KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Mechanics 2
Chapter 5 – Forces and Friction

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior Knowledge Check

1 A particle of mass 5 kg is acted on by two forces:

$\mathbf{F}_1 = (8\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (-3\mathbf{i} + 8\mathbf{j})$ N.

Find the acceleration of the particle in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-2}.

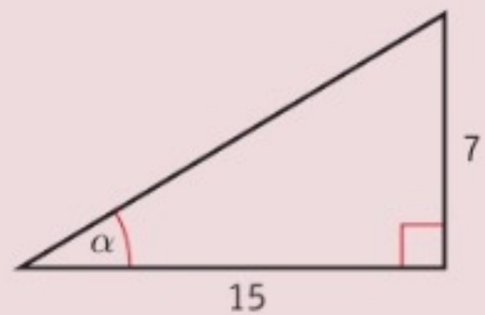
← Year 1, Chapter 10

2 In the diagram below, calculate

a the length of the hypotenuse

b the size of α.

Give your answers correct to 2 d.p.

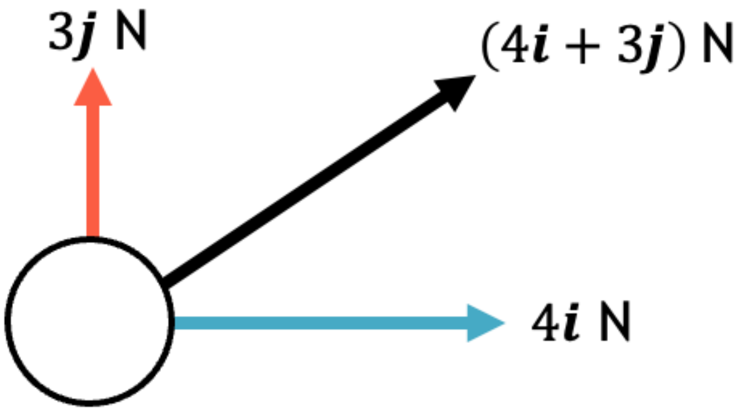


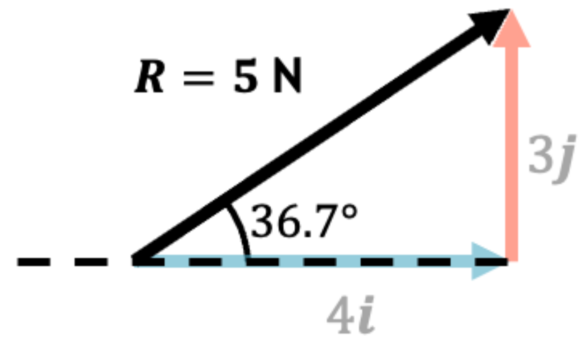
← GCSE Mathematics

5.1 Resolving Forces

Recap – Magnitude-Direction Form of a Vector

There are two useful perspectives to have of forces as vectors acting on a body.





Component Form

Knowing the individual components of a force is useful for:

- Performing calculations with vectors.
- Applying Newton's Laws in 1 Dimension.

Magnitude-Direction Form

Knowing the magnitude and the direction the vector makes with an axis.

This is useful for describing the overall effect of a vector like a force or acceleration and their directions.

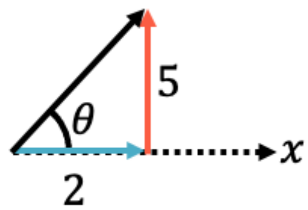
Recap – Magnitude-Direction Form of a Vector

The vector $\boldsymbol{a}$ is $2\boldsymbol{i} + 5\boldsymbol{j}$.
Find the magnitude of $\boldsymbol{a}$ and the angle it makes with the positive x −axis.

Magnitude:

$|\boldsymbol{a}| = \sqrt{5^2 + 2^2}$

$|\boldsymbol{a}| = \sqrt{29}$ or $\mathbf{5.38}$



Direction:

$\theta = \tan^{-1}\left(\frac{5}{2}\right)$

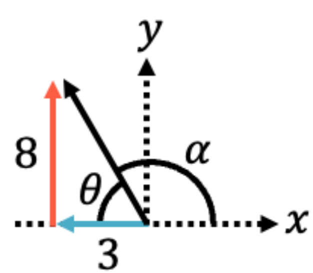
$\boldsymbol{\theta} = \mathbf{68.2°}$

Apply Pythagoras' Theorem for the magnitude, and right angled trigonometry for direction.

The vector $\boldsymbol{b}$ is $\begin{pmatrix} -3 \\ 8 \end{pmatrix}$.
Find the angle $\boldsymbol{b}$ makes with the positive x axis.

Direction:

$\theta = \tan^{-1}\left(\frac{8}{3}\right) = 69.4°$

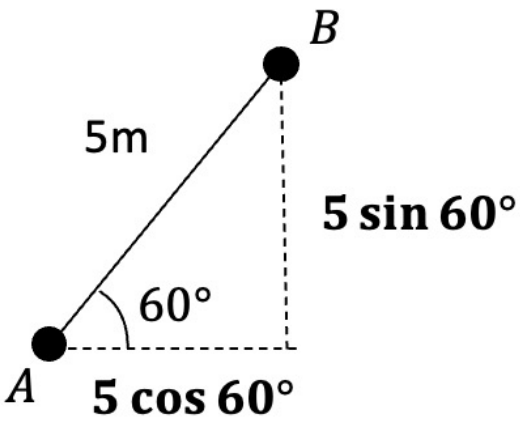


$\alpha = 180 - 69.4$

$\boldsymbol{\alpha} = \mathbf{110.6°}$

The angle that $\boldsymbol{b}$ makes with the positive x axis is α in this diagram.

Resolving Forces



In the last chapter/Year 1 we have already taken the '**components**' of a distance in particular directions, for example the horizontal and vertical components.

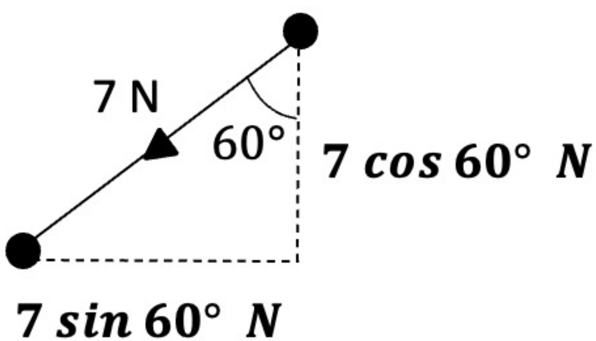
This allowed us for example to convert a displacement (from A to B) from scalar form to vector form:

$$\mathbf{5m} \Rightarrow \begin{pmatrix} 5\cos 60° \\ 5\sin 60° \end{pmatrix} = \begin{pmatrix} 2.5 \\ 4.33 \end{pmatrix} m$$

And we could convert back to scalar form by finding the magnitude of the displacement vector:

$$\sqrt{(5\cos 60°)^2 + (5\sin 60°)^2} = 5$$

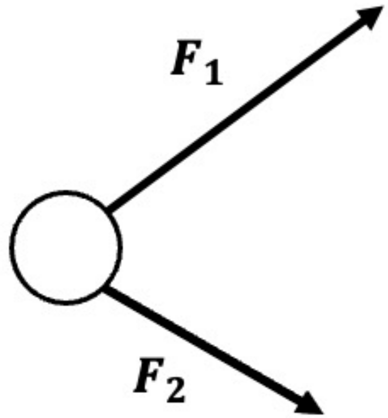
Speed Tip: If x is the magnitude/the hypotenuse, use $x \cos\theta$ for the side adjacent to the angle and $x \sin\theta$ for the side opposite it.

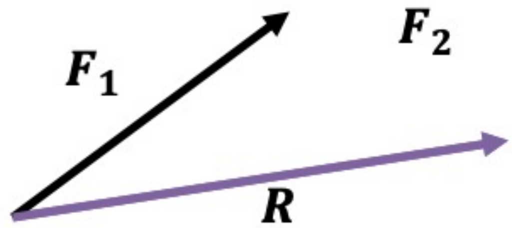


We can use exactly the same principle to find the components of a force, and convert between vector and scalar form.

Notes

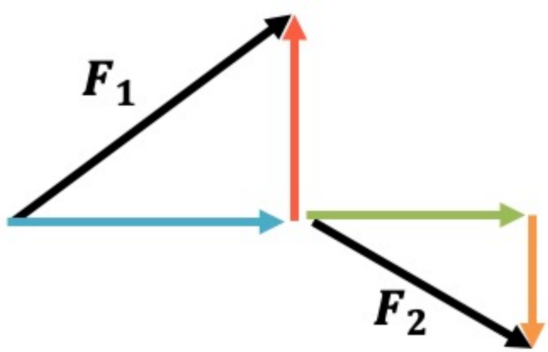
Determine the Resultant Vector of Multiple Forces

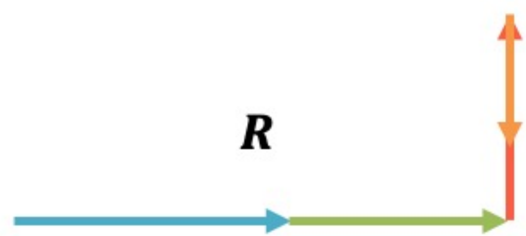




Previously we have found resultant force vectors by adding individual force vectors together tip-to-tail.

We will now do the same thing, but by breaking down a resultant vector into its horizontal and vertical components first.

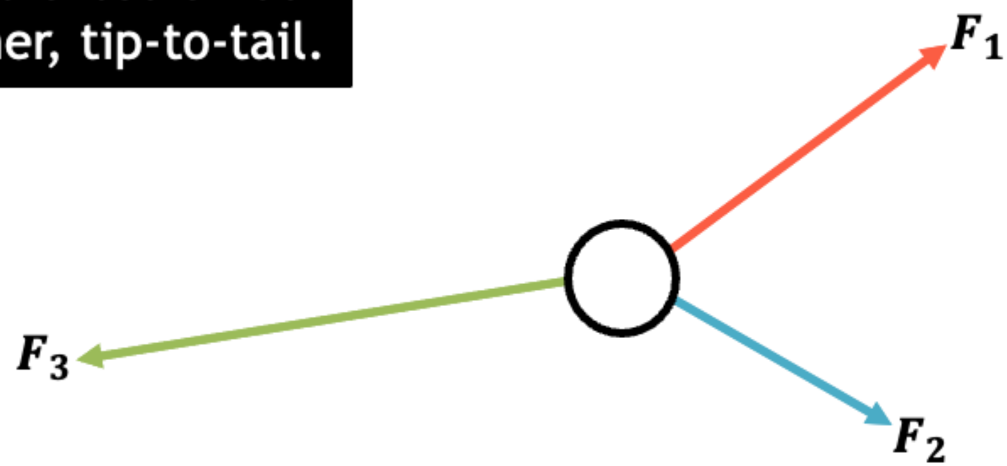




These approaches have the same result.

Find the Direction of a Force Required for Equilibrium

Consider what happens when we add these three forces together, tip-to-tail.



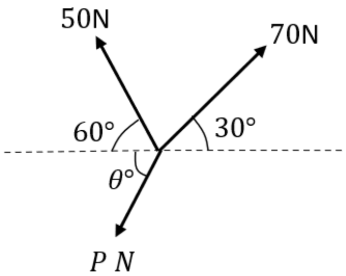
What is the implication of the three forces forming a closed triangle?

If all of the force vectors form a closed shape then the forces are in equilibrium.

Worked Example

Three forces act on a particle as shown.

Given that the particle is in equilibrium, calculate the magnitude of P



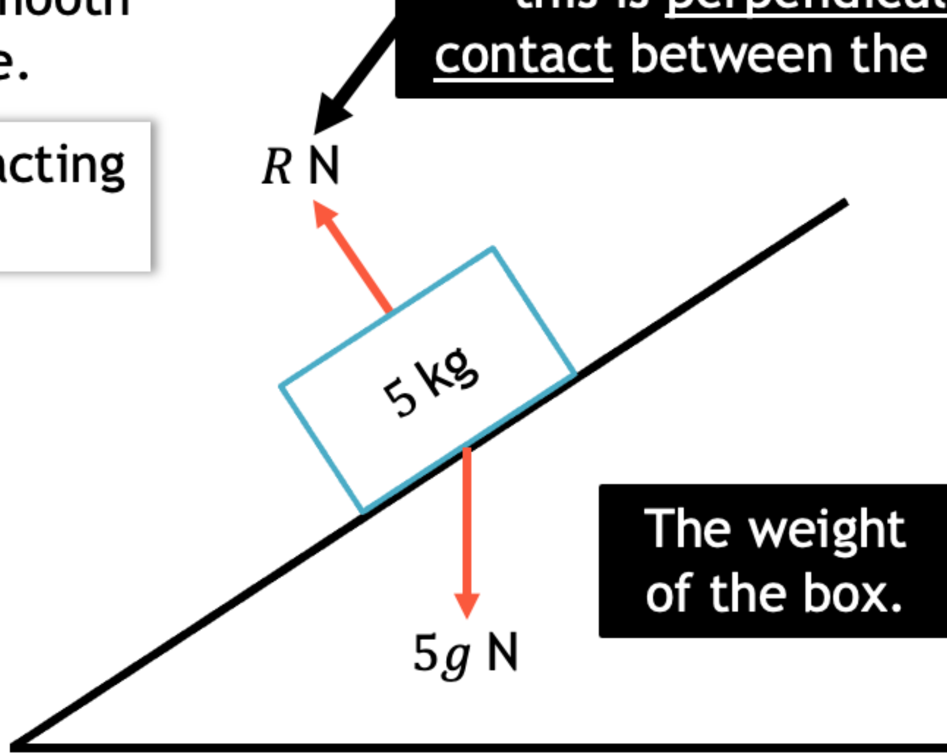
5.2 Inclined Planes

Forces on a Particle on a Slope

Consider a 5 kg box positioned on a smooth inclined plane.

What forces are acting on the box?

The normal reaction force – remember this is perpendicular to the point of contact between the box and the surface.

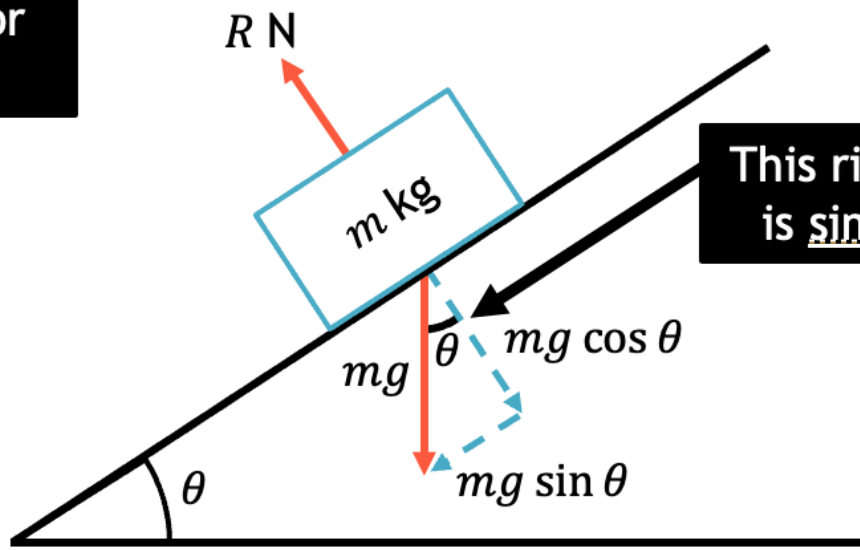


The weight of the box.

Resolving Forces on a Plane

In order to solve problems about forces on planes, we need to resolve forces into the same direction – we will resolve **parallel** to and **perpendicular** to the slope.

It is recommended to use dashed lines for resolved forces.



This right-angled triangle is similar to the slope.

Generally for a weight mg on an inclined plane, angle θ to the horizontal:

- Component parallel to the slope: $mg\sin\theta$
- Component perpendicular to the slope: $mg\cos\theta$

Notes

Worked Example

A particle of mass m is pushed up a smooth slope, inclined at $60°$ by a force of magnitude $10g$ N acting at angle of $30°$ to the slope, causing the particle to accelerate up the slope at 0.25 ms^{-2}

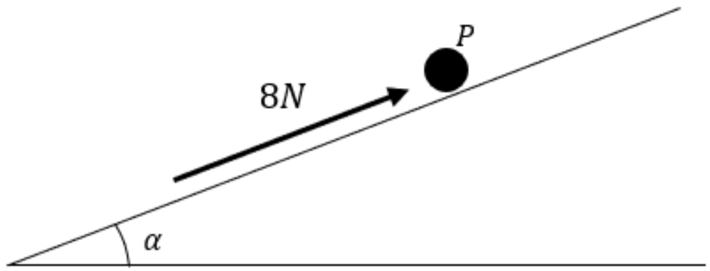
Show that the mass of the particle is $\left(\frac{20\sqrt{3}g}{1+2\sqrt{3}g}\right)$ kg

Worked Example

A particle P of mass 4kg is moving on a smooth slope and is being acted on by a force of 8N that acts parallel to the slope, as shown.

The slop is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{5}{12}$

Work out the acceleration of the particle.

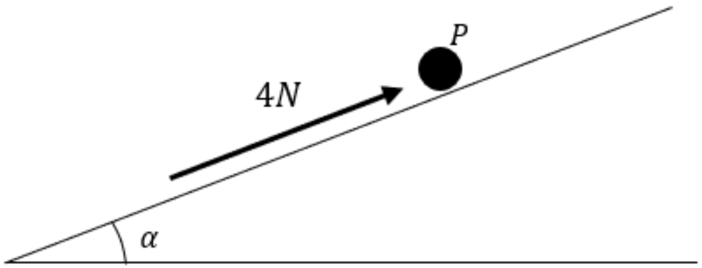


Your Turn

A particle P of mass 2kg is moving on a smooth slope and is being acted on by a force of 4N that acts parallel to the slope, as shown.

The slop is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$

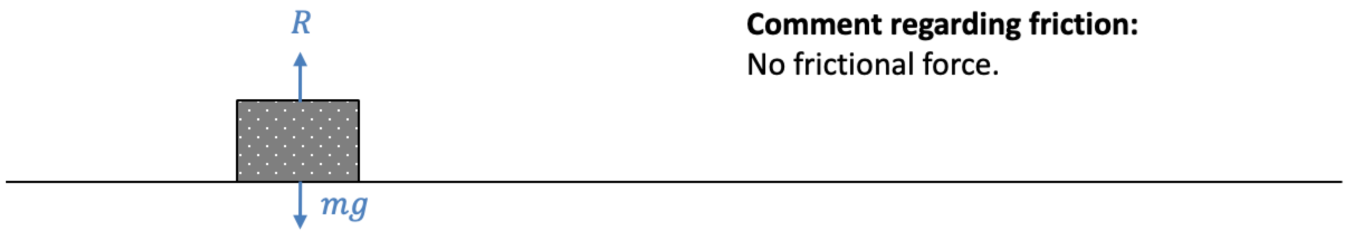
Work out the acceleration of the particle.



5.3 Friction

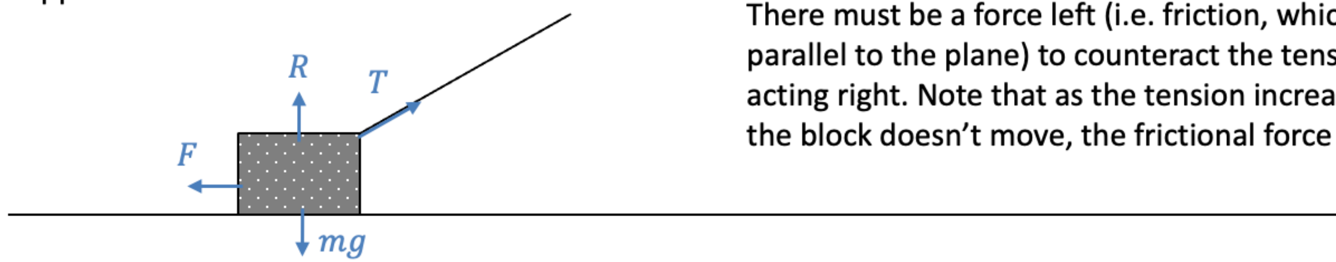
Friction

Scenario 1: A block is on a horizontal rough surface with no forces (other than gravity) acting on it.



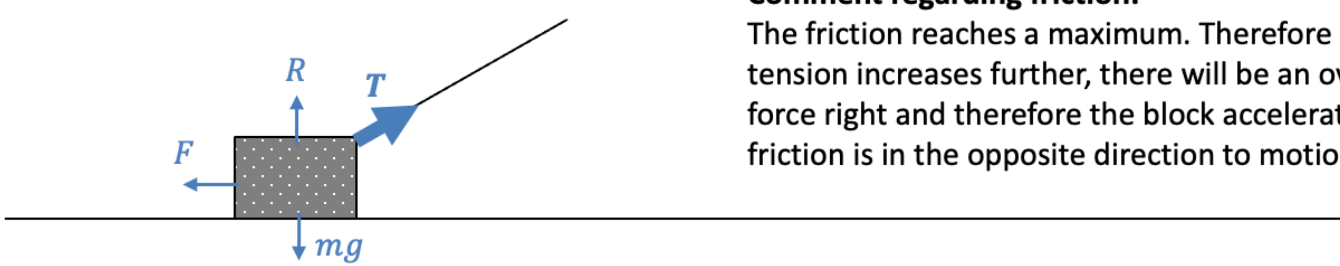
Comment regarding friction:
No frictional force.

Scenario 2: A cable is attached to the block and a force applied. The block doesn't move.



Comment regarding friction:
There must be a force left (i.e. friction, which acts parallel to the plane) to counteract the tension force acting right. Note that as the tension increases but the block doesn't move, the frictional force increases.

Scenario 3: The tension is increased until the block starts to move.



Comment regarding friction:
The friction reaches a maximum. Therefore if the tension increases further, there will be an overall force right and therefore the block accelerates. The friction is in the opposite direction to motion.

This 'maximum friction' depends on two things:

- How **rough** the surface is (i.e. the rougher the surface, the more force required before the block starts moving).
- How hard the block is pressing against the surface (and more formally, by application of Newton's 3rd Law, how large the **reaction force $\boldsymbol{R}$** is).

The maximum friction between two surfaces:

$$F_{max} = \mu R$$

where μ is the coefficient of friction and R is the normal reaction between two surfaces.

Example μ: (source physlink.com)

Materials	Coeff. of Static Friction μ_s
Steel on Steel	0.74
Aluminum on Steel	0.61
Copper on Steel	0.53
Rubber on Concrete	1.0
Wood on Wood	0.25-0.5
Glass on Glass	0.94
Waxed wood on Wet snow	0.14
Waxed wood on Dry snow	-
Metal on Metal (lubricated)	
Ice on Ice	0.1
Teflon on Teflon	0.04
Synovial joints in humans	0.01

Notes

Worked Example

A box of mass 4 kg is held in equilibrium on a fixed rough inclined plane by a rope.
The rope lies in a vertical plane containing a line of greatest slope of the inclined plane.
The ropes is inclined to the plane at an angle α, where $\tan\alpha = \frac{5}{12}$, and the plane is at an angle of $45°$ to the horizontal.
The coefficient of friction between the box and the inclined plane is $\frac{1}{4}$ and the box is on the point of slipping up the plane.
By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

Worked Example

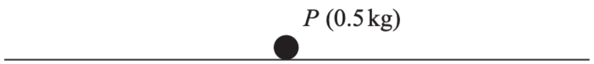


Figure 1

Figure 1 shows a particle P of mass 0.5 kg at rest on a rough horizontal plane.

(a) Find the magnitude of the normal reaction of the plane on P.

(1)

The coefficient of friction between P and the plane is $\frac{2}{7}$

A horizontal force of magnitude X newtons is applied to P.

Given that P is now in limiting equilibrium,

(b) find the value of X.

(2)

Your Turn

Figure 1

Figure 1 shows a particle P of mass 0.4 kg at rest on a rough horizontal plane.

(a) Find the magnitude of the normal reaction of the plane on P.

(1)

The coefficient of friction between P and the plane is $\frac{3}{7}$

A horizontal force of magnitude X newtons is applied to P.

Given that P is now in limiting equilibrium,

(b) find the value of X.

(2)

Worked Example

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.4\,\text{m}\,\text{s}^{-2}$

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

Your Turn

Figure 1

A particle P has mass 8 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 35 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at $1.05\,\text{m s}^{-2}$

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

Worked Example

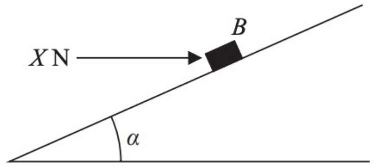


Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.5

(b) find the acceleration of B down the plane.

(6)

Your Turn

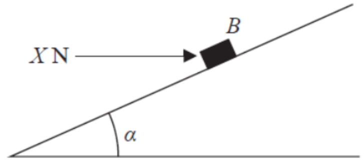


Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{4}{3}$

A toy truck B of mass 10 kg is held in equilibrium on the plane by a horizontal force exerted by a plastic crane of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 100 N.

Using the model,

(*a*) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.25

(*b*) find the acceleration of B down the plane.

(6)

Worked Example

A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A brick P of mass m is placed on the plane.

The coefficient of friction between P and the plane is μ

Brick P is in equilibrium and on the point of sliding down the plane.

Brick P is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick P

(2)

(b) show that $\mu = \frac{3}{4}$

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick P is now removed from the plane and a much heavier brick Q is placed on the plane.

The coefficient of friction between Q and the plane is also $\frac{3}{4}$

(c) Explain briefly why brick Q will remain at rest on the plane.

(1)

Brick Q is now projected with speed $0.5\,\text{m s}^{-1}$ down a line of greatest slope of the plane.

Brick Q is modelled as a particle.

Using the model,

(d) describe the motion of brick Q, giving a reason for your answer.

(2)

Your Turn

A rough plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{5}{12}$

A brick B of mass m is placed on the plane.

The coefficient of friction between B and the plane is μ

Brick B is in equilibrium and on the point of sliding down the plane.

Brick B is modelled as a particle.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on brick B

(2)

(b) show that $\mu = \frac{5}{12}$

(4)

For parts (c) and (d), you are not required to do any further calculations.

Brick B is now removed from the plane and a much heavier brick C is placed on the plane.

The coefficient of friction between C and the plane is also $\frac{5}{12}$

(c) Explain briefly why brick C will remain at rest on the plane.

(1)

Brick C is now projected with speed 0.6 m s^{-1} down a line of greatest slope of the plane.

Brick C is modelled as a particle.

Using the model,

(d) describe the motion of brick C, giving a reason for your answer.

(2)

Worked Example

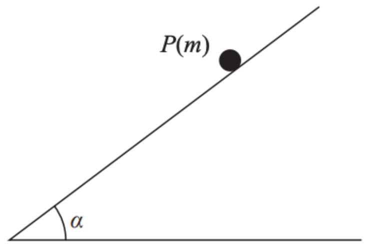


Figure 3

A particle P of mass m is held at rest at a point on a rough inclined plane, as shown in Figure 3.

It is given that

- the plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{5}{12}$
- the coefficient of friction between P and the plane is μ, where $\mu < \frac{5}{12}$

The particle P is released from rest and slides down the plane.
Air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on P,

(2)

(b) show that, as P slides down the plane, the acceleration of P down the plane is

$$\frac{1}{13}g(5 - 12\mu)$$

(4)

(c) State what would happen to P if it is released from rest but $\mu \geqslant \frac{5}{12}$

(1)

Your Turn

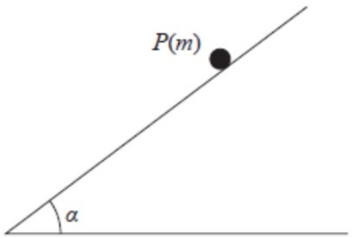


Figure 3

A particle P of mass m is held at rest at a point on a rough inclined plane, as shown in Figure 3.

It is given that

- the plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{7}{24}$
- the coefficient of friction between P and the plane is μ, where $\mu < \frac{7}{24}$

The particle P is released from rest and slides down the plane.
Air resistance is modelled as being negligible.

Using the model,

(a) find, in terms of m and g, the magnitude of the normal reaction of the plane on P,

(2)

(b) show that, as P slides down the plane, the acceleration of P down the plane is

$$\frac{1}{25}g(7 - 24\mu)$$

(4)

(c) State what would happen to P if it is released from rest but $\mu = \frac{3}{8}$

(1)

Summary

1. If a force is applied at an angle to the direction of motion, you can resolve it to find the component of the force that acts in the direction of motion.
2. The component of a force of magnitude F in a certain direction is $F \cos\theta$, where θ is the size of the angle between the force and the direction.
3. To solve problems involving inclined planes, it is usually easier to resolve parallel to and at right angles to the plane.
4. The maximum or limiting value of the friction between two surfaces, F_{MAX}, is given by $\mathbf{F_{MAX} = \mu R}$ where μ is the coefficient of friction and R is the normal reaction between the two surfaces.