KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Mathematics 2026

Unit 25 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

1 Exponential and Trigonometric Graphs

Worked Example	Your Turn
a) The equation of the curve is $y = 1.25^x$ M is the point where the curve intercepts the y-axis. State the coordinates of M b) The equation of the curve is $y = -\frac{1}{2} \times 5^x$ Y is the point where the curve intercepts the y-axis. State the coordinates of Y	a) The equation of the curve is $y = 0.25^x$ M is the point where the curve intercepts the y-axis. State the coordinates of M b) The equation of the curve is $y = 7 \times \left(\frac{1}{2}\right)^x$ K is the point where the curve intercepts the y-axis. State the coordinates of K

Dr Frost 424a and 424b

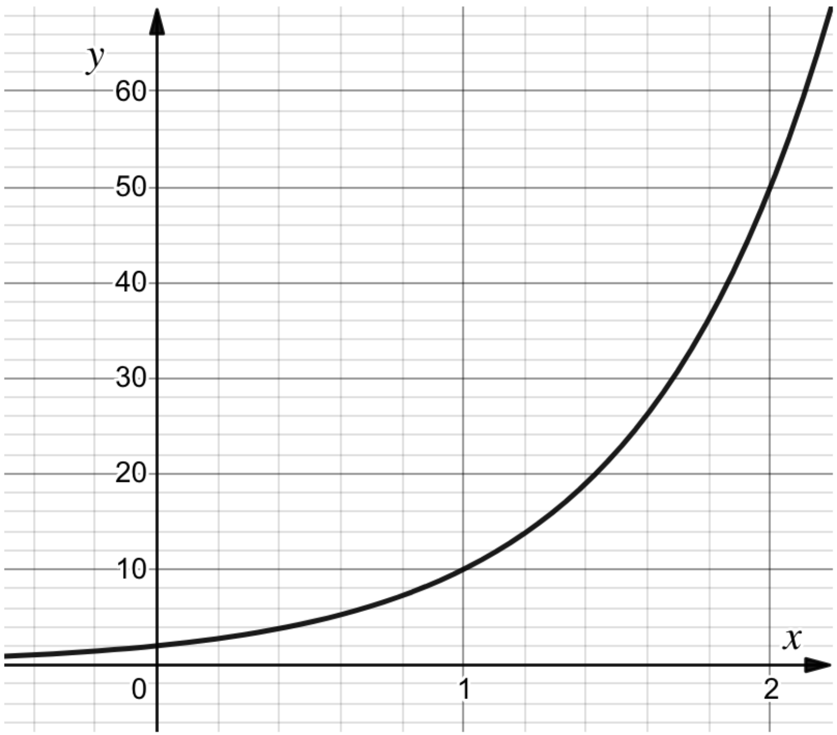
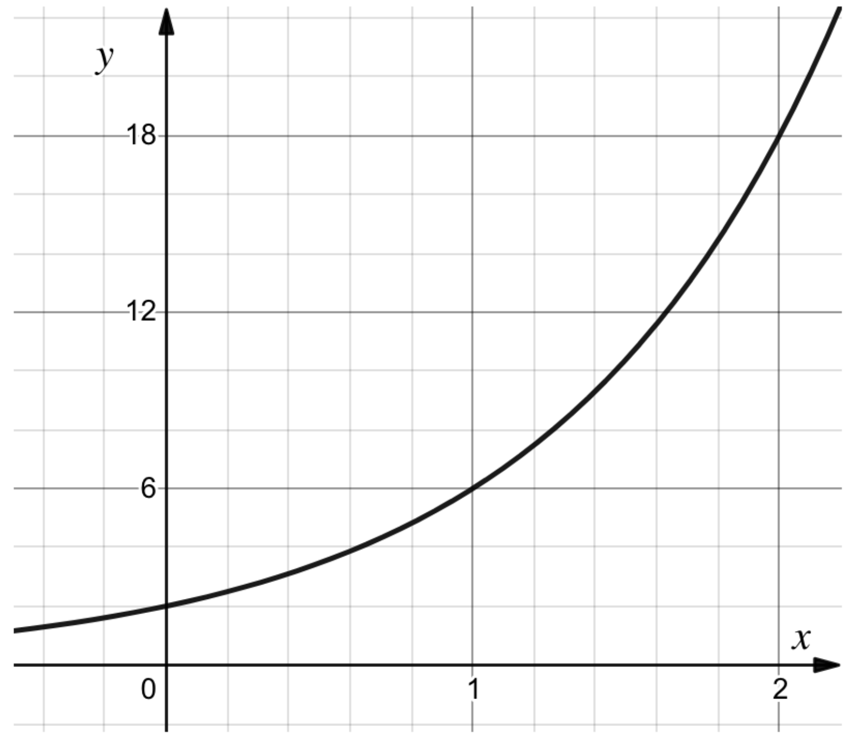
Worked Example	Your Turn
Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 1750\times(1.02)^t$ State the initial amount of money invested.	Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 3000\times(1.005)^t$ State the initial amount of money invested.

Dr Frost 424c

Worked Example	Your Turn
Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 500\times(1.04)^t$ Determine the interest rate offered by the bank.	Some money M has been invested in a bank. The value of the money after t years is modelled by the function $M(t) = 1250\times(1.025)^t$ Determine the interest rate offered by the bank.

Worked Example	Your Turn
The sketch graph shows a curve with equation $y = ab^x$ The curve passes through the points $(0, 3.25)$ and $(3, 87.75)$. Calculate the value of a and the value of b.	The sketch graph shows a curve with equation $y = ab^x$ The curve passes through the points $(0, 2.75)$ and $(2, 68.75)$. Calculate the value of a and the value of b.

Dr Frost 425a

Worked Example	Your Turn
This graph shows a curve with equation $y = pq^x$  Calculate the value of p and q.	This graph shows a curve with equation $y = pq^x$  Calculate the value of p and q.

Dr Frost 425b

Worked Example	Your Turn
At the start of an experiment, a petri dish contained $4{,}000{,}000$ bacteria. After 4 days, there were $6{,}000{,}000$ bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 7 days after the start of the experiment, giving your answer to 3 significant figures.	At the start of an experiment, a petri dish contained $4{,}000{,}000$ bacteria. After 5 days, there were $13{,}000{,}000$ bacteria. It is assumed that the number of bacteria is given by the formula $N = ar^t$ where N is the number of bacteria, t days after the start of the experiment. Calculate the number of bacteria 11 days after the start of the experiment, giving your answer to 3 significant figures.

Dr Frost 425c

Trigonometric Graphs

Angle (θ Degrees)	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
$\sin(\theta)$								
$\cos(\theta)$								
$\tan(\theta)$								

Worked Example

Sketch the graph $y = \sin(x)$ for $-360° \leq x \leq 360°$

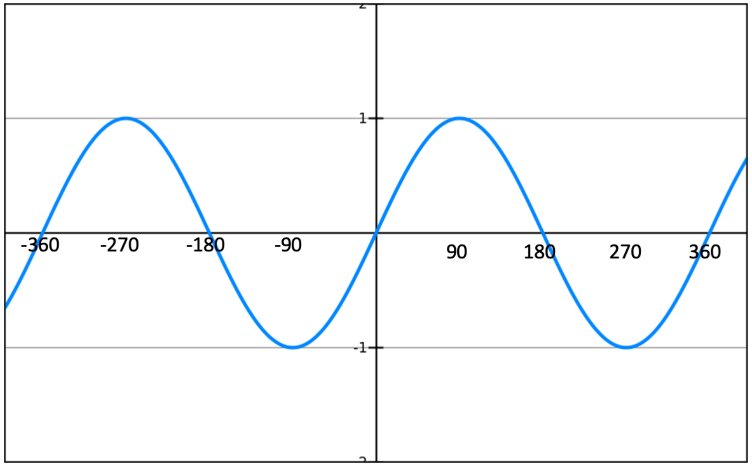
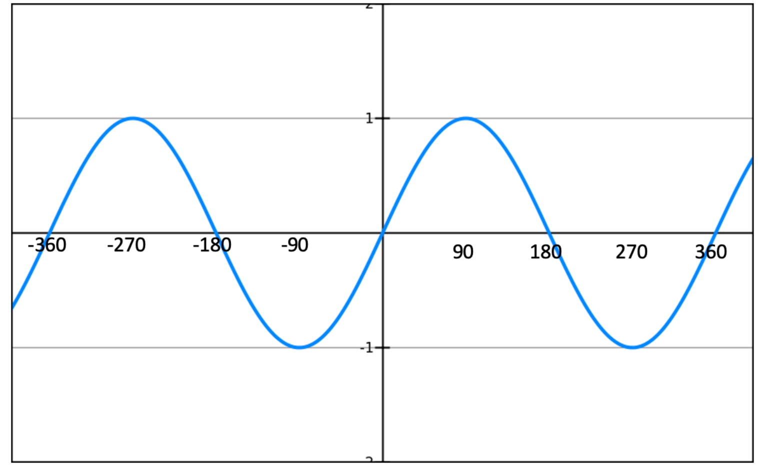
Worked Example

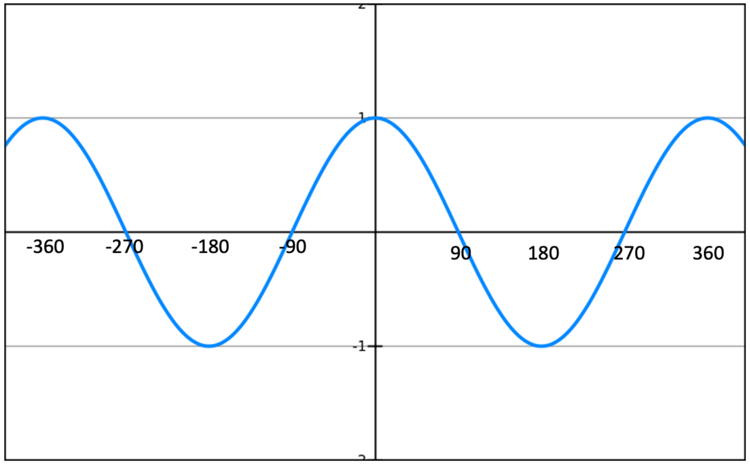
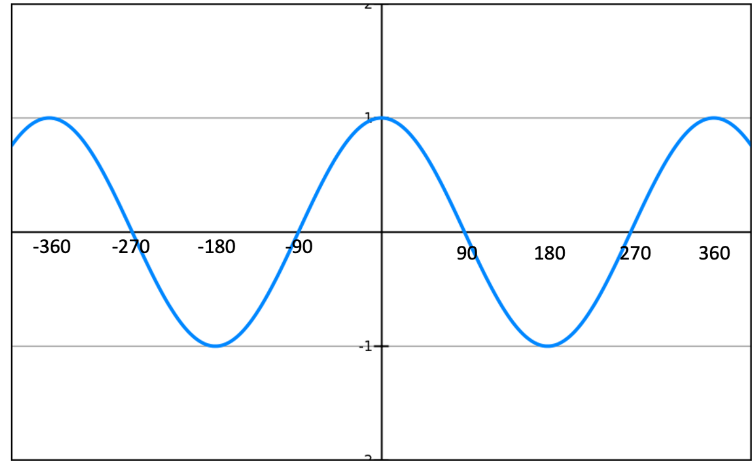
Sketch the graph $y = \cos(x)$ for $-360° \leq x \leq 360°$

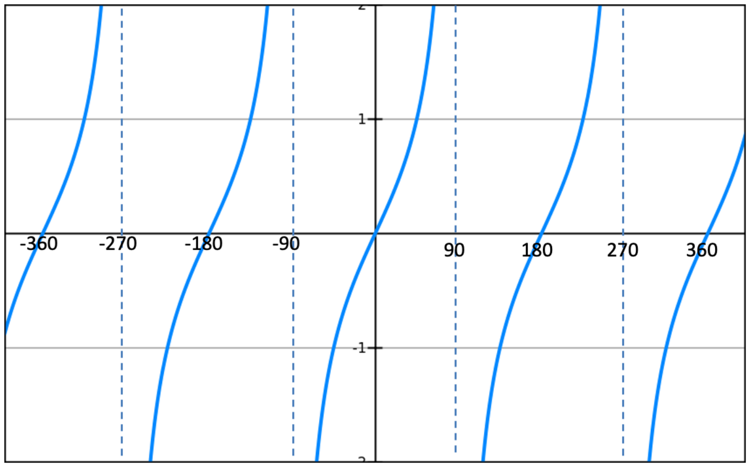
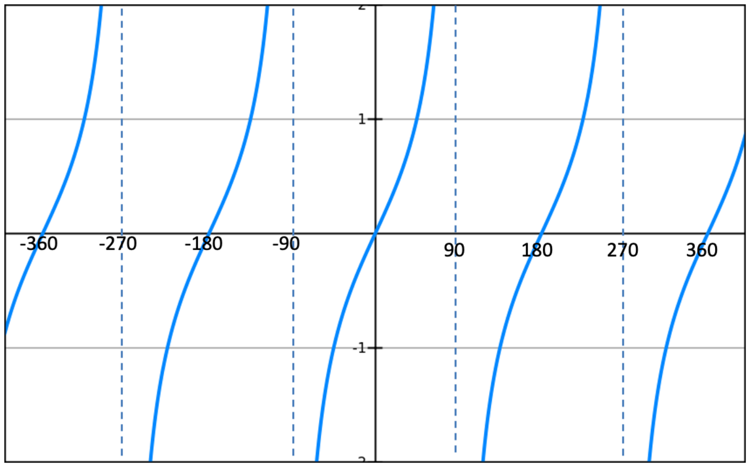
Dr Frost 431a

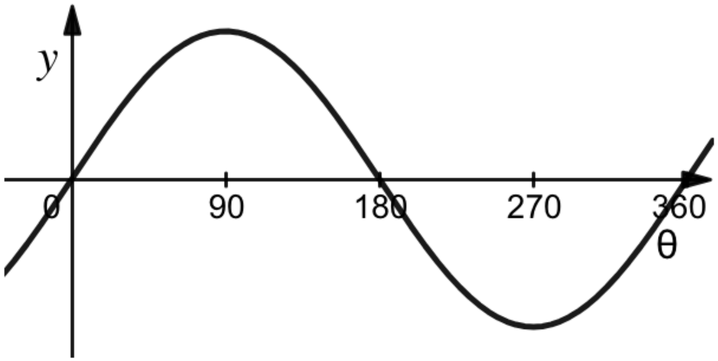
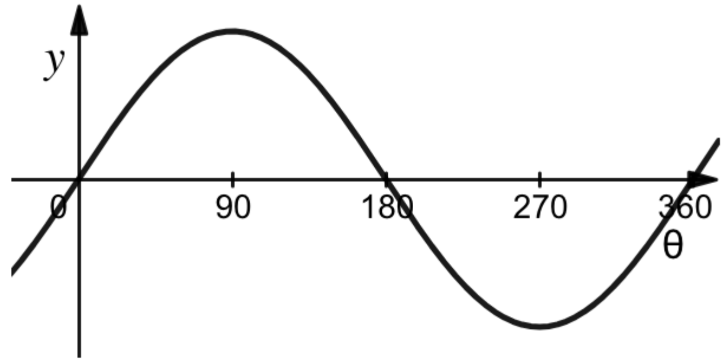
Worked Example

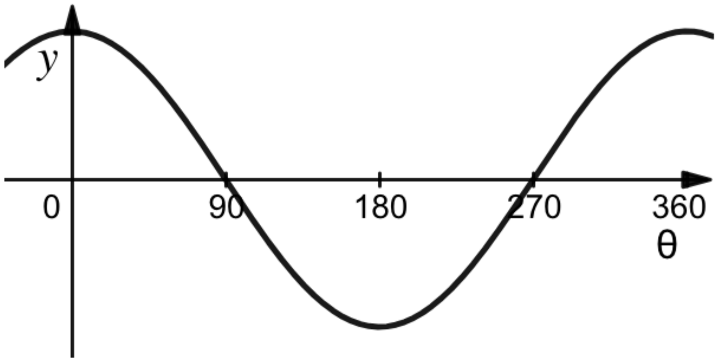
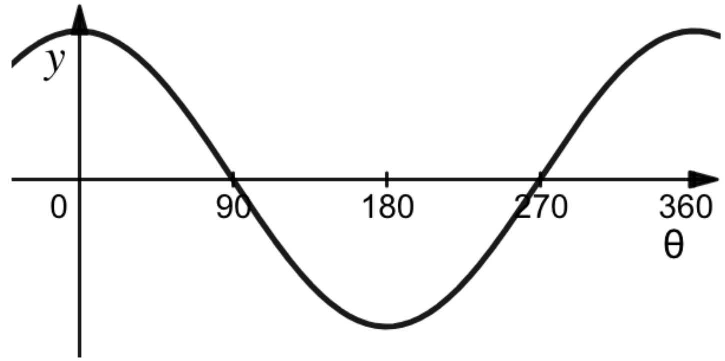
Sketch the graph $y = \tan(x)$ for $-360° \leq x \leq 360°$

Worked Example	Your Turn
	
Suppose we know that $\sin(30) = 0.5$. By thinking about symmetry in the graph, work out:	Suppose we know that $\sin(60) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out:
a) $\sin(150) =$	a) $\sin(240) =$
b) $\sin(-30) =$	b) $\sin(120) =$
c) $\sin(210) =$	c) $\sin(-60) =$

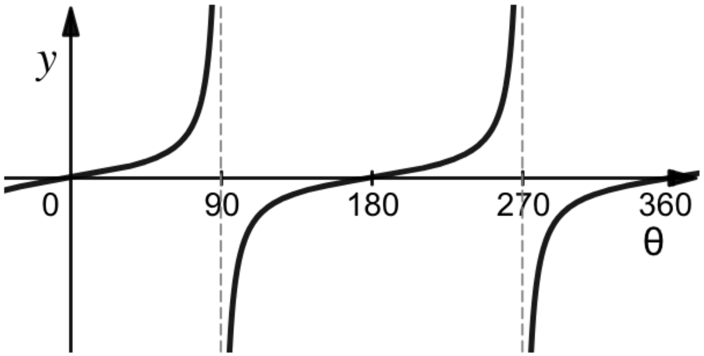
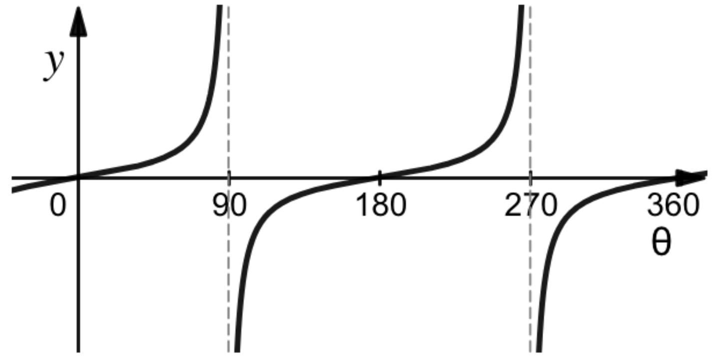
Worked Example	Your Turn
 Suppose we know that $\cos(60) = 0.5$. By thinking about symmetry in the graph, work out: a) $\cos(120) =$ b) $\cos(-60) =$ c) $\cos(240) =$	 Suppose we know that $\cos(30) = \frac{\sqrt{3}}{2}$. By thinking about symmetry in the graph, work out: a) $\cos(-30) =$ b) $\cos(210) =$ c) $\cos(150) =$

Worked Example	Your Turn
 Suppose we know that $\tan(30) = \frac{1}{\sqrt{3}}$. By thinking about symmetry in the graph, work out: a) $\tan(-30) =$ b) $\tan(150) =$	 Suppose we know that $\tan(60) = \sqrt{3}$. By thinking about symmetry in the graph, work out: a) $\tan(120) =$ b) $\tan(-60) =$

Worked Example	Your Turn
Here is a graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$.  Solve $\sin\theta = -0.8$ for $0° \leq \theta \leq 360°$. Give your solutions correct to 2 decimal places.	Here is a graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$.  Solve $\sin\theta = 0.3$ for $0° \leq \theta \leq 360°$. Give your solutions correct to 2 decimal places.

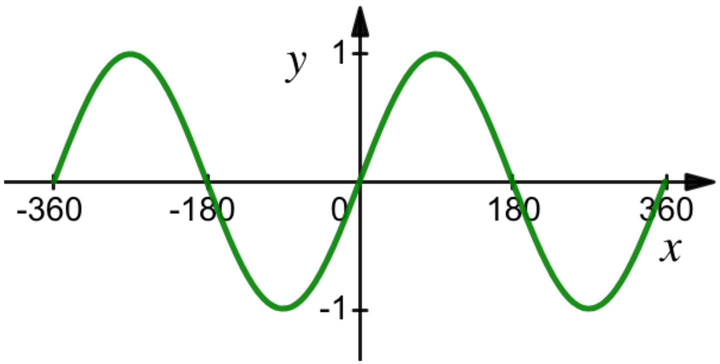
Worked Example	Your Turn
Here is a graph of $y = \cos\theta$ for $0° \leq \theta \leq 360°$.  Solve $\cos\theta = 0.7$ for $0° \leq \theta \leq 360°$. Give your solutions correct to 2 decimal places.	Here is a graph of $y = \cos\theta$ for $0° \leq \theta \leq 360°$.  Solve $\cos\theta = -0.2$ for $0° \leq \theta \leq 360°$. Give your solutions correct to 2 decimal places.

Dr Frost 434d

Worked Example	Your Turn
Here is a graph of $y = \tan\theta$ for $0° \leq \theta \leq 360°$.  Solve $\tan\theta = -7$ for $0° \leq \theta \leq 360°$. Give your solutions correct to 2 decimal places.	Here is a graph of $y = \tan\theta$ for $0° \leq \theta \leq 360°$.  Solve $\tan\theta = 6$ for $0° \leq \theta \leq 360°$. Give your solutions correct to 2 decimal places.

Worked Example

Here is the graph of $y = \sin x$ for the interval $-360 \leq x \leq 360°$.

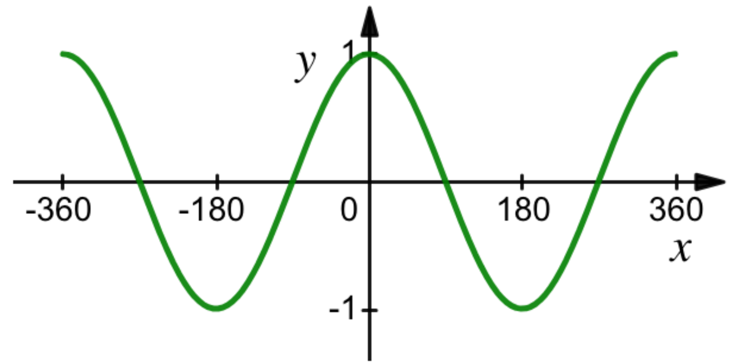


$x = 203.6$ is a solution to the equation $\sin x = -0.4$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$.
Give your solutions to one decimal place.

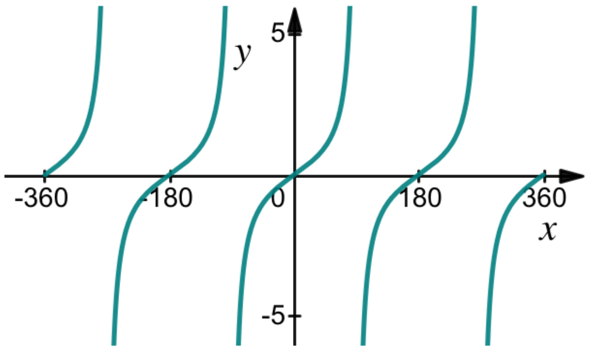
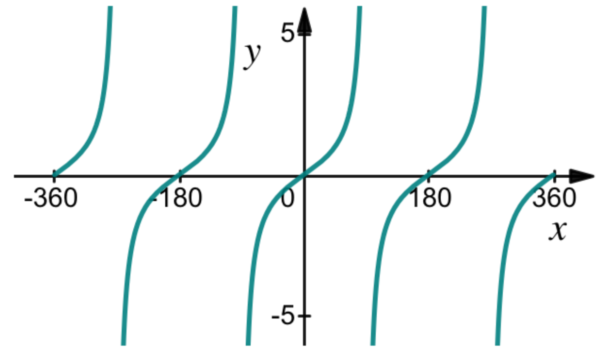
Your Turn

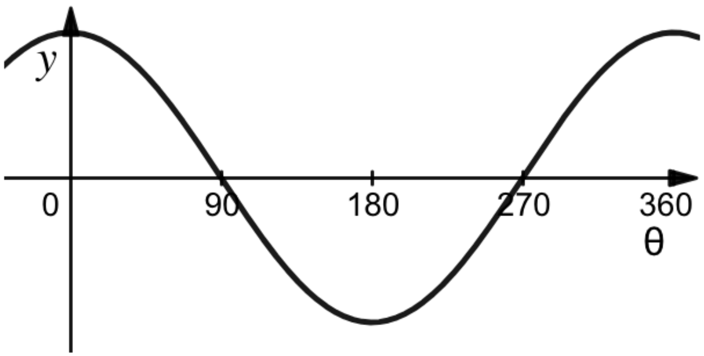
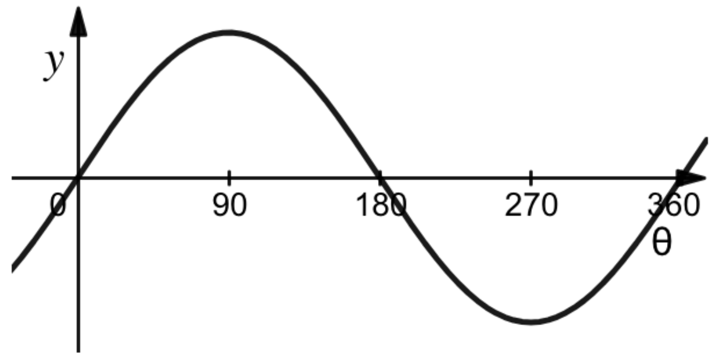
Here is the graph of $y = \cos x$ for the interval $-360 \leq x \leq 360°$.



$x = 107.5$ is a solution to the equation $\cos x = -0.3$

Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$.
Give your solutions to one decimal place.

Worked Example	Your Turn
Here is the graph of $y = tan\, x$ for the interval $-360 \leq x \leq 360°$.  $x = 105.95$ is a solution to the equation $\tan x = -3.5$ Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$. Give your solutions to two decimal places.	Here is the graph of $y = tan\, x$ for the interval $-360 \leq x \leq 360°$.  $x = 56.31$ is a solution to the equation $\tan x = 1.5$ Use the graph to find the other solutions to the equation in the interval $-360 \leq x \leq 360°$. Give your solutions to two decimal places.

Worked Example	Your Turn
Here is a graph of $y = \cos\theta$ for $0° \leq \theta \leq 360°$.  Solve $\cos\theta = -0.4$ for $0° \leq \theta \leq 720°$. Give your solutions correct to 2 decimal places.	Here is a graph of $y = \sin\theta$ for $0° \leq \theta \leq 360°$.  Solve $\sin\theta = -0.7$ for $0° \leq \theta \leq 720°$. Give your solutions correct to 2 decimal places.

Dr Frost 434h

Extra Notes

2 Graph Transformations

Worked Example	Your Turn
The point $A(2, 5)$ is on the graph of $y = f(x)$. Write the new coordinates of A after the transformation: a) $y = f(x) + 3$ b) $y = f(x + 3)$ c) $y = -f(x)$ d) $y = f(-x)$ e) $y = -f(x) + 3$ f) $y = f(-x) + 3$	The point $A(3, 4)$ is on the graph of $y = f(x)$. Write the new coordinates of A after the transformation: a) $y = f(x) - 4$ b) $y = f(x - 4)$ c) $y = f(-x)$ d) $y = -f(x)$ e) $y = -f(x) - 6$ f) $y = -f(-x) - 6$

Dr Frost 448d, 448c, 449c and 449d

Worked Example	Your Turn
Sketch $y = \cos(x) + 1$, $0 \leq x \leq 360°$	Sketch $y = \sin(x) - 2$, $0 \leq x \leq 360°$

Worked Example	Your Turn
Sketch $y = \sin(x - 45°)$, $0 \leq x \leq 360°$	Sketch $y = \cos(x + 45°)$, $0 \leq x \leq 360°$

Dr Frost 448f

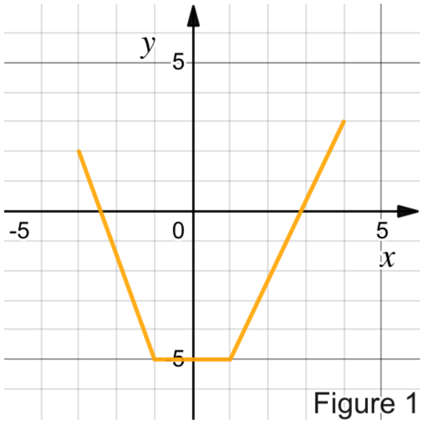
Worked Example	Your Turn
Sketch $y = -\sin(x)$, $0 \leq x \leq 360°$	Sketch $y = -\tan(x)$, $0 \leq x \leq 360°$

Worked Example	Your Turn
Sketch $y = \cos(-x)$, $0 \leq x \leq 360°$	Sketch $y = \tan(-x)$, $0 \leq x \leq 360°$

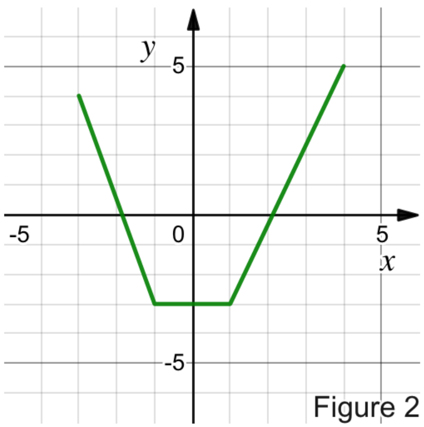
Dr Frost 449e

Worked Example

The graph of $y = f(x)$ is shown in Figure 1.



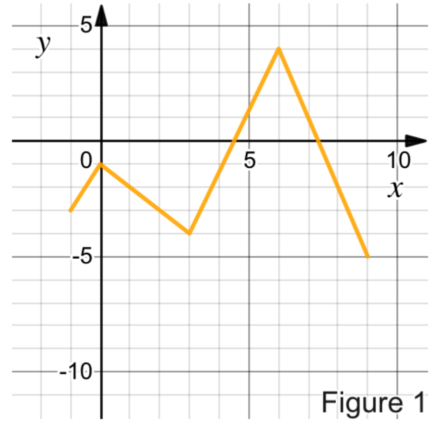
The graph of $y = f(x) + a$ is shown in Figure 2.



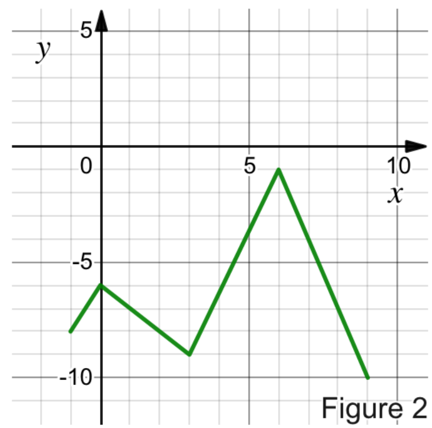
Determine the value of a.

Your Turn

The graph of $y = f(x)$ is shown in Figure 1.



The graph of $y = f(x) + a$ is shown in Figure 2.



Determine the value of a.

Worked Example

The graph of $y = f(x)$ is shown in Figure 1.

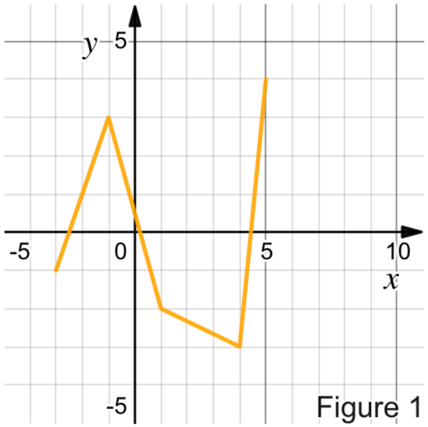


Figure 1

The graph of $y = f(x + a)$ is shown in Figure 2.

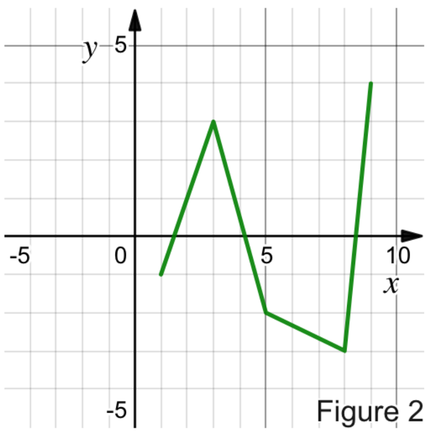


Figure 2

Determine the value of a.

Your Turn

The graph of $y = f(x)$ is shown in Figure 1.

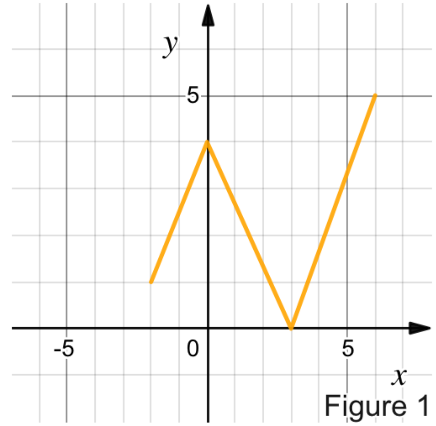


Figure 1

The graph of $y = f(x + a)$ is shown in Figure 2.

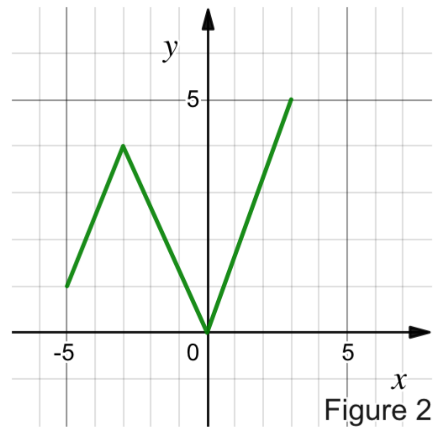


Figure 2

Determine the value of a.

Dr Frost 448g

Worked Example	Your Turn
a) The curve $y = \cos(4x + 90)$ is translated by $\binom{30}{0}$ State the equation of the new curve after this transformation. b) The curve $y = \frac{4}{2x-5}$ is translated by $\binom{0}{3}$ State the equation of the new curve after this transformation.	a) The curve $y = \tan(3x - 30)$ is translated by $\binom{0}{-3}$ State the equation of the new curve after this transformation. b) The curve $y = \frac{1}{3x-3}$ is translated by $\binom{-4}{0}$ State the equation of the new curve after this transformation.

Dr Frost 448h

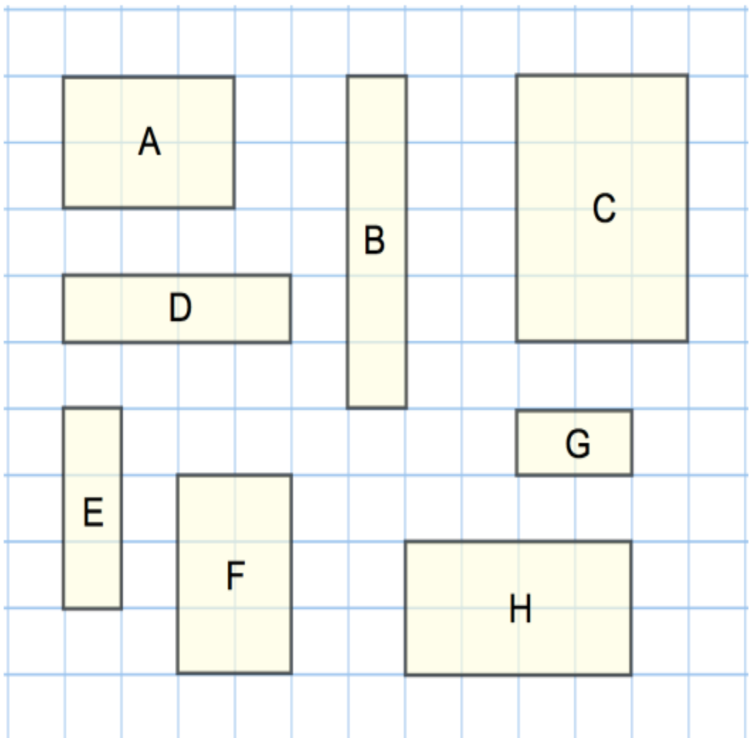
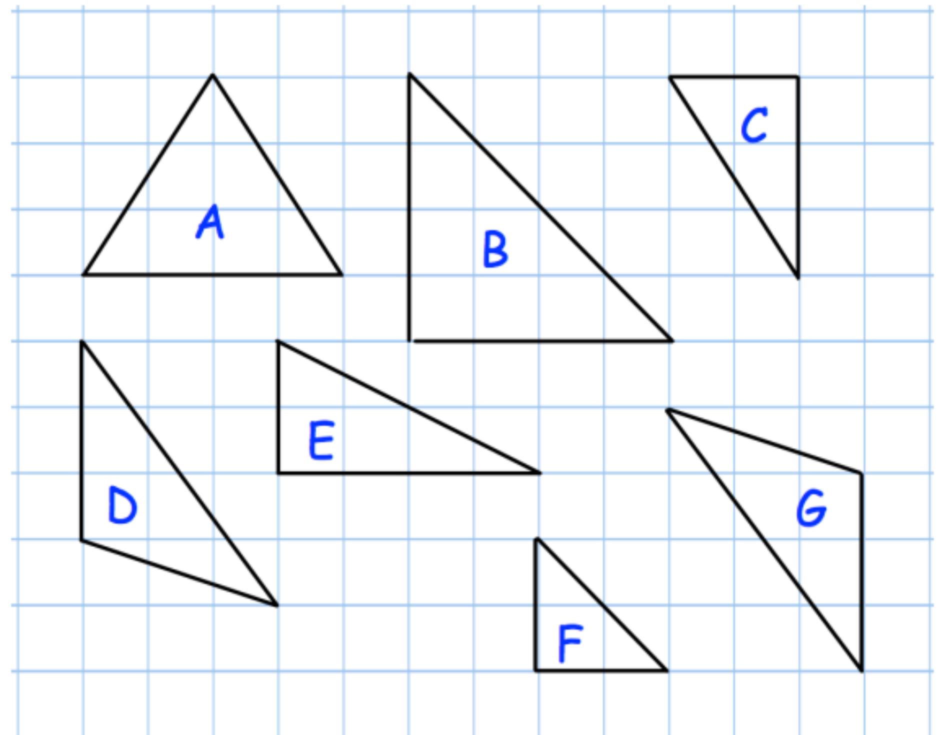
Worked Example	Your Turn
The curve $y = 2x^2 + 3x$ is translated by $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$ State the equation of the new curve after this transformation.	The curve $y = 2x^3 - x^2$ is translated by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ State the equation of the new curve after this transformation.

Fill in the Gaps

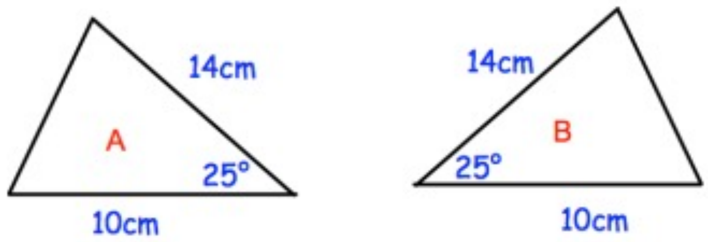
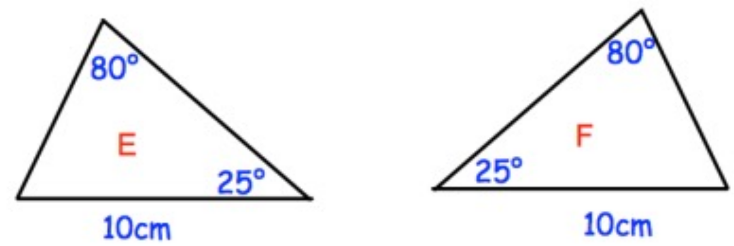
$f(x)$	Function notation	Description of translation	Vector of translation	New function
$2x+1$	$f(x-3)$	3 places right	$\begin{pmatrix}3\\0\end{pmatrix}$	$f(x-3) = 2(x-3)+1$ $= 2x-6+1$ $= 2x-5$
$3x+1$	$f(x-2)$			
x^2	$f(x-1)$			
x^2		2 places left		
x^2+5			$\begin{pmatrix}-3\\0\end{pmatrix}$	
				$4(x+5)+2$
x^2+2x-1		1 place left		
	$f(x-4)$			

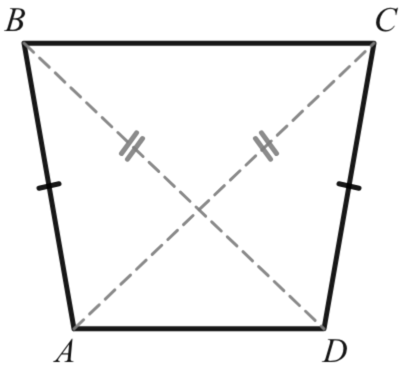
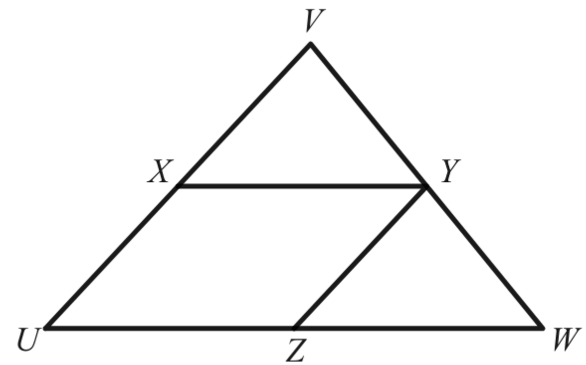
Extra Notes

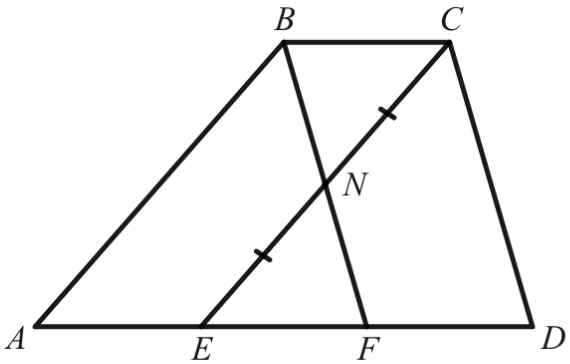
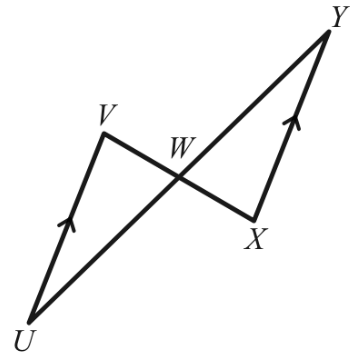
3 Congruence and Similarity Proofs

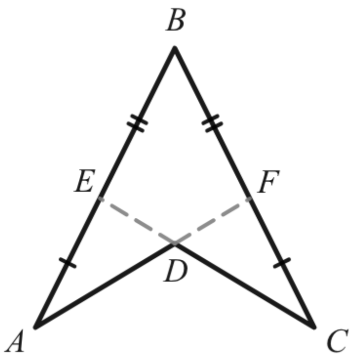
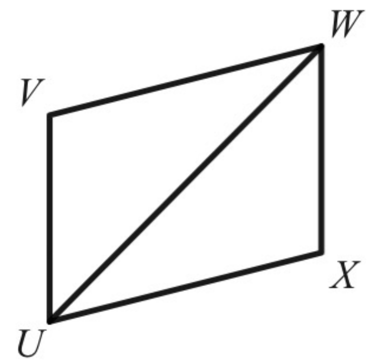
Worked Example	Your Turn
 a) Which two shapes are congruent? b) Which two shapes are similar?	 a) Which two shapes are congruent? b) Which two shapes are similar?

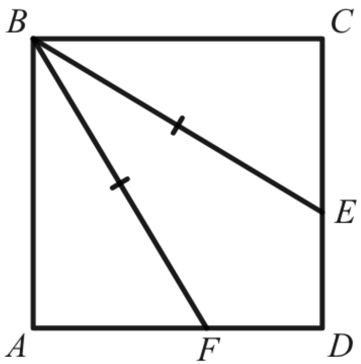
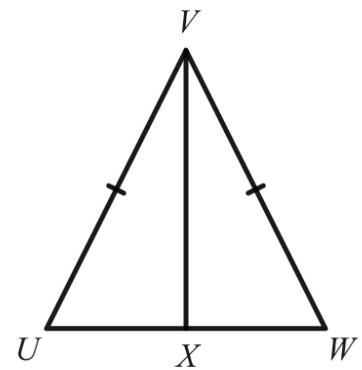
Dr Frost 290a and 291d

Worked Example	Your Turn
State the condition why these two triangles are congruent.  	State the condition why these two triangles are congruent. 

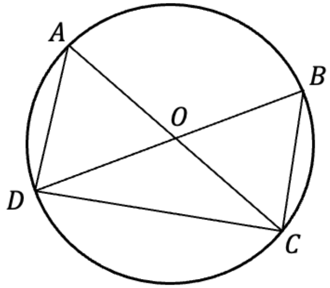
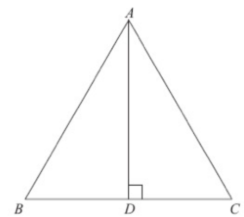
Worked Example	Your Turn
Below is a quadrilateral $ABCD$.  Prove that triangle ABD is congruent to triangle ACD.	The diagram shows triangle UVW.  $UXYZ$ is a parallelogram where X is the midpoint of UV, Y is the midpoint of VW, and Z is the midpoint of UW. Prove that triangle XVY and triangle ZYW are congruent.

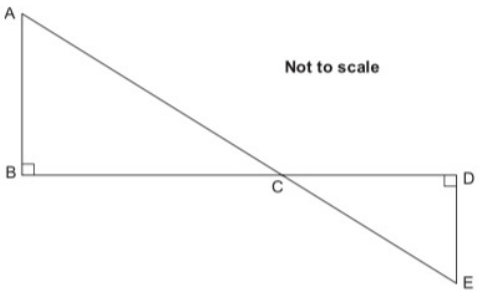
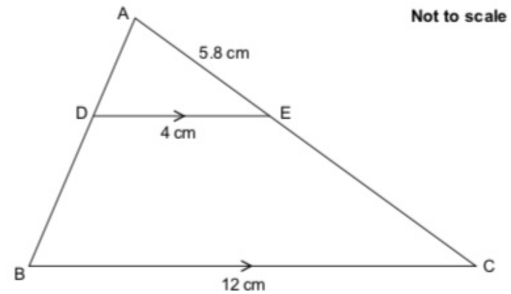
Worked Example	Your Turn
In the diagram below, $ABCE$ and $BCDF$ are parallelograms and N is the midpoint of CE.  Prove that triangle BCN and triangle EFN are congruent.	In the diagram below, UWY and VWX are straight lines.  UV and XY are parallel. W is the midpoint of UWY. Prove that triangle UVW and triangle WXY are congruent.

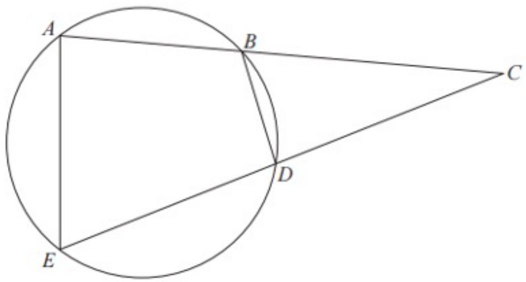
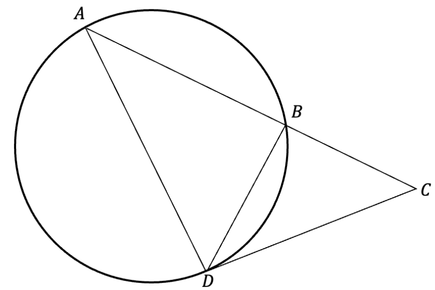
Worked Example	Your Turn
Below is a quadrilateral $ABCD$.  Prove that triangle ABF and triangle BCE are congruent.	The diagram below shows a parallelogram $UVWX$.  Prove that triangle UVW and triangle UWX are congruent.

Worked Example	Your Turn
Below is a square $ABCD$.  $BE = BF$ Prove that triangle ABF and triangle BCE are congruent.	Below is an isosceles triangle UVW.  VX is the perpendicular bisector of UW. Prove that triangle UVX and triangle VWX are congruent.

Dr Frost 471d

Worked Example	Your Turn
AOC and BOD are diameters of the circle with centre O. a) Prove that triangle ABD and triangle BCD are congruent. b) Show that $AD = BC$ 	ABC is an equilateral triangle. D lies on BC AD is perpendicular to BC a) Prove that triangle ADC is congruent to triangle ADB b) Hence prove that $BD = \frac{1}{2}AB$ 

Worked Example	Your Turn
In the diagram below, AE and BD are straight lines. Show that triangles ABC and EDC are similar. 	In the diagram BC is parallel to DE. Prove that triangle ABC is similar to triangle ADE. 

Worked Example	Your Turn
A, B, D and E are points on a circle. ABC and EDC are straight lines. Prove that triangle BCD is similar to triangle ECA You must give reasons for your working. 	The line CD is tangent to the circle. Prove that BCD is similar to ABC. 

Extra Notes

4 Circle Theorem Proofs

Worked Example

Prove angles in a semicircle are 90°.

Worked Example

Prove the angle at the centre of a circle is twice the angle at the circumference.

Worked Example

Prove angles in the same segment are equal.

Worked Example

Prove opposite angles of a cyclic quadrilateral add to 180°.

Worked Example

Prove the alternate segment theorem.

Extra Notes