KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Mathematics (L2FM) 2026

Unit 25 Tasks – Part 1

DO NOT WRITE INSIDE

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Unit 25 Tasks – Part 2

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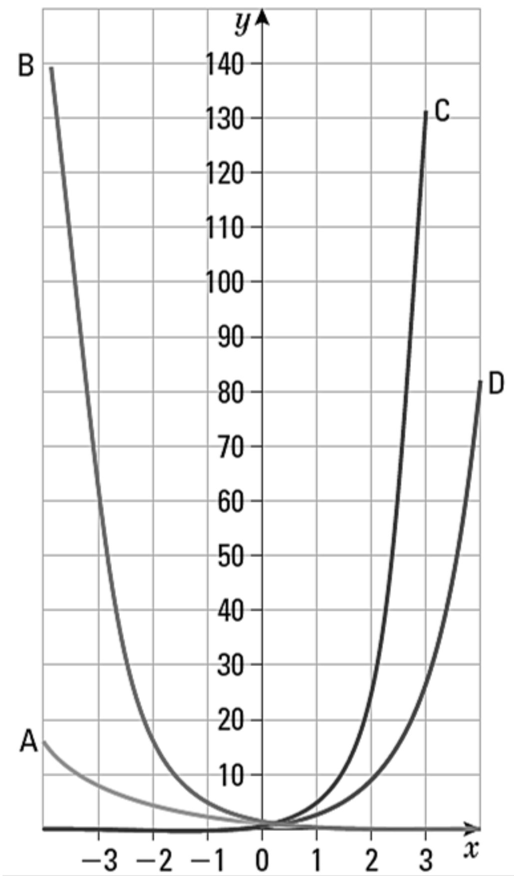
Contents

1 Exponential and Trigonometric Graphs

Purposeful Practice

3. The diagram shows the graphs of $y = 3^x$, $y = 2^{-x}$, $y = 5^x$ and $y = \left(\frac{1}{4}\right)^x$.



Match each graph to its equation.

4. The number of rabbits, n, in a particular population grows at a rate given by the equation $n = 5 \times 2^y$ where y is the number of years.

a) How many rabbits were there initially (when $y = 0$)?

b) How many rabbits are there after 6 years?

c) How many years will it take for the rabbit population to exceed 5000?

Purposeful Practice

(a) Sketch the graph of $y = 2^x$, marking the coordinates of any points that cross the axes.
(b) Sketch the graph of $y = 0.5^x$, marking the coordinates of any points that cross the axes.

The exponential growth of a bat population can be described by the equation $P = 20 \times 1.2^t$, where P is the population at time t in months.
(a) What is the initial bat population?
(b) Calculate the population of bats after 6 months.
(c) What is the percentage increase in the bat population per month?
(d) Sketch the graph of the bat population over time, marking the coordinates of any points where the graph crosses the axes.

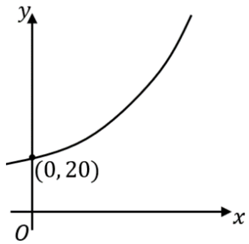
A radioactive element decays according to the equation $m = 500 \times 0.5^t$ where m is the mass of the element in kg and t is the time in days.
(a) What is the initial mass of the radioactive element?
(b) What is the mass of the element after 2 days?
(c) What is the mass of the element after 15 days? Give your answer in grams to 1 decimal place.
(d) What is the half-life of the element? The half-life is the time it takes to decay to half its original mass.
(e) Sketch the graph of the mass against time.

Fluency Practice

Finding the Equation of Exponential Graphs

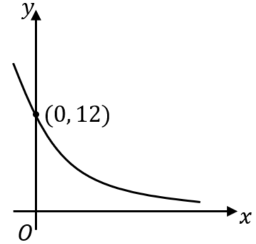
(a)

The curve has equation $y = ab^x$. Write down the value of a.



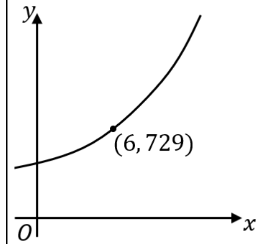
(b)

The curve has equation $y = ab^{-x}$. Write down the value of a.



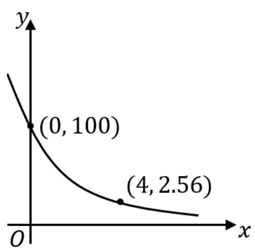
(c)

The curve has equation $y = k^x$. Find the value of k.



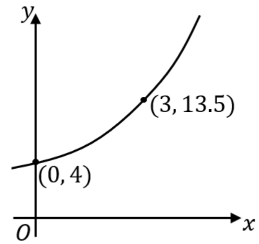
(d)

The curve has equation $y = ab^{-x}$. Find the values of a and b, where $b > 0$



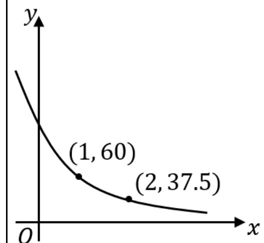
(e)

The curve has equation $y = ab^x$. Find the values of a and b.



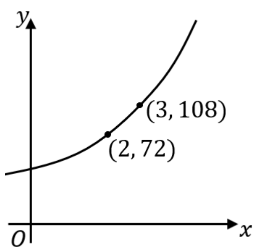
(f)

The curve has equation $y = ab^{-x}$. Find the values of a and b.



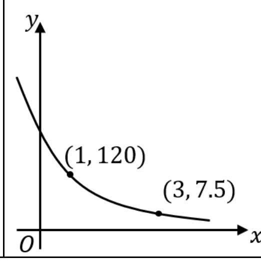
(g)

The curve has equation $y = ab^x$. Find the values of a and b.



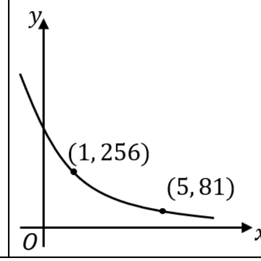
(h)

The curve has equation $y = ab^{-x}$. Find the values of a and b, where $b > 0$



(i)

The curve has equation $y = ab^{-x}$. Find the values of a and b, where $b > 0$



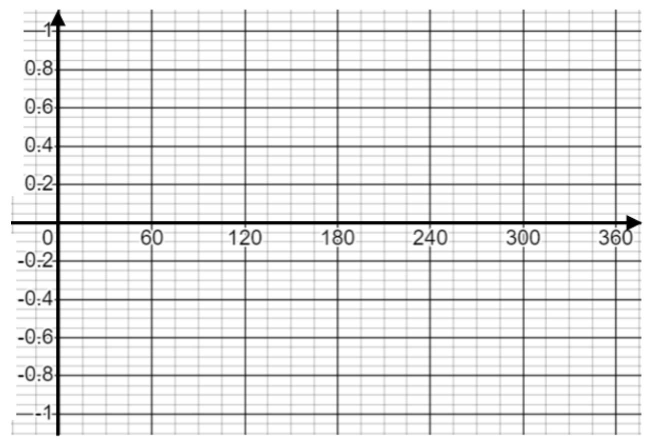
Fluency Practice

Plotting Trigonometric Graphs

$y = \sin x$

x	y
0	
30	
60	
90	
120	
150	
180	

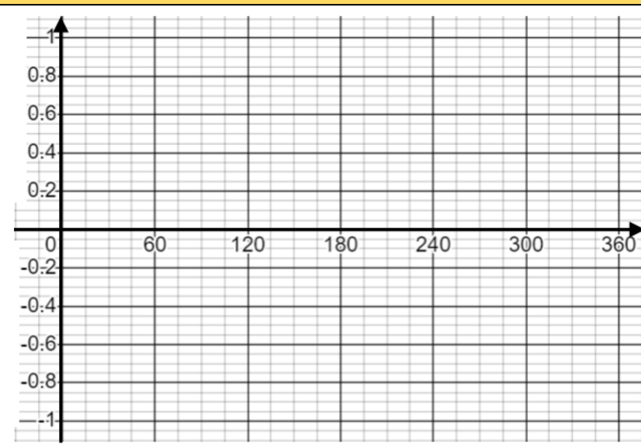
x	y
210	
240	
270	
300	
330	
360	



$y = \cos x$

x	y
0	
30	
60	
90	
120	
150	
180	

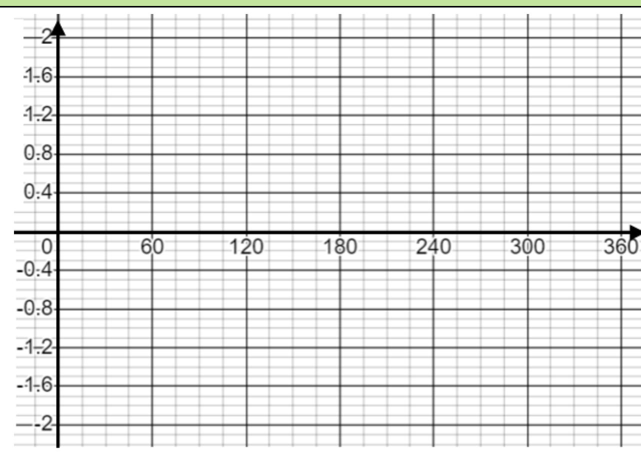
x	y
210	
240	
270	
300	
330	
360	



$y = \tan x$

x	y
0	
30	
60	
90	
120	
150	
180	

x	y
210	
240	
270	
300	
330	
360	

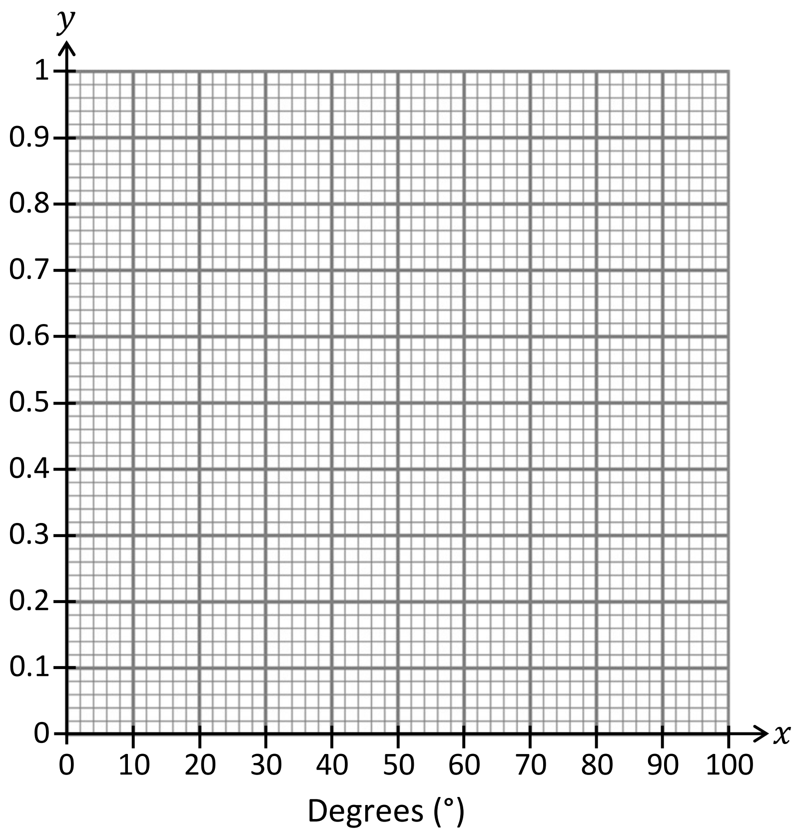


Fluency Practice

Graphing the Sine & Cosine Trigonometric Functions

Angle, x	$\sin(x)$
0°	
10°	
20°	
30°	
40°	
50°	
60°	
70°	
80°	
90°	
100°	

Angle, x	$\cos(x)$
0°	
10°	
20°	
30°	
40°	
50°	
60°	
70°	
80°	
90°	
100°	



Sketch & label the graphs:

$y = \sin x$

&

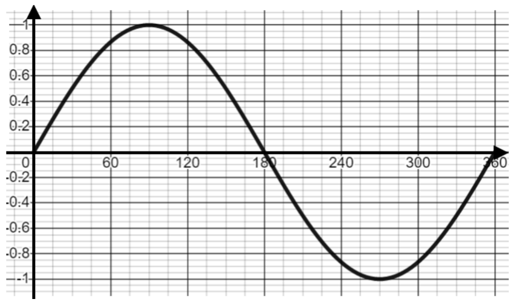
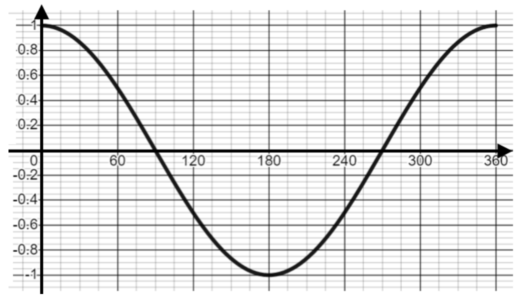
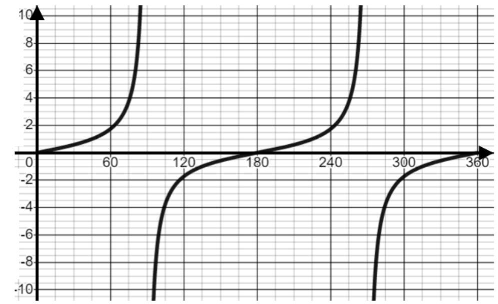
$y = cos\, x$

When does $\sin x = \cos x$? Why?

Fluency Practice

Solving Trigonometric Equations Using Graphs

Use your calculator and the trigonometric graphs to find all values of the angle between 0° and 360°

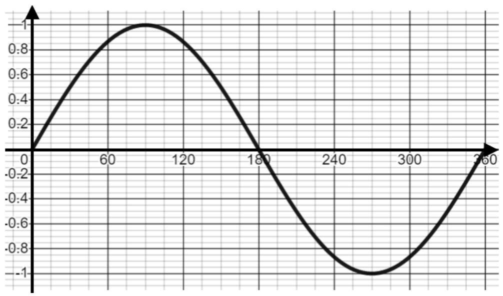
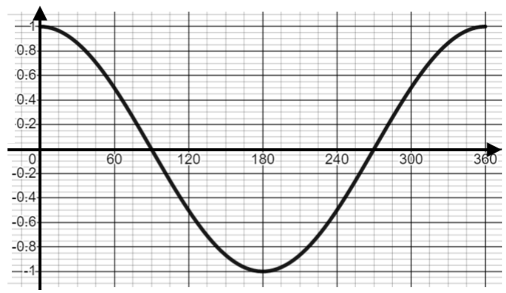
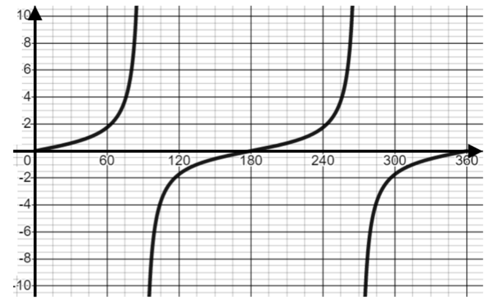
$y = \sin x$	$y = \cos x$	$y = \tan x$
		

$\sin x = 0.3$	$\cos x = 0.9$	$\tan x = 3.2$	$\cos x = 0.6$
$\sin x = 0.8$	$\cos x = 0.15$	$\tan x = 8$	$\sin x = 0.43$
$\cos x = 0.37$	$\sin x = 0.285$	$\tan x = 0$	$\cos x = 1$
$\sin x = 0.5$	$\tan x = 1$	$\sin x = 1$	$\cos x = 0$

Fluency Practice

More Solving Trigonometric Equations Using Graphs

Use your calculator and the trigonometric graphs to find all values of the angle between 0° and 360°

$y = \sin x$	$y = \cos x$	$y = \tan x$
		

$\sin x = -0.4$	$\cos x = -0.1$	$\tan x = -3.7$	$\cos x = -0.65$
$\sin x = -0.88$	$\cos x = -0.25$	$\tan x = -6$	$\sin x = -0.97$
$\cos x = -0.31$	$\sin x = -0.745$	$\tan x = -2.3$	$\cos x = -0.523$
$\sin x = -0.5$	$\tan x = -1$	$\sin x = -1$	$\cos x = -1$

Fluency Practice

1 Solve the following in the range $0 \leq x \leq 360$

a) $\sin(x) = 0.5 \rightarrow \boldsymbol{x} =$?

b) $\cos(x) = \frac{\sqrt{3}}{2} \rightarrow \boldsymbol{x} =$?

c) $\tan(x) = \sqrt{3} \rightarrow \boldsymbol{x} =$?

d) $\sin(x) = 0.1 \rightarrow \boldsymbol{x} =$?

e) $4\cos(x) = 3 \rightarrow$?

f) $6\tan(x) = 5 \rightarrow$?

g) $\sin(x) = 0.4 \rightarrow$?

2 Solve the following in the range $0 \leq x \leq 360$

a) $\sin(\theta) = -0.5 \rightarrow$?

b) $\cos(\theta) = -0.5 \rightarrow$?

c) $\tan(\theta) = -1 \rightarrow$?

d) $\sin(\theta) = -0.4 \rightarrow$?

e) $\cos(\theta) = -0.7 \rightarrow$?

f) $\tan(\theta) = -0.2 \rightarrow$?

Fluency Practice

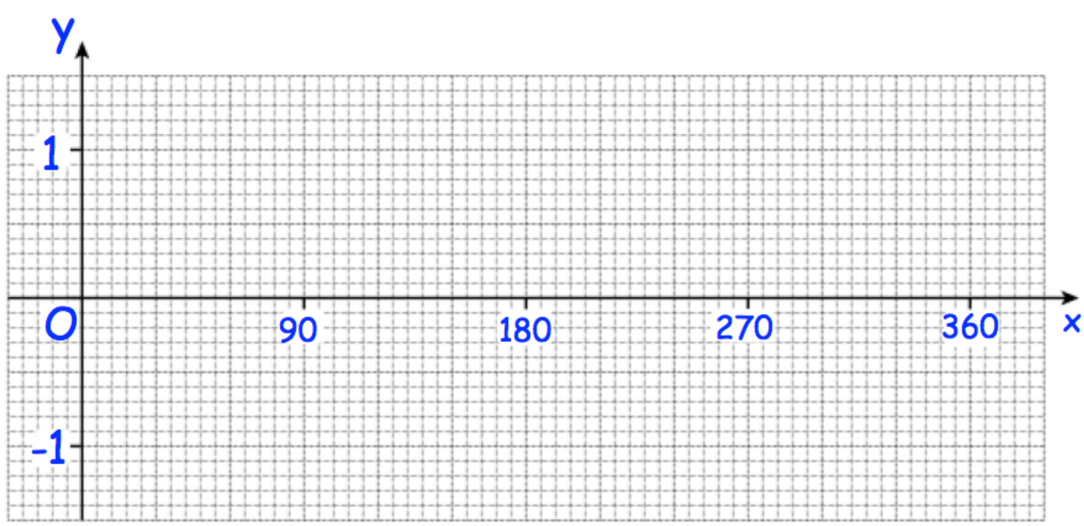
Question 1: (a) Complete the tables below for $y = sin(x)$

x	0°	30°	45°	60°	90°	120°
y						

x	135°	150°	180°	210°	225°	240°
y						

x	270°	300°	315°	330°	360°
y					

(b) Plot the points and draw the graph of $y = sin(x)$



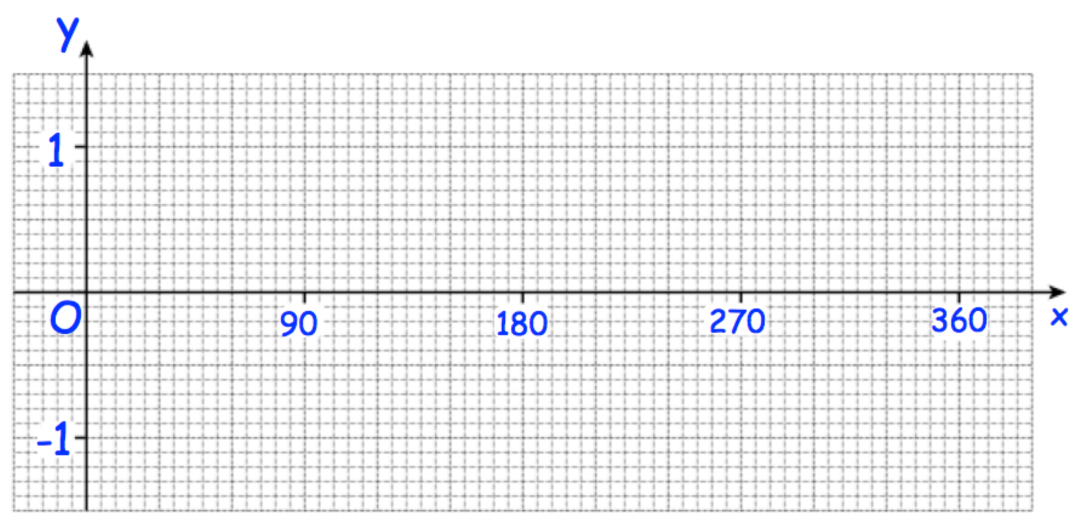
Question 2: (a) Complete the tables below for $y = cos(x)$

x	0°	30°	45°	60°	90°	120°
y						

x	135°	150°	180°	210°	225°	240°
y						

x	270°	300°	315°	330°	360°
y					

(b) Plot the points and draw the graph of $y = cos(x)$



Question 3: (a) Complete the tables below for $y = tan(x)$

x	0°	1°	15°	30°	45°	60°
y						

x	75°	89°	90°	91°	105°	120°
y						

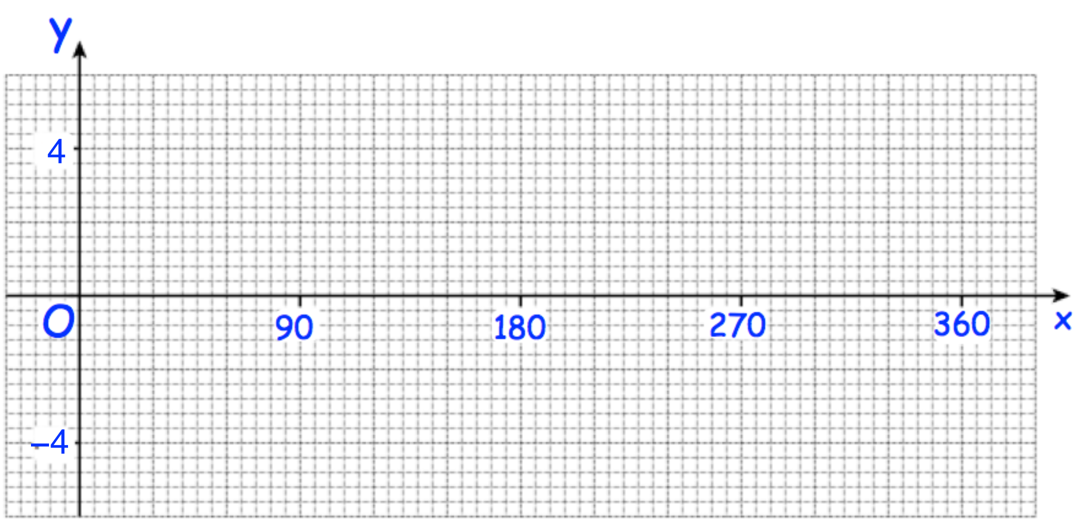
x	135°	150°	165°	179°	180°	181°
y						

x	195°	210°	225°	240°	255°	269°
y						

x	270°	271°	285°	300°	315°	330°
y						

x	345°	360°
y		

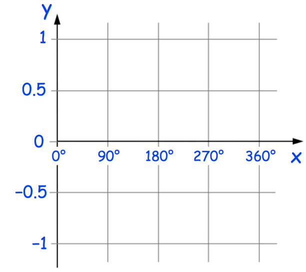
(b) Plot the points and draw the graph of $y = tan(x)$



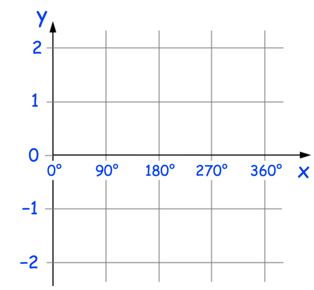
Purposeful Practice

Question 4: Sketch the graph of $y = cos(x)$ for $0^\circ \leq x \leq 360^\circ$

Question 5: Sketch the graph of $y = sin(x)$ for $0^\circ \leq x \leq 360^\circ$



Question 6: Sketch the graph of $y = tan(x)$ for $0^\circ \leq x \leq 360^\circ$



Apply

Question 1: Which of these values cannot be the sine of an angle?

0   −0.9   ⅔   1.2

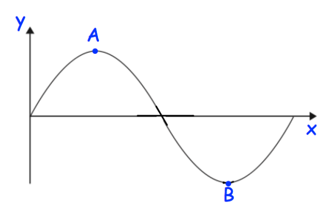
Question 2: Which of these values cannot be the cosine of an angle?

−1   3   0.7   −0.04

Question 3: Here is part of the curve $y = sin(x)$

(a) Write down the coordinates of the point A

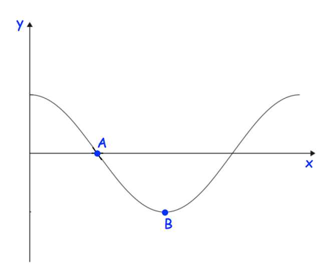
(b) Write down the coordinates of the point B



Question 4: Here is part of the curve $y = cos(x)$

(a) Write down the coordinates of the point A

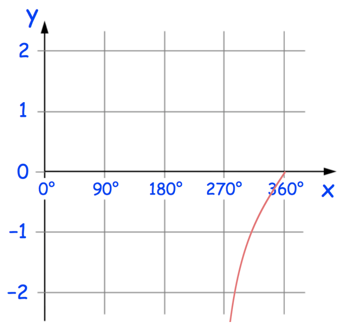
(b) Write down the coordinates of the point B



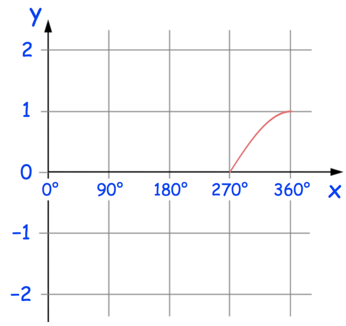
Purposeful Practice

Question 5: Here are three graphs for $270^{\circ} \leq x \leq 360^{\circ}$

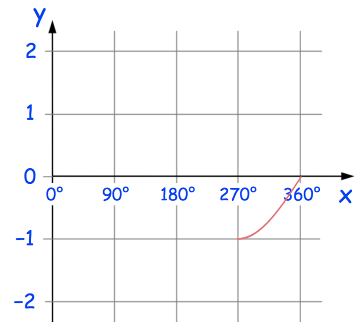
Graph 1



Graph 2



Graph 3



(a) Which graph is $y = sin(x)$?

(b) Which graph is $y = cos(x)$?

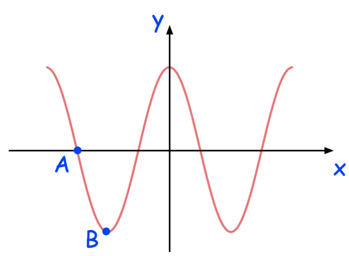
(c) Which graph is $y = tan(x)$?

Question 6: Write down the coordinates of the maximum point of $y = sin(x)$ for $180^{\circ} \leq x \leq 540^{\circ}$

Question 7: Write down the coordinates of the minimum point of $y = sin(x)$ for $360^{\circ} \leq x \leq 720^{\circ}$

Question 8: Write down the coordinates of the minimum point of $y = cos(x)$ for $360^{\circ} \leq x \leq 720^{\circ}$

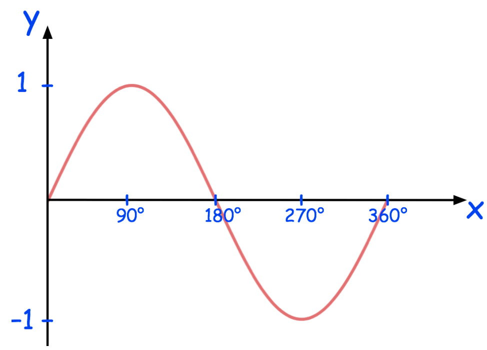
Question 9: Here is a sketch of $y = cos(x)$ for $-360^{\circ} \leq x \leq 360^{\circ}$



(a) Write down the coordinates of the point A

(b) Write down the coordinates of the point B

Question 10: Here is the graph of $y = sin(x)$ for $0^\circ \leq x \leq 360^\circ$

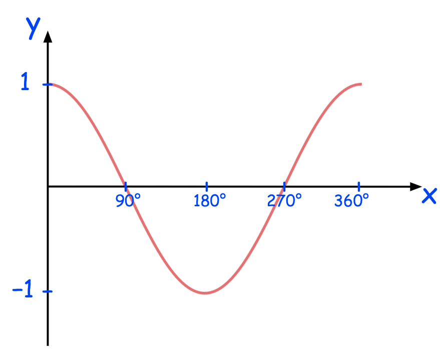


One solution of $sin(x^\circ) = -0.5$ is $x = 210^\circ$

(a) Find another solution of $sin(x^\circ) = -0.5$ for $0^\circ \leq x \leq 360^\circ$

(b) Find the solutions of $sin(x^\circ) = 0.5$ for $0^\circ \leq x \leq 360^\circ$

Question 11: Here is a sketch of $y = cos(x)$ for $0^\circ \leq x \leq 360^\circ$



$cos(x) = cos(30^\circ)$

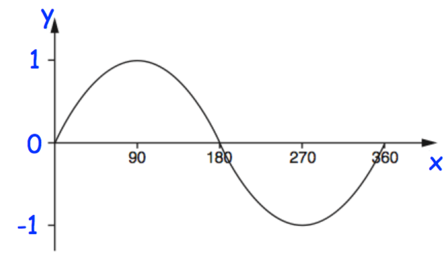
(a) Work out the value of x when $90^\circ \leq x \leq 360^\circ$

$cos(x) = -cos(30^\circ)$

(b) Find the two values of x for $0^\circ \leq x \leq 360^\circ$

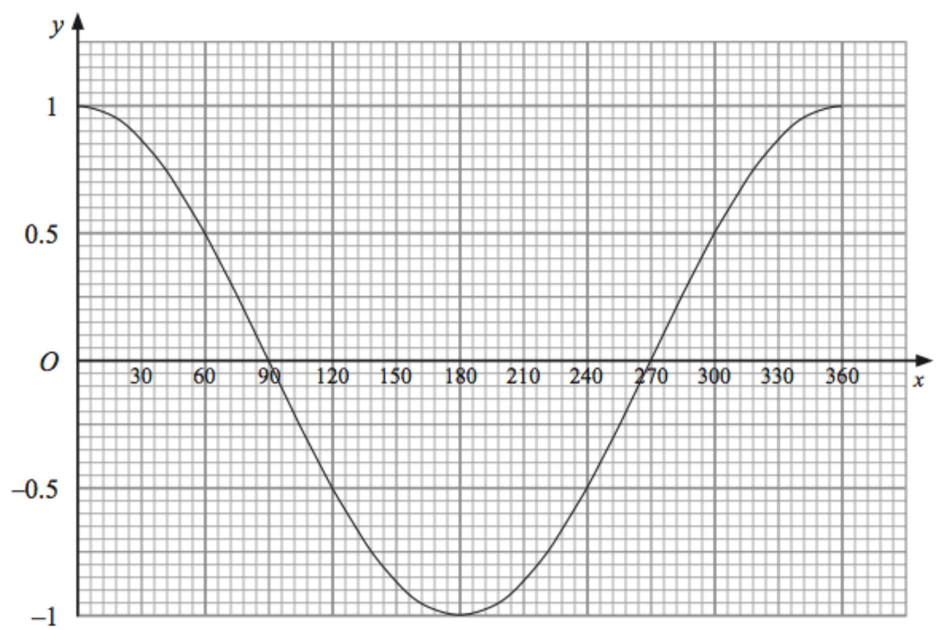
Purposeful Practice

Question 12: Here is the graph of $y = sin(x)$ for $0 \leq x \leq 360$



One solution of $sin x = -0.5$ is $x = 330^\circ$
Find another solution of $sin x = -0.5$

Question 13: Here is the graph of $y = cos(x)$



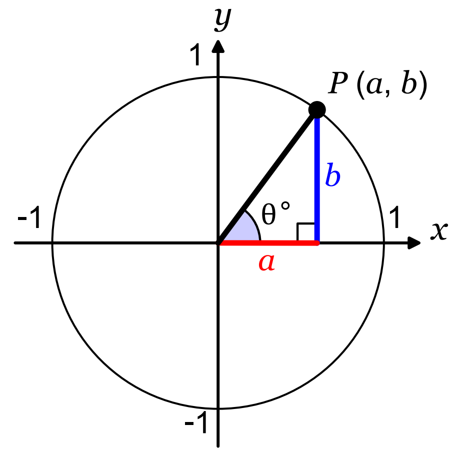
(a) Use the graph to solve $cos(x) = 0.75$

(b) Use the graph to solve $cos(x) = -0.75$

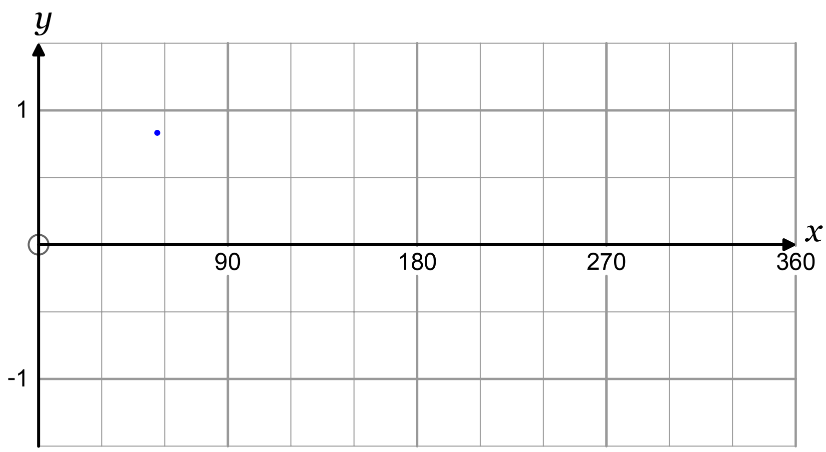
trigonometric graphs

The unit circle is centered on the origin and has a radius of 1.

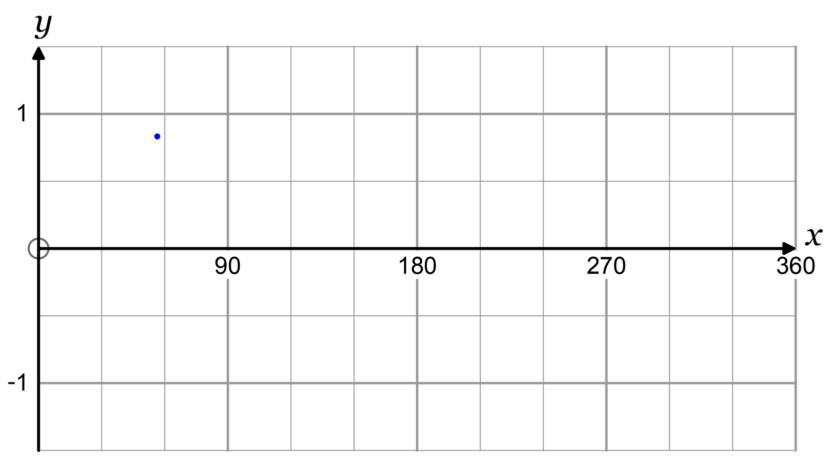
1. Work out a and b in terms of θ.



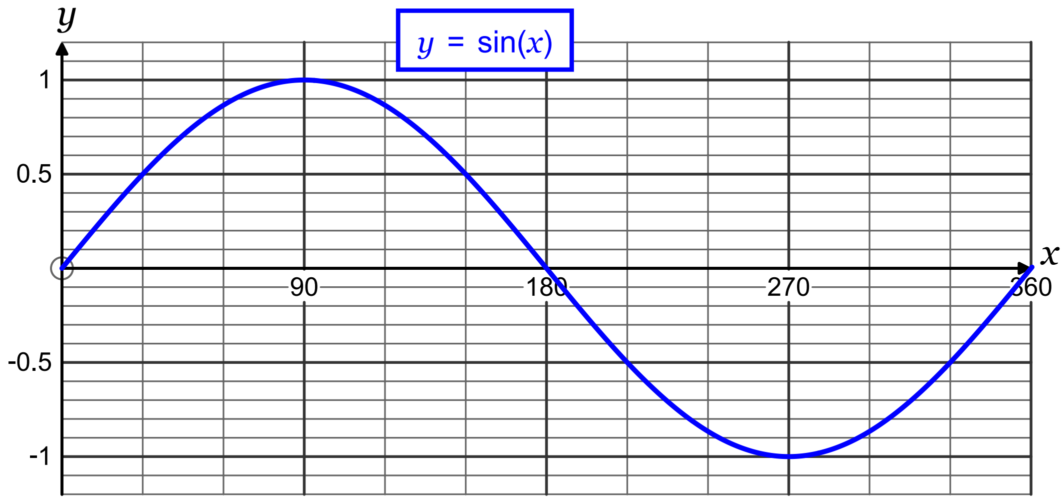
2. Sketch the graph of $y = \sin(x)$ for $0 \leq x \leq 360°$.



3. Sketch the graph of $y = \cos(x)$ for $0 \leq x \leq 360°$.



the sine graph



1. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place.

a) $\sin(x) = 0.7$ b) $\sin(x) = 0.4$

c) $\sin(x) = -0.3$ d) $\sin(x) = -0.8$

2. Solve for $0 \leq x \leq 360°$. Give your answers to 1 decimal place where necessary.

a) $\sin(x) = 0.55$ b) $\sin(x) = -0.9$ c) $\sin(x) = -0.5$

d) $\sin(x) = 1$ e) $\sin(x) = \frac{\sqrt{3}}{2}$ f) $\sin(x) = 0$

g) $2\sin(x) = 1$ h) $\sin(x) = 0.95$ i) $\sin(x) = -\frac{1}{3}$

3. a) Given $\sin(40°) = 0.643$, complete: $\sin(140°) =$ _______

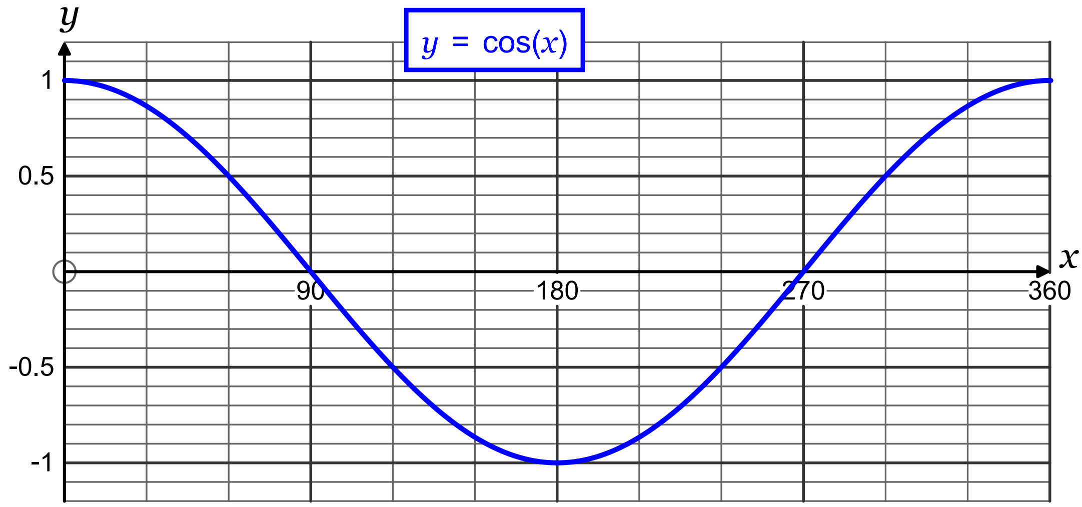
b) Given $\sin(25°) = 0.423$, complete: $\sin(205°) =$ _______

c) Given $\sin(165°) = 0.259$, complete: $\sin(15°) =$ _______

d) Given $\sin(315°) = -0.707$, complete: $\sin(45°) =$ _______

Fluency Practice

the cosine graph



1. Solve for $0 \le x \le 360°$. Give your answers to 1 decimal place.

a) $\cos(x) = 0.75$ b) $\cos(x) = 0.2$

c) $\cos(x) = -0.6$ d) $\cos(x) = -0.35$

2. Solve for $0 \le x \le 360°$. Give your answers to 1 decimal place where necessary.

a) $\cos(x) = 0.45$ b) $\cos(x) = -0.08$ c) $\cos(x) = 0.6$

d) $\cos(x) = -1$ e) $\cos(x) = \frac{1}{2}$ f) $\cos(x) = 0$

g) $4\cos(x) = 1$ h) $\cos(x) = -0.65$ i) $\cos(x) = \frac{7}{8}$

3. a) Given $\cos(25°) = 0.906$, complete: $\cos(335°) =$ _______

b) Given $\cos(80°) = 0.174$, complete: $\cos(100°) =$ _______

c) Given $\cos(160°) = -0.940$, complete: $\cos(20°) =$ _______

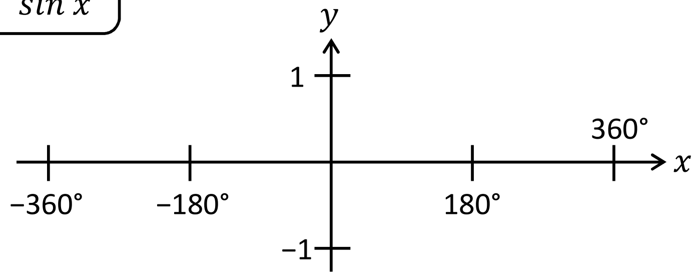
d) Given $\cos(235°) = -0.574$, complete: $\cos(125°) =$ _______

Trigonometry: Angles Between −360° and 360°

Input the values into your calculator to find coordinates and sketch these graphs.

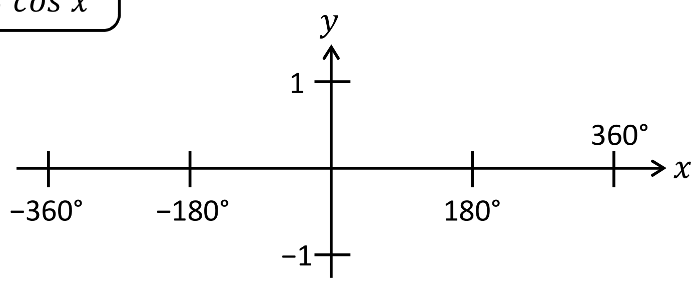
$y = \sin x$

1) sin 0° =
2) sin 90° =
3) sin 180° =
4) sin 270° =
5) sin 360° =
6) Use your own values to sketch the graph for $-360° \leqslant x \leqslant 0°$



$y = \cos x$

1) cos 0° =
2) cos 90° =
3) cos 180° =
4) cos 270° =
5) cos 360° =
6) Use your own values to sketch the graph for $-360° \leqslant x \leqslant 0°$



Using the graphs, and **without** using a calculator, answer these questions.

A) sin 30° = 0.5

For what 3 other values of x, between −360° and 360°, does y = 0.5?
(Sketch y = 0.5 on your graph. Find where it crosses the sin curve.)

1) ___________ 2) ___________ 3) ___________

B) sin −20° = −0.34

For what 3 other values of x, between −360° and 360°, does y = −0.34?

1) ___________ 2) ___________ 3) ___________

3) Use the cosine graph to complete this table.

y = 0.5			cos 60°	
y = 0.77				cos 320°
y = −0.94		cos −160°		
y = −0.09	cos −265°			

Fluency Practice

In each box, cross off pairs that are **equal in value**. Angles are in degrees.
Circle the value that is left over.

A

sin90	$\frac{1}{\sqrt{3}}$	sin30
tan30	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$
1	cos45	0

B

$\sqrt{3}$	tan60	1
cos0	sin60	0.5
0	$\frac{\sqrt{3}}{2}$	tan180

C

$\frac{1}{\sqrt{2}}$	tan45	sin60
cos60	$\frac{1}{2}$	1
cos30	sin45	tan90

D

sin50	sin40	sin100
sin150	sin60	sin130
sin140	sin80	sin30

E

cos50	cos260	cos10
cos250	cos110	cos130
cos310	cos100	cos350

F

tan15	tan275	tan165
tan105	tan195	tan135
tan315	tan95	tan285

G

sin150	1	$-\frac{1}{2}$
cos240	-1	$\frac{1}{2}$
cos180	$-\frac{\sqrt{3}}{2}$	tan225

H

$-\frac{1}{2}$	$\frac{1}{2}$	sin210
sin60	$-\frac{\sqrt{3}}{2}$	tan135
-1	cos210	$\frac{\sqrt{3}}{2}$

I

sin90	$-\frac{\sqrt{3}}{2}$	sin45
$\frac{\sqrt{3}}{2}$	tan45	cos120
cos45	$-\frac{1}{2}$	sin300

J

$-\frac{\sqrt{3}}{2}$	cos(-45)	$\frac{1}{2}$
cos135	cos45	cos(-40)
cos210	cos40	cos(-60)

K

sin(-30)	sin270	-sin45
$-\frac{\sqrt{3}}{2}$	sin(-60)	sin30
sin(-45)	sin150	sin(-90)

L

tan(-45)	-tan(-75)	tan45
tan(-15)	tan330	tan(-30)
tan75	-tan15	tan135

Purposeful Practice

Explain how we can know if these are true or false.

Use the circle and your graphs to help you decide.

1. $cos\emptyset = \cos(360 - \emptyset)$
2. $sin\emptyset = \sin(360 - \emptyset)$
3. $tan\emptyset = \tan(360 - \emptyset)$

4. $cos\emptyset = \cos(180 - \emptyset)$
5. $sin\emptyset = \sin(180 - \emptyset)$
6. $tan\emptyset = \tan(180 - \emptyset)$

7. $cos\emptyset = \cos(180 + \emptyset)$
8. $sin\emptyset = \sin(180 + \emptyset)$
9. $tan\emptyset = \tan(180 + \emptyset)$

10. $cos\emptyset = \cos(-\emptyset)$
11. $sin\emptyset = \sin(-\emptyset)$
12. $tan\emptyset = \tan(-\emptyset)$

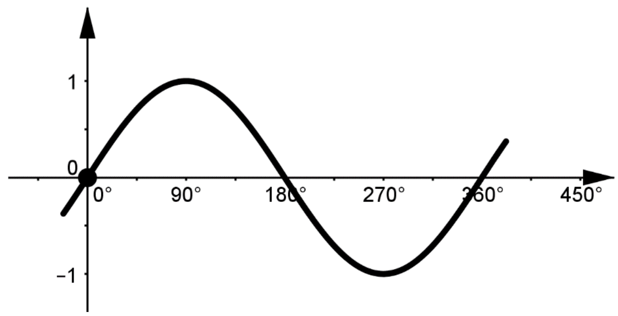
13. $cos\emptyset = -\cos(-\emptyset)$
14. $sin\emptyset = -\sin(-\emptyset)$
15. $tan\emptyset = -\tan(-\emptyset)$

16. $\sin(\emptyset + 90) = \cos(\emptyset)$
17. $\sin(\emptyset - 90) = \cos(\emptyset)$
18. $\sin(90 - \emptyset) = \cos(\emptyset)$

Symmetry in Trigonometric Graphs

NON-CALCULATOR TRIGONOMETRY

1. The graph shows part of the function $y = \sin x$. Decide whether each statement is true or false, explaining your thinking by referring to the graph.



a) $\sin(450°) = 1$ b) $\sin(900°) = 2$ c) $\sin(280°) > \sin(80°)$

d) $\sin(20°) = \sin(160°)$ e) $\sin(45°) = -\sin(315°)$ f) $\sin(-10) < \sin(190)$

2. a) Sketch the graph of $y = \cos x$ for $0 \le x \le 360$.
   b) Use the graph to help you list these values in order, starting with the smallest:

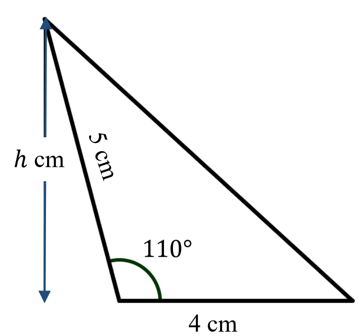
$\cos(75°)$ $\cos(90°)$ $\cos(180°)$ $\cos(330°)$

3. Given that $\sin(30°) = \frac{1}{2}$ and $\cos(30°) = \frac{\sqrt{3}}{2}$, state the exact values of

a) $\sin(150°)$ b) $\sin(330°)$ c) $\cos(390°)$

d) $\sin(-90°)$ e) $\cos(210°)$ f) $\cos(-210°)$

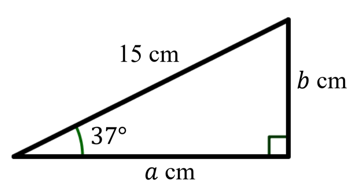
4. a) Which of these expressions can be used to calculate the height h cm of this triangle?



   i. $5\sin(70°)$
   ii. $\sqrt{5^2 + 4^2}$
   iii. $5\sin(110°)$
   iv. $5\cos(20°)$
   iv. $4\tan(35°)$

   b) Hence or otherwise find the area of the triangle given that $\sin(70°)$ to two decimal places is 0.94.

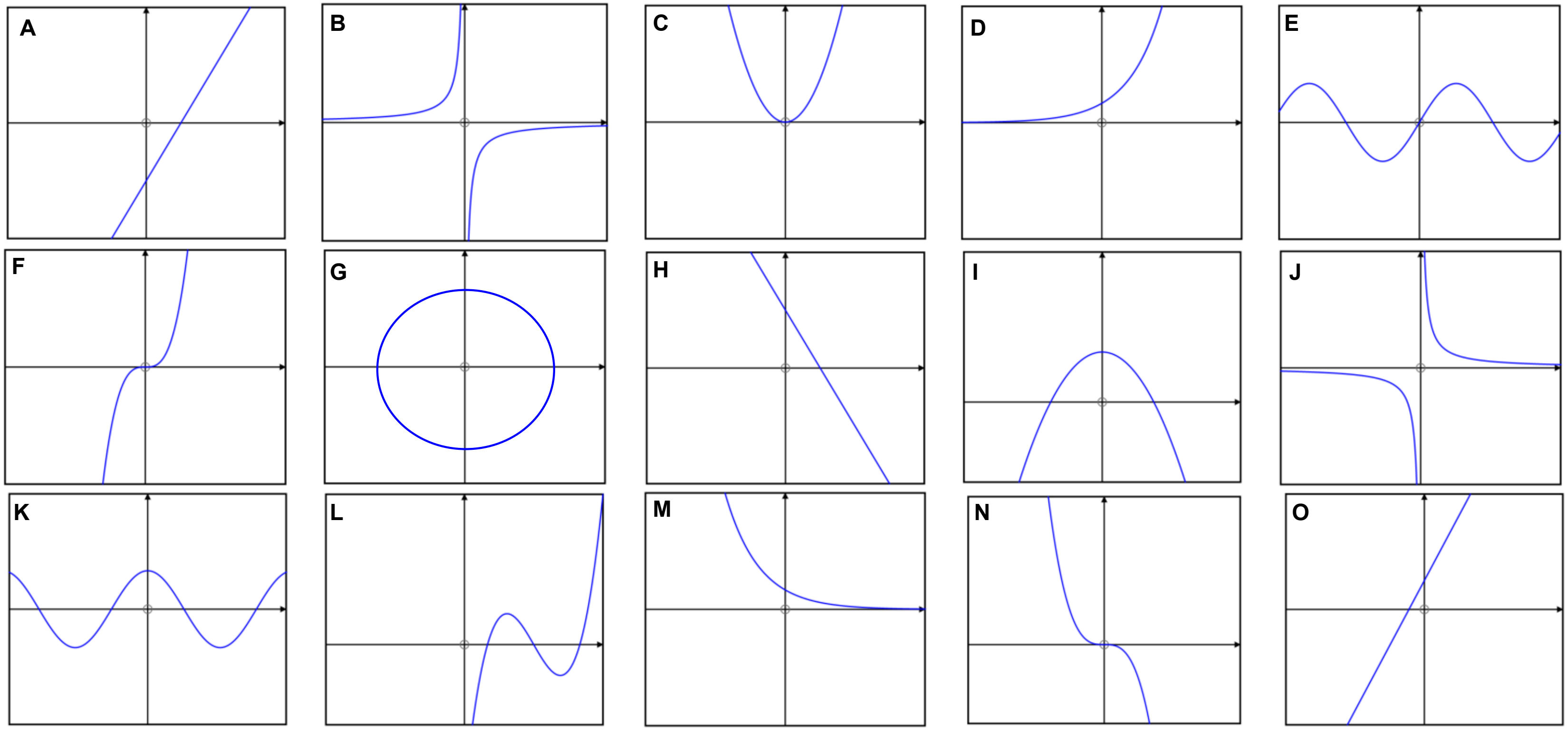
5. Given that the hypotenuse of this right-angled triangle is 15 centimetres and $\cos(143°) = -0.80$ (to two significant figures), find the lengths of the other two sides.

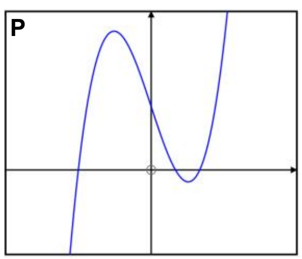


Fluency Practice

recognising graphs

Match these graphs to their equations at the bottom.



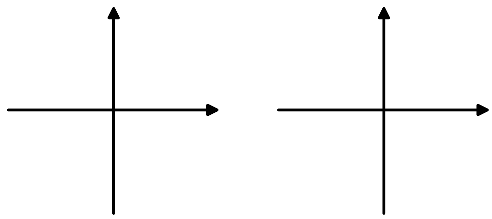


$y = 3x + 2$	$y = x^3$	$y = 3 - 2x$	$y = (x-2)(x-1)(x+3)$	
$y = x^2$	$y = 0.5^x$	$y = 5 - x^2$	$y = (x-5)(x-3)(x-1)$	
$y = \cos(x)$	$y = -x^3$	$y = \frac{-1}{x}$	$y = \frac{1}{x}$	$y = 2x - 3$
$y = \sin(x)$	$y = 2^x$			$x^2 + y^2 = 16$

Fluency Practice

1. Type:

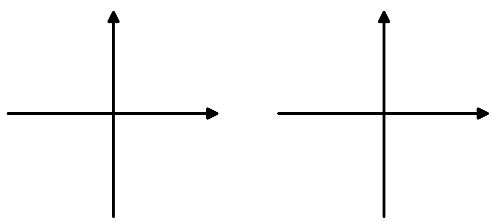
Shapes:



Equation forms:

2. Type:

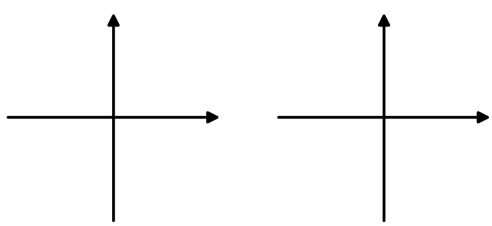
Shapes:



Equation forms:

3. Type:

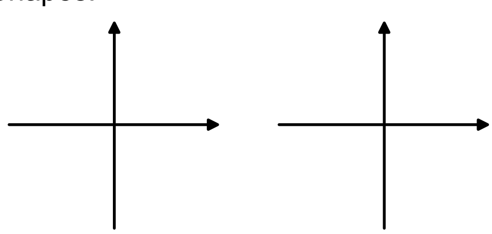
Shapes:



Equation forms:

4. Type:

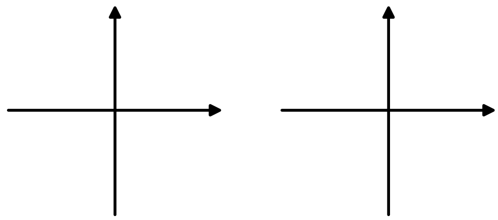
Shapes:



Equation forms:

5. Type:

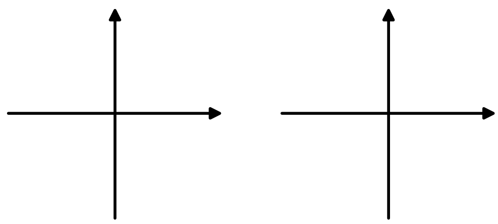
Shapes:



Equation forms:

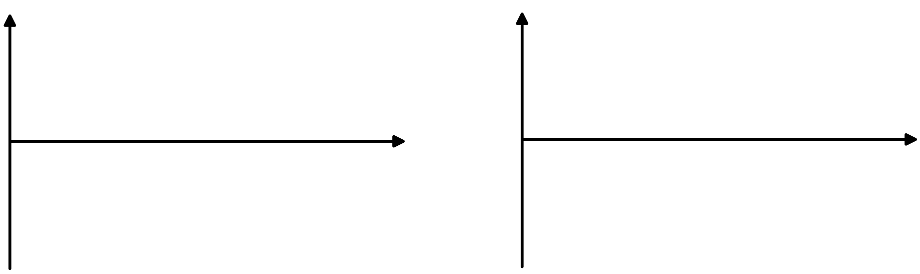
6. Type:

Shapes:

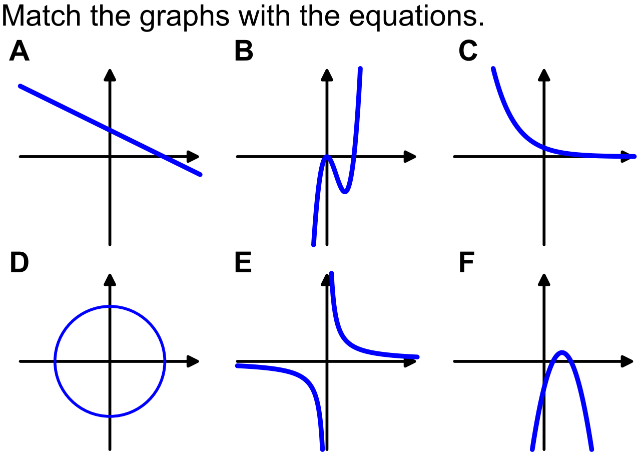


Equation forms:

7. Type:



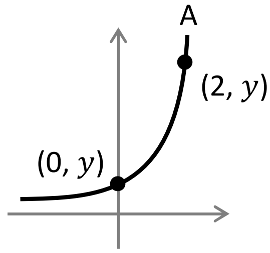
8. Match the graphs with the equations.

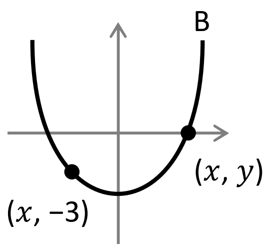


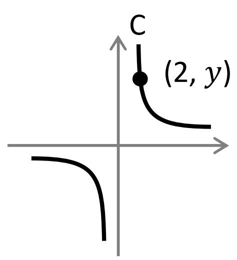
P	$y = x^2(x-3)$
Q	$y = (x-3)(1-x)$
R	$x^2 + y^2 = 49$
S	$y = \frac{5}{x}$
T	$y = 3 - \frac{1}{2}x$
U	$y = 3^{-x}$

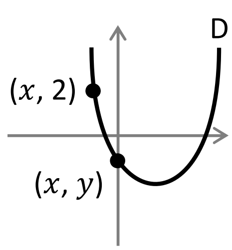
Non-Linear Graph Matchup

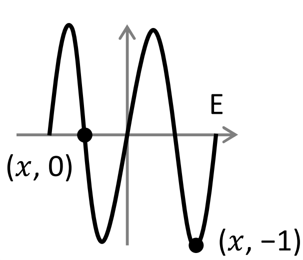
Match each graph to its equation.
Complete the missing coordinates on each graph.











$y = x^2 + 3x$

$y = x^2 - x - 4$

$y = \frac{5}{x}$

$y = \cos x$

$y = 2^x$

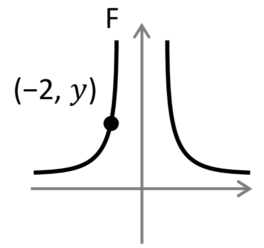
$y = x^3 + 2x$

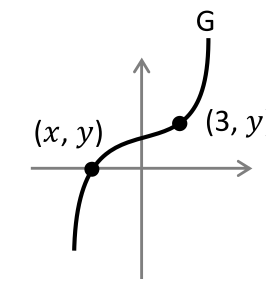
$y = \sin x$

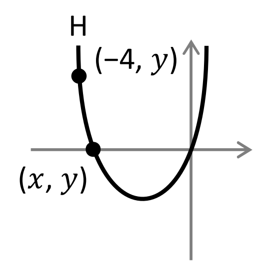
$y = x^2 - 4$

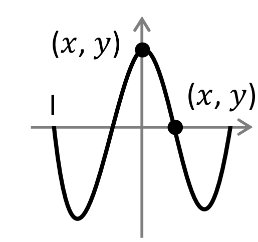
$y = x^3 + 2$

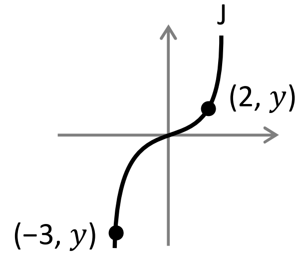
$y = \frac{12}{x^2}$







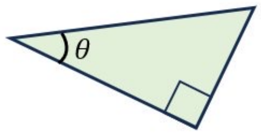




Purposeful Practice

1. Factual Recall

a. Label the triangle below Opposite (O), Adjacent (A), and Hypotenuse (H) with respect to the angle θ



b. Complete the three trigonometric ratios below:

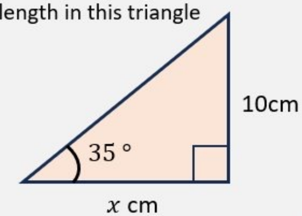
$\sin\theta = \dfrac{opposite}{h\_\_\_\_\_\_\_\_}$

$\cos\theta = \dfrac{a\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_}$

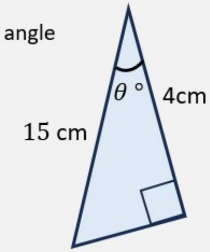
$\_\_\_\_\_\theta = \dfrac{opposite}{adjacent}$

2. Carry out a routine procedure

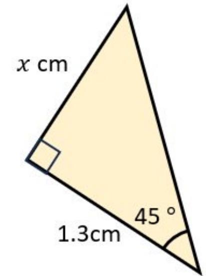
a. Find the missing side length in this triangle



b. Find the missing angle in this triangle

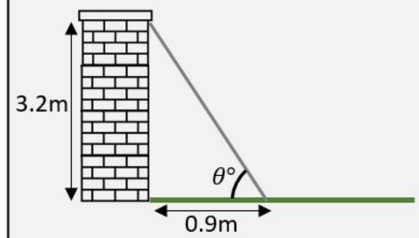


3. Classify a mathematical object



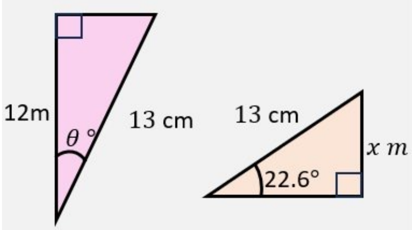
By calculation or otherwise, what type of triangle is this (other than a right angled triangle!) ?

4. Interpret a situation or answer



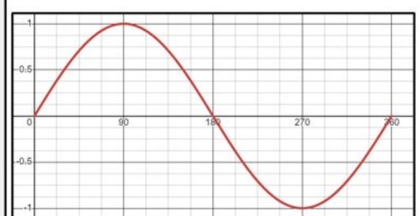
A ladder is placed 0.9m away from the base of a wall. The ladder reaches the top of the 3.2m tall wall.
The angle the ladder makes with the ground needs to be 75° or more to be safe. Is this ladder safe?

5. Prove, Show, or Justify



These two triangles are **congruent.** Explain why we are able to make this claim.

6. Extend a concept

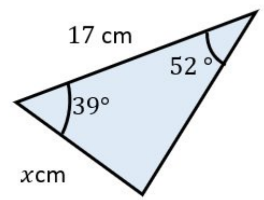


This is the graph of $y = \sin x$.
If you type $\sin(180)$ into your calculator, you will get a result of 0.
Using the graph, can you identify what other values you can put in to $\sin\theta$ to get a result of 0?

7. Construct a scenario

A helicopter leaves the heli-port from the point **H** , and flies 80 miles due East. It then flies 20 miles due North to reach its destination, **D.**
Construct a diagram for this situation, and find the **bearing** of **D** from **H.**

8. Criticise a fallacy



Jamie wants to use SOHCAHTOA to find the missing side length in this triangle. Why can't they?

2 Trigonometric Identities and Equations (L2FM Only)

Fluency Practice

(a) Show that

$$\cos^2\theta - \sin^2\theta \equiv 1 - 2\sin^2\theta$$

(b) Show that $\tan^2\theta \equiv \frac{\sin^2\theta}{1-\sin^2\theta}$

(a) Prove that

$$\frac{1}{\cos x} - \cos x \equiv \sin x \tan x$$

(b) Show that $1 + \tan^2\theta \equiv \frac{1}{\cos^2\theta}$

(a) Prove that

$$\frac{\sin^4 x + \sin^2 x \cos^2 x}{\cos^2 x - 1} \equiv -1$$

(b) Prove that

$$(2\sin x - \cos x)^2 + (\sin x + 2\cos x)^2 \equiv 5$$

(a) Show that

$$\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$$

(b) Show that

$$\frac{1}{\cos^2 x} - 1 \equiv \tan^2 x$$

27

THE GREEK PHILOSOPHER,

1	2	3	4	5	6	5	7	8

SAID THAT EVERYTHING WAS MADE UP OF THE FOUR SUBSTANCES,

8	1	2	5	9	1	3	2

10	3	2	8	1	11	12	13	1	5	8	2

Simplify the expressions below and find the answers in the code table. The letter with each answer and the question number give the code to find the puzzle solution.

1. $\dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta + \cos\theta}$ =

2. $\cos^4\theta - \sin^4\theta$ =

3. $\sin^3\theta + \sin\theta\cos^2\theta$ =

4. $\tan^2\theta + 1$ =

5. $\sin\theta\tan\theta + \cos\theta$ =

6. $\dfrac{\tan\theta\cos\theta}{\sin\theta}$ =

7. $\cos^2\theta\tan\theta\sin\theta + \cos^3\theta$ =

8. $\sin^2\theta + \sin^2\theta\cos\theta + \cos^2\theta + \cos^3\theta$ =

9. $\dfrac{\sin\theta\cos^2\theta}{\cos^3\theta}$ =

10. $(\sin^3\theta + \sin\theta\cos^2\theta)^2$ =

11. $\dfrac{\sin^2\theta - \cos^2\theta}{\sin\theta - \cos\theta}$ =

12. $\dfrac{\sin^2\theta}{\cos^2\theta}$ =

13. $2\cos^2\theta + 2\sin^2\theta$ =

F	$\sin^2\theta$
I	$\sin\theta$
E	$1 + \cos\theta$
L	$\cos\theta$
A	$\sin\theta - \cos\theta$
H	$\tan\theta$
O	1
N	$\cos\theta + \sin\theta$
R	$\cos^2\theta - \sin^2\theta$
S	$\dfrac{1}{\cos^2\theta}$
T	$\dfrac{1}{\cos\theta}$
D	$\tan^2\theta$
W	2

A MEMBER OF THE ANIMAL KINGDOM THAT HAS AN INTERNAL SKELETON.

26

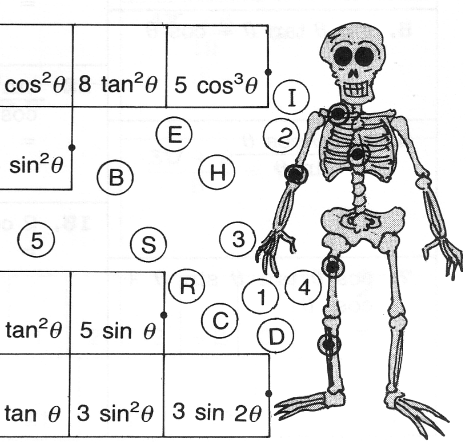
1	2	3	4	5

Simplify the trigonometric expressions. Each answer and the letter next to each question gives the code to find the human bone names. Join the dot next to each bone name to the dot on the bone on the skeleton. Each line will pass through a letter and number giving the puzzle answer code.

M	$2 \sin\theta + 3 \sin\theta =$	$5 \tan\theta - 2 \tan\theta =$	**C**
F	$2 \sin\theta \times 3 \sin\theta =$	$5 \cos^2\theta \times \cos\theta =$	**S**
O	$\cos^2\theta + 3 \cos^2\theta =$	$\cos\theta + 5 \cos\theta =$	**I**
H	$10 \tan^5\theta \div 2 \tan^4\theta =$	$5 \tan 2\theta - 4 \tan 2\theta =$	**R**
N	$5 \tan\theta + 3 \tan\theta =$	$6 \sin^2\theta - \sin^2\theta =$	**V**
U	$4 \tan\theta \times 2 \tan\theta =$	$12 \cos 2\theta - 8 \cos 2\theta =$	**T**
L	$4 \sin^2\theta - \sin^2\theta =$	$\sin 2\theta + 2 \sin 2\theta =$	**E**
B	$3 \cos 2\theta \times \cos 2\theta =$	$12 \sin^3\theta \div 3 \sin\theta =$	**A**

HUMAN BONES:

$5 \tan\theta$	$8 \tan^2\theta$	$5 \sin\theta$	$3 \sin 2\theta$	$\tan 2\theta$	$4 \cos^2\theta$	$8 \tan^2\theta$	$5 \cos^3\theta$
$6 \sin^2\theta$	$6 \cos\theta$	$3 \cos^2 2\theta$	$8 \tan^2\theta$	$3 \sin^2\theta$	$4 \sin^2\theta$		
$6 \sin^2\theta$	$3 \sin 2\theta$	$5 \sin\theta$	$8 \tan^2\theta$	$\tan 2\theta$			
$5 \cos^3\theta$	$4 \cos 2\theta$	$3 \sin 2\theta$	$\tan 2\theta$	$8 \tan\theta$	$8 \tan^2\theta$	$5 \sin\theta$	
$3 \tan\theta$	$3 \sin^2\theta$	$4 \sin^2\theta$	$5 \sin^2\theta$	$6 \cos\theta$	$3 \tan\theta$	$3 \sin^2\theta$	$3 \sin 2\theta$



Fluency Practice

1 Solve the following in the range

$$0° \leq x < 360°$$

a $\sin\theta = 3\cos\theta \quad \rightarrow \quad \boldsymbol{\theta} =$?

b $2\sin\theta = 3\cos\theta \quad \rightarrow \quad \boldsymbol{\theta} =$?

2 Solve the following by first factorising. $0° \leq x < 360°$

a $\cos^2\theta - \cos\theta = 0 \quad \rightarrow \quad \boldsymbol{\theta} =$?

b $\tan^2\theta - 3\tan\theta = 0 \quad \rightarrow \quad \boldsymbol{\theta} =$?

c $\sin x\cos x + \sin x = 0 \quad \rightarrow \quad \boldsymbol{\theta} =$?

3 Solve the following: $0° \leq x < 360°$

a $\sin^2\theta = \frac{3}{4} \quad \rightarrow \quad \boldsymbol{\theta} =$?

b $\cos^2\theta = \frac{3}{4} \quad \rightarrow \quad \boldsymbol{\theta} =$?

c $\tan^2\theta = 3 \quad \rightarrow \quad \boldsymbol{\theta} =$?

4 By factorising these 'quadratics', solve in the range $0 \leq x < 360$

a $3\cos^2\theta + 2\cos\theta - 1 = 0 \quad \rightarrow \quad \boldsymbol{\theta} =$?

b $6\sin^2\theta - \sin\theta - 1 = 0 \quad \rightarrow \quad \boldsymbol{\theta} =$?

☠ $\sin\theta\cos\theta + \sin\theta + \cos\theta = -1 \quad \rightarrow$?

Fluency Practice

1 Simplify $3\sin x(\sin x + 2) - 3(2\sin x - \cos^2 x)$

$=$?

$=$

2 Write out the following in terms of $\sin x$:

a) $\cos^2 x \tan^2 x \;=$?

b) $\tan x \cos^3 x \;=$?

c) $\cos x(2\cos x - 3\tan x) \;=$?

3 Prove the following:

a. $\tan x\sqrt{1 - \sin^2 x} \equiv \sin x$

b. $\frac{1-\cos^2 x}{1-\sin^2 x} \equiv \tan^2 x$

c. $(1 + \sin x)(1 - \sin x) \equiv \cos^2 x$

d. $\frac{2\sin x \cos x}{\tan x} \equiv 2 - 2\sin^2 x$

e. $\frac{1}{\cos\theta} - \cos\theta \equiv \sin\theta\tan\theta$

f. $(2\sin\theta - \cos\theta)^2 + (\sin\theta + 2\cos\theta)^2 \equiv 5$

Fluency Practice

Solve for $0 \leq x < 360°$, giving your answers to 1 decimal place.

(a) $\sin x = 0.78$

(b) $2\tan x = 5$

(c) $16\cos^2 x = 9$

(d) $\tan x + 2 = 5$

(e) $4\sin^2 x - 1 = 0$

Solve for $0 \leq x < 360°$, giving your answers to 1 decimal place.

(a) $5\sin x = \cos x$

(b) $\sin x + \cos x = 0$

(c) $7\sin x = 3\tan x$

Solve for $0 \leq \theta < 360°$, giving your answers to 1 decimal place.

(a) $\sin^2\theta - 3\cos^2\theta = 0$

(b) $2\cos^2\theta - \cos\theta - 1 = 0$

(c) $\tan^2\theta + 3\tan\theta = 0$

(d) $2\sin^2\theta + 7\sin\theta + 5 = 0$

Solve for $0 \leq \theta < 360°$, giving your answers to 1 decimal place.

(a) $2\cos^2\theta + 3\sin\theta = 3$

(b) $2\sin^2\theta = 3 - 3\cos\theta$

Fluency Practice

Patrons are reminded that drawing the relevant graph is a good idea and that $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$. Also, $(\cos \text{something})^2$ is written as $\cos^2 \text{something}$.

Give all answers as either exact or to 1 decimal place.

1. Solve $\sin\theta = \frac{1}{2}$ for $0° < \theta < 720°$.
2. Solve $\cos\theta = \frac{\sqrt{3}}{2}$ for $-360° < \theta < 360°$.
3. Solve $\sin\theta = \frac{1}{\sqrt{2}}$ for $-360° < \theta < 360°$.
4. Solve $\tan\theta = \frac{1}{\sqrt{3}}$ for $0° < \theta < 360°$.
5. Solve $2\sin\theta + 1 = 0$ for $0° < \theta < 720°$.
6. Solve $\cos^2\theta = 1$ for $0° < \theta < 720°$.
7. Solve $\sin^2\theta - 3 = 1$ for $-360° < \theta < 360°$.
8. Solve $\sin\theta = \frac{2}{3}$ for $0° < \theta < 360°$.
9. Solve $2\tan\theta + 1 = 6$ for $-360° < \theta < 360°$.
10. Solve $3\sin\theta + 1 = 0$ for $0° < \theta < 360°$.
11. Solve $3\sin\theta = 5\cos\theta$ for $0° < \theta < 720°$.
12. Solve $5\cos^2\theta = 1$ for $-360° < \theta < 0°$.

To spot a quadratic in disguise you are looking for an equation where the power on one of the variables is twice that on the other. For example

$$(\text{whatever})^{18} + 4(\text{whatever})^9 - 5.$$

This can then factorise to

$$((\text{whatever})^9 + 5)((\text{whatever})^9 - 1),$$

or you can complete the square to

$$((\text{whatever})^9 + 2)^2 - 9.$$

Solve the following:

1. $x^4 - 13x^2 + 36 = 0$.
2. $x^4 - 15x^2 - 16 = 0$.
3. $x^4 + 5x^2 + 6 = 0$.
4. $x^6 + 7x^3 = 8$.
5. $2x^4 = x^2 + 1$.
6. $x + 3 = 4\sqrt{x}$.
7. $12x^4 = 2x^2 + 4$.
8. $2(\sin x)^2 + \sin x - 1 = 0$ in range $0 < x < 360$.
9. $\frac{4x^4+144}{73} = x^2$.
10. $2(\cos x)^2 + (\cos x) = 6$.
11. $x = 2\sqrt{x} + 3$.
12. $6x^{2/3} + 5x^{1/3} - 4 = 0$.
13. $(x^2 - 4x + 1)^2 + (x^2 - 4x + 1) - 12 = 0$.
14. $2\theta + 15 = 11\sqrt{\theta}$.
15. $x^2 + \frac{72}{x^2} = 17$.
16. $\sqrt{z} - 2\sqrt[4]{z} = 3$.
17. $2^{2x} - 12 \times 2^x + 32 = 0$.
18. $t = 4\sqrt{t} + 1$.
19. $2^{2x} + 8 = 9 \times 2^x$.
20. $2\sqrt{x} + \frac{9}{\sqrt{x}} = 9$.
21. $\left(\frac{1}{x}\right)^2 + 1 = 8\left(\frac{1}{x}\right)$. [Do this question in two ways.]
22. $2(\cos\theta)^2 = 8\cos\theta + 21$.
23. $x^8 + 4x^4 = 3$.
24. $2^{2x} + 1 = 2^{x+1}$.
25. $2^{2x} + 128 = 3 \times 2^{x+3}$.
26. $81 + 3^{2x+1} = 4 \times 3^{x+2}$.
27. $a^{2x} + a^4 = a^{x+1} + a^{x+3}$.

Only attempt the following if you have studied logarithms.

28. $2^{2x} - 13 \times 2^x + 42 = 0$.
29. $4^{2x} - 9 \times 4^x + 14 = 0$.
30. $3^{2x} + 10 = 7 \times 3^x$.
31. $2^{2x} - 5 \times 2^{x+1} + 25 = 0$.
32. $3^{2x} - 3^{x+2} + 20 = 0$.

Purposeful Practice

Give answers to 1 decimal place where necessary.

① Solve the following equations for $0° \leqslant \theta \leqslant 360°$.

(i) $\cos\theta = 0.5$ (ii) $\tan\theta = 1$ (iii) $\sin\theta = \frac{\sqrt{3}}{2}$

(iv) $\sin\theta = -0.5$ (v) $\cos\theta = 0$ (vi) $\tan\theta = -5$

(vii) $\tan\theta = 0$ (viii) $\cos\theta = -0.54$ (ix) $\sin\theta = 1$

② Solve the following equations for $-180° \leqslant \theta \leqslant 180°$.

(i) $3\cos\theta = 2$ (ii) $7\sin\theta = 5$ (iii) $3\tan\theta = 8$

(iv) $6\sin\theta + 5 = 0$ (v) $5\cos\theta + 2 = 0$ (vi) $5 - 9\tan\theta = 10$

PS ③ Solve the following equations for $0° \leqslant \theta \leqslant 360°$.

(i) $\sin^2\theta = 0.75$ (ii) $\cos^2\theta = 0.5$ (iii) $\tan^2\theta = 1$

PS ④ (i) Factorise $2x^2 + x - 1$

(ii) Hence solve $2x^2 + x - 1 = 0$

(iii) Use your results to solve these equations for $-360° \leqslant \theta \leqslant 360°$.

(a) $2\sin^2\theta + \sin\theta - 1 = 0$

(b) $2\cos^2\theta + \cos\theta - 1 = 0$

(c) $2\tan^2\theta + \tan\theta - 1 = 0$

$\sin^2\theta$ is alternative notation for $(\sin\theta)^2$

PS ⑤ Solve the following equations for $-180° \leqslant x \leqslant 180°$.

(i) $\tan^2 x - 3\tan x = 0$ (ii) $1 - 2\sin^2 x = 0$

(iii) $3\cos^2 x + 2\cos x - 1 = 0$ (iv) $2\sin^2 x = \sin x + 1$

PS ⑥ *Do not use a calculator in this question.*

Solve the following equations for $-360° < x < 360°$.

(i) $\tan x = \sqrt{3}$ (ii) $2\sin x = 1$

(iii) $\sqrt{2}\cos x - 1 = 0$ (iv) $2\sin x = \sqrt{3}$

(v) $\tan^2 x - \tan x = 0$ (vi) $4\cos x = \sqrt{12}$

PS ⑦ Solve $(\cos\theta - 1)(\cos\theta + 2)(2\cos\theta - 1) = 0$ for $0° \leqslant \theta \leqslant 360°$.

PS ⑧ (i) Given that $f(x) = 2x^3 - x^2 - 3x - 1$, calculate $f\left(-\frac{1}{2}\right)$.

(ii) Hence solve $2\sin^3\theta - \sin^2\theta - 3\sin\theta - 1 = 0$ for $-180° \leqslant \theta \leqslant 180°$.

Purposeful Practice

① For each of the equations (i)–(v):

(a) use the identity $\sin^2\theta + \cos^2\theta \equiv 1$ to rewrite the equation in a form involving only one trigonometric function

(b) factorise, and hence solve, the resulting equation for $0° \leqslant \theta \leqslant 360°$.

(i) $2\cos^2\theta + \sin\theta - 1 = 0$

(ii) $\sin^2\theta + \cos\theta + 1 = 0$

(iii) $2\sin^2\theta - \cos\theta - 1 = 0$

(iv) $\cos^2\theta + \sin\theta = 1$

(v) $1 + \sin\theta - 2\cos^2\theta = 0$

② For each of the equations (i)–(iii):

(a) use the identity $\sin^2\theta + \cos^2\theta \equiv 1$ to rewrite the equation in a form involving only one trigonometric function

(b) use the quadratic formula to solve the resulting equation for $0° \leqslant \theta \leqslant 180°$.

(i) $\sin^2\theta - 2\cos\theta + 1 = 0$

(ii) $\cos^2\theta - \sin\theta = 0$

(iii) $\sin^2\theta - 3\cos\theta = 0$

③ (i) Use the identity

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

to rewrite the equation $\sin\theta = 2\cos\theta$ in terms of $\tan\theta$.

(ii) Hence solve the equation $\sin\theta = 2\cos\theta$ for $0° \leqslant \theta \leqslant 180°$.

④ Use the identity

$$\tan\theta \equiv \frac{\sin\theta}{\cos\theta}$$

to solve the following equations for $0° \leqslant \theta \leqslant 360°$.

(i) $2\sin\theta + \cos\theta = 0$

(ii) $\sqrt{3}\tan\theta = 2\sin\theta$

(ii) $4\cos\theta\tan\theta = 1$

PS ⑤ Write the following in terms of $\sin x$.

(i) $\cos^2 x\tan^2 x$ (ii) $\tan x\cos^3 x$ (iii) $\cos x\,(2\cos x - 3\tan x)$

PS ⑥ Show that $(3\sin x)(\sin x + 2) - 3(2\sin x - \cos^2 x)$ simplifies to an integer.

PS ⑦ Prove the following identities.

(i) $\tan x\sqrt{1 - \sin^2 x} \equiv \sin x$

(ii) $\dfrac{1 - \cos^2 x}{1 - \sin^2 x} \equiv \tan^2 x$

(iii) $(1 + \sin x)(1 - \sin x) \equiv \cos^2 x$

(iv) $\dfrac{2\sin x\cos x}{\tan x} \equiv 2 - 2\sin^2 x$

PS ⑧ Solve $5\sin x\left(\sin x + \cos x\right) = 3$ for $0° < x < 360°$.

Hint: Replace 3 with 3×1 and then replace the 1 with $\sin^2 x + \cos^2 x$.

Solving Trig Equations with... The Factor Theorem

Solve each equation in the given region:

Round answers to 1 decimal place, where appropriate.

1) $\mathbf{6\sin^3 x - 5\sin^2 x - 3\sin x + 2 = 0}$, for $0° \leq x < 360°$,
2) $\mathbf{12\cos^4 x - \cos^3 x - 18\cos^2 x + \cos x + 6 = 0}$, for $-180° \leq x < 180°$,
3) $\mathbf{6\tan^5 x + 35\tan^4 x + 62\tan^3 x + 35\tan^2 x + 6\tan x = 0}$, for $0° \leq x < 180°$.

4) $\mathbf{2\cos^3 x + 3\sin^2 x - 8\cos x - 6 = 0}$, for $0° \leq x < 720°$,
5) $\mathbf{-3\sin(x)\cos^2 x + 11\sin^2 x - 16\sin x + 5 = 0}$, for $-360° \leq x < 360°$,
6) $\mathbf{\tan(x)\sin^2 x - 3\sin^2 x - 10\sin(x)\cos x + 24\cos^2 x = 0}$, for $-360° \leq x < 0°$.

7) $\mathbf{6\sin^4 2x - 5\sin^3 2x - 14\sin^2 2x - \sin 2x + 2 = 0}$, for $0° \leq x < 180°$,
8) $\mathbf{5\cos^5 3x - 19\cos^4 3x - 9\cos^3 3x + 79\cos^2 3x - 44\cos 3x - 12 = 0}$, for $0° \leq x < 120°$,
9) $\mathbf{\tan^4(4x+5) - 27\tan^2(4x+5) - 14\tan(4x+5) + 120 = 0}$, for $0° \leq x < 90°$.

3 Domain and Range (L2FM Only)

Fluency Practice

Work out a suitable domain and the range of each function.
A sketch may help with each one.

1

Function	$f(x) = 2x$
Domain	?
Range	

2

Function	$f(x) = 2^x$
Domain	?
Range	

3

Function	$f(x) = \frac{1}{x}$
Domain	?
Range	

4

Function	$f(x) = \sin x$
Domain	?
Range	

5

Function	$f(x) = 2\cos x$
Domain	?
Range	

6

Function	$f(x) = x^3 + 1$
Domain	?
Range	

7

Function	$f(x) = \frac{1}{x-2} + 1$
Domain	?
Range	

8

Function	$f(x) = 2\cos\left(\sqrt{x+1}\right)$
Domain	?
Range	

Fluency Practice

Find the ranges for each of these functions and their domains:

(a) $f(x) = 5x + 1 \quad x = \{1, 2, 3\}$

(b) $g(x) = x^2 - 3 \quad x = \{3, 4, 5\}$

(c) $h(x) = \sqrt{2x + 1} \quad x = \{2, 4, 12\}$

The domain is $\{1, 2, 3, 4\}$. Find the ranges of these functions:

(d) $f: x \to x + 9$

(e) $g: x \to 2x^2$

(f) $h: x \to \frac{x}{x+1}$

What value of x must be excluded from the domains for the following functions?

(g) $f(x) = \frac{3}{x}$

(h) $g(x) = \frac{x}{x-2}$

(i) $h(x) = \frac{x+1}{x+2}$

What values of x must be excluded from the domains for the following functions?

(j) $f: x \to \sqrt{x}$

(k) $g: x \to \sqrt{x - 3}$

(l) $h: x \to \sqrt{x + 2}$

What values of x must be excluded from the domains for the following functions?

(m) $f(x) = \frac{2}{x-1} + \frac{3}{x+5}$

(n) $g(x) = \sqrt{2x - 1}$

Fluency Practice

1: Give the domain.

a) $g(x) = \frac{1 - 7x}{4x}$ ______

b) $f(x) = \frac{-7x - 5}{x - 1}$ ______

c) $h(x) = \frac{-5x - 9}{5x - 2}$ ______

d) $f(x) = \frac{-8}{x} - 5$ ______

2: Give the domain.

a) $g(x) = 7\sqrt{x} + 6$ ______

b) $h(x) = \sqrt{\left(\frac{-x + 10}{4}\right)}$ ______

c) $g(x) = \sqrt{(-3x - 9)}$ ______

d) $h(x) = \sqrt{\left(\frac{-x}{2}\right)} - 5$ ______

3: Give the domain.

a) $f(x) = \frac{2x - 5}{3x - 9}$ ______

b) $g(x) = \sqrt{\left(\frac{x - 9}{6}\right)}$ ______

c) $f(x) = 9\sqrt{x} - 3$ ______

d) $h(x) = \frac{x + 7}{x - 4}$ ______

Fluency Practice

4: What values must be excluded from the domain?

a) $h(x) = \dfrac{1}{-5 - 4x}$ ________

b) $g(x) = \dfrac{10}{x + 6}$ ________

c) $f(x) = \dfrac{1 - 4x}{6x}$ ________

d) $g(x) = \dfrac{2x - 3}{6x - 5}$ ________

5: What values must be excluded from the domain?

a) $f(x) = \sqrt{(-4x + 5)}$ ________

b) $h(x) = \sqrt{\left(\dfrac{-x}{2}\right)} + 3$ ________

c) $h(x) = 8\sqrt{x} - 4$ ________

d) $g(x) = \sqrt{(-5x - 3)}$ ________

6: What values must be excluded from the domain?

a) $f(x) = \dfrac{x + 4}{x - 3}$ ________

b) $g(x) = \sqrt{\left(\dfrac{x - 6}{8}\right)}$ ________

c) $h(x) = \sqrt{\left(\dfrac{x}{2}\right)} + 7$ ________

d) $f(x) = \dfrac{5x + 5}{x - 1}$ ________

Fluency Practice

A1 $f(x) = 3x - 5$ Find f(6)	**A2** $f(x) = x^2 - \frac{10}{x}$ Find f(–2)	**A3** $f(x) = \frac{3x+2}{x}$ Find f(0.5)	**A4** $f(x) = \frac{9}{x+2} + \frac{3}{x-1}$ Find f(0)
B1 $f(x) = \sqrt{8-x}$ State the values of x which must be excluded from the domain of f.	**B2** $f(x) = \frac{7}{3x+1}$ State the value of x which must be excluded from the domain of f.	**B3** $f(x) = \frac{5}{x+1} + \frac{2}{x-3}$ State the values of x which cannot be included in any domain of f.	**B4** $f(x) = \sqrt{x-4}$ State the values of x which cannot be included in any domain of f.
C1 $f(x) = 4x - 9$ Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$	**C2** $f(x) = \frac{2x}{x-1}$ Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$	**C3** $f(x) = \frac{x}{3x+1}$ Find $f^{-1}(x)$	**C4** $f(x) = \sqrt{2x-1}$ Express the inverse function f^{-1} in the form $f^{-1}(x) = \ldots$
D1 $f(x) = 2x - 7$ Given that $f(a) = 3$, work out the value of a	**D2** $f(x) = \frac{1}{2}x + 4$ $f(a) = -2$ Work out the value of a.	**D3** $f(x) = \frac{x}{x-1}$ Solve the equation $f(x) = 1.2$ Show your working clearly.	**D4** $f(x) = \frac{3}{x+1} + \frac{1}{x-2}$ Find the value of x for which $f(x) = 0$ Show your working clearly.

Fluency Practice

① Write down the range of f(x) in each of the following.

(i) $f(x) = 3x$ $\quad x < 2$

(ii) $f(x) = x + 4$ $\quad x \geqslant 1$

(iii) $f(x) = 2x + 4$ $\quad x \geqslant -1$

(iv) $f(x) = 10 - x$ $\quad x \leqslant 4$

② Write down the range of f(x) in each of the following.

(i) $f(x) = 2x$ $\quad 1 \leqslant x \leqslant 5$

(ii) $f(x) = x - 3$ $\quad 0 < x < 10$

(iii) $f(x) = 5 - 2x$ $\quad x \geqslant -3$

(iv) $f(x) = 3 - 4x$ $\quad -2 \leqslant x < 3$

③ Write down the range of f(x) in each of the following.

(i) $f(x) = \dfrac{x+5}{2}$ $\quad 0 \leqslant x \leqslant 5$

(ii) $f(x) = \dfrac{2x-3}{4}$ $\quad -2 \leqslant x \leqslant 2$

(iii) $f(x) = \dfrac{3-2x}{3}$ $\quad -3 \leqslant x \leqslant 5$

(iv) $f(x) = \dfrac{1-3x}{2}$ $\quad -3 \leqslant x \leqslant 5$

④ Write down the range of f(x) in each of the following.

(i) $f(x) = x^2$ $\quad -2 \leqslant x < 2$

(ii) $f(x) = x^2$ $\quad 0 < x < 4$

(iii) $f(x) = x^3$ $\quad x \geqslant 0$

(iv) $f(x) = x^3$ $\quad -1 \leqslant x \leqslant 3$

⑤ Write down the range of f(x) in each of the following.

(i) $f(x) = 2x^2 - 3$ $\quad 0 \leqslant x \leqslant 4$

(ii) $f(x) = 3x^2 - 2$ $\quad 0 \leqslant x \leqslant 4$

(iii) $f(x) = 3 - 2x^2$ $\quad -1 \leqslant x \leqslant 2$

(iv) $f(x) = 2 - 3x^2$ $\quad -1 \leqslant x \leqslant 2$

⑥ Write down the range of f(x) in each of the following.

(i) $f(x) = x^2 + 2x$ $\quad 0 \leqslant x \leqslant 3$

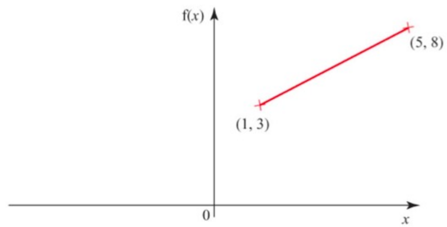
(ii) $f(x) = x^2 + 2x$ $\quad -2 \leqslant x \leqslant 3$

(iii) $f(x) = x^2 - 2x$ $\quad 0 \leqslant x \leqslant 3$

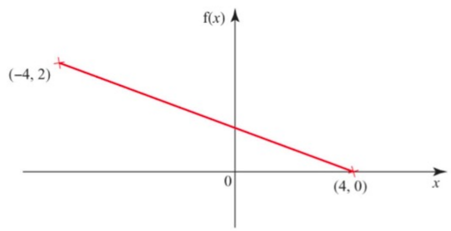
(iv) $f(x) = x^2 - 2x$ $\quad 2 \leqslant x \leqslant 3$

⑦ In each of the following, a sketch of a function, f(x), is shown. Write down the domain and the range for f(x).

(i)



(ii)



(iii)

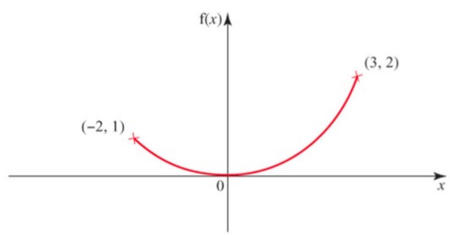


Figure 3.4

⑧ Sketch the graph of each of the following functions and write down the corresponding range.

(i) $f(x) = 3x - 2$ for $-1 \leqslant x \leqslant 3$

(ii) $f(x) = 2 - 3x$ for $-1 \leqslant x \leqslant 2$

(iii) $f(x) = x^2 + 3$ for $-2 \leqslant x \leqslant 2$

(iv) $f(x) = 4 - x^2$ for $-2 \leqslant x \leqslant 3$

Fluency Practice

1 Work out the range for each of these functions.

(a) $f(x) = x^2 + 6$ for all x

?

(b) $f(x) = 3x - 5,\ -2 \le x \le 6$

?

(c) $f(x) = 3x^4,\ \ x < -2$

?

2 (a) $f(x) = \frac{x+2}{x-3}$

Give a reason why $x > 0$ is not a suitable domain for $f(x)$.

?

(b) Give a possible domain for $f(x) = \sqrt{x-5}$

?

3

$$f(x) = 3 - 2x, \qquad a < x < b$$

The range of $f(x)$ is $-5 < f(x) < 5$

Work out a and b.

?

4 [Set 1 Paper 2] (a) The function $f(x)$ is defined as:

$$f(x) = 22 - 7x, \qquad -2 \le x \le p$$

The range of $f(x)$ is $-13 \le f(x) \le 36$

Work out the value of p.

?

(b) The function $g(x)$ is defined as $g(x) = x^2 - 4x + 5$ for all x.

(i) Express $g(x)$ in the form $(x-a)^2 + b$

?

(ii) Hence write down the range of $g(x)$.

?

5 [June 2012 Paper 1] $f(x) = 2x^2 + 7$ for all values of x.

(a) What is the value of $f(-1)$?

?

(b) What is the range of $f(x)$?

?

Fluency Practice

6 [Jan 2013 Paper 2]

$$f(x) = \sin x \qquad 180° \leq x \leq 360°$$
$$g(x) = \cos x \qquad 0° \leq x \leq \theta$$

(a) What is the range of $f(x)$?

?

(b) You are given that $0 \leq g(x) \leq 1$. Work out the value of θ.

?

7 By completing the square or otherwise, determine the range of the following functions:

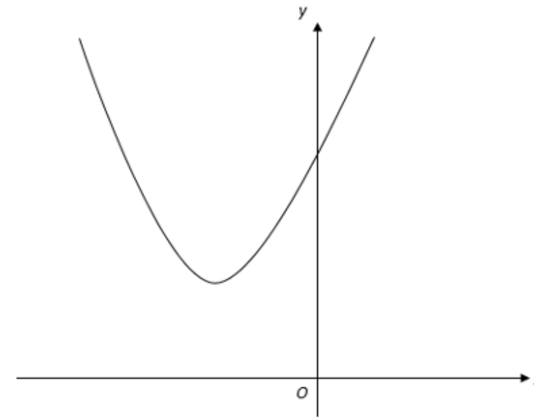
(a) $f(x) = x^2 - 2x + 5$, for all x

?

(b) $f(x) = x^2 + 6x - 2$, for all x

?

8

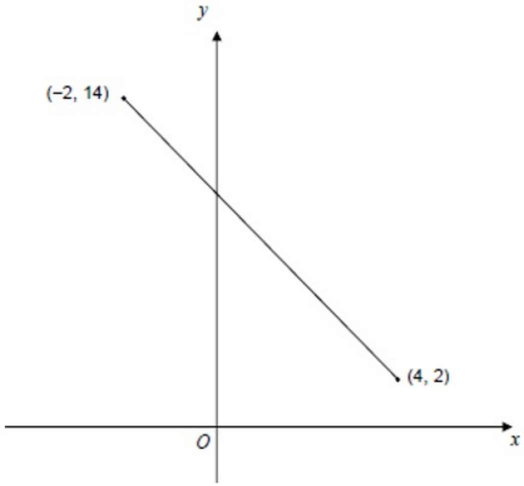


Here is a sketch of $f(x) = x^2 + 6x + a$ for all x, where a is a constant. The range of $f(x)$ is $f(x) \geq 11$. Work out the value of a.

?

Fluency Practice

9



The straight line shows a sketch of $y = f(x)$ for the full domain of the function.

(a) State the domain of the function.

$2 \le f(x) \le 14$

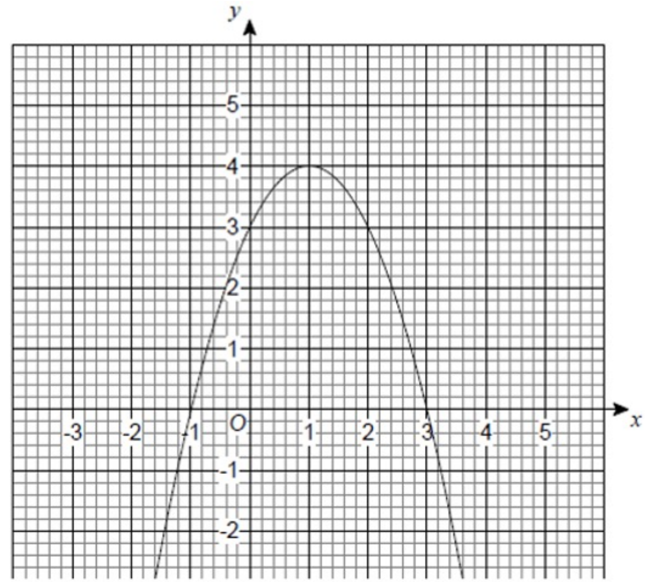
?

(b) Work ? he line.

$f(x) = -2x + 10$

?

10



$f(x)$ is a quadratic function with domain all real values of x. Part of the graph of $y = f(x)$ is shown.

(a) Write down the range of $f(x)$.

?

(b) Use the graph to find solutions of the equation $f(x) = 1$.

?

(c) Use the graph to solve $f(x) < 0$.

?

Special Functions

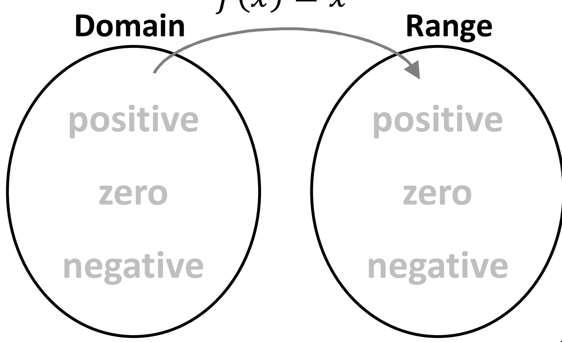
①

Think about these 4 functions.

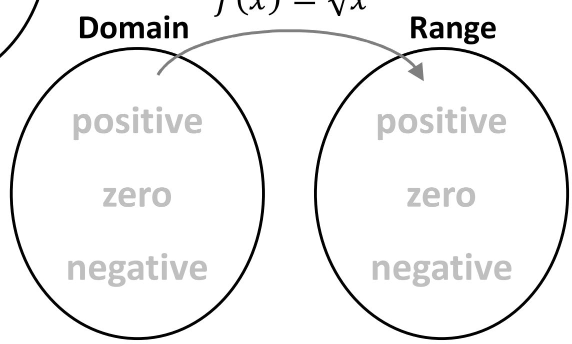
How do positive & negative numbers (& zero) in the Domain map onto the Range?

Which numbers **cannot map** between the domain & range?

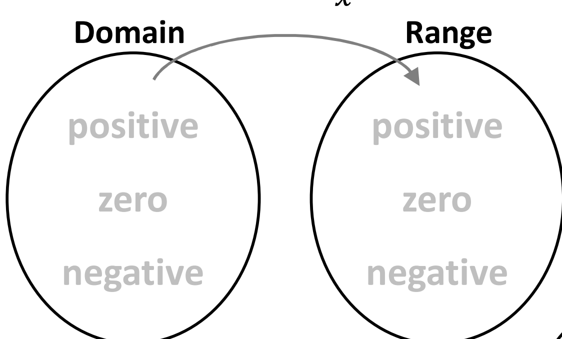
$f(x) = x^2$



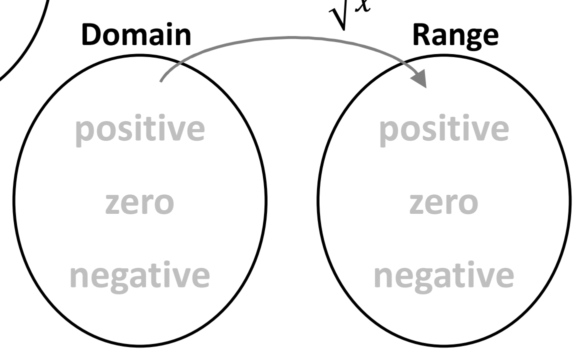
$f(x) = \sqrt{x}$



$f(x) = \frac{1}{x}$



$f(x) = \sqrt{\frac{1}{x}}$



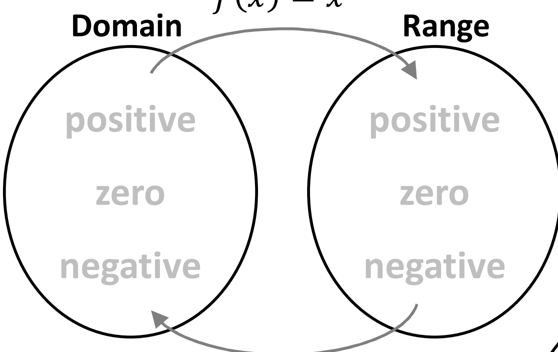
Special Functions ②

Think about these 4 functions.

How do positive & negative numbers (& zero) in the Domain map onto the Range?

Express the inverse function, how do numbers map in the reverse direction?

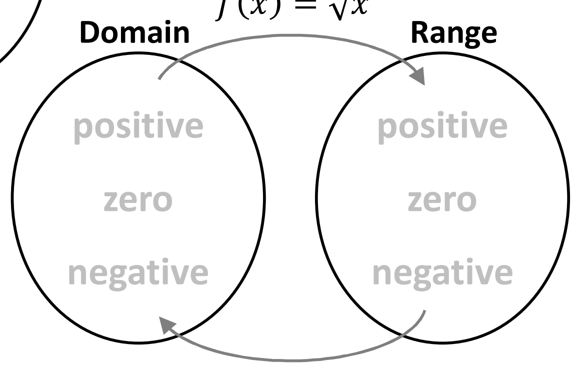
$f(x) = x^2$



$f^{-1}(x) =$

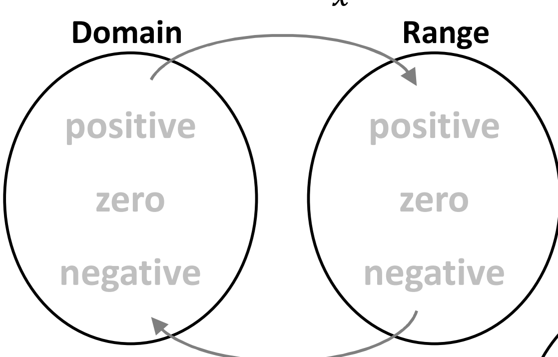
Which numbers **cannot map** between the domain & range?

$f(x) = \sqrt{x}$



$f^{-1}(x) =$

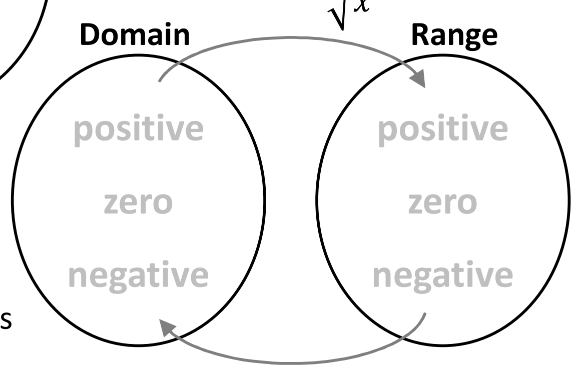
$f(x) = \frac{1}{x}$



$f^{-1}(x) =$

For specific values of x, how are the domain & range values connected for the normal & inverse operations?

$f(x) = \sqrt{\frac{1}{x}}$



$f^{-1}(x) =$

Fluency Practice

1. Domain is $1 \leq x < 3$. Range $1 \leq f(x) \leq 3$. $f(x)$ is an increasing function.

   ?

2. Domain is $1 \leq x \leq 3$. Range $1 \leq f(x) \leq 3$. $f(x)$ is a decreasing function.

   ?

3. Domain is $5 \leq x \leq 7$. Range $7 \leq f(x) \leq 11$. $f(x)$ is an increasing function.

   ?

4. Domain is $5 \leq x \leq 7$. Range $7 \leq f(x) \leq 11$. $f(x)$ is a decreasing function.

   ?

5. Domain is $-4 \leq x \leq 7$. Range $4 \leq f(x) \leq 8$. $f(x)$ is a decreasing function.

   ?

4 Piecewise Functions (L2FM Only)

Sketch the following functions:

(a) $f(x) = 2 \quad for \; -5 \leq x < 0$

$f(x) = 2 - x \quad for \; 0 \leq x \leq 3$

$f(x) = -1 \quad for \; 3 < x \leq 5$

(b) $f(x) = x \quad for \; -4 \leq x < 0$

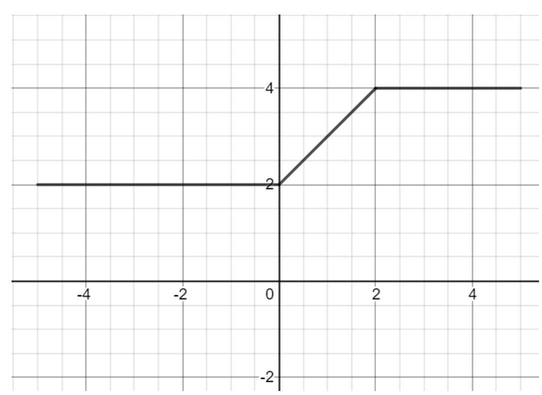
$f(x) = x^2 \quad for \; 0 \leq x \leq 4$

(c) $f(x) = x^2 + 1 \quad for \; -4 \leq x < 0$

$f(x) = 1 \quad for \; 0 \leq x \leq 2$

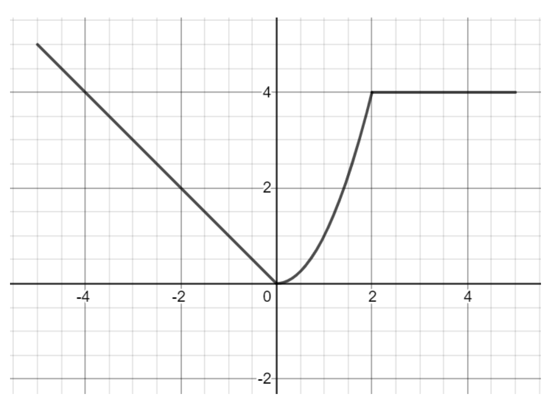
$f(x) = x - 1 \quad for \; 2 < x \leq 4$

(a) Given the graph of $y = f(x)$, define the function, stating the domain of each part clearly.



(b) Evaluate $f(1)$

(a) Given the graph of $y = f(x)$, define the function, stating the domain of each part clearly.



(b) Solve $f(x) = 1$

Fluency Practice

① Draw the graph of $y = f(x)$ where

$$f(x) = \begin{cases} 2 & -2 \leqslant x < 1 \\ 2x & 1 \leqslant x \leqslant 3 \end{cases}$$

② Draw the graph of $y = f(x)$ where

$$f(x) = \begin{cases} x^2 & 0 \leqslant x < 3 \\ 9 & 3 \leqslant x \leqslant 5 \end{cases}$$

③ Draw the graph of $y = g(x)$ where

$$g(x) = \begin{cases} x + 3 & -3 \leqslant x < 0 \\ 3 - x & 0 \leqslant x \leqslant 3 \end{cases}$$

④ Draw the graph of $y = f(x)$ where

$$f(x) = \begin{cases} 3x - 1 & -2 \leqslant x < 1 \\ 3 - x & 1 \leqslant x < 4 \\ -1 & 4 \leqslant x \leqslant 6 \end{cases}$$

⑤ Draw the graph of $y = f(x)$ where

$$f(x) = \begin{cases} 2x & -2 \leqslant x < 0 \\ \frac{1}{2}x & 0 \leqslant x < 2 \\ 5 - 2x & 2 \leqslant x \leqslant 4 \end{cases}$$

⑥ Draw the graph of $y = g(x)$ where

$$g(x) = \begin{cases} -4 & -3 \leqslant x < -2 \\ -x^2 & -2 \leqslant x < 2 \\ 3x - 10 & 2 \leqslant x \leqslant 4 \end{cases}$$

⑦ Figure 3.39 shows the graph of $y = f(x)$.

(i) Define $f(x)$, stating clearly the domain for each part.

(ii) State the range of $f(x)$.

(iii) Solve $f(x) = 5$.

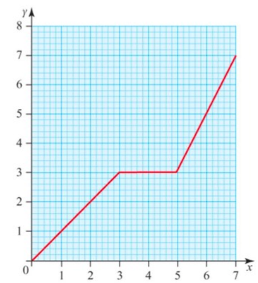


Figure 3.39

⑧ Figure 3.40 shows the graph of $y = f(x)$.

(i) Define $f(x)$, stating clearly the domain for each part.

(ii) State the range of $f(x)$.

(iii) Solve $f(x) = 3$.

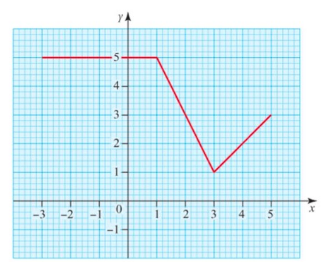


Figure 3.40

⑨ The graph of $y = f(x)$ is shown in Figure 3.41.

(i) Define $f(x)$, stating clearly the domain for each part.

(ii) Work out the area between the graph and the x-axis.

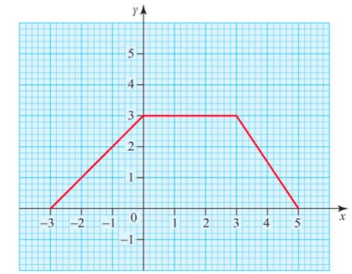


Figure 3.41

⑩ The graph of $y = g(x)$ is shown in Figure 3.42.

(i) Define $g(x)$, stating clearly the domain for each part.

(ii) Work out the area between the graph and the x-axis.

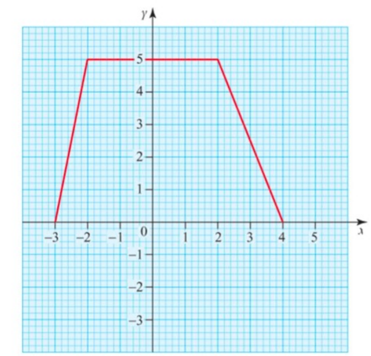


Figure 3.42

⑪ The distance–time graph in Figure 3.43 shows the relationship between distance travelled and time for a student who leaves home at 8:15 a.m., walks to the bus stop and then catches the bus to school.

(i) Describe what is happening between O and A.

(ii) Why do you think the distance doesn't change between A and B?

(iii) Think about the part of the graph from B to C. What does this tell you about the bus journey? How realistic is that?

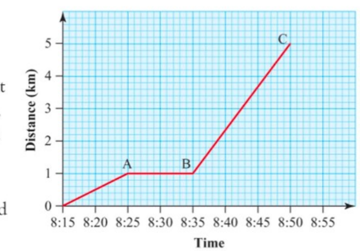


Figure 3.43

Fluency Practice

1 [Jan 2013 Paper 2] A function $f(x)$ is defined as:

$$f(x) = \begin{cases} 4 & x < -2 \\ x^2 & -2 \le x \le 2 \\ 12 - 4x & x > 2 \end{cases}$$

(a) Draw the graph of $y = f(x)$ for $-4 \le x \le 4$

(b) Use your graph to write down **how many** solutions there are to $f(x) = 3$

b ?

(c) Solve $f(x) = -10$

c ?

a ?

2 [June 2013 Paper 2] A function $f(x)$ is defined as:

$$f(x) = \begin{cases} x + 3 & -3 \le x < 0 \\ 3 & 0 \le x < 1 \\ 5 - 2x & 1 \le x \le 2 \end{cases}$$

Draw the graph of $y = f(x)$ for $-3 \le x < 2$

?

Fluency Practice

3 [Set 1 Paper 1] A function $f(x)$ is defined as:

$$f(x) = \begin{cases} 3 & 0 \leq x < 2 \\ x + 1 & 2 \leq x < 4 \\ 9 - x & 4 \leq x \leq 9 \end{cases}$$

Draw the graph of $y = f(x)$ for $0 \leq x \leq 9$.

?

4 [Specimen 1 Q4] A function $f(x)$ is defined as:

$$f(x) = \begin{cases} 3x & 0 \leq x < 1 \\ 3 & 1 \leq x < 3 \\ 12 - 3x & 3 \leq x \leq 4 \end{cases}$$

Calculate the area enclosed by the graph of $y = f(x)$ and the x −axis.

Sketch ?

Area = ?

Fluency Practice

5 [AQA Worksheet Q9]

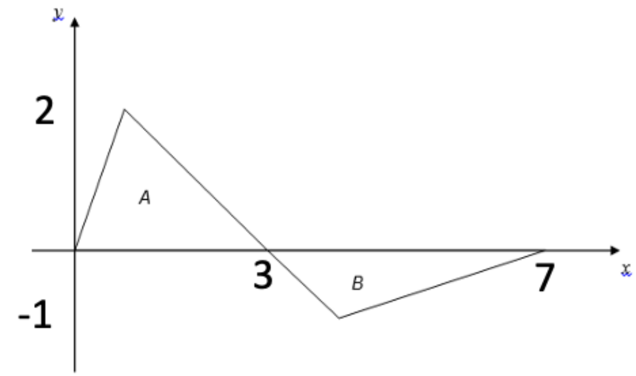
$$f(x) = \begin{cases} -x^2 & 0 \le x < 2 \\ -4 & 2 \le x < 3 \\ 2x - 10 & 3 \le x \le 5 \end{cases}$$

Draw the graph of $f(x)$ from $0 \le x \le 5$.

?

6 [AQA Worksheet Q10]

$$f(x) = \begin{cases} 2x & 0 \le x < 1 \\ 3 - x & 1 \le x < 4 \\ \dfrac{x-7}{3} & 4 \le x \le 7 \end{cases}$$



Show that $area\ of\ A : area\ of\ B = 3:2$

?

Fluency Practice

11 The function $f(x)$ is defined as:

$$f(x) = \begin{cases} x^2 - 4 & 0 \leq x < 3 \\ 14 - 3x & 3 \leq x \leq 5 \end{cases}$$

Work out the range of $f(x)$.

?

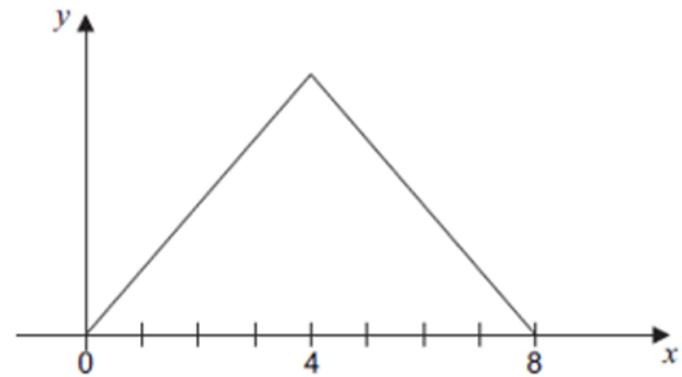
12 The function $f(x)$ has the domain $-3 \leq x \leq 3$ and is defined as:

$$f(x) = \begin{cases} x^2 + 3x + 2 & -3 \leq x < 0 \\ 2 + x & 0 \leq x \leq 3 \end{cases}$$

Work out the range of $f(x)$.

?

13



[June 2012 Paper 2] A sketch of $y = g(x)$ for domain $0 \leq x \leq 8$ is shown. The graph is symmetrical about $x = 4$. The range of $g(x)$ is $0 \leq g(x) \leq 12$. Work out the function $g(x)$.

$$g(x) = \begin{cases} ? & 0 \leq x \leq 4 \\ ? & 4 < x \leq 8 \end{cases}$$

?

5 Graph Transformations

Fluency Practice

What effect will the following transformations have on these points?

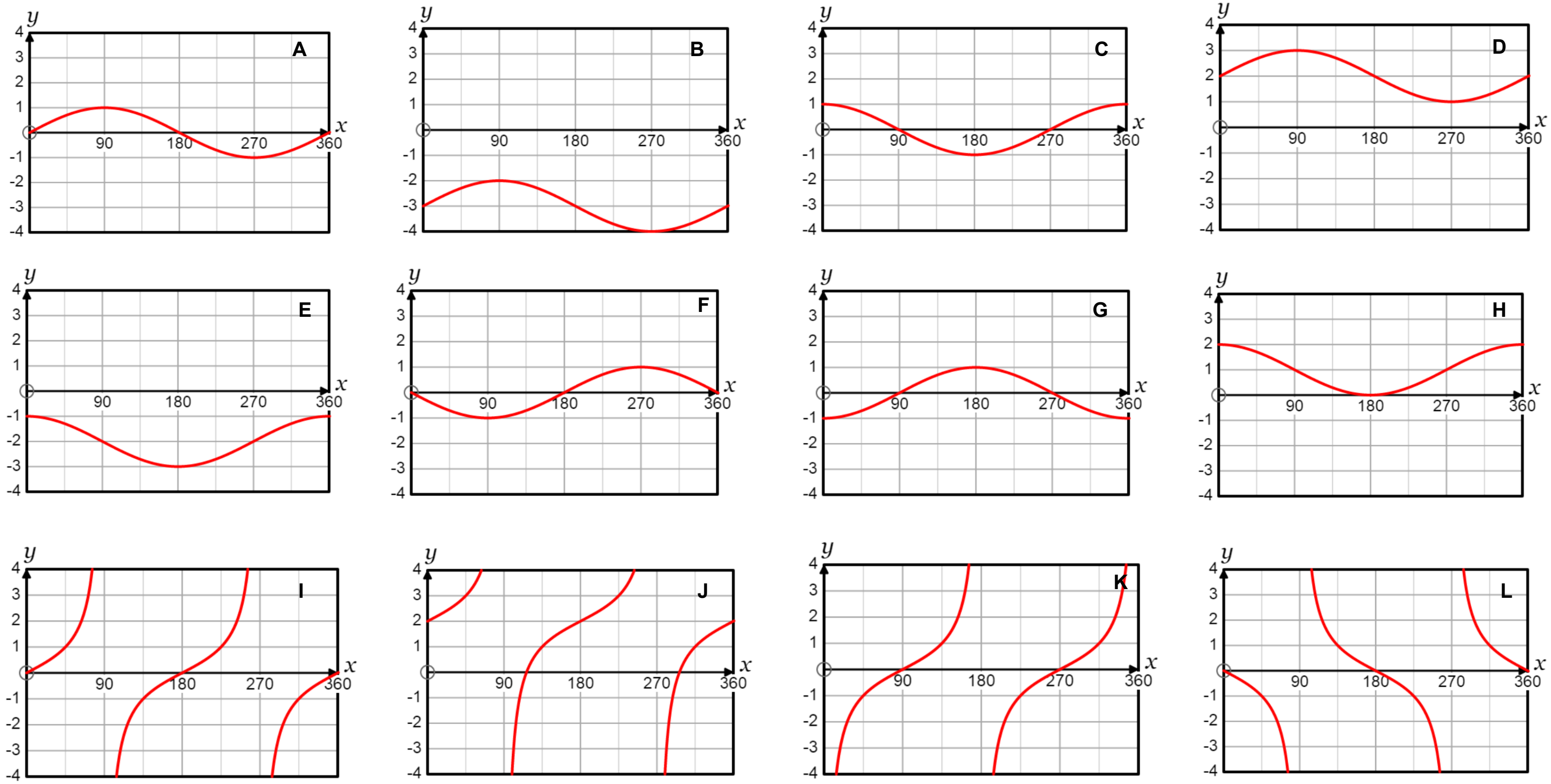
	$y = f(x)$	$(4, 3)$	$(1, 0)$	$(6, -4)$
a	$y = f(x+1)$	?	?	?
b	$y = f(x) - 1$	?	?	?
c	$y = f(-x)$	?	?	?
d	$y = -f(x)$	?	?	?

Fluency Practice

tRiG tRaNsfORmatiONs

1

Match the graphs to their equations below.



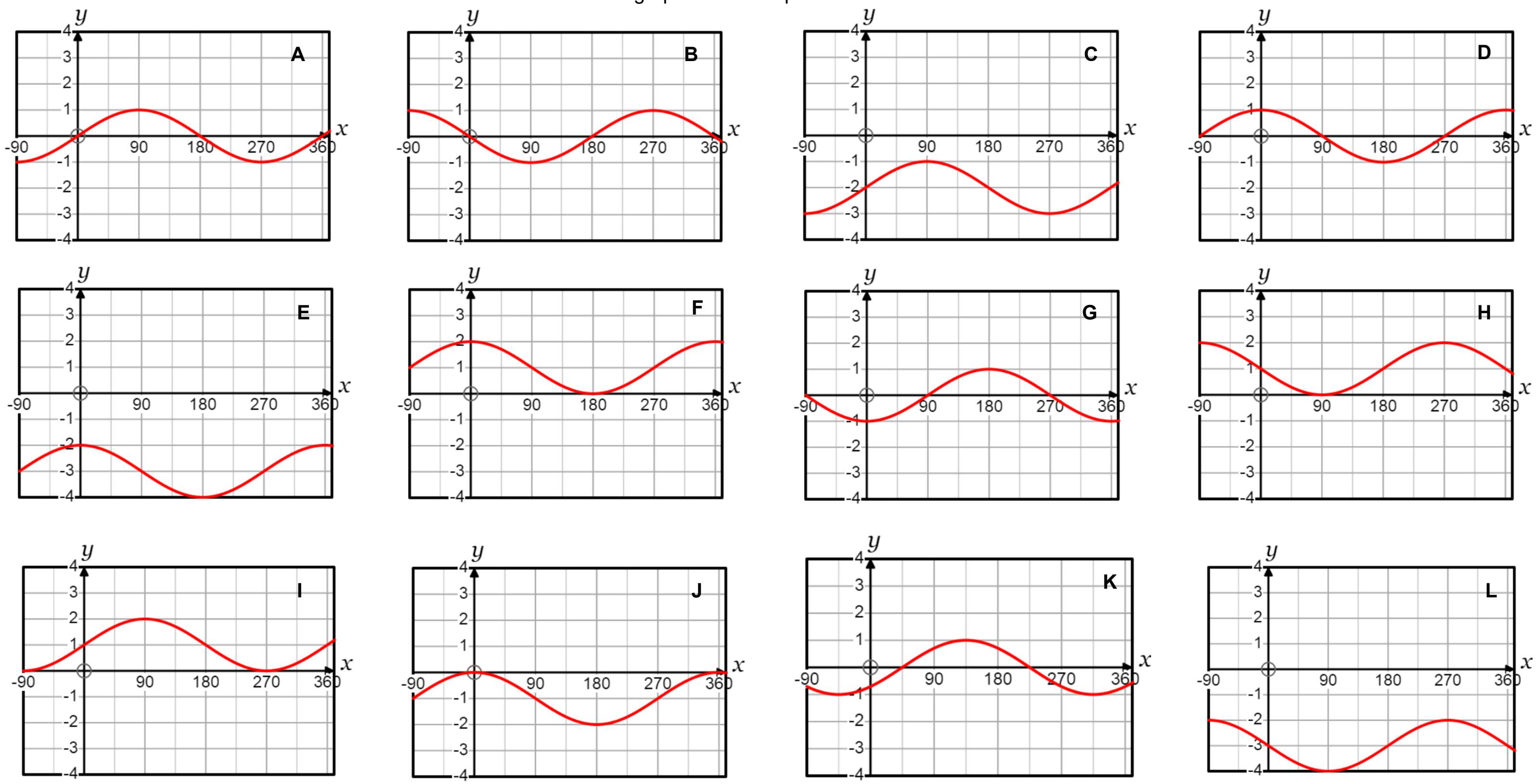
y = -cos(x)	y = -tan(x)	y = -sin(x)	y = sin(x) - 3	y = tan(x - 90)	y = cos(x)
y = tan(x)	y = tan(x) + 2	y = cos(x) - 2	y = sin(x)	y = sin(x) + 2	y = cos(x) + 1

Fluency Practice

TRIG TRANSFORMATIONS

2

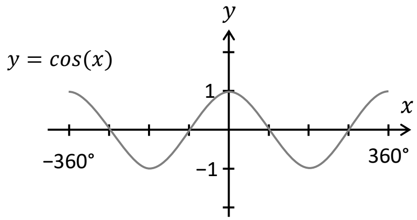
Match the graphs to their equations below.



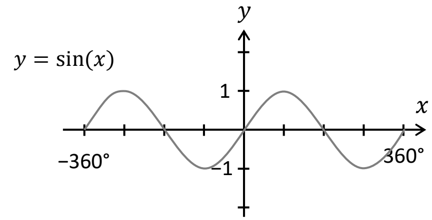
$y = \sin(-x) - 3$	$y = \sin(x - 45)$	$y = \cos(x - 90) + 1$	$y = \sin(x)$	$y = -\sin(x) + 1$	$y = \cos(x) + 1$
$y = \sin(x) - 2$	$y = \cos(x + 180)$	$y = \cos(-x) - 1$	$y = \cos(x)$	$y = \cos(x) - 3$	$y = \sin(x + 180)$

Fluency Practice

A) Sketch: $y = cos(x) + 1$



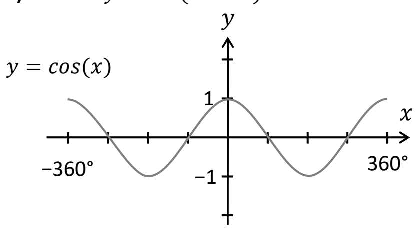
B) Sketch: $y = sin(x) - 1$



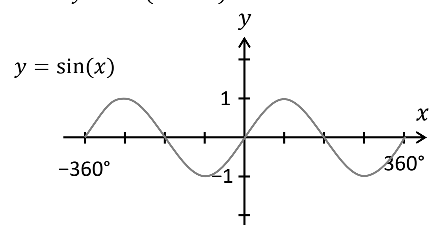
Transforming sin & cos **Graphs**

$y = f(x) + a$ $= \binom{0}{a}$	$y = f(x - a)$ $= \binom{a}{0}$
$y = -f(x)$ Reflection in x axis.	$y = f(-x)$ Reflection in y axis.

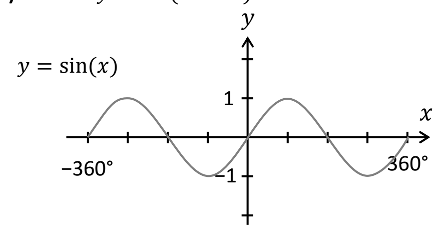
C) Sketch: $y = cos(x - 90)$



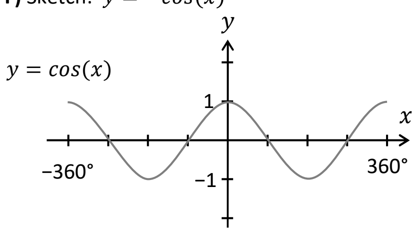
D) Sketch: $y = sin(x + 90)$



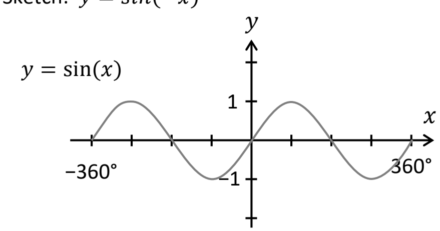
E) Sketch: $y = sin(x - 30)$



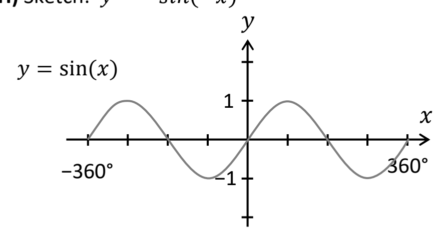
F) Sketch: $y = -cos(x)$



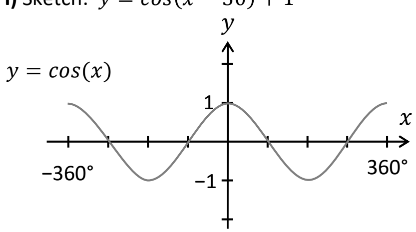
G) Sketch: $y = sin(-x)$



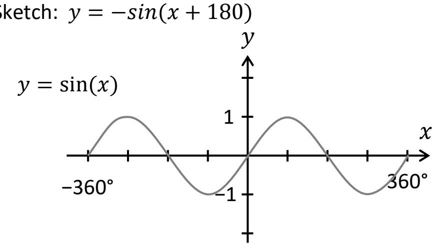
H) Sketch: $y = -sin(-x)$



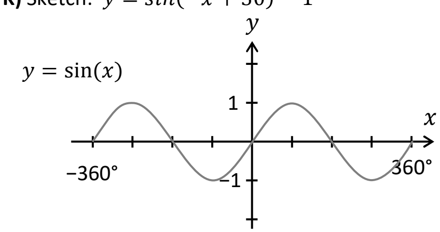
I) Sketch: $y = cos(x - 30) + 1$



J) Sketch: $y = -sin(x + 180)$



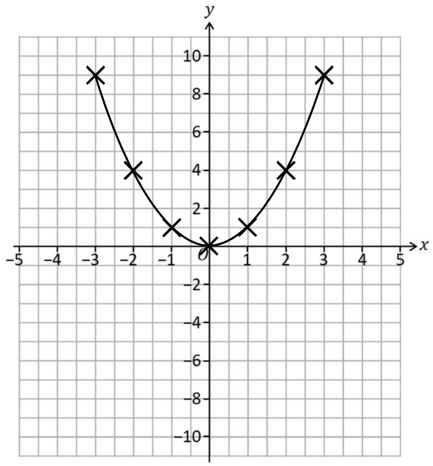
K) Sketch: $y = sin(-x + 30) - 1$



Fluency Practice

Transforming Graphs

① The graph $y = x^2$ has been plotted.
We will **transform** it by adding 2 to the function **output**.
Complete the table of values & plot the graph.

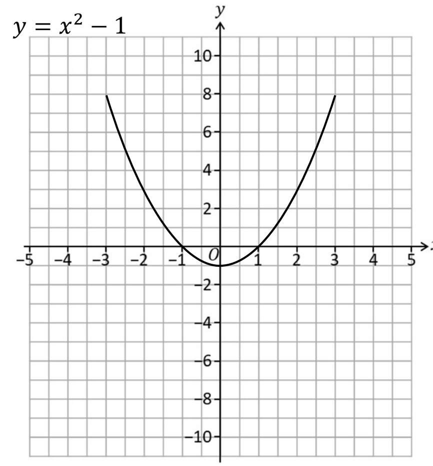


$f(x) = x^2$

x	$f(x)$	$f(x) + 2$
3	9	11
2	4	6
1	1	
0	0	
−1	1	
−2	4	
−3	9	

Describe how the original graph has moved (transformed).

② We will **transform** this graph by adding 2 to the function **input**.
Complete the table of values & plot the graph.



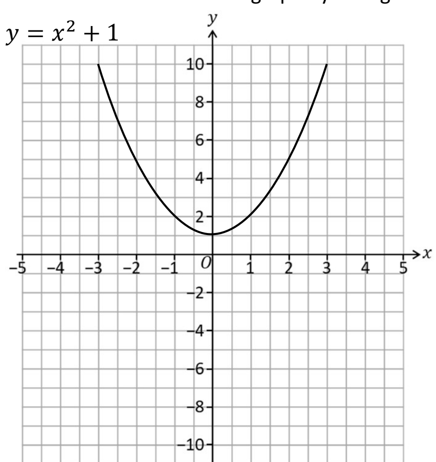
$f(x) = x^2 - 1$

x	$f(x)$	$x + 2$	$f(x + 2)$
3	8	5	24
2	3	4	15
1	0	3	8
0	−1	2	
−1	0		
−2	3		
−3	8		
−4	15		
−5	24		

Describe how the original graph has been transformed.

③ We will **transform** this graph by taking the negation of the **output**.

change
+ to −, or − to +

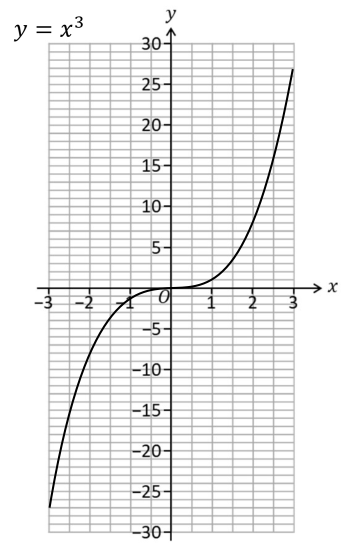


$f(x) = x^2 + 1$

x	$f(x)$	$-f(x)$
3	10	−10
2	5	
1	2	
0	1	
−1	2	
−2	5	
−3	10	

Describe how the original graph has been transformed.

④ We will **transform** this graph by taking the negation of the **input**.



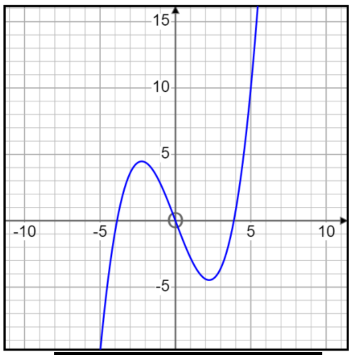
$f(x) = x^3$

x	$f(x)$	$-x$	$f(-x)$
3	27	−3	
2	8		
1	1		
0	0		
−1	−1		
−2	−8		
−3	−27		

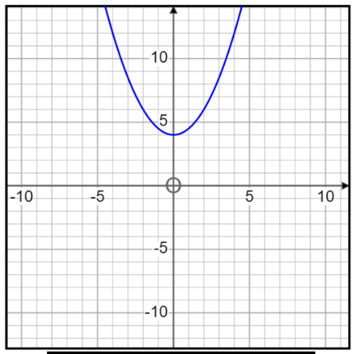
Describe how the original graph has been transformed.

transformations

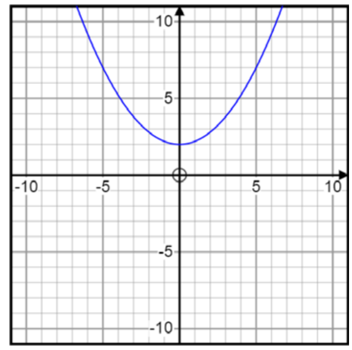
On each grid, y = f(x) is drawn.
Sketch the graph of the transformation indicated.



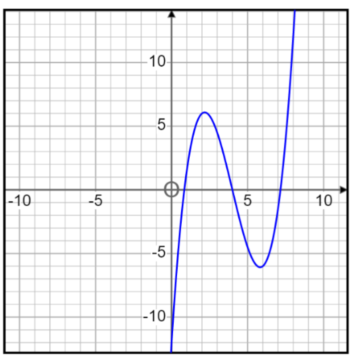
y = f(x) + 5



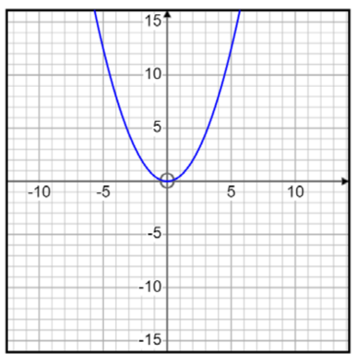
y = -f(x)



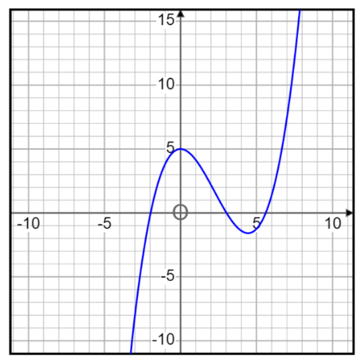
y = f(x + 3)



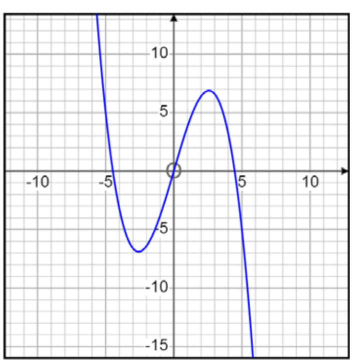
y = f(-x)



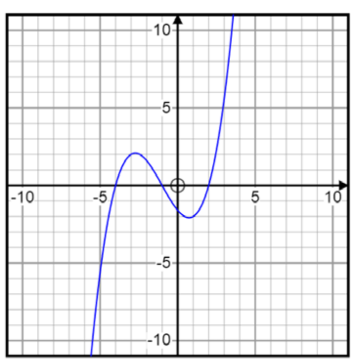
y = f(x-3)



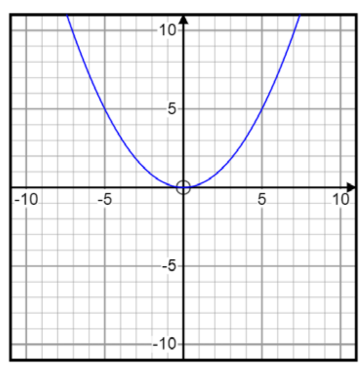
y = f(x+3)



y = f(x - 3) - 4



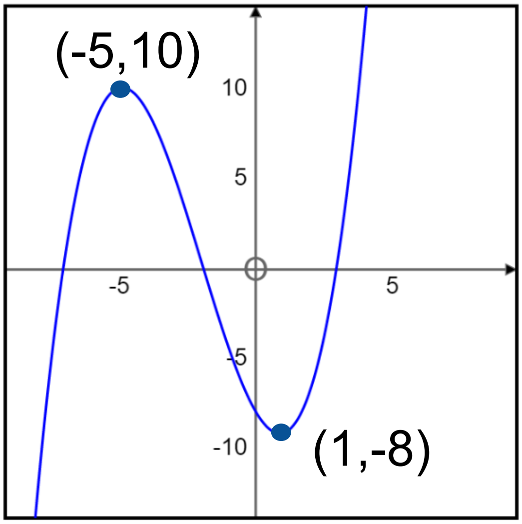
y = f(x - 3) + 4



y = -f(x - 5)

transformed functions

The minimum and maximum points of y = f(x) are shown.
Work out the maximum and minimum points after each of the transformations below.

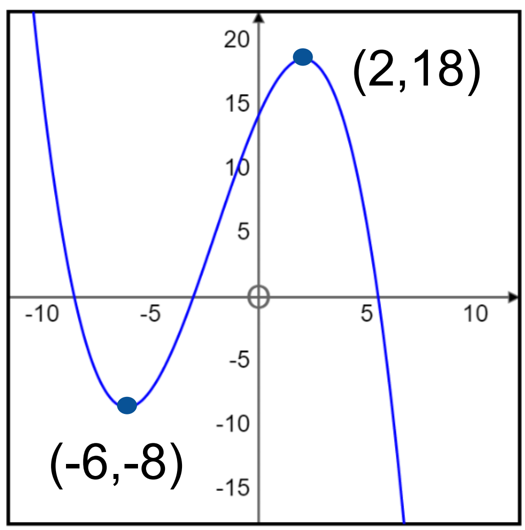


y = f(x)

y = f(-x) Max: Min:	y = f(x) + 2 Max: Min:
y = f(x - 2) Max: Min:	y = -f(x) Max: Min:

The boxes contain the minimum and maximum points of a transformation of y = f(x). Work out the equation of each of these transformations.

.... Max: (3,18) Min: (-5,-8)	.... Max: (2, 23) Min: (-6, -3)
.... Max: (5, 18) Min: (-3, -8)	.... Max: (-1, 18) Min: (-9, -8)

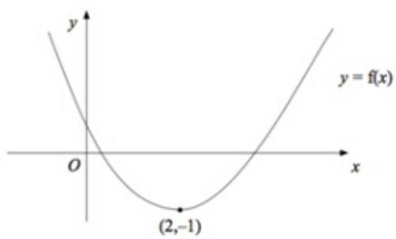


y = f(x)

Fluency Practice

Q	$A\ (3, 4)$ is a point on $y = f(x)$. Work out the new coordinates of A	Answers	Q	$A\ (-5, 2)$ is a point on $y = f(x)$. Work out the new coordinates of A	Answers
1	$y = f(x) + 1$		13	$y = f(x) + 3$	
2	$y = f(x) - 1$		14	$y = f(x) - 3$	
3	$y = f(x + 1)$		15	$y = f(x + 3)$	
4	$y = f(x - 1)$		16	$y = f(x - 3)$	
5	$y = -f(x)$		17	$y = -f(x)$	
6	$y = f(-x)$		18	$y = f(-x)$	
7	$y = f(x + 2) - 5$		19	$y = f(x + 4) - 1$	
8	$y = f(x - 2) + 5$		20	$y = f(x - 1) + 4$	
9	$y = -f(x - 5) + 2$		21	$y = -f(x - 4) + 1$	
10	$y = -f(x + 5) - 2$		22	$y = -f(x + 1) - 4$	
11	$y = -f(-x) - 7$		23	$y = -f(-x) - 7$	
12	$y = f(x + a) + b$		24	$y = f(x - a) - b$	

Question 1



The diagram shows part of the curve with equation $y = f(x)$.The minimum point of the curve is at (2,–1)

Write down the coordinates of the minimum point of the curve with equation $y = f(x + 2)$

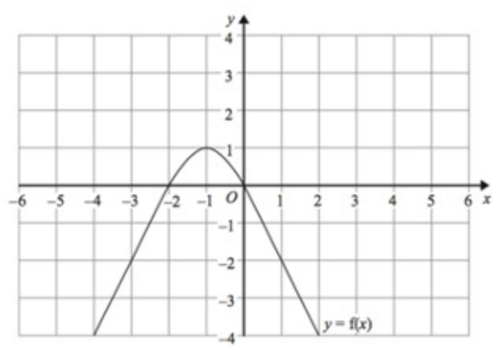
Question 4

The curve with equation $y = f(x)$ has a maximum point at $(2, -7)$.

Find the coordinates of the minimum point of the curve with equation $y = -f(x)$

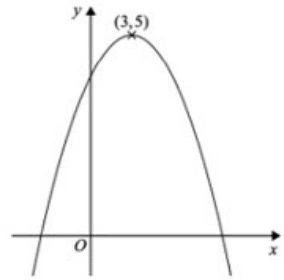
Question 6

The graph of $y = f(x)$ is shown on the grid.



The graph of $y = f(x)$ has a turning point at the point $(-1,1)$. Write down the coordinates of the turning point of the graph of $y = f(-x) + 2$

Question 7



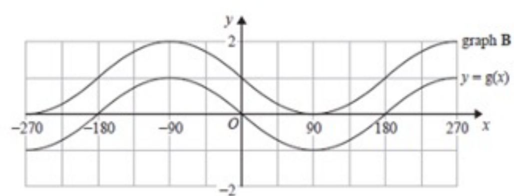
The diagram shows part of the curve with equation $y = f(x)$ The coordinates of the maximum point of the curve are (3,5). The curve with equation $y = f(x)$ is transformed to give the curve with equation

$$y = f(x) - 4$$

Describe the transformation.

Question 8

The graph of $y = g(x)$ is shown on the grid.

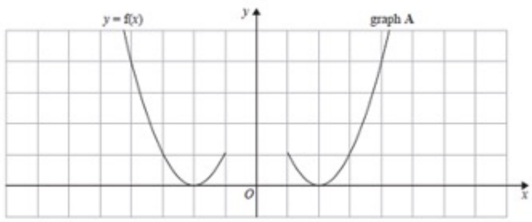


Graph B is a translation of the graph of $y = g(x)$.

Write down the equation of graph B.

Question 9

The graph of $y = f(x)$ is shown on the grid.

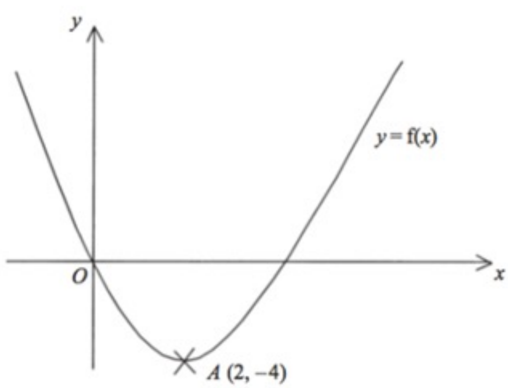


Graph A is a reflection of the graph of $y = f(x)$.

Write down the equation of graph A.

Question 10

This is a sketch of the curve with equation $y = f(x)$.
It passes through the origin O.



The only vertex of the curve is at $A(2, -4)$.

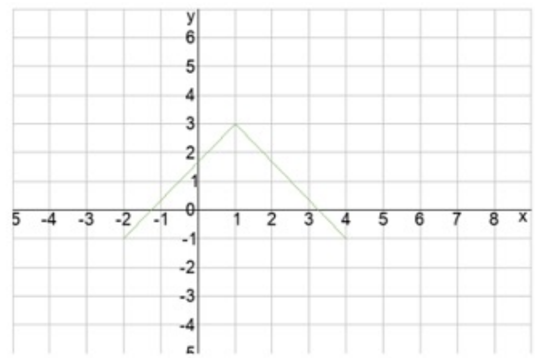
The curve with equation $y = x^2$ has been translated to give the curve $y = f(x)$.

Find $f(x)$ in terms of x.

Question 12

Here is the graph of $y = f(x)$

On the grid, draw the graph of $y = f(-x)$



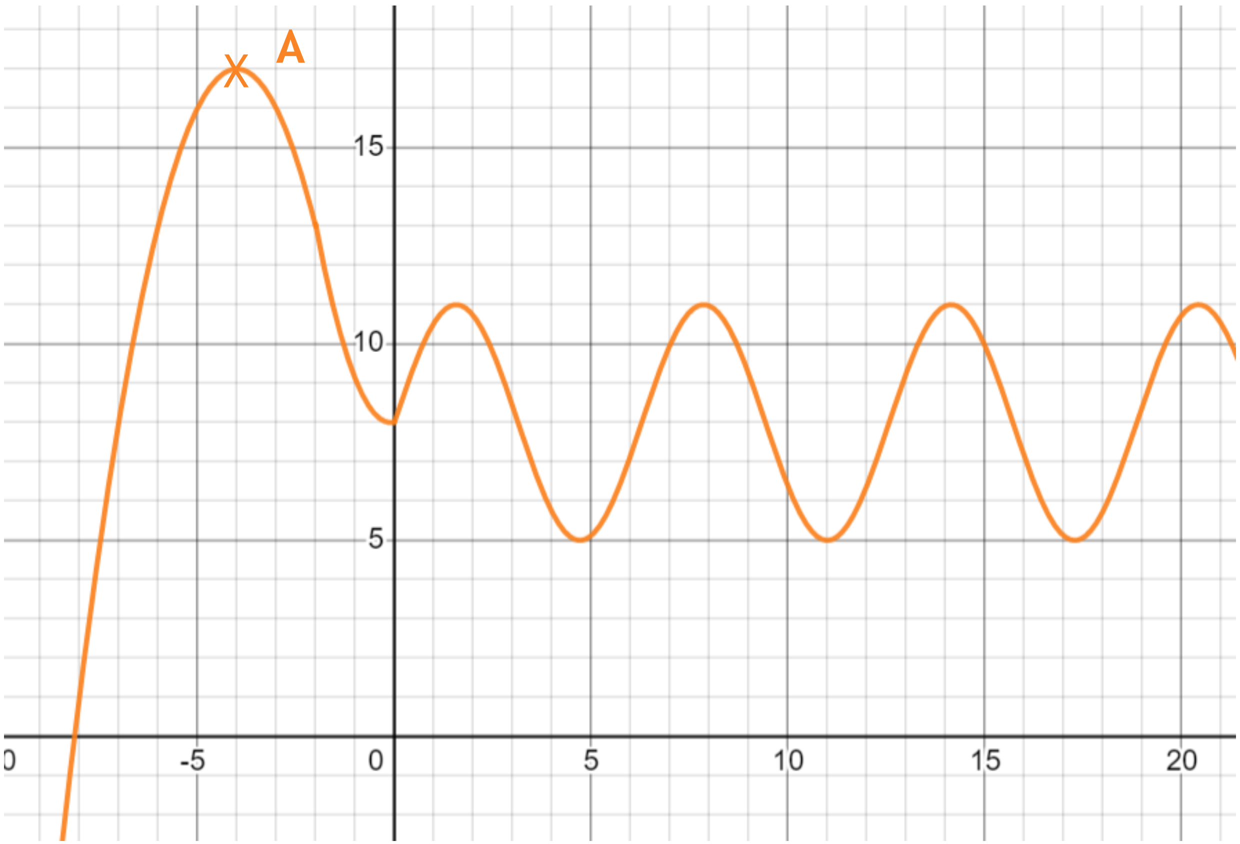
Question 13

The coordinates of the turning point of the graph of $y = x^2 - 8x + 25$ is $(4,9)$.

Hence describe the single transformation which maps the graph of $y = x^2$ onto the graph of $y = x^2 - 8x + 25$.

Fluency Practice

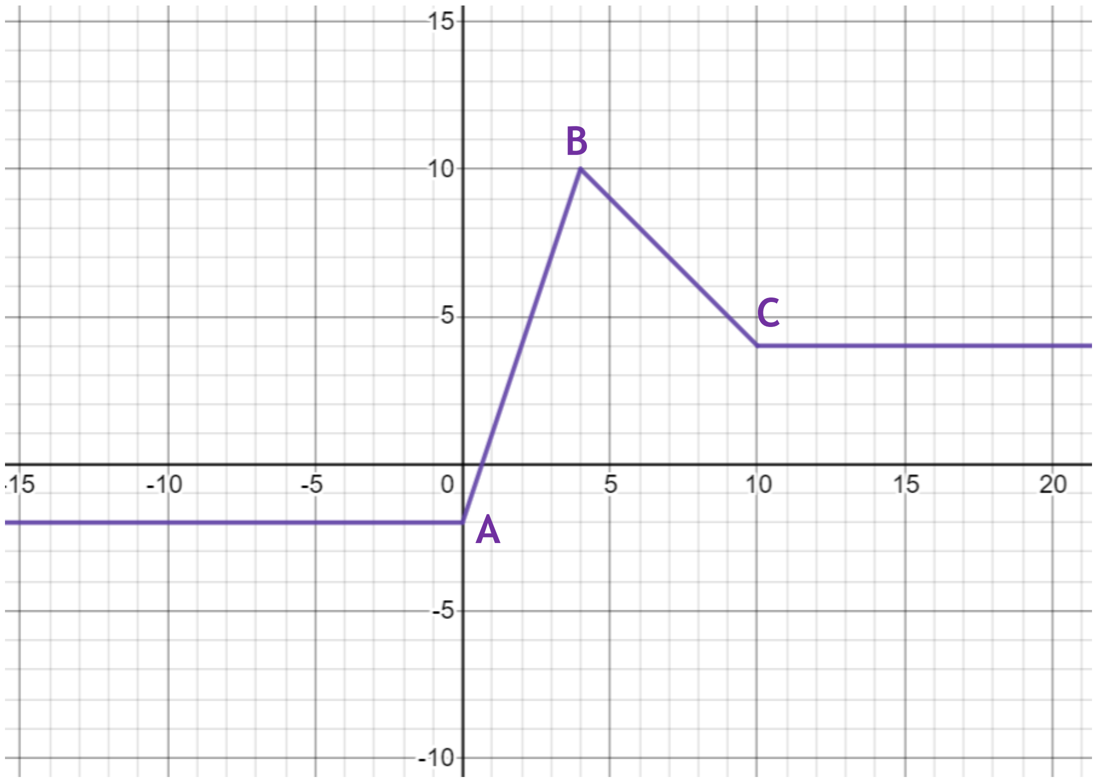
The graph shows the function $g(x)$



Transformation	Point A is now at ____
$g(x-7)$	
	$(-11, 17)$
$g(x)+7$	
$-g(x)$	
	$(4, -17)$
$g(-x)+7$	
$g(-x)-3$	
	$(2, 16)$
$-g(4-x)+2$	

Purposeful Practice

The graph shows the function, $f(x)$



1) Find length AC

2) Find the angle AB makes with the y-axis

3) Find the midpoint of BC

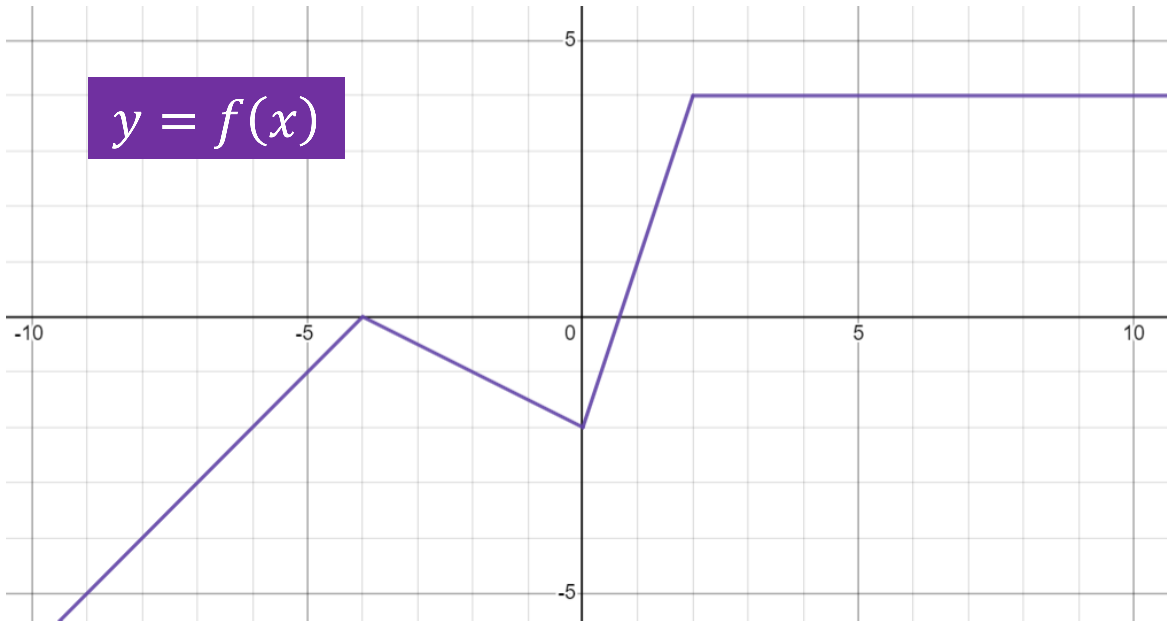
4) Sketch the following functions and write down the coordinates of A, B and C after the transformation:

a) $y = f(x) - 5$
b) $y = f(x - 5)$
c) $y = -f(x) + 5$
d) $y = f(x + 5)$
e) $y = f(5 - x)$
f) $y = -f(x + 2) - 1$
g) $y = -(f(x + 2) - 1)$

5) Find the area between $f(x)$ and the line $y = \frac{3}{5}x - 2$

6) Line segment BD is perpendicular to AB and intersects $f(x)$. Find the length of BD.

Fluency Practice



Sketch each translated graph:

$y - 4 = f(x)$

$y = f(x - 3) - 2$

$y = f(x - 4)$

$y = f(3 + x) + 2$

$y + 4 = f(x - 4)$

$y + 3 = f(x) + 4$

Fluency Practice

Match each vector representation with its equivalent translation in function notation.

Transformations in function notation

$y-3=f(x+2)$	$y=f(x+3)+3$	$y=f(x-3)+2$
$y-2=f(x)-3$	$y+2=f(x-3)$	$1+y=f(2+x)$
$y=f(1+x)+2$	$y+2=f(x-3)$	$y-2=f(3+x)$
$y=1+f(x)$	$y+3=f(x-2)+3$	$y=f(x-2)+3$
$y=f(x)+2$	$y+2=f(1+x)+2$	$y=f(1+x-2)$
	$y=f(x-2)-3$	

Vectors

$\begin{pmatrix}-2\\3\end{pmatrix}$	$\begin{pmatrix}1\\0\end{pmatrix}$	$\begin{pmatrix}-3\\2\end{pmatrix}$
$\begin{pmatrix}3\\-2\end{pmatrix}$	$\begin{pmatrix}-2\\-1\end{pmatrix}$	$\begin{pmatrix}-1\\0\end{pmatrix}$
$\begin{pmatrix}3\\2\end{pmatrix}$	$\begin{pmatrix}0\\2\end{pmatrix}$	$\begin{pmatrix}2\\0\end{pmatrix}$
$\begin{pmatrix}-3\\3\end{pmatrix}$	$\begin{pmatrix}3\\-2\end{pmatrix}$	$\begin{pmatrix}2\\3\end{pmatrix}$
$\begin{pmatrix}0\\-1\end{pmatrix}$	$\begin{pmatrix}0\\1\end{pmatrix}$	$\begin{pmatrix}-1\\2\end{pmatrix}$
	$\begin{pmatrix}2\\-3\end{pmatrix}$	

6 Congruence and Similarity Proofs

Conditions for Congruent Triangles

1. **SSS** (side, side, side)
   All three corresponding sides are equal in length.
2. **SAS** (side, angle, side)
   A pair of corresponding sides and the included angle are equal.

3. **ASA** (angle, side, angle)
   A pair of corresponding angles and the included side are equal.

4. **AAS** (angle, angle. corresponding side)
   A pair of corresponding angles and a non-included side are equal (the non-included side must be opposite one of the equal angles).

5. **RHS** (Right-angled triangle, hypotenuse, side)
   Two right-angled triangles are congruent if the hypotenuse and one side are equal.

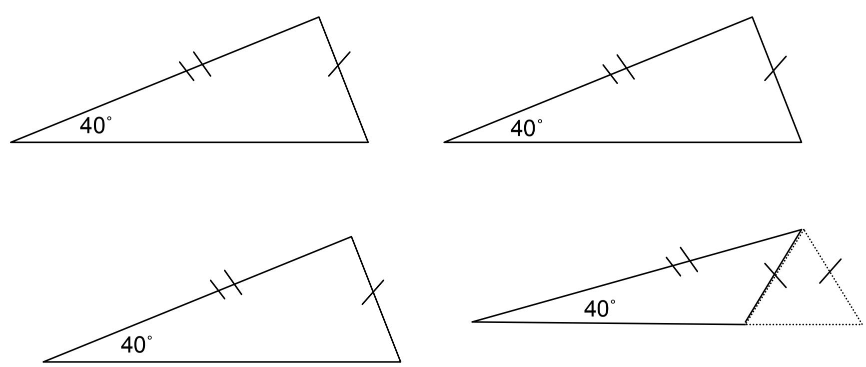
NOTES:

1. AAA does not work.

If all the corresponding angles of a triangle are the same, the triangles will be the same shape, but not necessarily the same size. **The triangles are said to be similar.**

2. SSA also does not work.

Given two sides and a non-included angle, it is possible to draw two different triangles that satisfy the values. It is therefore not sufficient to prove congruence.

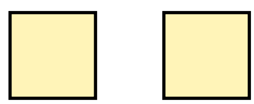


Diagrams not drawn to scale

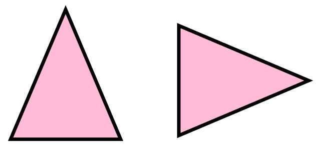
Fluency Practice

Question 1: For each pair of shapes, state whether they are congruent or not congruent

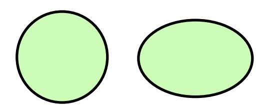
(a)



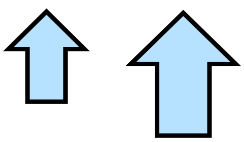
(b)



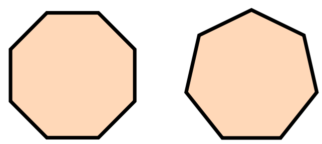
(c)



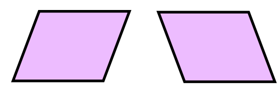
(d)



(e)

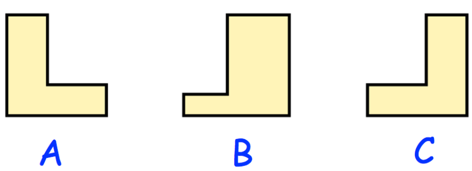


(f)

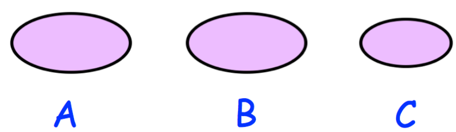


Question 2: Write down the shape that is not congruent to the others

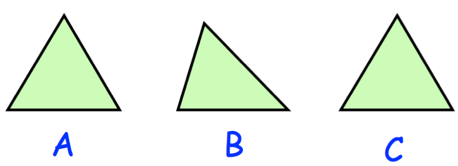
(a)



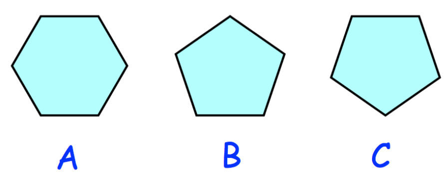
(b)



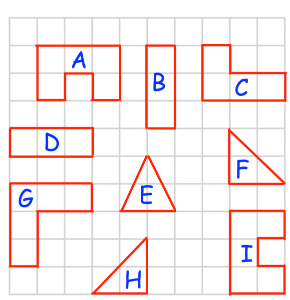
(c)



(d)

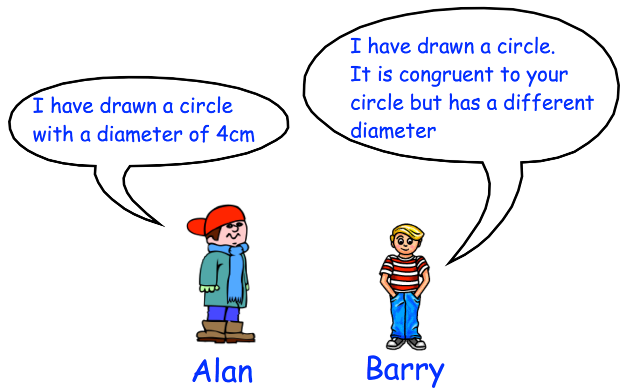


Question 3: Which pairs of shapes on the grid are congruent?



Apply

Question 1: Can Barry be correct?
Explain your answer.

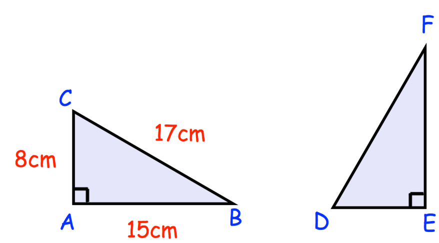


Question 2: Triangles ABC and DEF are congruent.

(a) Write down the length of DF

(b) Write down the length of AC

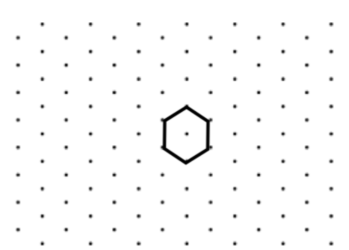
(c) Write down the length of DE



Question 3: Triangles A and B are congruent.
Tick the correct boxes.

	True	False	Maybe
If Triangle A is isosceles, Triangle B has to be isosceles.	☐	☐	☐
Triangles A and B have different size angles	☐	☐	☐
Triangle A has a larger area than Triangle B	☐	☐	☐

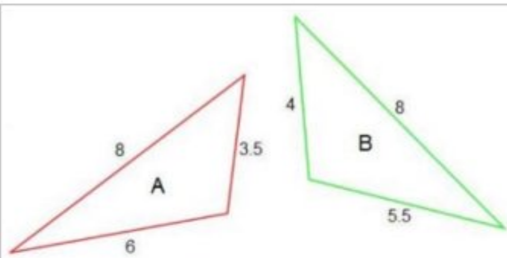
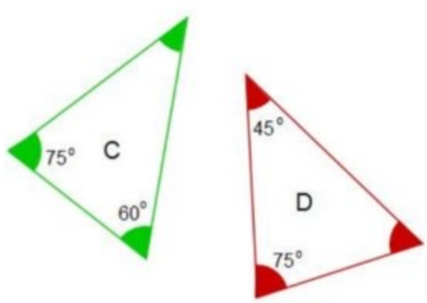
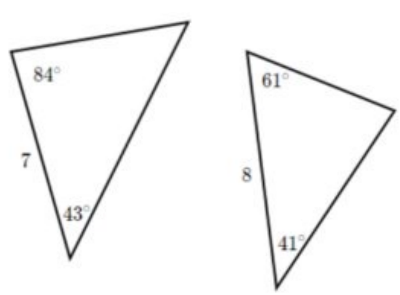
Question 4: A regular hexagon is drawn below.
Draw at least 8 more congruent hexagons
to show the hexagon tessellates.



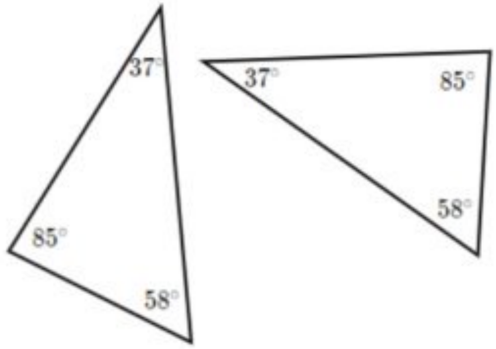
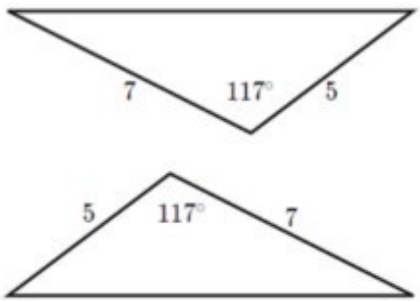
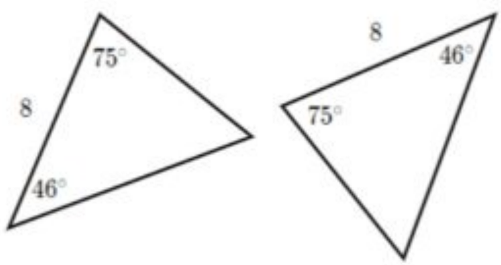
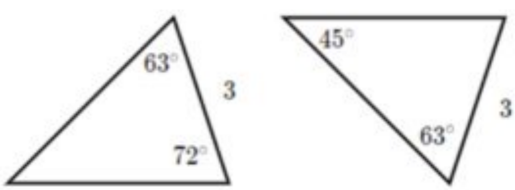
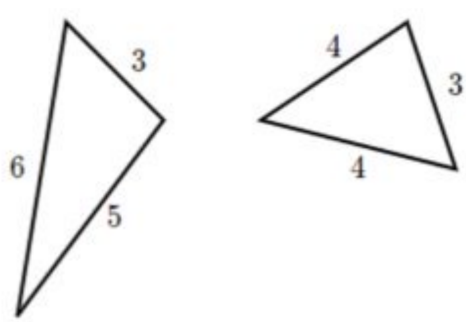
Fluency Practice

Exactly the same size and shape. All lengths and angles are the same.

For each of these pairs of triangles decide if they are Congruent, not congruent or there is not enough information

	Congruent Not congruent Not enough information
	Congruent Not congruent Not enough information
	Congruent Not congruent Not enough information

Fluency Practice

	Congruent Not congruent Not enough information
	Congruent Not congruent Not enough information
	Congruent Not congruent Not enough information
	Congruent Not congruent Not enough information
	Congruent Not congruent Not enough information

Fluency Practice

congruent triangles

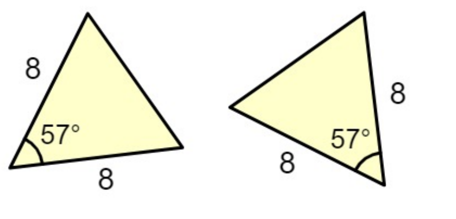
Diagrams are not drawn accurately

Are each pair of triangles congruent?

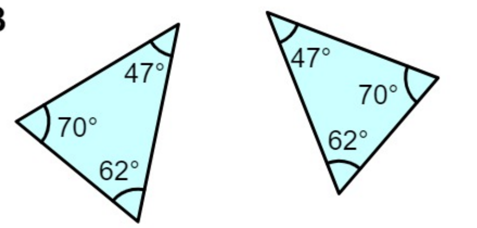
For each pair, decide whether...

- they are congruent, giving a reason (SSS, ...)
- they are not congruent
- there is not enough information to decide

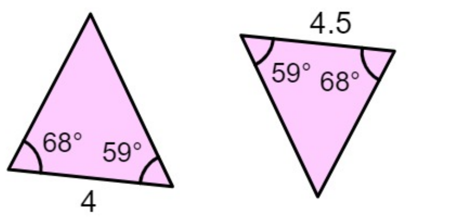
A



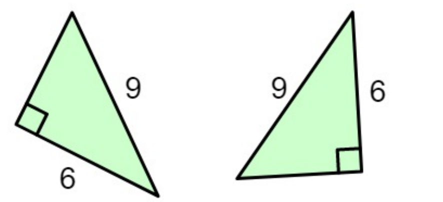
B



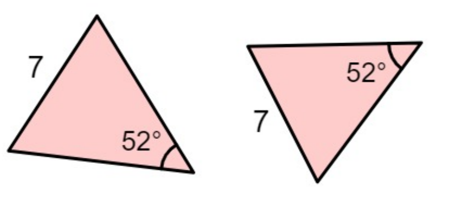
C



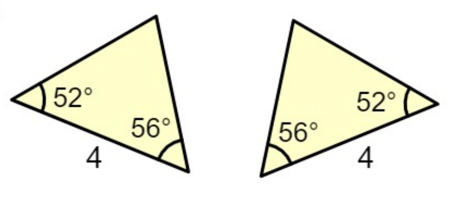
D



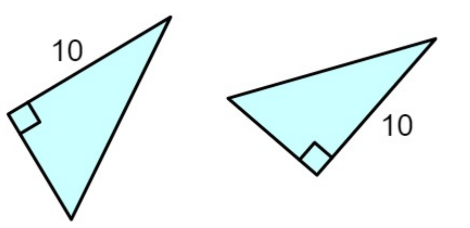
E



F



G



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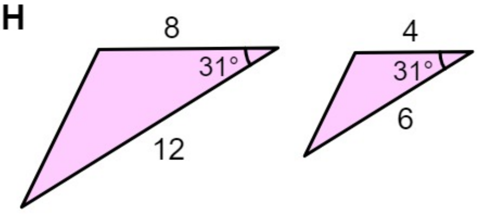

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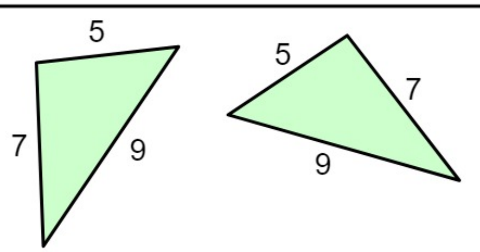

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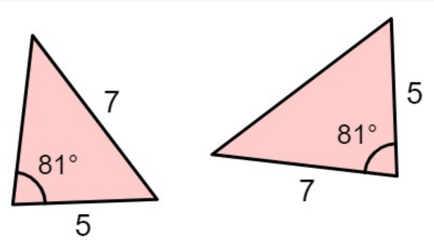
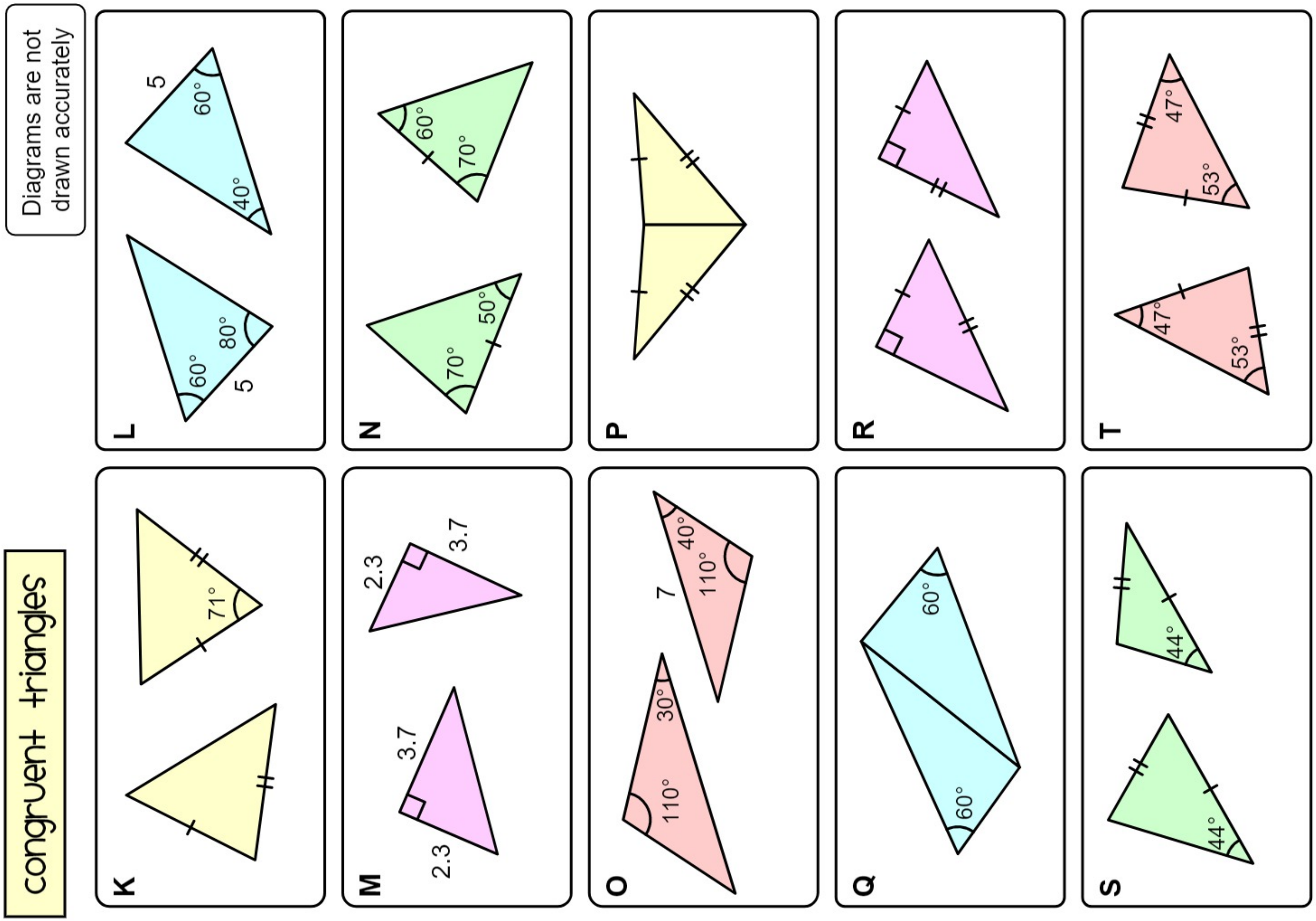


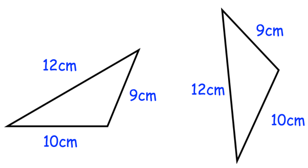
Fluency Practice



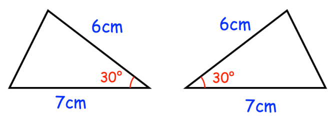
Fluency Practice

Question 1: The following pairs of triangles are congruent, state the condition that shows they are congruent.

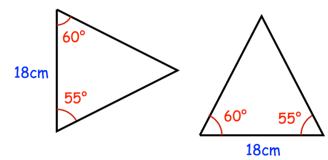
(a)



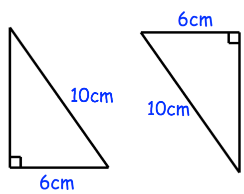
(b)



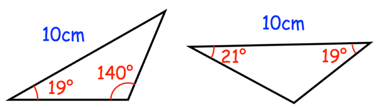
(c)



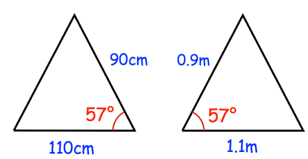
(d)



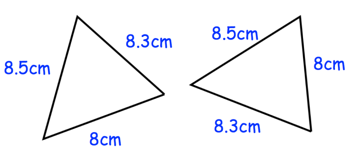
(e)



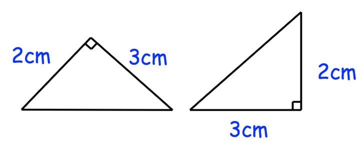
(f)



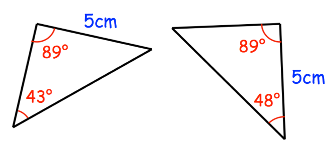
(g)



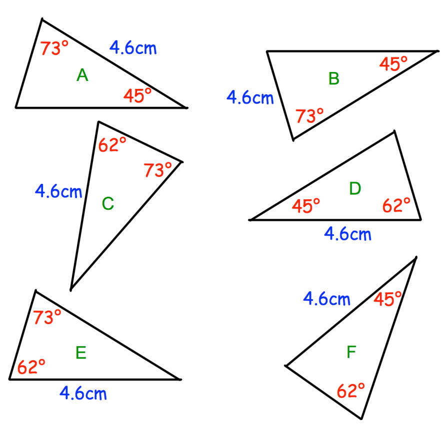
(h)



(i)



Question 2: Shown are six triangles.
Which triangles are congruent?



Question 3: In triangle ABC, AB = 7cm, ∠BAC = 50° and ∠ABC = 35°
In triangle DEF, EF = 7cm, ∠DEF = 35° and ∠DFE = 50°
Are triangles ABC and DEF congruent? If they are, state the condition.

Question 4: In triangle GHI, GH = 7cm, HI = 4cm and GI = 5cm.
In triangle JKL, JK = 7cm, KL = 4.5cm and JL = 5cm.
Are triangles GHI and JKL congruent? If they are, state the condition.

Question 5: In triangle MNO, ∠MNO = 50°, ∠NOM = 60° and ∠OMN = 70°
In triangle PQR, ∠PQR = 50°, ∠QRP = 60° and ∠RPQ = 70°
Are triangles MNO and PQR congruent? If they are, state the condition.

Question 6: In triangle STU, SU = 13cm, ∠TSU = 20° and ∠TUS = 30°
In triangle VWX, WX = 13cm, ∠WXV = 30° and ∠XVW = 20°
Are triangles STU and VWX congruent? If they are, state the condition.

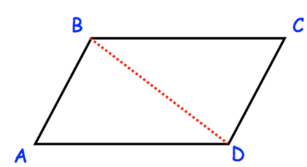
Apply

Question 1: Hannah and Chris each draw a triangle with one side of 3cm, one angle of 35° and one angle of 80°.
Hannah says their triangles **must** be congruent.
Is Hannah correct?

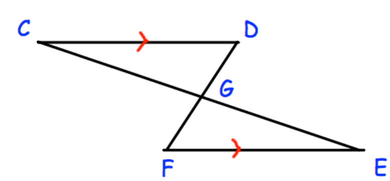
Question 2: Paul and Greg each draw a triangle with one side of 3cm, one side of 9cm and one side of 10cm.
Greg says their triangles **must** be congruent.
Is Greg correct?

Question 3: Carl and Michael each draw a triangle with one angle of 58°, one angle of 68° and one angle of 54°.
Carl says their triangles **must** be congruent.
Is Carl correct?

Question 4: ABCD is a parallelogram.
Prove that triangles ABD and BCD are congruent.

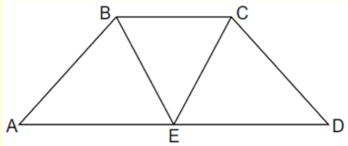


Question 5: In the diagram, the lines CE and DF intersect at G.
CD and FE are parallel and CD = FE.
Prove that triangles CDG and EFG are congruent.



Purposeful Practice

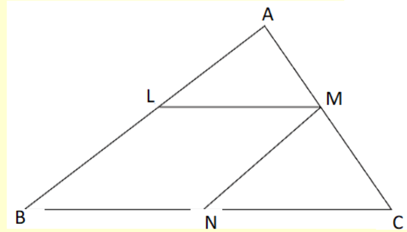
1) The diagram shows **trapezium** $ABCD$.
E is the **midpoint** of AD.
BCE is an **equilateral** triangle.



Prove that the triangle ABE is congruent to triangle DCE

2) The diagram shows a triangle ABC.
$LMNB$ is a **parallelogram** where
L is the **midpoint** of AB,
M is the **midpoint** of AC,
and N is the **midpoint** of BC.

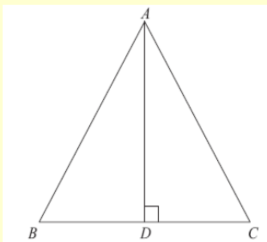
Prove that triangle ALM and triangle MNC are congruent.
You must give reasons for each stage of your proof.



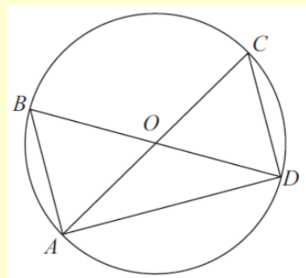
3) ABC is an **equilateral** triangle.
D lies on BC.
AD is **perpendicular** to BC.

a. Prove that triangle ADC is congruent to triangle ADB.

b. Hence, prove that $BD = \frac{1}{2}AB$



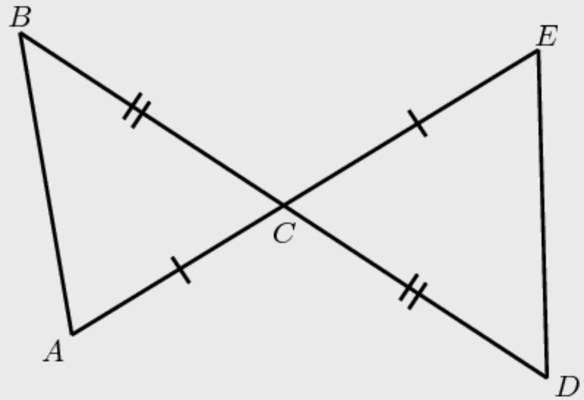
4) AOC and BOD are **diameters** of a circle, **centre** O.
Prove that triangle ABD and triangle DCA are congruent



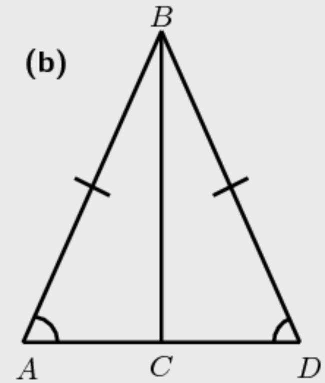
Fluency Practice

State whether the following pairs of triangles are congruent or not. Give reasons for your answers. If there is not enough information to make a decision, explain why.

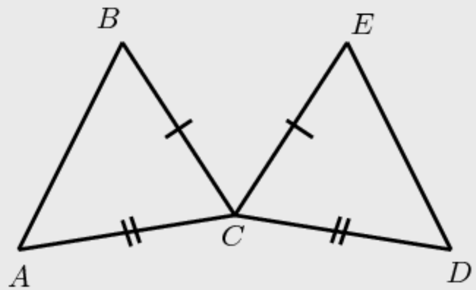
(a)



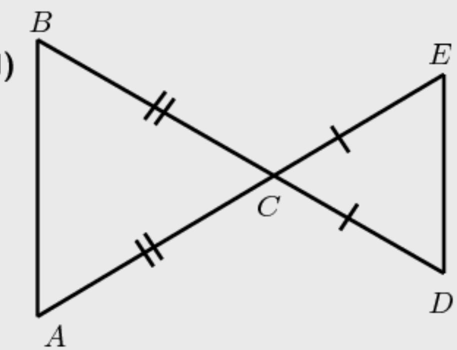
(b)



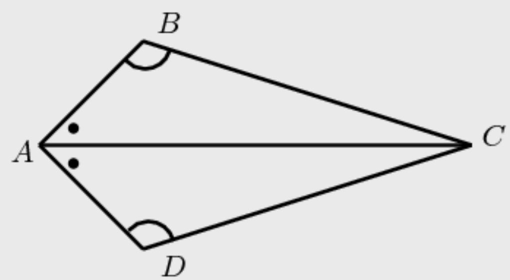
(c)



(d)



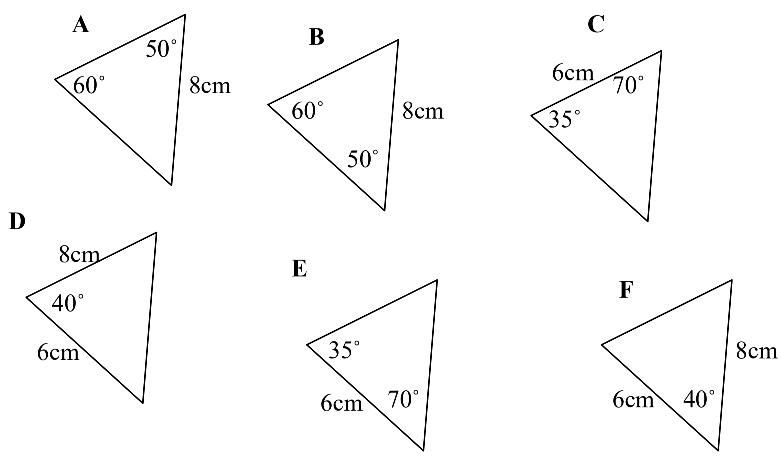
(e)



NOTE: ALL DIAGRAMS **NOT** DRAWN TO SCALE.

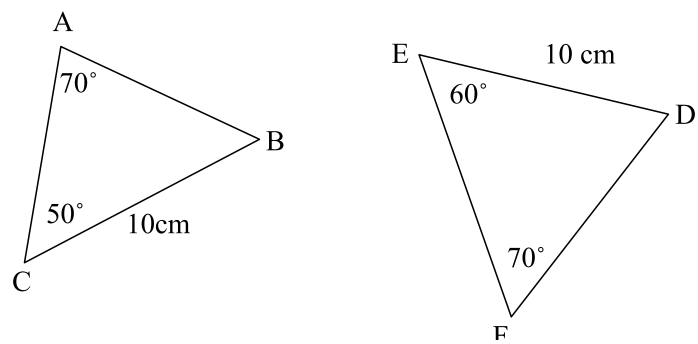
* means "may be challenging for some"

1. Which triangles are congruent? Give reasons.



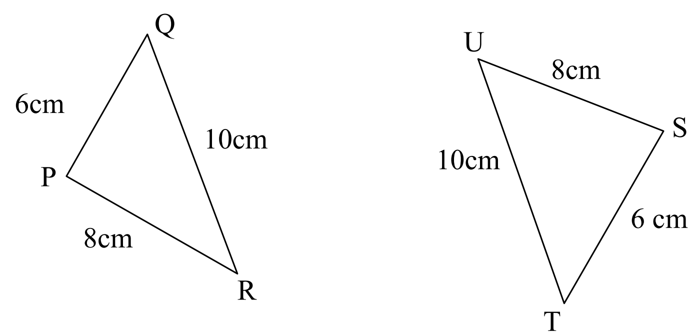
2. Prove that the triangles ABC and DEF are congruent.

Diagrams not drawn to scale



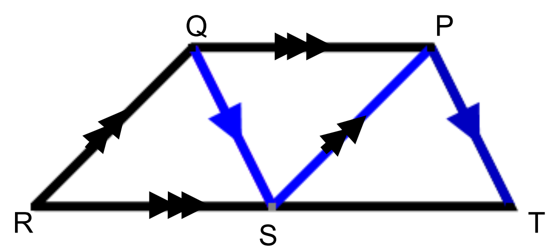
3. Prove that the triangles PQR and STU are congruent.

Diagrams not drawn to scale



4. PQRS is a parallelogram. The line drawn from P parallel to QS meets RS produced at T.
   Prove that TS = SR.

Diagram not drawn to scale



5. The triangle PQR is an isosceles triangle. PS is perpendicular to QR.
   (a) Use congruent triangles to prove that SQ = SR.
   (b) If PQ = 10cm and QR = 12cm, work out the area of the triangle PQR.

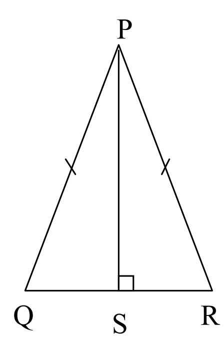


Diagram not drawn to scale

6. PQRS is a rectangle.
M is the mid-point of PQ, N is the midpoint of QR,
T is the midpoint of PS and U is the midpoint of RS.

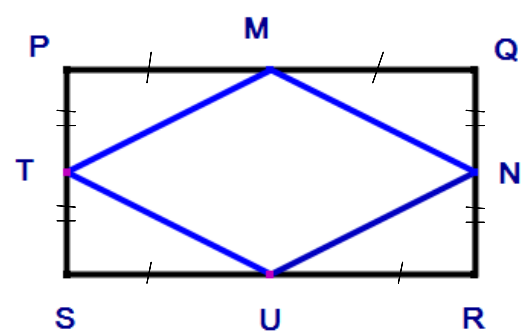


Diagram not drawn to scale

(a) Prove that the triangles PMT and RUN are congruent.
(b) Are the lines TM and UN equal? Why?
(c) Is the triangle STU congruent to the triangle RNU? Give reasons
(d) Are the lines TU and UN equal? Why?
(e) What is the special name given to the quadrilateral MNUT?
(f) If PQ = 8cm and PT = 6cm, what is the area of the quadrilateral MNUT?

7. ABCD is a kite, with AB = AD and BC = CD.
(a) Prove that triangles ABC and ADC are congruent.
The line joining B to D meets the diagonal AC at E.
(b) Prove that triangles ABE and ADE are congruent.
(c) Make a geometrical statement about the point E.
(d) If BD = 6cm and AC = 12cm, work out the area of the kite ABCD.

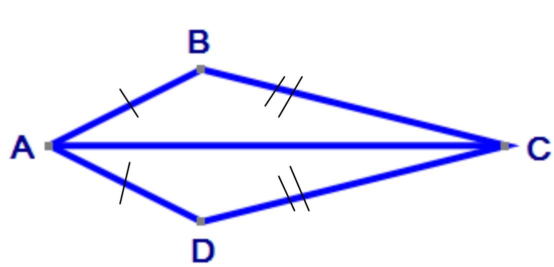
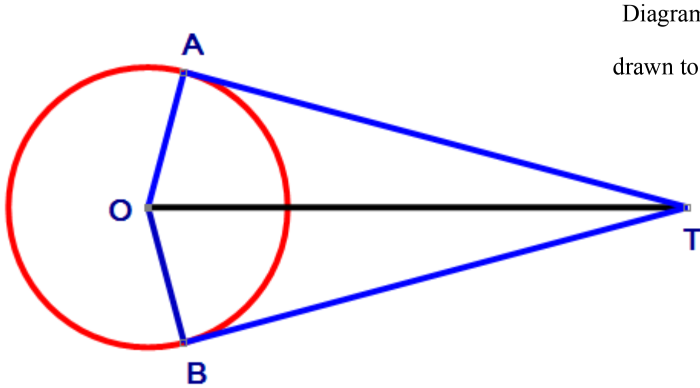


Diagram not drawn to scale

*8. TA and TB are tangents to the circle, centre, O.

(a) Use congruent triangles to prove that AT = BT.
(b) Which angle is equal to angle AOT?
(c) If angle AOT = 40˚, work out the size of angle OTB.
(d) If the radius of the circle is 6cm and OT = 10cm, work out the area of the quadrilateral BOAT.

Diagram not drawn to scale

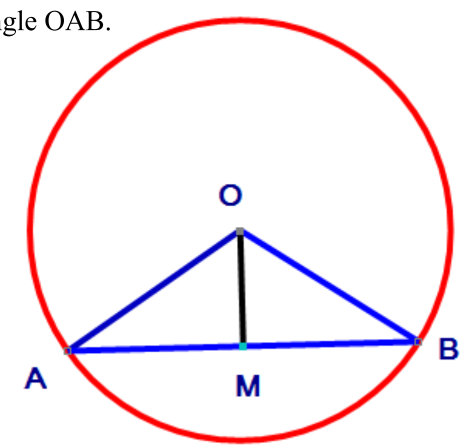


*9. The diagram below shows a circle, centre, O.

AB is a chord of the circle. M is the midpoint of AB.
(a) Use congruent triangles to prove that angle OMA is 90˚.

*(b) If the radius of the circle is 13cm and AB = 24cm, work out the area of triangle OAB.

Diagram not drawn to scale



10. PQRS is a rectangle. T and V are the midpoints of PS and QR respectively.

U and W are points on PQ and RS such that PU = SW.

(a) Prove that triangles PUT and SWT are congruent.
(b) Why is UQ = WR?
(c) Hence, or otherwise, prove that triangles QUV and RWV are congruent.
(d) Hence, or otherwise, prove that triangles TUV and TWV are congruent.
(e) What is the special name given to the quadrilateral TUVW?
(f) If PQ = 12cm and QR = 6cm, what is the area of TUVW?
(g) If TV = 20cm and UW = 8cm, what is area of TUVW?

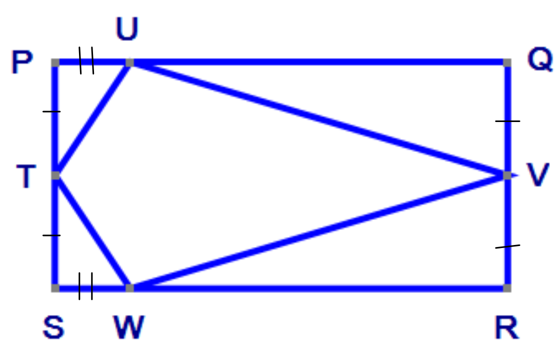


Diagram not

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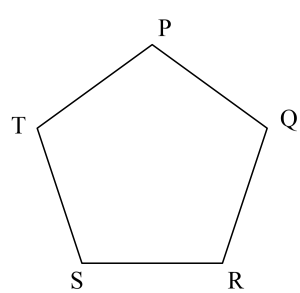
11. PQRST is a regular pentagon.

List all the triangles that are congruent to triangle RTQ.



12. PQRS is a square. M is the midpoint of PQ and N is the midpoint of PS.

(a) Use congruent triangles to prove that RM = QN.

* (b) Given that PQ = $2a$, where a is a positive integer,

use Pythagoras' Theorem to show that $RM = a\sqrt{5} = QN$.

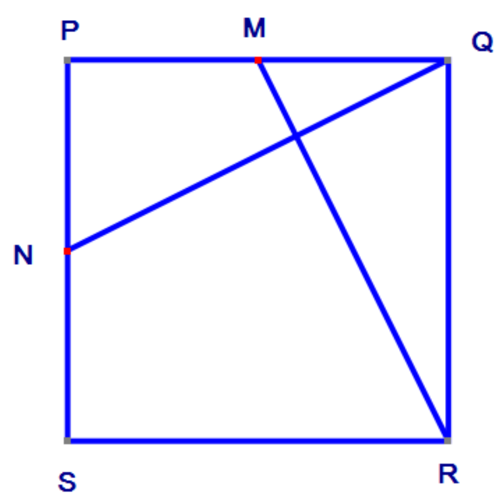


Diagram not drawn to scale

13. PQRS and PTUV are squares attached to the two sides of a triangle.
Prove that: (a) the triangle PQT and PSV are congruent.

** (b) QT is perpendicular to SV (that they meet at 90˚).

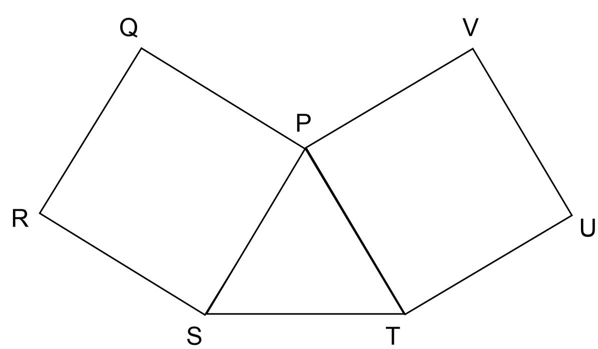


Diagram not drawn to scale

constructions and congruence in triangles

three sides the same [SSS].
(1) 9 cm, 8 cm and 5 cm.
(2) 11 cm, 6 cm and 4.5 cm.
Will everyone's triangles look the same?
Reasons?

three angles the same [AAA].
(3) 45^o, 70^o, 65^o.
(4) 80^o, 40^o, 70^o.
Will all the triangles that people have drawn be identical (be congruent)?
Reasons?

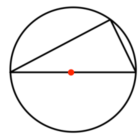
two angles and an included side (between them) the same [ASA]
(5) 60^o, 9cm, 40^o.
(6) 100^o, 8cm, 30^o.
Will all the triangles drawn be congruent?
Reasons?

two sides and an angle [SAS] and [ASS]
a triangle PQR with:
(7) PQ = 9cm, QR = 8cm, angle P = 50^o.
(8) PQ = 9cm, PR = 11cm, angle P = 50^o.
(9) PQ = 8cm, PR = 6cm, angle R = 90^o.
(10) PQ = 10cm, PR = 7cm, angle P = 50^o.
Will all the triangles drawn be congruent?
Reasons?

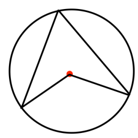
7 Circle Theorem Proofs

Fluency Practice

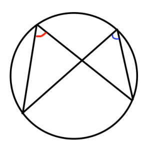
Question 1: Prove that the angle in a semi-circle is always 90°



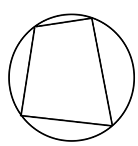
Question 2: Prove that the angle at the centre is twice the angle at the circumference.



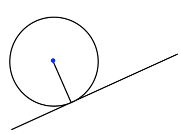
Question 3: Prove the angles in the same segment are equal.



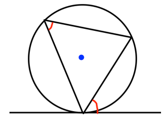
Question 4: Prove the opposite angles in a cyclic quadrilateral add to 180°



Question 5: Prove the angle between a tangent and the radius is 90°



Question 6: Prove the alternate segment theorem; that the angle between the tangent and the chord at the point of contact is equal to the angle in the alternate segment.

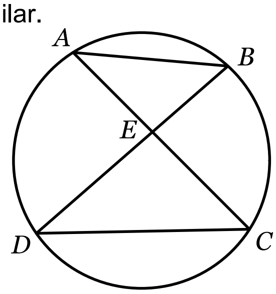


Fluency Practice

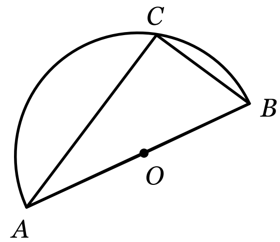
circle theorems and proof

Give reasons for each stage of your working.

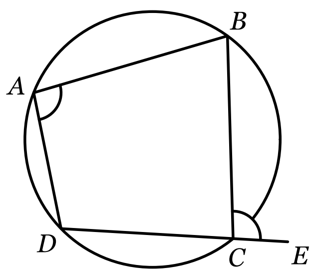
1. Prove that triangle AEB and triangle DEC are similar.



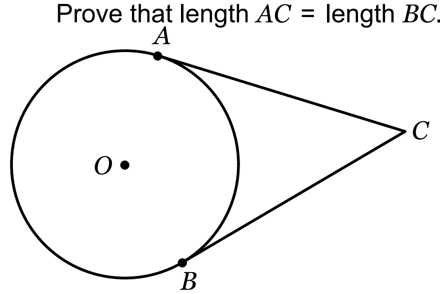
2. Point C lies on the arc of the semicircle. Prove that angle ACB is 90°.



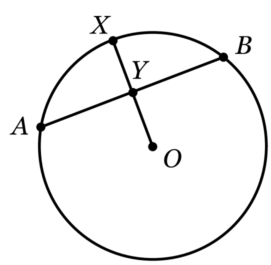
3. Prove that angle DAB is equal to angle BCE.



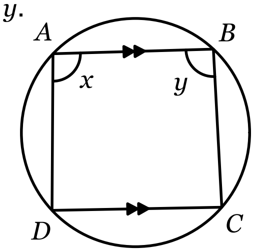
4. AC and BC are tangents to the circle. Prove that length AC = length BC.



5. OX is perpendicular to the chord AB. Prove that Y is the midpoint of AB.

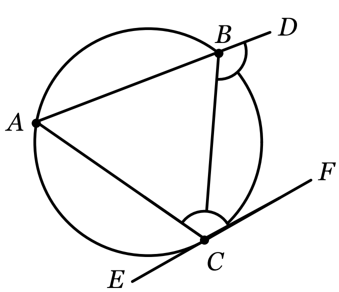


6. a) Prove that $x = y$.



b) Jake says that x and y could both measure 80°. Is he right?

7. Prove that angle CBD = angle ACF.



8. Prove that $y = x + 90$.