KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 **Level 2 Further Mathematics** 2026

Unit 27 Tasks

DO NOT WRITE INSIDE

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1 Binomial Expansion

Without using a calculator:

(a) write out the first seven rows of Pascal's triangle.

(b) Hence find the expansions of:

(i) $(1+x)^5$ (ii) $(1+x)^7$

Using your calculator, find:

(a) ${}_5C_3$ (b) ${}_8C_0$ (c) ${}_{10}C_5$

(d) ${}_6C_1$ (e) ${}_9C_9$ (f) ${}_7C_4$

Using your calculator, find:

(a) The first four terms, in ascending powers of x, in the expansion of $(1+x)^{10}$

(b) The first four terms, in ascending powers of x, in the expansion of $(1+x)^8$

(c) The first three terms, in ascending powers of x, in the expansion of $(1+x)^{13}$

(d) The first three terms, in ascending powers of x, in the expansion of $(1+x)^{16}$

Write down the combination you would use, and its value, for:

(a) The coefficient of the x^4 term in the expansion of $(1+x)^9$

(b) The coefficient of the x^2 term in the expansion of $(1+x)^6$

(c) The coefficient of the x term in the expansion of $(1+x)^{12}$

(d) The coefficient of the x^3 term in the expansion of $(1+x)^{20}$

Expand and simplify

(a) $(x+1)^5$

(b) $(x+2)^3$

(c) $(x-1)^3$

(d) $(a+b)^4$

(e) $(2x+y)^4$

(f) $(2x-3y)^5$

(a) Find the coefficient of the x^2 term in the expansion of $(2x-1)^4$

(b) Find the coefficient of the x term in the expansion of $(3-x)^3$

The first three numbers in the tenth row of Pascal's triangle are 1, 10, and 45.

(a) Write down the first three numbers in the eleventh row of Pascal's triangle.

(b) Hence write down the first three terms, in ascending powers of x, of $(1+x)^{11}$

(a) Expand and simplify $(x^2+2)^4$

(b) Work out the coefficient of the x term in the expansion of $\left(x+\frac{1}{x}\right)^5$

The coefficient of the x^2 term in the expansion of $(a+2x)^6$ is 960. Find the possible values of a.

(a) Find the coefficient of the x^2 term in the expansion of $(2+x)^3$

(b) Find the coefficient of the x^3 term in the expansion of $(1+2x)^4$

(c) Find the coefficient of the x term in the expansion of $(2+3x)^5$

(d) Find the coefficient of the x^2 term in the expansion of $(1-3x)^6$

(a) Find the coefficient of the x term in the expansion of $(4-3x)^5$

(b) Find the coefficient of the x^2 term in the expansion of $(x+5)^6$

(c) Find the coefficient of the x^3 term in the expansion of $(2x-1)^4$

(d) Find the coefficient of the x^2 term in the expansion of $(\sqrt{2}+x)^5$

(a) The coefficient of the x^5 term in the expansion of $(a+x)^7$ is 84. Given that a is positive, find its value.

(b) The coefficient of the x^3 term in the expansion of $(3-bx)^5$ is -2430. Find the value of b.

(a) In the expansion of $(2+ax)^4$ the coefficient of the x^2 term is three times the coefficient of the x term. Find the value of a.

(b) In the expansion of $\left(\frac{x}{2}+b\right)^5$ the coefficient of the x^2 term is 72 times the coefficient of the x^4 term. Find the two possible values of b.

Purposeful Practice

① Use Pascal's triangle to expand

(i) $(1+x)^3$ (ii) $(y+1)^4$ (iii) $(x+y)^5$

(iv) $(5+w)^3$ (v) $(p+4)^4$ (vi) $(2+m)^5$.

② Use Pascal's triangle to expand

(i) $(x-y)^3$ (ii) $(1-2x)^4$ (iii) $(2-y)^5$

(iv) $(5-2p)^3$ (v) $(3x-4)^4$ (vi) $(4x-1)^5$.

③ Work out the first three terms in ascending powers of x in the expansion of $(1+x)^6$.

④ Work out the first three terms in descending powers of x in the expansion of $(2+x)^7$.

⑤ Work out the coefficient of the x^3 term in the expansion of $(4-3x)^5$.

⑥ Write down the second number in the 10th row of Pascal's triangle.

⑦ Write down the last number in the 19th row of Pascal's triangle.

⑧ Write down the third number in the 9th row of Pascal's triangle.

⑨ Expand $\left(3x^2+\frac{1}{x}\right)^4$.

⑩ (i) Expand $(1+2x)^5$.

(ii) Hence write down the expansion of $(1-2x)^5$.

(iii) Hence simplify $(1+2x)^5-(1-2x)^5$.

⑪ (i) Expand $(3+w)^3$.

(ii) Hence, by replacing w with $x+2y$, write down the expansion of $(3+x+2y)^3$.

PS ⑫ The simplified expansion of $(mx+y)^n$ includes the term $240x^2y^4$.

(i) Write down the value of n.

(ii) Hence work out the possible values of m.

(iii) Hence work out the coefficient of the x^4y^2 term.

PS ⑬ In the expansion of $\left(x+\frac{2}{x}\right)^6$ work out the term which is independent of x.

PS ⑭ Given that the 10th row of Pascal's triangle is

1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1,

work out the coefficient of x^2 in the expansion of $\left(x-\frac{2}{x}\right)^{10}$.

2 Factor Theorem

(a) Show that $(x-2)$ is a factor of x^3+x^2-4x-4.

(b) Show that $(x-3)$ is a factor of $2x^3+x^2-18x-9$.

(c) Show that $(x-1)$ is a factor of $4x^3-3x^2-1$.

(d) Show that $(x+1)$ is a factor of $x^3-10x^2+19x+30$.

Factorise fully:

(a) x^3+x^2-4x-4

(b) $x^3-10x^2+19x+30$

(c) $x^3-4x^2-11x+30$

Solve:

(a) $2x^3+x^2-18x-9=0$

(b) $x^3-7x^2+2x+40=0$

(c) $x^3-5x^2+5x+3=0$

$(x+2)$ and $(x-3)$ are both factors of the cubic $x^3+ax^2+bx+18$. Find the values of a and b.

$(x+2)$ and $(x-4)$ are both factors of the cubic x^3+3x^2+ax+b. Find the third factor of this cubic.

(a) Show that $2x - 1$ is a factor of $2x^3 + 5x^2 - 7x + 2$

(b) Show that $3x + 1$ is a factor of $6x^3 + 21x^2 - 38x - 15$

(c) Show that $5x - 2$ is a factor of $5x^3 + 23x^2 + 40x - 20$

(a) Show that $2x + 1$ is a factor of $4x^3 + 4x^2 - 5x - 3$. Hence, fully factorise $4x^3 + 4x^2 - 5x - 3$.

(b) Show that $4x - 1$ is a factor of $4x^3 + 3x^2 - 25x + 6$. Hence, solve $4x^3 + 3x^2 - 25x + 6 = 0$.

(c) Show that $2x - 3$ is a factor of $6x^3 + 25x^2 - 31x - 30$. Hence, solve $6x^3 + 25x^2 - 31x - 30 = 0$.

(a) Given that $2x - 1$ is a factor of $4x^3 - 12x^2 + ax + 12$, find the value of a.

(b) Given that $3x + 2$ is a factor of $6x^3 + bx^2 + 27x + 14$, find the value of b.

(c) Given that $2x - 5$ is a factor of $cx^3 - 29x^2 + 16x - 15$, find the value of c.

(a) Given that both $x - 2$ and $2x + 1$ are factors of $6x^3 - ax^2 - 18x - b$, find the values of a and b.

(b) Given that $x - a$ is a factor of $3x^3 + 2x^2 - 12ax - 8a$, and that a is a non-zero integer, find the value of a.

Purposeful Practice

① Determine whether the following linear functions are factors of the given polynomials.

(i)	$x^3 - 8x + 7$	$(x - 1)$
(ii)	$x^3 + x^2 - 4x - 5$	$(x + 2)$
(iii)	$x^4 - 6x^2 + 10x - 12$	$(x - 2)$
(iv)	$x^5 + 32$	$(x + 2)$
(v)	$2x^4 - x^3 - 20$	$(x + 2)$
(vi)	$x^3 - ax^2 + a^2x - a^3$	$(x - a)$

Exam questions are unlikely to require students to find five different linear factors of an expression. However, students will be expected to feel comfortable working with polynomials of such a high degree.

② Factorise the following functions as a product of linear factors.

(i) $x^3 - 3x^2 - x + 3$ (ii) $x^3 - 7x - 6$

(iii) $x^3 - x^2 - 2x$ (iv) $x^3 - 2x^2 - 13x - 10$

(v) $x^3 - x^2 - 14x + 24$ (vi) $x^4 - 3x^3 - 11x^2 + 3x + 10$

(vii) $x^4 - 4x^3 + 6x^2 - 4x + 1$ (viii) $x^4 - 13x^2 + 36$

(ix) $x^5 - 4x^4 - 17x^3 + 24x^2 + 36x$ (x) $x^5 - 3x^4 - 23x^3 + 51x^2 + 94x - 120$

③ Solve the following equations.

(i) $x^3 - 2x^2 - 5x + 6 = 0$ (ii) $x^3 + 3x^2 - 6x - 8 = 0$

(iii) $x^3 - 2x^2 - 21x - 18 = 0$ (iv) $x^4 + 3x^3 - 5x^2 - 3x + 4 = 0$

(v) $2x^3 + x^2 - 7x + 4 = 0$ (vi) $x^5 - 3x^4 - 23x^3 + 51x^2 + 94x - 120 = 0$

④ $f(x) = x^3 + 2x^2 + ax - 76$

Given that $(x - 4)$ is a factor of $f(x)$, work out the value of a.

⑤ $f(x) = x^3 + px^2 + qx + 6$

(i) Given that $(x - 1)$ is a factor of $f(x)$, write an equation in p and q.

(ii) Given also that $(x + 3)$ is a factor of $f(x)$, write another equation in p and q.

(iii) Solve your simultaneous equations to work out the values of p and q.

PS ⑥ (i) Work out the value of k for which $x = 2$ is a root of $x^3 + kx + 6 = 0$

(ii) Work out the other roots when k takes this value.

PS ⑦ The diagram shows an open cuboid tank whose base is a square of side x metres and whose volume is 8 m^3.

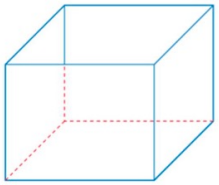

Figure 4.12

(i) Write down an expression in terms of x for the height of the tank.

(ii) Show that the surface area of the tank is $\left(x^2 + \frac{32}{x}\right)$ m^2.

(iii) Given that the surface area is 24 m^2, show that

$x^3 - 24x + 32 = 0$

(iv) Solve $x^3 - 24x + 32 = 0$ to work out the possible values of x.

PS ⑧ $(x - 1)$ and $(5x + 2)$ are factors of $5x^4 + px^3 - 10x^2 + qx + 2$

Solve $5x^4 + px^3 - 10x^2 + qx + 2 = 0$