KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Level 2 Further Mathematics 2026

Unit 28 Booklet

HGS Maths

Tasks

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

1 Matrices

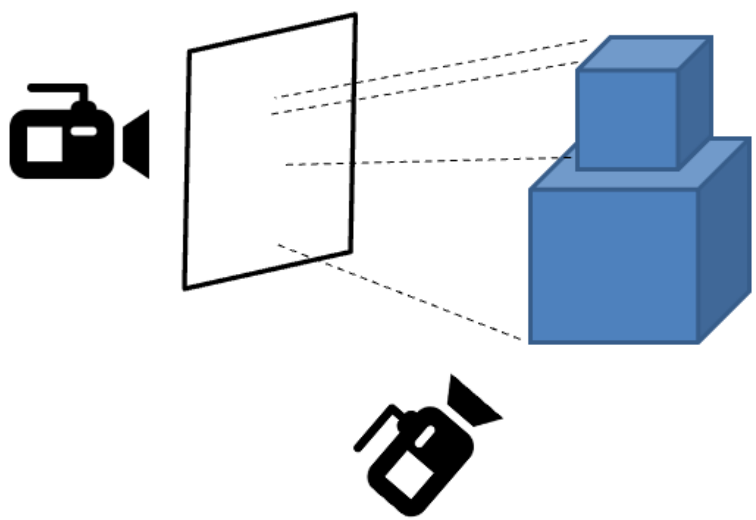
Introduction

A matrix (plural: matrices) is **simply an ‘array’ of numbers**, e.g.

$$\begin{pmatrix} 1 & 3 & -7 \\ 4 & 0 & 5 \end{pmatrix}$$

On a simple level, a matrix is simply a way to organise values into rows and columns, and represent these multiple values as a single structure.

For the purposes of IGCSE Further Maths, you should understand matrices as **a way to transform points**.



Matrices are particularly useful in 3D graphics, as matrices can be used to carry out rotations/enlargements (useful for changing the camera angle) or project into a 2D ‘viewing’ plane.

#1 Dimensions of Matrices

The dimension of a matrix is its size, in terms of its number of rows and columns.

Matrix	Dimensions
$\begin{pmatrix} 1 & 3 & -7 \\ 4 & 0 & 5 \end{pmatrix}$	2×3
$\begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$	3×1
$\begin{pmatrix} 1 & 6 & 0 \end{pmatrix}$	1×3

#2 Notation/Names for Matrices

A matrix can have square or curvy brackets*.

$$\begin{pmatrix} 7 & 1 & 2 \\ 6 & 1 & 5 \end{pmatrix} \qquad \begin{bmatrix} 1 \\ 6 \\ -3 \end{bmatrix} \qquad \begin{pmatrix} 1 & 6 & 0 \end{pmatrix}$$

Matrix | **Column Vector** (The vector you know and love) | **Row Vector**

So a matrix with one column is simply a vector in the usual sense.

#3 Variables for Matrices

If we wish a variable to represent a matrix, we use bold, capital letters.

$$\boldsymbol{A} = \begin{pmatrix} 1 \\ 6 \\ -3 \end{pmatrix}$$

$$\boldsymbol{C} = \boldsymbol{P}^2\boldsymbol{T}\boldsymbol{P}$$

#4 Adding/Subtracting Matrices

Simply add/subtract the corresponding elements of each matrix.
They must be of the same dimension.

$$\begin{pmatrix} 1 & 3 & -7 \\ 4 & 0 & 5 \end{pmatrix} + \begin{pmatrix} 6 & -2 & 9 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 7 & 1 & 2 \\ 6 & 1 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 0 \\ -1 & 2 \\ 0 & 3 \end{pmatrix} - \begin{pmatrix} q & -3 \\ 1 & 1 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 3-q & 3 \\ -2 & 1 \\ 4 & 2 \end{pmatrix}$$

#5 Scalar Multiplication

A scalar is a number which can 'scale' the elements inside a matrix/vector.

1

$$3\begin{pmatrix}1 & 3 & -7\\ 4 & 0 & 5\end{pmatrix} = \begin{pmatrix}3 & 9 & -21\\ 12 & 0 & 15\end{pmatrix}$$

2

$$\boldsymbol{A} = \begin{pmatrix}q & -3\\ 1 & 1\\ -4 & 1\end{pmatrix} \qquad 2\boldsymbol{A} = \begin{pmatrix}2q & -6\\ 2 & 2\\ -8 & 2\end{pmatrix}$$

3

$$\begin{pmatrix}-3\\ k\end{pmatrix} + k\begin{pmatrix}2k\\ 2k\end{pmatrix} = \begin{pmatrix}k\\ 6\end{pmatrix} \qquad k = \frac{3}{2}$$

#6 Matrix Multiplication

This is where things get slightly more complicated...

Now repeat for the next row of the left matrix...

$$\begin{bmatrix} 1 & 0 & 3 & -2 \\ 2 & 8 & 4 & 3 \\ 7 & -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 7 \\ 0 & 3 \\ 8 & -3 \end{bmatrix} \qquad \begin{bmatrix} -11 & 16 \\ 42 & 61 \\ 50 & -6 \end{bmatrix}$$

We start with this row and column, and sum the products of each pair.
(1 x 5) + (0 x 1) + (3 x 0) + (-2 x 8) = -11

Specification

Exam Note: In IGCSEFM, you will only have to multiply either a 2×2 by 2×1 or 2×2 by 2×2

Multiplication of Matrices

Worked Example	Your Turn
Find: $5\begin{pmatrix}-1 & 2\\ 3 & -4\end{pmatrix}$	Find: $4\begin{pmatrix}3 & 4\\ -2 & -5\end{pmatrix}$

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Worked Example	Your Turn
Given that $P = \begin{pmatrix} 2 & -4 \\ -5 & 0 \end{pmatrix}$ and $Q = \begin{pmatrix} -5 & 3 \\ -6 & 1 \end{pmatrix}$ Find $5P - Q$	Given that $A = \begin{pmatrix} -2 & -6 \\ -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 6 & 1 \\ -2 & 2 \end{pmatrix}$ Find $2A - 3B$

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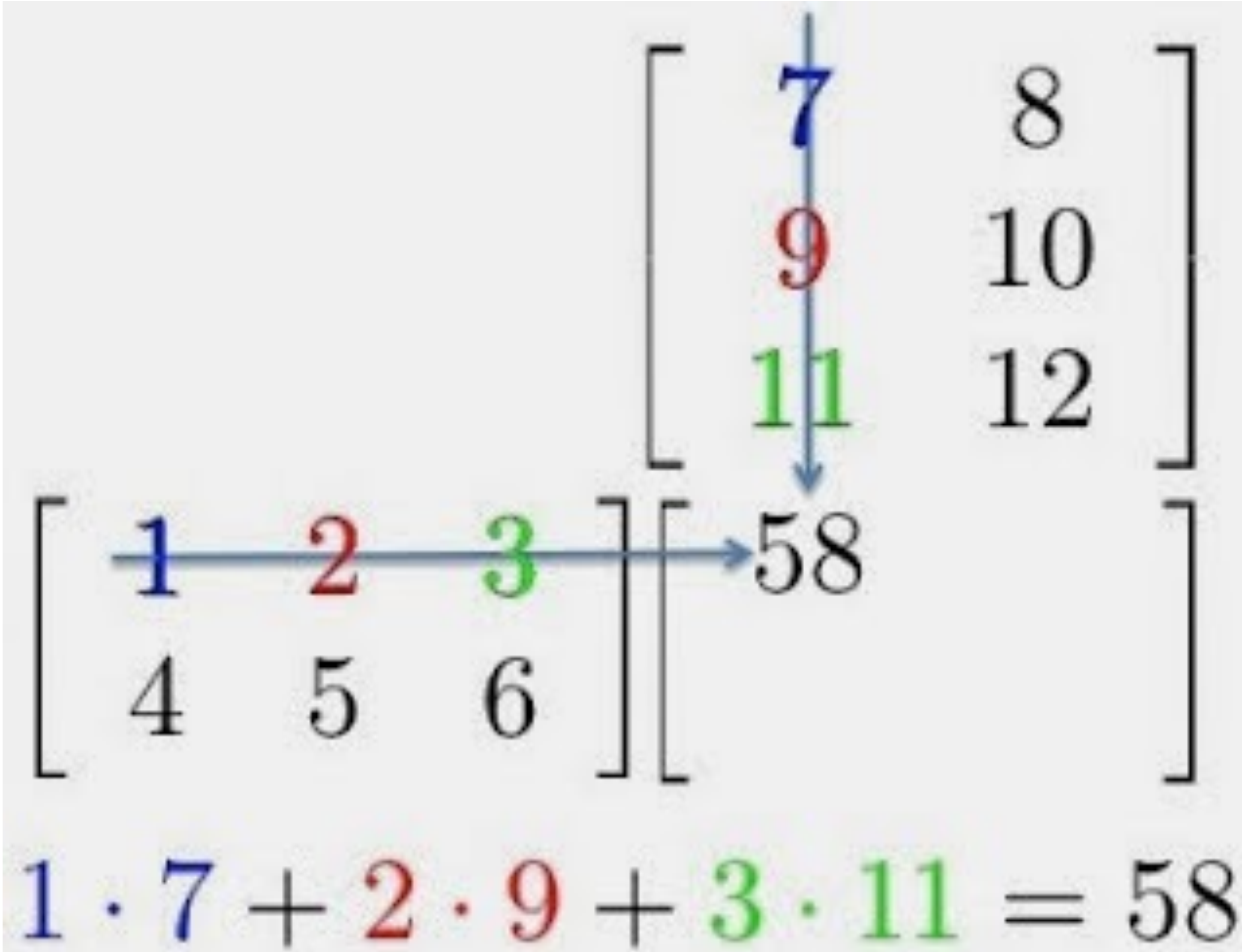
Method


Worked Example	Your Turn
Find: $\begin{pmatrix} -1 & 2 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 5 \\ -6 \end{pmatrix}$	Find: $\begin{pmatrix} 3 & 4 \\ -2 & -5 \end{pmatrix}\begin{pmatrix} 5 \\ -6 \end{pmatrix}$

Worked Example	Your Turn
Find: $\begin{pmatrix} 2 & 3 \\ -4 & -5 \end{pmatrix}\begin{pmatrix} -1 & 7 \\ 8 & -6 \end{pmatrix}$	Find: $\begin{pmatrix} 5 & -3 \\ -2 & -4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -7 & 6 \end{pmatrix}$

Worked Example	Your Turn
Find: $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}^4$	Find: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^4$

Worked Example	Your Turn
Matrices A and B are defined by $A = \begin{pmatrix} -1 & x \\ 3 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & x \\ -5 & -2 \end{pmatrix}$ Calculate AB, giving your answer in terms of x	Matrices U and V are defined by $U = \begin{pmatrix} 4 & 0 \\ a & 3 \end{pmatrix}$ and $B = \begin{pmatrix} a & 5 \\ 6 & -4 \end{pmatrix}$ Calculate UV, giving your answer in terms of a

Identity Matrix

Let $\boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

Determine:

$$AI = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$$

$$IA = \begin{pmatrix} \boldsymbol{a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{d} \end{pmatrix}$$

$\boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is known as the 'identity matrix'.
Multiplying by it has no effect, i.e. $AI = IA = A$ for any matrix A.

It may seem pointless to have such a matrix, but it'll have more importance when we consider matrices as 'transformations' later. Although admittedly you won't quite fully appreciate why we have it unless you do Further Maths A Level…

Worked Example	Your Turn
Find the value of t, given that $\begin{pmatrix} -5 & 3 \\ 7 & -4 \end{pmatrix}\begin{pmatrix} 4 & t \\ 7 & 5 \end{pmatrix} = I$	Find the value of t, given that $\begin{pmatrix} -7 & 4 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} -3 & -4 \\ -5 & t \end{pmatrix} = I$

Worked Example	Your Turn
$M = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$ Show that $M^3 = I$	$M = \begin{pmatrix} -2 & -1 \\ 3 & 1 \end{pmatrix}$ Show that $M^3 = I$

Worked Example	Your Turn
$2\begin{pmatrix} 5 & 0 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ Find the values of a and b	$3\begin{pmatrix} 2 & 1 \\ -5 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ -3 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ Find the values of a and b

Worked Example	Your Turn
Work out **all** solutions for x and y, given that $\begin{pmatrix} x & 1 \\ -4 & y \end{pmatrix}\begin{pmatrix} x \\ -3 \end{pmatrix} = \begin{pmatrix} 2x \\ 9 \end{pmatrix}$	Work out **all** solutions for x and y, given that $\begin{pmatrix} x & 3 \\ 1 & y \end{pmatrix}\begin{pmatrix} x \\ -4 \end{pmatrix} = \begin{pmatrix} 4x \\ 8 \end{pmatrix}$

Worked Example	Your Turn
$\begin{pmatrix} 2 & 1 \\ 8x & 5x \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \end{pmatrix}$ Work out the possible values for x and y	$\begin{pmatrix} 1 & 2 \\ x & 3x \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$ Work out the possible values for x and y

Fill in the Gaps

Question	Working		Answer
$\begin{pmatrix}3&0\\1&4\end{pmatrix}\times\begin{pmatrix}2&1\\5&0\end{pmatrix}$	$3\times2+0\times5$	$3\times1+0\times0$	$\begin{pmatrix}6&\square\\\square&\square\end{pmatrix}$
	$1\times2+4\times5$	$1\times1+4\times0$	
$\begin{pmatrix}4&2\\3&0\end{pmatrix}\times\begin{pmatrix}7&0\\1&1\end{pmatrix}$		$4\times0+2\times1$	$\begin{pmatrix}\square&2\\\square&\square\end{pmatrix}$
	$3\times7+0\times1$	$3\times0+0\times1$	
$\begin{pmatrix}2&-1\\1&3\end{pmatrix}\times\begin{pmatrix}0&4\\2&1\end{pmatrix}$			$\begin{pmatrix}\square&\square\\6&\square\end{pmatrix}$
	$1\times0+3\times2$		
$\begin{pmatrix}-3&1\\-2&0\end{pmatrix}\times\begin{pmatrix}0&-1\\4&1\end{pmatrix}$			$\begin{pmatrix}\square&\square\\\square&\square\end{pmatrix}$
$\begin{pmatrix}1.5&0\\-3&2\end{pmatrix}\times\begin{pmatrix}4&0.5\\2&-1\end{pmatrix}$			$\begin{pmatrix}\square&\square\\\square&\square\end{pmatrix}$
$\begin{pmatrix}\frac{3}{4}&-2\\0&3\end{pmatrix}\times\begin{pmatrix}-1&\frac{2}{3}\\4&2\end{pmatrix}$			$\begin{pmatrix}\square&\square\\\square&\square\end{pmatrix}$
$\begin{pmatrix}1&\sqrt{3}\\-2&0\end{pmatrix}\times\begin{pmatrix}-2&0\\\sqrt{3}&4\end{pmatrix}$			$\begin{pmatrix}\square&\square\\\square&\square\end{pmatrix}$
$\begin{pmatrix}\square&1\\0&2\end{pmatrix}\times\begin{pmatrix}2&1\\3&\square\end{pmatrix}$			$\begin{pmatrix}11&\square\\\square&10\end{pmatrix}$
$\begin{pmatrix}\square&\square\\\square&\square\end{pmatrix}\times\begin{pmatrix}2&-1\\1&3\end{pmatrix}$			$\begin{pmatrix}3&16\\7&0\end{pmatrix}$

Transformations

Matrices Representing Transformations

Matrices can represent transformations to points in 2D or 3D space.

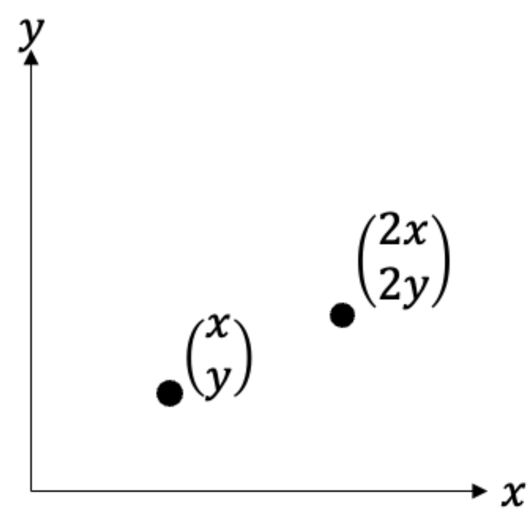
Let us represent a point as the vector $\begin{pmatrix} x \\ y \end{pmatrix}$

We can multiply it by a matrix:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \mathbf{2x} \\ \mathbf{2y} \end{pmatrix}$$

(Note: You're used to representing points as coordinates like (x, y) rather than vectors, but it allows us to apply matrix transformations to them more easily in this form)

Important Note: When we multiply by a matrix, it goes on the front, not after. This is a bit like how with composite functions, e.g. $gf(x)$, we applied f to x followed g. We go right to left.



What 'transformation' therefore does the matrix $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ represent?

An enlargement by scale factor 2 about the origin.

Investigate

In pairs or otherwise, determine the transformations that each of these matrices represents.

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$	?	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	?
$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	?	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	?
$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	?	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	?
$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	?		

Worked Example	Your Turn
Find a 2×2 matrix that represents: a) A reflection in the y-axis b) A reflection in the line $y = x$	Find a 2×2 matrix that represents: a) A reflection in the x-axis b) A reflection in the line $y = -x$

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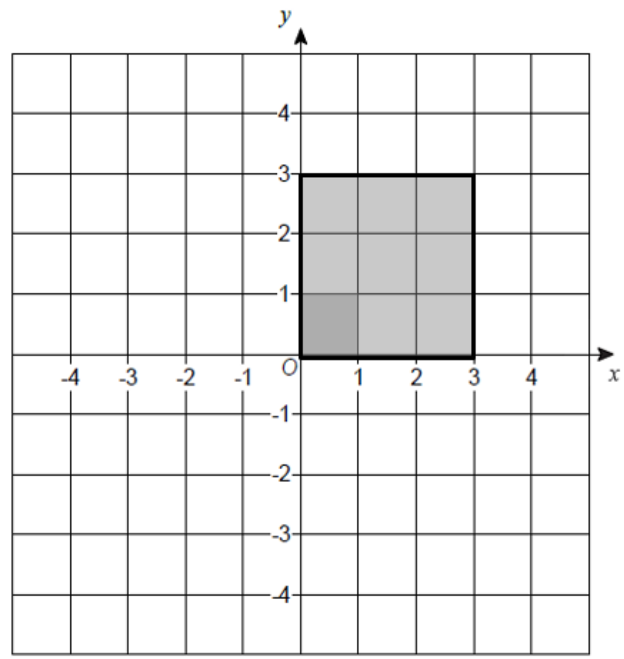
Worked Example	Your Turn
Find a 2×2 matrix that represents: Rotation $90°$ anticlockwise about the origin	Find a 2×2 matrix that represents: Rotation $270°$ anticlockwise about the origin

Transforming the Unit Square

Set 3 Paper 2 Q17

The unit square is shaded on the grid.



(a) On the grid, draw the image of the unit square after it is transformed using the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

For more complex transformations it's not sufficient to look at the effect on just one point: we can't fully see what the matrix is doing.

If we look at the effect on a **unit square** (with coordinates $(0,0), (1,0), (1,1), (0,1)$), we can better see the effect of a matrix transformation on a region in the x-y plane.

Just apply the transformation to each point of the unit square.

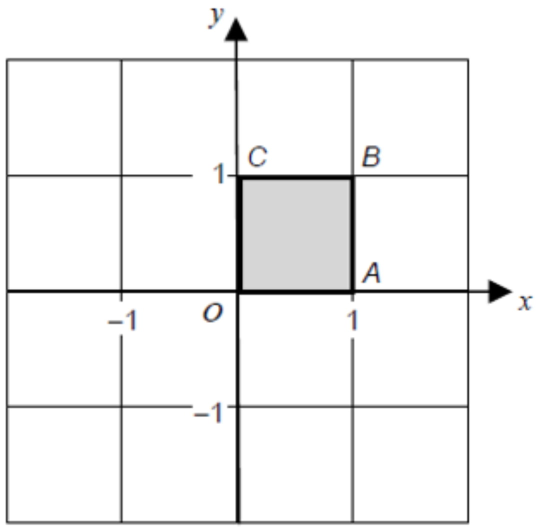
$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \end{pmatrix}$$
$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{0} \end{pmatrix}$$
$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{3} \end{pmatrix}$$
$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \mathbf{3} \\ \mathbf{3} \end{pmatrix}$$

Worked Example	Your Turn
Describe geometrically the effect of the following matrices: $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	Describe geometrically the effect of the following matrices: $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$

Worked Example	Your Turn
The transformation matrix $\begin{pmatrix} a & -2 \\ 1 & 1 \end{pmatrix}$ maps the point $(2, 5)$ onto the point $(3, b)$. Work out the values of a and b.	The transformation matrix $\begin{pmatrix} a & 2 \\ -1 & 1 \end{pmatrix}$ maps the point $(3, 4)$ onto the point $(2, b)$. Work out the values of a and b.

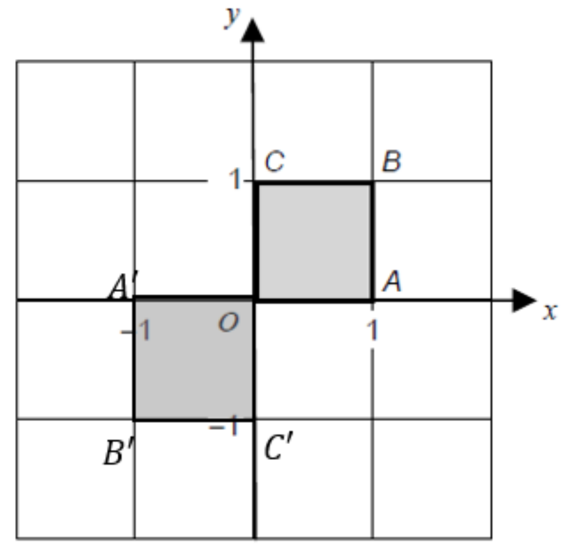
Worked Example	Your Turn
The matrix $\begin{pmatrix} -a & 3b \\ 2a & b \end{pmatrix}$ maps the point (2, 3) onto the point (-31, -1). Work out the values of a and b.	The matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point (5, 4) onto the point (1, 17). Work out the values of a and b.

Worked Example	Your Turn
The diagram shows the unit square $OABC$	The diagram shows the unit square $OABC$



The image of OABC after transformation by the matrix $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is $OA'B'C'$

Draw and label $OA'B'C'$

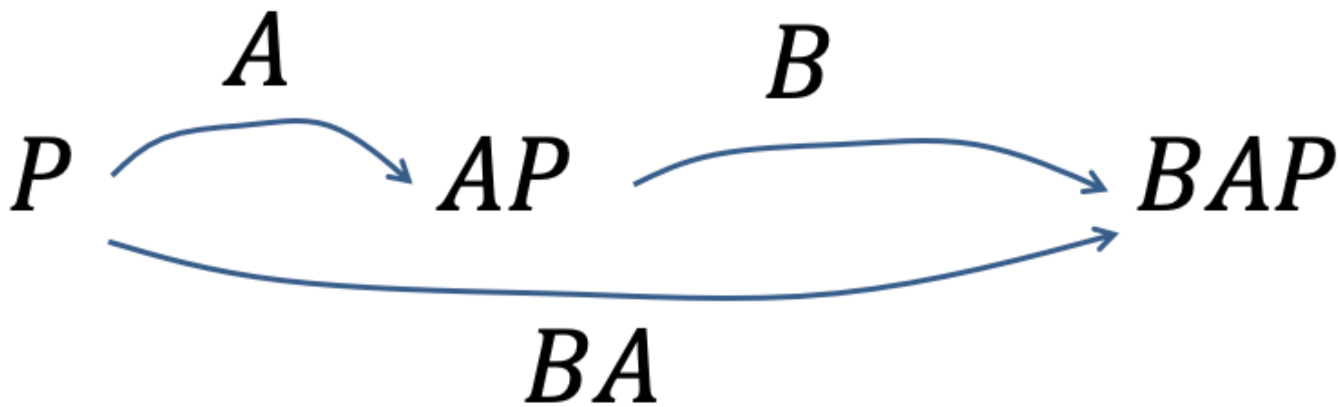


The image of OABC after transformation by the matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ is $OA'B'C'$

Draw and label $OA'B'C'$

Worked Example	Your Turn
A triangle T has vertices $(1, 1), (1, 2)$ and $(2, 2)$ a) Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$ b) Sketch T and its image, T' on a coordinate grid.	A triangle T has vertices $(1, 1), (1, 2)$ and $(2, 2)$ a) Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$ b) Sketch T and its image, T' on a coordinate grid.

Combinations of Transformations



If a point P is transformed by the matrix A followed by the matrix B, what calculation would get the new point?
Therefore what matrix represents the combined transformation of A followed by B?

The matrix BA represents the combined transformation of A followed by B.

Note

Note: The default direction of rotation is anticlockwise if not specified.

Worked Example	Your Turn
A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$. Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$.	A triangle T has vertices $(1, 1)$, $(1, 2)$ and $(2, 2)$. Find the vertices of the image of T under the transformation given by the matrix $\boldsymbol{M} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

Worked Example	Your Turn
A, B and C are transformations in the $x-y$ plane. A: Rotation through $270°$ anticlockwise about the origin. B: Reflection in the y-axis C: Transformation B followed by transformation A. Use matrix multiplication to show that C is equivalent to a single reflection.	A, B and C are transformations in the $x-y$ plane. A: Rotation through 90° anticlockwise about the origin. B: Reflection in the x-axis C: Transformation A followed by transformation B. Use matrix multiplication to show that C is equivalent to a single reflection.

Worked Example	Your Turn
Use **matrix multiplication** to show that, in the $x - y$ plane, a reflection in the line $y = x$, followed by a rotation, $90°$ clockwise about the origin, followed by a reflection in the x-axis is equivalent to a transformation by the identity matrix.	Use **matrix multiplication** to show that, in the $x - y$ plane, a reflection in the line $y = -x$, followed by a rotation, $90°$ anticlockwise about the origin, followed by a reflection in the x-axis is equivalent to a transformation by the identity matrix.

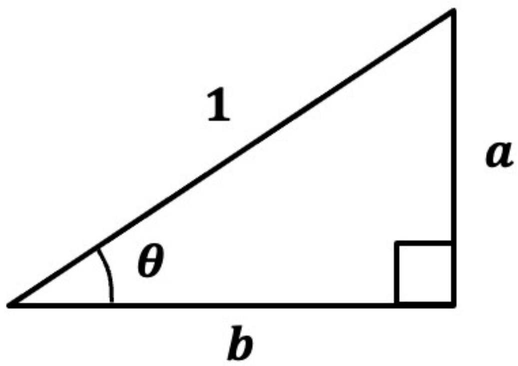
Extra Notes

2 Trigonometric Identities and Equations

Specification		
6.9	Know and use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$	Including expressions to be simplified, proofs of identities and equations solved
6.10	Solution of simple trigonometric equations in given intervals	Equations will be restricted to single angles: $\sin x = 0.5$; $\sqrt{2}\sin x = \cos x$ for $0^\circ \leqslant x \leqslant 360^\circ$; $\sin^2 x = \dfrac{1}{4}$ for $0^\circ \leqslant x \leqslant 360^\circ$

Trigonometric Identities

Proving that $sin^2\theta + cos^2\theta = 1$



Consider a triangle with **<u>hypotenuse of length 1</u>**. Determine a and b in terms of θ.

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{a}{1} = \boldsymbol{a}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{b}{1} = \boldsymbol{b}$$

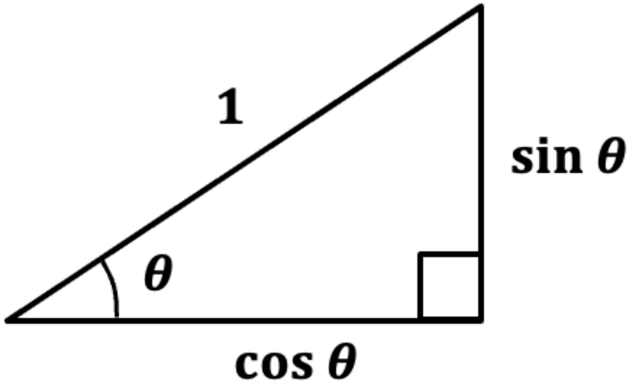
Hence use Pythagoras's theorem to determine and identity involving θ

$$a^2 + b^2 \equiv 1$$
$$(\sin\theta)^2 + (\cos\theta)^2 \equiv 1$$
$$\mathbf{\sin^2\theta + \cos^2\theta \equiv 1}$$

As $(\sin\theta)^2$ is quite cumbersome to write, we can write $\sin^2\theta$ as a notational convenience. It does <u>not</u> mean the "sin" is squared (sin is not a quantity!), but that all of $\sin\theta$ is squared.

The $\equiv$ indicates an <u>**identity**</u>, which means the equation is true for <u>all</u> values of θ, not just specific values.

Proving that $sin\theta / cos\theta = \tan\theta$



Earlier we considered a right-angled triangle where the hypotenuse is 1. By simple trigonometry, we found the two other sides were $\sin\theta$ and $\cos\theta$

What would $\tan\theta$ be?

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin\theta}{\cos\theta}$$

This gives us the second of our two key trigonometric identities.

$$\sin^2\theta + \cos^2\theta \equiv 1$$
$$\frac{\sin\theta}{\cos\theta} \equiv \tan\theta$$

Worked Example	Your Turn
It is given that α is acute and that $\cos\alpha = \frac{2}{3}$ Find the exact value of $\tan\alpha$	It is given that α is acute and that $\sin\alpha = \frac{4}{5}$ Find the exact value of $\tan\alpha$

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Worked Example	Your Turn
Prove that $1 - \frac{\tan\theta \cos^3\theta}{\sin\theta} \equiv \sin^2\theta$	Prove that $1 - \tan\theta \sin\theta \cos\theta \equiv \cos^2\theta$

Worked Example	Your Turn
Prove that $\tan\theta - \frac{1}{\tan\theta} \equiv \frac{1-2\cos^2\theta}{\sin\theta\, cos\theta}$	Prove that $\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$

Worked Example	Your Turn
Prove that $\frac{\cos\theta - \cos^3\theta}{\sin^3\theta} \equiv \frac{1}{\tan\theta}$	Prove that $\frac{\sin\theta - \sin^3\theta}{\cos^3\theta} \equiv \tan\theta$

Worked Example	Your Turn
Prove that $\frac{\sin^4\theta-\cos^4\theta}{\sin^2\theta}\equiv 1-\frac{1}{\tan^2\theta}$	Prove that $\frac{\cos^4\theta-\sin^4\theta}{\cos^2\theta}\equiv 1-\tan^2\theta$

Worked Example	Your Turn
Prove that $\frac{\frac{\sin x}{\tan x}}{\sqrt{1-\sin^2 x}} \equiv 1$	Prove that $\frac{\tan x \cos x}{\sqrt{1-\cos^2 x}} \equiv 1$

Worked Example	Your Turn
Prove that $\frac{1}{\tan^2 \theta} \equiv \frac{1}{\sin^2 \theta} - 1$	Prove that $\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1$

Dr Frost 502i and 502j

Trigonometric Equations

Laws of Trigonometric Functions

$$\sin(x) = \sin(180 - x)$$

$$\cos(x) = \cos(360 - x)$$

sin and cos repeat every 360°

tan repeats every 180°

Summary

1. $\sin(x) = \sin(180 - x)$
2. $\cos(x) = \cos(360 - x)$
3. sin, cos repeat every 360
4. tan repeats every 180

A. $\tan x = \dfrac{\sin(x)}{\cos(x)}$

B. $\sin^2 x + \cos^2 x = 1$

Worked Example	Your Turn
Solve $\cos x = -0.8$ in the interval $-180° \leq x \leq 540°$ Give your solution(s) correct to 1 decimal place where appropriate.	Solve $\sin x = -0.4$ in the interval $-180° \leq \theta \leq 360°$ Give your solution(s) correct to 1 decimal place where appropriate.

Worked Example	Your Turn
Solve $\tan x = 0.4$ in the interval $-180° \leq x \leq 180°$ Give your solution(s) correct to 1 decimal place where appropriate.	Solve $\tan x = -0.4$ in the interval $-180° \leq x \leq 180°$ Give your solution(s) correct to 1 decimal place where appropriate.

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Worked Example	Your Turn
Solve $\sin^2 x = 0.53$ in the interval $-180° \leq x \leq 360°$ Give your solution(s) correct to 1 decimal place where appropriate.	Solve $\cos^2 x = 0.62$ in the interval $-180° \leq \theta \leq 360°$ Give your solution(s) correct to 1 decimal place where appropriate.

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Worked Example	Your Turn
Solve $8\tan^2 x = -6\tan x - 1$ in the interval $0° < x < 540°$ Give your solution(s) correct to 1 decimal place where appropriate.	Solve $12\tan^2 x - \tan x = 1$ in the interval $0° < x < 540°$ Give your solution(s) correct to 1 decimal place where appropriate.

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Worked Example	Your Turn
Solve $6\sin^2 x - 5\cos x = 2$ in the interval $0° \leq x < 540°$ Give your solution(s) correct to 2 decimal places where appropriate.	Solve $8\cos^2 x = 2\sin x + 5$ in the interval $0° \leq x < 540°$ Give your solution(s) correct to 2 decimal places where appropriate.

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Extra Notes