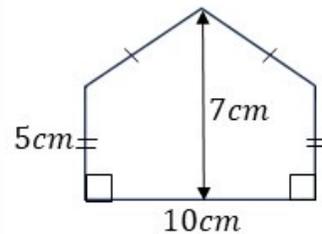
⑩ Calculate the shaded area to 2 d.p.

P, $PQ = 10cm$, $PR = 5cm$, Q, C, R
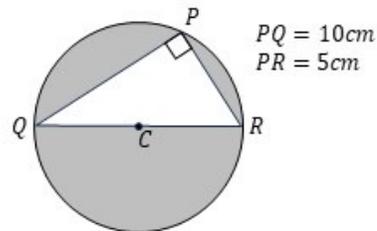
⑪ Calculate x to 2 d.p.

x, 9cm, 6cm, 15cm
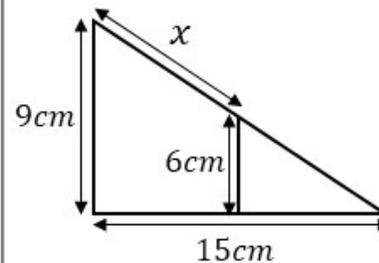
⑫ The area of the hexagon is $24cm^2$, calculate x to 2 d.p.
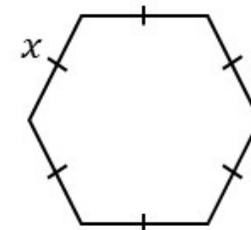
x

Fluency Practice

circle geometry and areas

C is the centre of a circle, radius 10 cm

(1)

B

A 12 cm

C

D

area ABD ?

(2)

B

64°

A

C

area ABC ?

(3)

A

C

26°

B

area ABC ?

(4)

B

A 30°

C

100°

D

area ABDC ?

(5)

B

60°

A

40° 35°

C

E

area ABDE ?

(6)

ABD is an equilateral triangle

A

E

C 30°

D B
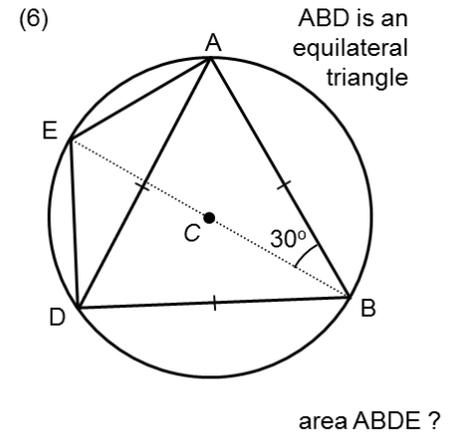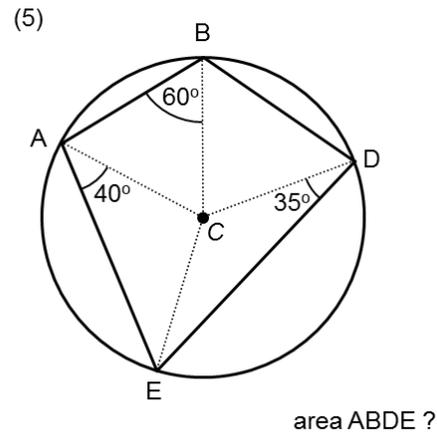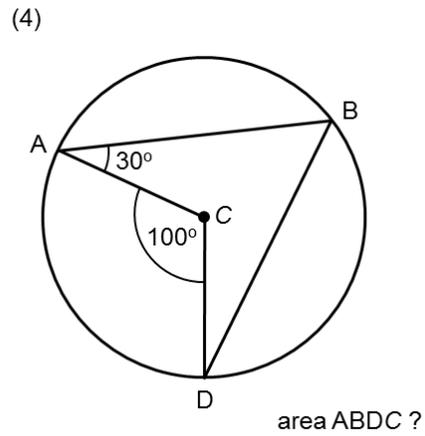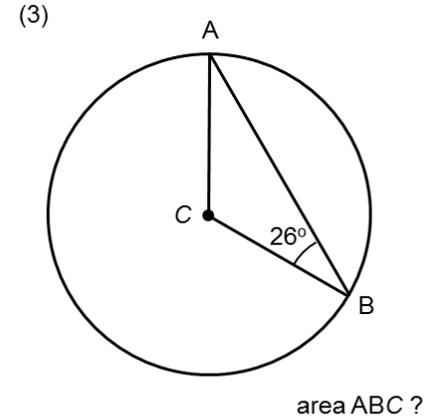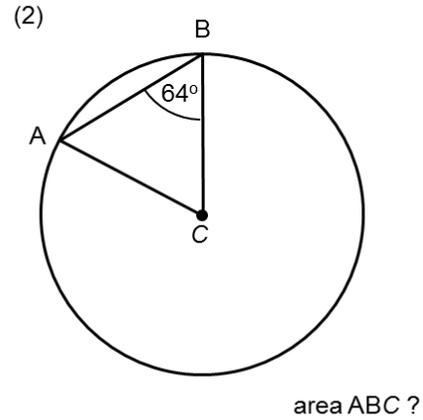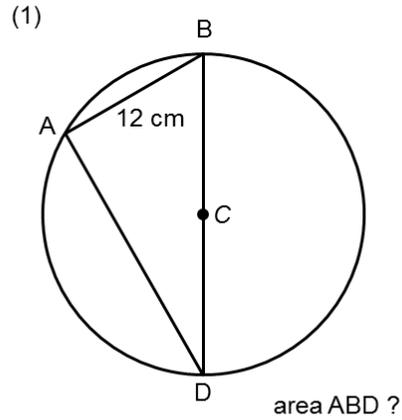
area ABDE ?

Fluency Practice

circle geometry and areas

C is the centre of a circle, radius 10 cm

(1)
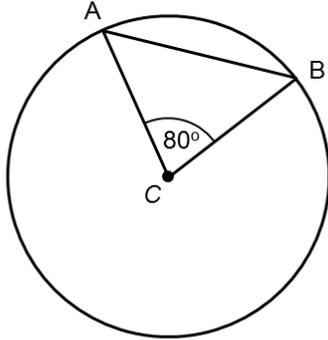
area ABC ?

(3)
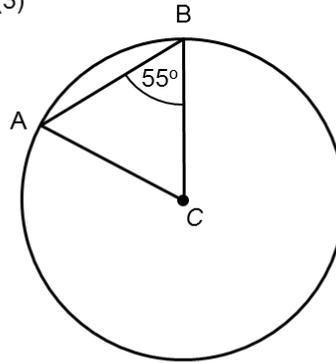
area ABC ?

(5)
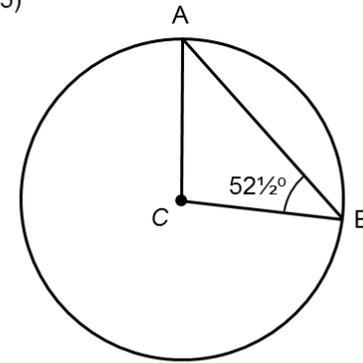
area ABC ?

(2)
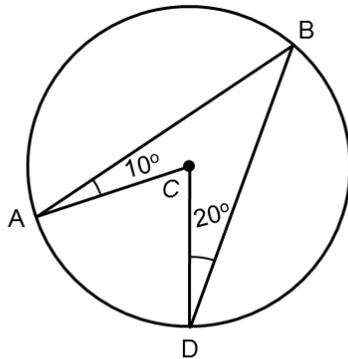
area ABDC ?

(4)
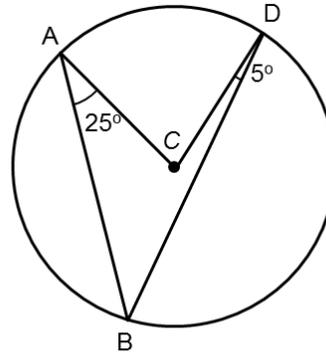
area ABDC ?

(6)
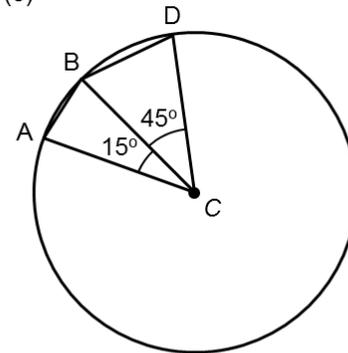
area ABDC ?

Fluency Practice

Find the area of each polygon.

Fluency Practice
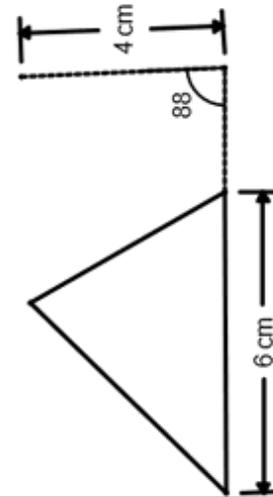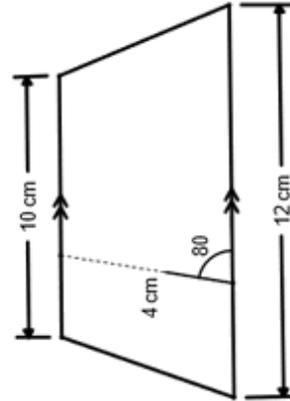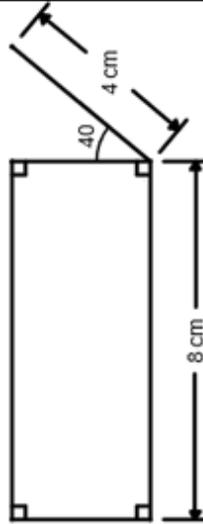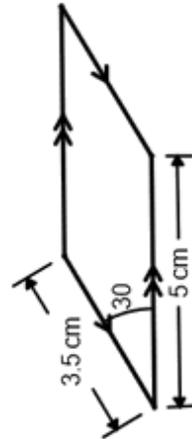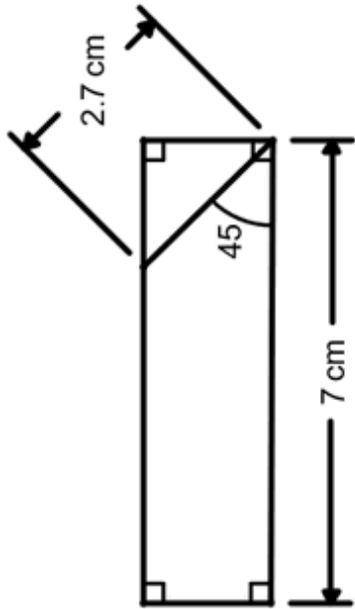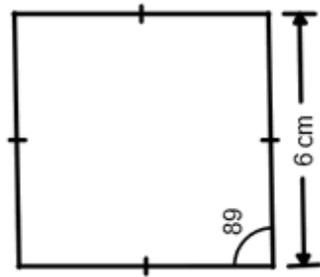
		Work out the exact value of x	Work out the exact value of x	Work out the size of angle y
Sine	State the value of sine of | 0°	30°	45°	60°	90° | |---|---|---|---|---| | | | | | |	16 cm, x, 30°	12 cm, 60°	y°, 15 cm, 7.5 cm
Cosine	State the value of cosine for | 0° | 30° | 45° | 60° | 90° | |---|---|---|---|---| | | | | | |	10 cm, 45°, x	x, 60°, 10 cm	9 cm, y°, $6\sqrt{3}$ cm
Tangent	State the value of tan for | 0° | 30° | 45° | 60° | |---|---|---|---| | | | | |	x, 30°, 12 cm	$4\sqrt{2}$ cm, 45°, x	4 cm, $4\sqrt{3}$ cm, y°
Challenge Questions	Show that 20 cos 30° + 4 sin 60° - 2 tan 60° can be written in the form $\sqrt{k}$ where k is an integer	Work out the size of angle y y°, $2\sqrt{3} - 2$ cm, $3 - \sqrt{3}$	Work out the area of the triangle 12 cm, 30°, 8 cm	The parallelogram has an area of $20\sqrt{3}$ cm². Find the value of x. x cm, 60°, x+3 cm
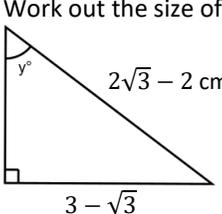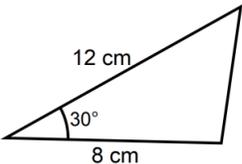
Fluency Practice

Complete the missing sides and angles for these special triangles:
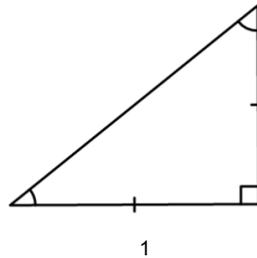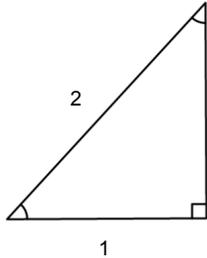
Complete the table to show the exact values of sin, cos and tan for these angles:

	30°	45°	60°
sin			
cos			
tan			

In this exercise, assume θ is an acute angle and rationalise the denominator of your answers, where appropriate.

1. Given that $\sin(\theta) = \frac{1}{\sqrt{2}}$, what is θ?

2. Given that $\tan(\theta) = \sqrt{3}$, what is $\cos(\theta)$?

3. Calculate the exact length of y in each triangle:

a)

b)

c)

d)
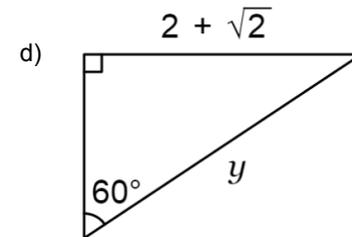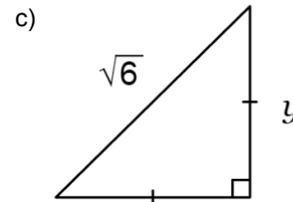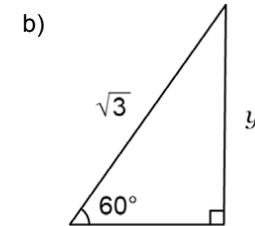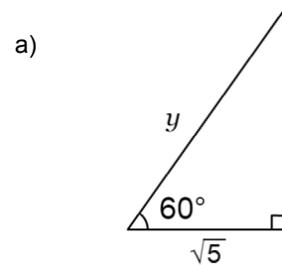
4. Simplify $4\sin(60°)\cos(60°)$

5. Simplify $2\sin(45°)\cos(45°)$

6. Write $6\tan(60°) + 3\tan(30°)$ in the form $a\sqrt{b}$

7. Show that $\tan(45°) = \frac{\sin(45°)}{\cos(45°)}$

8. Show that $\tan(30°) = \frac{\sin(30°)}{\cos(30°)}$

Page 72

Fluency Practice

1. From the triangle, write in fractional form, the values of:

   a) $sin(\theta)$ b) $tan(\theta)$ c) $cos(\theta)$
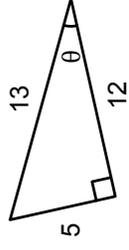
2. Which of the following are impossible?

   a) $sin(\theta) = 1.8$ b) $tan(\theta) = 2.5$ c) $cos(\theta) = 1.6$ d) $sin(\theta) = 0.25$

3. Use triangle A to work out the exact value of cos(30°).
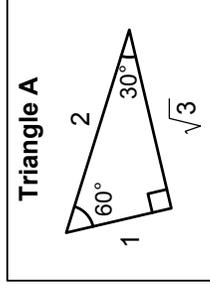
   Triangle A

   a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{3}}$ c) $\frac{2}{\sqrt{3}}$ d) $\frac{\sqrt{3}}{2}$

4. Use triangle A to work out the exact value of:

   a) sin(30°)   b) tan(30°)   c) sin(60°)

5. Use triangle B to work out the exact value of:

   Triangle B
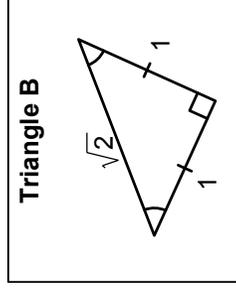
   a) sin(45°)   b) cos(45°)   c) tan(45°)

6. Complete the table of exact values:

	30°	45°	60°
sin(θ)			
cos(θ)			
tan(θ)			

7. Use the table above to work out the exact values of:

   a) $8sin(30)$ b) $cos^2(60)$ c) $sin(30) + cos(60)$

8. True or false?

   > If you double the side lengths of a right angled triangle, the value of tan(θ) remains the same.
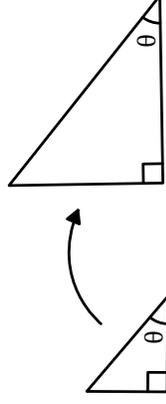
Fluency Practice

using exact trigonometric values for some angles

what are the values of these:

(1) $5\sin 30° - 11\cos 60° + 4\tan 45°$

(2) $\sin 45°.\cos 45° + \cos 30°.\tan 30°$

(3) $\sin 30°.\tan 45° + \tan 30°.\sin 60°$

(4) $2\sin 45°.\cos 45°$

(5) $\cos^2 45° + \sin^2 45°$

(6) $\cos 30°.\sin 60° + \sin 30°.\cos 60°$

(7) $\sin^2 30° + \cos^2 30°$

(8) $3\cos^2 30° - 5\sin^2 30°$

(9) $\dfrac{1}{\cos^2 30°} - \tan^2 30°$

(10) $\dfrac{2\tan^2 45°}{\tan^2 60°} + 2\tan^2 30°$
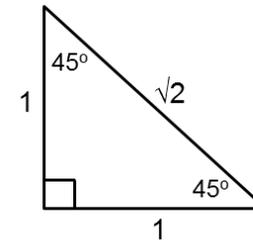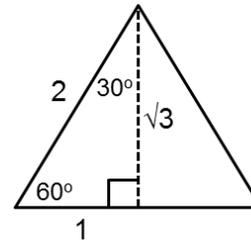
note that $\sin^2 30°$ is $(\sin 30°)^2$

(11) find angle A

(12) find angle B

Fluency Practice

Exact Trigonometric Values
Video 341 on www.corbettmaths.com

Scan here

Click here

Examples

Workout
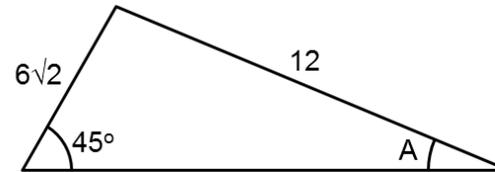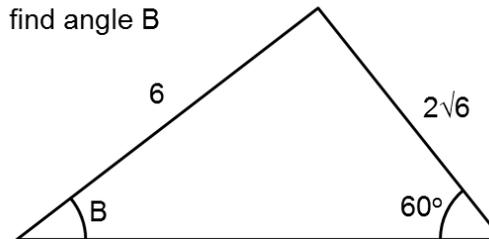
Question 1: Write down the exact values of each of the following

(a) sin 30°   (b) cos 0°   (c) tan 45°   (d) sin 90°   (e) sin 0°

(f) cos 60°   (g) tan 0°   (h) sin 45°   (i) cos 30°   (j) tan 60°

(k) cos 90°   (l) sin 60°   (m) cos 45°   (n) tan 30°

Question 2: Write down the exact values of each of the following

(a) cos 60° + sin 30°   (b) cos 0° + tan 45° + sin 90°

(c) sin 30° + sin 90°   (d) sin 45° + cos 45°

Question 3: Write down the exact values of each of the following

(a) sin 45° + cos 45°   (b) tan 30° + tan 60°   (c) cos 30° + sin 60°

Question 4: Write down the values of each of the following

(a) sin 180°   (b) cos 270°   (c) tan 360°   (d) sin 360°

(e) sin 270°   (f) cos 180°   (g) cos 360°   (h) cos 450°

(i) sin 720°   (j) sin 900°   (k) cos 990°   (l) sin 1710°

Question 5: Write down the exact values of each of the following

(a) sin 120°   (b) cos 150°   (c) tan 135°   (d) cos 210°

(e) tan 120°   (f) sin 240°   (g) sin 225°   (h) cos 315°

(i) sin 150°   (j) tan 405°   (k) sin 495°   (l) cos 870°

Purposeful Practice

Exact Trigonometric Values

Apply

Corbett mɑths

Question 1: Using the triangle below, explain each of the following.

(a) $sin(30°) = \dfrac{1}{2}$

(b) $cos(30°) = \dfrac{\sqrt{3}}{2}$

(c) $tan(30°) = \dfrac{\sqrt{3}}{3}$

(d) $sin(60°) = \dfrac{\sqrt{3}}{2}$

(e) $cos(60°) = \dfrac{1}{2}$

(f) $tan(60°) = \sqrt{3}$

Triangle with angles 30° and 60°, sides 2, 1.
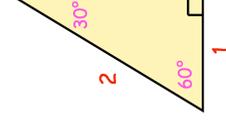
Question 2: Using the triangle below, explain each of the following.

(a) $tan(45°) = 1$

(b) $sin(45°) = \dfrac{\sqrt{2}}{2}$

(c) $cos(45°) = \dfrac{\sqrt{2}}{2}$

Triangle with angles 45° and 45°, sides 1, 1.
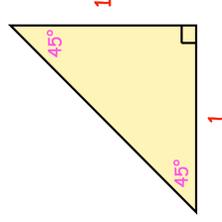
Question 3: Conor says that $cos(45°) = \dfrac{1}{\sqrt{2}}$

Is he correct?

Question 4: Find the size of the angle labeled x.

Triangle with sides 5√10, 10√10, angle x.
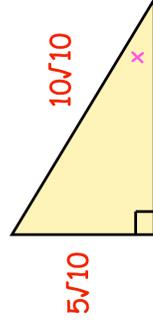
Question 5: Find the exact length of the side labelled w.

Triangle with 4 + 2√3, angle 60°, side w.

Fluency Practice

Q	Find the length	Answers	Q	Find the length	Answers	Q	Find the angle	Answers
1a	4 cm, x cm, 30°		**1b**	x cm, 4 cm, 30°		**1c**	6 cm, 3 cm, x°	
2a	4 cm, 30°, x cm		**2b**	x cm, 30°, 4 cm		**2c**	10 cm, x°, $5\sqrt{3}$ cm	
3a	x cm, 30°, 4 cm		**3b**	4 cm, 30°, x cm		**3c**	5 cm, x°, $5\sqrt{3}$ cm	

Fluency Practice

Q	Find the length	Answers	Q	Find the length	Answers	Q	Find the angle	Answers
1a	4 cm, x cm, 45°		**1b**	x cm, 4 cm, 45°		**1c**	8 cm, $4\sqrt{2}$ cm, x°	
2a	4 cm, 45°, x cm		**2b**	x cm, 45°, 4 cm		**2c**	10 cm, x°, $5\sqrt{2}$ cm	
3a	x cm, 45°, 4 cm		**3b**	4 cm, 45°, x cm		**3c**	4 cm, x°, 4 cm	

Fluency Practice

Q	Find the length	Answers	Q	Find the length	Answers	Q	Find the angle	Answers
1a	x cm, 4 cm, 60°		**1b**	4 cm, x cm, 60°		**1c**	$4\sqrt{3}$ cm, 8 cm, $x°$	
2a	4 cm, 60°, x cm		**2b**	x cm, 60°, 4 cm		**2c**	10 cm, $x°$, $5\sqrt{3}$ cm	
3a	x cm, 60°, 4 cm		**3b**	4 cm, 60°, x cm		**3c**	$4\sqrt{3}$ cm, $x°$, 4 cm	

Fluency Practice

Exact Trigonometric Values in Right-Angled Triangles

(a)

Find the exact value of x.
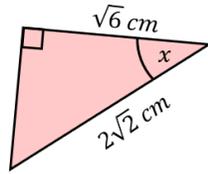
$\sqrt{6}\ cm$

x

$2\sqrt{2}\ cm$

(b)

Find the exact length of y.
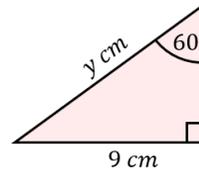
$y\ cm$

60

9 cm

(c)

Find the exact length of x.

$x\ mm$

12 mm

30

(d)

Find the exact length of y.

$y\ cm$

60

45

$4\sqrt{5}\ cm$

(e)

Find the exact length of x.

$2\sqrt{5}\ m$

45

30

$x\ m$
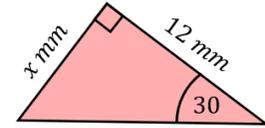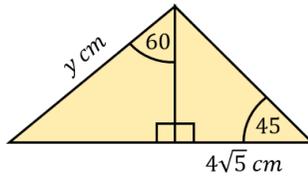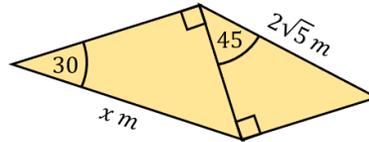
(f)

Find the exact length of x.

$4\sqrt{3}\ cm$

45

30

$x\ cm$

(g)

Find the area of the parallelogram.

9 cm

4 cm

60
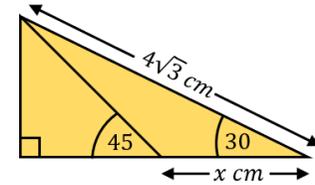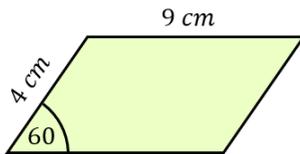
(h)

Find the exact value of the shaded area.

30

45

$4\sqrt{3}\ cm$
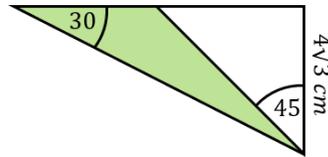
(i)

A triangle is joined to a parallelogram. Find the exact perimeter of the shape.

12 cm

60

6 cm

30
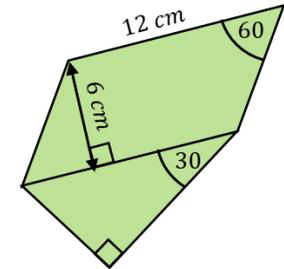
Fluency Practice

1 What is the exact solution of:

$$\sin(60)$$

2 What is the exact solution of:

$$\tan(90)$$

3 Calculate the value of x. Leave your answer in exact form.

12cm

60°

x
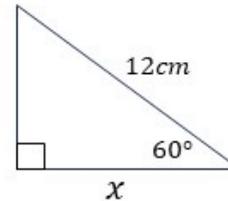
4 Calculate the value of x. Leave your answer in exact form.

$5\sqrt{2}cm$

10cm

x
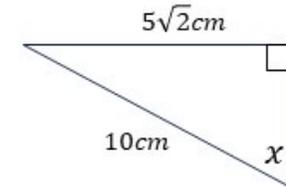
5 $$\cot(x) = \frac{\cos(x)}{\sin(x)}$$

Find the exact value of $\cot(45)$

6 $$\sec^2(x) = 1 + \tan^2(x)$$

Find the exact value of $\sec(30)$

7 Calculate the exact area of the triangle.

$\sqrt{2}\ cm$

60°

$2\sqrt{2}\ cm$
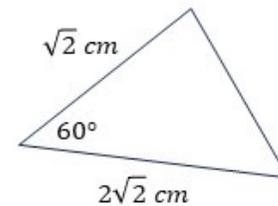
8 Calculate the area of the segment. Leave your answer in terms of pi.

4cm

45°
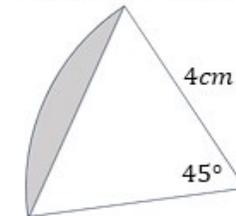
9 Calculate the value of x.
Leave your answer in the form $k\sqrt{a}$.

12cm   75°

x

60°
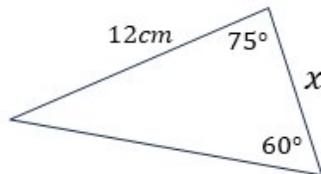
10 Calculate the value of x. Leave your answer as an integer.

45°   3cm

$3\sqrt{2}\ cm$

x
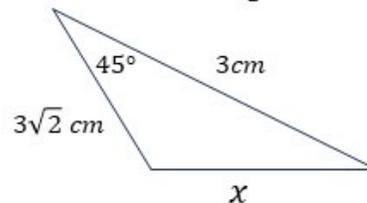
11 Calculate the area of the regular hexagon. Leave your answer in the form $k\sqrt{a}$.

4cm
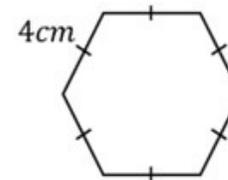
12 The following is a cyclic equilateral triangle. The radius of the circle is 5cm. Calculate the shaded area in terms of π.
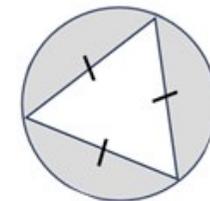
Page 87

Fluency Practice

find the exact values of the sides indicated, without using a calculator

(1)

30°

$12\sqrt{3}$

120°

d

(2)

105°   e

$6\sqrt{2}$

45°

(3)

f

8

30°

75°

a parallelogram with an isosceles triangle

(4)

$\sqrt{6}-1$

g

60°

45°
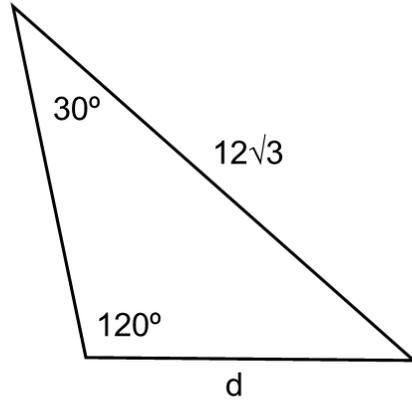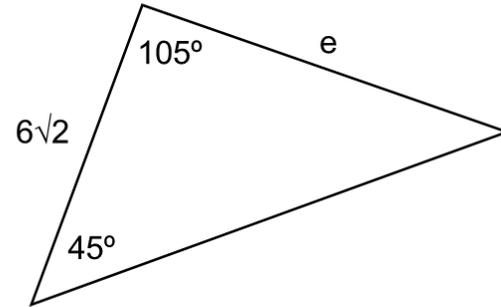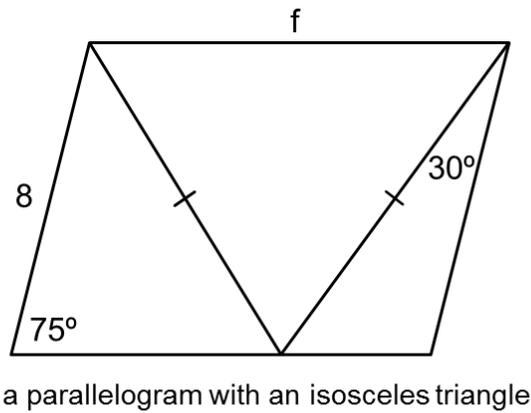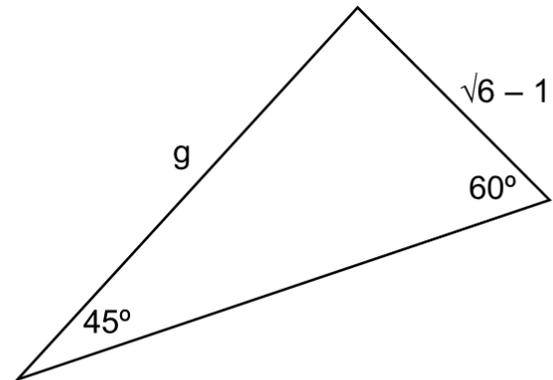
Fluency Practice

find the values of the angles, without using a calculator

(5) find angle A
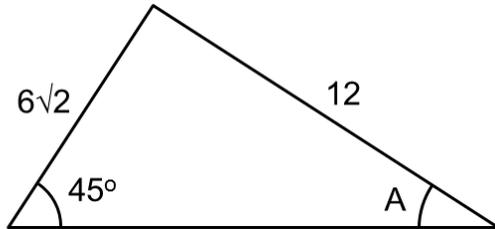
(6) find angle B
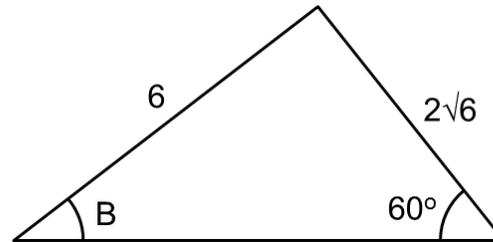
(7) find angle C
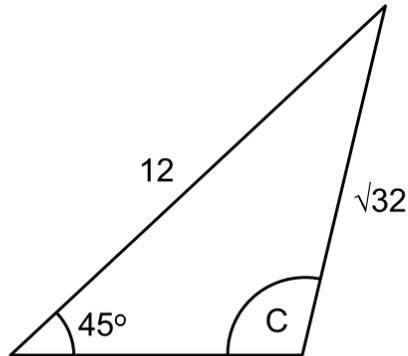
(8) find angle D
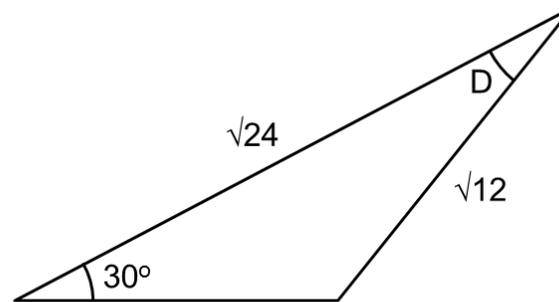
Fluency Practice

exact trigonometric forms

(5)
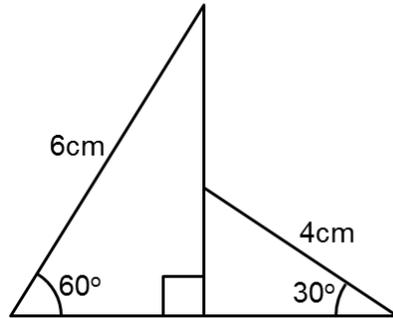
show that the perimeter of the
quadrilateral is 11 + 5√3

(6)
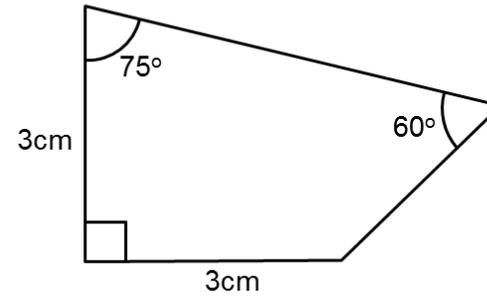
what is an exact value for
(i) the perimeter (ii) the area?
give your answers in the form a + b√c

(7)
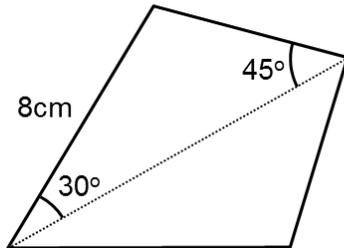
what is an exact value for the area of
the (symmetrical) kite?
give your answer in the form a + a√b

(8)
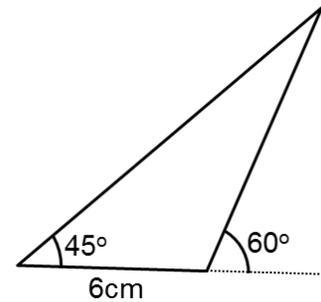
what is an exact value for
the area of the triangle?
give your answer in the form a + b√c

Fluency Practice

exact trigonometric forms

(1)

8m

60°
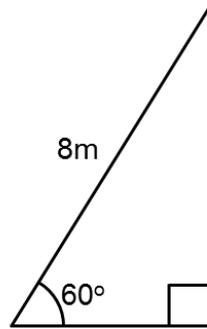
what is an exact value for the perimeter of the right angled triangle?

give your answer in the form a + b√c

(2)

12km

45°

30°
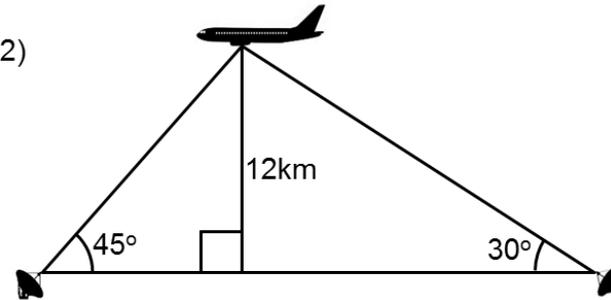
what is an exact value for the distance between the radar sites?

give your answer in the form a + b√c

(3)

H

9km

60°

45°

G
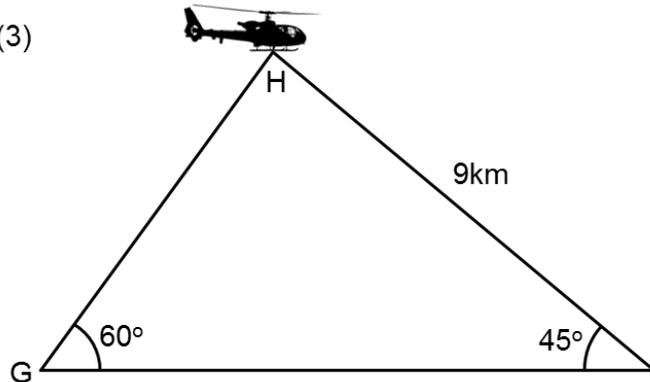
what is an exact value for the distance HG?

give your answer in the form a√b

(4)

d

60°

45°

9cm
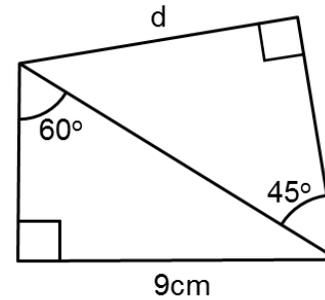
what is an exact value for the distance 'd' ?

give your answer in the form a√b

Purposeful Practice

using exact trigonometric values for some angles
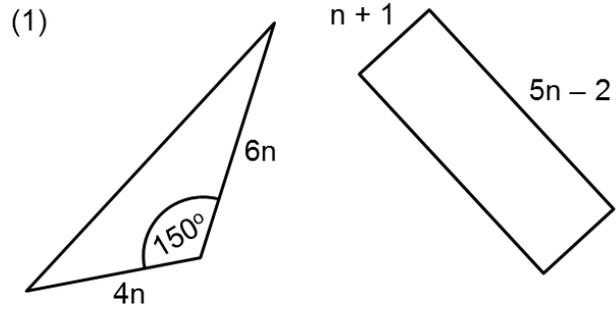
(1)

what values of 'n' make the area of the
triangle and the area of the rectangle equal?

(2)

what values of 'n' make the area of the
triangle and the area of the rectangle equal?

(3)

find the value of 'a'

(4)

find the value of 'b'

tan 15º + tan 75º = 4

tan 15º × tan 75º = 1

find tan 15º
and tan 75º
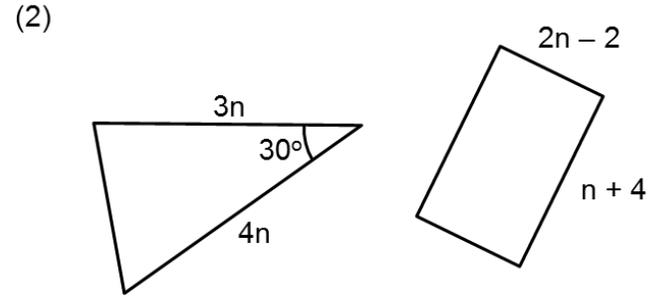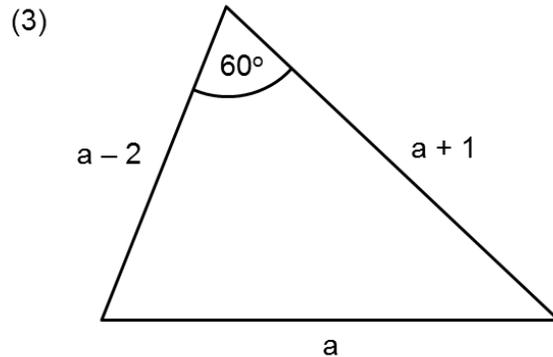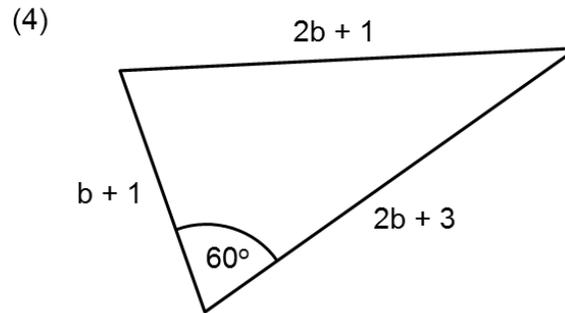
in surd
form

$$\cos(45°) \times \cos(46°) \times \cos(47°) \times \ldots \times \cos(135°) = ?$$

A circle with radius 1 touches the sides of a rhombus, as shown. Each of the smaller angles between the sides of the rhombus is 60°.

What is the area of the rhombus?

A 6   B 4   C $2\sqrt{3}$ D $3\sqrt{3}$ E $\dfrac{8\sqrt{3}}{3}$

Fluency Practice

Find the missing length or angle.

(a)

8 m 8 m 65 x

(b)

y 4 cm 35 6 cm
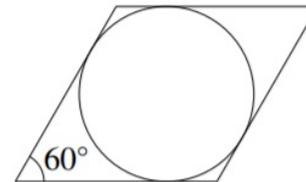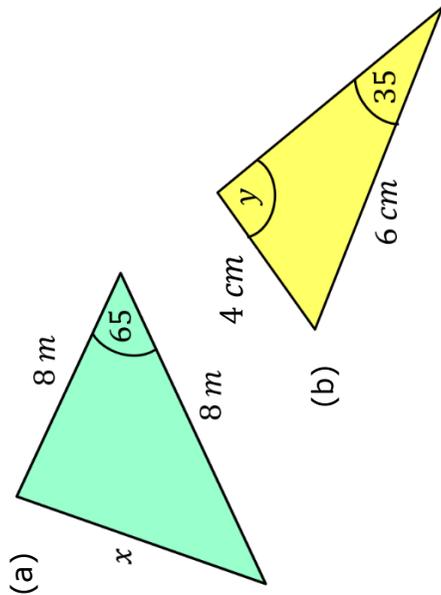
Find the area of the segment.

(c) 9 cm 145

(d) 2.5 m 128
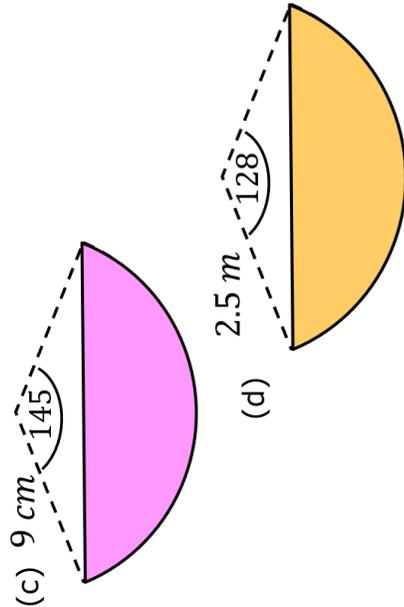
Find the perimeter of these shapes.

(e)

101 37 8.3 cm

(f)

4 m 101 6 m 81 27
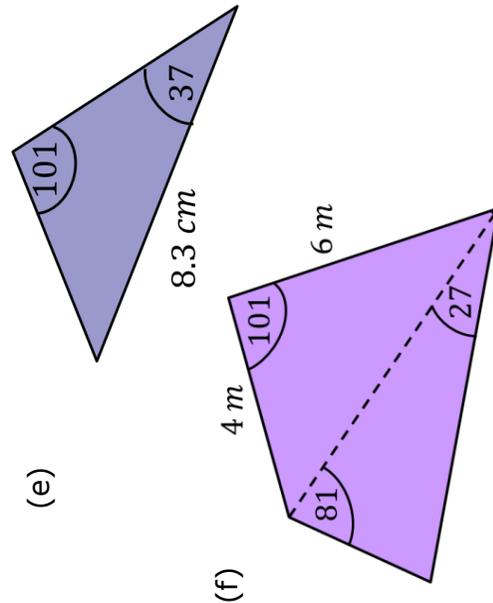
(g) The hands of a clock have lengths 8cm and 10 cm. The time on the clock is 5 o'clock. Find the distance between the tips of the two hands.

Fluency Practice

Sine and Cosine Rules Revision

(a)
Find the area of the triangle.

6 cm, 39, 5 cm

(b)
Find x to 1 decimal place.

81, 3.1 cm, x, 6.8 cm

(c)
Find y to 2 decimal places.

1.3 m, 61, 1.4 m, y
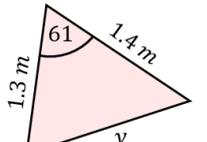
(d)
Find x to 3 significant figures.

23 mm, 79, 32, x

(e)
Find y to 1 decimal place.

11 cm, 10 cm, y, 7 cm
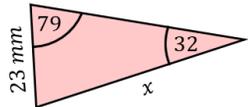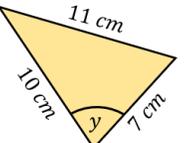
(f)
Find x to 1 decimal place.

13.5 cm, 42, x, 88

(g)
Find the area of the parallelogram.
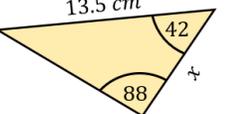
4.5 cm, 82, 8.8 cm
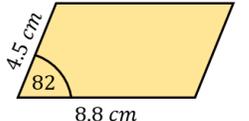
(h)
The area of the triangle is 4.6 m². Find y to 1 decimal place.

2.5 m, 63, y
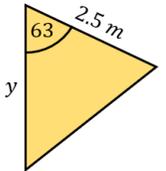
(i)
Find y to 3 significant figures.

50, 19 mm, y, 28 mm
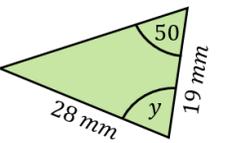
(j)
Find the area of the triangle.

8 cm, 73, 11 cm
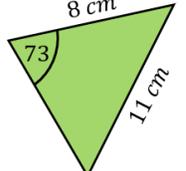
(k)
Find the area of the compound shape.

109, 58, 11 cm

(l)
Find the value of x.

x, $2\sqrt{7}$ cm, 60, $x + 2$

Fluency Practice

Q	Work out the missing length	Answers	Q	Work out the missing angle	Answers
1a	5 cm, x cm, 60°, 40°		1b	x acute — 7 cm, 5 cm, $x°$, 40°	
2a	5 cm, x cm, 40°, 7 cm		2b	5 cm, 9 cm, $x°$, 8 cm	
3a	x cm, 60°, 40°, 5 cm		3b	x obtuse — 7 cm, 5 cm, $x°$, 40°	
4a	7 cm, 5 cm, 40°, 60°, x cm		4b	5 cm, $x°$, 9 cm, 8 cm	

Fluency Practice

Sine & Cosine Rules   Lengths are centimetres.   not to scale

10   a   100°   44°

b°   8   89°   6

35°   c   15   113°

8   85°   10   d

e°   40°   20   24

21   14   f°   12

35   58°   30   g

16   h°   8   15

9   7   44°   i

20   15   30°   j°

37°   k   11   7

13   l°   10   6   30°   78°
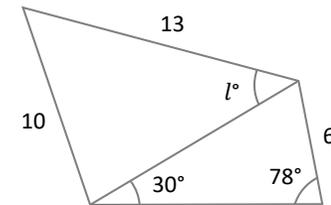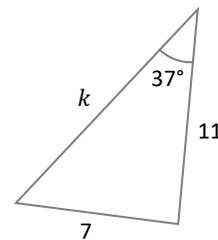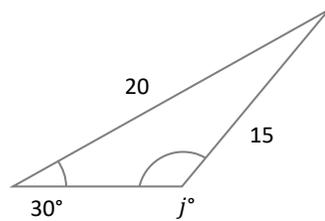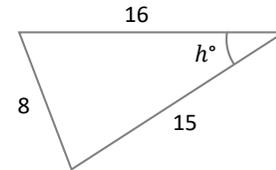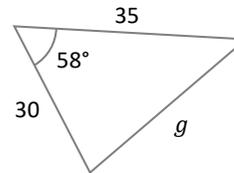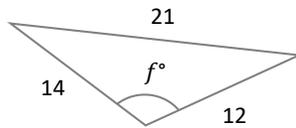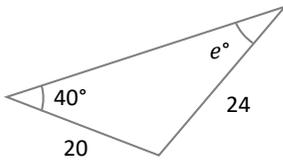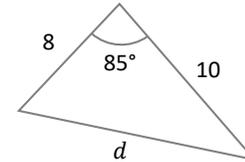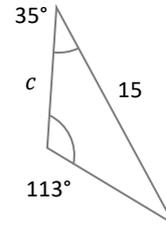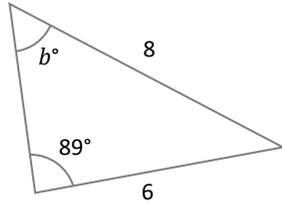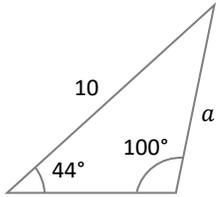
10 Answers	8.6	31.8	138.2	12.2	107.5
	9.6	48.6	32.4	29.7	7.1

Fluency Practice

1. Work out the value of x in each triangle. Give your answers correct to 2 significant figures.

a)

74°, 43°, 3cm, x

b)

x, 8cm, 52°

c)

9cm, 7cm, 8cm, x

d)

41°, 103°, x, 4cm

e)

x, 80°, 10cm

f)

3cm, 38°, x

g)

4cm, 98°, x, 5cm

h)

7cm, 60°, x

i)

x, 10cm, 53°, 6cm

2. Work out the value of x in each triangle. Give your answers correct to 2 significant figures.

a)

B, 6cm, 11cm, x, A, 7cm, C

b)

B, 58°, x, A, 39°, 4cm, C

c)

B, 52°, 12cm, 4cm, A, x, C

d)

B, 2cm, 56°, A, x, C

e)

Q, 3cm, x, P, 8cm, R

f)

V, 7cm, x, 76°, U, 5cm, W

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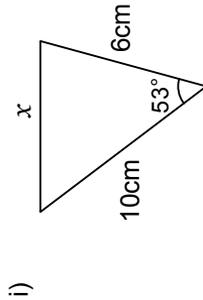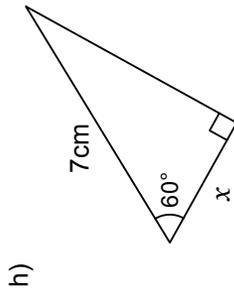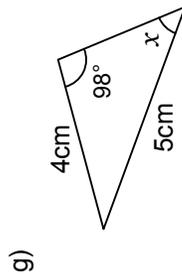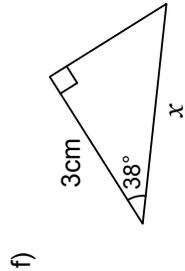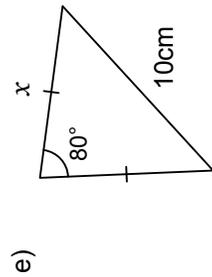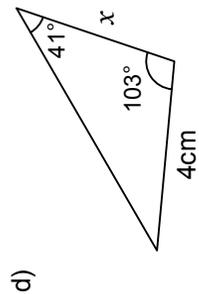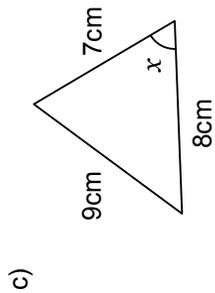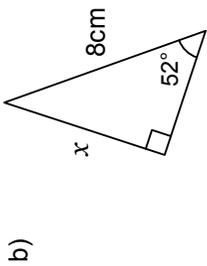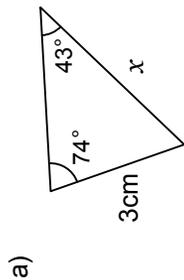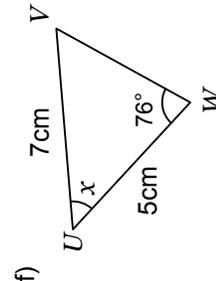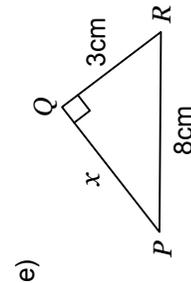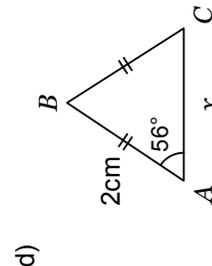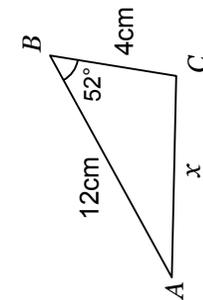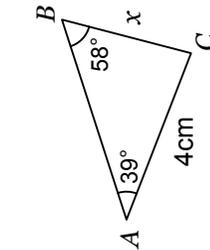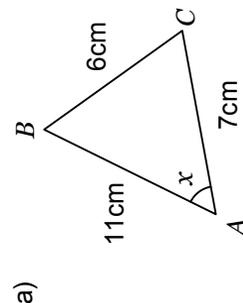
Purposeful Practice

3. Work out the length *CD*. Round your answer to the nearest 0.1cm.

B 66° 7cm *C*

39° *A* 5cm *D*
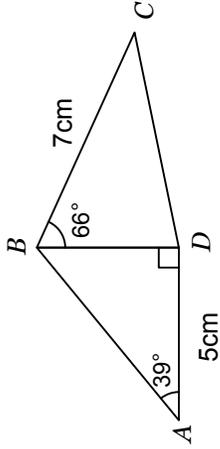
4. Work out $A\hat{B}D$. Round your answer to the nearest degree.

B 8cm *C* 96° 3cm *D*

64° *A* 9cm
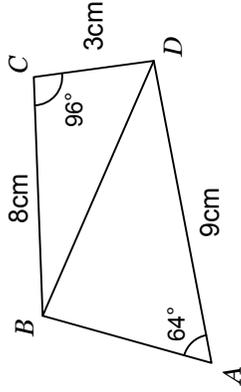
5. Work out the $B\hat{C}D$ correct to the nearest degree.

B 8cm *C*

67° *A* 4cm *D*
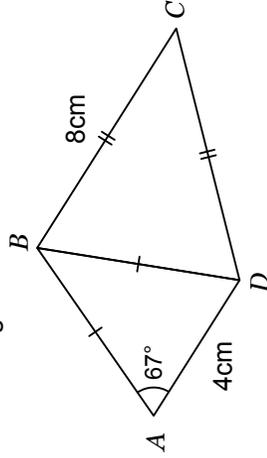
6. Work out *RS*. Round your answer to the nearest 0.1cm.

Q 50° 3cm *R* 59° 54° *S* 4cm *P*
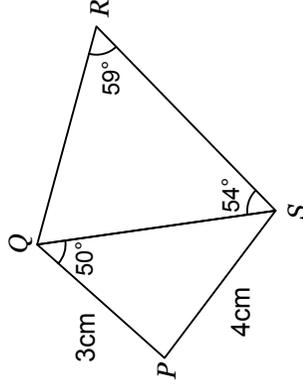
7. *BCD* and *AED* are straight lines. *AE* = 3cm and *ED* = 5cm. Work out *CE* correct to 2 significant figures.

B 6cm *C* 42° *A* 3cm *E* 5cm *D*

8. Show that $QR = y\sqrt{7}$.

Q 3y *P* 60° 2y *R*
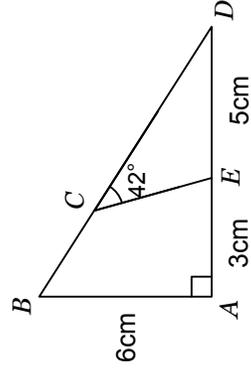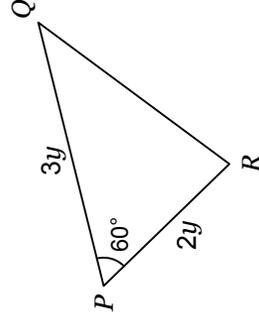
9. Each regular polygon is inscribed in a circle with radius 1cm. Work out the area of each polygon, correct to the nearest 0.01cm².

a)

b)

c)

Page 103
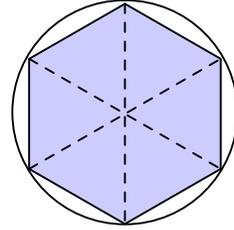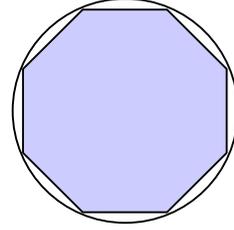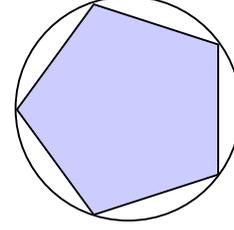
Purposeful Practice

Find the size of angle NMK

6.7 cm

35°

88°

2.4 cm

2.8 cm

35°

91°

59°

5.7 cm

2.2 cm

70°

4.4 cm

4.6 cm

78°

1 cm

1 cm

10°

76°

8 cm

111°

2.3 cm

M

N

K

F

I

D

E

L

J

H

G

C

A

B
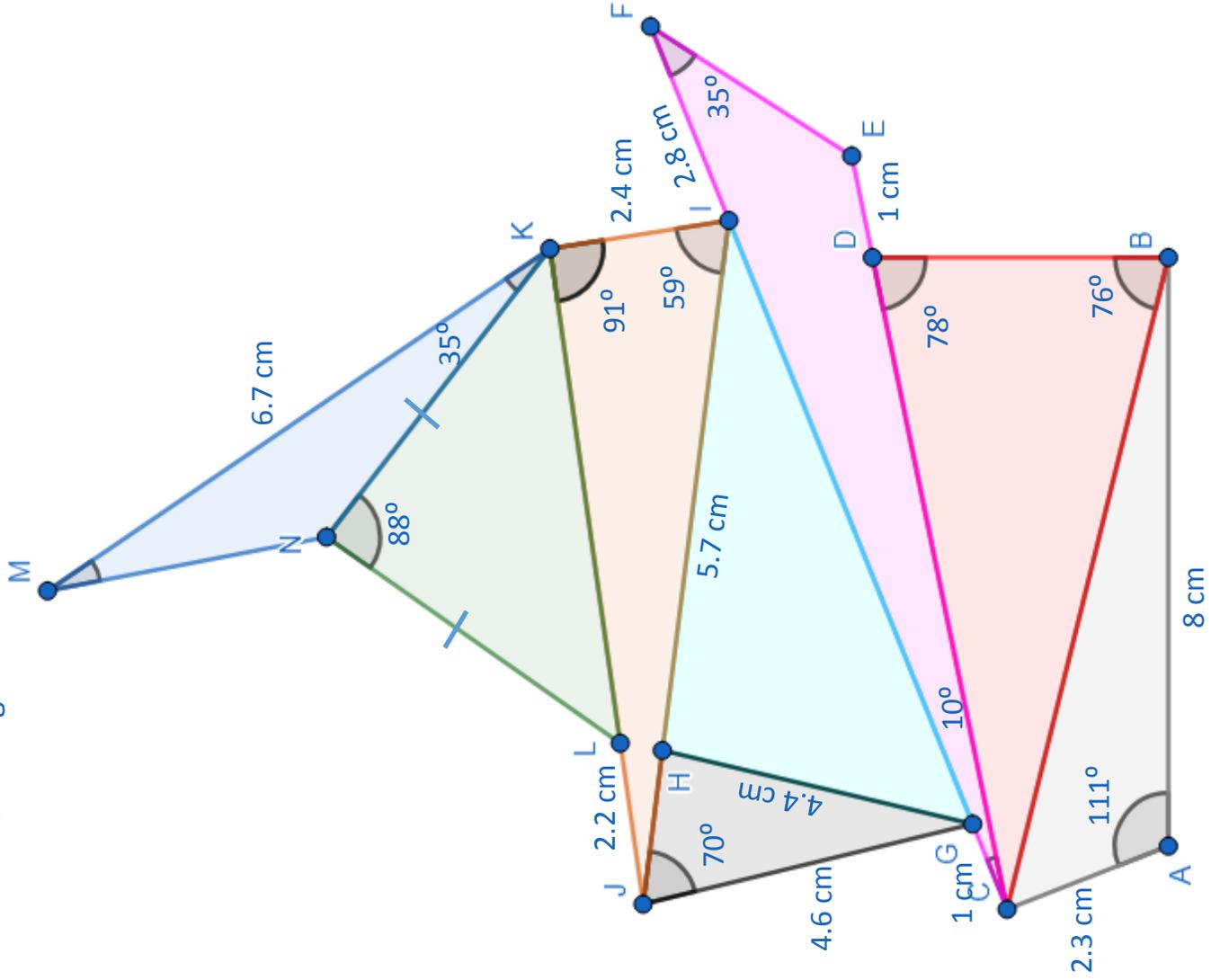
Problem Solving

Position the three numbers as either:
•a side length in cm
•an angle size in degrees.

90   40   50

Can you always work out
the other three measurements?

Try to create scenarios
that require one or more of:

•SOHCAHTOA

•Sine rule

•Cosine rule

•Pythagoras' theorem
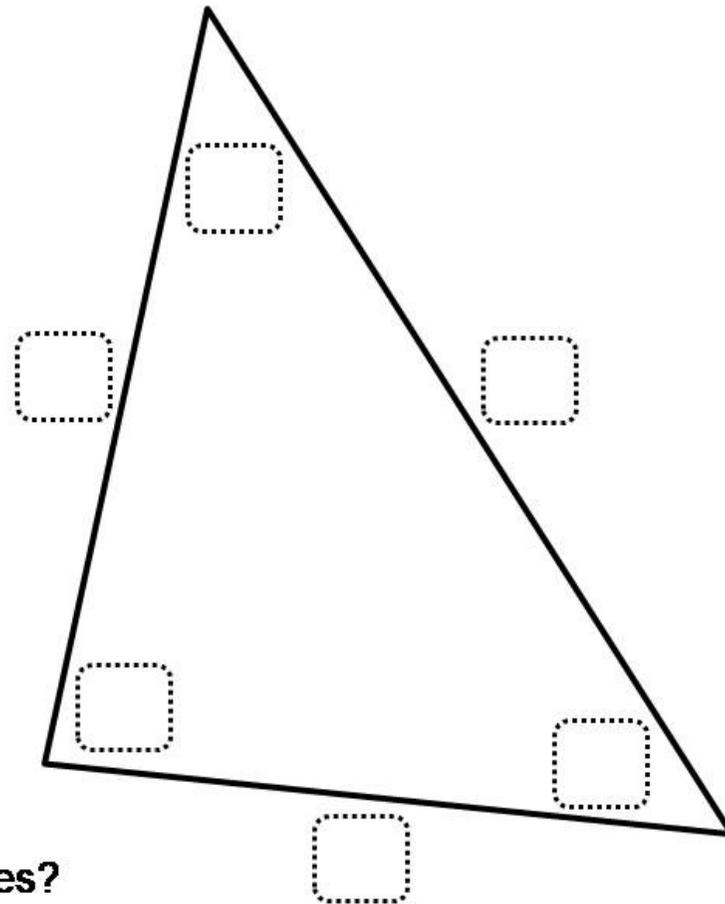
Are there any impossible triangles?

2 3D Pythagoras' Theorem and Trigonometry

Corbett mɑths

Examples

3D Pythagoras

Video 259 on www.corbettmaths.com

Click here   Scan here

Question 1: ABCDEFGH is a cube with side length 5cm.
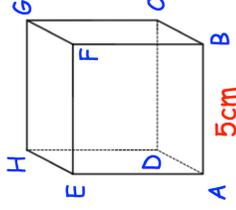
(a) Work out the length of AC

(b) Work out the length of AG

Question 2: ABCDEFGH is a cuboid.
AB = 6cm, BC = 2cm and CG = 3cm.
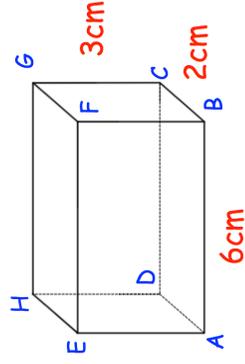
(a) Work out the length of BG

(b) Work out the length of BD

(c) Work out the length of HC

(d) Work out the length of AG

Question 3: Shown is a triangular prism.
Triangle ABC is a right angle triangle.
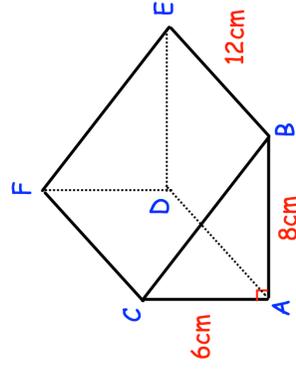
(a) Work out the length of BC

(b) Work out the length of CD

(c) Work out the length of BF

Question 4: Shown is a square based pyramid ABCDE.
F is the midpoint of CD.
M is the point on the base directly below the vertex E.
AD = 4cm   and   CE = 7cm
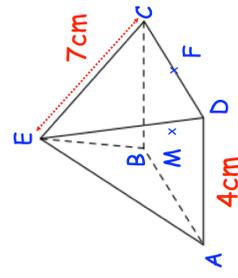
Calculate the length of

(a) AC   (b) AM   (c) EM   (d) EF

Corbett Mɑths

Apply

3D Pythagoras
Video 259 on www.corbettmaths.com

Question 1: A rectangular field is 30m long and 25m wide.
An obelisk is situated in the corner of the field.
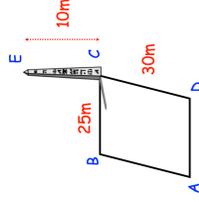
Calculate the length AE.

Question 2: Can a 1m rod fit inside this box?
Explain your answer.
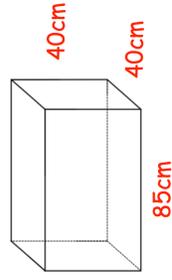
Question 3: Calculate the volume of this cone.
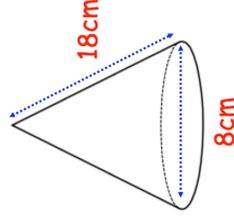
Question 4: ABCDEFGH is a cuboid

P is a point on AB, such that AP:PB is 1:2
Q is a point on GH, such that GQ:QH is 2:3

Calculate the distance PQ.

Answers

Scan here

Click here
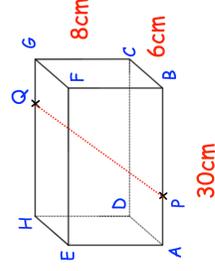
Fluency Practice

Find the length of the diagonal to 1dp.

(a)

8 cm
5 cm
5 cm

(b)

4 cm
4 cm
4 cm

(c)

10 m
3 m
3 m

(d)

3 cm
4 cm
6 cm

Find the distance AB to 1dp.

(a)

4 cm
B
7 cm
3 cm
A

(b)

2 m
B
5 m
1.5 m
A

(c)

6 cm
B
11 cm
5 cm
A

(d)

B
8 m
A
3 m
3 m

(e)

3.5 cm
8 cm
B
3.5 cm
A
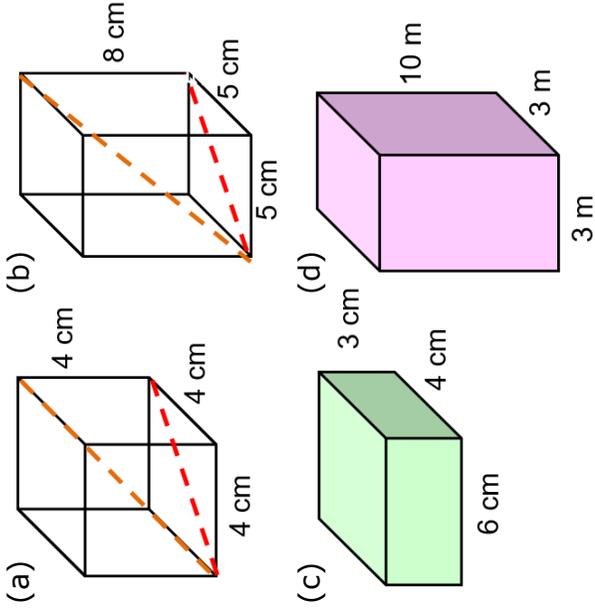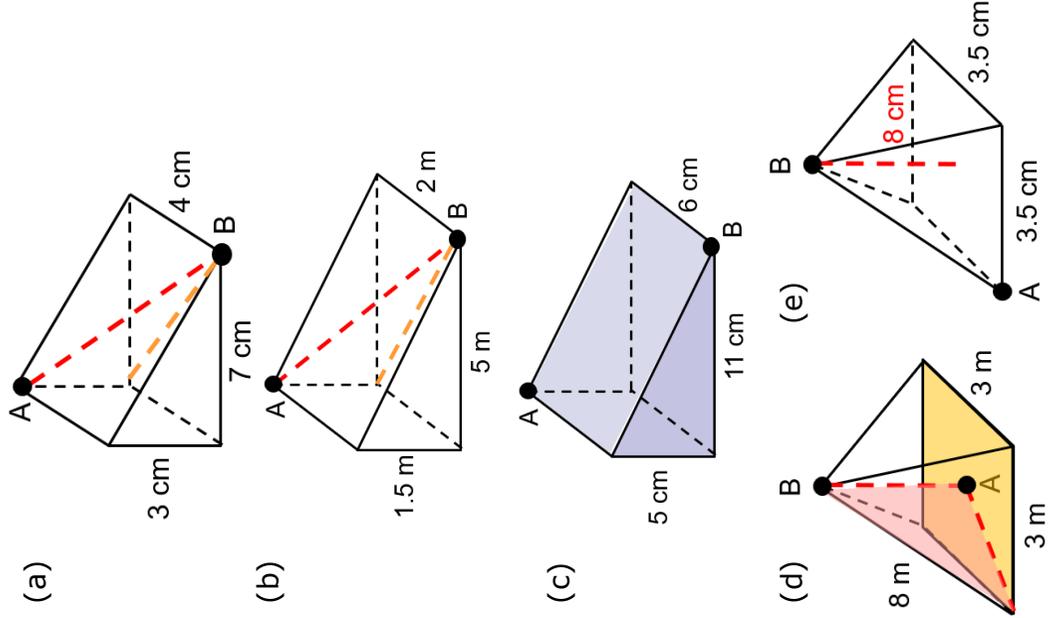
Purposeful Practice

Pythagoras' Theorem Revision

(a)	(b)	(c)	(d)
Find the value of x to 1 decimal place.	Find the value of y to 3 significant figures.	A triangle has sides of lengths $10.5\ cm$, $12\ cm$ and $16.5\ cm$. Is the triangle right-angled? Explain your answer.	Find the perimeter of the triangle to 3 significant figures.
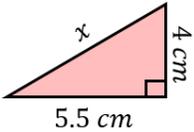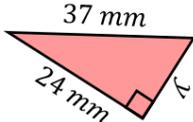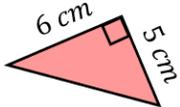
(e)	(f)	(g)	(h)
Find the distance between the coordinates $(1,6)$ and $(3,2)$, giving an exact answer.	Find the value of x to 3 significant figures.	Find the area of the isosceles triangle to 1 decimal place.	From point A, a boat sails 80 km east. It then turns and sails 110 km south to point B. Find the distance AB to the nearest km.
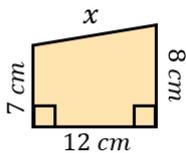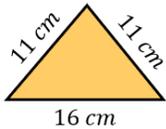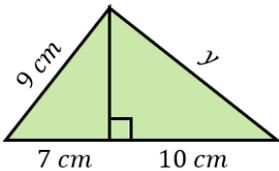
(i)	(j)	(k)
Find the value of y to 3 significant figures.	The area of the isosceles triangle is $40\ cm^2$. Find the perimeter of the triangle, to 1 decimal place.	Find the length of the line AB, giving your answer to 3 significant figures.
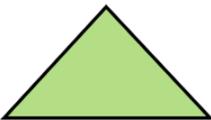
Purposeful Practice

3D-Pythag
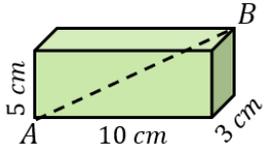
1)
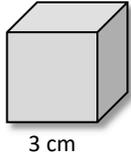
Find the diagonal length within this cube.

3 cm

2)
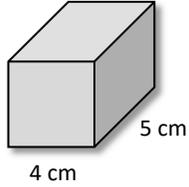
Find the diagonal length within this square-based prism.

5 cm
4 cm

3) 8 cm   2 cm
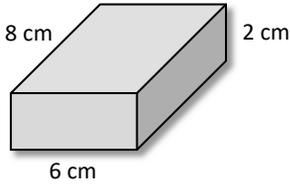
Find the diagonal length within this cuboid.

6 cm

4)
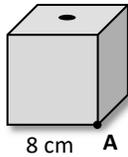
The centre of the top face of this cube has been marked. Find the distance from **A** to this point.

8 cm   **A**

5)   7 cm
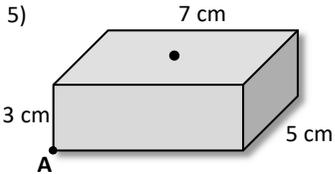
Find the distance from **A** to the centre of the top face.

3 cm
5 cm
A

6)

B

10 cm

• C

9 cm   **A**   6 cm
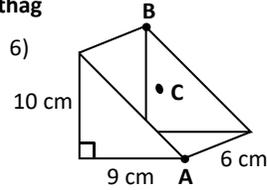
Point **C** is equidistant between **A** & **B** on the surface of this prism.

Find distance **AC**.

7)

15 cm

A

9 cm   12 cm
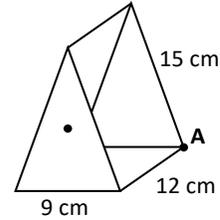
The point halfway up the front face of this isosceles-triangle-based prism has been marked.

Find its distance from **A**.

8)   B

9 cm

A   6 cm
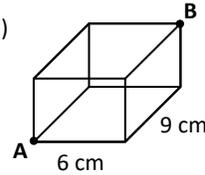
Length **AB** is 12 cm.
Find the volume of the cuboid.

9)

The diagonal within this cube is 12 cm.
Find the volume of the shape.

10)   The sides of a cuboid are in the ratio 1:2:3
The diagonal within the shape is 38 cm long.
Find the volume of the cuboid.

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Fluency Practice

1. *ABCDEFGH* is a cuboid.
Work out the length of *AG*.

4cm

7cm

3cm
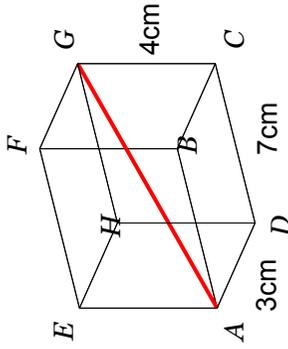
2. *ABCDEFGH* is a cube.
Work out the length of *CE*.

10cm
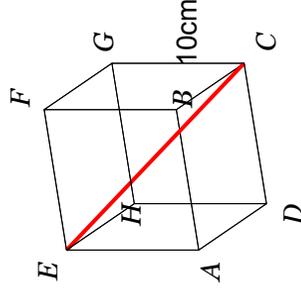
3. *ABCDE* is a square-based pyramid with perpendicular height 8cm.
The apex of the pyramid is directly above the centre of the square, *O*.
M is the midpoint of *BC*. Work out the length of *EM*.

8cm

6cm
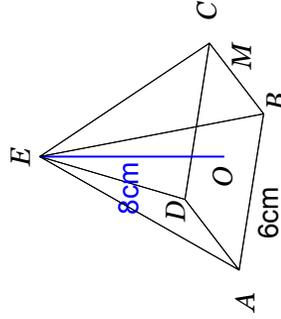
4. *ABCDE* is a square-based pyramid with perpendicular height 4cm.
The apex of the pyramid is directly above the centre of the square, *O*.
Work out the length of *EC*.

4cm

4cm
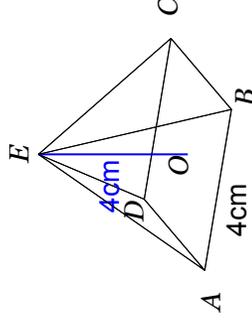
5. *ABCD* is a regular tetrahedron with each edge length equal to 5cm.
M is the midpoint of *AC*. Work out the exact length of *DM*.

5cm
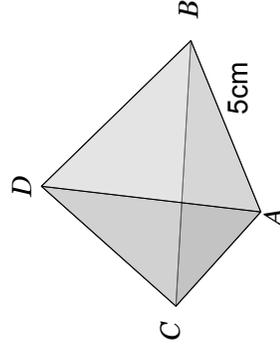
6. A triangular prism is shown.
Work out the length *AE*.

3cm

4cm

10cm
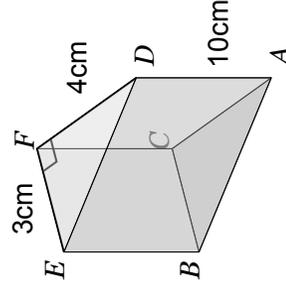
7. *ABCD* is a skew pyramid. *D* is directly above *C*. $A\hat{B}C = 90°$. Work out the length of *AD*.
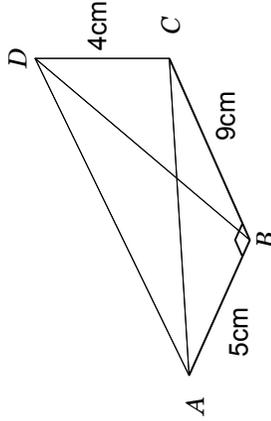
8. A rectangular-based pyramid has a perpendicular height 9cm. The apex of the pyramid is directly above the centre of the rectangle. Work out the length *BE*.
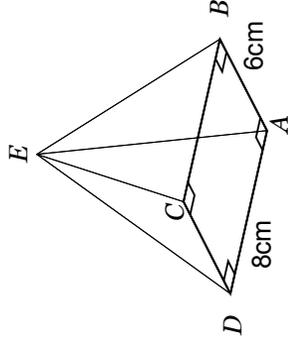
9. Square-based pyramid *ABCDE* has a perpendicular height of 15cm. The apex of the pyramid is directly above the centre of the square. *CE* = 17cm. Work out the length *AB*.
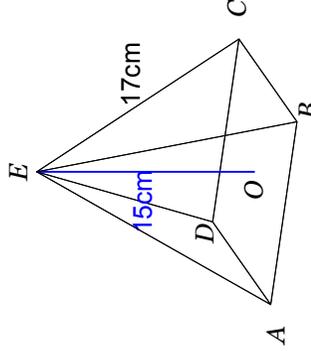
10. Four cubes, each with edge lengths of 3cm are arranged as shown. Work out the straight-line distance from *X* to *Y*.
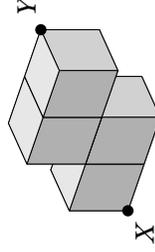
11. In the cuboid, *M* is the midpoint of *AB* and *N* is the midpoint of *FG*. Work out the length *MN*.
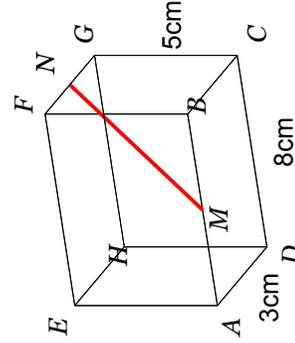
12. In the triangular prism, *AC* = 8cm, *BC* = 8cm, *ED* = 4cm, *BE* = 7cm and *M* is the midpoint of *AB*. Work out the length *MF*.

Fluency Practice

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Examples

Workout

Scan here
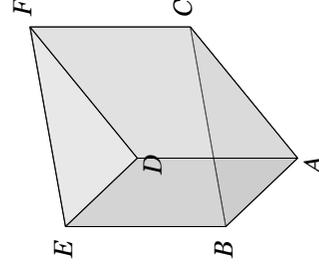
Click here

Question 1: Here is a cuboid.
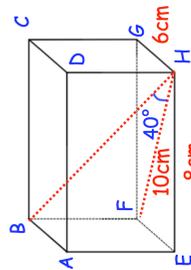
EH = 8cm, GH = 6cm, FH = 10cm and ∠FHB = 40°

(a) Work out the length BF
(b) Work out the length BH
(c) Work out the size of angle FBH

Question 2: Here is a cube with side length 4cm.
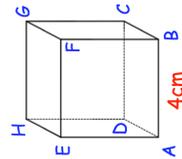
(a) Calculate the length AC
(b) Calculate the size of angle CAG

Question 3: Shown is a cube with side length 10cm
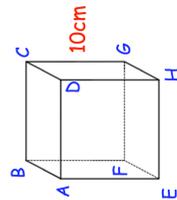
(a) Calculate the length of FH.
(b) Calculate the size of angle DFH

Question 4: Shown is a cuboid.
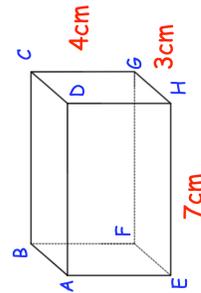
(a) Work out the size of angle GEH
(b) Work out the size of angle CHG
(c) Work out the length EG
(d) Work out the size of angle CEG

Question 5: ABCDEF is a triangular prism
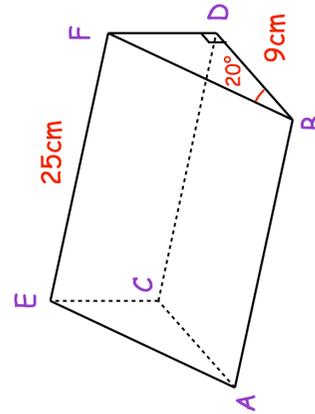
BD = 9cm, EF = 25cm, ∠BDF = 90° and ∠DBF = 20°

(a) Work out the length DF
(b) Work out the length BF
(c) Work out the length BC
(d) Work out the size of angle CBE

3D Trigonometry
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Question 6: Shown is a triangular prism.

M is the midpoint of BC.
AB = 16cm, CM = 5cm, CF = 8cm and ∠BMF = 90°

Calculate

(a) Length FM (b) Length BE (c) Length BD

(d) Angle CBD (e) Angle ABE (f) Angle DMF
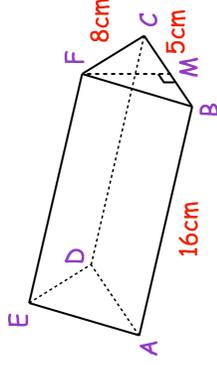
Question 7: Shown is a square-based pyramid.
The apex E is directly over the centre of the base.

AD = 10cm and CE = 18cm

(a) Work out the length of AC
(b) Calculate angle CAE
(c) Work out the height of the pyramid.
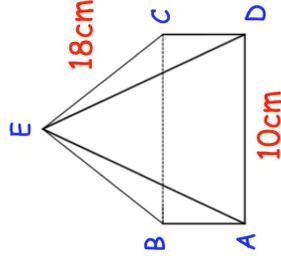
Question 8: Shown is a cuboid.

FG = 5.5cm DH = 6.2cm Angle FHG = 47°

Calculate the angle between DF and the plane EFGH.
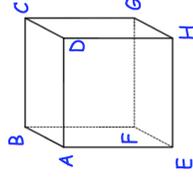
Question 9: Shown below is a rectangular-based pyramid.
The apex E is directly over the centre of the base.

AD = 8cm CD = 6cm CE = 11cm

(a) Calculate the height of the pyramid.
(b) Calculate the angle between face ABE and the base ABCD.
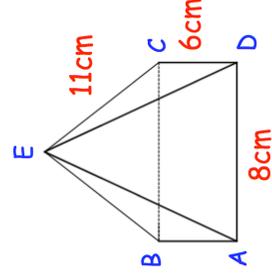
Apply

Question 1: A tree is located in the corner of a rectangular field.
The field is 30 metres long and 24 metres wide.
The tree is 15 metres tall.

Calculate angle CAE.

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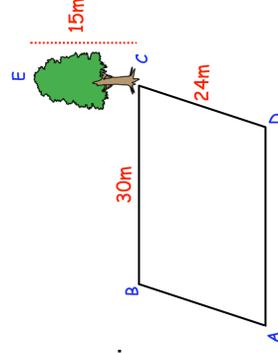
Question 2: Here is a square-based pyramid
The apex E is directly over the centre of the base.

Calculate the volume of the pyramid.

E

B   C

60°

A   D
40cm
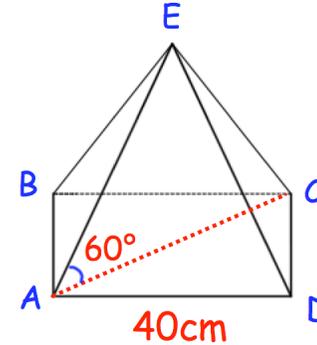
Question 3: The diagram shows a cuboid and a pyramid.
The apex of the pyramid, I, is directly above the centre, M, of ABCD.

AC = 7m   CD = 9m   DG = 3.5m
IM = 8m

(a) Calculate the angle between EI and the
    plane EFGH.

(b) Calculate the angle between AI and
    plane ABCD.

(c) Calculate the angle between the planes FGI
     and EFGH

(d) Calculate the angle between the planes EHI and ACEH

I

E   F

8m

A   B

H   G

7m   M

3.5m

C   9m   D
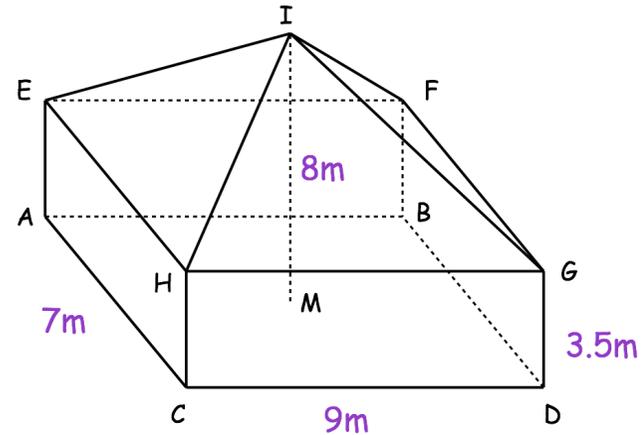
Fluency Practice

Find the angle $\overline{BAC}$

(a)

- B
- 3 cm
- C
- 4 cm
- y
- x
- A
- 10 cm

(b)

- B
- 2.5 cm
- C
- 5 cm
- x
- A
- 12.5 cm
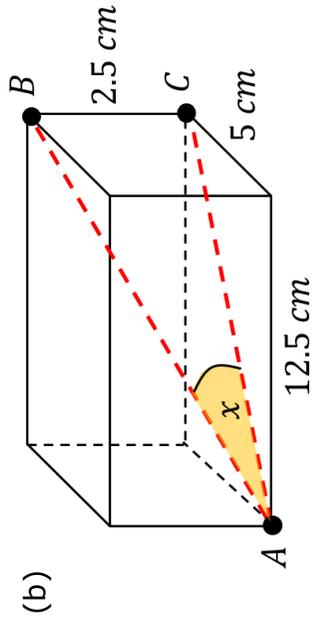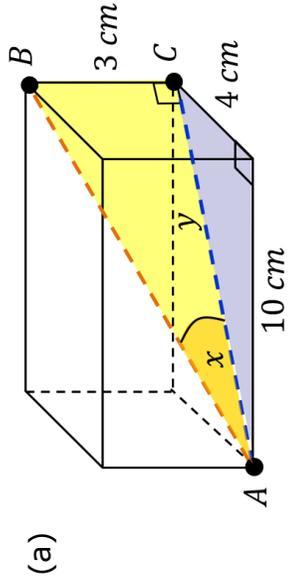
Find the angle between the line EA and the plane ABCD

(a)

- E
- 7 cm
- C
- 7 cm
- B
- D
- 12 cm
- A

(b)

- A
- 1.5 m
- B
- D
- 5 m
- E
- 1 m
- C

(c)

- C
- 8 cm
- B
- E
- 13 cm
- D
- A
- 8 cm
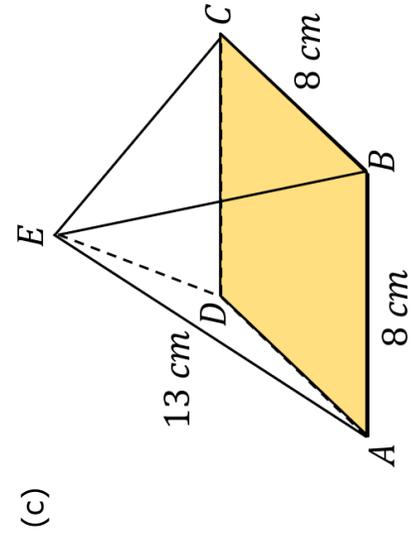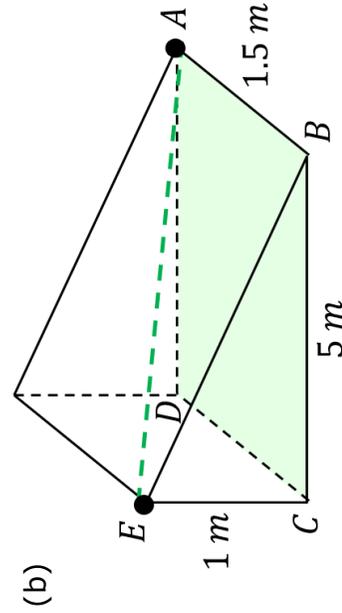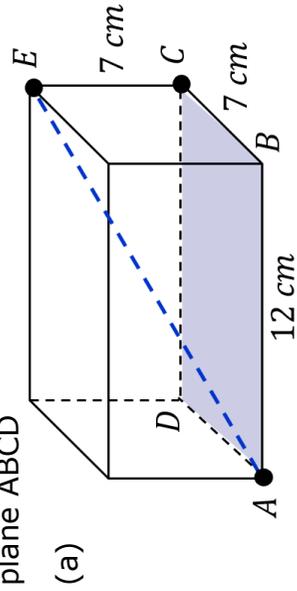
Purposeful Practice

3D Pythagoras and Trigonometry

(a)

The volume of the cuboid ABCDEFGH is $360\ cm^3$. Find:
(a) the length of AD
(b) the length of AG
(c) the angle between AG and the plane EFGH
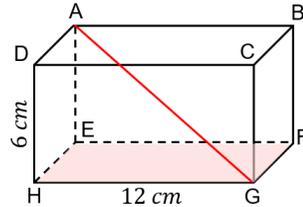
(b)

The point J splits the edge EF in the ratio $2 : 1$. Find:
(a) the length HJ
(b) the length DJ
(c) the angle HDJ
(d) The angle between DJ and the plane EFGH
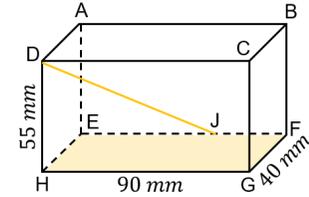
(c)

ABCDEF is a triangular prism with a surface area of $660\ cm^2$.
M is the midpoint of DE. Find:
(a) the length of MF
(b) the length of BE
(c) the length of CM
(d) the angle between CM and the plane ABED

(d)
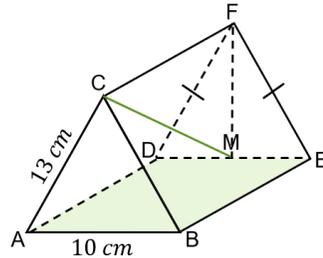
The volume of the square-based pyramid ABCDE is $180\ cm^3$.
M is the centre of the base and is vertically below E. Find:
(a) the height of the pyramid ME
(b) the length of AE
(c) the angle EAM
(d) the angle between the planes BCE and ABCD
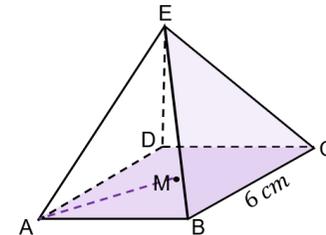
Purposeful Practice

Q1 ABCDEFGH is a cuboid. AB = 12cm and BC = CG = 5cm.
Calculate the angle between:
[a] the line BH and the plane DCGH.
[b] the line BH and the plane ADHE.
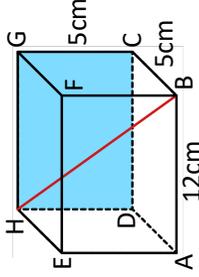
Q2 ABCDEFGH is a cuboid.
AB = 10cm, BC = 5cm and CG = 3cm.
[a] Calculate ∠FEG to 1dp.
[b] Calculate ∠DAH to 1dp.
[c] Calculate ∠DCH to 1dp.
[d] Calculate ∠CAG to 1dp.
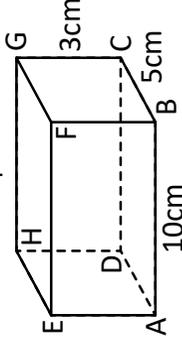
Q3 Shown is a triangular prism.
Triangle ABC is a right angle triangle.
[a] Calculate ∠ACB to 1dp.
[b] Calculate ∠EBF to 1dp.
[c] Calculate ∠AEC to 1dp.

Q4 Ruth says that because a cube has all equal sides, ∠CAG = 45°. Show that Ruth is incorrect.

Q5 Shown is a square based pyramid, ABCDE. F is the midpoint of CD. M is the point directly below the vertex E.
Calculate the following to the nearest integer:
[a] ∠MAE
[b] ∠MEC
[c] ∠AEC
[d] ∠BFC
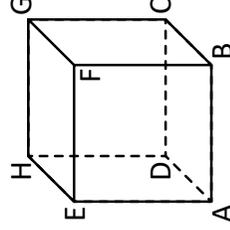
Q6 Shown is a square based pyramid, ABCDE. F is the midpoint of CD.
Calculate the length of:
[a] ∠BED
[b] ∠FBC
[c] ∠BFD

Q7 A lamppost is located in the corner of a rectangular car park. The car park is 40m long and 25m wide.
The lamppost is 3 metres tall.
Calculate ∠CAE to 1dp.
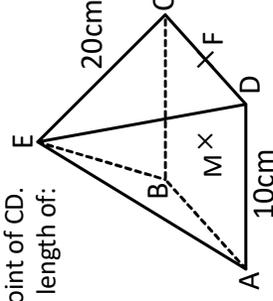
Q8 Shown is a cone with volume 30πcm³.
The diameter of the base is 6cm.
Calculate the angle, θ, made at the apex to 1dp.

Page 130
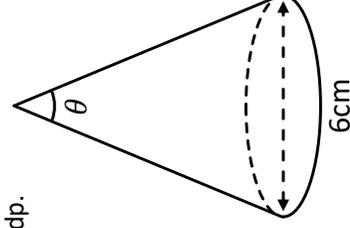
Purposeful Practice

Patrons are given questions on four types of shapes:

1. A CUBOID.

2. A PYRAMID. (A rectangular base such that the vertex lies above the mid-point of the rectangle.)

3. A YANGMA. (A rectangular based pyramid such that the vertex lies above one of the rectangle's corners.)

4. A PRISM. (A prism with triangular cross-section.)
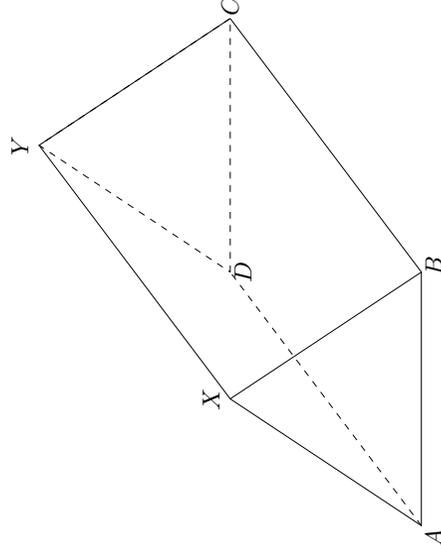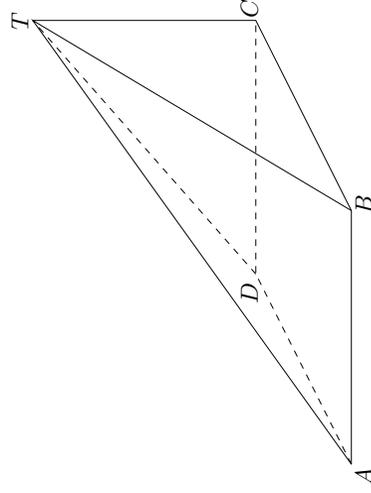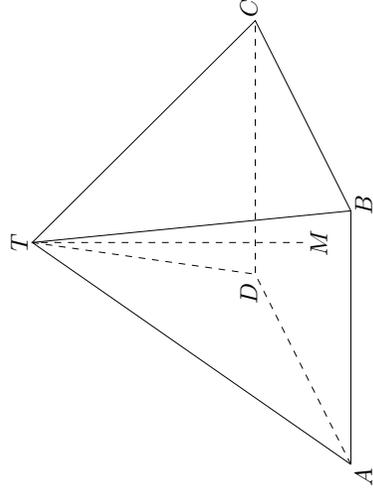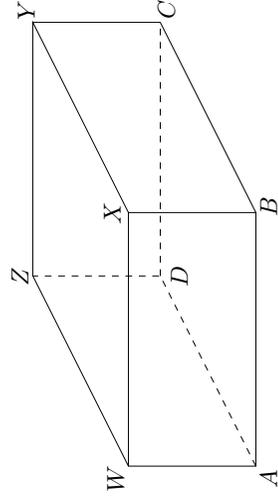
Please give all non-exact answers to three significant figures.

Questions

1. CUBOID. $AB = 7$ $BC = 6$ $CY = 4$.

   (a) Find the length AY.

   (b) Find the angle the diagonal AY makes with the plane $ABCD$.

   (c) Find the angle the diagonal AY makes with the plane $ADZW$.

2. PYRAMID. $AB = 8$ $BC = 6$ $CT = 9$.

   (a) Find the length AC.

   (b) Find the length MT.

   (c) Find the angle CT makes with the plane $ABCD$.

   (d) Find the angle $A\hat{T}C$.

   (e) Find the angle between the planes $ABCD$ and ADT.

3. YANGMA. $AB = 11$ $BC = 11$ $CT = 5$.

    (a) Find the length BT.

    (b) Find the length AT.

    (c) Find the angle AT makes with the plane $ABCD$.

    (d) Find the angle between the planes ABT and $ABCD$.

4. PRISM. $AB = BX = XA = 8$ $BC = 12$.

    (a) Find the length XC.

    (b) Find the angle $C\hat{X}A$.

    (c) Find the angle that XC makes with the plane $ABCD$.

5. CUBOID. $AB = 9$ $BC = 12$ $AYC = 65°$.

    (a) Find the length CY.

    (b) Find the length AY.

    (c) Find the angle the diagonal WC makes with the plane $BCYX$.

6. PYRAMID. $AB = 10$ $CAB = 50°$ $TAM = 40°$.

    (a) Find the length BC.

    (b) Find the length MT.

    (c) Find the length DT.

    (d) Find the angle between the planes $ABCD$ and ADT.

7. YANGMA. $AB = 12$ $BC = 15$ $TAC = 28°$.

    (a) Find the length TC.

    (b) Find the length AT.

    (c) Find the angle BT makes with the plane $ABCD$.

    (d) Find the angle between the planes ADT and $ABCD$.

8. PRISM. $AB = 6$ $BX = 7$ $ABX = 50°$ $BCX = 22°$.

    (a) Find the length BC.

    (b) Find the length XC.

    (c) Find the angle $A\hat{Y}B$.

    (d) Find the angle AY makes with the plane $ABCD$.

Fluency Practice

① *ABCDEFG* is a cuboid.

not to scale
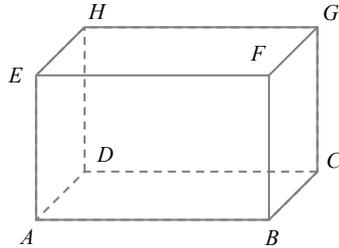
GH = 5 cm
AE = 3 cm

Calculate angle *ABE*.

② *ABCDEFG* is a cuboid.

FB = 2 cm
EG = 9 cm
DC = 7 cm

Calculate angle *AEB*.
Calculate angle *EGH*.

Give answers correct to 2 decimal places.

③ *ABCDEFG* is a cuboid.
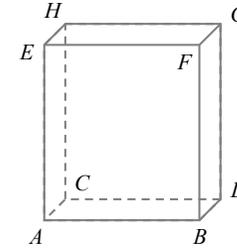
HE = 1 cm
AB = 4 cm
Angle *ABE* = 50°

Calculate the length *AE*.
Calculate angle *GBD*.

④ *ABCDEFG* is a cuboid.
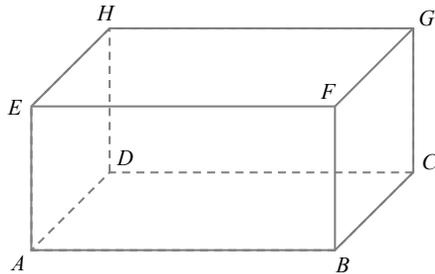
GC = 4 cm
Angle *ABD* = 22°
CB = 5 cm

Calculate the length *DB*.
Calculate angle *DBH*.

⑤ *ABCDEFG* is a cuboid.
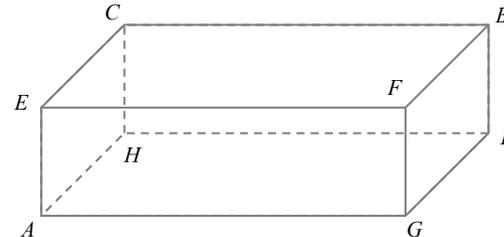
CF = 21 cm
GD = 6 cm
Angle *HGC* = 12°

Calculate angle *GCD*.

Purposeful Practice

Angles of Elevation & Depression
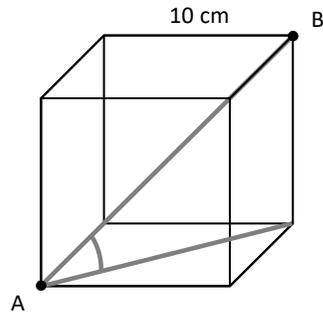
10 cm
B
A
Find the angle of elevation of
the diagonal (A to B) within this cube.
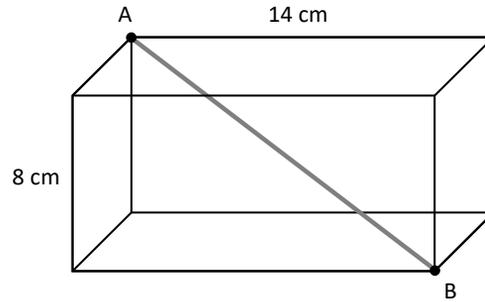
A   14 cm
8 cm
B
This is a square-based prism.
Find the angle of depression from A to B.
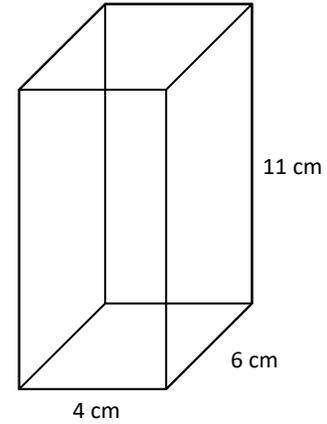
11 cm
6 cm
4 cm
Find the angle of elevation of
the diagonal within this cuboid.
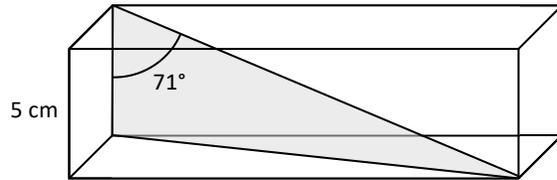
71°
5 cm
Find the volume of
this square-based prism.
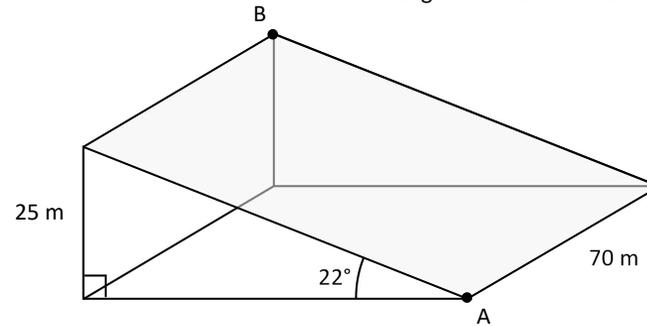
B
25 m
22°
70 m
A
This triangular prism represents a hill with a 22° incline.
John walks diagonally up the hill from A to B.
At what angle of elevation does he walk?
Explain why this gradient is smaller than that of the hill.

Purposeful Practice

3D Trigonometry

1) Ahmed is 25 metres from a tree.
The angle of elevation to the top of the tree (C) is 22°
Billy is 35 metres from the tree.
What is the angle of elevation to the top of the tree from where he is standing?

C

22°
25 m

35 m

A

B

2) ABCD is a football goal 5 metres wide.
The ball (G) is directly in front of the left post.
Calculate the height of the goal

A   5 m   B

C   D

50°

65°

G
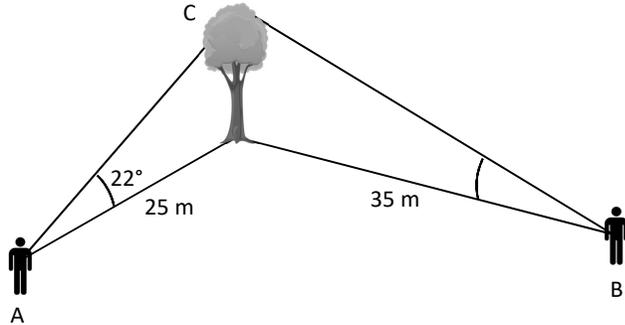
3) A UFO is flying at the same height for 4 km.
At A, it is 2 km away from C
How far away is the UFO from C when it is at B?

B   4 Km

A

2 Km

16°

28°

C
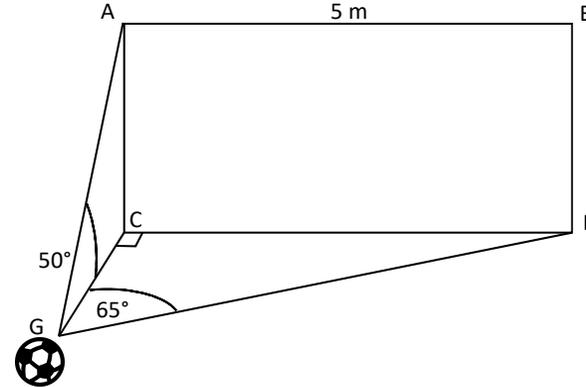
4) ABCDEF is an isosceles-triangle-based prism.
X is the midpoint of AC. Y is the midpoint of DF.
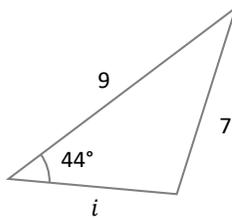Find the angle of depression from B to Y

E

B

8 cm   56°

D

Y   F

24 cm

A   X   C
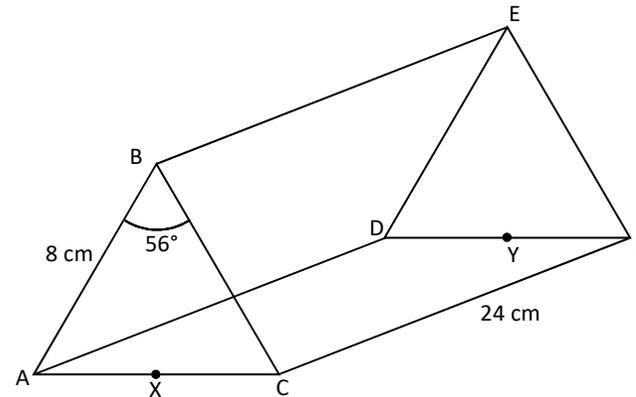
Fluency Practice

1. *ABCDEFGH* is a cuboid.
   Work out the value of $C\hat{A}G$.

   5cm
   8cm
   4cm
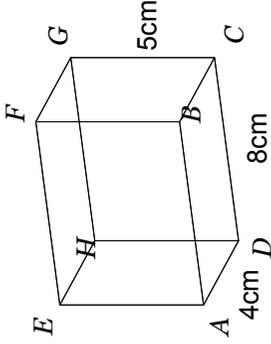
2. *ABCDEFGH* is a cube.
   Thomas says: "Angle ECA is 45°."
   Annabel says: "No, it is less than that."
   Work out who is correct.
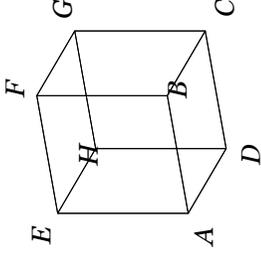
3. Point *E* is vertically above point *C*.
   CE = 3cm and *AE* = 8cm.
   Work out the angle between the line
   AE and the horizontal plane *ABCD*.

   8cm
   3cm
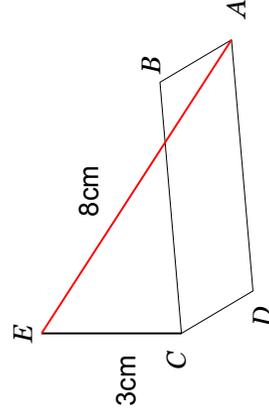
4. *ABCDEFGH* is a cuboid.
   Work out the angle between the
   line *DF* and the plane *DCGH*.

   4cm
   2cm   6cm
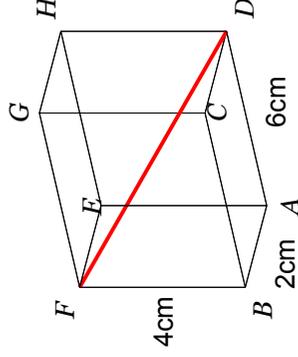
5. *ABCDE* is a square-based pyramid.
   The apex of the pyramid is directly
   above the centre of the square.
   EC = 10cm. Work out the angle $A\hat{E}C$.

   10cm
   5cm
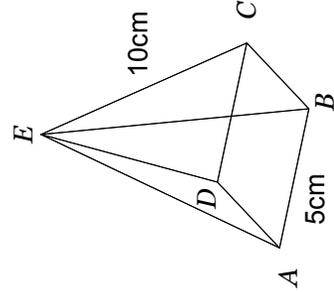
6. A triangular prism is shown.
   Work out $D\hat{E}A$.

   5cm
   6cm   2cm
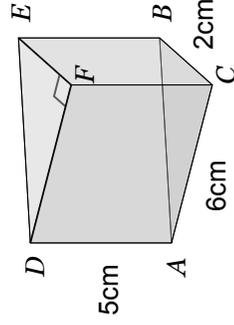
Fluency Practice

7. *ABCDEFGH* is a cuboid.
   Work out the angle between the line *BH* and the plane *ABCD*.
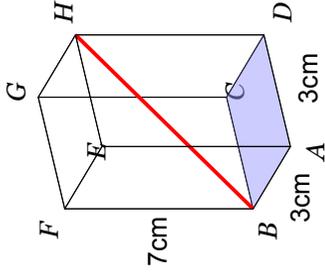
8. *ABCDEFGH* is a cuboid.
   Work out the angle between the line *FD* and the plane *BCGF*.
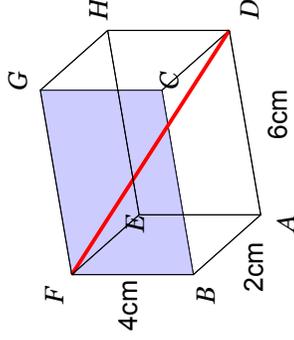
9. *ABCDEFGH* is a cuboid.
   Work out the angle between the line *FD* and the plane *ADEH*.
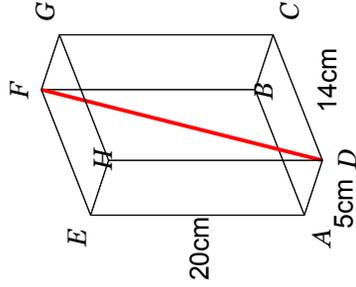
10. *ABCDE* is a square based pyramid.
    O is the centre of the square.
    E is directly above *O*. *M* is the midpoint of *AB*, *EO* = 12cm and *EC* = 15cm.
    Work out the angle *EMO*.
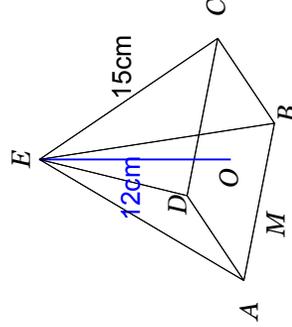
11. A rectangular-based pyramid has a perpendicular height 8cm. The apex of the pyramid is directly above the centre of the rectangle. Work out *DB̂E*.
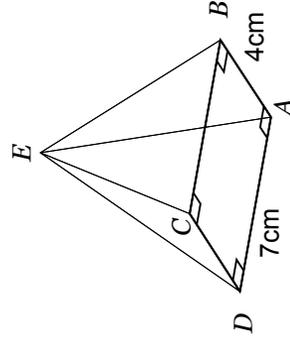
12. In the triapezoidal prism, *AD* = 6cm, *AB* = 9cm, *CD* = 2cm and *DH* = 4cm.
    Work out the angle between the line that passes through the points *B* and *G* and the infinite plane that includes *ADHE*.
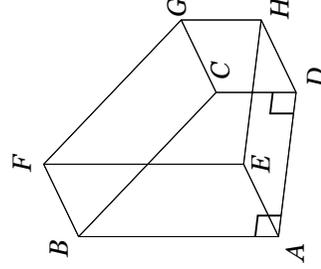
Fluency Practice

1			$AE = 3m$ $ED = 6m$ $DC = 4m$ Find the length AC.
2			$AC = 10m$ $ED = 7m$ $DC = 4m$ Find the length AE.
3			$AC = 12m$ $ED = 7m$ $DC = 4m$ Find the angle $\angle ACE$.
4			Angle $\angle ACE = 40°$ $ED = 7m$ $DC = 5m$ Find the length AC.
5			Angle $\angle DCE = 40°$ $ED = 7m$ $AE = 5m$ Find the length AC.

Fluency Practice
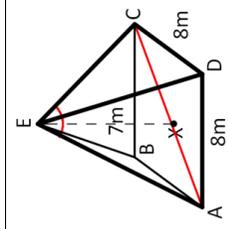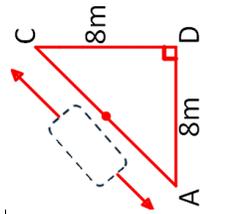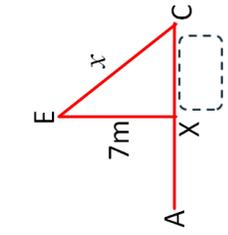
6

$AD = 8m$
$DC = 8m$
E is $7m$ above the midpoint of AC.

Find the length EC.
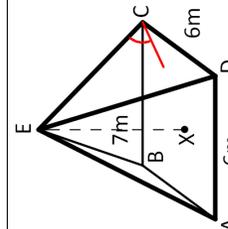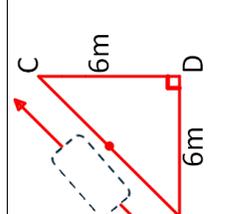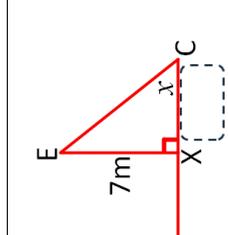
7

$AD = 6m$
$DC = 6m$
E is $7m$ above the midpoint of AC.

Find the angle $\angle ACX$.
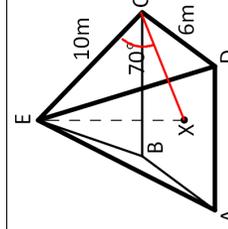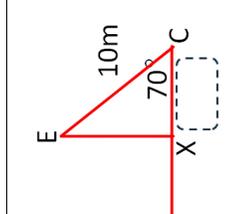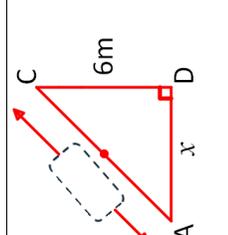
8

E is above the midpoint of AC.
Angle $\angle ECX = 70°$
$DC = 6m$
$EC = 10m$

Find the length AD.
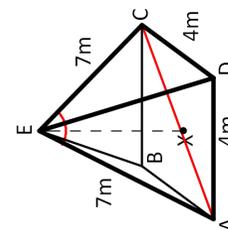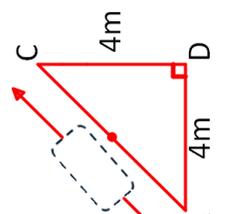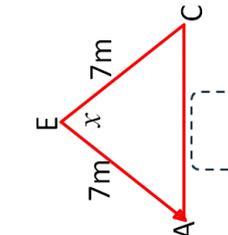
9

E is above the midpoint of AC.
$AD = DC = 4m$
$EC = AE = 7m$

Find the angle $\angle AEC$.

HINT: COSINE RULE

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

1	2	3	4	5	6	7	8	9
7.81	5.92	47.8°	11.2m	12.0m	9m	58.8°	3.28	47.7°

Purposeful Practice

the angle that the space diagonal makes
with the base is 30°

a
a
30°
b
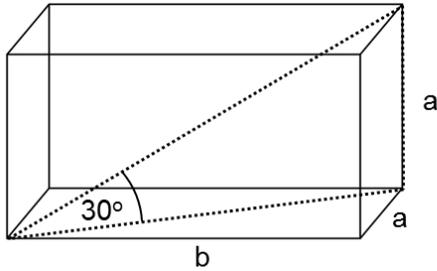
what is the relationship between 'a' and 'b'?

a cuboid with a square cross section is
twice as long as it is wide

a   a
a
A
a
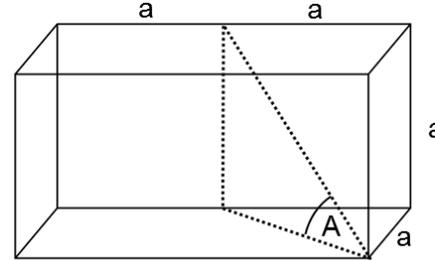
what is angle A (to 2 dp)?

a cuboid with a square cross section is
twice as long as it is wide

2a
B
a
a
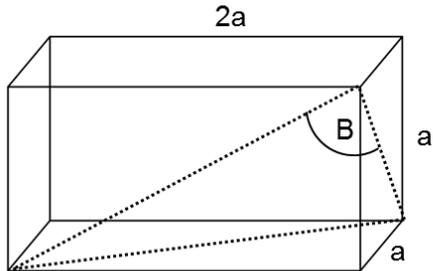
what is angle B (to 2 dp)?

the cuboid has a space diagonal of length 21
the width of the cuboid is 4

21
b
4
c
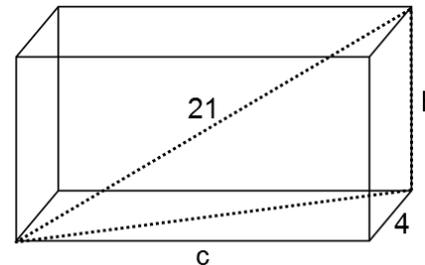
what are b and c if:
(i) b + c = 25   (ii) b + c = 27   (iii) b + c = 29

Purposeful Practice

(1)
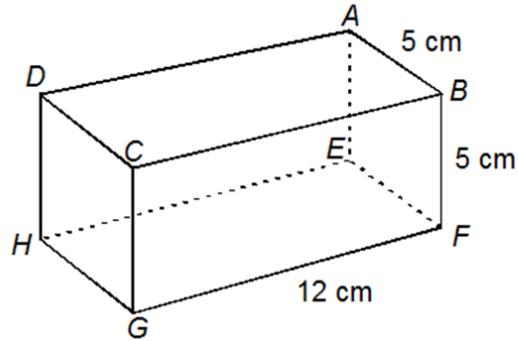
calculate angle *DFH* for a cuboid
with sides of 5 cm, 5 cm and 12 cm

(2)
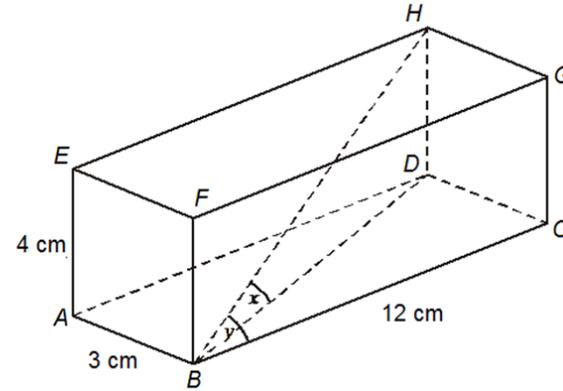
what are the sizes of the angles marked x (angle
HBD) and y (angle *HBC*) for a cuboid
with sides of 3 cm, 4 cm and 12 cm ?

(3)
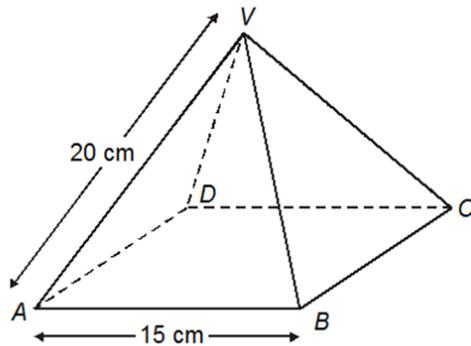
for the right, square-based pyramid shown,
calculate the angle between
the edge *VA* and the base, *ABCD*

(4)
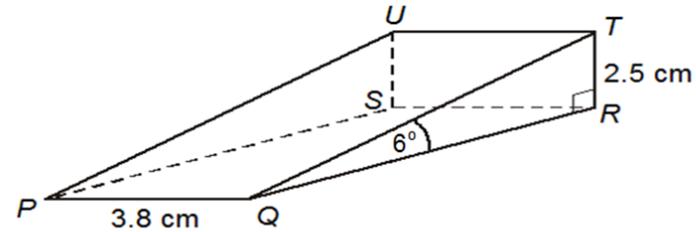
for the door wedge shape (triangular prism)
calculate the length of the diagonal *PT*

Purposeful Practice

a square base pyramid has a height of 6 and a base length of 16
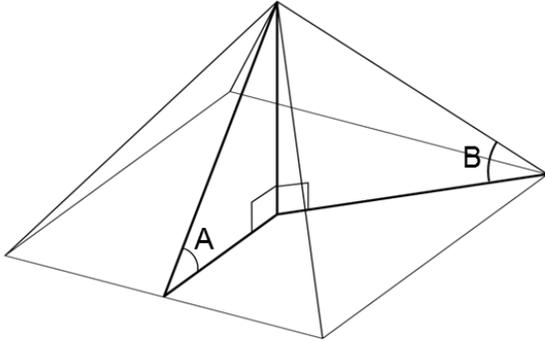
show that A − B is roughly 9°

a special square base pyramid with four equilateral triangles

show that the obtuse angle between two of the triangular faces is 109.57°

square base length = 12, height = h
apothem = a, lateral edge = L

create a formula for L in terms of h
find L if (i) h = 3 (ii) h = 7 (iii) h = 16

a special square base pyramid with four equilateral triangles
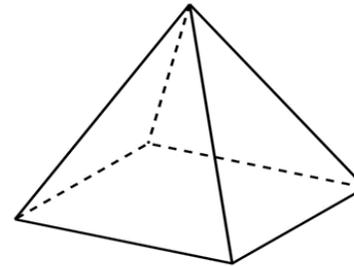
create formulas for (i) the total surface area
(ii) the volume

Fluency Practice

1 A cube has sides 8cm. Find:
a) The length AB. ?
b) The angle between AB and the horizontal plane. ?
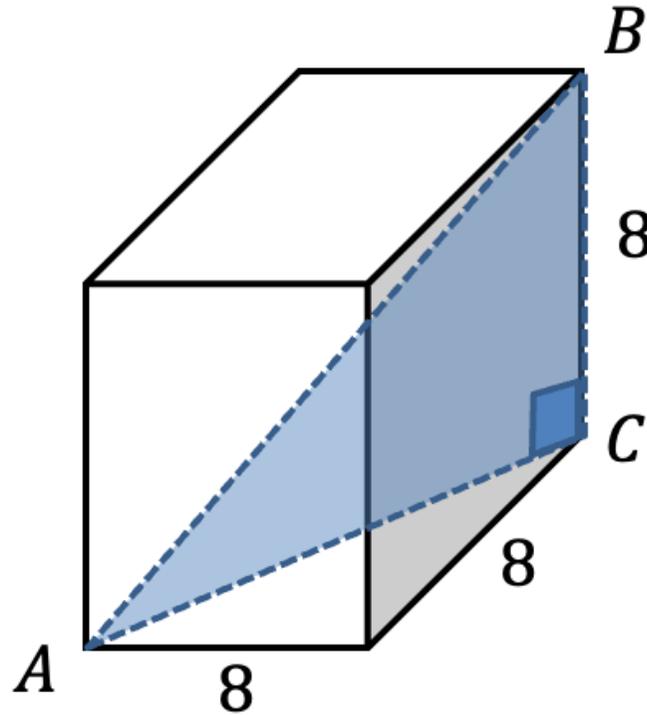
2 For the following cuboid, find:
a) The length AB. ?
b) The angle between AB and the horizontal plane. ?

3

A

16

$\sqrt{87}$

E

13

24

B

D   10   C
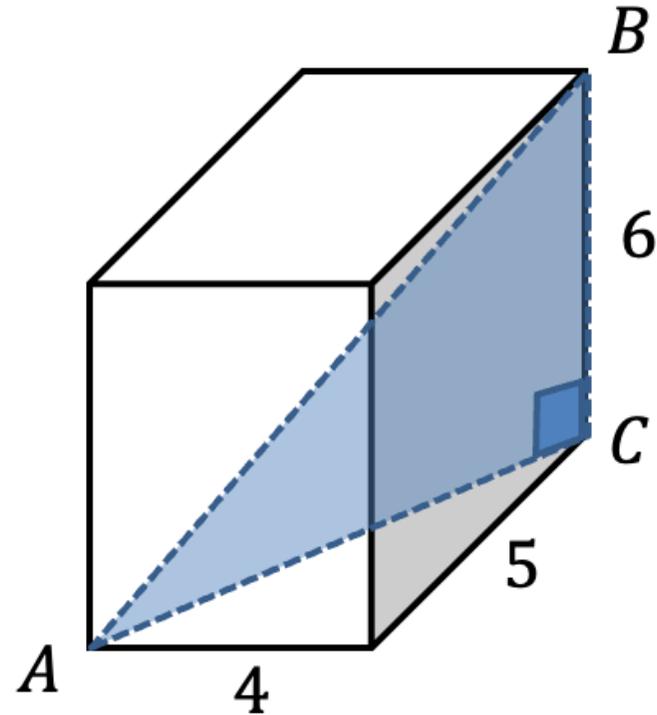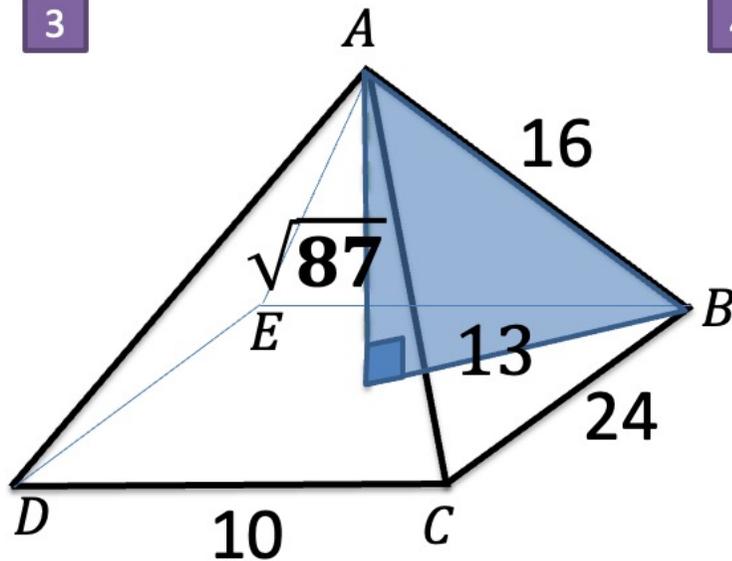
a) Determine the height of the pyramid.

?

b) Determine the angle between the line AB and the base $BCDE$ of the pyramid.

?

4 A radio tower 150m tall has two support cables running 300m due East and some distance due South, anchored at A and B. The angle of inclination to the horizontal of the latter cable is 50° as indicated.

150m

50°

300m   A

B
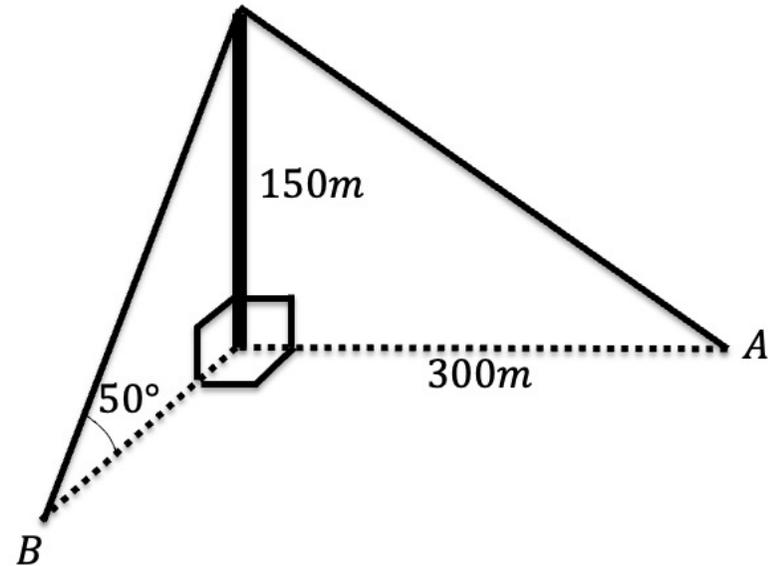
a) Find the angle between the cable attached to A and the horizontal plane.   ?

b) Find the distance between A and B.   ?

5 *[AQA FM Mock Set 1 Paper 2]* The diagram shows a vertical mast, AB, 12 metres high. Points B, C, D are on a horizontal plane.

Point C is due West of B.

a) Calculate CD

b) Calculate the bearing of D from C to the nearest degree.

a)

?

b)

?
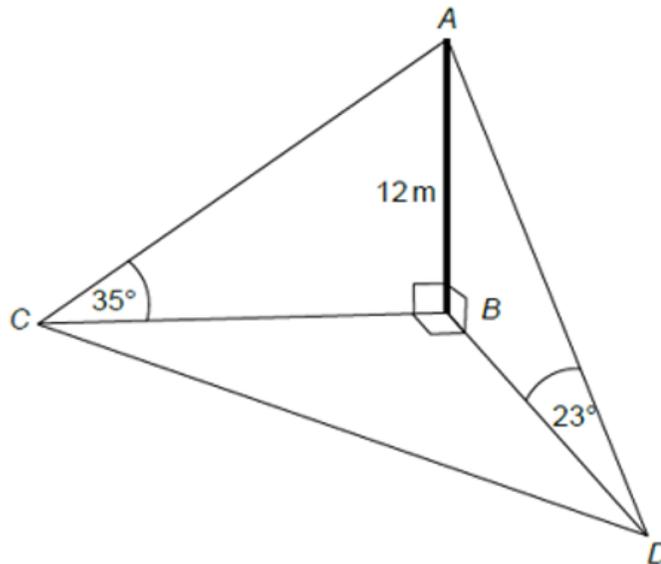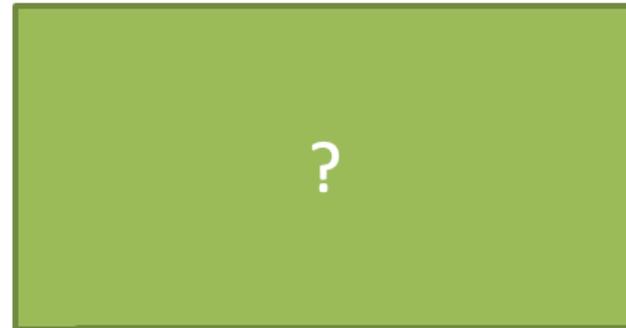
Fluency Practice

6
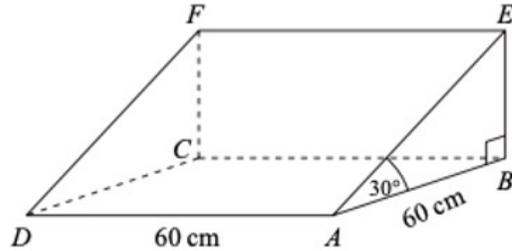
[Edexcel GCSE June 2004 6H Q18] The diagram represents a prism.
$AEFD$ is a rectangle. $ABCD$ is a square.
EB and FC are perpendicular to plane $ABCD$.
$AB = 60$ cm. $AD = 60$ cm. Angle $ABE = 90°$.
Angle $BAE = 30°$.
Calculate the size of the angle that the line DE makes with the plane $ABCD$.
Give your answer correct to 1 decimal place.

?

7
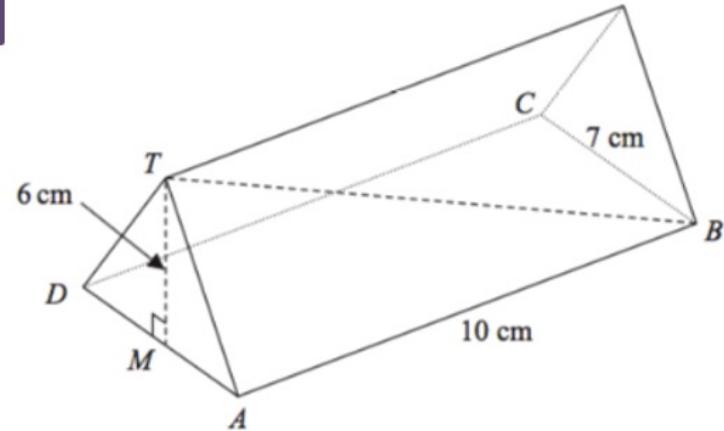
[Edexcel IGCSE May2013-4H Q22]
The diagram shows a triangular prism with a horizontal rectangular base $ABCD$.
$AB = 10$ cm. $BC = 7$ cm.
M is the midpoint of AD.
The vertex T is vertically above M.
$MT = 6$ cm. Calculate the size of the angle between TB and the base $ABCD$.
Give your answer correct to 1 decimal place.

?

8

I

J

G

H

F

E

12m

8m

D

C

7m

A 24m B
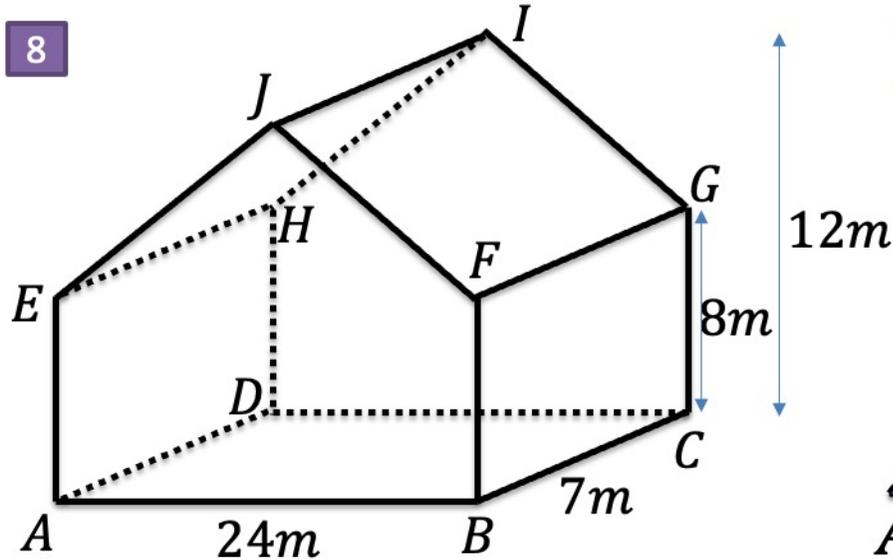
Frost Manor is as pictured, with $EFGH$ horizontally level.

a) Find the angle between the line AG and the plane $ABCD$. ?

b) Find the angle between the planes $FGIJ$ and $EFGH$. ?

9

E

20cm

D

C

M

X

24cm

A 24cm B
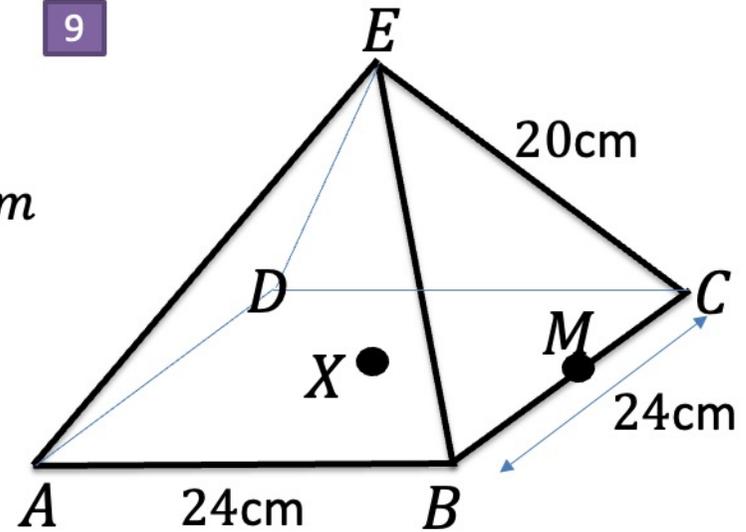
A school buys a set of new 'extra comfort' chairs with its seats pyramid in shape. X is at the centre of the base of the pyramid, and M is the midpoint of BC.

a) By considering the triangle EBC, find the length EM. ?

b) Hence determine the angle between the triangle EBC and the plane $ABCD$. ?

Fluency Practice

10 [AQA FM June 2013 Paper 2] $ABCDEFGH$ is a cuboid. M is the midpoint of HG. N is the midpoint of DC.

a) Show that $BN = 7.5$m
b) Work out the angle between the line MB and the plane $ABCD$.

> ?

c) Work out the **obtuse** angle between the planes MNB and $CDHG$.

> ?
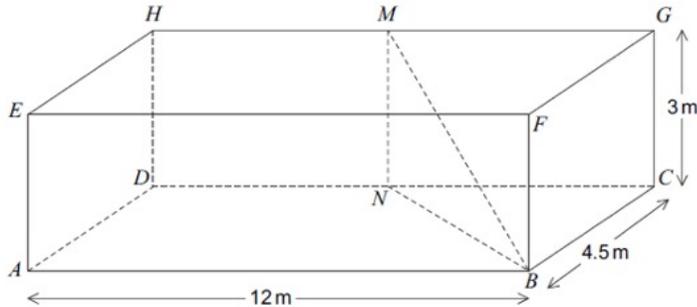
11 [AQA FM Set 3 Paper 2] The diagram shows part of a skate ramp, modelled as a triangular prism. $ABCD$ represents horizontal ground. The vertical rise of the ramp, CF, is 7 feet. The distance $BC = 24$ feet.
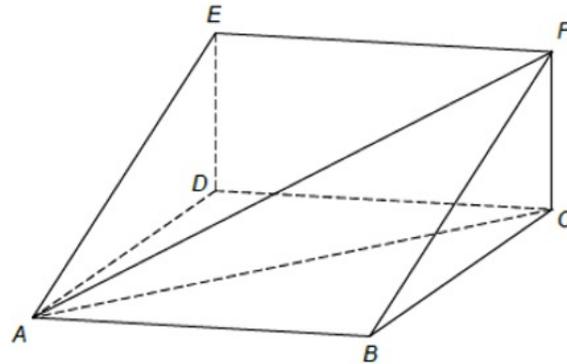
You are given that $gradient = \dfrac{vertical\ rise}{horizontal\ distance}$

a) The gradient of BF is twice the gradient of AF. Write down the distance AC. > ?

b) Greg skates down the ramp along FB. How much further would he travel if he had skated along FA.

> ?

Page 159

12
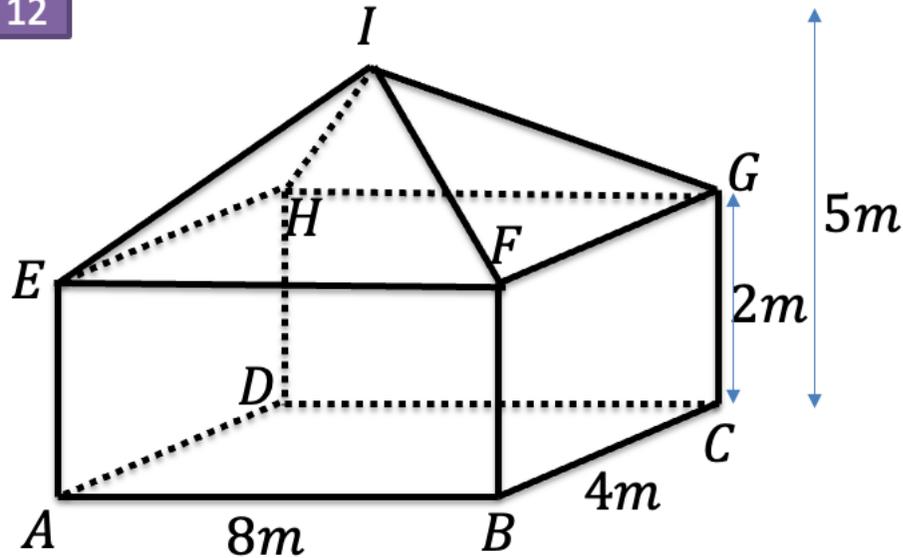
a) Determine the angle between the line AI and the plane $ABCD$.

> ?

b) Determine the angle between the planes FHI and $EFGH$.

> ?

13
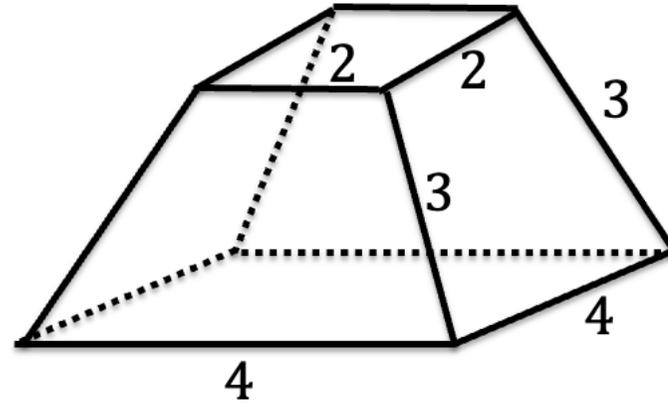
A 'truncated pyramid' is formed by slicing off the top of a square-based pyramid, as shown. The top and bottom are two squares of sides 2 and 4 respectively and the slope height 3. Find the angle between the sloped faces with the bottom face.

> ?

14 *(Knowledge of sine/cosine rules required)*
[Edexcel GCSE Nov2007-6H Q25] The diagram shows a tetrahedron.
AD is perpendicular to both AB and AC.
AB = 10 cm.
AC = 8 cm.
AD = 5 cm.
Angle BAC = 90°.
Calculate the size of angle BDC.
Give your answer correct to 1 decimal place.
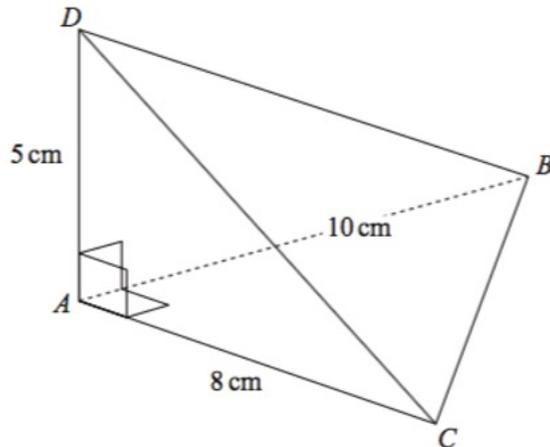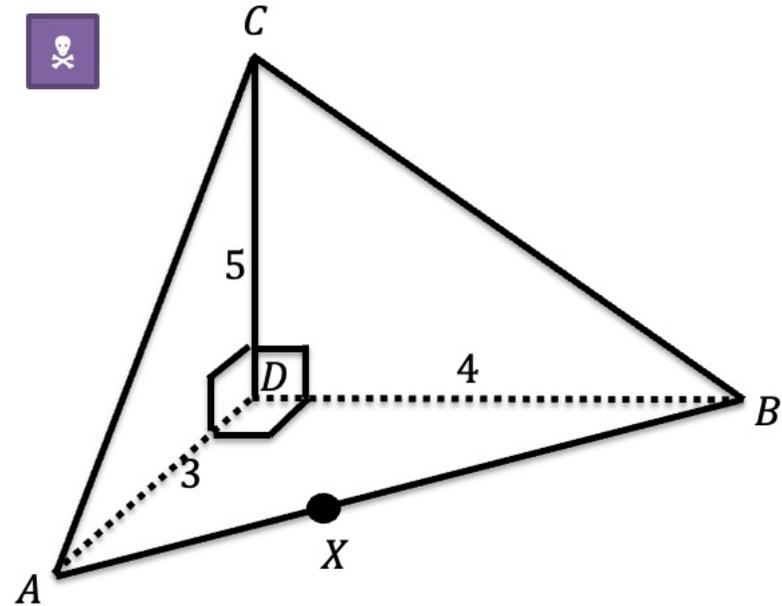
Determine the angle between the planes *ABC* and *ABD*.

?

3 Bearings

Fluency Practice

Measuring Bearings

For each question, find the bearings of B from A:

(a)
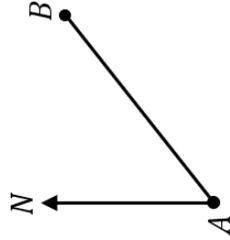
(b)
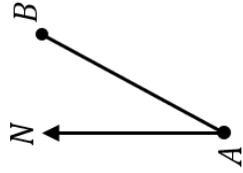
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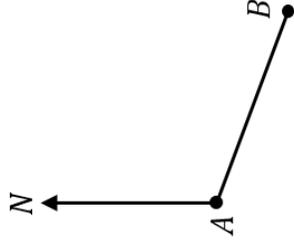
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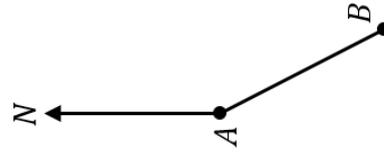
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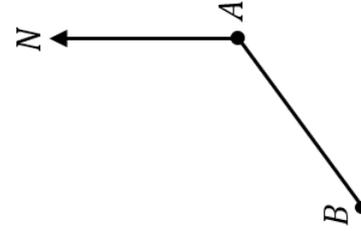
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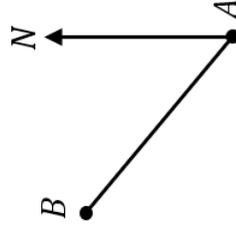
(g)
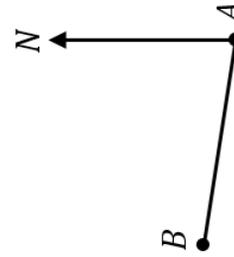
(h)
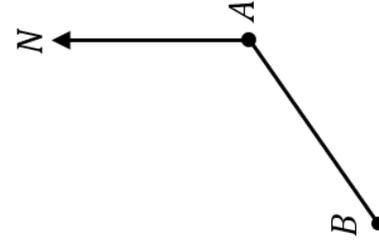
(i)
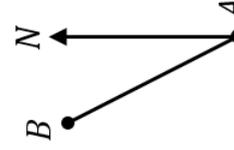
Fluency Practice

Drawing Bearings

For each question, draw a line on the bearing given:
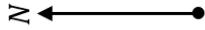
(a) 085°	**(b)** 040°	**(c)** 115°
(d) 155°	**(e)** 200°	**(f)** 310°
(g) 262°	**(h)** 148°	**(i)** 327°

Fluency Practice

Measure the bearing of B from A.

(a)

(b)

(c)

(d)

(e)

(f)
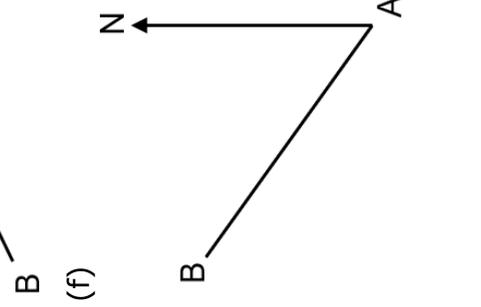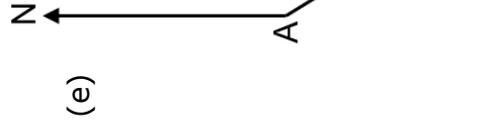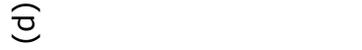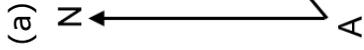
Draw the following bearings:

(a) A point B on a bearing of 135° from A
(b) A point B on a bearing of 080° from A
(c) A point B on a bearing of 162° from A

Draw the following bearings:

(a) A point B on a bearing of 300° from A
(b) A point B on a bearing of 205° from A
(c) A point B on a bearing of 330° from A

Edenfield

Bolton •

Rochdale •

• Prestwich

N ←——————•

Edenfield •
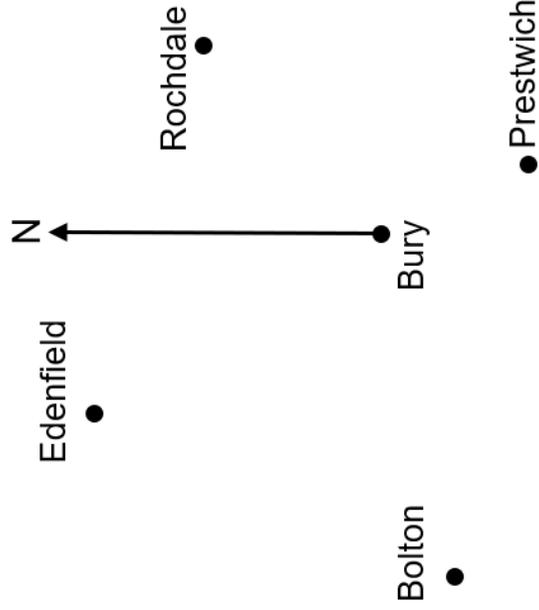
Bury

Bolton •

Using a scale of 1 cm to 2 km, find the distance and bearings of the following:

(a) Rochdale from Bury
(b) Prestwich from Bury
(c) Edenfield from Bury
(d) Bolton from Bury

Using the same scale diagram, find the bearing of:

(a) Bury from Edenfield
(b) Bury from Rochdale

A ship sails from a port on a bearing of 100° for 50 km. It then turns and sails on a bearing of 240° for 80 km. Using a scale of 1 cm to 10 km, draw a scale diagram. Find the distance of the ship from the port, and the bearing it must head on to return to the port.

(a) The bearing of B from A is 110°. Find the bearing of A from B.
(b) The bearing of B from A is $x°$. Find the bearing of A from B.

Fluency Practice

C

B

A

D

Find the bearing of…		
B from A:		
C from A:		
D from A:		
A from B:		
C from B:		
D from B:		
A from C:		
B from C:		
D from C:		
A from D:		
B from D:		
C from D:		

What do you notice about opposite pairs of bearings? (eg B from A and A from B)

Why do you think this might be?

Fluency Practice

Measuring Bearings
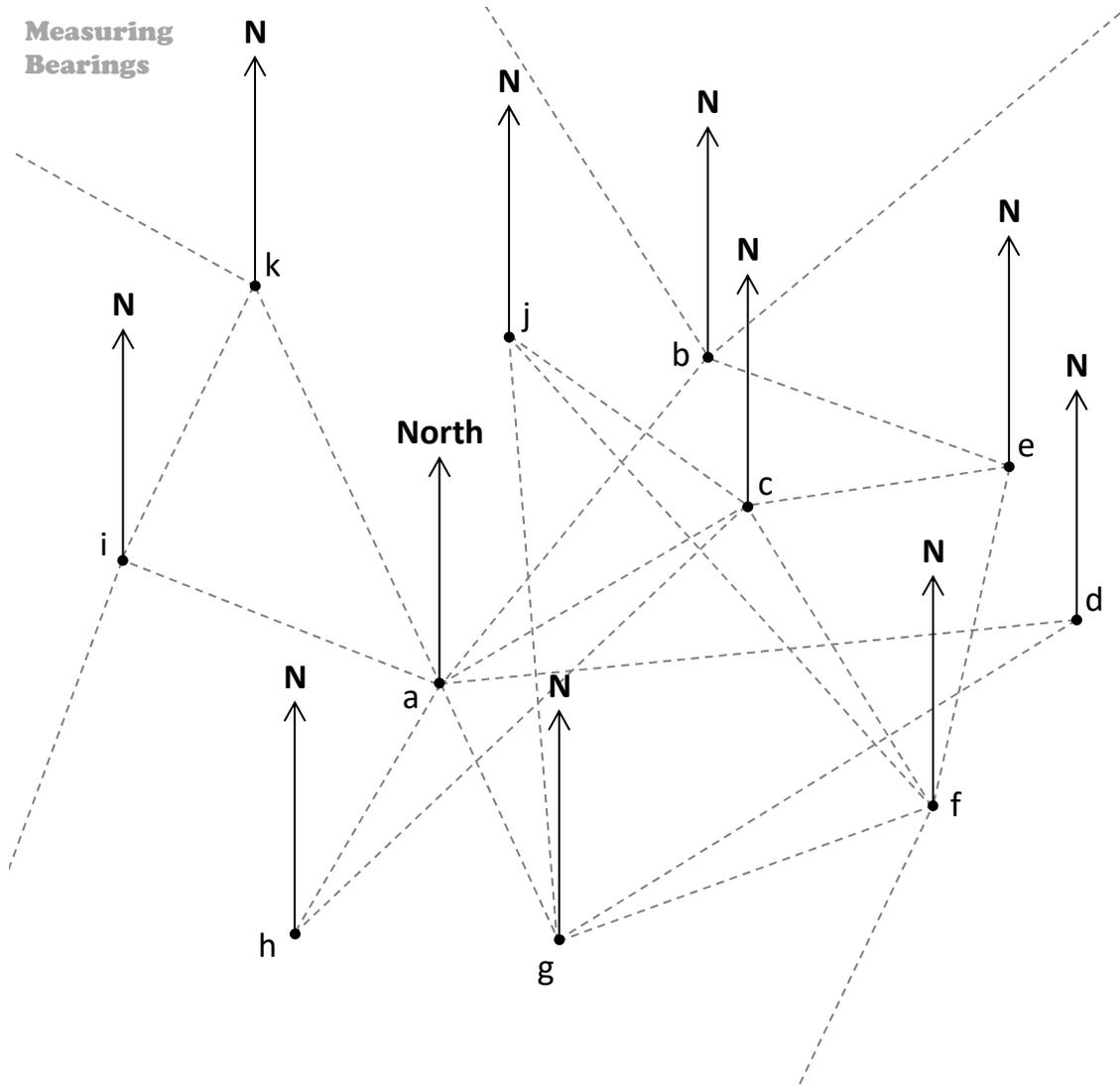
Complete this table:
(try to be accurate to within 5°)

	Bearing
a to **b**	
a to **c**	
a to **d**	
c to **f**	
a to **g**	
a to **h**	
f to **g**	
a to **i**	
c to **j**	
f to **j**	
k to **a**	
k from **a**	
i from **k**	
c from **e**	
e from **f**	
g from **d**	
h from **c**	

Point **X** is on a bearing of 050° from **b**, and a bearing of 330° from **e**. Mark the position of point **X**.

Point **Y** is on a bearing of 215° from **e**, and a bearing of 115° from **a**. Mark the position of point **Y**.

Point **Z** is on a bearing of 282° from **h**, and a bearing of 187° from **i**. Find the bearing of **Z** from point **j**.

Fluency Practice

A1 Find the bearing of *B* from *A*

North

North

45°

A

B

A2 Find the bearing of *B* from *A*

North

North

A

285°

B

A3 Find the bearing of *B* from *A*

North

North

105°

A

B

A4 Find the bearing of *B* from *A*

North

North

A

255°

B

B1 Find the bearing of *A* from *B*

North

North

B

230°

A

B2 Find the bearing of *A* from *B*

North

North

110°

B

A

B3 Find the bearing of *A* from *B*

North

North

A

B

280°

B4 Find the bearing of *A* from *B*

North

North

A

80°

B

C1 Find the bearing of *A* from *B*

North

North

105°

A

B

C2 Find the bearing of *B* from *A*

North

North

110°

B

A

C3 Find the bearing of *B* from *A*

North

North

B

230°

A

C4 Find the bearing of *A* from *B*

North

North

B

A

285°

A1 Find the bearing of *A* from *B*

North

North

B

A

280º

A2 Find the bearing of *A* from *B*

North

North

95º

B

A

A3 Find the bearing of *B* from *A*

North

125º

North

A

B

A4 Find the bearing of *B* from *A*

North

North

A

B

260º

B1 Find the bearing of *B* from *A*

North

North

105º

A

B

B2 Find the bearing of *A* from *B*

North

North

A

230º

B

B3 Find the bearing of *B* from *A*

North

North

95º

A

B

B4 Find the bearing of *A* from *B*

North

North

A

70º

B

C1 Find the bearing of *A* from *B*

North

North

B

A

260º

C2 Find the bearing of *B* from *A*

North

North

A

255º

B

C3 Find the bearing of *A* from *B*

North

North

110º

B

A

C4 Find the bearing of *B* from *A*

North

North

A

230º

B

Fluency Practice

A1 Find the bearing of *B* from *A*

North

North

105°

A

B

A2 Find the bearing of *B* from *A*

North

North

120°

B

A

A3 Find the bearing of *A* from *B*

North

North

B

245°

A

A4 Find the bearing of *A* from *B*

North

North

A

70°

B

B1 Find the bearing of *A* from *B*

North

North

A

240°

B

B2 Find the bearing of *B* from *A*

North

North

50°

B

A

B3 Find the bearing of *A* from *B*

North

North

105°

A

B

B4 Find the bearing of *B* from *A*

North

North

120°

B

A

C1 Find the bearing of *A* from *B*

North

North

A

B

280°

C2 Find the bearing of *B* from *A*

North

North

B

245°

A

C3 Find the bearing of *B* from *A*

North

North

A

240°

B

C4 Find the bearing of *B* from *A*

North

North

A

B

285°

Fluency Practice

A1 Find the bearing of *A* from *B*

North

57º

B

A

A2 Find the bearing of *A* from *B*

North

B

A

277º

A3 Find the bearing of *B* from *A*

North

B

67º

A

A4 Find the bearing of *B* from *A*

North

A

B

284º

B1 Find the bearing of *B* from *A*

North

A

B

293º

B2 Find the bearing of *B* from *A*

North

103º

A

B

B3 Find the bearing of *A* from *B*

North

B

238º

A

B4 Find the bearing of *A* from *B*

North

B

A

292º

C1 Find the bearing of *A* from *B*

North

87º

B

A

C2 Find the bearing of *B* from *A*

North

A

249º

B

C3 Find the bearing of *A* from *B*

North

76º

B

A

C4 Find the be bearing of *A* from *B*

North

118º

A

B

Fluency Practice

A1 The bearing of *S* from *L* is 285°. Find the bearing of *L* from *S*

North

S

C   *L*
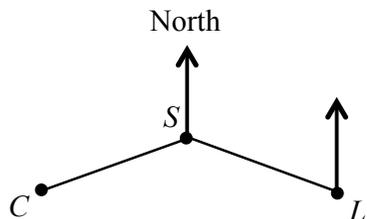
A2 The bearing of *S* from *C* is 76°. Find the bearing of *C* from *S*

North

S

C   *L*
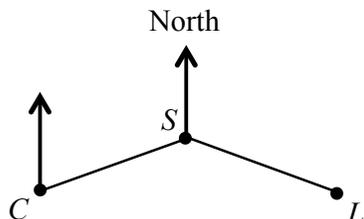
A3 The bearing of *L* from *S* is 118°. Find the bearing of *S* from *L*

North

S

C   *L*
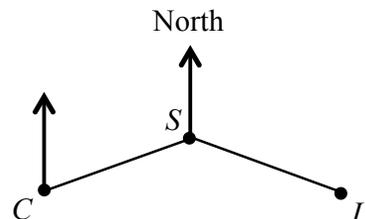
A4 The bearing of *C* from *S* is 252°. Find the bearing of *S* from *C*

North

S

C   *L*
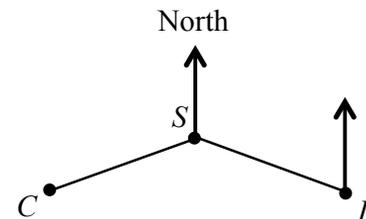
B1 The bearing of *S* from *C* is 68°. Find the bearing of *C* from *S*

North

S

C   *L*
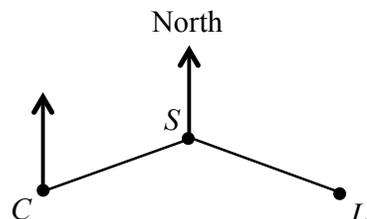
B2 The bearing of *L* from *S* is 106°. Find the bearing of *S* from *L*

North

S

C   *L*
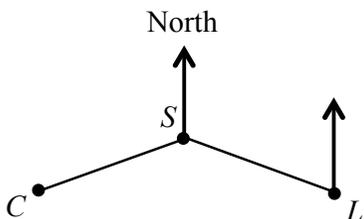
B3 The bearing of *S* from *L* is 289°. Find the bearing of *L* from *S*

North

S

C   *L*
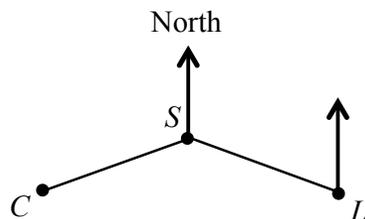
B4 The bearing of *C* from *S* is 258°. Find the bearing of *S* from *C*

North

S

C   *L*
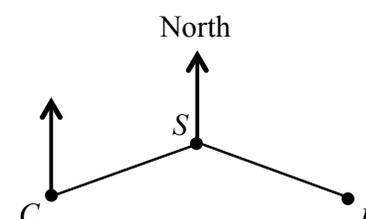
C1 The bearing of *S* from *L* is 294°. Find the bearing of *L* from *S*

North

S

C   *L*
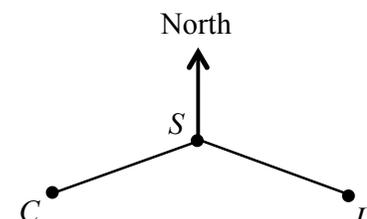
C2 The bearing of *C* from *S* is 248°. Find the bearing of *S* from *C*

North

S

C   *L*
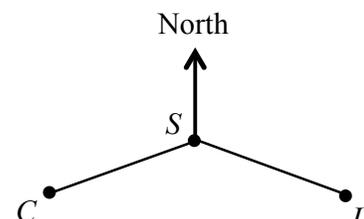
C3 The bearing of *S* from *L* is 289°. Find the bearing of *L* from *S*

S

C   *L*
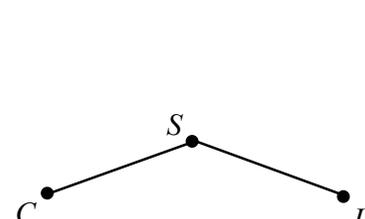
C4 The bearing of *C* from *S* is 241°. Find the bearing of *S* from *C*

S

C   *L*

North

57°   • *B*

N

103°

North

57°   • *B*

North

57°   • *B*
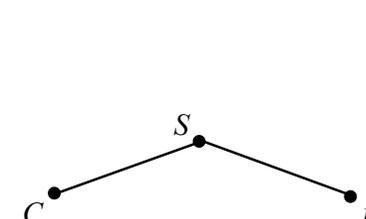
Fluency Practice

Calculating Bearings　　　　　　　　How can you label the diagrams to help?　　　　　　　not to scale
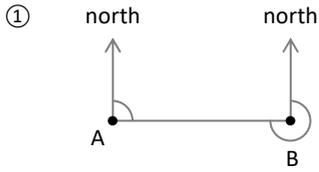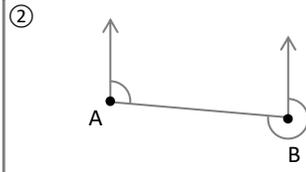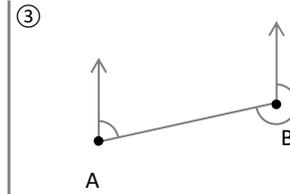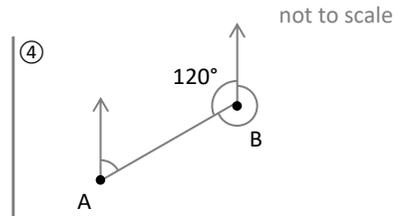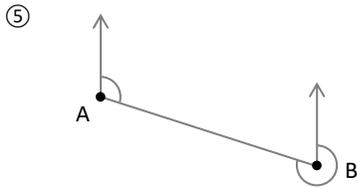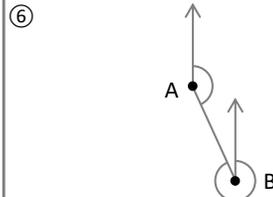
① north　　　north

A to B = 090°　B to A =

②

A to B = 100°　B to A =

③

A to B = 080°　B to A =

④ 120°

A to B =　　B to A =

⑤

A to B = 112°　B to A =

⑥

B to A = 340°　A to B = °

⑦

A to B = 180°　B to A =

⑧ 281°

A to B =　　B to A =

⑨

A to B = 089°　A from B =

⑩ 68°

A to B =　　A from B =

⑪

B from A = 163°　B to A =

⑫

A to B = 272°　A from B =

⑬ 140°

C to B = 290°　B to A =

⑭ 130°

B from C = 305°　A to B =

⑮ 120°
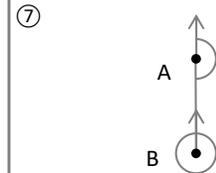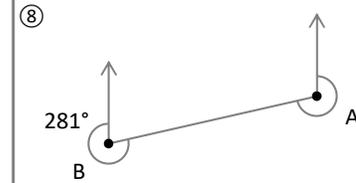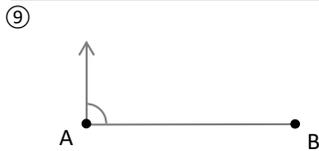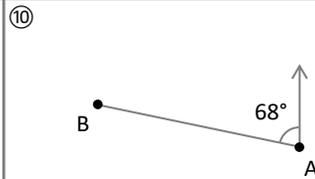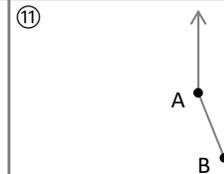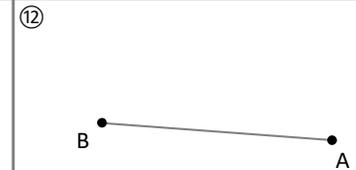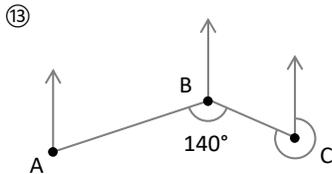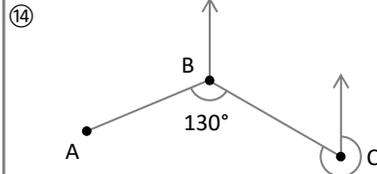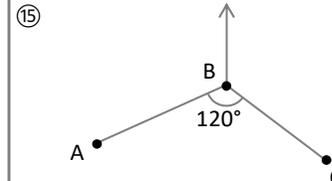
B from A = 065°
B to C =　　B from C =

⑯ a
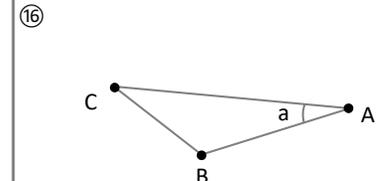
B to A = 072°
A from C = 094°　Angle a =

15 Answers	000°	160°	260°	292°	075°	079°	292°	060°
	280°	269°	250°	240°	259°	270°	112°	

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Fluency Practice

Calculating Bearings

Diagram NOT drawn to scale
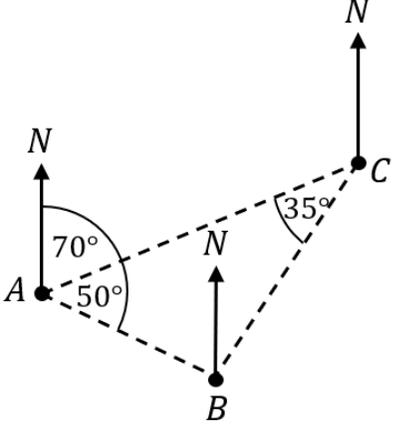
(a)	(b)	(c)
Find the bearing of C from A	Find the bearing of B from A	Find the bearing of A from C
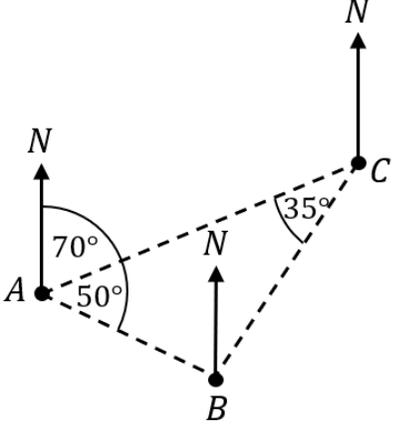
(d)	(e)	(f)
Find the bearing of C from B	Find the bearing of A from B	Find the bearing of B from C

Diagram NOT drawn to scale
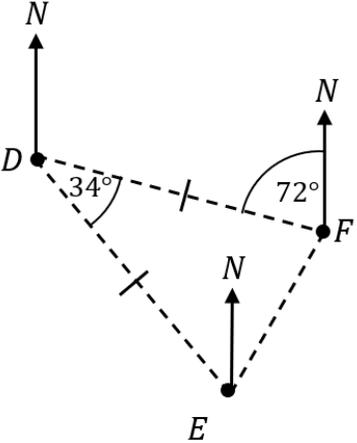
(g)	(h)	(i)
Find the bearing of D from F	Find the bearing of F from D	Find the bearing of E from D
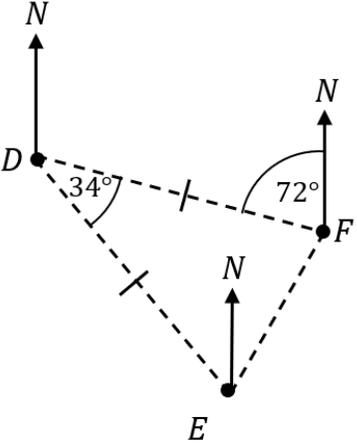
(j)	(k)	(l)
Find the bearing of E from F	Find the bearing of D from E	Find the bearing of F from E

Fluency Practice

Work out the bearings

68°   a

113°   b

c   242°

Give the bearing to A from B

63°
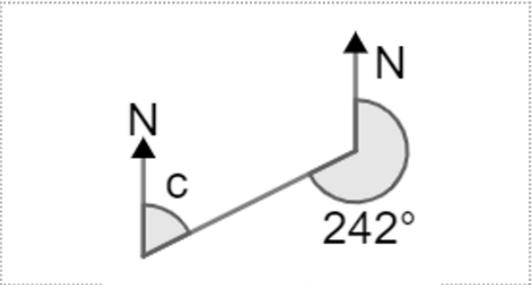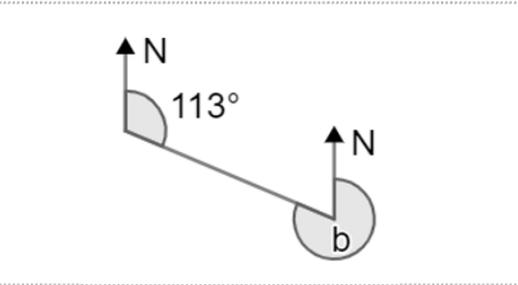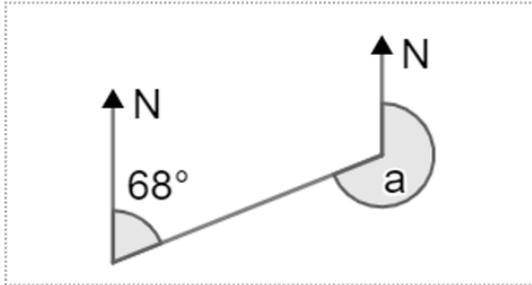
N   A   B
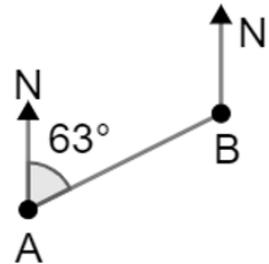
Give the bearing to B from A

A   B   315°
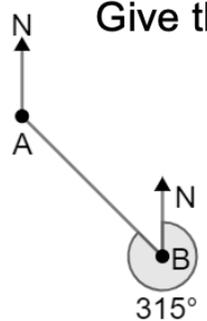
Give the bearing to A from B

244   A   B

Give the bearing of B from A

A   B   80°   C   144°
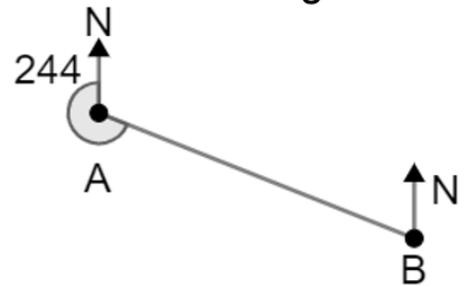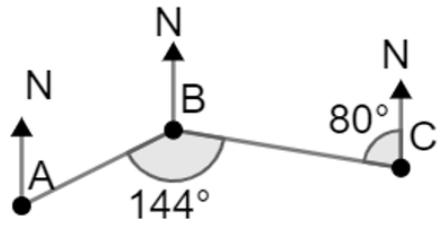
AC is straight line. Give the bearing of B from A

117°   C   B   A

63°   B   45   C   45°   D
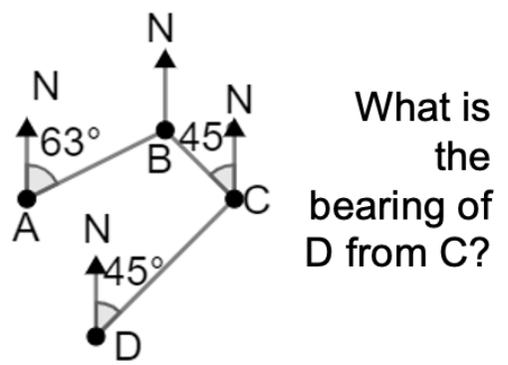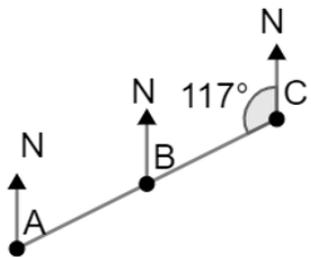
What is the bearing of D from C?

1.

2.

3.

4.
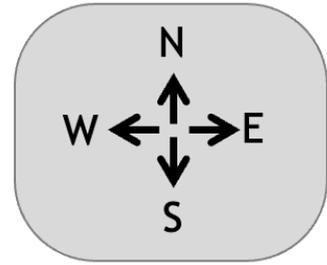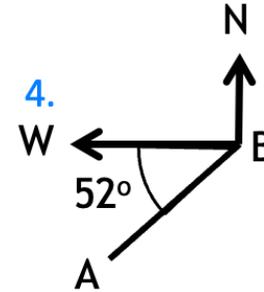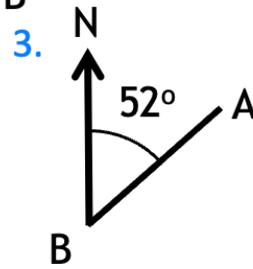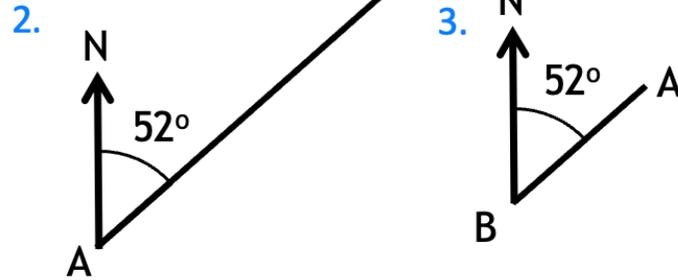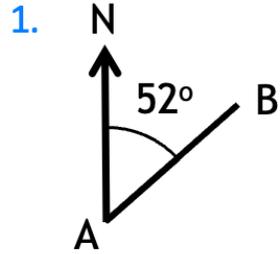
Each question contains point A, point B and directions indicated with an arrow.

5.

6.

7.

8.
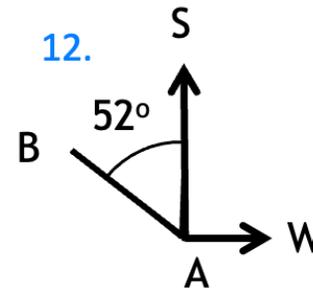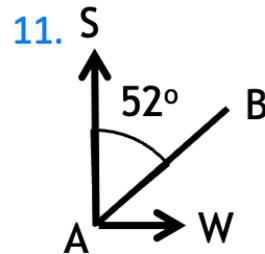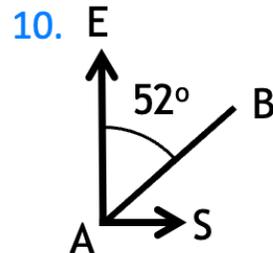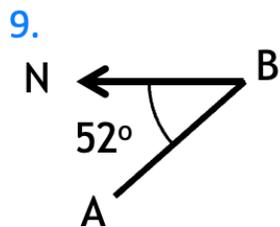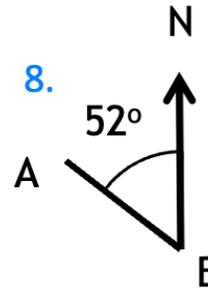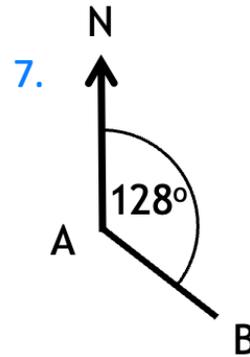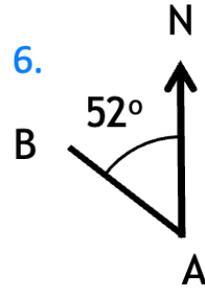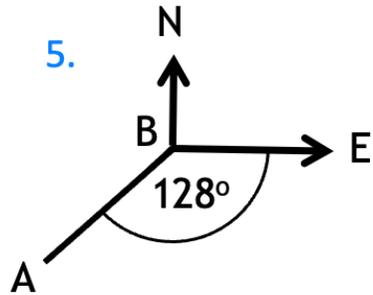
In each case, find the bearing of B from A.

9.

10.

11.

12.

Fluency Practice

With words

1) The bearing from A to B is 060°. What is the bearing to A from B?

2) The bearing from C to D is 100°. What is the bearing to C from D?

3) E is on a bearing of 300° from F. What is the bearing of F from E?

4) G is southwest from H? What bearing is H from G?

5) A boat is equidistant from two lighthouses. It is on a bearing of 80 from lighthouse A and a bearing of 300 from light house B. What is the bearing from lighthouse A to lighthouse B?

Fluency Practice

the bearing of P from Q and the bearing of Q from P

(1)

140°

P
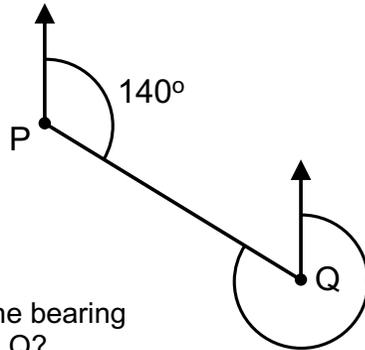
what is the bearing
of P from Q?

Q

(2)

what is the bearing
of P from Q?

75°

Q

P

(3)

260°

P

Q
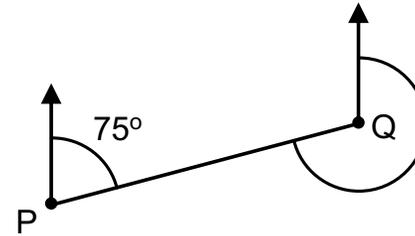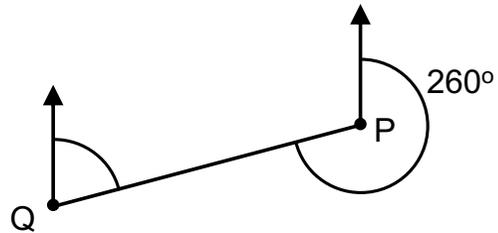
what is the bearing
of P from Q?

(4)

what is the bearing
of P from Q?

Q

300°

P
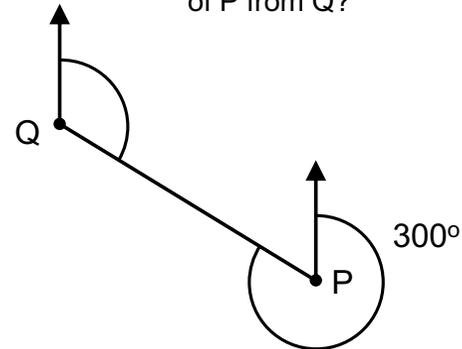
Fluency Practice

(1)

what is the relationship
between angle A and
angle B ?

(2)

what is the relationship
between angle A and
angle B ?
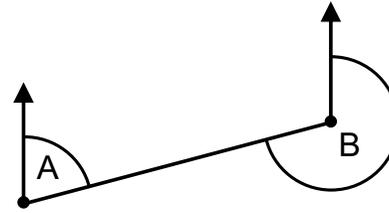
(3)
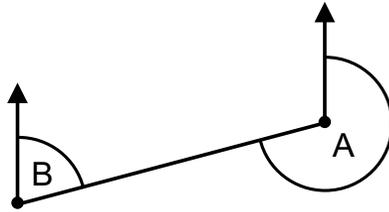
what is the relationship
between angle A and
angle B ?

(4)
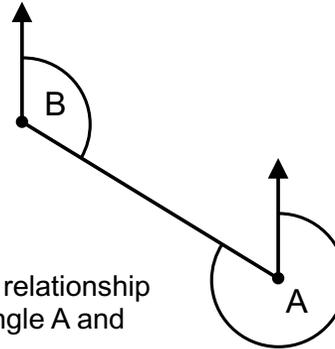
what is the relationship
between angle A and
angle B ?

Fluency Practice

what is the bearing of Q from P?

(1)

20°

PC = CQ = QP

(2)

60°

30°

PC = CQ

(3)

120°

150°

PC = CQ

(4)

120°

30°

PC = CQ
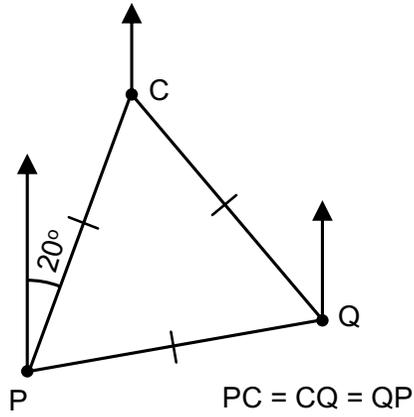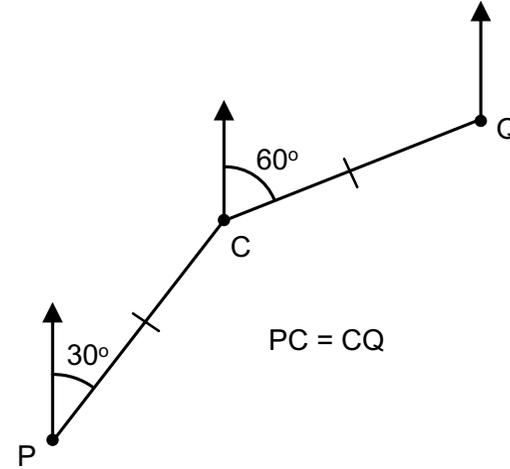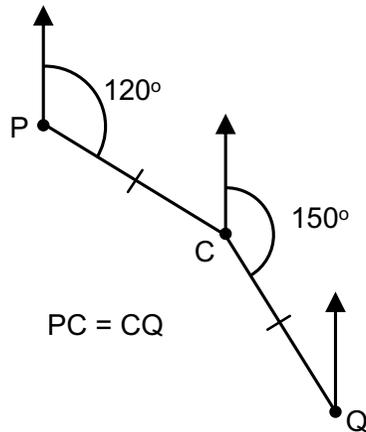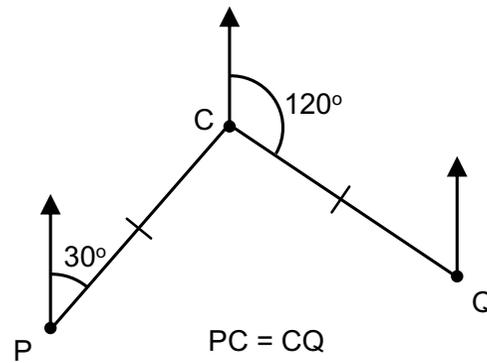
what are the bearings of P from Q?

Fluency Practice

right-angles triangles

(1)

triangle ABC is right-angled at A
($\forall$ BAC = 90°)

the bearing of B from A is 050°
the bearing of C from B is 180°
what is the bearing of A from C?

(2)

triangle ABC is right-angled at A
($\forall$ BAC = 90°)

the bearing of B from A is 050°
the bearing of C from B is 200°
what are the angles in the triangle?

(3)

triangle ABC is right-angled at B
($\forall$ ABC = 90°)

$\forall$ CAB = 25°

the bearing of B from A is 130°
the bearing of C from B is 220°
what is the bearing of A from C?

(4)

triangle ABC is right-angled at B
($\forall$ ABC = 90°)

the bearing of B from A is 010°
the bearing of C from B is 100°
what is the bearing of C from A?

Fluency Practice

a square

(1)

30°
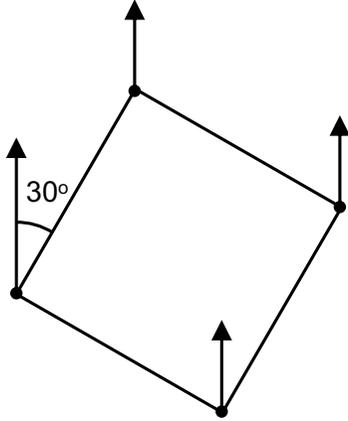
what do the
four bearings
add up to?

(2)

150°
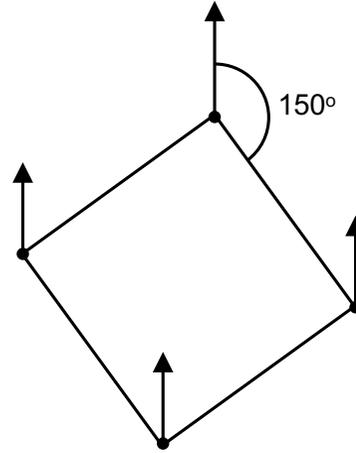
what do the
four bearings
add up to?

(3)

340°
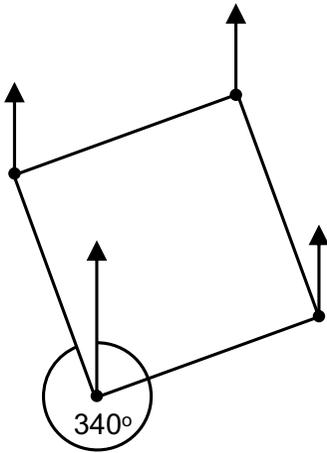
what do the
four bearings
add up to?

(4)

A
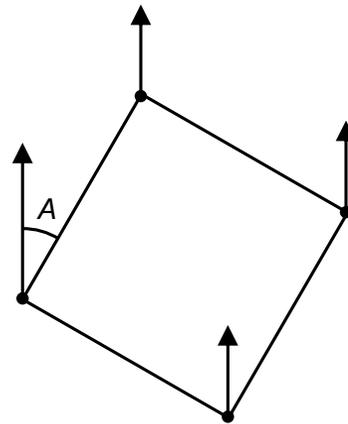
what do the
four bearings
add up to?

Purposeful Practice

1. Each diagram shows the bearing of A from B.
Calculate the bearing of B from A.

a)

70°

b)

135°

c)

230°
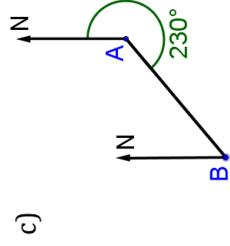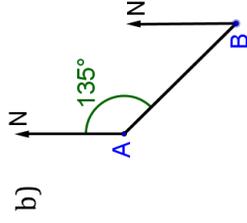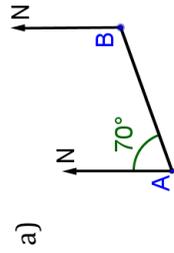
2. Determine the shaded angle ABC in each diagram.
You do not need a scale drawing or protractor.

a)

40° 150°

b)

75° 130°

c)

100° 135°

d)

80° 200°
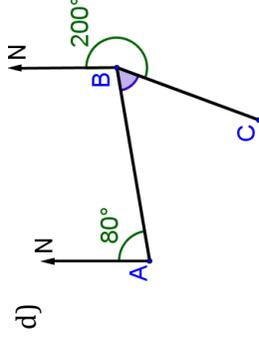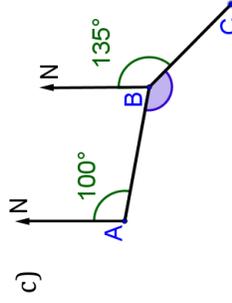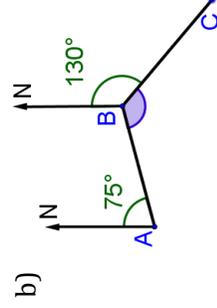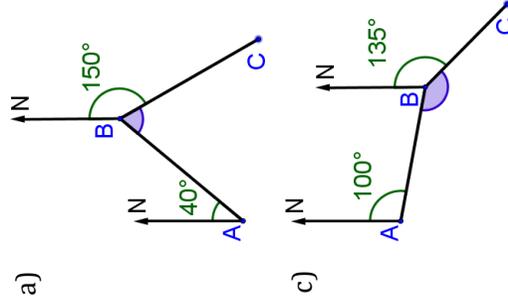
3. In each diagram, a boat sails from A to B, then turns through the marked angle and
sails to C. When the boat is at B, calculate:

i. the bearing of A from B
ii. the bearing of C from B

a)

60° 120°

b)

110° 40°
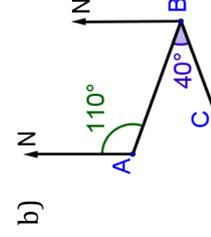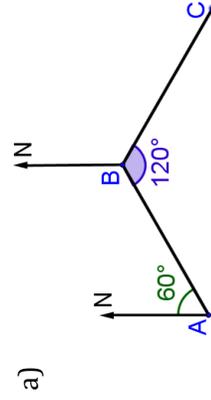
Purposeful Practice

4. A boat sails a triangular course clockwise as shown.
   Calculate the internal angles A, B and C in each triangle.

a)

130° B
60° A
270° C

b)

120° B
50° A
275° C
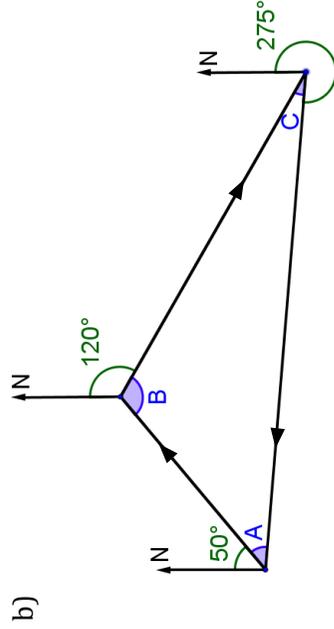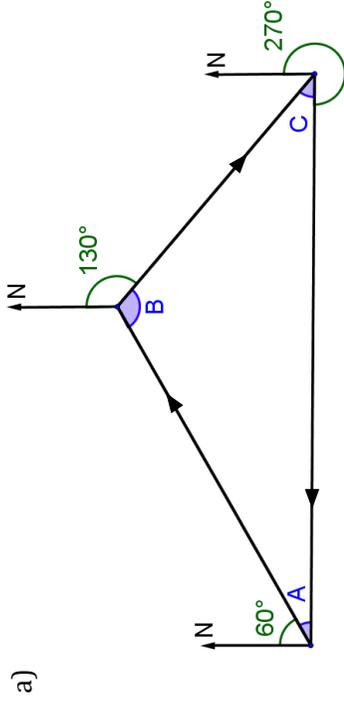
*(Hint: angle A is not 40°.
Find angles B and C first.)*

c)

157° B
50° A
303° C
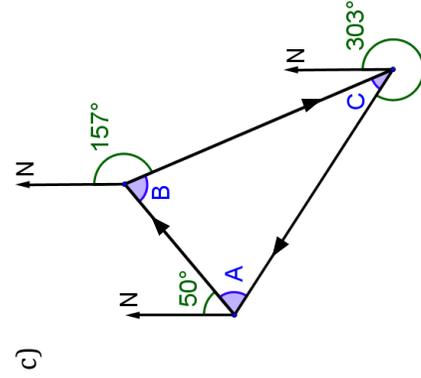
What do you notice about this triangle?

5. Use your answer for question 4 to calculate the bearings needed to sail each course anticlockwise instead, i.e. the bearings from A to C, then C to B and finally B to A.

6. The organisers decide that the course ABC below is too long. They make a new course ABM, where M is halfway between A and C.
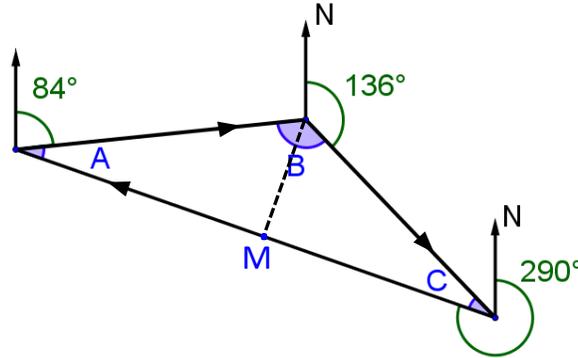
a) Calculate the internal angles A, B and C in the original course and hence explain why the turn at M must be exactly 90 degrees.

b) Find the two new bearings needed to complete this course, i.e.
   i.   the bearing of M from B
   ii. the bearing of A from M

7. a) Sketch a clockwise triangular course where the first leg is on a bearing of 040° and all three legs are the same distance. Calculate the other two bearings.

   b) State the second two bearings if the first leg is instead on a bearing of 051°.

8. A particularly mathematical sailing club want a course in the shape of an isosceles triangle. If the first leg is on a bearing of 040° and the second leg is 188°, sketch two possible courses and calculate the bearing of the third leg.

Purposeful Practice

(diagrams not to scale!)

Give your answers to 1 d.p.

Q1

A ship travels from A to B on a bearing of 050 degrees for a distance of 12km. How far <u>east</u> did it travel?

B

x

12km

50°

A
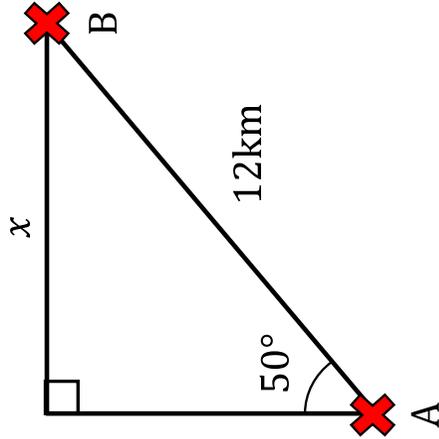
Q2

A ship travels from A to B on a bearing of 070 degrees for a distance of 23km. How far <u>north</u> did it travel?

B

A
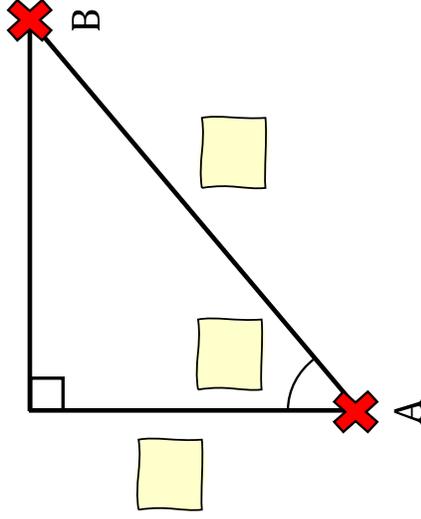
Q3

A ship travels from A to B on a bearing of 130 degrees for a distance of 5km. How far <u>south</u> did it travel?

130°

A

50°

5km

x

B
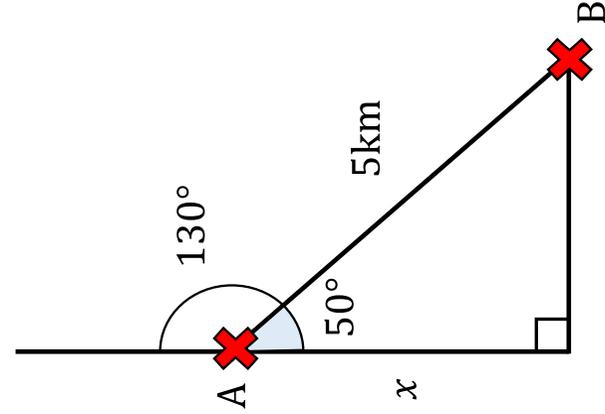
Q4

A ship travels from A to B on a bearing of 145 degrees for a distance of 19km. How far <u>east</u> did it travel?

B

A
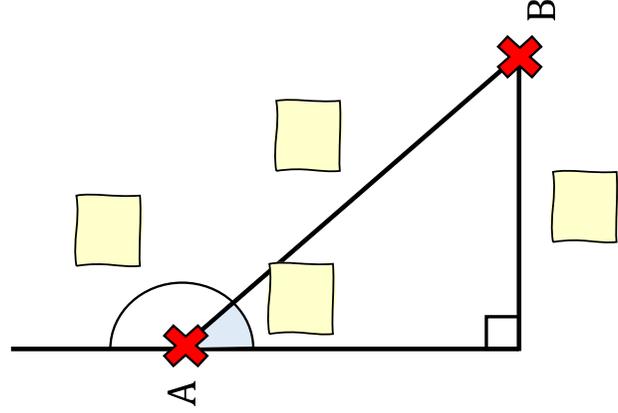
Purposeful Practice

(diagrams not to scale!)

Give your answers to 1 d.p.

Q5 A ship travels from A to B on a bearing of 200 degrees for a distance of 10km. How far <u>west</u> did it travel?

x

70°

10km

A

B

200°
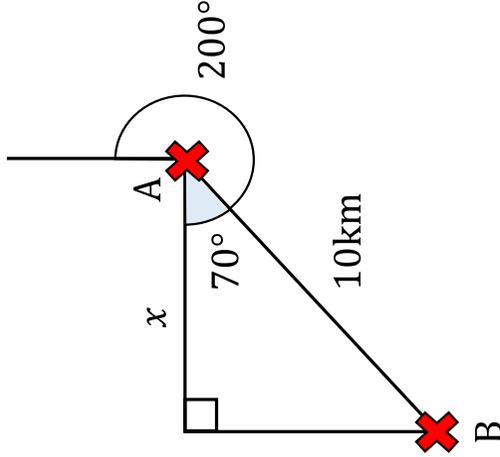
Q6 A ship travels from A to B on a bearing of 210 degrees for a distance of 17km. How far <u>south</u> did it travel?

A

B
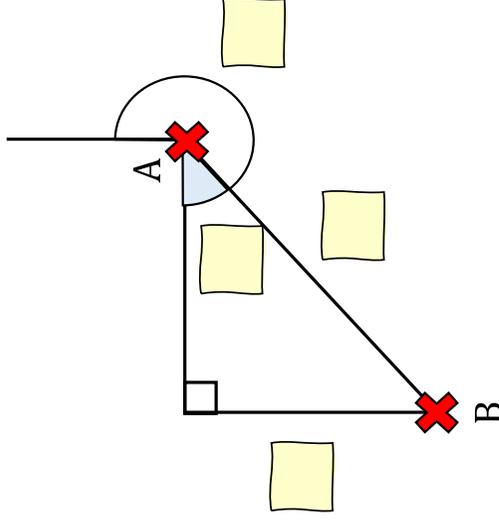
Q7 A ship travels from A to B on a bearing of 310 degrees for a distance of 44km. How far <u>north</u> did it travel?

44km

40°

A

B

x

270°
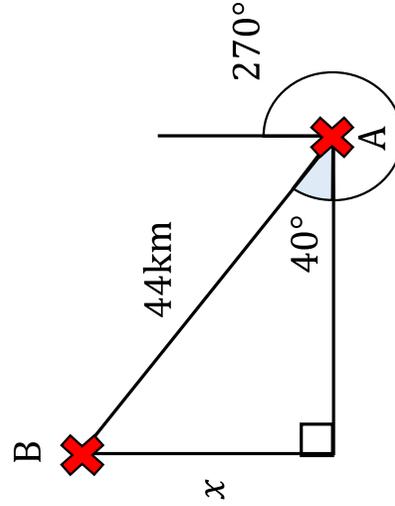
Q8 A ship travels from A to B on a bearing of 302 degrees for a distance of 49km. How far <u>west</u> did it travel?

A

B

Purposeful Practice
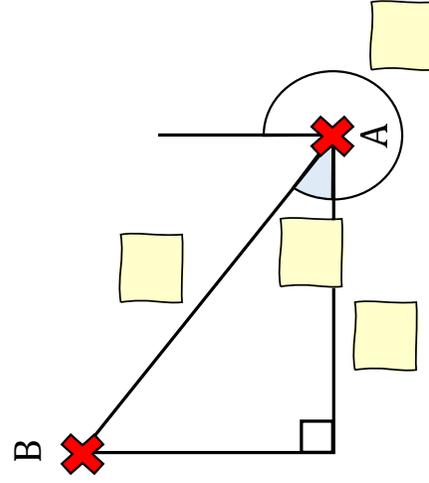
Finding Bearings using Trigonometry

① Point B is 400 km away from point A.
Point B is 350 km east of A.
Find the bearing of B from A.

② Point B is 12 km north of point A.
Point B is 150 km from A.
Find the bearing of B from A.
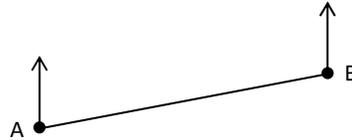
③ Point B is 18 km east of point A.
Point B is 7 km north of A.
Find the bearing of B from A.

④ Buckford is 60 km east of Adley. The cities
are 70 km away from each other.
Find the bearing of Buckford from Adley.
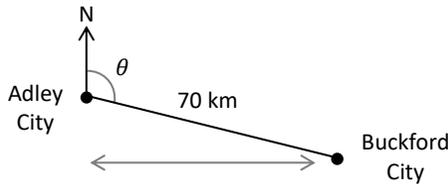
⑤ Flight AN-898 is 58 km from
its destination airport. The plane is
40 km west of the airport.
What bearing does the plane need to fly on
to reach the airport?
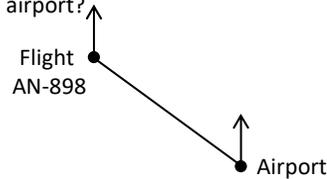
⑥ A plane flies 100 km south and a strong
wind also pushes the plane 8 km east.
On what bearing does
the plane actually travel?

⑦ A boat will travel 77 km direct to its harbour.
The boat is currently 56 km east of the harbour.
On what bearing should the boat travel?
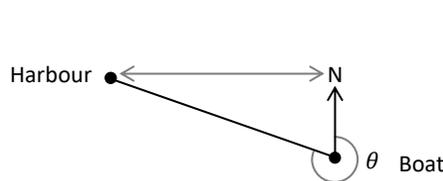
⑧ A cruise ship heads 12 km south but an
ocean current from the east pushes the
boat west. In total the cruise ship
travels 16 km. On what bearing did the
ship actually travel?
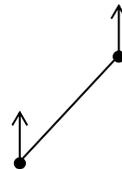
⑨ Delia hikes from Whitclif to Mardsden.
At the end of the day they are
4 times further south than they are west.
What bearing did they hike on?

5 Answers	313.3	121.0	175.4
	085.4	061.0	

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Purposeful Practice

① A ship travels on a bearing of 070° for 400 km. How far west does it travel (x)?
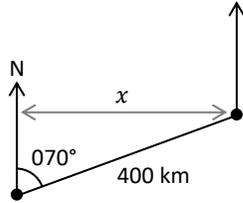
② A plane flies 250 km on a bearing of 062°. How far west does it travel (x)?
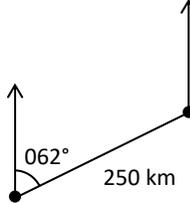
③ A boat sails on a bearing of 048° for 120 metres. How far north does it travel?
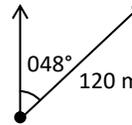
④ Point B is 217 metres away from point A on a bearing of 075°. How far north is point B from point A?

⑤ A satellite travels 370 km on a bearing of 110°. How far east does it travel (x)?
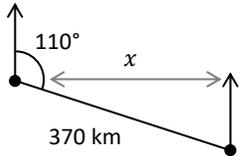
⑥ A runway is 158 km from flight GA-226 on a bearing of 140°. How far south is the runway?
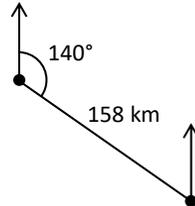
⑦ Middleford is 87 km north, and 121 km east of Gredwich. What is the distance between the two cities?

⑧ Yarmud is 87 km away from Hilltown on a bearing of 240°. How far east is Yarmud from Hilltown?
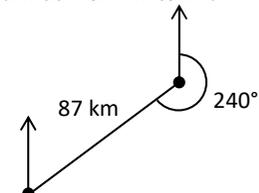
⑨ A ship travels 116 km on a bearing of 262°. How far west does it travel? How far south does it travel?
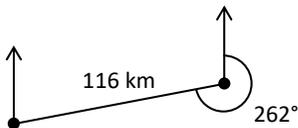
⑩ Point B is 150 m north of point A and on a bearing of 065°. How far east is point B from point A?
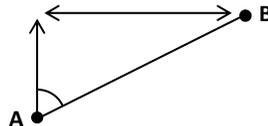
⑪ Drekham is 300 km west of Greyford on a bearing of 251°. How far south is Drekham from Greyford?
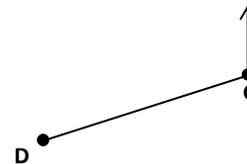
⑫ Waypoint ZOLOT is 400 km east of BETTA on a bearing of 075°. How far north is ZOLOT from BETTA?

8 Answers	221	103	102	80
	149	348	115	376

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Purposeful Practice

Wind

An airplane heads (aims) 12 km directly east but a northerly wind pushes the plane 2 km south. What bearing does the plane actually travel on?

wind ∫∫∫

12 km

2 km

The pilots must steer into the wind to make sure the plane goes directly east.

wind ∫∫∫

heading

actual flight path

1) wind ∫∫∫

N

θ

20 km

4 km

Over 20 km a northerly wind will push a plane 4 km south.
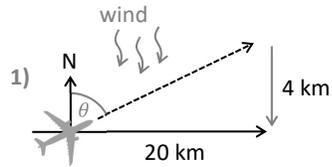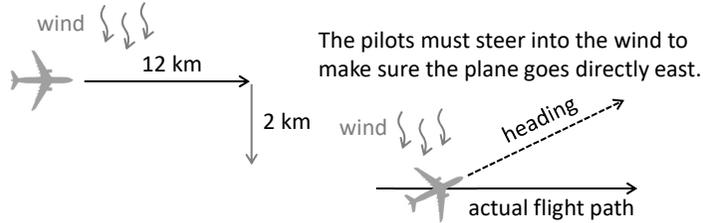
On what bearing should the pilots fly (their heading) so the flight path is directly east?

2) 3 km

30 km

Memphis airport is 30 km directly north of flight AT-808 but a westerly wind will push the plane 3 km east. On what bearing should the pilots fly to reach the airport?
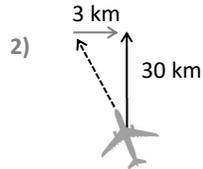
3) Dallas airport is 58 km away from flight AX-377 on a bearing of 270°. The wind is at 180° and will push the plane 7 km off course. Find the required heading to reach the airport.

4) A plane heads directly south for 17 km, but the wind pushes the plane 3 km west. What bearing does the plane actually fly on? How far does the plane travel above the ground?

Current

A motorboat travels up a river. The engines push the boat forward 10 km, but the current of the river pushes the boat backwards by 2 km. How far does the boat actually travel?

10 km

current

2 km

If the boat uses 100 ml of fuel per km. How much fuel does it use?

5) On the open ocean a motorboat heads 27 km due east but is pushed 8 km north by the current. Find the direction and distance of the boat's path.

6) A boat's safe harbour is 24 km due west. The current will push the boat 2 km off course to the south. Find the required heading to reach safety.

The boat uses 2 litres of fuel per kilometre. How much fuel will it use to reach the harbour?

7) A speedboat heads south at 10 m/s. The current pushes the boat east at 1 m/s. On what bearing does the boat travel? How far does the boat travel in 3 seconds?

8) A boat heads east but is pushed 3 km north by the current. In total the boat travels 31 km. On what bearing did the boat travel?

9) A boat's engines propel it at 10 m/s west. With the current pushing the boat north it actually travels at 14 m/s. On what bearing does the boat travel? How fast is the current?

Purposeful Practice

Q1 A scout troop are hiking in a forest. Starting from their base, they walk 4.2km south followed by 7.1km west. They want to walk the shortest distance back to their base.
On what bearing should the scouts walk?

Q2 A dragon has been wreaking havoc in a local village. It is then chased away by a knight. The dragon flies 3 miles due south followed by 4 miles due west.

[a] Work out the distance between the dragon and the village.

[b] Find the bearing of the dragon from the village to the nearest degree.

Q3 The diagram shows the positions of a tower and a tree. The tree is 2.1km South of the tower and 4.5km East of the tower.

[a] Work out the distance between the tower and the tree to one decimal place.

[b] Work out the bearing of the tree from the tower to the nearest degree.
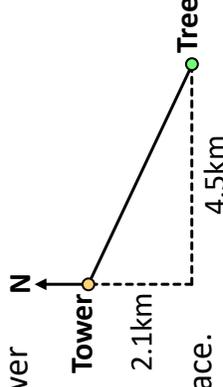
Q4 A aeroplane flies 22 miles due south followed by 34 miles due east. On what bearing is the aeroplane's starting position from its current position to the nearest degree?

Q5 A buoy is located 214m west and 185m south of a ship. Calculate the bearing of the buoy from the ship to the nearest degree.

Q6 Two ships sail away from port. Ship A sails 6.4miles due south followed by 12.4miles due west. Ship B sails 3miles due south followed by 10.1 miles due east.

[a] Work out the distance between the two ships to one decimal place.

[b] Work out the bearing of Ship B from Ship A to the nearest degree.

Purposeful Practice

Q1 The following diagram represents the positions and bearings of two helicopters. Helicopter B is 1.2km away from helicopter A on a bearing of 122°. How far north is helicopter A from helicopter B?

Helicopter A

N

122°

1.2km

N

Helicopter B
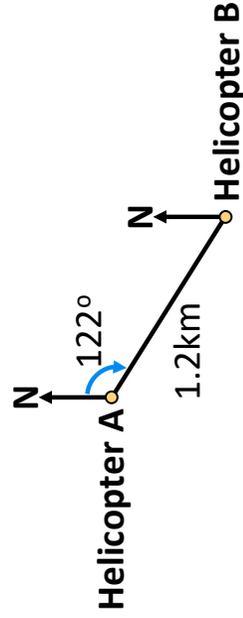
Q2 Animal conservationists are tracking the position of a polar bear. The polar bear's position is 3km in an eastward direction and is on a bearing of 104°. What is the shortest distance between the polar bear and the conservationists.

Q3 The following diagram represents the positions and bearings of three buoys floating in the ocean.

[a] How far west is Buoy B from Buoy C.
[b] How far east is Buoy A from Buoy B.
[c] How far north is Buoy C from Buoy A.

N

Buoy C

345m

N

077°

Buoy B

240m

N

314°

Buoy A
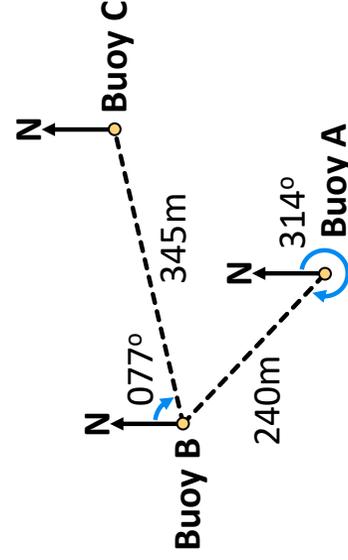
Q4 A ship sails on a bearing of 074° for 10 miles followed by a bearing of 131° for 15 miles. Work out the bearing of the ship from its starting position to the nearest degree.

Purposeful Practice

1. Amanda and Michael leave the same campsite and set off in different directions.
Michael walks 7 kilometres on a bearing of 035^0 and Amanda walks 8 kilometres due east.
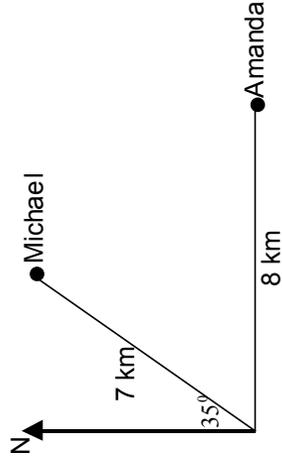
How far apart are Amanda and Michael?

Michael •
N
7 km
35^0
8 km
• Amanda

2. Two yachts leave harbour H.
Yacht A sails on a bearing of 072^0 for 30 km and then stops.
Yacht B sails on a bearing of 140^0 for 50 km and then stops
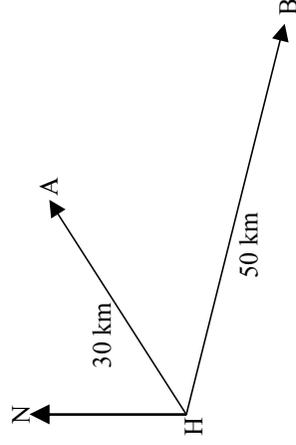
How far apart are the two yachts when they both have stopped?

A
N
30 km
H
50 km
B

3. Debbie is taking part in an orienteering walk. She leaves checkpoint A and walks 6.8 km due south to checkpoint B.
From B she walks 4.7 km on a bearing of 080^0.

How far is Debbie from checkpoint A?
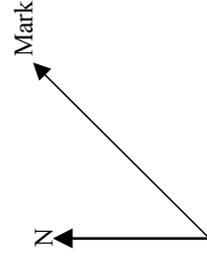
4. Mark and Rabnawaz leave a hostel at the same time.
Mark walks on a bearing of 045^0 at a speed of 4.4 kmph.
Rabnawaz walks on a bearing of 100^0 at a speed of 4.8 kmph.

If both walk at steady speeds, how far apart will they be after 2 hours?

Mark
N
Rabnawaz

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5. The sketch below shows the journey of a ship from a harbour.
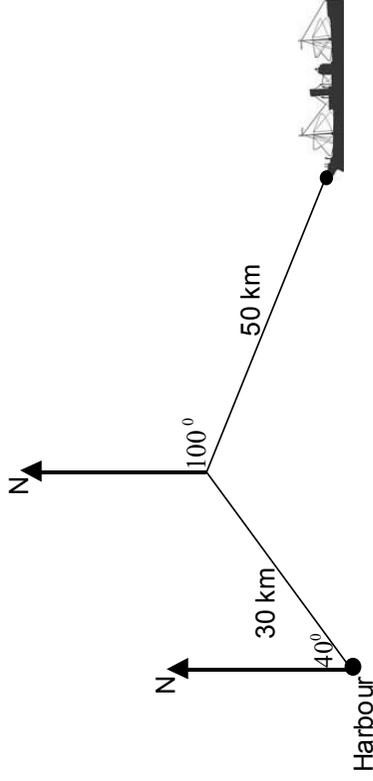
The ship leaves the harbour and sails 30 km on a bearing of 040^0 and then sails 50 km on a bearing of 100^0.

Calculate the distance the ship is from the harbour.

6. Two ships leave Liverpool at the same time.
One of them travels north-west at an average speed of 10.5 kmph while the other travels at an average speed of 14 kmph on a bearing of 280^0.

How far apart are these ships after 3 hours?

7. Three oil platforms, Alpha, Beta and Gamma are situated in the North Sea.
Beta is 12 km from Alpha and is on a bearing of 145^0 from Alpha.
Gamma is 16 km from Alpha and is on a bearing of 220^0 from Alpha.
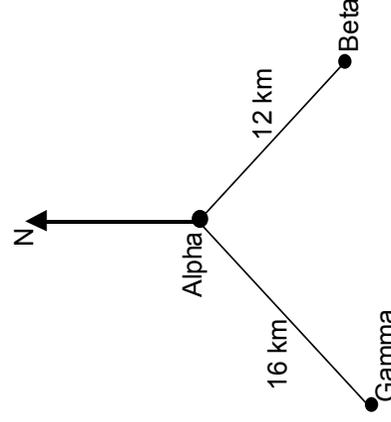
Find the distance of Beta from Gamma.

Purposeful Practice

8. A submarine is positioned at A.
Its radar detects another ship 50 km away on a bearing of 145^0, at C.
The submarine informs its base at B, 40 km away on a bearing of 210^0, of this ship.

The base informs the submarine to intercept this ship.
How far is the ship at C from the base B?
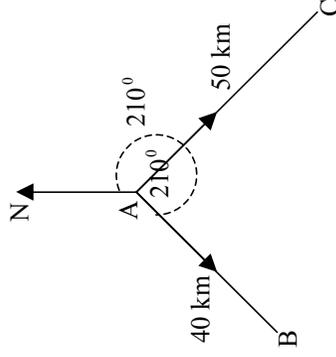
9. David takes part in a cross-country race.
He leaves the starting point of the race and runs due west for 5.5 km before changing direction and running 6.3 km in a north-west direction.
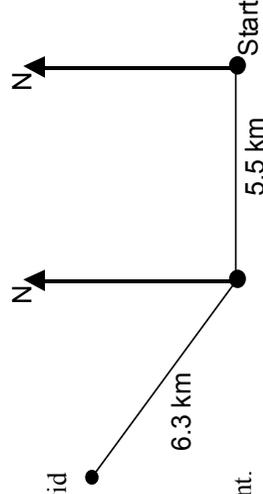
Calculate how far David is from the starting point.

10. The diagram shows the position of 3 airports, A, E and G.
G is 200km from A.
E is 160 km from A
From G the bearing of A is 052^0.
From A the bearing of E is 216^0.
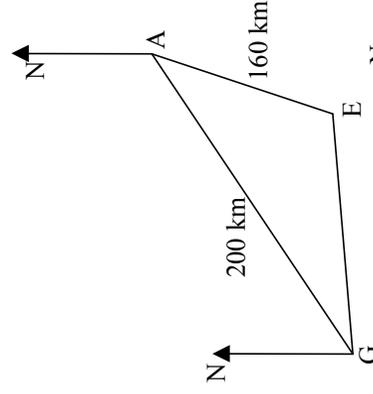
How far apart are airports G and E?

11. The diagram shows the position of 3 towns E, B and F.

B is 300 km from E and is on a bearing of 046^0 from E.
From E, the bearing of F is 116^0.
F is due south of B.
Calculate the distance from E to F.
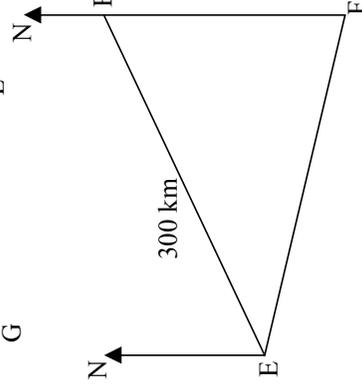
Purposeful Practice

12. A submarine leaves Faslane in Scotland and sails into the Atlantic Ocean on a bearing of 250^0 for a distance of 120 km before changing course and sailing on a bearing of 190^0 for 90 km.

Calculate the distance the submarine is from Faslane.
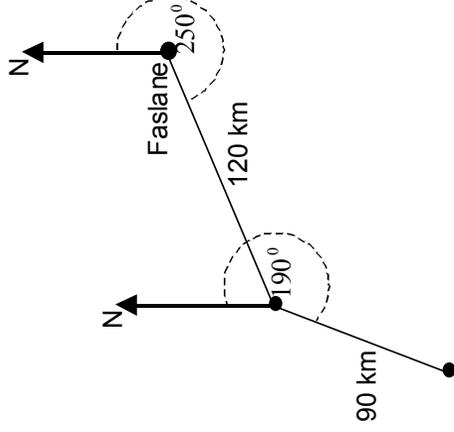
13. An aeroplane flying to Edinburgh is 100 km due south of Edinburgh when the pilot is told to change direction because of fog.
He is told to fly to Prestwick on a bearing of 320^0.
The bearing of Prestwick from Edinburgh is 240^0.

(a) Find the distance of the aeroplane from Prestwick.

(b) Find the bearing of Edinburgh from Prestwick.
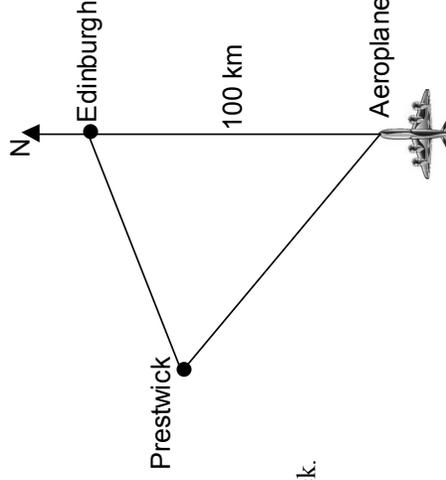
14. A church is 600 metres due west of a flagpole.
A statue is on a bearing of 160^0 from the church and on a bearing of 220^0 from the flagpole.

Find the distance of the church from the statue.
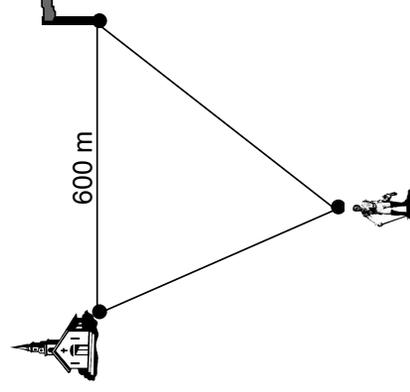
Purposeful Practice

15. Three radio masts, K, W and P are situated in the Australian outback.
K is 250km due south of W. W is 410km from P.
P is on a bearing of 130° from K.

Calculate the bearing of P from W.
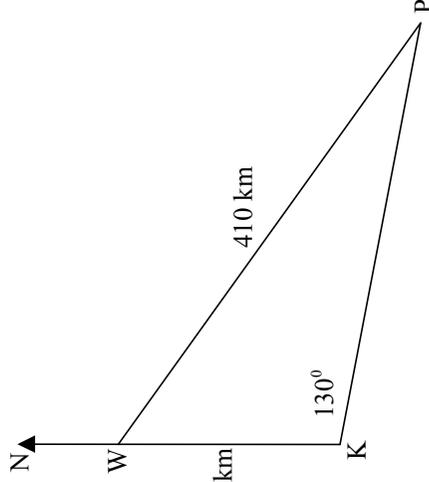
16. A ship sails from harbour H on a bearing of 084° for 340km until it reaches point P.
It then sails on a bearing of 210° for 160km until it reaches point Q.
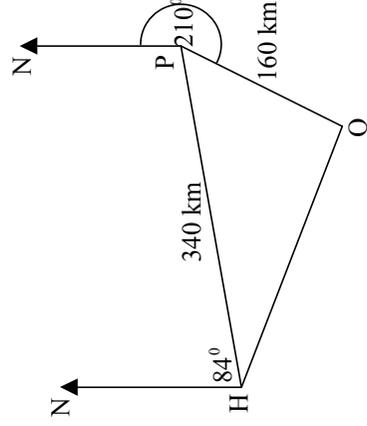
Calculate the distance between the point Q and the harbour.

17. Two ships A and B are anchored at sea. B is 65km due east of A. A lighthouse is positioned on a bearing of 055° from A and on a bearing of 310° from B.
Calculate how far the lighthouse is from ship B.

Purposeful Practice

18. A ship, at position P, observes a lighthouse at position Q on a bearing of 040°. The ship travels 30km on a bearing of 125° to position R. From position R, the ship observes the lighthouse on a bearing of 340°.
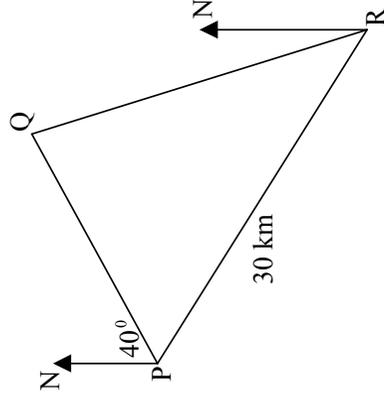
(a) Find the bearing of R from the lighthouse.

(b) Calculate the distance from R to the lighthouse

19. A boat sails on a bearing of 160° for 600km before changing course and sailing 700km on a bearing of 050°.

(a) How far the boat is from its starting point?

(b) Find the bearing the ship is from its starting point.

20. A surveyor is walking due west when he comes to a marsh. To avoid the marsh he turns at P and walks for 60m on a bearing of 215° and then for 85m on a bearing of 290°.
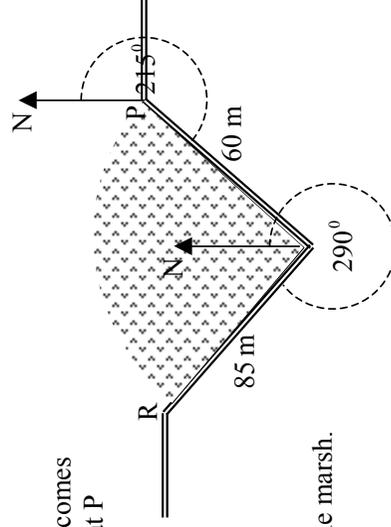
Calculate PR the direct distance across the marsh.

4 Advanced Ratio

Fluency Practice

Ratio & Formulae

$a : b = 3 : 4$

$\dfrac{a}{b} = \dfrac{3}{4}$ $\dfrac{b}{a} = \dfrac{4}{3}$

$a = \dfrac{3}{4}b$ $b = \dfrac{4}{3}a$

① $a : b = 2 : 5$

$\dfrac{a}{b} =$ $\dfrac{b}{a} =$

$a =$ $b =$

② $a : b = 1 : 3$

$\dfrac{a}{b} =$ $\dfrac{b}{a} = \dfrac{}{1}$

$a =$ $b =$

③ $a : b = 6 : 5$

$\dfrac{a}{b} =$ $\dfrac{b}{a} =$

$a =$ $b =$

④ $a : 4 = 3 : b$

$a =$ $= b$

⑥ $a : 2 = b : 9$

$\dfrac{a}{} =$

$a =$ $= b$

⑦ $3 : a = 4 : b$

$a =$ $= b$

⑧ $a : 2 = 3 : 7$

$a =$

⑨ $5 : a = 2 : 3$

$a =$

⑩ $4 : 7 = 3 : 5a$

$= \dfrac{}{3}$

$= a$

⑪ If, $a = \dfrac{4}{5}b$

$a : b =$

⑫ If, $b = \dfrac{8}{7}a$

$a : b =$

⑬ If, $a = 1.25b$

$a : b =$

⑭ If, $b = 1.8a$

$a : b =$

Fluency Practice

Comparing Ratios   For each ratio, express a in terms of b… and b in terms of a.   Complete each sentence without using fractions.

① $a : b = 1 : 4$

$a : b$

$\dfrac{a}{b} = \dfrac{1}{}$ $\dfrac{b}{a} = \dfrac{4}{}$

$a = \dfrac{1}{}b$ $b = \quad a$

b is _____ times the size of a.

② $a : b =$

$a : b$

$\dfrac{a}{b} = \dfrac{1}{}$ $\dfrac{b}{a} = \dfrac{}{1}$

$a = \quad b$ $b =$

a is _____ times the size of b.

③ $a : b =$

$a : b$

$\dfrac{a}{b} =$ $\dfrac{b}{a} = \dfrac{}{2}$

$a =$ $b =$

b is _____ times the size of a.

④ $a : b =$

$a : b$

$\dfrac{a}{b} =$ $\dfrac{b}{a} =$

$a =$ $b =$
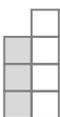
a is _____ times the size of b.

⑤ $a : b =$

$a : b$

$\dfrac{a}{b} =$ $\dfrac{b}{a} = \dfrac{}{4}$

$a =$ $b =$

b is _____ times the size of a.

a is _____ times the size of b.

⑥ $a : b =$

$a : b$

$\dfrac{a}{b} =$

$a = \dfrac{4}{}b$ $b =$
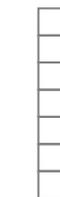
b is _____ % larger than a.

⑦ $a : b = 5 : 2$

$a : b$

$a =$ $b =$

a is _____ times the size of b.

b is _____ % smaller than a.

⑧ $a : b =$

$a : b$

$a = \dfrac{5}{7}b$ $b =$

a is _____ times the size of b.

⑨ $a : b$ $a : b =$

$a =$ $b =$

b is $2\dfrac{2}{3}$ times the size of a.

a is _____ % smaller than b.

Fluency Practice
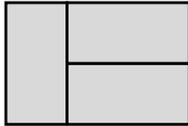
Ratios & Formulas

This compound shape is made from 3 congruent rectangles.

For one small rectangle...

a) What is the formula for the length in terms of width?

b) What is the ratio **length : width** ?
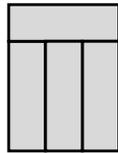
1)

Formula _____

Ratio _____
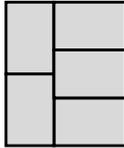
2)

Formula for length

length : width =

⇧   ⇧

3)
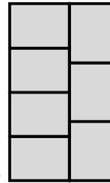
Formula _____

Ratio _____

4)
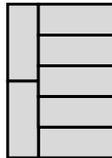
Formula _____

Ratio _____

5)
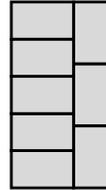
Formula _____

Ratio _____

6)
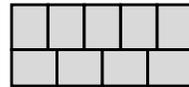
Formula _____

Ratio _____

For these **squares**, write a formula for **a** in terms of **b**.
Convert the formula into the ratio **a : b**

7)
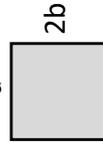
a

2b

Formula _____

a : b _____

8)

2a

3b

Formula _____

a : b _____

9)

4a

3b

Formula _____

a : b _____

10)

8a
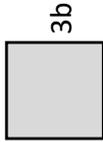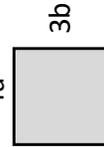
10b

Formula _____

a : b _____

Fluency Practice

Ratios & Formulas

These rectangles are **similar**.

What can we **deduce** about the **relationship** between x and y?

Length : Width

4 : 1

4 cm

1 cm

Length : Width

$x : y$

x

y

The length and width are in the same ratio.

$4 : 1 = x : y$

Therefore,

$$\frac{4}{1} = \frac{x}{y} \qquad \text{and} \qquad \frac{1}{4} = \frac{y}{x}$$

$$x = 4y \qquad\qquad y = \frac{x}{4}$$

We can rearrange these equations to express x in terms of y and vice-versa.

For each similar pair, express their equal ratios and convert this into a formula.

Rearrange the formula to express x in terms of y.

A)

3 cm

1 cm

x

y

B)

3 cm

2 cm

x

y

C)

5 cm

2 cm

x

y

D)

3 cm

4 cm

x

y

E)

9 cm

5 cm

x

y

x

F)

7 cm

x

2 cm

y

G)

6 cm

8 cm

x

y
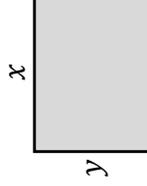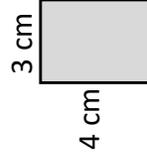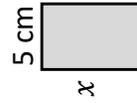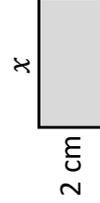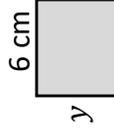
Find the value of x when $y = 12$

H)

2 cm

5 cm

x

y

Find the value of y when $x = 8$

Fluency Practice

Show each of the formulas using the equivalent ratios below.

$(y+x) : (y-x)$ is equivalent to k : 1

Show that $y = \dfrac{x(k+1)}{k-1}$

$(b+m) : (b-m)$ is equivalent to k : 1

Show that $b = \dfrac{m(k+1)}{k-1}$

$(p+y) : (p-y)$ is equivalent to k : 3

Show that $p = \dfrac{y(k+3)}{k-3}$

$(p+q) : (p-q)$ is equivalent to k : 5

Show that $p = \dfrac{q(k+5)}{k-5}$

$(3x+y) : (x-3y)$ is equivalent to k : 2

Show that $x = \dfrac{y(3k+2)}{k-6}$

$(x+2y) : (3x-4y)$ is equivalent to k : 2

Show that $x = \dfrac{4y(k+1)}{(3k-2)}$

Fluency Practice

Changing Ratios   scaling

Jia and Jen share some money in the ratio 1:2
Jen spends £3. The money is now in the ratio 2:3

	then	now	
	1:2	**2:3**	Jen's money
equalise the			change
constant part	×2 ↓	↓	
(Jia's money)	**2:4**	**2:3**	−1
	↓	↓ ×3	↓
	6:12	6:9	−3

−£3

① Rosh and Roe share some money in the ratio 1:3
Roe spends £5. The money is now in the ratio 2:5
How much money does Roe have now?

	then	now	
	1:3	**2:5**	
equalise the	×2 ↓	↓	change
constant part			
	↓	↓	↓

② Pen and Pol share some money in the ratio 3:5
Pol spends £16. The money is now in the ratio 9:7
How much money is there in total now?

then	now
3:5	**9:7**
↓	↓

③ Mia and May share some money in the ratio 3:4
May spends £20. The money is now in the ratio 2:1
How much money does May have now?

3:4   **2**:1

④ Ken and Kel share some money in the ratio 4:3
Kel spends £3. The money is now in the ratio 3:2
How much money did they start with?

⑤ Ann and Ai share some money in the ratio 3:2
Ann spends £18. The money is now in the ratio 3:5
How much money did Ann start with?

⑥ A bag contains red and blue disks in the ratio 4:5
After 4 blue disks are removed the ratio is now 2:3
How many disks are there now?

⑦ In a store there are 50% more bananas than apples.
15 bananas are added so the apple:banana ratio = 3:7
How many apples are there?

⑧ A bag has 25% more triangles than circles.
After 15 triangles are added the
circle:triangle ratio is 5:7
How many triangles are there now?

Purposeful Practice

1. The ratio of boys to girls in a group is 7 : 5.
   15 more boys join the group.
   The ratio is now 12 : 5.
   How many students are there in total?

2. The ratio of green pens to red pens in a drawer is 5 : 1.
   15 red pens are added to the drawer.
   The ratio becomes 5 : 4.
   How many pens of each colour are now in the drawer?

3. In an animal sanctuary the ratio of dogs : cats : donkeys is 5 : 4 : 2
   8 more cats arrive at the sanctuary.
   The ratio is now 5 : 8 : 2.
   How many donkeys are there?

4. Jim likes to buy shirts of particular colours.
   The ratio of blue, black and white shirts is 1 : 3 : 4.
   There are 6 more black shirts than blue shirts.
   How many shirts are there in total?

5. Julia keeps pigs, sheep and goats in the ratio of 2 : 3 : 5.
   She has 9 more goats than pigs.
   How many sheep does she have?

6. Bill buys sweets for his friends, Tom, John and Kevin.
   He gives out the sweets in the ratio 4 : 5 : 2.
   John has 12 more sweets than Kevin.
   How many sweets does each friend have?

7. Green, white and yellow counters are placed in a bag in the ratio of 4 : 3 : 2.
   20 green counters are removed and the ratio becomes 2 : 3 : 2.
   How many of each counter are left in the bag?

8. Burgers, hot-dogs, and sausage rolls are sold in the ratio of 5 : 6 : 7.
   10 more hot-dogs than burgers were sold.
   How many items were sold in total?

Fluency Practice

Person A and Person B share money in a ratio. A gives B some money. Now they have the same amount …

A	B	Ratio	A gives B	Question
James	Jessie	5 : 3	$4	How much did Jessie have to start?
Jessie	Brock	9 : 7	$2	How much did Jessie have to start?
Brock	Ash	3 : 1	$8	How much did they start with altogether?
Ash	Misty	5 : 1	$6	How much did Ash start with?
Misty	Brock	3 : 1	$1	How much do they both have now?
Ash	James	9 : 5	$1	How much did Ash start with?
Brock	Jessie	3 : 1	$0.50	How much do they both have now?

Purposeful Practice

A bag contains black and white cubes in the ratio 1:2
3 black cubes are added and the ratio becomes 3:5

| then | $x : 2x$ | $= 1 : 2$ | $15 : 30$ |
| now | $x + 3 : 2x$ | $= 3 : 5$ | $18 : 30$ |

$$\frac{x + 3}{2x} = \frac{3}{5}$$

$$5(x + 3) = 2x(3)$$

$$5x + 15 = 6x$$

$$15 = x$$

① A bag contains black and white cubes in the ratio 1:2
4 black cubes are added and the ratio becomes 3:5
How many white cubes are there?

| then | $x : 2x$ | $= 1 : 2$ |
| now | | $= 3 : 5$ |

② In a bag, black cubes : grey cubes = 1:3
2 black cubes are added and
the ratio becomes 2:5
How many black cubes are there now?

$$x : 3x$$

③ In a bag, red cubes : grey cubes = 2:3
10 red cubes are added and
the ratio becomes 3:2
How many red cubes are there now?

④ In a bag, red cubes : blue cubes = 3:4
6 red cubes are added and
the ratio becomes 4:5
How many red cubes are there now?

⑤ In a bag, green : blue cubes = 4:5
2 green cubes are **removed** and
the ratio becomes 3:4
How many cubes are there now?

$$4x - 2 :$$

⑥ In a bag, grey : blue cubes = 3:7
4 grey cubes are removed and
the ratio becomes 2:5
How many blue cubes are there?

⑦ In a sports club, adults : teens = 5:8
2 adults join **and** 8 teens join and
the ratio becomes 1:2
How many adults are there now?

$$5x + 2 :$$

⑧ In a sports club, adults : teens = 2:3
2 adults join and 4 teens **leave** and
the ratio becomes 5:4
How many people are in the club now?

⑨ There are 80% more adults than teens in a sports club.
9 adults leave and 2 teens join and
the adult:teen ratio becomes 3:2
Now 3 teens join the club. As a percentage,
how many more adults than teens are there?

Purposeful Practice

(a) The ratio of white socks to black socks is 1 : 4 in a drawer. Four white socks are added to the drawer and the ratio of white socks to black socks becomes 5 : 12. Find the number of white socks and black socks there were initially.

(b) Lily and Mary have marbles in the ratio 5 : 6. Lily gets two more marbles and now the ratio is 7 : 8. How many marbles did each girl have initially?

(c) The ratio of the number of boys to girls at a party is 3 : 4 Six boys leave the party. The ratio of the number of boys to girls at the party is now 5 : 8 Work out the number of girls at the party.

(d) The ratio of pigeons to ducks in a park is 3 : 2. When 5 pigeons fly away, the ratio of pigeons to ducks becomes 5 : 4. How many ducks and pigeons were there originally?

(e) Bill and Chuck share some sweets in the ratio 7 : 3. Bill gives 3 sweets to Chuck and now the ratio is 5 : 3. How many sweets did each have initially?

(f) There are two bags containing counters, bag A and bag B. The ratio of counters in bag A to bag B is 3 : 4. Twelve counters are taken from bag B and added to bag A and the number of counters in each bag is now the same. How many counters were there originally in each bag?

Purposeful Practice

A A bag contains red and blue counters in the ratio 1 : 3.

When four red counters are added to the bag, the ratio changes to 1 : 2.

Work out the number of blue counters in the bag.

B A fruit bowl contains apples and bananas in the ratio 4 : 5.

Two apples are removed, changing the ratio to 2 : 3.

Work out the total number of pieces of fruit that remain in the bowl.

C In a group of people, the ratio of right-handed to left-handed people is 8 : 1.

Two right-handed people join the group. This changes the ratio to 25 : 3.

Work out the total number of people that are now in the group.

D At a company, 75% of the employees were men.

After six women joined the company, the ratio of men to women was 2 : 1.

Work out how many men work at the company.

E After a certain number of matches, the ratio of wins, draws and losses for a football team is 3 : 1 : 2.

If the team win both of their next two matches, they will have won 55% of their matches.

Work out how many matches the team has played so far.

F The colours of cars in a car park are shown in the table.

	Red	Blue	Grey	White	Black
Freq.	3	5	2	1	4

Work out the smallest number of cars that would need to enter the car park (without any cars leaving) for the ratio of blue to white cars to change to 2 : 5.

G There are 108 people in a hall. The ratio of adults to children is 5 : 4.

How many adults must leave to hall for the ratio to change to 1 : 1 ?

H The ratio of red pencils to green pencils in a box is 4 : 1.

What is the smallest possible number of pencils that would have to be added to change the ratio to 5 : 2 ?

I A box contains gold, silver and bronze tokens in the ratio 5 : 7 : 6.

Six gold tokens are added.

A token is then selected at random from the box.

The probability that it is gold is $\frac{7}{20}$.

Work out the probability that it is silver.

J Helen has 1.5 litres of pink paint, made from a mixture of red and white paint in the ratio 1 : 4.

She adds enough red paint to change the ratio to 1 : 3.

How much white paint must she add to change the ratio back to 1 : 4 ?

Purposeful Practice

A In a company, the ratio of the number of men to the number of women is 2 : 3.

$\frac{1}{4}$ of the men are part-time.

$\frac{1}{3}$ of the women are part-time.

There are 12 part-time workers at the company. Work out how many workers there are in total.

B On a bookshelf, the ratio of fiction to non-fiction books is 2 : 5.

The ratio of paperback books to hardback books on the shelf is 3 : 7.

Work out the smallest possible number of books on the shelf.

C At a gallery, there is a photography exhibition. The ratio of portrait to landscape photographs on show is 5 : 3.

60% of the portrait photographs are in colour. 24 of the portrait photographs are not in colour.

Work out the total number of photographs at the exhibition.

D The ratio of men to women in an orchestra is 3 : 2.

35% of the men play a stringed instrument. 45% of the women play a stringed instrument.

What percentage of all of the members of the orchestra play a stringed instrument?

E The table shows the prices for hiring boats at a lake.

	Rowing boat	Pedal boat
30 mins	£10	£12
1 hour	£16	£18

One day, the ratio of rowing boats to pedal boats hired was 2 : 3.

$\frac{5}{8}$ of the rowing boats hired were for 1 hour.

$\frac{5}{9}$ of the pedal boats hired were for 1 hour.

Given that there were 16 pedal boats hired for 30 minutes, work out the total sales income for the day.

F A box contains a number of tokens, of 4 different types. Each token is either blue or white, and either round or square.

The ratio of blue to white tokens in the box is 3 : 4.

The ratio of round blue tokens to square blue tokens is 1 : 2.

The ratio of round white tokens to square white tokens is 3 : 1.

What fraction of all the tokens are round?

G Pupils at a school all study exactly one foreign language.
The ratio of pupils who study French, German and Spanish is 4 : 3 : 2.

One of the French classes has 24 pupils, which is $\frac{2}{9}$ of all the pupils who study French.

Work out the number of pupils who study Spanish at the school.

Purposeful Practice

ratio problems (i)

(1) the ratio of shares owned by Asterix and Cleopatra is in the ratio 3 : 5

if Asterix gives Cleopatra 2 of his shares then the ratio will be 1 : 3

how many shares do they each have initially?

(2) Jan and Kim own numbers of marbles that are in the ratio 5 : 6

Jan gains 2 more marbles and the ratio is now 7 : 8

how many marbles do each own initially?

(3) the ratio of Ann's age to Bob's age is 3 : 4

in 7 years time this ratio will be 4 : 5

(i)   what are their ages now?
(ii) after how many years (from now) will the ratio be 5 : 6?

(4) the ratio of cockles to winkles in a bag of sea shells is 1 : 2

when 6 winkles are accidentally spilt from the bag the ratio is 3 : 5

how many of each shell were there initially?

(5) the ratio of geese to ducks on a lake is 4 : 9

when 10 extra geese land on the lake (and there are no duck changes) the ratio is 2 : 3

how many of each were there initially?

(6) the ratio of the width to depth (when they are measured in metres) of a dam is 5 : 7

when 10 extra metres is added to the width of the dam (and the depth remains the same) the ratio is 5 : 6

how wide and deep was the dam initially?

Purposeful Practice

ratio problems (ii)

(1)
bag A contains 7n counters
bag B contains 3n counters

3 counters are taken from bag A
and put in bag B

the ratio of counters in bag A to
bag B is now 5 : 3

how many counters were in the
bags initially?

(2)
box C contains 5k pens
box D contains 2k pens

8 pens are taken from box C and
placed in box D

the ratio of pens in box C to box D
is now 3 : 2

how many pens were there in
each box initially?

(3)
the ratio of pairs of white socks to
pairs of red socks is 9 : 5

9 pairs of the white socks are
dyed red

the ratio of pairs of white socks to
pairs of red socks is now 3 : 2

how many pairs of each colour
were there initially?

(4)
Sapna and Rob have many
paperweights, in the ratio of 8 : 3

Sapna, being a generous person, gives
Rob 14 of her paperweights

the ratio is now 6 : 5

how many paperweights did each have
initially?

explore what other ratios could have been
created

Purposeful Practice

ratio problems (iii) (simplest form involves integers)

(1) the ratio **a : b** = 3 : 4

what is the ratio

(i) b : 3a – b in its simplest form?

(ii) 2a + b : 4a – b in its simplest form?

(2) the ratio **a : b** = 5 : 6

the ratio **b : c** = 9 : 10

what is the ratio
(i) a : c in its simplest form?

(ii) a + b : 9b – 2a in its simplest form?

(3) the ratio **a : b** = 4 : 5

what is the ratio

(i) a + 2b : 3a – 2b in its simplest form?

(ii) 3b – 2a : a + 9b in its simplest form?

(4) the ratio **a : b** = 4 : 3

the ratio **a : c** = 2 : 5

what is the ratio
(i) b : c in its simplest form?

(ii) 2a – b : 2b + a in its simplest form?

(5) the ratio **a : b** = 2 : 5

the ratio **d : e** = 10a : 3b

what is the ratio
(i) d : e in its simplest form?

(ii) a + 2b : 2a + b in its simplest form?

(6) the ratio **a : b** = 3 : 2

the ratio **b : c** = 5 : 6

what is the ratio
(i) a : c in its simplest form?

(ii) 2a + b : 2b + c in its simplest form?

Purposeful Practice

ratio problems (iv)

(1)

the ratio

(a number add 2 : the number add 4)

is the same as the ratio

(the same number add 6 : this number add 9)

what is the number?

(2)

the ratio

(a number take 1 : the number add 3)

is the same as the ratio

(the same number add 1 : this same number add 6)

what is the number?

(3)

the ratio

(twice a number add 1 : this number take 1)

is the same as the ratio

(twice the same number add 8 : this same number add 2)

what is the number?

(4)

the ratio

(a number add 1 : the number add 4)

is the same as the ratio

(twice the same number take 5 : this same number add 20)

what is the number?

[find two solutions if you can]

Purposeful Practice

Cube Ratios

① red : blue

some blue cubes added

2:5 → 2:7

What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were added?

② red : blue

some blue cubes added

2:3 → 1:5

What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were added?

③ red : blue

some blue cubes added

2:3 → 4:9

What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were added?

If 6 cubes were added, how many are there now?

④ red : blue

some blue cubes added

3:2 → 5:3

Why is this impossible?

⑤ red : blue

some blue cubes removed

1:3 → 3:5

What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were removed?

If 12 cubes were removed, how many are there now?

⑥ red : blue

some red cubes removed

2:3 → 2:5

What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were removed?

If 20 cubes were removed, how many are there now?

⑦ red : blue

some red cubes added

3:4 → 6:7

What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were added?

If 30 cubes were added, how many were there?

⑧ red : blue

some red cubes added

5:6 → 4:5

Why is this impossible?

⑨ red : blue

12 red cubes added

2:5 → 4:7

How many cubes are there now?

ratio problems involving algebra

(1) if $a : 7 = b : 4$ then $(a + b) : 11 = (a - b) : 3$

(2) if $a : b = c : d$ then $a : c = (a + b) : (c + d)$

(3) if $a : b = c : d$ then $(a + b) : (a - b) = (c + d) : (c - d)$

(4) if $3a : 5b : 6c = p : q : r$ then $a : b : c = 10p : 6q : 5r$

(5) if $a : b = b : c$ then $(b + c) : c = (a - c) : (b - c)$

(6) if $a : b = b : c$ then $(a + b) : a = (a - c) : (a - b)$

Purposeful Practice

Ratio Worded Problems

(a)	(b)	(c)	(d)
A pencil case contains 5 blue pencils, 3 red pencils and 7 black pencils. What fraction of the pencils are red? Give your answer in its simplest form.	In a florist shop, the ratio of roses to tulips to gerberas is 7 : 2 : 3. What fraction of the flowers in the shop are roses or tulips? Give your answer in its simplest form.	The ratio of the number of boys to the number of girls in a school is 4 : 7. There are 91 girls in the school. Work out the number of boys in the school.	The books in a library are fiction or non-fiction in the ratio 3 : 2. If there are 573 fiction books, how many books are there in the library in total?

(e)	(f)	(g)	(h)
A map has a scale of 1 : 500. Martha measures a distance of 5 cm on the map. What actual distance in m does this correspond to?	Madeira cake mixes flour, sugar and butter in the ratio 5 : 3 : 2. How much flour and butter is required to mix with 75 g sugar?	In a car park there are twice as many white cars as black cars, and three times as many red cars as white cars. Write the ratio of cars red : white : black in its simplest form.	A recipe to make 8 pancakes needs 200 ml milk and 120 g flour. How much milk and flour is need to make 20 pancakes?

(i)	(j)	(k)	(l)
The angles in a quadrilateral are in the ratio 5 : 7 : 4 : 2. Find the size of each of the angles.	120 ml of orange drink is made with cordial and water in the ratio 1 : 4. To make the drink stronger 10 ml more of cordial is added. What is the ratio of cordial to water for the new drink?	Ishaq builds a model rocket. The height of the model rocket is 16 cm, and the actual height of the model is 48 m. Find the ratio of the model rocket to the real rocket.	Jim mixes 1 litre of red and white paint in the ratio 1 : 3. How many ml of red paint should be added so that the ratio becomes 8 : 15 ?

Purposeful Practice

Ratio 1

① Some money is shared in the ratio 2 : 3 : 5

The largest share is £45 larger than the smallest.

How much money is there in total?

---

② Trees in a forest, evergreen : deciduous = 5 : 2

For ever-green trees, healthy : unhealthy = 4 : 7

There were 35 unhealthy evergreen trees. How many deciduous trees were there?

---

③ Write the ratio 9 : 4 in the form n : 1 where n Is a decimal

$$a : b = 5 : 3$$

How many times larger is a than b?

---

④ The value of y is 50% more than the value of x.

Express the ratio $x : y$ using integers.

$$a : b = 3 : 2$$
$$a : c = 4 : 7$$

Express the ratio a : b : c

---

⑤ There are triangles, hexagons & circles in a bag.

triangles to hexagons = 2:3
hexagons to circles = 6:7

If there are less than 100 shapes in the bag, what is the greatest number of hexagons in the bag?

---

⑥ Jan has some money.
Caleb has £6 more than four times the money Jan has.

In total they have £51.

Express this as a ratio Jan : Caleb

---

⑦ $$\frac{1}{7} : \frac{3}{5} = x : 1$$

Calculate the value of x.

---

⑧ The points A, B, C and D lie in order on a straight line.
AB : BD = 1:3
AC : CD = 5:7
Work out AB : BC : CD

---

⑨ Greg scored 20 more than Kim.
Tan scored triple Kim's score.
In total they scored
80 points at archery.

Find the ratio
Kim's score : Greg's score

---

⑩ a : b = 2 : 5 and 4b = 5c
Work out a : c in its simplest form.

---

⑪ b is one-quarter of c
$$2a = 3c$$
Express the ratio $a : b : c$ using integers.

---

⑫ Ann has £28 in savings.
Bill has no savings.

Each week from now,
Ann saves £6 & Bill saves £4.

How many weeks will it take
for Ann & Bill to
have savings in the ratio 5 : 2?

Purposeful Practice

Ratio 1

① A distance is split in the ratio 5 : 7 : 2

The smallest distance is 3.5 km smaller than the largest.

What is the total distance?

② Electricity generation, renewable : non-renewable = 9 : 5

For renewable energy, solar : wind = 11 : 8

18 MW of power was generated from wind power. How much energy was from non-renewable sources?

③ Write the ratio 4 : 2.5 in the form n : 1 where n is a decimal

$a : b = 3 : \frac{2}{3}$

How many times larger is a than b?

④ The value of y is 20% more than the value of x.
Express the ratio $x : y$ using integers.

$a : b = 2 : 3$
$b : c = 7 : 8$

Express the ratio $a : c$

⑤ There are octagons, pentagons & squares in a bag.

octagons to pentagons = 4:9
octagons to squares = 5:8

If there are less than 1000 shapes in the bag, what is the greatest number of squares in the bag?

⑥ On a test, Will scored double the lowest score.

Dan scored 140 marks more than the lowest score.

Will's score : Dans score = 1:4

What was the lowest score?

⑦ $$\frac{2}{5} : \frac{6}{7} = x : 1$$

Calculate the value of x.

⑧ The points A, B, C and D lie in order on a straight line.
AB : BD = 1:2
AC : CD = 3:1
Work out AB : BC : CD

⑨ Wai has £8 less than Toby. Cindy has 4 times the money Wai has. In total they have £260.

Find the ratio of money
Toby : Cindy

⑩ $a : b = 4 : 5$ and $3b = 10c$
Work out a : c in its simplest form.

⑪ b is two-fifths of c
$3a = 7c$
Express the ratio $a : b : c$ using integers.

⑫ Cane has £27 in savings. Dale has £28 savings.

Each week from now, Cane saves £6 & Dale saves £3.

How many weeks will it take for Cane & Dale to have savings in the ratio 3 : 2?

Purposeful Practice

A. Work out the ratio $a : b : c$ in the simplest form.
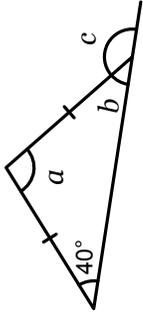
B. Shapes A and B have the same height and the same area. Work out the ratio *perimeter of A : perimeter of B* in the simplest form.
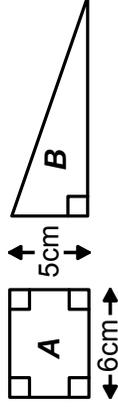
C. Work out the ratio $x : y$ in the form $1 : n$.
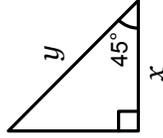
D. Given that:

$$2x - 9 : x - 4 = x : 4$$

Work out the value of x.

E. A shop sells a type of cereal in two packet sizes. The larger packets cost 40% more than the smaller packets.

Work out the ratio of the cost of a smaller packet to the cost of a larger packet in the form $1 : n$.

F. Jason sells red, blue and green raffle tickets. He starts with 120 of each colour.

Jason managed to sell red, blue and green tickets in the ratio $3 : 2 : 1$. 168 of the tickets remained **unsold**.

Work out how many of each colour ticket **were sold**.

G. Penny has a small block of iron and a small block of silver.

The iron is in the shape of a cuboid, with dimensions 31mm, 25mm and 43mm.

The silver has a mass of 324g and has a density of $10.5 \text{g}/\text{cm}^3$.

Work out the ratio of the volume of iron to the volume of silver in the form $1 : n$.

H. The prices of two games are in the ratio $a : b$.

If both prices were decreased by £10, the ratio would become $3 : 5$.

If both prices were increased by £5, the ratio would become $3 : 4$.

Work out the ratio $a : b$ in the simplest form.

I. In the diagram, the dots represent points on a number line.
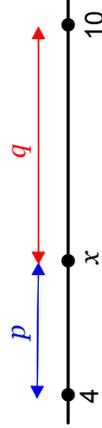
Given that $p : q = 1 : 3$, work out x.

J. In the diagram, the dots represent points on a number line.
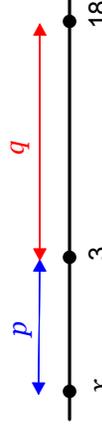
Given that $p : q = 2 : 5$, work out x.

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Examples

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Corbett mCths

Question 1: The table below shows information about the number of medals won by 6 countries in the 2016 Olympic Games.

Country	Gold	Silver	Bronze	Total
Spain	7	4	6	17
France	10	18	14	42
Germany	17	10	15	42
Italy	8	12	8	28
Japan	12	8	21	41
Australia	8	11	10	29

(a) Write down the ratio of the total number of medals won by Italy to the total number of medals won by France.
Give your answer in its simplest form.

(b) Write down the ratio of the total number of gold medals won to the total number of bronze medals won.
Give your answer in its simplest form.

Question 2: There are between 30 and 40 sweets in a bag.
Ben and Daisy share the sweets in the ratio 4:5.
There are no sweets remaining in the bag.

How many sweets were in the bag to begin with?

Question 3: George buys some first class and second class stamps to put on some letters.
Each first class stamp costs 67p
Each second class stamp costs 58p
The ratio of first class stamps to second class stamps that he buys is 1:4
George bought 20 stamps.

How much did the stamps cost George?

Question 4: 1,935 people visit a library during one week.
The ratio children : adults is 1 : 4
How many more adults than children visited the library?

Ratio: Problem Solving
Video 271e on www.corbettmaths.com

Question 5: The sizes of the interior angles of a triangle are in the ratio 1:3:8
Calculate the difference in size between the largest and smallest angles.

Question 6: Mr Rodgers is organising a trip to a football match for 84 children.
The ratio of teachers on the trip to children on the trip is 1:6
Each bus holds 30 passengers.
Show four buses are needed.

Question 7: Isaac and Victoria share money in the ratio 2:5
Victoria receives £6.60
Work out the difference between how much money Isaac and Victoria receive.

Question 8: Charlotte and Melissa booked theatre tickets costing £180.
They have a voucher that entitles them to 20% off the total price.
Charlotte and Melissa share the total cost of the tickets in the ratio 5:7.

Work out how much more Melissa pays than Charlotte.

Question 9: Barnaby has 288 counters in a bag.
The counters are red, yellow and white.

$\frac{3}{8}$ of the counters are red.

The other counters are yellow and white in the ratio 1:4

Work out how many counters of each colour there are.

Question 10: The sizes of the interior angles of a pentagon are in the ratio 1:2:5:5:7
Calculate the size of the largest

Question 11: Jack is 10 years old.
Kylie is 17 years old.
Vanessa is 23 years old.
Kylie and Vanessa share £16 in the ratio of their ages.
Kylie gives 20% of her share to Jack.
Vanessa gives a quarter of her share to Jack.
How much money does Jack receive?
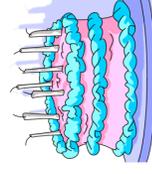
Question 12: Two numbers are in the ratio 3:7
One of the numbers is 42
There are two possible values for the other number.
What are the two possible values?

Question 13: Alex is 12 years older than George.
Carl is three times older than Alex
The sum of their ages is 68
Find the ratio of George's age to Carl's age to Alex's age.

Purposeful Practice

Question 14: Felicity has two boxes of counters, each with an equal ratio of black and white beads.
In box A, 54 of the beads of black and 36 are white.
There are 162 white beads in box B.

How many beads are there in the two boxes in total?

Question 15: Sophie is making 400 scones.
She uses butter, sugar and flour in the ratio 2:1:9
Here are the costs of those ingredients.

Butter	£2.20 per 500g
Sugar	£1.60 per kilogram
Flour	60p per 1.5kg

The total mass of the butter, sugar and flour in each scone is 30g

Work out the total cost of these ingredients for the 400 scones.

Question 16: Copper costs £5 per kilogram.
Zinc costs £3.20 per kilogram.
Copper and zinc are mixed in the ratio 4:1 to make brass.

Work out the cost of 7 kilograms of brass.

Question 17: Geraint has 2p and 50p coins in the ratio 20 : 3

Write the ratio of the value of the 2p coins to the value of 50p coins in its simplest form.

Question 18: Mrs Chambers is organising a school trip to a museum for year 7 and year 8.
She needs to work out the total cost of the museum tickets and bus hire.
The table below shows the museum ticket prices.

Visitor Age	Price
0 - 3	free
4 - 12	£4.50
13 - 17	£6.50
18+	£11.50

Each bus has 51 seats and costs £125

Altogether 300 students want to go on the trip.
The ratio of the number of students to the number of teachers is 25:1
The ratio of the number of students in year 7 to the number of students in year 8 is 8:7

At the time of the trip, all of the students in year 7 are 11 or 12 years old.
Of year 8 students, the ratio of number of 12 year olds to 13 year olds is 2:3.
Work out the total price of the school trip.

Ratio: Problem Solving
Video 271e on www.corbettmaths.com

Question 19: Alannah, Colin and Hannah visit the park and collect conkers and acorns.

In total they collect the same number of conkers and acorns.

Alannah, Colin and Hannah collect conkers in the ratio 4:7:9
Alannah, Colin and Hannah collect acorns in the ratio 3:1:5

(a) Did Hannah collect more conkers or acorns?

(b) What is the lowest possible number of conkers they could have collected?

Question 20: A bag contains red, yellow and blue beads.

The ratio of red beads to yellow beads is 2:3
The ratio of yellow beads to blue beads is 5:4

Work out what fraction of the beads are red.

Question 21: In a box

the number of blue counters and the number green counters are in the ratio 7:4
the number of green counters and the number of red counters are in the ratio 3:1

The total number of counters in the bag is 444.

How many green counters are in the bag?

Question 22: The ratio of the red cards to black cards in a deck is 3:10
2 more red cards are added to the deck.
The ratio of red cards to black cards is now 1:3

Work out the number of black cards in the deck.
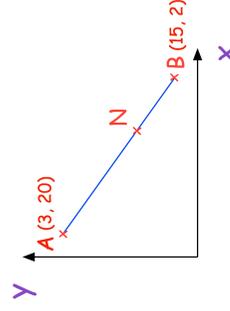
Question 23: A is the point with coordinates (3, 20)
B is the point with coordinates (15, 2)
N is a point of the line AB such that AN : NB = 2 : 1

Find the coordinates of the point N.

Question 24: Bag A contains 5x coins.
Bag B contains 3x coins.
8 coins are taken from Bag B and put into Bag A
The ratio of coins in Bag A to Bag B is now 11:5

Work out the total number of coins.

Purposeful Practice

Question 25: A cone has radius 4cm and perpendicular height of 9cm.
A sphere has a radius of 6cm
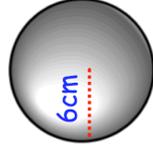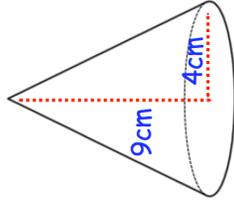
Volume of a sphere	$\frac{4}{3}\pi r^3$
Volume of a cone	$\frac{1}{3}\pi r^2 h$

9cm   4cm

6cm

Calculate the ratio of the volume of the cone to the volume of the sphere.

Question 26: On 1st March 2001, the ratio of Freddie's age to his mother's age was 1:11
On 1st March 2018, the ratio of Freddie's age to his mother's age was 2:5
Write the ratio of Freddie's age to his mother's age on 1st March 2030

Question 27: The ratio 25 000 000 000 : 500 can be written in the form $n : 1$
Work out the value of n

Give your answer in standard form

Question 28: The distance of Mercury from the Sun is 5.7×10^7 km
The distance of Neptune from the Sun is 4.3×10^9 km

Work out the ratio of the distance of Mercury from the Sun to the distance of Neptune from the Sun.

Question 29: Class 10D make some cakes using milk chocolate, dark chocolate or white chocolate.
Some of the cakes contain nuts and the rest do not.

The ratio of the number of milk chocolate cakes to dark chocolate cakes is 10:3
The ratio of the number of white chocolate cakes to milk chocolate cakes is 1:6

Of the milk chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 1:8

Of the dark chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 3:2

Of the white chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 2:5

What percentage of the cakes contain nuts?

Answers

Click here

Purposeful Practice

gcse ratio problems (based on Edexcel)

(1) Riley and Dan share some jelly beans
Dan gets 40% of the jelly beans
write down the ratio of the number of jelly beans
Riley gets to the number that Dan gets

(2) there are 20 boxes of eggs with 6 eggs in each box
write, as a ratio, the number of eggs in four of the
boxes to the number of eggs that there are altogether
what does this ratio simplify to?

(3) Harry, Ryan and Kara share £405 in the ratio 3 : 2 : 4
how much money does Kara get?

(4) Arnold and Lara share some money in the ratio 7 : 13
what percentage of the money does Lara get?

(5) a path is made of white tiles and grey and
¼ of the tiles are white
(i) what is the ratio of white tiles to grey tiles ?

there are 72 tiles altogether
(ii) work out the number of grey tiles

(6) there are only red buttons, yellow buttons and orange
buttons in a jar
these are in the ratio 2 : 4 : 9 respectively

what percentage of buttons in the jar that are orange?

(7) 2.5 kg of plums cost £5.10
work out the cost of 3.5 kg of plums at the same price

(8) there are only red, orange and green tennis balls in a bag
there are three times as many orange as red
and four times as many green as orange

Hannah takes a tennis ball at random from the bag
work out the probability that Hannah picks a green one

(9) the ratio of the cost of one metre of cotton to the
cost of one metre of silk (same widths) is 2 : 7

if 2m of cotton cost £8,
how much does 9m of silk cost?

(10) this recipe for flapjacks makes 12 pieces

    120 g butter
    140 g light brown sugar
    250 g porridge oats
    2 tablespoons syrup

how much of each ingredient is needed to make
18 pieces?

(11) Kim is 6 years older than Jim
Tim is twice as old as Kim
the sum of their three ages is 90 years

find the ratio of Jim's age : Kim's age : Tim's age
in the simplest possible form

Purposeful Practice

gcse ratio problems (based on Edexcel)

(12) one Saturday, some adults and some children were in a the Grand theatre, Wolverhampton

the ratio of the number of adults to the number of children was 7 : 4

each person sat in the circle or the stalls

¾ of the children had seats in the stalls
68 children had seats in the circle

there are exactly 1200 seats in the theatre

on this particular Saturday, were there people on more than 60% of the seats?

(13) Jay bakes 280 cakes,

four different kinds of cake are baked:
vanilla, banana, lemon and chocolate

- 2 out of every 7 of the cakes are vanilla cakes
- 35% of the cakes are banana cakes
- the ratio of the number of lemon cakes to the number of chocolate cakes is 1 : 5

are there more chocolate cakes than vanilla cakes?

you must show your working to support your answer

(14) it normally takes 5 cleaners 4½ hours to clean all of the rooms in a hotel

today there are only 3 cleaners to clean all of the rooms

each cleaner is paid at a rate of £8.10 per hour (or part of an hour) that he/she works

how much will **each** cleaner be paid today?

(15) Theo drove 54 km from Liverpool to Manchester
he then drove 62 km from Manchester to Sheffield

Theo's average speed from Liverpool to Manchester was 72 km/h

Theo then took 75 minutes to drive from Manchester to Sheffield

work out Theo's average speed for his total drive from Liverpool to Sheffield

(16) a gold bullion bar has a mass of 12.4 kg

the density of gold is 19.32 g/cm^3

work out the volume of the gold bar

give your answer correct to 1 decimal place

Purposeful Practice

gcse ratio problems (based on Edexcel)

(17) the density of apple juice is 1.04 grams per cm^3
the density of fruit syrup is 1.4 grams per cm^3
the density of carbonated water is 0.99 grams per cm^3

30 cm^3 of apple juice are mixed with 10 cm^3 of fruit syrup and 280 cm^3 of carbonated water to make a drink with a volume of 320 cm^3

work out the density of the drink

give your answer correct to 2 decimal places

(18) there are only blue, black and red pens in a box

the ratio of the number of blue to the number of black is 2 : 5

the ratio of the number of black to the number of red is 4 : 1

there are less than 100 pens altogether in the box

what is the greatest possible number of red pens in the box?

(19) Jack and Jill cycled along the same 40 km route
Jack took 2½ hours to cycle the 40 km

Jill started to cycle 10 minutes after Jack started to cycle
Jill caught up with Jack when they had both cycled 24 km
Jack and Jill both cycled at constant speeds

work out Jill's speed

(20) the points *A*, *B*, *C* and *D* lie in order along a straight line
$$AB : BD = 2 : 5$$
$$AC : CD = 11 : 10$$

work out the ratio   *AB* : *BC* : *CD*

(21) white shapes and black shapes are used in a game

some shapes are circles and the other shapes are squares

the ratio of white shapes to black shapes is 3 : 7

the ratio of white circles to white squares is 4 : 5

the ratio of black circles to black squares is 2 : 5

work out what fraction of all the shapes are circles

Exam Questions

1

(a) A box contains only red pens and blue pens.

The ratio   red : blue = 3 : 2

There are 24 **red** pens.

Work out the number of pens in the box.

[2 marks]

(b) Another box contains only green pens and black pens.

$\frac{3}{4}$ of the pens are **green**.

What is the ratio of green pens to black pens?

[1 mark]

2 Amy has some red beads and some blue beads.
She uses the beads to make two necklaces, A and B.

Necklace A 24 beads 3 times as many red beads as blue beads
Necklace B 35 beads number of red beads : number of blue beads = 3 : 2

How many **more** red beads than blue beads does she use?

[4 marks]

3 There are 60 balls in a bag.
The balls are either blue, red or yellow.

The ratio of blue balls to red balls is   5 : 3
The ratio of red balls to yellow balls is   1 : 4

Work out how many **red** balls are in the bag.

[3 marks]

4 Ahmed, Ben and Carla share some money.

Ahmed gets $\frac{3}{7}$ of the money.

Ben and Carla share the rest of the money in the ratio 3 : 2

Carla gets £64

Work out how much Ahmed gets.

[5 marks]

5 Jack sells packs of envelopes.
There are three types of pack, Economy (E), Standard (S) and Luxury (L).

One week he sells 270 packs in the ratio

E : S : L = 5 : 3 : 2

He makes a profit of

12p a pack on Economy packs
15p a pack on Standard packs
20p a pack on Luxury packs.

Work out the **total** profit he makes selling envelopes.

[3 marks]

6 Three of the notes on a piano keyboard are C, F and G.

Each note has a frequency, measured in Hertz (Hz)

frequency of C : frequency of F = 3 : 4

frequency of F : frequency of G = 8 : 9

(a) Show that   frequency of C : frequency of G = 2 : 3

[2 marks]

(b) The frequency of C is 261.6 Hz

Work out the frequency of G.

[2 marks]

Exam Questions

7 A blend of tea contains Darjeeling tea and Assam tea in the ratio

Darjeeling : Assam = 4 : 1

Darjeeling tea costs £14 per kilogram.
Assam tea costs £42 per kilogram.

Work out the cost of 250 grams of this blend.

[5 marks]

8 A, B and C are such that

$$A : B = 1 : 5$$

B is $\frac{3}{4}$ of C

Work out the ratio A : C

[3 marks]

9 $0.4a = 0.75b$

Work out the ratio $\quad a : b$
Give your answer in its simplest form.

[3 marks]

10 Bob and Kai share £x in the ratio $\qquad$ 2 : 3

How much money does Kai receive?
Circle the correct expression.

£$\frac{2}{3}x$ £$\frac{2}{5}x$ £$\frac{3}{5}x$ £$\frac{5}{8}x$

[1 mark]

Bob gives Kai £8 from his share.
The amount of money they each have now is in the ratio $\qquad$ 2 : 5

Work out the value of x.
You **must** show your working.
Do **not** use Trial and Improvement.

[4 marks]