KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025 Level 2 Further Mathematics 2026

Unit 30 Booklet

HGS Maths

Tasks

Dr Frost Course

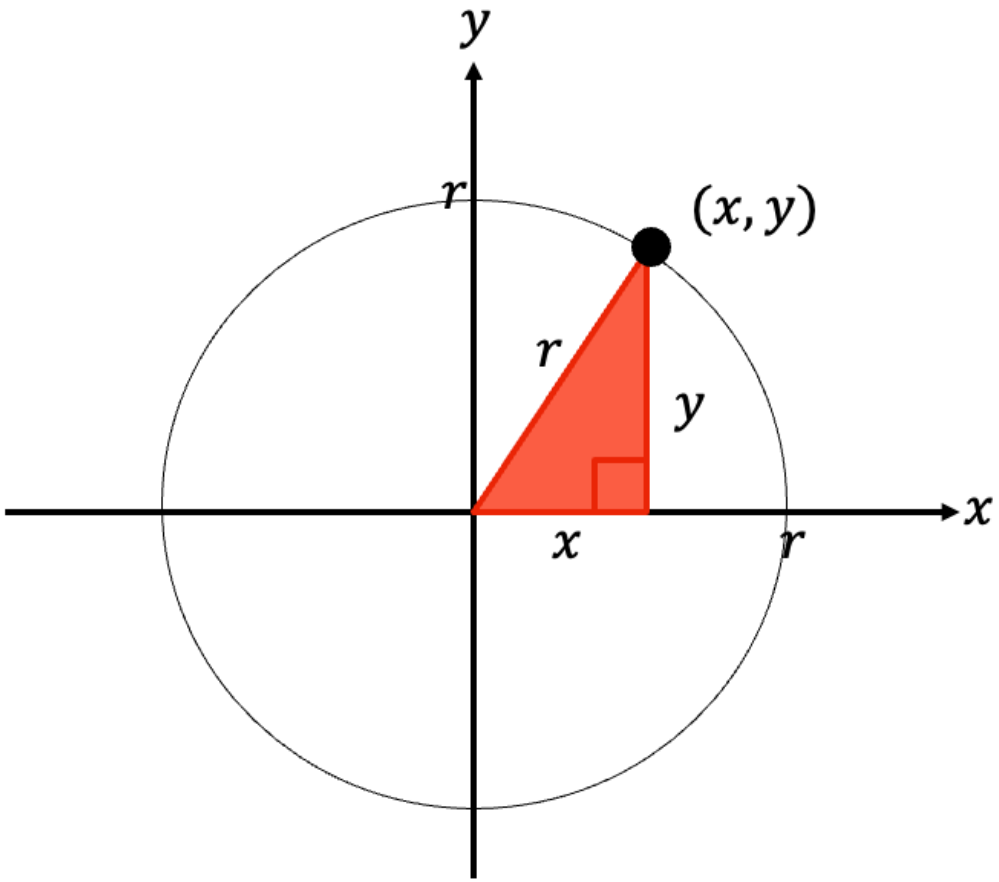
Name: _____

Class: _____

Contents

1 Advanced Equations of Circles

Equation of a Circle Centered at the Origin



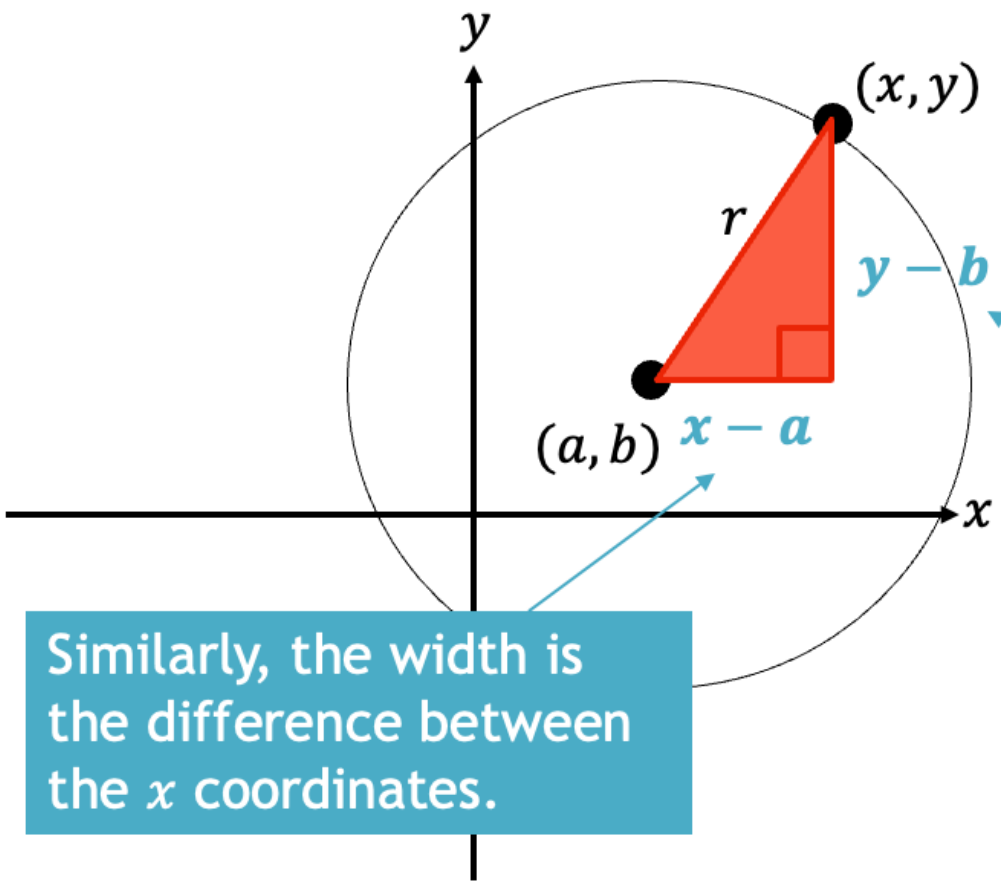
Recall that a graph is a set of points (x, y) that satisfy some equation.

Suppose we have a generic point (x, y) on a circle centred at the origin, with radius r. What equation must (x, y) satisfy?

By Pythagoras's theorem:

$$x^2 + y^2 = r^2$$

Equation of a Circle with Any Centre



But what if the centre had an arbitrary centre (a, b)? Consider the sides of the right-angled triangle now.

The height of the triangle is the difference between the y coordinates.

Similarly, the width is the difference between the x coordinates.

By Pythagoras's theorem:

$$(x-a)^2 + (y-b)^2 = r^2$$

The equation of a circle with centre (a, b) and radius r is:

$$(x - a)^2 + (y - b)^2 = r^2$$

Worked Example	Your Turn
Find an equation of the circle with radius $3\sqrt{2}$ and centre $(-2, 4)$.	Find an equation of the circle with radius $2\sqrt{11}$ and centre $(2, -5)$.

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Worked Example	Your Turn
Find the centre and exact value of the radius of the circle with equation $(x+1)^2+(y-4)^2=18$ where the centre of the circle is (a,b) and the radius is given in its simplest form.	Find the centre and exact value of the radius of the circle with equation $(x-4)^2+(y+5)^2=12$ where the centre of the circle is (a,b) and the radius is given in its simplest form.

Worked Example	Your Turn
Write down the equation of a circle with centre $(-1, 5)$ and diameter of $2\sqrt{10}$. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants to be found.	Write down the equation of a circle with centre $(-6, -4)$ and diameter of $2\sqrt{15}$. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants to be found.

Worked Example	Your Turn
Determine whether the point $(-4, -5)$ is inside the circle, on the circle, or outside the circle with equation $(x+1)^2 + (y-1)^2 = 47$	Determine whether the point $(-4, 9)$ is inside the circle, on the circle, or outside the circle with equation $(x+2)^2 + (y-4)^2 = 28$

Worked Example	Your Turn
Find the centre and radius of the circle with equation $x^2 + y^2 + 24x - 14y + 150 = 0$ giving your answer in exact form.	Find the centre and radius of the circle with equation $x^2 + y^2 - 20x + 18y = -89$ giving your answer in exact form.

Worked Example	Your Turn
A circle C has equation $x^2 + y^2 + 2kx + 4ky + 80 = 0$ where k is a constant. By considering the radius of C, state the range of possible values for k.	A circle C has equation $x^2 + y^2 + 2kx + 2ky = -18$ where k is a constant. By considering the radius of C, state the range of possible values for k.

Fill in the Gaps

Equation in Factorised Form	Equation in Expanded Form	Centre of Circle	Radius of Circle
$(x+2)^2+(y+5)^2=9$	$x^2+y^2+4x+10y+20=0$	$(-2,-5)$	3
$(x-3)^2+(y+2)^2=25$	$x^2+y^2-6x+4y-12=0$		
$x^2+(y-1)^2=4$			
		$(-1,4)$	10
		$(-6,0)$	5
		$(4,2)$	$\sqrt{15}$
	$x^2+y^2+2x+6y-6=0$		
	$x^2+y^2-8x+10y-40=0$		
		$\left(\frac{1}{2},\frac{3}{2}\right)$	2
	$x^2+y^2-5x-12y+30=0$		

Worked Example	Your Turn
The line PQ is the diameter of a circle where P and Q have coordinates $(-3, -2)$ and $(9, -18)$ respectively. Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$	The line CD is the diameter of a circle where C and D have coordinates $(-9, -4)$ and $(-3, -12)$ respectively. Find the equation of the circle, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$

Fill in the Gaps

Each circle has a diameter AB, a centre C and a radius r

Point A	Point B	Gradient of AB	Equation of AB	Centre C	Radius r	Equation of Circle
$(3, 4)$	$(-3, -4)$				5	$x^2 + y^2 = 25$
$(0, 5)$	$(6, -3)$					
$(4, 0)$				$(2, -1)$		
	$(2, -2)$			$(4, 2)$		
$(-12, 4)$						$(x + 9)^2 + y^2 = 25$
		1		$(1, -1)$	$\sqrt{2}$	
			$y = 3x - 17$	$(4, -5)$	$\sqrt{10}$	
			$y = \frac{3}{4}x - \frac{9}{4}$			$(x + 1)^2 + (y + 3)^2 = 100$

Worked Example	Your Turn
The circle C has the equation $x^2 + y^2 - 22x - 14y + 40 = 0$ Find the coordinates of the points where the circle C crosses the x or y axes.	The circle C has the equation $x^2 + y^2 + 2x - 22y - 48 = 0$ Find the coordinates of the points where the circle C crosses the x or y axes.

Fill in the Gaps

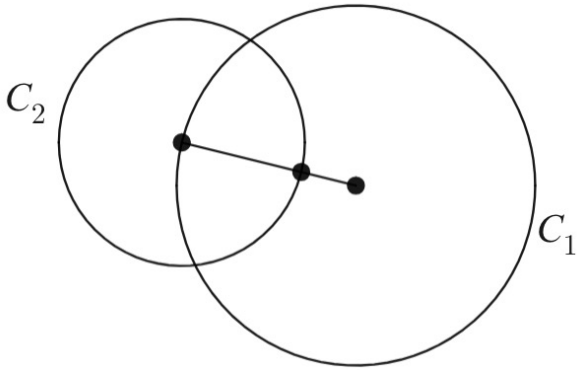
Complete the table. Cells will be filled with either coordinates, values, inequalities, or words.

SKETCH IT.

	Circle Equation C	Centre	Radius	Does it cross the x-axis?	Does it cross the y-axis?	Does it cross $y = 2x + 1$?
1)	$(x+2)^2+(y+3)^2=100$					
2)	$(x+2)^2+(y+3)^2=3$					
3)	$(x+2)^2+(y+3)^2=k$			No, but the x-axis is a tangent to C.		
4)	$(x+2)^2+(y+3)^2=k$				No, but the y-axis is a tangent to C.	
5)	$(x+a)^2+(y+b)^2=16$			No, but the x-axis is a tangent to C.	No, but the y-axis is a tangent to C.	
6)	$x^2+y^2=k$					No, but $y = 2x + 1$ is a tangent to C.
7)	$x^2+y^2=k$					No
8)	$(x+a)^2+(y+b)^2=16$			No, but the x-axis is a a tangent to C.		No, but $y = 2x + 1$ is a tangent to C.
9)	$x^2+(y+b)^2=16$					No

Worked Example

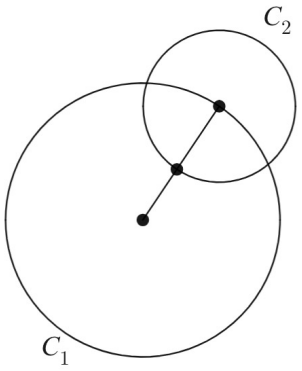
Circle C_1 has equation $(x-5)^2+(y-3)^2=a$
Circle C_2 has equation $x^2+y^2-2x-8y-25=0$



The centre of C_2 lies on the circumference of C_1
Find the value of a.

Your Turn

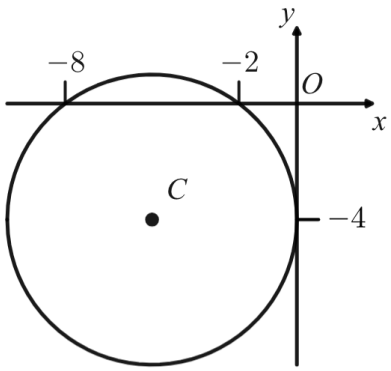
Circle C_1 has equation $(x-3)^2+(y-2)^2=h$
Circle C_2 has equation $x^2+y^2-10x-10y-54=0$



The centre of C_2 lies on the circumference of C_1
Find the value of h.

Worked Example

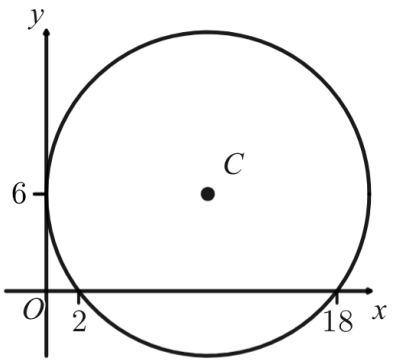
The diagram shows a circle with centre C.
The circle intersects the x-axis at $(-2, 0)$ and $(0, -4)$
The circle touches the y-axis at $(-8, 0)$



Work out the equation of the circle.

Your Turn

The diagram shows a circle with centre C.
The circle intersects the x-axis at $(18, 0)$ and $(0, 6)$
The circle touches the y-axis at $(2, 0)$



Work out the equation of the circle.

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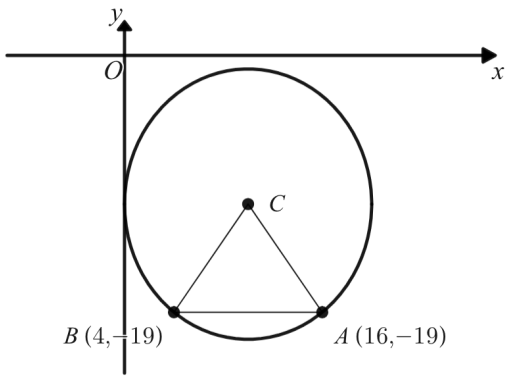
Worked Example

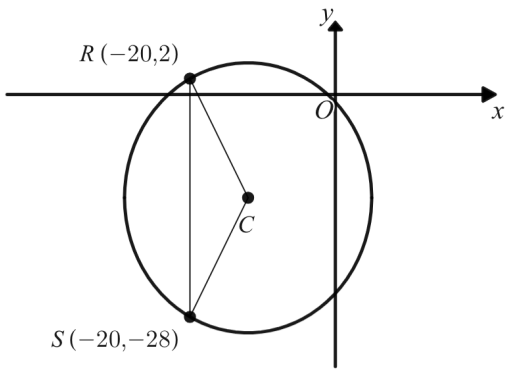
The diagram shows a circle, centre C with radius 10. The circle passes through the points $A(16, -19)$ and $B(4, -19)$.



Work out the equation of the circle.

Your Turn

The diagram shows a circle, centre C with radius 17. The circle passes through the points $R(-20, 2)$ and $S(-20, -28)$.



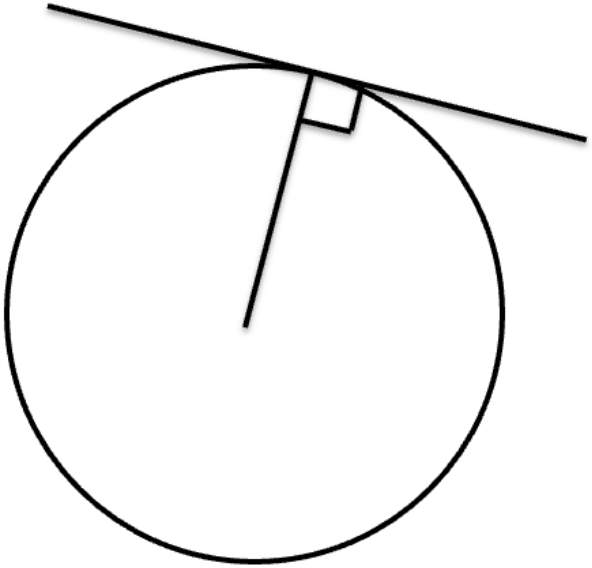
Work out the equation of the circle.

Worked Example	Your Turn
Circle A has equation $x^2 + y^2 - 8x - 6y - 84 = 0$ Circle B has equation $x^2 + y^2 + 10x + 30y + 184 = 0$ Select the correct statement. • The circles overlap • The circles touch • The circles do not touch or overlap	Circle A has equation $x^2 + y^2 + 40x + 42 + 517 = 0$ Circle B has equation $x^2 + y^2 - 8x - 6y - 169 = 0$ Select the correct statement. • The circles overlap • The circles touch • The circles do not touch or overlap

Worked Example	Your Turn
Circle A has equation $x^2 + y^2 + 6x - 8y - 24 = 0$ Circle B has equation $x^2 + y^2 - 30x + 16y + 120 = 0$ Find the exact shortest distance between the two circles.	Find the exact shortest distance between the two circles. Circle A has equation $x^2 + y^2 + 6x + 8y - 56 = 0$ Circle B has equation $x^2 + y^2 - 18x - 24y + 144 = 0$ Find the exact shortest distance between the two circles.

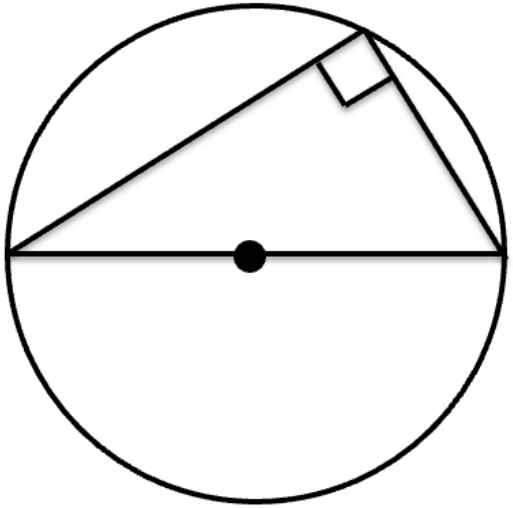
Circle Theorems

There are three circle theorems relevant to L2FM circle questions.



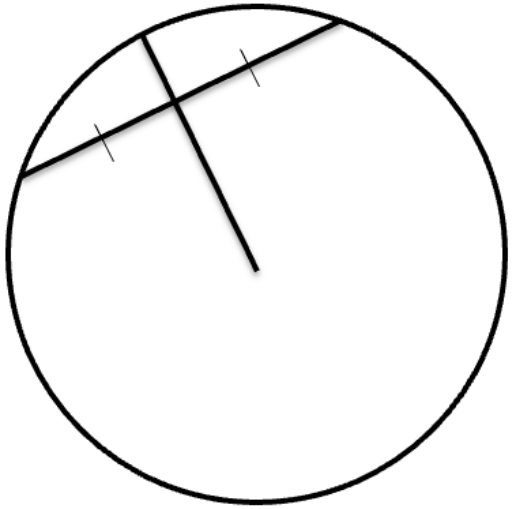
Tangent is at right-angles to the radius.

e.g. "Find equation of the tangent at the point (2,4)"



Angle in a semi-circle is $90°$ (so two top lines are perpendicular)

e.g. Use fact lines are perpendicular to find other side of diameter.

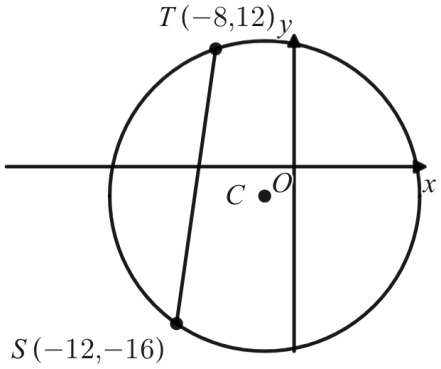


The perpendicular bisector of any chord passes through the centre of the circle.

e.g. Given two points on the circumference of the circle, if the centre of the circle is (3, k), find k.

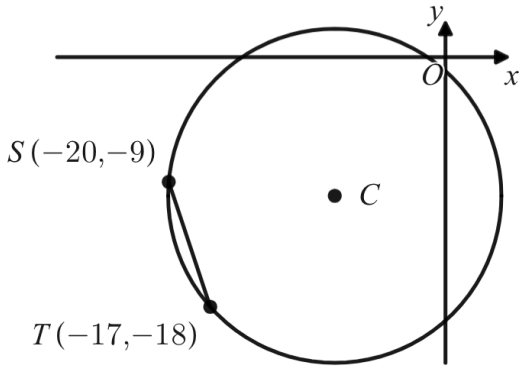
Worked Example

A circle has centre C and equation
$(x+3)^2+(y+3)^2=250$.
There are two points $S(-12,-16)$ and $T(-8,12)$ which lie on the circle. Find the length of the shortest distance from C to the chord ST.



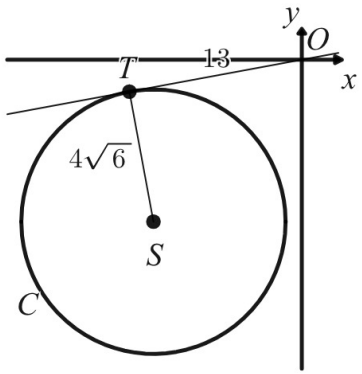
Your Turn

A circle has centre C and equation
$(x+8)^2+(y+10)^2=145$.
There are two points $S(-20,-9)$ and $T(-17,-18)$ which lie on the circle. Find the length of the shortest distance from C to the chord ST.



Worked Example

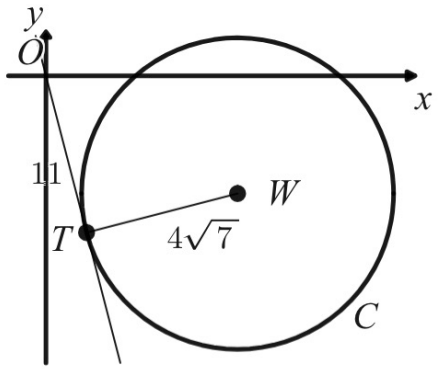
The diagram shows a circle C with centre S and radius $4\sqrt{6}$ and the point T which lies on C.
The tangent to C at point T passes through the origin O and $OT = 13$



Given that the coordinates of S are $(m, -12)$, find the value of m.

Your Turn

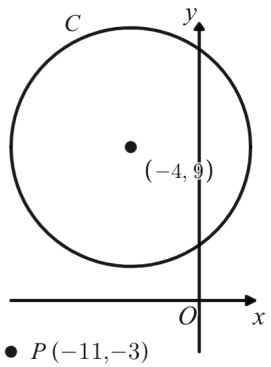
The diagram shows a circle C with centre W and radius $4\sqrt{7}$ and the point T which lies on C.
The tangent to C at point T passes through the origin O and $OT = 11$



Given that the coordinates of W are $(13, k)$, find the value of k.

Worked Example

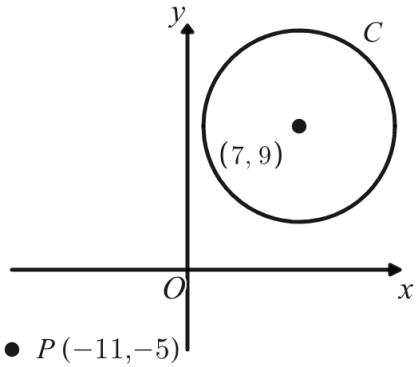
A circle C has radius 7 and centre $(-4, 9)$, as shown in the figure.



A line through the point $P(-11, -3)$ is a tangent to the circle C at the point T.
Find the length of PT.

Your Turn

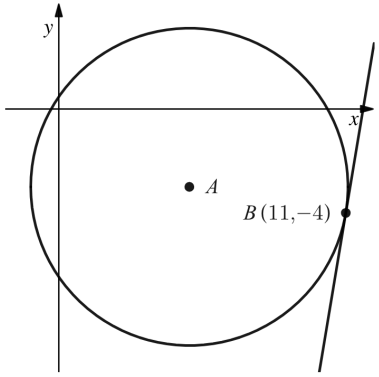
A circle C has radius 6 and centre $(7, 9)$, as shown in the figure.



A line through the point $P(-11, -5)$ is a tangent to the circle C at the point T.
Find the length of PT.

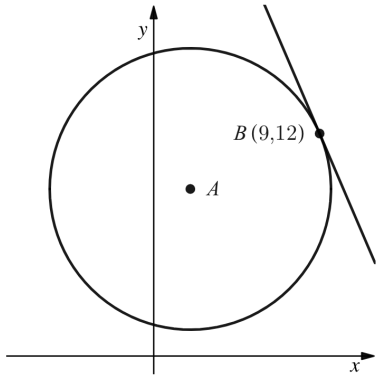
Worked Example

A circle has equation
$x^2 + y^2 - 10x + 6y - 3 = 0$
Find the equation of the tangent to the circle at the point $(11, -4)$. Give your answer in the form $ax + by = c$, where a, b and c are integers to be found.



Your Turn

A circle has equation
$x^2 + y^2 - 4x - 18y + 27 = 0$
Find the equation of the tangent to the circle at the point $(9, 12)$. Give your answer in the form $ax + by = c$, where a, b and c are integers to be found.



Extra Notes

2 Limiting Values of Sequences

Limiting Values

The nth term of a sequence is $\frac{n-3}{n+1}$

As $n \to \infty$, what is the limiting value of the sequence?

Notation: $n \to \infty$ means "as n tends towards infinity".

The first 20 terms are as following (and the 100th). What do you notice?

$$-\frac{2}{2}, -\frac{1}{3}, \frac{0}{4}, \frac{1}{5}, \frac{2}{6}, \frac{3}{7}, \frac{4}{8}, \frac{5}{9}, \frac{6}{10}, \frac{7}{11}, \frac{8}{12}, \frac{9}{13}, \frac{10}{14}, \frac{11}{15}, \frac{12}{16}, \frac{13}{17}, \frac{14}{18}, \frac{15}{19}, \frac{16}{20}, \frac{17}{21}, \dots, \frac{97}{101}, \dots$$

We can see that as n becomes larger, the -3 and +1 become increasingly inconsequential. We could write:

"As n becomes large, $\frac{n-3}{n+1} \to \frac{n}{n} = 1$"

As $n \to \infty$, what is the limiting value of the sequences with nth term:

$\frac{2n+3}{n}$ **As n becomes large, $\frac{2n+3}{n} \to \frac{2n}{n} = 2$**

$\frac{3n-1}{4n+1}$ **As n becomes large, $\frac{3n-1}{4n+1} \to \frac{3n}{4n} = \frac{3}{4}$**

$\frac{n^2+n}{2n^2-4}$ **As n becomes large, $\frac{n^2+n}{2n^2-4} \to \frac{n^2}{2n^2} = \frac{1}{2}$**

Worked Example	Your Turn
As $n \to \infty$: a) $\frac{5}{n} \to$ b) $\frac{3n^2}{n} \to$ c) $\frac{4n}{3n+1} \to$	As $n \to \infty$: a) $\frac{8}{n} \to$ b) $\frac{n^2}{n} \to$ c) $\frac{7n}{2n+1} \to$

Worked Example	Your Turn
The nth term of a sequence is $\frac{5n-1}{2n-4}$ Write down the limiting value of the sequence as $n \to \infty$	The nth term of a sequence is $\frac{4n+6}{5n+3}$ Write down the limiting value of the sequence as $n \to \infty$

Worked Example	Your Turn
The nth term of a sequence is $\frac{6n^2+n+5}{n^2+4n-5}$ Write down the limiting value of the sequence as $n \to \infty$	The nth term of a sequence is $\frac{3n^2+4n+6}{2n^2+5n+3}$ Write down the limiting value of the sequence as $n \to \infty$

Worked Example	Your Turn
a) Find the nth term of the sequence $9, \frac{13}{6}, \frac{17}{11}, \frac{21}{16}, \ldots$ b) Hence find the limit of the sequence as $n \to \infty$	a) Find the nth term of the sequence $\frac{1}{5}, \frac{4}{7}, \frac{7}{9}, \frac{10}{11}, \ldots$ b) Hence find the limit of the sequence as $n \to \infty$

Extra Notes