Please check the examination d	etails below before entering your	candidate information
Candidate surname	Other na	mes
Pearson Edexcel Level 3 GCE	Centre Number	Candidate Number
Specimen Paper)
	Paper Reference	e 8FM0/22
Further Mathe Advanced Subsidiary 22: Further Pure Mather		
You must have: Mathematical Formulae and S	tatistical Tables, calculator	Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
 there may be more space than you need.
- You should show sufficient working to make your methods clear.
 Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 4 questions in this question paper. The total mark for this paper is 40.
- The marks for each question are shown in brackets
 use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶



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Answer ALL questions. Write your answers in the spaces provided.

- 1. (i) The set $G = \{1,3,4,9,10,12\}$ forms a group under the operation of multiplication modulo 13
 - (a) Complete the Cayley table below, Table 1, for this group.

(3)

(b) Find a subgroup of (G, \times_{13}) of order 3

(1)

(c) Explain why there can be no subgroup of (G, \times_{13}) of order 4

(1)

(ii) Determine whether the set {1,3,6,9,12} under the operation of multiplication modulo 15 forms a group.

(3)

× ₁₃	1	3	4	9	10	12
1		3				
3			12			
4						
9			10			
10	10					
12					3	

Table 1

A spare table can be found on page 3 if you need to rewrite your Cayley table.

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Question 1 continued	
Question I continued	

Only use this grid if you need to rewrite the Cayley table.

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(Total for Question 1 is 8 marks)



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2. The two sequences x_n and y_n satisfy the recurrence relations

$$x_1 = 2$$
 $x_{n+1} = 5x_n - y_n$ $x_{n+1} = 3x_n + y_n$ $x_n \ge 1$

(a) Find the value of x_3

(2)

These recurrence relations can be written in matrix form as

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \qquad \begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_n \\ y_n \end{pmatrix} \qquad n \geqslant 1$$

where $\mathbf{A} = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix}$

- (b) Find
 - (i) the eigenvalues of A,
 - (ii) the corresponding eigenvectors of A.

(5)

(c) Hence write down matrices **P** and **D** such that $P^{-1}AP = D$, where **D** is a diagonal matrix.

2)

New sequences u_n and v_n can be formed from x_n and y_n using the transformation

$$\begin{pmatrix} u_n \\ v_n \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

(d) Show that $\begin{pmatrix} u_{n+1} \\ v_{n+1} \end{pmatrix} = \mathbf{D} \begin{pmatrix} u_n \\ v_n \end{pmatrix}$

(2)

(e) Hence find closed form expressions for the original sequences $\boldsymbol{x_n}$ and $\boldsymbol{y_n}$

(5)

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Question 2 continued



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Question 2 continued
(Total for Question 2 is 16 marks)



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3. (a) Sketch, on an Argand diagram, the arc given by the locus of points z in the complex plane satisfying

$$\arg\left(\frac{z}{z - 25i}\right) = \frac{11\pi}{12}$$

(2)

The centre of the circle containing this arc is x + yi where

$$x = -\frac{25}{2} \left(2 + \sqrt{3} \right)$$

(b) Find the radius of this circle, giving your answer to 3 decimal places.

(4)

In a game, players take turns to roll balls along a horizontal surface from a fixed starting position to a fixed target point 25 metres away.

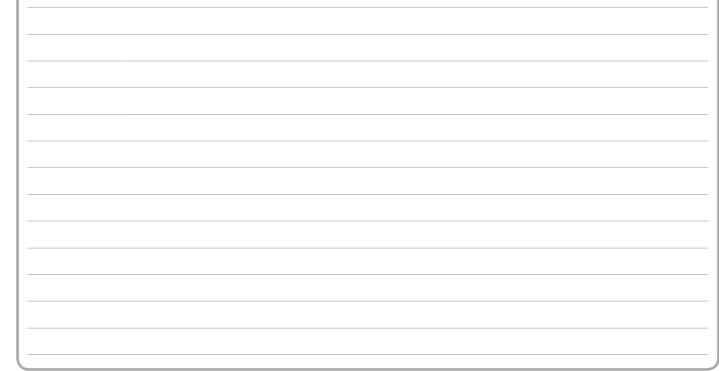
The game can be modelled in the complex plane, with the starting position at the origin and the target at the point 25i, where the units are metres.

In a particular game, the first player's ball has come to rest with its centre at 0.5 + 23.25i The second player's ball is following the path sketched in part (a).

Given that the radius of each ball is approximately 5 centimetres,

(c) use this model to determine whether the second player's ball will hit the first player's ball.

(4)



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Question 3 continued



Question 3 continued	
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Question 3 continued
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4. A positive integer n satisfies the inequality 99 < n < 1000 The integer n is written as a three digit number abc such that

$$n = 100a + 10b + c$$

where the digits a, b, and c are each integers between 0 and 9 inclusive, $a \neq 0$

The integer n has the following three properties:

- *n* is divisible by 11
- the sum of the digits of n is odd
- $n \equiv 5 \pmod{9}$

Use this information to find all possible values for n, making your reasoning clear.

(6)

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Question 4 continued



Question 4 continued	

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Question 4 continued		



Question 4 continued	
	(Total for Question 4 is 6 marks)
	TOTAL FOR PAPER IS 40 MARKS
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