KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Statistics 1
Chapter 5 – Probability

HGS Maths

Dr Frost Course

Name: ________________________________

Class: ________________________________

Contents

Prior Knowledge Check

1 A bag contains three red balls, four yellow balls and two blue balls. A ball is chosen at random from the bag. Write down the probability that the ball is:

a blue　　**b** yellow

c not red　　**d** green.

← **GCSE Mathematics**

2 Three coins are flipped. Write down all the possible outcomes. ← **GCSE Mathematics**

3 Poppy rolls a dice. She keeps rolling until she rolls a 6. Work out the probability that Poppy rolls the dice:

a exactly three times

b fewer than three times

c more than three times.

← **GCSE Mathematics**

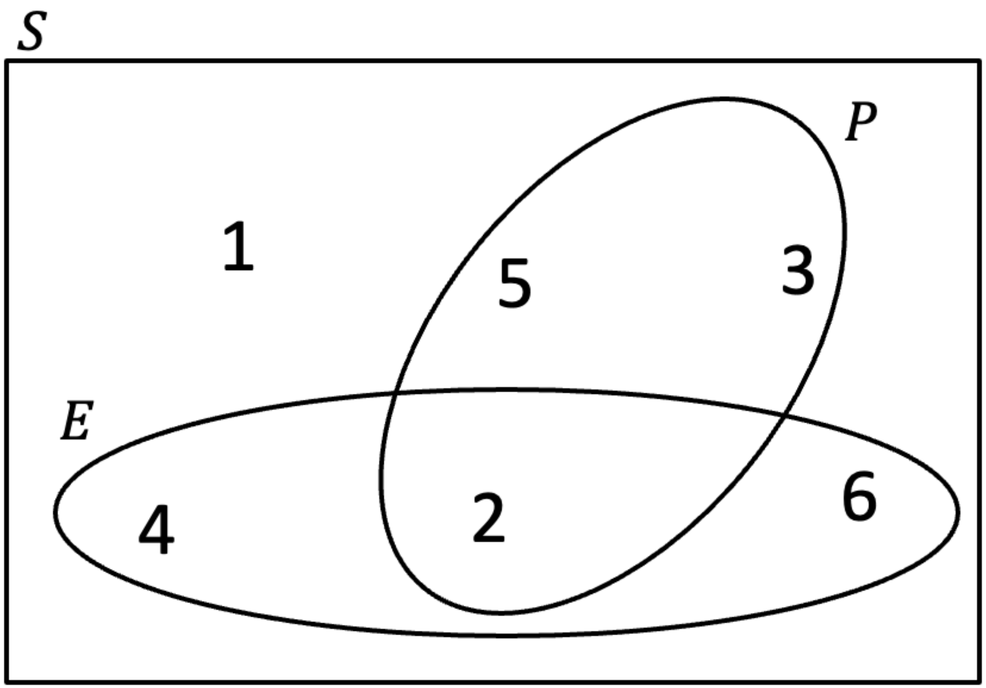
5.1 Calculating Probabilities

Probability Concepts

An **experiment** is a repeatable process that gives rise a number a number of **outcomes**.

An **event** is a set of <u>one or more</u> of these outcomes.

(We often use capital letters to represent them)



$E =$ "rolling an even number"
$P =$ "rolling a prime number"

A **sample space** is the set of all possible outcomes.

Because we are dealing with sets, we can use a **Venn diagram**, where

- the numbers are the individual outcomes,
- the sample space is a rectangle and
- the events are sets, each a subset of the sample space.

You do not need to use set notation like ∩ and ∪ in this module (but ordinarily you would!)

Notes

Worked Example

The table shows the times taken, in minutes, for a group of students to complete a number puzzle.

Time, t (min)	$5 \leq t < 8$	$8 \leq t < 11$	$11 \leq t < 12$	$12 \leq t < 14$	$14 \leq t < 15$
Frequency	4	16	7	9	5

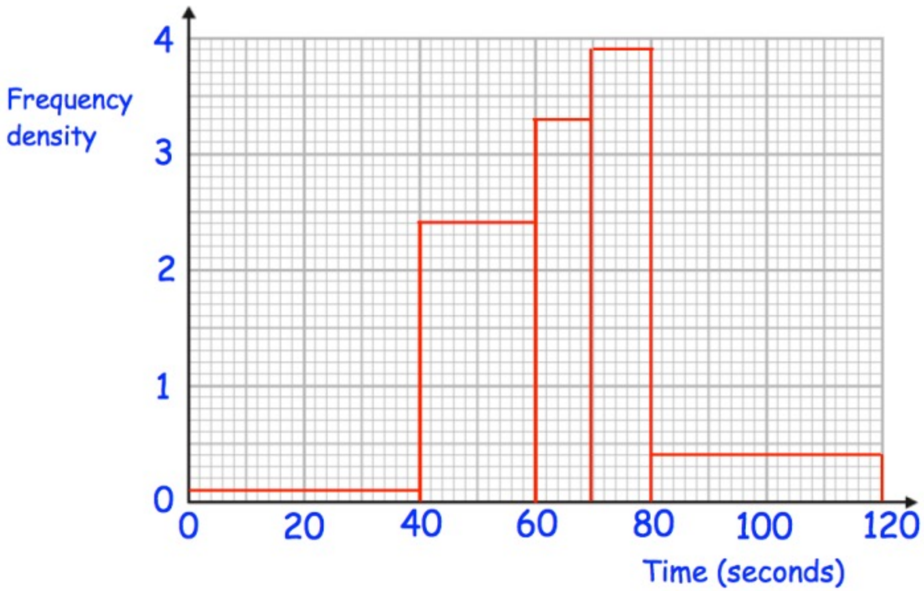
A student is chosen at random. Find the probability that they completed the number puzzle in:

a) under 12 minutes

b) over 9.5 minutes

Worked Example

A participant is chosen at random.
What is the probability they took longer than 60 seconds?



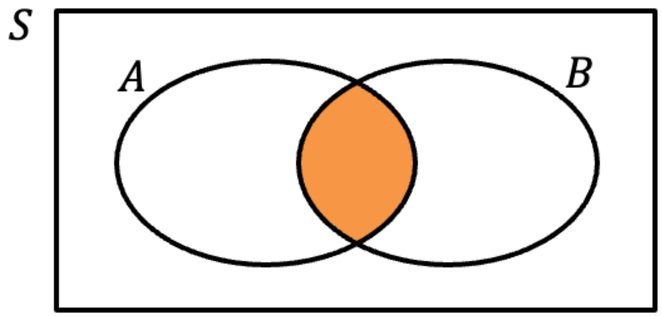
Your Turn

A participant is chosen at random.
What is the probability they weigh more than $14\ kg$?

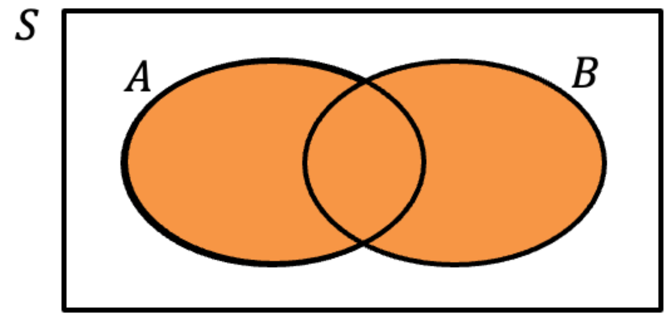
5.2 Venn Diagrams

Venn Diagrams

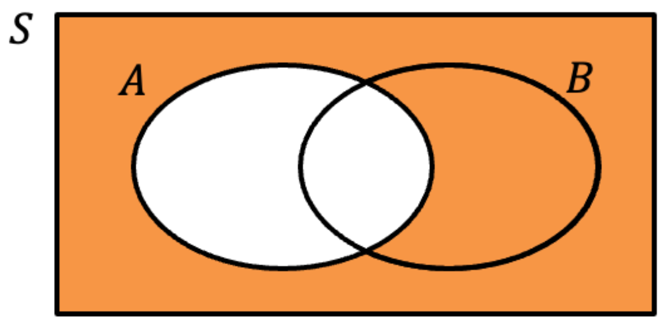
Venn Diagrams allow us to combine events, e.g. "A happened **and** B happened".



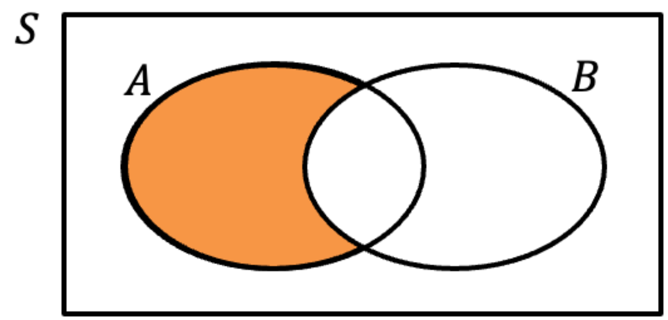
The event "A **and** B"
Known as the **intersection** of A and B.



The event "A **or** B"
Known as the **union** of A and B.



The event "not A"
Known as the **union** of A and B.

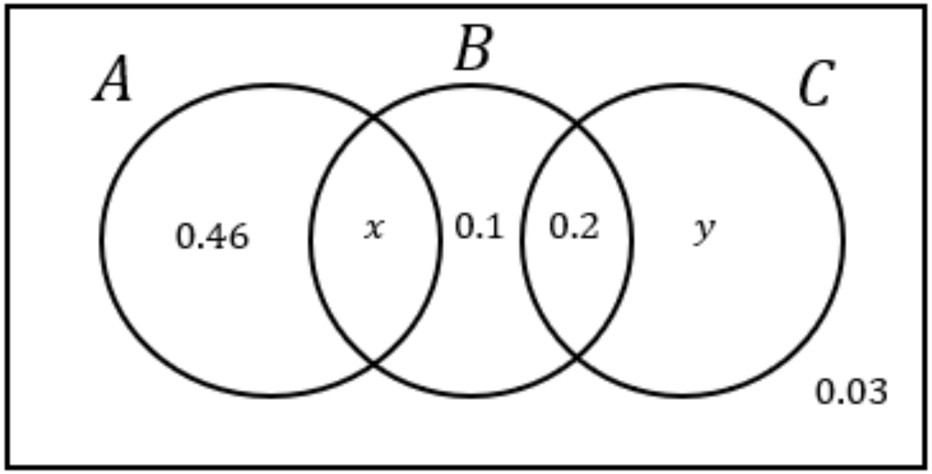


These can be combined,
e.g. "A and not B".

Notes

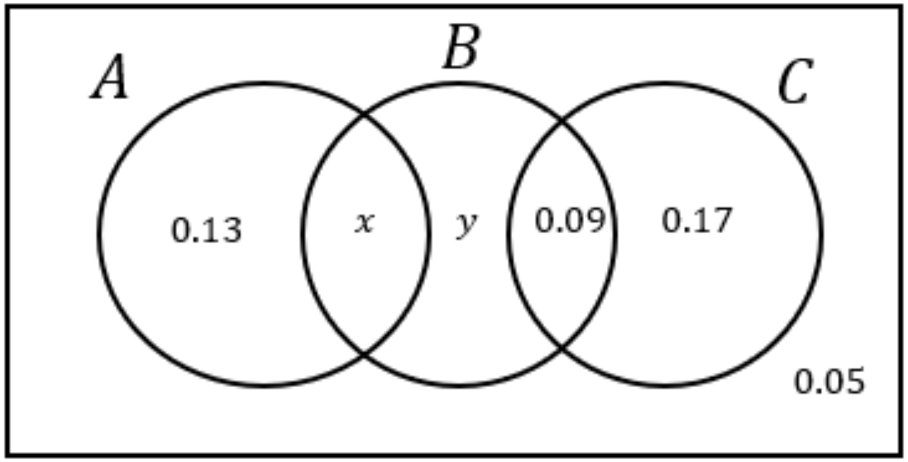
Worked Example

The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
Given that $P(B) = 0.39$, find $P(C)$



Worked Example

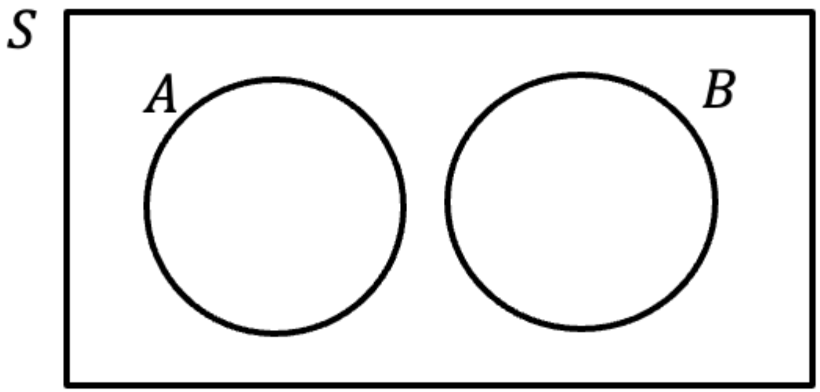
The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
Given that $P(A) = P(B)$, find the values of x and y



5.3 Mutually Exclusive and Independent Events

Mutually Exclusive Events

- If two events are mutually exclusive **they can't happen at the same time.**
- If A and B are mutually exclusive then:
  - $\boldsymbol{P(A\ and\ B) = 0}$
  - $\boldsymbol{P(A\ or\ B) = P(A) + P(B)}$
- The Venn Diagram would look like:



Since $P(A\ and\ B) = 0$, there can't be any outcomes in the overlap, so we don't have an overlap!

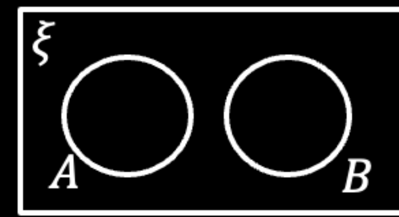
Probabilities of Mutually Exclusive Events

For **mutually exclusive events** (which means they can't happen at the same time):

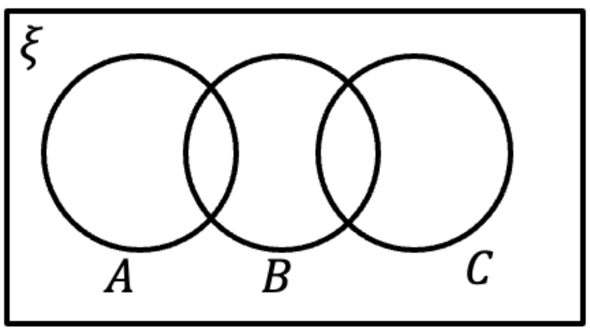
$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cap B) = 0$$

and $A \cap B = \emptyset$



$\emptyset$ is the 'empty set', so $A \cap B = \emptyset$ means "there are no outcomes which are in A and in B"



In this Venn diagram, which events are mutually exclusive?

A and C because they do not overlap.

If A and B are **not** mutually exclusive, we can find $P(A \cup B)$ using the **addition rule**.

What is Independence?

Events are **independent** if **the outcome of one event does not affect the other**.

If B happened and the number was divisible by 4, then A **must also happen**. So one event does affect the other. This is why $P(A \text{ and } B) \neq P(A) \times P(B)$

Let A be the event that the card is divisible by 2.

Let B be the event that the card is divisible by 4.

Let H be the event of flipping at Heads.

However, what card was picked will clearly have no effect on the separate coin toss. These events are independent.

Two events A and B are independent if (and only if) $P(A \cap B) = P(A) \times P(B)$

Recall that **events are sets of outcomes** and $\cap$ gives the **intersection** of the two, i.e. what outcomes are in **both** sets. This is equivalent to using the word 'and'.

Independent Events

- If two events are independent
  then whether one event happens does not affect the probability of the other happening.
- If A and B are independent then:
  - $\boldsymbol{P(A\ and\ B) = P(A) \times P(B)}$

Note: Independence does not affect how the circles interact in a Venn Diagram.

Example

1 2 3 4

1 I pick one of the four numbers 1, 2, 3, 4 at random. What's the probability that:

a) I pick a multiple of 2: $\frac{1}{2}$

b) I pick a multiple of 4: $\frac{1}{4}$

2 Explain (conceptually) why these two events are not independent.
If it is a multiple of 4, then it must also be a multiple of 2. But if it wasn't a multiple of 4 then it may or may not be a multiple of 2. So the events are linked and whether one happened influences the probability of the other happening.

3 Show that the events are not independent.

$$P(multiple\ of\ 2) \times P(multiple\ of\ 4) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

$$P(multiple\ of\ 2\ \boldsymbol{and}\ multiple\ of\ 4) = \frac{1}{4}$$

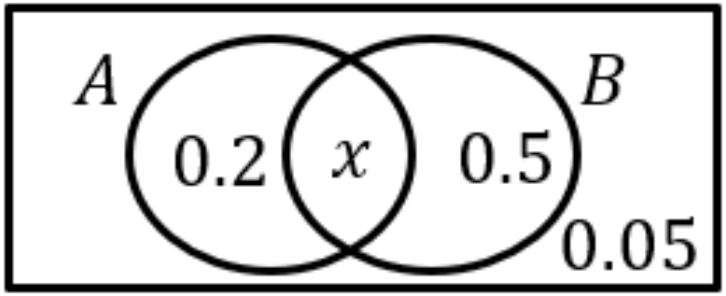
But $\frac{1}{8} \neq \frac{1}{4}$ therefore not independent.

This is a common exam question. Either show that $P(A\ and\ B) = P(A) \times P(B)$ or that $P(A\ and\ B) \neq P(A) \times P(B)$

Notes

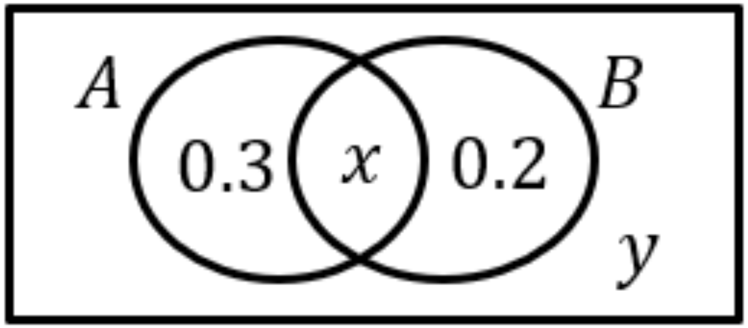
Worked Example

Determine if events A and B are independent.

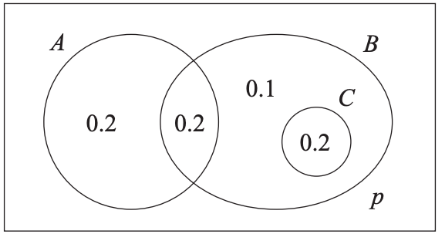


Worked Example

Given that A and B are independent, determine the possible values for x and y



Worked Example



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

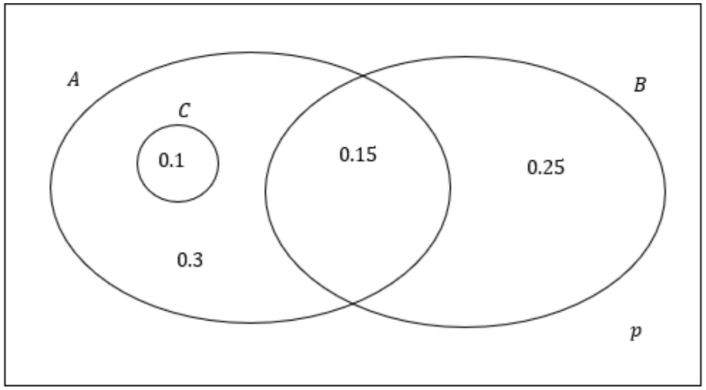
(a) Find the value of p.

(1)

(b) Write down a pair of mutually exclusive events from A, B and C.

(1)

Your Turn



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p.

(1)

(b) Write down a pair of mutually exclusive events from A, B and C.

(1)

Worked Example

In an after-school club, students can choose to take part in Art, Music, both or neither.

There are 45 students that attend the after-school club. Of these

- 25 students take part in Art
- 12 students take part in both Art and Music
- the number of students that take part in Music is x

(a) Find the range of possible values of x

(2)

One of the 45 students is selected at random.

Event A is the event that the student selected takes part in Art.

Event M is the event that the student selected takes part in Music.

(b) Determine whether or not it is possible for the events A and M to be independent.

(4)

Your Turn

In an after-school club, students can choose to take part in Drama, Geography, both or neither.

There are 55 students that attend the after-school club. Of these

- 30 students take part in Drama
- 16 students take part in both Drama and Geography
- the number of students that take part in Geography is x

(a) Find the range of possible values of x

(2)

One of the 55 students is selected at random.

Event D is the event that the student selected takes part in Drama.

Event G is the event that the student selected takes part in Geography.

(b) Determine whether or not it is possible for the events D and G to be independent.

(4)

5.4 Tree Diagrams

Notes

Worked Example

The probability I hit a target on each shot is 0.4. I keep firing until I hit the target. Determine the probability I hit the target on the 6^{th} shot.

Worked Example

Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 4 red, 3 yellow and n green balls.
Bag **B** contains 5 red, 3 yellow and 1 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.
A ball is then selected at random from bag **B** and placed into bag **A**.

The probability that bag **A** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(5)

Your Turn

Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 5 red, 4 yellow and 2 green balls.
Bag **B** contains 5 red, n yellow and 4 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.

A ball is then selected at random from bag **B** and placed into bag **A**.
The probability that bag **B** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(5)

Summary

1 A **Venn diagram** can be used to represent events graphically. Frequencies or probabilities can be placed in the regions of the Venn diagram.

2 For **mutually exclusive** events, $P(A \text{ or } B) = P(A) + P(B)$.

3 For **independent** events, $P(A \text{ and } B) = P(A) \times P(B)$.

4 A **tree diagram** can be used to show the outcomes of two (or more) events happening in succession.