KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 10 Trigonometric Identities and Equations
Booklet

HGS Maths

Dr Frost Course

Name: ________________________________

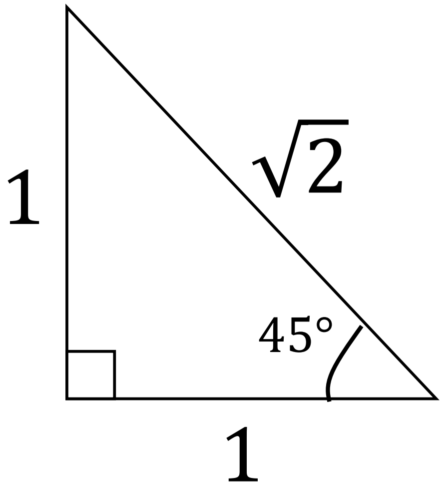
Class: ________________________________

Contents

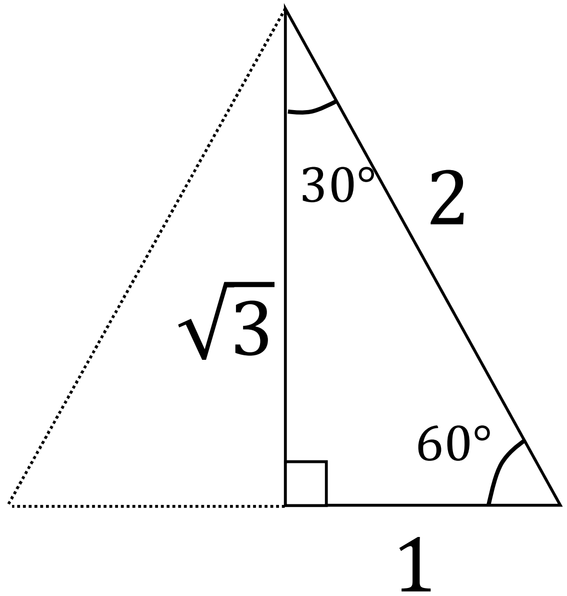
Prerequisite Knowledge

sin/cos/tan of $30°, 45°, 60°$

You will frequently encounter angles of $30°, 60°, 45°$ in geometric problems. Why?
We see these angles in equilateral triangles and half squares:



$$\sin(45°) = \frac{1}{\sqrt{2}}$$
$$\cos(45°) = \frac{1}{\sqrt{2}}$$
$$\tan(45°) = 1$$



$$\sin(30°) = \frac{1}{2}$$
$$\cos(30°) = \frac{\sqrt{3}}{2}$$
$$\tan(30°) = \frac{1}{\sqrt{3}}$$
$$\sin(60°) = \frac{\sqrt{3}}{2}$$
$$\cos(60°) = \frac{1}{2}$$
$$\tan(60°) = \sqrt{3}$$

Activity

Sort It Out	Exact Trigonometric Values

Sort the trigonometric values into the correct group below.

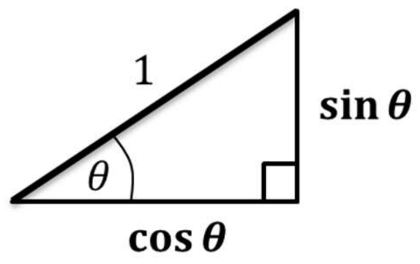
1	$\sin 45$	**2**	$\cos 30$	**3**	$\tan 45$
4	$\cos 0$	**5**	$\sin 60$	**6**	$\tan 360$
7	$\sin 90$	**8**	$\cos 45$	**9**	$\sin 30$
10	$\cos 60$	**11**	$\tan 0$	**12**	$\sin 0$
13	$\sin 360$	**14**	$\tan 60$	**15**	$\sin 270$
16	$\tan 180$	**17**	$\cos 90$	**18**	$\tan 30$
19	$\sin 135$	**20**	$\cos 360$	**21**	$\tan 225$
22	$\cos 180$	**23**	$\tan 210$	**24**	$\cos 300$
25	$\tan 240$	**26**	$\cos 330$	**27**	$\sin 120$
28	$\sin 150$	**29**	$\cos 315$	**30**	$\tan 135$

A	Equal to $\frac{1}{2}$	**B**	Equal to $\frac{\sqrt{3}}{2}$	**C**	Equal to $\frac{\sqrt{2}}{2}$	**D**	Equal to -1
E	Equal to 1	**F**	Equal to 0	**G**	Equal to $\frac{\sqrt{3}}{3}$	**H**	Equal to $\sqrt{3}$

Notes

The Unit Circle and Trigonometry

For values of θ in the range $0 < \theta < 90°$, you know that $\sin\theta$ and $\cos\theta$ are lengths on a right-angled triangle:



And what would be the **gradient** of the bold line?

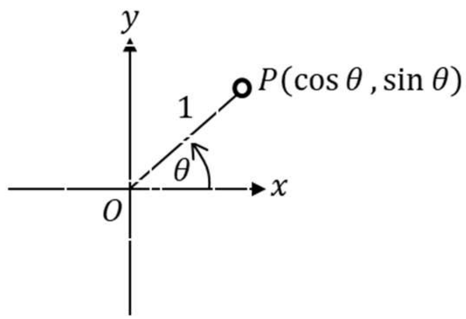
$$m = \frac{\Delta y}{\Delta x} = \frac{\sin\theta}{\cos\theta}$$

But also: $\tan\theta = \frac{opp}{adj} = \frac{\sin\theta}{\cos\theta}$

$$\therefore m = \tan\theta$$

But how do we get the rest of the graph for sin, cos and tan when $90° \leq \theta \leq 360°$?

The point P on a unit circle, such that OP makes an angle θ with the positive x-axis, has coordinates $(\cos\theta, \sin\theta)$.
OP has gradient $\tan\theta$.



Angles are always measured **anticlockwise**.
(Further Mathematicians will encounter the same when they get to Complex Numbers)

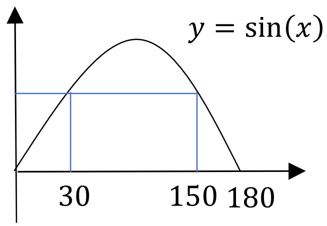
We can consider the coordinate $(\cos\theta, \sin\theta)$ as θ increases from 0 to $360°$...

Notes

A Few Trigonometric Angle Laws

1 $\sin(x) = \sin(180° - x)$

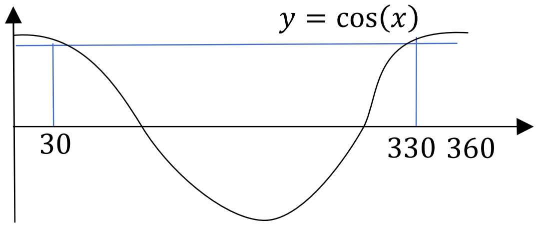
e.g. $\sin(150°) = \sin(30°)$



We saw this in the previous chapter when covering the 'ambiguous case' when using the sine rule.

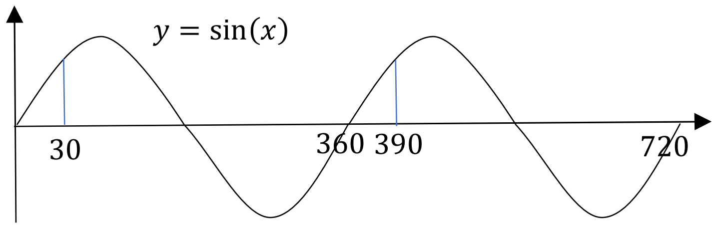
2 $\cos(x) = \cos(360° - x)$

e.g. $\cos(330°) = \cos(30°)$



3 *sin* and *cos* repeat every 360° but *tan* every 180°

e.g. $\sin(390°) = \sin(30°)$



4 $\sin(x) = \cos(90° - x)$

e.g. $\sin(50°) = \cos(40°)$

Remember from the previous chapter that "cosine" by definition is the sine of the "complementary" angle. This was/is never covered in the textbook but caught everyone by surprise when it came up in a C3 exam.

Examples

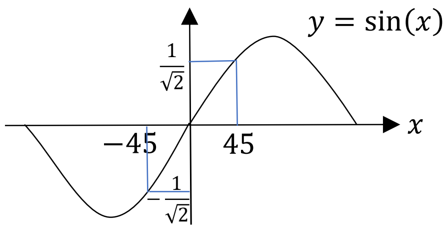
$\tan(225°) = \mathbf{\tan(45°)} = \mathbf{1}$ ← *tan* repeats every 180° so can subtract 180°

$\tan(210°) = \mathbf{\tan(30°)} = \mathbf{\frac{1}{\sqrt{3}}}$

$\sin(150°) = \mathbf{\sin(30°)} = \mathbf{\frac{1}{2}}$ ← For *sin* we can subtract from 180°.

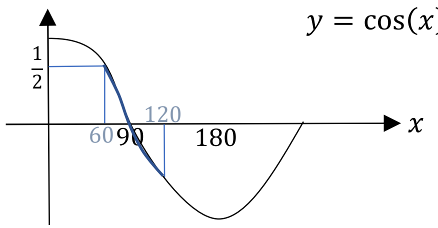
$\cos(300°) = \mathbf{\cos(60°)} = \mathbf{\frac{1}{2}}$ ← For *cos* we can subtract from 360°.

$\sin(-45°) = -\mathbf{\sin(45°)} = -\mathbf{\frac{1}{\sqrt{2}}}$ ← We have to resort to a sketch for this one.



$\cos(750°) = \mathbf{\cos(30°)} = \mathbf{\frac{\sqrt{3}}{2}}$ ← cos repeats every 360°.

$\cos(120°) = -\mathbf{\cos(60°)} = -\mathbf{\frac{1}{2}}$ ← Again, let's just use a graph.



The graph is rotationally symmetric about 90°. Since 120° is 30° above 90°, we get the same y value for $90° - 30° = 60°$, except negative.

Use the 'laws' where you can, but otherwise just draw out a quick sketch of the graph.

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- sin, cos repeat every 360° but tan every 180°

reflections: It's not hard to see from the graph that in general, $\sin(-x) = -\sin(x)$. Even more generally, a function f is known as an '**odd function**' if $f(-x) = -f(x)$. tan is similarly 'odd' as $\tan(-x) = -\tan(x)$.

A function is **even** if $f(-x) = f(x)$. Examples are $f(x) = \cos(x)$ and $f(x) = x^2$ as $\cos(-x) = \cos(x)$ and $(-x)^2 = (x)^2$. You do not need to know this for the exam.

Test Your Understanding

Without a calculator, work out the value of each below.

$\cos(315°) =$

$\sin(420°) =$

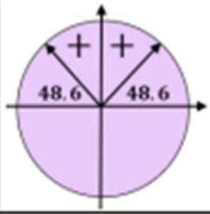
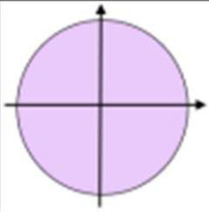
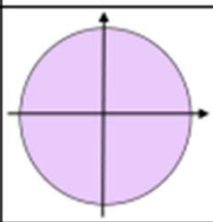
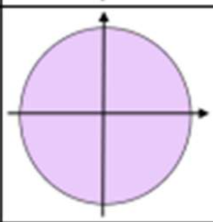
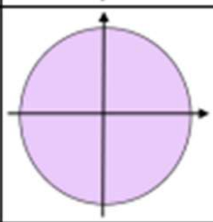
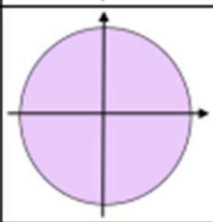
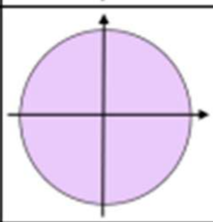
$\tan(-120°) =$

$\tan(-45°) =$

$\sin(135°) =$

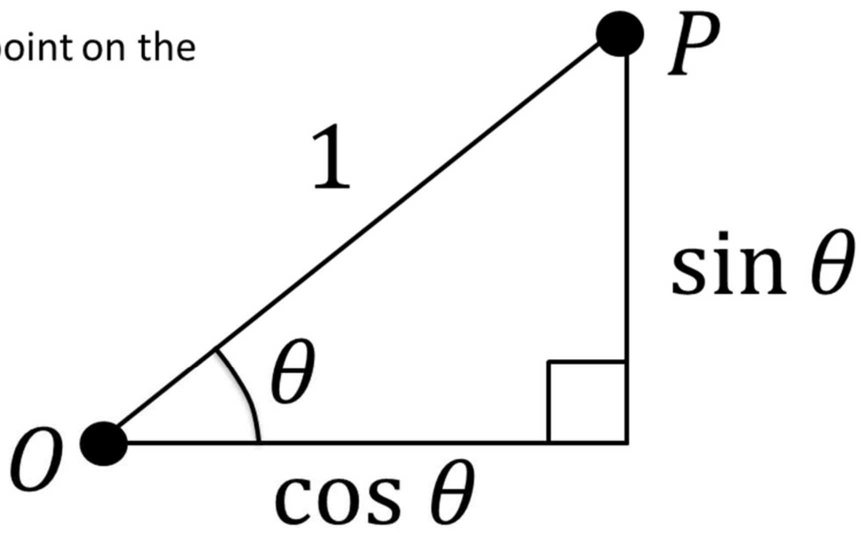
Prerequisite Knowledge

Fill in the Blanks — Solving Trigonometric Equations

Question	Rearranged Equation	Acute Angle	Unit Circle	Solutions
Solve $4\sin\theta = 3$ for $0° \leq \theta < 360°$	$\sin\theta = \frac{3}{4}$	$\theta = \sin^{-1}\left(\frac{3}{4}\right)$ $\theta = 48.6°$		
Solve $5\cos\theta = 1$ for $0° \leq \theta < 360°$				
Solve $5\tan\theta = -8$ for $0° \leq \theta < 360°$				
Solve $\tan\theta - 6 = 0$ for $-180° \leq \theta < 180°$				
Solve $3\sin\theta + 2 = 0$ for $-180° \leq \theta < 180°$				
Solve $16\cos^2\theta = 9$ for $0° \leq \theta < 360°$				
Solve $4\tan^2\theta = 25$ for $-180° \leq \theta < 180°$				

10.3 Trigonometric Identities

Returning to our point on the unit circle...



1 Then $tan\ \theta = \dfrac{sin\ \theta}{cos\ \theta}$

2 Pythagoras gives you...

$$sin^2\theta + cos^2\theta = 1$$

$\sin^2\theta$ is a shorthand for $(\sin\theta)^2$. It does NOT mean the sin is being squared – this does not make sense as sin is a function, and not a quantity that we can square!

Notes

Worked Example

Simplify:

$\sin^2 3x + \cos^2 3x$

Worked Example

Prove that $1 - \tan\theta \sin\theta \cos\theta \equiv \cos^2\theta$

Worked Example

Prove that $\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$

Worked Example

Simplify $5 - 5\sin^2\theta$

Worked Example

Simplify:

$$\frac{\sin 2\theta}{\sqrt{1-\sin^2 2\theta}}$$

Worked Example

Prove that

$$\frac{\cos^4\theta - \sin^4\theta}{\cos^2\theta} \equiv 1 - \tan^2\theta$$

Worked Example

Prove that

$$\frac{\tan x \cos x}{\sqrt{1 - \cos^2 x}} \equiv 1$$

Worked Example

Prove that

$$\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1$$

Worked Example K502a

Given that $\sin\theta = \frac{2}{5}$ and that θ is obtuse, find the exact value of $\cos\theta$

Worked Example

502b: Determine exact values of $\sin$, $\cos$ and $\tan$ using another known trigonometric ratio, for obtuse or reflex angles.

It is given that B is reflex and that $\sin B = -\frac{1}{2}$

Find the exact value of $\cos B$.

Worked Example

502c: Determine exact values of $\sin$, $\cos$ and $\tan$ using another known trigonometric ratio in terms of an algebraic expression.

Given that

$$\sin\theta = \frac{k-1}{k+1}$$

where k is a positive constant and θ is acute.

Find an expression for $\cos\theta$ in terms of k.

$\cos\theta =$

Worked Example

502f: Write an expression given in the form $a\sin^2 x + b\cos^2 x$ in terms of just $\sin$ or $\cos$

Given that:

$$2\sin^2 3\alpha + 8\cos^2 3\alpha \equiv A + B\sin^2 3\alpha$$

where A and B are integers.

Work out the values of A and B.

Worked Example

502i: Simplify a trigonometric expression by using the identity $\sin^2 x + \cos^2 x \equiv 1$ and $\tan x = \frac{\sin x}{\cos x}$

Simplify

$$1 - \sin\alpha\cos\alpha\tan\alpha$$

giving your answer as a single trigonometric function.

Worked Example

502j: Simplify a trigonometric expression involving $\tan^2 x$ or $\tan^3 x$

Simplify

$$1 - \cos^2\theta \tan^2\theta$$

giving your answer as a single trigonometric function.

Worked Example

502k: Simplify an algebraic fraction in terms of $\sin$ and $\cos$ using quadratic factorisation.

Simplify

$$\frac{12 + 3\sin x}{2\cos^2 x - 5\sin x + 10}$$

Giving your answer in the form $\dfrac{a}{b + c\sin x}$

Worked Example

Given that $p = 3\cos\theta$ and $q = 2\sin\theta$, show that $4p^2 + 9q^2 = 36$

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta \equiv \frac{\cos\theta}{1 - \sin\theta} \qquad \theta \neq (2n+1)90^\circ \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

Your Turn

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Show that

$$\frac{1}{\sin\theta} + \frac{1}{\tan\theta} \equiv \frac{\sin\theta}{1-\cos\theta} \qquad \theta \neq 180n° \quad n \in \mathbb{Z}$$

(3)

12. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4\tan x = 5\cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0$$

(3)

Your Turn

12. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Show that the equation

$$\tan x \cos x - \sin x \cos^2 x = \frac{\sin x + \cos^3 x}{\tan x}$$

can be written as

$$\tan^2 x - \tan x - 1 = 0$$

(3)

10.4 Simple Trigonometric Equations

Notes

Worked Example

503a: Solve a trigonometric equation given in the form $\sin x = k$ where x is in degrees.

Solve $\sin x = -0.8$ in the interval $0^\circ \leq x \leq 720^\circ$

Give your solution(s) correct to 1 decimal place where appropriate.

$x =$ ______ $^\circ$

$x =$ ______ $^\circ$

$x =$ ______ $^\circ$

$x =$ ______ $^\circ$

Worked Example

503f: Solve trigonometric equations given in the form $\sin^2 x = a$ where x is in degrees.

Solve $\sin^2 x = 0.49$ in the interval $0° \leq x \leq 360°$

Give your solution(s) correct to 1 decimal place where appropriate.

$x =$ ______ °

$x =$ ______ °

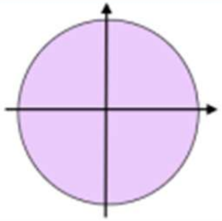
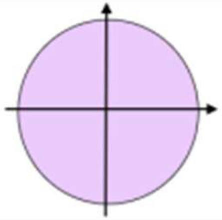
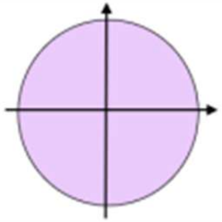
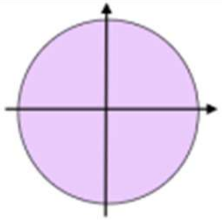
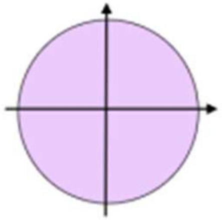
$x =$ ______ °

$x =$ ______ °

10.5 Harder Trigonometric Equations

Notes

Fill in the Blanks — Solving Trigonometric Equations with Multiple Angles

Question	Substitute $x = \cdots$	Rearrange Equation	Acute Angle	Range for x	Unit Circle	Solutions for x	Solutions for θ
Solve $7\cos(2\theta) = 5$ for $0° \le \theta < 180°$	$x = 2\theta$ $7\cos x = 5$	$\cos x = \frac{5}{7}$	$x = \cos^{-1}\left(\frac{5}{7}\right)$ $x = 44.415°$	$0° \le x < 360°$			
Solve $8\sin(3\theta) - 7 = 0$ for $-90° \le \theta < 90°$	$x = 3\theta$ $8\sin x - 7 = 0$						
Solve $\frac{4}{\tan(\theta + 25)} = 3$ for $-180° \le \theta < 180°$							
$9\cos(2\theta - 15) = 4$ for $0° \le \theta < 360°$							
$\frac{\tan(3\theta + 70)}{2} + 3 = 0$ for $-90° \le \theta < 90°$							

503e: Solve a trigonometric equation given in the form $\sin(ax+b)=k$ where x is in degrees and k is negative.

Solve $\sin(2x+90) = -0.4$ in the interval $0° \leq x \leq 360°$

Give your solution(s) correct to 1 decimal place where appropriate.

$x =$ ____ °

$x =$ ____ °

$x =$ ____ °

$x =$ ____ °

Worked Example

503i: Solve a trigonometric equation given in the form $p\sin(ax+b) = q\cos(ax+b)$

Solve $4\sin(\frac{1}{2}\theta + 60) = \sqrt{3}\cos(\frac{1}{2}\theta + 60)$ in the interval $0° < \theta < 720°$

Give your solutions correct to 2 decimal places where appropriate.

$\theta =$ ______ °

$\theta =$ ______ °

Worked Example

503n: Solve a trigonometric equation from a modelled scenario.

The depth of water, H meters, in a harbour on a particular day is modelled by the formula

$$H = 10 + 2\sin\left(\tfrac{1}{2}\theta\right), \qquad 0 \leq \theta \leq 1440$$

where θ is the number of minutes after midnight.

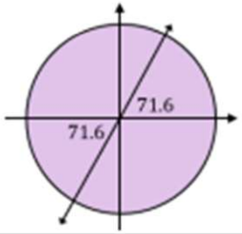
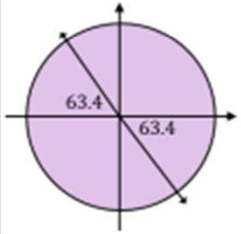
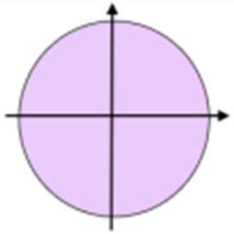
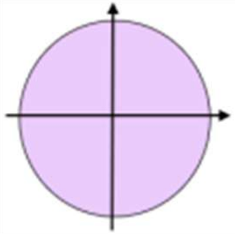
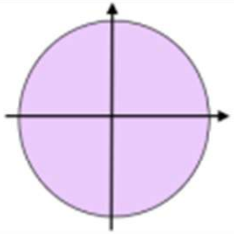
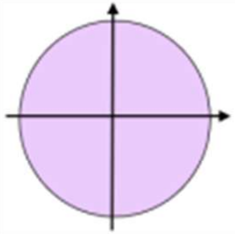
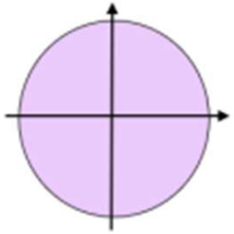
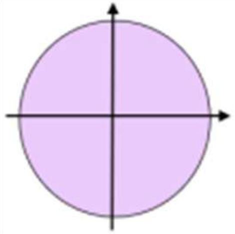
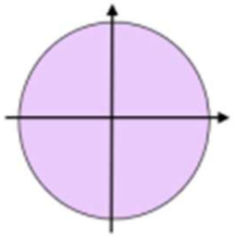
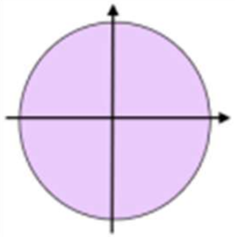
Freddie needs to leave the harbour by 1 pm at the latest. Find the last time the water is at a depth of the water is **11.96** meters before Freddie needs to leave.
Give your answer to the nearest minute in 24 hour clock.

☐ : ☐

10.6 Equations and Identities

Fill in the Blanks

Solving Quadratic Trigonometric Equations

Question	Factorise and Solve	Solve 1st Trig Equation	Solve 2nd Trig Equation
Solve $\tan^2\theta - \tan\theta - 6 = 0$ for $0° \leq \theta < 360°$	$(\tan\theta - 3)(\tan\theta + 2) = 0$ $\tan\theta = 3$ or $\tan\theta = -2$	 $\theta = \tan^{-1}(3)$ $\theta = 71.6°$ $\boldsymbol{\theta = 71.6°, 251.6°}$	 $\theta = \tan^{-1}(2)$ $\theta = 63.4°$ $\boldsymbol{\theta = 116.6°, 296.6°}$
Solve $6\sin^2\theta - \sin\theta - 1 = 0$ for $-180° \leq \theta < 180°$			
Solve $2\cos^2\theta + 7\cos\theta + 3 = 0$ for $0° \leq \theta < 360°$			
Solve $3\tan^2\theta - 8\tan\theta + 4 = 0$ for $-180° \leq \theta < 180°$			
Solve $9\sin^2\theta - 1 = 0$ for $0° \leq \theta < 360°$			

Worked Example

Solve in the interval $0 \leq x < 360°$:

$5\sin^2 x + 3\sin x - 2 = 0$

Worked Example

503j: Solve a quadratic equation involving a single trigonometric function.

Solve $2\cos^2 x = 15\cos x - 7$ in the interval $0^\circ < x < 360^\circ$

Give your solution(s) correct to 2 decimal places where appropriate.

$x =$ ______ $^\circ$

$x =$ ______ $^\circ$

Worked Example

503k: Solve a trigonometric equation involving a mixture of $\sin$ and $\cos$ where one is squared.

Solve $2\cos^2\theta = 3(-3\sin\theta + 2)$ in the interval $0^\circ \leq \theta < 360^\circ$

Give your solution(s) correct to 2 decimal places where appropriate.

$\theta =$ $^\circ$

$\theta =$ $^\circ$

Worked Example

Solve in the interval $0 \leq x \leq 360°$:

$\sin^2(x - 30°) = \frac{1}{2}$

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos\theta} + \tan\theta = \frac{\cos\theta}{1-\sin\theta} \qquad \theta \neq (2n+1)90° \quad n \in \mathbb{Z}$$

(3)

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90°$

$$\frac{1}{\cos 2x} + \tan 2x = 3\cos 2x$$

giving your answers to one decimal place.

(5)

Your Turn

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Show that

$$\frac{1}{\sin\theta}+\frac{1}{\tan\theta}\equiv\frac{\sin\theta}{1-\cos\theta} \qquad \theta\neq 180n^\circ \quad n\in\mathbb{Z}$$

(3)

Given that $\sin 2x$, $\tan 2x \neq 0$

(*b*) solve for $0 \leq x \leq 180^\circ$

$$\frac{1}{\sin 2x}+\frac{1}{\tan 2x}=4\sin 2x$$

(5)

(Total for Question 13 is 8 marks)

12. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4\tan x = 5\cos x$$

can be written as

$$5\sin^2 x + 4\sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leqslant 360°$

$$4\tan x = 5\cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4\tan 3x = 5\cos 3x$$

in the interval $0 < x \leqslant 1800°$, explaining briefly the reason for your answer.

(2)

Your Turn

12. **In this question you must show detailed reasoning.**

Solutions relying entirely on calculator technology are not acceptable.

(*a*) Show that the equation

$$\tan x \cos x - \sin x \cos^2 x = \frac{\sin x + \cos^3 x}{\tan x}$$

can be written as

$$\tan^2 x - \tan x - 1 = 0$$

(3)

(*b*) Hence solve, for $-90° < x < 90°$

$$\tan x \cos x - \sin x \cos^2 x = \frac{\sin x + \cos^3 x}{\tan x}$$

giving your answers to one decimal place.

(4)

(*c*) Hence find the **number of solutions** of the equation

$$\tan \frac{1}{2}x \cos \frac{1}{2}x - \sin \frac{1}{2}x \cos^2 \frac{1}{2}x = \frac{\sin \frac{1}{2}x + \cos^3 \frac{1}{2}x}{\tan \frac{1}{2}x}$$

in the interval $-900° < x < 900°$, explaining briefly the reason for your answer.

(2)

(Total for Question 12 is 9 marks)

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29\cos(9t + \alpha)° + \beta \qquad 0 \leqslant \alpha < 360°$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

Your Turn

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 42 m
- a carriage starts a circuit at a vertical height of 1.5 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 15)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 25$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 20.25 \cos(7t + \alpha)^\circ + \beta \qquad 0 \leqslant \alpha < 360^\circ$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

(Total for Question 13 is 7 marks)

12. (a) Solve, for $-180° \leqslant x < 180°$, the equation

$$3\sin^2 x + \sin x + 8 = 9\cos^2 x$$

giving your answers to 2 decimal places.

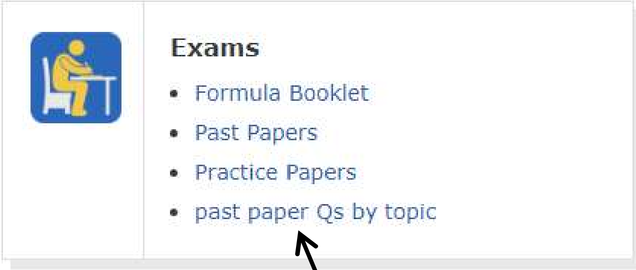
(6)

(b) Hence find the smallest positive solution of the equation

$$3\sin^2(2\theta - 30°) + \sin(2\theta - 30°) + 8 = 9\cos^2(2\theta - 30°)$$

giving your answer to 2 decimal places.

(2)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

12(a)	Uses $\cos^2 x = 1 - \sin^2 x \Rightarrow 3\sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$	M1	3.1
	$\Rightarrow 12\sin^2 x + \sin x - 1 = 0$	A1	1.1
	$\Rightarrow (4\sin x - 1)(3\sin x + 1) = 0$	M1	1.1
	$\Rightarrow \sin x = \frac{1}{4}, -\frac{1}{3}$	A1	1.1
	Uses arcsin to obtain two correct values	M1	1.1
	All four of $x = 14.48°, 165.52°, -19.47°, -160.53°$	A1	1.1
		(6)	
(b)	Attempts $2\theta - 30° = -19.47°$	M1	3.1
	$\Rightarrow \theta = 5.26°$	A1ft	1.1
		(2)	
		(8 marks)	

Summary of Key Points

5 For all values of θ, $\sin^2\theta + \cos^2\theta \equiv 1$

6 For all values of θ such that $\cos\theta \neq 0$, $\tan\theta \equiv \dfrac{\sin\theta}{\cos\theta}$

7
- Solutions to $\sin\theta = k$ and $\cos\theta = k$ only exist when $-1 \leqslant k \leqslant 1$
- Solutions to $\tan\theta = p$ exist for all values of p.

8 When you use the inverse trigonometric functions on your calculator, the angle you get is called the **principal value**.

9 Your calculator will give principal values in the following ranges:
- $\cos^{-1}$ in the range $0 \leqslant \theta \leqslant 180°$
- $\sin^{-1}$ in the range $-90° \leqslant \theta \leqslant 90°$
- $\tan^{-1}$ in the range $-90° \leqslant \theta \leqslant 90°$