KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 9 Trigonometric Ratios
Booklet

HGS Maths

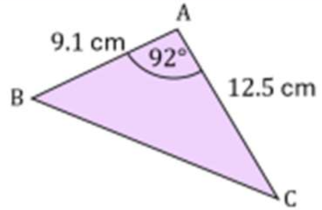
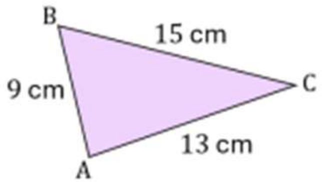
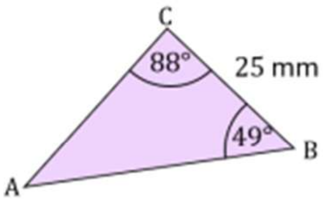
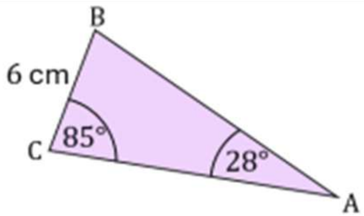
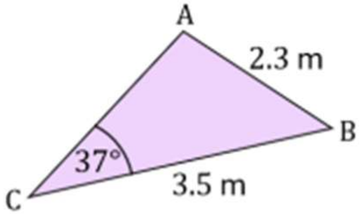
Dr Frost Course

Name: ___________________________________

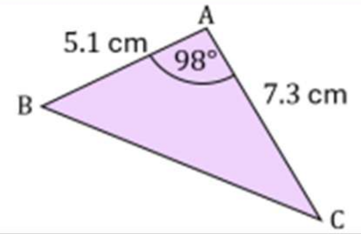
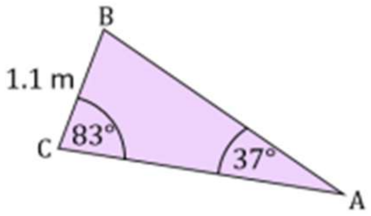
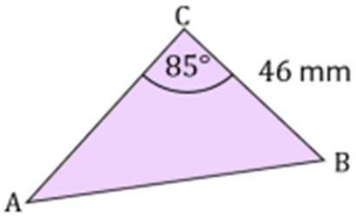
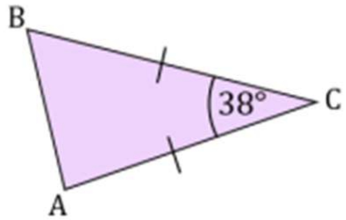
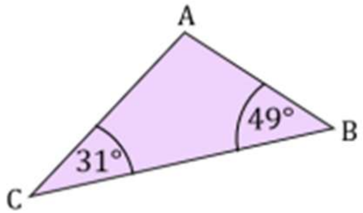
Class: ___________________________________

Contents

Fill in the Blanks	Sine and Cosine Rules

Triangle	AB	BC	AC	$A\hat{B}C$	$A\hat{C}B$	$B\hat{A}C$
	9.1 cm		12.5 cm			92°
	9 cm	15 cm	13 cm			
		25 mm		49°	88°	
		6 cm			85°	28°
	2.3 m	3.5 m			37°	

Fill in the Blanks — Area of a Non Right-Angled Triangle

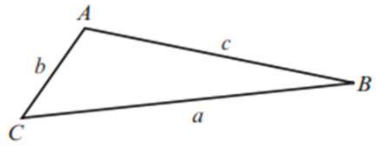
Triangle	Area	AB	BC	AC	$A\hat{B}C$	$A\hat{C}B$	$B\hat{A}C$
		5.1 cm		7.3 cm			98°
			1.1 m			83°	37°
	1489 mm^2		46 mm			85°	
	33.9 cm^2					38°	
	9.67 cm^2				49°	31°	

9.4 Solving Triangle Problems

These are **not** in formulae booklet:

- **This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:**

  $a^2 = b^2 + c^2 - 2bc \cos A$

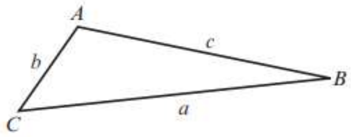


Watch out You can exchange the letters depending on which side you want to find, as long as each side has the same letter as the **opposite** angle.

The sine rule can be used to work out missing sides or angles in triangles.

- **This version of the sine rule is used to find the length of a missing side:**

  $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

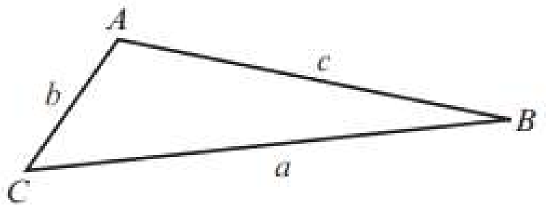


- **This version of the sine rule is used to find a missing angle:**

  $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

You can use the standard trigonometric ratios for right-angled triangles to prove the sine rule:

- **Area** $= \frac{1}{2}ab \sin C$

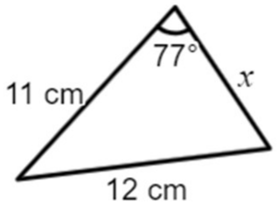


Notes

Worked Example

466e: **Use the sine rule/Law of Sines and cosine rule/Law of Cosines within a single triangle.**

Find the value of x.

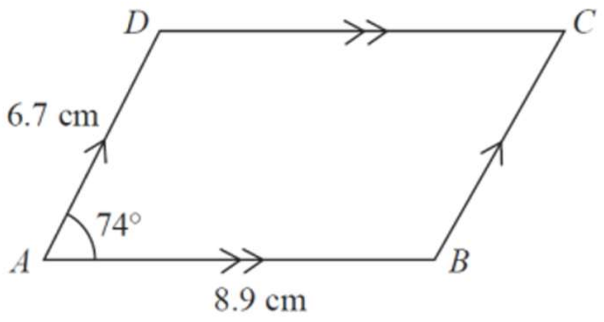


Give your answer correct to 1 decimal place.

T.189 9E: Qs 1-5, P.72 9.4: Qs 1-2

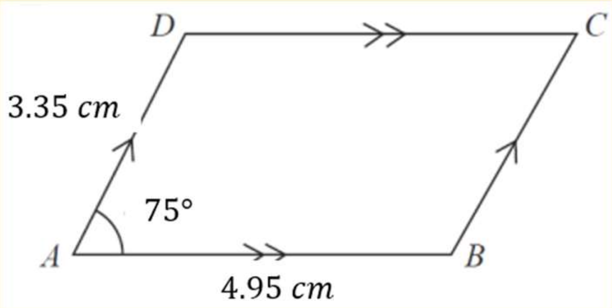
Worked Example

Calculate the area of the parallelogram.



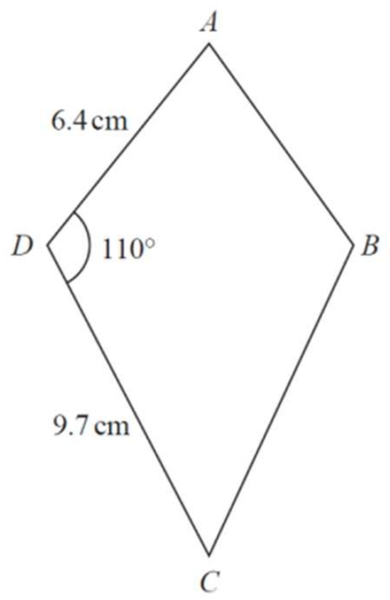
Your Turn

Calculate the area of the parallelogram.



Worked Example

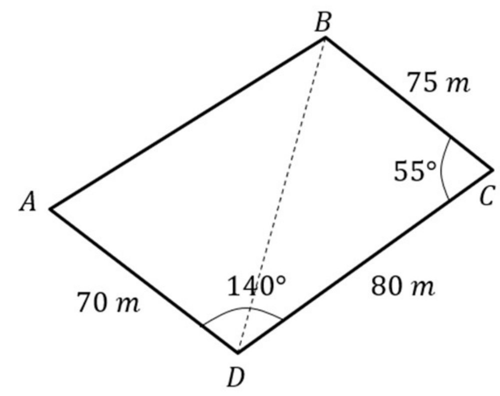
Calculate the area of the kite.



Worked Example

The diagram shows the locations of four mobile phone masts in a field.
$BC = 75\ m$, $CD = 80m$, angle $BCD = 55°$ and angle $ADC = 140°$.
In order that the masts do not interfere with each other, they must be at least 70m apart.
Given that A is the minimum distance from D, find:

a) The distance A is from B
b) The angle BAD
c) The area enclosed by the four masts.



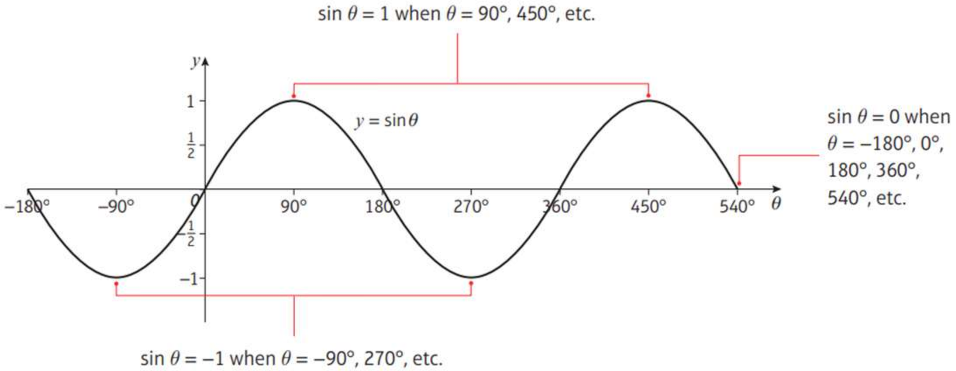
T.189 9E: Qs 6+, P.72 9.4: Qs 3+

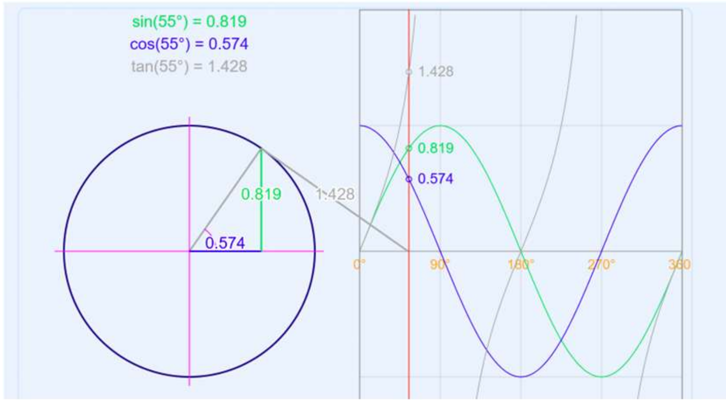
9.5 Graphs of Sine, Cosine and Tangent

- **The graphs of sine, cosine and tangent are periodic. They repeat themselves after a certain interval.**

You need to be able to draw the graphs for a given range of angles.

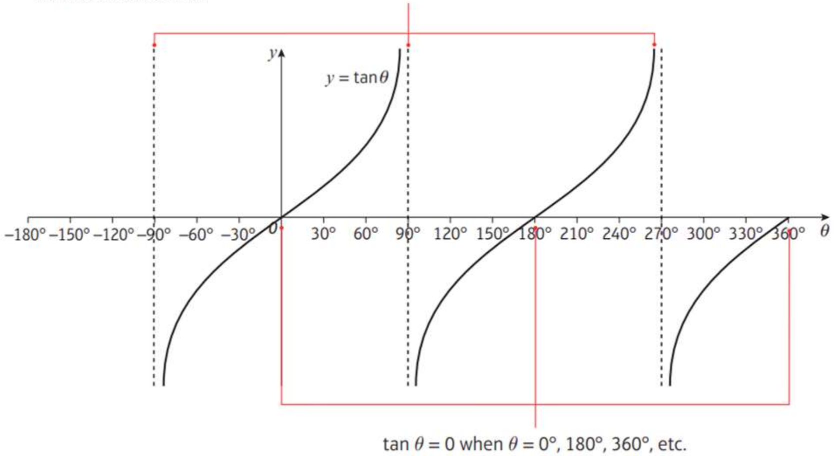
- **The graph of $y = \sin\theta$:**
  - **repeats every 360° and crosses the x-axis at ..., −180°, 0, 180°, 360°, ...**
  - **has a maximum value of 1 and a minimum value of −1.**



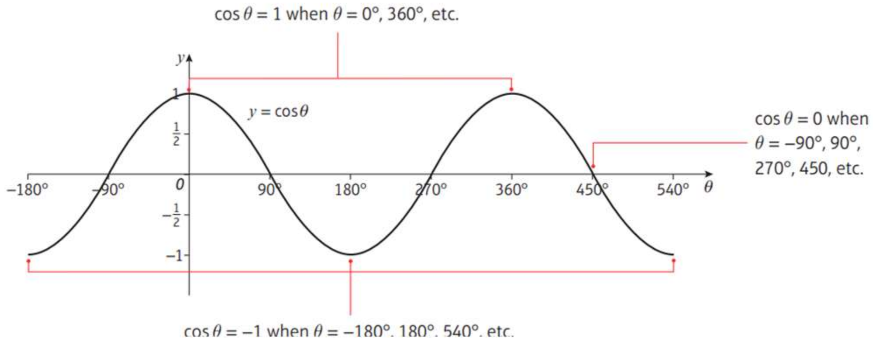


- **The graph of $y = \tan\theta$:**
  - **repeats every 180° and crosses the x-axis at ... −180°, 0°, 180°, 360°, ...**
  - **has no maximum or minimum value**
  - **has vertical asymptotes at $x = -90°$, $x = 90°$, $x = 270°$, ...**

tan θ does **not** have maximum and minimum points but approaches negative or positive infinity as the curve approaches the **asymptotes** at −90°, 90°, 270°, etc. tan θ is **undefined** for these values of θ.



- **The graph of $y = \cos\theta$:**
  - **repeats every 360° and crosses the x-axis at ..., −90°, 90°, 270°, 450°, ...**
  - **has a maximum value of 1 and a minimum value of −1.**



Worked Example

a) Sketch the graph of $y = \cos\theta$ in the interval $-360° \leq \theta \leq 360°$

b) i) Sketch the graph of $y = \sin x$ in the interval $-180° \leq x \leq 270°$

ii) $\sin(-30°) = -0.5$. Use your graph to determine two further values of x for which $\sin x = -0.5$

T.194 9E: Qs all, P.73 9.5: Qs all

9.6 Transforming Trigonometric Graphs

Reminders from chapter 4:

The graph of $y = \text{f}(x) + a$ is a translation of the graph $y = \text{f}(x)$ by the vector $\begin{pmatrix} 0 \\ a \end{pmatrix}$.

The graph of $y = \text{f}(x + a)$ is a translation of the graph $y = \text{f}(x)$ by the vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$.

When you translate a function, any asymptotes are also translated.

The graph of $y = a\text{f}(x)$ is a stretch of the graph $y = \text{f}(x)$ by a scale factor of a in the vertical direction.

The graph of $y = \text{f}(ax)$ is a stretch of the graph $y = \text{f}(x)$ by a scale factor of $\frac{1}{a}$ in the horizontal direction.

The graph of $y = -\text{f}(x)$ is a reflection of the graph of $y = \text{f}(x)$ in the x-axis.

The graph of $y = \text{f}(-x)$ is a reflection of the graph of $y = \text{f}(x)$ in the y-axis.

OR SIMPLY:

OUTSIDE to y and <u>DIRECT</u>

INSIDE to x and <u>OPPOSITE</u>

Notes

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = 3\sin x\,, 0 \leq x \leq 360°$

b) $y = -\tan\theta\,, -180° \leq \theta \leq 180°$

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = -1 + \sin x, 0 \leq x \leq 360°$

b) $y = \frac{1}{2} + \cos x, 0 \leq x \leq 360°$

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = \tan(\theta + 45°)\,, 0 \leq \theta \leq 360°$

b) $y = \cos(\theta - 90°), -360° \leq \theta \leq 360°$

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = \sin 2x\,, 0 \leq x \leq 360°$

b) $y = \cos\frac{\theta}{3}, -540° \leq \theta \leq 540°$

c) $y = \tan(-x)\,, -360° \leq x \leq 360°$

T.197 9G: Qs all, P.74 9.6: Qs all

7. A parallelogram $PQRS$ has area $50\,\text{cm}^2$

Given

- PQ has length $14\,\text{cm}$
- QR has length $7\,\text{cm}$
- angle SPQ is obtuse

find

(a) the size of angle SPQ, in degrees, to 2 decimal places,

(3)

(b) the length of the diagonal SQ, in cm, to one decimal place.

(2)

Your Turn

7. A parallelogram *PQRS* has area 40 cm^2

Given

- *PQ* has length 14 cm
- *QR* has length 7 cm
- angle *SPQ* is obtuse

find

(*a*) the size of angle *SPQ*, in degrees, to 2 decimal places,

(3)

(*b*) the length of the diagonal *SQ*, in cm, to one decimal place.

(2)

(Total for Question 7 is 5 marks)

3.

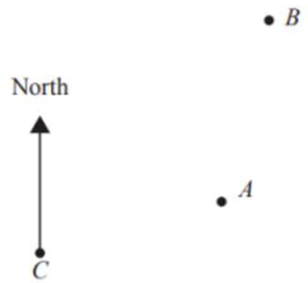


Figure 1

Figure 1 is a sketch showing the position of three phone masts, A, B and C.

The masts are identical and their bases are assumed to lie in the same horizontal plane.

From mast C

- mast A is 8.2 km away on a bearing of 072°
- mast B is 15.6 km away on a bearing of 039°

(a) Find the distance between masts A and B, giving your answer in km to one decimal place.

(3)

An engineer needs to travel from mast A to mast B.

(b) Give a reason why the answer to part (a) is unlikely to be an accurate value for the distance the engineer travels.

(1)

Your Turn

3.

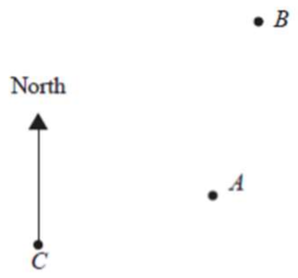


Figure 1

Figure 1 is a sketch showing the position of three lighthouses, A, B and C.

The lighthouses are identical and their bases are assumed to lie in the same horizontal plane.

From lighthouse A, lighthouse C is 10.4 km away on a bearing of 251°

From lighthouse C, lighthouse B is 20.3 km away on a bearing of 042°

(a) Find the distance between lighthouses A and B, giving your answer in km to one decimal place.

(3)

The bearing of lighthouse B from lighthouse A is found to be 018°. A ship needs to travel from lighthouse A to lighthouse B in the most direct route.

(b) Give a reason why the captain of the ship may need to set the ship in a direction with a different bearing to 018°.

(1)

(Total for Question 3 is 4 marks)

4.

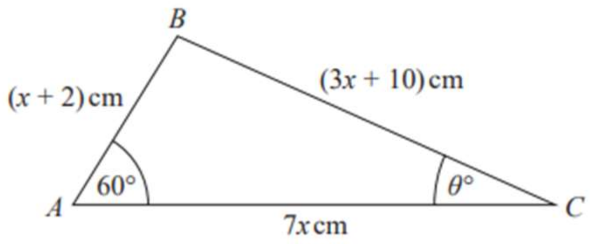


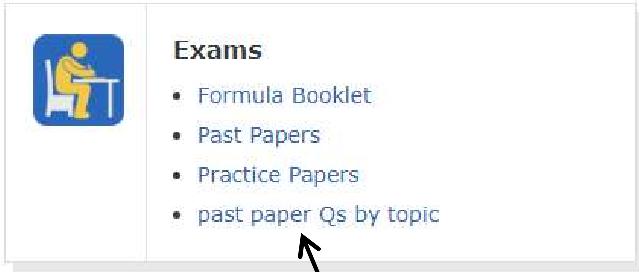
Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$ **(3)**

(ii) Hence find the value of x. **(1)**

(b) Hence find the value of θ giving your answer to one decimal place. **(2)**



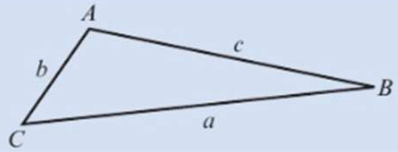
Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

4(a)(i)	$(3x+10)^2=(x+2)^2+(7x)^2-2(x+2)(7x)\cos 60°$ or	M1	3.1a
	Uses $\cos 60° = \frac{1}{2}$, expands the brackets and proceeds to a 3 term quadratic equation	dM1	1.1b
	$17x^2-35x-48=0$ *	A1*	2.1
		(3)	
(ii)	$x=3$	B1	2.2a
		(1)	
(b)	$\frac{\sin ACB}{5}=\frac{\sin 60°}{19} \Rightarrow \sin ACB = \ldots\left(\frac{5\sqrt{3}}{38}\right)$ or e.g. $5^2=21^2+19^2-2\times19\times21\cos ACB \Rightarrow \cos ACB = \ldots\left(\frac{37}{38}\right)$	M1	1.1b
	θ = awrt 13.2	A1	1.1b
		(2)	
		(6 marks)	

Summary of Key Points

1 This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:

$$a^2 = b^2 + c^2 - 2bc\cos A$$



2 This version of the cosine rule is used to find an angle if you know all three sides:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

3 This version of the sine rule is used to find the length of a missing side:

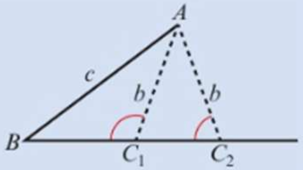
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

4 This version of the sine rule is used to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

5 The sine rule sometimes produces two possible solutions for a missing angle:

$$\sin\theta = \sin(180° - \theta)$$



6 Area of a triangle $= \frac{1}{2}ab\sin C$.

7 The graphs of sine, cosine and tangent are **periodic**. They repeat themselves after a certain interval.

- The graph of $y = \sin\theta$: repeats every 360° and crosses the x-axis at …, −180°, 0, 180°, 360°, …
  has a maximum value of 1 and a minimum value of −1.
- The graph of $y = \cos\theta$: repeats every 360° and crosses the x-axis at …, −90°, 90°, 270°, 450°, …
  has a maximum value of 1 and a minimum value of −1
- The graph of $y = \tan\theta$: repeats every 180° and crosses the x-axis at … −180°, 0°, 180°, 360°, …
  has no maximum or minimum value
  has vertical asymptotes at $x = -90°$, $x = 90°$, $x = 270°$, …

T.198 mixed ex, P.76 BSG