KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 12
Pure Mathematics
P1 5 Straight Line graphs
Booklet

HGS Maths

Dr Frost Course

Name: ______________________________________

Class: ______________________________________

Contents

Prior knowledge check

Prior knowledge check

1 Find the point of intersection of the following pairs of lines.

a $y = 4x + 7$ and $5y = 2x - 1$

b $y = 5x - 1$ and $3x + 7y = 11$

c $2x - 5y = -1$ and $5x - 7y = 14$

← GCSE Mathematics

2 Simplify each of the following:

a $\sqrt{80}$ **b** $\sqrt{200}$ **c** $\sqrt{125}$

← Section 1.5

3 Make y the subject of each equation:

a $6x + 3y - 15 = 0$ **b** $2x - 5y - 9 = 0$

c $3x - 7y + 12 = 0$

← GCSE Mathematics

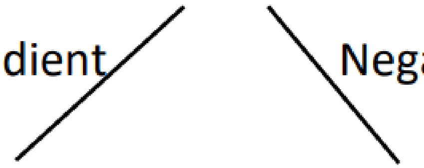
5.2) Equations of straight lines

GRAPHS OF LINEAR FUNCTIONS

$$y = mx + c$$

the line intercepts the y axis at (0, c)

Gradient = $\frac{change\ in\ y}{change\ in\ x}$



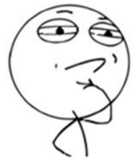
Finding the equation of a line with gradient m through point (x_1,y_1)

Use the equation $(y - y_1) = m(x - x_1)$

If necessary rearrange to the required form ($ax + by = c$ or $y = mx - c$)

notes

Why might we want to put a straight line equation in the form $ax + by + c = 0$?

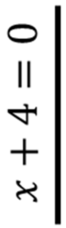


$y = mx + c$
"Slope-Intercept Form"

$ax + by + c = 0$
"Standard Form"

Coverage

$y = mx + c$ doesn't allow you to represent vertical lines. Standard form allows us to do this by just making b zero.



Symmetry

In general, the '**linear combination**' of two variables x and y is $ax + by$, i.e. "some amount of x and some amount of y". There is a greater elegance and symmetry to this form over $y = mx + c$ because x and y appear similarly within the expression.

Usefulness

This more 'elegant' form also means it ties in with vectors and matrices. In FM, you will learn about the '**dot product**' of two vectors:

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = ax + by$$

thus since $ax + by + c = 0$, we can represent a straight line using:

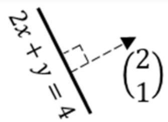
$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + c = 0 \qquad (1)$$

We can extend to 3D points to get the equation of a **plane**:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d = 0 \qquad (2)$$

Conveniently, in equation (1), the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ is **perpendicular to the line**. And in equation (2), the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is perpendicular to the plane. Nice!

$$2x + y = 4 \;\Rightarrow\; \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 4 \;\Rightarrow$$



Fill in the blank

Line	x-intercept	y-intercept
$y = 2x + 3$		
$y = 3x + 2$		
$y = 3x - 2$		
$y = 2x - 3$		
$y = 3 - 2x$		
$y = 2 - 3x$		
$2x + 3y = 6$		
$3x + 2y = 6$		
$y = ax + b$		

Worked Example

The lines $y = 2x - 7$ and $3x + 2y - 21 = 0$ intersect at the point A.
The point B has coordinates $(2, -8)$.
Find the equation of the line that passes through the points A and B.
Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

T.94: 5C Qs 3+ and 5D all Qs, P.38: 5.2 Qs 5+

5.3) Parallel and perpendicular lines

Finding the equation of a line with gradient m through point (x_1,y_1)

Use the equation $(y - y_1) = m(x - x_1)$

If necessary rearrange to the required form ($ax + by = c$ or $y = mx - c$)

Parallel and Perpendicular Lines

$y = m_1x + c_1$ $y = m_2x + c_2$

If $\mathbf{m_1 = m_2}$ then the lines are **PARALLEL**

If $\mathbf{m_1 \times m_2 = -1}$ then the lines are **PERPENDICULAR**

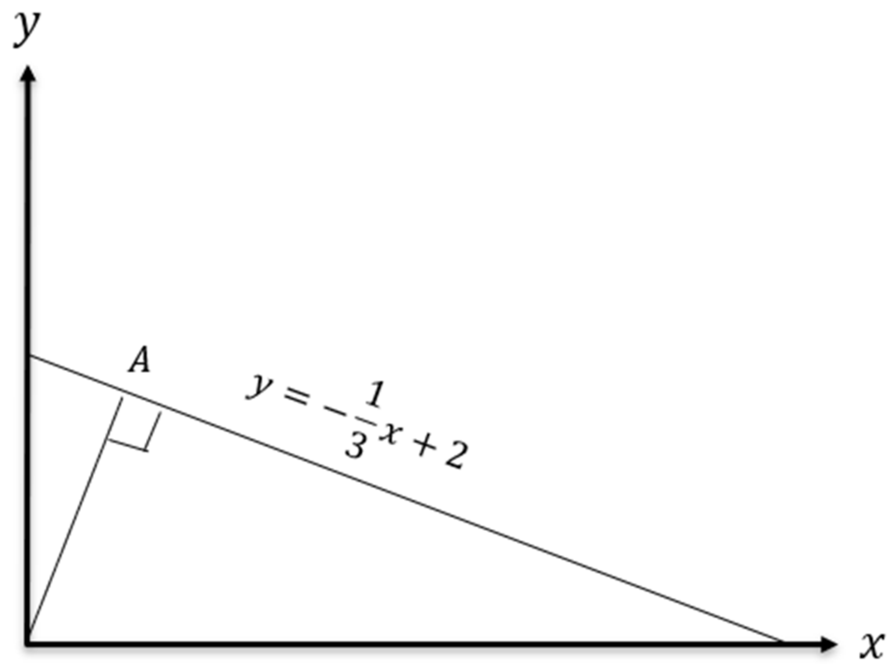
notes

Worked Example

The points A, B and C have coordinates $(0, 12)$, $(-3, 0)$ and $(0, c)$ respectively.
The line through points A and B is perpendicular to the line through points B and C.
Find the value of c

Worked Example

Determine the coordinates of A



T.99: 5F Qs 7+, P.39: 5.3 Qs 5+

Past Paper Q

AS2019

1. The line l_1 has equation $2x + 4y - 3 = 0$

   The line l_2 has equation $y = mx + 7$, where m is a constant.

   Given that l_1 and l_2 are perpendicular,

   (a) find the value of m.

   (2)

   The lines l_1 and l_2 meet at the point P.

   (b) Find the x coordinate of P.

   (2)

Your Turn

1.

The line L_1 has equation $4x + 2y - 3 = 0$

(a) Find the gradient of L_1.

(1)

The line L_2 is perpendicular to L_1 and passes through the point (2, 5).

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants.

(3)

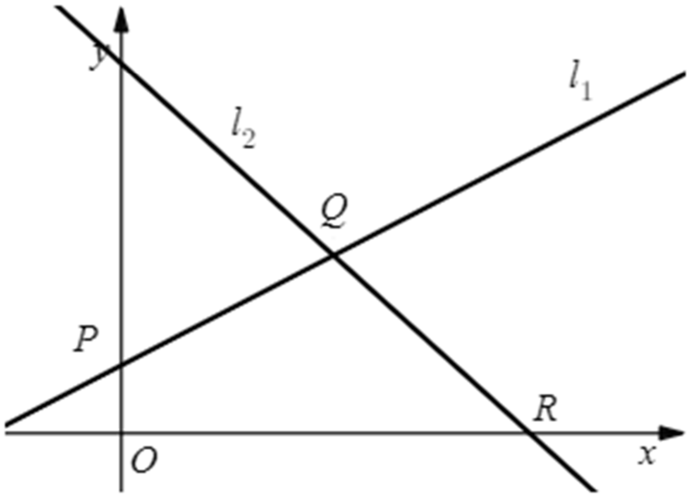
5.4) Length and area

Worked Example

495g: Determine the area of a quadrilateral enclosed by both axes and two intersecting lines.

The line l_1 has equation $y = 4x + 5$
The line l_2 has equation $y = -7x + 27$

The line l_1 passes through Q and intersects the y-axis at P.
The line l_2 passes through Q and intersects the x-axis at R



Find the exact area of the quadrilateral $OPQR$.

AS2023

Q1.

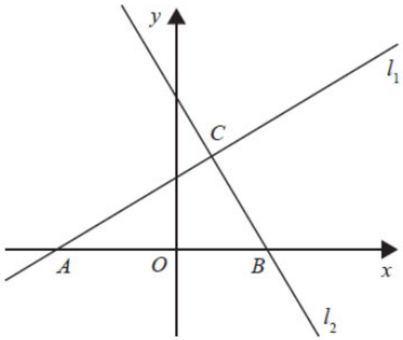


Figure 4

The line l_1 has equation $y = \frac{3}{5}x + 6$

The line l_2 is perpendicular to l_1 and passes through the point $B(8,0)$, as shown in the sketch in Figure 4.

[redacted] equation for line l_2 is

$$5x + 3y = 40$$

(3)

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

(b) find the exact area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{p}{q}$

(5)

Your Turn

10.

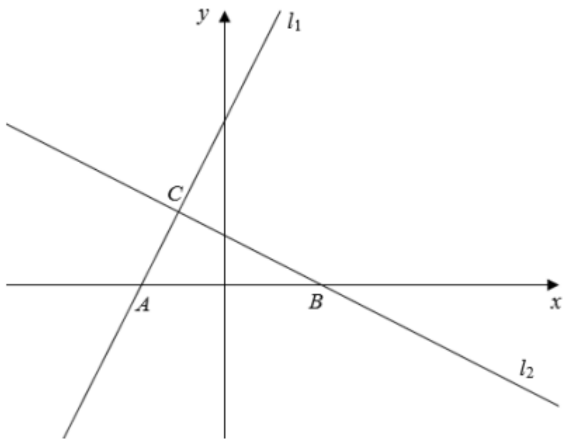


Figure 4

The line l_1 has equation $y = \frac{7}{3}x + 9$

The line l_2 is perpendicular to l_1 and passes through the point $B\left(\frac{77}{15}, 0\right)$, as shown in the sketch in Figure 4.

(a) Show that an equation for line l_2 is

$$15x + 35y = 77$$

(3)

Given that

- lines l_1 and l_2 intersect at the point C
- line l_1 crosses the x-axis at the point A

(b) find the area of triangle ABC, giving your answer as a fully simplified fraction in the form $\frac{a}{b}$ or a decimal rounded to 3 significant figures.

(5)

(Total for Question 10 is 8 marks)

5.5) Modelling with straight lines

notes

Worked Example

In 2010 the population of rabbits in an area was 200. Locals projected that the number of rabbits would increase by 4 per year.

a) Write a linear model for the population, p, of rabbits t years after 2010

b) Write down a reason why this might not be a realistic model.

T.106: 5H Qs all, P.42: 5.5 Qs all

AS2019

4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

Your Turn

4.
A large plant pot shown in figure A initially has some water in it. Jim then starts to pour more water into it at a constant rate.

The depth of water, D cm, was measured t seconds after Jim started pouring more water in.

Exactly 5 seconds after Jim starts pouring, the depth was 13.52cm
Exactly 8 seconds after he started pouring the depth was 18.68cm

Figure A

Using a **linear** model

(a) Find an equation for D in terms of t. (3)

(b) Find the initial depth of the water. (1)

(c) Explain why this model may not be suitable. (1)

8.

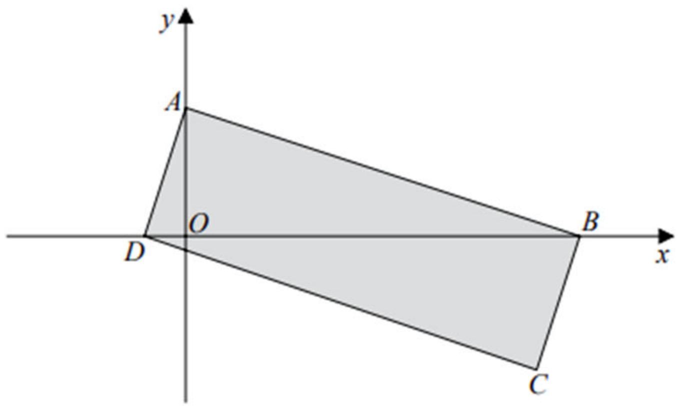


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD. Either $\sqrt{5^2+2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2+2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	

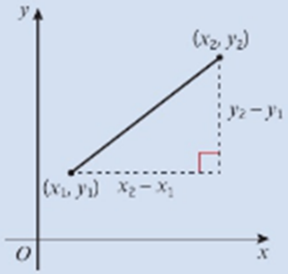
(7 marks)

Summary of Key Points

Summary of key points

1 The gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



2 • The equation of a straight line can be written in the form

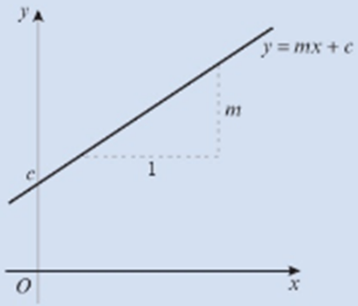
$y = mx + c$,

where m is the gradient and $(0, c)$ is the y-intercept.

• The equation of a straight line can also be written in the form

$ax + by + c = 0$,

where a, b and c are integers.



3 The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as $y - y_1 = m(x - x_1)$.

4 Parallel lines have the same gradient.

5 If a line has a gradient m, a line perpendicular to it has a gradient of $-\frac{1}{m}$

6 If two lines are perpendicular, the product of their gradients is -1.

7 You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

8 The point of intersection of two lines can be found using simultaneous equations.

9 Two quantities are in direct proportion when they increase at the same rate. The graph of these quantities is a straight line through the origin.

10 A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problems in order to create a model.

T.108: mixed ex. , P.44: BSG