KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 13
Pure Mathematics
P2 2 Functions

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

Prior knowledge check

1 Make y the subject of each of the following:

a $5x = 9 - 7y$ **b** $p = \frac{2y + 8x}{5}$

c $5x - 8y = 4 + 9xy$ ← GCSE Mathematics

2 Write each expression in its simplest form.

a $(5x - 3)^2 - 4$ **b** $\frac{1}{2(3x - 5) - 4}$

c $\frac{\frac{x + 4}{x + 2} + 5}{\frac{x + 4}{x + 2} - 3}$ ← GCSE Mathematics

3 Sketch each of the following graphs. Label any points where the graph cuts the x- or y-axis.

a $y = e^x$ **b** $y = x(x + 4)(x - 5)$

c $y = \sin x, 0 \leqslant x \leqslant 360°$ ← Year 1

4 $f(x) = x^2 - 3x$. Find the values of:

a $f(7)$ **b** $f(3)$ **c** $f(-3)$ ← Year 1

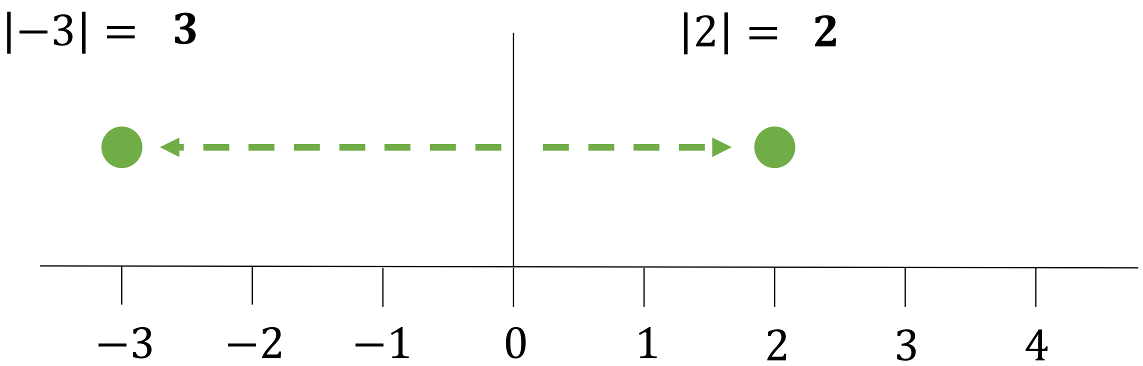
2.1) The modulus function

The **modulus** $|x|$ of a value is its **magnitude/size**. It is also known as the **absolute function**, sometimes written $\text{abs}(x)$

For real numbers, the modulus is simply the **distance from 0**, i.e. the **value with its sign ignored**.

Formal definition (for $x \in \mathbb{R}$):

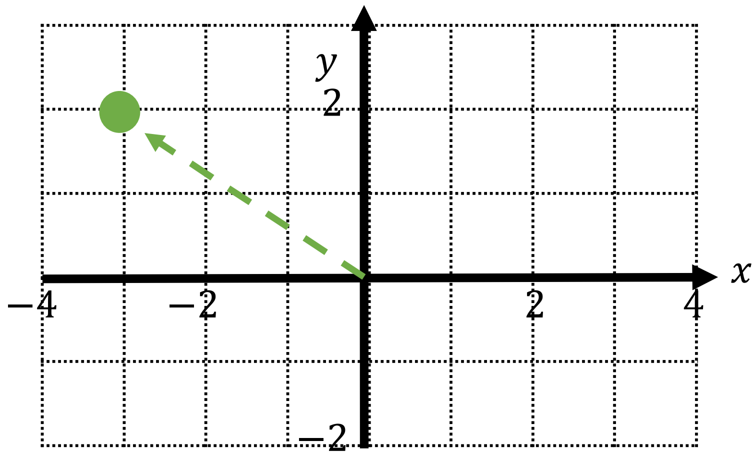
$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



This is useful in Mechanics. e.g. if a displacement is −2m (i.e. $2m$ backwards), the **distance** is simply $|-2| = 2$, i.e. the modulus of displacement.

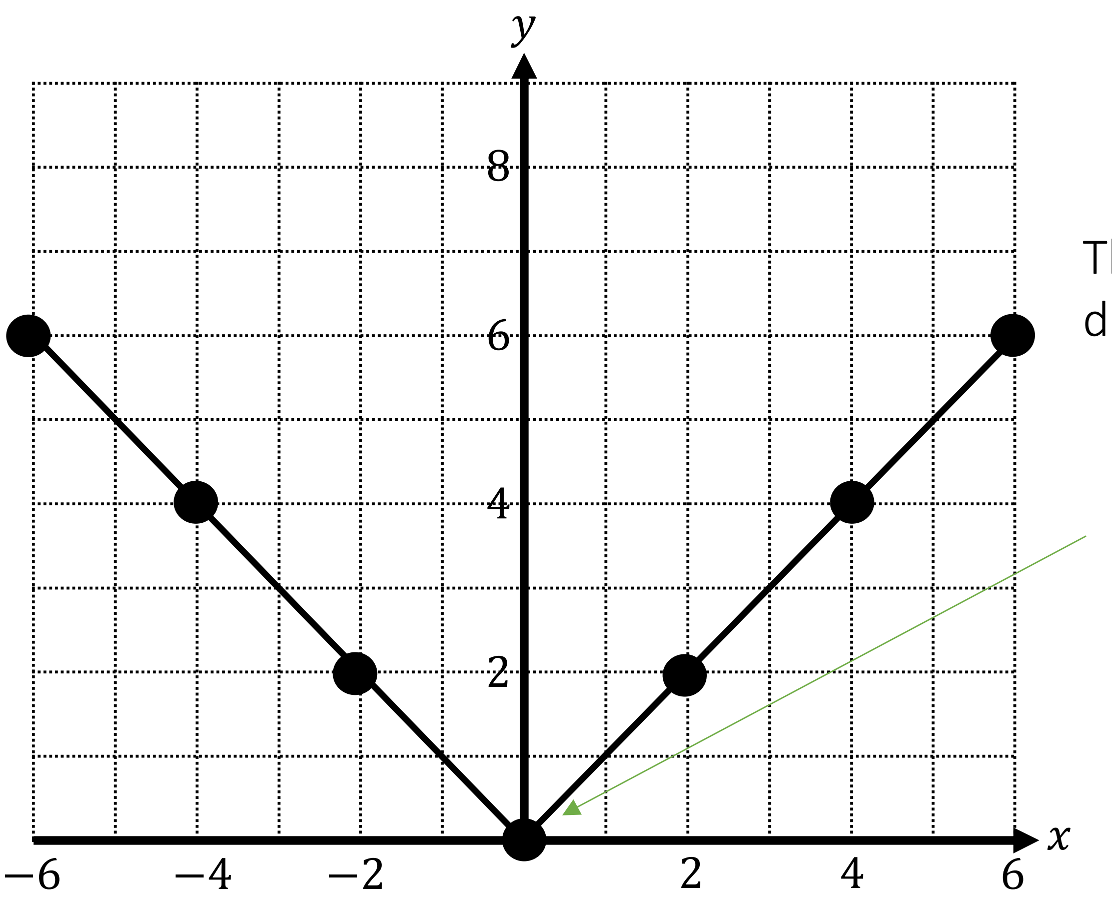
In **2D**, the modulus is the magnitude (size) of the vector, i.e. the distance from $(0,0)$.

$$\left|\begin{pmatrix} -3 \\ 2 \end{pmatrix}\right| = \sqrt{(-3)^2 + 2^2}$$
$$= \sqrt{13}$$



Graph of $y = |x|$

x	-6	-4	-2	0	2	4	6
y	6	4	2	0	2	4	6



The graph of $y = |x|$ has a distinctive 'V' shape.

A point where a graph suddenly changes gradient is known as a cusp.

Like a parabola, we also call this the vertex.

Quickfire Modulus Function

a $|-6| =$?
b $|8| =$?
c $|-7| - 8 =$?
d $|0| =$?
e $\sqrt{x^2} =$?

If $f(x) = |3x - 1| + 2$, determine:

$f(0)$

$f(-5)$

$f(10)$

T.26 : 2A Qs 1-3, P.6: 2.1 Qs 1,2

Worked Example

570c: **Sketch graphs given in the form $y = |ax + b|$**

Plot the graph of $y = \left|-\frac{3}{2}x + 1\right|$ from $x = -2$ to $x = 4$

T.26 : 2A Qs 4, P.6: 2.1 Qs 3

Worked Example

Solve:

$$|3x - 2| = 7$$

Your Turn

Solve:

a) $|5x - 2| = 3 - \frac{1}{3}x$

b) $hence\ solve\colon\ |5x - 2| < 3 - \frac{1}{3}x$

T.26 : 2A Qs 11-13, P.6: 2.1 Qs 5-6

Worked Example

Solve:

$$|x + 3| = 5x + 2$$

T.26 : 2A Qs 5+, P.6: 2.1 Qs 4,7+

Figure 2 shows a sketch of the graph with equation

$$y = 2|x+4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x+4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x+4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

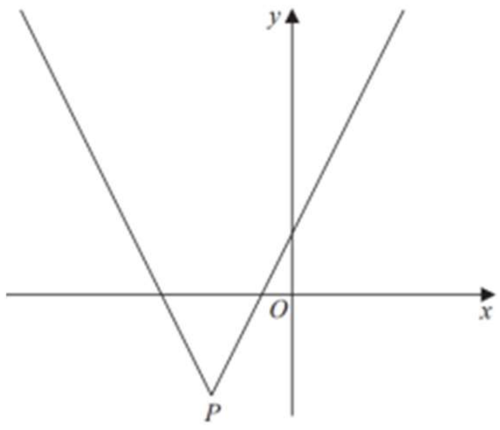


Figure 2

Your Turn

11.

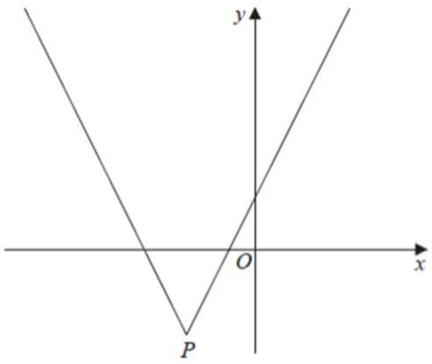


Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 4\,|\,x + 3\,| - 7$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$5x + 60 = 4\,|\,x + 3\,| - 7$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 4\,|\,x + 3\,| - 7$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

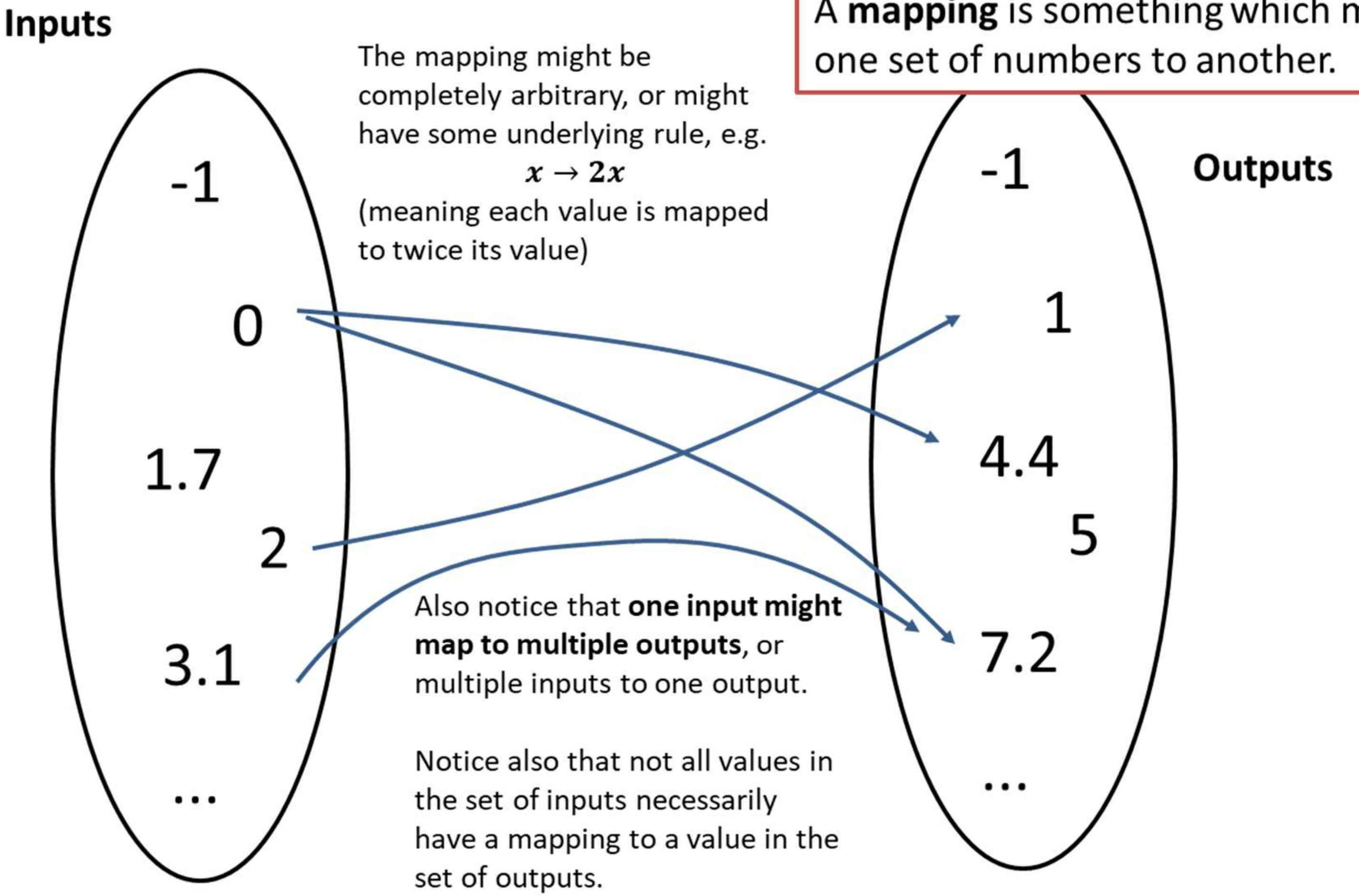
(3)

(Total for Question 11 is 7 marks)

2.2) Functions and mappings

What is a mapping?

> A **mapping** is something which maps one set of numbers to another.



The mapping might be completely arbitrary, or might have some underlying rule, e.g.

$$x \to 2x$$

(meaning each value is mapped to twice its value)

Also notice that **one input might map to multiple outputs**, or multiple inputs to one output.

Notice also that not all values in the set of inputs necessarily have a mapping to a value in the set of outputs.

> The **domain** is the set of possible inputs.

> The **range** is the set of possible outputs.

Notes

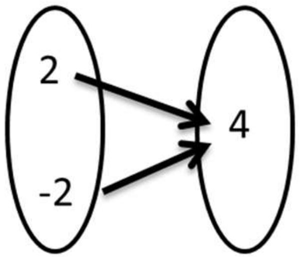
A function is: a mapping such that every element of the domain is mapped to exactly one element of the range.

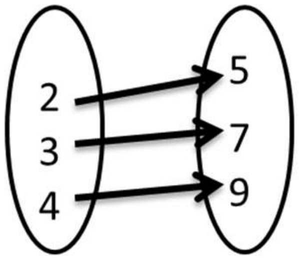
Notation: $f(x) = 2x + 1$ $\qquad$ $f: x \to 2x + 1$

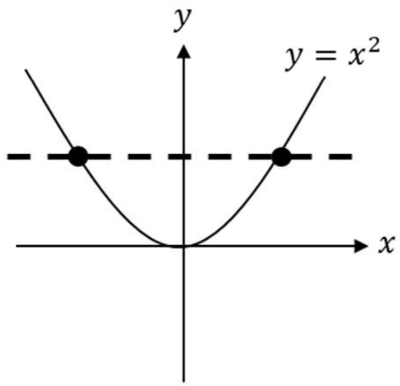
$f(x)$ refers to the **output** of the function.

One-to-one vs Many-to-one

While functions permit an input only to be mapped to one output, there's nothing stopping multiple different inputs mapping to the same output.

<table>
<tr><th>Type</th><th>Description</th><th>Example</th></tr>
<tr><td>Many-to-one function</td><td>Multiple inputs can map to the same output.

</td><td>$f(x) = x^2$

e.g. $f(2) = 4$
$f(-2) = 4$</td></tr>
<tr><td>One-to-one function</td><td>Each output has one input and vice versa.

</td><td>$f(x) = 2x + 1$</td></tr>
</table>



You can use the 'horizontal ray test' to see if a function is one-to-one or many-to-one.

Summary of Domain/Range

It is important that you can identify the range for **<u>common graphs</u>**, **using a suitable sketch**:

$f(x) = x^2, \qquad x \in \mathbb{R}$
Range: $\boldsymbol{f(x) \geq 0}$

$f(x) = \frac{1}{x}, \qquad x \in \mathbb{R}, x \neq 0$
Range: $\boldsymbol{f(x) \neq 0}$

$f(x) = \ln x, \qquad x \in \mathbb{R}, x > 0$
Range: $\boldsymbol{f(x) \in \mathbb{R}}$

$f(x) = e^x, \qquad x \in \mathbb{R},$
Range: $\boldsymbol{f(x) > 0}$

$f(x) = x^2 + 2x + 9, \qquad x \in \mathbb{R}$
Range: $\boldsymbol{f(x) \geq 8}$

Be careful in noting the domain – it may be 'restricted', which similarly restricts the range. Again, use a sketch!

$f(x) = x^2, \qquad x \in \mathbb{R}, -1 \leq x \leq 4$
Range: $\boldsymbol{0 \leq f(x) \leq 16}$

Notes

Exercise

State whether:

- the mapping is one-to-one, many-to-one, or one-to-many
- the mapping is a function

$$f(x) = 2x - 3, \qquad x \in \mathbb{R}$$

$$p(x) = x^3, \qquad x \in \mathbb{R}$$

$$g(x) = x^2, \qquad x \in \mathbb{R}$$

$$q(x) = \left|\frac{1}{x}\right|, \qquad x \in \mathbb{R}$$

$$h(x) = \frac{1}{x}, \qquad x \in \mathbb{R}$$

$$r(x) = \sqrt{x}, x \in \mathbb{R}, x \geq 0, \qquad x \in \mathbb{R}$$

$$i(x) = \sqrt{x}, \qquad x \in \mathbb{R}$$

$$s(x) = \pm\sqrt{x}, x \in \mathbb{R}, x \geq 0$$

T.30 : 2B Qs 1-2, P.7: 2.2 Qs 1

Worked example

Find the range of the following functions:

$$f(x) = \frac{1}{x}, \qquad x = \{-1, -2, -3, -4\}$$

$$g(x) = \frac{1}{x-2}, \qquad x \in \mathbb{R}, x \leq 1$$

$$h(x) = \frac{1}{x+3}, \qquad x \in \mathbb{R}, -2 \leq x < 5$$

Your turn

Find the range of the following functions:

$$p(x) = \frac{1}{x}, \qquad x = \{1, 2, 3, 4\}$$

$$q(x) = \frac{1}{x+2}, \qquad x \in \mathbb{R}, x > -1$$

$$r(x) = \frac{1}{x-5}, \qquad x \in \mathbb{R}, -3 < x \leq 4$$

Worked example

Find the range of the following functions:

$$f(x) = \frac{1}{x}, \qquad x \in \mathbb{R}, x \neq 0$$

$$g(x) = \frac{1}{x} + 2, \qquad x \in \mathbb{R}, x \neq 0$$

Your turn

Find the range of the following functions:

$$h(x) = \frac{1}{x} - 3, \qquad x \in \mathbb{R}, x \neq 0$$

T.30 : 2B Qs 3-5, P.7: 2.2 Qs 2-3

Worked Example

564i: Determine the range of a rational function $f(x) = \frac{ax+b}{cx+d}$

$$h(x) = \frac{3x+6}{x-3}, \quad x \geq 8$$

Worked Example

Find the range of the following functions:

$g(x) = e^x - 4, \quad x \in \mathbb{R}, x > 0$

$f(x) = \ln x + 5, \quad x \in \mathbb{R}, x > 0$

$h(x) = -e^x - 3, \quad x \in \mathbb{R}, x \leq 0$

$g(x) = \ln x - 4, \quad x \in \mathbb{R}, x > 0$

Worked Example

564g: Determine the range of a quadratic function by completing the square.

$f(x) = x^2 + 4x - 7, \qquad x \in \mathbb{R}$

Find the range of $f(x)$

Worked Example

The function f is defined by $f(x) = x^2 - 8x + 27$ and has domain $x \geq a$. Given that $f(x)$ is a one-to-one function, find the smallest possible value of the constant a

Worked Example

The function $f(x)$ is defined by

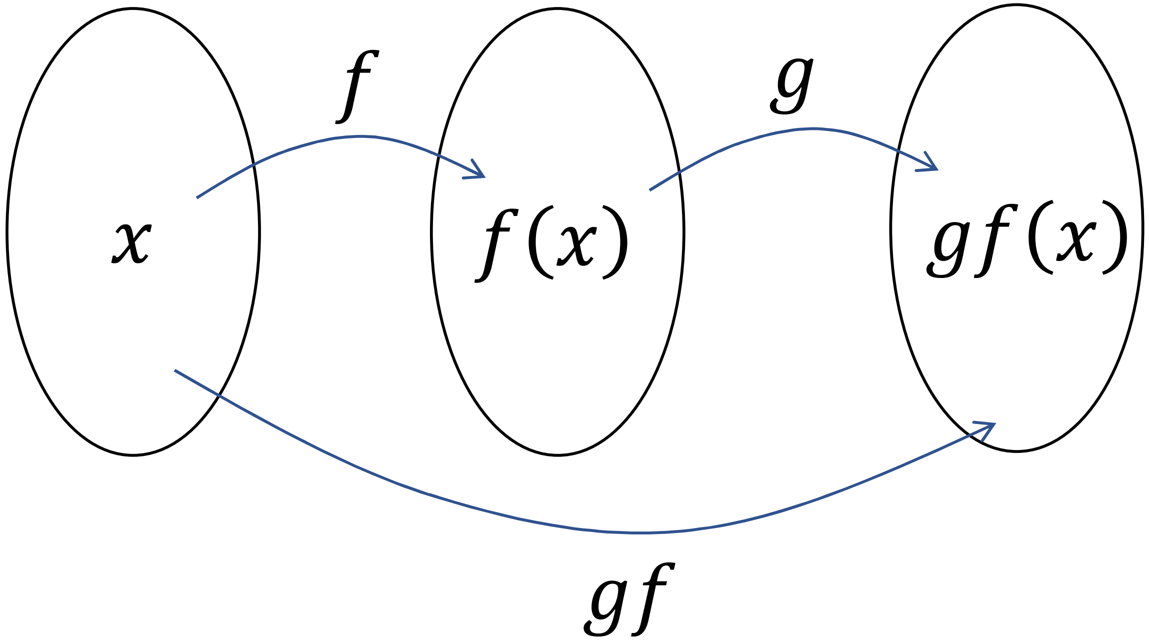
$$f: x \to \begin{cases} 2 - 5x, & x < 1 \\ x^2 - 3, & x \geq 1 \end{cases}$$

a) Sketch $y = f(x)$, and state the range of $f(x)$.

b) Solve $f(x) = 22$

T.30 : 2B Qs 6+, P.7: 2.2 Qs 4+

2.3) Composite functions



$gf(x)$ means $g(f(x))$, i.e. f is applied first, then g.

Sometimes we may apply multiple functions in succession to an input.
These combined functions are known as a **composite function**.

Notes

Worked Example

The functions f and g are defined by

$$f: x \to |3x - 12|$$
$$g: x \to \frac{x+2}{3}$$

a) Find $fg(2)$

b) Solve $fg(x) = x$

T.34 : 2C Qs 3-5, P.8: 2.3 Qs 3-4

Worked Example

The function g is defined by

$$g: x \to 4 - 3x, \qquad x \in \mathbb{R}$$

Solve the equation

$$g^2(x) + [g(x)]^2 = 0$$

Worked Example

The functions f and g are defined by

$$f: x \to e^{3x} - 2, \qquad x \in \mathbb{R}$$
$$g: x \to 4\ln(x+1), \qquad x > -1$$

Find $fg(x)$, giving your answer in its simplest form.

Worked Example

The functions f and g are defined by

$$f: x \to 3^{2x} - 1, \qquad x \in \mathbb{R}$$
$$g: x \to 4\log_3(x+5), \qquad x > -5$$

Find $fg(x)$, giving your answer in its simplest form.

Worked Example

$$f(x) = \frac{1}{x-1}, x \neq 1$$

Find an expression for $f^2(x)$ and $f^3(x)$

Worked Example

A function f has domain $-3 \leq x \leq 12$ and is linear from $(-3, 9)$ to $(0, 6)$ and from $(0, 6)$ to $(12,10)$.
Find the value of $f^2(0)$

T.34 : 2C Qs 6+, P.8: 2.3 Qs 5+

10. The function f is defined by

$$f(x) = \frac{8x + 5}{2x + 3} \qquad x > -\frac{3}{2}$$

(a) Find $f^{-1}\left(\frac{3}{2}\right)$

(2)

(b) Show that

$$f(x) = A + \frac{B}{2x + 3}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$g(x) = 16 - x^2 \qquad 0 \leqslant x \leqslant 4$$

(c) State the range of g^{-1}

(1)

(d) Find the range of $f g^{-1}$

(3)

Your Turn

10. The function f is defined by

$$f(x) = \frac{9x+8}{3x+5} \qquad x > -\frac{5}{3}$$

(*a*) Find $f^{-1}\left(\frac{5}{3}\right)$

(2)

(*b*) Show that

$$f(x) = A + \frac{B}{3x+5}$$

where A and B are constants to be found.

(2)

The function g is defined by

$$g(x) = 9 - x^2 \qquad 0 \leq x \leq 3$$

(*c*) State the range of g^{-1}

(1)

(*d*) Find the range of $f\,g^{-1}$

(3)

(Total for Question 10 is 8 marks)

7. The function f is defined by

$$f(x) = 3 + \sqrt{x-2} \qquad x \in \mathbb{R} \quad x > 2$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{15}{x-3} \qquad x \in \mathbb{R} \quad x \neq 3$$

(c) Find gf(6)

(2)

(d) Find the exact value of the constant a for which

$$f(a^2 + 2) = g(a)$$

(2)

Your Turn

7. The function f is defined by

$$f(x) = 2 + \sqrt{x-3} \qquad x \in \mathbb{R} \qquad x > 3$$

(a) State the range of f

(1)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = \frac{10}{x-2} \qquad x \in \mathbb{R} \qquad x \neq 2$$

(c) Find gf(4)

(2)

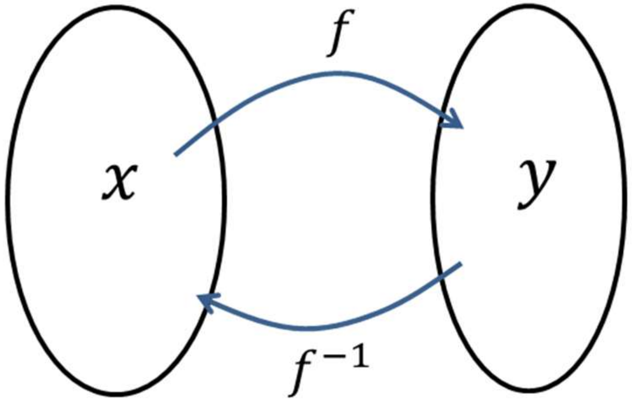
(d) Find the exact value of the constant a for which

$$f(a^2 + 3) = g(a)$$

(2)

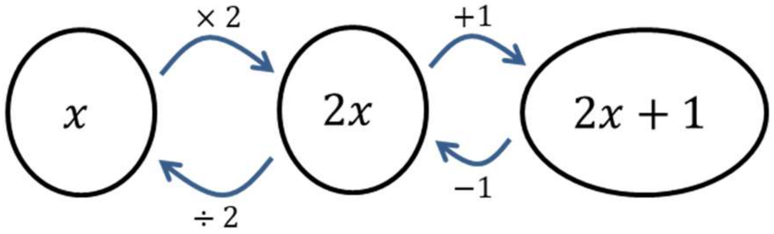
(Total for Question 7 is 8 marks)

2.4) Inverse functions



An inverse function f^{-1} **does the opposite of the original function**. For example, if $f(4) = 2$, then $f^{-1}(2) = 4$.

If $f(x) = 2x + 1$, we could do the opposite operations within the function in reverse order to get back to the original input:



Thus $f^{-1}(x) = \frac{x-1}{2}$

Notation: Just like f^2 means "apply f twice", f^{-1} means "apply f -1 times", i.e. once backwards!
This is why we write $\sin^{-1}(x)$ to mean "inverse sin".

This has appeared in exams before.

Explain why the function must be one-to-one for an inverse function to exist:
If the mapping was many-to-one, then the inverse mapping would be one-to-many. But this is not a function!

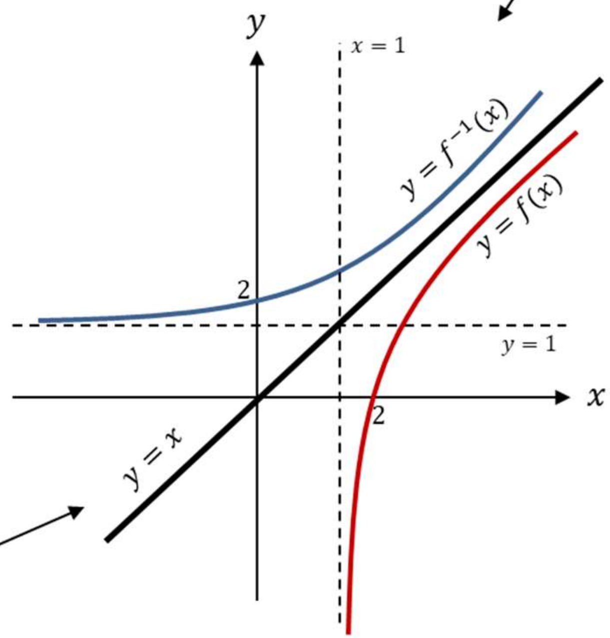
Graphing an Inverse Function

We saw that the inverse function effectively swaps the input x and output y. Thus the $\boldsymbol{x}$ **and** $\boldsymbol{y}$ **axis are swapped** when sketching the original function and its inverse.

And since the set of inputs and set of outputs is swapped…

The domain of $f(x)$ is the range of $f^{-1}(x)$ and vice versa.

Notice that x-intercepts become y-intercepts, and vertical asymptotes become horizontal ones.



$y = f(x)$ and $y = f^{-1}(x)$ have the line $y = x$ as a line of symmetry.

Domain of f: $\boldsymbol{x > 1}$

Range of f^{-1}: $\boldsymbol{f^{-1}(x) > 1}$

The domain of the function is the same as the range of the inverse, but remember that we write a domain in terms of x, but a range in terms of $f(x)$ or $f^{-1}(x)$.

Notes

Worked Example

REMEMBER TO DEFINE THE DOMAIN TOO!

Find the inverse functions:

$f(x) = 3x^2 - 5, \qquad x \geq 0$

$g(x) = 4x^2 + 6, \qquad x \geq 0$

T.38: 2D Qs 1-4, P.9 2.4 Qs 1

Worked Example

Find the inverse functions:

$f(x) = x^2 + 4x + 3, \qquad x \geq -2$

$g(x) = x^2 - 8x - 5, \qquad x \geq 5$

Worked Example

Find the inverse functions:

$f(x) = \dfrac{2}{x-5}, \qquad x \in \mathbb{R}, x \neq 5$

$g(x) = \dfrac{7}{x+2}, \qquad x \in \mathbb{R}, x \neq -2$

Worked Example

568a: Determine the inverse of exponential and logarithmic functions.

Given that $f(x) = 4 + \ln(x - 4)$, find $f^{-1}(x)$.

Worked Example

Find the inverse functions:

$$f(x) = \frac{x-2}{2x+1}, \qquad x \neq \frac{1}{2}$$

$$g(x) = \frac{2x+3}{4x-5}, \qquad x \neq \frac{5}{4}$$

Worked Example

$$f(x) = \sqrt{x-3}\ \{x \in \mathbb{R}, x \geq 3\}$$

a) State the range of $f(x)$

b) Find the function $f^{-1}(x)$ and state its domain and range

c) Sketch $y = f(x)$, $y = f^{-1}(x)$ and $y = x$

T.38: 2D Qs 5+ P.9 2.4 Qs 2+

2019 P2 Q6

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

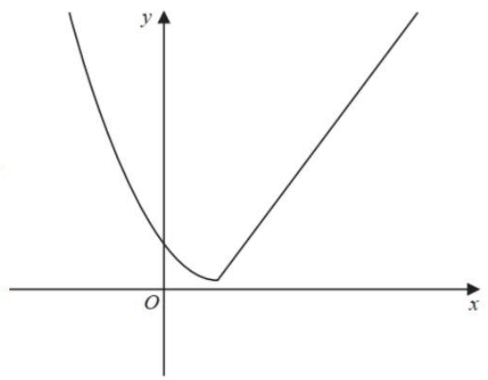


Figure 4

Your Turn

6. A function, g(x), is defined as:

$$g(x) = \begin{cases} (x-3)^2 + 1 & x \le 3, \\ 3x - 8 & x > 3 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which $g(x) > 9$.

(4)

The function h is defined by $h(x) = (x - 6)^2 + 6,\ x \le 6$.

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation $h^{-1}(x) = -2$.

(3)

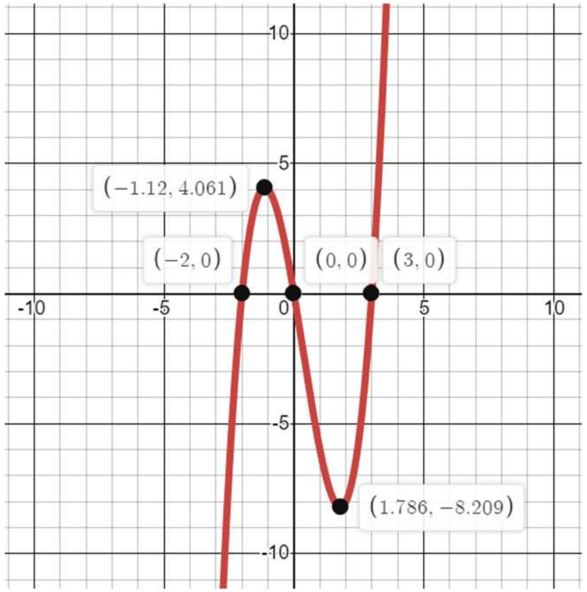
(Total for Question 6 is 10 marks)

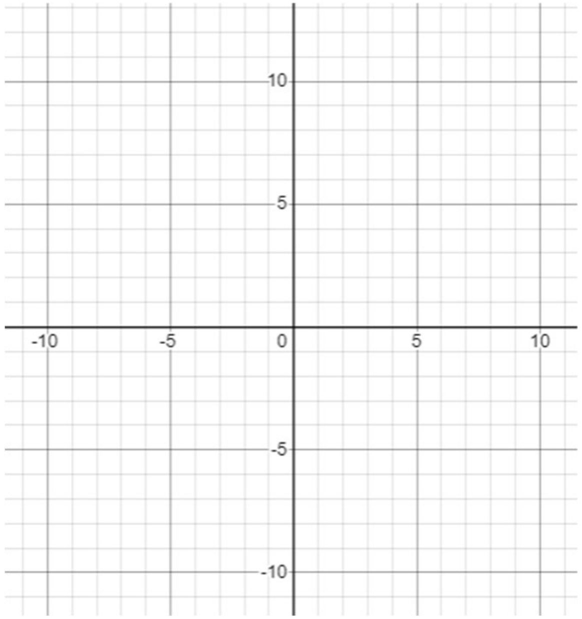
2.5) $y=|f(x)|$ and $y=f(|x|)$

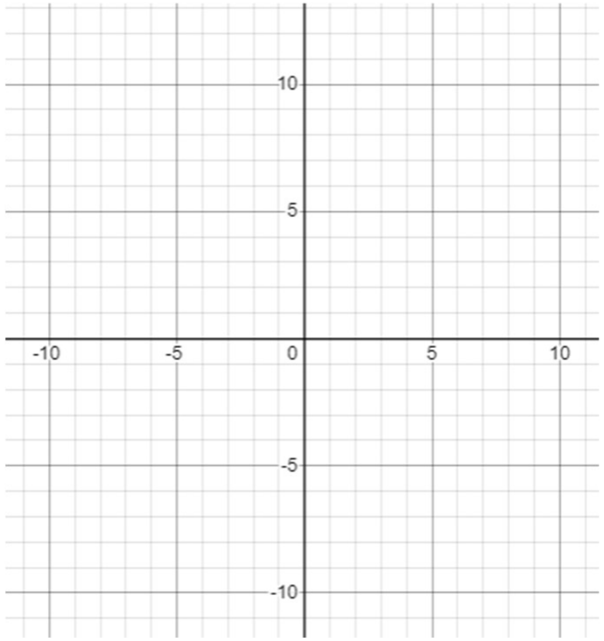
Notes

Worked Example

A sketch of $y = f(x)$ is shown.
Sketch $y = |f(x)|$ and $y = f(|x|)$
on separate axes.

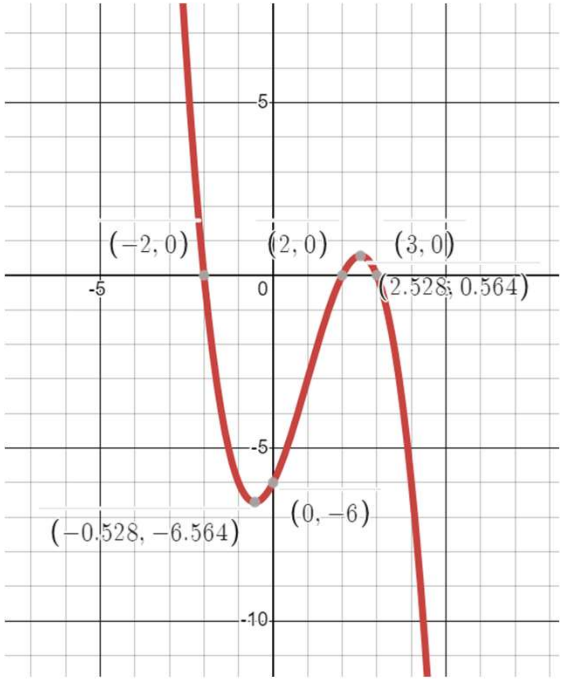


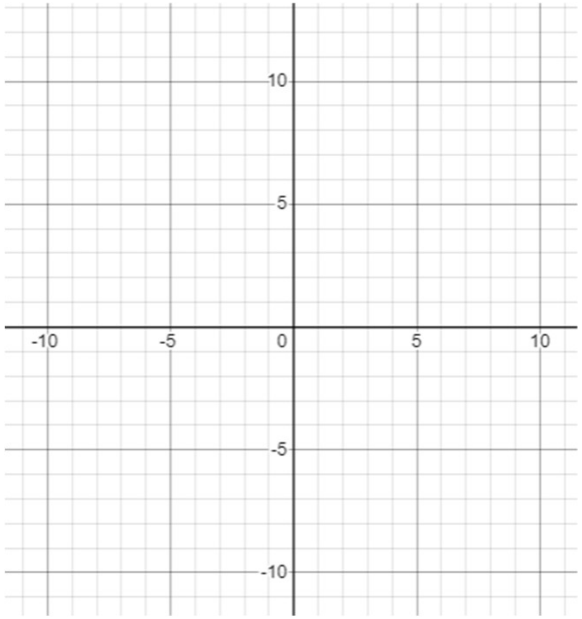


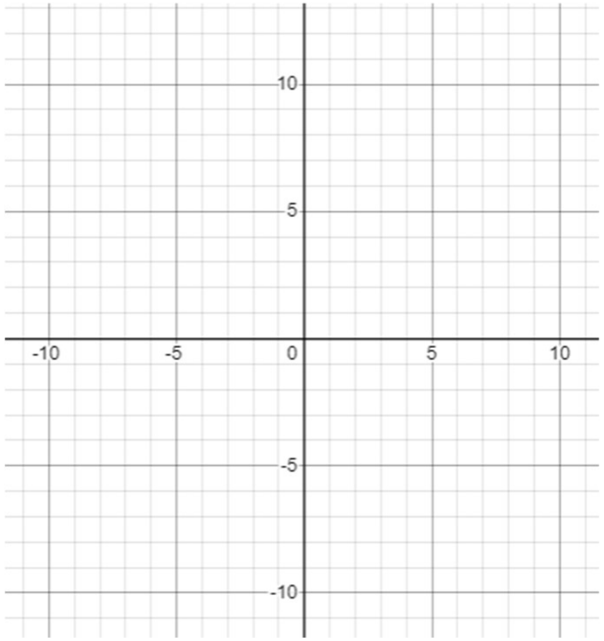


Your Turn

A sketch of $y = f(x)$ is shown.
Sketch $y = |f(x)|$ and $y = f(|x|)$ on separate axes.







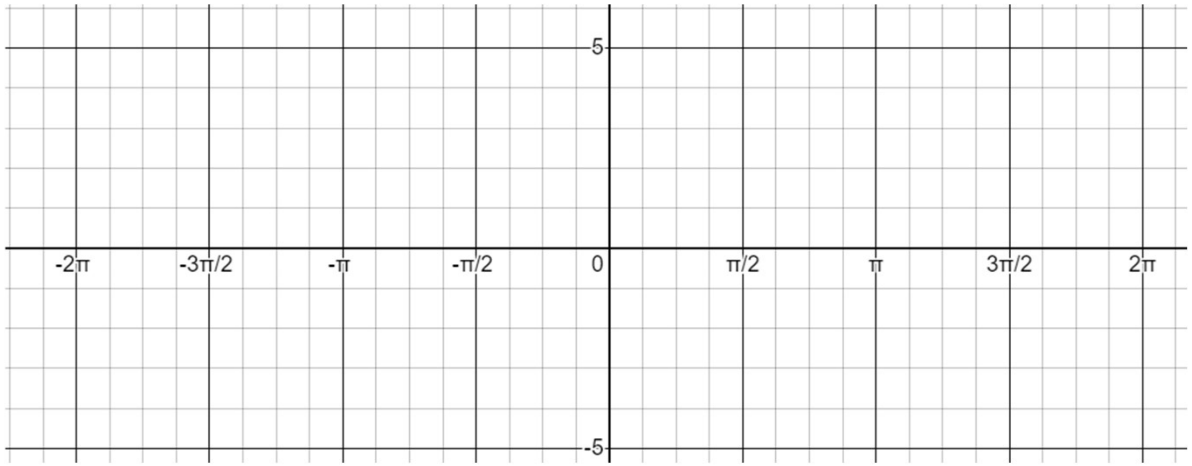
T.42: 2E Qs 1,3-10 P.10 2.5 Qs 3-7

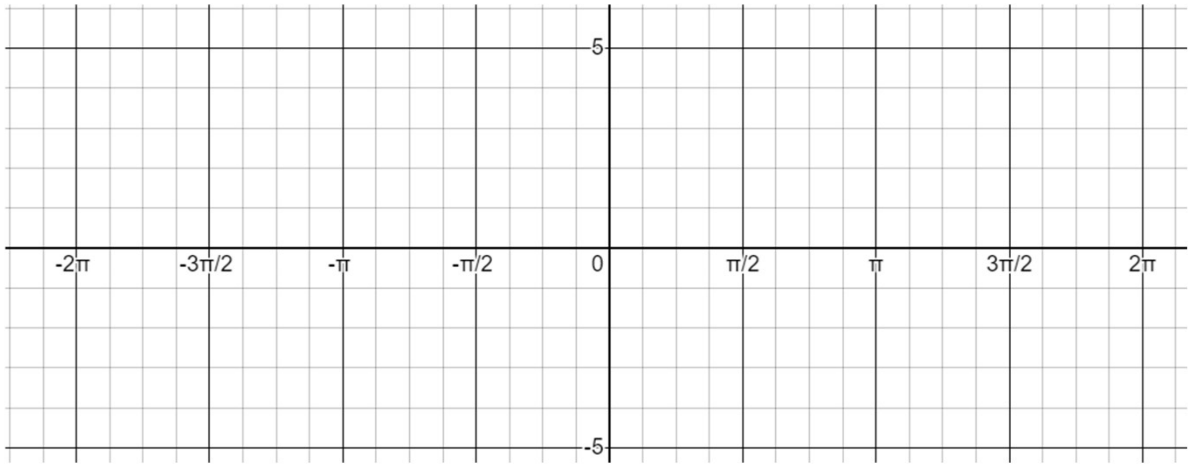
Worked Example

$$y = \cos x, \qquad -2\pi \le x \le 2\pi$$

Sketch:

a) $y = |\cos x|$ b) $y = \cos|x|$



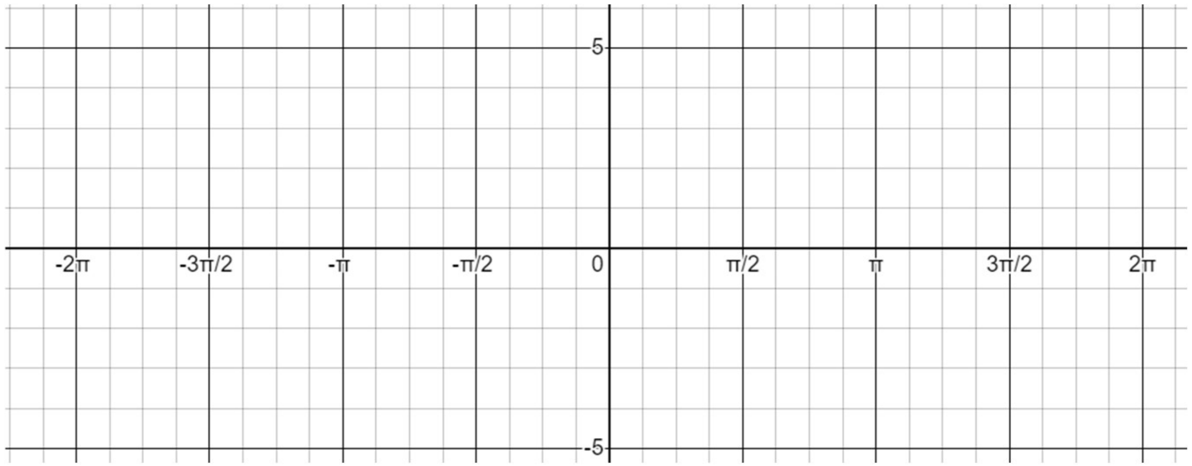


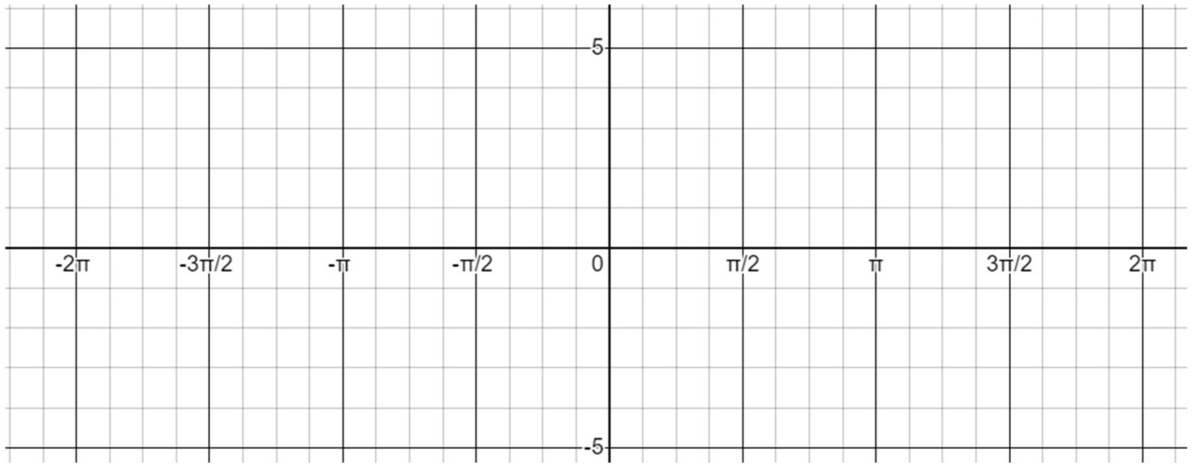
Your Turn

$$y = tan\, x\,, \qquad -2\pi \leq x \leq 2\pi$$

Sketch:

a) $y = |tan x|$ b) $y = \tan|x|$





T.42: 2E Qs 2 P.10 2.5 Qs 1-2

2.6) Combining transformations

	Affects which axis?	What we expect or opposite?
Change **inside** $f(\,)$	x	Opposite
Change **outside** $f(\,)$	y	What we expect

What if two x changes or two y changes?

$$y = 2f(x) + 1$$

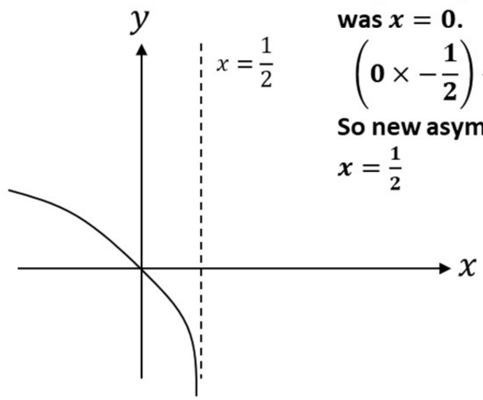
The y values are multiplied by 2, and then 1 is added.

$$y = f(2x + 1)$$

The easiest way is the think of the **inverse function** of $2x + 1$, i.e. $\frac{x-1}{2}$.
This gives us the changes to the x values, and in the correct order! In this case, we would -1 from the x values (translation 1 left) and then halve the x values (stretch on x-axis of scale factor $\frac{1}{2}$)

Sketch $y = \ln(1 - 2x)$

Inverse of $1 - 2x$ is $\frac{1-x}{2} = -\frac{1}{2}x + \frac{1}{2}$
So multiply x values by $-\frac{1}{2}$ and then add $\frac{1}{2}$.

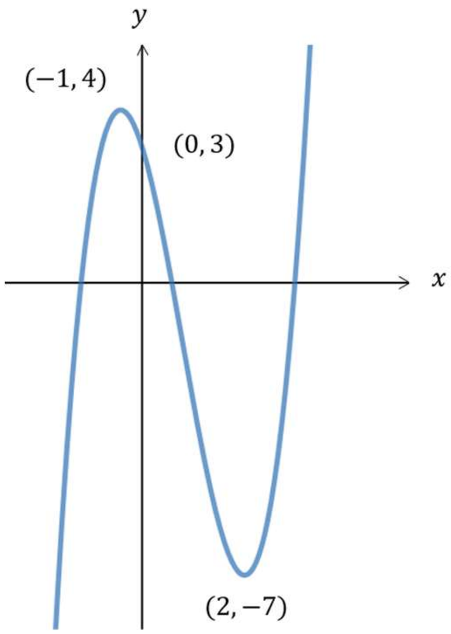


Original asymptote was $x = 0$.
$\left(0 \times -\frac{1}{2}\right) + \frac{1}{2} = \frac{1}{2}$
So new asymptote is $x = \frac{1}{2}$

Notes

Worked Example

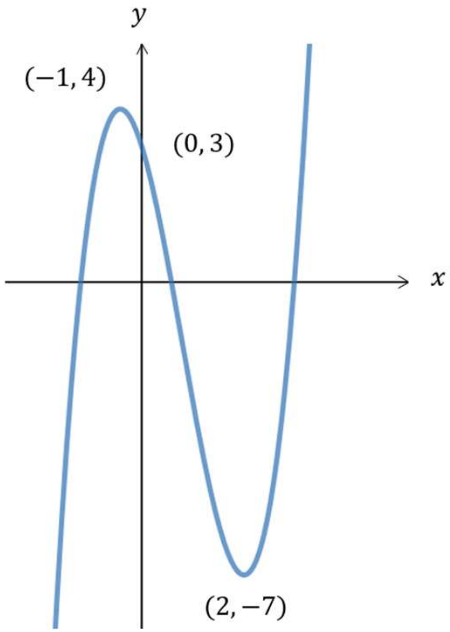
A sketch of the graph $y = f(x)$ is shown:



Sketch the graph of $y = -f(x) + 3$

Worked Example

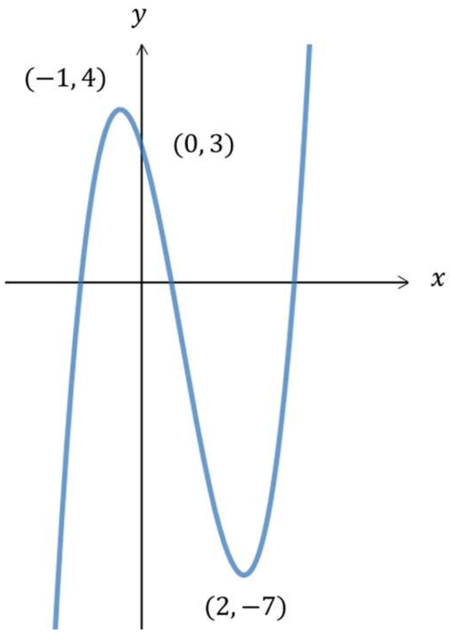
A sketch of the graph $y = f(x)$ is shown:



Sketch the graph of $y = f(-x) - 3$

Worked Example

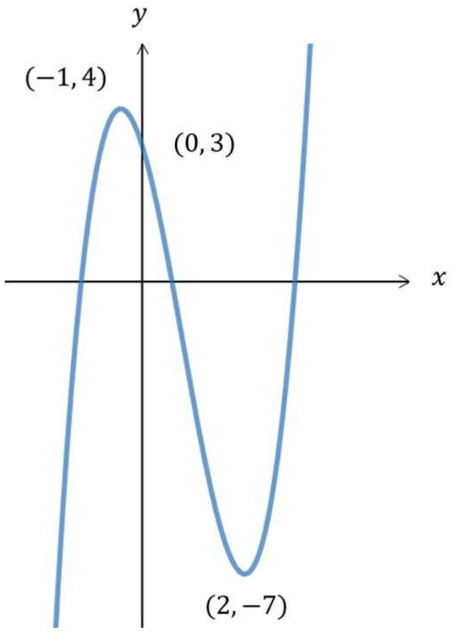
A sketch of the graph $y = f(x)$ is shown:



Sketch the graph of $y = -5f(x+2) + 3$

Worked Example

A sketch of the graph $y = f(x)$ is shown:



Sketch the graph of $y = -f(|x|)$

T.47: 2F Qs all, P.12 2.6 Qs all

1. The point $P(-2, -5)$ lies on the curve with equation $y = f(x), \quad x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = f(x)$ is transformed to the curve with equation

(a) $y = f(x) + 2$

(b) $y = |f(x)|$

Your Turn

1. The point $P(-5, -2)$ lies on the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(*a*) $y = \mathrm{f}(x) - 4$

(1)

(*b*) $y = |\,\mathrm{f}(x)\,|$

(1)

(*c*) $y = 5\mathrm{f}(x + 4) - 4$

(2)

(Total for Question 1 is 4 marks)

2.7) Solving modulus problems

Notes

Worked Example

$$f(x) = 2|x+1| - 3, x \in \mathbb{R}$$

(a) Sketch the graph of $y = f(x)$

(b) State the range of f.

(c) Solve the equation $f(x) = \frac{1}{3}x + 2$

Worked Example

$$f(x) = 6 - 2|x + 3|, x \in \mathbb{R}$$

(a) Sketch the graph of $y = f(x)$

(b) State the range of f.

(c) Solve the inequality $f(x) > 5$

Worked Example

$$f(x) = 6 + 3|x - 2|, x \in \mathbb{R}$$

State the range of values of k for which $f(x) = k$ has:

a) no solutions

b) exactly one solution

c) two distinct solutions

T.51: 2G Qs all, P.14 2.7 Qs all

Figure 4 shows a sketch of the graph with equation

$$y = |2x - 3k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = f(x)$ where

$$f(x) = k - |2x - 3k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$k - |2x - 3k| > x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 3 - 5f\left(\frac{1}{2}x\right)$$

(2)

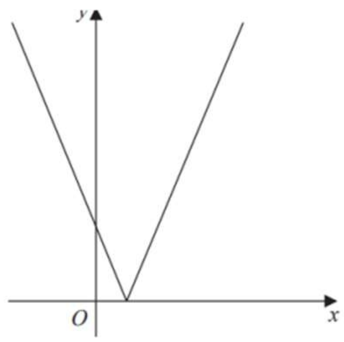


Figure 4

Your Turn

Figure 4 shows a sketch of the graph with equation

$$y = |4x - 5k|$$

where k is a positive constant.

(a) Sketch the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 3k - |4x - 5k|$$

stating

- the coordinates of the maximum point
- the coordinates of any points where the graph cuts the coordinate axes

(4)

(b) Find, in terms of k, the set of values of x for which

$$3k - |4x - 5k| > 2x - k$$

giving your answer in set notation.

(4)

(c) Find, in terms of k, the coordinates of the minimum point of the graph with equation

$$y = 5 - 2\mathrm{f}\left(\frac{1}{4}x\right)$$

(2)

(Total for Question 11 is 10 marks)

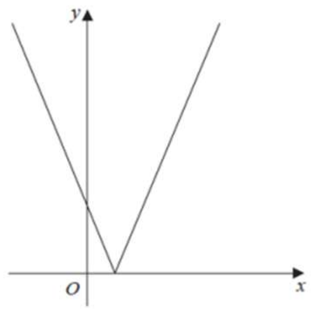


Figure 4

Past Paper Questions

Figure 4 shows a sketch of the graph of $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \begin{cases} (x-2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

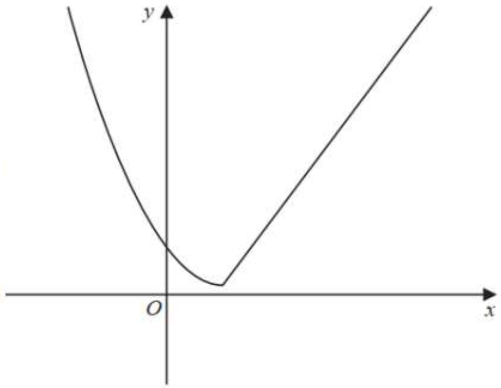


Figure 4

(a) Find the value of $\mathrm{gg}(0)$. **(2)**

(b) Find all values of x for which

$$\mathrm{g}(x) > 28$$

(4)

The function h is defined by

$$\mathrm{h}(x) = (x-2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not. **(1)**

(d) Solve the equation

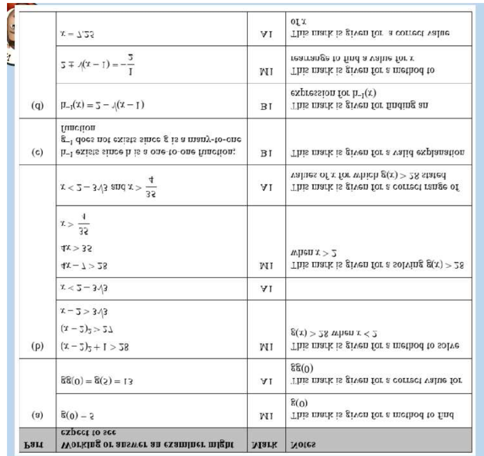
$$\mathrm{h}^{-1}(x) = -\frac{1}{2}$$

(3)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com



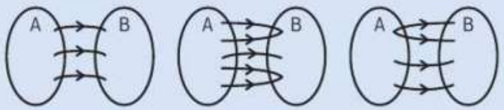
Summary of Key Points

1 A modulus function is, in general, a function of the type $y = |f(x)|$.

- When $f(x) \geqslant 0$, $|f(x)| = f(x)$
- When $f(x) < 0$, $|f(x)| = -f(x)$

2 To sketch the graph of $y = |ax + b|$, sketch $y = ax + b$ then reflect the section of the graph below the x-axis in the x-axis.

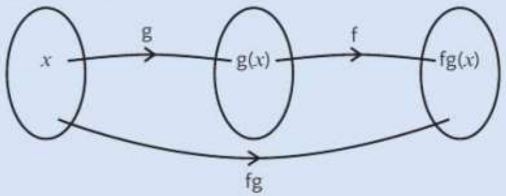
3 A mapping is a **function** if every input has a distinct output. Functions can either be **one-to-one** or **many-to-one**.



one-to-one function | many-to-one function | not a function

4 $fg(x)$ means apply g first, then apply f.

$fg(x) = f(g(x))$



5 Functions $f(x)$ and $f^{-1}(x)$ are inverses of each other. $ff^{-1}(x) = x$ and $f^{-1}f(x) = x$.

6 The graphs of $y = f(x)$ and $y = f^{-1}(x)$ are reflections of each another in the line $y = x$.

7 The domain of $f(x)$ is the range of $f^{-1}(x)$.

8 The range of $f(x)$ is the domain of $f^{-1}(x)$.

9 To sketch the graph of $y = |f(x)|$

- Sketch the graph of $y = f(x)$
- Reflect any parts where $f(x) < 0$ (parts below the x-axis) in the x-axis
- Delete the parts below the x-axis

10 To sketch the graph of $y = f(|x|)$

- Sketch the graph of $y = f(x)$ for $x \geqslant 0$
- Reflect this in the y-axis

11 $f(x + a)$ is a horizontal translation of $-a$.

12 $f(x) + a$ is a vertical translation of $+a$.

13 $f(ax)$ is a horizontal stretch of scale factor $\frac{1}{a}$

14 $af(x)$ is a vertical stretch of scale factor a.

15 $f(-x)$ reflects $f(x)$ in the y-axis.

16 $-f(x)$ reflects $f(x)$ in the x-axis.