KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 11

2025 Mathematics (L2FM) 2026

Unit 24 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: ______________________________

Class: ______________________________

KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
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Unit 24 Booklet – Part 2

HGS Maths **Tasks** **Dr Frost Course**

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Contents

1 Equations of Circles and Tangents

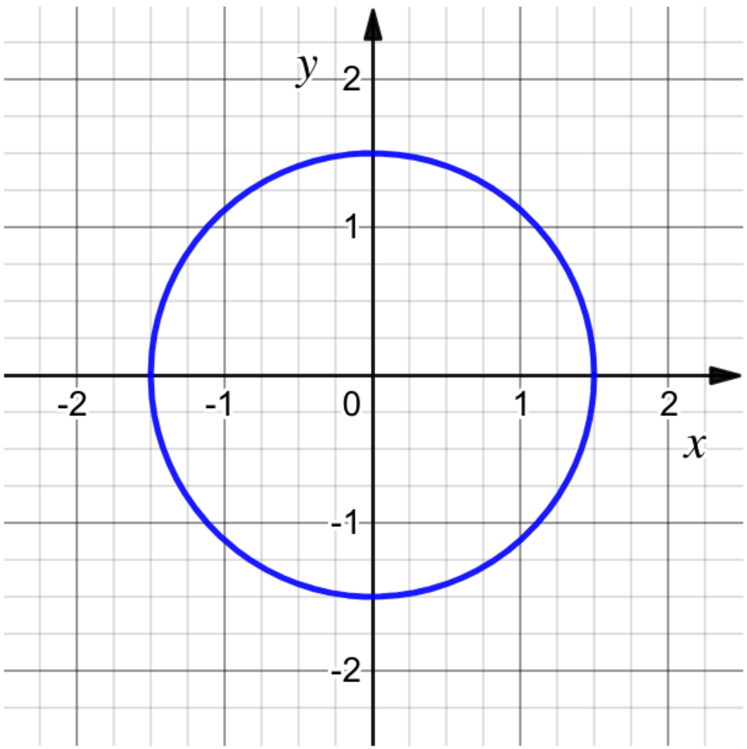
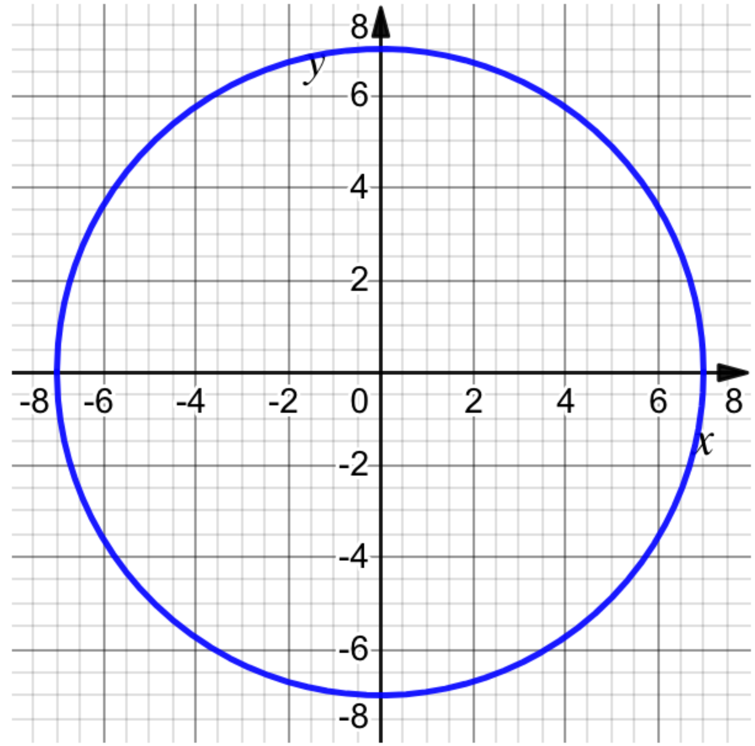
Worked Example	Your Turn
Determine whether the point with coordinates $(-5, 7)$ lies on the circle with the equation $x^2 + y^2 = 85$.	Determine whether the point with coordinates $(6, -8)$ lies on the circle with the equation $x^2 + y^2 = 100$.

Dr Frost 446a

Worked Example	Your Turn
Find the radius of the circle with equation: a) $x^2 + y^2 = 196$ b) $x^2 + y^2 = 326$	Find the radius of the circle with equation: a) $x^2 + y^2 = 169$ b) $x^2 + y^2 = 362$

Worked Example	Your Turn
Find an equation of the circle with radius $3\sqrt{5}$ and centre $(0, 0)$.	Find an equation of the circle with radius $5\sqrt{2}$ and centre $(0, 0)$.

Dr Frost 446d

Worked Example	Your Turn
Find an equation of the circle drawn below. 	Find an equation of the circle drawn below. 

Dr Frost 446e

Worked Example	Your Turn
The point $(-5, 3)$ lies on a circle centered on the origin. Find an equation for this circle.	The point $(-7, -2)$ lies on a circle centered on the origin. Find an equation for this circle.

Dr Frost 446f

Worked Example	Your Turn
The circle below is given by the equation $x^2 + y^2 = 16$. a) Calculate its circumference, C. b) Calculate the shaded area, A. Give your answers correct to 2 decimal places.	The circle below is given by the equation $x^2 + y^2 = 64$. a) Calculate its circumference, C. b) Calculate the shaded area, A. Give your answers correct to 2 decimal places.

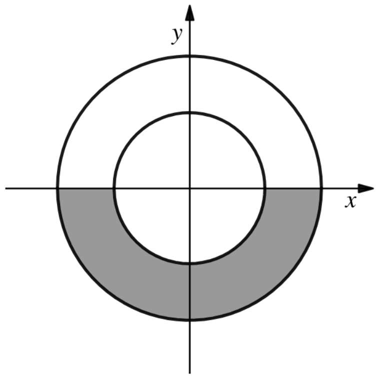
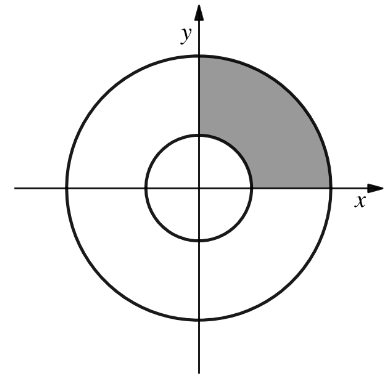
Dr Frost 446g

Worked Example	Your Turn
a) A circle has a circumference of 6π. Find an equation for the circle. b) A circle has an area of 49π. Find an equation for the circle.	a) A circle has a circumference of 12π. Find an equation for the circle. b) A circle has an area of 25π. Find an equation for the circle.

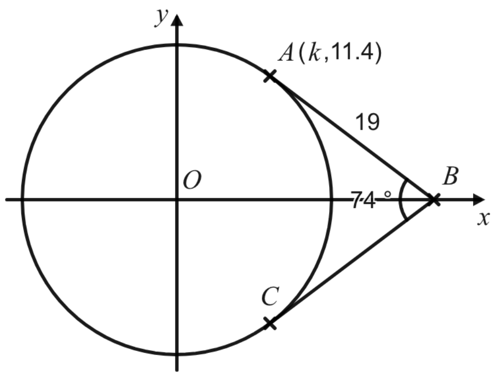
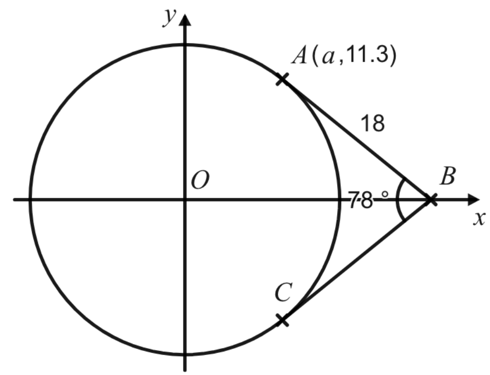
Dr Frost 446h

Fill in the Gaps

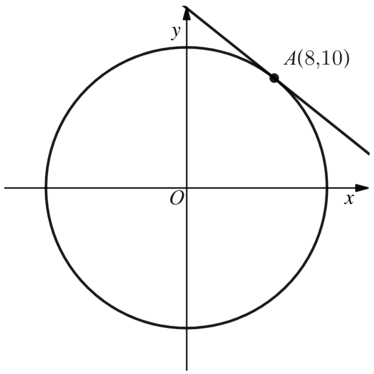
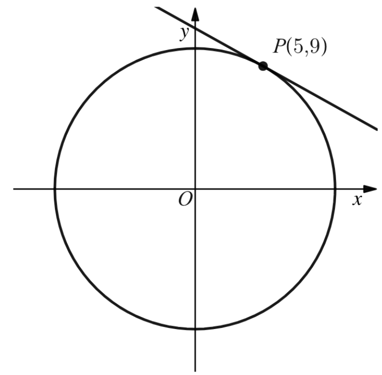
Equation	Radius	Area	Point 1	Point 2	Where is $(3, 7)$?
$x^2 + y^2 = 25$			$(3,$ ____$)$	$($____$, 0)$	Outside
$x^2 + y^2 = 50$			$(-5,$ ____$)$	$($____$, 7)$	
$x^2 + y^2 = 65$			$(1,$ ____$)$	$($____$, 7)$	
	15		$(9,$ ____$)$	$($____$, 0)$	
	$5\sqrt{5}$		$(-5,$ ____$)$	$($____$, 11)$	
		130π	$(-7,$ ____$)$	$($____$, 11)$	
		2042	$(19,$ ____$)$	$($____$, 11)$	
			$(-4,$ ____$)$	$(8, 11)$	
			$(1,$ ____$)$	$(-7, 11)$	
			$(-7,$ ____$)$	$($____$, \sqrt{22})$	On the circle

Worked Example	Your Turn
The annulus below is formed of two circles centred on the origin. The equations of the circles are: $x^2 + y^2 = 49$ $x^2 + y^2 = 16$  a) Calculate the perimeter of the shaded shape. b) Calculate the area of the shaded shape. Give your answers correct to 2 decimal places.	The annulus below is formed of two circles centred on the origin. The equations of the circles are: $x^2 + y^2 = 25$ $x^2 + y^2 = 4$  a) Calculate the perimeter of the shaded shape. b) Calculate the area of the shaded shape. Give your answers correct to 2 decimal places.

Dr Frost 446i

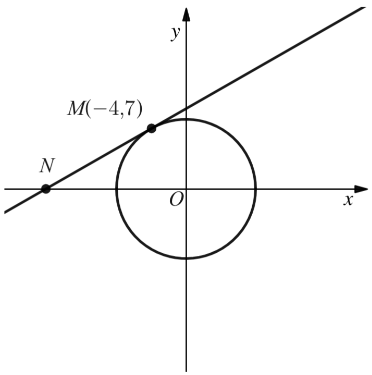
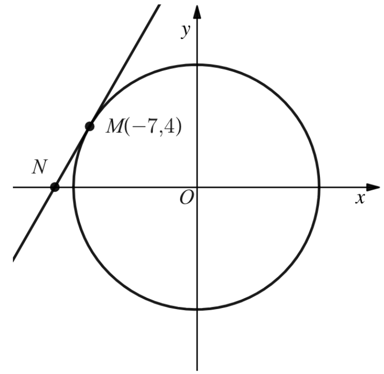
Worked Example	Your Turn
The diagram shows a circle, centre O  AB and BC are tangents to the circle. Angle $ABC = 74°$ AB has a length of 19 units. Calculate the value of k Give your answer correct to 1 decimal place.	The diagram shows a circle, centre O  AB and BC are tangents to the circle. Angle $ABC = 78°$ AB has a length of 18 units. Calculate the value of a Give your answer correct to 1 decimal place.

Dr Frost 446j

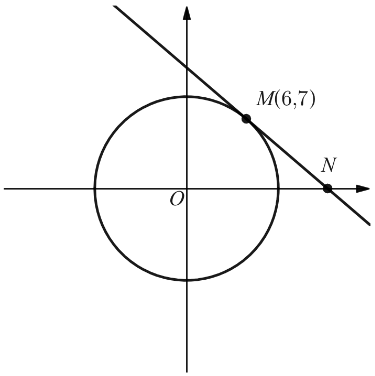
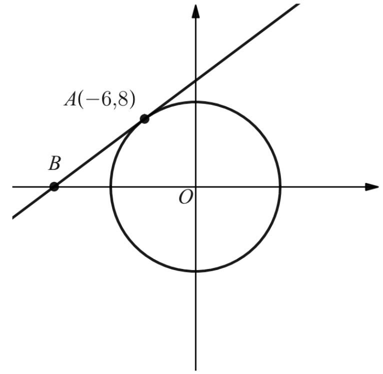
Worked Example	Your Turn
The diagram shows the circle with equation $x^2 + y^2 = 164$	The diagram shows the circle with equation $x^2 + y^2 = 106$
	
A tangent to the circle is drawn at point A with coordinates $(8, 10)$. Find an equation of the tangent at A.	A tangent to the circle is drawn at point P with coordinates $(5, 9)$. Find an equation of the tangent at P.

Fill in the Gaps

Equation of Circle	Point on Circle	Gradient of Radius	Gradient of Tangent	Equation of Tangent
$x^2 + y^2 = 45$	$(3, 6)$	2	$-\frac{1}{2}$	
$x^2 + y^2 = 10$	$(3, -1)$	$m = -\frac{1}{3}$		
$x^2 + y^2 = 68$	$(-2, -8)$			
$x^2 + y^2 = 25$	$(-4, 3)$			
$x^2 + y^2 = 73$	$(8, 3)$			
$x^2 + y^2 = \frac{53}{2}$	$\left(\frac{5}{2}, -\frac{9}{2}\right)$			
$x^2 + y^2 = 6$	$\left(-2, \sqrt{2}\right)$			
$x^2 + y^2 = 100$				$y = \frac{3}{4}x - \frac{25}{2}$

Worked Example	**Your Turn**
A circle has equation $x^2 + y^2 = 65$ M is the point on the circle with coordinates $(-4, 7)$  The tangent to the circle at M intersects the x-axis at point N. Work out the x-coordinate of N.	The diagram shows a circle with centre $(0, 0)$ and a tangent at the point $M(-7, 4)$  The tangent to the circle at M intersects the x-axis at point N. Work out the x-coordinate of N.

Dr Frost 447b

Worked Example	Your Turn
A circle has equation$x^2 + y^2 = 85$ M is the point on the circle with coordinates $M(6, 7)$  The tangent to the circle at M intersects the x-axis at point N. Work out the area of triangle OMN.	A circle has equation $x^2 + y^2 = 100$ A is the point on the circle with coordinates $A(-6, 8)$  The tangent to the circle at A intersects the x-axis at point B. Work out the area of triangle OAB.

Dr Frost 447c

Extra Notes

2 Advanced Equations of Circles (L2FM Only)

Worked Example	Your Turn
Find an equation of the circle with radius $3\sqrt{2}$ and centre $(-2, 4)$.	Find an equation of the circle with radius $2\sqrt{11}$ and centre $(2, -5)$.

Dr Frost 496c and 496a

Worked Example	Your Turn
Find the centre and exact value of the radius of the circle with equation $(x+1)^2+(y-4)^2=18$ where the centre of the circle is (a,b) and the radius is given in its simplest form.	Find the centre and exact value of the radius of the circle with equation $(x-4)^2+(y+5)^2=12$ where the centre of the circle is (a,b) and the radius is given in its simplest form.

Dr Frost 496d

Worked Example	Your Turn
Write down the equation of a circle with centre $(-1, 5)$ and diameter of $2\sqrt{10}$. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants to be found.	Write down the equation of a circle with centre $(-6, -4)$ and diameter of $2\sqrt{15}$. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants to be found.

Dr Frost 496e

Worked Example	Your Turn
Determine whether the point $(-4, -5)$ is inside the circle, on the circle, or outside the circle with equation $(x+1)^2 + (y-1)^2 = 47$	Determine whether the point $(-4, 9)$ is inside the circle, on the circle, or outside the circle with equation $(x+2)^2 + (y-4)^2 = 28$

Dr Frost 496g

Worked Example	Your Turn
Find the centre and radius of the circle with equation $x^2 + y^2 + 24x - 14y + 150 = 0$ giving your answer in exact form.	Find the centre and radius of the circle with equation $x^2 + y^2 - 20x + 18y = -89$ giving your answer in exact form.

Dr Frost 496k

Worked Example	Your Turn
A circle C has equation $x^2 + y^2 + 2kx + 4ky + 80 = 0$ where k is a constant. By considering the radius of C, state the range of possible values for k.	A circle C has equation $x^2 + y^2 + 2kx + 2ky = -18$ where k is a constant. By considering the radius of C, state the range of possible values for k.

Dr Frost 496l

Fill in the Gaps

Equation in Factorised Form	Equation in Expanded Form	Centre of Circle	Radius of Circle
$(x+2)^2+(y+5)^2=9$	$x^2+y^2+4x+10y+20=0$	$(-2,-5)$	3
$(x-3)^2+(y+2)^2=25$	$x^2+y^2-6x+4y-12=0$		
$x^2+(y-1)^2=4$			
		$(-1,4)$	10
		$(-6,0)$	5
		$(4,2)$	$\sqrt{15}$
	$x^2+y^2+2x+6y-6=0$		
	$x^2+y^2-8x+10y-40=0$		
		$\left(\frac{1}{2},\frac{3}{2}\right)$	2
	$x^2+y^2-5x-12y+30=0$		

Worked Example	Your Turn
The line PQ is the diameter of a circle where P and Q have coordinates $(-3,-2)$ and $(9,-18)$ respectively. Find the equation of the circle, giving your answer in the form $x^2+y^2+ax+by+c=0$	The line CD is the diameter of a circle where C and D have coordinates $(-9,-4)$ and $(-3,-12)$ respectively. Find the equation of the circle, giving your answer in the form $x^2+y^2+ax+by+c=0$

Dr Frost 496j

Fill in the Gaps

Each circle has a diameter AB, a centre C and a radius r

Point A	Point B	Gradient of AB	Equation of AB	Centre C	Radius r	Equation of Circle
$(3, 4)$	$(-3, -4)$				5	$x^2 + y^2 = 25$
$(0, 5)$	$(6, -3)$					
$(4, 0)$				$(2, -1)$		
	$(2, -2)$			$(4, 2)$		
$(-12, 4)$						$(x + 9)^2 + y^2 = 25$
		1		$(1, -1)$	$\sqrt{2}$	
			$y = 3x - 17$	$(4, -5)$	$\sqrt{10}$	
			$y = \frac{3}{4}x - \frac{9}{4}$			$(x + 1)^2 + (y + 3)^2 = 100$

Worked Example	Your Turn
The circle C has the equation $x^2 + y^2 - 22x - 14y + 40 = 0$ Find the coordinates of the points where the circle C crosses the x or y axes.	The circle C has the equation $x^2 + y^2 + 2x - 22y - 48 = 0$ Find the coordinates of the points where the circle C crosses the x or y axes.

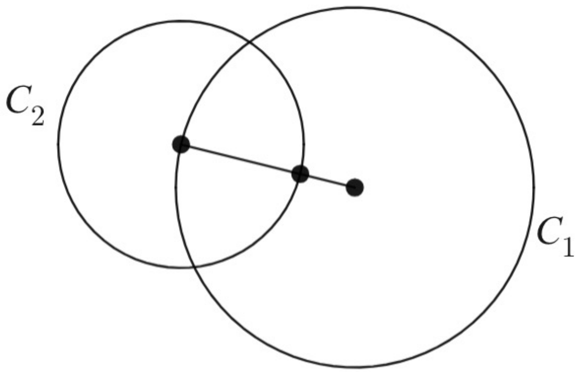
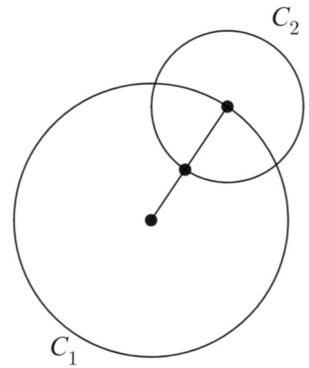
Dr Frost 496h

Fill in the Gaps

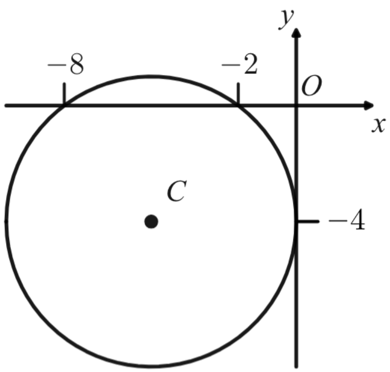
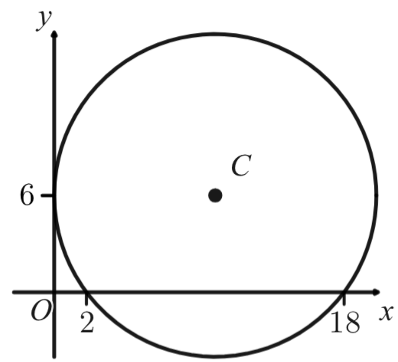
Complete the table. Cells will be filled with either coordinates, values, inequalities, or words.

SKETCH IT.

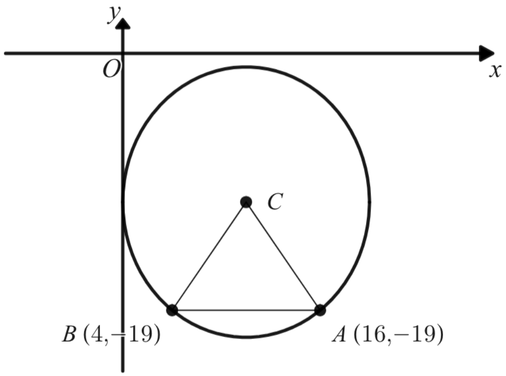
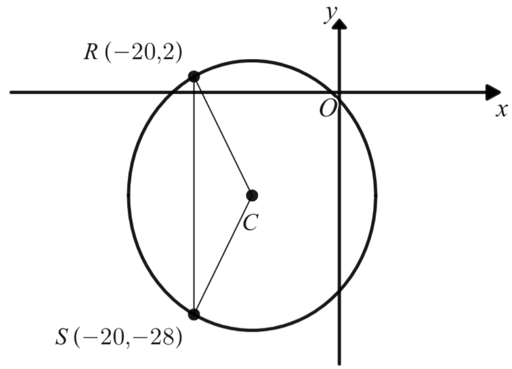
	Circle Equation C	Centre	Radius	Does it cross the x-axis?	Does it cross the y-axis?	Does it cross $y = 2x + 1$?
1)	$(x+2)^2+(y+3)^2=100$					
2)	$(x+2)^2+(y+3)^2=3$					
3)	$(x+2)^2+(y+3)^2=k$			No, but the x-axis is a tangent to C.		
4)	$(x+2)^2+(y+3)^2=k$				No, but the y-axis is a tangent to C.	
5)	$(x+a)^2+(y+b)^2=16$			No, but the x-axis is a tangent to C.	No, but the y-axis is a tangent to C.	
6)	$x^2+y^2=k$					No, but $y=2x+1$ is a tangent to C.
7)	$x^2+y^2=k$					No
8)	$(x+a)^2+(y+b)^2=16$			No, but the x-axis is a a tangent to C.		No, but $y=2x+1$ is a tangent to C.
9)	$x^2+(y+b)^2=16$					No

Worked Example	Your Turn
Circle C_1 has equation $(x-5)^2+(y-3)^2=a$ Circle C_2 has equation $x^2+y^2-2x-8y-25=0$  The centre of C_2 lies on the circumference of C_1 Find the value of a.	Circle C_1 has equation $(x-3)^2+(y-2)^2=h$ Circle C_2 has equation $x^2+y^2-10x-10y-54=0$  The centre of C_2 lies on the circumference of C_1 Find the value of h.

Dr Frost 496i

Worked Example	Your Turn
The diagram shows a circle with centre C. The circle intersects the x-axis at $(-2, 0)$ and $(0, -4)$ The circle touches the y-axis at $(-8, 0)$  Work out the equation of the circle.	The diagram shows a circle with centre C. The circle intersects the x-axis at $(18, 0)$ and $(0, 6)$ The circle touches the y-axis at $(2, 0)$  Work out the equation of the circle.

Dr Frost 496m

Worked Example	Your Turn
The diagram shows a circle, centre C with radius 10. The circle passes through the points $A(16, -19)$ and $B(4, -19)$.  Work out the equation of the circle.	The diagram shows a circle, centre C with radius 17. The circle passes through the points $R(-20, 2)$ and $S(-20, -28)$.  Work out the equation of the circle.

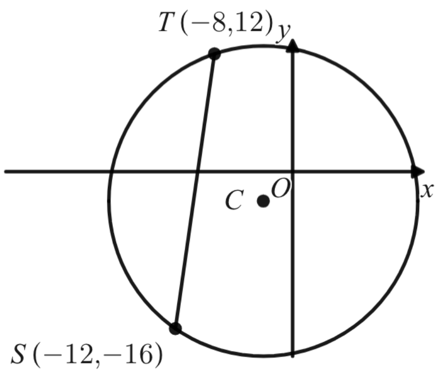
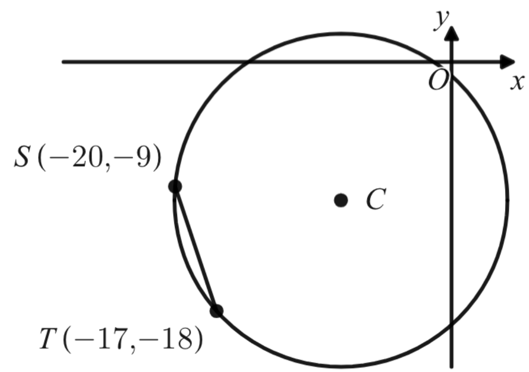
Dr Frost 496n

Worked Example	Your Turn
Circle A has equation $x^2 + y^2 - 8x - 6y - 84 = 0$ Circle B has equation $x^2 + y^2 + 10x + 30y + 184 = 0$ Select the correct statement. • The circles overlap • The circles touch • The circles do not touch or overlap	Circle A has equation $x^2 + y^2 + 40x + 42 + 517 = 0$ Circle B has equation $x^2 + y^2 - 8x - 6y - 169 = 0$ Select the correct statement. • The circles overlap • The circles touch • The circles do not touch or overlap

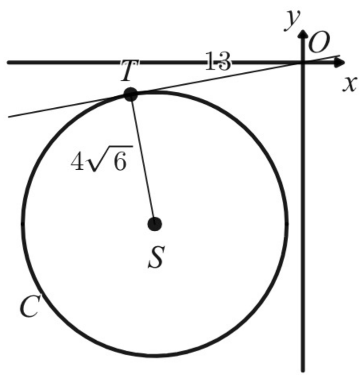
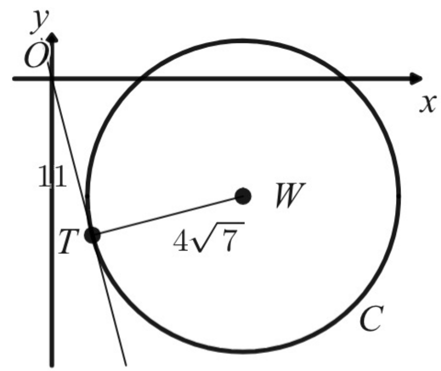
Dr Frost 496r

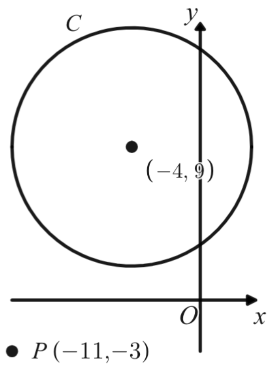
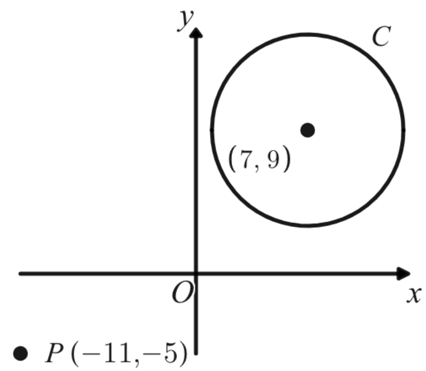
Worked Example	Your Turn
Circle A has equation $x^2 + y^2 + 6x - 8y - 24 = 0$ Circle B has equation $x^2 + y^2 - 30x + 16y + 120 = 0$ Find the exact shortest distance between the two circles.	Find the exact shortest distance between the two circles. Circle A has equation $x^2 + y^2 + 6x + 8y - 56 = 0$ Circle B has equation $x^2 + y^2 - 18x - 24y + 144 = 0$ Find the exact shortest distance between the two circles.

Dr Frost 496q

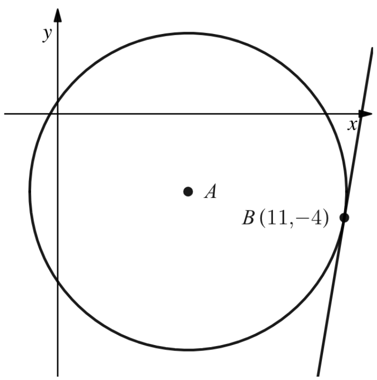
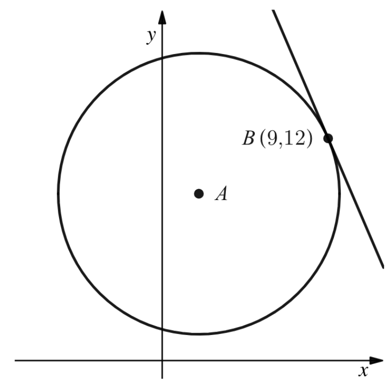
Worked Example	Your Turn
A circle has centre C and equation $(x+3)^2+(y+3)^2=250$. There are two points $S(-12,-16)$ and $T(-8,12)$ which lie on the circle. Find the length of the shortest distance from C to the chord ST.	A circle has centre C and equation $(x+8)^2+(y+10)^2=145$. There are two points $S(-20,-9)$ and $T(-17,-18)$ which lie on the circle. Find the length of the shortest distance from C to the chord ST.
	

Dr Frost 496t

Worked Example	Your Turn
The diagram shows a circle C with centre S and radius $4\sqrt{6}$ and the point T which lies on C. The tangent to C at point T passes through the origin O and $OT = 13$  Given that the coordinates of S are $(m, -12)$, find the value of m.	The diagram shows a circle C with centre W and radius $4\sqrt{7}$ and the point T which lies on C. The tangent to C at point T passes through the origin O and $OT = 11$  Given that the coordinates of W are $(13, k)$, find the value of k.

Worked Example	Your Turn
A circle C has radius 7 and centre $(-4, 9)$, as shown in the figure.  A line through the point $P(-11, -3)$ is a tangent to the circle C at the point T. Find the length of PT.	A circle C has radius 6 and centre $(7, 9)$, as shown in the figure.  A line through the point $P(-11, -5)$ is a tangent to the circle C at the point T. Find the length of PT.

Dr Frost 496s

Worked Example	Your Turn
A circle has equation $x^2 + y^2 - 10x + 6y - 3 = 0$ Find the equation of the tangent to the circle at the point $(11, -4)$. Give your answer in the form $ax + by = c$, where a, b and c are integers to be found.  	A circle has equation $x^2 + y^2 - 4x - 18y + 27 = 0$ Find the equation of the tangent to the circle at the point $(9, 12)$. Give your answer in the form $ax + by = c$, where a, b and c are integers to be found. 

Dr Frost 496p

Extra Notes

3 Advanced Simultaneous Equations

Worked Example	Your Turn
Solve the following pair of simultaneous equations: $y = x^2 + x - 2$ $y = 2x + 4$	Solve the following pair of simultaneous equations: $y = x^2 + 7x - 2$ $y = 2x + 4$

Dr Frost 419b

Worked Example	Your Turn
Solve the following pair of simultaneous equations: $xy = 2$ $y = x + 1$	Solve the following pair of simultaneous equations: $xy = 2$ $y = x - 1$

Worked Example	Your Turn
Solve the following pair of simultaneous equations: $9x^2 - xy - 6 = 0$ $y = 7x - 1$	Solve the following pair of simultaneous equations: $4x^2 + xy - 6 = 0$ $y = 5x - 3$

Dr Frost 419c

Worked Example	Your Turn
Solve the following pair of simultaneous equations: $x^2 + y^2 = 9$ $y = x + 3$	Solve the following pair of simultaneous equations: $x^2 + y^2 = 9$ $y = x - 3$

Dr Frost 419d

Worked Example	Your Turn
Solve the following pair of simultaneous equations: $3x + 4y = 5$ $x^2 + y^2 = 17$	Solve the following pair of simultaneous equations: $4x - 5y = 1$ $x^2 + y^2 = 61$

Dr Frost 419e

Worked Example	Your Turn
Solve: $3y^2 - 2x^2 = 19$ $2y + 3x = 15$	Solve: $2y^2 - 3x^2 = 38$ $3y + 2x = 19$

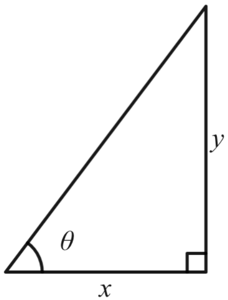
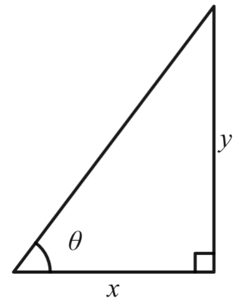
Worked Example	Your Turn
Solve the following pair of simultaneous equations: $\frac{12}{y} - \frac{4}{5x} + 1 = 0$ $y = 2x - 2$	Solve the following pair of simultaneous equations: $\frac{9}{2y} - \frac{3}{xy} = -1$ $y = 2x - 7$

Dr Frost 419f

Fill in the Gaps

Question	State $x =$/ $y =$ substitution	Substitute and rearrange to give quadratic equation	Solve the quadratic equation	Find corresponding y or x values
$y = x^2 - 5x + 3$ $y = 2x - 7$	$y = 2x - 7$	$2x - 7 = x^2 - 5x + 3$ $0 = x^2 - 7x + 10$	$(x-2)(x-5) = 0$ $x = 2$ or $x = 5$	
$x^2 + 2y = 13 - 4x$ $x + y = 5$	$y = 5 - x$	$x^2 + 2(5 - x) = 13 - 4x$ $x^2 + 10 - 2x = 13 - 4x$ $x^2 + 2x - 3 = 0$		
$x^2 + y^2 = 20$ $x - y = 2$	$x = y + 2$			
$y + 10 = x^2 + x$ $x - y - 1 = 0$				
$3x^2 - 2y = 7x - 8$ $3x = y - 2$				
$x^2 + y^2 + xy = 31$ $x + y + 1 = 0$				

Worked Example	Your Turn
A rectangle with length x cm and width y cm, where $x > y$, has a perimeter of 26 cm and an area of 40 cm^2 Find the values of x and y	A rectangle with length x cm and width y cm, where $x > y$, has a perimeter of 20 cm and an area of 24 cm^2 Find the values of x and y

Worked Example	Your Turn
A right-angled triangle is shown below with all lengths in centimetres.  Given that $\tan\theta = \frac{4}{3}$ and the area of the triangle is 24 cm², find the values of x and y	A right-angled triangle is shown below with all lengths in centimetres.  Given that $\tan\theta = \frac{5}{3}$ and the area of the triangle is 120 cm², find the values of x and y

Worked Example	Your Turn
A tennis club has 11 members. There are more men than women at the club. The total number of possible mixed doubles pairings is 5 more than the total number of men-only and women-only doubles pairings. Calculate the number of men and women who are members of the club.	A tennis club has 9 members. There are more women than men at the club. The total number of possible mixed doubles pairings is 8 less than the total number of men-only and women-only doubles pairings. Calculate the number of men and women who are members of the club.

Dr Frost 419g

Extra Notes

4 Advanced Sequences

Geometric Sequences

Worked Example	Your Turn
Generate the first 5 terms of the following geometric sequence: $4 \times 3^{n-1}$	Generate the first 5 terms of the following geometric sequence: $5 \times 4^{n-1}$

Dr Frost 204d

Worked Example	Your Turn
Write down the nth term of the following geometric sequences: a) $4, 12, 36, 108$ b) $4, -12, 36, -108$ c) $108, 36, 12, 4$ d) $\sqrt{7}, 7, 7\sqrt{7}, 49$ e) $3p^4, 6p^4q^4, 12p^4q^8$	Write down the nth term of the following geometric sequences: a) $5, 20, 80, 320$ b) $5, -20, 80, -320$ c) $320, 80, 20, 5$ d) $\sqrt{3}, 3, 3\sqrt{3}, 9$ e) $2x^4, \frac{8x^4}{y^4}, \frac{32x^4}{y^8}$

Dr Frost 370a, 370b, 370c, 370d, 370e, 370f and 370g

Worked Example	Your Turn
The second term of a geometric sequence is 78. The sixth term of the same sequence is $101{,}088$. Calculate the value of the common ratio.	A geometric sequence has second and fifth terms 108 and 4, respectively. Calculate the value of the common ratio.

Worked Example	Your Turn
A geometric sequence has first term $(x-3)$, second term $(x+1)$ and third term $(4x-2)$. Find the two possible values of x.	The first three terms of a geometric sequence are $4p, (3\mathrm{p}+15)$ and $(5p+20)$ respectively, where p is a positive constant. Find the value of p.

Quadratic Sequences

Worked Example	Your Turn
Generate the first 5 terms of the following quadratic sequence: $3n^2 + 2n - 5$	Generate the first 5 terms of the following quadratic sequence: $3n^2 - 2n + 5$

Dr Frost 204c

Worked Example	Your Turn
By comparison with the first four terms of the sequence n^2, find the nth term of the second sequence:	By comparison with the first four terms of the sequence n^2, find the nth term of the second sequence:

Worked Example

a)

First four terms				nth term
1	4	9	16	n^2
7	28	63	112	____

b)

First four terms				nth term
1	4	9	16	n^2
-1	2	7	14	____

Your Turn

a)

First four terms				nth term
1	4	9	16	n^2
4	7	12	19	____

b)

First four terms				nth term
1	4	9	16	n^2
-7	-28	-63	-112	____

Worked Example	Your Turn
Find the n^{th} term of the following sequence: 0, 11, 28, 51, 80	Find the n^{th} term of the following sequence: 6, 13, 26, 45, 70

Dr Frost 369h, 369b, 369c, 369d, 369e, 369f and 369g

Worked Example	Your Turn
Here are the first five terms of a quadratic sequence $6, -4, -22, -48, -82$ Find an expression, in terms of n, for the nth term of the sequence.	Here are the first five terms of a quadratic sequence $-14, -25, -38, -53, -70$ Find an expression, in terms of n, for the nth term of the sequence.

Dr Frost 369i

Worked Example	Your Turn
The nth term of a sequence is given by $an^2 + bn + c$ The second term is 23, the fourth term is 57 and the sixth term is 107. Find the values of a, b and c.	The nth term of a sequence is given by $an^2 + bn + c$ The fourth term is 34, the seventh term is 124 and the eleventh term is 328. Find the values of a, b and c.

Dr Frost 369j

Worked Example	Your Turn
A quadratic sequence has an nth term of $-3n^2 + 2n - 2$ A term in this sequence is equal to -343. Find the position of this term.	A sequence has an nth term of $-2n^2 - 5n + 1$ A term in this sequence is equal to -816. Find the position of this term.

Dr Frost 369k

Worked Example	Your Turn
Here are the first five terms of a sequence. $-11, -14, -13, -81$ An expression for the nth term of this sequence is $2n^2 - 9n - 4$. Find an expression for the nth term of a sequence whose first five terms are $-99, -126, -117, -729$	Here are the first five terms of a sequence. $-8, -5, 2, 13,28$ An expression for the nth term of this sequence is $2n^2 - 3n - 7$. Find an expression for the nth term of a sequence whose first five terms are $56, 35, -14, -91, -196$

Dr Frost 369l

Fill in the Gaps

Sequence	Type	n^{th} term	10^{th} term	11^{th} term	12^{th} term	30^{th} term	Is 60 in the sequence?
8, 11, 14, 17, …							
4, 11, 20, 31, …							
			67	74	81		
-4, -10, -16, -22, …							
0, 11, 28, 51, …							
		$n^2 + 12n - 4$					
3, 7, 15, 27, …							
		$4n - 8$					
		$4n^2 + n$					
-3, 0, 5, 12, …							
	Linear		56		66		
	Linear				70	178	

Fill in the Gaps

	u_1	u_2	u_3	u_4	...	u_k		
eg	-1	2	5	8	...	71	$u_n =$	$k =$
(a)	12	17	22	27	...	162	$u_n =$	$k =$
(b)	8	6	4	2	...	-96	$u_n =$	$k =$
(c)	3.5	6.5	9.5	12.5	...	123.5	$u_n =$	$k =$
(d)	-0.3	-0.1	0.1	0.3	...	2.7	$u_n =$	$k =$
(e)					...	430	$u_n = 8n - 2$	$k =$
(f)					...	-7.5	$u_n = 4 - 0.5n$	$k =$
(g)					...	$8\frac{1}{8}$	$u_n = \frac{1}{4}n + \frac{3}{8}$	$k =$
(h)					...	-194	$u_n = -3n - 2$	$k =$
(i)	5.8		7.4		...	20.2	$u_n =$	$k =$
(j)		-5.2		-15.6	...	-130	$u_n =$	$k =$

Extra Notes

5 Limiting Values of Sequences (L2FM Only)

Worked Example	Your Turn
As $n \to \infty$: a) $\frac{5}{n} \to$ b) $\frac{3n^2}{n} \to$ c) $\frac{4n}{3n+1} \to$	As $n \to \infty$: a) $\frac{8}{n} \to$ b) $\frac{n^2}{n} \to$ c) $\frac{7n}{2n+1} \to$

Worked Example	Your Turn
The nth term of a sequence is $\frac{5n-1}{2n-4}$ Write down the limiting value of the sequence as $n \to \infty$	The nth term of a sequence is $\frac{4n+6}{5n+3}$ Write down the limiting value of the sequence as $n \to \infty$

Dr Frost 510e

Worked Example	Your Turn
The nth term of a sequence is $\frac{6n^2+n+5}{n^2+4n-5}$ Write down the limiting value of the sequence as $n \to \infty$	The nth term of a sequence is $\frac{3n^2+4n+6}{2n^2+5n+3}$ Write down the limiting value of the sequence as $n \to \infty$

Dr Frost 510f

Worked Example	Your Turn
a) Find the n[th] term of the sequence $9, \frac{13}{6}, \frac{17}{11}, \frac{21}{16}, \ldots$ b) Hence find the limit of the sequence as $n \to \infty$	a) Find the n[th] term of the sequence $\frac{1}{5}, \frac{4}{7}, \frac{7}{9}, \frac{10}{11}, \ldots$ b) Hence find the limit of the sequence as $n \to \infty$

Extra Notes

6 Algebraic Proof

Fluency Practice

Forming Expressions	If n is any integer, how can we form algebraic expressions to describe these types, sequences, sums & products of numbers?
A number	
An even number	
An odd number	
Two consecutive numbers	
Two consecutive even numbers	
Two consecutive odd numbers	
The sum of two consecutive numbers	
The sum of two consecutive even numbers	
The sum of two consecutive odd numbers	
A number squared	
The square of an even number	
The square of an odd number	
The product of two consecutive numbers	
The product of two consecutive even numbers	
The product of two consecutive odd numbers	

Worked Example	Your Turn
A number is given as $5n - 8$ where n is an integer. Write down the expression for the next consecutive integer.	A number is given as $-2n + 13$ where n is an integer. Write down the expression for the next consecutive integer.

Dr Frost 454h

Worked Example	Your Turn
a) An odd number is given as $-2n+13$ where n is an integer. Write down the expression for the next consecutive odd number. b) An even number is given as $-2n-4$ where n is an integer. Write down the expression for the next consecutive even number.	a) An even number is given as $-2n+14$ where n is an integer. Write down the expression for the next consecutive even number. b) An odd number is given as $-2n-9$ where n is an integer. Write down the expression for the next consecutive odd number.

Dr Frost 454k

Worked Example	Your Turn
a) Given that n is an integer. Prove that $(2n-5)(4n-9)-10$ is always an odd number. b) Given that n is an integer. Prove that $(4n-3)^2+9$ is always an even number.	a) Given that n is an integer. Prove that $(4n+9)(4n-1)-3$ is always an even number. b) Given that n is an integer. Prove that $(2n-7)^2+2$ is always an odd number.

Dr Frost 454d and 454c

Worked Example	Your Turn
Given that n is a positive integer. Prove that $(2m+3)^2-(2m+2)^2-1$ is always divisible by 4.	Given that n is a positive integer. Prove that $(2m+2)^2-(2m-4)^2-12$ is always divisible by 24.

Dr Frost 454f

Worked Example	Your Turn
Prove that $(2x-7)(4x-7)-(2x-2)(2x-7)+1$ is a perfect square.	Prove that $(4y-7)(5y-1)-(2y+3)(2y-3)+7y$ is a perfect square.

Dr Frost 454q

Worked Example	Your Turn
Prove algebraically that the sum of any four consecutive integers is not divisible by 4.	Prove algebraically that the sum of any six consecutive integers is divisible by 3.

Dr Frost 454i

Worked Example	Your Turn
Prove algebraically that the sum of the squares of any three consecutive integers is always two more than a multiple of 3.	Prove algebraically that the sum of the squares of any four consecutive integers is always two more than a multiple of 4.

Dr Frost 454j

Worked Example	Your Turn
Prove algebraically that the sum of four consecutive even integers is always divisible by 4.	Prove algebraically that the sum of three consecutive odd integers is always divisible by 3.

Dr Frost 454l

Worked Example	Your Turn
a) Prove algebraically that the sum of the squares of two consecutive even integers is always divisible by 4. b) Prove algebraically that the sum of the squares of two consecutive odd integers is always 2 more than a multiple of 4.	a) Prove algebraically that the sum of the squares of three consecutive odd integers is always 1 less than a multiple of 12. b) Prove algebraically that the sum of the squares of three consecutive even integers always has a remainder of 8 when divided by 12.

Worked Example	Your Turn
a) Prove algebraically that the sum of any two odd integers is always even. b) Prove algebraically that the difference of any two even integers is always even.	a) Prove algebraically that the sum of any two even integers is always even. b) Prove algebraically that the difference of any two odd integers is always even.

Dr Frost 454o

Worked Example	Your Turn
Given that the difference of the squares of 2 consecutive even integers equals 68, find the value of the smallest integer.	Given that the difference of the squares of 2 consecutive odd integers equals 112, find the value of the smallest integer.

Worked Example	Your Turn
A sequence has the nth term $n^2 - 6n + 10$. By completing the square, show that every term is positive.	A sequence has the nth term $n^2 - 10n + 27$. By completing the square, show that every term is positive.

Worked Example	Your Turn
Show that for any integer n, $n^2 + n$ is always even.	Prove that $n(n-1)+1$ is odd for all integers n.

Worked Example	Your Turn
I think of a two-digit number. I then reverse the digits. Prove that the difference between the two numbers is a multiple of 9.	Prove that the sum of a four-digit number and its reverse is a multiple of 11.

Worked Example	Your Turn
Given that $4bx - 3a + 7 - 10ax \equiv -30x - 8$ Find the values of a and b.	Given that $ax + 5b - 8ax + 4bx \equiv -23x + 15$ Find the values of a and b.

Worked Example	Your Turn
Given that $3(4px - q) + 5(px + 3q) \equiv 68x - 60$ Find the values of p and q.	Given that $5(4ax + 3b) - 2(3ax + 2b) \equiv -84x + 66$ Find the values of a and b.

Dr Frost 455b

Worked Example	Your Turn
Given that $2x^2 - 4x + p \equiv (x-3)(qx+r) + 1$ where p, q and r are integers, find the values of p, q, and r	Given that $(x+3)(lx+m) + 9 \equiv 3x^2 + 5x + k$ where k, l and m are integers, find the values of k, l, and m

Dr Frost 455c

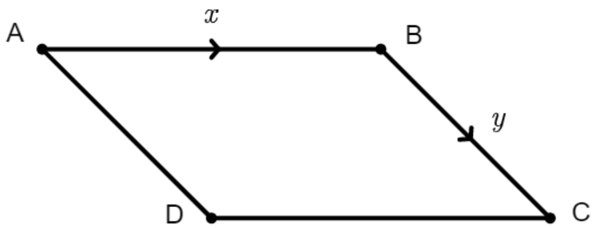
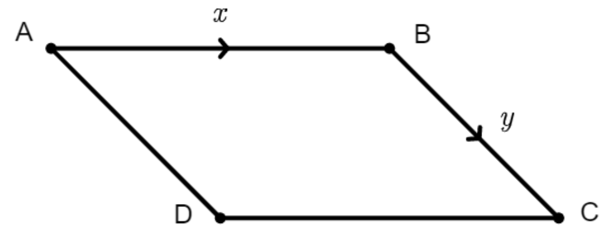
Worked Example	Your Turn
Given that $(2y-1)^2 + ay + 7 = (2y+b)(2y+4)$ where a and b are integers, find the value of a and the value of b.	Given that $(2x+1)^2 - 12x + r = (2x+s)(2x-2)$ where r and s are integers, find the value of r and the value of s.

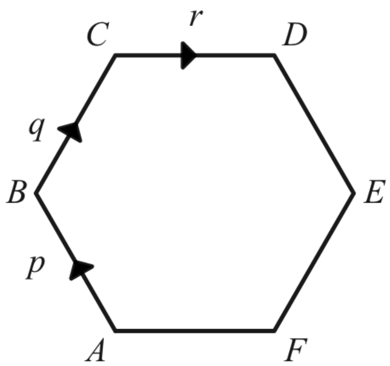
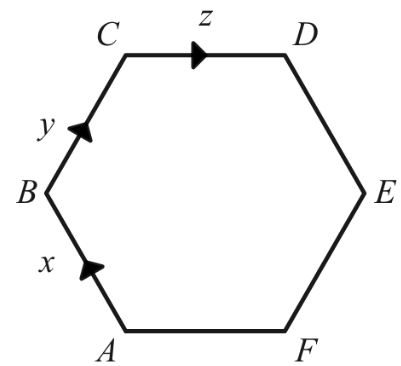
Dr Frost 455d

Worked Example	Your Turn
$3x^2 - 3bx + 16a \equiv 3(x-a)^2 + 5$ Work out the two possible pairs of values of a and b	$2x^2 - 2bx + 7a \equiv 2(x-a)^2 + 3$ Work out the two possible pairs of values of a and b

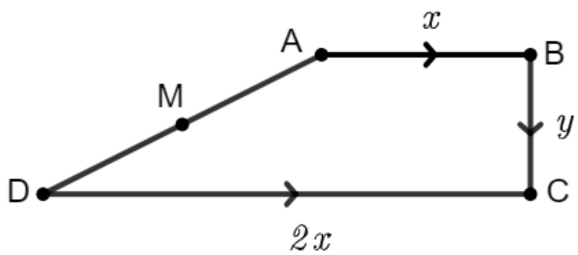
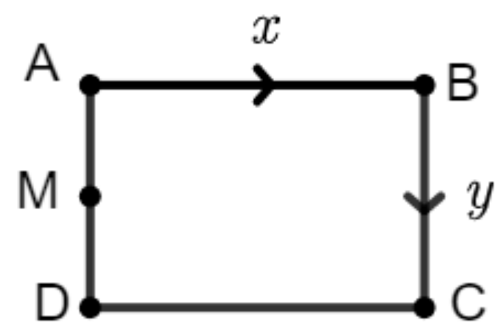
Extra Notes

7 Advanced Vectors

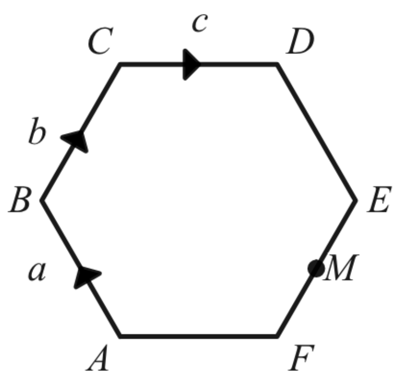
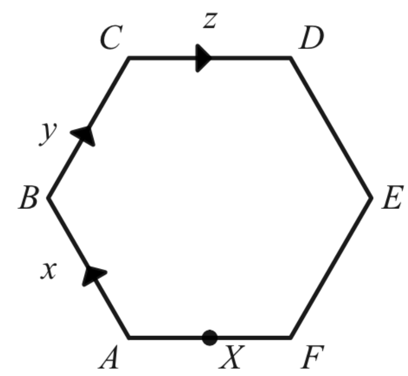
Worked Example	Your Turn
ABCD is a parallelogram. Express $\overrightarrow{DB}$ in terms of x and y. 	ABCD is a parallelogram. Express $\overrightarrow{CA}$ in terms of x and y. 

Worked Example	Your Turn
Express $\overrightarrow{DF}$ in terms of p, q and r. 	Express $\overrightarrow{BF}$ in terms of x, y and z. 

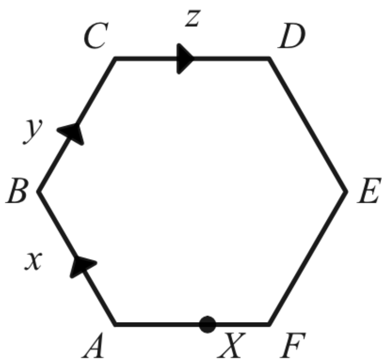
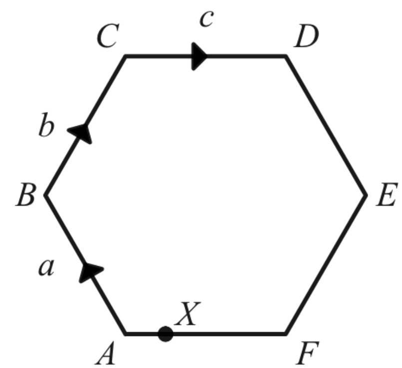
Dr Frost 473a

Worked Example	Your Turn
ABCD is a trapezium. M is the midpoint of AD. Find $\overrightarrow{MA}$ in terms of x and y.  	ABCD is a rectangle. M is the midpoint of AD. Find $\overrightarrow{MA}$ in terms of x and y. 

Dr Frost 419d

Worked Example	Your Turn
The point M is the midpoint of EF. Express $\overrightarrow{DM}$ in terms of a, b and c. 	The point x is the midpoint of FA. Express $\overrightarrow{EX}$ in terms of x, y and z. 

Dr Frost 473b

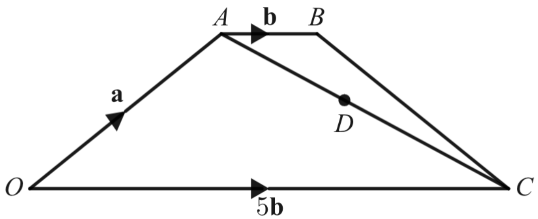
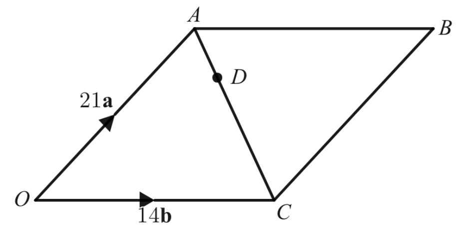
Worked Example	Your Turn
The point X shares the line segment FA in the ratio $2 : 3$. Express $\overrightarrow{EX}$ in terms of x, y and z.  	The point X shares the line segment FA in the ratio $3 : 1$. Express $\overrightarrow{CX}$ in terms of a, b and c. 

Dr Frost 473c

Fill in the Gaps

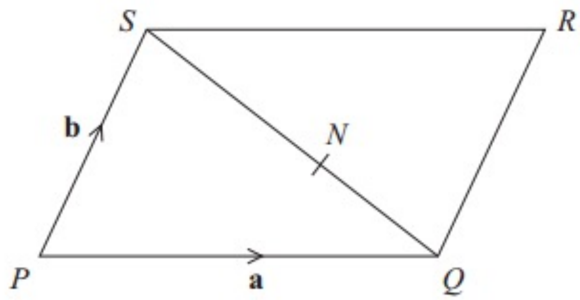
Point X divides the vector $\overrightarrow{AB}$ in the ratio given to create vectors $\overrightarrow{AX}$ and $\overrightarrow{XB}$.

$\overrightarrow{AB}$	Ratio $AX : XB$	$\overrightarrow{AX}$	$\overrightarrow{XB}$
$3\boldsymbol{a}$	$1 : 2$	$\boldsymbol{a}$	$2\boldsymbol{a}$
$3\boldsymbol{a} + 3\boldsymbol{b}$	$2 : 1$	$2\boldsymbol{a} + 2\boldsymbol{b}$	
$4\boldsymbol{a} - 4\boldsymbol{b}$	$3 : 1$		
$5\boldsymbol{a} + 10\boldsymbol{b}$	$3 : 2$		
$10\boldsymbol{a} - 15\boldsymbol{b}$	$1 : 4$		
$\boldsymbol{a}$	$2 : 1$	$\frac{2}{3}\boldsymbol{a}$	
$\boldsymbol{a} + \boldsymbol{b}$	$1 : 2$		$\frac{2}{3}\boldsymbol{a} + \frac{2}{3}\boldsymbol{b}$
$\boldsymbol{a} - \boldsymbol{b}$	$3 : 1$		
$2\boldsymbol{a} + \boldsymbol{b}$	$4 : 1$		
$\boldsymbol{a} - 4\boldsymbol{b}$	$3 : 2$		
	$1 : 3$	$\frac{1}{4}\boldsymbol{a} - \frac{1}{4}\boldsymbol{b}$	
$2\boldsymbol{a} - 3\boldsymbol{b}$			$\frac{4}{3}\boldsymbol{a} - 2\boldsymbol{b}$
		$\frac{6}{5}\boldsymbol{a} + \frac{3}{10}\boldsymbol{b}$	$\frac{4}{5}\boldsymbol{a} + \frac{1}{5}\boldsymbol{b}$

Worked Example	Your Turn
$OABC$ is a trapezium $\overrightarrow{OA} = a$ $\overrightarrow{AB} = b$ $\overrightarrow{OB} = 5b$ D is the point on AC such that $AD : DC = 3 : 4$  Find, in terms of a and b, the vector $\overrightarrow{OD}$	$OABC$ is a parallelogram $\overrightarrow{OA} = 21a$ $\overrightarrow{OB} = 14b$ D is the point on AC such that $AD : DC = 2 : 5$  Find, in terms of a and b, the vector $\overrightarrow{DB}$

Dr Frost 473e

Worked Example



PQRS is a parallelogram.
N is the point on *SQ* such that $SN : NQ = 3 : 2$

$\overrightarrow{PQ} = \mathbf{a}$ $\overrightarrow{PS} = \mathbf{b}$

(a) Write down, in terms of **a** and **b**, an expression for $\overrightarrow{SQ}$.

(b) Express $\overrightarrow{NR}$ in terms of **a** and **b**.

Your Turn

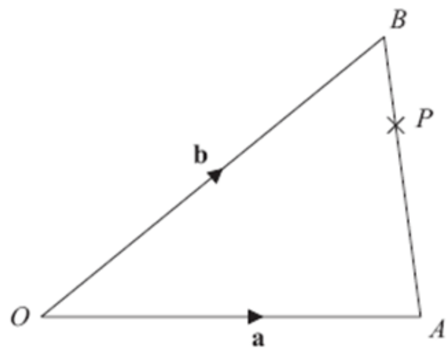


Diagram **NOT** accurately drawn

OAB is a triangle.

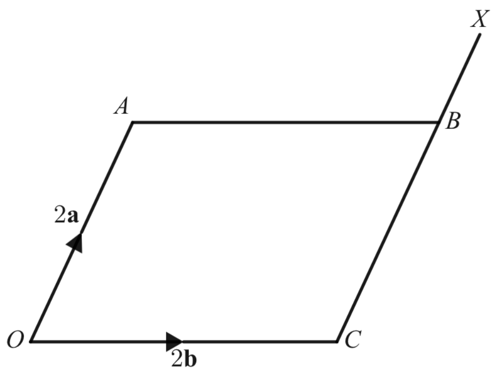
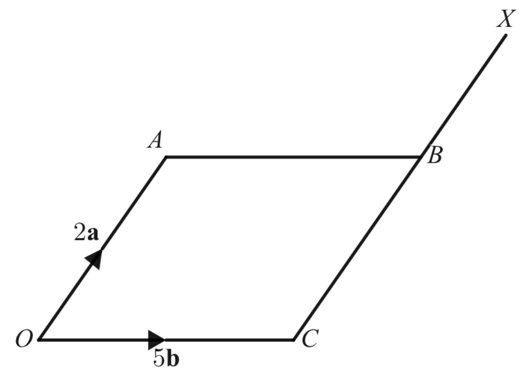
$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

....
(1)

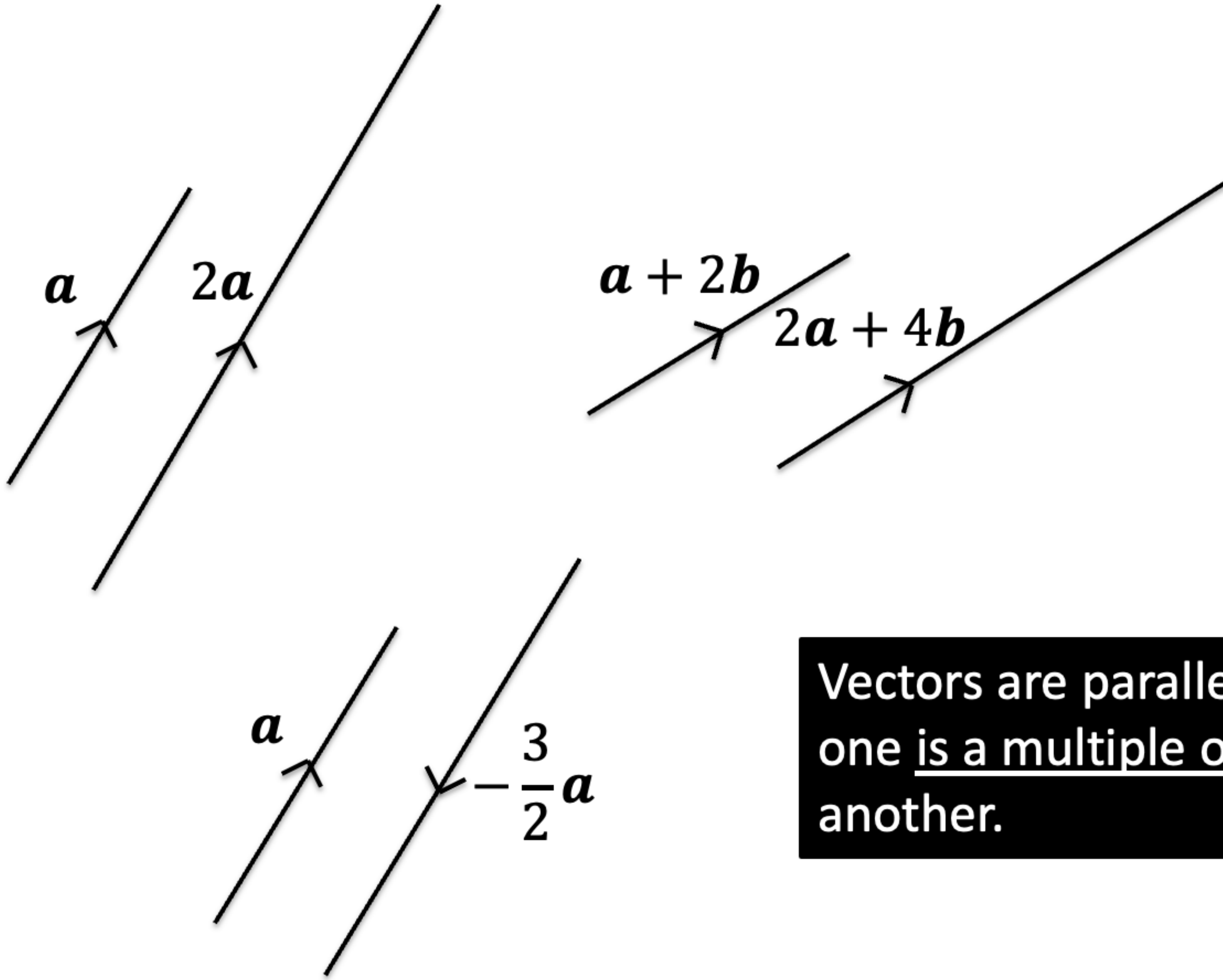
P is the point on *AB* such that $AP : PB = 3 : 1$

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

Worked Example	Your Turn
$OABC$ is a parallelogram $\overrightarrow{OA} = 2a$ $\overrightarrow{OB} = 2b$ CBX is a straight line $CB : BX = 5 : 2$ The point Y is such that $\overrightarrow{CY} = 5\overrightarrow{AX}$  Find, in terms of a and b, the vector $\overrightarrow{OY}$	$OABC$ is a parallelogram $\overrightarrow{OA} = 2a$ $\overrightarrow{OB} = 5b$ CBX is a straight line $CB : BX = 3 : 2$ The point Y is such that $\overrightarrow{CY} = 4\overrightarrow{AX}$  Find, in terms of a and b, the vector $\overrightarrow{OY}$

Dr Frost 473f

Parallel Vectors



Vectors are parallel if one <u>is a multiple of</u> another.

Parallel Vectors

Two vectors are parallel if they are ***multiples*** of each other.

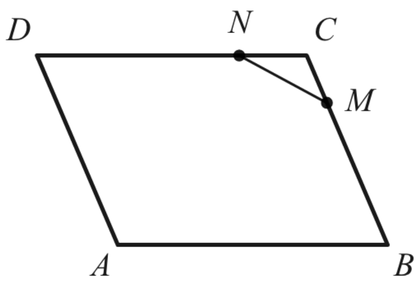
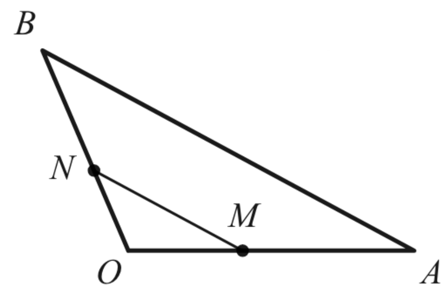
Vector 1	Vector 2	Parallel?	
a	-**a**	Yes	No
a + **b**	2**a** + 2**b**	Yes	No
a + **b**	**a** + 2**b**	Yes	No
$\frac{1}{2}$**a** + **b**	**a** + 2**b**	Yes	No
2**a** + 5**b**	4**a** + 10**b**	Yes	No
a + **b**	**a** - **b**	Yes	No
a + **b**	-**a** - **b**	Yes	No
a - **b**	-**a** + **b**	Yes	No
2**a** + 3**b**	$\frac{2}{3}$**a** + **b**	Yes	No

Dr Frost 474a

Worked Example	Your Turn
a) The vectors $-3a - b$ and $6a + nb$ are parallel. Given that a and b are not parallel, find the value of n b) Vector $p = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and vector $q = \begin{pmatrix} n \\ 7 \end{pmatrix}$ Vector $3p + q$ is parallel to $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ Find the value of n	a) The vectors $12p - 12q$ and $-6p + dq$ are parallel. Given that p and q are not parallel, find the value of d b) Vector $a = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ and vector $b = \begin{pmatrix} l \\ 6 \end{pmatrix}$ Vector $2a + b$ is parallel to $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ Find the value of l

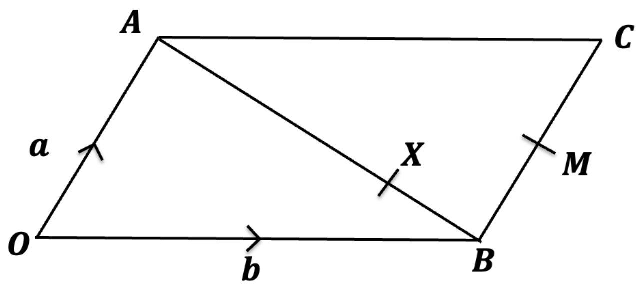
Worked Example	Your Turn
D, E and F are three points on a straight line such that $\overrightarrow{DE} = 6x - 3y$ $\overrightarrow{EF} = 8x - 4y$ Find the ratio length of EF : length of DE Give your answer in its simplest form.	U, V and W are three points on a straight line such that $\overrightarrow{UV} = 7a - 3b$ $\overrightarrow{VW} = \frac{35}{2}a - \frac{15}{2}b$ Find the ratio length of UV : length of UW Give your answer in its simplest form.

Dr Frost 474c

Worked Example	**Your Turn**
The diagram shows parallelogram $ABCD$  $\overrightarrow{AB} = a$ $\overrightarrow{AC} = b$ $BM : MC = 3 : 1$ $DN : NC = 3 : 1$ Use a vector method to prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{BD}$	The diagram shows triangle OAB  $\overrightarrow{OA} = a$ $\overrightarrow{OB} = b$ $OM : MA = 2 : 3$ $ON : NB = 2 : 3$ Use a vector method to prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{AB}$

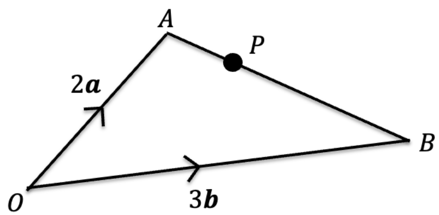
Dr Frost 474d

Worked Example



X is a point on AB such that $AX : XB = 3 : 1$. M is the midpoint of BC.
Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

Your Turn



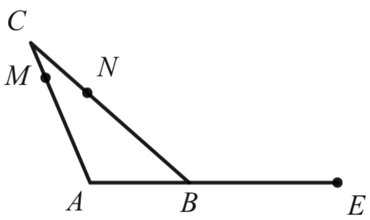
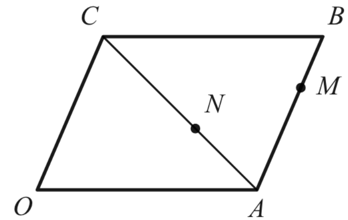
a) Find $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

b) P is the point on AB such that $AP:PB = 2:3$.
Show that $\overrightarrow{OP}$ is parallel to the vector $\boldsymbol{a} + \boldsymbol{b}$.

Worked Example	Your Turn
The quadrilateral $OABC$ has vertices O, A, B, C where $\overrightarrow{OA} = 2a + b$, $\overrightarrow{OB} = 3b$ and $\overrightarrow{OC} = -4a + b$ By showing that $\overrightarrow{OA}$ and $\overrightarrow{BC}$ are parallel, prove that the quadrilateral $OABC$ is a trapezium.	The quadrilateral $OABC$ has vertices O, A, B, C where $\overrightarrow{OA} = a + b$, $\overrightarrow{OB} = 2b$ and $\overrightarrow{OC} = -3a - b$ By showing that $\overrightarrow{OA}$ and $\overrightarrow{BC}$ are parallel, prove that the quadrilateral $OABC$ is a trapezium.

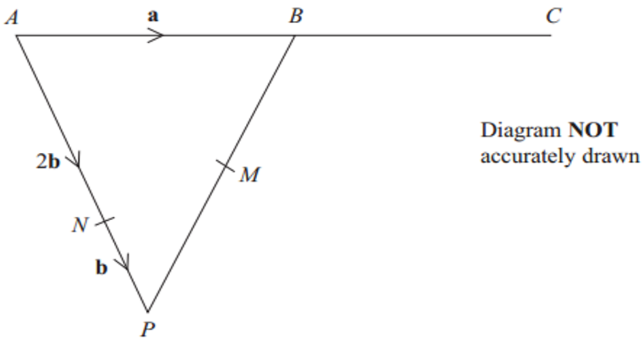
Dr Frost 474e

Straight Lines

Worked Example	Your Turn
The diagram shows triangle ABC The point E lies on the straight line through A and B  $\overrightarrow{AB} = a$ $\overrightarrow{AC} = b$ $\overrightarrow{AE} = \frac{5}{2}a$ $AM : MC = 3 : 1$ $BN : NC = 9 : 5$ Use a vector method to prove that M, N and E are collinear.	The diagram shows parallelogram OABC  $\overrightarrow{OA} = 5a$ $\overrightarrow{OC} = 5b$ $AM : MB = 2 : 1$ $AN : NC = 2 : 3$ Use a vector method to prove that O, N and M are collinear.

Dr Frost 474f

Worked Example



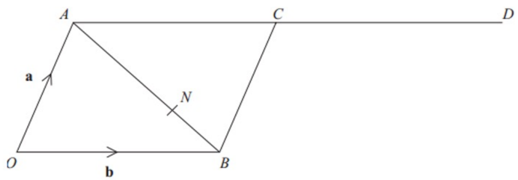
$$\overrightarrow{AN} = 2\boldsymbol{b}, \qquad \overrightarrow{NP} = \boldsymbol{b}$$

B is the midpoint of AC. M is the midpoint of PB.

a) Find $\overrightarrow{PB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

b) Show that NMC is a straight line.

Your Turn



$\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$
D is the point such that $\overrightarrow{AC} = \overrightarrow{CD}$
The point N divides AB in the ratio 2: 1.

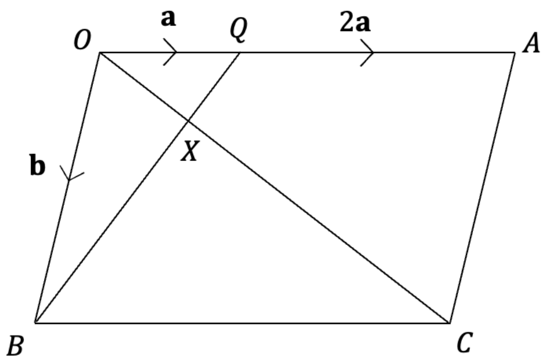
(a) Write an expression for $\overrightarrow{ON}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(b) Prove that OND is a straight line.

Vector Proofs

Worked Example

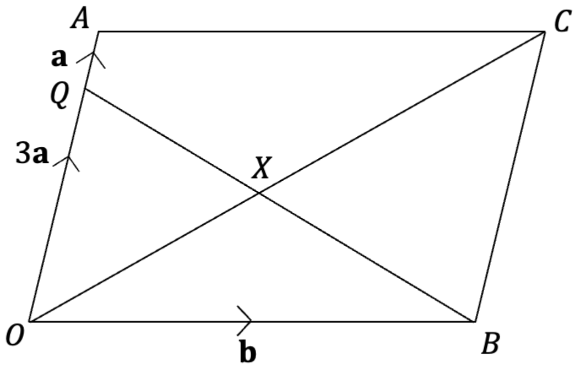
$OACB$ is a parallelogram. Given that OXC and BXQ are straight lines, determine the ratio $OX : XC$.



Dr Frost 475a

Your Turn

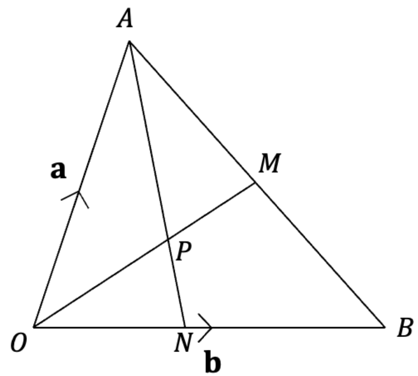
$OACB$ is a parallelogram. Given that OXC and BXQ are straight lines, determine the ratio $OX : XC$.



Worked Example

OAB is a triangle.
OPM and APN are straight lines.
M is the midpoint of AB.

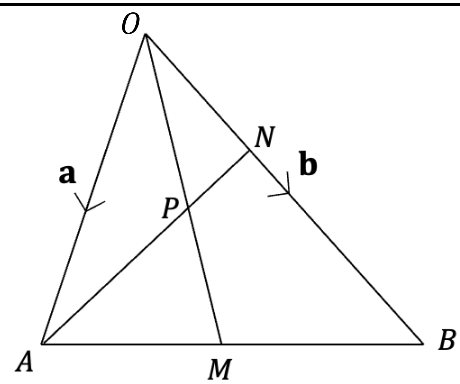
$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$
$OP : PM = 3 : 2$
Work out the ratio $ON : NB$



Your Turn

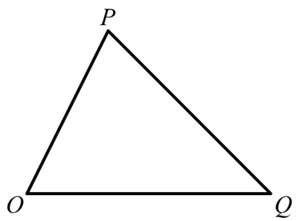
OAB is a triangle.
OPN and APN are straight lines.
M is the midpoint of OB.

$\overrightarrow{OA} = \mathbf{a}$
$\overrightarrow{OB} = \mathbf{b}$
$OP : PM = 5 : 3$
Work out the ratio $ON : NB$



Worked Example

The diagram below shows a sketch of triangle OPQ



The point R is such that $OP : PR = 1 : 2$
The point M is such that $PM : MQ = 2 : 3$

The straight line through R to M cuts OQ at the point N

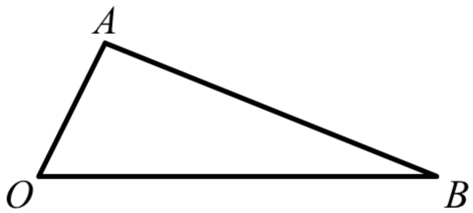
Let $\overrightarrow{OP} = a$ and $\overrightarrow{OQ} = b$

By first finding $\overrightarrow{RM}$ in terms of a and b, and letting $\overrightarrow{RN} = \lambda\overrightarrow{RM}$, find $ON : NQ$

Dr Frost 475c

Your Turn

The diagram below shows a sketch of triangle OAB



The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$
The point M is such that $AM : MB = 3 : 2$

The straight line through C to M cuts OB at the point N

Let $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$

By first finding $\overrightarrow{CM}$ in terms of a and b, and letting $\overrightarrow{CN} = \lambda\overrightarrow{CM}$, find $ON : NB$

Extra Notes