KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Mathematics (L2FM) 2026

Unit 26 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: ______________________________

Class: ______________________________

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Unit 26 Booklet – Part 2

HGS Maths

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Contents

1 Differentiation (L2FM Only)

Basic Differentiation

Worked Example	Your Turn
Differentiate with respect to x: a) x^4 b) $5x^4$ c) $4x - 5$ d) $\frac{1}{x^3}$ e) $\frac{6}{5x^3}$	Differentiate with respect to x: a) x^5 b) $-3x^5$ c) $6x - 7$ d) $\frac{1}{x^4}$ e) $\frac{7}{8x^4}$

Worked Example	Your Turn
Differentiate with respect to x: a) $y = 4x^3 + 3x^2 + 2x + 1$ b) $y = (5x - 3)^2$ c) $f(x) = x^3(x + 2)$	Differentiate with respect to x: a) $y = 5x^4 - 2x^7 + 12345 - x^5$ b) $y = (3x - 5)^2$ c) $f(x) = x^2(x - 3)$

Dr Frost 512a and 512b

Worked Example	Your Turn
a) A curve has equation $y = 2x^3 + kx^2 - 6x - 4$, where k is a constant. Find $\frac{dy}{dx}$ b) A curve has equation $y = f(x)$, where $f(x) = x^3 + bx^2 - 6x + 6$ Given that $f'(-1) = -11$ Find the value of b	a) A curve has equation $y = px^2 + 5x - 8$, where p is a constant. Find $\frac{dy}{dx}$ b) A curve has equation $y = f(x)$, where $f(x) = rx^2 - 2x + 6$ Given that $f'(4) = 38$ Find the value of r

Worked Example	Your Turn
Find an expression for the rate of change of y with respect to x: $y = (x-3)(x-4)^2$	Find an expression for the rate of change of y with respect to x: $y = (x+1)(x+2)^2$

Dr Frost 512i

Worked Example	Your Turn
Differentiate with respect to x: a) $f(x) = \frac{(3x-2)^2}{5x}$ b) $y = \frac{3x^4(2x^5-11x)}{x^2}$	Differentiate with respect to x: a) $f(x) = \frac{(2x+3)^2}{5x}$ b) $f(x) = \frac{2x^2(3x^3-7x)}{x}$

Worked Example	Your Turn
a) Find the gradient of the curve: $y = 2x^3 - x + 5$ at $(-1, 4)$ b) Find the coordinates of the point(s) where the gradient is 4: $y = 5x^2 - x + 7$	a) Find the gradient of the curve: $y = 3x^2 - 2x + 1$ at $(-2, 17)$ b) Find the coordinates of the point(s) where the gradient is 3: $y = 3x^2 - 9x + 7$

Worked Example	Your Turn
A curve has gradient function $\frac{dy}{dx} = 3x^2 + 7$ Work out the values of x for which the rate of change of y with respect to x is 55	A curve has gradient function $\frac{dy}{dx} = 5x^2 - 7$ Work out the values of x for which the rate of change of y with respect to x is 38

Worked Example

Let $f(x) = 4x^2 - 8x + 3$

a) Find the gradient of $y = f(x)$ at the point $\left(\frac{1}{2}, 0\right)$

b) Find the coordinates of the point on the graph of $y = f(x)$ where the gradient is 8

c) Find the gradient of $y = f(x)$ at the points where the curve meets the line $y = 4x - 5$

Your Turn

Let $f(x) = x^2 - 4x + 2$

a) Find the gradient of $y = f(x)$ at the point $(1, -1)$

b) Find the coordinates of the point on the graph of $y = f(x)$ where the gradient is 5

c) Find the gradient of $y = f(x)$ at the points where the curve meets the line $y = 2 - x$

Worked Example	Your Turn
Find the coordinates of the point(s) where the gradient is 10: $y = x^3 + 6x^2 - 26x + 7$	Find the coordinates of the point(s) where the gradient is 2: $y = x^3 - 3x^2 - 7x + 8$

Worked Example	Your Turn
$y = 3x^2 + bx$ The rate of change of y with respect to x when $x = 4$ is triple the rate of change of y with respect to x when $x = -2$ Work out the value of b	$y = 2x^3 + ax$ The rate of change of y with respect to x when $x = 2$ is twice the rate of change of y with respect to x when $x = -1$ Work out the value of a

Fill in the Gaps

Equation	Gradient Function	Point P	Gradient at P
$y = x^2$	$\frac{dy}{dx} = 2x$	$(2, 4)$	4
$y = x^3 + x$		$(1, 2)$	
$y = 6x - x^2$		$(4, 8)$	
$y = x^3 - 3x^2 + 4x$		$(-1, 0)$	
$y = 5x^2 - 7x + 1$		$(-2, 36)$	
$y = (2x + 5)(x - 3)$		$(3, 0)$	
$y = 3x(x - 1)^2$		$(-1, -12)$	
$y = \frac{1}{x^2}$		$\left(2, \frac{1}{4}\right)$	
$y = \frac{x^4 - 5x^3}{x}$		$(1, -4)$	
$y = \frac{2x^3 + x}{x^2}$		$\left(3, \frac{19}{3}\right)$	
$y = 10 - 2x - x^2$			-10
$y = x^4 + 3$			32
$y = (x + 4)(3x - 5)$			1
$y = x^2 + \frac{54}{x}$			0
	$\frac{dy}{dx} = 3x^2 + 6x - 1$	$(1, 3)$	

Tangents and Normals

Worked Example	Your Turn
a) Find the equation of the tangent to the curve $y = x^4$ when $x = 2$ b) Find the equation of the normal to the curve $y = x^4$ when $x = 2$	a) Find the equation of the tangent to the curve $y = x^3$ when $x = 2$ b) Find the equation of the normal to the curve $y = x^3$ when $x = 2$

Worked Example	Your Turn
Find the equation of the tangent to the curve with equation $y = x^3 - 5x^2 - 3x + 2$ at the point $(5, -13)$	Find the equation of the normal to the curve with equation $y = x^3 - 3x^2 + 2x - 1$ at the point $(3, 5)$

Dr Frost 520c and 520d

Worked Example	Your Turn
The tangent to the curve $y = \frac{1}{4}x^3 + x^2 - 3x$ at the point P is parallel to the line with equation $y = -\frac{17}{4}x + 2$. Find the two possible values for the x-coordinate of the point P.	The tangent to the curve $y = \frac{1}{5}x^3 - x^2 - x$ at the point P is parallel to the line with equation $y = 4x - 9$. Find the two possible values for the x-coordinate of the point P.

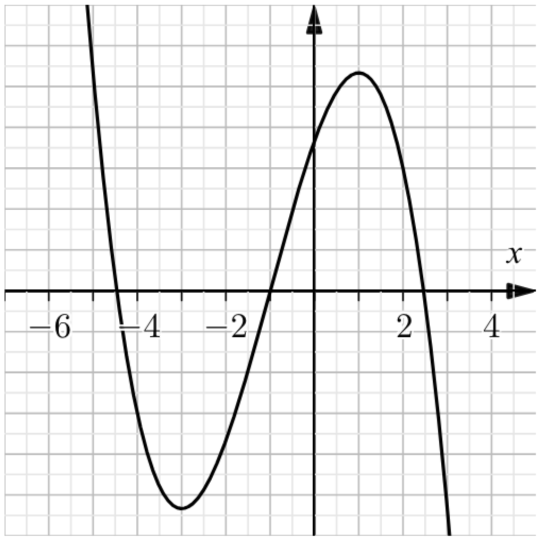
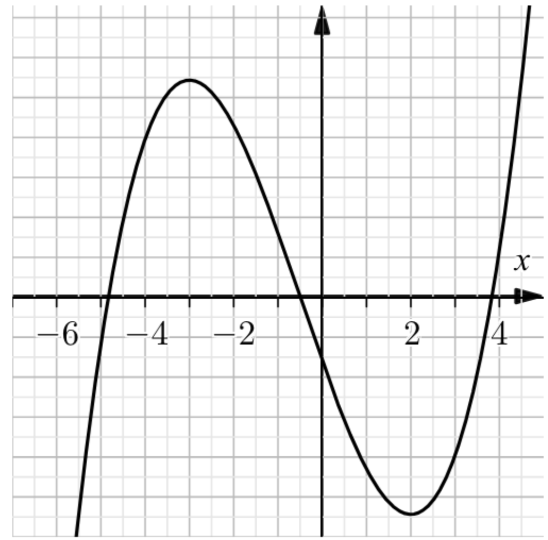
Dr Frost 520f and 520e

Worked Example	Your Turn
The point P with x-coordinate $\frac{1}{4}$ lies on the curve with equation $y = 2x^2$. The normal to the curve at P intersects the curve at points P and Q. Find the coordinates of Q	The point P with x-coordinate $\frac{1}{2}$ lies on the curve with equation $y = 4x^2$. The normal to the curve at P intersects the curve at points P and Q. Find the coordinates of Q

Worked Example	Your Turn
The curve $y = 4x^3 - 7$ intersects the y-axis at C. The tangent to the curve at $P(3, 101)$ intersects the y-axis at D Work out the length of CD	The curve $y = 2x^3 - 5$ intersects the y-axis at C. The tangent to the curve at $P(2, 11)$ intersects the y-axis at D Work out the length of CD

Worked Example	Your Turn
Point B lies on the curve $y = x^3 - 5x + 4$ The x-coordinate of B is -8 Show that the equation of the normal to the curve at B is $187y + x = -87524$	Point A lies on the curve $y = x^2 + 5x + 8$ The x-coordinate of A is -4 Show that the equation of the normal to the curve at A is $3y - x = 16$

Increasing and Decreasing Functions

Worked Example	Your Turn
A sketch of the curve $y = f(x)$ is shown below.  Using the graph, find the interval in which $f(x)$ is decreasing.	A sketch of the curve $y = f(x)$ is shown below.  Using the graph, find the interval in which $f(x)$ is increasing.

Dr Frost 516a

Worked Example	Your Turn
Find the values of x for which the function $f(x) = x^2 - 5x + 8$ is increasing	Find the values of x for which the function $f(x) = x^2 + 8x - 5$ is decreasing

Dr Frost 516b

Worked Example	Your Turn
A cubic function is given by the equation $f(x) = -3x^3 - 6x^2 - 3x + 5$ Determine whether is increasing, decreasing, or neither increasing nor decreasing, when $x = -2$	A cubic function is given by the equation $f(x) = -x^3 + 8x^2 + 35x + 4$ Determine whether is increasing, decreasing, or neither increasing nor decreasing, when $x = 7$

Dr Frost 516c

Worked Example	Your Turn
Find the interval(s) on which the function $f(x) = x^3 - 6x^2 - 135x + 1$ is increasing	Find the interval(s) on which the function $f(x) = x^3 + 6x^2 - 135x - 2$ is increasing

Worked Example	Your Turn
Find the interval on which the function $f(x) = x^3 - 3x^2 - 9x - 10$ is decreasing	Find the interval on which the function $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing

Dr Frost 516d

Worked Example	Your Turn
Show that the function $f(x) = x^3 + 26x - 1$ is increasing for all real values of x	Show that the function $f(x) = x^3 + 16x - 2$ is increasing for all real values of x

Worked Example	Your Turn
Show that the function $f(x) = x^3 - 3x^2 + 8x - 5$ is increasing for all real values of x	Show that the function $f(x) = x^3 + 6x^2 + 21x + 2$ is increasing for all real values of x

Worked Example	Your Turn
Show that the function $5 - x(4x^2 + 3)$ is decreasing for all real values of x	Show that the function $3 + 4x(-x^2 - 5)$ is decreasing for all real values of x

Dr Frost 516e

Worked Example	Your Turn
A function is given by the equation $f(x) = 2x + \frac{32}{x} - 19, x \neq 0$ Find the exact interval on which is increasing.	A function is given by the equation $f(x) = -2x - \frac{49}{x} - 4, x \neq 0$ Find the exact interval on which is decreasing.

Dr Frost 516f

Stationary Points

Worked Example	Your Turn
Find the least value of $f(x) = x^2 + 6x - 9$	Find the least value of $f(x) = x^2 - 4x + 9$

Dr Frost 517a

Worked Example	Your Turn
Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = 2x^3 + 6x^2 - 4$	Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = x^3 + 3x^2 - 4$

Worked Example	Your Turn
Find the stationary points on the curve $y = \frac{5}{3}x^3 - 80x$	Find the stationary points on the curve $y = x^3 - 12x$

Worked Example	Your Turn
Find the coordinates of the turning points of $y = x^3 - 6x^2 - 15x$	Find the coordinates of the turning points of $y = x^3 + 6x^2 - 135x$

Worked Example	Your Turn
Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = \frac{2}{3}x^3 - 3.5x^2 + 3x + 5$	Find the coordinates of the turning/stationary point(s) of the curves by differentiation: $y = x^3 + \frac{1}{2}x^2 - 2x + 4$

Dr Frost 517b

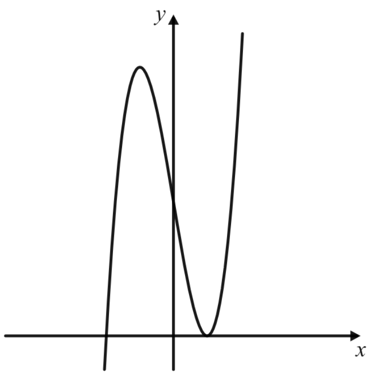
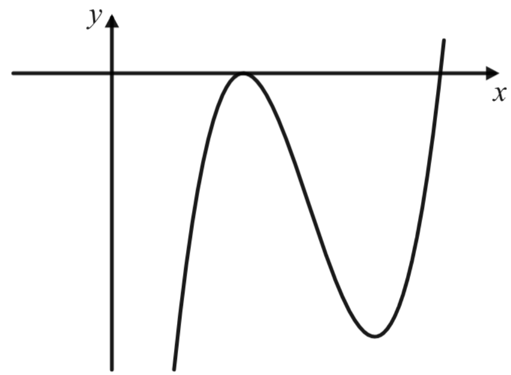
Worked Example	Your Turn
A graph has equation $y = 4 - \frac{27}{x} - 3x$ Determine the x-coordinates of the stationary points.	A graph has equation $y = 5 - \frac{2}{x} - 2x$ Determine the x-coordinates of the stationary points.

Dr Frost 517f

Worked Example	Your Turn
$y = 4ax^3 + \frac{3}{x}$ y has a minimum when $x = \frac{1}{3}$ Work out the value of a	$y = 8ax^3 + \frac{6}{x}$ y has a minimum when $x = \frac{1}{2}$ Work out the value of a

Worked Example	Your Turn
The curve $y = x^3 + ax + b$ has a stationary point at $(-2, 3)$ Work out the values of a and b	The curve $y = x^3 + ax^2 + b$ has a stationary point at $(3, -8)$ Work out the values of a and b

Dr Frost 517j

Worked Example	Your Turn
The curve sketched below has equation $y = 2x^3 + ax^2 - 6x + b$  The x-axis is a tangent to the curve at its minimum point where $x = 1$ Determine the coordinate of the curve's maximum point.	The curve sketched below has equation $y = 2x^3 - 18x^2 + ax + b$  The x-axis is a tangent to the curve at its maximum point where $x = 2$ Determine the coordinate of the curve's minimum point.

Second Derivative and Maxima/Minima

Worked Example	Your Turn
$y = 5x^3 - 7x^2 - x + 4$ Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$	$y = 2x^3 - 3x^2 + x + 9$ Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$

Dr Frost 515b and 515a

Worked Example	Your Turn
Show that one of the stationary points of the curve with equation $y = x^3 - 3x^2 + 45x$ is $(-3, -159)$, and by testing the gradient of the curve either side of the stationary point, determine whether it is a maximum or a minimum.	Show that one of the stationary points of the curve with equation $y = x^3 + 3x^2 - 45x$ is $(3, -81)$, and by testing the gradient of the curve either side of the stationary point, determine whether it is a maximum or a minimum.

Worked Example	Your Turn
Find the coordinates of the stationary points on the curve with equation $y = 4x^3 + 30x^2 + 48x - 3$ and use the second derivative to determine their nature.	Find the coordinates of the stationary points on the curve with equation $y = 2x^3 - 15x^2 + 24x + 6$ and use the second derivative to determine their nature.

Dr Frost 517d

Fill in the Gaps

Equation of Curve	$\frac{dy}{dx}$	$\frac{dy}{dx} = 0$	x-coordinate	y-coordinate	Maximum or Minimum Point
$y = x^2 - 10x + 2$				$y = -23$	$Minimum$
$y = 3x^2 + 12x + 20$					
$y = 15 - 2x - x^2$					
$y = 3 + 8x - 2x^2$					
$y = x^2 + 12x + \square$				$y = -6$	$Minimum$
$y = x^2 - 9x + \square$				$y = -\frac{21}{4}$	
$y = x^2 - \square x + 15$			$x = 4$		
$y = \square + \square\ x - x^2$			$x = 2$	$y = 10$	

Fill in the Gaps

Function	Derivative	Derivative at $x = 2$	2nd Derivative	Function at $x = 1$	Stationary Point(s)
$y = x^2 - 2x + 10$					
	$6x^2$			2	
	$5x$			$-\frac{3}{2}$	
		8	$6x - 2$	-10	
	$x^2 - 3x + 2$			1	
			-2	7	$x = 3$
	3			5	

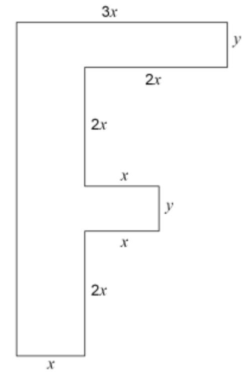
Optimisation

Worked Example	Your Turn
$U = 81y + \frac{49}{y}, y > 0$ Use calculus to show that V has a minimum value and work out the minimum value of V	$V = 49x + \frac{81}{x}, x > 0$ Use calculus to show that V has a minimum value and work out the minimum value of V

Worked Example

An F shape is made from rectangles.
All lengths are in centimetres.
The perimeter of the shape is $128\ cm$
The area of the shape is $A\ cm^2$

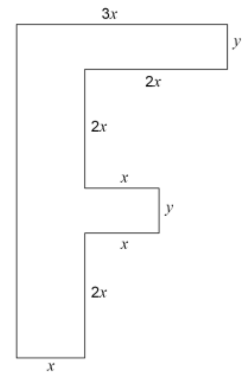
a) Find an expression for y
b) Hence find an expression for A.
c) Use calculus to derive an expression for the rate of change of A as x varies.
d) Hence work out the maximum area



Your Turn

An F shape is made from rectangles.
All lengths are in centimetres.
The perimeter of the shape is $64\ cm$
The area of the shape is $A\ cm^2$

a) Find an expression for y
b) Hence find an expression for A.
c) Use calculus to derive an expression for the rate of change of A as x varies.
d) Hence work out the maximum area



Worked Example

A cuboid is to be made with volume $81\ cm^3$

The cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm

The volume of the cuboid is $81\ cm^3$

a) Show that the total length, L, of the twelve edges of the cuboid is given by $L = 12x + \frac{162}{x^2}$

b) Given that x can vary, use differentiation to find the maximum or minimum value of L

c) Justify that the value of L you have found is a minimum

Your Turn

A cuboid is to be made from 54m^2 of sheet metal.
The cuboid has a horizontal base and no top.
The height of the cuboid is x metres.
Two of the opposite vertical faces are squares.

a) Show that the volume, V m^3, of the tank is given by $V = 18x - \frac{2}{3}x^3$

b) Given that x can vary, use differentiation to find the maximum or minimum value of V

c) Justify that the value of V you have found is a maximum

Graph Sketching

Worked Example	Your Turn
Sketch the following graph, labelling all intercept(s) and any turning point(s): $y = 2x^3 - 3x^2 - 11x + 6$	Sketch the following graph, labelling all intercept(s) and any turning point(s): $y = 2x^3 - 3x^2 - 11x + 6$

Dr Frost 517i

Extra Notes

2 Matrices (L2FM Only)

Multiplication of Matrices

Worked Example	Your Turn
Find: $5\begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}$	Find: $4\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix}$

Dr Frost 679b

Worked Example	Your Turn
Given that $P = \begin{pmatrix} 2 & -4 \\ -5 & 0 \end{pmatrix}$ and $Q = \begin{pmatrix} -5 & 3 \\ -6 & 1 \end{pmatrix}$ Find $5P - Q$	Given that $A = \begin{pmatrix} -2 & -6 \\ -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 1 \\ -2 & 2 \end{pmatrix}$ Find $2A - 3B$

Dr Frost 679c

Worked Example	Your Turn
Find: $\begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 5 \\ -6 \end{pmatrix}$	Find: $\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix}\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

Worked Example	Your Turn
Find: $\begin{pmatrix} 2 & 3 \\ -4 & -5 \end{pmatrix}\begin{pmatrix} -1 & 7 \\ 8 & -6 \end{pmatrix}$	Find: $\begin{pmatrix} 5 & -3 \\ -2 & -4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -7 & 6 \end{pmatrix}$

Worked Example	Your Turn
Find: $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}^4$	Find: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^4$

Worked Example	Your Turn
Matrices A and B are defined by $A = \begin{pmatrix} -1 & x \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & x \\ -5 & -2 \end{pmatrix}$ Calculate AB, giving your answer in terms of x	Matrices U and V are defined by $U = \begin{pmatrix} 4 & 0 \\ a & 3 \end{pmatrix}$ and $B = \begin{pmatrix} a & 5 \\ 6 & -4 \end{pmatrix}$ Calculate UV, giving your answer in terms of a

Dr Frost 680j

Worked Example	Your Turn
Find the value of t, given that $\begin{pmatrix} -5 & 3 \\ 7 & -4 \end{pmatrix}\begin{pmatrix} 4 & t \\ 7 & 5 \end{pmatrix} = I$	Find the value of t, given that $\begin{pmatrix} -7 & 4 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} -3 & -4 \\ -5 & t \end{pmatrix} = I$

Worked Example	Your Turn
$M = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ Show that $M^3 = I$	$M = \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$ Show that $M^3 = I$

Worked Example	Your Turn
$2\begin{pmatrix} 5 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ Find the values of a and b	$3\begin{pmatrix} 2 & 1 \\ -5 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ Find the values of a and b

Worked Example	Your Turn
Work out **all** solutions for x and y, given that $\begin{pmatrix} x & 1 \\ -4 & y \end{pmatrix}\begin{pmatrix} x \\ -3 \end{pmatrix} = \begin{pmatrix} 2x \\ 9 \end{pmatrix}$	Work out **all** solutions for x and y, given that $\begin{pmatrix} x & 3 \\ 1 & y \end{pmatrix}\begin{pmatrix} x \\ -4 \end{pmatrix} = \begin{pmatrix} 4x \\ 8 \end{pmatrix}$

Worked Example	Your Turn
$\begin{pmatrix} 2 & 1 \\ 8x & 5x \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$ Work out the possible values for x and y	$\begin{pmatrix} 1 & 2 \\ x & 3x \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$ Work out the possible values for x and y

Fill in the Gaps

Question	Working		Answer
$\begin{pmatrix} 3 & 0 \\ 1 & 4 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 5 & 0 \end{pmatrix}$	$3 \times 2 + 0 \times 5$ $1 \times 2 + 4 \times 5$	$3 \times 1 + 0 \times 0$ $1 \times 1 + 4 \times 0$	$\begin{pmatrix} 6 & \square \\ \square & \square \end{pmatrix}$
$\begin{pmatrix} 4 & 2 \\ 3 & 0 \end{pmatrix} \times \begin{pmatrix} 7 & 0 \\ 1 & 1 \end{pmatrix}$	 $3 \times 7 + 0 \times 1$	$4 \times 0 + 2 \times 1$ $3 \times 0 + 0 \times 1$	$\begin{pmatrix} \square & 2 \\ \square & \square \end{pmatrix}$
$\begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 0 & 4 \\ 2 & 1 \end{pmatrix}$	 $1 \times 0 + 3 \times 2$		$\begin{pmatrix} \square & \square \\ 6 & \square \end{pmatrix}$
$\begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix} \times \begin{pmatrix} 0 & -1 \\ 4 & 1 \end{pmatrix}$			$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$
$\begin{pmatrix} 1.5 & 0 \\ -3 & 2 \end{pmatrix} \times \begin{pmatrix} 4 & 0.5 \\ 2 & -1 \end{pmatrix}$			$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$
$\begin{pmatrix} \frac{3}{4} & -2 \\ 0 & 3 \end{pmatrix} \times \begin{pmatrix} -1 & \frac{2}{3} \\ 4 & 2 \end{pmatrix}$			$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$
$\begin{pmatrix} 1 & \sqrt{3} \\ -2 & 0 \end{pmatrix} \times \begin{pmatrix} -2 & 0 \\ \sqrt{3} & 4 \end{pmatrix}$			$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix}$
$\begin{pmatrix} \square & 1 \\ 0 & 2 \end{pmatrix} \times \begin{pmatrix} 2 & 1 \\ 3 & \square \end{pmatrix}$			$\begin{pmatrix} 11 & \square \\ \square & 10 \end{pmatrix}$
$\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \times \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$			$\begin{pmatrix} 3 & 16 \\ 7 & 0 \end{pmatrix}$

Transformations

Worked Example	Your Turn
Find a 2×2 matrix that represents: a) A reflection in the y-axis b) A reflection in the line $y = x$	Find a 2×2 matrix that represents: a) A reflection in the x-axis b) A reflection in the line $y = -x$

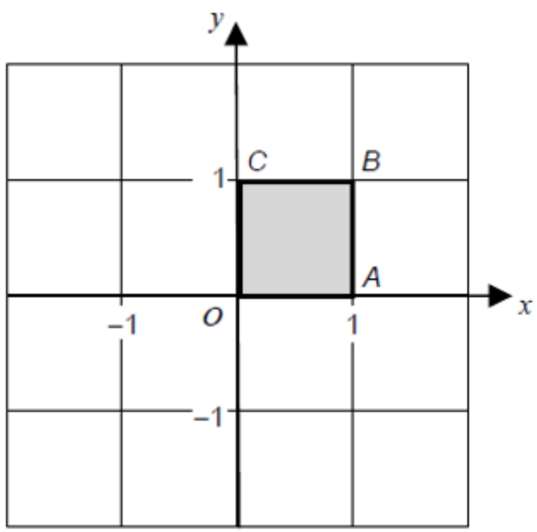
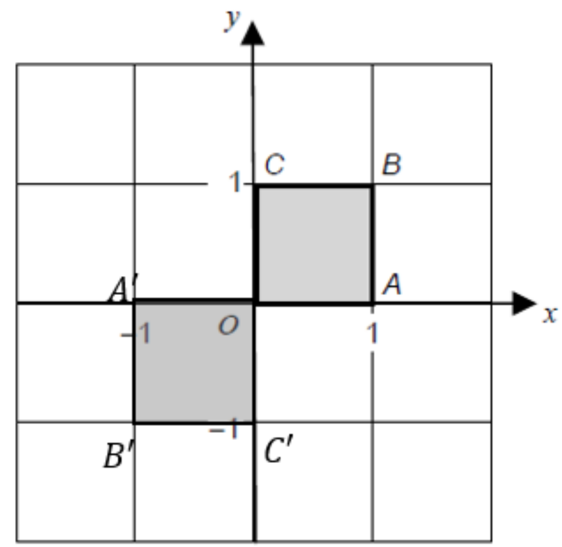
Dr Frost 692b

Worked Example	Your Turn
Find a 2×2 matrix that represents: Rotation $90°$ anticlockwise about the origin	Find a 2×2 matrix that represents: Rotation $270°$ anticlockwise about the origin

Worked Example	Your Turn
Describe geometrically the effect of the following matrices: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Describe geometrically the effect of the following matrices: $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

Worked Example	Your Turn
The transformation matrix $\begin{pmatrix} a & -2 \\ 1 & 1 \end{pmatrix}$ maps the point $(2, 5)$ onto the point $(3, b)$. Work out the values of a and b.	The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point $(3, 4)$ onto the point $(2, b)$. Work out the values of a and b.

Worked Example	Your Turn
The matrix $\begin{pmatrix} -a & 3b \\ 2a & b \end{pmatrix}$ maps the point (2, 3) onto the point (-31, -1). Work out the values of a and b.	The matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point (5, 4) onto the point (1, 17). Work out the values of a and b.

Worked Example	Your Turn
The diagram shows the unit square $OABC$  The image of OABC after transformation by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is $OA'B'C'$ Draw and label $OA'B'C'$	The diagram shows the unit square $OABC$  The image of OABC after transformation by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is $OA'B'C'$ Draw and label $OA'B'C'$

Worked Example	Your Turn
A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$ a) Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ b) Sketch T and its image, T' on a coordinate grid.	A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$ a) Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$ b) Sketch T and its image, T' on a coordinate grid.

Combinations of Transformations

Worked Example	Your Turn
A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$. Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.	A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$. Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

Worked Example	Your Turn
A, B and C are transformations in the $x-y$ plane. A: Rotation through $270°$ anticlockwise about the origin. B: Reflection in the y-axis C: Transformation B followed by transformation A. Use matrix multiplication to show that C is equivalent to a single reflection.	A, B and C are transformations in the $x-y$ plane. A: Rotation through 90° anticlockwise about the origin. B: Reflection in the x-axis C: Transformation A followed by transformation B. Use matrix multiplication to show that C is equivalent to a single reflection.

Worked Example	Your Turn
Use **matrix multiplication** to show that, in the $x - y$ plane, a reflection in the line $y = x$, followed by a rotation, $90°$ clockwise about the origin, followed by a reflection in the x-axis is equivalent to a transformation by the identity matrix.	Use **matrix multiplication** to show that, in the $x - y$ plane, a reflection in the line $y = -x$, followed by a rotation, $90°$ anticlockwise about the origin, followed by a reflection in the x-axis is equivalent to a transformation by the identity matrix.

Extra Notes