



KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS



KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM

# Year 11

## 2025

# Mathematics (L2FM)

## 2026

# Unit 25 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_



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# Unit 25 Booklet – Part 2

HGS Maths



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Name: \_\_\_\_\_

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## Contents

- 1 [Exponential and Trigonometric Graphs](#)
- 2 [Trigonometric Identities and Equations \(L2FM Only\)](#)
- 3 [Domain and Range \(L2FM Only\)](#)
- 4 [Piecewise Functions \(L2FM Only\)](#)
- 5 [Graph Transformations](#)
- 6 [Congruence and Similarity Proofs](#)
- 7 [Circle Theorem Proofs](#)

# 1 Exponential and Trigonometric Graphs

### Worked Example

- a) The equation of the curve is  $y = 1.25^x$   
 $M$  is the point where the curve intercepts the  $y$ -axis.  
State the coordinates of  $M$
- b) The equation of the curve is  $y = -\frac{1}{2} \times 5^x$   
 $Y$  is the point where the curve intercepts the  $y$ -axis.  
State the coordinates of  $Y$

### Your Turn

- a) The equation of the curve is  $y = 0.25^x$   
 $M$  is the point where the curve intercepts the  $y$ -axis.  
State the coordinates of  $M$
- b) The equation of the curve is  $y = 7 \times \left(\frac{1}{2}\right)^x$   
 $K$  is the point where the curve intercepts the  $y$ -axis.  
State the coordinates of  $K$

### Worked Example

Some money  $M$  has been invested in a bank. The value of the money after  $t$  years is modelled by the function

$$M(t) = 1750 \times (1.02)^t$$

State the initial amount of money invested.

### Your Turn

Some money  $M$  has been invested in a bank. The value of the money after  $t$  years is modelled by the function

$$M(t) = 3000 \times (1.005)^t$$

State the initial amount of money invested.

### Worked Example

Some money  $M$  has been invested in a bank. The value of the money after  $t$  years is modelled by the function  
 $M(t) = 500 \times (1.04)^t$   
Determine the interest rate offered by the bank.

### Your Turn

Some money  $M$  has been invested in a bank. The value of the money after  $t$  years is modelled by the function  
 $M(t) = 1250 \times (1.025)^t$   
Determine the interest rate offered by the bank.

### Worked Example

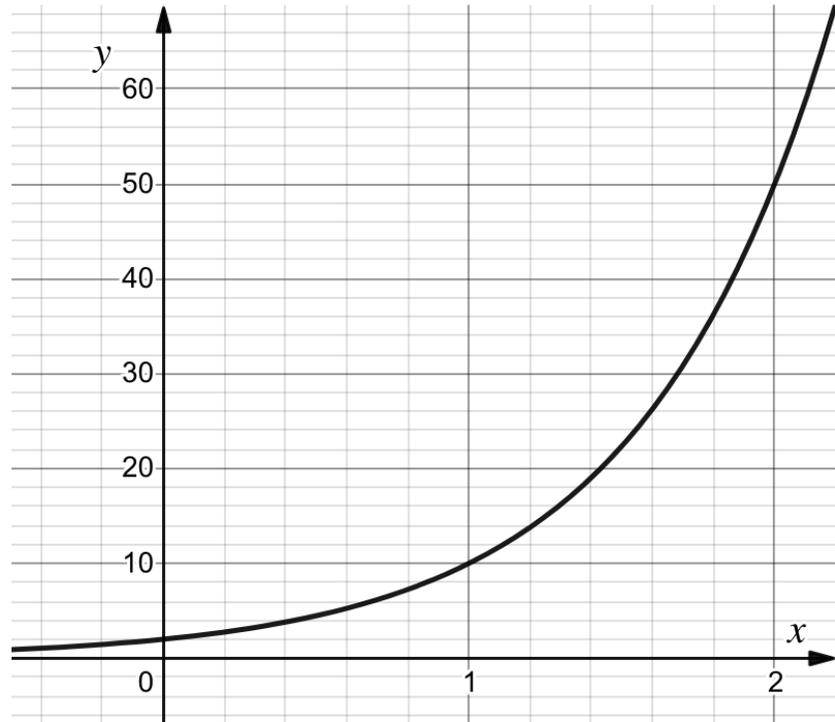
The sketch graph shows a curve with equation  $y = ab^x$   
The curve passes through the points  $(0, 3.25)$  and  $(3, 87.75)$ .  
Calculate the value of  $a$  and the value of  $b$ .

### Your Turn

The sketch graph shows a curve with equation  $y = ab^x$   
The curve passes through the points  $(0, 2.75)$  and  $(2, 68.75)$ .  
Calculate the value of  $a$  and the value of  $b$ .

## Worked Example

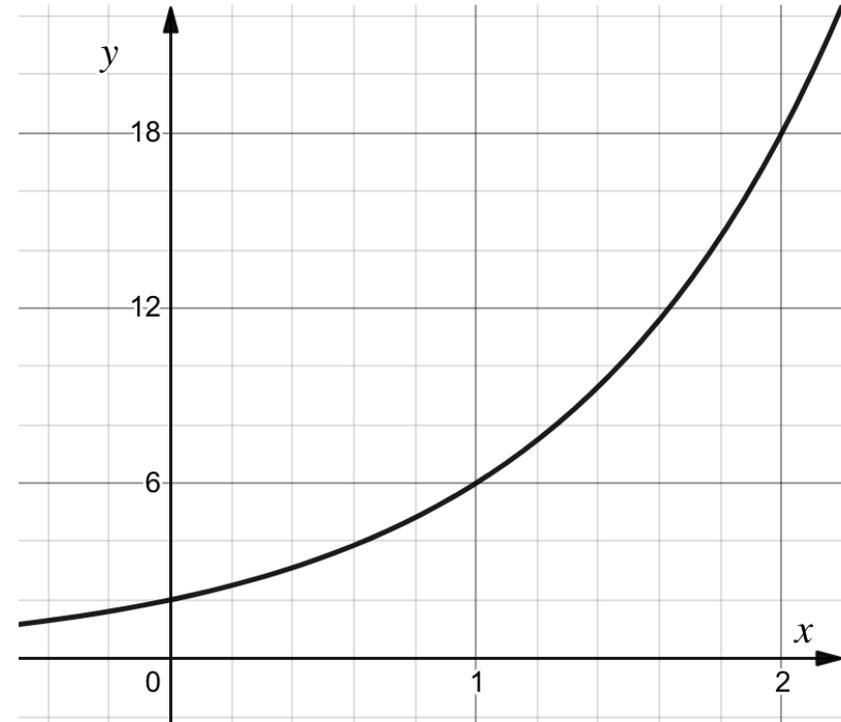
This graph shows a curve with equation  $y = pq^x$



Calculate the value of  $p$  and  $q$ .

## Your Turn

This graph shows a curve with equation  $y = pq^x$



Calculate the value of  $p$  and  $q$ .

### Worked Example

At the start of an experiment, a petri dish contained 4,000,000 bacteria. After 4 days, there were 6,000,000 bacteria. It is assumed that the number of bacteria is given by the formula  $N = ar^t$  where  $N$  is the number of bacteria,  $t$  days after the start of the experiment. Calculate the number of bacteria 7 days after the start of the experiment, giving your answer to 3 significant figures.

### Your Turn

At the start of an experiment, a petri dish contained 4,000,000 bacteria. After 5 days, there were 13,000,000 bacteria. It is assumed that the number of bacteria is given by the formula  $N = ar^t$  where  $N$  is the number of bacteria,  $t$  days after the start of the experiment. Calculate the number of bacteria 11 days after the start of the experiment, giving your answer to 3 significant figures.

## Trigonometric Graphs

Angle ( $\theta$ Degrees)	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$180^\circ$	$270^\circ$	$360^\circ$
$\sin(\theta)$								
$\cos(\theta)$								
$\tan(\theta)$								

## Worked Example

Sketch the graph  $y = \sin(x)$  for  $-360^\circ \leq x \leq 360^\circ$

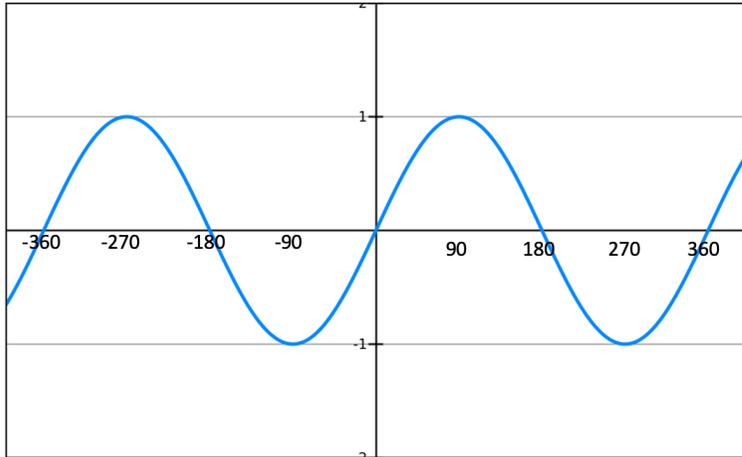
## Worked Example

Sketch the graph  $y = \cos(x)$  for  $-360^\circ \leq x \leq 360^\circ$

## Worked Example

Sketch the graph  $y = \tan(x)$  for  $-360^\circ \leq x \leq 360^\circ$

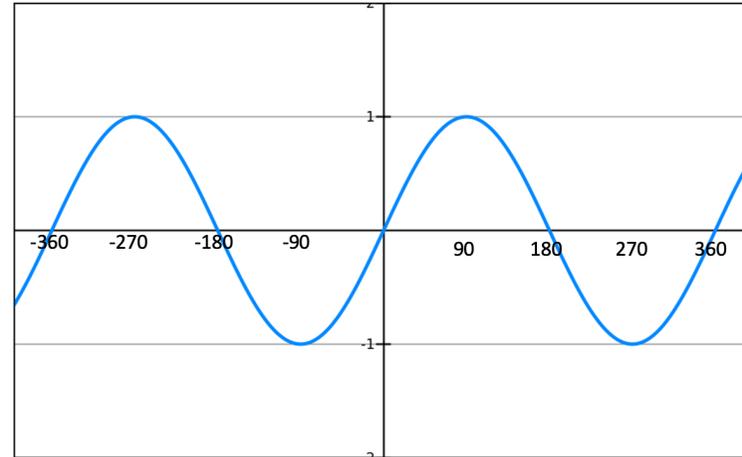
## Worked Example



Suppose we know that  $\sin(30) = 0.5$ . By thinking about symmetry in the graph, work out:

- a)  $\sin(150) =$
- b)  $\sin(-30) =$
- c)  $\sin(210) =$

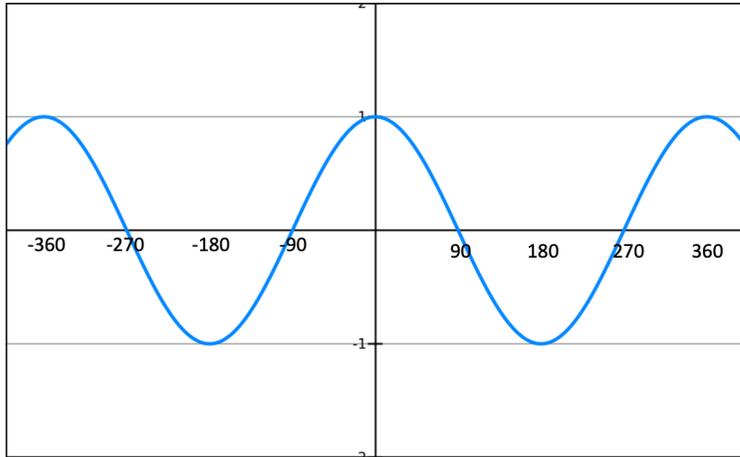
## Your Turn



Suppose we know that  $\sin(60) = \frac{\sqrt{3}}{2}$ . By thinking about symmetry in the graph, work out:

- a)  $\sin(240) =$
- b)  $\sin(120) =$
- c)  $\sin(-60) =$

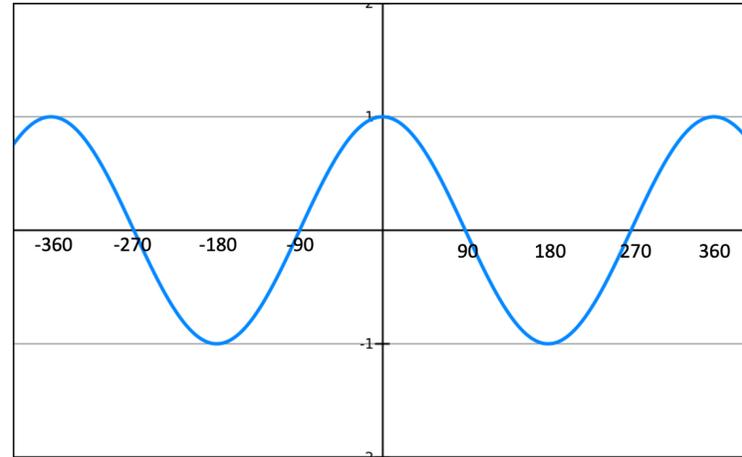
## Worked Example



Suppose we know that  $\cos(60) = 0.5$ . By thinking about symmetry in the graph, work out:

- a)  $\cos(120) =$
- b)  $\cos(-60) =$
- c)  $\cos(240) =$

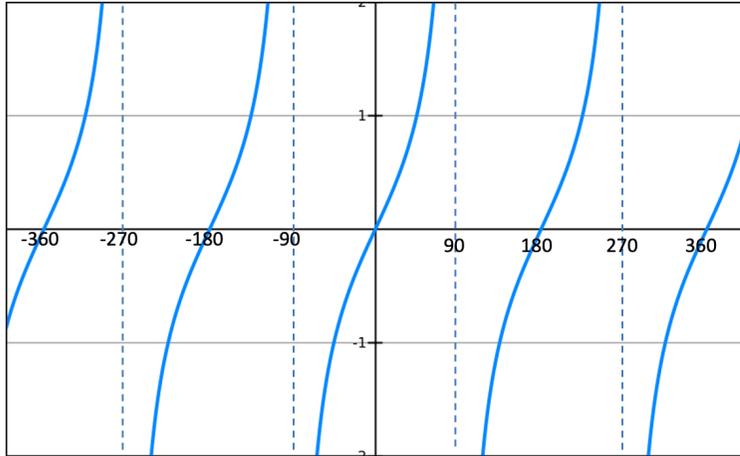
## Your Turn



Suppose we know that  $\cos(30) = \frac{\sqrt{3}}{2}$ . By thinking about symmetry in the graph, work out:

- a)  $\cos(-30) =$
- b)  $\cos(210) =$
- c)  $\cos(150) =$

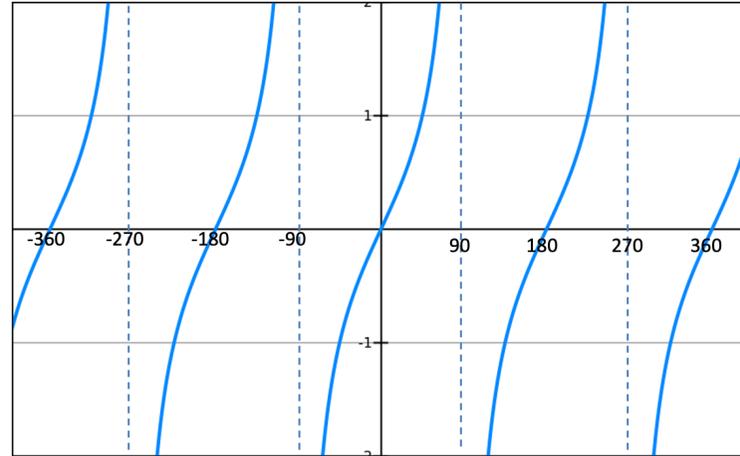
## Worked Example



Suppose we know that  $\tan(30) = \frac{1}{\sqrt{3}}$ . By thinking about symmetry in the graph, work out:

- a)  $\tan(-30) =$
- b)  $\tan(150) =$

## Your Turn

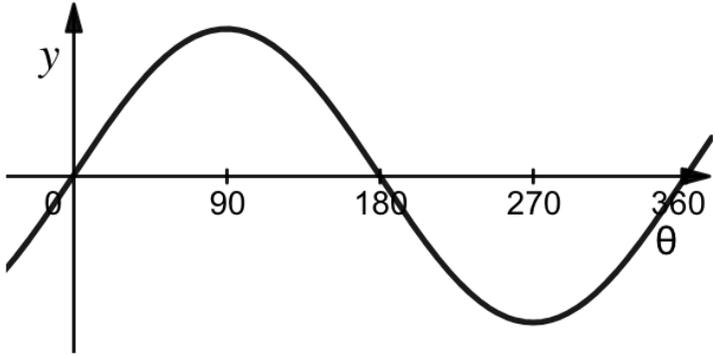


Suppose we know that  $\tan(60) = \sqrt{3}$ . By thinking about symmetry in the graph, work out:

- a)  $\tan(120) =$
- b)  $\tan(-60) =$

## Worked Example

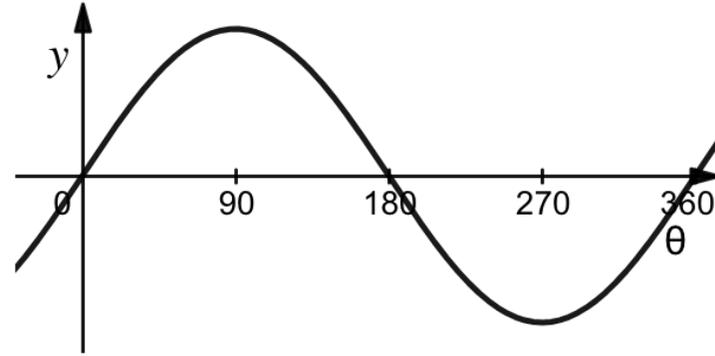
Here is a graph of  $y = \sin \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\sin \theta = -0.8$  for  $0^\circ \leq \theta \leq 360^\circ$ .  
Give your solutions correct to 2 decimal places.

## Your Turn

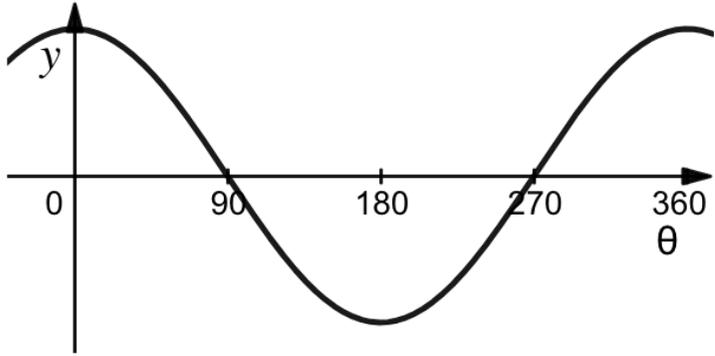
Here is a graph of  $y = \sin \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\sin \theta = 0.3$  for  $0^\circ \leq \theta \leq 360^\circ$ .  
Give your solutions correct to 2 decimal places.

## Worked Example

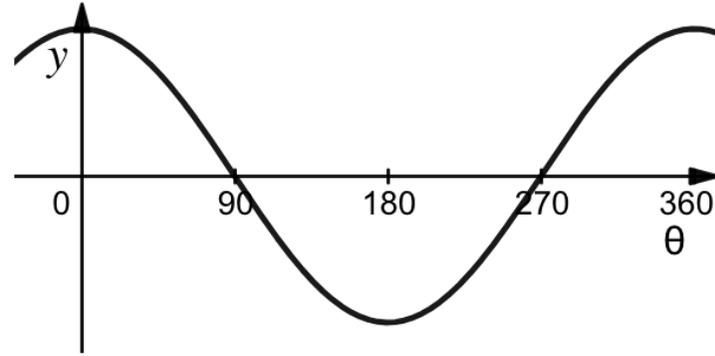
Here is a graph of  $y = \cos \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\cos \theta = 0.7$  for  $0^\circ \leq \theta \leq 360^\circ$ .  
Give your solutions correct to 2 decimal places.

## Your Turn

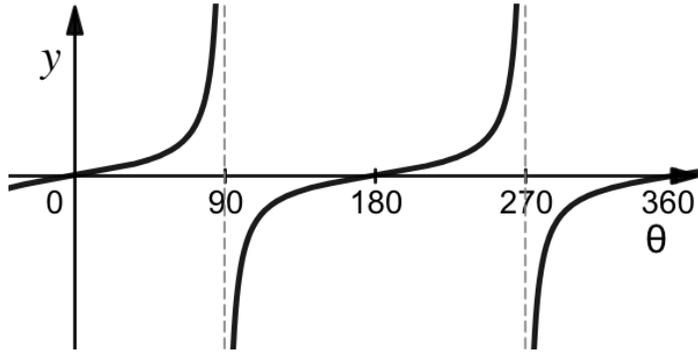
Here is a graph of  $y = \cos \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\cos \theta = -0.2$  for  $0^\circ \leq \theta \leq 360^\circ$ .  
Give your solutions correct to 2 decimal places.

### Worked Example

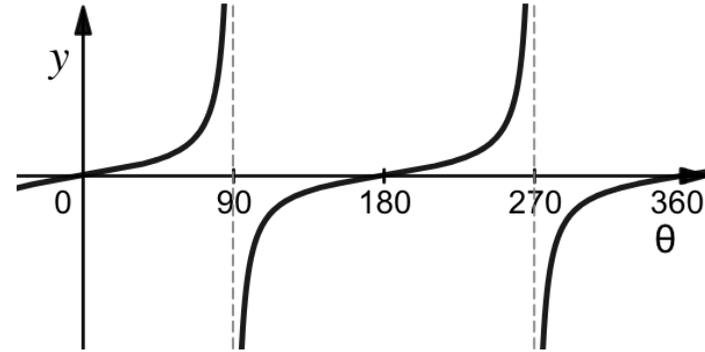
Here is a graph of  $y = \tan \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\tan \theta = -7$  for  $0^\circ \leq \theta \leq 360^\circ$ .  
Give your solutions correct to 2 decimal places.

### Your Turn

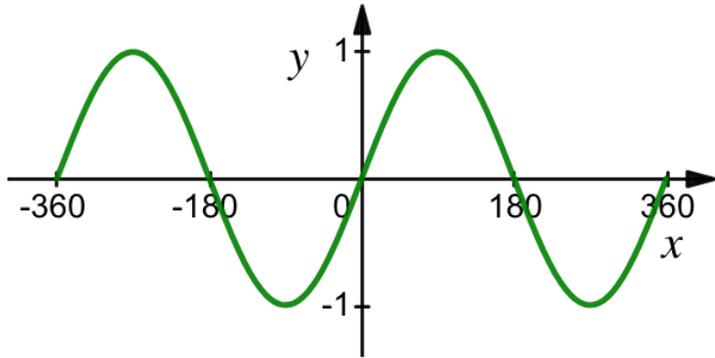
Here is a graph of  $y = \tan \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\tan \theta = 6$  for  $0^\circ \leq \theta \leq 360^\circ$ .  
Give your solutions correct to 2 decimal places.

## Worked Example

Here is the graph of  $y = \sin x$  for the interval  $-360 \leq x \leq 360^\circ$ .



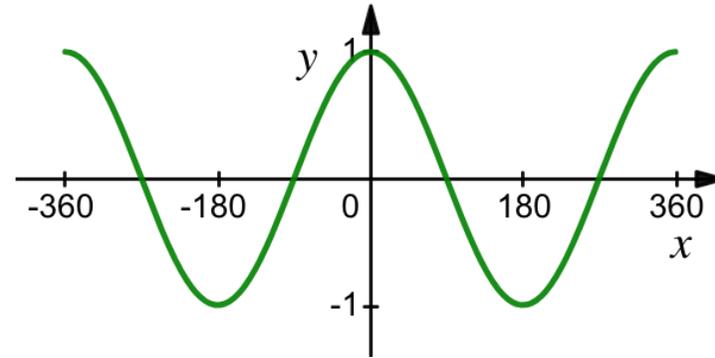
$x = 203.6$  is a solution to the equation  $\sin x = -0.4$

Use the graph to find the other solutions to the equation in the interval  $-360 \leq x \leq 360^\circ$ .

Give your solutions to one decimal place.

## Your Turn

Here is the graph of  $y = \cos x$  for the interval  $-360 \leq x \leq 360^\circ$ .



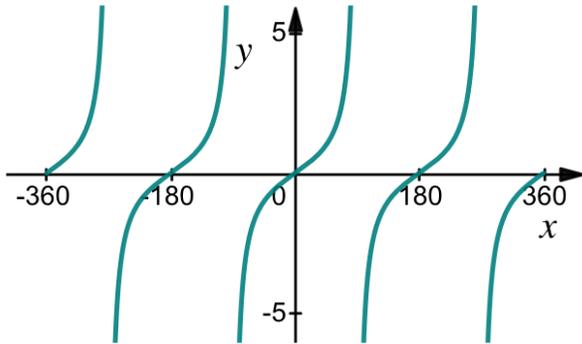
$x = 107.5$  is a solution to the equation  $\cos x = -0.3$

Use the graph to find the other solutions to the equation in the interval  $-360 \leq x \leq 360^\circ$ .

Give your solutions to one decimal place.

## Worked Example

Here is the graph of  $y = \tan x$  for the interval  $-360 \leq x \leq 360^\circ$ .



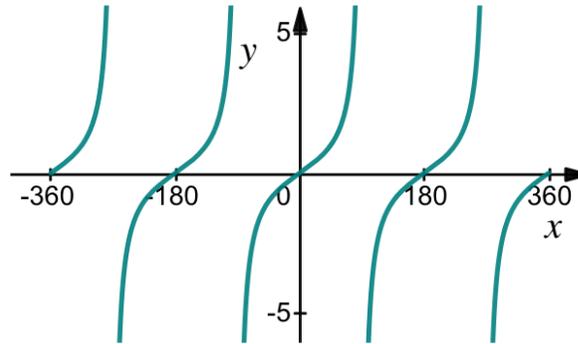
$x = 105.95$  is a solution to the equation  $\tan x = -3.5$

Use the graph to find the other solutions to the equation in the interval  $-360 \leq x \leq 360^\circ$ .

Give your solutions to two decimal places.

## Your Turn

Here is the graph of  $y = \tan x$  for the interval  $-360 \leq x \leq 360^\circ$ .



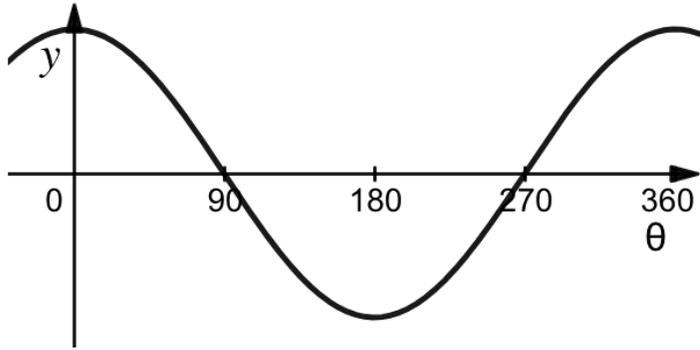
$x = 56.31$  is a solution to the equation  $\tan x = 1.5$

Use the graph to find the other solutions to the equation in the interval  $-360 \leq x \leq 360^\circ$ .

Give your solutions to two decimal places.

## Worked Example

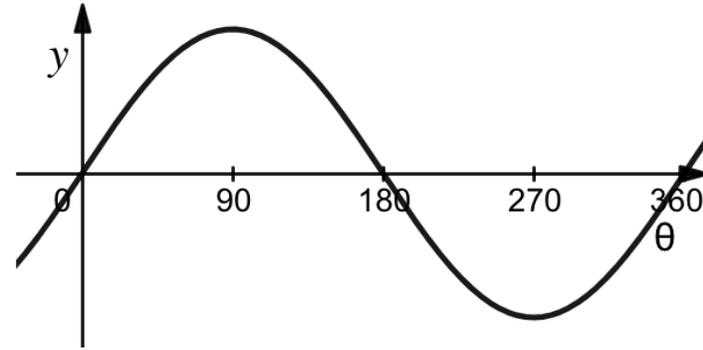
Here is a graph of  $y = \cos \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\cos \theta = -0.4$  for  $0^\circ \leq \theta \leq 720^\circ$ .  
Give your solutions correct to 2 decimal places.

## Your Turn

Here is a graph of  $y = \sin \theta$  for  $0^\circ \leq \theta \leq 360^\circ$ .



Solve  $\sin \theta = -0.7$  for  $0^\circ \leq \theta \leq 720^\circ$ .  
Give your solutions correct to 2 decimal places.

## Extra Notes

## 2 Trigonometric Identities and Equations (L2FM Only)

# Trigonometric Identities

### Worked Example

It is given that  $\alpha$  is acute and that  $\cos \alpha = \frac{2}{3}$   
Find the exact value of  $\tan \alpha$

### Your Turn

It is given that  $\alpha$  is acute and that  $\sin \alpha = \frac{4}{5}$   
Find the exact value of  $\tan \alpha$

### Worked Example

Prove that  $1 - \frac{\tan \theta \cos^3 \theta}{\sin \theta} \equiv \sin^2 \theta$

### Your Turn

Prove that  $1 - \tan \theta \sin \theta \cos \theta \equiv \cos^2 \theta$

### Worked Example

Prove that  $\tan \theta - \frac{1}{\tan \theta} \equiv \frac{1-2 \cos^2 \theta}{\sin \theta \cos \theta}$

### Your Turn

Prove that  $\tan \theta + \frac{1}{\tan \theta} \equiv \frac{1}{\sin \theta \cos \theta}$

## Worked Example

Prove that  $\frac{\cos \theta - \cos^3 \theta}{\sin^3 \theta} \equiv \frac{1}{\tan \theta}$

## Your Turn

Prove that  $\frac{\sin \theta - \sin^3 \theta}{\cos^3 \theta} \equiv \tan \theta$

### Worked Example

Prove that  $\frac{\sin^4 \theta - \cos^4 \theta}{\sin^2 \theta} \equiv 1 - \frac{1}{\tan^2 \theta}$

### Your Turn

Prove that  $\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} \equiv 1 - \tan^2 \theta$

## Worked Example

Prove that  $\frac{\frac{\sin x}{\tan x}}{\sqrt{1-\sin^2 x}} \equiv 1$

## Your Turn

Prove that  $\frac{\tan x \cos x}{\sqrt{1-\cos^2 x}} \equiv 1$

### Worked Example

Prove that  $\frac{1}{\tan^2 \theta} \equiv \frac{1}{\sin^2 \theta} - 1$

### Your Turn

Prove that  $\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1$

# Trigonometric Equations

### Worked Example

Solve  $\cos x = -0.8$  in the interval  $-180^\circ \leq x \leq 540^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Your Turn

Solve  $\sin x = -0.4$  in the interval  $-180^\circ \leq \theta \leq 360^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Worked Example

Solve  $\tan x = 0.4$  in the interval  $-180^\circ \leq x \leq 180^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Your Turn

Solve  $\tan x = -0.4$  in the interval  $-180^\circ \leq x \leq 180^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Worked Example

Solve  $\sin^2 x = 0.53$  in the interval  $-180^\circ \leq x \leq 360^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Your Turn

Solve  $\cos^2 x = 0.62$  in the interval  $-180^\circ \leq \theta \leq 360^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Worked Example

Solve  $8 \tan^2 x = -6 \tan x - 1$  in the interval  $0^\circ < x < 540^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Your Turn

Solve  $12 \tan^2 x - \tan x = 1$  in the interval  $0^\circ < x < 540^\circ$   
Give your solution(s) correct to 1 decimal place where appropriate.

### Worked Example

Solve  $6 \sin^2 x - 5 \cos x = 2$  in the interval  $0^\circ \leq x < 540^\circ$   
Give your solution(s) correct to 2 decimal places where appropriate.

### Your Turn

Solve  $8 \cos^2 x = 2 \sin x + 5$  in the interval  $0^\circ \leq x < 540^\circ$   
Give your solution(s) correct to 2 decimal places where appropriate.

## Extra Notes

### 3 Domain and Range (L2FM Only)

### Worked Example

$$f(x) = 3x^2 - 2$$

The domain of  $f(x)$  is  $\{1, 2, 3, 4\}$ .

What is the range?

### Your Turn

$$g(x) = 2x^3 + 1$$

The domain of  $g(x)$  is  $\{1, 2, 3, 4\}$ .

What is the range?

### Worked Example

- a) Work out a suitable domain and the range of  $f(x) = x^2$
- b) Work out a suitable domain and the range of  $f(x) = \sqrt{x}$

### Your Turn

- a) Work out a suitable domain and the range of  $f(x) = x^3$
- b) Work out a suitable domain and the range of  $f(x) = \sqrt[3]{x}$

### Worked Example

$$f(x) = \frac{x - 3}{5x - 6}$$

State the value of  $x$  that cannot be in the domain of  $f(x)$

### Your Turn

$$f(x) = \frac{3 - x}{6x - 5}$$

State the value of  $x$  that cannot be in the domain of  $f(x)$

**Worked Example**

$g(x) = x^2 - 6x + 5, x \in \mathbb{R}$   
Determine the range of  $g(x)$

**Your Turn**

$f(x) = x^2 - 4x + 7, x \in \mathbb{R}$   
Determine the range of  $f(x)$

**Worked Example**

$g(x) = 3x^2 - 2x + 4, x \in \mathbb{R}$   
Determine the range of  $g(x)$

**Your Turn**

$f(x) = 2x^2 + 7x - 7, x \in \mathbb{R}$   
Determine the range of  $f(x)$

### Worked Example

$g(x) = 14 + 2x - x^2, x \in \mathbb{R}$   
Determine the range of  $g(x)$

### Your Turn

$f(x) = 21 + 4x - x^2, x \in \mathbb{R}$   
Determine the range of  $f(x)$

### Worked Example

$f(x)$  is a function with domain all values of  $x$   
 $f(x) = \sqrt{x^2 + 12x - a}$  where  $a$  is a constant  
Work out the possible values of  $a$

### Your Turn

$f(x)$  is a function with domain all values of  $x$   
 $f(x) = \sqrt{x^2 + 6x - a}$  where  $a$  is a constant  
Work out the possible values of  $a$

**Worked Example**

$g(x) = x^2 + 6x + 5, x \geq 2$   
Determine the range of  $g(x)$

**Your Turn**

$f(x) = x^2 + 4x + 3, x \geq 1$   
Determine the range of  $f(x)$

**Worked Example**

$f(x) = 23 - 5x$  with domain  $-3 < x \leq 1$   
Work out the range of  $f(x)$

**Your Turn**

$g(x) = 32 - 3x$  with domain  $-5 \leq x < 2$   
Work out the range of  $g(x)$

### Worked Example

Determine the range of:

$$g(x) = \sin x, 180 \leq x < 360^\circ$$

### Your Turn

Determine the range of:

$$f(x) = \cos x, 180 < x \leq 360^\circ$$

### Worked Example

A function  $f$  is defined by

$$f(x) = \sin x, 0 \leq x \leq k^\circ$$

Given that the range is  $0 \leq f(x) \leq 1$ , determine the minimum value of  $k$

### Your Turn

A function  $f$  is defined by

$$f(x) = \cos x, k \leq x \leq 360^\circ$$

Given that the range is  $0 \leq f(x) \leq 1$ , determine the minimum value of  $k$

### Worked Example

$$g(x) = \frac{8x-2}{x-1}, x \geq 7$$

Work out the range of  $g$

### Your Turn

$$f(x) = \frac{5x+3}{x-4}, x \geq 5$$

Work out the range of  $f$

### Worked Example

$g(x)$  is an increasing function with domain  $1 \leq x \leq 5$  and range  $3 \leq g(x) \leq 11$ .

Construct a suitable function.

### Your Turn

$f(x)$  is a decreasing function with domain  $4 \leq x \leq 6$  and range  $7 \leq f(x) \leq 19$ .

Construct a suitable function.

## Extra Notes

## 4 Piecewise Functions (L2FM Only)

### Worked Example

$$f(x) = \begin{cases} x^2 + 4, & -8 \leq x \leq 0 \\ 3x + 4, & 0 < x \leq 7 \end{cases}$$

Work out the value of  $f(-3)$

### Your Turn

$$f(x) = \begin{cases} x^2 + 1, & 0 \leq x \leq 3 \\ 2x + 4, & 3 < x \leq 8 \end{cases}$$

Work out the value of  $f(7)$

### Worked Example

$$f(x) = \begin{cases} (x-2)^2 + 1, & 0 \leq x < 3 \\ \frac{1}{4}x + \frac{5}{4}, & 3 \leq x \leq 7 \end{cases}$$

Sketch the graph of  $y = f(x)$

### Your Turn

$$f(x) = \begin{cases} (x-1)^2 + 2, & 0 \leq x < 2 \\ \frac{1}{3}x + \frac{7}{3}, & 2 \leq x \leq 5 \end{cases}$$

Sketch the graph of  $y = f(x)$

### Worked Example

$$f(x) = \begin{cases} 3, & 0 \leq x < 1 \\ x^2 + 2, & 1 \leq x < 2 \\ 8 - x, & 2 \leq x < 3 \end{cases}$$

Sketch the graph of  $y = f(x)$

### Your Turn

$$f(x) = \begin{cases} x^2, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \\ 3 - x, & 2 \leq x < 3 \end{cases}$$

Sketch the graph of  $y = f(x)$

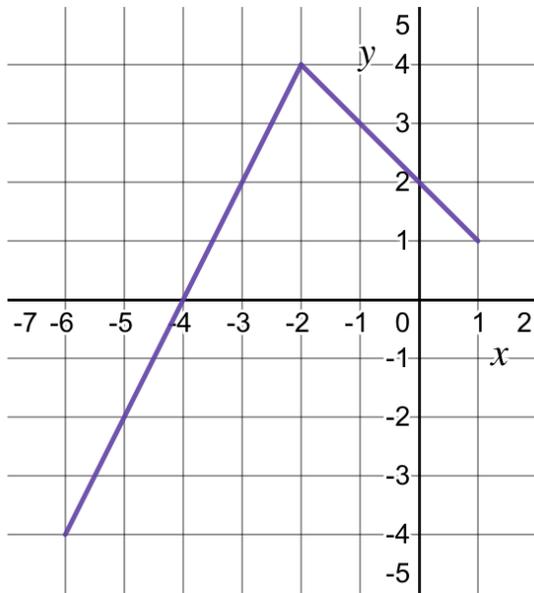
## Worked Example

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} p, & -6 \leq x \leq -2 \\ q, & -2 < x \leq 1 \end{cases}$$

where  $p$  and  $q$  are unknown expressions.

The graph of  $y = f(x)$  is shown below.



Find the expressions represented by  $p$  and  $q$ .

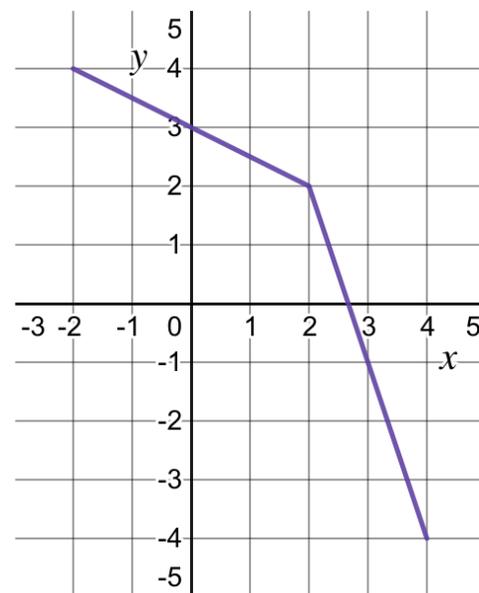
## Your Turn

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} p, & -2 \leq x \leq 2 \\ q, & 2 < x \leq 4 \end{cases}$$

where  $p$  and  $q$  are unknown expressions.

The graph of  $y = f(x)$  is shown below.



Find the expressions represented by  $p$  and  $q$ .

### Worked Example

$$f(x) = \begin{cases} (x - a)^2 + b, & 0 \leq x < 3 \\ cx + d, & 3 \leq x \leq 7 \end{cases}$$

The graph of  $y = f(x)$  passes through the points  $(0, 5)$ ,  $(2, 1)$ ,  $(3, 2)$  and  $(7, 3)$

Find the values of  $a$ ,  $b$ ,  $c$  and  $d$

### Your Turn

$$f(x) = \begin{cases} (x - a)^2 + b, & 0 \leq x < 2 \\ cx + d, & 2 \leq x \leq 5 \end{cases}$$

The graph of  $y = f(x)$  passes through the points  $(0, 3)$ ,  $(1, 2)$ ,  $(2, 3)$  and  $(5, 3)$

Find the values of  $a$ ,  $b$ ,  $c$  and  $d$

### Worked Example

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} x^2 + 7, & -2 \leq x \leq 2 \\ 15 - 2x, & 2 < x \leq 7 \end{cases}$$

Solve  $f(x) = 8$

### Your Turn

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} -2x - 10, & -4 \leq x \leq -2 \\ -x^2 - 2, & -2 < x \leq 2 \end{cases}$$

Solve  $f(x) = -3$

### Worked Example

The function  $f(x)$  is defined for all  $x$ :

$$f(x) = \begin{cases} 9, & x < -3 \\ x^2, & -3 \leq x \leq 3 \\ 15 - 2x, & x > 3 \end{cases}$$

Determine the range of  $f(x)$

### Your Turn

The function  $f(x)$  is defined for all  $x$ :

$$f(x) = \begin{cases} 4, & x < -2 \\ x^2, & -2 \leq x \leq 2 \\ 12 - 4x, & x > 2 \end{cases}$$

Determine the range of  $f(x)$

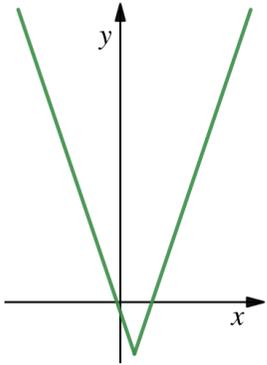
## Worked Example

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} p, & -7 \leq x \leq 1 \\ q, & 1 < x \leq 9 \end{cases}$$

where  $p$  and  $q$  are unknown expressions.

The graph of  $y = f(x)$  is shown below.



The graph is symmetrical about  $x = 1$ .

The range of  $f(x)$  is  $-6 \leq f(x) \leq 34$ .

Find the expressions represented by  $p$  and  $q$ .

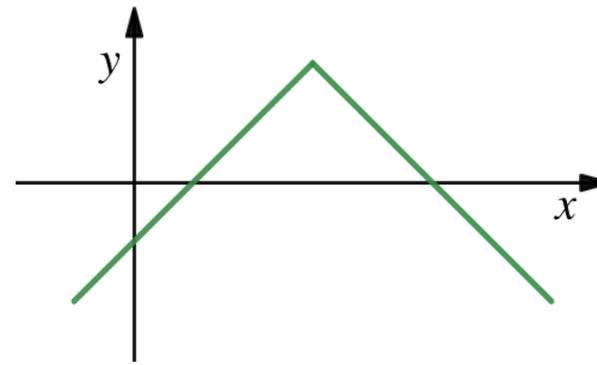
## Your Turn

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} p, & -1 \leq x \leq 3 \\ q, & 3 < x \leq 7 \end{cases}$$

where  $p$  and  $q$  are unknown expressions.

The graph of  $y = f(x)$  is shown below.



The graph is symmetrical about  $x = 3$ .

The range of  $f(x)$  is  $-2 \leq f(x) \leq 2$ .

Find the expressions represented by  $p$  and  $q$ .

### Worked Example

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} 2, & 0 \leq x \leq 5 \\ 2x - 8, & 5 < x \leq 7 \\ 27 - 3x, & 7 < x \leq 9 \end{cases}$$

Find the area enclosed by the graph of  $y = f(x)$ , the  $y$ -axis and the  $x$ -axis.

### Your Turn

The function  $f(x)$  is defined as

$$f(x) = \begin{cases} 2, & 0 \leq x \leq 5 \\ x - 3, & 5 < x \leq 8 \\ 45 - 5x, & 8 < x \leq 9 \end{cases}$$

Find the area enclosed by the graph of  $y = f(x)$ , the  $y$ -axis and the  $x$ -axis.

## Extra Notes

## 5 Graph Transformations

### Worked Example

The point  $A(2, 5)$  is on the graph of  $y = f(x)$ . Write the new coordinates of  $A$  after the transformation:

- a)  $y = f(x) + 3$
- b)  $y = f(x + 3)$
- c)  $y = -f(x)$
- d)  $y = f(-x)$
- e)  $y = -f(x) + 3$
- f)  $y = f(-x) + 3$

### Your Turn

The point  $A(3, 4)$  is on the graph of  $y = f(x)$ . Write the new coordinates of  $A$  after the transformation:

- a)  $y = f(x) - 4$
- b)  $y = f(x - 4)$
- c)  $y = f(-x)$
- d)  $y = -f(x)$
- e)  $y = -f(x) - 6$
- f)  $y = -f(-x) - 6$

**Worked Example**

Sketch  $y = \cos(x) + 1, 0 \leq x \leq 360^\circ$

**Your Turn**

Sketch  $y = \sin(x) - 2, 0 \leq x \leq 360^\circ$

### Worked Example

Sketch  $y = \sin(x - 45^\circ)$ ,  $0 \leq x \leq 360^\circ$

### Your Turn

Sketch  $y = \cos(x + 45^\circ)$ ,  $0 \leq x \leq 360^\circ$

**Worked Example**

Sketch  $y = -\sin(x)$ ,  $0 \leq x \leq 360^\circ$

**Your Turn**

Sketch  $y = -\tan(x)$ ,  $0 \leq x \leq 360^\circ$

### Worked Example

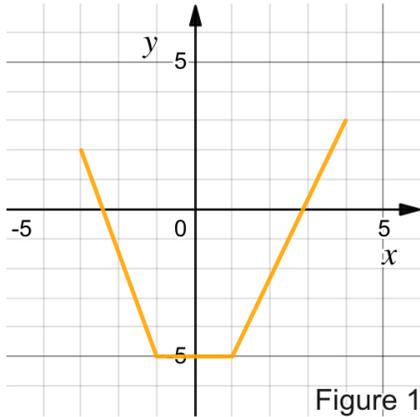
Sketch  $y = \cos(-x)$ ,  $0 \leq x \leq 360^\circ$

### Your Turn

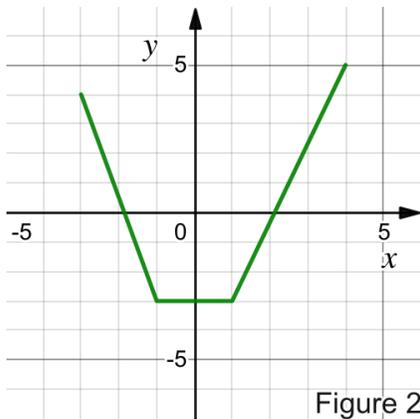
Sketch  $y = \tan(-x)$ ,  $0 \leq x \leq 360^\circ$

## Worked Example

The graph of  $y = f(x)$  is shown in Figure 1.



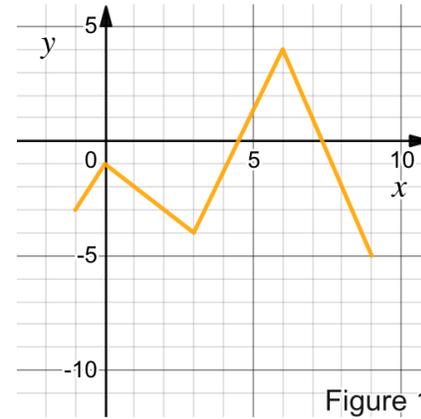
The graph of  $y = f(x) + a$  is shown in Figure 2.



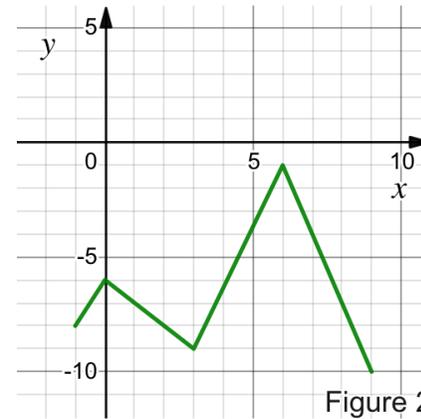
Determine the value of  $a$ .

## Your Turn

The graph of  $y = f(x)$  is shown in Figure 1.



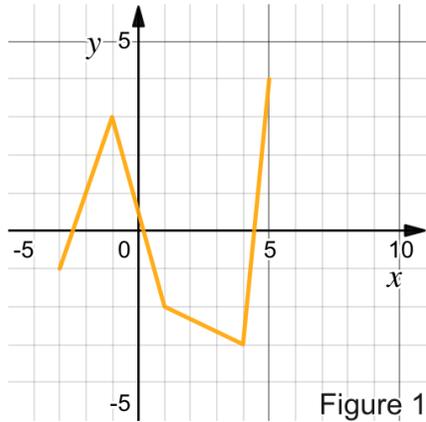
The graph of  $y = f(x) + a$  is shown in Figure 2.



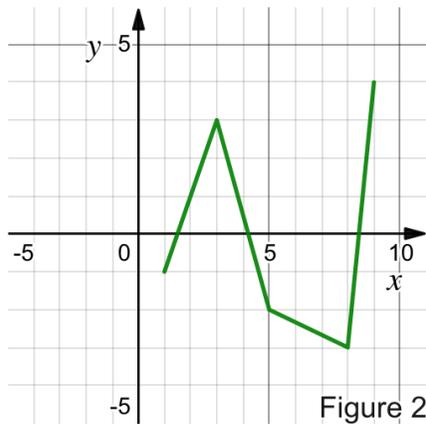
Determine the value of  $a$ .

## Worked Example

The graph of  $y = f(x)$  is shown in Figure 1.



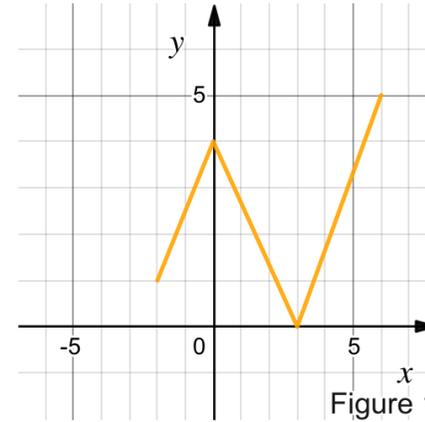
The graph of  $y = f(x + a)$  is shown in Figure 2.



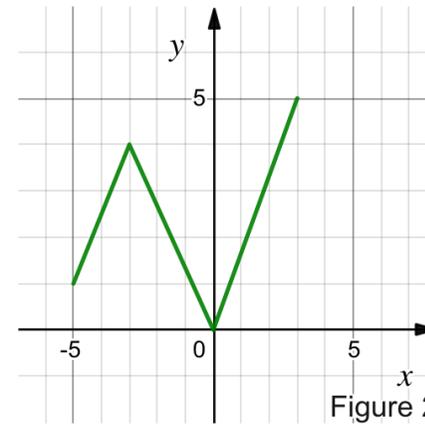
Determine the value of  $a$ .

## Your Turn

The graph of  $y = f(x)$  is shown in Figure 1.



The graph of  $y = f(x + a)$  is shown in Figure 2.



Determine the value of  $a$ .

### Worked Example

- a) The curve  $y = \cos(4x + 90)$  is translated by  $\begin{pmatrix} 30 \\ 0 \end{pmatrix}$   
State the equation of the new curve after this transformation.
- b) The curve  $y = \frac{4}{2x-5}$  is translated by  $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$   
State the equation of the new curve after this transformation.

### Your Turn

- a) The curve  $y = \tan(3x - 30)$  is translated by  $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$   
State the equation of the new curve after this transformation.
- b) The curve  $y = \frac{1}{3x-3}$  is translated by  $\begin{pmatrix} -4 \\ 0 \end{pmatrix}$   
State the equation of the new curve after this transformation.

### Worked Example

The curve  $y = 2x^2 + 3x$  is translated by  $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$   
State the equation of the new curve after this transformation.

### Your Turn

The curve  $y = 2x^3 - x^2$  is translated by  $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$   
State the equation of the new curve after this transformation.

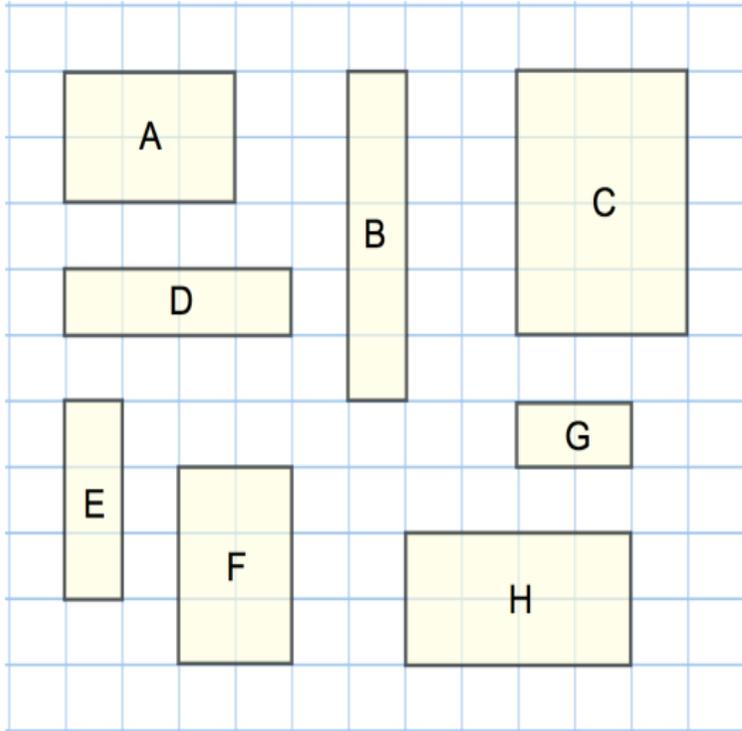
## Fill in the Gaps

$f(x)$	Function notation	Description of translation	Vector of translation	New function
$2x + 1$	$f(x - 3)$	3 places right	$\begin{pmatrix} 3 \\ 0 \end{pmatrix}$	$f(x - 3) = 2(x - 3) + 1$ $= 2x - 6 + 1$ $= 2x - 5$
$3x + 1$	$f(x - 2)$			
$x^2$	$f(x - 1)$			
$x^2$		2 places left		
$x^2 + 5$			$\begin{pmatrix} -3 \\ 0 \end{pmatrix}$	
				$4(x + 5) + 2$
$x^2 + 2x - 1$		1 place left		
	$f(x - 4)$			

## Extra Notes

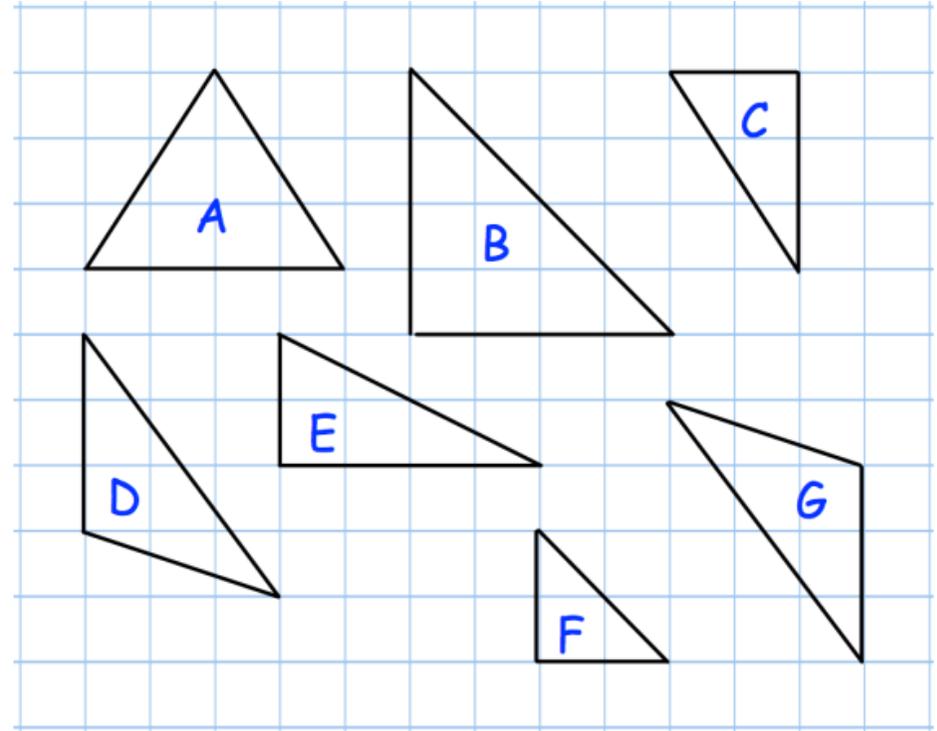
## 6 Congruence and Similarity Proofs

## Worked Example



- Which two shapes are congruent?
- Which two shapes are similar?

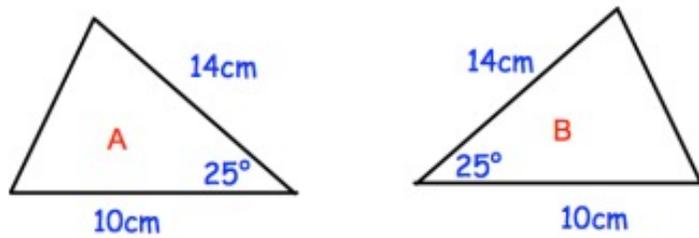
## Your Turn



- Which two shapes are congruent?
- Which two shapes are similar?

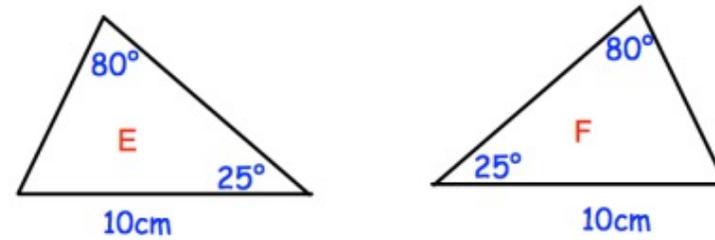
## Worked Example

State the condition why these two triangles are congruent.



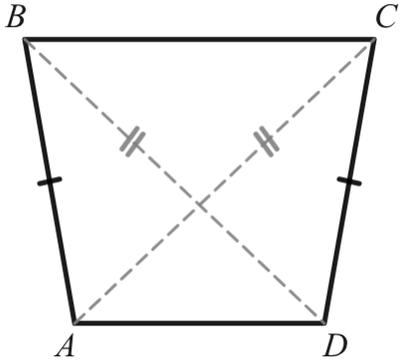
## Your Turn

State the condition why these two triangles are congruent.



## Worked Example

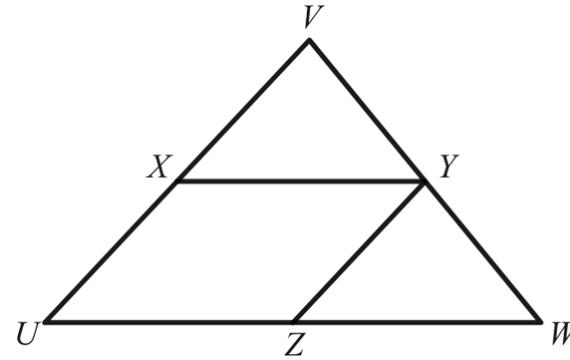
Below is a quadrilateral  $ABCD$ .



Prove that triangle  $ABD$  is congruent to triangle  $ACD$ .

## Your Turn

The diagram shows triangle  $UVW$ .



$WXYZ$  is a parallelogram where

$X$  is the midpoint of  $UV$ ,

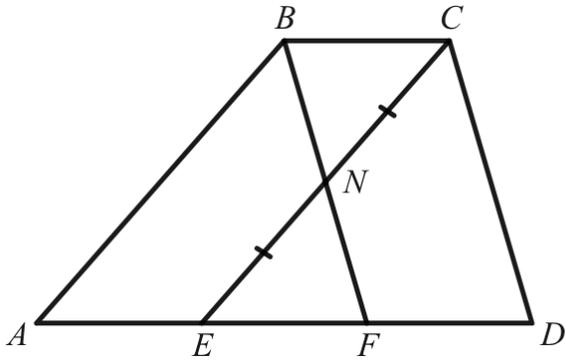
$Y$  is the midpoint of  $VW$ ,

and  $Z$  is the midpoint of  $UW$ .

Prove that triangle  $XVY$  and triangle  $ZYW$  are congruent.

## Worked Example

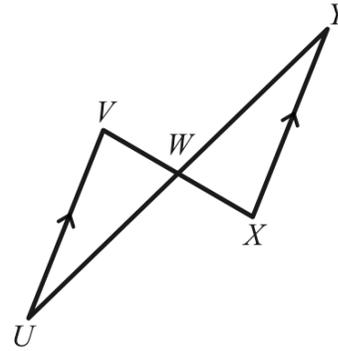
In the diagram below,  $ABCE$  and  $BCDF$  are parallelograms and  $N$  is the midpoint of  $CE$ .



Prove that triangle  $BCN$  and triangle  $EFN$  are congruent.

## Your Turn

In the diagram below,  $UWY$  and  $VWX$  are straight lines.



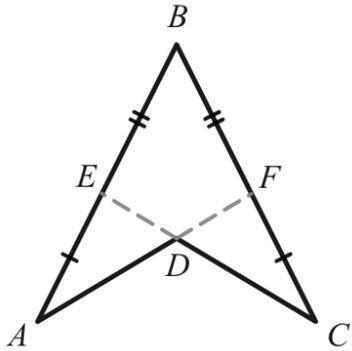
$UV$  and  $XY$  are parallel.

$W$  is the midpoint of  $UWY$ .

Prove that triangle  $UVW$  and triangle  $WXY$  are congruent.

## Worked Example

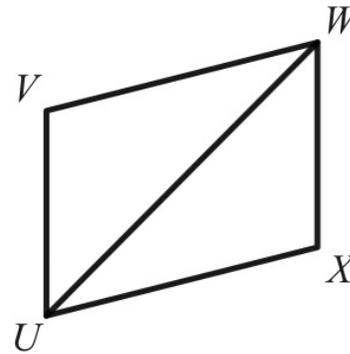
Below is a quadrilateral  $ABCD$ .



Prove that triangle  $ABF$  and triangle  $BCE$  are congruent.

## Your Turn

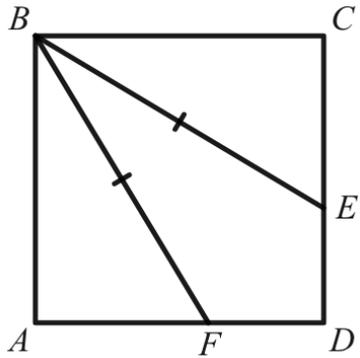
The diagram below shows a parallelogram  $UVWX$ .



Prove that triangle  $UVW$  and triangle  $UWX$  are congruent.

## Worked Example

Below is a square  $ABCD$ .

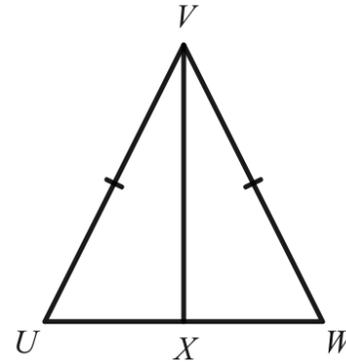


$$BE = BF$$

Prove that triangle  $ABF$  and triangle  $BCE$  are congruent.

## Your Turn

Below is an isosceles triangle  $UVW$ .



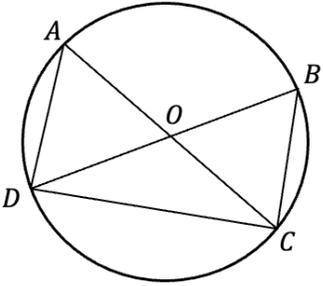
$VX$  is the perpendicular bisector of  $UW$ .

Prove that triangle  $UVX$  and triangle  $VWX$  are congruent.

## Worked Example

$AOC$  and  $BOD$  are diameters of the circle with centre  $O$ .

- Prove that triangle  $ABD$  and triangle  $BCD$  are congruent.
- Show that  $AD = BC$



## Your Turn

$ABC$  is an equilateral triangle.

$D$  lies on  $BC$

$AD$  is perpendicular to  $BC$

- Prove that triangle  $ADC$  is congruent to triangle  $ADB$
- Hence prove that  $BD = \frac{1}{2}AB$

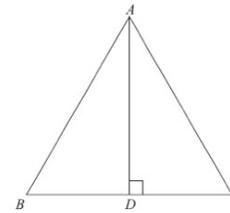
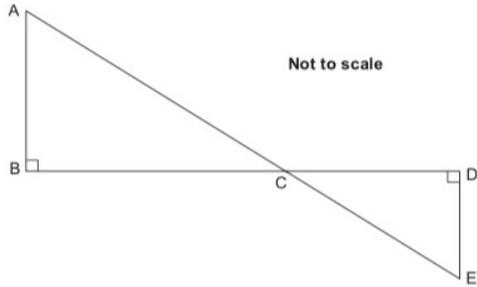


Diagram NOT  
accurately drawn

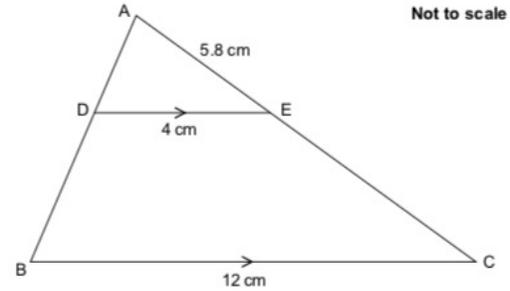
## Worked Example

In the diagram below,  $AE$  and  $BD$  are straight lines.  
Show that triangles  $ABC$  and  $EDC$  are similar.



## Your Turn

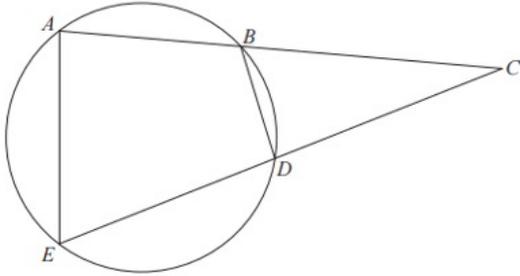
In the diagram  $BC$  is parallel to  $DE$ .  
Prove that triangle  $ABC$  is similar to triangle  $ADE$ .



## Worked Example

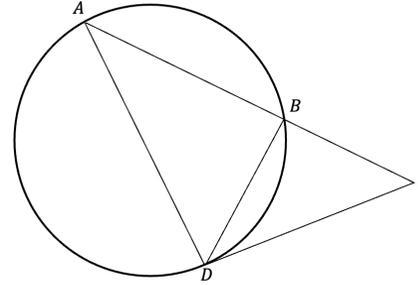
$A$ ,  $B$ ,  $D$  and  $E$  are points on a circle.  
 $ABC$  and  $EDC$  are straight lines.

Prove that triangle  $BCD$  is similar to triangle  $ECA$   
You must give reasons for your working.



## Your Turn

The line  $CD$  is tangent to the circle. Prove that  $BCD$  is similar to  $ABC$ .



## Extra Notes

## 7 Circle Theorem Proofs

## Worked Example

Prove angles in a semicircle are  $90^\circ$ .

## Worked Example

Prove the angle at the centre of a circle is twice the angle at the circumference.

## Worked Example

Prove angles in the same segment are equal.

## Worked Example

Prove opposite angles of a cyclic quadrilateral add to  $180^\circ$ .

## Worked Example

Prove the alternate segment theorem.

## Extra Notes