



KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS



KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM

# Year 11

## 2025 Mathematics 2026

### Unit 23 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_



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# Year 11

## 2025 Mathematics 2026

### Unit 23 Booklet – Part 2

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_

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# 1 Completing the Square

### Worked Example

- a) Express  $x^2 + 12x + 36$  in the form  $(x + a)^2$
- b) Express  $4x^2 + 12x + 9$  in the form  $(ax + b)^2$

### Your Turn

- a) Express  $x^2 - 14x + 49$  in the form  $(x + a)^2$
- b) Express  $9x^2 - 12x + 4$  in the form  $(ax + b)^2$

### Worked Example

Complete the square on the following expressions:

a)  $x^2 + 10x$

b)  $x^2 - 10x$

### Your Turn

Complete the square on the following expressions:

a)  $x^2 + 6x$

b)  $x^2 - 6x$

### Worked Example

Write  $x^2 + 4x + 6$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are integers.

### Your Turn

Write  $x^2 - 2x + 5$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are integers.

### Worked Example

Write  $x^2 + 5x - 7$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are constants to be found.

### Your Turn

Write  $x^2 - 11x - 4$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are constants to be found.

### Worked Example

Write  $x^2 + 2px + 4$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are in terms of  $p$ .

### Your Turn

Write  $x^2 + 8kx + 5$  in the form  $(x + a)^2 + b$  where  $a$  and  $b$  are in terms of  $k$ .

### Worked Example

Write  $x^2 + 4\sqrt{5}x + 10$  in the form  $(x + a)^2 + b$  where  $a$  is a surd and  $b$  is an integer.

### Your Turn

Write  $x^2 - 8\sqrt{3}x + 5$  in the form  $(x + a)^2 + b$  where  $a$  is a surd and  $b$  is an integer.

## Fill in the Gaps

Quadratic Expression	$\left(x + \frac{b}{2}\right)^2$	$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$	$\pm c$	Completed Square
$x^2 + 10x$	$(x + 5)^2$	$(x + 5)^2 - 25$	$(x + 5)^2 - 25$	$(x + 5)^2 - 25$
$x^2 + 8x - 2$	$(x + 4)^2$	$(x + 4)^2 - 16$	$(x + 4)^2 - 16 - 2$	
$x^2 + 2x + 5$	$(x + 1)^2$	$(x + 1)^2 - 1$		
$x^2 + 4x + 7$	$(x + 2)^2$			
$x^2 + 6x - 11$				
$x^2 + 18x + 50$				
$x^2 - 12x$	$(x - 6)^2$	$(x - 6)^2 - 36$	$(x - 6)^2 - 36$	$(x - 6)^2 - 36$
$x^2 - 2x + 5$	$(x - 1)^2$	$(x - 1)^2 - 1$	$(x - 1)^2 - 1 + 5$	
$x^2 - 6x - 1$	$(x - 3)^2$	$(x - 3)^2 - 9$		
$x^2 - 8x + 16$	$(x - 4)^2$			
$x^2 - 4x + 5$				
$x^2 + 7x + 1$	$\left(x + \frac{7}{2}\right)^2$	$\left(x + \frac{7}{2}\right)^2 - \frac{49}{4}$	$\left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + 1$	$\left(x + \frac{7}{2}\right)^2 - \frac{45}{4}$
$x^2 - 5x - 3$				
$x^2 + x + 6$				
$x^2 - 3x + 2$				
				$(x + 5)^2 - 7$
				$\left(x - \frac{3}{2}\right)^2 + \frac{3}{4}$

### Worked Example

Write the expression  $4x^2 - 8x - 9$  in the form  $r(x + p)^2 + q$  where  $r, p,$  and  $q$  are integers.

### Your Turn

Write the expression  $3x^2 + 12x + 19$  in the form  $r(x + p)^2 + q$  where  $r, p,$  and  $q$  are integers.

## Fill in the Gaps

Quadratic Expression	Take out Common Factor	Complete the Square	Multiply by Common Factor	Completed Square
$2x^2 + 16x$	$2[x^2 + 8x]$	$2[(x + 4)^2 - 16]$	$2(x + 4)^2 - 32$	$2(x + 4)^2 - 32$
$3x^2 - 18x$	$3[x^2 - 6x]$			
$2x^2 + 12x + 1$	$2[x^2 + 6x] + 1$	$2[(x + 3)^2 - 9] + 1$	$2(x + 3)^2 - 18 + 1$	$2(x + 3)^2 - 17$
$2x^2 - 20x - 7$	$2[x^2 - 10x] - 7$	$2[(x - 5)^2 - 25] - 7$		
$3x^2 + 6x - 5$	$3[x^2 + 2x] - 5$			
$4x^2 + 16x - 1$				
$5x^2 - 30x + 11$				
$2x^2 - 10x + 3$	$2[x^2 - 5x] + 3$	$2\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4}\right] + 3$	$2\left(x - \frac{5}{2}\right)^2 - \frac{25}{2} + 3$	$2\left(x - \frac{5}{2}\right)^2 - \frac{19}{2}$
$2x^2 + 6x - 1$				
$3x^2 - 9x + 2$				

### Worked Example

Write  $-3x^2 + 2x + 6$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are rational numbers.

### Your Turn

Write  $-3x^2 + 8x + 5$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are rational numbers.

## Worked Example

Solve the equation:  
 $(x + 1)^2 = 4$

## Your Turn

Solve the equation:  
 $(x - 4)^2 = 9$

### Worked Example

Solve the equation:  
 $(2x + 3)^2 = 4$

### Your Turn

Solve the equation:  
 $(3x - 2)^2 = 9$

### Worked Example

Solve the following quadratic equation, leaving your answer in exact form:  $x^2 - 6x + 2 = 0$

### Your Turn

Solve the following quadratic equation, leaving your answer in exact form:  $x^2 + 4x - 11 = 0$

### Worked Example

Solve the following quadratic equation, leaving your answer in exact form:  $3x^2 - 7x + 1 = 0$

### Your Turn

Solve the following quadratic equation, leaving your answer in exact form:  $4x^2 - 7x + 3 = 0$

## Fill in the Gaps

Quadratic in the form $f(x) = ax^2 + bx + c$	Quadratic in factorised form	Quadratic in completed square form	Solutions to quadratic equation $f(x) = 0$
$f(x) = x^2 - 2x - 15$			
$f(x) = x^2 + 6x + 8$			
	$f(x) = (x + 5)(x - 2)$		
		$f(x) = (x + 5)^2 - 9$	
		$f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$	
			$x = -6, x = 4$
			$x = 3, x = -8$
			$x = q, x = 3q$

## Fill in the Gaps

Quadratic in the form $f(x) = ax^2 + bx + c$	Quadratic in factorised form	Quadratic in completed square form	Solutions to quadratic equation $f(x) = 0$
$f(x) = 4x^2 + 16x + 7$			
$f(x) = 4x^2 - 4x - 3$			
	$f(x) = 2x(x + 4)$		
		$f(x) = 3\left(x + \frac{5}{3}\right)^2 - \frac{49}{3}$	
		$f(x) = 2\left(x - \frac{1}{4}\right)^2 - \frac{25}{8}$	
			$x = -\frac{1}{3}, x = -2$
			$x = \frac{3}{2}, x = -\frac{3}{4}$
			$x = \frac{p}{2}, x = -\frac{3p}{2}$

## Worked Example

Make  $x$  the subject:

a)  $y = x^2 + 6x - 2$

b)  $y = 2x^2 + 16x - 2$

## Your Turn

Make  $x$  the subject:

a)  $y = x^2 + 8x - 3$

b)  $y = 2x^2 + 12x - 3$

## Extra Notes

## 2 Advanced Functions

### Worked Example

$$\text{If } f(x) = 3x + 4,$$

$$g(x) = 2x - 5$$

$$\text{a) } fg(6) =$$

$$\text{b) } gf(6) =$$

### Your Turn

$$\text{If } f(x) = 4x - 3,$$

$$g(x) = 5x + 2$$

$$\text{a) } fg(8) =$$

$$\text{b) } gf(8) =$$

### Worked Example

$$\text{If } f(x) = 5x^2,$$

$$g(x) = 2x + 3$$

$$\text{a) } fg(2) =$$

$$\text{b) } gf(2) =$$

### Your Turn

$$\text{If } f(x) = 4x^2,$$

$$g(x) = 3x + 2$$

$$\text{a) } fg(4) =$$

$$\text{b) } gf(4) =$$

### Worked Example

$$\text{If } f(x) = x + 3,$$

$$g(x) = \frac{1}{x-2}$$

$$\text{a) } fg(5) =$$

$$\text{b) } gf(5) =$$

### Your Turn

$$\text{If } f(x) = x - 5,$$

$$g(x) = \frac{1}{x+4}$$

$$\text{a) } fg(8) =$$

$$\text{b) } gf(8) =$$

## Fill in the Gaps

Question	Input	1 <sup>st</sup> Function	2 <sup>nd</sup> Function	Output	Answer
$f(x) = 3x$ $g(x) = x - 1$ Find $fg(2)$	<input type="text" value="2"/> →	<input type="text" value="-1"/> →	<input type="text" value="× 3"/> →	<input type="text" value="3"/>	$fg(2) =$
$f(x) = 5x$ $g(x) = x + 3$ Find $gf(6)$	<input type="text" value="6"/> →	<input type="text" value="× 5"/> →	<input type="text"/> →	<input type="text"/>	
$f(x) = x - 1$ $g(x) = x^2$ Find $fg(3)$	<input type="text"/> →	<input type="text"/> →	<input type="text"/> →	<input type="text"/>	
$f(x) = x + 9$ $g(x) = \sqrt{x}$ Find $gf(-5)$	<input type="text"/> →	<input type="text"/> →	<input type="text"/> →	<input type="text"/>	
$f(x) = \frac{x}{2}$ $g(x) = x + 7$ Find $fg(4)$	<input type="text"/> →	<input type="text"/> →	<input type="text"/> →	<input type="text"/>	
$g(x) = \sqrt{x}$ $h(x) = x - 3$ Find $gh(3.25)$	<input type="text"/> →	<input type="text"/> →	<input type="text"/> →	<input type="text"/>	
$f(x) = \frac{1}{x}$ $g(x) = x^2$ Find $gf(0.4)$	<input type="text"/> →	<input type="text"/> →	<input type="text"/> →	<input type="text"/>	

## Fill in the Gaps

Question	Input	1 <sup>st</sup> Function	2 <sup>nd</sup> Function	Output	Answer
$f(x) = x^2 + 2$ $g(x) = 3x - 1$ Find $fg(4)$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">4</div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;">× 3</div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;">- 1</div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;">square</div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div>	$fg(4) =$
$f(x) = 3\sqrt{x}$ $g(x) = 2x + 5$ Find $gf(9)$	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;">square root</div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div>	$gf(9) =$
$f(x) = \frac{1}{x} - 3$ $g(x) = 2x + 4$ Find $fg(-1)$	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div>	$fg(-1) =$
$g(x) = \frac{x}{2} + 1$ $h(x) = 4x^2$ Find $hg(0.5)$	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div>	
$f(x) = x^2 + 3$ $g(x) = 2x - 7$ Find $fg(5)$	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> → <div style="border: 1px solid black; padding: 5px; display: inline-block; margin-right: 10px;"></div> →	<div style="border: 1px solid black; width: 60px; height: 30px; margin: 0 auto;"></div>	

### Worked Example

$$\text{If } f(x) = 3x + 4,$$

$$g(x) = 2x - 5$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

### Your Turn

$$\text{If } f(x) = 4x - 3,$$

$$g(x) = 5x + 2$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

**Worked Example**

$$\text{If } f(x) = 2x - 3$$
$$ff(x) =$$

**Your Turn**

$$\text{If } g(x) = 3x - 2$$
$$gg(x) =$$

### Worked Example

$$\text{If } f(x) = 5x^2,$$
$$g(x) = 2x + 3$$

a)  $fg(x) =$

b)  $gf(x) =$

### Your Turn

$$\text{If } f(x) = 4x^2,$$
$$g(x) = 3x + 2$$

a)  $fg(x) =$

b)  $gf(x) =$

**Worked Example**

$$\text{If } f(x) = 4x^2 - 3$$
$$ff(x) =$$

**Your Turn**

$$\text{If } g(x) = 5x^2 - 2$$
$$gg(x) =$$

**Worked Example**

$$\text{If } f(x) = x + 3,$$

$$g(x) = \frac{1}{x-2}$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

**Your Turn**

$$\text{If } f(x) = x - 5,$$

$$g(x) = \frac{1}{x+4}$$

$$\text{a) } fg(x) =$$

$$\text{b) } gf(x) =$$

**Worked Example**

$$\text{If } f(x) = \frac{x+2}{3}$$
$$ff(x) =$$

**Your Turn**

$$\text{If } g(x) = \frac{x-3}{2}$$
$$gg(x) =$$

### Worked Example

The function  $f$  is defined by:

$$f(x) = \frac{5x - 2}{2x - 3}$$

Find  $ff(x)$

Give your answer in the form  $\frac{Ax+B}{C+Dx}$

### Your Turn

The function  $f$  is defined by:

$$f(x) = \frac{2x - 5}{5 + 3x}$$

Find  $ff(x)$

Give your answer in the form  $\frac{A+Bx}{C+Dx}$

## Fill in the Gaps

Question	Input	1 <sup>st</sup> Function	2 <sup>nd</sup> Function	Output	Answer
$f(x) = 5x$ $g(x) = x + 2$ Find $fg(x)$	$x$ →	+2 →	× 5 →	$fg(x)$	$fg(x) = 5(x + 2)$
$f(x) = 5x$ $g(x) = x + 2$ Find $gf(x)$	$x$ →	× 5 →	+2 →	□	
$f(x) = x - 1$ $g(x) = x^2$ Find $fg(x)$	$x$ →	square →	□ →	□	
$f(x) = x + 3$ $g(x) = \sqrt{x}$ Find $gf(x)$	$x$ →	□ →	□ →	□	
$f(x) = \frac{x}{2}$ $g(x) = x + 7$ Find $fg(x)$	$x$ →	□ →	□ →	□	
$g(x) = x - 4$ $h(x) = \sqrt{x}$ Find $gh(x)$	$x$ →	□ →	□ →	□	
$f(x) = \frac{1}{x}$ $g(x) = x^2$ Find $gf(x)$	$x$ →	□ →	□ →	□	

## Fill in the Gaps

Question	Input	1 <sup>st</sup> Function	2 <sup>nd</sup> Function	Output	Answer
$f(x) = 3x - 1$ $g(x) = x^2 + 5$ Find $fg(x)$	$x$ →	square → +5 →	$\times 3$ → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	$fg(x)$	$fg(x)$ $= 3(x^2 + 5) - 1$ $= 3x^2 + 14$
$f(x) = 2\sqrt{x}$ $g(x) = 4x - 3$ Find $gf(x)$	$x$ →	square root → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	<span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	$gf(x)$	$= gf(x)$
$f(x) = \frac{x}{2} + 1$ $g(x) = 3x^2$ Find $fg(x)$	$x$ →	<span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	<span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	$fg(x)$	$= fg(x)$
$g(x) = \frac{1}{x-2}$ $h(x) = 4\sqrt{x}$ Find $hg(x)$	$x$ →	<span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	<span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	$hg(x)$	
$f(x) = 5x^2$ $g(x) = 2x - 1$ Find $fg(x)$	$x$ →	<span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	<span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> → <span style="border: 1px solid black; display: inline-block; width: 40px; height: 20px; vertical-align: middle;"></span> →	$fg(x)$	

## Fill in the Gaps

$f(x)$	$g(x)$	$fg(x)$	$gf(x)$
$f(x) = x - 3$	$g(x) = x^2$	$fg(x) = x^2 - 3$	$gf(x) = (x - 3)^2$
$f(x) = \frac{x}{5}$	$g(x) = x + 1$		
$f(x) = 3x$	$g(x) = 7 - x$		
$f(x) = \sqrt{x}$	$g(x) = \frac{x}{4}$		
$f(x) = 2x + 9$	$g(x) = x - 3$	$fg(x) = 2x + 3$	
$f(x) = x^2 - 1$	$g(x) = \frac{x}{3}$		
$f(x) = \sqrt{x}$	$g(x) = 4 - 3x$		
$f(x) = \frac{2x}{5}$	$g(x) = x^2$		$gf(x) = \frac{4x^2}{25}$
$f(x) = \frac{1}{x}$	$g(x) = 2x - 3$		
$f(x) = 9 - x$	$g(x) = \sqrt{2x}$		
$f(x) = 3x - 1$	$g(x) = \frac{2}{x+1}$		
	$g(x) = x - 3$	$fg(x) = \frac{x-3}{10}$	
$f(x) = 2x + 1$		$fg(x) = 2x^3 + 1$	
		$fg(x) = \frac{1}{x^2} + 2$	$gf(x) = \frac{1}{x^2} + 2$

### Worked Example

Given that  $f(x) = x^2 - 6x - 4$ , find the possible values of  $x$  when  $f(x) = 3$ .

### Your Turn

Given that  $g(x) = x^2 - 3x - 12$ , find the possible values of  $x$  when  $g(x) = -8$ .

### Worked Example

Given  $f(x) = \sqrt{-2x + 17}$

Solve  $f(x^2) = 3$

### Your Turn

Given  $g(x) = \sqrt{-3x + 12}$

Solve  $g(x^2) = 3$

## Worked Example

Given

$$f(x) = 2x^2 + 3x + 1$$

$$g(x) = 6x^2 + 11x - 11$$

Solve  $f(x) = g(x)$

## Your Turn

Given

$$g(x) = x^2 + 2x - 1$$

$$h(x) = 5x^2 + 18x - 1$$

Solve  $g(x) = h(x)$

### Worked Example

Find the inverse function:

$$f(x) = x^2 + 4x - 5$$

### Your Turn

Find the inverse function:

$$g(x) = x^2 + 8x - 5$$

### Worked Example

Find the inverse function:

$$f(x) = 2x^2 - 10x + 9$$

### Your Turn

Find the inverse function:

$$g(x) = 2x^2 - 12x + 3$$

## Extra Notes

## 3 Quadratic Graphs

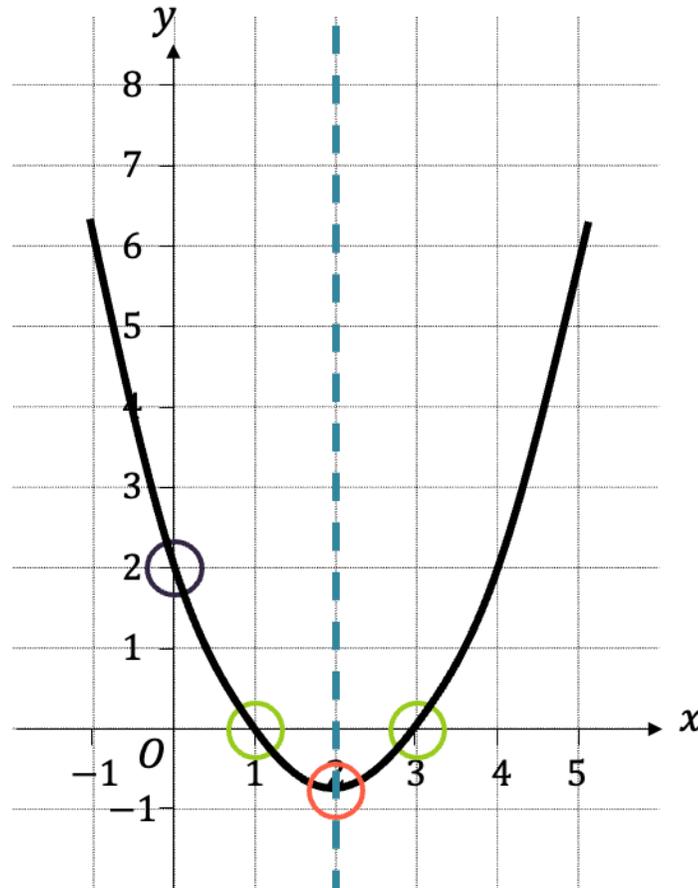
## Key Features of Quadratic Graphs

There are some key features of quadratic graphs that we can use if we want to **sketch** a quadratic rather than plot it.

1 The shape of the graph - U or  $\cap$

2 Where the graph crosses the  $y$ -axis

3 Where the graph crosses the  $x$ -axis



4 The line of symmetry of the graph

5 The turning point of the graph

### Worked Example

A quadratic graph has equation  $y = (8 - 2x)(3x + 7)$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.

### Your Turn

A quadratic graph has equation  $y = (2x - 5)(8 + 3x)$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.

### Worked Example

A quadratic graph has equation  $y = 5x^2 + 4x - 1$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.

### Your Turn

A quadratic graph has equation  $y = 5x^2 - 24x - 5$

Calculate the

- a)  $y$ -intercept
- b)  $x$ -intercepts of the graph.

### Worked Example

Sketch the following graph:  
 $y = (x - 7)(x + 10)$

### Your Turn

Sketch the following graph:  
 $y = (x + 3)(x + 8)$

### Worked Example

Sketch the following graph:

$$y = x(x + 2)$$

### Your Turn

Sketch the following graph:

$$y = x(x - 3)$$

## Worked Example

Sketch the following graph:

$$y = x^2 + 5x - 36$$

## Your Turn

Sketch the following graph:

$$y = x^2 + 6x + 8$$

### Worked Example

Sketch the following graph:

$$y = (3x + 7)(2x - 3)$$

### Your Turn

Sketch the following graph:

$$y = (2x + 3)(3x - 5)$$

### Worked Example

Sketch the following graph:  
 $y = 2x^2 - 13x + 15$

### Your Turn

Sketch the following graph:  
 $y = 5x^2 + 13x + 6$

### Worked Example

Sketch the following graph:

$$y = (2x - 1)(7 - 2x)$$

### Your Turn

Sketch the following graph:

$$y = (1 - 2x)(2x + 5)$$

### Worked Example

Sketch the following graph:

$$y = -3x^2 - 2x + 8$$

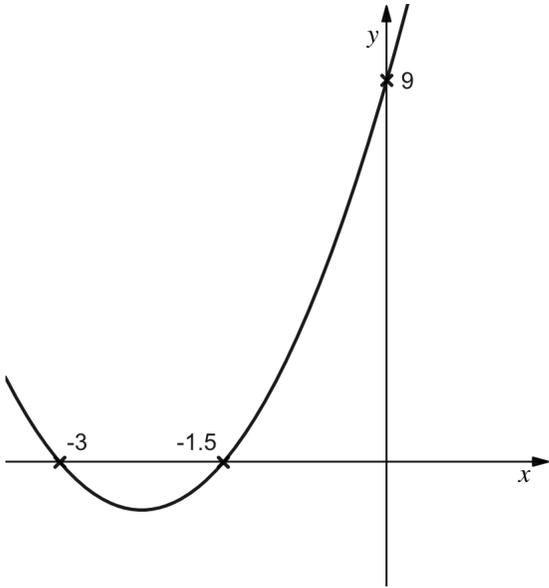
### Your Turn

Sketch the following graph:

$$y = -2x^2 + 5x + 7$$

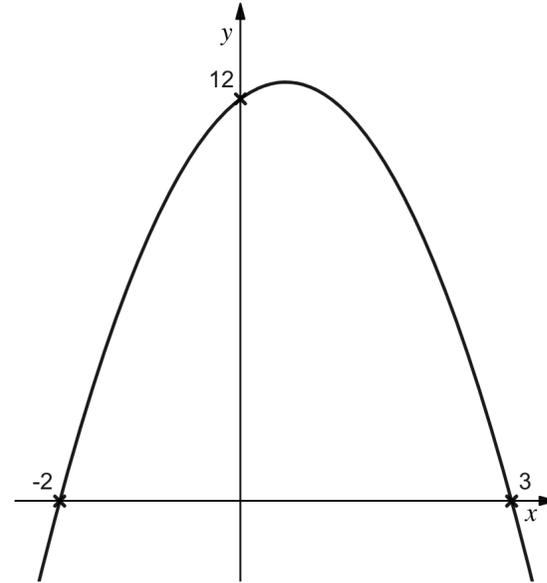
### Worked Example

Find the equation of the line of symmetry of the quadratic graph.



### Your Turn

Find the equation of the line of symmetry of the quadratic graph.



### Worked Example

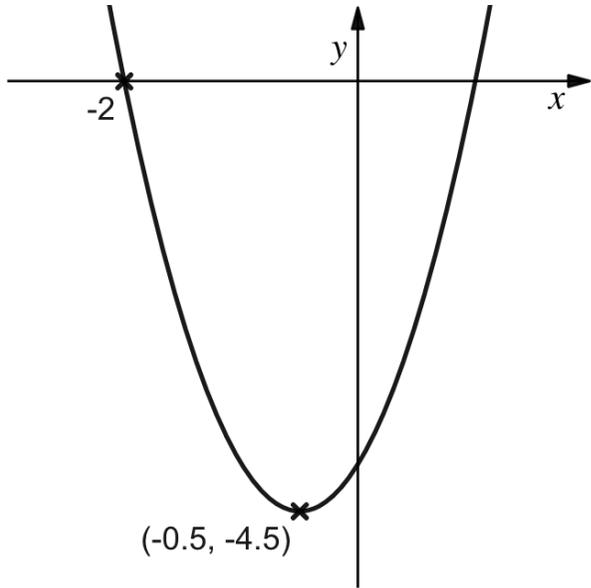
Find the equation of the line of symmetry of the quadratic graph with the equation  $y = -2(x - 1)(x + 5)$

### Your Turn

Find the equation of the line of symmetry of the quadratic graph with the equation  $y = 2(x + 4)(x - 3)$

### Worked Example

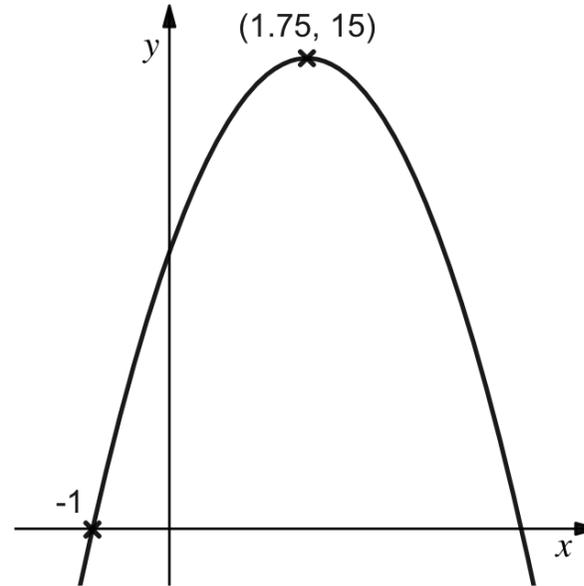
The graph of a quadratic function passed through the point  $(-2, 0)$  and has a turning point at  $(-0.5, -4.5)$



Find the coordinate of the other root.

### Your Turn

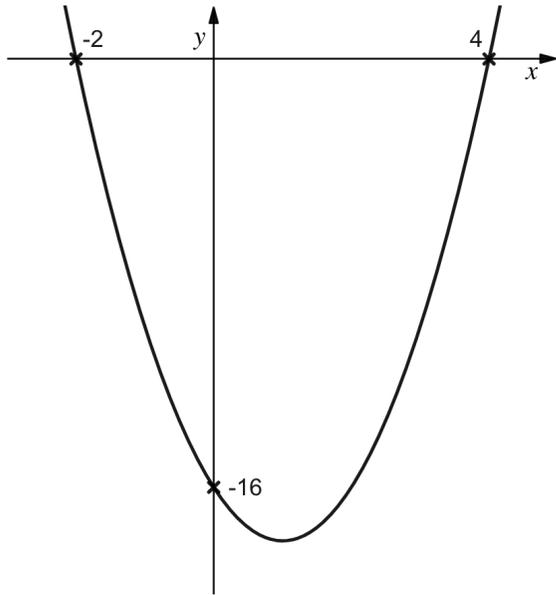
The graph of a quadratic function passed through the point  $(-1, 0)$  and has a turning point at  $(1.75, 15)$



Find the coordinate of the other root.

### Worked Example

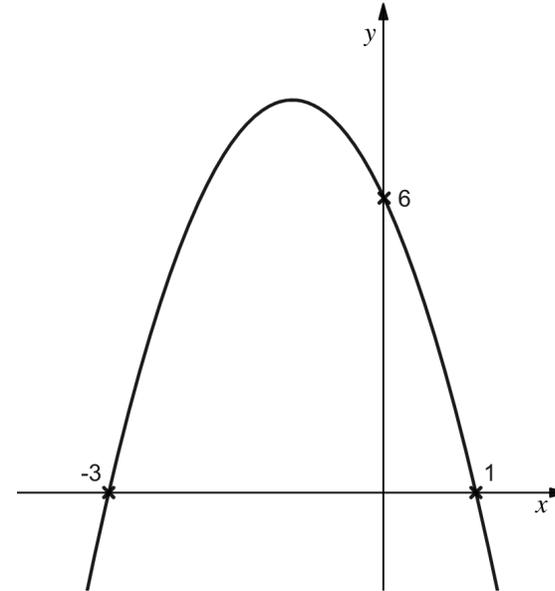
The graph below has equation  $y = 2(x + 2)(x - 4)$



Find the coordinate of the turning point.

### Your Turn

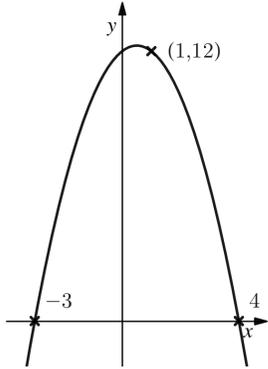
The graph below has equation  $y = -2(x - 1)(x + 3)$



Find the coordinate of the turning point.

## Worked Example

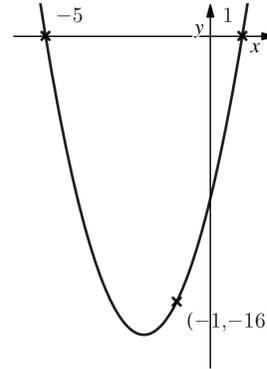
The graph below shows a quadratic curve.



Find the equation of the curve, giving your answer in the form  $y = ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are integers.

## Your Turn

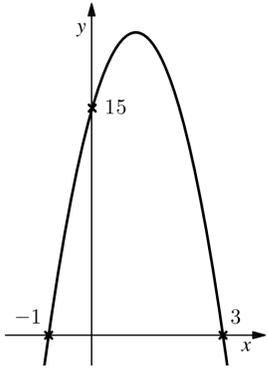
The graph below shows a quadratic curve.



Find the equation of the curve, giving your answer in the form  $y = ax^2 + bx + c$  where  $a$ ,  $b$  and  $c$  are integers.

## Worked Example

The graph below shows a quadratic curve.

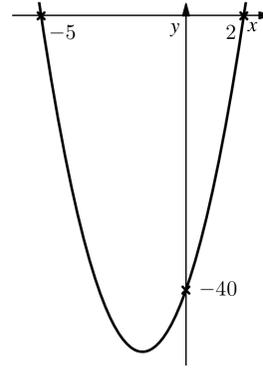


The equation of the curve is  $y = -5x^2 + bx + 15$

Find the value of  $b$

## Your Turn

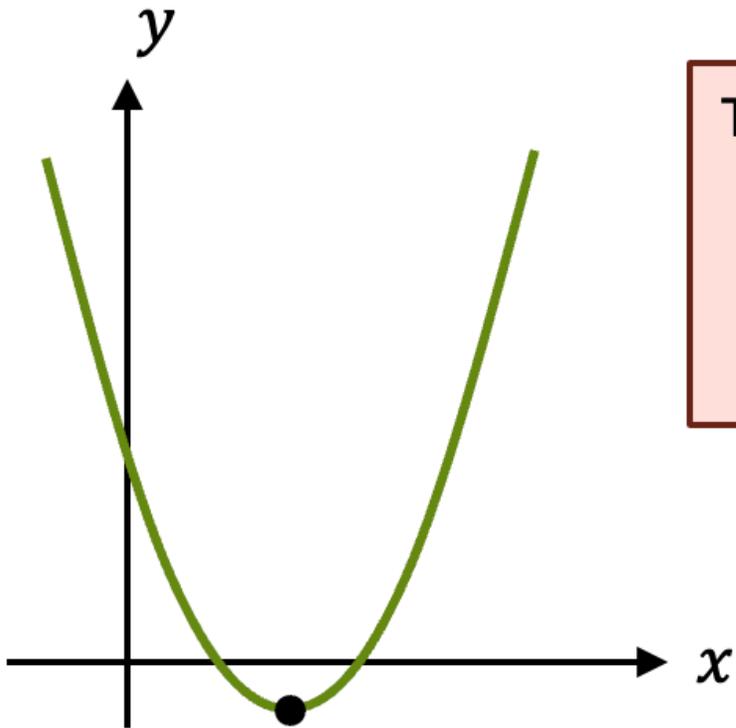
The graph below shows a quadratic curve.



The equation of the curve is  $y = 4x^2 + bx - 40$

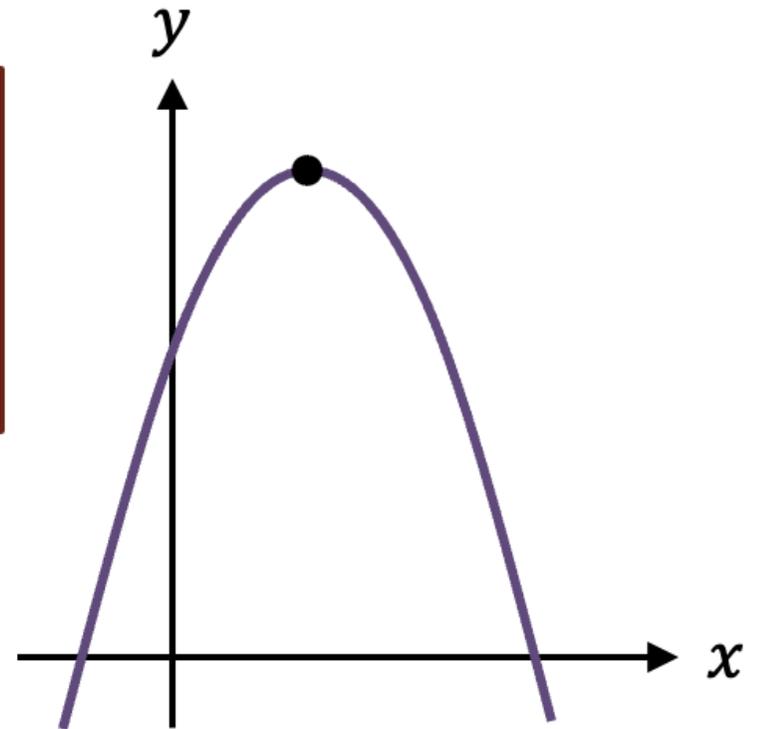
Find the value of  $b$

## Turning Point



The turning point of a quadratic graph is its maximum or minimum point

For a **positive** quadratic, the turning point is a **minimum**



For a **negative** quadratic, the turning point is a **maximum**

### Worked Example

$x^2 + 8x + 11$  can be expressed as  $(x + 4)^2 - 5$ .

Hence, or otherwise, state the coordinates of the minimum point of the graph  $y = x^2 + 8x + 11$ .

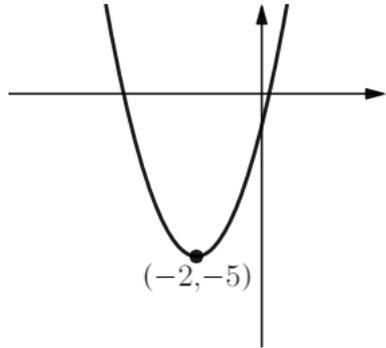
### Your Turn

$x^2 + 6x + 15$  can be expressed as  $(x + 3)^2 + 6$ .

Hence, or otherwise, state the coordinates of the minimum point of the graph  $y = x^2 + 6x + 15$

## Worked Example

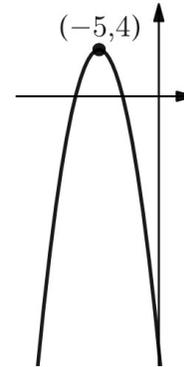
The graph below shows part of the parabola with equation of the form  $y = (x + a)^2 + b$



The minimum turning point  $(-2, -5)$  is shown on the diagram. State the values of  $a$  and  $b$ .

## Your Turn

The graph below shows part of the parabola with equation of the form  $y = -(x + a)^2 + b$



The maximum turning point  $(-5, 4)$  is shown on the diagram. State the values of  $a$  and  $b$ .

### Worked Example

$-x^2 - 6x - 10$  can be expressed as  $-(x + 3)^2 - 1$ .

Hence, or otherwise, state the coordinates of the maximum point of the graph  $y = -x^2 - 6x - 10$ .

### Your Turn

$-x^2 + 8x - 19$  can be expressed as  $-(x - 4)^2 - 3$ .

Hence, or otherwise, state the coordinates of the maximum point of the graph  $y = -x^2 + 8x - 19$ .

### Worked Example

Find the minimum point of the graph with equation  
 $y = x^2 - 2x - 3$

### Your Turn

Find the minimum point of the graph with equation  
 $y = x^2 + 6x + 8$

### Worked Example

Find the maximum point of the graph with equation  
 $y = -x^2 + 2x - 4$

### Your Turn

Find the maximum point of the graph with equation  
 $y = -x^2 - 4x + 1$

### Worked Example

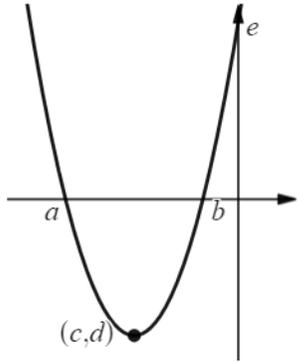
A graph has equation  $y = x^2 + 10x - 9$   
Find the equation of its line of symmetry.

### Your Turn

A graph has equation  $y = x^2 - 9x - 16$   
Find the equation of its line of symmetry.

### Worked Example

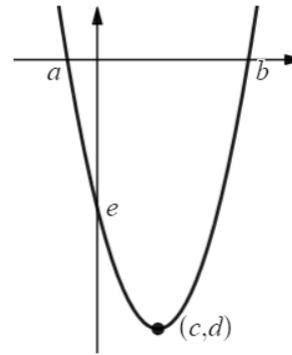
A quadratic graph has equation  $y = x^2 + 6x + 5$



Find the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$

### Your Turn

A quadratic graph has equation  $y = x^2 - 4x - 5$



Find the values of  $a$ ,  $b$ ,  $c$ ,  $d$  and  $e$

### Worked Example

The graph of  $y = ax^2 + bx + c$  has a minimum at  $(3, -5)$  and passes through  $(4, 0)$ . Find the values of  $a$ ,  $b$  and  $c$

### Your Turn

The graph of  $y = ax^2 + bx + c$  has a minimum at  $(7, -2)$  and passes through  $(8, 0)$ . Find the values of  $a$ ,  $b$  and  $c$

### Worked Example

By completing the square, explain why the curve  $y = 2x^2 - 8x + 9$  does not intersect the  $x$ -axis

### Your Turn

By completing the square, explain why the curve  $y = 2x^2 - 20x + 51$  does not intersect the  $x$ -axis

## Worked Example

$$y = x^2 + 2x - 15$$

Factorise:

Complete the square:

Intercepts:

Turning point:

Sketch:

## Your Turn

$$y = x^2 + 2x - 24$$

Factorise:

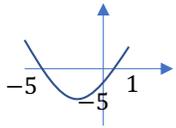
Complete the square:

Intercepts:

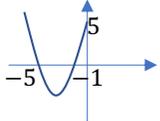
Turning point:

Sketch:

## Fill in the Gaps

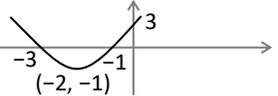
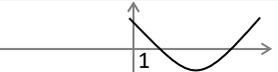
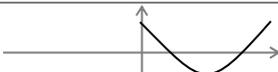
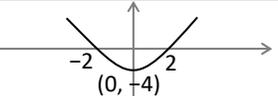
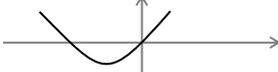
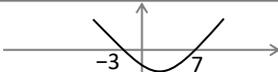
	$x^2 + bx + c$	$(x + d)(x + e)$	$(x + p)^2 + q$	x intercepts	y intercept	Turning point	Sketch
1.	$x^2 - 2x - 3$						
2.		$(x - 1)(x + 3)$					
3.							
4.			$(x + 1)^2 - 9$				
5.				(-2, 0) (-4, 0)			
6.		$(x - 4)(x + 4)$					
7.				(4, 0) (0, 0)			

## Fill in the Gaps

	$x^2 + bx + c$	$(x + d)(x + e)$	$(x + p)^2 + q$	x intercepts	y intercept	Turning point	Sketch
8.	$x^2 + 4x$						
9.						(-2, 0)	
10.		$(x + 3)(x + 3)$					
11.							
12.			$(x + 3)^2 + 4$				
13.		$-(x + 1)(x + 5)$					
14.					(0, 5)		

# Fill in the Gaps

## ① Significant Points of a Quadratic Curve Using Factorisation & Symmetry

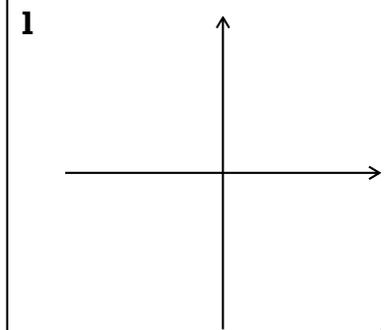
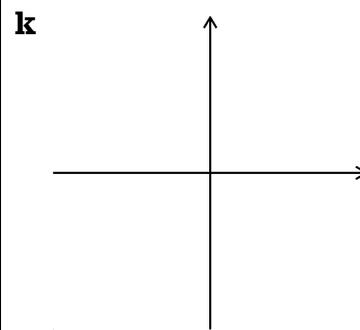
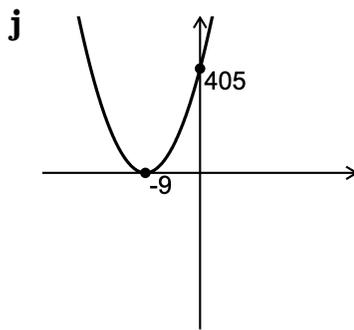
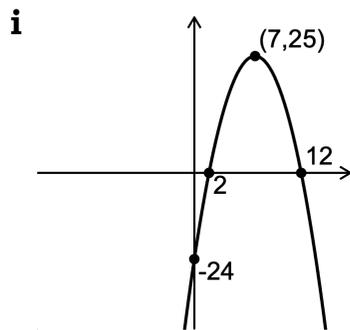
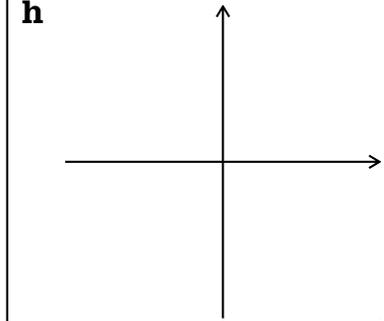
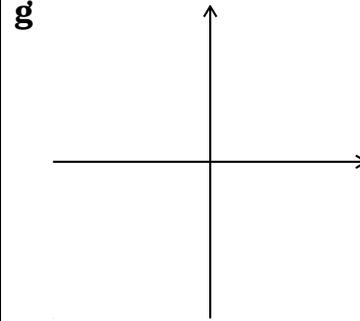
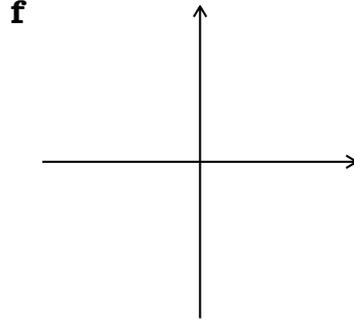
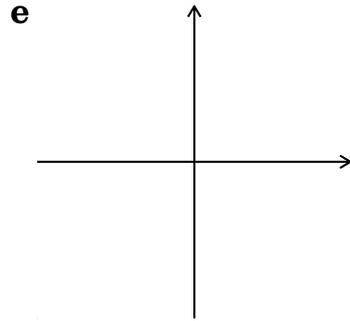
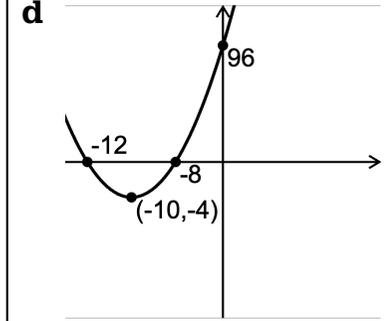
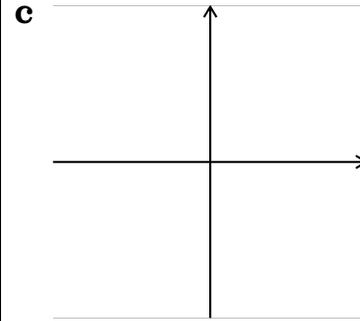
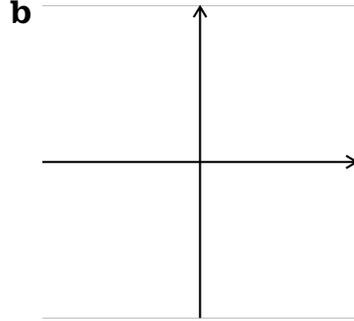
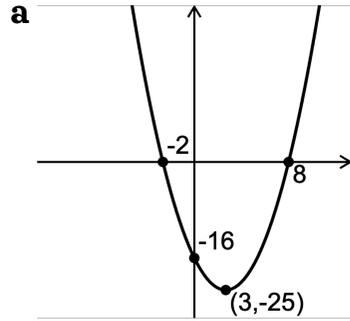
Equation	Factorisation	1 <sup>st</sup> Root	Turning Point	2 <sup>nd</sup> Root	y-intercept	Sketch
$y = x^2 + 4x + 3$	$(\quad)(x + 1) = 0$			$(-1, 0)$	$+3$	
$y = x^2 + 8x + 7$	$(x + 1) = 0$	$(-7, 0)$	$(x, -9)$			
$y = x^2 - 6x + 5$	$\quad = 0$		$(3, y)$			
$y = x^2 - 8x + 12$						
	$(x + 3)(x - 5) = 0$				$-15$	
$y = x^2 + 2x - 24$						
		$(-2, 0)$		$(2, 0)$	$-4$	
$y = x^2 + 5x$			$(x, -6.25)$			
$y = x^2 + 8x + 16$						
						

## Fill in the Gaps

	Expanded	Factorised	Completed	Roots	y-intercept	Vertex	Sketch
<b>a</b>	$y = x^2 - 6x - 16$	$y = (x + 2)(x - 8)$	$y = (x - 3)^2 - 25$	$x = -2, x = 8$	$(0, -16)$	$(3, -25)$	<input checked="" type="checkbox"/>
<b>b</b>			$y = (x - 5)^2 - 4$				
<b>c</b>		$y = (x + 13)(x - 1)$					
<b>d</b>							<input checked="" type="checkbox"/>
<b>e</b>	$y = x^2 - 8x + 16$						
<b>f</b>				$x = -3, x = 5$	$(0, -15)$		
<b>g</b>		$y = 3(x + 3)(x - 5)$					
<b>h</b>					$(0, 24)$	$(7, -25)$	
<b>i</b>							<input checked="" type="checkbox"/>
<b>j</b>							<input checked="" type="checkbox"/>
<b>k</b>	$y = -2x^2 + 20x + 22$						
<b>l</b>				$x = -1, x = 10$	$(0, 20)$		

# Fill in the Gaps

## Sketches

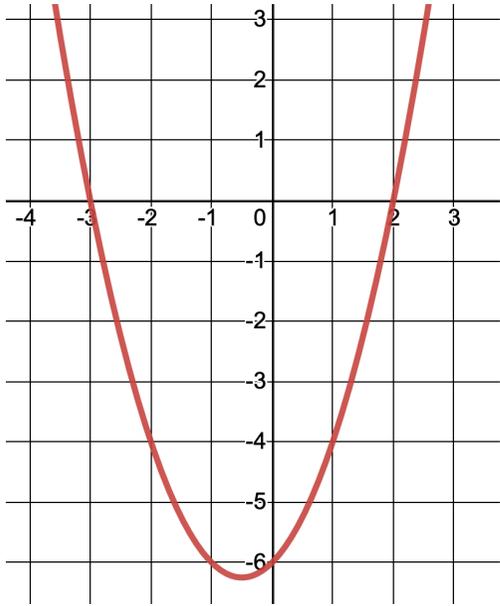


## Extra Notes

## 4 Quadratic Inequalities

## Worked Example

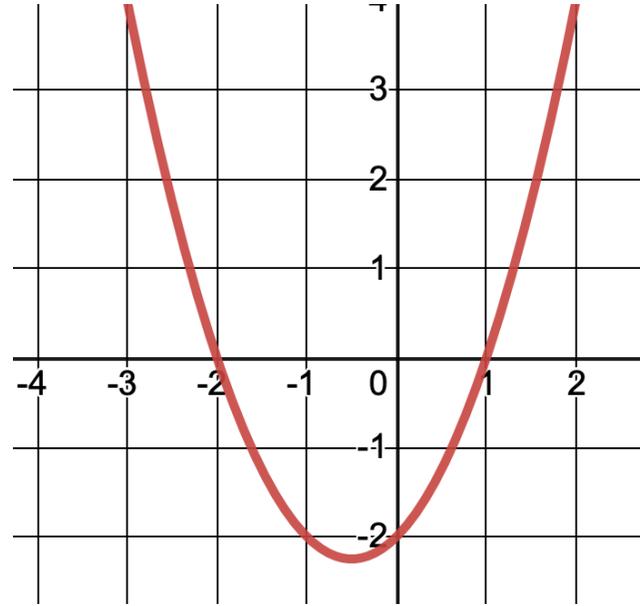
Here is the graph of  
 $y = (x + 3)(x - 2)$



- Show which part of the graph that represents  $(x + 3)(x - 2) < 0$
- State the solution set of  $(x + 3)(x - 2) < 0$

## Your Turn

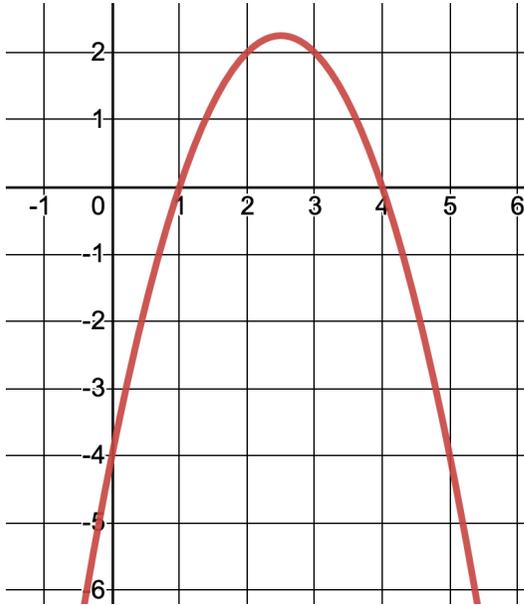
Here is the graph of  
 $y = (x + 2)(x - 1)$



- Show which part of the graph that represents  $(x + 2)(x - 1) < 0$
- State the solution set of  $(x + 2)(x - 1) < 0$

## Worked Example

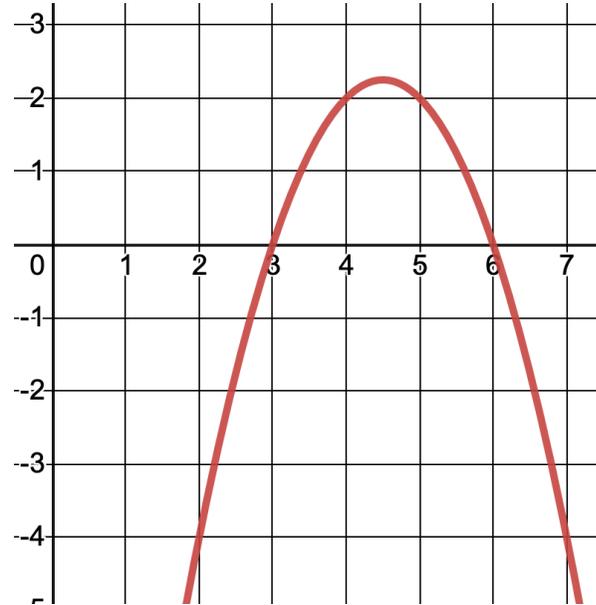
Here is the graph of  
 $y = (4 - x)(x - 1)$



- Show which part of the graph that represents  $(4 - x)(x - 1) < 0$
- State the solution set of  $(4 - x)(x - 1) < 0$

## Your Turn

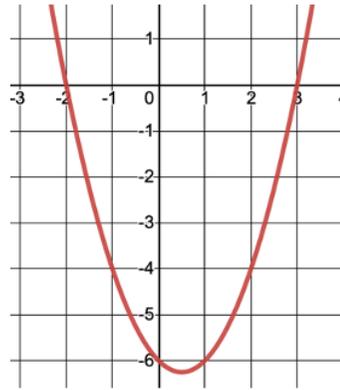
Here is the graph of  
 $y = (3 - x)(x - 6)$



- Show which part of the graph that represents  $(3 - x)(x - 6) < 0$
- State the solution set of  $(3 - x)(x - 6) < 0$

## Fluency Practice

1) Here is the graph of  
 $y = (x + 2)(x - 3)$

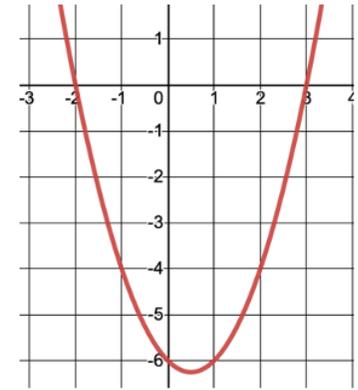


Show which part of the graph  
that represents

$$(x + 2)(x - 3) < 0$$

State the solution set of  
 $(x + 2)(x - 3) < 0$

2) Here is the graph of  
 $y = (x + 2)(x - 3)$

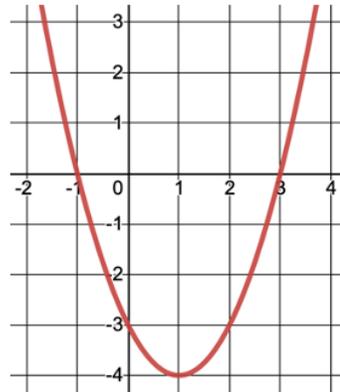


Show which part of the graph  
that represents

$$(x + 2)(x - 3) > 0$$

State the solution set of  
 $(x + 2)(x - 3) > 0$

3) Here is the graph of  
 $y = (x + 1)(x - 3)$

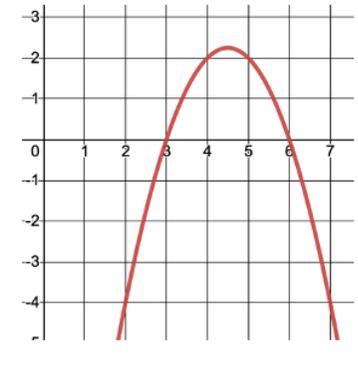


Show which part of the graph  
that represents

$$(x + 1)(x - 3) < 0$$

State the solution set of  
 $(x + 1)(x - 3) < 0$

4) Here is the graph of  
 $y = (-x + 6)(x - 3)$



Show which part of the graph  
that represents

$$(-x + 6)(x - 3) > 0$$

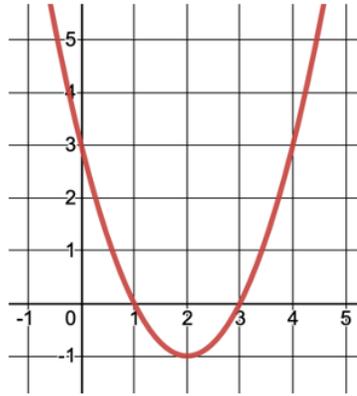
State the solution set of  
 $(-x + 6)(x - 3) > 0$

## Fluency Practice

5) Here is the graph of  
 $y = (x - 1)(x - 3)$

Show which part of the graph  
that represents  
 $(x - 1)(x - 3) < 0$

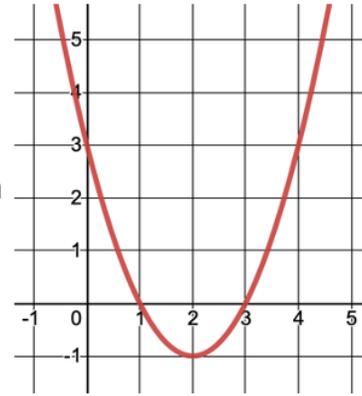
State the solution set of  
 $(x - 1)(x - 3) < 0$



6) Here is the graph of  
 $y = (x - 1)(x - 3)$

Show which part of the graph  
that represents  
 $(x - 1)(x - 3) > 0$

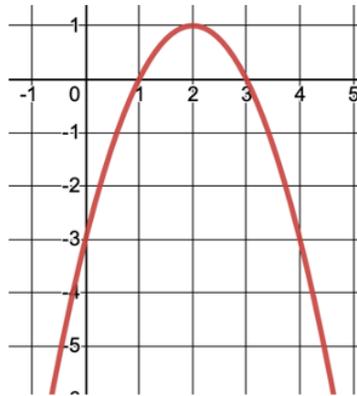
State the solution set of  
 $(x - 1)(x - 3) > 0$



7) Here is the graph of  
 $y = (-x + 1)(x - 3)$

Show which part of the graph  
that represents  
 $(-x + 1)(x - 3) < 0$

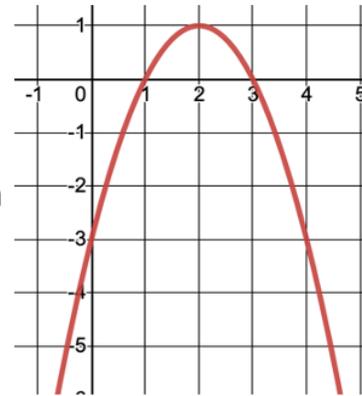
State the solution set of  
 $(-x + 1)(x - 3) < 0$



8) Here is the graph of  
 $y = (-x + 1)(x - 3)$

Show which part of the graph  
that represents  
 $(-x + 1)(x - 3) > 0$

State the solution set of  
 $(-x + 1)(x - 3) > 0$



**Worked Example**

Solve  $x^2 + 3x - 10 < 0$

**Your Turn**

Solve  $x^2 - 7x + 12 \geq 0$

**Worked Example**

Solve  $x^2 - 6x \geq 0$

**Your Turn**

Solve  $x^2 + 3x \leq 0$

**Worked Example**Solve  $x^2 - 1 > 0$ **Your Turn**Solve  $x^2 - 16 < 0$

**Worked Example**

Solve the inequality  $x(x - 5) > 2(x - 6)$

**Your Turn**

Solve the inequality  $x(x - 3) < 2(x - 2)$

**Worked Example**

Solve  $3x^2 - 13x + 4 < 0$

**Your Turn**

Solve  $5x^2 + 9x - 2 > 0$

**Worked Example**

Solve the inequality  $(5x + 4)(x + 4) \leq 21$

**Your Turn**

Solve the inequality  $(5x + 2)(x - 2) > -7$

**Worked Example**

Solve the inequality  $3x^2 \leq 8x$

**Your Turn**

Solve the inequality  $2x^2 > 3x$

**Worked Example**

Solve the inequality  $9x^2 \geq 4$

**Your Turn**

Solve the inequality  $3x^2 > 27$

**Worked Example**

Solve  $5 - 4x - x^2 > 0$

**Your Turn**

Solve  $3x - 2 - x^2 < 0$

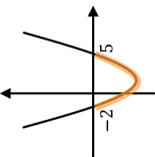
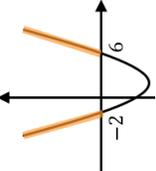
## Worked Example

Solve  $2 \leq \frac{5x}{x^2+1}$

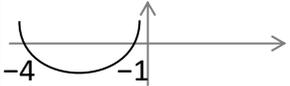
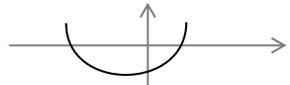
## Your Turn

Solve  $1 > \frac{7x}{x^2+6}$

## Fill in the Gaps

Quadratic Inequality	Factorised Quadratic	Critical Values	Sketch and Shade	Solution
$x^2 - 3x - 10 < 0$	$(x - 5)(x + 2) < 0$	$x = 5,$ $x = -2$		$-2 < x < 5$
$x^2 + 4x - 12 < 0$	$(x + 6)(x - 2) < 0$			
$x^2 - 6x + 5 < 0$				
$x^2 + 5x + 6 \leq 0$				
$x^2 - 4x - 12 > 0$	$(x - 6)(x + 2) > 0$	$x = 6,$ $x = -2$		$x < -2,$ $x > 6$
$x^2 + 4x + 3 \geq 0$				
$x^2 - x - 6 \geq 0$				
				$-4 < x < 1$

## Fill in the Gaps

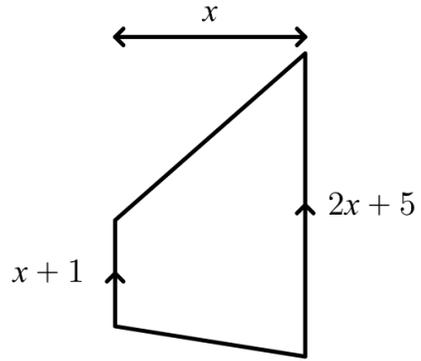
Quadratic Inequalities					
Inequality	Factorisation	Critical Values	Sketch	Solution(s)	Integer Answers (?)
$x^2 + 5x + 4 > 0$	$(x + 1)(x + 4) = 0$	-1		$x >$ $x <$	
$x^2 + 7x + 10 > 0$		-2			
$x^2 + 2x - 15 > 0$				$x > 3$	
	$(x + 3)(x - 4) = 0$				
$x^2 + 7x + 6 < 0$					
	$(x - 2)(x + 4) = 0$				
$x^2 + x - 20 \leq 0$					
				$3 < x < 4$	
$x^2 - 8x + 16 \leq 0$					
$x^2 - 12x + 35 \geq 0$					

## Fill in the Gaps

Quadratic Inequality	Rearrange	Factorise	Critical Values	Sketch and Shade	Solution
$x^2 < 7x - 12$	$x^2 - 7x + 12 < 0$				
$3x^2 + 5 \geq 16x$					
$x^2 > 3x$					
$2x^2 < 6 + 11x$					
$\frac{4x^2 + 5x}{3} \leq 2$					

### Worked Example

The maximum area of the trapezium shown below is  $24 \text{ cm}^2$

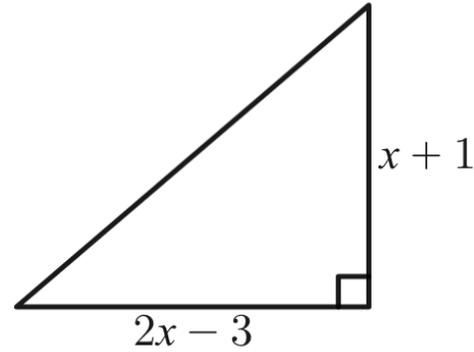


All lengths are in centimetres.

Determine the range of possible values for  $x$

### Your Turn

The maximum area of the triangle shown below is  $21 \text{ cm}^2$



All lengths are in centimetres.

Determine the range of possible values for  $x$

### Worked Example

Using algebra, solve the inequality:

$$5m^2 + 26m - 24 < 0$$

Write your answer in set notation.

### Your Turn

Using algebra, solve the inequality:

$$3 - 4m^2 - 11m > 0$$

Write your answer in set notation.

### Worked Example

$n$  is an integer such that  $7n + 4 < 18$  and  $\frac{5n}{n^2+1} \geq 2$   
Find all the possible values of  $n$ .

### Your Turn

$n$  is an integer such that  $7n + 1 > 8$  and  $2n^2 - 7n + 5 \leq 0$   
Find all the possible values of  $n$ .

**Worked Example**

Solve  $44 < 5x^2 - 1 < 79$

**Your Turn**

Solve  $76 < 3x^2 + 1 < 148$

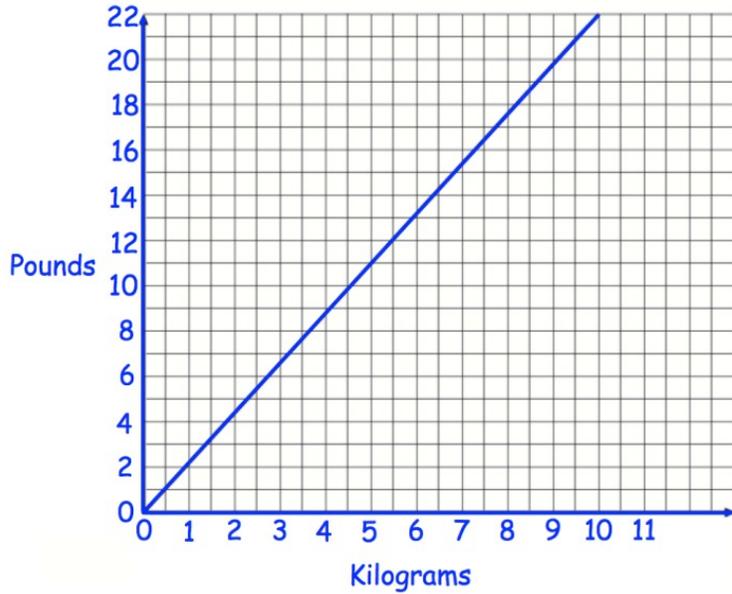
## Extra Notes

## 5 Real-Life Graphs

## Conversion Graphs

## Worked Example

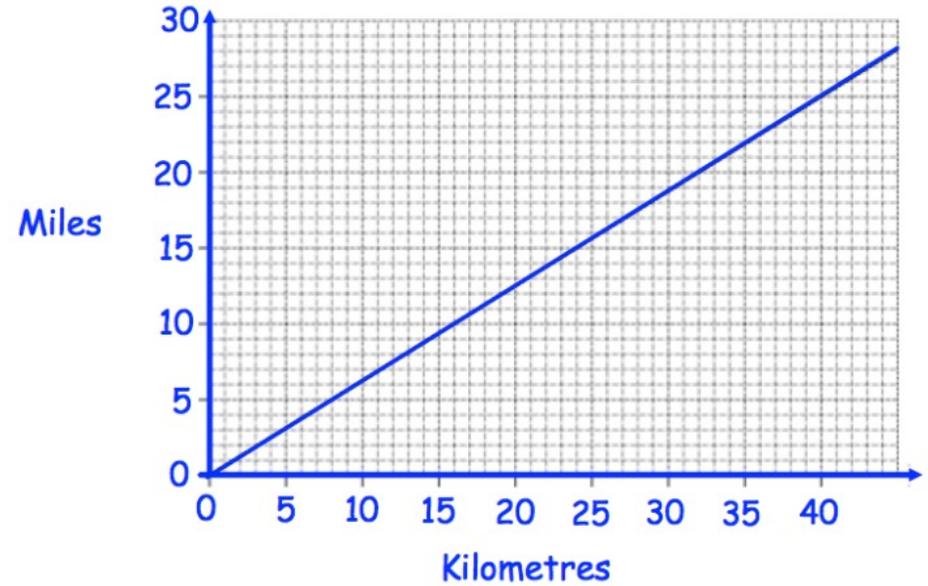
Use the graph to convert:



- a) 5 kilograms to pounds
- b) 8 pounds to kilograms
- c) 150 kilograms to pounds

## Your Turn

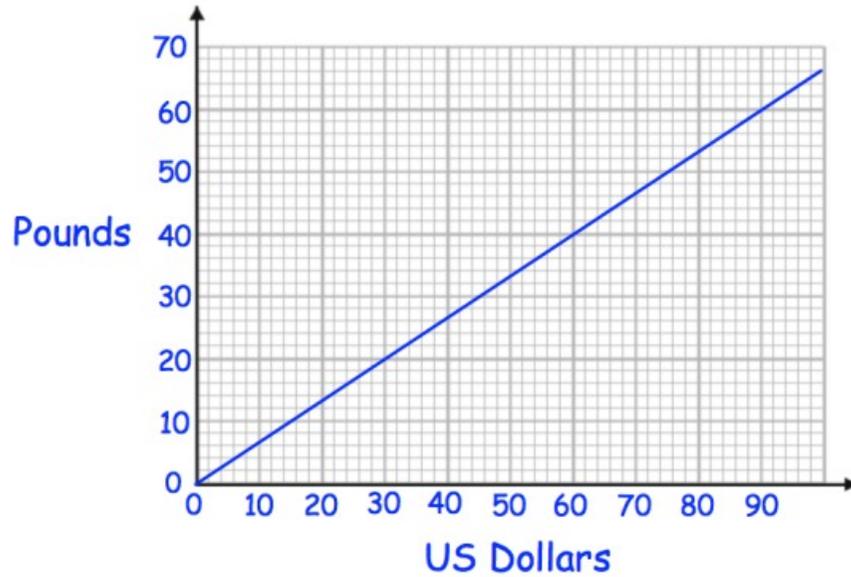
Use the graph to convert:



- a) 40 kilometres to miles
- b) 10 miles to kilometres
- c) 800 kilometres to miles

## Worked Example

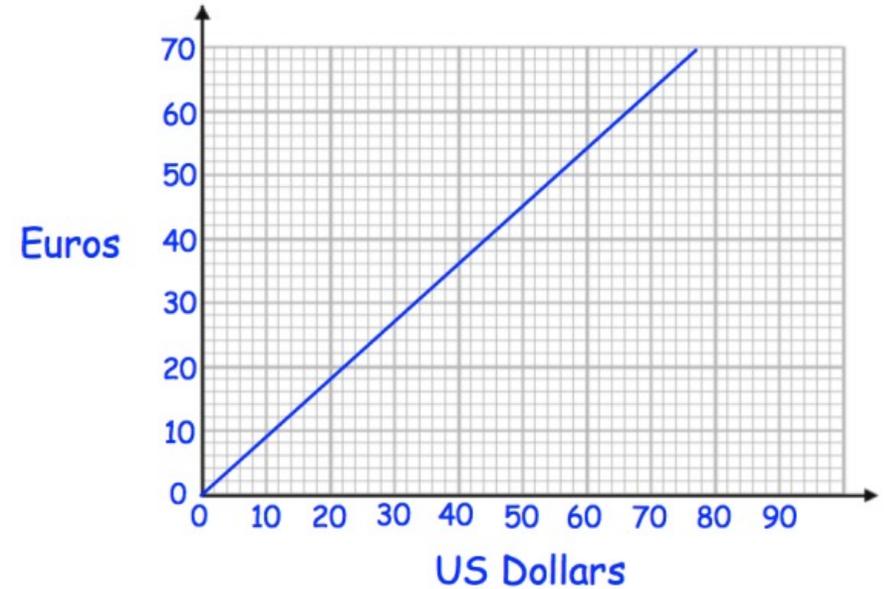
Use the graph to convert:



- a) 60 US Dollars to Pounds
- b) 20 Pounds to US Dollars
- c) 1800 US dollars to Pounds

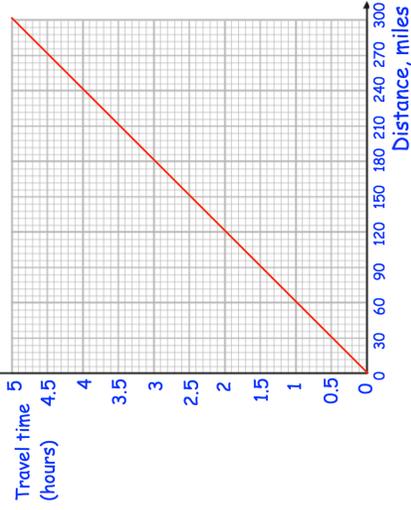
## Your Turn

Use the graph to convert:



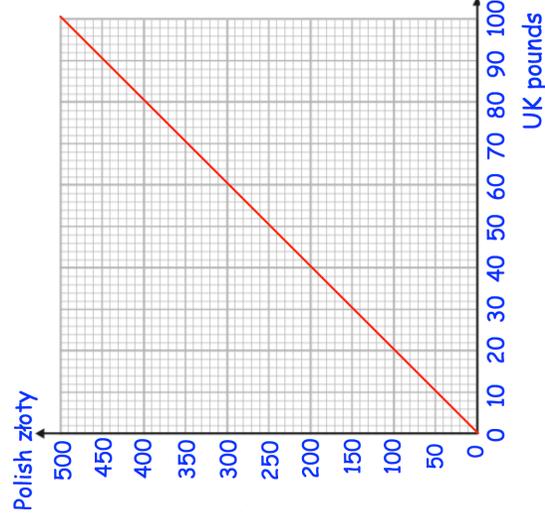
- a) 60 US Dollars to Euros
- b) 20 Euros to US Dollars
- c) 1800 US dollars to Euros

# Fluency Practice



**Question 1:**

- (a) How long should a 120 mile journey take?
- (b) How long should a 270 mile journey take?
- (c) Carlos has spent 1 hour travelling. What distance is he expected to have travelled?
- (d) Rosie has spent 3.5 hours travelling. What distance is she expected to have travelled?

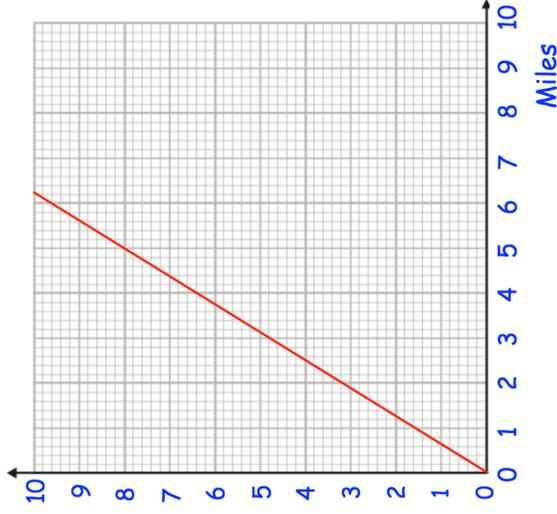


**Question 2:**

- (a) Change £20 into Polish zloty
- (b) Change £90 into Polish zloty
- (c) Change 300zł into UK pounds
- (d) Change 450zł into UK pounds
- (e) Change £50 into Polish zloty
- (f) Change £200 into Polish zloty
- (g) Change 800zł into UK pounds

# Fluency Practice

Question 3: This conversion graph can be used to change between miles and kilometres.



- (a) Change 5 miles into kilometres
- (b) Change 1 mile into kilometres
- (c) Change 6km into miles
- (d) Change 4.8km into miles
- (e) Change 20 miles into kilometres
- (f) Change 16km into miles

Question 4: Draw conversion graphs for the following sets of information.

- (a) A conversion graph to convert between UK pound and South African Rand.

Horizontal axis: UK pound from £0 to 10

Vertical axis: South African Rand from 0 to 200 Rand

Pounds	0	1	2	3
SA Rand	0	20	40	60

- (b) A conversion graph for hours and cost of driving lessons.

Horizontal axis: Hours from 0 to 10

Vertical axis: Cost (decide scale yourself)

Hours	1	4
Cost	£15	£60

- (c) A conversion graph to show how to change from kilograms to pounds.

Horizontal axis: Kilograms from 0kg to 10kg

Vertical axis: Pounds (decide scale yourself)

Kilograms, kg	5
Pounds, lb	11

- (d) A conversion graph to convert between Euros and US Dollars.

Horizontal axis: Euros from 0 to €100

Vertical axis: US Dollar (decide scale yourself)

US Dollars	\$ 77
Euros	€ 70

# Purposeful Practice

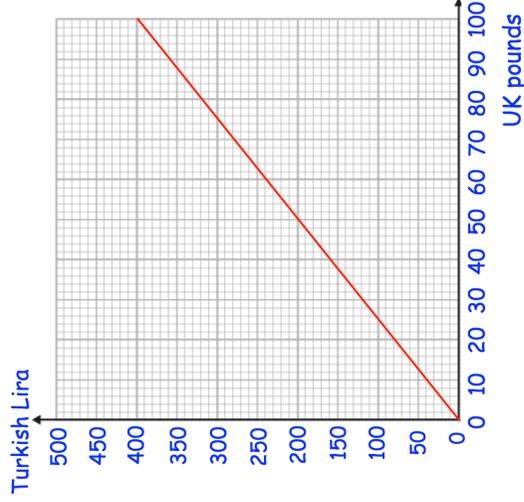
## Apply

Question 1:

Richard has ₺300 and £800.  
He buys a flight that costs ₺900

He pays use the ₺300 and some of the pounds.

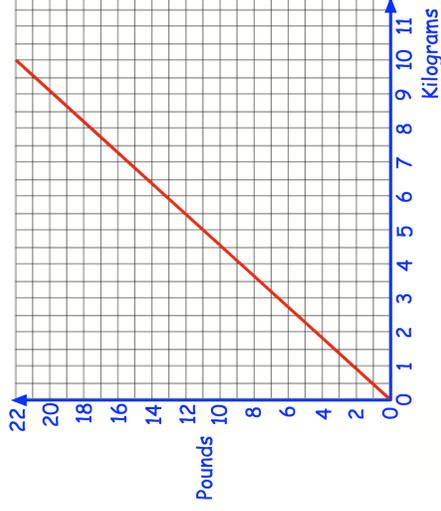
Work out how many pounds he has left.



Question 2:

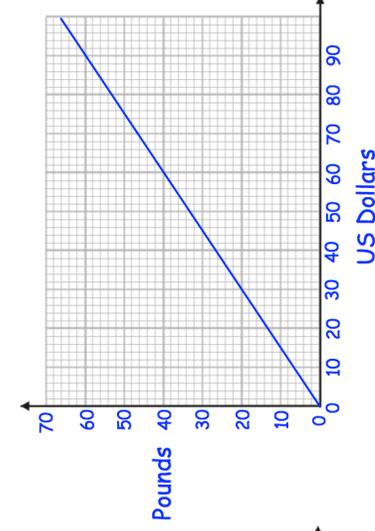
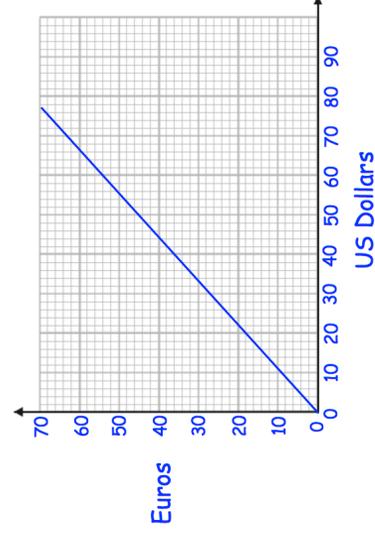
Jenny's weight is 65kg.  
1 stone = 14 pounds.

What is Jenny's weight in stones and pounds?



Question 3: The two conversion graphs below show how to change between some currencies.

Convert £800 into Euros.



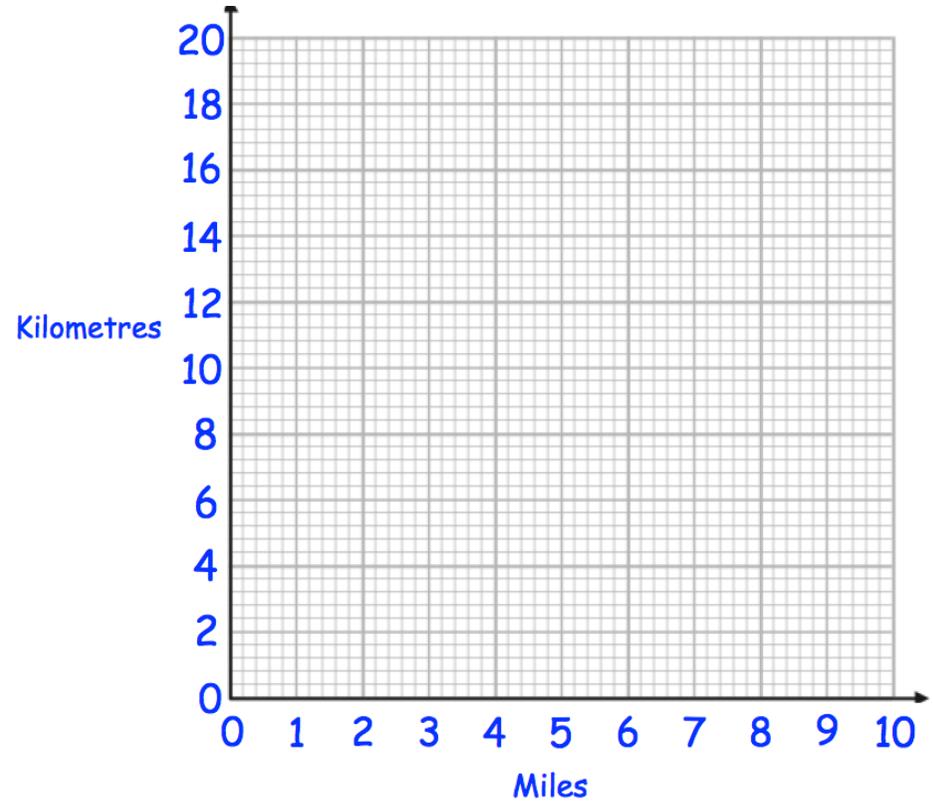
## Purposeful Practice

Question 4: 5 miles  $\approx$  8 kilometres

(a) Use this information to draw a conversion graph.

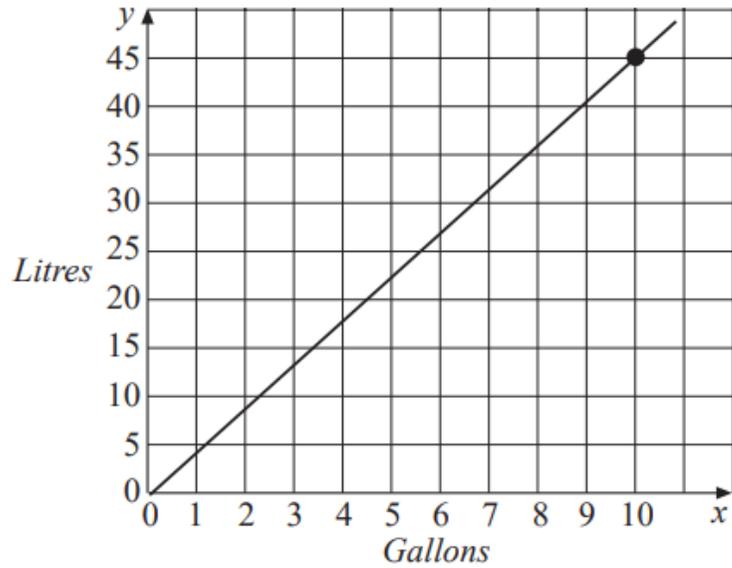
A car is travelling 60km/h.

(b) Use the graph to convert this into mph.



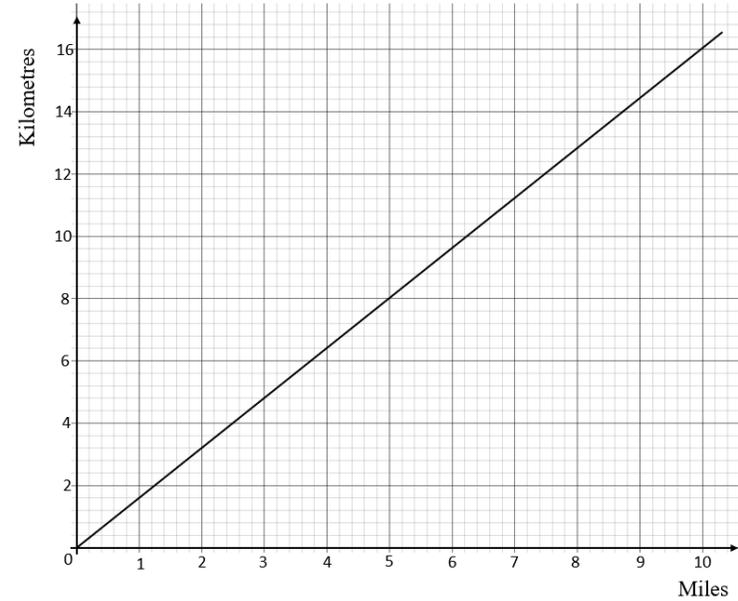
## Worked Example

Interpret the gradient and equation of this graph.



## Your Turn

Interpret the gradient and equation of this graph.



### Worked Example

The A Level Maths mark,  $y$  %, and GCSE Maths mark,  $x$  %, is recorded for several students.

Assume the line goes through  $(0, 40)$  and  $(60, 80)$ .

- a) Write a linear model.
- b) Interpret the gradient and  $y$ -intercept in this context.

### Your Turn

The temperature  $y$  at different points on a mountain is recorded at different altitudes  $x$ .

Assume the line goes through  $(0, 70)$  and  $(250, 20)$ .

- a) Write a linear model.
- b) Interpret the gradient and  $y$ -intercept in this context.

## Fill in the Gaps

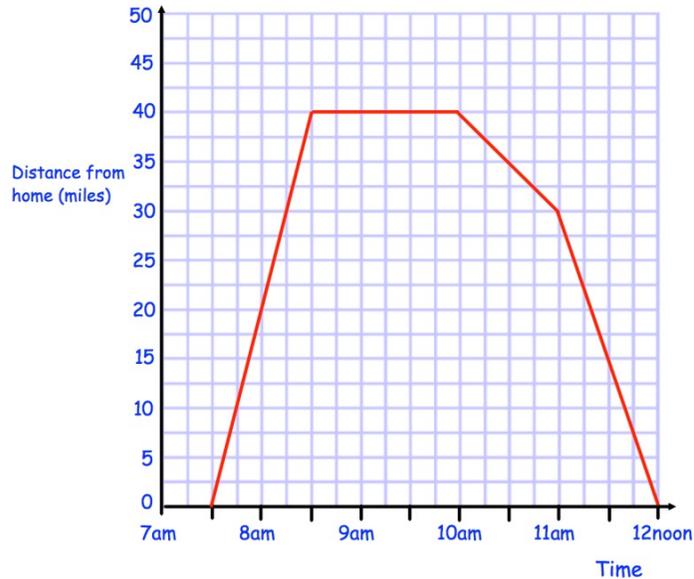
Independent variable	Response variable	Gradient & intercept	Equation of line	Sketch of graph (with labelled axes)	Interpretation of the gradient	Interpretation of the intercept
$t$ : time (s)	$M$ : mass (kg)	$m = 3.5$ $c = -0.04$				
$V$ : Volume of water in kettle (litres)	$t$ : time taken to boil (s)		$t = 167V + 12.6$			
$t$ :	$h$ :				Each day the shoot grows by 0.0206mm.	Initially the shoot is 0.312mm tall.
$x$ :	$v$ :	$m =$ $c = 2.73$			Walking speed decreases by $0.011\text{ms}^{-1}$ for each year of age.	
$r$ : altitude of runway (m)			$t = 0.258r + \dots$			Take off distance at sea level (0m altitude) is 623m.
$x$ : amount of fertilizer (kg/hectare)	$y$ : yield of maize (tonnes/hectare)	$m = 0.76$ $c = 0.058$				

## Distance-Time Graphs

## Worked Example

Study the distance-time graph below.

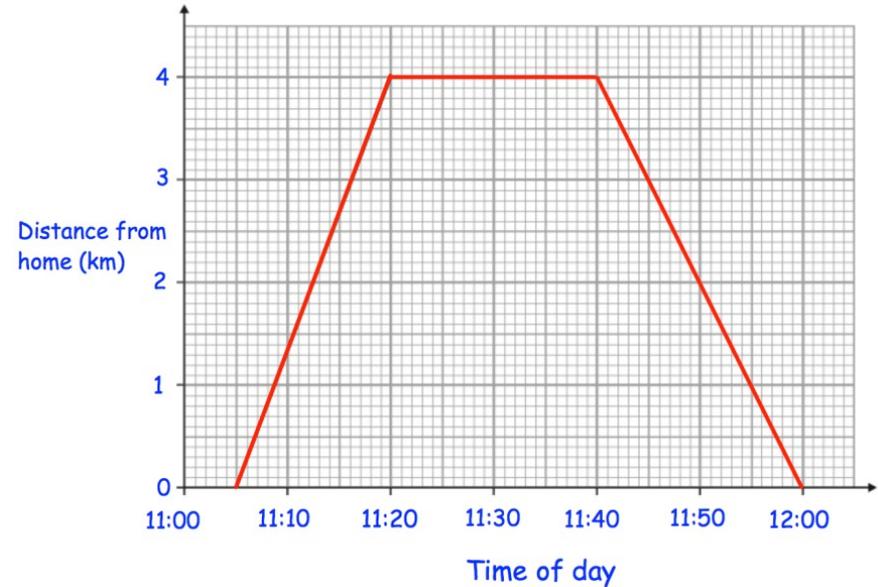
- How far was the person from home at 8.30am?
- How long did the person stop for?
- How far did the person travel in total?



## Your Turn

Study the distance-time graph below

- How far was the person from home at 11.20?
- How long did the person stop for?
- How far did the person travel in total?

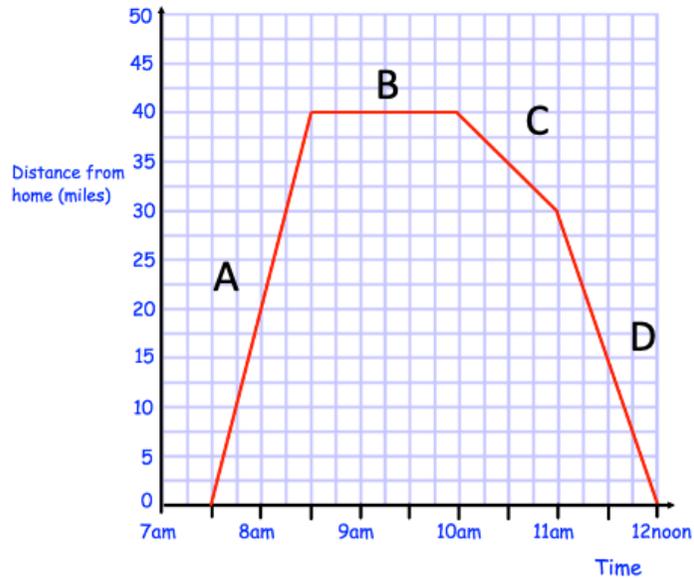


## Worked Example

Study the distance-time graph below.

What was the person's speed in each section of the journey:

- A
- B
- C
- D

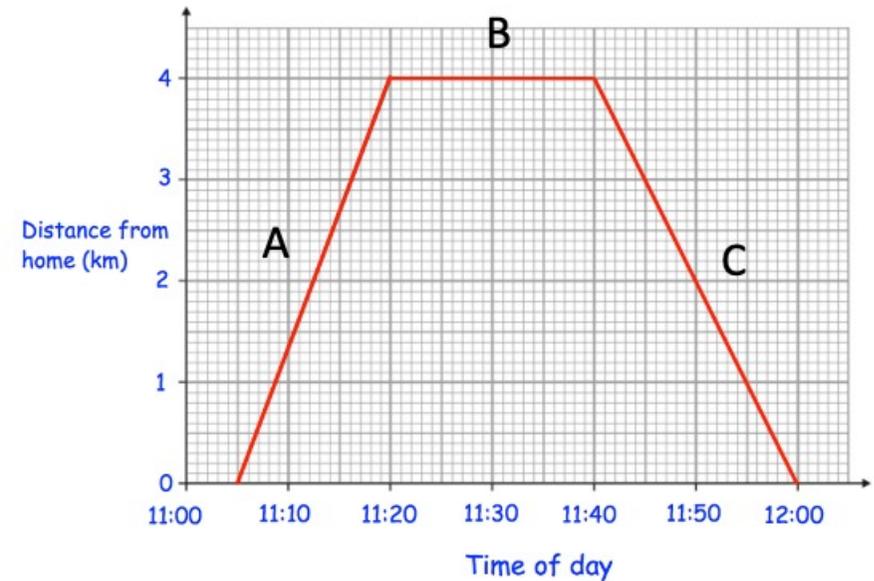


## Your Turn

Study the distance-time graph below.

What was the person's speed in each section of the journey:

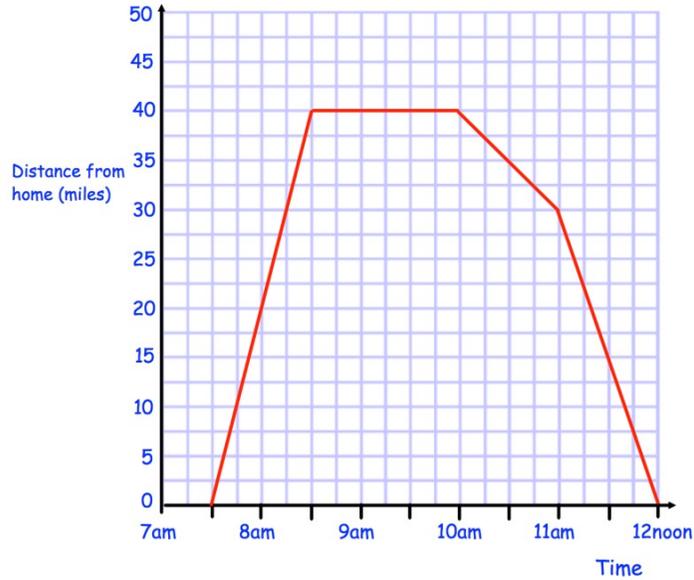
- A
- B
- C



## Worked Example

Study the distance-time graph below.

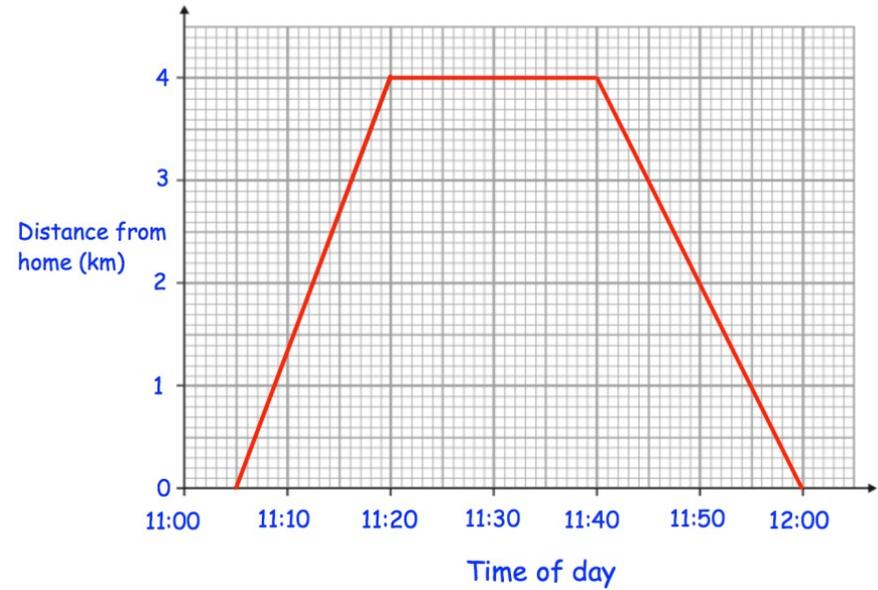
What was the person's average speed for the whole journey?



## Your Turn

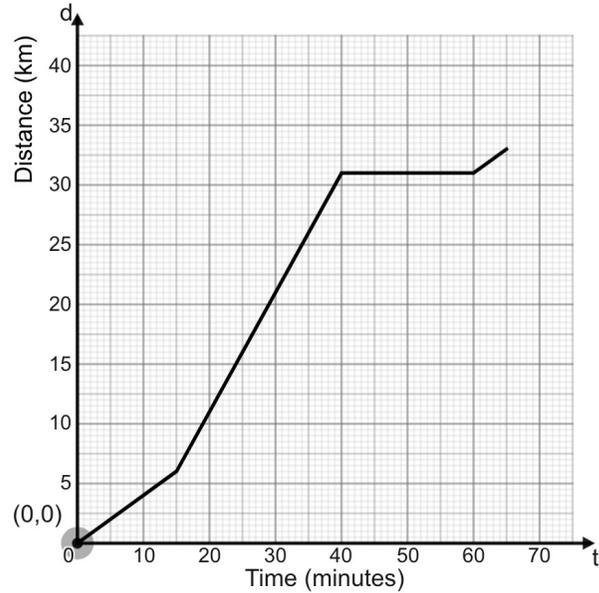
Study the distance-time graph below.

What was the person's average speed for the whole journey?



## Worked Example

Here is a travel graph showing Kabir's journey from his house to the airport.

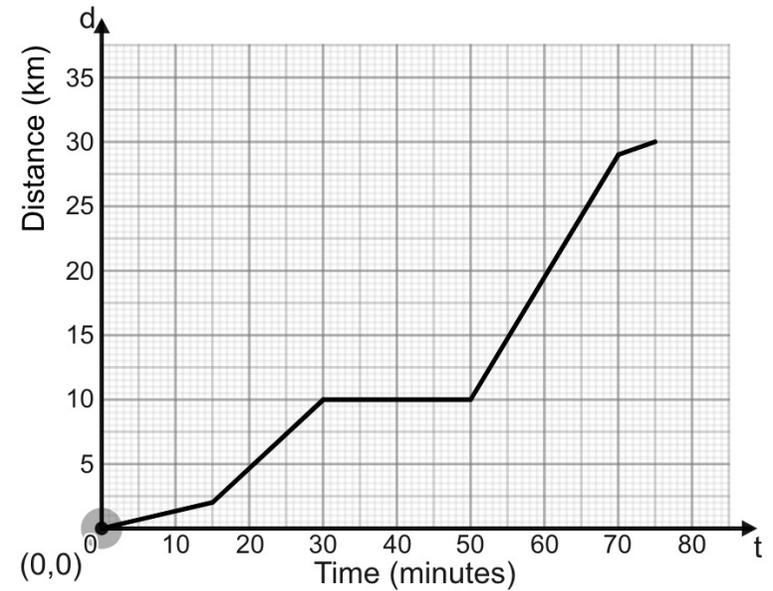


Kabir drives the same route back at an average speed of 50 km/h.

Work out how long the return journey took, giving your answer to the nearest whole number.

## Your Turn

Here is a travel graph showing Gaetan's journey from his house to the football match.

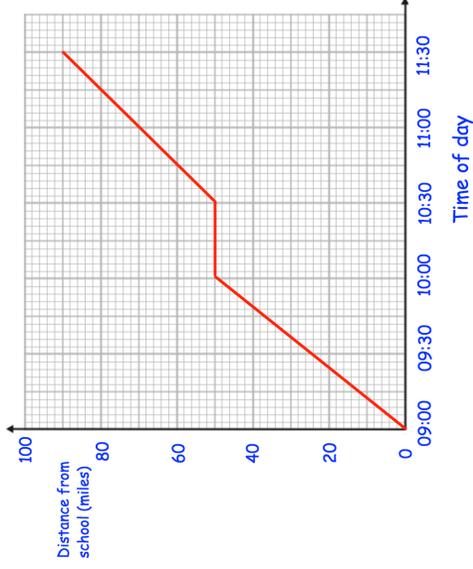


Gaetan drives the same route back at an average speed of 55 km/h.

Work out how long the return journey took, giving your answer to the nearest whole number.

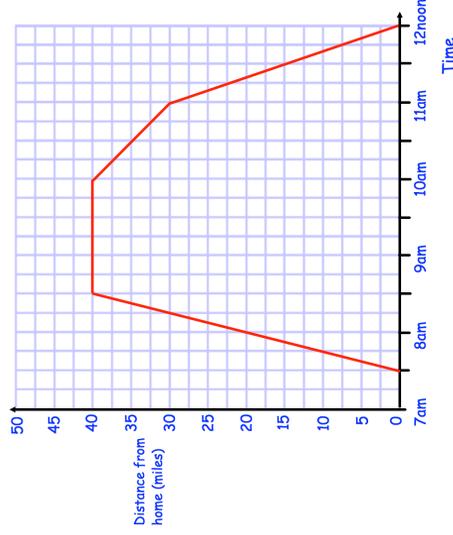
# Fluency Practice

Question 1: The distance-time graph shows class 8A's journey to the zoo. They stopped for a picnic on the way to the zoo.



- (a) What time did the bus leave school?
- (b) What time did they stop for a picnic?
- (c) How far had they travelled when they stopped for a picnic?
- (d) How long did they stop for?
- (e) What time did they arrive at the zoo?
- (f) How far is the zoo from school?

Question 2: Emma travelled to her Grandmother's house and back. The distance-time graph shows information about her journey.



- (a) What time did Emma begin her journey?
- (b) How far was Emma from home at 8am?
- (c) How long did Emma stay at her Grandmother's house?
- (d) What time did Emma leave her Grandmother's house?
- (e) How far was Emma from home at 11:45?
- (f) How far did Emma travel in total?

# Fluency Practice

Question 3: A train travels from Milton to Redville, stops for 30 minutes, then travels to Leek.

(a) How long did it take the train to travel from Milton to Redville?

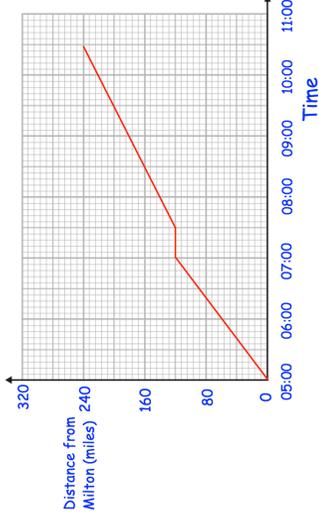
(b) How far is Redville from Milton?

(c) Work out the speed of the train for the journey from Milton to Redville.

(d) How long did it take the train to travel from Redville to Leek?

(e) How far is Leek from Redville?

(f) Work out the speed of the train for the journey from Redville to Leek.

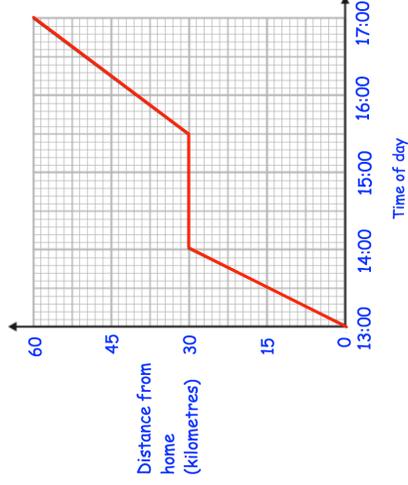


Question 4: Ben drove 60 kilometres, from his home to Liverpool. He stopped and visited his friend Tim on the way.

(a) Work out Ben's speed for the first part of his journey.

(b) How long did Ben spend visiting Tim?

(c) Work out Ben's speed for the last part of his journey.



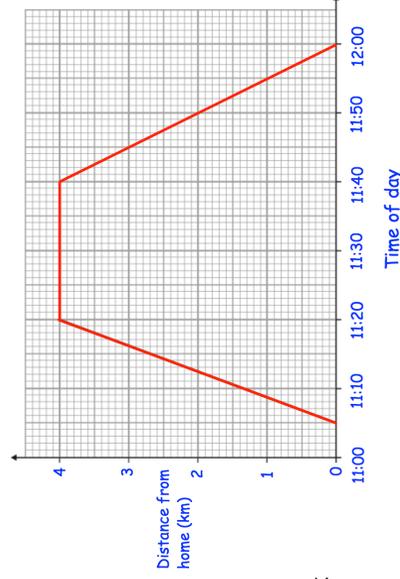
Question 5: Laura goes for a cycle from her house to the post office, 4 km away.

(a) How long did it take Laura to cycle to the post office?

(b) Work out Laura's speed cycling to the post office.

(c) How long did Laura spend at the post office?

(d) Work out Laura's speed cycling back home.



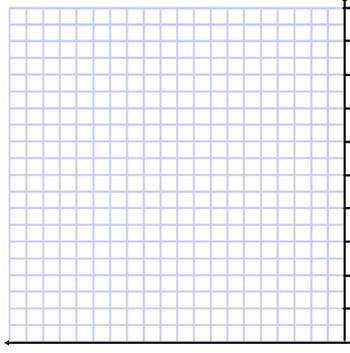
# Purposeful Practice

## Apply

Question 1:

Erin leaves home at 11am.  
She cycles at a speed of 16 miles per hour for 90 minutes.  
She stops for half an hour.  
Erin then cycles home and arrives at 3pm.

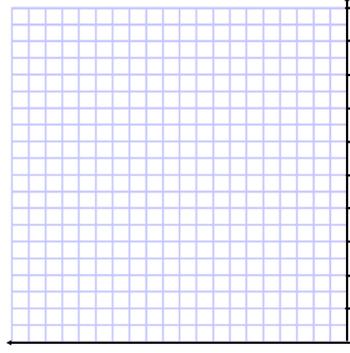
- (a) Draw a distance-time graph to show Erin's journey.  
(b) What is Erin's average speed on the return part of her cycle?



Question 2:

Thomas leaves home at 14:00  
He drives at an average speed of 40mph for 3½ hours  
Thomas stops for 30 minutes.  
He then drives home at an average speed of 70mph.

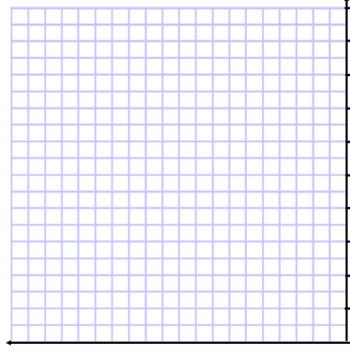
Draw a distance-time graph to show Thomas's journey



Question 3:

A helicopter leaves Bristol at 10:00.  
It flies for 45 minutes at 80km/h.  
It lands for 30 minutes and then flies a further  
65 kilometres in 30 minutes.  
The helicopter then immediately returns to  
its base in Bristol, flying at 100km/h.

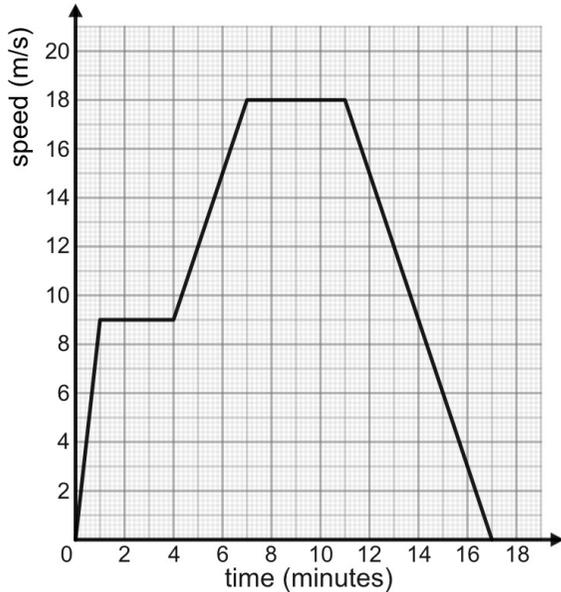
Draw a distance-time graph to show the journey.



## Speed-Time Graphs

## Worked Example

Edmund drives home from work. The graph shows the speed of his car during the journey.



Write down the car's maximum speed.  
Give your answer in metres per second.

## Your Turn

The graph shows Dave's speed as he cycles to school.

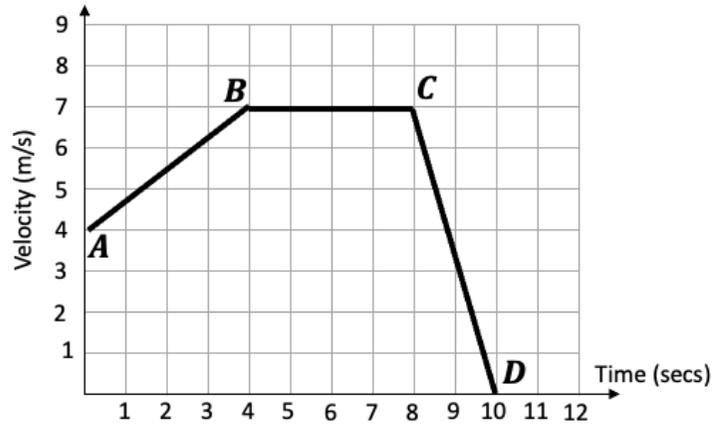


Write down Dave's maximum speed on his journey to school.  
Give your answer in kilometres per hour.

## Worked Example

State and describe the acceleration between:

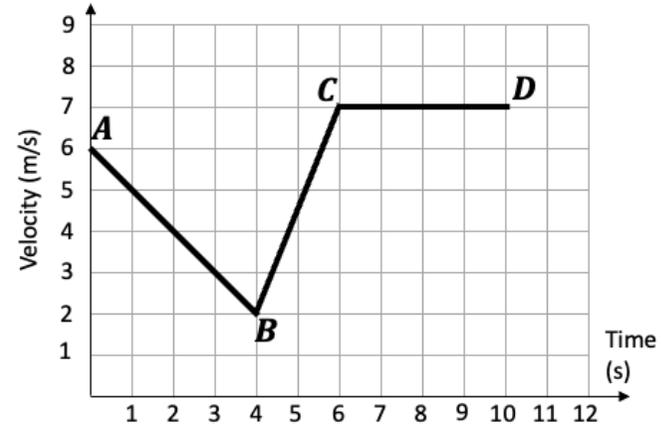
- a) *A* and *B*
- b) *B* and *C*
- c) *C* and *D*



## Your Turn

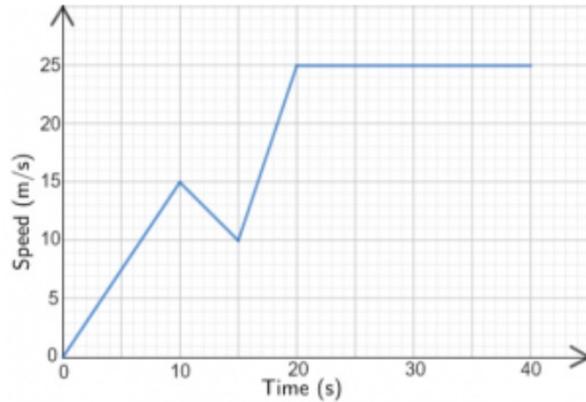
Determine the acceleration from:

- a) *A* to *B*
- b) *B* to *C*
- c) *C* to *D*



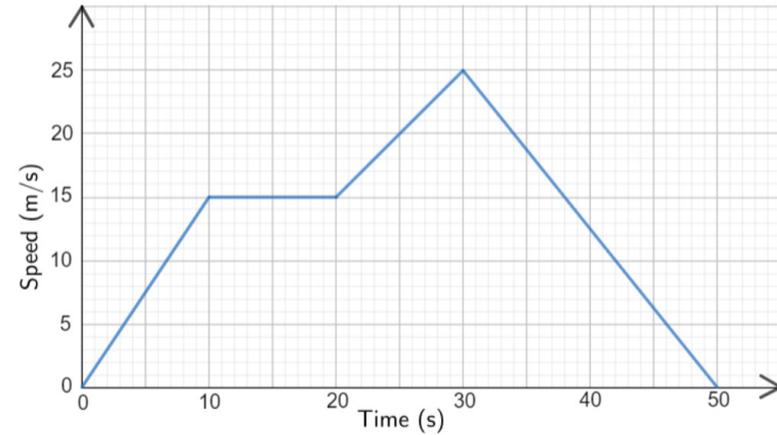
## Worked Example

Below is the speed-time graph of 40-second cycle journey. Work out the total distance travelled by the cyclist over the journey.



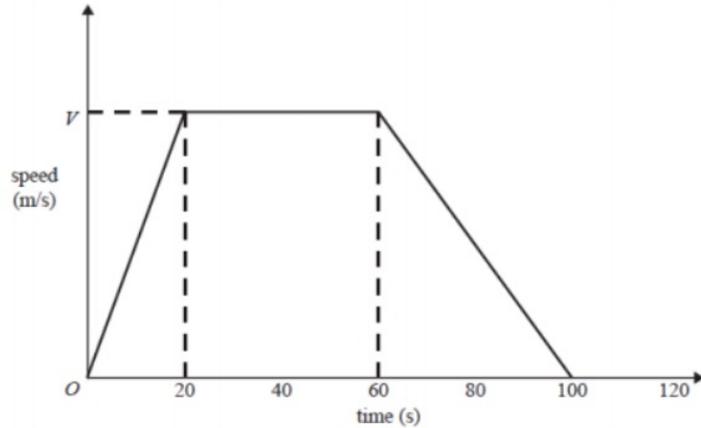
## Your Turn

Below is the speed-time graph of a 50-second car journey. Work out the total distance travelled by the car over the journey.



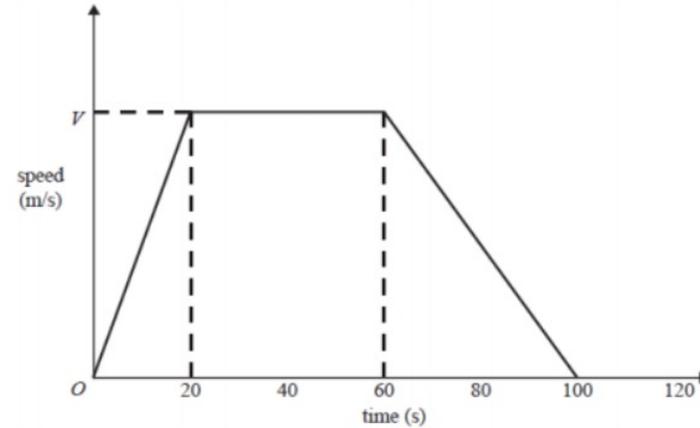
## Worked Example

Below is a speed-time graph for a car journey.  
The car travelled 3500 m in 100 seconds.  
Calculate the value of  $V$ .



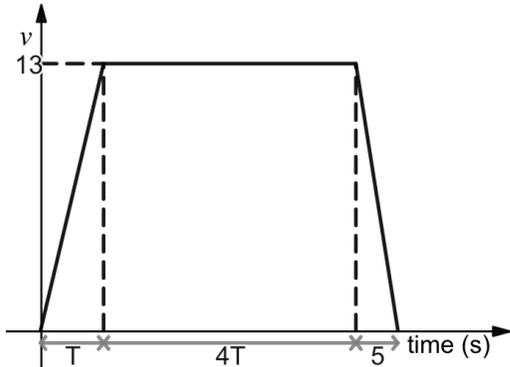
## Your Turn

Below is a speed-time graph for a car journey.  
The car travelled 1750 m in 100 seconds.  
Calculate the value of  $V$ .



## Worked Example

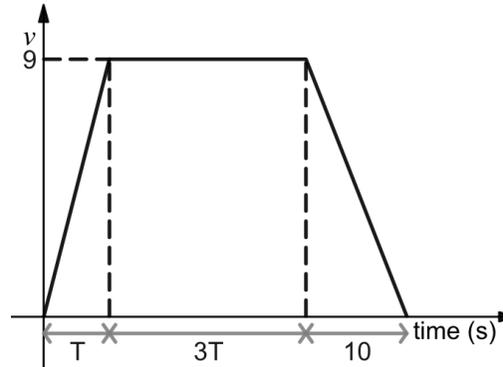
The velocity-time graph illustrate the motion of a vehicle moving in a straight line.



Given that the total displacement of the vehicle is 471.25 m, find the value of  $T$ .

## Your Turn

The velocity-time graph illustrate the motion of a particle moving in a straight line.



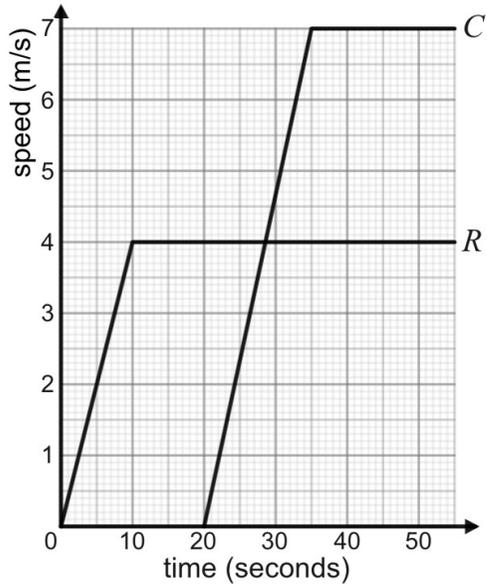
Given that the total displacement of the particle is 249.75 m, find the value of  $T$ .

## Worked Example

A runner sets off to complete a circuit of a racecourse.

After 20 seconds a cyclist sets off after them.

This speed-time graph shows the speed of the runner  $R$  and the cyclist  $C$  over the first 55 seconds.



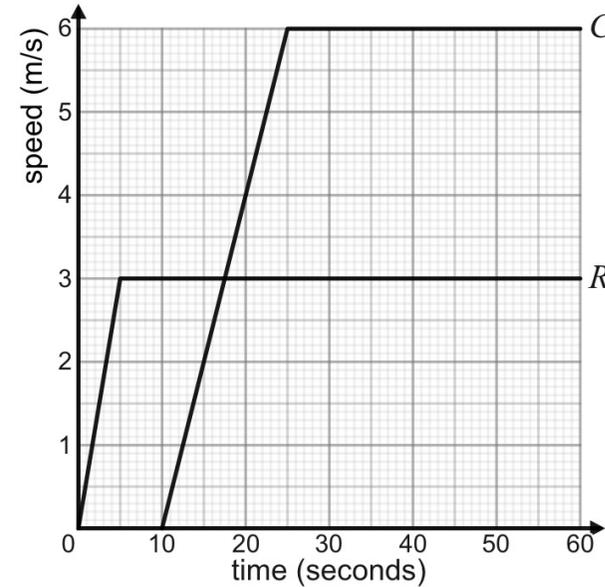
Show that the cyclist does not overtake the runner.

## Your Turn

A runner sets off to complete a circuit of a racecourse.

After 10 seconds a cyclist sets off after them.

This speed-time graph shows the speed of the runner  $R$  and the cyclist  $C$  over the first 60 seconds.



Show that the cyclist overtakes the runner.

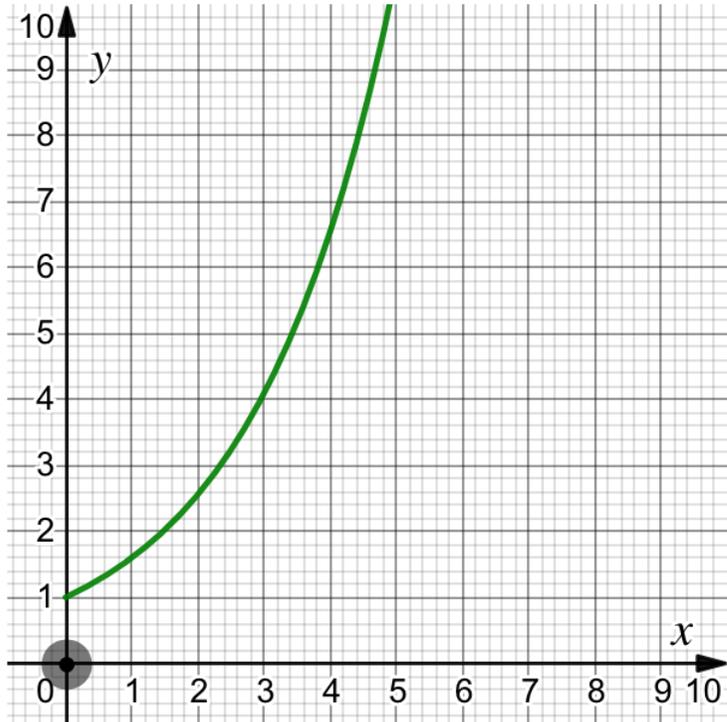
## Extra Notes

## 6 Gradients and Areas of Graphs

## Gradients of Curves

## Worked Example

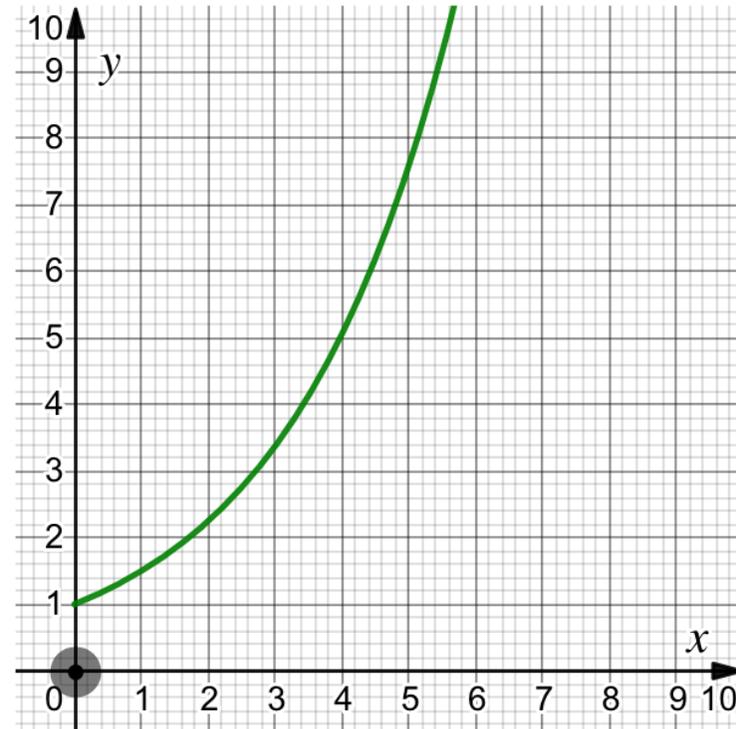
The diagram below shows part of the graph of  $y = f(x)$ .



$P$  is the point on the graph of  $y = f(x)$  where  $x = 3$ .  
Work out an estimate for the gradient of the graph at the point  $P$ .

## Your Turn

The diagram below shows part of the graph of  $y = f(x)$ .

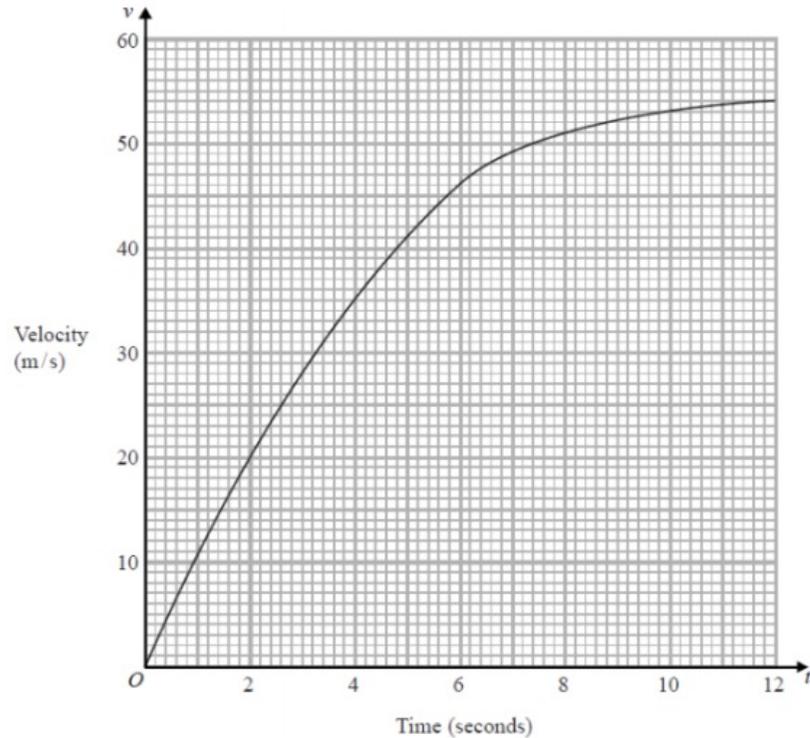


$P$  is the point on the graph of  $y = f(x)$  where  $x = 4$ .  
Calculate an estimate for the gradient of the graph at the point  $P$ .

## Worked Example

The graph shows the velocity,  $v$  m/s, of a parachutist  $t$  seconds after leaving a plane.

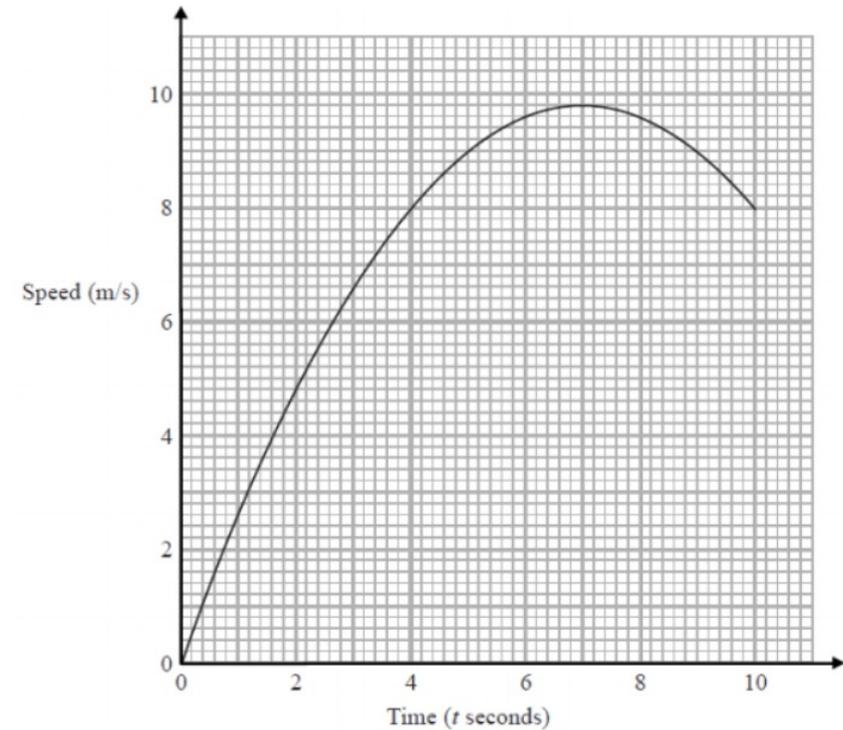
Calculate an estimate for the acceleration of the parachutist at  $t = 6$ .



## Your Turn

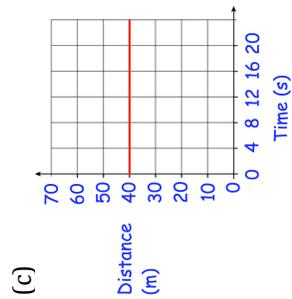
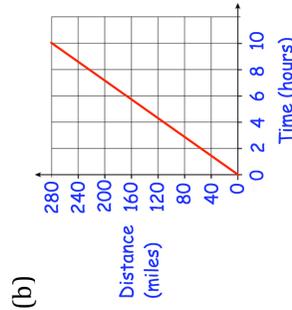
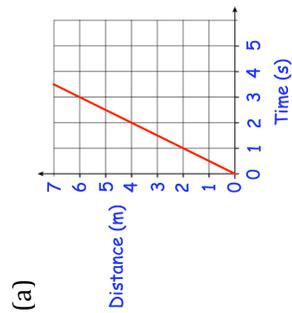
The graph shows the speed, m/s, of a runner,  $t$  seconds after the start of a race.

Calculate an estimate for the acceleration of the runner at  $t = 4$ .

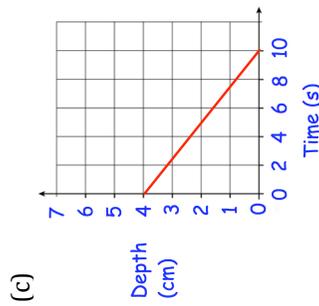
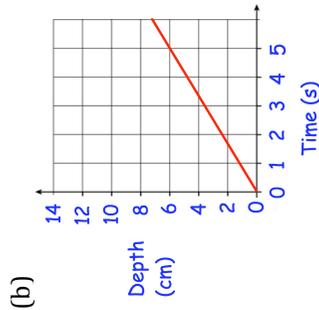
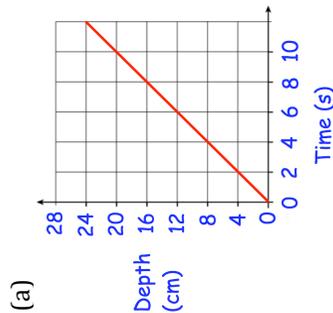


# Fluency Practice

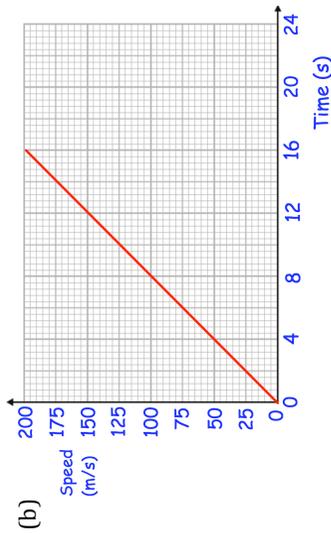
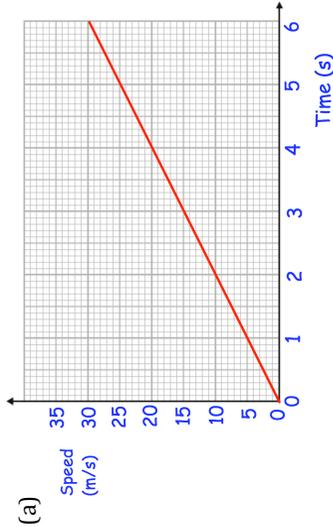
Question 1: For each graph below, work out the speed.



Question 2: For each graph below, work out the rate of change of depth.  
Give each answer in  $cm/s$

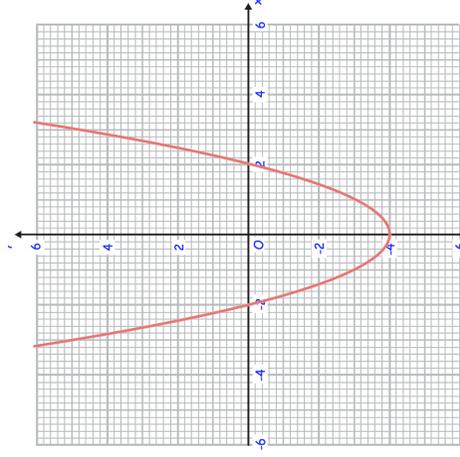


Question 3: For each graph below, work out the acceleration.  
Give each answer in  $m/s^2$



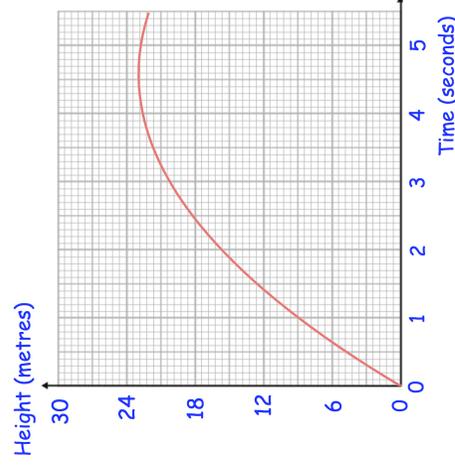
# Fluency Practice

Question 4: Here is part of a quadratic graph



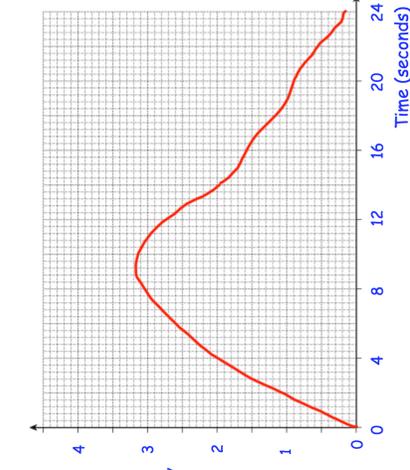
- (a) What is the gradient of the graph at the point  $(0, -4)$ ?
- (b) Calculate an estimate of the gradient of the graph at the point  $(2, 0)$
- (c) Calculate an estimate of the gradient of the graph at the point  $(-1, -3)$

Question 5: The graph shows the height of a ball above the ground.



- (a) Use the graph to work out an estimate of the speed of the ball at 1 second.
- (b) When was the speed  $0 \text{ m/s}$ ?
- (c) Use the graph to work out an estimate of the speed of the ball at 4 seconds.

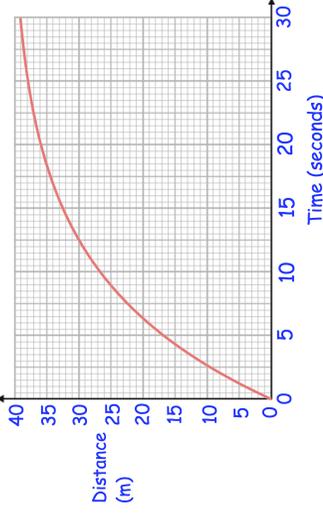
Question 6: The graph shows the velocity of object.



- (a) Use the graph to work out an estimate of the acceleration of the object at 2 seconds.
- (b) Use the graph to work out an estimate of the acceleration of the object at 8 seconds.
- (c) Use the graph to work out an estimate of the deceleration of the object at 16 seconds

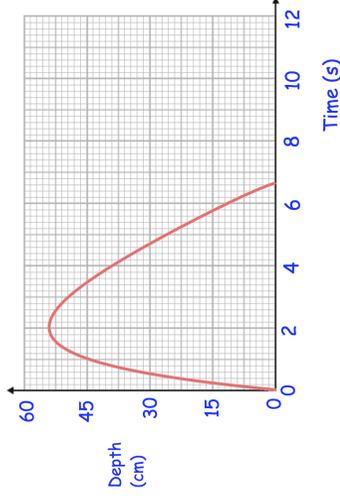
# Fluency Practice

Question 7: Shown is a distance-time graph of part of a journey.



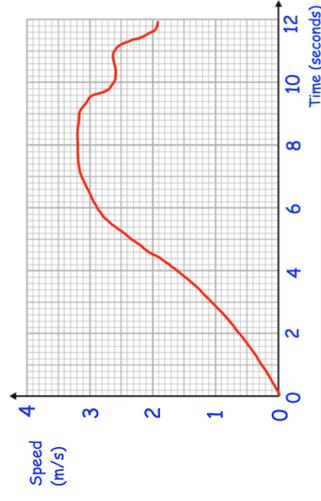
- (a) Work out the average speed over the first 5 seconds of the journey.
- (b) Work out the average speed between 15 and 30 seconds.

Question 8: Hugh has a bucket with holes in it. Hugh fills the bucket with water and records the depth of water. The graph shows the depth of water in the bucket.



- (a) Work out the average rate of change of depth of water between 0 and 2 seconds.
- (b) Work out the average rate of change of depth of water between 2 and 6 seconds.

Question 9: Here is the speed of a toy car during 12 seconds.



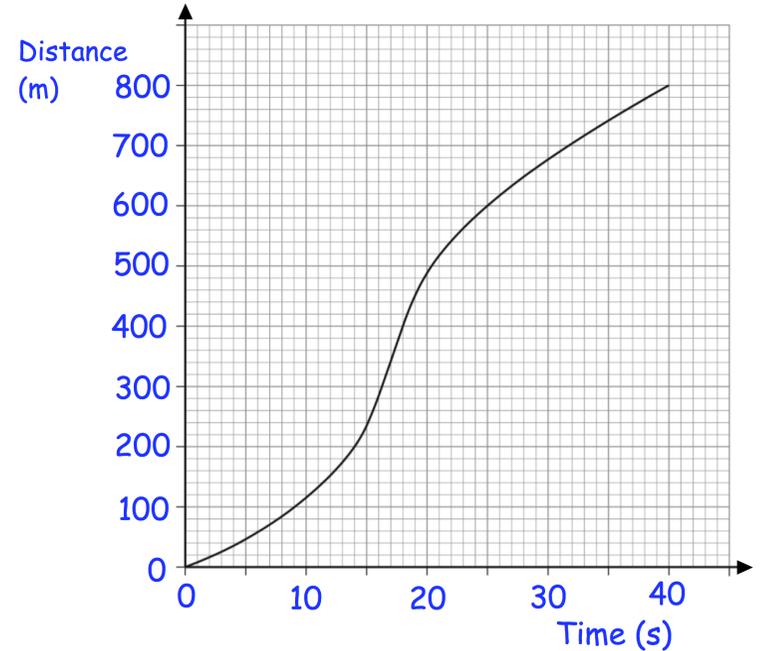
- (a) Work out the average acceleration of the toy car between 1 and 5 seconds.
- (b) Work out the average acceleration of the toy car between 8 and 12 seconds.

## Purposeful Practice

### Apply

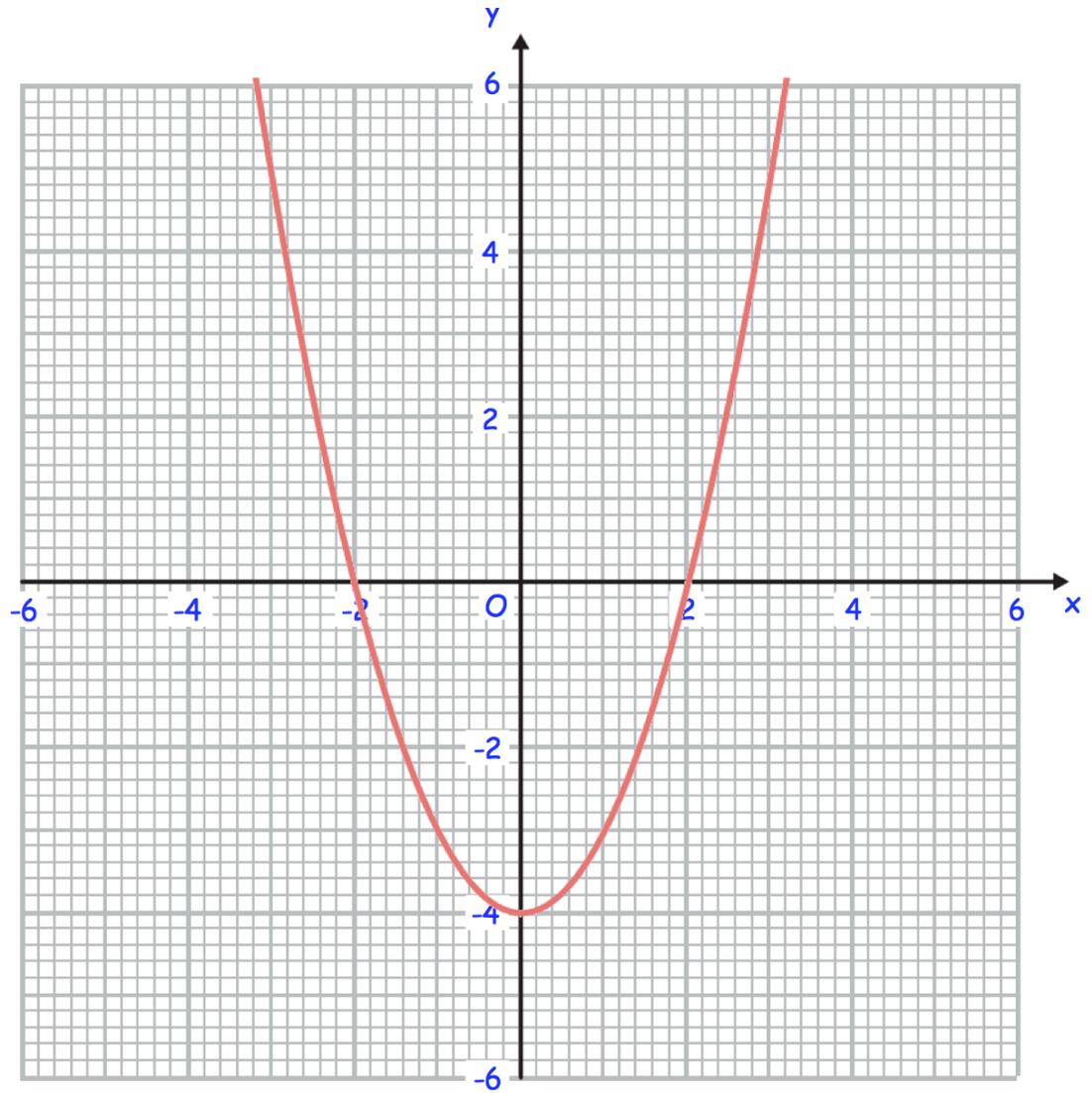
Question 1: Below is the distance-time graph for the first 40 seconds of a train journey.

- Use the graph to calculate an estimate for the speed of the train at 30 seconds.
- Explain why your answer to (a) is only an estimate.
- Estimate the highest speed reached by the train on the journey.



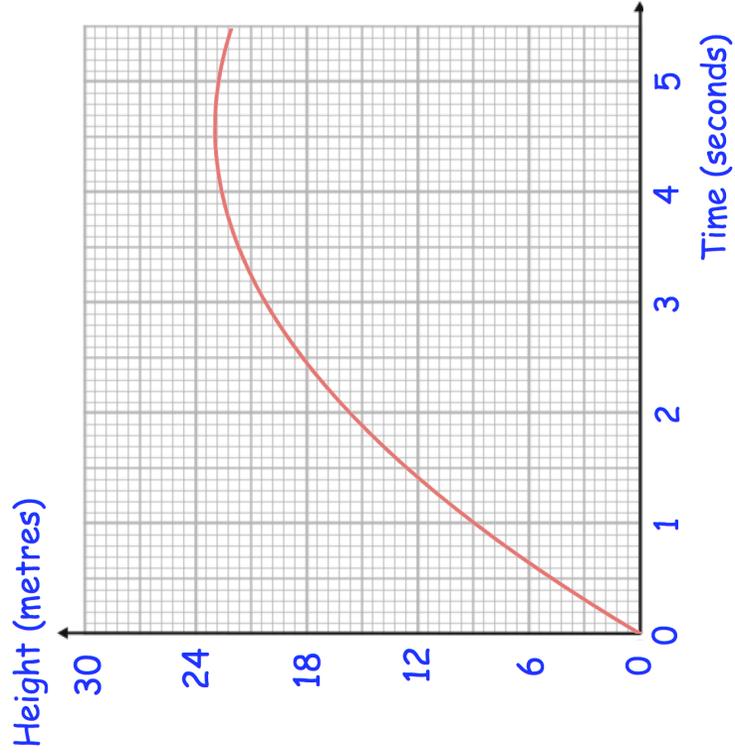
# Templates

Question 4:

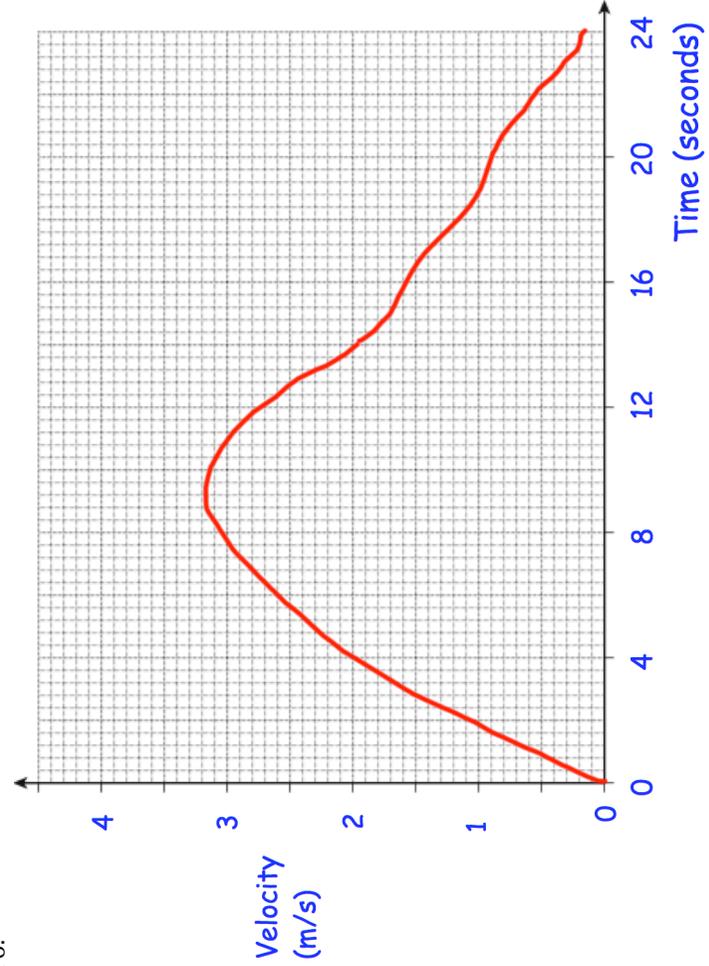


# Templates

Question 5:

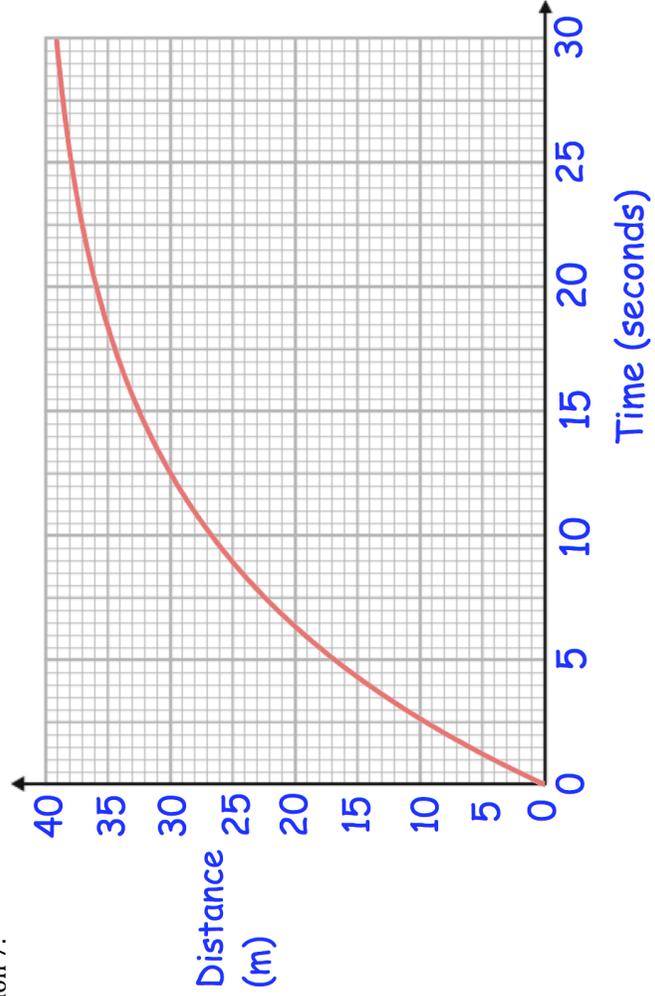


Question 6:

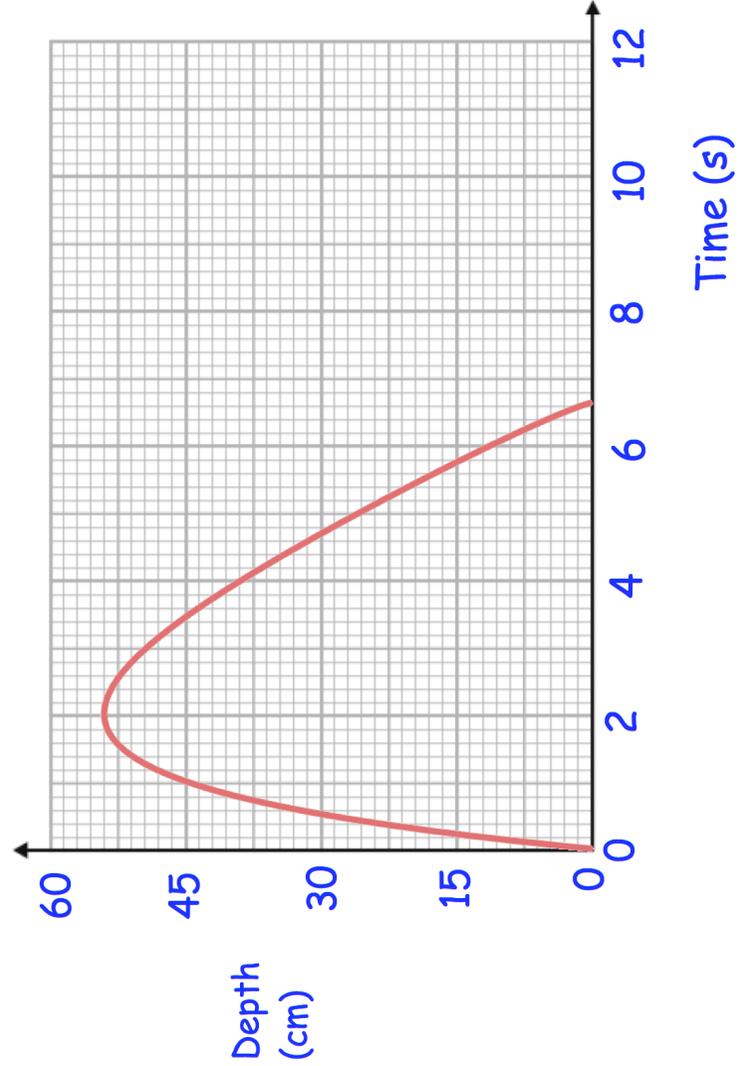


# Templates

Question 7:

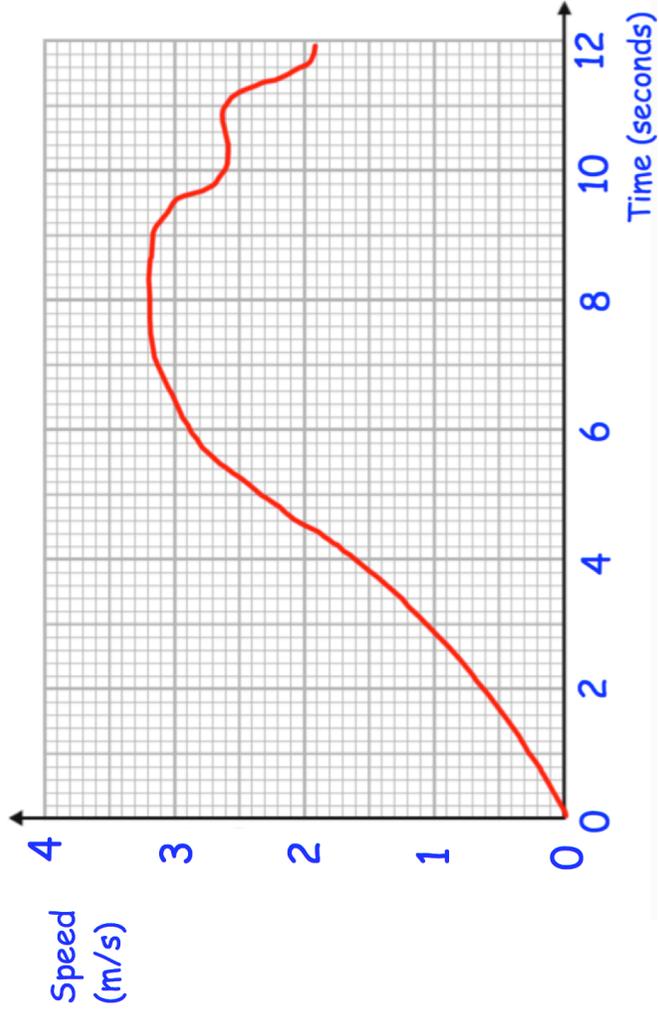


Question 8:

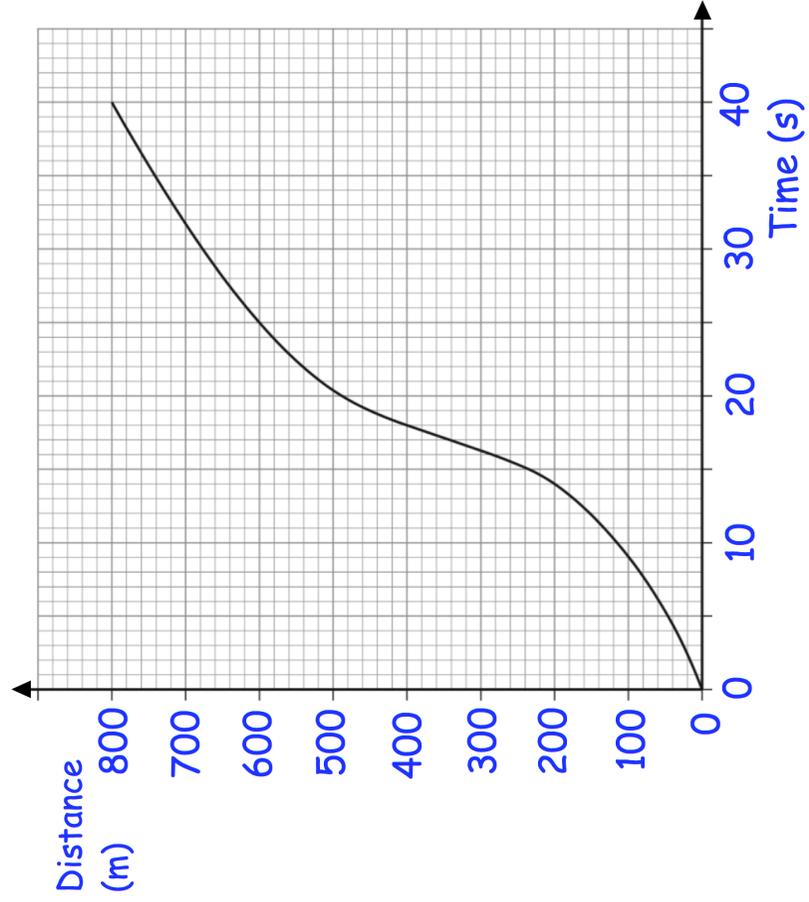


# Templates

Question 9:



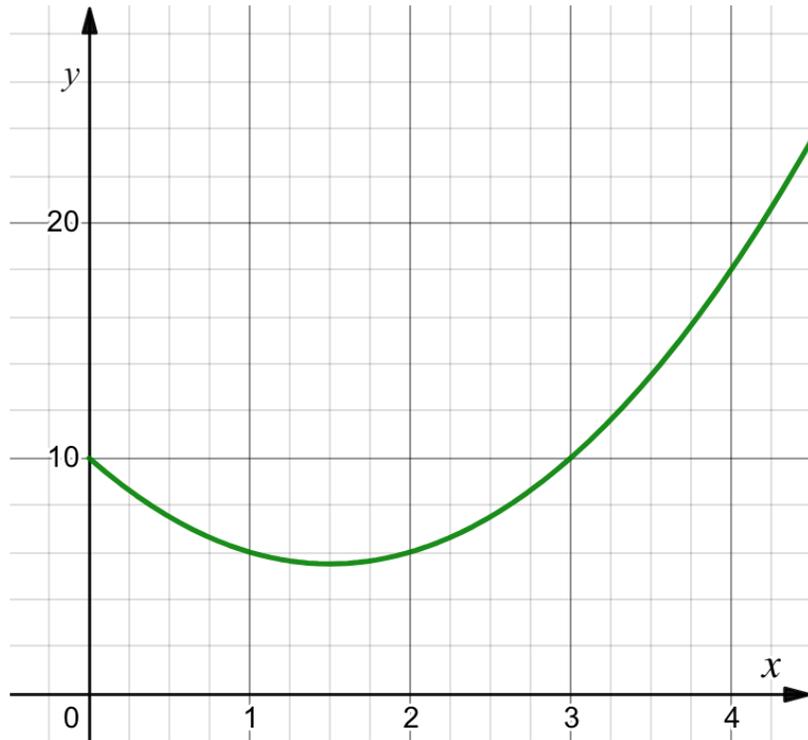
Apply 1:



## Area under Curves

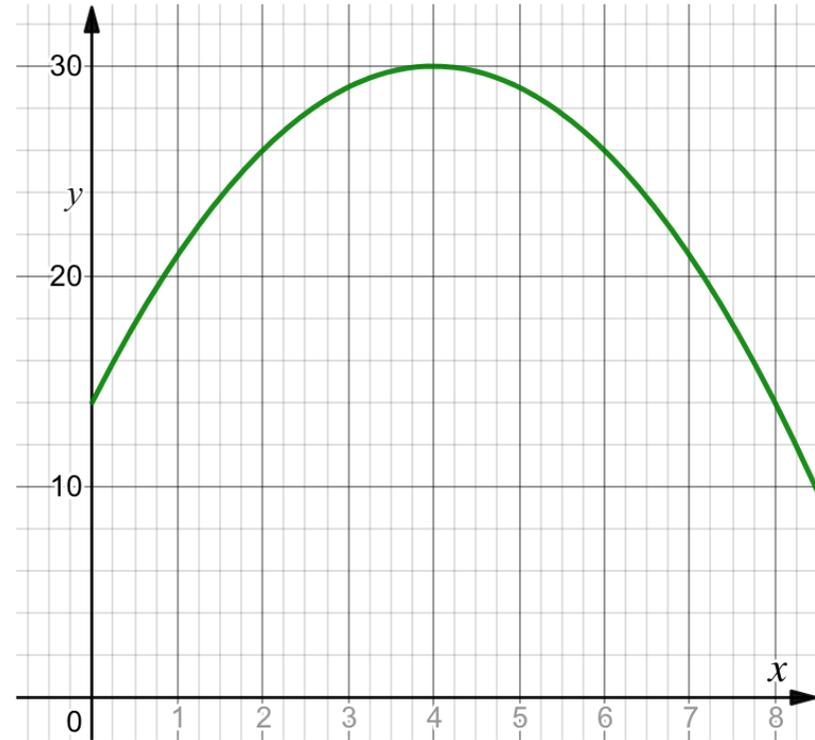
## Worked Example

Work out an estimate of the area between the curve, the axes and  $x = 4$ .  
Use 4 strips of equal width.



## Your Turn

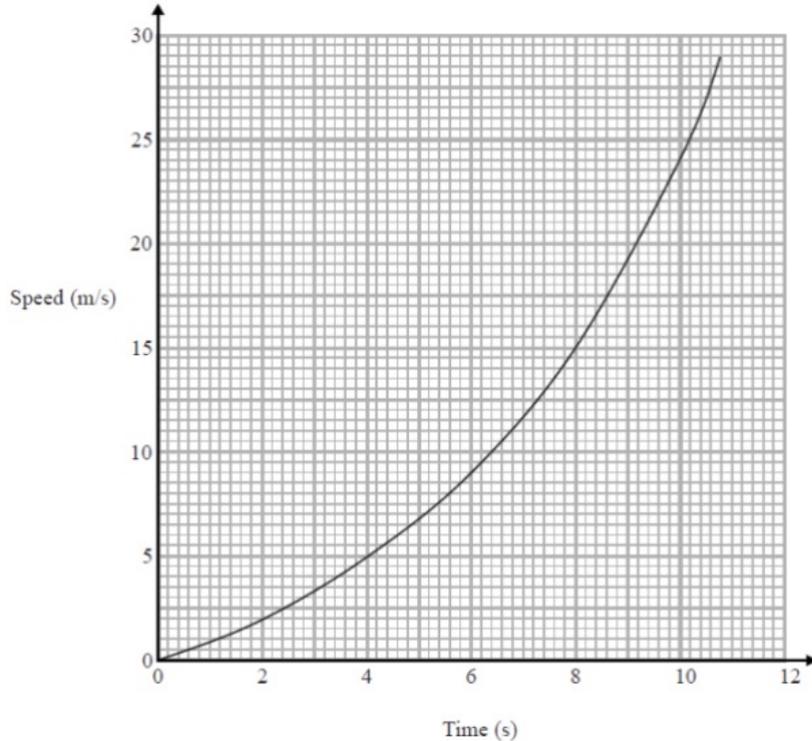
Work out an estimate of the area between the curve, the axes and  $x = 8$ .  
Use 4 strips of equal width.



## Worked Example

Below is a speed-time graph showing the speed, in metres per second, of an object  $t$  seconds after it started to move.

- a) Use 5 strips of equal width to find an estimate for the distance travelled in the first 10 seconds.

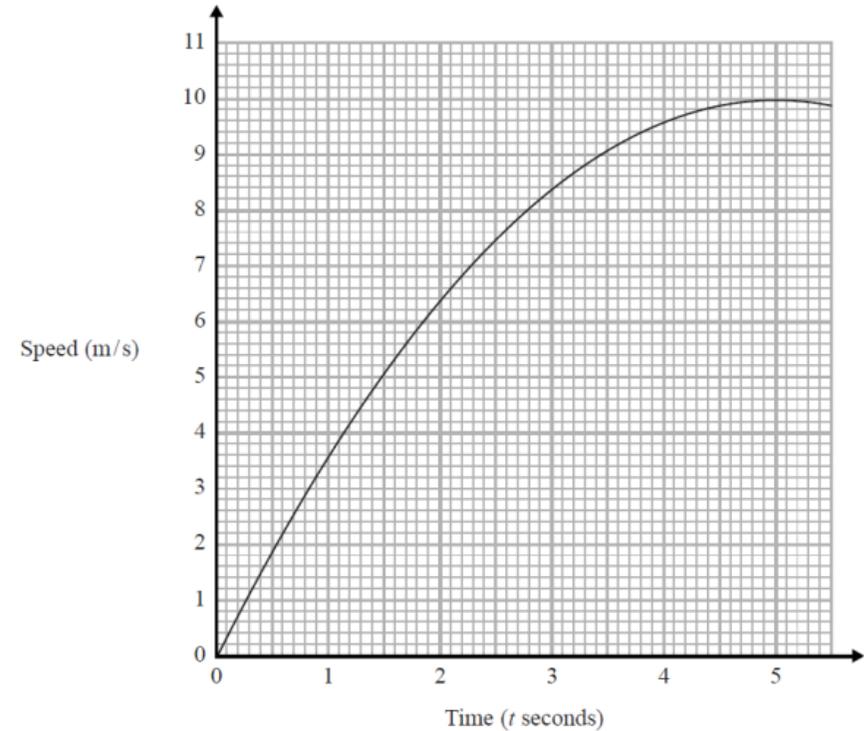


- b) Is your estimate an overestimate or underestimate?

## Your Turn

Below is a speed-time graph showing the speed, in metres per second, of an object  $t$  seconds after it started to move.

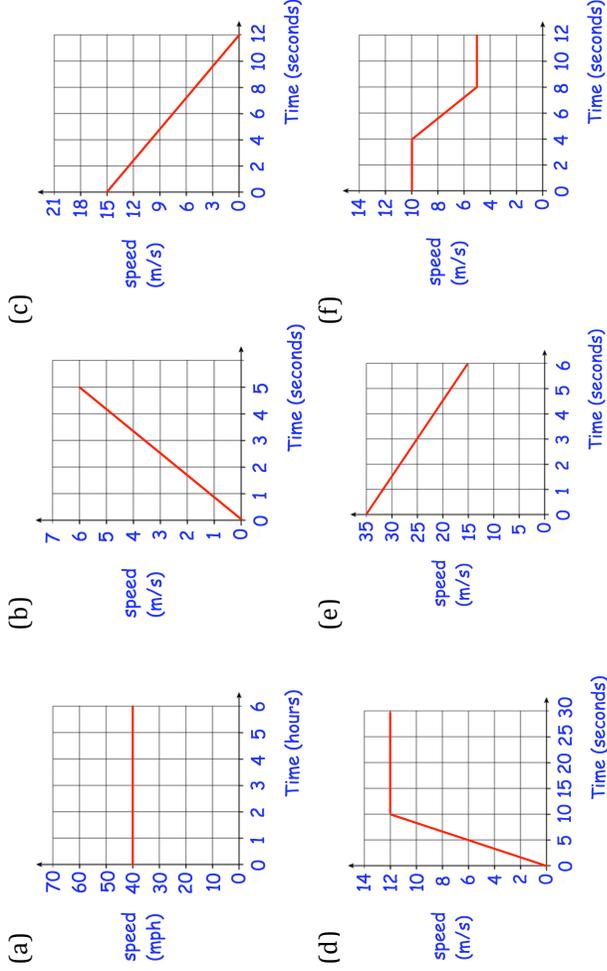
- a) Use 3 strips of equal width to find an estimate for the distance travelled between  $t = 1$  and  $t = 4$ .



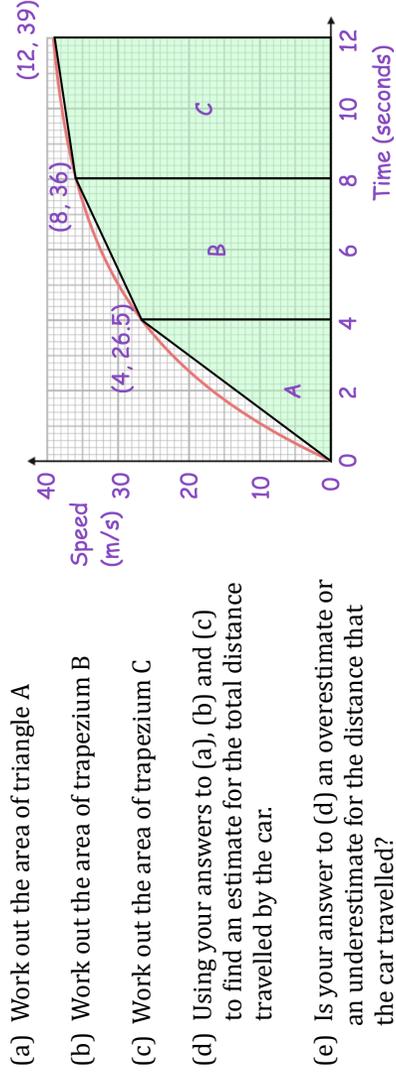
- b) Is your estimate an overestimate or underestimate?

# Fluency Practice

**Question 1:** Shown below are speed-time graphs for some journeys. For each journey, calculate the total distance travelled.



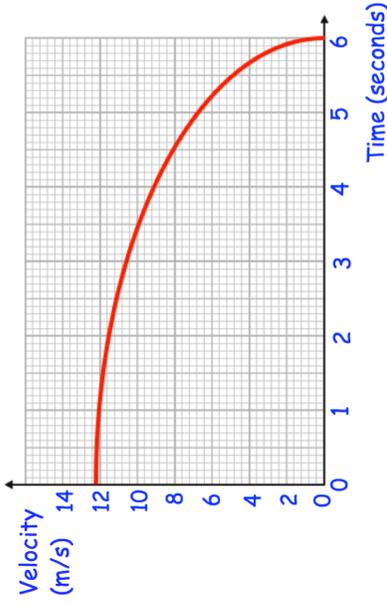
**Question 2:** Here is the speed-time graph for a car's journey.



- (a) Work out the area of triangle A
- (b) Work out the area of trapezium B
- (c) Work out the area of trapezium C
- (d) Using your answers to (a), (b) and (c) to find an estimate for the total distance travelled by the car.
- (e) Is your answer to (d) an overestimate or an underestimate for the distance that the car travelled?

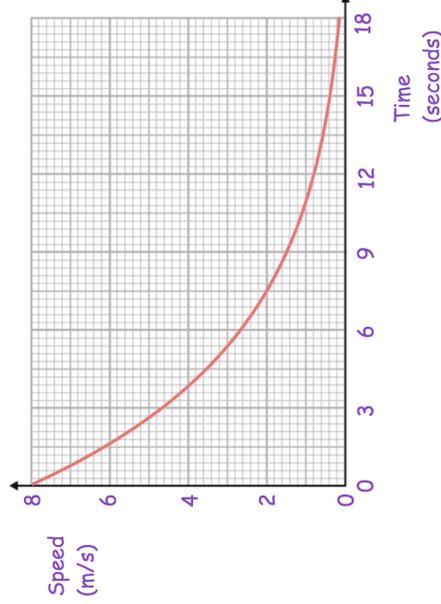
# Fluency Practice

Question 3: Here is a velocity-time graph for 6 seconds of a journey.



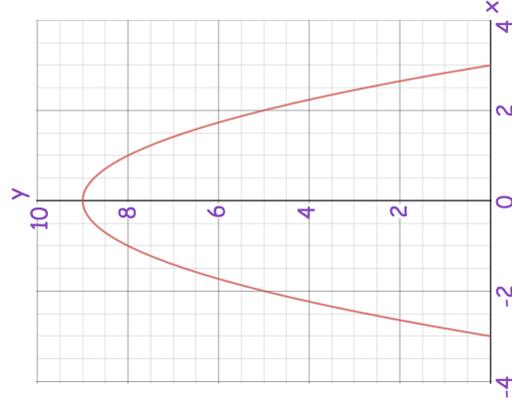
- (a) Work out an estimate for the distance travelled over 6 seconds. Use 3 strips of equal width.
- (b) Is your answer to (a) an overestimate or an underestimate of the actual distance travelled?

Question 4: Here is a speed-time graph for a remote-controlled car



- (a) Work out an estimate for the distance travelled over the first 12 seconds of the journey. Use 4 strips of equal width.
- (b) Is your answer to (a) an overestimate or an underestimate of the actual distance travelled?

Question 5: Here is a sketch of  $y = 9 - x^2$

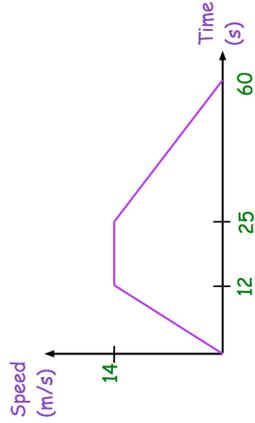


The graph is used to model the cross section of a tunnel.

Calculate an estimate of the area under the graph.

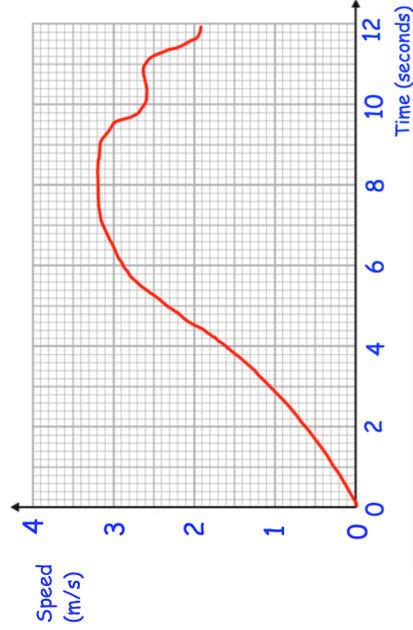
# Fluency Practice

Question 1: The graph shows the speed of a bicycle between two houses.



Calculate the distance between the houses.

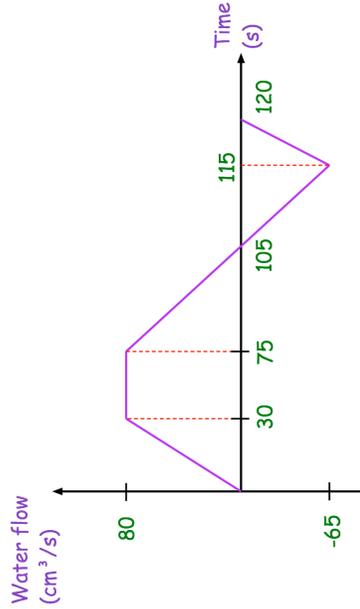
Question 2: Here is a speed-time graph for the first 12 seconds of a journey.



Calculate an estimate for the total distance travelled over the 12 seconds.

Question 3: The graph below shows information on how an empty container is being filled with water.

How much water is in the container after 120 seconds?

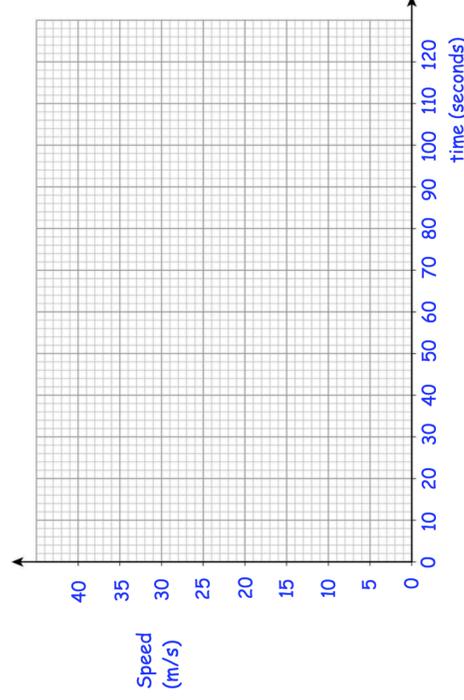


# Fluency Practice

Question 4: Finn is driving his car in a straight line.

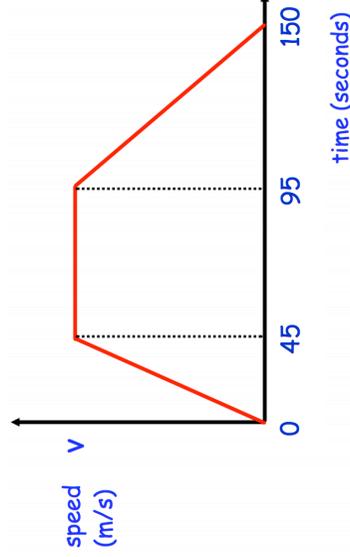
The car begins at rest.  
 He accelerates uniformly at  $2\text{m/s}^2$  for 15 seconds.  
 Finn drives at the same speed for the next 25 seconds.  
 He then accelerates uniformly to a speed of  $40\text{m/s}$  by 80 seconds.  
 The remainder of the 2 minute journey is spent decelerating to rest.

(a) Draw a speed-time graph for his journey.



(b) Write down the average speed for the total journey.

Question 5: Here is a speed-time graph for a train journey.

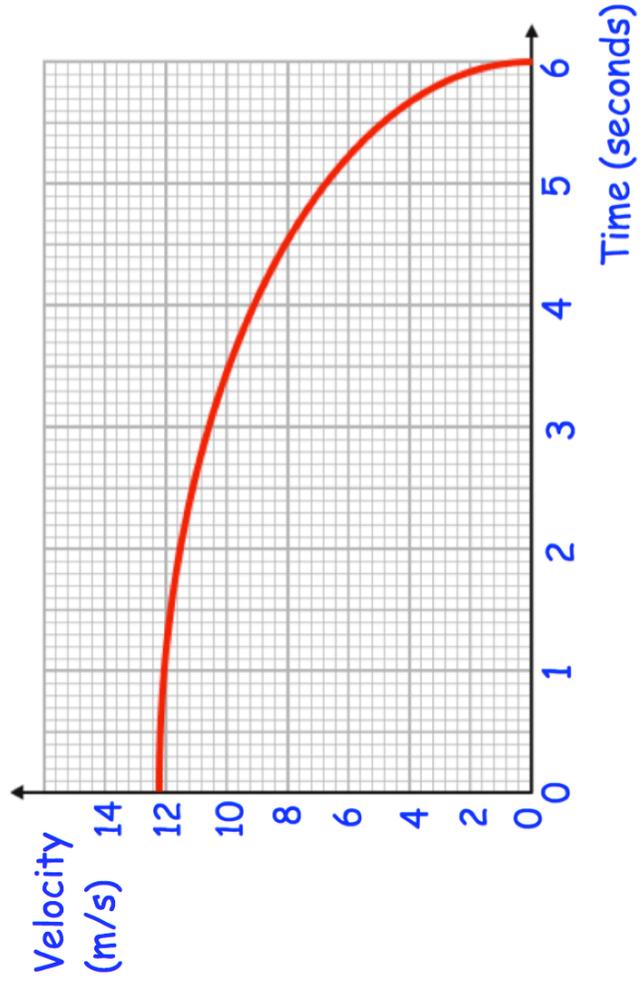


The journey took 150 seconds.  
 The train travelled 1.53km in the 150 seconds.

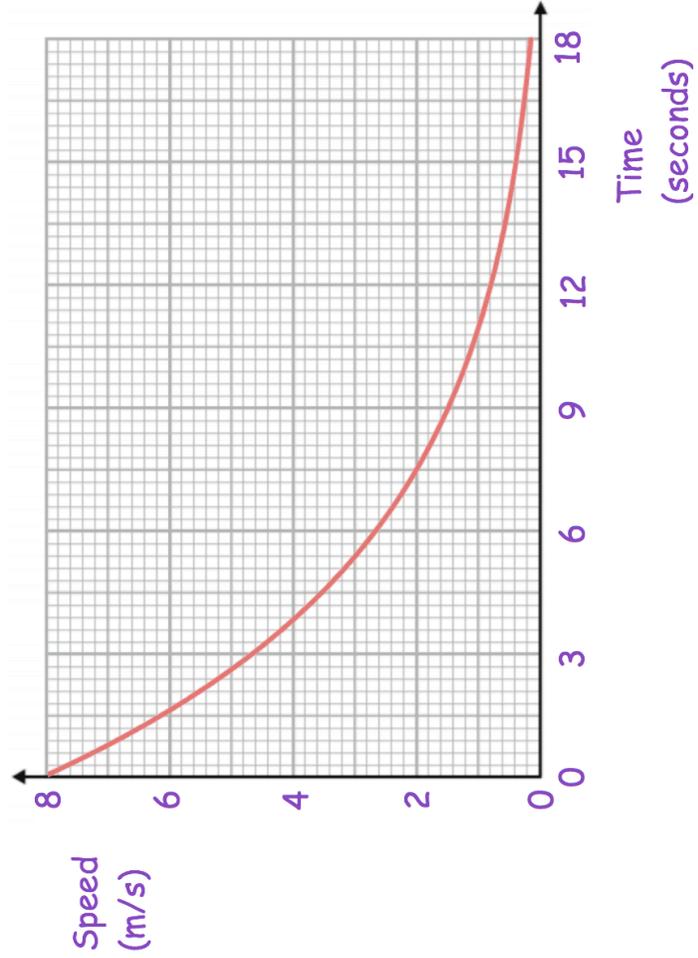
Work out the value of  $v$ .

# Templates

Question 3:

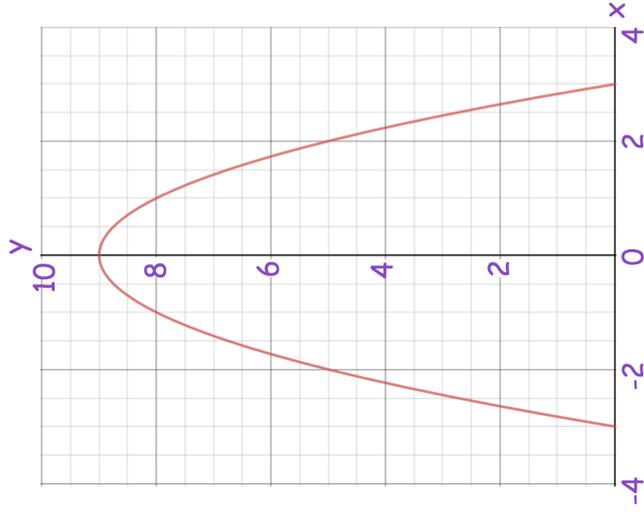


Question 4:

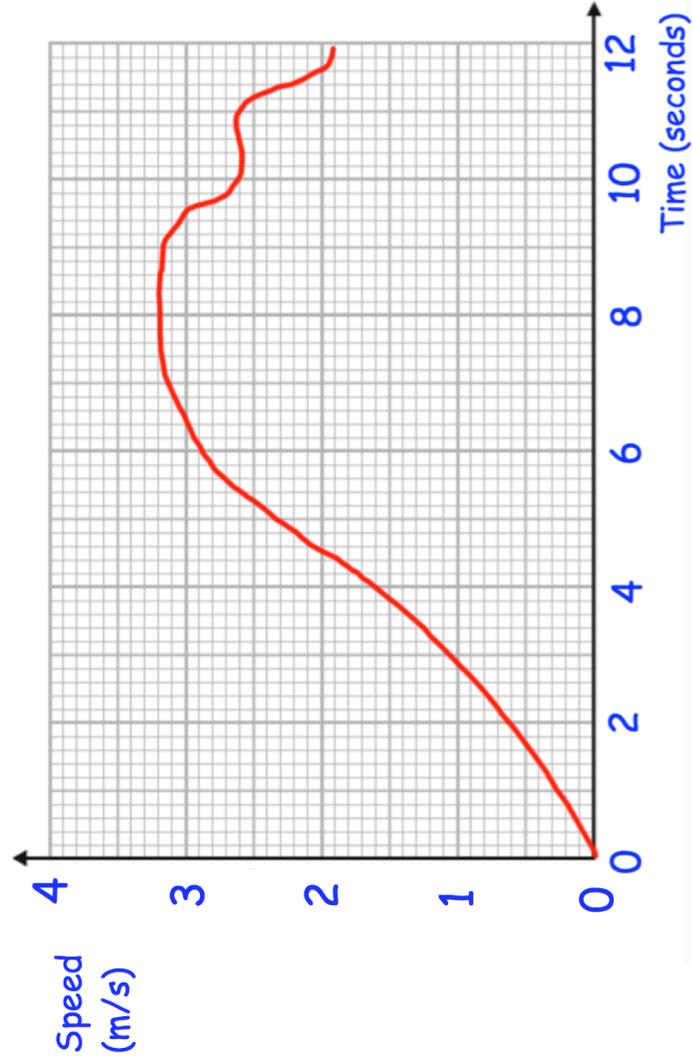


# Templates

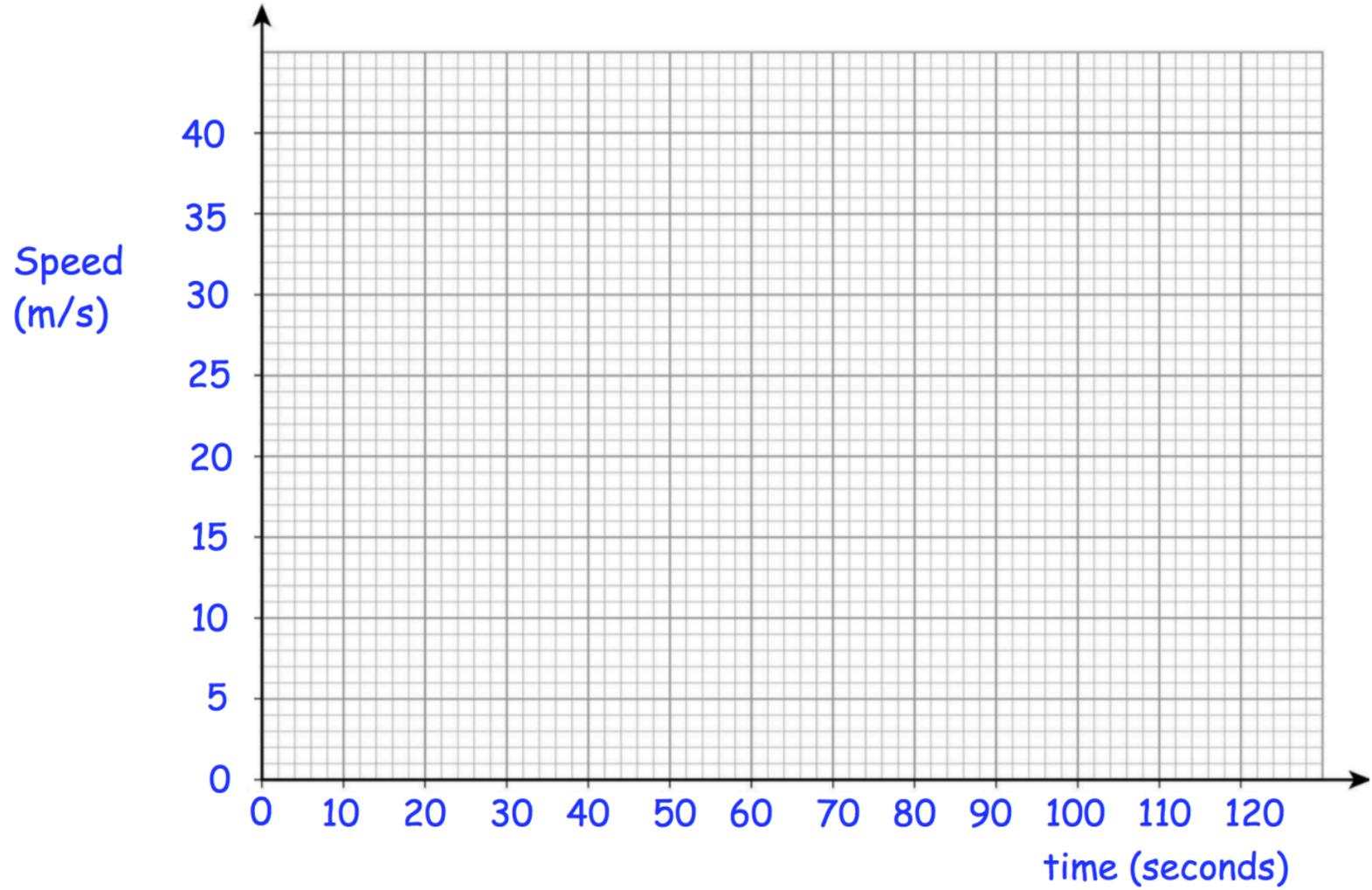
Question 5:



Apply Question 2:



# Templates



## Extra Notes

## 7 Iterations

### Worked Example

Show that the equation  $3 + 5x - 2x^2 - x^3 = 0$  has a solution between  $-4$  and  $-3$ .

### Your Turn

Show that the equation  $x^3 + 4x^2 - x - 5 = 0$  has a solution between  $1$  and  $2$ .

### Worked Example

Show that the equation  $\sqrt{2x + 4} - 3 = 0$  has a solution between 2 and 3.

### Your Turn

Show that the equation  $\frac{3}{\sqrt{2+x}} - 4 = 0$  has a solution between  $-1.5$  and  $-1$ .

### Worked Example

Find the solution to the equation  $x^3 + x^2 - 0.5 = 0$  between 0 and 1, to 1 decimal place.

### Your Turn

Find the solution to the equation  $x^3 - 5x - 1 = 0$  between 2 and 3, to 1 decimal place.

### Worked Example

An approximate solution to the equation  $x^3 - 2x^2 - x - 1 = 0$  can be found using the iteration formula  $x_{n+1} = \sqrt[3]{2x_n^2 + x_n + 1}$ . Starting with  $x_0 = 5$ , find the value of  $x_6$ . Give your answer correct to 3 decimal places.

### Your Turn

The equation  $3x^3 + 8x^2 + 7x + 2 = 0$  can be rearranged to give the iterative formula  $x_{n+1} = \frac{-3(x_n)^3 - 8(x_n)^2 - 2}{7}$

Starting with  $x_0 = -2$ , find the value of  $x_5$ . Give your answer correct to 3 decimal places.

### Worked Example

Use the iterative formula  $x_{n+1} = \sqrt{3 - x_n}$  to find a solution to the equation  $x^2 + x - 3 = 0$  to 3 decimal places. Starting with  $x_0 = 1$ .

### Your Turn

Use the iterative formula  $x_{n+1} = \frac{1-4x_n^3}{3}$  to find a solution to the equation  $4x^3 + 3x - 1 = 0$  to 2 decimal places. Starting with  $x_0 = 0$ .

### Worked Example

Show that  $x^3 - 2x^2 - x - 1 = 0$  can be rearranged to  
 $x = \sqrt[3]{2x^2 + x + 1}$

### Your Turn

Show that  $x^3 - 2x + 5 = 0$  can be rearranged to give  
 $x = \sqrt[3]{2x - 5}$

### Worked Example

The value of a car at the start of year  $n$  is  $V_n$ . The value at the start of the following year is  $V_{n+1}$  where  $V_{n+1} = kV_n$ . A car was purchased as new in 2020 for £3,200. The same car was sold in 2022 for £2,048. Work out the value of the depreciation constant  $k$ .

### Your Turn

At the start of year  $n$ , the number of animals in a population is  $P_n$ . At the start of the following year, the number of animals in the population is  $P_{n+1}$  where  $P_{n+1} = kP_n$ . At the start of 2017 the number of animals in the population was 4000. At the start of 2019 the number of animals in the population was 3610. Find the value of the constant  $k$ .

## Extra Notes