KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 14 Exponentials and Logarithms – Part 1
Booklet

HGS Maths

Dr Frost Course

Name: ________________________________

Class: ________________________________

Contents

Prior knowledge check

Prior knowledge check

1 Given that $x = 3$ and $y = -1$, evaluate these expressions without a calculator.

a 5^x **b** 3^y **c** 2^{2x-1} **d** 7^{1-y} **e** 11^{x+3y}

← GCSE Mathematics

2 Simplify these expressions, writing each answer as a single power.

a $6^8 \div 6^2$ **b** $y^3 \times (y^9)^2$ **c** $\frac{2^5 \times 2^9}{2^8}$ **d** $\sqrt{x^8}$

← Sections 1.1, 1.4

3 Plot the following data on a scatter graph and draw a line of best fit.

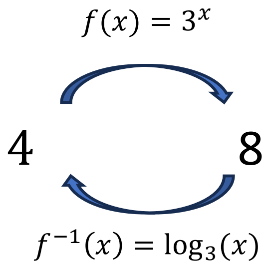
x	1.2	2.1	3.5	4	5.8
y	5.8	7.4	9.4	10.3	12.8

Determine the gradient and intercept of your line of best fit, giving your answers to one decimal place.

← GCSE Mathematics

14.4 Logarithms

Logarithms are the inverse of exponentials, i.e.



$log_a(n) = x$ is equivalent to $a^x = n$

Example	Non-Example
$3^2 = 9 \Leftrightarrow \log_3(9) = 2$	$3^x \neq x^3$

The output of a log can be a negative, but you <u>cannot</u> log a negative, i.e.

Example	Non-Example
$\log_4\left(-\frac{1}{16}\right) = -4$	$\log_4(-4) = -1$

Notes

Worked Example (DFM 527b)

Write each statement as a logarithm.

a) $3^2 = 9$

b) $2^7 = 128$

c) $64^{\frac{1}{2}} = 8$

Worked Example

Rewrite each statement using a power.

a) $\log_3 81 = 4$

b) $\log_2 \left(\frac{1}{8}\right) = -3$

14.5 Laws of Logarithms

From indices facts: $x^0 = 1 \quad x^1 = x$: $\log_{\mathrm{x}}(1) = 0 \qquad \log_{\mathrm{x}}(x) = 1$

By default $\log(x) \equiv \log_{10} x$ and $\ln(\mathrm{x}) \equiv \log_e x$

<u>LAWS:</u> *Opposite of indices*

$$\mathbf{\log_{x}(a) + \log_{x}(b) \equiv \log_{x}(ab)}$$

$$\mathbf{\log_{\boldsymbol{x}}(\boldsymbol{a}) - \log_{x}(b) \equiv \log_{x}\left(\frac{a}{b}\right)}$$

$$\mathbf{\log_{a}(x^{\boldsymbol{n}}) \equiv \boldsymbol{n}\log_{a}(x)}$$

<u>ANTI LAWS:</u>

$$\log_{\mathrm{x}}(\mathrm{a} \pm \mathrm{b}) \neq \log_{\mathrm{x}}(\mathrm{a}) \pm \log_{\mathrm{x}}(\mathrm{b})$$
$$(\log_{x}(a))^2 \neq 2\log_{\mathrm{x}}(\mathrm{a})$$

Notes

Worked Example (DFM 527f)

Write as a single logarithm.

a) $\log_3 6 + \log_3 7$

b) $\log_2 15 - \log_2 3$

c) $2\log_5 3 + 3\log_5 2$

d) $\log_{10} 3 - 4\log_{10}\left(\frac{1}{2}\right)$

T.323 4E: Qs 1-2, P.11. 4.5: Qs 1-2

Worked Example

Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$.

a) $\log_a(x^2yz^3)$

b) $\log_a\left(\frac{x}{y^3}\right)$

c) $\log_a\left(\frac{x\sqrt{y}}{z}\right)$

d) $\log_a\left(\frac{x}{a^4}\right)$

T.323 4E: Qs 3, P.11. 4.5: Qs 3

Worked Example

527c: **Solve logarithmic equations given in the form** $\log_a x = b$

Find the exact solution of

$$2\log_4(9y+8)+3=9$$

$y =$

Worked Example

Solve the equation $\log_3(x+11) - \log_3(x-5) = 2$

T.323 4E: Qs 4+, P.11. 4.5: Qs 4+

Past Paper Q P2 2021 Q3

Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(Total for question = 3 marks)

Your Turn

3. Using the laws of logarithms, solve the equation

$$\log_2(10y+6)-\log_2(1-5y)=3$$

(3)

(Total for Question 3 is 3 marks)

Past Paper Q AS 2023 Q9

Using the laws of logarithms, solve the equation

$$2\log_5(3x-2) - \log_5 x = 2$$

(Total for question = 5 marks)

Your Turn

9. (a) Show, using the laws of logarithms, that:

$$\log_b 15x - 2\log_b (2x - 3) = 1 \text{ , where } b \text{ is a positive integer}$$

can be written in the form $kx = b\left(px^2 + qx + r\right)$ and k, p, q and r are constants to be found.

(3)

(b) Hence or otherwise, solve $\log_5 15x - 2\log_5 (2x - 3) = 1$

(2)

(Total for Question 9 is 5 marks)

$a = \log_2 x \qquad b = \log_2(x + 8)$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$

(1)

(b) $\log_2(x^2 + 8x)$

(2)

(c) $\log_2\left(8 + \dfrac{64}{x}\right)$

Give your answer in simplest form.

(3)

(Total for question = 6 marks)

Your Turn

6. $a = \log_2 x \qquad b = \log_2(x + 4)$

Express in terms of a and/or b

(a) $\log_2 \dfrac{1}{\sqrt[3]{x}}$

(1)

(b) $\log_2(2x^2 + 8x)$

(2)

(c) $\log_2 \left(\dfrac{16}{x} + 4\right)$

Give your answer in simplest form.

(3)

(Total for Question 6 is 6 marks)

14.6 Solving Equations using Logarithms

Just as you would with undoing a ‘trapped’ root, you must isolate e^x terms and any $\log_a b$ terms, BEFORE using the inverse operation.

Example:

$$3 + \log_2 x = 5$$
$$\log_2 x = 2$$
$$x = 2^2 = 4$$

Non-Example:

$$2^x - 3 = 5$$
$$\log_2(2^x) - 3 = \log_2 5$$
$$x = 3 + \log_2 5$$

Notes

Worked Example

Solve the following equations, giving your answers to 3 decimal places.

a) $3^x = 20$

b) $5^{4x-1} = 61$

T.325 4F: Qs 1, P.118 4.6: Qs 1-2

Worked Example

Solve the equation $5^{2x} - 12(5^x) + 20 = 0$, giving your answer to 3 significant figures.

T.325 4F: Qs 2, P.118 4.6: Qs 3-4

Worked Example

Find the solution to the equation $3^x = 2^{x+1}$, giving your answer to four decimal places.

Worked Example

Solve the equation $3^{x+1} = 4^{x-1}$. Round your answer to 3 decimal places.

Worked Example

Solve the equation $2^x 3^{x+1} = 5$. Give your answer in exact form.

T.325 4F: Qs 3+, P.118 4.6: Qs 5+

Past Paper Q AS 2021 Q2

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x, writing your answer in simplest form.

(Total for question = 3 marks)

Your Turn

2. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{8^{x-1}}{2^{y+2}} = 64$$

express y in terms of x, writing your answer in simplest form.

(3)

(Total for Question 2 is 3 marks)

3. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)

Your Turn

3. (*a*) Given that

$$2 \log (5 - x) = \log (x + 7)$$

show that

$$x^2 - 11x + 18 = 0$$

(3)

(*b*) (i) Write down the roots of the equation

$$x^2 - 11x + 18 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2 \log (5 - x) = \log (x + 7)$$

giving a reason for your answer.

(2)

(Total for Question 3 is 5 marks)

Past Paper Q 2020 P2

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

Your Turn

5. The curve with equation $y = 6 \times 3^x$ meets the curve with equation $y = 60 - 3^{x+2}$ at the point P

Find, using algebra, the exact x coordinate of P.

(4)

(Total for Question 5 is 4 marks)

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

A2 2021 Paper 2

Exponentials and Logs

3. Using the laws of logarithms, solve the equation

$$\log_3(12y+5) - \log_3(1-3y) = 2$$

(3)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

Question	Scheme	Marks	AOs
3	$\log_3(12y+5) - \log_3(1-3y) = 2 \Rightarrow \log_3 \frac{12y+5}{1-3y} = 2$ or e.g. $2 = \log_3 9$	B1 M1 on EPEN	1.1b
	$\log_3 \frac{12y+5}{1-3y} = 2 \Rightarrow \frac{12y+5}{1-3y} = 3^2 \Rightarrow 9 - 27y = 12y + 5 \Rightarrow y = \ldots$ or e.g. $\log_3(12y+5) = \log_3(3^2(1-3y)) \Rightarrow (12y+5) = 3^2(1-3y) \Rightarrow y = \ldots$	M1	2.1
	$y = \frac{4}{39}$	A1	1.1b
		(3)	
			(3 marks)

Summary of Key Points

3 $\log_a n = x$ is equivalent to $a^x = n$ $(a \neq 1)$

4 The laws of logarithms:

- $\log_a x + \log_a y = \log_a xy$ (the multiplication law)
- $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ (the division law)
- $\log_a (x^k) = k \log_a x$ (the power law)

5 You should also learn to recognise the following special cases:

- $\log_a \left(\frac{1}{x}\right) = \log_a (x^{-1}) = -\log_a x$ (the power law when $k = -1$)
- $\log_a a = 1$ $(a > 0, a \neq 1)$
- $\log_a 1 = 0$ $(a > 0, a \neq 1)$

6 Whenever $f(x) = g(x)$, $\log_a f(x) = \log_a g(x)$