KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 12
Applied Mathematics
M1 10 Forces and Motion Booklet

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

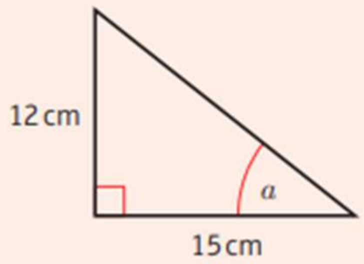
Prior knowledge check

1 Calculate:

a $(2\mathbf{i} + \mathbf{j}) + (3\mathbf{i} - 4\mathbf{j})$ **b** $(-\mathbf{i} + 3\mathbf{j}) - (3\mathbf{i} - \mathbf{j})$

← Section 8.4

2 The diagram shows a right-angled triangle.



Work out:

a the length of the hypotenuse

b the size of the angle a.

Give your answers correct to 1 d.p.

← GCSE Mathematics

3 A car starts from rest and accelerates constantly at 1.5 m s^{-2}.

a Work out the velocity of the car after 12 seconds.

After 12 seconds, the driver brakes, causing the car to decelerate at a constant rate of 1 m s^{-2}.

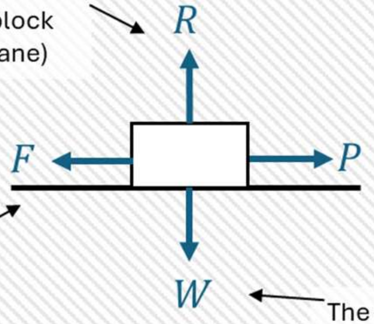
b Calculate the distance the car travels from the instant the driver brakes until the car comes to rest.

← Chapter 9

10.1) Force diagrams

The **reaction force** of the plane on the block (i.e. resisting the block sinking into the plane)

We consider the forces acting on each object one at a time.



Resistances to motion, in this case the **friction** between the block and the plane.

Force pulling the block. When a string/cable is involved, this is tension T.

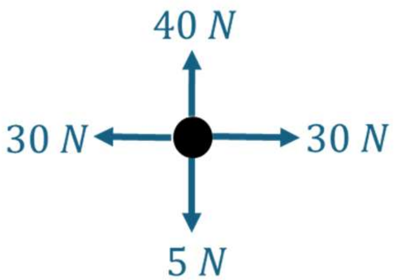
The weight of the block.

Recall that we often model an object as a **particle**, i.e. a point with negligible dimensions.

Newton's 1st Law of Motion states than an object at rest will stay at rest and that an object moving with constant velocity will remain at that velocity unless an unbalanced force acts on the object.

In other words, if the object is not accelerating, the **forces are balanced in every direction**, e.g. forces up = forces down and forces left = forces right.

The '**resultant force**' is the overall force acting on the object. An object will accelerate in the direction of the resultant force.



$R(\uparrow)$: $40 - 5 = 35\ N$

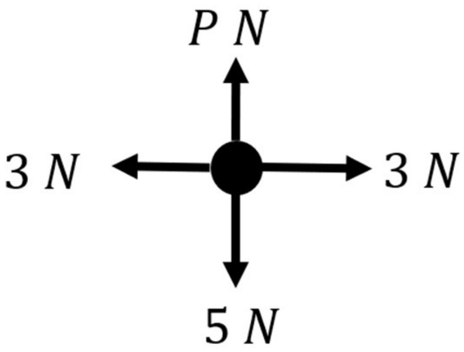
$R(\rightarrow)$: $30 - 30 = 0\ N$

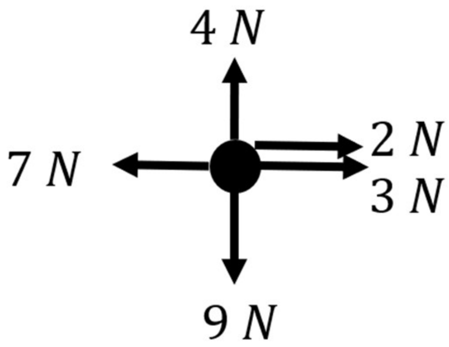
Therefore a 'resultant' force of 35 N upwards and the object will accelerate upwards.

Notes

Worked Example

Draw a force diagram to represent the resultant force:

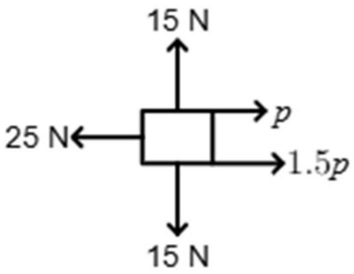




Worked Example

553a: Find the magnitude of unknown forces when a particle is in equilibrium.

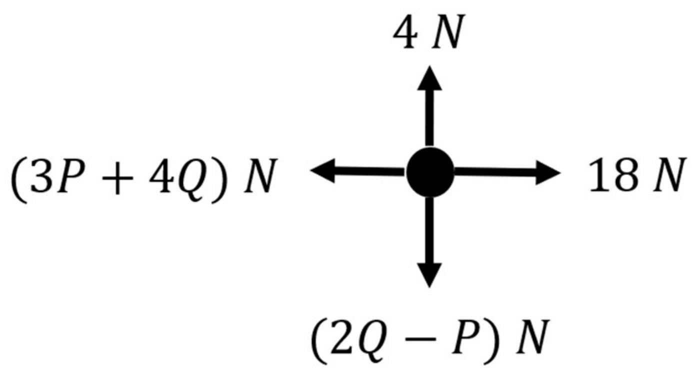
The particle is in equilibrium.



Find the magnitude of p.

Worked Example

Given that the particle is moving with constant velocity, find the values of P and Q



10.2) Forces as vectors

Notes

Worked Example

The forces $3\boldsymbol{i} - 2\boldsymbol{j}$, $-4\boldsymbol{i} + \boldsymbol{j}$, $-\mathbf{2}\boldsymbol{i} - 3\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b.

Worked Example

The vector i is due east and j due north.
A particle begins at rest at the origin.
It is acted on by three forces $(3\boldsymbol{i} - \boldsymbol{j})$ N, $(2\boldsymbol{i} + 3\boldsymbol{j})$ N and $(-4\boldsymbol{i} + \boldsymbol{j})$ N.

(a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$.

(b) Work out the magnitude and bearing of the resultant force.

Worked Example

Three forces F_1, F_2 and F_3 acting on a particle P are:

$$F_1 = (9\boldsymbol{i} - 7\boldsymbol{j})\ N$$
$$F_2 = (6\boldsymbol{i} + 5\boldsymbol{j})\ N$$
$$F_3 = (p\boldsymbol{i} + q\boldsymbol{j})\ N$$

where p and q are constants.
Given that P is in equilibrium,
a) Find the value of p and the value of q
The force F_3 is now removed. The resultant of F_1 and F_2 is R. Find:
b) The magnitude of R
c) The angle, to the nearest degree, that the direction of R makes with $\boldsymbol{j}$.

Worked Example

554f: Determine unknown coefficients when a resultant vector is parallel to a given vector.

A particle is acted upon by two forces $\mathbf{F_1}$ and $\mathbf{F_2}$, given by

$$\mathbf{F}_1 = 9\mathbf{i} + 2\mathbf{j} \text{ N}$$
$$\mathbf{F}_2 = 10p\mathbf{i} + 8p\mathbf{j} \text{ N}$$

The resultant of $\mathbf{F_1}$ and $\mathbf{F_2}$ is $\mathbf{R}$.

Given that $\mathbf{R}$ is parallel to $\mathbf{i}$, find the value of p.

Worked Example

Two forces F_1 and F_2 acting on a particle P are:

$$F_1 = (3\boldsymbol{i} - 2\boldsymbol{j})\ N$$
$$F_2 = (p\boldsymbol{i} + 3p\boldsymbol{j})\ N$$

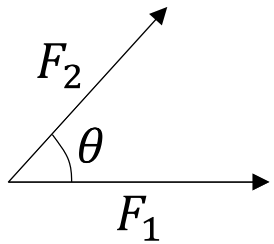
where p is a positive constant.
The resultant of F_1 and F_2 is R.
Given that R is parallel to $13\boldsymbol{i} + 10\boldsymbol{j}$, find the value of p

Exercise 10B

Pearson Stats/Mechanics Year 1
Pages 161-162

Extension

A force F_1 acts in the direction of i and a force F_2 acts at an angle of θ to i, as shown. Show that the resultant force has magnitude $\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$



Force vector: $\begin{pmatrix} \boldsymbol{F_1 + F_2 \cos\theta} \\ \boldsymbol{F_2 \sin\theta} \end{pmatrix}$

Magnitude:

$$\sqrt{(\boldsymbol{F_1 + F_2 cos\theta})^2 + \boldsymbol{F_2^2 \sin^2\theta}}$$

$$= \sqrt{\boldsymbol{F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta}}$$

$$= \sqrt{\boldsymbol{F_1^2 + 2F_1F_2\, cos\,\theta + F_2^2(\cos^2\theta + sin^2\,\theta)}}$$

$$= \sqrt{\boldsymbol{F_1^2 + F_2^2 + 2F_1F_2\, cos\,\theta}}$$

10.3) Forces and acceleration

Newton's 2nd Law of Motion: $F = ma$
(where the force F and acceleration a are in the same direction)

Notes

Worked Example

A car of 1000kg has a driving force of 1600N and forces of 400N resisting its motion.
Determine its acceleration.

Worked Example

555a: Calculate the acceleration of a particle.

The particle is accelerating upwards.

Find the acceleration of the particle.

Worked Example

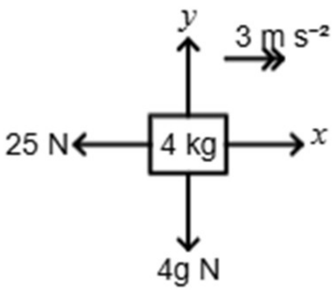
An object of mass 140kg experiences air resistance of 600 N. Determine the object's acceleration as it falls towards the ground.

Worked Example

An adult has a mass of 100kg. What is the gravitational force (weight) acting on the adult?

555b: Determine the magnitude of missing forces when a particle is accelerating.

The particle is accelerating as shown.



Find the magnitude of x and y.

Worked Example

A body of mass 10kg is pulled along a rough horizontal table by a horizontal force of magnitude 40N against a constant friction force of magnitude 8N. Given that the body is initially at rest, find:

(a) the acceleration of the body

(b) the distance travelled by the body in the first 2 seconds

(c) the magnitude of the normal reaction between the body and the table

Worked Example

An objects of mass $8\ kg$ hits soft ground at a speed of $14\ ms^{-1}$ and sinks vertically downwards before coming to rest. The ground is assumed to exert a constant resistive force of magnitude $5000\ N$.
Find the vertical distance that the object sinks into the ground before coming to rest.

Worked Example

A lift of mass $500\ kg$ is lowered or raised by a metal cable attached to its top. The lift contains passengers whose total mass is $100\ kg$. The lift starts from rest and accelerates at a constant rate, reaching a speed of $5\ ms^{-1}$ after moving a distance of $4\ m$. Find:

a) The acceleration of the lift

b) The tension in the cable if the lift is moving vertically downwards

c) The tension in the cable if the lift is moving vertically upwards

10.4) Motion in 2 dimensions

Can be a vector: Force, acceleration, velocity, displacement
Scalar only: Mass, area, volume

Notes

Worked Example

A boat is modelled as a particle of mass 30 kg being acted on by three forces.

$$F_1 = \begin{pmatrix} 25 \\ 40 \end{pmatrix} N,$$

$$F_2 = \begin{pmatrix} 5q \\ 10q \end{pmatrix} N,$$

$$F_3 = \begin{pmatrix} 50 \\ -37.5 \end{pmatrix} N$$

Given that the boat is accelerating at a rate of $\begin{pmatrix} -0.75 \\ 0.4 \end{pmatrix}$ ms^{-2}, find the values of p and q.

Worked Example

A particle of mass 5 kg start from rest and is acted upon by a force R of $(4\boldsymbol{i} + k\boldsymbol{j})\ N$. R acts on a bearing of $45°$. Find the value of k

Worked Example

Two forces, $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ N and $\begin{pmatrix} p \\ q \end{pmatrix}$ N act on a particle of mass m kg. The resultant of the two forces is R.

a) Given that R acts in a direction which is parallel to the vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$, show that $2p + q + 12 = 0$

b) Given also that $p = 1$ and that P moves with an acceleration of magnitude $10\sqrt{5}\ ms^{-2}$, find the value of m

10.5) Connected particles

When you have a system of connected particles we model as follows:

- Typically assume that the string/rope/bar connecting particles is light and inelastic (i.e. has zero mass and doesn't compress, therefore there is a uniform constant tension through connector)
- We apply **F = ma** to each particle separately forming two equations
- *There is also an option of applying F=ma to the system as a whole*

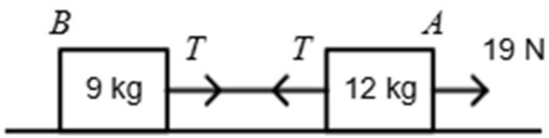
- For some problems we also require N III law:

Newton's 3rd Law: For every action there is an equal and opposite reaction.

Notes

556a: Determine the tension in the string connecting two particles on smooth horizontal ground.

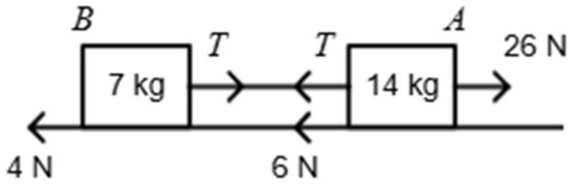
The particles A and B are accelerating as shown.



Find the tension in the string.

556b: Determine the tension in the string connecting two particles on rough horizontal ground.

The particles A and B are accelerating as shown.

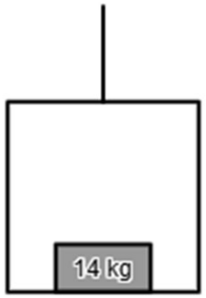


Find the tension in the string.

556c: Determine the tension in the string attached to a lift.

A vertical rope has its end attached to the top of a small lift cage.

The lift cage has mass 95 kg and carries a block of mass 14 kg, as shown.



The lift cage is raised vertically by moving the rope vertically upwards with constant acceleration 2.3 m s^{-2}

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model, find the tension in the rope.

Worked Example

Two particles, P and Q, of masses 6kg and 4kg respectively, are connected by a light inextensible string. Particle Q is pulled by a horizontal force of magnitude 20N along a rough horizontal plane. Particle Q experiences a frictional force of 5N and particle P experiences a frictional force of 3N.

(a) Find the acceleration of the particles.

(b) Find the tension in the string.

(c) Explain how the modelling assumptions that the string is light and inextensible have been used.

Worked Example

A light scale-pan is attached to a vertical light inextensible string.
The scale-pan carries two masses A and B.
The mass of A is 300g and the mass of B is 200g.
A rests on top of B.
The scale-pan is raised vertically, using the string, with acceleration 0.25 ms^{-2}.

(a) Find the tension in the string.
(b) Find the force exerted on mass B by mass A.
(c) Find the force exerted on mass B by the scale-pan.

Worked Example

A person travels in a lift. The mass of the person is $40\ kg$ and the mass of the lift is $860\ kg$.
The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration $4\ ms^{-2}$. By modelling the cable as being light and inextensible, find:

a) The tension in the cable
b) The magnitude of the force exerted on the woman by the floor of the lift

Worked Example

A car of mass $1200\ kg$ pulls a trailer of mass $400\ kg$ along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes $400\ N$ and $200\ N$ respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude $2000\ N$.

a) Find the acceleration of the car and trailer

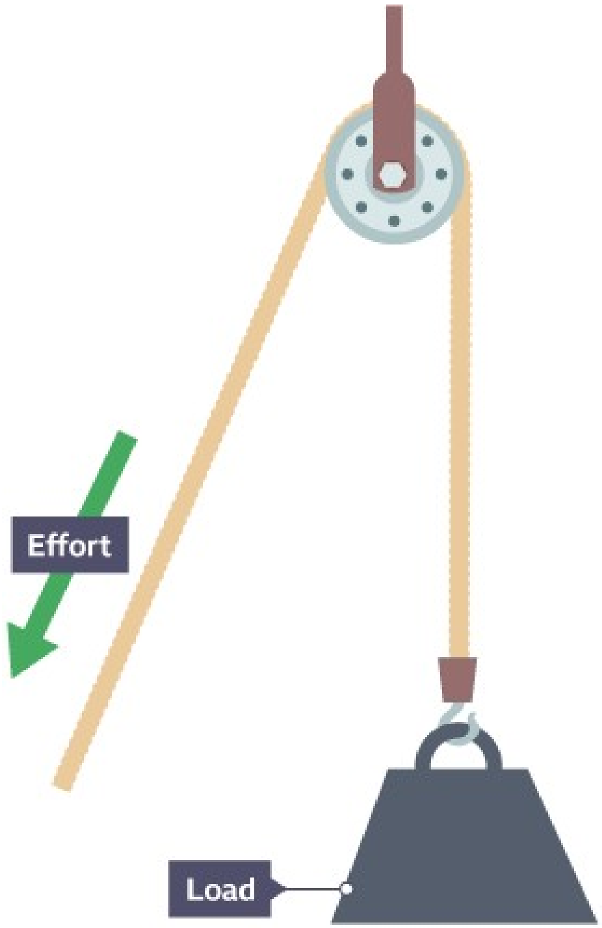
b) Find the magnitude of the tension in the tow-bar

The engine cuts out, reducing the force produced by the engine to zero and the brakes are applied. The brakes produce a force on the car of magnitude F Newtons and the car and trailer decelerate.

Given that the resistances to motion are unchanged, and the magnitude of the thrust in the towbar is $300\ N$, find the value of F

10.6) Pulleys

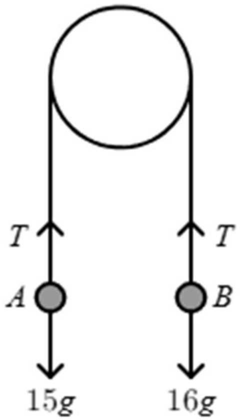
- A pulley is a wheel on which a rope/string/cable passes.
- We usually assume the pully is smooth (meaning no friction acting on the rope)



Notes

556e: Determine the acceleration of two particles connected by a string passing over a smooth pulley.

Two particles A and B of masses 15 kg and 16 kg respectively are connected by a light inextensible string which passes over a light smooth pulley.

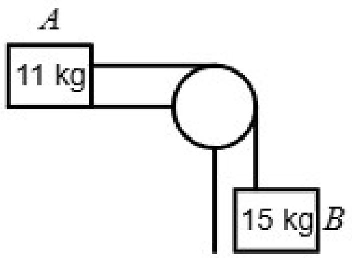


Find the acceleration of the system and the tension in the string.

Worked Example

556g: Determine the acceleration of two particles connected by a string passing over a smooth pulley on the corner of a table.

Two particles A and B of masses 11 kg and 15 kg respectively are connected by a light inextensible string which passes over a light smooth pulley.

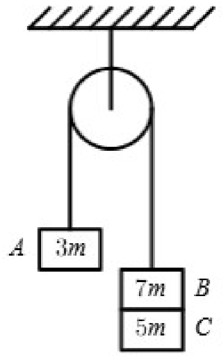


Find the acceleration of the system and the tension in the string.

Worked Example

556h: Determine the tension in the string passing over a smooth pulley connecting particles whose mass are variables.

Three particles, A, B and C, have masses $3m$, $7m$ and $5m$. Particles A and B are connected by a light inextensible string that passes over a smooth light fixed pulley. Particle C is attached to particle B. The system is held at rest with the string taut and the hanging parts of the string vertical, as shown. The system is released from rest.

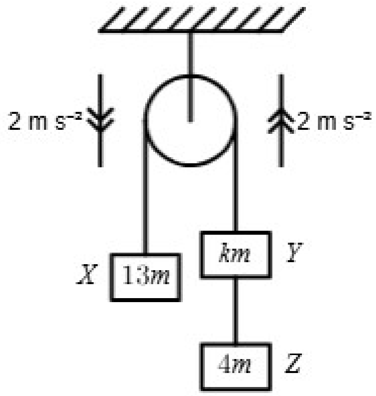


Find the tension in the string.
Give your answer in terms of m and g.

Worked Example

556i: Determine the tension, acceleration and mass when three particles are joined by strings with one string passing over a smooth pulley and the masses are variable.

Three particles, X, Y and Z, of masses $13m$, km and $4m$ respectively are connected by light inextensible strings as shown in the diagram. The string connecting X and Y passes over a smooth pulley. The system is accelerating at 2 m s^{-2} such that X is moving downwards.



Find the tensions in the two strings, giving your answers as exact values in terms of m and g and the value of k correct to 2 significant figures.

Let T_1 be the tension in the string going over the pulley from X to Y, and T_2 be the tension in the string from Y to Z.

Worked Example

Particles P and Q, of masses $5m$ and $4m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

(a) Write down an equation of motion for P and for Q.

(b) Find the acceleration of each mass.

(c) Find the tension in the string.

(d) Find the force exerted on the pulley by the string.

(e) Find the distance moved by P in the first 2 s, assuming that Q does not reach the pulley.

Worked Example

Two particles A and B of masses 0.8kg and 1.6kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 9m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 1m above horizontal ground. A frictional force of magnitude 0.16g opposes the motion of particle A. The system is released from rest. Find:

(a) The acceleration of the system

(b) The time taken for B to reach the ground

(c) The total distance travelled by A before it first comes to rest.

Worked Example

Two particles A and B have masses $10m$ and km respectively, where $k < 10$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane.
The system is released from rest.
After release, A descends with acceleration $\frac{1}{2}g$.
After descending for 2.4 s, the particle A reaches the plane.
It is immediately brought to rest by the impact with the plane.
The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.
Find the greatest height reached by B above the plane.

4.

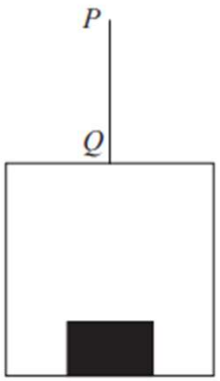


Figure 1

A vertical rope PQ has its end Q attached to the top of a small lift cage.

The lift cage has mass 40 kg and carries a block of mass 10 kg, as shown in Figure 1.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration $0.2\,\text{m s}^{-2}$

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

(a) find the tension in the rope PQ

(3

(b) find the magnitude of the force exerted on the block by the lift cage.

(3

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

4(a)	Translate situation into the model and set up the **equation** of motion for the cage and the block to obtain an equation in T only.	M1	3.3
	$T - 40g - 10g = 50\times0.2$	A1	1.1b
	500 (N) Must be positive	A1	1.1b
	Some examples: $T - 50 = 50\times0.2$ and $T - 40g - 10g = 50g\times0.2$ both score M1A0A0		
		(3)	
(b)	Use the model to set up the equation of motion for the block to obtain an equation in R only.	M1	3.4
	$R - 10g = 10\times0.2$ Allow $-R$ instead of R	A1	1.1b
	100 (N) Must be positive.	A1	1.1b
	OR: Use the model to set up the equation of motion for the cage to obtain an equation in R only.	M1	3.4
	$T - 40g - R = 40\times0.2$ with their T substituted	A1	1.1b
	100 (N) Must be positive	A1	1.1b
		(3)	

(6 marks)

Summary of Key Points

Summary of key points

1 **Newton's first law** of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object.

2 A **resultant** force acting on an object will cause the object to **accelerate in the same direction** as the resultant force.

3 You can find the **resultant** of two or more forces given as vectors by adding the vectors.

4 **Newton's second law** of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced: $F = ma$.

5 $W = mg$

6 You can use $\mathbf{F} = m\mathbf{a}$ to solve problems involving vector forces acting on particles.

7 You can solve problems involving connected particles by considering the particles separately or, if they are moving in the same straight line, as a single particle.

8 **Newton's third law** states that for every action there is an equal and opposite reaction.