KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Mechanics 1
Chapter 10 – Forces and Motion

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

8.1 Constructing a Model

Notes

8.2 Modelling Assumptions

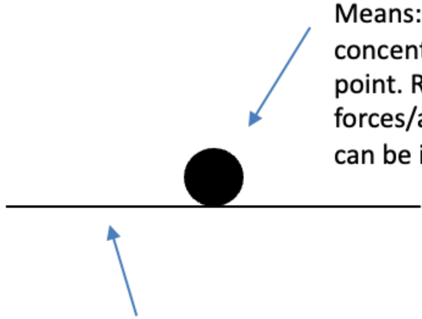
Notes

Modelling Assumptions

As with many areas of applied maths, we often have to make various modelling assumptions, to make the maths cleaner or to use well-known mathematical approaches.
Here are common modelling assumptions often made in Mechanics:

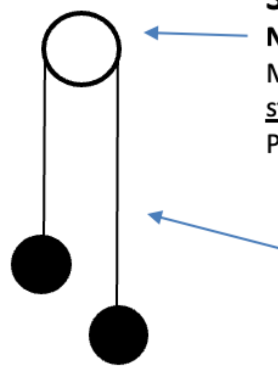
Particle

Dimensions of object are negligible
Means: Mass of object concentrated at single point. Rotational forces/air resistance can be ignored.



Smooth/light pulley

No friction.
Means: <u>Tension the same in string either side of pulley</u>. Pulley has no mass.



Inextensible string

String does not stretch under load.
Means: <u>Acceleration the same</u> in any connected objects.

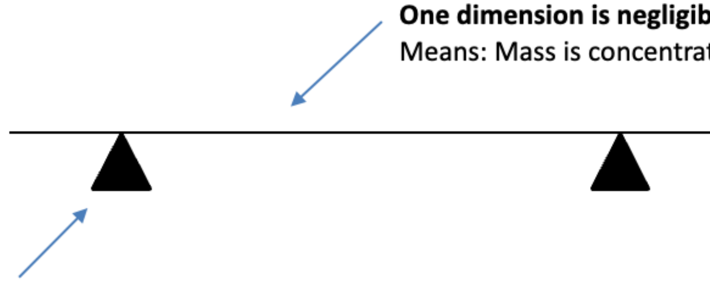
Tip: Particularly make note of underlined text!

Rough/Smooth surface

Means: Objects in contact with surface does/does not experience friction.

Rod

One dimension is negligible, like a pole or beam.
Means: Mass is concentrated along line. Rigid.



Peg/Support

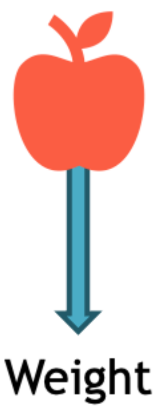
A support from which a body can be suspended or rested.
Means: Dimensionless and fixed. Can be rough or smooth depending on question.

10.1 Force Diagrams

Notes

Newton's 1st Law

In 1687, **Isaac Newton** first stated three laws of motion. In this lesson we will just focus on the first law.



Newton's 1st Law of Motion:
A body remains at rest, or in motion at a constant speed in a straight line, unless there is a resultant force on the object.

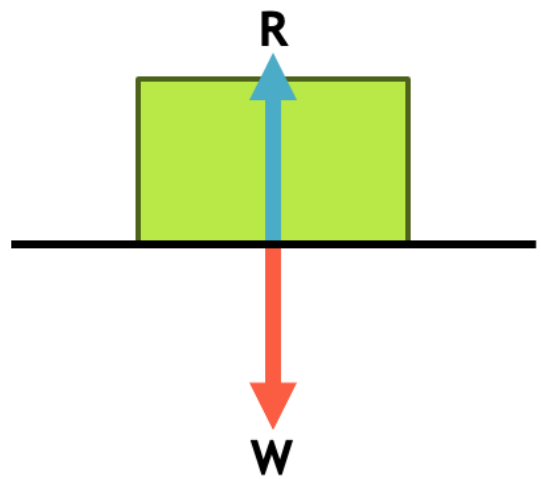
An apple falls off a tree- why is it that the apple accelerates towards the ground?

Any object that has mass also has a weight, which points downwards towards the Earth.

This means there is a resultant force (remember, weight is a force) on the apple, so it accelerates downwards.

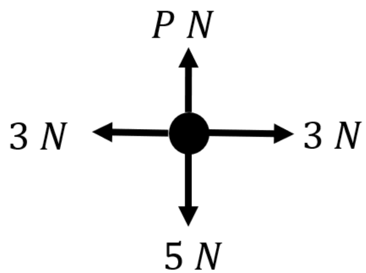
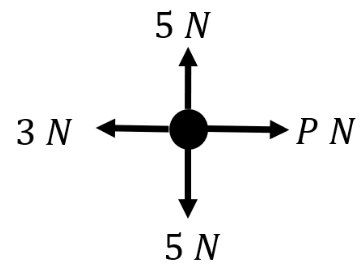
Equilibrium

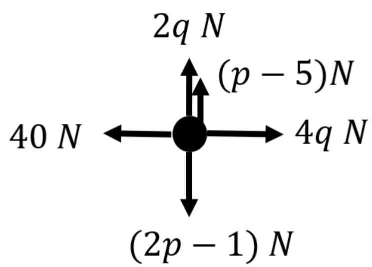
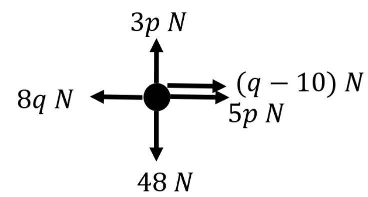
If a box is stationary on the floor, there are two forces acting on it.



Newton's 1st Law of Motion:
A body remains at rest, or in motion at a constant speed in a straight line, unless there is a resultant force on the object.

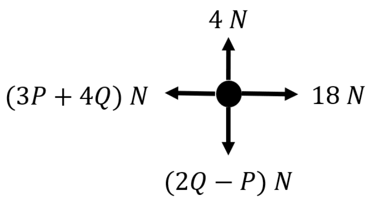
For an object to remain at rest or at a constant speed, it must be in equilibrium (i.e. there is no resultant force).

Worked Example	Your Turn
Draw a force diagram to represent the resultant force: 	Draw a force diagram to represent the resultant force: 

Worked Example	Your Turn
A particle is acted on a by a set of forces. Given that the particle is at rest, find the values of p and q 	A particle is acted on a by a set of forces. Given that the particle is at rest, find the values of p and q 

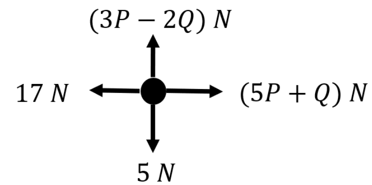
Worked Example

Given that the particle is moving with constant velocity, find the values of P and Q



Your Turn

Given that the particle is moving with constant velocity, find the values of P and Q



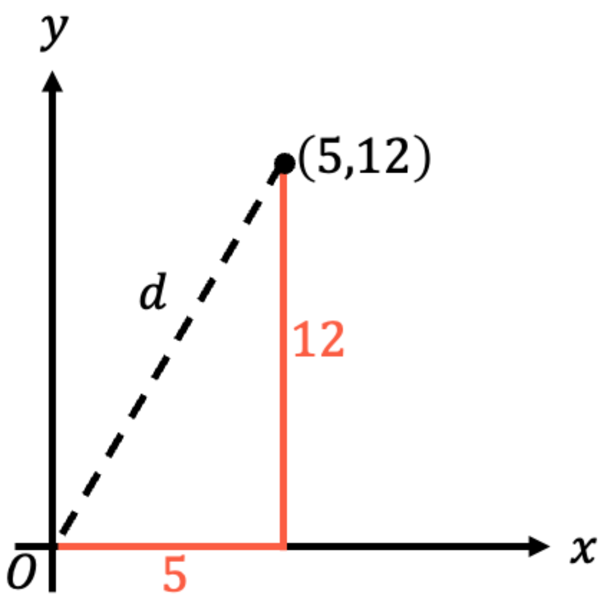
10.2 Forces as Vectors

Notes

Worked Example	Your Turn
The forces $3\boldsymbol{i} - 2\boldsymbol{j}$, $-4\boldsymbol{i} + \boldsymbol{j}$, $-\mathbf{2}\boldsymbol{i} - 3\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b	The forces $2\boldsymbol{i} + 3\boldsymbol{j}$, $4\boldsymbol{i} - \boldsymbol{j}$, $-3\boldsymbol{i} + 2\boldsymbol{j}$ and $a\boldsymbol{i} + b\boldsymbol{j}$ act on an object which is in equilibrium. Find the values of a and b

Dr Frost 554b

Magnitude of a Vector in 2D



Determine the distance of the point with coordinates (5,12) from the origin.

Using Pythagoras' Theorem:

$$\boldsymbol{d = \sqrt{5^2 + 12^2} = 13}$$

We can use the same principle to find the magnitude, i.e. the size/length, of a vector $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$.

$$|\mathbf{a}| = \sqrt{x^2 + y^2}$$

Since force is a vector, then we can find the magnitude of a force vector $\boldsymbol{F} = \begin{pmatrix} x \\ y \end{pmatrix} = x\boldsymbol{i} + y\boldsymbol{j}$

$$|\boldsymbol{F}| = \sqrt{x^2 + y^2}$$

Worked Example	Your Turn
The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(3\boldsymbol{i} - \boldsymbol{j})$ N, $(2\boldsymbol{i} + 3\boldsymbol{j})$ N and $(-4\boldsymbol{i} + \boldsymbol{j})$ N a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$ b) Work out the magnitude and bearing of the resultant force.	The vector i is due east and j due north. A particle begins at rest at the origin. It is acted on by three forces $(2\boldsymbol{i} + \boldsymbol{j})$ N, $(3\boldsymbol{i} - 2\boldsymbol{j})$ N and $(-\boldsymbol{i} + 4\boldsymbol{j})$ N a) Find the resultant force in the form $p\boldsymbol{i} + q\boldsymbol{j}$ b) Work out the magnitude and bearing of the resultant force.

Dr Frost 554a and 554c

Worked Example	Your Turn
Three forces F_1, F_2 and F_3 acting on a particle P are: $F_1 = (9\boldsymbol{i} - 7\boldsymbol{j})\ N$ $F_2 = (6\boldsymbol{i} + 5\boldsymbol{j})\ N$ $F_3 = (p\boldsymbol{i} + q\boldsymbol{j})\ N$ where p and q are constants. Given that P is in equilibrium, a) Find the value of p and the value of q The force F_3 is now removed. The resultant of F_1 and F_2 is R. Find: b) The magnitude of R c) The angle, to the nearest degree, that the direction of R makes with $\boldsymbol{j}$	Three forces F_1, F_2 and F_3 acting on a particle P are: $F_1 = (7\boldsymbol{i} - 9\boldsymbol{j})\ N$ $F_2 = (5\boldsymbol{i} + 6\boldsymbol{j})\ N$ $F_3 = (p\boldsymbol{i} + q\boldsymbol{j})\ N$ where p and q are constants. Given that P is in equilibrium, a) Find the value of p and the value of q The force F_3 is now removed. The resultant of F_1 and F_2 is R. Find: b) The magnitude of R c) The angle, to the nearest degree, that the direction of R makes with $\boldsymbol{j}$

Dr Frost 554d

Parallel Vectors

For vectors to be parallel, they must be <u>scalar multiples</u> of one another.

$$\begin{pmatrix}-3\\5\end{pmatrix}\times -1 = \begin{pmatrix}3\\-5\end{pmatrix}$$

$$\begin{pmatrix}-3\\5\end{pmatrix}\times -2 = \begin{pmatrix}6\\-10\end{pmatrix}$$

$$\begin{pmatrix}-3\\5\end{pmatrix}\times 3 = \begin{pmatrix}-9\\15\end{pmatrix}$$

Any vector in the form $k\begin{pmatrix}-3\\5\end{pmatrix}$ is parallel to $\begin{pmatrix}-3\\5\end{pmatrix}$

Generally,
A vector parallel to $\begin{pmatrix}a\\b\end{pmatrix}$ can be written as $k\begin{pmatrix}a\\b\end{pmatrix}$
Where k is a constant.

Worked Example	Your Turn
Two forces F_1 and F_2 acting on a particle P are: $F_1 = (3\boldsymbol{i} - 2\boldsymbol{j})\ N$ $F_2 = (p\boldsymbol{i} + 3p\boldsymbol{j})\ N$ where p is a positive constant. a) Find the angle between F_2 and $\boldsymbol{i}$ The resultant of F_1 and F_2 is R b) Given that R is parallel to $\boldsymbol{j}$, find the value of p	Two forces F_1 and F_2 acting on a particle P are: $F_1 = (\boldsymbol{i} - 3\boldsymbol{j})\ N$ $F_2 = (p\boldsymbol{i} + 2p\boldsymbol{j})\ N$ where p is a positive constant. a) Find the angle between F_2 and $\boldsymbol{j}$ The resultant of F_1 and F_2 is R b) Given that R is parallel to $\boldsymbol{i}$, find the value of p

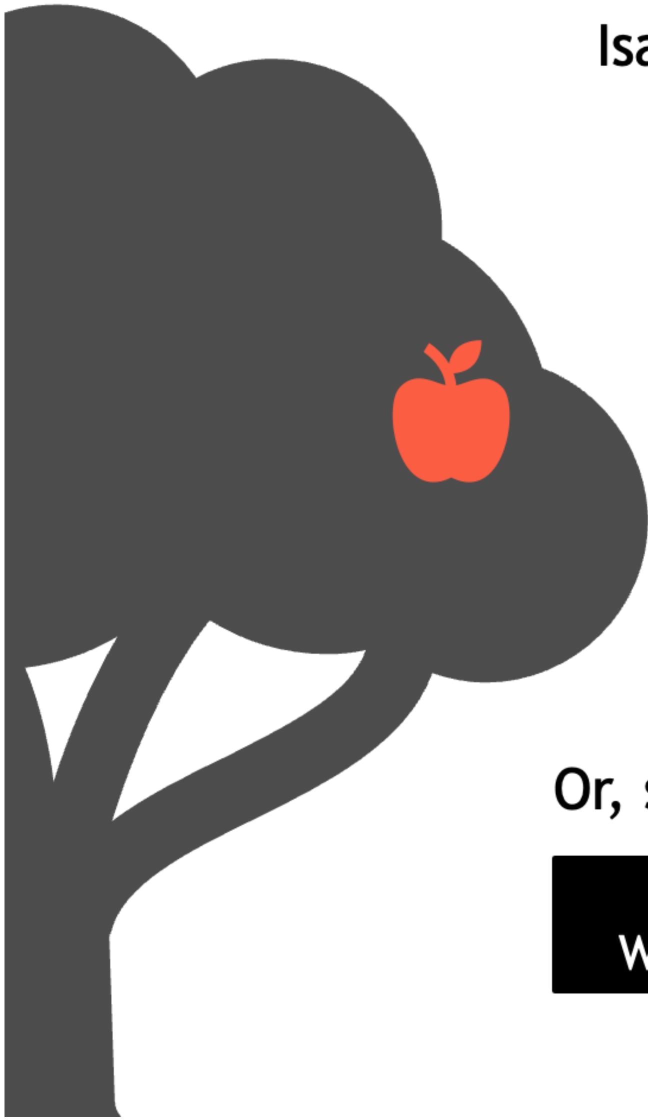
Worked Example	Your Turn
Two forces F_1 and F_2 acting on a particle P are: $F_1 = (3\boldsymbol{i} - 2\boldsymbol{j})\ N$ $F_2 = (p\boldsymbol{i} + 3p\boldsymbol{j})\ N$ where p is a positive constant. The resultant of F_1 and F_2 is R Given that R is parallel to $13\boldsymbol{i} + 10\boldsymbol{j}$, find the value of p	Two forces F_1 and F_2 acting on a particle P are: $F_1 = (2\boldsymbol{i} - 3\boldsymbol{j})\ N$ $F_2 = (p\boldsymbol{i} + 2p\boldsymbol{j})\ N$ where p is a positive constant. The resultant of F_1 and F_2 is R Given that R is parallel to $12\boldsymbol{i} + 11\boldsymbol{j}$, find the value of p

Dr Frost 554e

10.3 Forces and Acceleration

Notes

Newton's 2nd Law



Isaac Newton was famously inspired to write his theory of gravitation after watching an apple fall from a tree.

Why is it that the apple falls downwards?

There is a resultant force acting on the apple that causes it to accelerate.

Newton's 2nd Law of Motion:
The resultant force on a body is equal to its mass multiplied by its acceleration.

Or, simply:

$$F = ma$$

Where F is the resultant force on the body.

Worked Example	Your Turn
A car of 1000kg has a driving force of 1600N and forces of 400N resisting its motion. Determine its acceleration.	A car of 2000kg has a driving force of 800N and forces of 200N resisting its motion. Determine its acceleration.

Dr Frost 555a

Worked Example	Your Turn
An object of mass 140kg experiences air resistance of 600 N. Determine the object's acceleration as it falls towards the ground.	An object of mass 70kg experiences air resistance of 300 N. Determine the object's acceleration as it falls towards the ground.

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Worked Example	Your Turn
An adult has a mass of 100kg. What is the gravitational force (weight) acting on the adult?	A child has a mass of 50kg. What is the gravitational force (weight) acting on the child?

Worked Example	Your Turn
A body of mass 10kg is pulled along a rough horizontal table by a horizontal force of magnitude 40N against a constant friction force of magnitude 8N. Given that the body is initially at rest, find: a) the acceleration of the body b) the distance travelled by the body in the first 2 seconds c) the magnitude of the normal reaction between the body and the table	A body of mass 5kg is pulled along a rough horizontal table by a horizontal force of magnitude 20N against a constant friction force of magnitude 4N. Given that the body is initially at rest, find: a) the acceleration of the body b) the distance travelled by the body in the first 4 seconds c) the magnitude of the normal reaction between the body and the table

Dr Frost 555d

Worked Example	Your Turn
An objects of mass $8\ kg$ hits soft ground at a speed of $14\ ms^{-1}$ and sinks vertically downwards before coming to rest. The ground is assumed to exert a constant resistive force of magnitude $5000\ N$. Find the vertical distance that the object sinks into the ground before coming to rest.	An objects of mass $4\ kg$ hits soft ground at a speed of $28\ ms^{-1}$ and sinks vertically downwards before coming to rest. The ground is assumed to exert a constant resistive force of magnitude $5000\ N$. Find the vertical distance that the object sinks into the ground before coming to rest.

Worked Example	Your Turn
A lift of mass $500\ kg$ is lowered or raised by a metal cable attached to its top. The lift contains passengers whose total mass is $100\ kg$. The lift starts from rest and accelerates at a constant rate, reaching a speed of $5\ ms^{-1}$ after moving a distance of $4\ m$. Find: a) The acceleration of the lift b) The tension in the cable if the lift is moving vertically downwards c) The tension in the cable if the lift is moving vertically upwards	A lift of mass $400\ kg$ is lowered or raised by a metal cable attached to its top. The lift contains passengers whose total mass is $200\ kg$. The lift starts from rest and accelerates at a constant rate, reaching a speed of $4\ ms^{-1}$ after moving a distance of $5\ m$. Find: a) The acceleration of the lift b) The tension in the cable if the lift is moving vertically downwards c) The tension in the cable if the lift is moving vertically upwards

10.4 Motion in 2 Dimensions

Notes

Newton's 2nd Law in Vector Form

Previously, we stated Newton's 2nd Law as:

$$F = ma$$

We can extend this to apply to vectors:

$$\underline{F} = m\underline{a}$$

As before, remember:

$\underline{F}$ is the <u>resultant force</u> vector.

Worked Example	Your Turn
Let $\boldsymbol{i}$ represent East and $\boldsymbol{j}$ North. A resultant force of $(2\boldsymbol{i} + 7\boldsymbol{j})\ N$ acts upon a particle of mass 0.25 kg. a) Find the acceleration of the particle in the form $(p\boldsymbol{i} + q\boldsymbol{j})$ ms^{-2} b) Find the magnitude and bearing of the acceleration of the particle.	Let $\boldsymbol{i}$ represent East and $\boldsymbol{j}$ North. A resultant force of $(3\boldsymbol{i} + 8\boldsymbol{j})\ N$ acts upon a particle of mass 0.5 kg. a) Find the acceleration of the particle in the form $(p\boldsymbol{i} + q\boldsymbol{j})\ ms^{-2}$ b) Find the magnitude and bearing of the acceleration of the particle.

Worked Example	Your Turn
A boat is modelled as a particle of mass 30 kg being acted on by three forces. $F_1 = \begin{pmatrix} 25 \\ 40 \end{pmatrix} N$, $F_2 = \begin{pmatrix} 5q \\ 10q \end{pmatrix} N$, $F_3 = \begin{pmatrix} 50 \\ -37.5 \end{pmatrix} N$ Given that the boat is accelerating at a rate of $\begin{pmatrix} -0.75 \\ 0.4 \end{pmatrix}$ ms^{-2}, find the values of p and q.	A boat is modelled as a particle of mass 60 kg being acted on by three forces. $F_1 = \begin{pmatrix} 80 \\ 50 \end{pmatrix} N$, $F_2 = \begin{pmatrix} 10q \\ 20q \end{pmatrix} N$, $F_3 = \begin{pmatrix} -75 \\ 100 \end{pmatrix} N$ Given that the boat is accelerating at a rate of $\begin{pmatrix} 0.8 \\ -1.5 \end{pmatrix}$ ms^{-2}, find the values of p and q.

Worked Example	Your Turn
A particle of mass $5\ kg$ start from rest and is acted upon by a force R of $(4\boldsymbol{i} + k\boldsymbol{j})\ N$. R acts on a bearing of $45°$. Find the value of k	A particle of mass $4\ kg$ start from rest and is acted upon by a force R of $(5\boldsymbol{i} + k\boldsymbol{j})\ N$. R acts on a bearing of $135°$. Find the value of k

Worked Example	Your Turn
Two forces, $\begin{pmatrix}5\\2\end{pmatrix}$ N and $\begin{pmatrix}p\\q\end{pmatrix}$ N act on a particle of mass m kg. The resultant of the two forces is R a) Given that R acts in a direction which is parallel to the vector $\begin{pmatrix}-1\\2\end{pmatrix}$, show that $2p + q + 12 = 0$ b) Given also that $p = 1$ and that P moves with an acceleration of magnitude $10\sqrt{5}\ ms^{-2}$, find the value of m	Two forces, $\begin{pmatrix}3\\4\end{pmatrix}$ N and $\begin{pmatrix}p\\q\end{pmatrix}$ N act on a particle of mass m kg. The resultant of the two forces is R a) Given that R acts in a direction which is parallel to the vector $\begin{pmatrix}-2\\1\end{pmatrix}$, show that $2q + p + 11 = 0$ b) Given also that $p = 5$ and that P moves with an acceleration of magnitude $40\sqrt{5}\ ms^{-2}$, find the value of m

10.5 Connected Particles

Notes

Newton's Laws So Far

We have previously met Newton's 1st and 2nd Laws

Newton's 1st Law of Motion:
A body remains at rest, or in motion at a constant speed in a straight line, unless there is a resultant force on the object.

Newton's 2nd Law of Motion:
The resultant force on a body is equal to its mass multiplied by its acceleration.

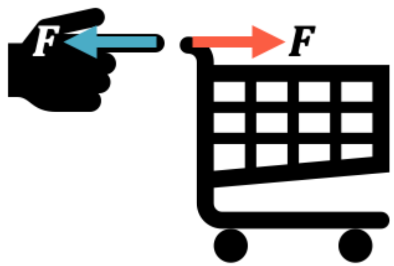
Or:

$$F = ma$$

Where F is the <u>resultant force</u> on an object.

Newton's 3rd Law

When you push a shopping trolley, you exert a force on the handle.

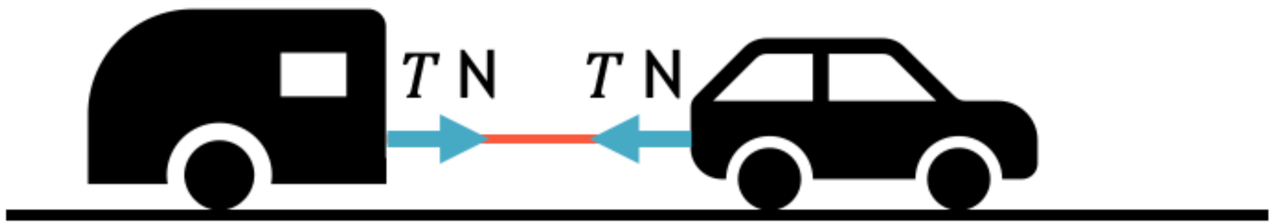


You would feel a force on your hand when you push the trolley- this is an equally sized force in the opposite direction acting on your hand.

This is Newton's 3rd Law in action:

Newton's 3rd Law of Motion:
When object A exerts a force on object B, object B exerts a force of equal size on object A, in the opposite direction.

We can use Newton's 3rd Law to better understand tension in a connection.



If the car exerts a force T on the caravan, the caravan must exert a force T on the car, in an equal and opposite direction.

Worked Example	Your Turn
Two particles, P and Q, of masses 6kg and 4kg respectively, are connected by a light inextensible string. Particle Q is pulled by a horizontal force of magnitude 20N along a rough horizontal plane. Particle Q experiences a frictional force of 5N and particle P experiences a frictional force of 3N. a) Find the acceleration of the particles. b) Find the tension in the string.	Two particles, P and Q, of masses 5kg and 3kg respectively, are connected by a light inextensible string. Particle P is pulled by a horizontal force of magnitude 40N along a rough horizontal plane. Particle P experiences a frictional force of 10N and particle Q experiences a frictional force of 6N. (a) Find the acceleration of the particles. (b) Find the tension in the string.

Dr Frost 556a and 556b

Worked Example	Your Turn
A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 300g and the mass of B is 200g. A rests on top of B. The scale-pan is raised vertically, using the string, with acceleration 0.25 ms^{-2}. a) Find the tension in the string. b) Find the force exerted on mass B by mass A. c) Find the force exerted on mass B by the scale-pan.	A light scale-pan is attached to a vertical light inextensible string. The scale-pan carries two masses A and B. The mass of A is 400g and the mass of B is 600g. A rests on top of B. The scale-pan is raised vertically, using the string, with acceleration 0.5 ms^{-2}. (a) Find the tension in the string. (b) Find the force exerted on mass B by mass A. (c) Find the force exerted on mass B by the scale-pan.

Dr Frost 556c

Worked Example	Your Turn
A person travels in a lift. The mass of the person is $40\ kg$ and the mass of the lift is $860\ kg$. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration $4\ ms^{-2}$. By modelling the cable as being light and inextensible, find: a) The tension in the cable. b) The magnitude of the force exerted on the woman by the floor of the lift.	A person travels in a lift. The mass of the person is $50\ kg$ and the mass of the lift is $950\ kg$. The lift is being raised vertically by a vertical cable which is attached to the top of the lift. The lift is moving upwards and has constant deceleration $2\ ms^{-2}$. By modelling the cable as being light and inextensible, find: a) The tension in the cable. b) The magnitude of the force exerted on the woman by the floor of the lift.

Worked Example

A car of mass $1200\ kg$ pulls a trailer of mass $400\ kg$ along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes $400\ N$ and $200\ N$ respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude $2000\ N$.

a) Find the acceleration of the car and trailer.

b) Find the magnitude of the tension in the tow-bar.

The engine cuts out, reducing the force produced by the engine to zero and the brakes are applied. The brakes produce a force on the car of magnitude F Newtons and the car and trailer decelerate.

c) Given that the resistances to motion are unchanged, and the magnitude of the thrust in the towbar is $300\ N$, find the value of F

Your Turn

A car of mass $600\ kg$ pulls a trailer of mass $200\ kg$ along a straight horizontal road using a light tow-bar which is parallel to the road. The horizontal resistances to motion of the car and the trailer have magnitudes $300\ N$ and $100\ N$ respectively. The engine of the car produces a constant horizontal driving force on the car of magnitude $1600\ N$.

a) Find the acceleration of the car and trailer.

b) Find the magnitude of the tension in the tow-bar.

The engine cuts out, reducing the force produced by the engine to zero and the brakes are applied. The brakes produce a force on the car of magnitude F Newtons and the car and trailer decelerate.

c) Given that the resistances to motion are unchanged, and the magnitude of the thrust in the towbar is $200\ N$, find the value of F

10.6 Pulleys

Notes

Pulleys – Modelling Assumptions

EXAM ADVICE

Ensure it is clear in your mind what the following assumptions mean:

Light	**Tension is equal** either side of the pulley.
Inextensible	**Acceleration is equal** throughout the system

If you are asked to provide one modelling assumption, you will often lose a mark for stating more than one assumption.

Worked Example

Particles P and Q, of masses $5m$ and $4m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

a) Write down an equation of motion for P and for Q

b) Find the acceleration of each mass.

c) Find the tension in the string.

d) Find the force exerted on the pulley by the string.

e) Find the distance moved by P in the first 2 s, assuming that Q does not reach the pulley.

Your Turn

Particles P and Q, of masses $2m$ and $3m$, are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.

a) Write down an equation of motion for P and for Q

b) Find the acceleration of each mass.

c) Find the tension in the string.

d) Find the force exerted on the pulley by the string.

e) Find the distance moved by Q in the first 4 s, assuming that P does not reach the pulley.

Worked Example

Two particles A and B of masses 0.8kg and 1.6kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 9m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 1m above horizontal ground. A frictional force of magnitude 0.16g opposes the motion of particle A. The system is released from rest. Find:

a) The acceleration of the system.

b) The time taken for B to reach the ground.

c) The total distance travelled by A before it first comes to rest.

Dr Frost 556h and 557b

Your Turn

Two particles A and B of masses 0.4kg and 0.8kg respectively are connected by a light inextensible string. Particle A lies on a rough horizontal table 4.5m from a small smooth pulley which is fixed at the edge of the table. The string passes over the pulley and B hangs freely, with the string taut, 0.5m above horizontal ground. A frictional force of magnitude $0.08g$ opposes the motion of particle A. The system is released from rest. Find:

a) The acceleration of the system.
b) The time taken for B to reach the ground.
c) The total distance travelled by A before it first comes to rest.

Worked Example

Two particles A and B have masses $10m$ and km respectively, where $k < 10$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane.

The system is released from rest.

After release, A descends with acceleration $\frac{1}{2}g$

After descending for 2.4 s, the particle A reaches the plane.

It is immediately brought to rest by the impact with the plane.

The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

Find the greatest height reached by B above the plane.

Dr Frost 557c and 557d

Your Turn

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane.

The system is released from rest.

After release, A descends with acceleration $\frac{1}{4}g$

After descending for 1.2 s, the particle A reaches the plane.

It is immediately brought to rest by the impact with the plane.

The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

Find the greatest height reached by B above the plane.

Worked Example

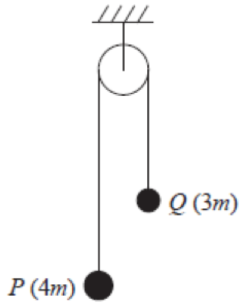


Figure 1

One end of a string is attached to a small ball P of mass $4m$.

The other end of the string is attached to another small ball Q of mass $3m$.

The string passes over a fixed pulley.

Ball P is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

Ball P is released.

The string is modelled as being light and inextensible, the balls are modelled as particles, the pulley is modelled as being smooth and air resistance is ignored.

(*a*) Using the model, find, in terms of m and g, the magnitude of the force exerted on the pulley by the string while P is falling and before Q hits the pulley.

(8)

(*b*) State one limitation of the model, apart from ignoring air resistance, that will affect the accuracy of your answer to part (a).

(1)

Your Turn

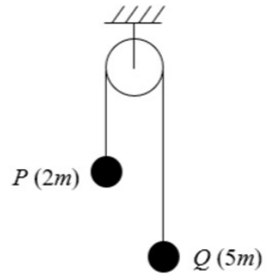


Figure 1

One end of a string is attached to a small ball P of mass $2m$.

The other end of the string is attached to another small ball Q of mass $5m$.

The string passes over a fixed pulley.

Ball Q is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

Ball Q is released.

The string is modelled as being light and inextensible, the balls are modelled as particles, the pulley is modelled as being smooth and air resistance is ignored.

(*a*) Using the model, find, in terms of m and g, the magnitude of the force exerted on the pulley by the string while Q is falling towards the floor and before P hits the pulley.

(8)

(*b*) State one limitation of the model, apart from ignoring air resistance, that will affect the accuracy of your answer to part (a).

(1)

Worked Example

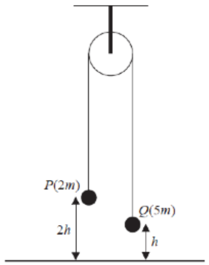


Figure 1

A ball P of mass $2m$ is attached to one end of a string.

The other end of the string is attached to a ball Q of mass $5m$.

The string passes over a fixed pulley.

The system is held at rest with the balls hanging freely and the string taut.

The hanging parts of the string are vertical with P at a height $2h$ above horizontal ground and with Q at a height h above the ground, as shown in Figure 1.

The system is released from rest.

In the subsequent motion, Q does not rebound when it hits the ground and P does not hit the pulley.

The balls are modelled as particles.

The string is modelled as being light and inextensible.

The pulley is modelled as being small and smooth.

Air resistance is modelled as being negligible.

Using this model,

(*a*) (i) write down an equation of motion for P,

(ii) write down an equation of motion for Q,

(4)

(*b*) find, in terms of h only, the height above the ground at which P first comes to instantaneous rest.

(7)

(*c*) State one limitation of modelling the balls as particles that could affect your answer to part (*b*).

(1)

In reality, the string will not be inextensible.

(*d*) State how this would affect the accelerations of the particles.

(1)

Your Turn

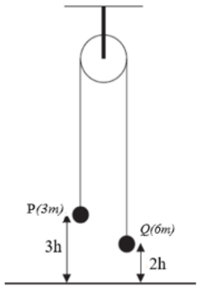


Figure 1

Two particles P and Q of masses $6m$ and $3m$ respectively are attached with a light inextensible string.

The string passes over a fixed pulley. The pulley is modelled as being small and smooth.

The system is held at rest with the balls hanging freely and the string taut.

The hanging parts of the string are vertical with P at a height $3h$ above horizontal ground and with Q at a height $2h$ above the ground, as shown in Figure 1.

The system is released from rest.

In the subsequent motion, Q does not rebound when it hits the ground and P does not hit the pulley.

Air resistance is ignored.

Using this model,

(*a*) (i) write down an equation of motion for P,

(ii) write down an equation of motion for Q,

(4)

(*b*) find, in terms of h only, the height above the ground at which P first comes to instantaneous rest.

(7)

(*c*) State how taking into account air resistance would (or could) affect your answer to part (b).

(1)

In reality, the string will not be inextensible.

(*d*) State how this would affect the accelerations of the particles.

(1)

Worked Example

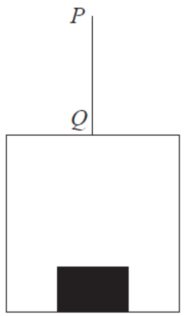


Figure 1

A vertical rope PQ has its end Q attached to the top of a small lift cage.

The lift cage has mass 40 kg and carries a block of mass 10 kg, as shown in Figure 1.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration 0.2 m s^{-2}

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

(*a*) find the tension in the rope PQ

(3)

(*b*) find the magnitude of the force exerted on the block by the lift cage.

(3)

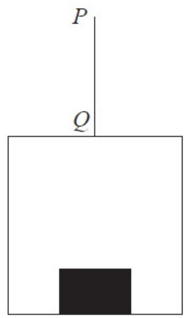


Figure 1

A vertical rope PQ has its end Q attached to the top of a small lift cage.

The lift cage has mass 100 kg and carries a child of mass 50 kg, as shown in Figure 1.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration 0.3 m s^{-2}

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

(*a*) find the tension in the rope PQ

(3)

(*b*) find the magnitude of the force exerted on the child by the lift cage.

(3)

Worked Example

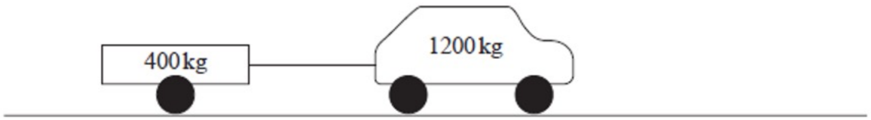


Figure 2

A car of mass 1200 kg is towing a trailer of mass 400 kg along a straight horizontal road using a tow rope, as shown in Figure 2.
The rope is horizontal and parallel to the direction of motion of the car.

- The resistance to motion of the car is modelled as a constant force of magnitude $2R$ newtons
- The resistance to motion of the trailer is modelled as a constant force of magnitude R newtons
- The rope is modelled as being light and inextensible
- The acceleration of the car is modelled as a m s^{-2}

The driving force of the engine of the car is 7400 N and the tension in the tow rope is 2400 N.

Using the model,

(*a*) find the value of a

(5)

In a refined model, the rope is modelled as having mass and the acceleration of the car is found to be a_1 m s^{-2}

(*b*) State how the value of a_1 compares with the value of a

(1)

(*c*) State one limitation of the model used for the resistance to motion of the car.

(1)

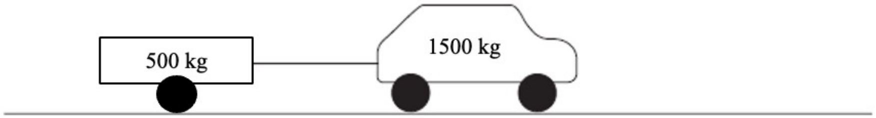


Figure 2

A car of mass 1500 kg is towing a trailer of mass 500 kg along a straight horizontal road using a tow rope, as shown in Figure 2.

The rope is horizontal and parallel to the direction of motion of the car.

- The resistance to motion of the car is modelled as a constant force of magnitude $2.5R$ newtons
- The resistance to motion of the trailer is modelled as a constant force of magnitude $1.5R$ newtons
- The rope is modelled as being light and inextensible
- The acceleration of the car is modelled as $a\,\mathrm{m\,s^{-2}}$

The driving force of the engine of the car is 8000 N and the tension in the tow rope is 2500 N.

Using the model,

(a) find the value of a

(5)

In a refined model, the rope is modelled as having mass and the acceleration of the car is found to be $a_1\,\mathrm{m\,s^{-2}}$

(b) State how the value of a_1 compares with the value of a

(1)

(c) State one limitation of the model used for the resistance to motion of the car.

(1)

Worked Example

[*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*.]

A particle P is moving on a smooth horizontal surface under the action of two forces.

Given that

- the mass of P is 2 kg
- the two forces are $(2\mathbf{i} + 4\mathbf{j})$ N and $(c\mathbf{i} - 2\mathbf{j})$ N, where c is a constant
- the magnitude of the acceleration of P is $\sqrt{5}$ m s^{-2}

find the two possible values of c.

(5)

Your Turn

[*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A particle P is moving on a smooth horizontal surface under the action of two forces.

Given that

- the mass of P is 4 kg
- the two forces are $(-3\mathbf{i} + 3\mathbf{j})$ N and $(c\mathbf{i} - 7\mathbf{j})$ N, where c is a constant
- the magnitude of the acceleration of P is $\sqrt{5}$ m s^{-2}

find the two possible values of c.

(5)

Worked Example

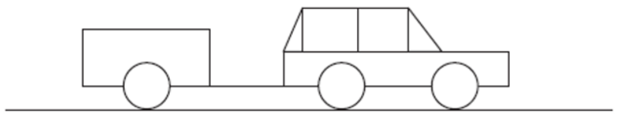


Figure 2

Figure 2 shows a car towing a trailer along a straight horizontal road.

The mass of the car is 800 kg and the mass of the trailer is 600 kg.

The trailer is attached to the car by a towbar which is parallel to the road and parallel to the direction of motion of the car and the trailer.

The towbar is modelled as a light rod.

The resistance to the motion of the car is modelled as a constant force of magnitude 400 N.

The resistance to the motion of the trailer is modelled as a constant force of magnitude R newtons.

The engine of the car is producing a constant driving force that is horizontal and of magnitude 1740 N.

The acceleration of the car is 0.6 m s^{-2} and the tension in the towbar is T newtons.

Using the model,

(*a*) show that $R = 500$

(3)

(*b*) find the value of T.

(3)

At the instant when the speed of the car and the trailer is 12.5 m s^{-1}, the towbar breaks.

The trailer moves a further distance d metres before coming to rest.

The resistance to the motion of the trailer is modelled as a constant force of magnitude 500 N.

Using the model,

(*c*) show that, after the towbar breaks, the deceleration of the trailer is $\frac{5}{6} \text{ m s}^{-2}$

(1)

(*d*) find the value of d.

(3)

In reality, the distance d metres is likely to be different from the answer found in part (*d*).

(*e*) Give two **different** reasons why this is the case.

(2)

Your Turn

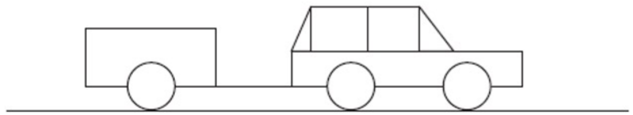


Figure 2

Figure 2 shows a car towing a trailer along a straight horizontal road.

The mass of the car is 900 kg and the mass of the trailer is 500 kg.

The trailer is attached to the car by a towbar which is parallel to the road and parallel to the direction of motion of the car and the trailer.

The towbar is modelled as a light rod.

The resistance to the motion of the car is modelled as a constant force of magnitude 450 N.

The resistance to the motion of the trailer is modelled as a constant force of magnitude R newtons.

The engine of the car is producing a constant driving force that is horizontal and of magnitude 1610 N.

The acceleration of the car is 0.4 m s^{-2} and the tension in the towbar is T newtons.

Using the model,

(*a*) show that $R = 600$

(3)

(*b*) find the value of T.

(3)

At the instant when the speed of the car and the trailer is 9 m s^{-1}, the towbar breaks.

The trailer moves a further distance d metres before coming to rest.

The resistance to the motion of the trailer is modelled as a constant force of magnitude 600 N.

Using the model,

(*c*) show that, after the towbar breaks, the deceleration of the trailer is 1.2 m s^{-2}

(1)

(*d*) find the value of d.

(3)

In reality, the distance d metres is likely to be different from the answer found in part (*d*).

(*e*) Give two **different** reasons why this is the case.

(2)

Summary

1 Mathematical models can be constructed to simulate real-life situations.

2 Modelling assumptions can be used to simplify your calculations.

1 **Newton's first law** of motion states that an object at rest will stay at rest and that an object moving with constant velocity will continue to move with constant velocity unless an unbalanced force acts on the object.

2 A **resultant** force acting on an object will cause the object to **accelerate in the same direction** as the resultant force.

3 You can find the **resultant** of two or more forces given as vectors by adding the vectors.

4 **Newton's second law** of motion states that the force needed to accelerate a particle is equal to the product of the mass of the particle and the acceleration produced: $F = ma$.

5 $W = mg$

6 You can use $\mathbf{F} = m\mathbf{a}$ to solve problems involving vector forces acting on particles.

7 You can solve problems involving connected particles by considering the particles separately or, if they are moving in the same straight line, as a single particle.

8 **Newton's third law** states that for every action there is an equal and opposite reaction.