KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 12

2025 Pure Mathematics 1 2026

2 Quadratics Booklet

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

A hidden quadratic expression is one of the form

$$\boldsymbol{a}[\ f(x)]^2 + \boldsymbol{b}[f(x)\ ] + \boldsymbol{c}$$

where $f(x)$ is some function of x and a, b and c are constants

$$x^4 - 13x^2 + 36$$

can be written as

$$(x^2)^2 - 13(x^2) + 36$$

So $f(x) = x^2$

We can use index laws to rewrite x^4 as $(x^2)^2$

$$x - 13\sqrt{x} + 36$$

can be written as

$$(\sqrt{x})^2 - 13(\sqrt{x}) + 36$$

So $f(x) = \sqrt{x}$

Recall that $(\sqrt{x})^2 = \sqrt{x} \times \sqrt{x} = x$

Notes

Factorise $x^4 - 13x^2 + 36$

Using the substitution $y = x^2$
we can rewrite the hidden quadratic as:

$$y^2 - 13y + 36$$

Recall, we need two factors of $+36$ that sum to -13

We can now factorise:

$$(y-9)(y-4)$$

These are -9 and -4

Finally, we can write the factorised expression in terms of x:

$$x^4 - 13x^2 + 36 = \boldsymbol{(x^2 - 9)(x^2 - 4)}$$

Factorise $4 \times 3^{2x} - 11 \times 3^x - 3$

Using the substitution $y = 3^x$
we can rewrite the hidden quadratic as:

$$4y^2 - 11y - 3$$

We can now factorise by splitting the middle term:

$$4y^2 - 12y + y - 3$$
$$= 4y(y-3) + 1(y-3)$$
$$= (4y+1)(y-3)$$

Since $a \times c = 4 \times -3 = -12$, we need two factors of -12 that sum to -11

These are -12 and $+1$

Finally, we can write the factorised expression in terms of x:

$$4 \times 3^{2x} - 11 \times 3^x - 3 = \boldsymbol{(4 \times 3^x + 1)(3^x - 3)}$$

Worked Example

Solve $6x^{\frac{2}{3}} + 5x^{\frac{1}{3}} - 4 = 0$

Worked Example

Solve the equations

a) $2x^2 + \frac{40}{x^2} - 21 = 0$

b) $2x^2 - \frac{20}{x^2} = -3$

Worked Example

Solve the equations

a) $2 \times 2^{2x+1} = -15 \times 2^x + 4$

b) $8 \times 4^x - 9 \times 2^x + 1 = 0$

Fill in the Gaps

Hidden Quadratic	Substitution $y = \cdots$	Quadratic in terms of y	Factorise and Solve Quadratic	Solutions to Hidden Quadratic
$x^4 - 6x^2 + 8 = 0$	$y = x^2$	$y^2 - 6y + 8 = 0$	$(y-4)(y-2) = 0$ $y = 4, y = 2$	$x^2 = 4, x^2 = 2$ $x = \pm 2, x = \pm\sqrt{2}$
$a^6 - 28a^3 + 27 = 0$	$y = a^3$			
$b + \sqrt{b} - 12 = 0$				
$2^{2x} - 5 \times 2^x + 4 = 0$				
$4w^4 - 13w^2 + 9 = 0$				
$9 \times 3^{2z} - 82 \times 3^z + 9 = 0$				
$6t^{2/3} - 5t^{1/3} - 4 = 0$				

Fill in the Gaps

Original Equation in x	Substitution	Quadratic in y	Solutions for y	Solutions for x
$x^4 - 10x^2 + 21 = 0$	$y = x^2$	$y^2 - 10y + 21 = 0$	$y = 7, y = 3$	$x = \pm\sqrt{7}, \pm\sqrt{3}$
$x^6 = 7x^3 + 8$	$y = x^3$		$y = 8, y = -1$	
$x - 3\sqrt{x} - 10 = 0$				$x = 25, x = 4$
$2^{2x} - 6 \times 2^x + 8 = 0$	$y = 2^x$			
$\sqrt{x} + \frac{1}{\sqrt{x}} = 2$				
$9^x - 28 \times 3^x + 27 = 0$				
$x\sqrt[3]{x} - 13x^{\frac{2}{3}} + 36 = 0$	$y = x^{\frac{2}{3}}$			
$x^3 + 9x + \frac{20}{x} = 0$				
$\left(x - \frac{6}{x}\right)^2 - 6\left(x - \frac{6}{x}\right) + 5 = 0$	$y = \left(x - \frac{6}{x}\right)$			

Fluency Practice

1 Identify a suitable substitution of the form $y = f(x)$ for each of these hidden quadratic expressions:

a) $x^6 - 6x^3 + 8$?

b) $x + 5\sqrt{x} + 4$?

c) $2x^{10} - 9x^5 + 10$?

d) $(3^x)^2 + 8(3^x) + 7$?

e) $4\times 2^{2x} - 33\times 2^x + 8$?

f) $2(x-3)^2 + 5(x-3) + 3$?

2 Factorise:

a) $z + \sqrt{z} - 12$?

b) $a^4 - 9a^2 + 20$?

c) $3^{2x} - 4\times 3^x - 21$?

d) $2d^6 + 11d^3 + 15$?

e) $5(3x-7)^2 - 3(3x-7) - 2$?

f) $10\times 4^{2w} - 7\times 4^w + 1$?

3 Factorise fully:

a) $a^4 - 9$?

b) $16 - w^6$?

c) $7^{2x} - 121$?

d) $4y^8 - 49$?

e) $z^4 - 81$?

4 Solve:

a) $w^4 - 17w^2 + 16 = 0$?

b) $x - 9\sqrt{x} + 20 = 0$?

c) $z^6 - 28z^3 + 27 = 0$?

d) $4d^4 - 17d^2 + 4 = 0$?

e) $6t + 5\sqrt{t} - 4 = 0$?

Fluency Practice

5 *[OCR C1 June 2006 Q6i]*

Solve the equation $x^4 - 10x^2 + 25 = 0$

?

6 *[OCR C1 June 2010 Q5]*

Find the real roots of the equation $4x^4 + 3x^2 - 1 = 0$

?

7 Solve:

a) $2^{2x} - 5\times2^x + 4 = 0$?

b) $3^{2y} - 4\times3^y + 3 = 0$?

c) $16^w - 17\times4^w + 16 = 0$?

d) $9\times9^x - 28\times3^x + 3 = 0$?

8 *[OCR A2 June 2018 P1 Q3]*

Find the two real roots of the equation $x^4 - 5 = 4x^2$ Give the roots in an exact form.

?

9 *[OCR C1 Jan 2008 Q10ii]*

Given that $f(x) = 8x^3 + \frac{1}{x^3}$, solve the equation $f(x) = -9$

?

10 *[WJEC AS Level June 2018 Unit 1 Q17c]*

Express $4^x - 10\times2^x$ in terms of y, where $y = 2^x$. Hence solve the equation $4^x - 10\times2^x = -16$

?

☠ Solve:

a) $3z^{2/3} + z^{1/3} - 2 = 0$?

b) $2^{2w+1} - 17\times2^w + 8 = 0$

?

Fluency Practice

To spot a quadratic in disguise you are looking for an equation where the power on one of the variables is twice that on the other. For example

$$(\text{whatever})^{18} + 4(\text{whatever})^9 - 5.$$

This can then factorise to

$$((\text{whatever})^9 + 5)((\text{whatever})^9 - 1),$$

or you can complete the square to

$$((\text{whatever})^9 + 2)^2 - 9.$$

Solve the following:

1. $x^4 - 13x^2 + 36 = 0$.
2. $x^4 - 15x^2 - 16 = 0$.
3. $x^4 + 5x^2 + 6 = 0$.
4. $x^6 + 7x^3 = 8$.
5. $2x^4 = x^2 + 1$.
6. $x + 3 = 4\sqrt{x}$.
7. $12x^4 = 2x^2 + 4$.
8. $2(\sin x)^2 + \sin x - 1 = 0$ in range $0 < x < 360$.
9. $\frac{4x^4+144}{73} = x^2$.
10. $2(\cos x)^2 + (\cos x) = 6$.
11. $x = 2\sqrt{x} + 3$.
12. $6x^{2/3} + 5x^{1/3} - 4 = 0$.
13. $(x^2 - 4x + 1)^2 + (x^2 - 4x + 1) - 12 = 0$.
14. $2\theta + 15 = 11\sqrt{\theta}$.
15. $x^2 + \frac{72}{x^2} = 17$.
16. $\sqrt{z} - 2\sqrt[4]{z} = 3$.
17. $2^{2x} - 12 \times 2^x + 32 = 0$.
18. $t = 4\sqrt{t} + 1$.
19. $2^{2x} + 8 = 9 \times 2^x$.
20. $2\sqrt{x} + \frac{9}{\sqrt{x}} = 9$.
21. $\left(\frac{1}{x}\right)^2 + 1 = 8\left(\frac{1}{x}\right)$. [Do this question in two ways.]
22. $2(\cos\theta)^2 = 8\cos\theta + 21$.
23. $x^8 + 4x^4 = 3$.
24. $2^{2x} + 1 = 2^{x+1}$.
25. $2^{2x} + 128 = 3 \times 2^{x+3}$.
26. $81 + 3^{2x+1} = 4 \times 3^{x+2}$.
27. $a^{2x} + a^4 = a^{x+1} + a^{x+3}$.

Only attempt the following if you have studied logarithms.

28. $2^{2x} - 13 \times 2^x + 42 = 0$.
29. $4^{2x} - 9 \times 4^x + 14 = 0$.
30. $3^{2x} + 10 = 7 \times 3^x$.
31. $2^{2x} - 5 \times 2^{x+1} + 25 = 0$.
32. $3^{2x} - 3^{x+2} + 20 = 0$.

T. 27 2E Qs 7,8 P.9 2.3 Qs 6,7

Exam Q AS 2019

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

Your Turn

Find, using algebra, all real solutions to the equation

$$a^4 - 2a^2 - 80 = 0$$

(4)

2.4 Quadratic Graphs

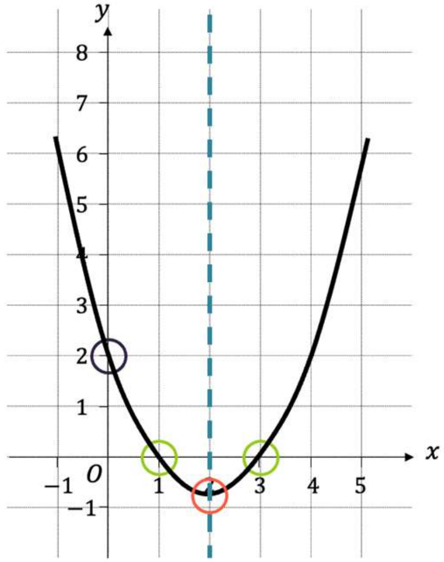
Determine the equation of a quadratic function from its graph only

There are some key features of quadratic graphs that we can use if we want to **sketch** a quadratic rather than plot it.

1 The shape of the graph - ∪ or ∩

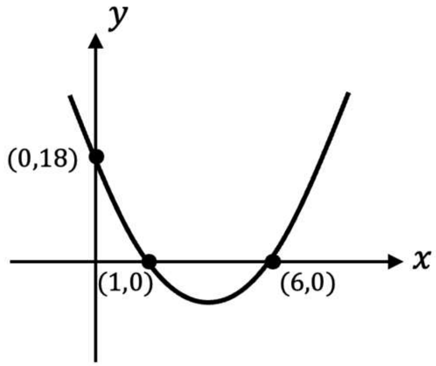
2 Where the graph crosses the y-axis

3 Where the graph crosses the x-axis



4 The line of symmetry of the graph

5 The turning point of the graph



Start by considering where the graph crosses the x-axis:

$x = 6$ and $x = 1$

So, the factorised equation must be:

$$y = a(x-6)(x-1)$$

To find the value of a, we use the coordinate where the graph crosses the y-axis:

$$y = a(x^2 - 7x + 6)$$

When $x = 0, y = 18$

$$18 = a \times 6$$

$$a = 3$$

Finally, we can find the equation:

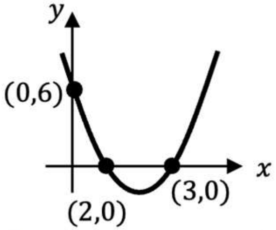
$$y = 3(x^2 - 7x + 6)$$

$$\mathbf{y = 3x^2 - 21x + 18}$$

Fluency Practice

2 Find the equations of the quadratics from their sketches.

a



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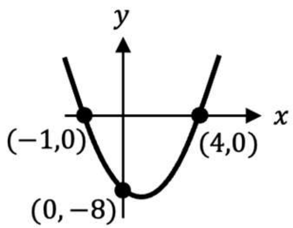
b



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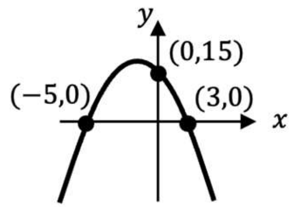
c



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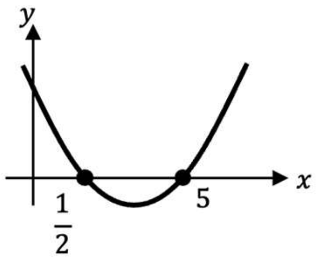
3 *[AQA IGCSE FM Practice Paper Set 1 P2 Q15]*

The diagram shows a quadratic graph that intersects the x-axis when $x = \frac{1}{2}$ and $x = 5$.
Work out the equation of the graph.



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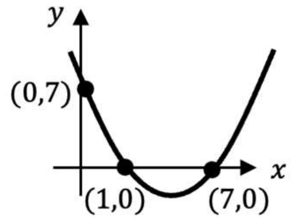
4 Given the sketch of the quadratic, find the equation of the line of symmetry and the coordinates of the turning point.

a



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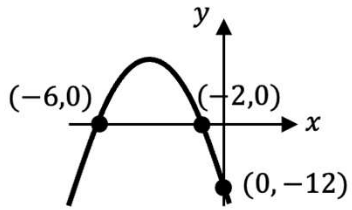
b



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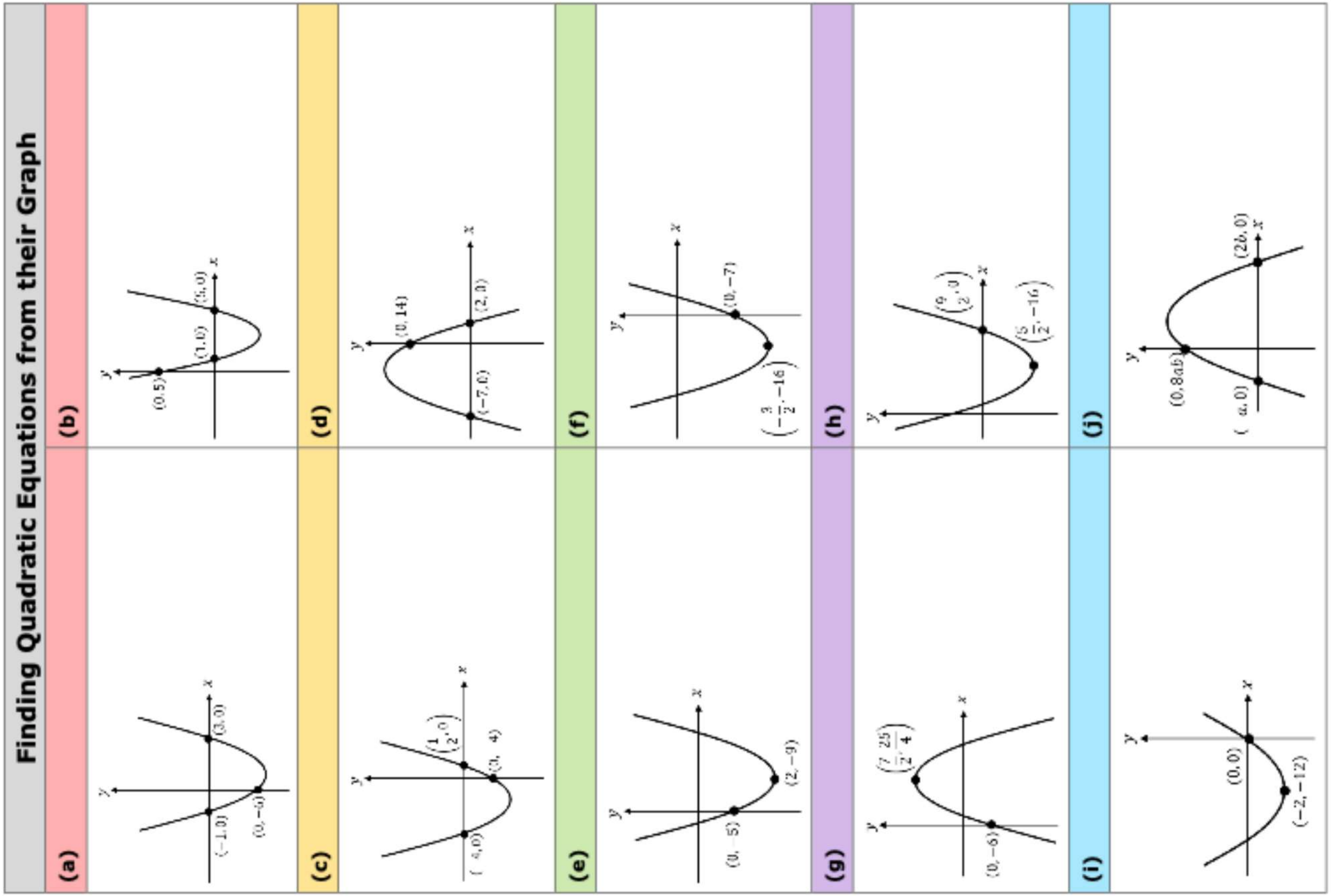
The graph of a quadratic function passes through the point $(-8,0)$ and has a turning point at $(-2,18)$.
Find the equation of the quadratic graph.

?

Worked Example

The graph of $y = ax^2 + bx + c$ has a minimum at $(3, -5)$ and passes through $(4, 0)$. Find the values of a, b and c

Fluency Practice



T.30 2F Qs 2,3 P.10 2.4 Qs 5+

Exam Q

A2 2021 Paper 1

Quadratics

2. Given that

$$f(x) = x^2 - 4x + 5 \qquad x \in \mathbb{R}$$

(a) express f(x) in the form $(x + a)^2 + b$ where a and b are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the y-axis at the point P
- has a minimum turning point at the point Q

(b) Write down

(i) the coordinates of P

(ii) the coordinates of Q

(2)

Your Turn

2. Given that

$$f(x) = x^2 - 6x + 7 \qquad x \in \mathbb{R}$$

(*a*) express f(*x*) in the form $(x + a)^2 + b$ where *a* and *b* are integers to be found.

(2)

The curve with equation $y = f(x)$

- meets the *y*-axis at the point *P*
- has a minimum turning point at the point *Q*

(*b*) Write down

(i) the coordinates of *P*

(ii) the coordinates of *Q*

(2)

(Total for Question 2 is 4 marks)

2.5 The Discriminant

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The term '**root**' refers to a **solution of an equation** in the form $f(x) = 0$, i.e. when one side of the equation is 0.

Reflecting on the quadratic formula you just used, **what part of the formula** do you think **distinguishes** between when we have 0 real roots, 1 distinct real root and 2 distinct real roots?

$$\frac{\ldots \pm \sqrt{\square}}{\ldots}$$

When this was **negative**, we couldn't square root it, so there were no real solutions.

$$\frac{\ldots \pm \sqrt{\square}}{\ldots}$$

When this was **positive**, adding or subtracting the square root gave different, i.e. distinct, solutions.

$$\frac{\ldots \pm \sqrt{\mathbf{0}}}{\ldots}$$

When this was **zero**, adding or subtracting had no effect, so the solution was the same.

✎ The discriminant $\Delta = b^2 - 4ac$ of a quadratic equation $ax^2 + bx + c = 0$ allows us to 'discriminate' between the different **number of roots/solutions**:

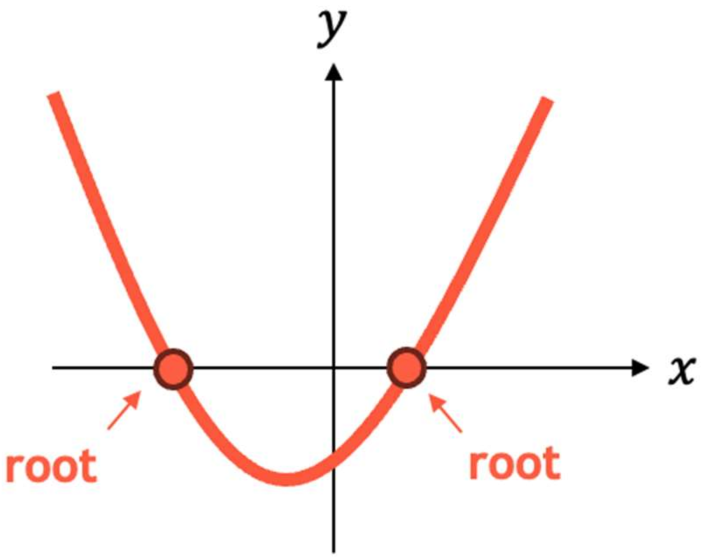
$b^2 - 4ac > 0$	Distinct real roots
$b^2 - 4ac < 0$	No real roots
$b^2 - 4ac = 0$	Equal roots

Δ is capital delta in the Greek alphabet. It is unrelatedly also used to mean 'change in', e.g. within the formula for gradient:

$$m = \frac{\Delta y}{\Delta x}$$

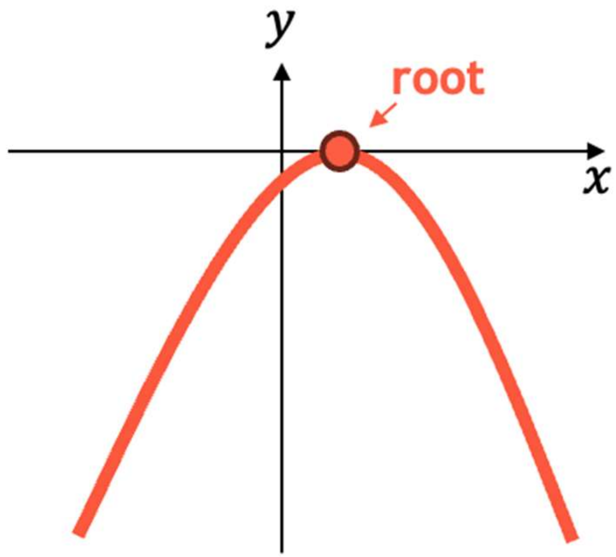
Quadratic Graphs and the Discriminant

When solving $ax^2 + bx + c = 0$, the solutions correspond to the **<u>x-intercepts/roots</u>** of the graph $y = ax^2 + bx + c$



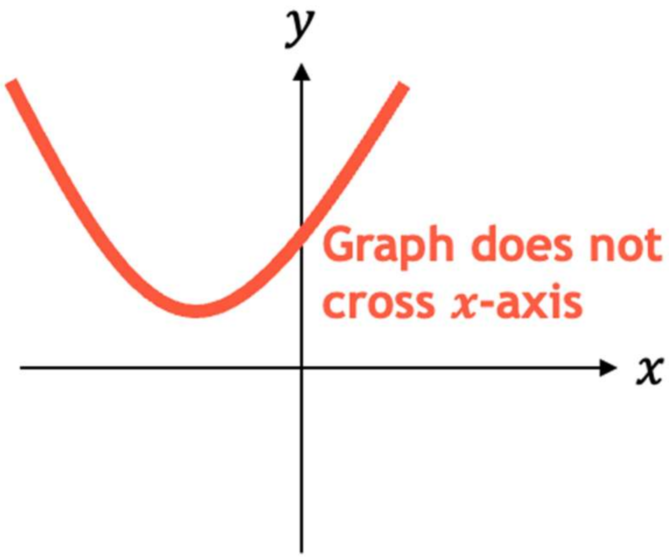
2 distinct real roots so:

$$\Delta > 0$$



1 distinct root so:

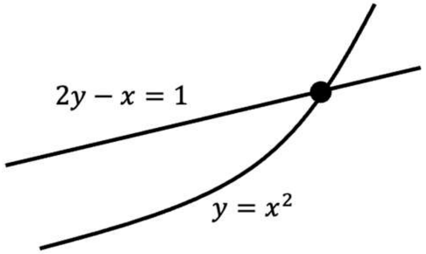
$$\Delta = 0$$



0 real roots so:

$$\Delta < 0$$

Using the Discriminant for Points of Intersection



How would you normally determine the points at which two lines, with given equations, intersect?

The **points of intersection** **correspond to the solutions** to the system of equations:

$$y = x^2$$
$$2y - x = 1$$

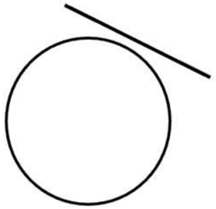
We could solve this by substitution:

$$2x^2 - x = 1$$

The number of solutions to this equation therefore corresponds to the number of points of intersection.
We can use the discriminant to determine this.

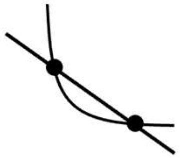
If a line is **tangent to a curve**, then there will be 1 point of intersection, and therefore **1 distinct root** to the equation obtained by substitution.

$$b^2 - 4ac = 0$$



Similarly, **if the line never meets the curve**, then the equation obtained by substitution will have no real roots.

$$b^2 - 4ac < 0$$



How many solutions would you expect there be to the equation obtained by substitution of this line and curve?

There will be 2 points of intersection, and therefore **2 distinct roots.**

$$b^2 - 4ac > 0$$

Notes

Questions

Tip: Write out $a = \cdots, b = \cdots, c = \cdots$ explicitly before calculating the discriminant.

$$\Delta = b^2 - 4ac$$

Equation	a, b, c	Δ	Number of distinct real roots
$x^2 + 3x + 4 = 0$			
$x^2 - 4x + 1 = 0$			
$x^2 - 4x + 4 = 0$			
$2x^2 - 6x - 3 = 0$			
$x - 4 - 3x^2 = 0$			
$1 - x^2 = 0$			

Fill in the Gaps

Quadratic Equation	a	b	c	Discriminant	Nature of Solutions
$x^2 + 5x + 3 = 0$	1	5	3	$5^2 - 4 \times 1 \times 3 = 13$	Two real solutions
$3x^2 - 2x + 5 = 0$	3	-2	5		
$x^2 + 4x + 4 = 0$					
$6x^2 + 5 = 0$					
$16 - x - 3x^2 = 0$					
$9x^2 - 2x = 0$					
	5	4	1		
	1	8		44	
		-9	8	-111	
		-12	9		One real solution
$5x^2 + 7x - 3 = 0$	5	7		109	
	-1		-25		One real solution
		6	5	-24	

Fluency Practice

	Factorises (with integers)	**Does not factorise**
Has two real and different roots $b^2 - 4ac > 0$ Graph crosses the x-axis		
Has one real root (2 identical roots) $b^2 - 4ac = 0$ Graph just touches the x-axis		***If the quadratic equation has one root then it always factorises***
Has no real roots $b^2 - 4ac < 0$ Graph does not touch or cross the x-axis	***If the quadratic equation has no real roots then it never factorises***	

$x^2 - 8x + 16 = 0$	$3x^2 + 7x - 6 = 0$	$x^2 + 2x + 4 = 0$	$x^2 - 25 = 0$
$x^2 + 3x - 11 = 0$	$x^2 + 7x + 12 = 0$	$-x^2 - 8x + 2 = 0$	$-x^2 + x - 2 = 0$
$-2x^2 + 9x - 4 = 0$	$x^2 - 2x + 1 = 0$	$6x^2 + 11x - 10 = 0$	$x^2 - 3x + 6 = 0$
$2x^2 + 10x - 14 = 0$	$2x^2 - 12x + 18 = 0$	$x^2 + 4x + 4 = 0$	$x^2 + 6x + 10 = 0$
$-2x^2 + 4x - 6 = 0$	$x^2 + 6x + 4 = 0$	$3x^2 + 60x + 300 = 0$	$2x^2 + 3x - 1 = 0$

Fluency Practice

	Factorises	**Does not factorise**
Has two real and different roots $b^2-4ac>0$ Graph crosses the x-axis		
Has one real root (2 identical roots) $b^2-4ac=0$ Graph just touches the x-axis		*If the quadratic equation has one root then it always factorises*
Has no real roots $b^2-4ac<0$ Graph does not touch or cross the x-axis	*If the quadratic equation has no real roots then it never factorises*	

$x^2-p^2=0$	$x^2+p^2=0$	$x^2-2px+p^2=0$	$q^2x^2+p^2=0$
$x^2+2px+p^2=0$	$x^2+(p+q)x+pq=0$	$qx^2+2pqx+qp^2=0$	$q^2x^2-p^2=0$
$x^2+(p-q)x-pq=0$	$x^2-(p+q)x+pq=0$	$q^2x^2-p^2=0$	$q^2x^2-2pqx+p^2=0$

$p,q \in$ ~ and $p,q \neq 0$

Worked Example

The equation $(3k-3)x-(2k-2)x^2-k+2=0$ has equal roots. Find the possible values of k

Dr Frost 492f

Worked Example

The equation $(k-1)x-(2k^2-2)x^2+1=0$ has real roots. Find the possible range of values of k

Dr Frost 492g

Worked Example

The equation $k^2x^2 - 3kx = 2x - 4$ has real roots. Find the possible values of k

Worked Example

The straight line with equation $y = 7px + 9p$ touches the curve with equation $y = 6px^2 - x + 3$, where p is a constant. Find the set of possible values of p

Dr Frost 492i

Worked Example

Prove that the function $f(x) = 4x^2 + (k + 8)x - k$ has two distinct real roots for all values of k

T. 32 2G Qs 2+, P.11 2.5 Qs 4+

2.6 Modelling with Quadratics

Notes

Worked Example

A spear is thrown over level ground from the top of a tower.

The height, in metres, of the spear above the ground after t seconds is modelled by the function: $h(t) = 1.65 + 24.5t - 4.9t^2$, $t \geq 0$

a) Interpret the meaning of the constant term 12.25 in the model.

b) After how many seconds does the spear hit the ground?

c) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.

d) Using your answer to part c or otherwise, find the maximum height of the spear above the ground, and the time at which this maximum height is reached?

T.34 2H Qs all, P.11 2.6 Qs all

Exam Q

A2 2021 Paper 1

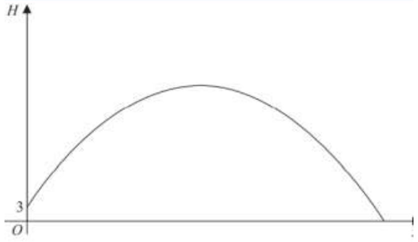


Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 3 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 90 m
- is at a vertical height of 27 m above the ground after travelling a horizontal distance of 120 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

Your Turn

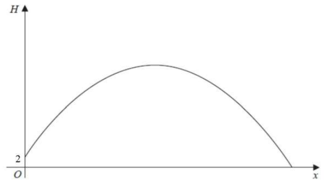


Figure 3

Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, H metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

- is hit from a point on the top of a platform of vertical height 2 m above the ground
- reaches its maximum vertical height after travelling a horizontal distance of 100 m
- is at a vertical height of 34 m above the ground after travelling a horizontal distance of 160 m

Given also that H is modelled as a **quadratic** function in x

(a) find H in terms of x

(5)

(b) Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first hits the ground, giving your answer to the nearest metre.

(3)

(c) The possible effects of wind or air resistance are two limitations of the model. Give one other limitation of this model.

(1)

(Total for Question 12 is 9 marks)

Exam Question

6.

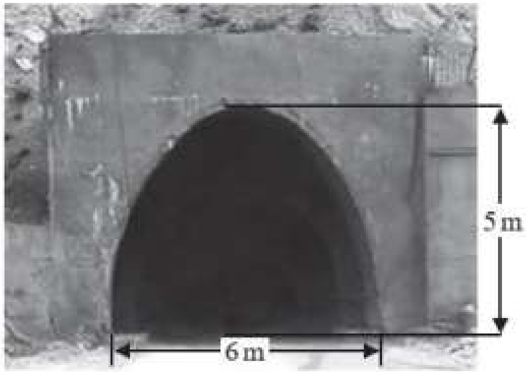


Figure 2

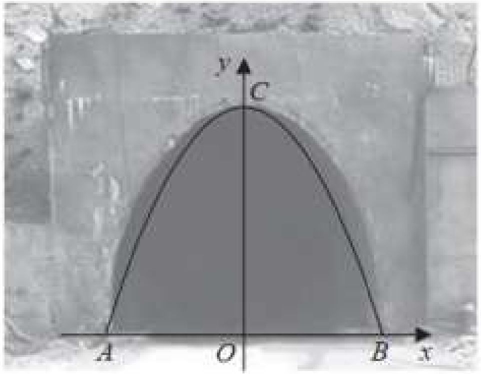


Figure 3

Figure 2 shows the entrance to a road tunnel. The maximum height of the tunnel is measured as 5 metres and the width of the base of the tunnel is measured as 6 metres.

Figure 3 shows a quadratic curve BCA used to model this entrance.

The points A, O, B and C are assumed to lie in the same vertical plane and the ground AOB is assumed to be horizontal.

(a) Find an equation for curve BCA. **(3)**

A coach has height 4.1 m and width 2.4 m.

(b) Determine whether or not it is possible for the coach to enter the tunnel. **(2)**

(c) Suggest a reason why this model may not be suitable to determine whether or not the coach can pass through the tunnel. **(1)**

Exams

- Past Papers
- More Past Papers (Login Required)
- Practice Papers
- Past Paper Questions by Topic

Past paper questions by topic. Both new and old specification can be found via this link on https://hgsmaths.com/year-12/maths/

Question	Scheme	Marks	AOs
6 (a)	Attempts to use an appropriate model; e.g. $y = A(3-x)(3+x)$ or $y = A(9-x^2)$	M1	3.3
	e.g. $y = A(9-x^2)$ Substitutes $x = 0, y = 5 \Rightarrow 5 = A(9-0) \Rightarrow A = \frac{5}{9}$	M1	3.1b
	$y = \frac{5}{9}(9-x^2)$ or $y = \frac{5}{9}(3-x)(3+x)$, $\{-3 \leqslant x \leqslant 3\}$	A1	1.1b
		(3)	
(b)	Substitutes $x = \frac{2.4}{2}$ into their $y = \frac{5}{9}(9-x^2)$	M1	3.4
	$y = \frac{5}{9}(9-x^2) = 4.2 > 4.1 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(b) Alt 1	$4.1 = \frac{5}{9}(9-x^2) \Rightarrow x = \frac{9\sqrt{2}}{10}$, so maximum width $= 2\left(\frac{9\sqrt{2}}{10}\right)$	M1	3.4
	$= 2.545... > 2.4 \Rightarrow$ Coach can enter the tunnel	A1	2.2b
		(2)	
(c)	E.g. • Coach needs to enter through the centre of the tunnel. This will only be possible if it is a one-way tunnel • In real-life the road may be cambered (and not horizontal) • The quadratic curve BCA is modelled for the entrance to the tunnel but we do not know if this curve is valid throughout the entire length of the tunnel • There may be overhead lights in the tunnel which may block the path of the coach	B1	3.5b
		(1)	

(6 marks)

Summary

1 To solve a quadratic equation by factorising:

- Write the equation in the form $ax^2 + bx + c = 0$
- Factorise the left-hand side
- Set each factor equal to zero and solve to find the value(s) of x

2 The solutions of the equation $ax^2 + bx + c = 0$ where $a \neq 0$ are given by the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3 $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

4 $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

5 The set of possible inputs for a function is called the **domain**.

The set of possible outputs of a function is called the **range**.

6 The **roots** of a function are the values of x for which $f(x) = 0$.

7 You can find the coordinates of a **turning point** of a quadratic graph by completing the square. If $f(x) = a(x + p)^2 + q$, the graph of $y = f(x)$ has a turning point at $(-p, q)$.

8 For the quadratic function $f(x) = ax^2 + bx + c = 0$, the expression $b^2 - 4ac$ is called the **discriminant**. The value of the discriminant shows how many roots $f(x)$ has:

- If $b^2 - 4ac > 0$ then a quadratic function has two distinct real roots.
- If $b^2 - 4ac = 0$ then a quadratic function has one repeated real root.
- If $b^2 - 4ac < 0$ then a quadratic function has no real roots.

9 Quadratics can be used to model real-life situations.

T.35 mixed ex, P.14 BSG