KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Mechanics 1
Chapter 9 – Constant Acceleration

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

8.3 Quantities and Units

Notes

SI Units

The SI units are a standard system of units, used internationally ("Système International d'unités"). These are the ones you will use:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/Displacement	metre	m
Time	Seconds	s
Speed/Velocity	metres per second	$m\ s^{-1}$
Acceleration	metres per second per second	$m\ s^{-2}$
Force/Weight	newton	N (= $kg\ m\ s^{-2}$)

This unit is consistent with force being mass × acceleration

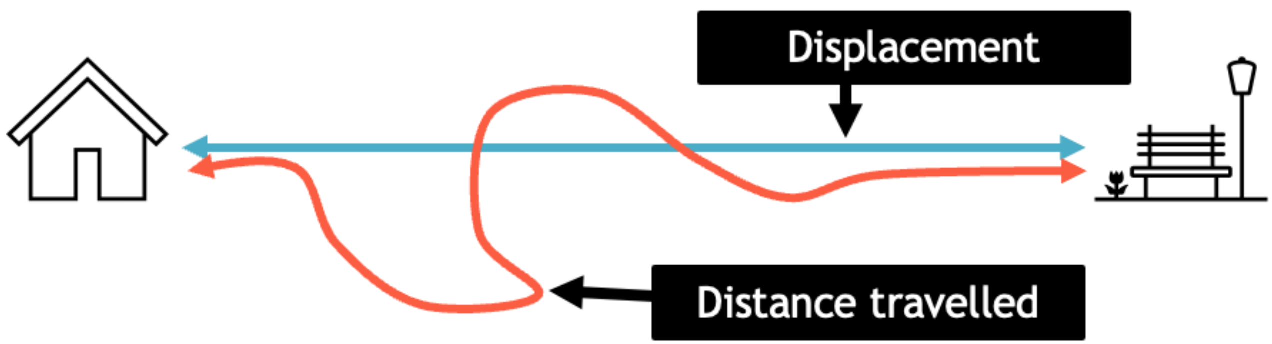
9.1 Displacement-Time Graphs

Notes

Displacement

We often think of journeys as a very simplified model, in a straight line.

In reality, a walk to the park may involve twists and turns, and sometimes even going in the direction back towards home.



The total length that you might walk to the park is the **distance travelled**.

This is different to the **distance from home**, which is known as **displacement**.

Displacement

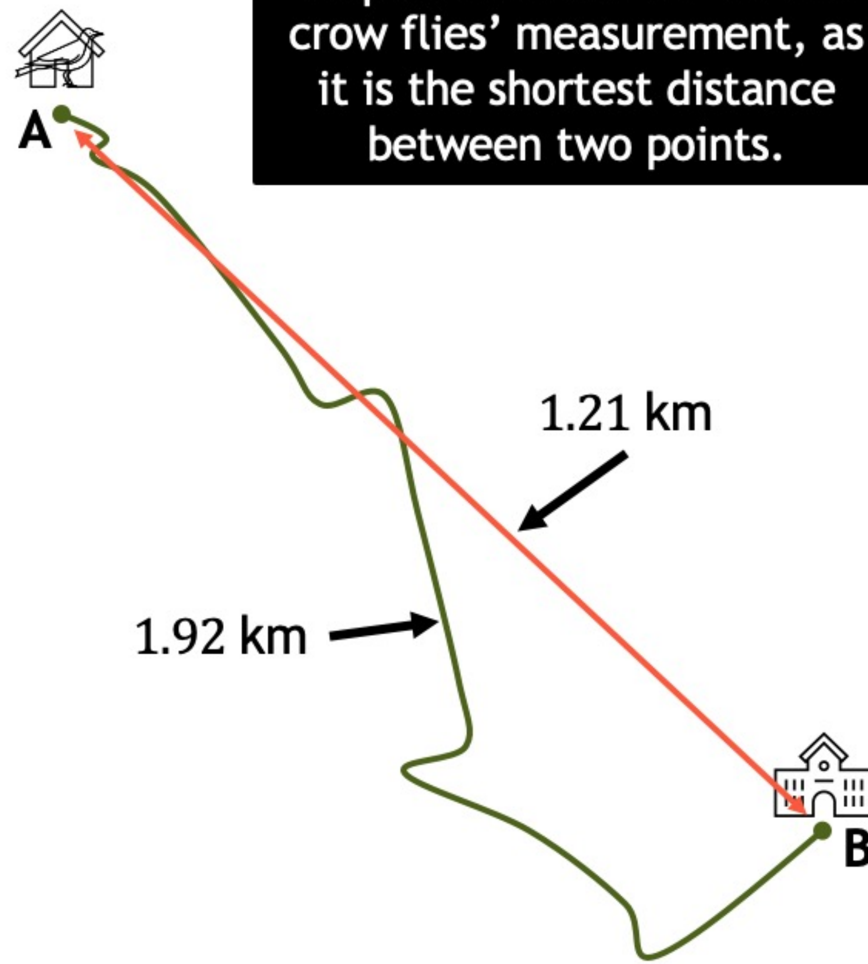
Dr Frost records his walk to work one day on his smart watch, which tells him that his journey is 1.92 km.

This is his **distance travelled.**

When he gets home, he checks the distance using a map.
The straight line distance is 1.21 km.

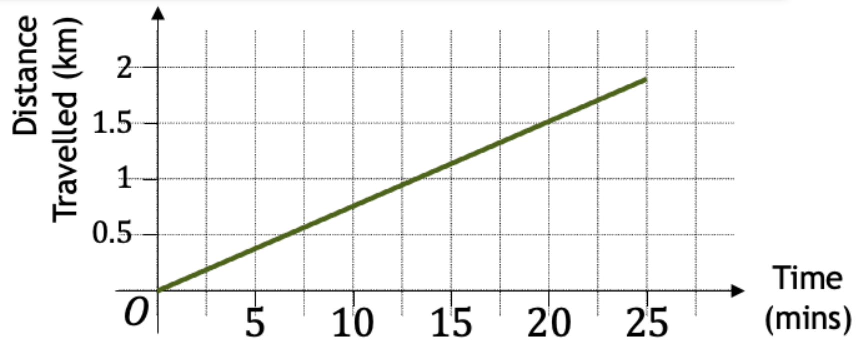
This is his **displacement** between his home (A) and work (B).

Some people might refer to displacement as an 'as the crow flies' measurement, as it is the shortest distance between two points.

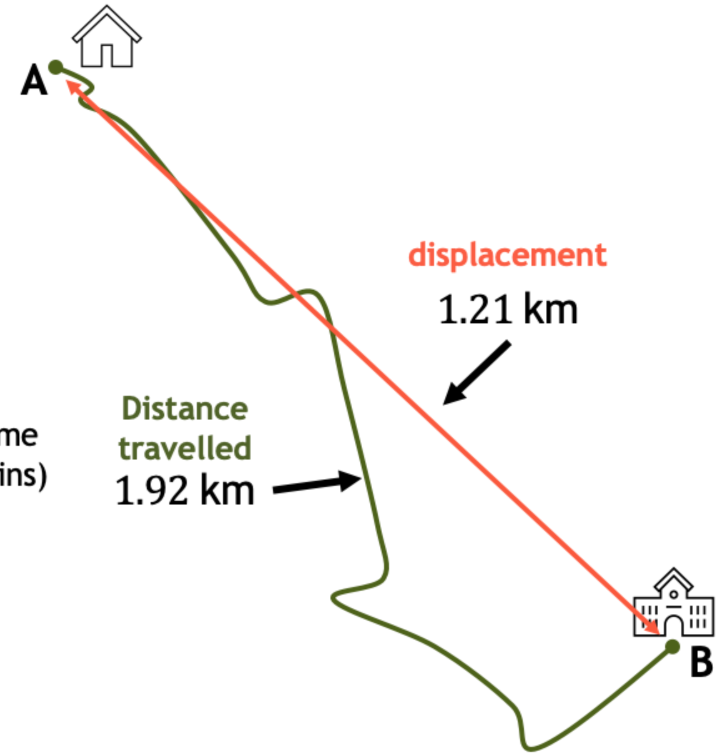


Displacement

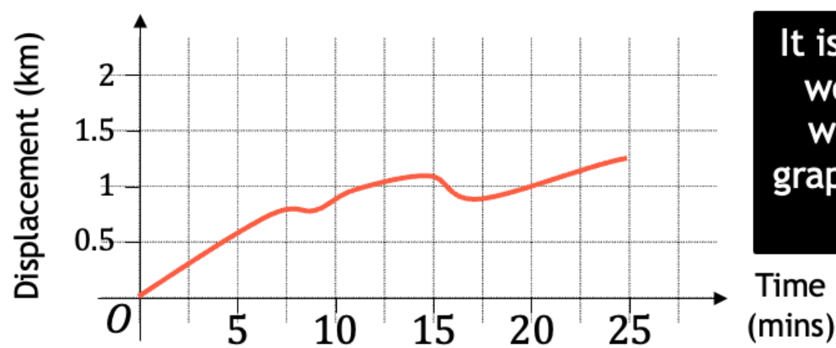
Assuming Dr. Frost walks at a constant speed to from his house to work, what might his distance-time graph look like?



It would be a graph with **constant speed** so a **constant** gradient.



How might the **displacement-time** graph of this journey compare?

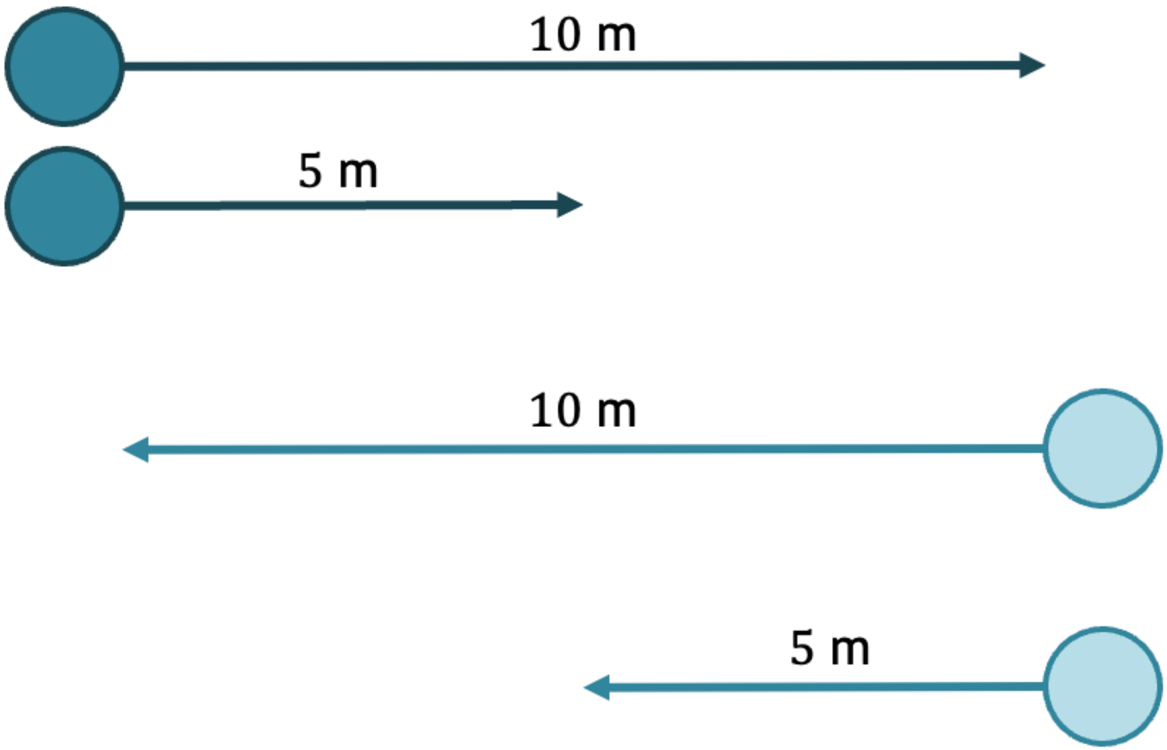


It is difficult to know exactly what this graph would look like as the journey has a lot of winding parts, but the displacement-time graph would reflect the winding nature of the journey.

Displacement – A Note on Signs

When we talk about displacement, the positive and negative direction are often defined by the context.

If we take right as the positive direction:



We say this has been displaced 10 m.

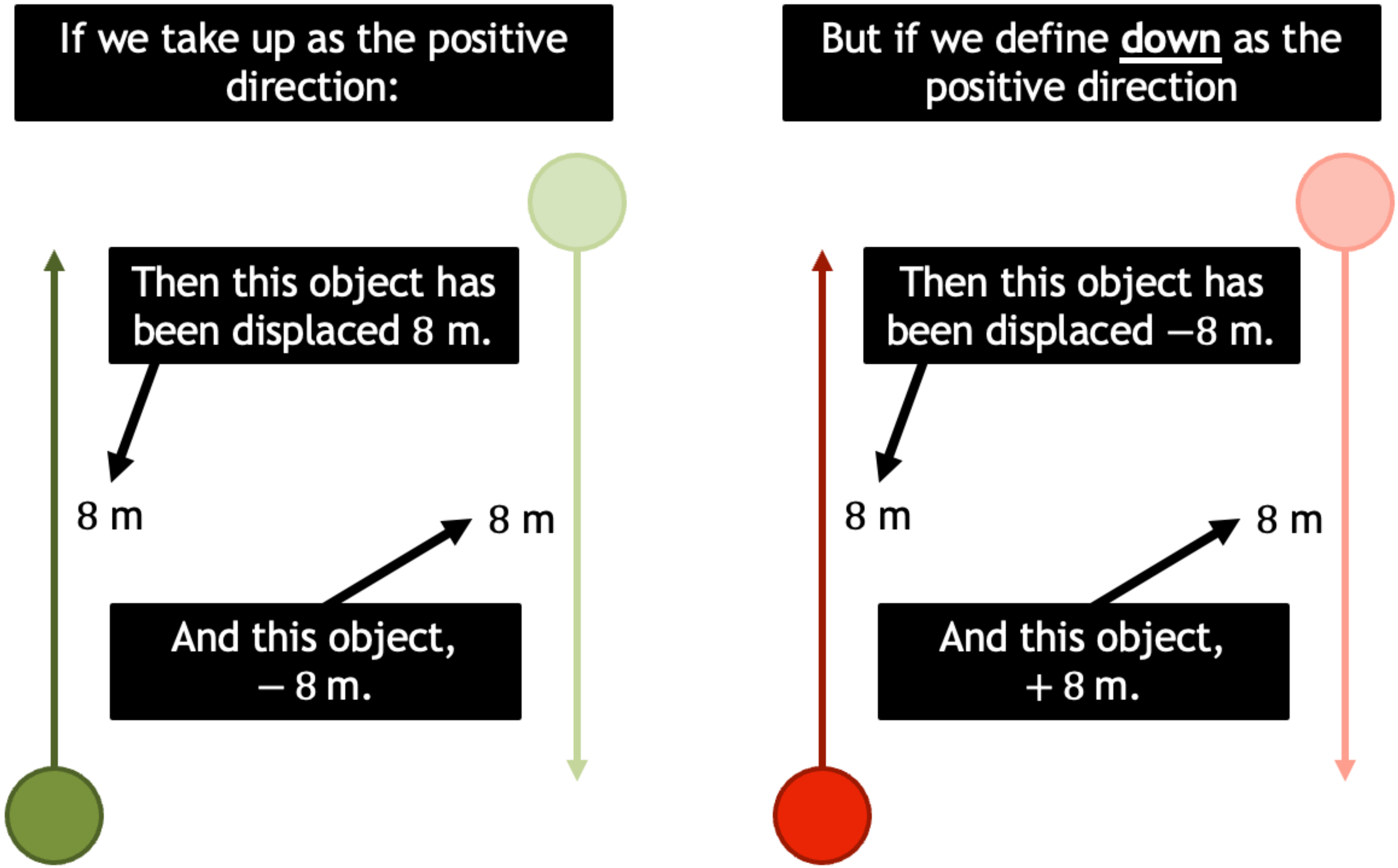
And this would have been displaced 5 m.

However, this shape has been displaced – 10 m as it travels to the left.

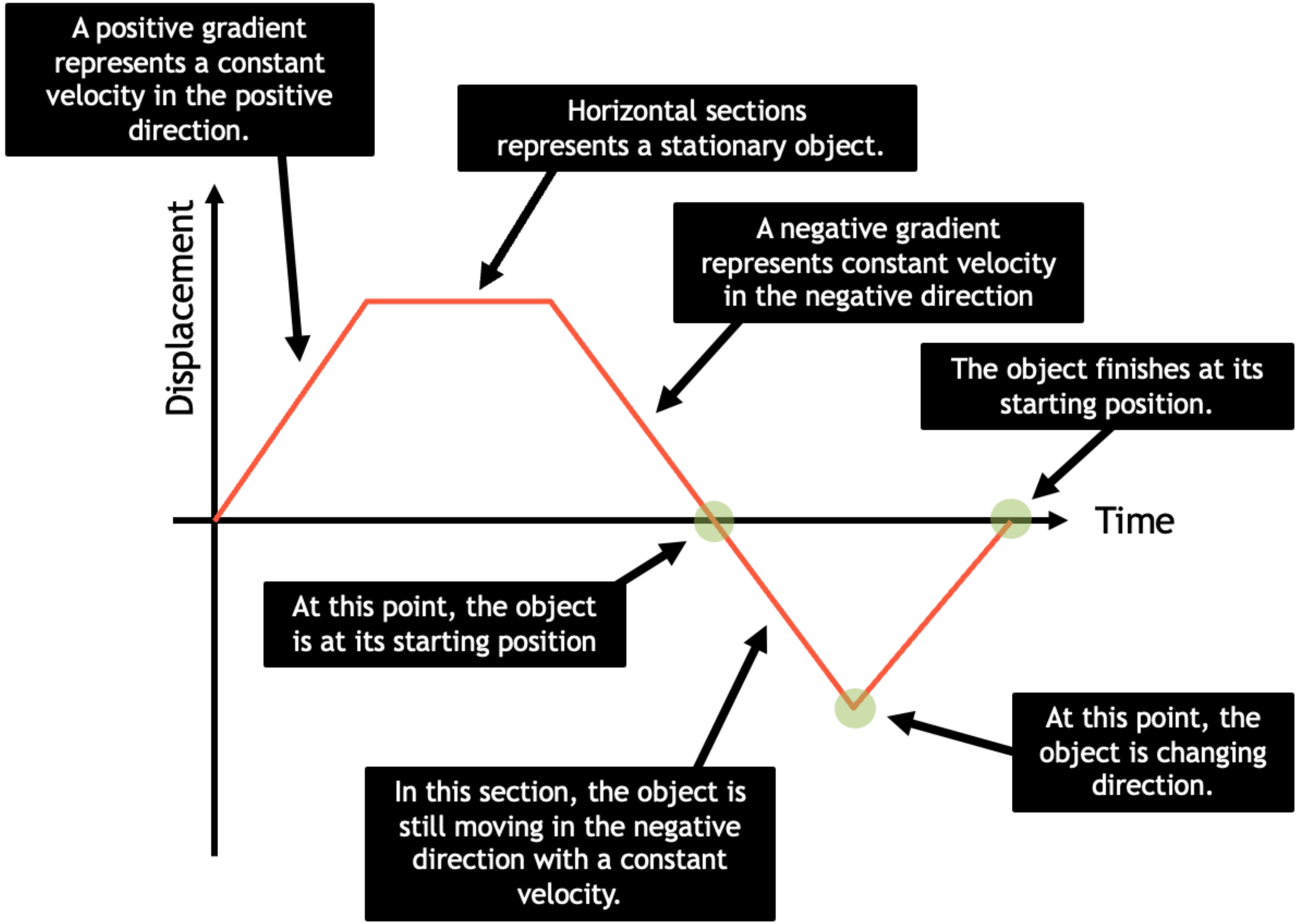
And this shape, – 5 m.

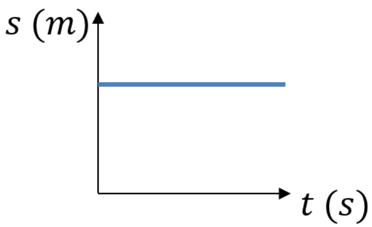
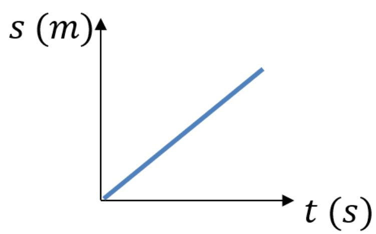
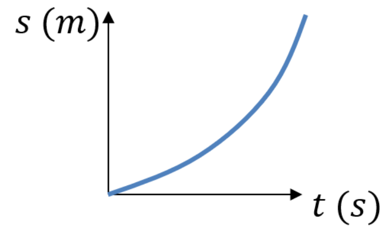
Displacement – A Note on Signs

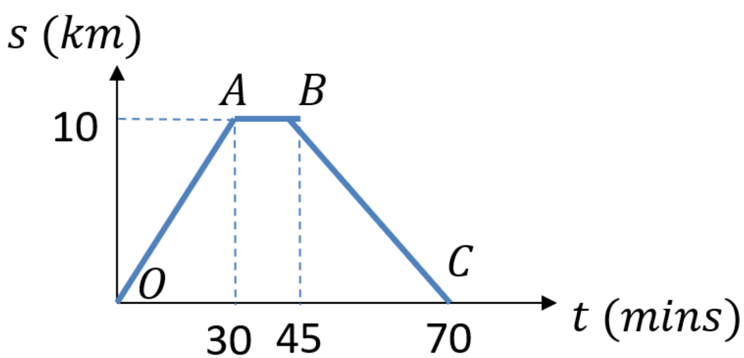
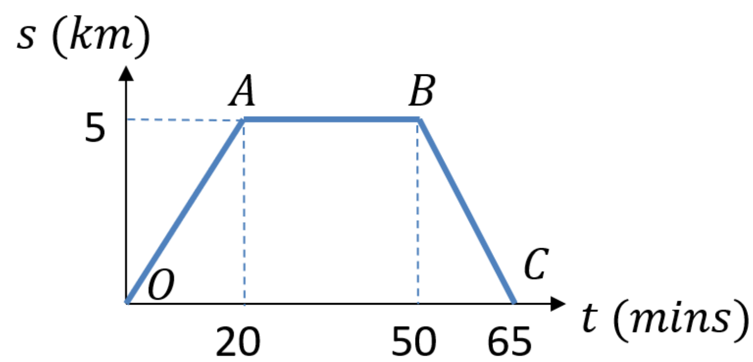
When we talk about displacement, the positive and negative direction are often defined by the context.



Key Features of Displacement-Time Graphs



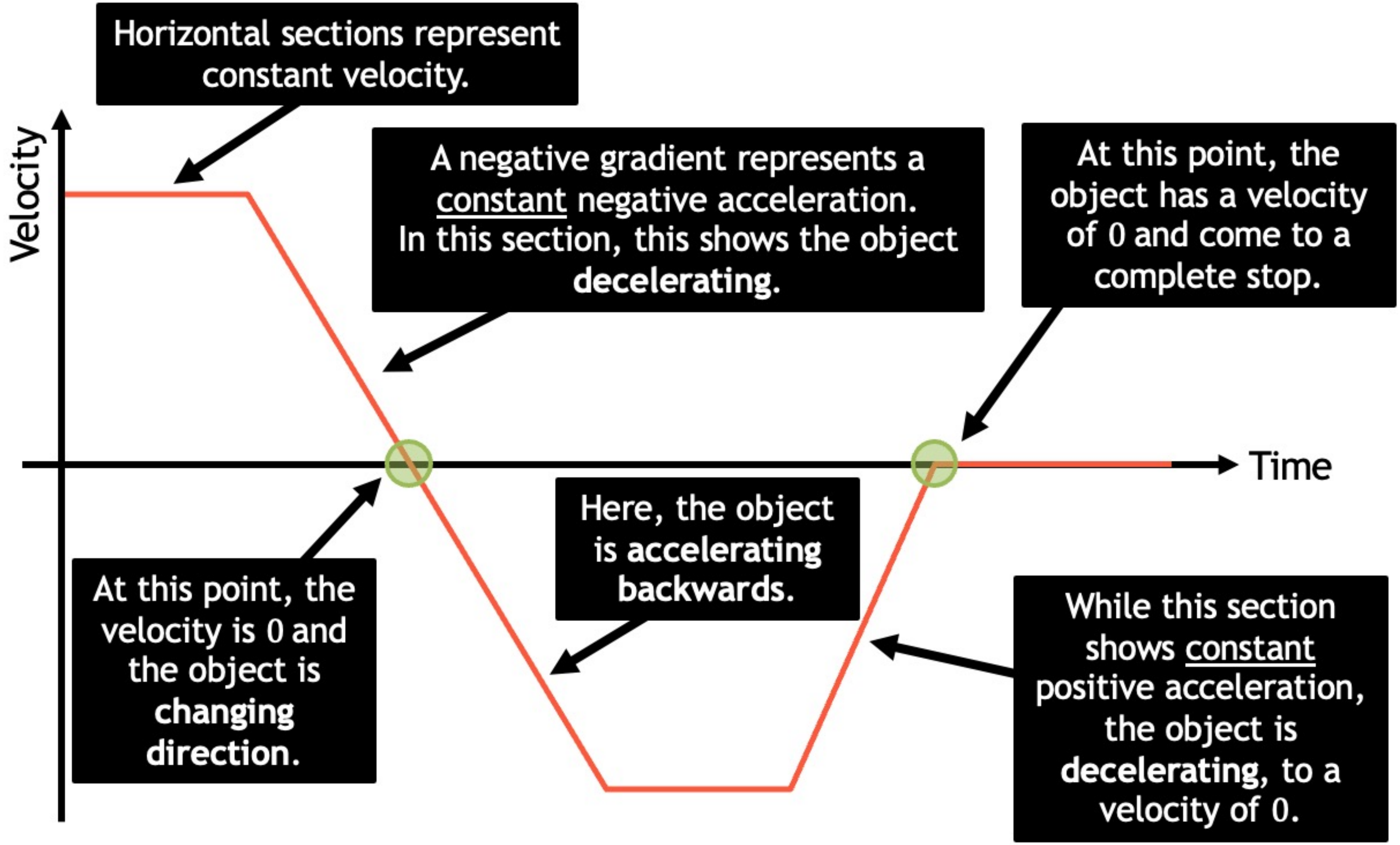
Worked Example	Your Turn
Describe the motion of each object from the displacement-time graph:   	Describe the motion of each object from the displacement-time graph: 

Worked Example	Your Turn
A cyclist rides in a straight line for 30 minutes. She waits for a quarter of an hour, then returns in a straight line to her starting point in 25 minutes. a) Work out the average velocity for each stage of the journey in km h^{-1} b) Write down the average velocity for the whole journey. c) Work out average speed for the whole journey. 	A cyclist rides in a straight line for 20 minutes. She waits for half an hour, then returns in a straight line to her starting point in 15 minutes. a) Work out the average velocity for each stage of the journey in km h^{-1} b) Write down the average velocity for the whole journey. c) Work out average speed for the whole journey. 

9.2 Velocity-Time Graphs

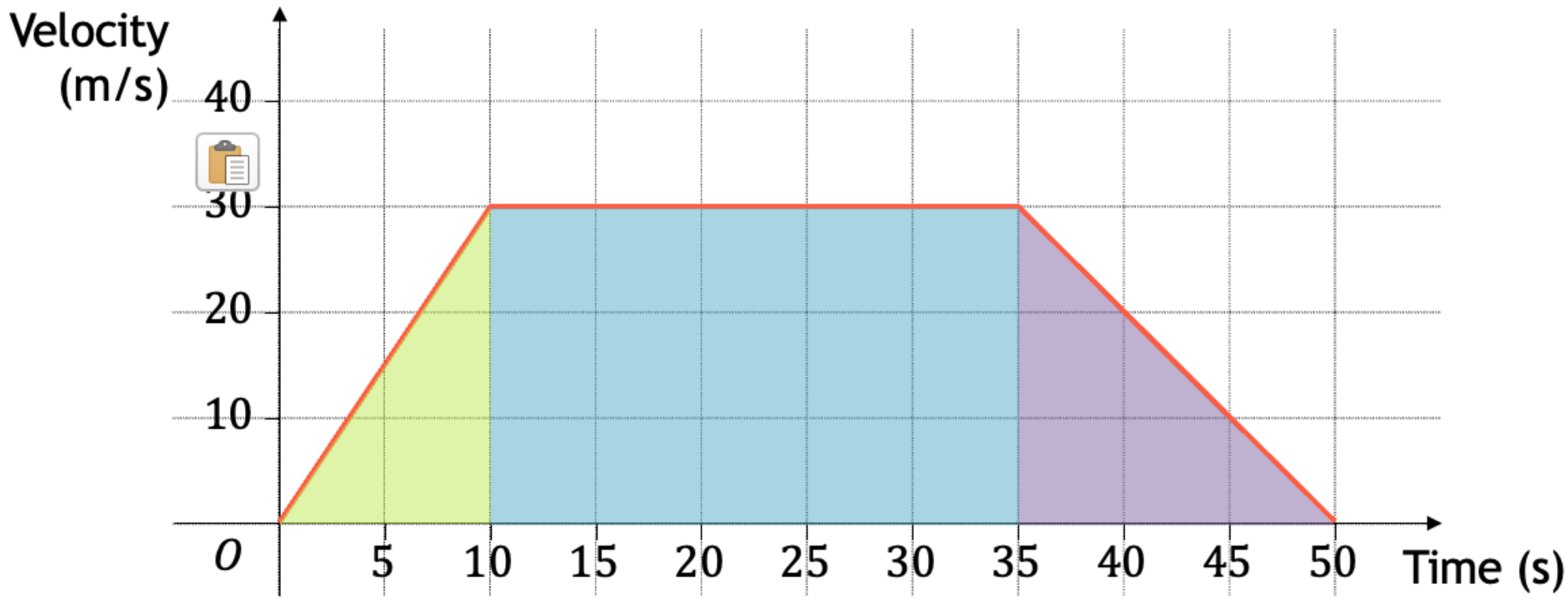
Notes

Key Features of Velocity-Time Graphs



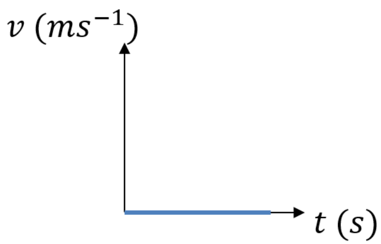
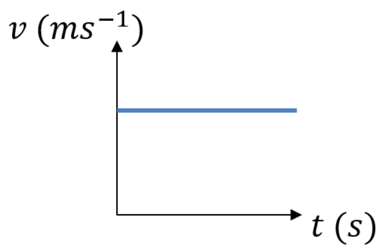
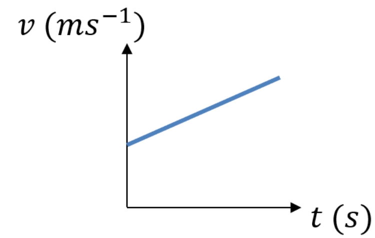
Distanced Travelled on a Velocity-Time Graph

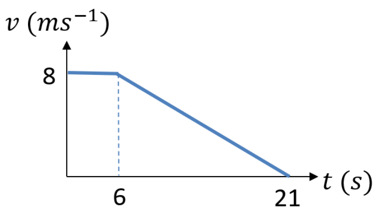
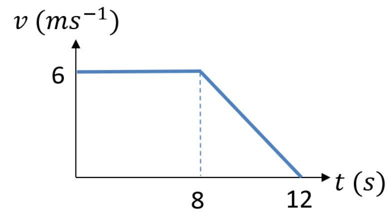
You will have previously seen that the area under a speed-time graph represents the distance travelled.



It is therefore true that the area under a velocity-time graph represents the displacement.

The area under a velocity-time graph represents displacement.

Worked Example	Your Turn
Describe the motion of each object from the velocity-time graph:	Describe the motion of each object from the velocity-time graph:
  	

Worked Example	Your Turn
A cyclist is moving along a straight road for a period of 21 seconds. For the first 6 seconds, she moves at a constant speed of 8 ms^{-1}. She then decelerates at a constant rate, stopping after a further 15 seconds. a) Find the displacement from the starting point of the cyclist after this 21 second period. b) Work out the rate at which the cyclist decelerates. 	A cyclist is moving along a straight road for a period of 12 seconds. For the first 8 seconds, she moves at a constant speed of 6 m s^{-1}. She then decelerates at a constant rate, stopping after a further 4 seconds. a) Find the displacement from the starting point of the cyclist after this 12 second period. b) Work out the rate at which the cyclist decelerates. 

Worked Example	Your Turn
A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of $16\ ms^{-1}$ in T seconds. The particle then travels at a constant velocity of $16\ ms^{-1}$ for $3T$ seconds. The particle then decelerates uniformly to rest in a further 4 s. a) Sketch a velocity-time graph to illustrate the motion of the particle. Given then the total displacement of the particle is 592m. b) Find the value of T.	A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 8 ms^{-1} in T seconds. The particle then travels at a constant velocity of 8 ms^{-1} for $5T$ seconds. The particle then decelerates uniformly to rest in a further 40 s. a) Sketch a velocity-time graph to illustrate the motion of the particle. Give then the total displacement of the particle is 600m. b) Find the value of T.

Worked Example	Your Turn
A car is travelling along a straight horizontal road. The car takes $60\ s$ to travel between two sets of traffic lights which are $1072.5\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $15\ s$ until its speed is $11\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights. a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights b) Find the value of T	A car is travelling along a straight horizontal road. The car takes $120\ s$ to travel between two sets of traffic lights which are $2145\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $30\ s$ until its speed is $22\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights. a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights b) Find the value of T

Worked Example	Your Turn
A motorcycle leaves the first set of traffic lights 15 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration and passes the car at the point A which is $495\ m$ from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed $11\ ms^{-1}$ c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A	A motorcycle leaves the first set of traffic lights 10 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration and passes the car at the point A which is $990\ m$ from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed $22\ ms^{-1}$ c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A

Worked Example	Your Turn
A car is moving along a straight horizontal road. At time $t = 0$, the car is moving with speed $10\ ms^{-1}$ and is at the point A. The car maintains this speed for $50\ s$. The car then moves with constant deceleration $0.6\ ms^{-2}$, reducing its speed from $10\ ms^{-1}$ to $4\ ms^{-1}$. The car then moves with constant speed $4\ ms^{-1}$ for $30\ s$. The car then moves with constant acceleration until it is moving with speed $10\ ms^{-1}$ at the point B. Given that the distance from A to B is $980\ m$, find the time taken for the car to move from A to B	A car is moving along a straight horizontal road. At time $t = 0$, the car is moving with speed $20\ ms^{-1}$ and is at the point A. The car maintains this speed for $25\ s$. The car then moves with constant deceleration $0.4\ ms^{-2}$, reducing its speed from $20\ ms^{-1}$ to $8\ ms^{-1}$. The car then moves with constant speed $8\ ms^{-1}$ for $60\ s$. The car then moves with constant acceleration until it is moving with speed $20\ ms^{-1}$ at the point B. Given that the distance from A to B is $1960\ m$, find the time taken for the car to move from A to B

9.3 Constant Acceleration Formulae 1

Notes

The Constant Acceleration Formulae

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

These are the **constant acceleration formulae**. The are sometimes known as the **kinematics equations** or as the **'suvat' equations.** These can be found in your exam formula booklet.

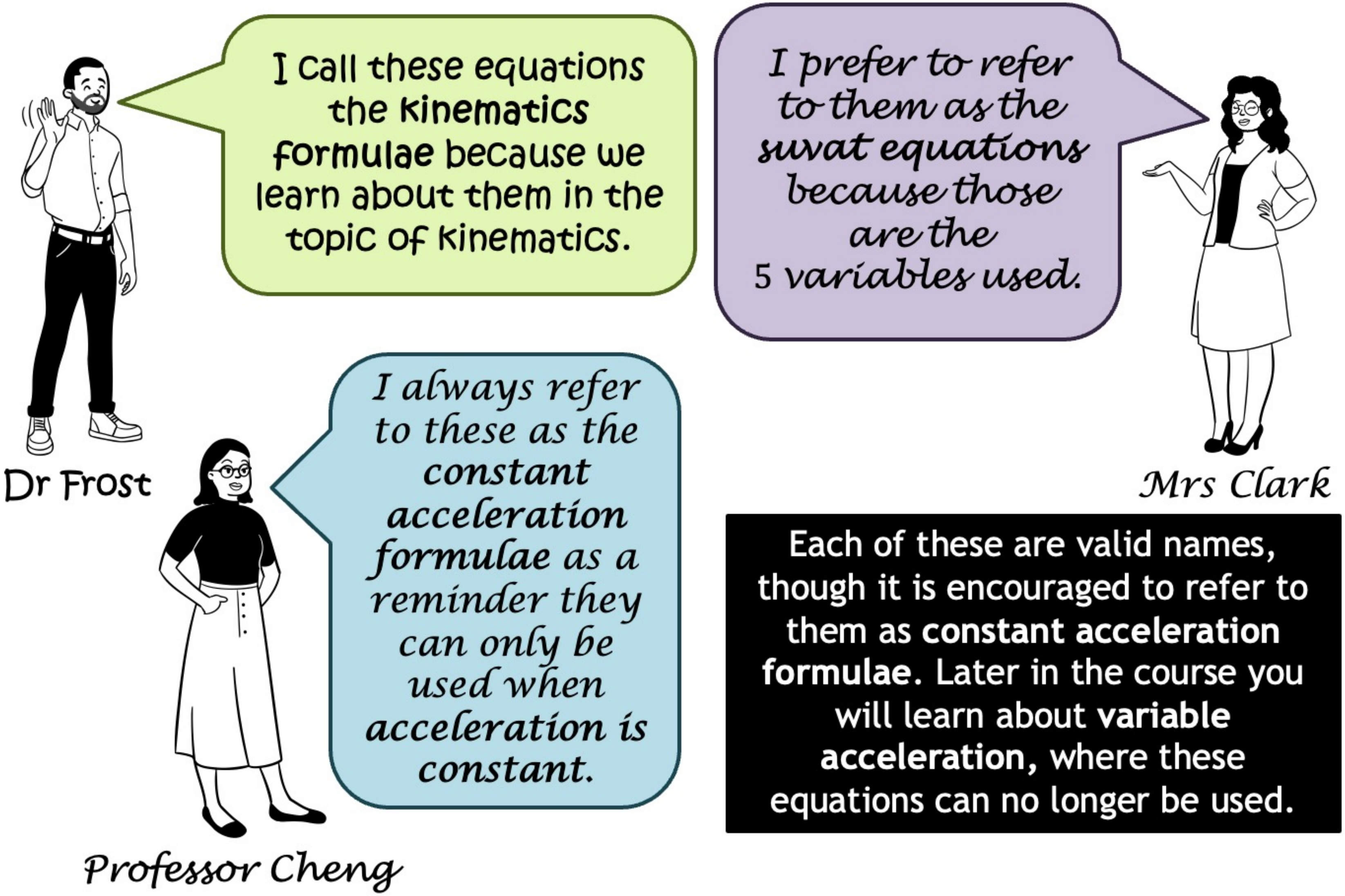
There are **5 variables** $s, u, v, a,$ and t and **5 equations.**

Can you notice anything missing from each of the equations?

Each equation only contains **4** of the variables, and each is missing a different variable.

If we know 3 variables, we can find the 4th by choosing the correct equation.

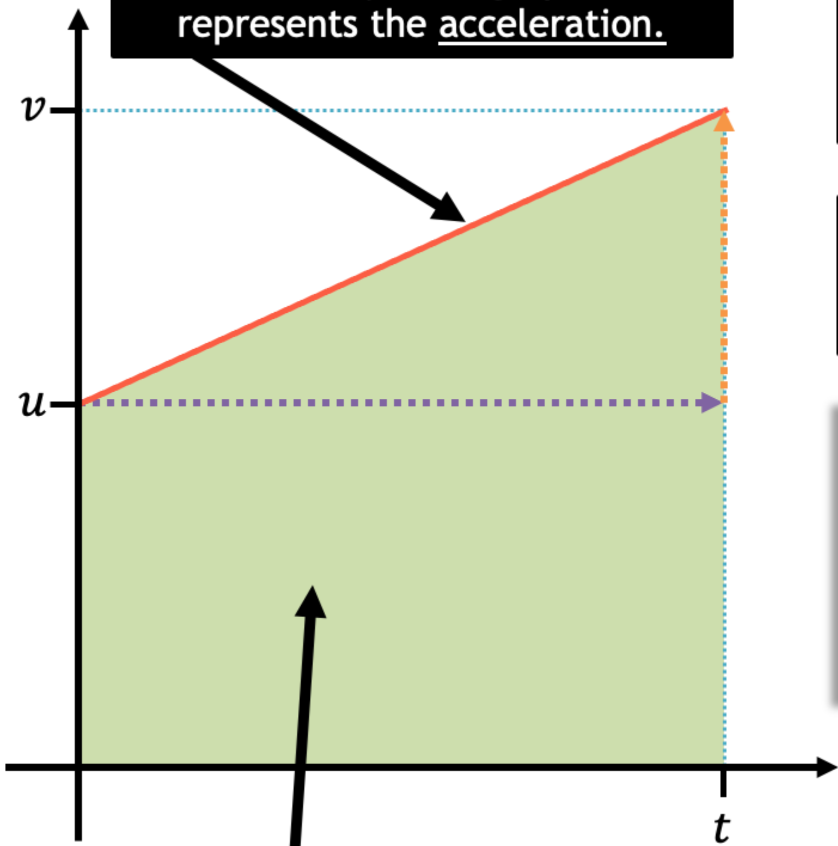
Referring to these Equations



Each of these are valid names, though it is encouraged to refer to them as **constant acceleration formulae**. Later in the course you will learn about **variable acceleration,** where these equations can no longer be used.

Deriving the Constant Acceleration Formulae

We could calculate the gradient of the velocity-time graph, which represents the acceleration.



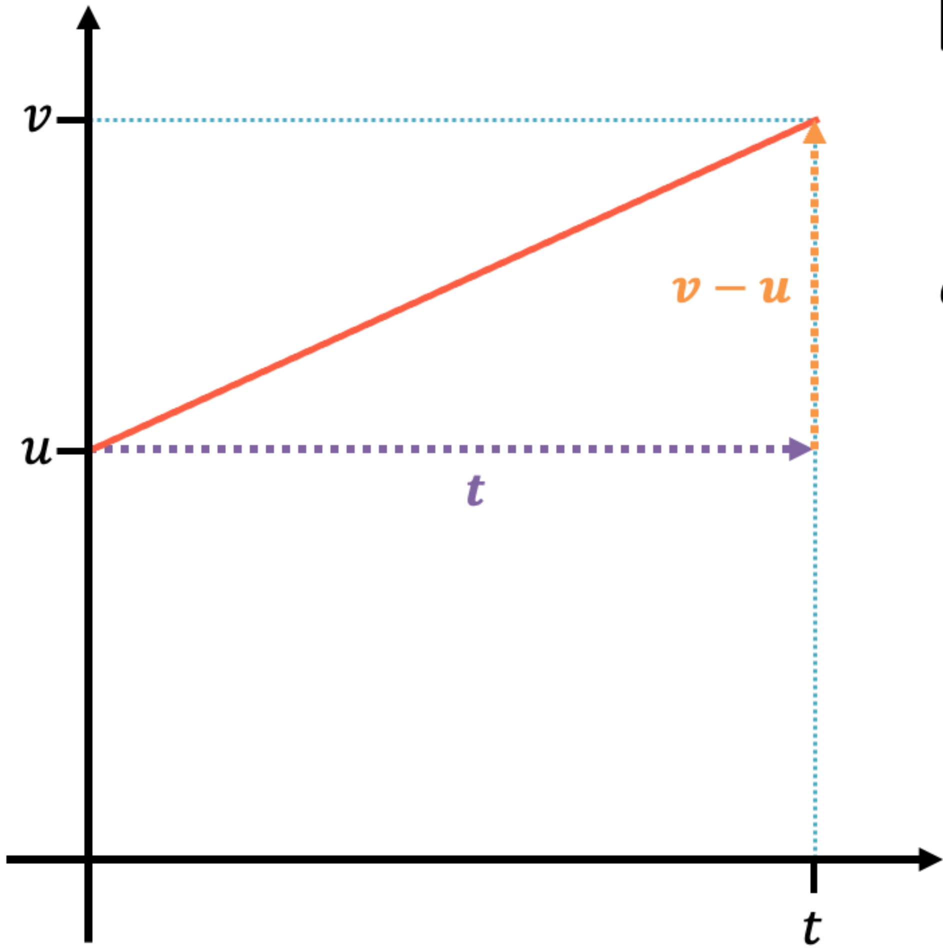
We could calculate the area under the velocity-time graph, which represents the displacement.

Four of the constant acceleration formulae can be derived by considering a simple **velocity-time** graph.

While the formulae are given in an exam **formula booklet,** you can be expected to produce the derivations in an exam.

This graph shows a general **initial velocity** (u), a **final velocity** (v), and a **time** (t).
Which quantities are missing, and how might we calculate them from a velocity-time graph?

Deriving the Constant Acceleration Formulae



Consider the <u>gradient</u> of the velocity-time graph.

$$gradient = \frac{\Delta y}{\Delta x}$$

$$acceleration = \frac{\Delta velocity}{\Delta time} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v-u}{t}$$

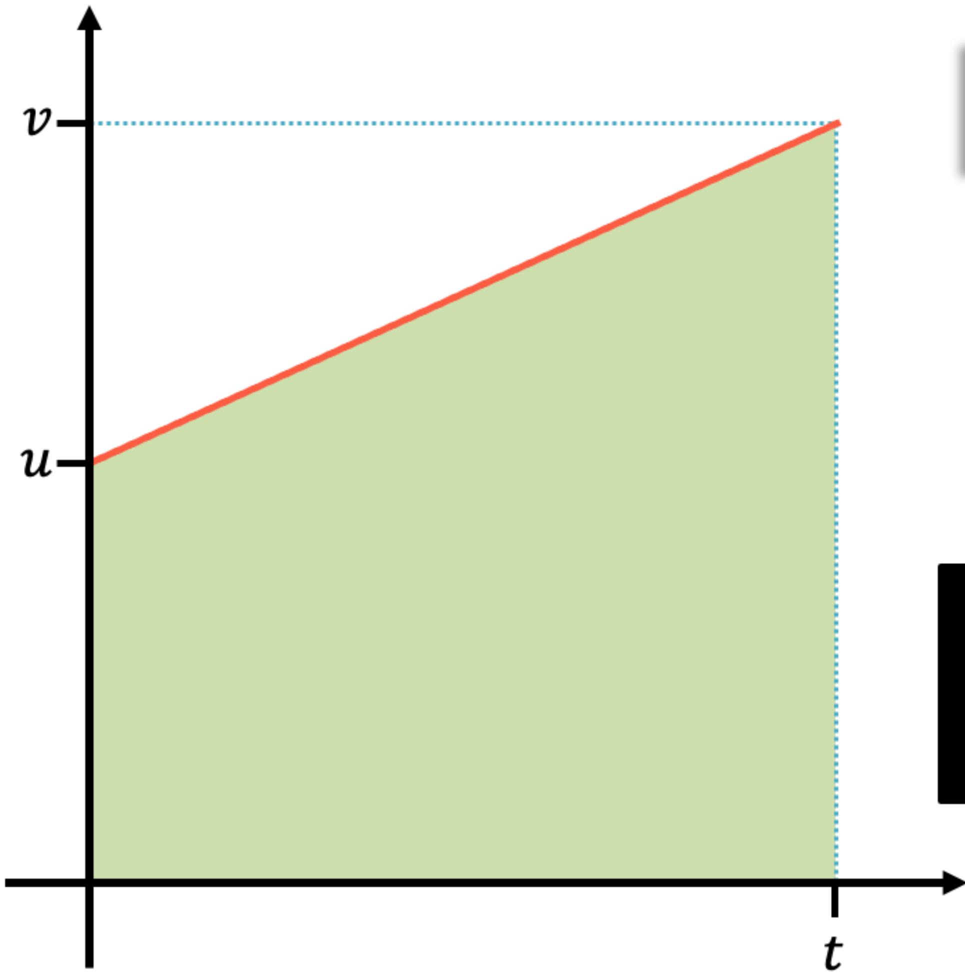
$$at = v-u$$

$$u + at = v$$

$$\boldsymbol{v = u + at}$$

This is usually written with v on the left hand side.

Deriving the Constant Acceleration Formulae



Consider the <u>area</u> under the velocity-time graph.

What shape is formed under this velocity-time graph?

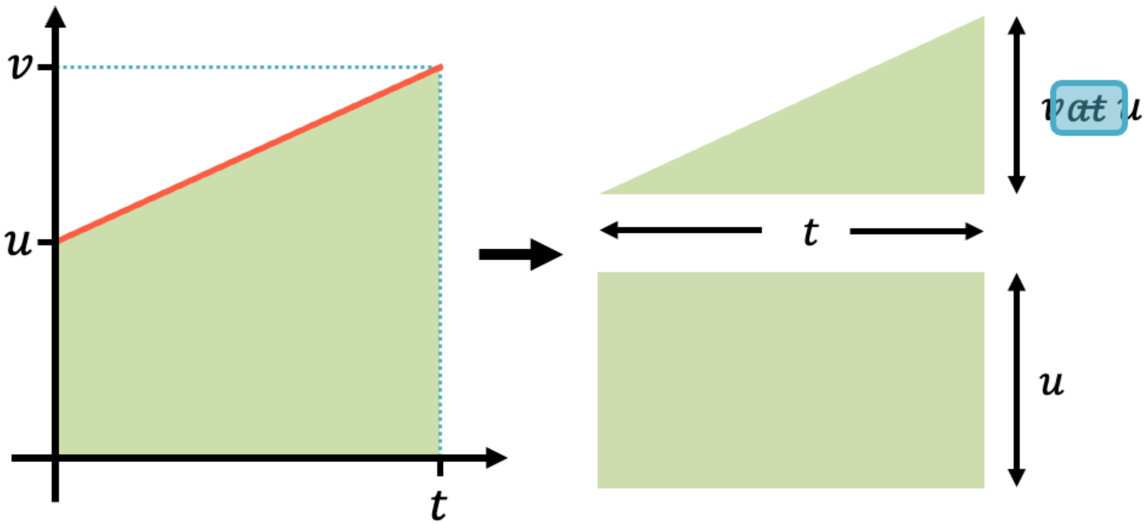
This shape is a **trapezium**, so apply for the formula for the area of a trapezium.

$$Area = \frac{1}{2}(a+b)\times h$$

The area represents **displacement**, s, the parallel sides are u and v, and the separation (h) between these sides is t, therefore:

$$\boldsymbol{s = \frac{1}{2}(u+v)\times t}$$

Deriving the Constant Acceleration Formulae



Earlier we showed

$$v = u + at$$

$-u$ from both sides:

$$v - u = at$$

We could therefore write the height of the triangle as at.

Find the area of the rectangle:

$$Area = u \times t$$
$$= ut$$

Find the area of the triangle:

$$Area = \frac{1}{2} \times t \times at$$
$$= \frac{1}{2} at^2$$

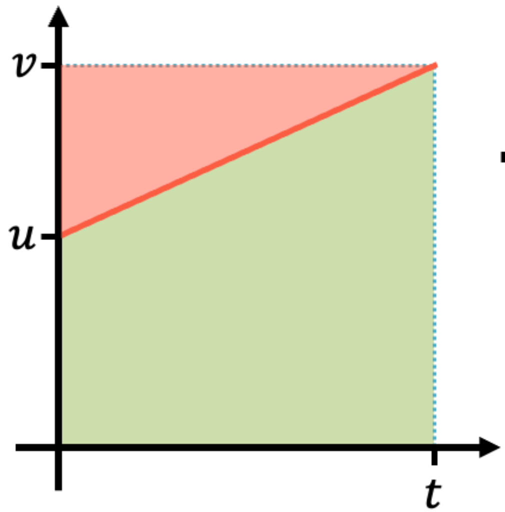
Add the areas to find the total displacement:

$$Displacement = ut + \frac{1}{2} at^2$$

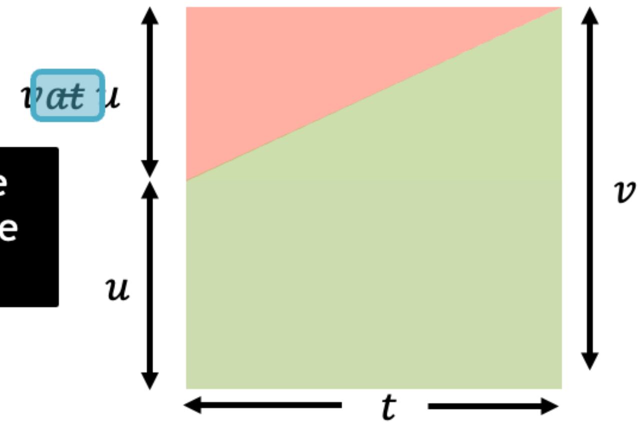
Therefore the third constant acceleration equation is:

$$\boldsymbol{s = ut + \frac{1}{2} at^2}$$

Deriving the Constant Acceleration Formulae



As before, write the height of the triangle as at.



Find the area of the whole rectangle:

$$Area = v \times t$$
$$= vt$$

Find the area of the red triangle:

$$Area = \frac{1}{2} \times t \times at$$
$$= \frac{1}{2}at^2$$

By repeating similar steps - show that this can be used to derive $s = vt - \frac{1}{2}at^2$

Subtract the red area from the whole area to find an equation for displacement:

$$Displacement = vt - \frac{1}{2}at^2$$

$\boldsymbol{s = vt - \frac{1}{2}at^2}$ as required.

Deriving the Constant Acceleration Formulae

Given the following formulae:

① $v = u + at$

② $s = \frac{1}{2}(u + v)t$

Show that:

$$v^2 = u^2 + 2as$$

There are two ways to show the result $v^2 = u^2 + 2as$ using the results we have so far. This method is algebraically more straightforward.

Notice that $v^2 = u^2 + 2as$ is independent of t - this means that t is not in this equation.

Start by rearranging ① for t.

① $v = u + at$

$$t = \frac{v - u}{a}$$

Substitute this equation into ②.

② $s = \frac{1}{2}(u + v)\left(\frac{v-u}{a}\right)$

$$s = \frac{(v + u)(v - u)}{2a}$$

Notice the numerator is a difference of two squares.

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

Rerrange for v^2 to achieve the result as required.

$$\boldsymbol{v^2 = u^2 + 2as}$$

Deriving the Constant Acceleration Formulae

Given the following formulae:

(1) $v = u + at$

(2) $s = ut + \frac{1}{2}at^2$

Show that:

$v^2 = u^2 + 2as$

This is another valid method to achieve the same result.

As before, rearrange (1) for t.

$$(1)\quad v = u + at$$

$$t = \frac{v-u}{a}$$

Substitute this equation into (2).

$$(2)\quad s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$$

Expand the square term.

$$s = \frac{uv-u^2}{a} + \frac{a(v^2-2uv+u^2)}{2a^2}$$

Simplify the terms.

$$s = \frac{uv-u^2}{a} + \frac{v^2-2uv+u^2}{2a}$$

Multiply through by a to clear this from the right hand side.

$$as = \cancel{uv} - u^2 + \frac{v^2}{2} - \cancel{uv} + \frac{u^2}{2}$$

Simplify the terms.

$$as = \frac{v^2}{2} - \frac{u^2}{2}$$

Multiply through by 2 and write in terms of v^2 to achieve the required result.

$$2as = v^2 - u^2$$

$$\mathbf{v^2 = u^2 + 2as}$$

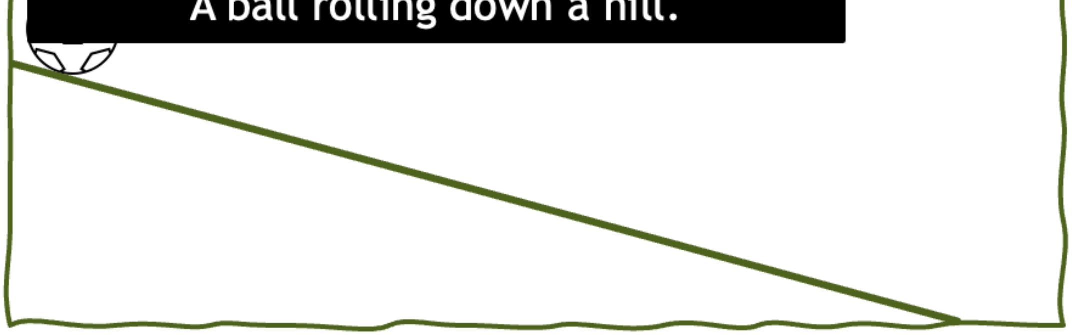
When Can We Use the Constant Acceleration Formulae?

Constant acceleration formulae can only be used when acceleration is constant. For example:

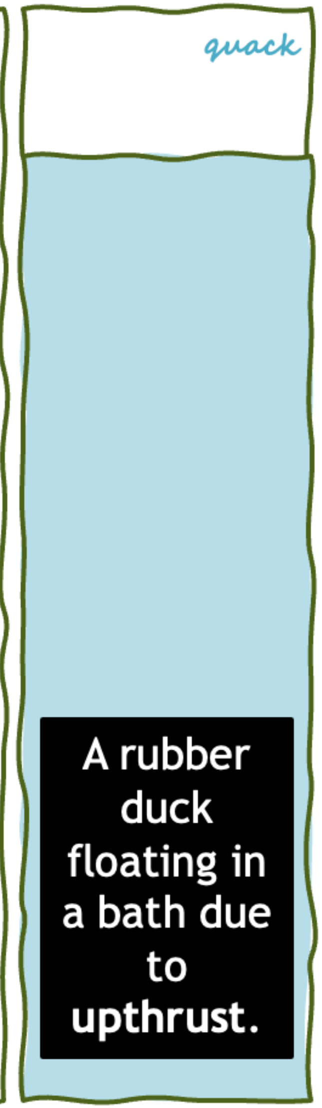
A car accelerating along a road.

A train decelerating to a stop.

A ball rolling down a hill.



A ball falling freely due to gravity.



A rubber duck floating in a bath due to upthrust.

When Acceleration is Not Constant?

Falling objects where there is <u>air</u> <u>**resistance**</u>.

Air resistance <u>increases</u> as an object gets faster, changing acceleration throughout the fall.

You cannot always apply the constant acceleration formulae.

Can you think of some scenarios where gravity is <u>not</u> constant?

Circular motion, like the moon around the Earth. While the moon stays at roughly the same <u>speed</u>, the <u>velocity</u> changes direction, therefore the moon is always <u>accelerating.</u>

Rocket launches. Rockets burn a lot of fuel but maintain the same thrust, accelerating faster when they have less fuel.

Worked Example	Your Turn
A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of $5\ ms^{-1}$ to a velocity of $7.4\ ms^{-1}$ in 50 seconds. Find: a) the distance she travels in these 50 seconds b) her acceleration in these 50 seconds.	A cyclist is travelling along a straight road. She accelerates at a constant rate from a velocity of $4\ ms^{-1}$ to a velocity of $7.5\ ms^{-1}$ in 40 seconds. Find: a) the distance she travels in these 40 seconds b) her acceleration in these 40 seconds.

Worked Example	Your Turn
A particle moves in a straight line from a point A to a point B with a constant deceleration $3\ ms^{-2}$ The velocity of the particle at A is $16\ ms^{-1}$ and the velocity of the particle at B is $4\ ms^{-1}$ Find: a) the time taken for the particle to move from A to B b) the distance from A to B After reaching B the particle continues to move along the straight line with constant deceleration $3\ ms^{-2}$ The particle is at the point C 12 seconds after passing through the point A Find: c) the velocity of the particle at C d) The distance from A to C	A particle moves in a straight line from a point A to a point B with a constant deceleration $1.5\ ms^{-2}$ The velocity of the particle at A is $8\ ms^{-1}$ and the velocity of the particle at B is $2\ ms^{-1}$ Find: a) the time taken for the particle to move from A to B b) the distance from A to B After reaching B the particle continues to move along the straight line with constant deceleration $1.5\ ms^{-2}$ The particle is at the point C 6 seconds after passing through the point A Find: c) the velocity of the particle at C d) The distance from A to C

Worked Example	Your Turn
A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 20 seconds later the car passes a speed-trap where it is registered as travelling at $54\ km\ h^{-1}$. Find: a) the acceleration of the car b) the distance between the traffic lights and the speed-trap.	A car moves from traffic lights along a straight road with constant acceleration. The car starts from rest at the traffic lights and 30 seconds later the car passes a speed-trap where it is registered as travelling at $45\ km\ h^{-1}$. Find: a) the acceleration of the car b) the distance between the traffic lights and the speed-trap.

Worked Example	Your Turn
A particle moves in a straight horizontal line with constant acceleration from A to B, then B to C $AB = 3\ km$ and $BC = 12\ km$ It takes 2 hours from A to B and 4 hours from B to C Find: a) The acceleration of the particle b) The particle's speed as it passes A	A particle moves in a straight horizontal line with constant acceleration from A to B, then B to C $AB = 4\ km$ and $BC = 12\ km$ It takes 2 hours from A to B and 3 hours from B to C Find: a) The acceleration of the particle b) The particle's speed as it passes A

9.4 Constant Acceleration Formulae 2

Notes

Selecting the Correct Formula

Recall that each of the kinematics equations are **independent** of one of the variables - meaning they do not contain one variable.

Equation	
$s = ut + \frac{1}{2}at^2$	Independent of v
$s = vt - \frac{1}{2}at^2$	Independent of u
$v = u + at$	Independent of s
$s = \frac{1}{2}(u + v)t$	Independent of a
$v^2 = u^2 + 2as$	Independent of t

Worked Example	Your Turn
A particle is moving along a straight line from A to B with constant acceleration $3\ ms^{-2}$ The velocity of the particle is $5\ ms^{-1}$ in the direction $\overrightarrow{AB}$ The velocity of the particle at B is $81\ ms^{-1}$ in the same direction. Find the distance from A to B	A particle is moving along a straight line from A to B with constant acceleration $5\ ms^{-2}$ The velocity of the particle is $3\ ms^{-1}$ in the direction $\overrightarrow{AB}$ The velocity of the particle at B is $18\ ms^{-1}$ in the same direction. Find the distance from A to B

Worked Example	Your Turn
A particle is moving in a straight horizontal line with constant deceleration 6 ms^{-2} At time $t = 0$ the particle passes through a point O with speed 23 ms^{-1} travelling towards a point A, where $OA = 40$ m Find: a) the times when the particle passes through A b) the value of t when the particle returns to O	A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2} At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1} travelling towards a point A, where $OA = 20$ m Find: a) the times when the particle passes through A b) the value of t when the particle returns to O

Worked Example	Your Turn
A particle is moving in a straight horizontal line with constant deceleration 6 ms^{-2} At time $t = 0$ the particle passes through a point O with speed 23 ms^{-1} Find the total distance travelled by the particle between when it first passes O and returns to O	A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2} At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1} Find the total distance travelled by the particle between when it first passes O and returns to O

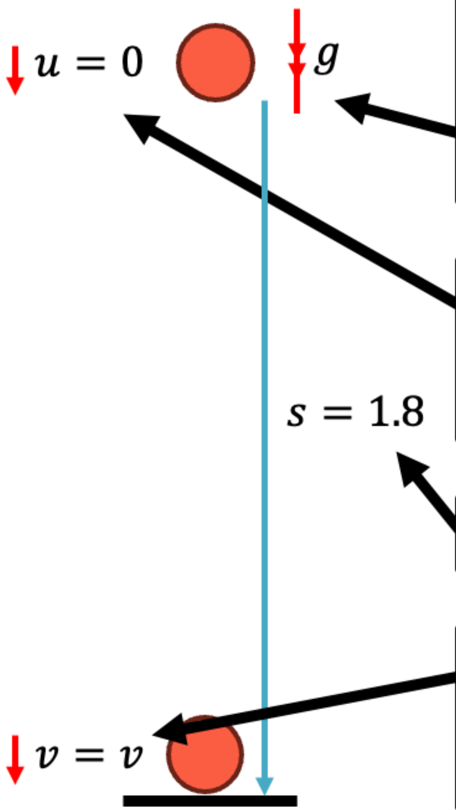
Worked Example	Your Turn
Two particles P and Q are moving along the same straight horizontal line with constant accelerations 2 and $4\ ms^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed $12\ ms^{-1}$. One second later Q passes through A with speed $6\ ms^{-1}$, moving in the same direction as P. a) Find the value of t where the particles meet. b) Find the distance of A from the point where the particles meet.	Two particles P and Q are moving along the same straight horizontal line with constant accelerations 6 and $8\ ms^{-2}$ respectively. At time $t = 0$, P passes through a point A with speed $10\ ms^{-1}$. One second later Q passes through A with speed $5\ ms^{-1}$, moving in the same direction as P. a) Find the value of t where the particles meet. b) Find the distance of A from the point where the particles meet.

9.5 Vertical Motion under Gravity

Notes

Determining the Final Speed of a Falling Object

A cat knocks a book off the top of a bookshelf. The book falls a total distance of 1.8m.
Find the speed at which the book hits the floor.



Conventionally, accelerations are drawn with a double arrow.

Include information you know about the motion, like the acceleration, the displacement, initial velocity, and final velocity.

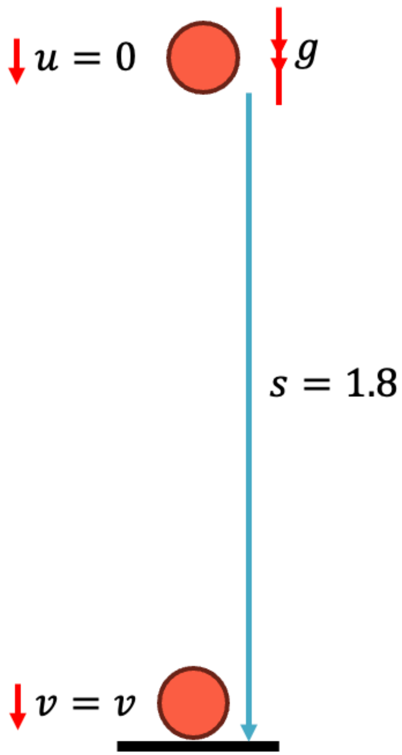
The initial velocity is 0 when the cat knocks it off the shelf.

The displacement is 1.8 m

We are looking to find the final velocity, which we can just label as v for now.

Determining the Final Speed of a Falling Object

A cat knocks a book off the top of a bookshelf. The book falls a total distance of 1.8m.
Find the speed at which the book hits the floor.



Acceleration is constant, which means we can apply the constant acceleration formulae.

$$s = ut + \frac{1}{2}at^2$$
$$s = vt - \frac{1}{2}at^2$$
$$v = u + at$$
$$s = \frac{1}{2}(u + v)t$$
$$v^2 = u^2 + 2as$$

Write down the *suvat* variables.

$s =$
$u =$
$v =$
$a =$
$t =$

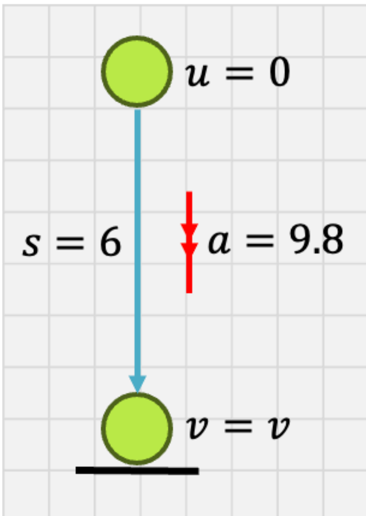
Choose the appropriate formula and solve for v.

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2\times 9.8\times 1.8$$
$$v^2 = 35.28$$
$$v = \pm 5.93969\ldots$$
$$v = 5.94 \text{ ms}^{-1}$$

We want the speed so can ignore any sign for now.

A Note on Signs

A ball is dropped from a height of 6 m above the ground.
Find the speed at which the ball hits the floor.



Maria

Let's work through each of their calculations.

Down as positive

$s = 6$

$u = 0$

$v = v$

$a = g = 9.8$

$t = $ ✗

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2\times 9.8\times 6$$
$$v^2 = 117.6$$
$$v = \pm 10.844\ldots$$
$$v = \mathbf{10.8\ ms^{-1}}$$

Up as positive

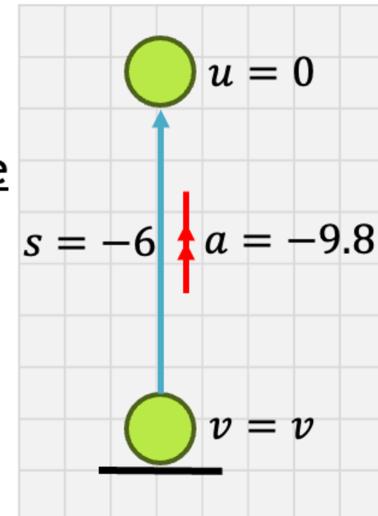
$s = -6$

$u = 0$

$v = v$

$a = -g = -9.8$

$t = $ ✗



Nathan

$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2\times(-9.8)\times(-6)$$
$$v^2 = 117.6$$
$$v = \pm 10.844\ldots$$
$$v = \mathbf{10.8\ ms^{-1}}$$

Both solutions arrive at the same result.

Worked Example	Your Turn
A book falls off the top shelf of a bookcase. The shelf is 2.8 m above a wooden floor. Find: a) the time the book takes to reach the floor, b) the speed with which the book strikes the floor.	A book falls off the top shelf of a bookcase. The shelf is 1.4 m above a wooden floor. Find: a) the time the book takes to reach the floor, b) the speed with which the book strikes the floor.

Find the Greatest Height of a Vertically Projected Particle

Between moving up and moving down, the ball instantaneously has no velocity.

$v = 0$

Let's replay that motion, but this time marking on the velocity throughout the journey.

The ball **decelerates** as it moves upwards.

The ball **accelerates** downwards.

When an object projected upwards is at its **greatest height**, it has an **instantaneous velocity** of 0.

The ball is **projected** with an initial velocity up.

The ball is at its **fastest** when it is about to hit the ground again.

Time of Flight

Previously we thought about the height of this football being kicked into the air.

We could also think about the time of flight of the ball.

Time of flight is the **total time** an object spends in the air.

Worked Example	Your Turn
A ball is projected vertically upwards, from a point X which is 5m above the ground, with speed 15 ms^{-1} Find a) the greatest height above the ground reached by the ball, b) the time of flight of the ball	A ball is projected vertically upwards, from a point X which is 7m above the ground, with speed 21 ms^{-1} Find a) the greatest height above the ground reached by the ball, b) the time of flight of the ball

Determine the Times a Particle is at a Given Height

When an object is projected vertically upwards, it passes through the points on its path twice.

Take this line, 2 m above the ground.

The ball passes it on its way up.

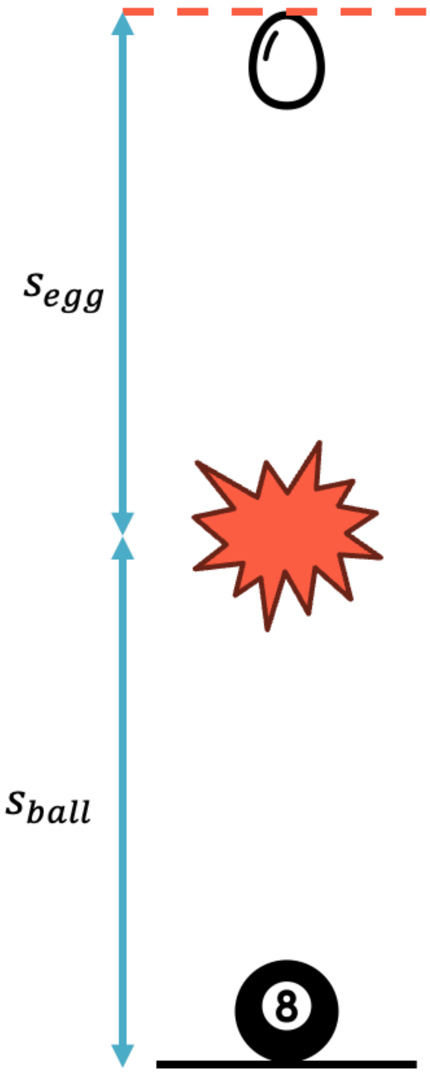
And again on the way down.

Worked Example	Your Turn
A ball is projected vertically upwards from ground level at a speed of $40\ ms^{-1}$ Determine the amount of time the ball is at least 20m above ground level.	A ball is projected vertically upwards from ground level at a speed of $20\ ms^{-1}$ Determine the amount of time the ball is at least 10m above ground level.

Worked Example	Your Turn
A ball is projected vertically upwards with initial speed of $20ms^{-1}$ It hits the ground 5 s later. Find the height above the ground from which the ball was thrown.	A ball is projected vertically upwards with initial speed of $15ms^{-1}$ It hits the ground 5 s later. Find the height above the ground from which the ball was thrown.

Worked Example	Your Turn
A stone is thrown vertically upward from a point which is $8\ m$ above the ground with speed $5\ ms^{-1}$ Find: a) The time of flight of the stone b) The total distance travelled by the stone	A stone is thrown vertically upward from a point which is $5\ m$ above the ground with speed $8\ ms^{-1}$ Find: a) The time of flight of the stone b) The total distance travelled by the stone

Colliding Particles



An egg falls from the sky, 60 m above the ground, at the same time as a snooker ball is projected vertically upwards.

Inevitably, they collide.

What do we know about any of the *suvat* quantities?

The <u>**time**</u> at which the particles collide must be equal.

The displacement of the snooker ball **added to** the displacement of the egg must equal 60 m.

Worked Example	Your Turn
Ball A falls vertically from rest from the top of a tower $48\ m$ high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed $12\ ms^{-1}$. The balls collide. Find the distance to the point where the balls collide from the bottom of the tower.	Ball A falls vertically from rest from the top of a tower $63\ m$ high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed $21\ ms^{-1}$. The balls collide. Find the distance to the point where the balls collide from the bottom of the tower.

Worked Example	Your Turn
At time $t = 0$, two balls A and B are projected vertically upwards. Ball A is projected upwards with speed $3\ ms^{-1}$ from a point $40\ m$ above the horizontal ground. Ball B is projected vertically upwards from the ground with speed $30\ ms^{-1}$. The balls are modelled as particles moving freely under gravity. Find the time and the height at which the balls are at the same vertical height.	At time $t = 0$, two balls A and B are projected vertically upwards. Ball A is projected upwards with speed $2\ ms^{-1}$ from a point $50\ m$ above the horizontal ground. Ball B is projected vertically upwards from the ground with speed $20\ ms^{-1}$. The balls are modelled as particles moving freely under gravity. Find the time and the height at which the balls are at the same vertical height.

Worked Example	Your Turn
A ball is released from rest at a point which is $20\ m$ above a wooden floor. Each time the ball strikes the floor, it rebounds with $\frac{2}{3}$ of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball: a) The first time it rebounds from the floor b) The second time it rebounds from the floor	A ball is released from rest at a point which is $10\ m$ above a wooden floor. Each time the ball strikes the floor, it rebounds with $\frac{3}{4}$ of the speed with which it strikes the floor. Find the greatest height above the floor reached by the ball: a) The first time it rebounds from the floor b) The second time it rebounds from the floor

Worked Example

At time $t = 0$, a small ball is projected vertically upwards with speed U m s^{-1} from a point A that is 16.8 m above horizontal ground.

The speed of the ball at the instant immediately before it hits the ground for the first time is 19 m s^{-1}

The ball hits the ground for the first time at time $t = T$ seconds.

The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

Using the model,

(*a*) show that $U = 5$
(2)

(*b*) find the value of T,
(2)

(*c*) find the time from the instant the ball is projected until the instant when the ball is 1.2 m below A.
(4)

(*d*) Sketch a velocity-time graph for the motion of the ball for $0 \leq t \leq T$, stating the coordinates of the start point and the end point of your graph.
(2)

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

(*e*) State, with a reason, how this new value of U would compare with the value found in part (a), using the initial unrefined model.
(1)

(*f*) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.
(1)

Your Turn

At time $t = 0$, a small stone is projected vertically upwards at a point, A, over the edge of a building with speed U m s^{-1}. The point is 19.6 m above horizontal ground.

The speed of the stone at the instant immediately before it hits the ground is 21 m s^{-1}.

The stone hits the ground at time $t = T$ seconds.

The motion of the stone, from the instant it is projected until the instant just before it hits the ground, is modelled as that of a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

Using the model,

(*a*) show that $U = 7$

(2)

(*b*) find the value of T,

(2)

(*c*) find the time from the instant the ball is projected until the instant when the ball is 1.6 m below A.

(4)

(*d*) Sketch a velocity-time graph for the motion of the ball for $0 \leq t \leq T$, stating the coordinates of the start point and the end point of your graph.

(2)

In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

(*e*) State, with a reason, how this new value of U would compare with the value found in part (a), using the initial unrefined model.

(1)

(*f*) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.

(1)

Worked Example

At time $t = 0$, a small stone is thrown vertically upwards with speed 14.7 m s^{-1} from a point A.

At time $t = T$ seconds, the stone passes through A, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(*a*) find the value of T,

(2)

(*b*) find the total distance travelled by the stone in the first 4 seconds of its motion.

(4)

(*c*) State one refinement that could be made to the model, apart from air resistance, that would make the model more realistic.

(1)

Your Turn

At time $t = 0$, a ball is thrown vertically upwards with speed 24.5 m s^{-1} from a point A, above the ground.

At time $t = T$ seconds, the ball passes through A, moving downwards.

The stone is modelled as a particle moving freely under gravity throughout its motion.

Using the model,

(*a*) find the value of T,

(2)

(*b*) find the total distance travelled by the ball in the first 6 seconds of its motion.

(4)

(*c*) State one assumption that you have made, apart from ignoring air resistance, to create this simple model.

(1)

Worked Example

The point A is 1.8 m vertically above horizontal ground.

At time $t = 0$, a small stone is projected vertically upwards with speed U m s^{-1} from the point A.

At time $t = T$ seconds, the stone hits the ground.

The speed of the stone as it hits the ground is 10 m s^{-1}

In an initial model of the motion of the stone as it moves from A to where it hits the ground

• the stone is modelled as a particle moving freely under gravity

• **the acceleration due to gravity is modelled as having magnitude 10 m s^{-2}**

Using the model,

(*a*) find the value of U,

(3)

(*b*) find the value of T.

(2)

(*c*) Suggest one refinement, apart from including air resistance, that would make the model more realistic.

(1)

In reality the stone will not move freely under gravity and will be subject to air resistance.

(*d*) Explain how this would affect your answer to part (a).

(1)

Your Turn

The point A is 17.5 m vertically above horizontal ground.

At time $t = 0$, a small stone is projected vertically upwards with speed U m s^{-1} from the point A.

At time $t = T$ seconds, the stone hits the ground.

The speed of the stone as it hits the ground is 28 m s^{-1}

In an initial model of the motion of the stone as it moves from A to where it hits the ground the stone is modelled as a particle moving freely under gravity

Using the model,

(*a*) find the value of U,

(3)

(*b*) find the value of T.

(2)

(*c*) Suggest one refinement, apart from including air resistance, that would make the model more realistic.

(1)

In reality the stone will not move freely under gravity and will be subject to air resistance.

(*d*) Explain how this would affect your answer to part (a).

(1)

Worked Example

A train travels along a straight horizontal track from station P to station Q.

In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of 25 m s^{-1}

The train then travels at this constant speed of 25 m s^{-1} before finally moving with constant deceleration until it comes to rest at Q.

The time spent decelerating is four times the time spent accelerating.

The journey from P to Q takes 700 s.

Using the model,

(*a*) sketch a speed-time graph for the motion of the train between the two stations P and Q.
(1)

The distance between the two stations is 15 km.

Using the model,

(*b*) show that the time spent accelerating by the train is 40 s,
(3)

(*c*) find the acceleration, in m s^{-2}, of the train,
(1)

(*d*) find the speed of the train 572 s after leaving P.
(2)

(*e*) State one limitation of the model which could affect your answers to parts (*b*) and (c).
(1)

Your Turn

A train travels along a straight horizontal track from station P to station Q.

In a model of the motion of the train, at time $t = 0$ the train starts from rest at P, and moves with constant acceleration until it reaches its maximum speed of 20 m s^{-1}

The train then travels at this constant speed of 20 m s^{-1} before finally moving with constant deceleration until it comes to rest at Q.

The time spent decelerating is six times the time spent accelerating.

The journey from P to Q takes 810 s.

Using the model,

(a) sketch a speed-time graph for the motion of the train between the two stations P and Q.
(1)

The distance between the two stations is 14.1 km

Using the model,

(b) show that the time spent accelerating by the train is 30 s,
(3)

(c) find the acceleration, in m s^{-2}, of the train,
(1)

(d) find the speed of the train 675 s after leaving P.
(2)

(e) State one limitation of the model which could affect your answers to parts (b) and (c).
(1)

Worked Example

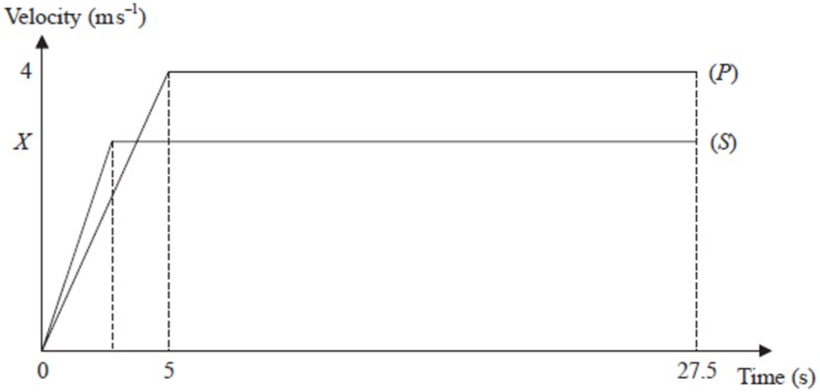


Figure 1

Two children, Pat (P) and Sam (S), run a race along a straight horizontal track.

Both children start from rest at the same time and cross the finish line at the same time.

In a model of the motion:

Pat accelerates at a constant rate from rest for 5 s until reaching a speed of 4 m s^{-1} and then maintains a constant speed of 4 m s^{-1} until crossing the finish line.

Sam accelerates at a constant rate of 1 m s^{-2} from rest until reaching a speed of X m s^{-1} and then maintains a constant speed of X m s^{-1} until crossing the finish line.

Both children take 27.5 s to complete the race.

The velocity-time graphs shown in Figure 1 describe the model of the motion of each child from the instant they start to the instant they cross the finish line together.

Using the model,

(*a*) explain why the areas under the two graphs are equal,

(1)

(*b*) find the acceleration of Pat during the first 5 seconds,

(1)

(*c*) find, in metres, the length of the race,

(2)

(*d*) find the value of X, giving your answer to 3 significant figures.

(4)

Your Turn

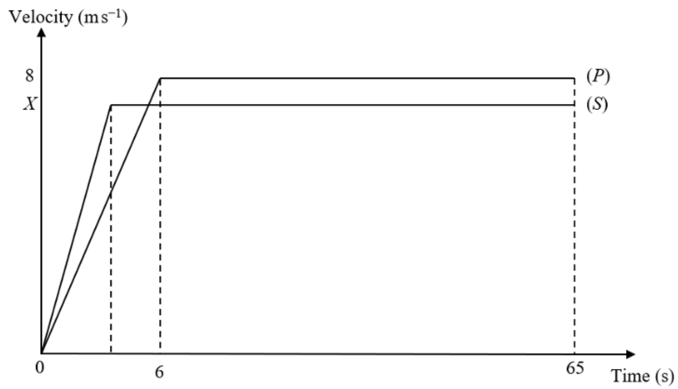


Figure 1

Two cyclists, Peter (P) and Steve (S), take part in a cycling race along a straight horizontal racing track.

Both cyclists start from rest at the same time and cross the finish line at the same time.

In a model of the motion:

Peter accelerates at a constant rate from rest for 6 s until reaching a speed of $8\,\text{m s}^{-1}$ and then maintains a constant speed of $8\,\text{m s}^{-1}$ until crossing the finish line.

Steve accelerates at a constant rate of $1.5\,\text{m s}^{-2}$ from rest until reaching a speed of $X\,\text{m s}^{-1}$ and then maintains a constant speed of $X\,\text{m s}^{-1}$ until crossing the finish line.

Both cyclists take 65 s to complete the race.

The velocity-time graphs shown in Figure 1 describe the model of the motion of each cyclist from the instant they start to the instant they cross the finish line together.

Using the model,

(a) explain the relationship between the areas of each graph, **(1)**

(b) find the acceleration of Peter during the first 6 seconds, **(1)**

(c) find, in metres, the length of the race, **(2)**

(d) find the value of X, giving your answer to 3 significant figures. **(4)**

Worked Example

A small stone is projected vertically upwards with speed 39.2 m s^{-1} from a point O.

The stone is modelled as a particle moving freely under gravity from when it is projected until it hits the ground 10 s later.

Using the model, find

(*a*) the height of O above the ground,

(3)

(*b*) the total length of time for which the speed of the stone is less than or equal to 24.5 m s^{-1}

(3)

(*c*) State one refinement that could be made to the model that would make your answer to part (*a*) more accurate.

(1)

Your Turn

A small stone is projected vertically upwards with speed $53.9\,\text{m}\,\text{s}^{-1}$ from a point O.

The stone is modelled as a particle moving freely under gravity from when it is projected until it hits the ground 12 s later.

Using the model, find

(a) the height of O above the ground,

(3)

(b) the total length of time for which the speed of the stone is less than or equal to $29.4\,\text{m}\,\text{s}^{-1}$

(3)

(c) State one refinement that could be made to the model that would make your answer to part (a) more accurate.

(1)

Worked Example

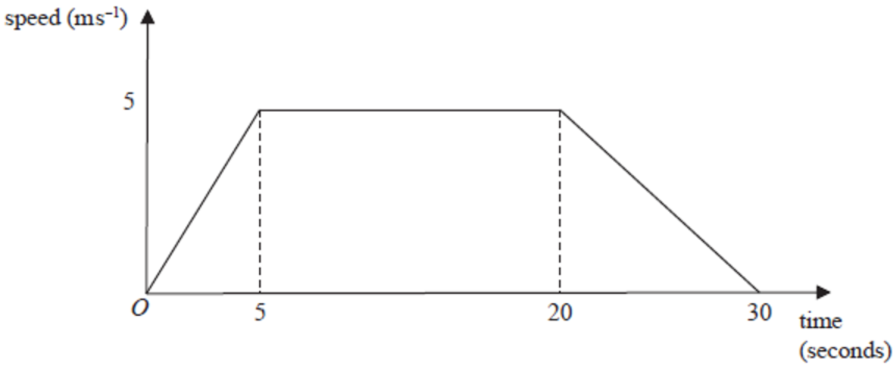


Figure 1

Figure 1 shows the speed-time graph for the journey of a car moving in a long queue of traffic on a straight horizontal road.

At time $t = 0$, the car is at rest at the point A.

The car then accelerates uniformly for 5 seconds until it reaches a speed of 5 m s^{-1}

For the next 15 seconds the car travels at a constant speed of 5 m s^{-1}

The car then decelerates uniformly until it comes to rest at the point B.

The total journey time is 30 seconds.

(*a*) Find the distance AB.

(3)

(*b*) Sketch a distance-time graph for the journey of the car from A to B.

(3)

Your Turn

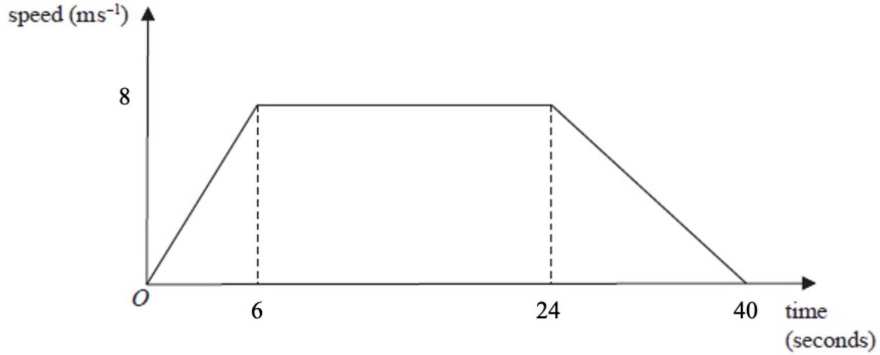


Figure 1

Figure 1 shows the speed-time graph for the journey of a motorbike moving in a long queue of traffic on a straight horizontal road.
At time $t = 0$, the car is at rest at the point A.

The motorbike then accelerates uniformly for 6 seconds until it reaches a speed of 8 m s^{-1}

For the next 18 seconds the motorbike travels at a constant speed of 8 m s^{-1}

The motorbike then decelerates uniformly until it comes to rest at the point B.

The total journey time is 40 seconds.

(*a*) Find the distance AB.

(3)

(*b*) Sketch a distance-time graph for the journey of the motorbike from A to B.

(3)

Worked Example

A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of 3.2 m s^{-2}

Find

(*a*) the speed of the car after 5 s,

(1)

(*b*) the distance travelled by the car in the first 5 s.

(2)

Your Turn

A car is initially at rest on a straight horizontal road.

The car then accelerates along the road with a constant acceleration of $2.8\,\text{m}\,\text{s}^{-2}$

Find

(a) the speed of the car after 6 s,

(1)

(b) the distance travelled by the car in the first 6 s.

(2)

Worked Example

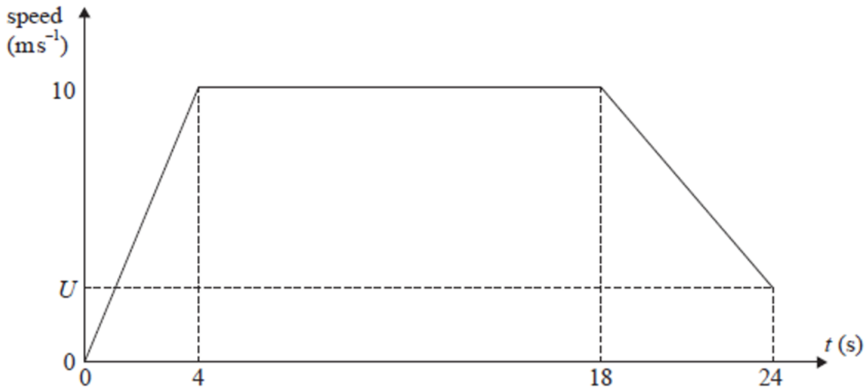


Figure 2

Figure 2 shows a speed-time graph for a model of the motion of an athlete running a **200 m** race in 24 s.

The athlete

- starts from rest at time $t = 0$ and accelerates at a constant rate, reaching a speed of $10\,\text{m}\,\text{s}^{-1}$ at $t = 4$
- then moves at a constant speed of $10\,\text{m}\,\text{s}^{-1}$ from $t = 4$ to $t = 18$
- then decelerates at a constant rate from $t = 18$ to $t = 24$, crossing the finishing line with speed $U\,\text{m}\,\text{s}^{-1}$

Using the model,

(*a*) find the acceleration of the athlete during the first 4 s of the race, stating the units of your answer,

(2)

(*b*) find the distance covered by the athlete during the first 18 s of the race,

(3)

(*c*) find the value of U.

(3)

Your Turn

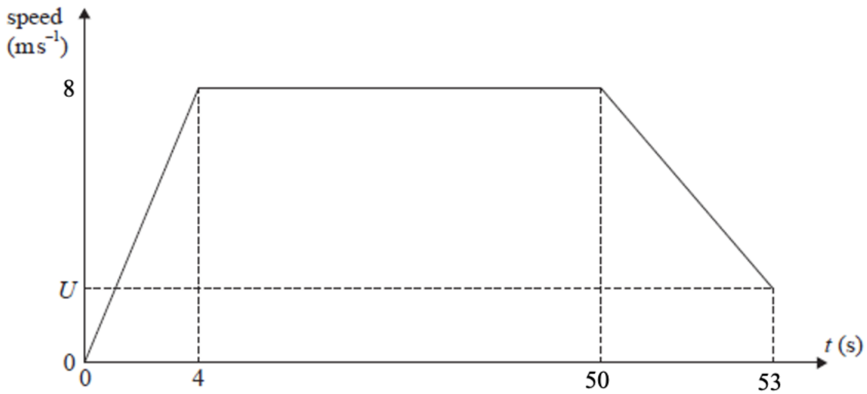


Figure 2

Figure 2 shows a speed-time graph for a model of the motion of an athlete running a **400 m** race in 53 s.

The athlete

- starts from rest at time $t = 0$ and accelerates at a constant rate, reaching a speed of $8\,\text{m s}^{-1}$ at $t = 4$
- then moves at a constant speed of $8\,\text{m s}^{-1}$ from $t = 4$ to $t = 50$
- then decelerates at a constant rate from $t = 50$ to $t = 53$, crossing the finishing line with speed $U\,\text{m s}^{-1}$

Using the model,

(a) find the acceleration of the athlete during the first 4 s of the race, stating the units of your answer,

(2)

(b) find the distance covered by the athlete during the first 50 s of the race,

(3)

(c) find the value of U.

(3)

Kinematics

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

Summary

3 The base SI units most commonly used in mechanics are:

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/displacement	metre	m
Time	second	s

1 Velocity is the **rate of change** of displacement.
On a displacement–time graph the **gradient** represents the velocity.
If the displacement–time graph is a straight line, then the velocity is constant.

2 Average velocity $= \dfrac{\text{displacement from starting point}}{\text{time taken}}$

3 Average speed $= \dfrac{\text{total distance travelled}}{\text{time taken}}$

4 Acceleration is the **rate of change** of velocity.
In a velocity–time graph the **gradient** represents the acceleration.
If the velocity–time graph is a straight line, then the acceleration is constant.

5 The area between a velocity–time graph and the horizontal axis represents the distance travelled.
For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.

6 You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- $v = u + at$
- $s = \left(\dfrac{u + v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

7 The force of **gravity** causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.

8 An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\ \text{m s}^{-2}$.