KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 11 Vectors Booklet

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

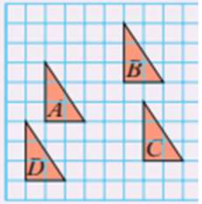
Prior knowledge check

1 Write the column vector for the translation of shape

a A to B

b A to C

c A to D



← GCSE Mathematics

2 P divides the line AB in the ratio $AP:PB = 7:2$.

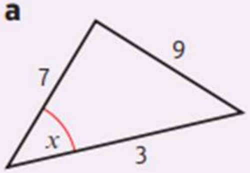
Find:

a $\frac{AP}{AB}$ **b** $\frac{PB}{AB}$ **c** $\frac{AP}{PB}$

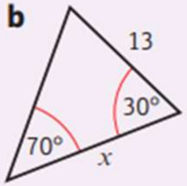
← GCSE Mathematics

3 Find x to one decimal place.

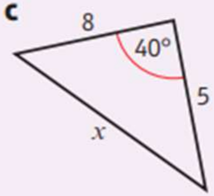
a



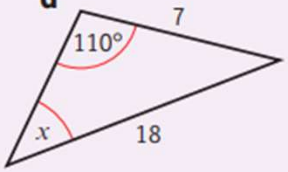
b



c



d



← Sections 9.1, 9.2

11.1) Vectors

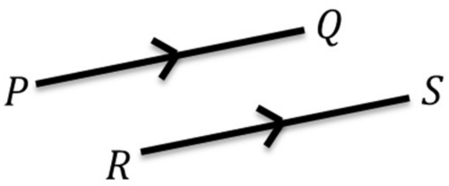
A Whereas a **coordinate** represents a **position** in space, a **vector** represents a **displacement** in space.

A vector has 2 properties:
- Direction
- Magnitude (i.e. length)

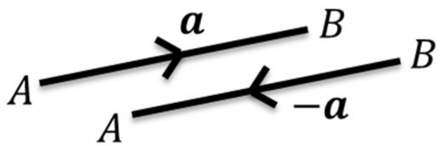
If P and Q are points then $\overrightarrow{PQ}$ is the vector between them.

B If two vectors $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have the same magnitude and direction, **they're the same vector** and are **parallel**.

This might seem obvious, but students sometimes think the vector is different because the movement occurred at a different point in space. Nope!

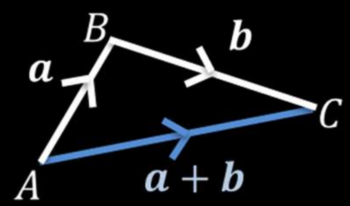


C $\overrightarrow{AB} = -\overrightarrow{BA}$ and the two vectors are parallel, equal in magnitude but in **opposite directions**.



D Triangle Law for vector addition:

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

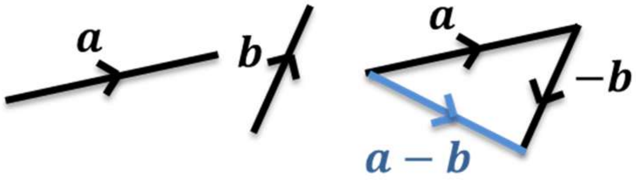


The vector of multiple vectors is known as the **resultant vector.**
(you will encounter this term in Mechanics)

Notes

E Vector **subtraction** is defined using vector addition and negation:

$$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$$



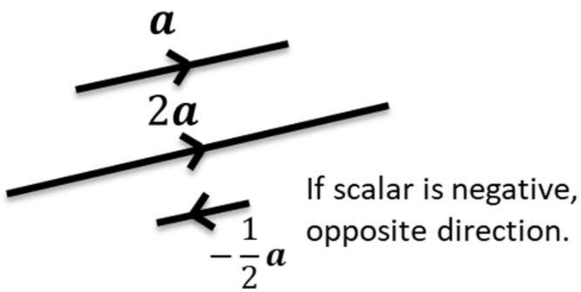
F The zero vector $\mathbf{0}$ (a bold 0), represents no movement.

$$\overrightarrow{PQ} + \overrightarrow{QP} = \mathbf{0}$$

In 2D: $\mathbf{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

G A **scalar** is a normal number, which can be used to 'scale' a vector.

- The **direction** will be the **same**.
- But the **magnitude** will be **different** (unless the scalar is 1).



H Any vector parallel to the vector $\boldsymbol{a}$ can be written as $\lambda\boldsymbol{a}$, where λ is a scalar.

The implication is that if we can write one vector **as a multiple of** another, then we can show they are parallel.

"Show $2\boldsymbol{a} + 4\boldsymbol{b}$ and $3\boldsymbol{a} + 6\boldsymbol{b}$ are parallel".

$3\boldsymbol{a} + 6\boldsymbol{b} = \frac{3}{2}(\boldsymbol{a} + 2\boldsymbol{b}) \therefore$ parallel

T.234: 11A Qs 3-5, P.86: 11.1 Qs 1-4

Worked Example

$PQRS$ is a parallelogram.
N is the point on SQ such that $SN : NQ = 3 : 4$
$\overrightarrow{PQ} = \boldsymbol{b}$ and $\overrightarrow{PS} = \boldsymbol{a}$
Express $\overrightarrow{NR}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

Worked Example

OAB is a triangle.
$\overrightarrow{OA} = \boldsymbol{b}$ and $\overrightarrow{OB} = \boldsymbol{a}$
P is the point on AB such that $AP : PB = 2 : 3$.
Find $\overrightarrow{OP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

Worked Example

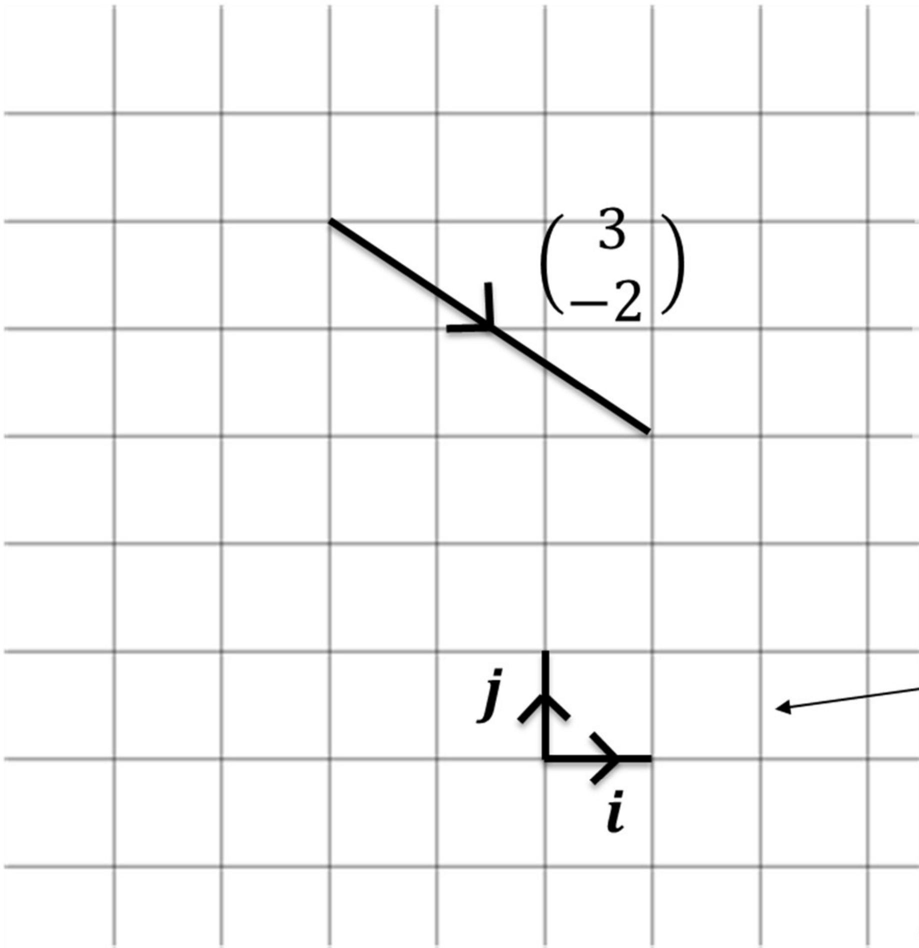
Show that the vectors are parallel:

$3\boldsymbol{a} + 4\boldsymbol{b}$ and $15\boldsymbol{a} + 20\boldsymbol{b}$

$3\boldsymbol{a} + 4\boldsymbol{b}$ and $-0.75\boldsymbol{a} - \boldsymbol{b}$

T.234: 11A Qs 4+, P.86: 11.1 Qs 5+

11.2) Representing Vectors



You should already be familiar that the value of a vector is the **displacement** in the x and y direction (if in 2D).

$$\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}, \qquad \boldsymbol{b} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\boldsymbol{a} + \boldsymbol{b} = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$$

$$2\boldsymbol{a} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

✏ A **unit vector** is a vector of magnitude 1. $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the x-axis and y-axis respectively.

$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

e.g. $\begin{pmatrix} 4 \\ 3 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 4\boldsymbol{i} + 3\boldsymbol{j}$

Side Notes: This allows us to write any vector algebraically without using vector notation. **Any point in 2D space,** as a vector from the origin, can be obtained using a linear combination of $\boldsymbol{i}$ and $\boldsymbol{j}$, e.g. if $P(5,-1)$, $\overrightarrow{OP} = 5\boldsymbol{i} - \boldsymbol{j}$. For this reason, $\boldsymbol{i}$ and $\boldsymbol{j}$ are known as **basis vectors** of 2D coordinate space. In fact, any two non-parallel/non-zero vectors can be used as basis vectors, e.g. if $\boldsymbol{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, it's possible to get any vector $\begin{pmatrix} x \\ y \end{pmatrix}$ using a linear combination of these, i.e. we can always find scalars p and q such that $\begin{pmatrix} x \\ y \end{pmatrix} = p\begin{pmatrix} 5 \\ 2 \end{pmatrix} + q\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

Notes

Worked Example

Given $\boldsymbol{a} = 8\boldsymbol{i} - 6\boldsymbol{j}$ and $\boldsymbol{b} = 9\boldsymbol{i} + 7\boldsymbol{j}$, find:

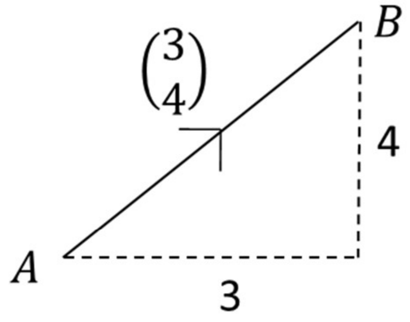
- $4\boldsymbol{b} - 2\boldsymbol{a}$
- $-\boldsymbol{b} + \frac{1}{4}\boldsymbol{a}$

T.235: 11B Qs 5+, P.87: 11.2 Qs 4+

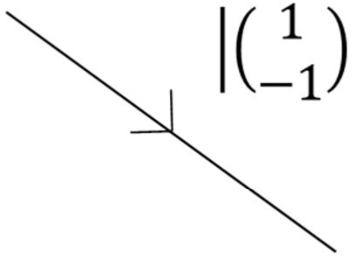
11.3) Magnitude and direction

The magnitude $|a|$ of a vector a is its length.

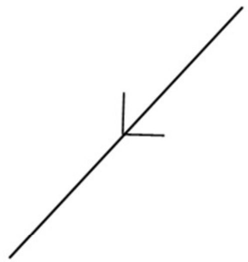
✏ If $\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad |\boldsymbol{a}| = \sqrt{x^2 + y^2}$



$$\left|\overrightarrow{AB}\right| = \sqrt{3^2 + 4^2} = 5$$



$$\left|\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right| = \sqrt{2}$$



$$\left|\begin{pmatrix} -5 \\ -12 \end{pmatrix}\right| = 13$$

$$\boldsymbol{a} = \begin{pmatrix} 4 \\ -1 \end{pmatrix} \quad |\boldsymbol{a}| = \sqrt{4^2 + 1^2} = \sqrt{17}$$

$$\boldsymbol{b} = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \quad |\boldsymbol{b}| = \sqrt{2^2 + 0^2} = 2$$

A unit vector is a vector whose magnitude is 1

There's certain operations on vectors that require the vectors to be 'unit' vectors. We just scale the vector so that its magnitude is now 1.

$$\boldsymbol{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$|\boldsymbol{a}| = \sqrt{3^2 + 4^2} = 5$$

$$\hat{\boldsymbol{a}} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$$

If $\boldsymbol{a}$ is a vector, then the unit vector $\hat{\boldsymbol{a}}$ in the same direction is

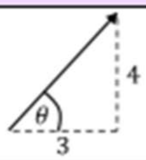
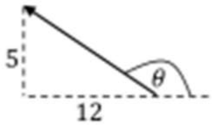
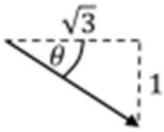
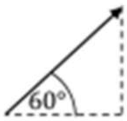
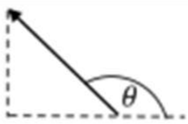
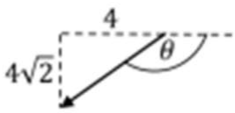
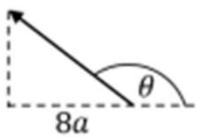
$$\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$

Worked Example

506a: **Determine the angle between a vector and the x or y axis.**

Find the angle between the vector $\begin{pmatrix} 13 \\ -12 \end{pmatrix}$ and the positive x-axis.
Give your answer correct to 1 decimal place.

Fill in the Blanks

Column Vector	Component Form	Sketch	Magnitude	Angle made with $+i$
$\begin{pmatrix}3\\4\end{pmatrix}$	$3\boldsymbol{i}+4\boldsymbol{j}$			
	$-12\boldsymbol{i}+5\boldsymbol{j}$			
$\begin{pmatrix}4\\-4\end{pmatrix}$				
	$-2\sqrt{5}\boldsymbol{i}-4\boldsymbol{j}$			
				
			12	
			$\sqrt{10}$	$135°$
$\begin{pmatrix}-4\\-4\sqrt{2}\end{pmatrix}$				
	$10\boldsymbol{i}-\square\boldsymbol{j}$		$2\sqrt{29}$	
$\begin{pmatrix}-8a\\\square\end{pmatrix}$			$10a$	

Worked Example

A vector $\boldsymbol{a} = p\boldsymbol{i} + q\boldsymbol{j}$ has magnitude 68 and makes an angle θ with the positive x-axis where $\sin\theta = \frac{8}{17}$. Find all the possible vectors

Worked Example

In triangle PQR, $\overrightarrow{PQ} = \boldsymbol{i} + 2\boldsymbol{j}$ and
$\overrightarrow{PR} = 8\boldsymbol{i} - 15\boldsymbol{j}$.
Find the area of triangle PQR

T.241: 11C Qs 3+, P.88: 11.3 Qs 5+

3. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\overrightarrow{QR}$

(2)

(b) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS:SR = 3:2$

(c) Find $\overrightarrow{PS}$

(2)

Your Turn

3. The triangle PQR is such that $\overrightarrow{PQ} = 2\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 10\mathbf{i} - 7\mathbf{j}$

(*a*) Find $\overrightarrow{QR}$

(2)

(*b*) Hence find $|\overrightarrow{QR}|$ giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS : SR = 1 : 3$

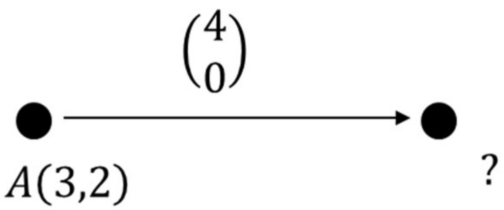
(*c*) Find $\overrightarrow{PS}$

(2)

(Total for Question 3 is 6 marks)

11.4) Position vectors

Suppose we started at a point $(3,2)$ and translated by the vector $\begin{pmatrix}4\\0\end{pmatrix}$:

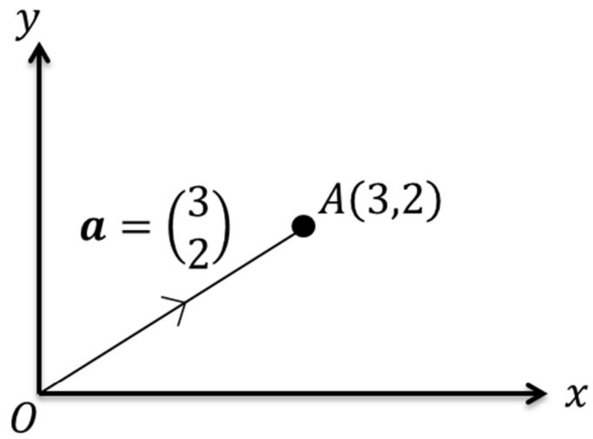


You might think we can do something like:

$$(3,2) + \begin{pmatrix}4\\0\end{pmatrix} = (7,2)$$

But only vectors can be added to other vectors. **If we treated the point $(3,2)$ as a vector**, then this solves the problem:

$$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}4\\0\end{pmatrix} = \begin{pmatrix}7\\2\end{pmatrix}$$



A vector used to represent a position is unsurprisingly known as a **position vector**.
A position can be thought of as a translation from the origin, as per above. It enables us to use positions in all sorts of vector (and matrix!) calculations.

The position vector of a point A is the vector $\overrightarrow{OA}$, where O is the origin. $\overrightarrow{OA}$ is usually written as $\boldsymbol{a}$.

Notes

Worked Example

$\overrightarrow{OA} = 4\boldsymbol{i} + 3\boldsymbol{j}$ and $\overrightarrow{AB} = 2\boldsymbol{i} - 5\boldsymbol{j}$. Find:

a) The position vector of B.

b) The exact value of $\left|\overrightarrow{OB}\right|$ in simplified surd form.

T.243: 11D Qs 4+, P.89: 11.4 Qs 3+

Past Paper Qs AS 2019 Q16

16. (i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$
The angle between vector **m** and vector **n** is 30°

Find the angle between vector **m** and vector **m** – **n**, giving your answer, in degrees, to one decimal place.

(4)

Your Turn

16.

(a) Two non zero vectors, **a** and **b**, are such that

$$|\boldsymbol{a}+\boldsymbol{b}| = |\boldsymbol{a}| - |\boldsymbol{b}|$$

Explain, geometrically, the significance of this statement.

(1)

(b) Two different vectors, $\boldsymbol{m}$ and $\boldsymbol{n}$, are such that $|\boldsymbol{m}| = 4$ and $|\boldsymbol{m}-\boldsymbol{n}| = 6$.

The angle between $\boldsymbol{m}$ and $\boldsymbol{m}-\boldsymbol{n}$ is $30°$.

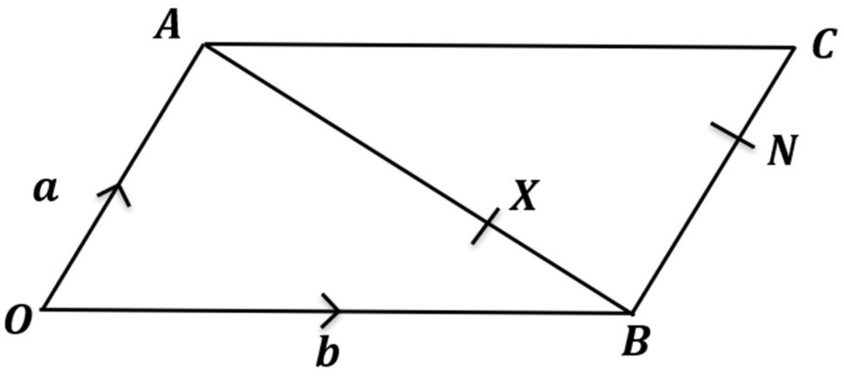
Find $|\boldsymbol{n}|$

(4)

11.5) Solving geometric problems

Worked Example

$OACB$ is a parallelogram.
X is a point on AB such that $AX: XB = 2: 1$. N is the point such that NC is half of BN.
Show that $\overrightarrow{XN}$ is parallel to $\overrightarrow{OC}$.



T.246: 11E Qs 3+, P.89: 11.5 Qs 2+

Worked Example

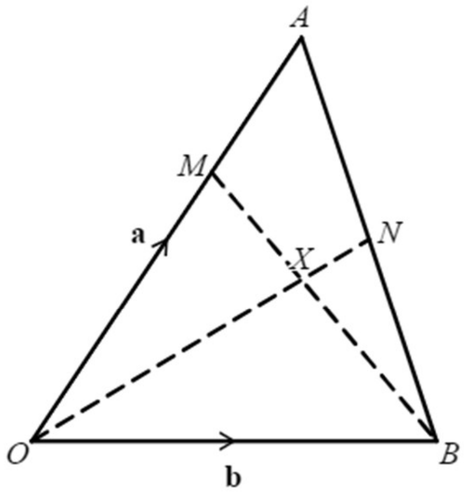
$\overrightarrow{AB} = 2\boldsymbol{i} - 5\boldsymbol{j}$ and $\overrightarrow{AC} = 3\boldsymbol{i} - 7\boldsymbol{j}$.

Determine $\angle BAC$.

Worked Example

475a: Determine a vector by equating coefficients for two different scalars/routes.

OAB is a triangle.



The vector $\overrightarrow{OA} = \mathbf{a}$.

The vector $\overrightarrow{OB} = \mathbf{b}$.

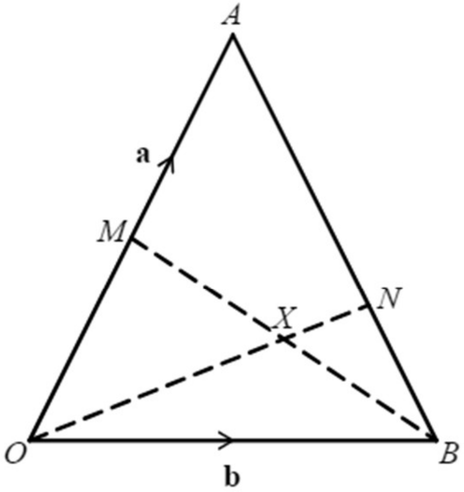
The ratio $OM : MA$ is $2 : 1$.
N is the midpoint of AB.

Given that $\overrightarrow{OX} = \lambda\overrightarrow{ON}$ and $\overrightarrow{MX} = \mu\overrightarrow{MB}$
use a vector method to find the value of λ and μ.

Worked Example

475b: Determine a ratio by writing a vector using two different scalars/routes.

OAB is a triangle.



The vector $\overrightarrow{OA} = \mathbf{a}$.

The vector $\overrightarrow{OB} = \mathbf{b}$.

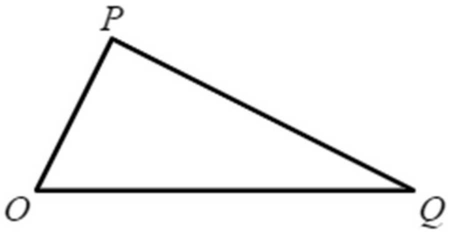
M is the midpoint of OA.
The ratio $BN : NA$ is $1 : 2$.

Find the ratio $OX : XN$.
Give your answer in its simplest form.

Worked Example

475c: **Determine a ratio of vectors by extending out a vector.**

The diagram below shows a sketch of triangle OPQ



The point R is such that $OP : PR = 2 : 3$

The point M is such that $PM : MQ = 3 : 2$

The straight line through R to M cuts OQ at the point N

Let $\vec{OP} = \mathbf{a}$ and $\vec{OQ} = \mathbf{b}$

By first finding $\vec{RM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, and letting $\vec{RN} = \lambda\vec{RM}$, find $ON : NQ$.

Extra Exercises

19) The position vectors of A, B and C respectively are $\mathbf{a} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 1 \\ k \end{pmatrix}$. There is a further point $D\ (p, 8)$ such that $\overrightarrow{AB} = \overrightarrow{CD}$
   a) Find the value of p and k
   b) Determine the shape of $ABDC$.

20) The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. A third point C forms the parallelogram $OABC$.
   a) In terms of $\mathbf{a}$ and $\mathbf{b}$,
      i) find $\overrightarrow{OC}$
      ii) find $\overrightarrow{OP}$, where P is the midpoint of AB
      iii) find $\overrightarrow{OQ}$, where Q is the midpoint of OC
   b) Show that the midpoint of $\overrightarrow{PQ}$ is at the centre of the parallelogram.

21) A quadrilateral is given by $ABCD$. It is known that $\overrightarrow{AB} = 4\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{DC}$ is the same length and direction as $\overrightarrow{AB}$. Two points C and D have position vectors $\mathbf{c} = 5\mathbf{i} + \mathbf{j}$ and $\mathbf{d} = k\mathbf{i} - 4\mathbf{j}$ respectively. Find
   a) the value of k
   b) the coordinates of A and B given $ABCD$ is a square.

22) The position vectors of A, B and C respectively are $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$. The midpoint of A and B is P and the midpoint of B and C is Q
   a) Find the coordinates of the points P and Q
   b) Show that $\overrightarrow{PQ} = \frac{1}{2}\overrightarrow{AC}$

   A further point D exists such that $|\overrightarrow{DB}| = \sqrt{74}$ and $\overrightarrow{BC}$ is parallel to $\overrightarrow{AD}$
   c) find the possible coordinates of D

23) The points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ relative to the origin O where $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$. Find the possible coordinates of a third point C such that $\overrightarrow{AB}$ is the hyponenuse of a right-angled isosceles triangle.

13. Relative to a fixed origin O

- point A has position vector $10\mathbf{i} - 3\mathbf{j}$
- point B has position vector $-8\mathbf{i} + 9\mathbf{j}$
- point C has position vector $-2\mathbf{i} + p\mathbf{j}$ where p is a constant

(a) Find $\overrightarrow{AB}$

(2)

(b) Find $\left|\overrightarrow{AB}\right|$ giving your answer as a fully simplified surd.

(2)

Given that points A, B and C lie on a straight line,

(c) (i) find the value of p,

(ii) state the ratio of the area of triangle AOC to the area of triangle AOB.

(3)

Your Turn

13. Relative to a fixed origin O

- point P has position vector $11\mathbf{i} + 43\mathbf{j}$
- point Q has position vector $2\mathbf{i} + 13\mathbf{j}$
- point R has position vector $k\mathbf{i} - 7\mathbf{j}$ where k is a constant

(*a*) Find $\overrightarrow{PQ}$

(2)

(*b*) Find $|\overrightarrow{PQ}|$ giving your answer as a fully simplified surd.

(2)

Given that points P, Q and R lie on a straight line,

(*c*) find the value of k

(2)

Given that the point X has position vector $a\mathbf{i} + b\mathbf{j}$ and is not on the same line as P, Q and R,

(*d*) state the ratio of the area of triangle PXQ to the area of triangle QXR.

(1)

(Total for Question 13 is 7 marks)

10.

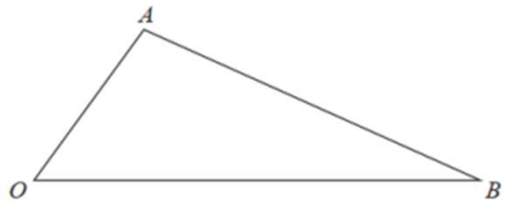


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**

(2)

(b) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(c) Hence prove that $ON:NB = 2:1$

(2)

Your Turn

10.

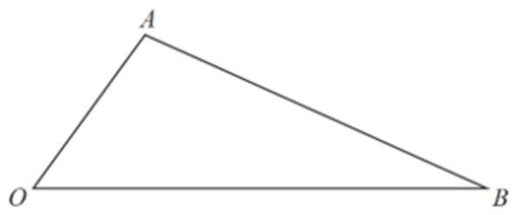


Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M lies on AB such that $AM{:}MB = 2{:}1$.

The straight line through C and M cuts OB at the point N.

Given that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$,

(a) Find $\overrightarrow{CM}$ in terms of **a** and **b**.

(2)

(b) Find $\overrightarrow{ON}$ in terms of k, where k is a scalar constant and a coefficient to $\overrightarrow{CM}$.

(2)

(c) Hence find the ratio $ON : NB$.

(2)

(Total for Question 10 is 6 marks)

Past Paper Qs 2020 P2 Q2

2. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)

Your Turn

2. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.
Given that

- P, Q and R lie on a straight line
- Q lies one fifth of the way from P to R

show that

$$\mathbf{q} = \frac{1}{5}(\mathbf{r} + 4\mathbf{p})$$

(3)

(Total for Question 2 is 3 marks)

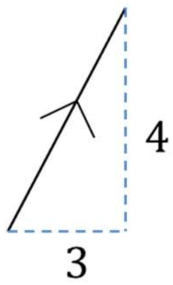
11.6) Modelling with vectors

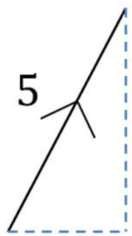
In Mechanics, you will see certain things can be represented as a simple number (without direction), or as a vector (with direction):

> Remember a 'scalar' just means a normal number (in the context of vectors). It can be obtained using the **magnitude** of the vector.

Vector Quantity	Equivalent Scalar Quantity
Velocity e.g. $\begin{pmatrix} 3 \\ 4 \end{pmatrix}\ km/h$	Speed $= 5\ km/h$
Displacement e.g. $\begin{pmatrix} -5 \\ 12 \end{pmatrix}\ km$	Distance $= 13\ km$

> This means the position vector of the object changes by $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ each hour.





> ...which is equivalent to moving 5km each hour.

Fill in the blanks

Quantity	Vector or Scalar	Non-SI Units	SI Units
Mass		528 g	0.528 kg
Time		13 minutes	
Displacement		28.3 km	
Speed		$75\ cm\ s^{-1}$	
	Scalar	4.3×10^{5} mm	
Velocity		$90\ km\ h^{-1}$	
Acceleration		$885\ cm\ s^{-2}$	
		5 h 18 min	
	Scalar	5.4 m per minute	
	Vector	7560 mm	
	Vector	$-0.083\ km\ s^{-1}$	
	Scalar	39000 mg	
		$480000\ cm^{3}$	
Pressure		$1.5\ N\ cm^{-2}$	
Density		$1.12\ g\ cm^{-3}$	

Worked Example

In an orienteering exercise, a cadet leaves the starting point O and walks 30 km on a bearing of $150°$ to reach A, the first checkpoint.
From A she walks 18 km on a bearing of $210°$ to the second checkpoint, at B.
From B she returns directly to O.
Find:

a) the position vector of A relative to O

b) $|\overrightarrow{OB}|$

c) the bearing of B from O

d) the position vector of B relative O.

T.251: 11F Qs 5+, P.90: 11.6 Qs 4+

2. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O.

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

Your Turn

2 [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard station O monitors the movements of a small boat.

At 09:00 the boat is at the point $(6\mathbf{i} - \mathbf{j})$ km relative to O.

At 12:15 the boat is at the point $(-2\mathbf{i} - 7\mathbf{j})$ km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(*a*) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

(3)

(*b*) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

(Total for Question 2 is 6 marks)

4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

(3)

Your Turn

4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-48\mathbf{i} - 14\mathbf{j})$ m relative to a fixed point O.

After 5 seconds the stone is at the point $B(24\mathbf{i} + 7\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(*a*) prove that the stone passes through O,

(2)

(*b*) calculate the speed of the stone.

(3)

(Total for Question 4 is 5 marks)

2. Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$,

the point B has position vector $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 5\mathbf{j} - 2\mathbf{k})$, where a is a constant and $a < 0$

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$.

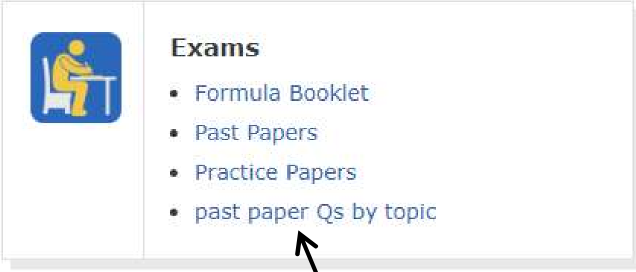
(a) Find the position vector of D.

(2)

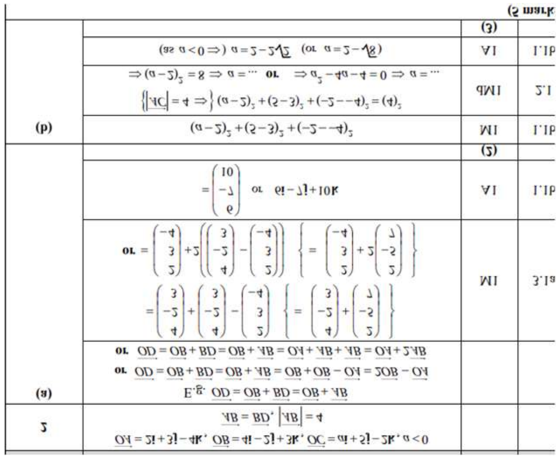
Given $|\overrightarrow{AC}| = 4$

(b) find the value of a.

(3)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com



Summary of Key Points

Summary of key points

1 If $\overrightarrow{PQ} = \overrightarrow{RS}$ then the line segments PQ and RS are equal in length and are parallel.

2 $\overrightarrow{AB} = -\overrightarrow{BA}$ as the line segment AB is equal in length, parallel and in the opposite directi
to BA.

3 **Triangle law for vector addition:** $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

If $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$, then $\mathbf{a} + \mathbf{b} = \mathbf{c}$

4 Subtracting a vector is equivalent to 'adding a negative vector': $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$

5 Adding the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ gives the zero vector **0**: $\overrightarrow{PQ} + \overrightarrow{QP} = 0$.

6 Any vector parallel to the vector **a** may be written as $\lambda\mathbf{a}$, where λ is a non-zero scalar.

7 To multiply a column vector by a scalar, multiply each component by the scalar: $\lambda\begin{pmatrix} p \\ q \end{pmatrix} = ($

8 To add two column vectors, add the x-components and the y-components $\begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} r \\ s \end{pmatrix} = ($

9 A unit vector is a vector of length 1. The unit vectors along the x- and y-axes are usually denoted by **i** and **j** respectively. $\mathbf{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\mathbf{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

10 For any two-dimensional vector: $\begin{pmatrix} p \\ q \end{pmatrix} = p\mathbf{i} + q\mathbf{j}$

11 For the vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j} = \begin{pmatrix} x \\ y \end{pmatrix}$, the magnitude of the vector is given by: $|\mathbf{a}| = \sqrt{x^2 + y^2}$

12 A unit vector in the direction of **a** is $\dfrac{\mathbf{a}}{|\mathbf{a}|}$

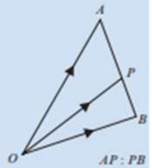
13 In general, a point P with coordinates (p, q) has position vector:

$$\overrightarrow{OP} = p\mathbf{i} + q\mathbf{j} = \begin{pmatrix} p \\ q \end{pmatrix}$$

14 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$, where $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are the position vectors of A and B respectively.

15 If the point P divides the line segment AB in the ratio $\lambda : \mu$, then

$$\overrightarrow{OP} = \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}\overrightarrow{AB}$$
$$= \overrightarrow{OA} + \frac{\lambda}{\lambda + \mu}(\overrightarrow{OB} - \overrightarrow{OA})$$



T.251: mixed ex, P.91: BSG