KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Mathematics 2026

Unit 22 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

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Unit 22 Booklet – Part 2

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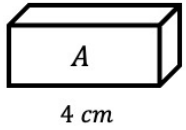
Contents

1 Similarity with Area and Volume
2 Arcs, Sectors and Segments
3 Volume and Surface Area of Non-Prisms
4 Advanced Probability
5 Capture-Recapture

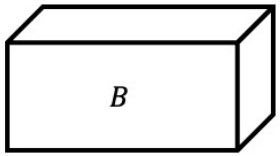
1 Similarity with Area and Volume

Worked Example

Cuboids A and B are similar.



4 cm



8 cm

Write down the scale factor for:

Length $A \rightarrow B$

Length $B \rightarrow A$

Surface Area $A \rightarrow B$

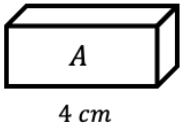
Surface Area $B \rightarrow A$

Volume $A \rightarrow B$

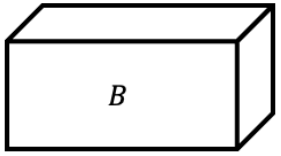
Volume $B \rightarrow A$

Your Turn

Cuboids A and B are similar.



4 cm



12 cm

Write down the scale factor for:

Length $A \rightarrow B$

Length $B \rightarrow A$

Surface Area $A \rightarrow B$

Surface Area $B \rightarrow A$

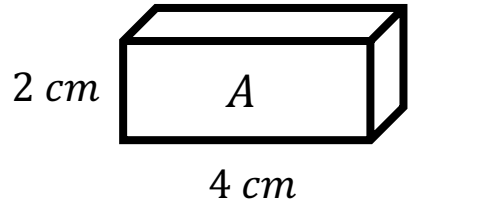
Volume $A \rightarrow B$

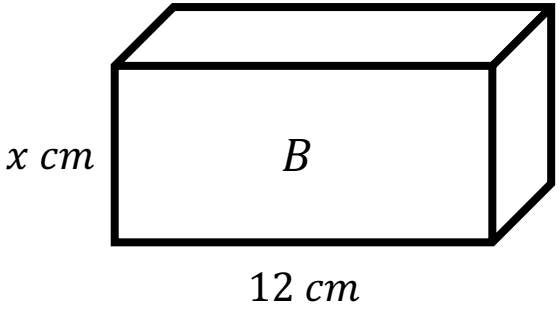
Volume $B \rightarrow A$

Dr Frost 324a and 324d

Worked Example

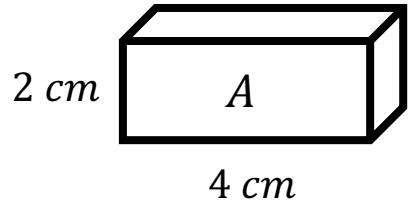
Cuboids A and B are similar.
Find x.

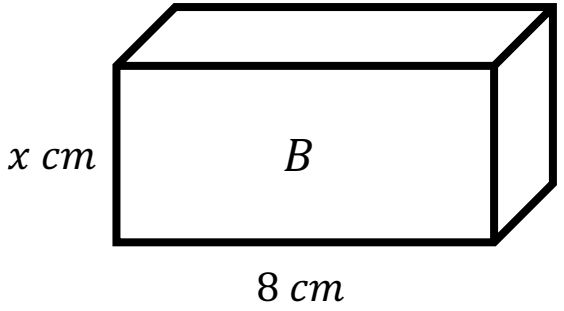




Your Turn

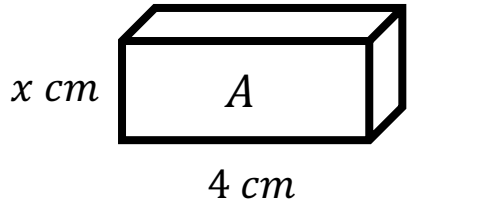
Cuboids A and B are similar.
Find x.

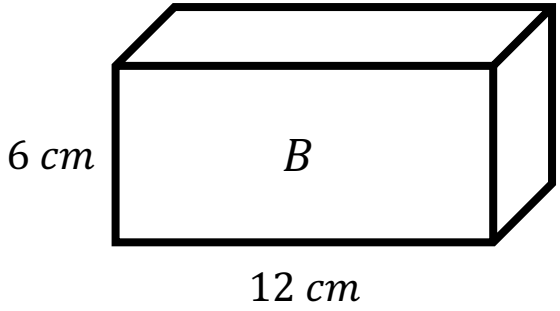




Worked Example

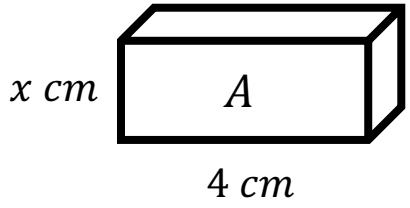
Cuboids A and B are similar.
Find x.

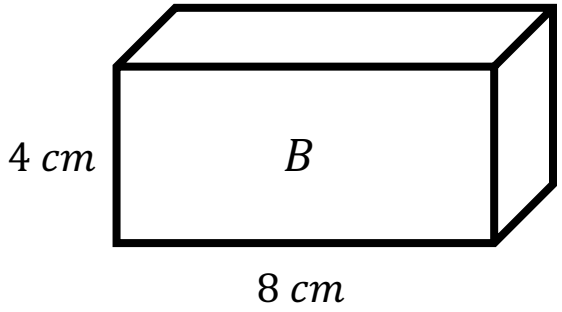




Your Turn

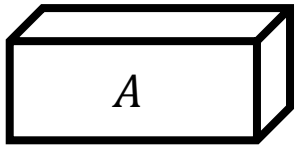
Cuboids A and B are similar.
Find x.



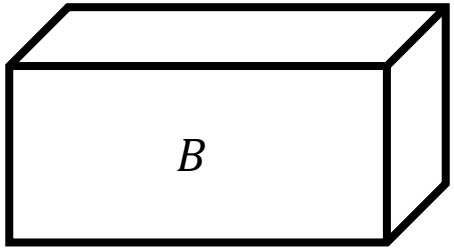


Worked Example

Cuboids A and B are similar.
The surface area of cuboid A is $72\ cm^2$. What is the surface area of cuboid B?



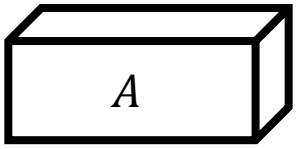
$4\ cm$



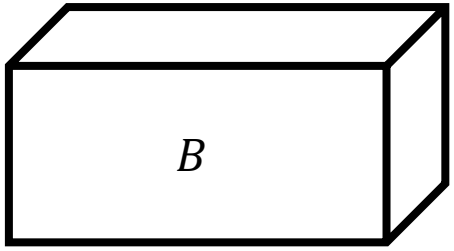
$12\ cm$

Your Turn

Cuboids A and B are similar.
The surface area of cuboid A is $72\ cm^2$. What is the surface area of cuboid B?



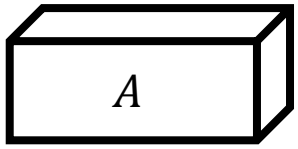
$4\ cm$



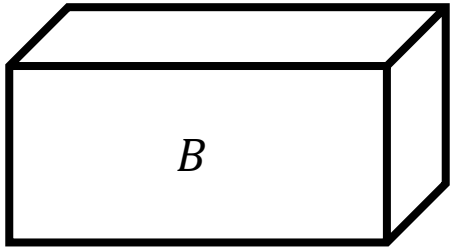
$8\ cm$

Worked Example

Cuboids A and B are similar.
The surface area of cuboid B is $72\ cm^2$. What is the surface area of cuboid A?



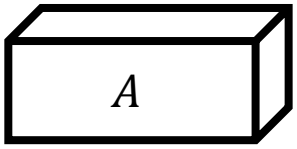
$4\ cm$



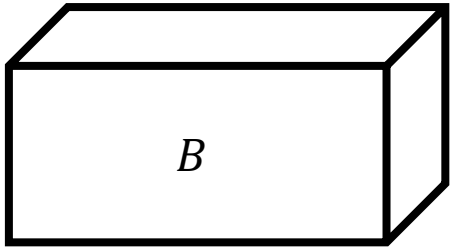
$12\ cm$

Your Turn

Cuboids A and B are similar.
The surface area of cuboid B is $72\ cm^2$. What is the surface area of cuboid A?



$4\ cm$

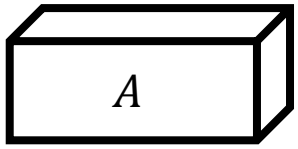


$8\ cm$

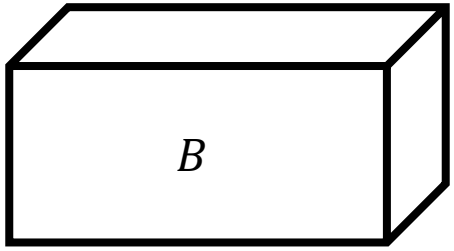
Dr Frost 324b and 324c

Worked Example

Cuboids A and B are similar.
The volume of cuboid A is $432\ cm^3$. What is volume of cuboid B?



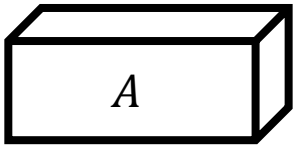
$4\ cm$



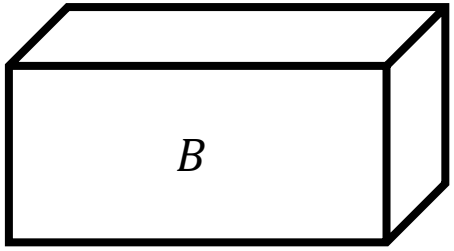
$12\ cm$

Your Turn

Cuboids A and B are similar.
The volume of cuboid A is $432\ cm^3$. What is volume of cuboid B?



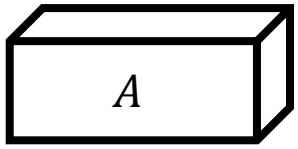
$4\ cm$



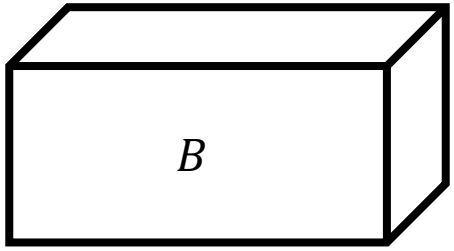
$8\ cm$

Worked Example

Cuboids A and B are similar.
The volume of cuboid B is $432\ cm^3$. What is volume of cuboid A?



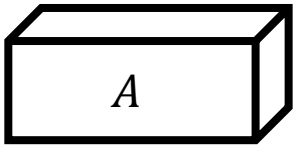
$4\ cm$



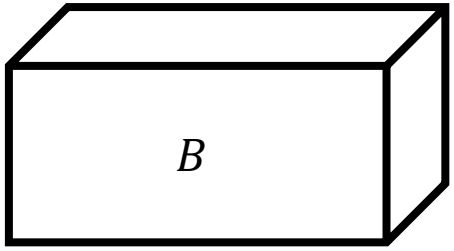
$12\ cm$

Your Turn

Cuboids A and B are similar.
The volume of cuboid B is $432\ cm^3$. What is volume of cuboid A?



$4\ cm$



$8\ cm$

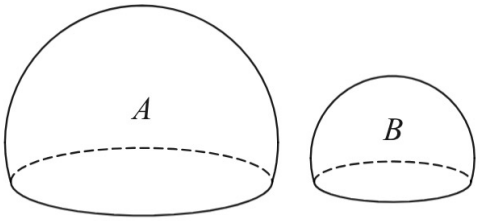
Dr Frost 324e and 324f

Worked Example	Your Turn
A and B are mathematically similar solids. The surface area of A is $100\ cm^2$. The surface area of B is $64\ cm^2$. Work out the ratio of the volume of A to the volume of B.	A and B are mathematically similar solids. The surface area of A is $120\ cm^2$. The surface area of B is $480\ cm^2$. Work out the ratio of the volume of A to the volume of B.

Worked Example	Your Turn
A and B are mathematically similar solids. The volume of A is $500\ cm^3$. The volume of B is $256\ cm^3$. Work out the ratio of the surface area of A to the surface area of B.	A and B are mathematically similar solids. The volume of A is $120\ cm^3$. The volume of B is $960\ cm^3$. Work out the ratio of the surface area of A to the surface area of B.

Worked Example

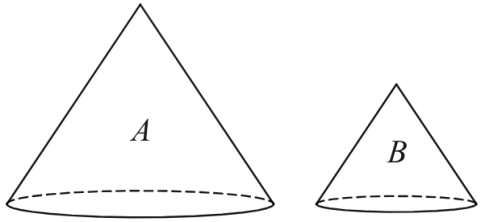
A and B are two solid shapes that are mathematically similar.
The shapes are made from the same material.



The total surface area of shape A is 62.5 cm^2.
The total surface area of shape B is 22.5 cm^2.
The mass of A is 375 g. Find the mass of B.

Your Turn

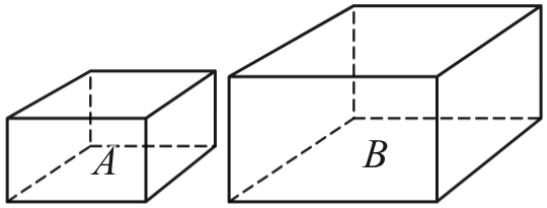
A and B are two solid cones that are mathematically similar.
The cones are made from the same material.



The total surface area of shape A is 75 cm^2.
The total surface area of shape B is 27 cm^2.
The mass of B is 81 g. Find the mass of A.

Worked Example

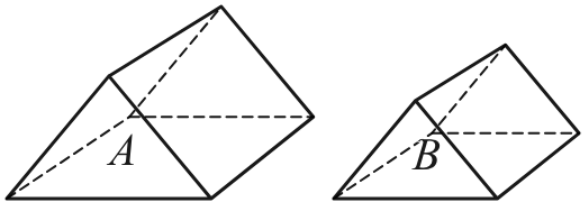
A and B are two solid cuboids that are mathematically similar.
The surface area scale factor from A to B is 2.25



Find the ratio of the volume of cuboid A to the volume of cuboid B

Your Turn

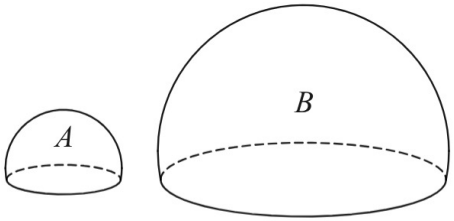
A and B are two solid prisms that are mathematically similar.
The surface area scale factor from A to B is 0.64



Find the ratio of the volume of prism A to the volume of prism B

Worked Example

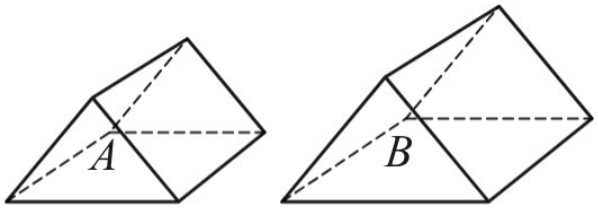
Shapes A and B are similar.



The volume of B is 1462.5% greater than the volume of A. Find the percentage increase from the surface area of shape A to the surface area of shape B.

Your Turn

Prisms A and B are similar.



The surface area of B is 44% greater than the surface area of A. Find the percentage increase from the volume of prism A to the volume of prism B.

Worked Example	Your Turn
The surface area of two mathematically similar solids are in the ratio $16:49$. The volume of the smaller solid is $128\ cm^3$. Work out the volume of the larger solid.	The surface area of two mathematically similar solids are in the ratio $9:25$. The volume area of the smaller solid is $108\ cm^3$. Work out the volume of the larger solid.

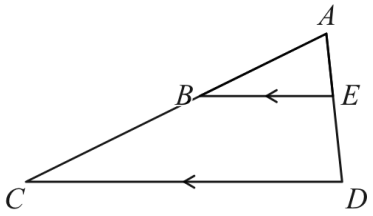
Worked Example	Your Turn
The volume of two mathematically similar solids are in the ratio $64: 343$. The surface area of the smaller solid is $32\ cm^2$. Work out the surface area of the larger solid.	The volume of two mathematically similar solids are in the ratio $27: 125$. The surface area of the smaller solid is $36\ cm^2$. Work out the surface area of the larger solid.

Worked Example	Your Turn
a) The scale of a map is $1 : 3000000$ On the map, the area of Toonhill Valley is 6.4 cm^2. Calculate the actual area of Toonhill Valley. Give your answer in square kilometres. b) The scale of a map is $1 : 400000$ The area of Lake Troycou is 646.4 km^2. Calculate the area of Lake Troycou on the map in cm^2.	a) The scale of a map is $1 : 70000$ On the map, the area of Fort Prisetomb is 50.6 cm^2. Calculate the actual area of Fort Prisetomb. Give your answer in square kilometres. b) The scale of a map is $1 : 60000$ The area of Saint Ralhay is 10.8 km^2. Calculate the area of Saint Ralhay on the map in cm^2.

Dr Frost 324m and 324n

Worked Example

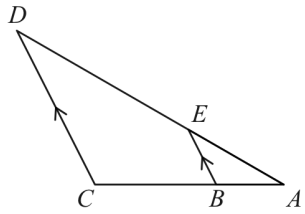
In the diagram ABC and AED are straight lines and BE is parallel to CD.



The length AB is 20 cm and the length BC is 27.5 cm.
The area of triangle ABE is 83.2 cm^2.
Work out the area of triangle ACD.

Your Turn

In the diagram ABC and AED are straight lines and BE is parallel to CD.

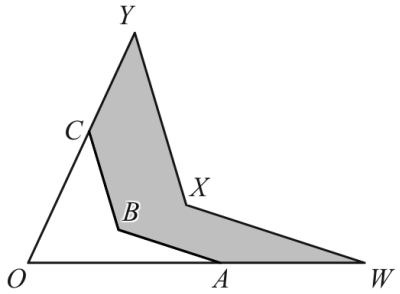


The length AB is 10.5 cm and the length BC is 18.9 cm.
The area of triangle ABE is 45 cm^2.
Work out the area of triangle ACD.

Dr Frost 324o

Worked Example

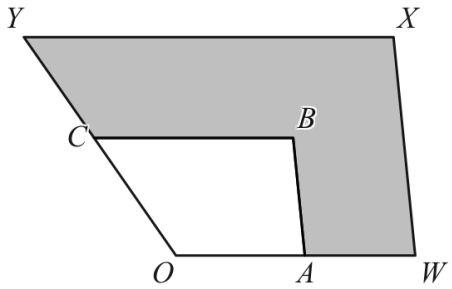
The diagram shows two similar quadrilaterals $OABC$ and $OWXY$.



The length of AB is 4 cm and the length of WX is 7 cm.
The area of quadrilateral $OABC$ is 9.6 cm^2.
Find the shaded area.

Your Turn

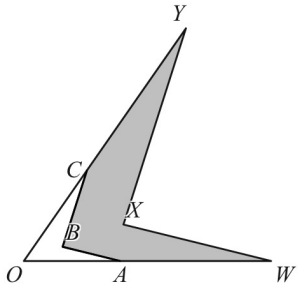
The diagram shows two similar trapeziums $OABC$ and $OWXY$.



The length of OA is 7 cm and the length of AW is 6 cm.
The area of trapezium $OABC$ is 27.44 cm^2.
Find the shaded area.

Worked Example

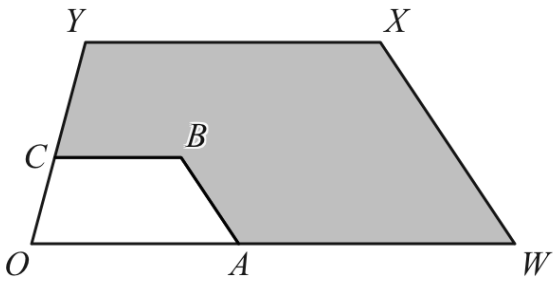
The diagram shows two similar quadrilaterals $OABC$ and $OWXY$.



The length of AB is 7 cm and the length of WX is 18 cm.
The area of quadrilateral $OWXY$ is 324 cm².
Find the shaded area.

Your Turn

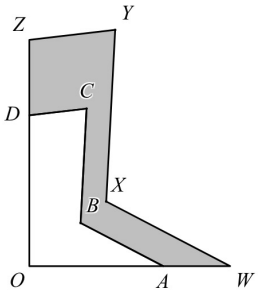
The diagram shows two similar trapeziums $OABC$ and $OWXY$.



The length of OA is 6 cm and the length of AW is 8 cm.
The area of trapezium $OWXY$ is 235.2 cm².
Find the shaded area.

Worked Example

The diagram shows two similar pentagons $OABCD$ and $OWXYZ$

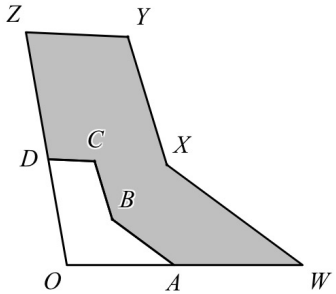


The length of OA is 8 cm and the length of AW is 4 cm
The shaded area is 176 cm^2

Find the area of pentagon $OWXYZ$

Your Turn

The diagram shows two similar pentagons $OABCD$ and $OWXYZ$



The length of OA is 5 cm and the length of AW is 6 cm
The shaded area is 192 cm^2

Find the area of pentagon $OABCD$

Extra Notes

2 Arcs, Sectors and Segments

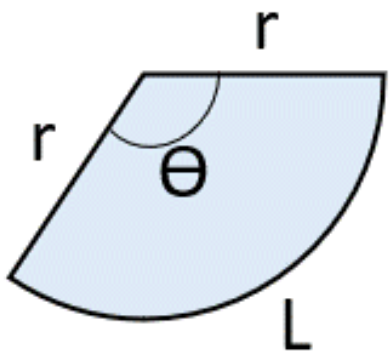
Arc Length and Perimeter of Sectors and Segments

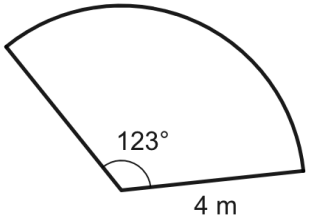
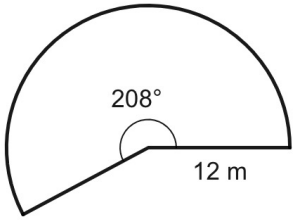
Arc Length of a Sector $= \frac{\text{Angle}}{360} \times \pi \times \text{Diameter}$

$$\mathbf{L = \frac{\theta}{360} \times \pi d}$$

Perimeter of a Sector $= \frac{\text{Angle}}{360} \times \pi \times \text{Diameter} + 2 \times \text{Radius}$

$$\mathbf{P = \frac{\theta}{360} \times \pi d + 2r}$$

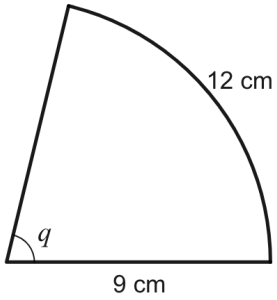


Worked Example	Your Turn
Find the arc length of the sector.  Give your answer correct to 1 decimal place.	Find the arc length of the sector.  Give your answer correct to 1 decimal place.

Dr Frost 318a

Worked Example

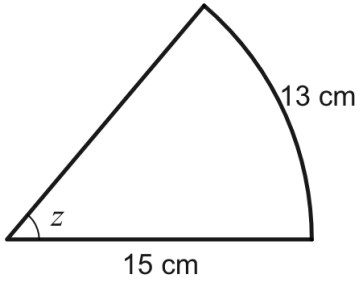
Given that the arc length of sector below is 12 cm, work out its angle, marked q on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

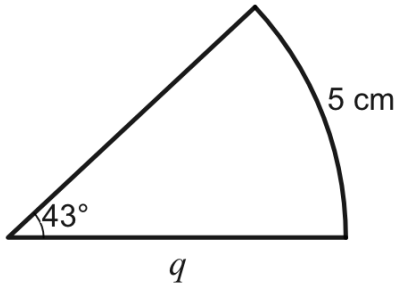
Given that the arc length of sector below is 13 cm, work out its angle, marked z on the diagram.



Give your answer correct to 1 decimal place.

Worked Example

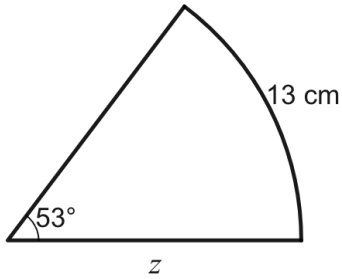
Given that the arc length of sector below is 5 cm, work out its radius, marked q on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

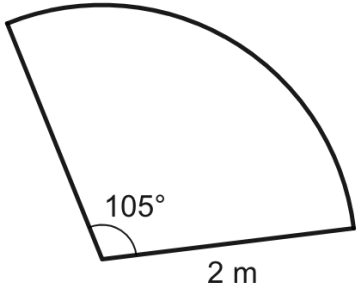
Given that the arc length of sector below is 13 cm, work out its radius, marked z on the diagram.



Give your answer correct to 1 decimal place.

Worked Example

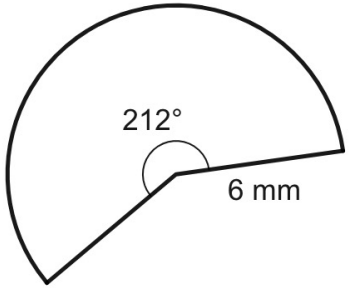
Work out the perimeter of the sector.



Give your answer correct to 1 decimal place.

Your Turn

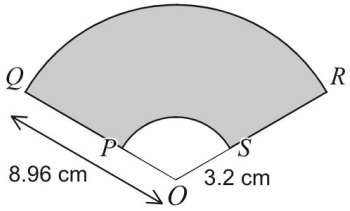
Work out the perimeter of the sector.



Give your answer correct to 1 decimal place.

Worked Example

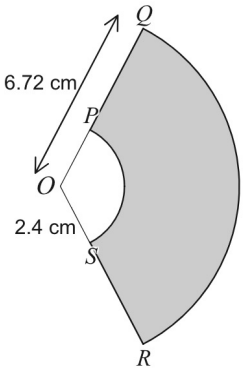
A shape is formed from the sectors of two circles with centre O.
OPQ and OSR are straight lines.



The length of OS is 3.2 cm.
The length of OQ is 8.96 cm.
Angle POS is $120°$.
Calculate the perimeter of the shaded region.
Give your answer to 1 decimal place.

Your Turn

A shape is formed from the sectors of two circles with centre O.
OPQ and OSR are straight lines.

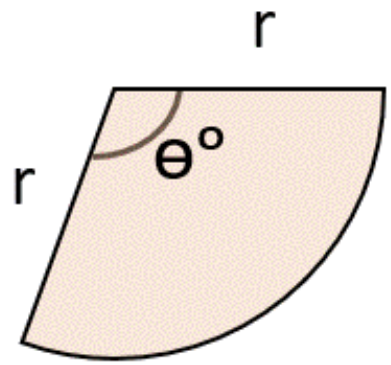


The length of OS is 2.4 cm.
The length of OQ is 6.72 cm.
Angle POS is $125°$.
Calculate the perimeter of the shaded region.
Give your answer to 1 decimal place.

Area of Sectors and Segments

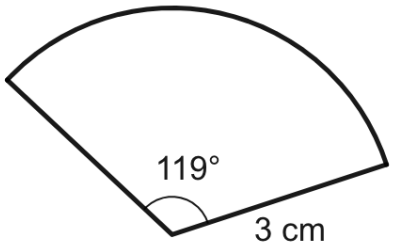
$$\text{Area of a Sector} = \frac{\text{Angle}}{360} \times \pi \times \text{Radius}^2$$

$$\mathbf{A = \frac{\theta}{360} \times \pi r^2}$$



Worked Example

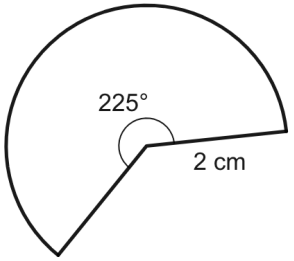
Find the area of the sector.



Give your answer correct to 1 decimal place.

Your Turn

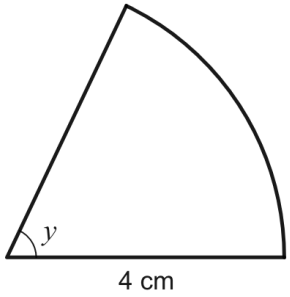
Find the area of the sector.



Give your answer correct to 1 decimal place.

Worked Example

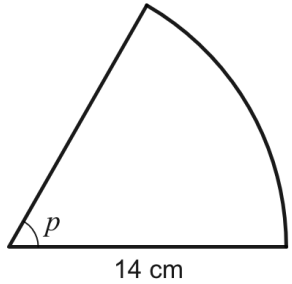
Given that the area of sector below is 9 cm^2, work out its angle, marked y on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

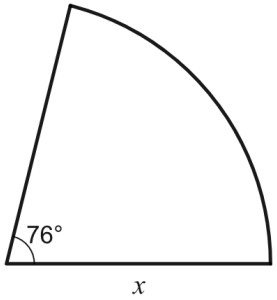
Given that the area of sector below is 103 cm^2, work out its angle, marked p on the diagram.



Give your answer correct to 1 decimal place.

Worked Example

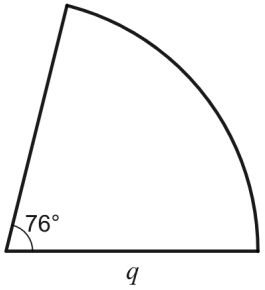
Given that the area of sector below is 17 cm^2, work out its radius, marked x on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

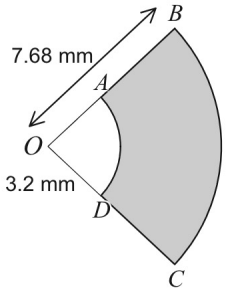
Given that the area of sector below is 97 cm^2, work out its radius, marked q on the diagram.



Give your answer correct to 1 decimal place.

Worked Example

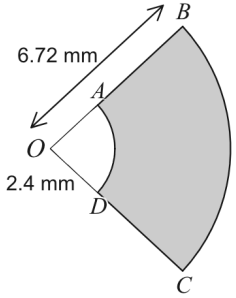
A metal component for a machine is formed from the sectors of two circles with centre O. The area of sector OAD is 7.7 mm^2.



OAB and ODC are straight lines.
The length of OD is 3.2 mm.
The length of OB is 7.68 mm.
Calculate the area of the shaded region.

Your Turn

A metal component for a machine is formed from the sectors of two circles with centre O. The area of sector OAD is 4.3 mm^2.

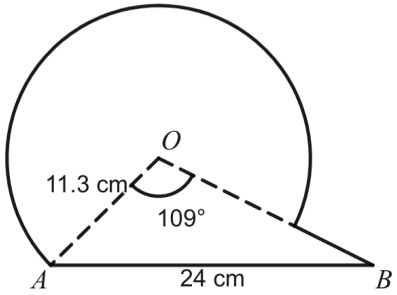


OAB and ODC are straight lines.
The length of OD is 2.4 mm.
The length of OB is 6.72 mm.
Calculate the area of the shaded region.

Dr Frost 319d

Worked Example

A shape is made from triangle AOB and a sector of a circle, centre O and radius OA

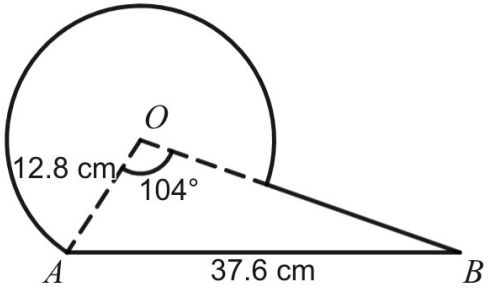


angle $AOB = 109°$
$OA = 11.3$ cm
$AB = 24$ cm

Work out the area of the shape.
Give your answer correct to 3 significant figures.

Your Turn

A shape is made from triangle AOB and a sector of a circle, centre O and radius OA



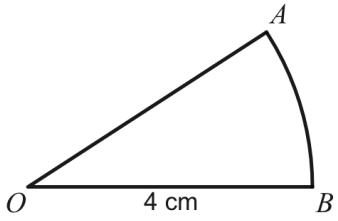
angle $AOB = 104°$
$OA = 12.8$ cm
$AB = 37.6$ cm

Work out the area of the shape.
Give your answer correct to 3 significant figures.

Dr Frost 467m

Worked Example

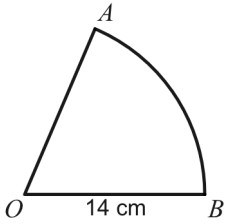
AOB is a sector of a circle, centre O and radius 4 cm.



The perimeter of the sector is 10 cm.
Work out the area of the sector.

Your Turn

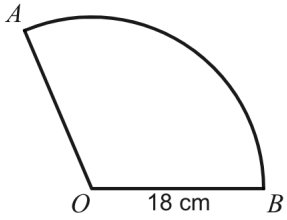
AOB is a sector of a circle, centre O and radius 14 cm.



The perimeter of the sector is 44 cm.
Work out the area of the sector.

Worked Example

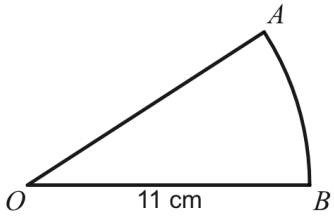
AOB is a sector of a circle, centre O and radius 18 cm.



The area of the sector is 320 cm^2.
Work out the perimeter of the sector.
Give your answer correct to 2 decimal places.

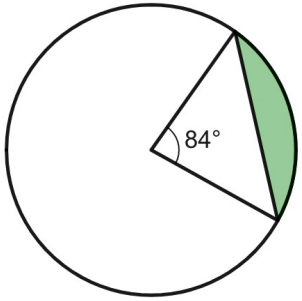
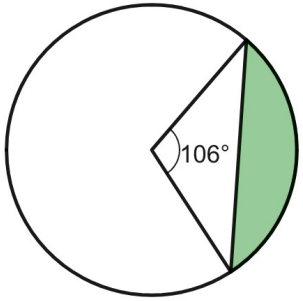
Your Turn

AOB is a sector of a circle, centre O and radius 11 cm.

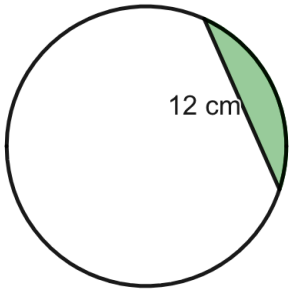
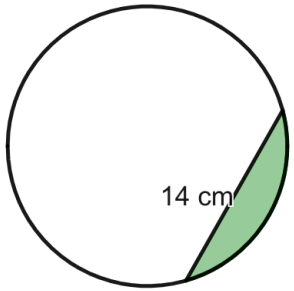


The area of the sector is 35 cm^2.
Work out the perimeter of the sector.
Give your answer correct to 2 decimal places.

Dr Frost 319f

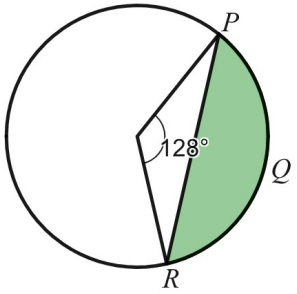
Worked Example	Your Turn
The radius of the circle is 6 cm.  Find the area of the shaded region.	The radius of the circle is 15 cm.  Find the area of the shaded region.

Dr Frost 467i

Worked Example	Your Turn
The radius of the circle is 9 cm.  Find the area of the shaded region.	The radius of the circle is 10 cm.  Find the area of the shaded region.

Worked Example

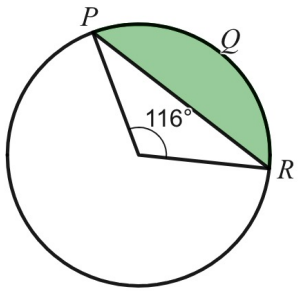
The area of the shaded region is 289 cm^2.



Find the length of the arc PQR.

Your Turn

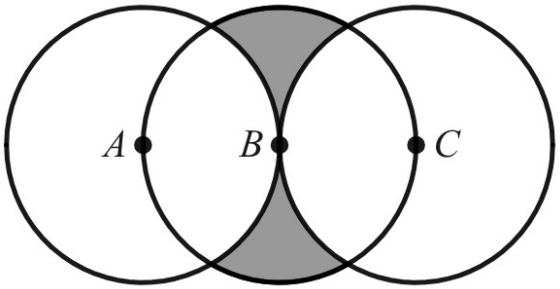
The area of the shaded region is 225 cm^2.



Find the length of the arc PQR.

Worked Example

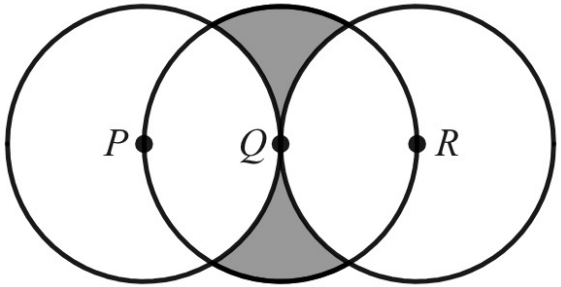
The diagram shows three circles, each of radius 9 cm.
The centres of the circles are A, B and C such that ABC is a straight line and $AB = BC = 9$ cm.



Work out the total area of the two shaded regions.
Give your answer in terms of π.

Your Turn

The diagram shows three circles, each of radius 6 cm.
The centres of the circles are P, Q and R such that PQR is a straight line and $PQ = QR = 6$ cm.



Work out the total area of the two shaded regions.
Give your answer in terms of π.

Dr Frost 467r

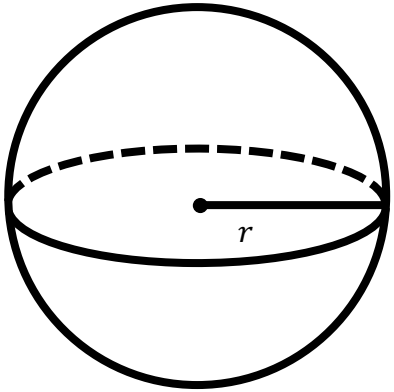
Extra Notes

3 Volume and Surface Area of Non-Prisms

Volume of Spheres

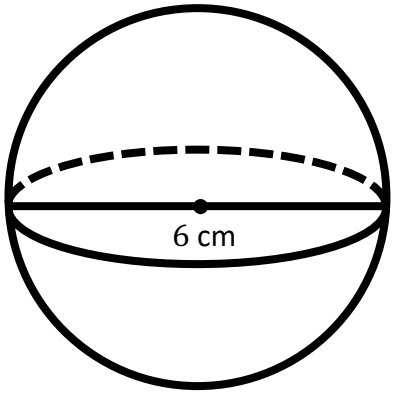
$$\text{Volume of Sphere} = \frac{4}{3} \times \pi \times \text{Radius}^3$$

$$V = \frac{4}{3}\pi r^3$$



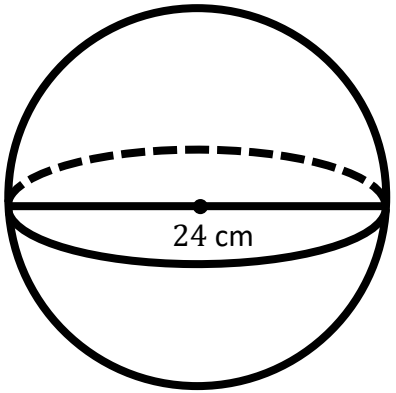
Worked Example

Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place.



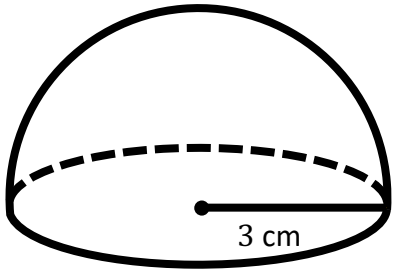
Your Turn

Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place.



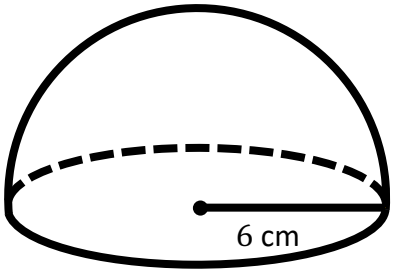
Worked Example

Calculate the volume of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



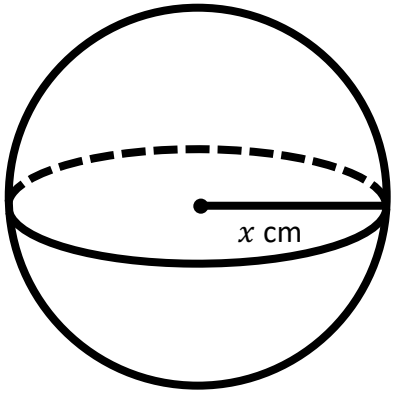
Your Turn

Calculate the volume of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



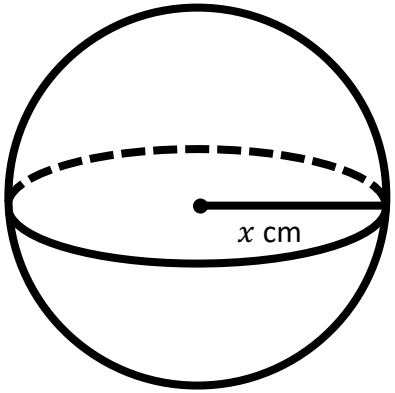
Worked Example

Find the radius, x, given that the volume of the following sphere is $113.1\ cm^3$. Give your answer to 1 decimal place.



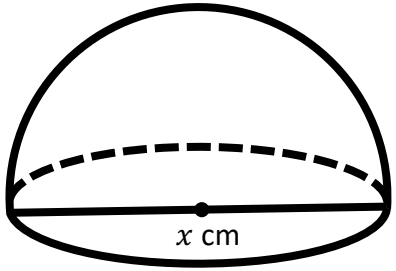
Your Turn

Find the radius, x, given that the volume of the following sphere is $904.8\ cm^3$. Give your answer to 1 decimal place.



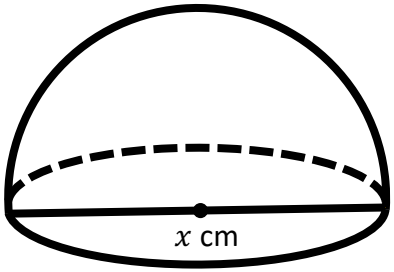
Worked Example

Find the diameter, x, given that the volume of the following hemisphere is $56.5\ cm^3$. Give your answer to 1 decimal place.



Your Turn

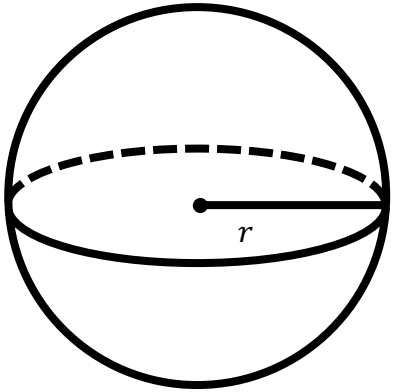
Find the diameter, x, given that the volume of the following hemisphere is $452.4\ cm^3$. Give your answer to 1 decimal place.



Surface Area of Spheres

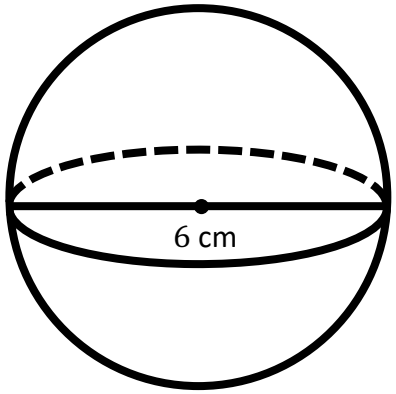
Surface Area of Sphere $= 4 \times \pi \times \text{Radius}^2$

$SA = 4\pi r^2$



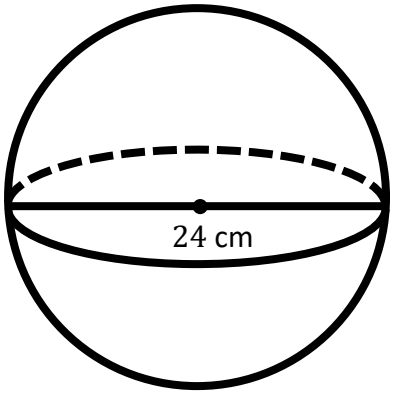
Worked Example

Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place.



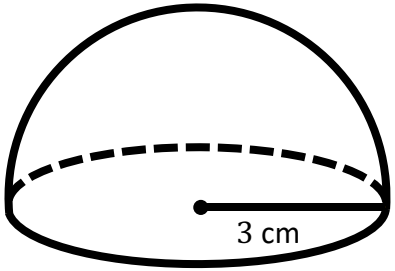
Your Turn

Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place.



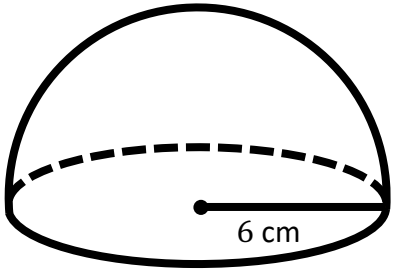
Worked Example

Calculate the total surface area of the following hemisphere.
Give your answer in terms of π and to 1 decimal place.



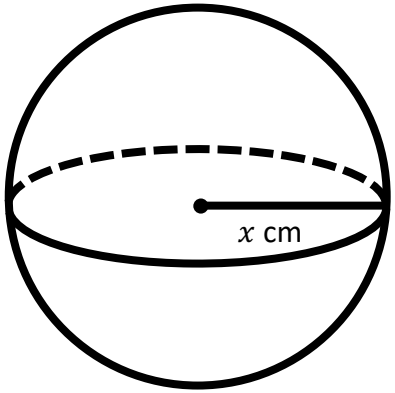
Your Turn

Calculate the total surface area of the following hemisphere.
Give your answer in terms of π and to 1 decimal place.



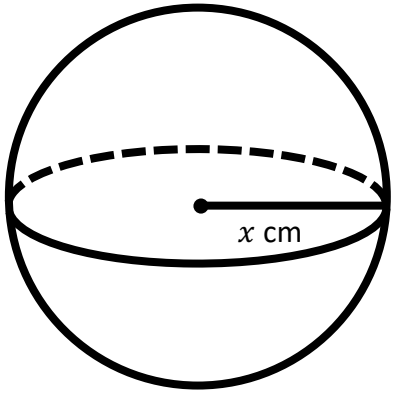
Worked Example

Find the radius, x, given that the surface area of the following sphere is $113.1\ cm^2$. Give your answer to 1 decimal place.



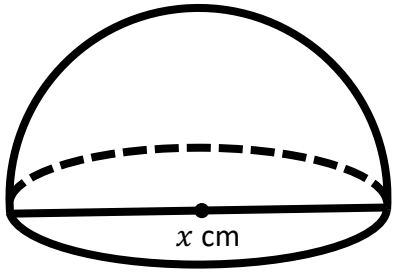
Your Turn

Find the radius, x, given that the surface area of the following sphere is $452.4\ cm^2$. Give your answer to 1 decimal place.



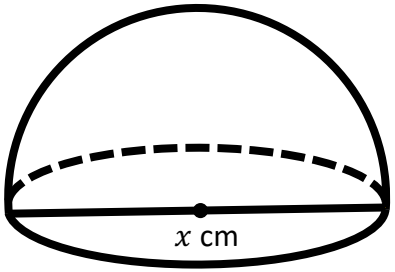
Worked Example

Find the diameter, x, given that the total surface area of the following hemisphere is $84.8\ cm^2$. Give your answer to 1 decimal place.



Your Turn

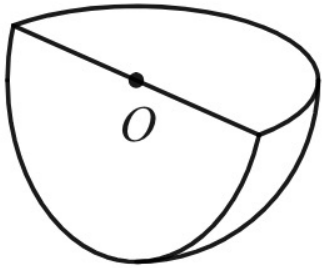
Find the diameter, x, given that the total surface area of the following hemisphere is $500\ cm^2$. Give your answer to 1 decimal place.



Worked Example	Your Turn
a) A sphere has a surface area of $36\pi\ cm^2$. Work out the volume of the sphere. Give your answer in terms of π and to 1 decimal place.	a) A sphere has a surface area of $144\pi\ cm^2$. Work out the volume of the sphere. Give your answer in terms of π and to 1 decimal place.
b) A sphere has a volume of $36\pi\ cm^3$. Work out the surface area of the sphere. Give your answer in terms of π and to 1 decimal place.	b) A sphere has a volume of $288\pi\ cm^3$. Work out the surface area of the sphere. Give your answer in terms of π and to 1 decimal place.

Worked Example

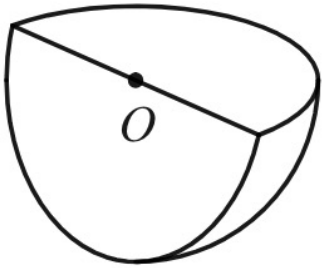
The diagram shows a solid quarter sphere, centre O.



The volume of the solid is 9750 cm^3.
Work out the surface area of the solid.
Give your answer to 1 decimal place.

Your Turn

The diagram shows a solid quarter sphere, centre O.

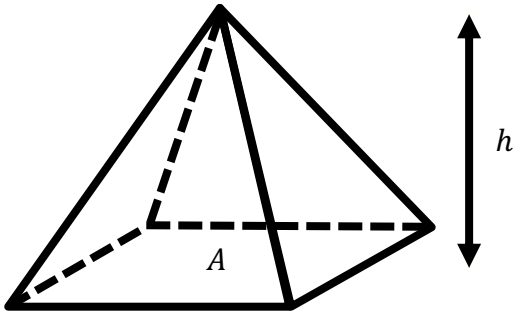


The volume of the solid is 3500 cm^3.
Work out the surface area of the solid.
Give your answer to 1 decimal place.

Volume of Pyramids

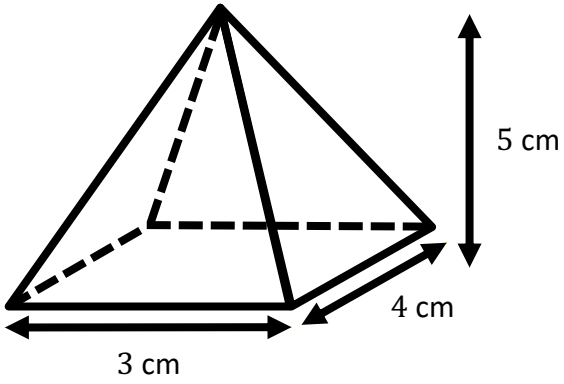
$$\text{Volume of Pyramid} = \frac{1}{3} \times \text{Area of Base} \times \text{Vertical Height}$$

$$V = \frac{1}{3}Ah$$



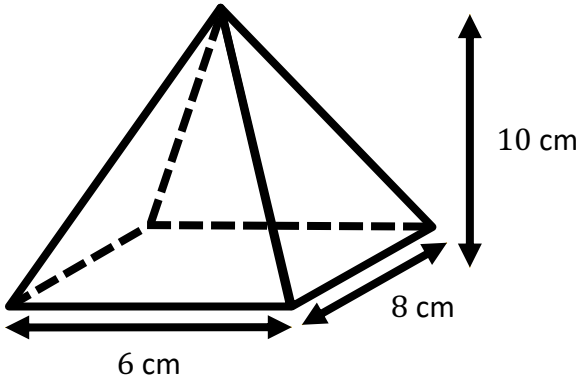
Worked Example

Calculate the volume of the following rectangular-based pyramid.



Your Turn

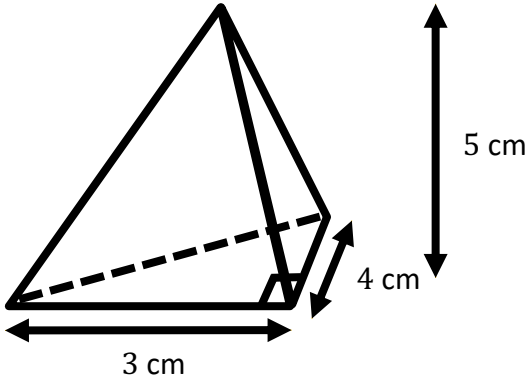
Calculate the volume of the following rectangular-based pyramid.



Dr Frost 405a

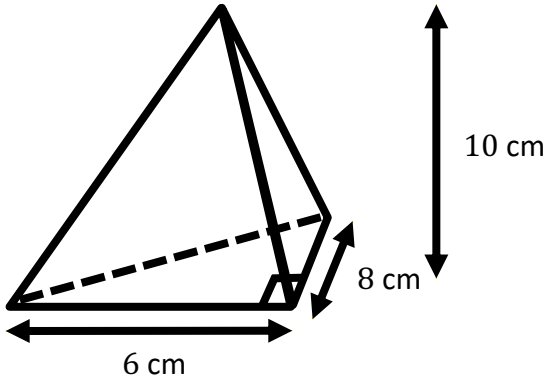
Worked Example

Calculate the volume of the following triangular-based pyramid.



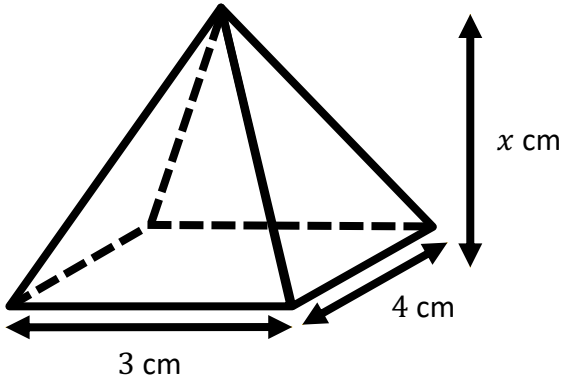
Your Turn

Calculate the volume of the following triangular-based pyramid.



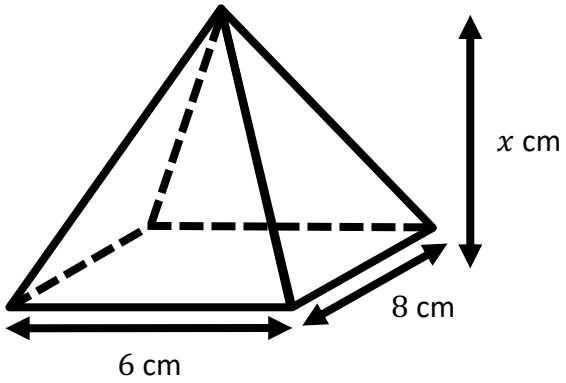
Worked Example

Find the height, x, given that the volume of the following rectangular-based pyramid is $20\ cm^3$.



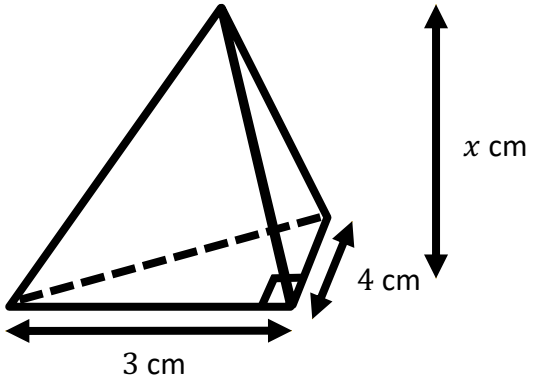
Your Turn

Find the height, x, given that the volume of the following rectangular-based pyramid is $160\ cm^3$.



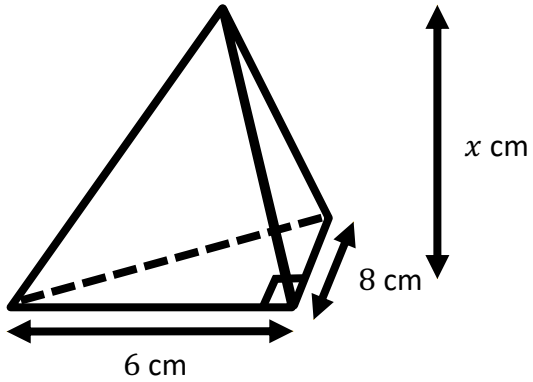
Worked Example

Find the height, x, given that the volume of the following triangular-based pyramid is $10\ cm^3$.



Your Turn

Find the height, x, given that the volume of the following triangular-based pyramid is $80\ cm^3$.

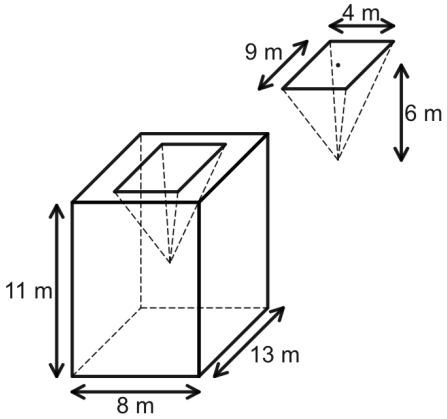


Worked Example

The diagram shows a compound solid comprising of a cuboid with a pyramidal indentation in the top.

The cuboid has length 8 m, width 13 m and height 11 m.

The rectangular based pyramid that has been removed has length 4 m, width 9 m and height 6 m as shown on the diagram.

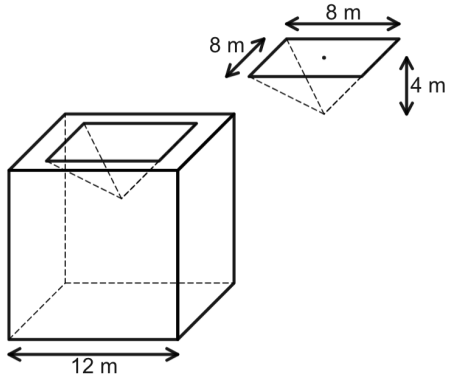


Calculate the volume of the remaining solid.
Give your answer in m^3 to the nearest integer.

Your Turn

The diagram shows a compound solid comprising of a cube with a pyramidal indentation in the top.

The cube has length 12 m.
The square based pyramid that has been removed has length and width of 8 m and a height 4 m as shown on the diagram.

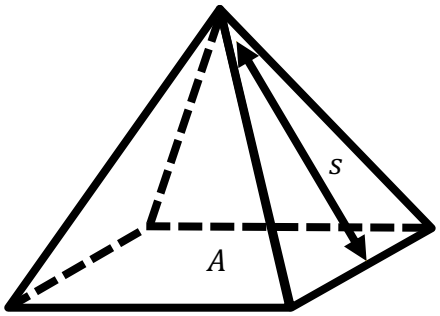


Calculate the volume of the remaining solid.
Give your answer in m^3 to the nearest integer.

Surface Area of Pyramids

$$\text{Total Surface Area of Pyramid} = \frac{1}{2} \times \text{Perimeter of Base} \times \text{Slant Height} + \text{Area of Base}$$

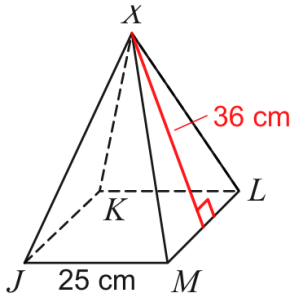
$$\text{TSA} = \frac{1}{2}\text{Ps} + \text{A}$$



Note: This formula only works for regular pyramids (e.g., pyramids with a square base). To find the surface area of other types of pyramids, you need to calculate the area of each face and the base separately, then add them together.

Worked Example

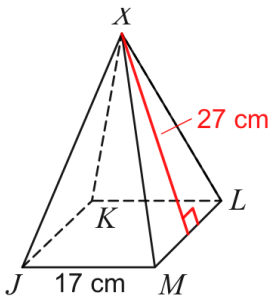
The diagram shows a square based pyramid $JKLMX$. The vertex, X, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 25 cm. The distance from X to the midpoint of each of the sides of the base is 36 cm. Find the total surface area of the pyramid.

Your Turn

The diagram shows a square based pyramid $JKLMX$. The vertex, X, of the pyramid is directly above the centre of the square base.

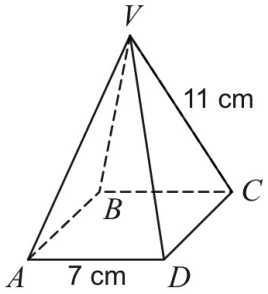


The length of each side of the base is 17 cm. The distance from X to the midpoint of each of the sides of the base is 27 cm. Find the total surface area of the pyramid.

Dr Frost 406a

Worked Example

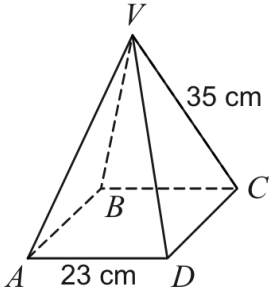
The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 7 cm. The length of each slanted edge is 11 cm. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Your Turn

The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.

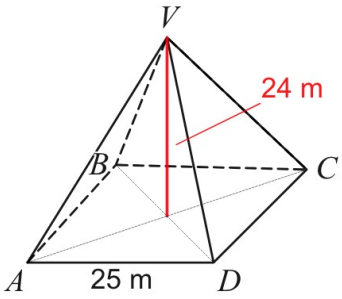


The length of each side of the base is 23 cm. The length of each slanted edge is 35 cm. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Dr Frost 406b

Worked Example

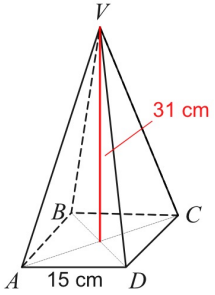
The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 25 m. The height of the pyramid is 24 m. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Your Turn

The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.

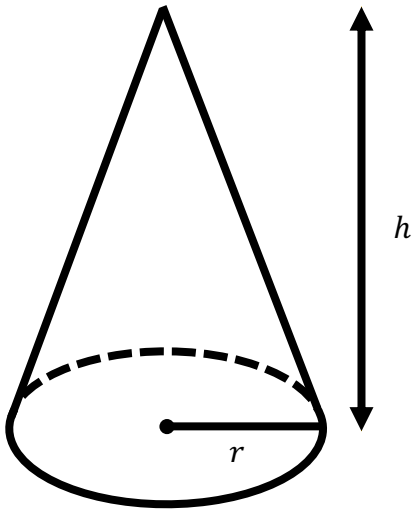


The length of each side of the base is 15 cm. The height of the pyramid is 31 cm. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Volume of Cones

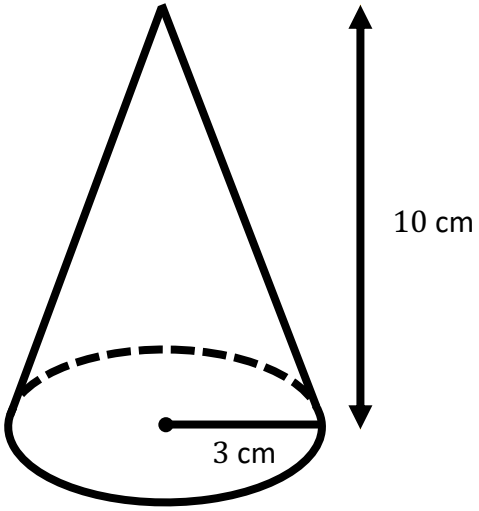
$$\text{Volume of Cone} = \frac{1}{3} \times \text{Area of Circle} \times \text{Height}$$

$$V = \frac{1}{3}\pi r^2 h$$



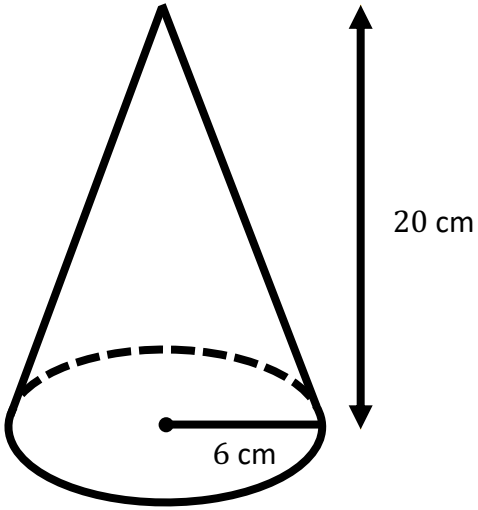
Worked Example

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



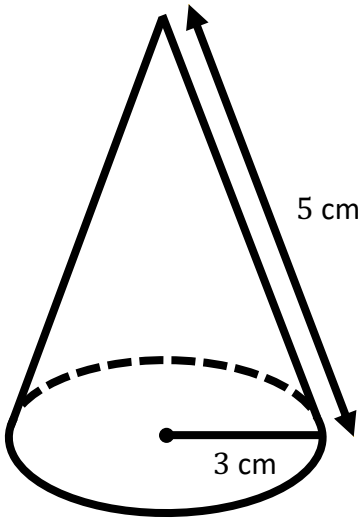
Your Turn

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



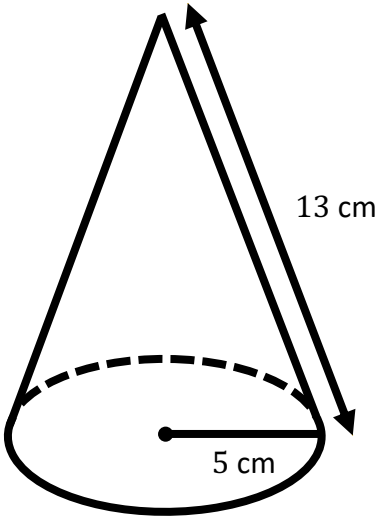
Worked Example

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



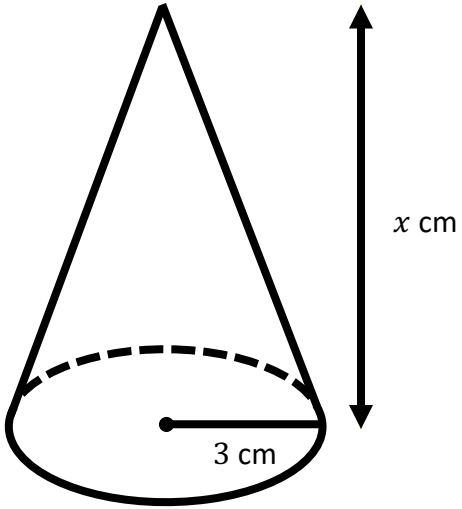
Your Turn

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



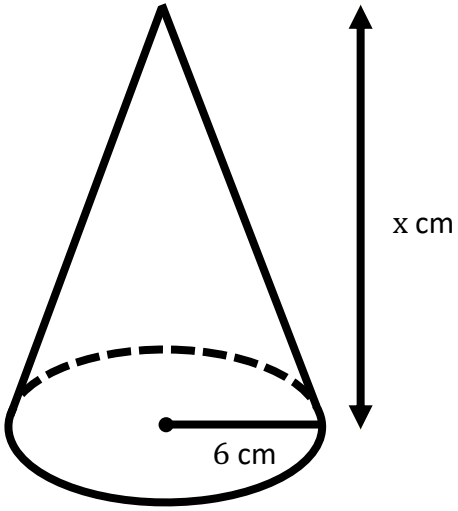
Worked Example

Find the height, x, given that the volume of the following cone is $94.2\ cm^3$. Give your answer to 1 decimal place.



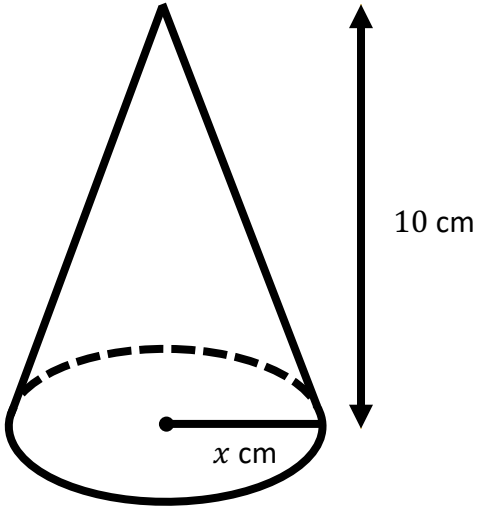
Your Turn

Find the height, x, given that the volume of the following cone is $754.0\ cm^3$. Give your answer to 1 decimal place.



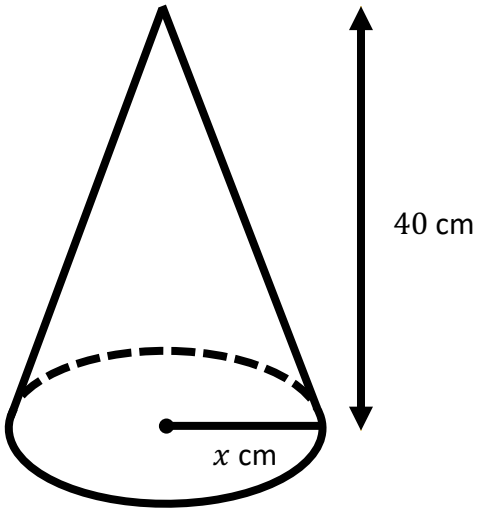
Worked Example

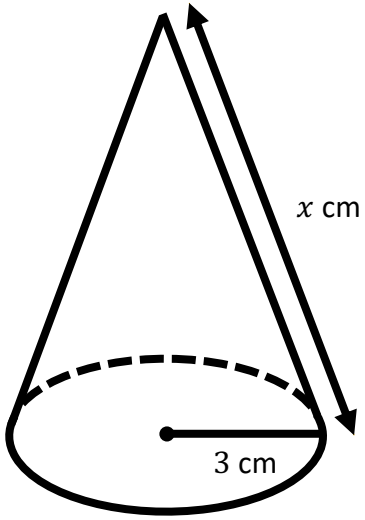
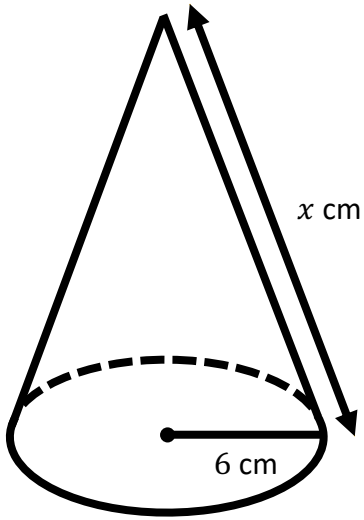
Find the radius, x, given that the volume of the following cone is $94.2\ cm^3$. Give your answer to 1 decimal place.

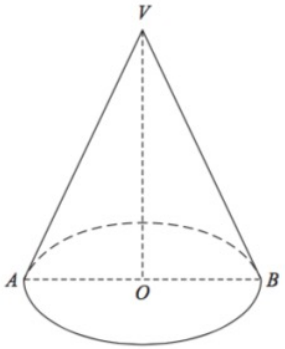
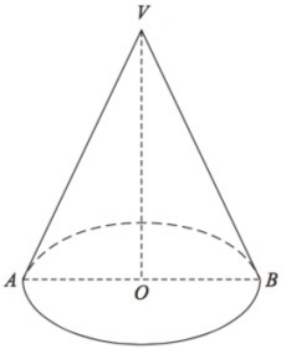


Your Turn

Find the radius, x, given that the volume of the following cone is $754.0\ cm^3$. Give your answer to 1 decimal place.

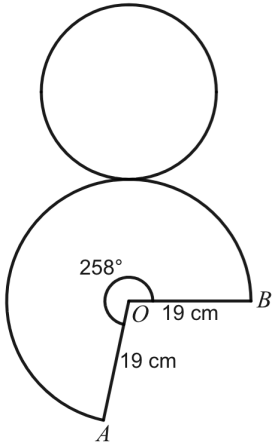


Worked Example	Your Turn
Find the slanted height, x, given that the volume of the following cone is $37.7\ cm^3$. Give your answer to 1 decimal place. 	Find the slanted height, x, given that the volume of the following cone is $500\ cm^3$. Give your answer to 1 decimal place. 

Worked Example	Your Turn
The diagram shows a solid cone. The base of the cone is a horizontal circle, centre O, with radius $9\ cm$. The curved surface area of the cone is $260\ cm^2$. Calculate the size of angle AVB.  	The diagram shows a solid cone. The base of the cone is a horizontal circle, centre O, with radius $4.5\ cm$. The curved surface area of the cone is $130\ cm^2$. Calculate the size of angle AVB. 

Worked Example

Here is the net of a cone.



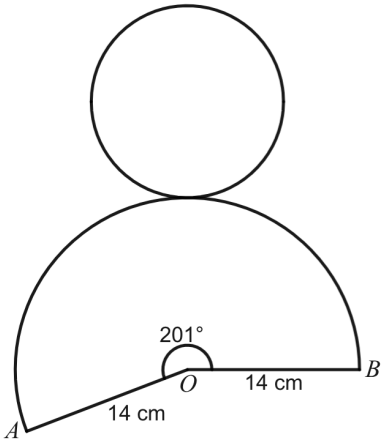
The net is formed from a sector of a circle, centre O, and radius 19 cm and an additional circle for the base.
Reflex angle $AOB = 258°$

The net makes a cone of slant height of 19 cm.

Calculate the volume of the cone.
Give your answer correct to 1 decimal place.

Your Turn

Here is the net of a cone.



The net is formed from a sector of a circle, centre O, and radius 14 cm and an additional circle for the base.
Reflex angle $AOB = 201°$

The net makes a cone of slant height of 14 cm.

Calculate the volume of the cone.
Give your answer correct to 1 decimal place.

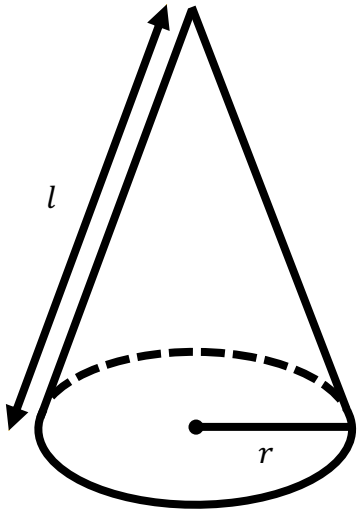
Surface Area of Cones

Curved Surface Area of Cone = π × Radius × Slanted Height

$CSA = \pi rl$

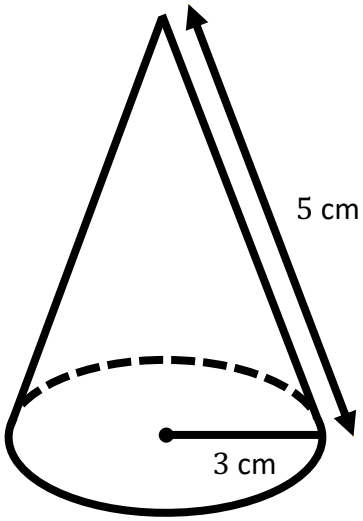
Total Surface Area of Cone = π × Radius × Slanted Height + π × Radius^2

$TSA = \pi rl + \pi r^2$



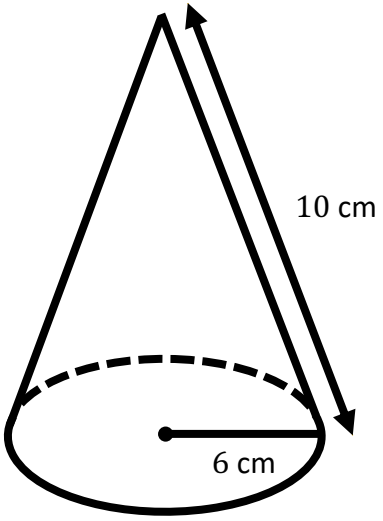
Worked Example

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



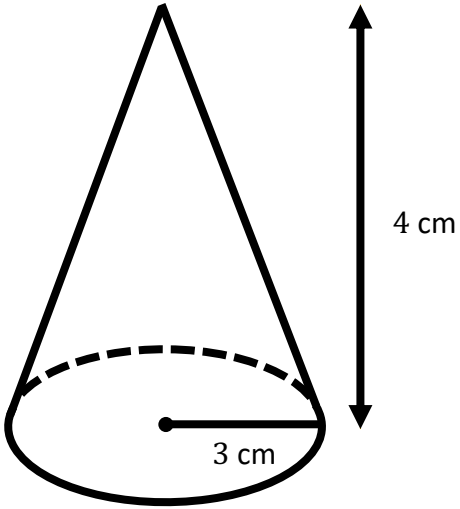
Your Turn

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



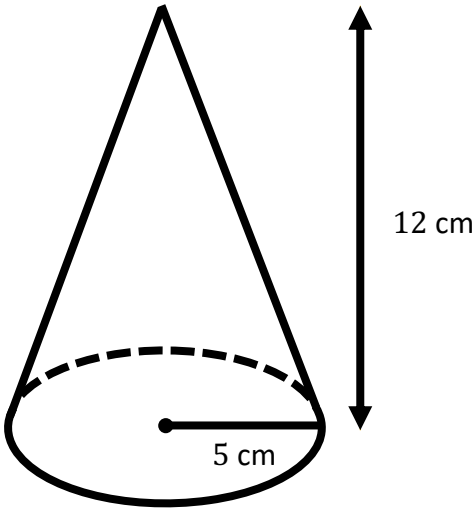
Worked Example

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



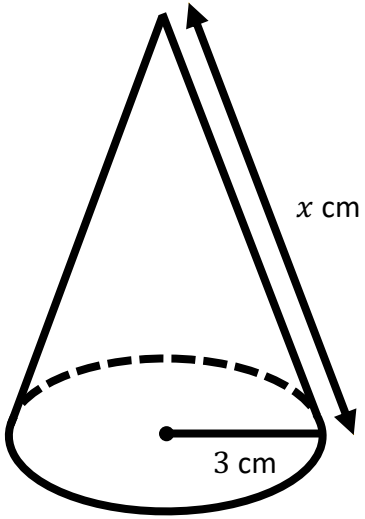
Your Turn

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



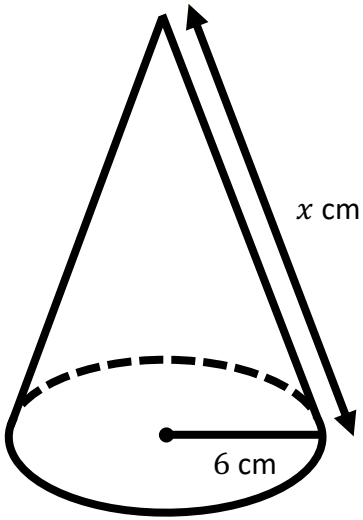
Worked Example

Find the slanted height, x, given that the total surface area of the following cone is $75.4\ cm^2$. Give your answer to 1 decimal place.



Your Turn

Find the slanted height, x, given that the total surface area of the following cone is $301.6\ cm^2$. Give your answer to 1 decimal place.



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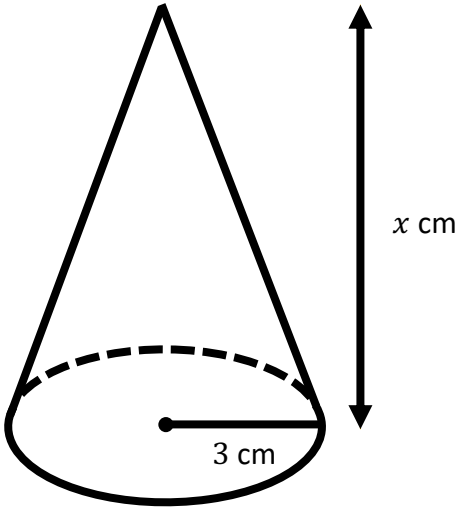
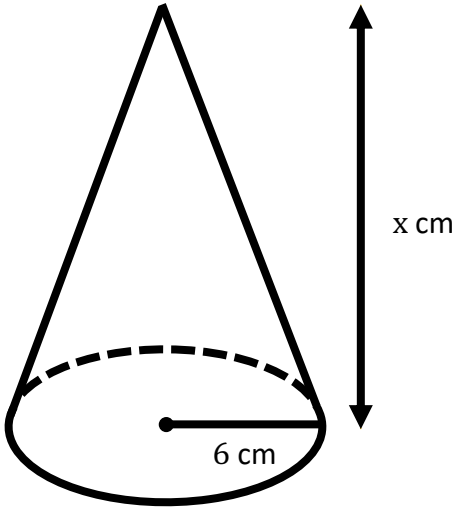
Worked Example

Find the perpendicular height, x, given that the total surface area of the following cone is $75.4\ cm^2$. Give your answer to 1 decimal place.



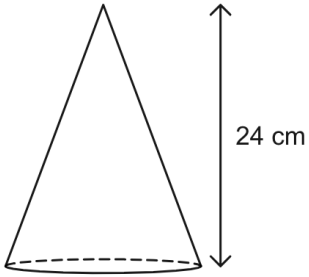
Your Turn

Find the perpendicular height, x, given that the total surface area of the following cone is $400\ cm^2$. Give your answer to 1 decimal place.



Worked Example

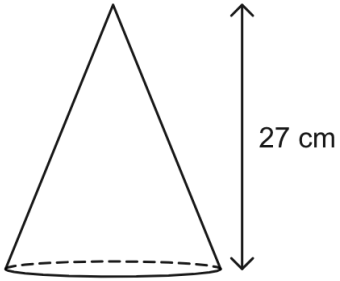
The diagram shows a cone with height 24 cm.



The volume of the cone is 2000 cm^3.
Find the total surface area of the cone.
Give your answer to one decimal place.

Your Turn

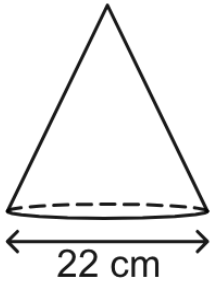
The diagram shows a cone with height 27 cm.



The volume of the cone is 3400 cm^3.
Find the total surface area of the cone.
Give your answer to one decimal place.

Worked Example

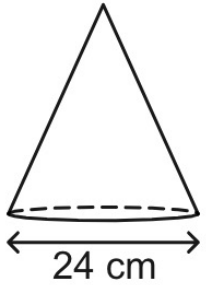
The diagram shows a cone with diameter 22 cm.



The total surface area of the cone is 1200 cm^2.
Find the volume of the cone.
Give your answer to one decimal place.

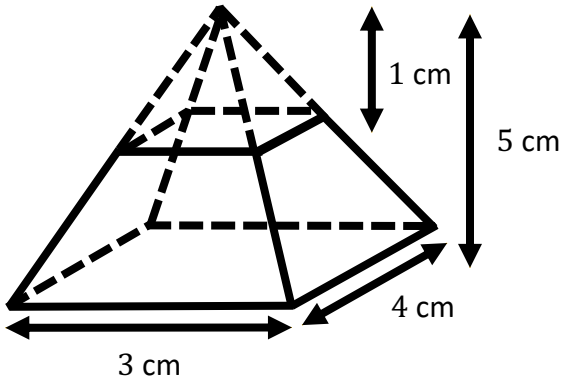
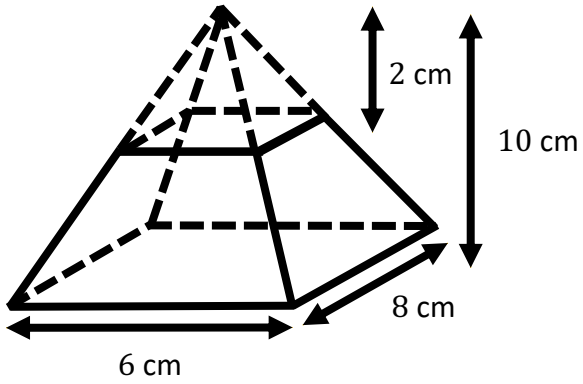
Your Turn

The diagram shows a cone with diameter 24 cm.



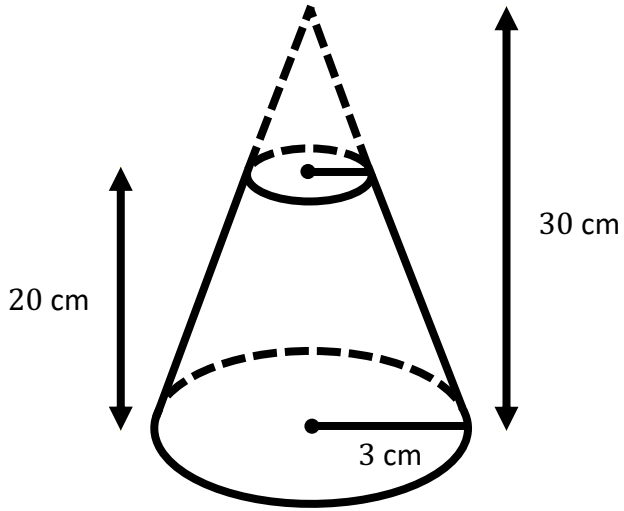
The total surface area of the cone is 1500 cm^2.
Find the volume of the cone.
Give your answer to one decimal place.

Volume of Frustums

Worked Example	Your Turn
Calculate the volume of the following frustum.  	Calculate the volume of the following frustum. 

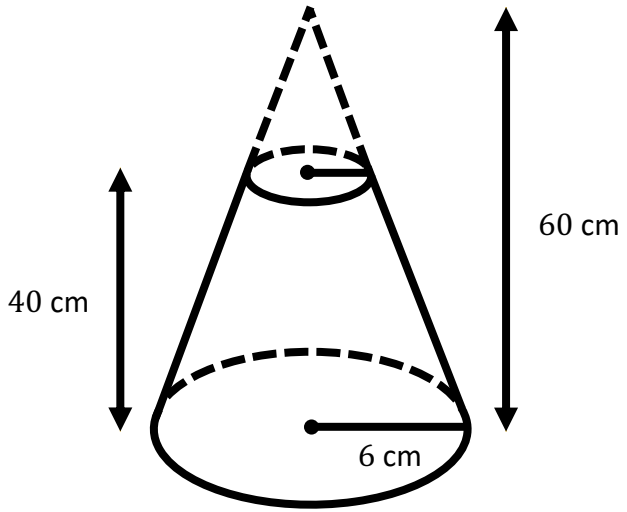
Worked Example

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



Your Turn

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



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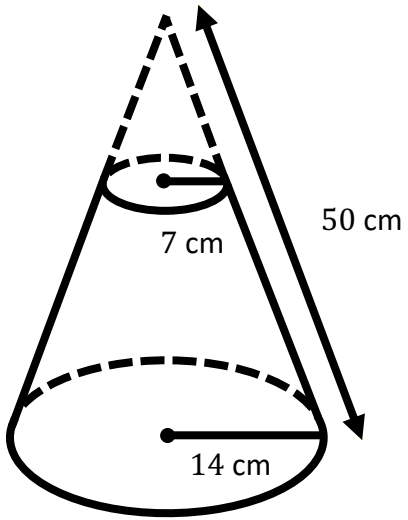
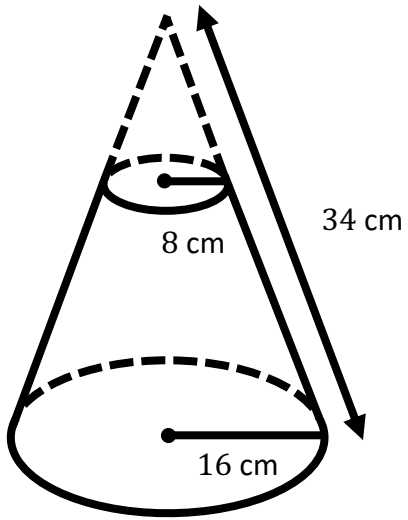
Worked Example

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



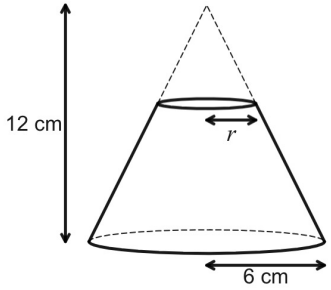
Your Turn

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



Worked Example

A cone of radius r cm is removed from a cone of radius 6 cm and height 12 cm to give a frustum.

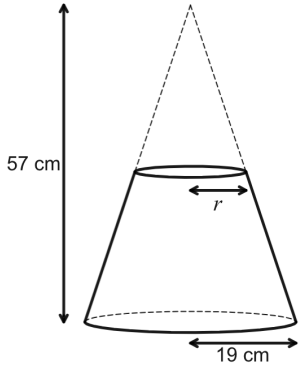


The volume of the frustum is 420 cm^3

Calculate the value of r
Give your answer correct to 1 decimal place

Your Turn

A cone of radius r cm is removed from a cone of radius 19 cm and height 57 cm to give a frustum.



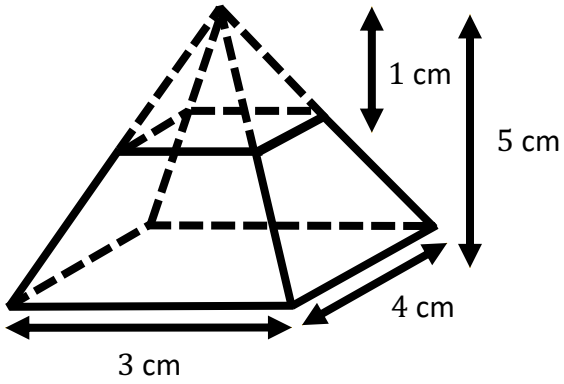
The volume of the frustum is 18407 cm^3

Calculate the value of r
Give your answer correct to 1 decimal place

Surface Area of Frustums

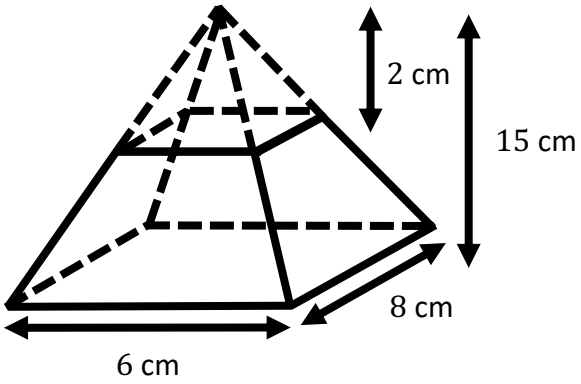
Worked Example

Calculate the total surface area of the following frustum.



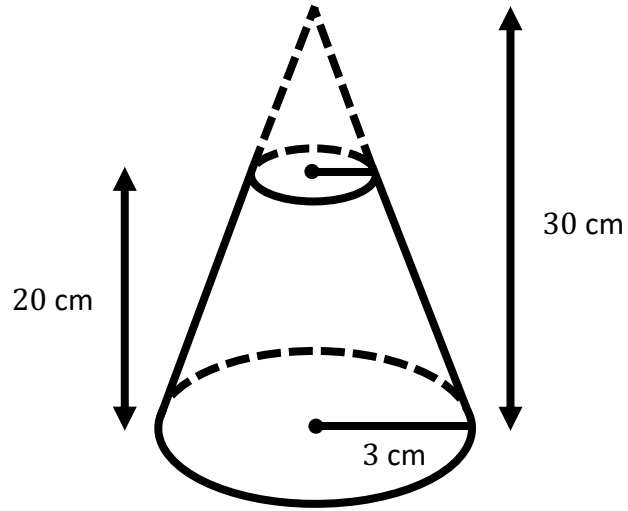
Your Turn

Calculate the total surface area of the following frustum.



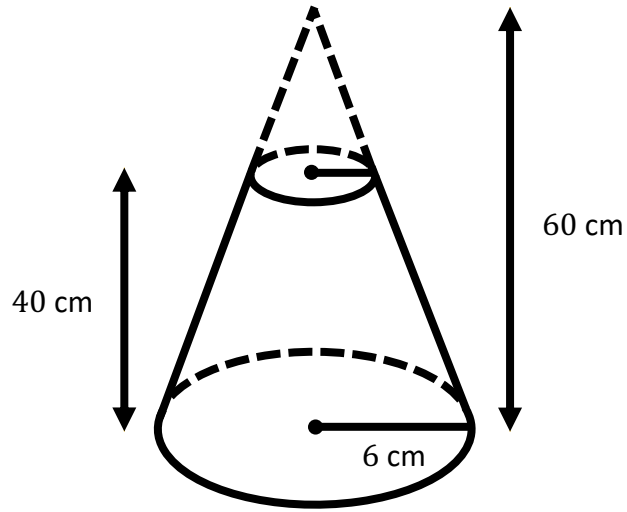
Worked Example

Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



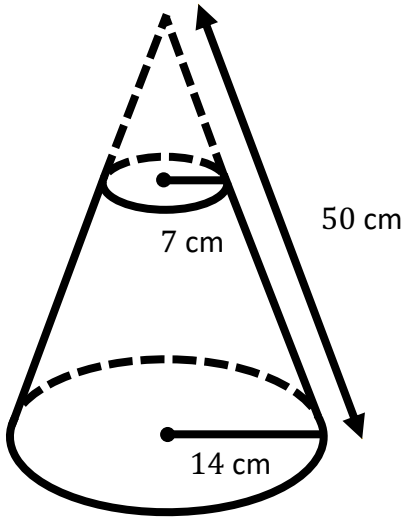
Your Turn

Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



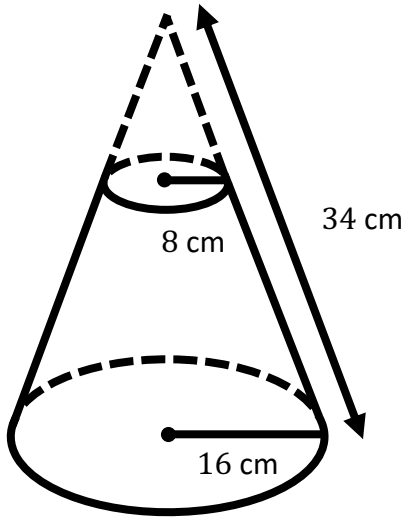
Worked Example

Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



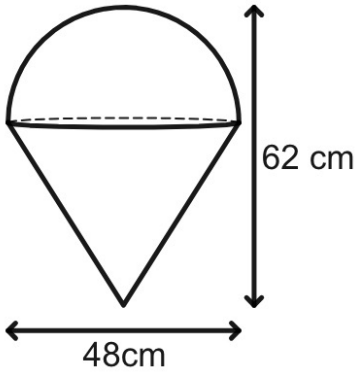
Your Turn

Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



Worked Example

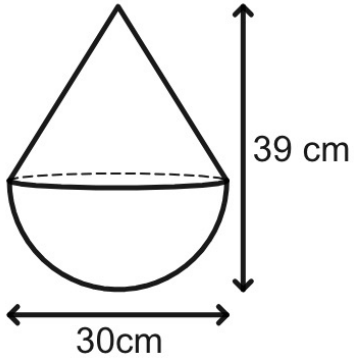
A child's toy is in the shape of a cone with a hemisphere on top, as shown below. The toy is 48 cm wide and 62 cm high.



Calculate the volume of the toy. Give your answer correct to two significant figures.

Your Turn

A child's toy is in the shape of a hemisphere with a cone on top, as shown below. The toy is 30 cm wide and 39 cm high.

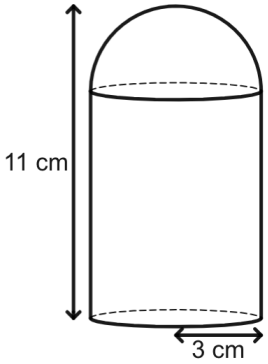


Calculate the volume of the toy. Give your answer correct to two significant figures.

Worked Example

The diagram shows a storage jar.

The storage jar consists of a cylinder with a hemisphere on top.



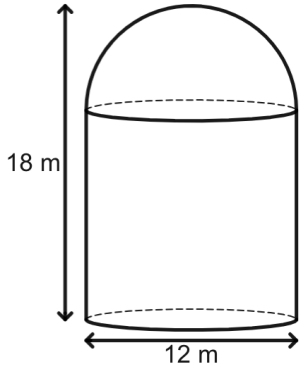
The height of the storage jar is 11 cm.
The radius is 3 cm.

Calculate the volume of the storage jar.
Give your answer in cm^3, in terms of π

Your Turn

The diagram shows an observatory.

The observatory consists of a cylinder with a hemisphere on top.

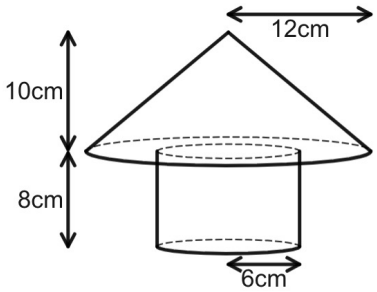


The height of the observatory is 18 m.
The radius is 12 m.

Calculate the volume of the observatory.
Give your answer in m^3, in terms of π

Worked Example

The diagram below shows a toy house made from a cylinder and a cone.

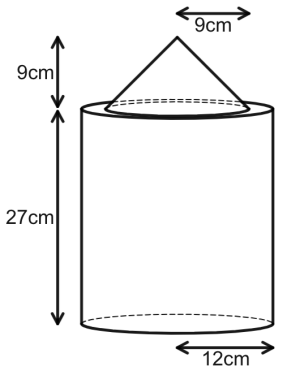


The cylinder has a height 8 cm and a radius 6 cm.
The cone has a height 10 cm and a radius 12 cm.

Calculate the volume of the house in cm^3.
Give your answer correct to 3 significant figures.

Your Turn

The diagram below shows a toy candle made from a cylinder and a cone.



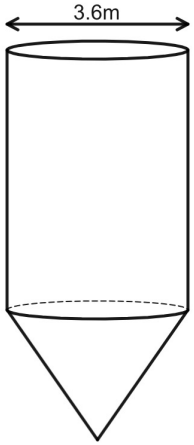
The cylinder has a height 27 cm and a radius 12 cm.

The cone has a height 9 cm and a radius 9 cm.

Calculate the volume of the candle in cm^3
Give your answer in terms of π

Worked Example

A water storage tank is constructed with a conical base and a cylindrical top section as shown in the diagram below.



The diameter of the top of the cone and the cylinder is 3.6 m.

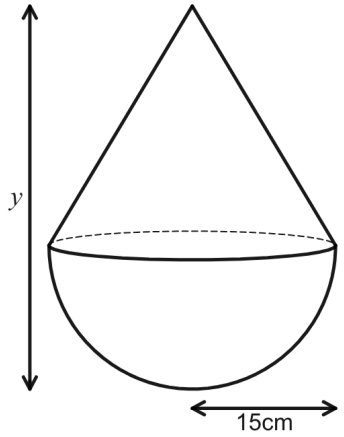
The total volume of the storage tank is 56 m^3

The height of the cylinder is two times the height of the cone.

Calculate the total height of storage tank.
Give your answer correct to 1 decimal place.

Your Turn

A buoy is constructed with a hemispherical base and a conical top section as shown in the diagram below.



The radius of the hemisphere and the base of the cone is 15 cm.

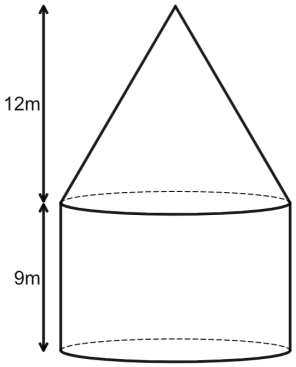
The total volume of the buoy is 4125π cm^3

The total height of the buoy is y cm

Calculate the value of y

Worked Example

A solid shape is made from a cylinder and a cone.



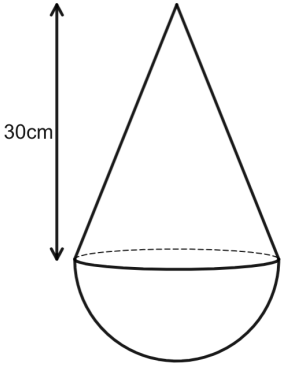
The cone exactly fits on top of the cylinder.
The cylinder has a height of 9 m.
The cone has a height of 12 m.

The volume of the cone is 196π m^3

Work out the total volume of the solid in m^3
Give your answer in terms of π

Your Turn

A solid shape is made from a cone and a hemisphere.



The radius of the hemisphere is equal to the radius of the base of the cone.

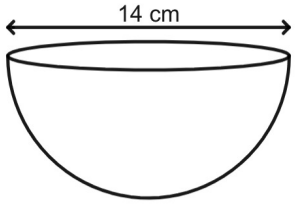
The cone has a height of 30 cm.

The volume of the hemisphere is 1152π cm^3

Work out the total volume of the solid in cm^3
Give your answer in terms of π

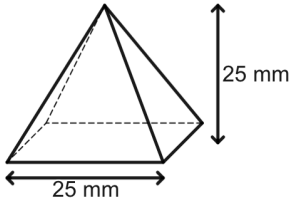
Worked Example

Kaitlyn has a hemispherical bowl filled with melted chocolate. The bowl has an internal diameter of 14 cm



The chocolate is poured into square-based pyramidal moulds to create chocolates with a height and base length of 25 mm

The chocolate must fill the pyramid-shaped moulds completely or they are rejected.



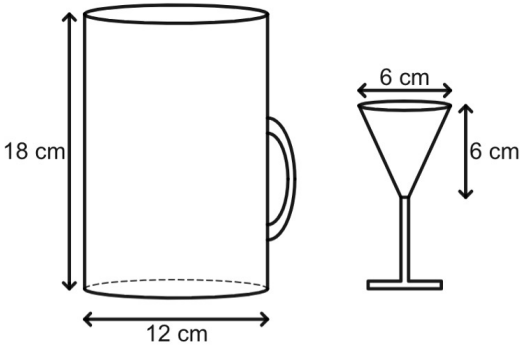
Calculate the number of chocolates that can be made from the bowl of melted chocolate.

Your Turn

Abdallah has a cylindrical jug filled with lemonade. The jug has an internal diameter of 12 cm and internal height of 18 cm

The lemonade is then poured into cone-shaped glasses with an internal diameter and bowl height of 6 cm

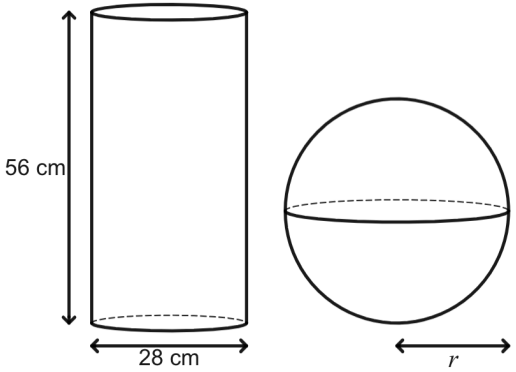
The lemonade fills the cone-shaped glasses completely.



Calculate the number of full glasses of lemonade that can be filled from the jug.

Worked Example

The diagram shows a cylinder and a sphere.



The cylinder has a diameter 28 cm and a height 56 cm
The cylinder and the sphere have the same volume.

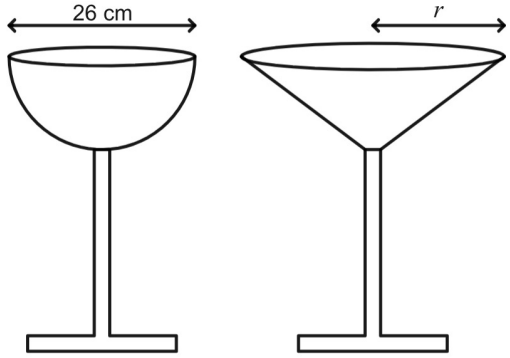
Work out the radius r of the sphere.
Give your answer in cm correct to 1 decimal place.

Your Turn

A hemispherical glass with a diameter of 26 cm is filled with lemonade.

The lemonade is then poured into a cone-shaped glass with the same height.

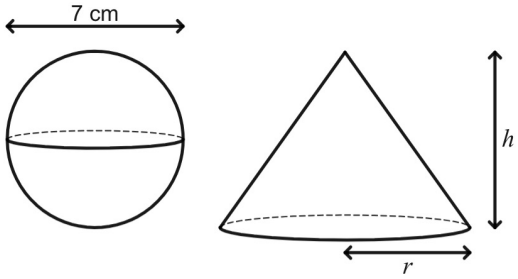
The lemonade fills the cone-shaped glass completely.



Calculate the radius r of the cone-shaped glass.
Give your answer in cm correct to 1 decimal place.

Worked Example

The diagram shows a sphere and a cone.



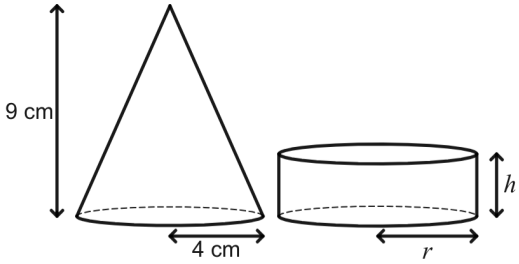
The sphere has a diameter 7 cm
The cone has a radius r cm and height h cm
The ratio $r : h$ is $5 : 7$

The volume of the cone is equal to the volume of the sphere.

Calculate the values of r and h
Give your answers in cm correct to 2 decimal places.

Your Turn

The diagram shows a cone and a cylinder.



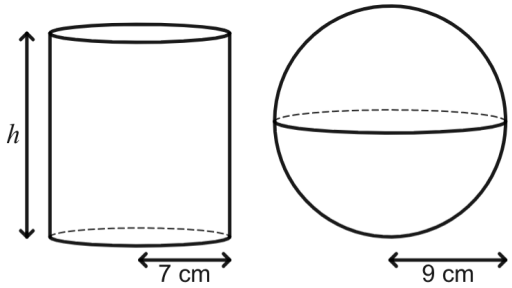
The cone has a radius 4 cm and height 9 cm
The cylinder has a radius r cm and height h cm
The ratio $r : h$ is $8 : 5$

The volume of the cylinder is equal to the volume of the cone.

Calculate the values of r and h
Give your answers in cm correct to 2 decimal places.

Worked Example

Shown below is a cylinder and a sphere.

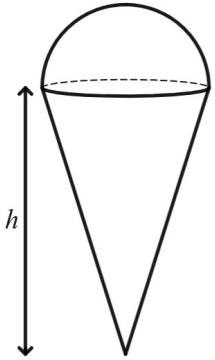


The ratio of the volume of the cylinder to the volume of the sphere is $4 : 5$

Calculate the height h of the cylinder.
Give your answer correct to 2 decimal places.

Your Turn

The diagram below shows an icecream made from a filled cone of internal radius 4.4 cm and height h topped with a hemisphere of icecream of the same radius.

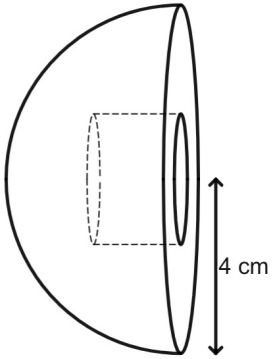


The ratio of the volume of the hemisphere to the volume of the cone is $5 : 8$

Calculate the height h of the cone.
Give your answer correct to 2 decimal places.

Worked Example

The diagram below shows a wooden end-piece for a curtain pole.
It is in the shape of a hemisphere with a radius of 4 cm.



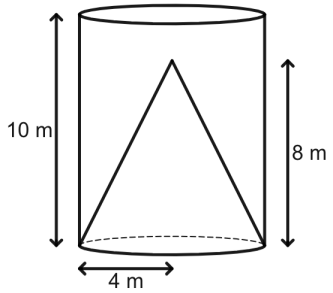
The curtain pole sits in a cylindrical hole that has been drilled into the end-piece.
The hole has a radius of 1.5 cm and depth 2 cm.

Calculate the volume of wood that remains.
Give your answer in cm^3 correct to the nearest integer.

Your Turn

A cylindrical tank with a radius of 4 metres and a height of 10 metres is partially filled with a conical heap of sand.

The cone has the same base radius of 4 metres and a height of 8 metres.

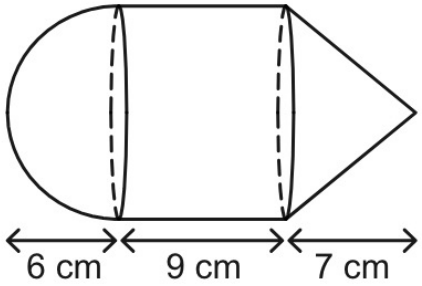


Calculate the volume of the space inside the cylindrical tank but outside the conical heap of sand.

Give your answer in m^3 correct to the nearest integer.

Worked Example

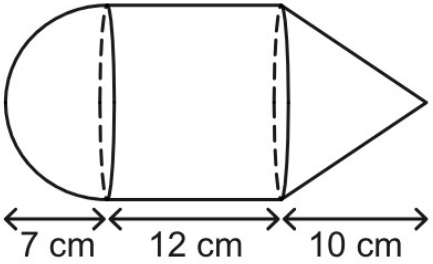
The diagram shows a solid formed from a hemisphere, a cylinder and a cone.



Find the total surface area of the solid.
Give your answer to one decimal place.

Your Turn

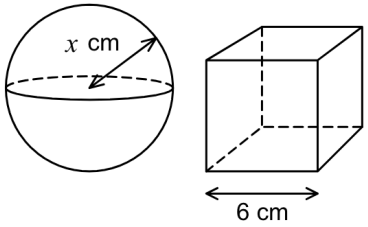
The diagram shows a solid formed from a hemisphere, a cylinder and a cone.



Find the total surface area of the solid.
Give your answer to one decimal place.

Worked Example

The diagram shows a solid sphere and a solid cube.

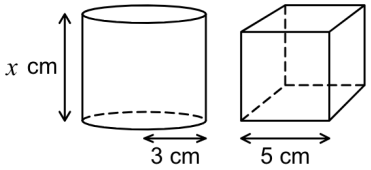


The sphere has radius x cm.
The cube has side length 6 cm.
The surface area of the sphere is equal to the surface area of the cube.

Find the value of x
Give your answer to 1 decimal place.

Your Turn

The diagram shows a solid cylinder and a solid cube.

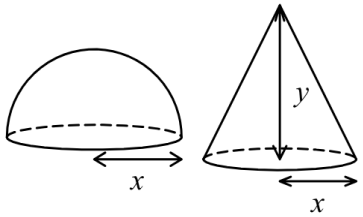


The cylinder has radius 3 cm and height x cm.
The cube has side length 5 cm.
The surface area of the cylinder is equal to the surface area of the cube.

Find the value of x
Give your answer to 1 decimal place.

Worked Example

The diagram shows a solid hemisphere and a solid cone.

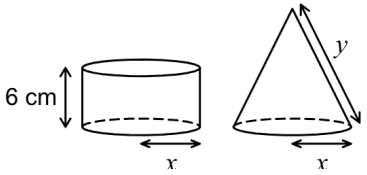


The hemisphere has radius x cm.
The cone has radius x cm and height y cm.
The surface area of the hemisphere is equal to the surface area of the cone.

Find an expression for y in terms of x

Your Turn

The diagram shows a solid cylinder and a solid cone.

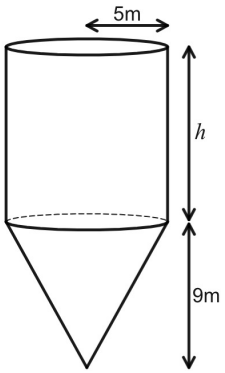


The cylinder has radius x cm and height 6 cm.
The cone has radius x cm and slant height y cm.
The surface area of the cylinder is equal to the surface area of the cone.

Find an expression for y in terms of x

Worked Example

The diagram shows a container for grain.



The container is a cylinder on top of a cone.
The cylinder has a radius of 5 m and a height of h m.
The cone has a base radius of 5 m and a vertical height of 9 m.

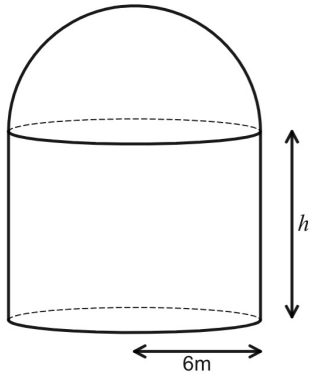
The container is initially empty.
The container is then filled with grain at a constant rate.

After 15 hours the depth of the grain is 15 m above the vertex of the cone.
After 23 hours the container is full of grain.

Work out the value of h

Your Turn

The diagram shows a water tank.



The container is a hemisphere on top of a cylinder.
The cylinder has a radius of 6 m and a height of h m.
The hemisphere has a base radius of 6 m.

The container is initially full.
The container is then emptied at a constant rate.

After 10 hours the height of the water has dropped by 12 m from the top of the hemisphere.
After 13 hours the container is empty of water.

Work out the value of h

Dr Frost 407k

Extra Notes

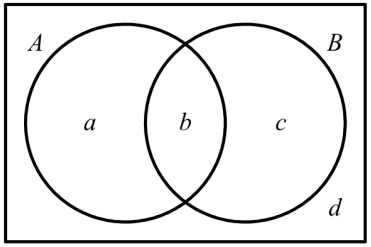
4 Advanced Probability

Venn Diagrams

Worked Example

Set A represents people who have a cat.

Set B represents people who have a dog.



a) Identify the region that represents people who have a cat but do not have a dog.

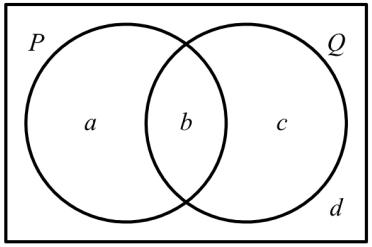
b) Identify the regions that, together, represent people who do not have a dog.

c) Identify the regions that, together, represent people who have a cat or a dog or both.

Your Turn

Set P represents people who own a bicycle.

Set Q represents people who work from home.



a) Identify the region that represents people who do not own a bicycle and do not work from home.

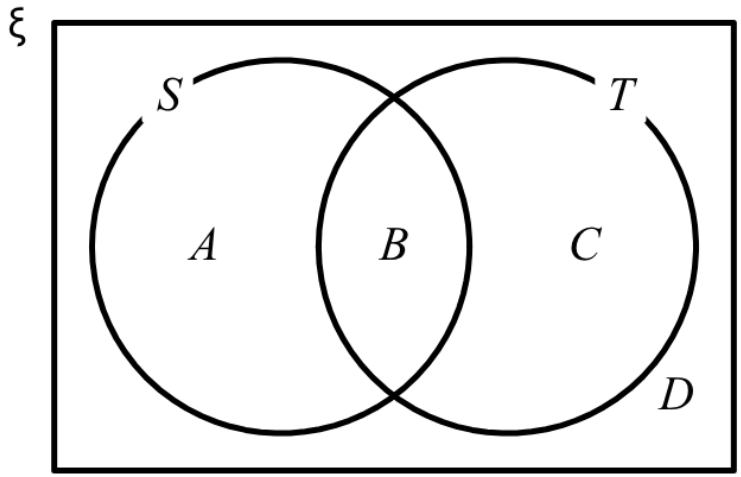
b) Identify the regions that, together, represent people who do not own a bicycle.

c) Identify the regions that, together, represent people who work from home.

Worked Example

$\xi = \{\text{Prime numbers less than } 18\}$
$S = \{5, 7, 13\}$
$T = \{5, 7, 17\}$

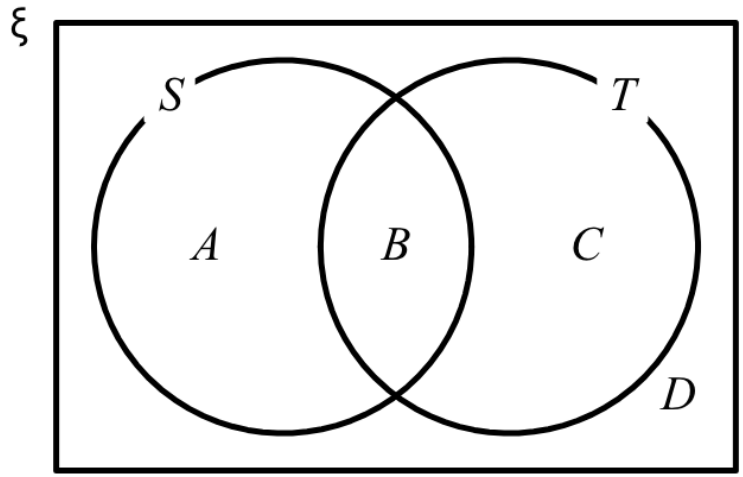
Complete the Venn diagram to represent this information.



Your Turn

$\xi = \{\text{Square numbers less than } 50\}$
$S = \{9, 16, 49\}$
$T = \{1, 4, 49\}$

Complete the Venn diagram to represent this information.



Worked Example	Your Turn
$\xi = \{1, 2, 3, \ldots, 10\}$ $A = \{2, 4, 6, 8, 10\}$ $B = \{3, 6, 9\}$ a) $A \cap B =$ b) $A \cup B =$ c) $A' =$ d) $B' =$ e) $A \cap B' =$ f) $A' \cap B =$ g) $A' \cap B' =$	$\xi =$ { all whole numbers } $A =$ { factors of 60 } $B =$ { multiples of 3 } a) $A \cap B =$ b) $A \cup B =$ c) $A' =$ d) $B' =$ e) $A \cap B' =$ f) $A' \cap B =$ g) $A' \cap B' =$

Fluency Practice

Complement: ‘

The <u>opposite</u> of a set.

B’ = everywhere not in B

Intersection: ∩

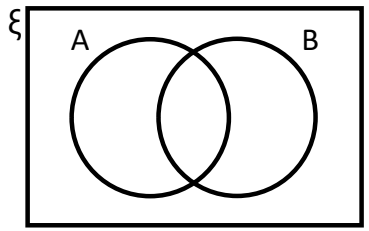
The <u>overlap</u> of regions.

A ∩ **B** = everywhere A and B overlap

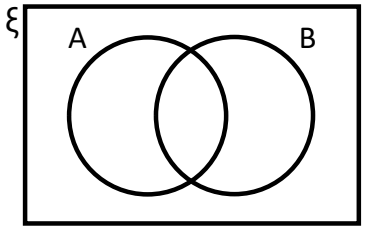
Union: ∪

The <u>sum</u> of regions.

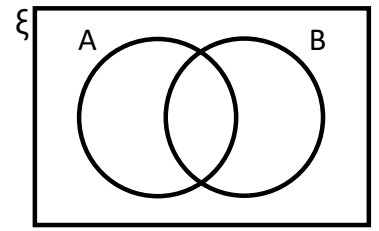
A ∪ **B** = A added to B



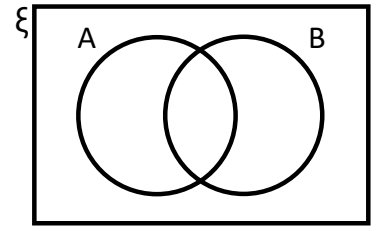
A



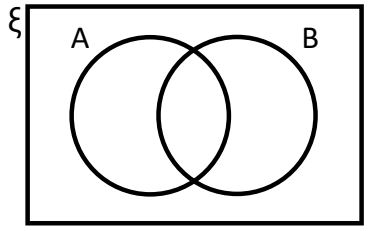
B



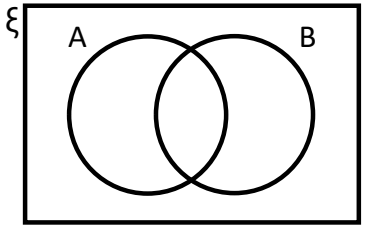
A’



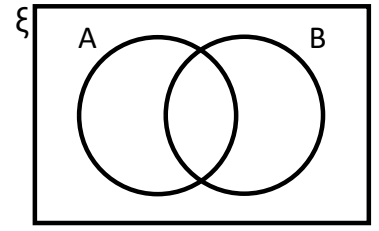
B’



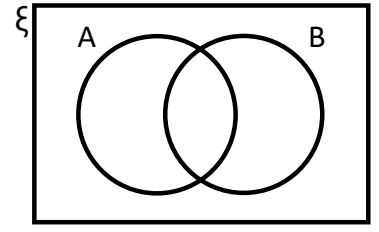
A ∪ B



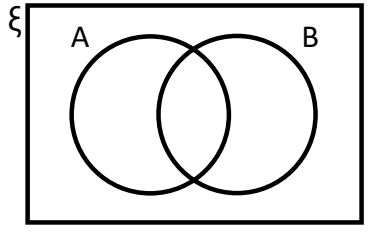
A ∪ B’



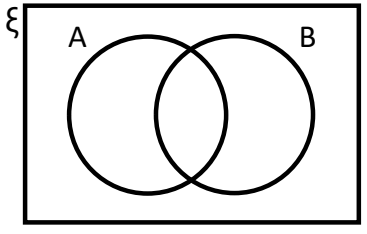
A’ ∪ B



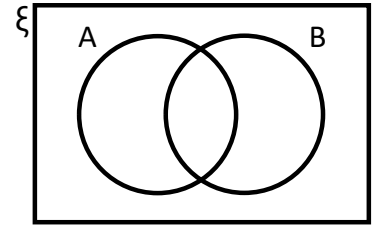
A’ ∪ B’



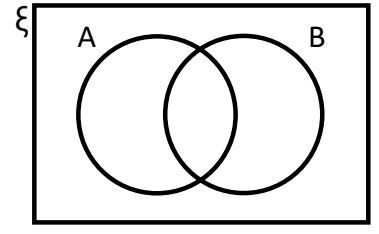
A ∩ B



A ∩ B’



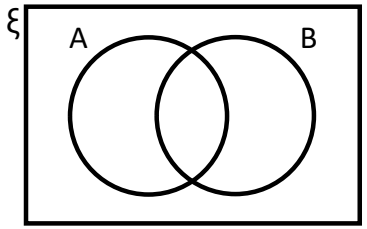
A’ ∩ B



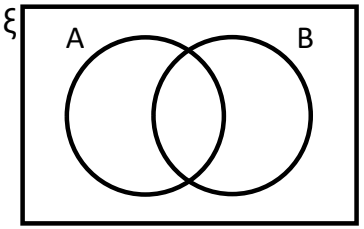
A’ ∩ B’

Fluency Practice

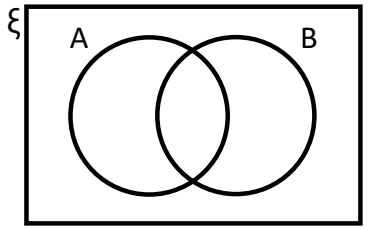
Shade the Venn Diagrams according to the notation. Remember! ∪ = sum, ∩ = overlap



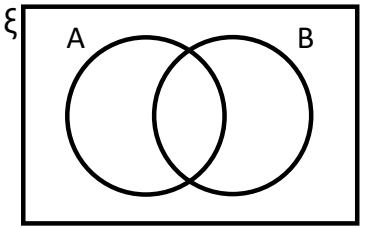
$A \cup B$



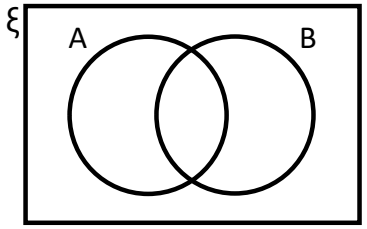
$A \cap B$



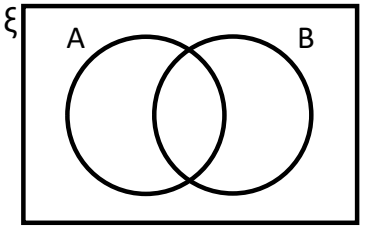
$A' \cup B$



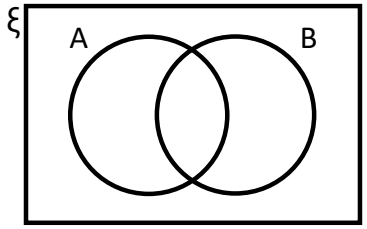
$A \cap B'$



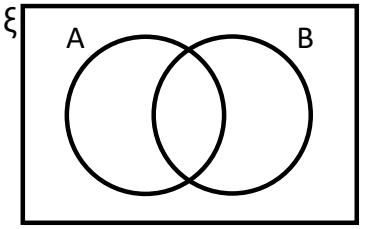
$A \cup B'$



$A' \cap B$



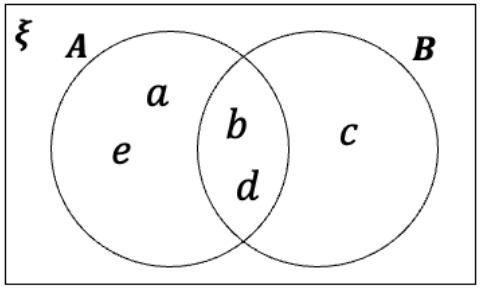
$A' \cup B'$



$A' \cap B'$

Dr Frost 243a and 243d

Worked Example



a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

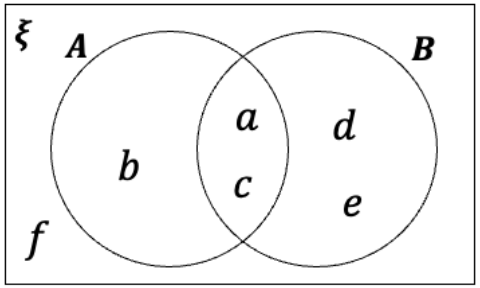
d) $B' =$

e) $A \cap B' =$

f) $A' \cap B =$

g) $A' \cap B' =$

Your Turn



a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

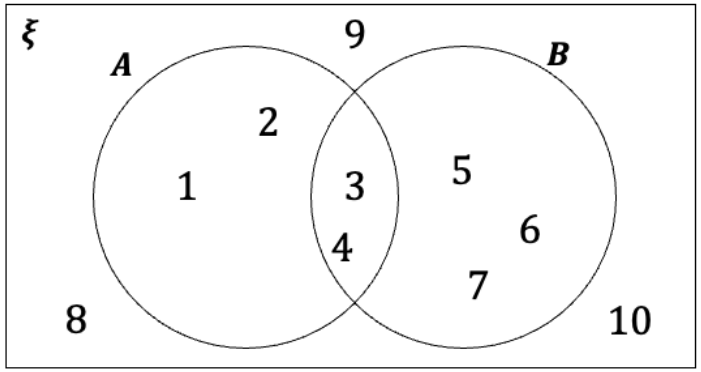
d) $B' =$

e) $A \cap B' =$

f) $A' \cap B =$

g) $A' \cap B' =$

Worked Example



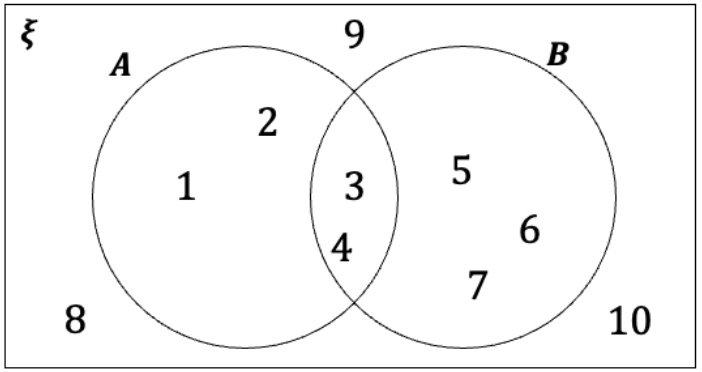
a) $n(A) =$

b) $n(A \cap B) =$

c) $n(A' \cap B) =$

d) $n(A' \cup B) =$

Your Turn



a) $n(B) =$

b) $n(A \cup B) =$

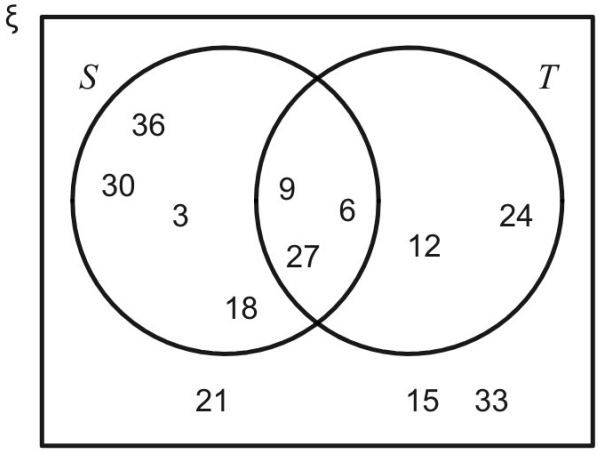
c) $n(A \cup B') =$

d) $n(A \cap B') =$

Worked Example

The Venn diagram represents the information from the following sets.

$\xi = \{\text{Multiples of 3 between 1 and 37}\}$
$S = \{3, 6, 9, 18, 27, 30, 36\}$
$T = \{6, 9, 12, 24, 27\}$



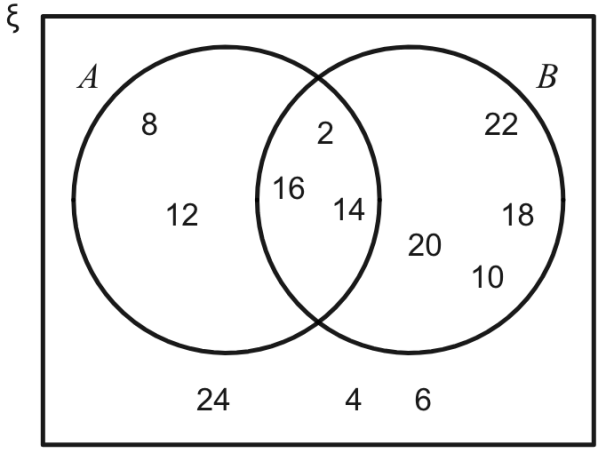
Find:

a) $P(S \cap T)$
b) $P(S')$
c) $P(S \cup T)$
d) $P(T)$

Your Turn

The Venn diagram represents the information from the following sets.

$\xi = \{\text{Multiples of 2 between 1 and 25}\}$
$S = \{2, 8, 12, 14, 16\}$
$T = \{2, 10, 14, 16, 18, 20, 22\}$



Find:

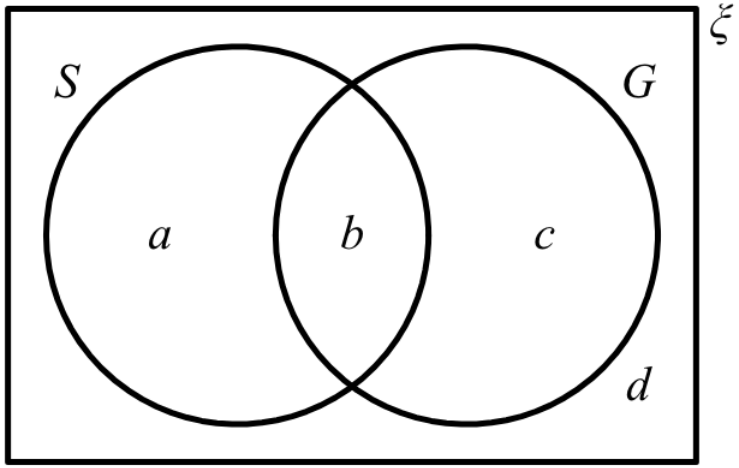
a) $P(A \cup B)$
b) $P(A)$
c) $P(A \cap B)$
d) $P(B')$

Worked Example

50 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

17 pupils use the swimming pool and the gym
25 pupils use the swimming pool
23 pupils use the gym

Complete the Venn diagram.

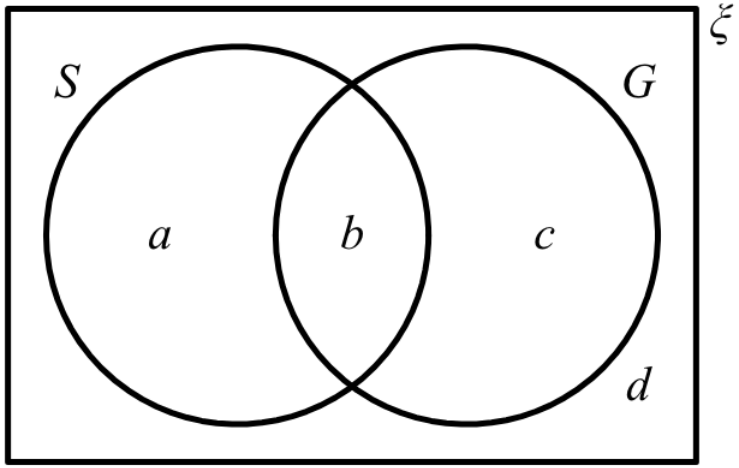


Your Turn

50 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

5 pupils use the swimming pool and the gym
28 pupils use the swimming pool
10 pupils use the gym

Complete the Venn diagram.

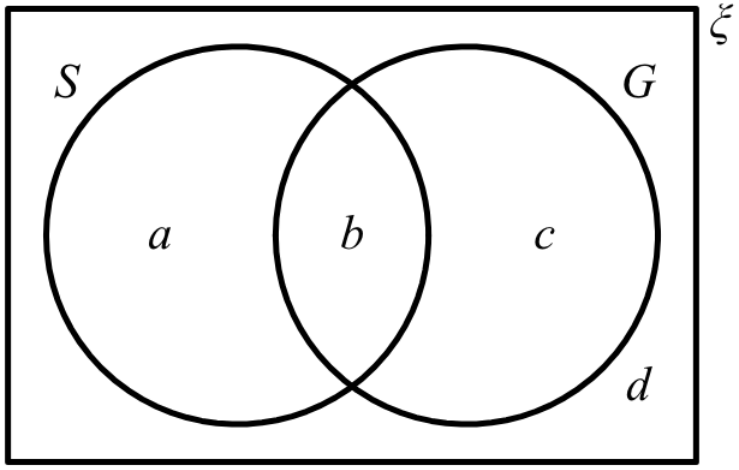


Worked Example

70 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

40 pupils use the swimming pool
39 pupils use the gym
21 pupils use neither the swimming pool nor the gym

Complete the Venn diagram.

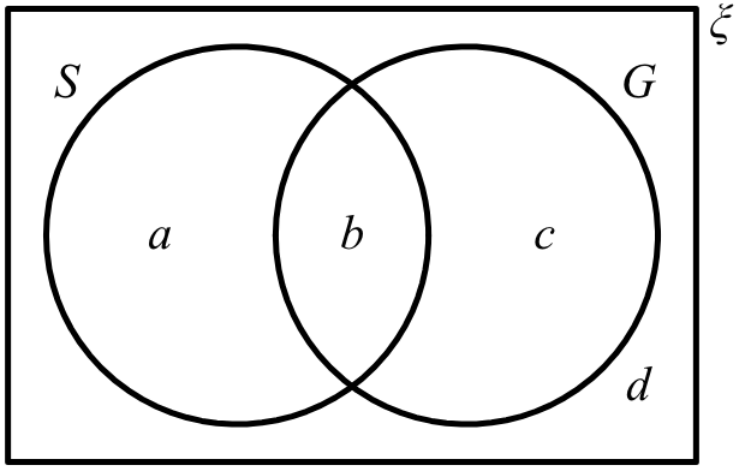


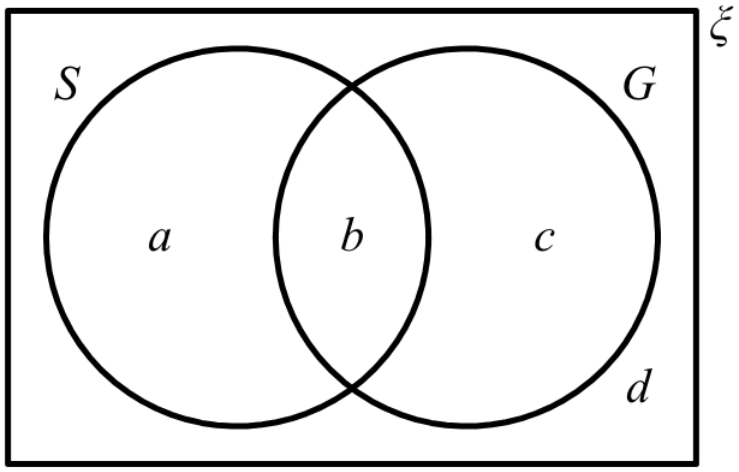
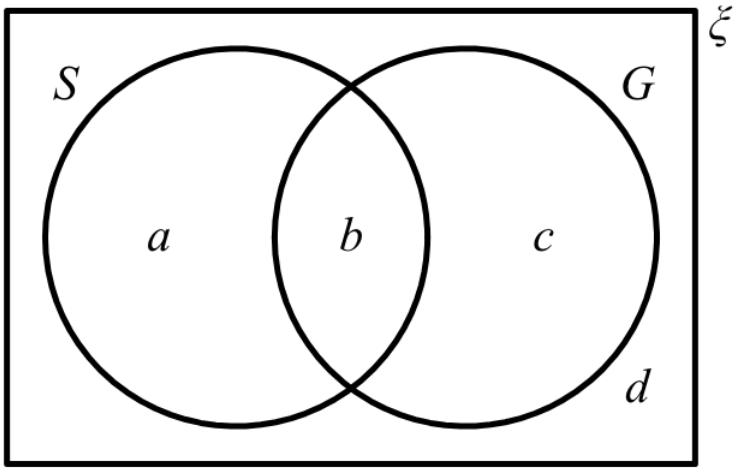
Your Turn

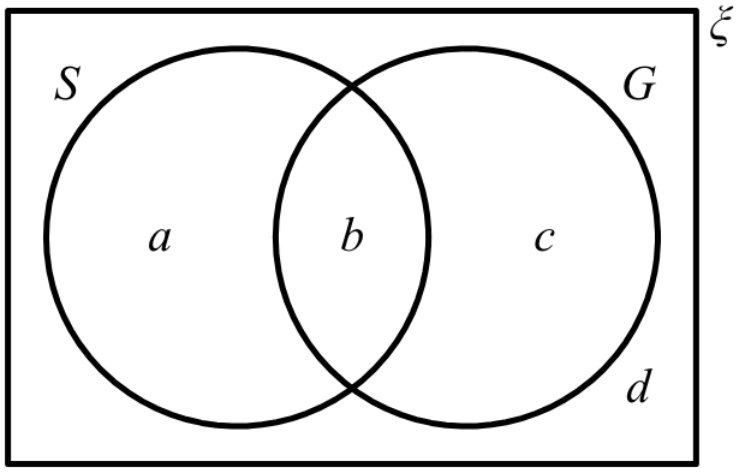
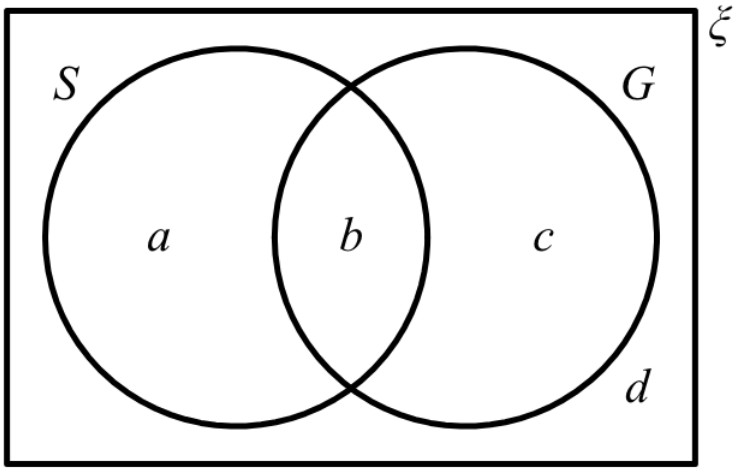
70 pupils in a sports centre are surveyed.
Some pupils use the swimming pool (S) and some use the gym (G).

26 pupils use the swimming pool
28 pupils use the gym
10 pupils use neither the swimming pool nor the gym

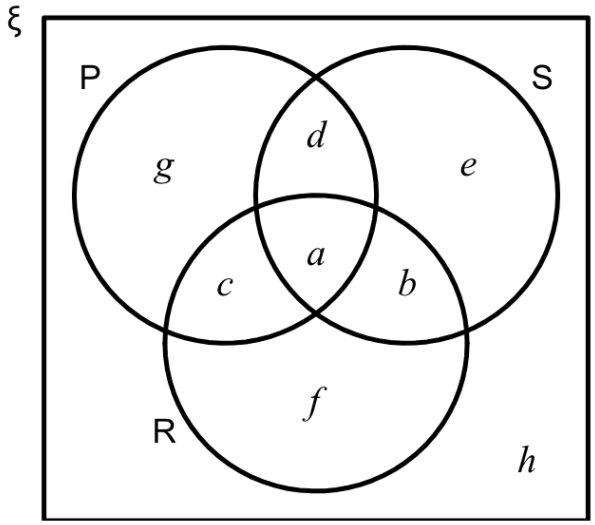
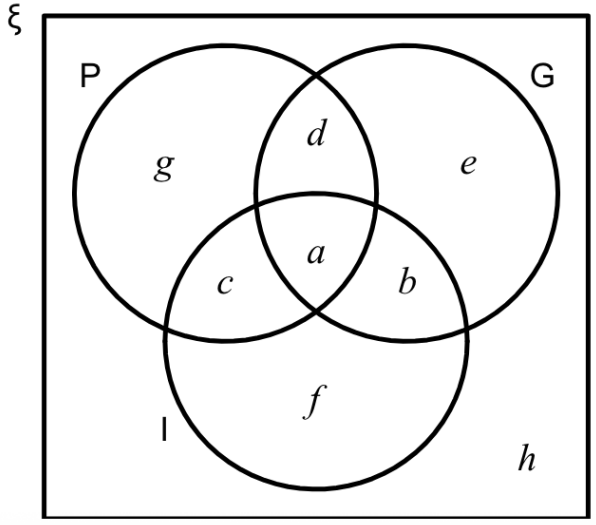
Complete the Venn diagram.



Worked Example	Your Turn
120 members visited a sports centre. 10 members used the gym (G) and 15 members used the swimming pool (S) There were no members who used both the gym and the swimming pool. Complete the Venn diagram.  	75 members visited a sports centre. 30 members used the gym (G) and 15 members used the swimming pool (S) There were no members who used both the gym and the swimming pool. Complete the Venn diagram. 

Worked Example	Your Turn
90 members visited a sports centre. 35 members used the gym (G) and 5 members used the swimming pool (S) Each member who used the swimming pool also used the gym. Complete the Venn diagram.  	80 members visited a sports centre. 35 members used the gym (G) and 15 members used the swimming pool (S) Each member who used the swimming pool also used the gym. Complete the Venn diagram. 

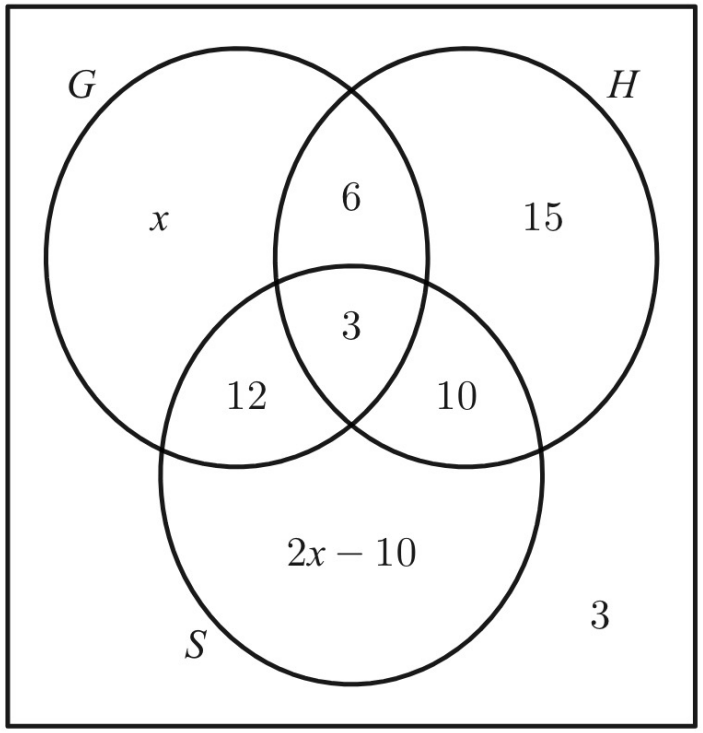
Dr Frost 242g

Worked Example	Your Turn
There are 130 pupils in a group. The only languages available for the group to study are Spanish, Russian and Portuguese. 11 pupils study the three languages. 27 pupils study both Spanish and Russian. 24 pupils study both Russian and Portuguese. 39 pupils study both Portuguese and Spanish. 75 pupils study Spanish. 57 pupils study Russian. 59 pupils study Portuguese.  Complete the Venn diagram.	There are 140 pupils in a group. The only languages available for the group to study are German, Italian and Portuguese. 33 pupils study the three languages. 49 pupils study both German and Italian. 40 pupils study both Italian and Portuguese. 53 pupils study both Portuguese and German. 86 pupils study German. 70 pupils study Italian. 85 pupils study Portuguese.  Complete the Venn diagram.

Worked Example

Some people are wearing gloves (G), some are wearing a hat (H) and some are wearing a scarf (S).

The Venn diagram shows information about a group of people.

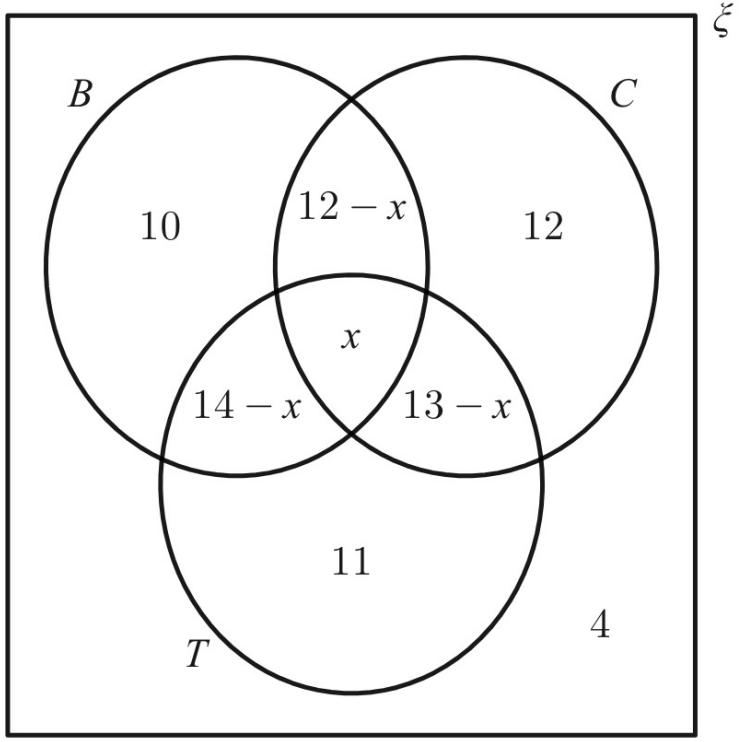


Given that 75 people are wearing **at least one** of these items of clothing, find the value of x

Your Turn

Some pupils play basketball (B), some play cricket (C) and some play tennis (T).

The Venn diagram shows information about a group of pupils.



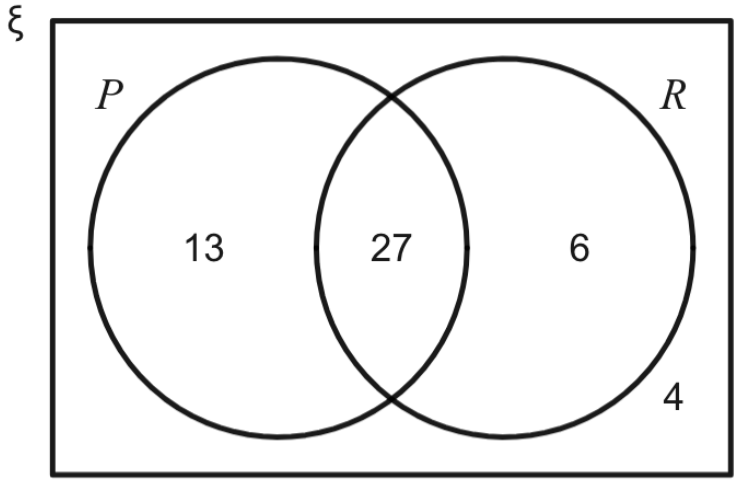
Given that 51 pupils play basketball or cricket or both, find the value of x

Dr Frost 242j

Worked Example

There are 50 pupils in a group.

The Venn diagram shows which languages they study from Portuguese and Russian.



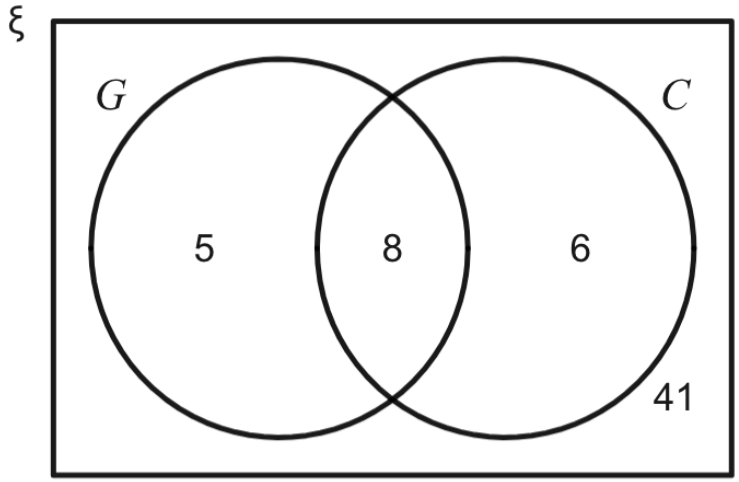
A pupil is selected at random.

a) Find the probability that this pupil studies Russian but not Portuguese.
b) Find the probability that this pupil studies Russian

Your Turn

There are 60 pupils in a group.

The Venn diagram shows which languages they study from German and Chinese.



A pupil is selected at random.

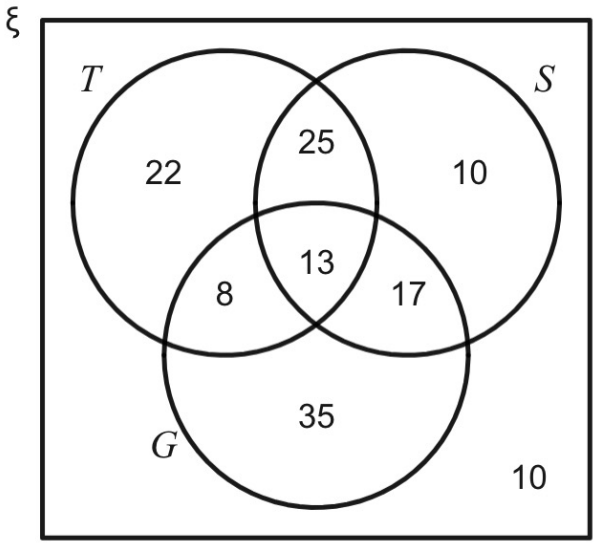
a) Find the probability that this pupil studies German but not Chinese.
b) Find the probability that this pupil studies Chinese.

Dr Frost 355c and 355d

Worked Example

140 pupils in a sports centre are surveyed.
The pupils can only use the swimming pool (S), the gym (G) and the tennis courts (T)

The Venn diagram shows the information about what facilities they used.



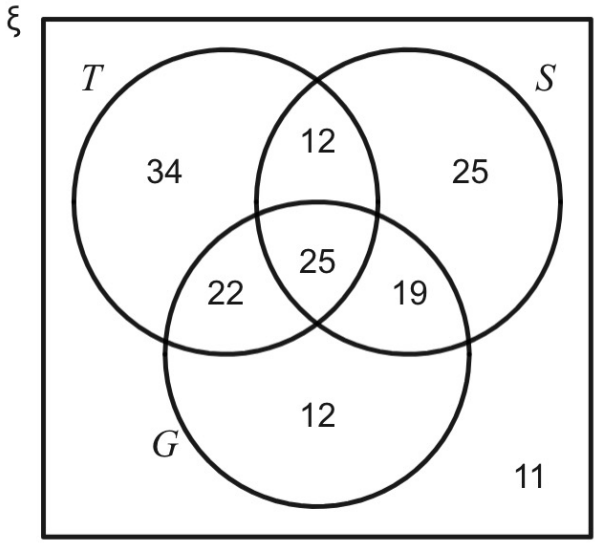
A pupil is selected at random.

Find the probability that this pupil uses both the swimming pool and the gym, but not the tennis courts.

Your Turn

160 pupils in a sports centre are surveyed.
The pupils can only use the swimming pool (S), the gym (G) and the tennis courts (T)

The Venn diagram shows the information about what facilities they used.



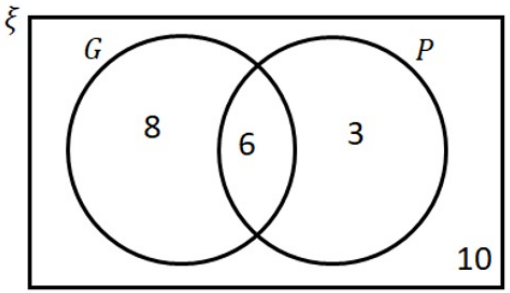
A pupil is selected at random.

Find the probability that this pupil uses both the tennis courts and the swimming pool, but not the gym.

Conditional Probability

Worked Example

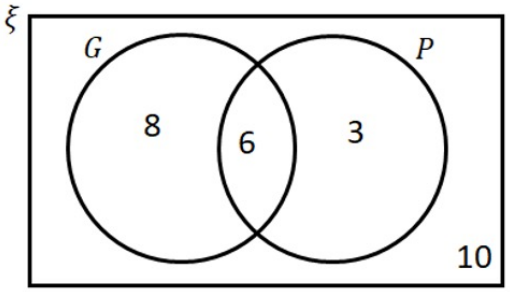
The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the guitar, given that they play the piano.

Your Turn

The Venn diagram shows a sample of people who play the guitar (G) or piano (P).

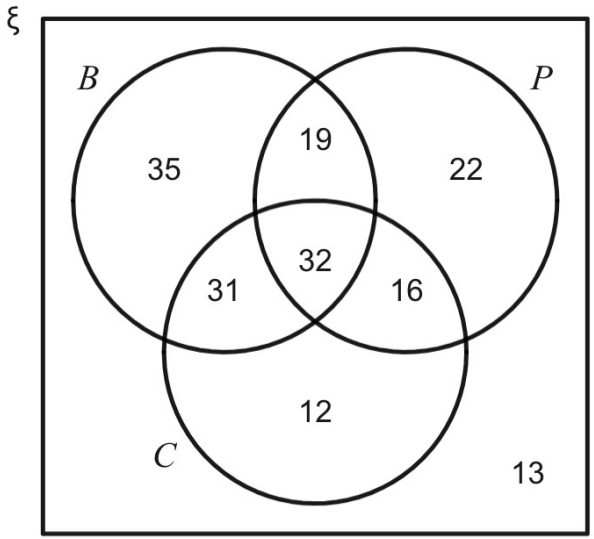


Find the probability that a student plays the piano, given that they play the guitar.

Worked Example

There are 180 pupils in a group who recently took three science tests.

The Venn diagram shows which tests they passed out of Physics, Chemistry and Biology.



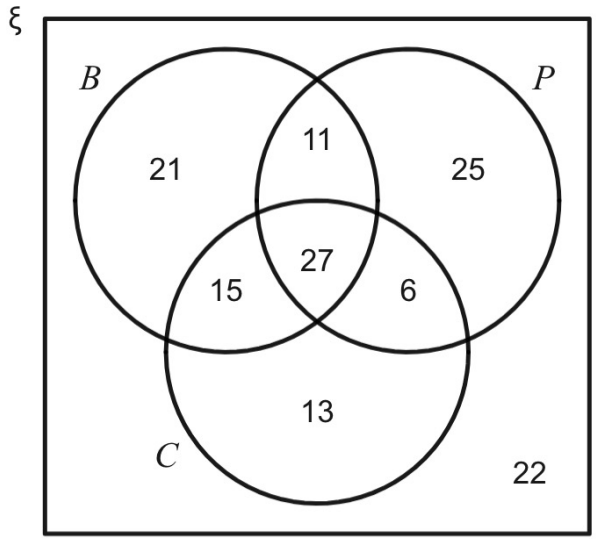
One of the pupils who passed Biology is selected at random.

Find the probability that this pupil also passed Chemistry but not Physics.

Your Turn

There are 140 pupils in a group who recently took three science tests.

The Venn diagram shows which tests they passed out of Physics, Chemistry and Biology.



One of the pupils who passed Chemistry is selected at random.

Find the probability that this pupil also passed Physics and Biology.

Worked Example

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

Draw a Venn Diagram, and hence answer the following questions:

a) $\mathrm{P}(\text{owns dog only})$
b) $\mathrm{P}(\text{does not own tropical fish})$
c) $\mathrm{P}(\text{does not own dogs, cats, or tropical fish})$
d) Given that a randomly chosen person owns a cat, what's the probability they own a dog?

Your Turn

The following shows the results of a survey on the types of exercise taken by a group of 100 people.
65 run, 48 swim, 60 cycle, 40 run and swim, 30 swim and cycle, 35 run and cycle and 25 do all three.

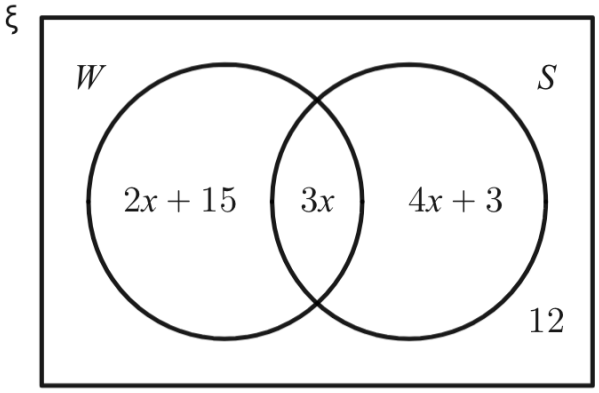
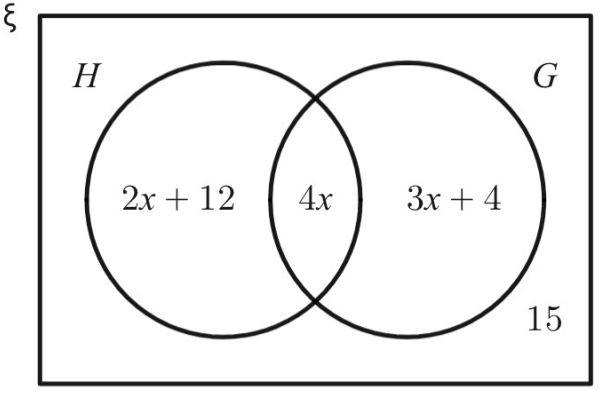
a) Draw a Venn Diagram to represent these data.

Find the probability that a randomly selected person from the survey

b) takes none of these types of exercise,
c) swims but does not run,
d) takes at least two of these types of exercise.

Jason is one of the above group. Given that Jason runs,

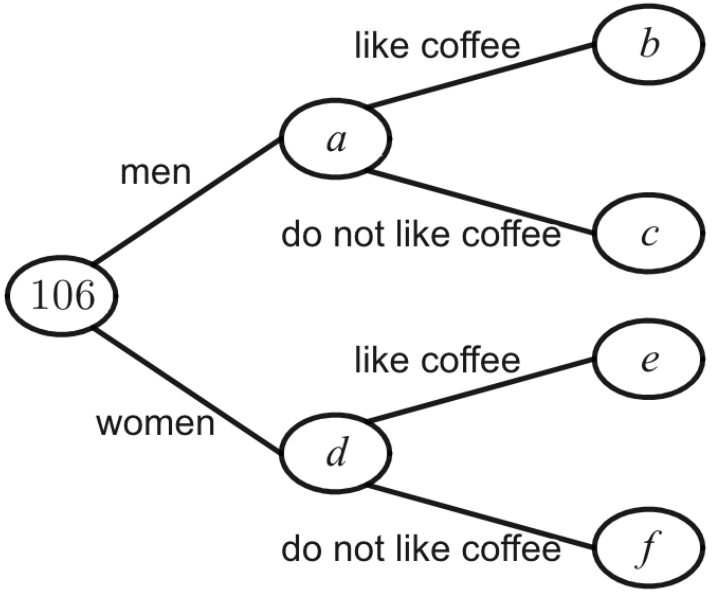
e) find the probability that he swims but does not cycle.

Worked Example	Your Turn
Yasmine has a collection of medals. The Venn diagram gives information about the number of windsurfing (W) medals and silver (S) medals in her collection.  Yasmine takes a windsurfing medal from her collection at random. The probability that the medal is not a silver medal is $\frac{5}{8}$ Work out the total number of medals in Yasmine's collection.	Arthur has a collection of medals. The Venn diagram gives information about the number of hockey (H) medals and gold (G) medals in her collection.  Arthur takes a hockey medal from his collection at random. The probability that the medal is not a gold medal is $\frac{5}{9}$ Work out the total number of medals in Arthur's collection.

Frequency Trees

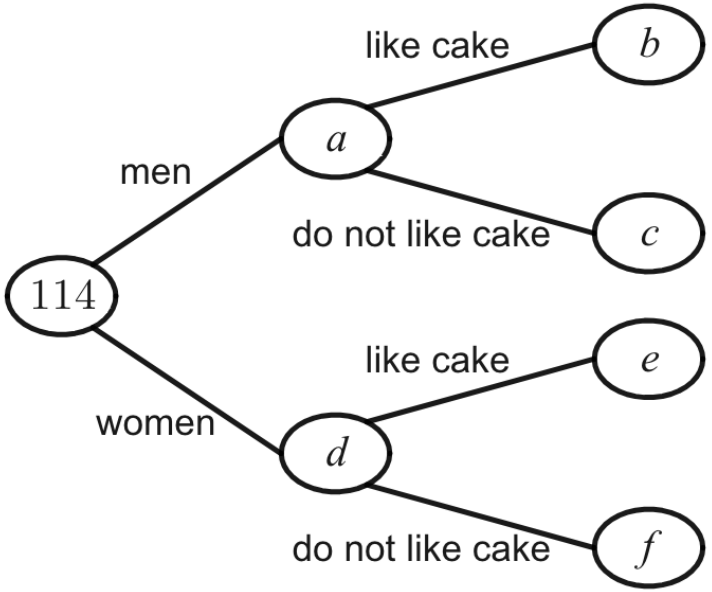
Worked Example

106 people are asked if they like coffee.
63 of these people are men.
60 of the 106 people like coffee.
21 of the women do not like coffee.
Use this information to complete the frequency tree.



Your Turn

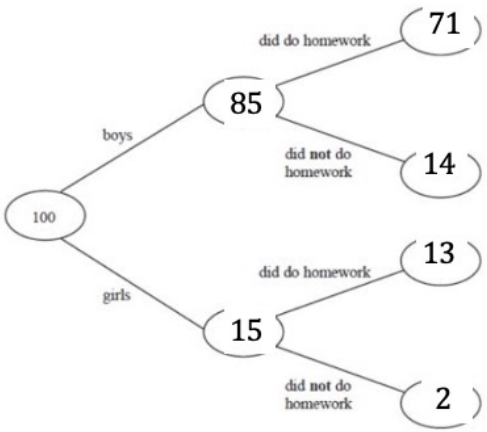
114 people are asked if they like cake.
58 of these people are men.
57 of the 114 people like cake.
24 of the women do not like cake.
Use this information to complete the frequency tree.



Dr Frost 241b and 241a

Worked Example

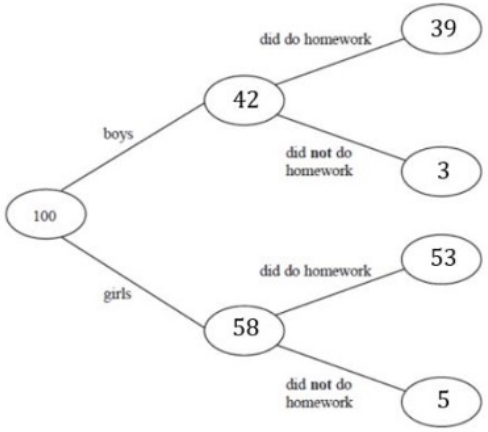
100 students had some homework.
The frequency tree shows the number of students who did the homework.



A boy is chosen at random.
Write down the probability that they did not do their homework.

Your Turn

100 students had some homework.
The frequency tree shows the number of students who did the homework.



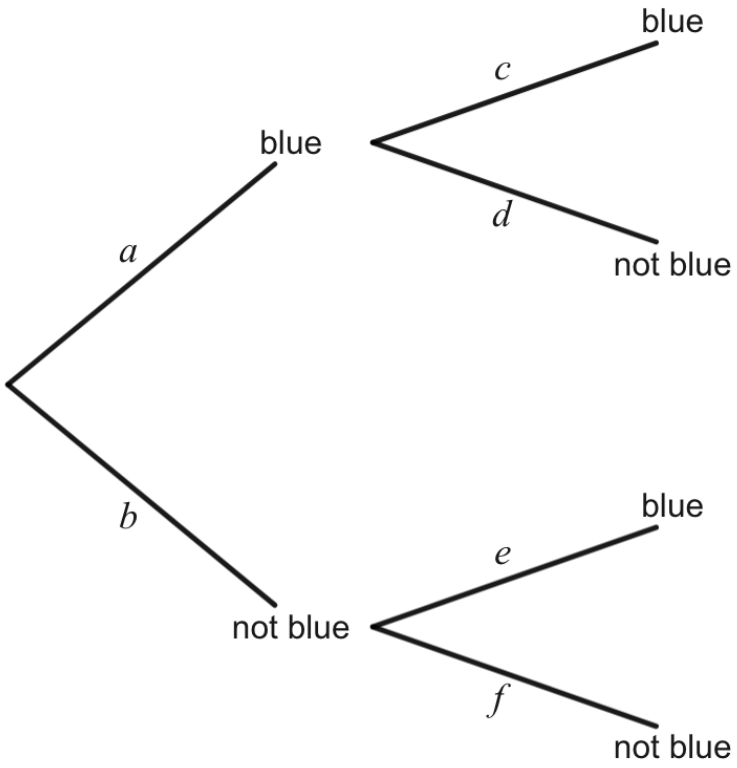
A girl is chosen at random.
Write down the probability that they did do their homework.

Tree Diagrams

Worked Example

Ruby has 2 blue balls, 2 purple balls and 1 black ball in a box.

Ruby takes one ball at random from the box, puts it back, and takes another ball from the box.

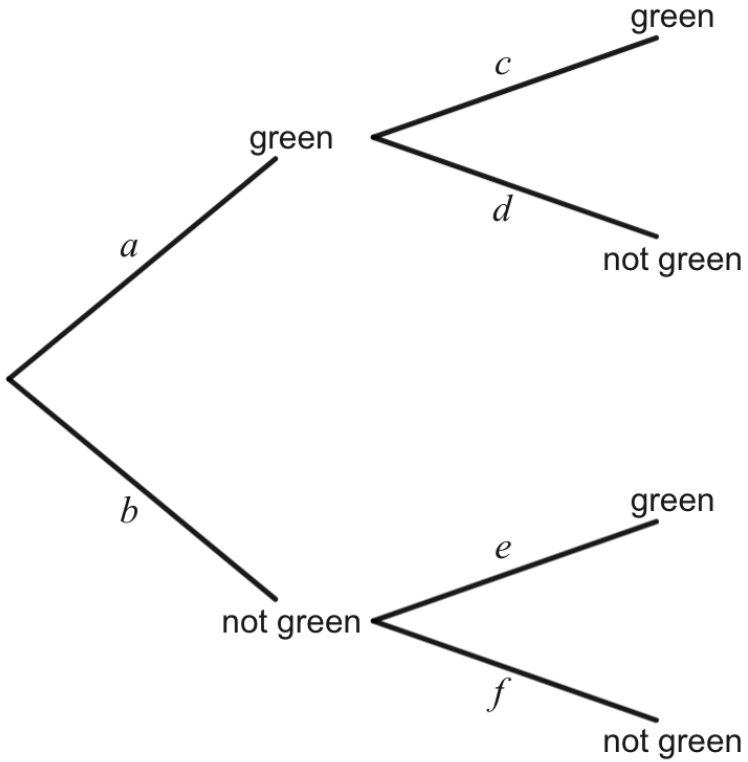


Complete the tree diagram.

Your Turn

Raphaël has 3 green socks, 1 blue sock and 1 yellow sock in a bag.

Raphaël takes one sock at random from the bag, puts it back, and takes another sock from the bag.



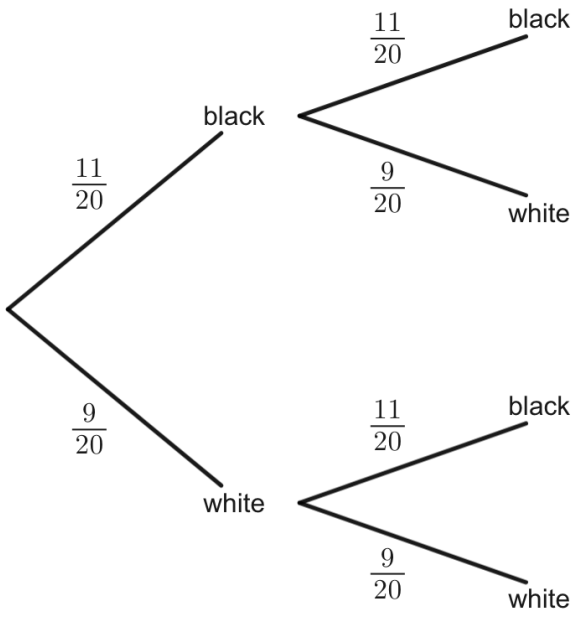
Complete the tree diagram.

Worked Example

Zion has 11 black counters and 9 white counters in a box.

Zion takes one counter at random from the box, puts it back, and takes another counter from the box.

A tree diagram is drawn to represent this information.



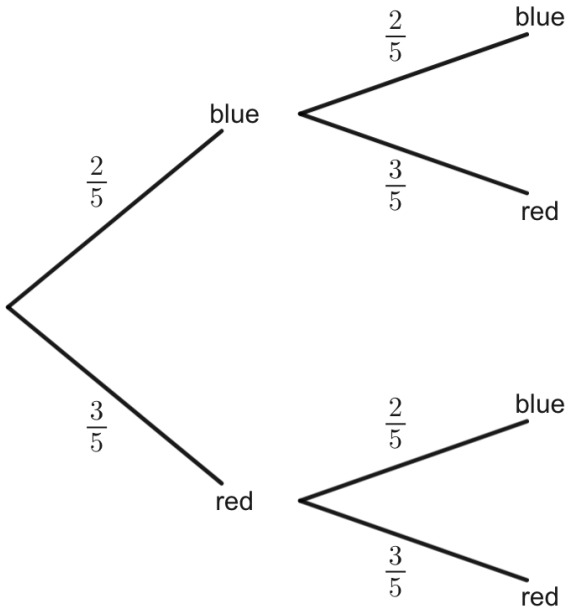
Find the probability that Zion takes two black counters.

Your Turn

Pierre has 2 blue balls and 3 red balls in a bag.

Pierre takes one ball at random from the bag, puts it back, and takes another ball from the bag.

A tree diagram is drawn to represent this information.



Find the probability that Pierre takes two balls that are not blue.

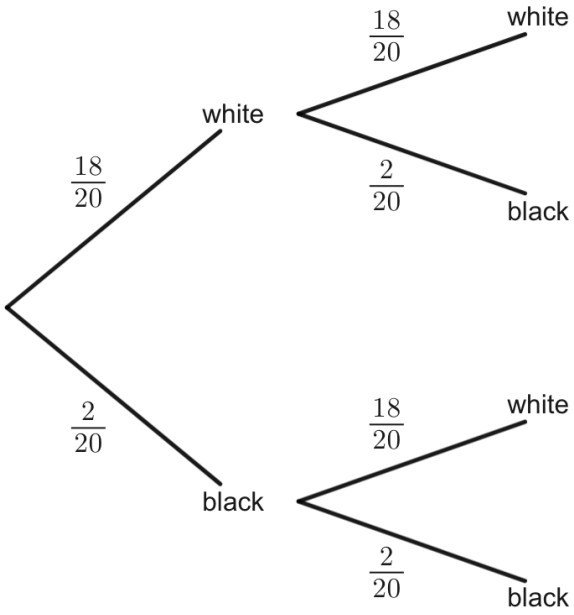
Worked Example	Your Turn
Diana has 3 red socks, 1 purple sock and 6 black socks in a bag. Diana takes one sock at random from the bag, puts it back, and takes another sock from the bag. Find the probability that Diana takes two socks that are not red.	Talula has 1 white counter, 4 yellow counters and 5 blue counters in a box. Talula takes one counter at random from the box, puts it back, and takes another counter from the box. Find the probability that Talula takes two counters that are not white.

Worked Example

Sophia has 18 white counters and 2 black counters in a box.

Sophia takes one counter at random from the box, puts it back, and takes another counter from the box.

A tree diagram is drawn to represent this information.



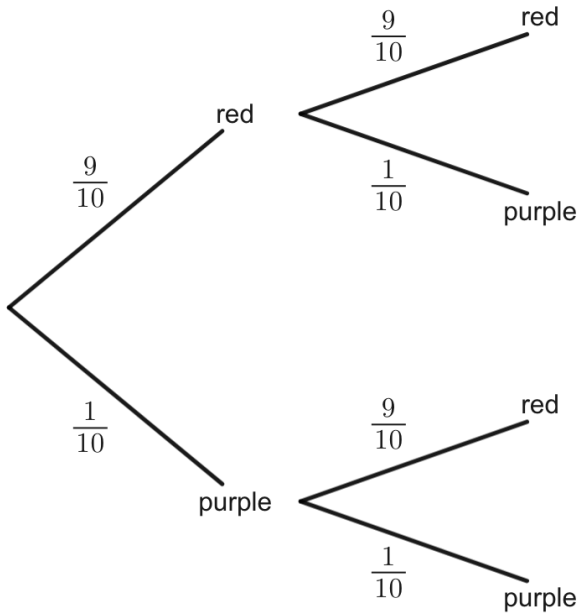
Find the probability that Sophia takes at least one white counter.

Your Turn

Alessia has 9 red balls and 1 purple ball in a box.

Alessia takes one ball at random from the box, puts it back, and takes another ball from the box.

A tree diagram is drawn to represent this information.



Find the probability that Alessia takes at least one red ball.

Worked Example	Your Turn
Holly has 18 red socks, 1 yellow sock and 1 white sock in a bag. Holly takes one sock at random from the bag, puts it back, and takes another sock from the bag. Find the probability that Holly takes at least one red sock.	Vamika has 4 purple socks, 1 red sock and 5 black socks in a box. Vamika takes one sock at random from the box, puts it back, and takes another sock from the box. Find the probability that Vamika takes at least one purple sock.

Worked Example

Astrid has a box of 30 nails.

The table shows the length of each nail.

Length of nail (cm)	3	7	8	9
Number of nails	9	10	6	5

She takes at random one of the nails and records its length.

She replaces the nail in the box.

She then takes at random a second nail from the box and records its length.

Calculate the probability that the two nails she takes have a total length of 16 cm.

Your Turn

Xavier has a box of 20 nails.

The table shows the length of each nail.

Length of nail (cm)	3	4	5	7	8	9
Number of nails	1	1	4	7	3	4

He takes at random one of the nails and records its length.
He replaces the nail in the box.

He then takes at random a second nail from the box and records its length.

Calculate the probability that the two nails he takes have a total length of 14 cm.

Worked Example

Imani has a bag of 30 coloured counters.

The table shows how many of each colour she has.

green	black	blue	purple
9	6	7	8

She picks at random one of the counters and records its colour and then puts it back in the bag.

Calculate the probability that Imani does not pick a black counter until her second pick.

Your Turn

Étienne has a bag of 20 coloured counters.

The table shows how many of each colour he has.

blue	purple	green	red
8	4	3	5

He picks at random one of the counters and records its colour and then puts it back in the bag.

Calculate the probability that Étienne does not pick a purple counter until his third pick.

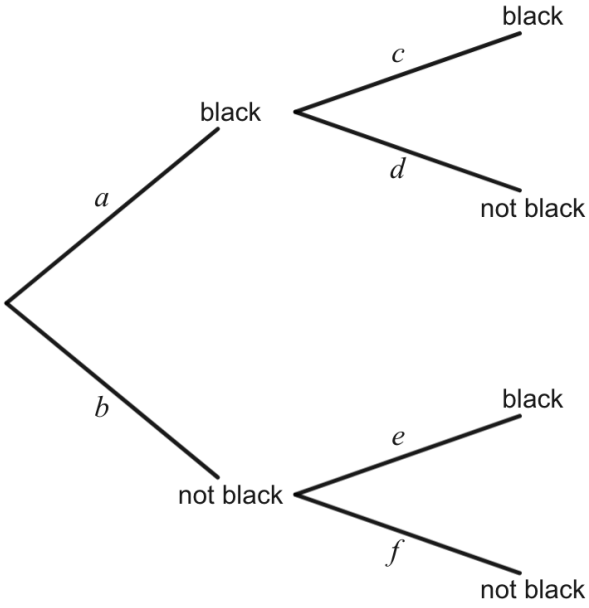
Worked Example	Your Turn
Alessia is going to play one game of Othello and one game of Tennis. The probability that she will win the game of Othello is 0.3 The probability that she will win both games is 0.15 Find the probability that she will win just one game.	Zara is going to play one game of Hearts and one game of Chess. The probability that she will win the game of Hearts is 0.7 The probability that she will win both games is 0.35 Find the probability that she will win just one game.

Worked Example	Your Turn
Kadiatou has a bag containing a large number of counters, some of which are red and the rest are green. She draws counters from the bag one at a time, notes the colour of the counter and returns it to the bag. The probability of Kadiatou getting a red counter on any one draw is $\frac{4}{5}$ Find the probability that Kadiatou will draw exactly 3 green counters **before** a red counter is drawn.	Mohammed has a bag containing a large number of counters, some of which are purple and the rest are orange. He draws counters from the bag one at a time, notes the colour of the counter and returns it to the bag. The probability of Mohammed getting a purple counter on any one draw is $\frac{3}{10}$ Find the probability that Mohammed will draw exactly 4 orange counters **before** a purple counter is drawn.

Worked Example

Emma has 14 black counters, 5 green counters and 1 blue counter in a box.

Emma takes one counter at random from the box, keeps it, and takes another counter from the box.

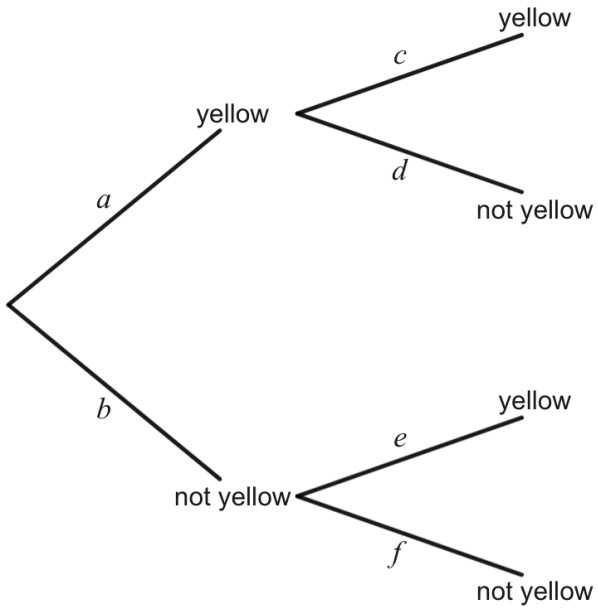


Complete the tree diagram.

Your Turn

Ámbar has 2 yellow balls, 4 green balls and 1 blue ball in a bag.

Ámbar takes one ball at random from the bag, keeps it, and takes another ball from the bag.



Complete the tree diagram.

Worked Example	Your Turn
Dhruv has 4 green socks, 3 yellow socks and 3 black socks in a box. Dhruv takes one sock at random from the box, keeps it, and takes another sock from the box. Find the probability that Dhruv takes two socks that are not green.	Thiago has 3 red socks, 2 green socks and 2 white socks in a box. Thiago takes one sock at random from the box, keeps it, and takes another sock from the box. Find the probability that Thiago takes two red socks.

Worked Example	Your Turn
Scarlet has 17 purple socks, 2 white socks and 1 blue sock in a box. Scarlet takes one sock at random from the box, keeps it, and takes another sock from the box. Find the probability that Scarlet takes at least one purple sock.	Ronan has 2 blue socks, 3 purple socks and 1 white sock in a bag. Ronan takes one sock at random from the bag, keeps it, and takes another sock from the bag. Find the probability that Ronan takes at least one blue sock.

Worked Example	Your Turn
Rose has 16 white counters and 4 yellow counters in a bag. Rose takes three counters at random from the bag. Work out the probability that she takes at least one white counter.	Hamza has 6 purple socks and 4 yellow socks in a bag. Hamza takes three socks at random from the bag. Work out the probability that he takes at least one purple sock.

Worked Example

Flynn has a box of 20 silk ribbons.

The table shows the length of each silk ribbon.

Length of silk ribbon (cm)	10	11	12	13	14	16
Number of silk ribbons	6	1	4	3	4	2

He takes at random two of the silk ribbons from the box, without replacement.

Calculate the probability that the two silk ribbons he takes have a total length of 24 cm.

Your Turn

Hugo has a deck of 40 cards each with different numbers on them.

The table shows the information about the cards.

Number on card	2	3	4	5	6	7	8	9
Frequency	5	5	5	8	3	5	5	4

He takes at random two of the cards from the deck, without replacement.

Calculate the probability that the two cards he takes can be multiplied together to give a product of 16

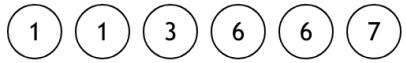
Worked Example

There are 8 counters in a bag.
There is a number on each counter.

Martha takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **odd** number.

Your Turn

There are 6 counters in a bag.
There is a number on each counter.



Emily takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **even** number.

Worked Example	Your Turn
There are 9 counters in a bag. There is a number on each counter.  Zoe takes at random three of the counters. She works out the **sum** of the numbers on the three counters. Work out the probability that the sum is an **odd** number.	There are 7 tiles in a sack. There is a number on each tile.  Jules takes at random three of the tiles. She **adds** together the numbers on the three counters to get a total. Find the probability that her total is an **even** number.

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Worked Example	Your Turn
There are q pens in a box, of which 4 are yellow and the rest are purple. Avery picks two pens at random. What is the probability that they pick a purple pen first and a yellow pen second? Give your answer as a single fraction.	There are w balls in a bucket, of which 5 are blue and the rest are purple. Giulia picks two balls at random. What is the probability that she picks two blue balls? Give your answer as a single fraction.

Worked Example	Your Turn
There are u green pens and 4 orange pens in a box. Said picks two pens at random. What is the probability that he picks two pens of different colours? Give your answer as a single fraction.	There are x beads in a box, of which 3 are yellow and the rest are purple. Abdul picks two beads at random. What is the probability that he picks two beads of the same colour? Give your answer as a single fraction.

Worked Example	Your Turn
A person throws a biased coin two times. The probability of flipping a heads on any throw is p a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails	A person throws a biased coin two times. The probability of flipping a heads on any throw is q a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails

Worked Example	Your Turn
Neha has n sweets, of which 6 are red. The remainder of the sweets are blue. Neha eats a sweet, does not regurgitate it, and then eats another sweet. The probability that she eats two red sweets is $\frac{15}{28}$ Show that $n^2 + an + b = 0$, where a and b are constants to be found.	Neha has 7 marbles, of which k are green. The remainder of the marbles are red. Neha takes a marble, does not replace it, and then takes another marble. The probability that she takes two green marbles is $\frac{2}{7}$ Show that $k^2 + ak + b = 0$, where a and b are constants to be found.

Worked Example	Your Turn
There are n sweets in a bag. 4 of the sweets are orange. The rest are yellow. Hannah takes at random a sweet from the bag. She eats the sweet. Hannah then takes at random another sweet from the bag and eats it. The probability that Hannah eats two orange sweets is $\frac{2}{5}$. Find n	There are n sweets in a bag. 6 of the sweets are orange. The rest are yellow. Hannah takes at random a sweet from the bag. She eats the sweet. Hannah then takes at random another sweet from the bag and eats it. The probability that Hannah eats two orange sweets is $\frac{1}{3}$. Find n

Worked Example	Your Turn
There are counters in a bag. At the start, 7 are red and the rest are blue. A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random. The probability that the first counter is blue, and the second counter is red is $\frac{21}{90}$ Work out the number of blue counters in the bag at the start.	There are counters in a bag. At the start, 7 are red and the rest are blue. A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random. The probability that the first counter is blue, and the second counter is red is $\frac{21}{80}$ Work out the number of blue counters in the bag at the start.

Worked Example	Your Turn
There are only green pens and blue pens in a box. There are three more blue pens than green pens in the box. There are more than 12 pens in the box. Simon is going to take at random two pens from the box. The probability that Simon will take two pens of the same colour is $\frac{27}{55}$ Work out the number of green pens in the box.	A bag contains red and blue balls. There is 2 more blue balls than red balls. The total number of balls is at least 7. Sophie takes two balls from the bag. The probability that they are different colours is $\frac{8}{15}$ Determine the number of balls in the bag.

Extra Notes

5 Capture-Recapture

Worked Example	Your Turn
I catch 12 fish in a pond and mark them with a red dot. The next day I catch 8 fish, of which, 3 have red dots on them. Estimate the population of fish in the pond.	I catch 18 fish in a pond and mark them with a red dot. The next day I catch 7 fish, of which, 2 have red dots on them. Estimate the population of fish in the pond.

Extra Notes