KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Level 2 Further Mathematics 2026

Unit 26 Booklet

HGS Maths

Tasks

Dr Frost Course

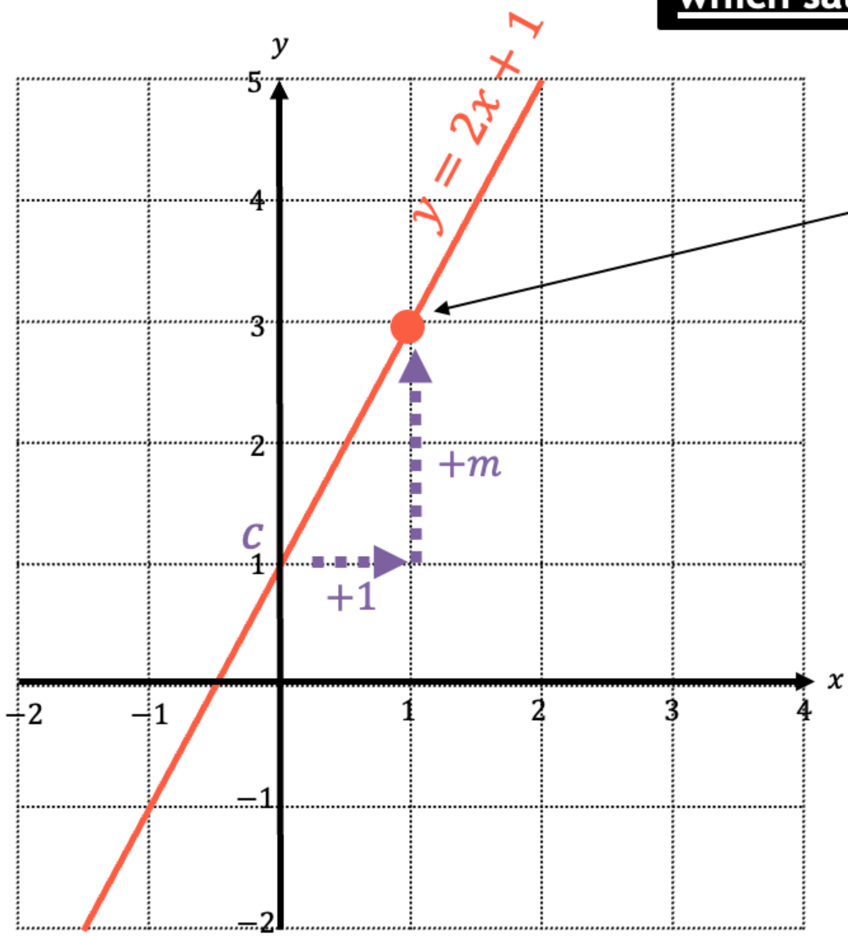
Name: ______________________________

Class: ______________________________

Contents

1 Advanced Straight Line Graphs

$y = mx + c$



For straight line graphs and more generally for curved graphs, **<u>the line consists of all points which satisfy the equation</u>**.

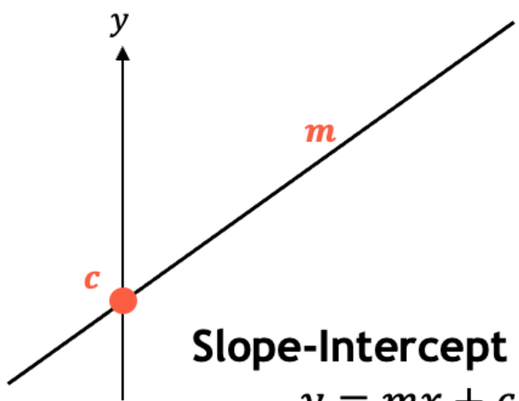
For example, $(1,3)$ is on the line because when substituting into $y = 2x + 1$:

$$3 = 2(1) + 1$$
$$3 = 3$$

If a straight line is in the form $y = mx + c$ then:

- m is the **<u>gradient</u>**. This gives a measure of the **<u>steepness</u>** of the line, and defined as the change in y each time x increases by 1. Here, $m = 2$
- c is the y-intercept. It's the value of y for which the line cuts the y-axis. Here $c = 1$

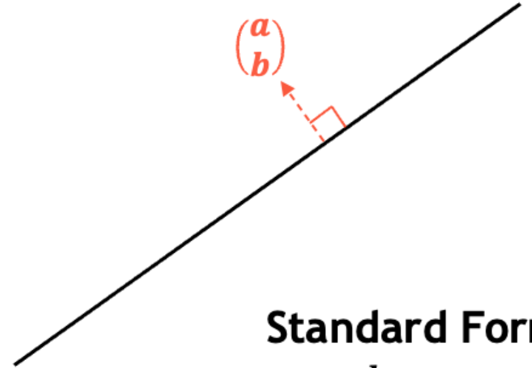
Different Straight Line Equation Forms



Slope-Intercept Form

$$y = mx + c$$

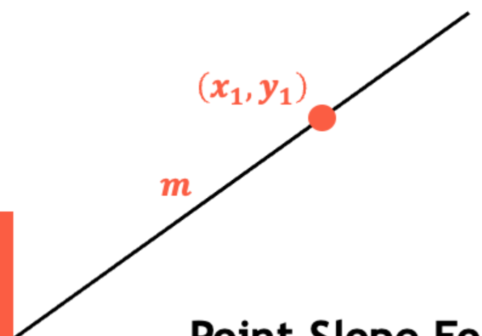
In this form, we use the <u>gradient</u> m and the <u>y-intercept</u> <u>c</u> of the line to construct the equation of the line.



Standard Form

$$ax + by + c = 0$$

The advantage of this form is that we can represent vertical lines (e.g. if $a = 1, b = 0, c = 3$, we get $x = -3$), whereas $y = mx + c$ cannot.
This form also nicely generalises to 3D; in skill **709** we will encounter the equations of planes.

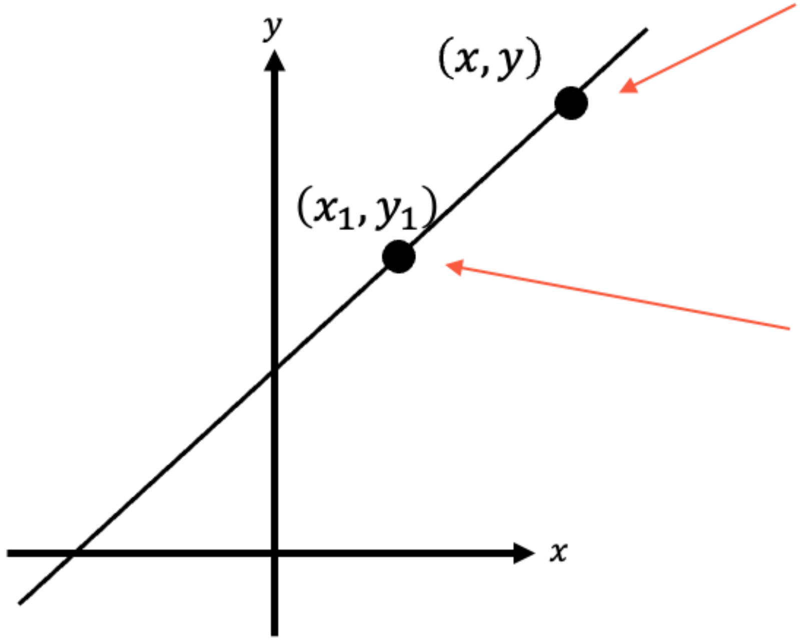


Point-Slope Form

$$y - y_1 = m(x - x_1)$$

This form, which we explore in this lesson, uses a fixed point (x_1, y_1) on the line, which is often more useful if the y-intercept is unknown.

Proving the Equation $y - y_1 = m(x - x_1)$



x and y are variables, which change as we consider different points on the line. (x, y) therefore represents a <u>general point</u> on the line.

x_1 and y_1 are constants, so (x_1, y_1) represents a <u>fixed point</u> on the line.
If we hypothetically wanted a second fixed point, we could refer to it as (x_2, y_2)

Then finding the gradient using the general point and fixed point:

$$m = \frac{\Delta y}{\Delta x} = \frac{y - y_1}{x - x_1}$$

Rearranging:

$$\boldsymbol{y - y_1 = m(x - x_1)}$$

Using $y - y_1 = m(x - x_1)$

The equation of a line that has gradient m and passes through a point (x_1, y_1) is:

$$y - y_1 = m(x - x_1)$$

This is known as **point-slope form** because it uses a point and the gradient/slope.

Determine the equation of a line that has gradient 3 and passes through the point $(2,5)$

Old Way

$$y = 3x + c$$
$$5 = 3(2) + c$$
$$c = -1$$
$$y = 3x - 1$$

Substitute known gradient into $y = mx + c$

Substitute point $(2,5)$ to work out the c.

New Way

You might find it helpful to write out the information.

$m = 3, \qquad (x_1, y_1) = (2,5)$

$$y - 5 = 3(x - 2)$$

Reflections

- This required only **one step**!
- If we expanded out and simplified, we would get the same equation as on the left.
- For this simple example, **you might be tempted to stick with the method on the left**. But its usefulness becomes apparent when you have more arbitrary expressions for the gradient or the point, e.g. the equation of a line with gradient π that passes through $(1, e^2 - 1)$

Worked Example	Your Turn
Find an equation of the line with gradient $-\frac{8}{3}$ and that passes through the point $(-10, -18)$.	Determine an equation of the line with gradient $-\frac{2}{5}$ and that passes through the point $(-17, 18)$.

Fluency Practice

	Gradient	Point	(Unsimplified) Equation
a	3	$(1,2)$	?
b	5	$(3,0)$	?
c	2	$(-3,4)$	?
d	$\frac{1}{2}$	$(1,-5)$	?
e	9	$(-4,-4)$	?

Worked Example	Your Turn
Find an equation of the line that passes through the points $(-20, 6)$ and $\left(-18, \frac{38}{5}\right)$. Write your answer in the form $ax + by = c$, where a, b and c are integers.	Find an equation of the line that passes through the points $(-16, 15)$ and $\left(-15, \frac{78}{5}\right)$. Write your answer in the form $ax + by = c$, where a, b and c are integers.

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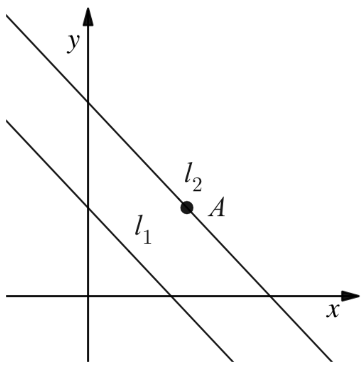
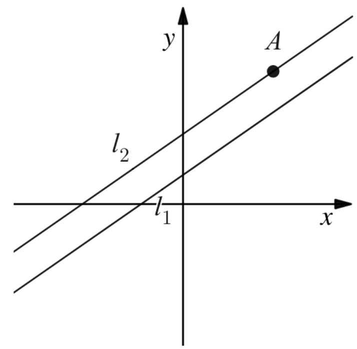
Worked Example	Your Turn
The line l_1 has the equation $4x + 3y - 30 = 0$. The line l_2 is parallel to l_1 and passes through the point $A(9, 10)$ as shown in the diagram below.  Determine the equation of l_2. Give your answer in the form $ax + by = c$, where a,b and c are integers.	The line l_1 has the equation $10x - 12y + 18 = 0$. The line l_2 is parallel to l_1 and passes through the point $A(4, 7)$ as shown in the diagram below.  Determine the equation of l_2. Give your answer in the form $ax + by = c$, where a,b and c are integers.

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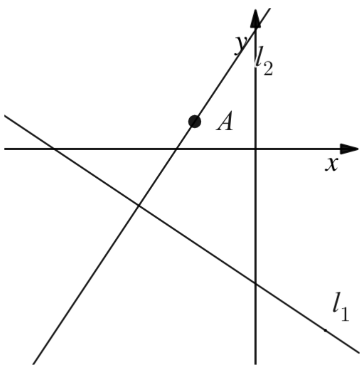
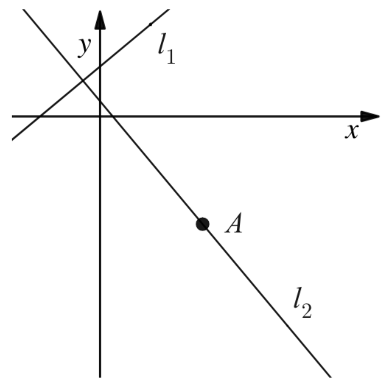
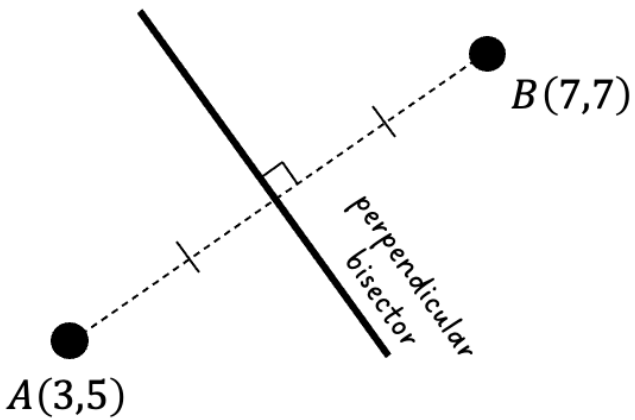
Worked Example	Your Turn
The line l_1 has the equation $2x + 3y + 30 = 0$. The line l_2 is perpendicular to l_1 and passes through the point $A\left(-\frac{9}{2}, 2\right)$ as shown in the diagram below.  Find the equation of l_2. Give your answer in the form $ax + by = c$, where a, b and c are integers.	The line l_1 has the equation $10x - 12y + 54 = 0$. The line l_2 is perpendicular to l_1 and passes through the point $A\left(\frac{19}{2}, -10\right)$ as shown in the diagram below.  Find the equation of l_2. Give your answer in the form $ax + by = c$, where a, b and c are integers.

Perpendicular Bisectors



Determine the equation of the perpendicular bisector of $A(3,5)$ and $B(7,7)$

Recall that a perpendicular bisector is the line which is perpendicular to the line connecting the two points, and cuts it in half. Every point on the perpendicular bisector is the same distance from A and B.

Midpoint:

$$\left(\frac{3+7}{2}, \frac{5+7}{2}\right) = (5,6)$$

Determine the midpoint of AB. This gives us a point we know the perpendicular bisector passes through. In this case we could use inspection, e.g. 5 is halfway between 3 and 7.

$$m_{AB} = \frac{7-5}{7-3} = \frac{1}{2}$$

Determine the gradient of the line passing through A and B.

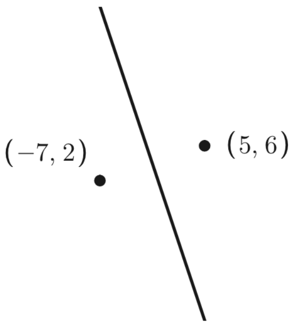
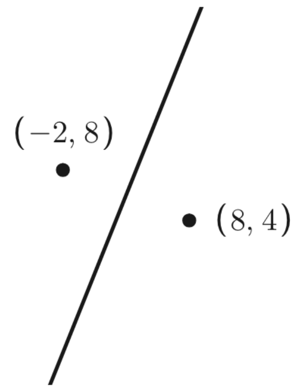
$$m_{\perp} = -2$$

Hence find the gradient of the line perpendicular to it.

$$\boldsymbol{y - 6 = -2(x - 5)}$$

Now we have a point on the perpendicular bisector and its gradient, use

$$y - y_1 = m(x - x_1)$$

Worked Example	Your Turn
A straight line passes through the points $P(-7,2)$ and $Q(5,6)$. Find the equation of the perpendicular bisector of PQ. Give your answer in the form $ax+by+c=0$, where a, b and c are integers. Simplify your answer where possible. 	A straight line passes through the points $A(8,4)$ and $B(-2,8)$. Find the equation of the perpendicular bisector of AB. Give your answer in the form $ax+by+c=0$, where a, b and c are integers. Simplify your answer where possible. 

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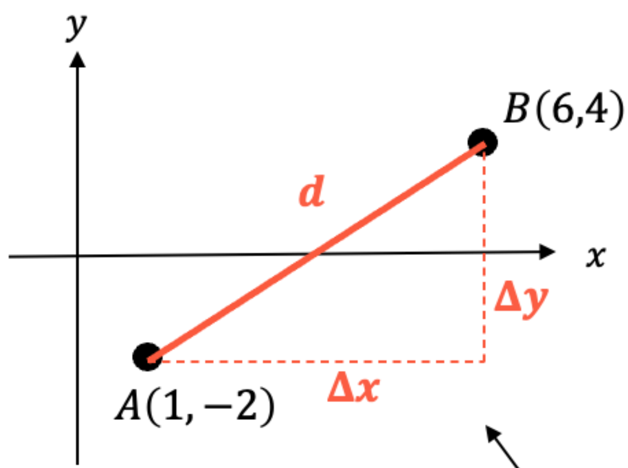
Distance Between Two Points



We can use Pythagoras' theorem, using the change in x (Δx) and the change in y (Δy) between the points.

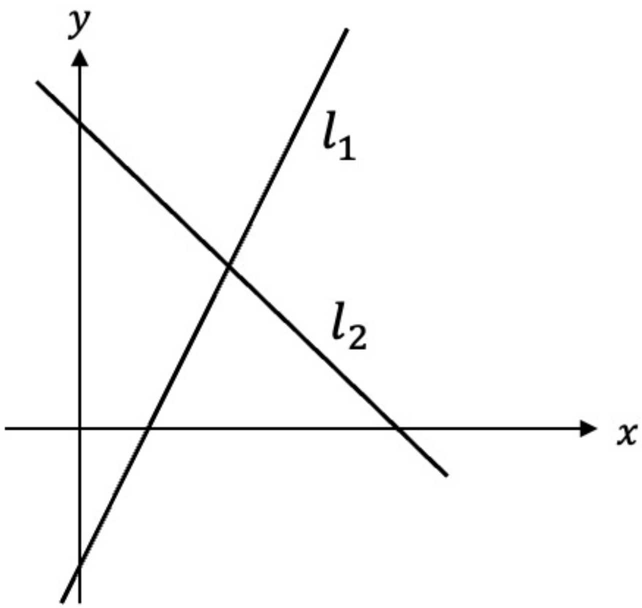
Determine the distance between the points A and B.

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$
$$= \sqrt{5^2 + 6^2}$$
$$= \sqrt{61}$$

Distance between two points:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Point of Intersection Between Two Lines



Determine the coordinates of the point for which the lines l_1, with equation $y = 3x - 5$ and the line l_2, with equation $y = 7 - x$, intersect.

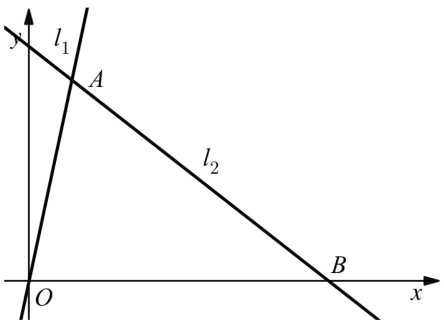
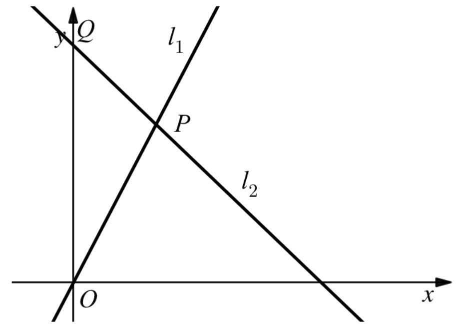
$$3x - 5 = 7 - x$$
$$4x = 12$$
$$x = 3$$

Solve the equations simultaneously. If they are both in the form $y = \cdots$, we can simply equate the right-hand-sides.

Substitute this value of x back into one of the equations, e.g. $y = 7 - x$, to determine y.

$y = 7 - 3 = 4$

Intersect at $(3, 4)$

Worked Example	Your Turn
The line l_1 has equation $y = 3x$ The line l_2 has equation $y = -\frac{1}{2}x + 6$ The lines l_1 and l_2 intersect at A The line l_2 intersects the x-axis at B  Find the exact area of triangle OAB	The line l_1 has equation $y = 3x$ The line l_2 has equation $2y + 3x = 12$ The lines l_1 and l_2 intersect at P The line l_2 intersects the y-axis at Q  Find the exact area of triangle OPQ

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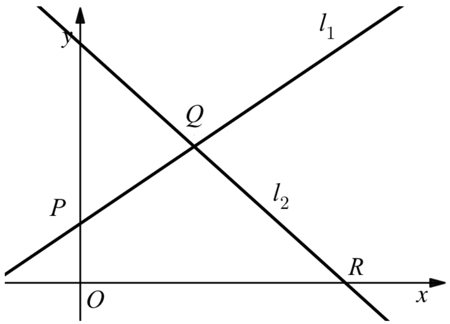
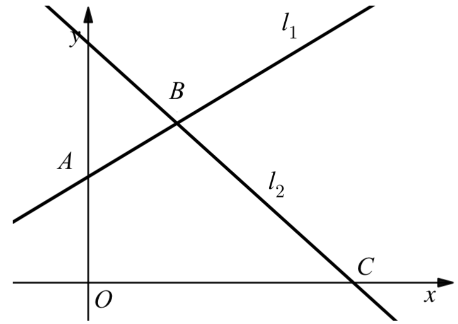
Worked Example	Your Turn
The line l_1 has equation $y = 3x + 7$ The line l_2 has equation $y + 4x = 28$ The lines l_1 passes through Q and intersects the y-axis at P The lines l_2 passes through Q and intersects the x-axis at R  Find the exact area of quadrilateral $OPQR$	The line l_1 has equation $y = 2x + 8$ The line l_2 has equation $y = -3x + 18$ The lines l_1 passes through B and intersects the y-axis at A The lines l_2 passes through B and intersects the x-axis at C  Find the exact area of quadrilateral $OABC$

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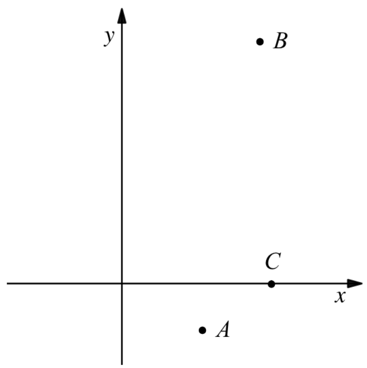
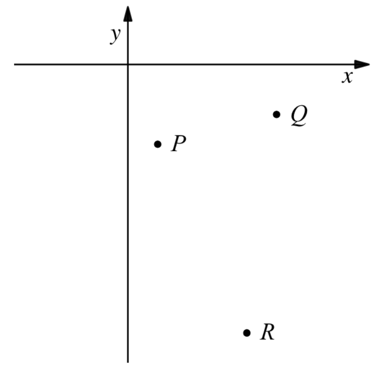
Worked Example	Your Turn
The line l_1 passes through the points $A(7,-4)$ and $B(12,21)$ The line l_2 passes through the point $C(13,0)$ and is perpendicular to l_1 The lines l_1 and l_2 intersect at the point D  By first using the equations of l_1 and l_2 to find the coordinates of D, work out the area of the triangle ABC	The line l_1 passes through the points $P(3,-8)$ and $Q(15,-5)$ The line l_2 passes through the point $R(12,-27)$ and is perpendicular to l_1 The lines l_1 and l_2 intersect at the point S  By first using the equations of l_1 and l_2 to find the coordinates of S, work out the area of the triangle PQR

Extra Notes

2 Three Linear Simultaneous Equations

Equations Involving 3 Variables

We can apply the techniques we have used when solving linear simultaneous equations to solve systems of equations involving 3 variables.

If we eliminate **the same** variable from two of the equations, we can solve that pair of equations

$$x + y + z = 6 \quad (1)$$
$$2x + 3y - z = 4 \quad (2)$$
$$x + 5y + 2z = 13 \quad (3)$$

We can now solve equations (4) & (5) simultaneously to find x & y

(1) + (2)

$$x + y + z = 6$$
$$2x + 3y - z = 4$$
$$3x + 4y + 0 = 10 \quad (4)$$

We can also manipulate the equations to eliminate a variable

We should label our newly formed equations

$2\times$(2) + (3)

$$4x + 6y - 2z = 8$$
$$x + 5y + 2z = 13$$
$$5x + 11y + 0 = 21 \quad (5)$$

$$3x + 4y = 10 \quad (4) \quad \times 5$$
$$5x + 11y = 21 \quad (5) \quad \times 3$$
$$15x + 20y = 50 \quad (6)$$
$$15x + 33y = 63 \quad (7)$$

(7) − (6)

$$13y = 13$$
$$\boldsymbol{y = 1}$$

$$\therefore 3x + 4(1) = 10$$
$$3x = 6$$
$$\boldsymbol{x = 2}$$

To find the value of z, we can substitute the values of x & y into any of the 3 original equations

Substitution into (1)

$$2 + 1 + z = 6$$
$$\boldsymbol{z = 3}$$

$$\therefore \boldsymbol{x = 2, y = 1, z = 3}$$

Worked Example	Your Turn
Solve the following simultaneous equations: $x - y + 3z = 5$ $x + y + 6z = 12$ $3x - 2y + 2z = 10$	Solve the following simultaneous equations: $2a + 2b - c = 12$ $2a + b + 2c = 16$ $2a + 3b + c = 25$

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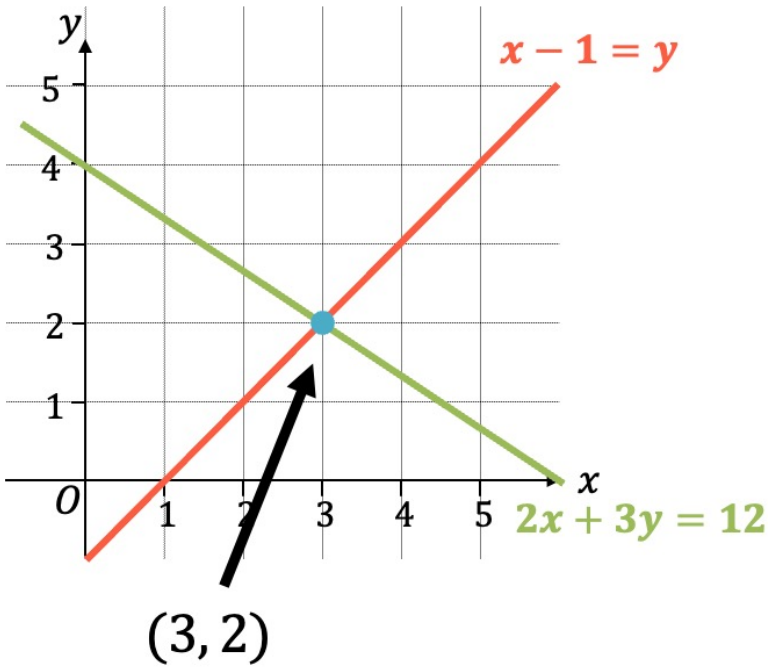
Visual Representation

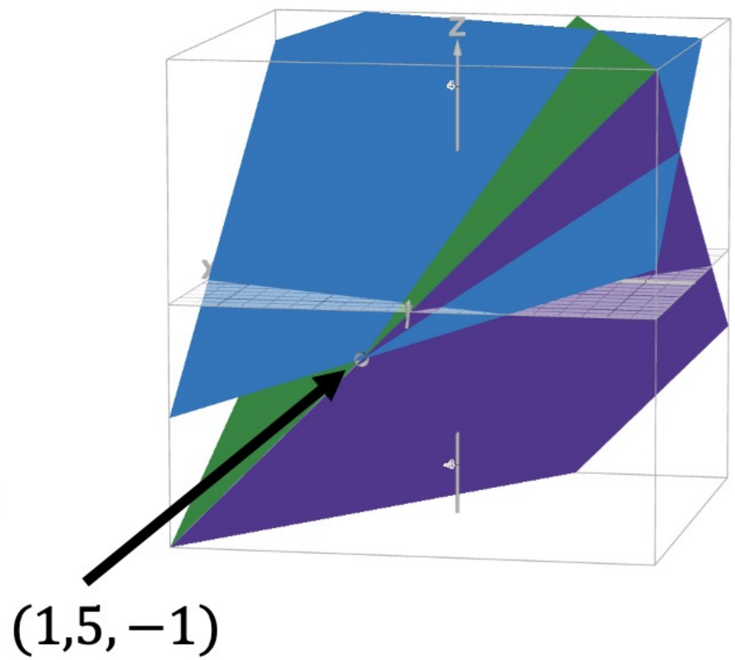
When solving a pair of simultaneous equations in two variables, we are ultimately finding the intersection points of two lines or curves in 2D space.

$$x - 1 = y$$
$$2x + 3y = 12$$



When instead solving three simultaneous equations in three variables, each equation represents a plane in 3D space, and the solution represents the single point of intersection of all three planes.

$$x + 2y + 3z = 8$$
$$3x + y + 3z = 5$$
$$3x - 2y + 3z = -10$$



Visual Representation

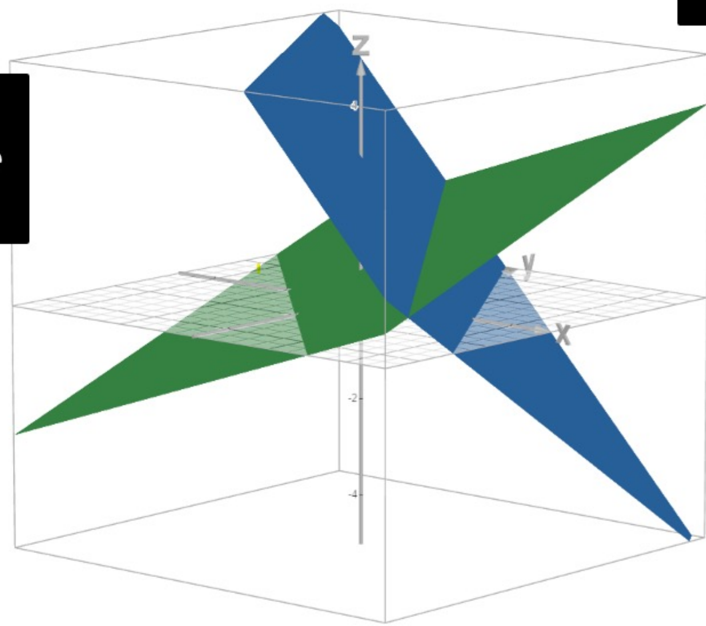
By looking at the following simultaneous equations, can you explain why it is not possible to solve this system of equations?

$$2x + 2y - 6z = -4$$
$$3x + 2y + 3z = 9$$
$$x + y - 3z = -2$$

Here the first equation is a multiple of the last equation

Therefore, there is no single point where the planes intersect

This means that they represent the same plane



Solving by Substitution

As possible when solving pairs of simultaneous equations, we can also use substitution to solve systems of equations with 3 variables.

Solve the following simultaneous equations:

$$3x + y + 2z = 25 \quad (1)$$
$$3x + 3y + 2z = 33 \quad (2)$$
$$2x + y + 3z = 23 \quad (3)$$

If we make a single variable the subject of any of the equations, we can substitute this into the other two equations to eliminate a variable

We can easily make y the subject of either (1) or (3)

Rearranging (1) $\rightarrow y = 25 - 3x - 2z$

Substitute into (2)

$$3x + 3(25 - 3x - 2z) + 2z = 33$$
$$3x + 75 - 9x - 6z + 2z = 33$$
$$6x + 4z = 42 \quad (4)$$

Substitute into (3)

$$2x + (25 - 3x - 2z) + 3z = 23$$
$$-x + z = -2 \quad (5)$$

Now we can solve (4) & (5) simultaneously to find x & z

$$6x + 4z = 42 \quad (4)$$
$$-x + z = -2 \quad (5) \quad \times 6$$

$$6x + 4z = 42 \quad (4)$$
$$-6x + 6z = -12 \quad (6)$$

(4) + (6) $\quad 10z = 30$

$$\mathbf{z = 3}$$

$$\therefore 6x + 4(3) = 42$$
$$6x = 30$$
$$\mathbf{x = 5}$$

Substitution into rearranged (1)

$$y = 25 - 3(5) - 2(3) = \mathbf{4}$$
$$\therefore \mathbf{x = 5, y = 4, z = 3}$$

Worked Example	Your Turn
Solve the following simultaneous equations: $3x + y + 2z = 25$ $3x + 3y + 2z = 33$ $2x + y + 3z = 23$	Solve the following simultaneous equations: $a - 3b - 2c = 10$ $a + 2b + 2c = -5$ $a + b + 3c = -11$

Worked Example	Your Turn
Imani, Hannah and Maria and buying books from their local charity shop. Imani bought one t-shirt, one hat, and two jumpers costing £24. Hannah bought one t-shirt, two hats, and three jumpers costing £40. Maria bought three t-shirts, one hat, and two jumpers for £32. How much does each item cost individually?	Logan, Abdi and Viktor went shopping in Floating Lion. Logan bought three bags of mints, one candle and two plant pots for £17. Abdi bought two bags of mints, three candles and one plant pot for £14. Viktor bought one bag of mints, one candle and three plant pots for £21. Find the cost of each item.

Extra Notes