KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Statistics 1
Chapter 6 – Statistical Distributions – Part 1

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior Knowledge Check

1 Three coins are flipped. Calculate the probability that:

a all the coins land on tails

b all the coins land on heads

c exactly one of the coins lands on tails

d at least two coins land on heads.

← Chapter 5

2 Two fair dice are rolled. Calculate the probability that the sum of the scores on the dice is:

a five

b even

c odd

d a multiple of 3

e a prime number.

← Chapter 5

6.1 Probability Distributions

What are Discrete Random Variables?

You may already be familiar with a **variable** in statistics representing a **collection of values**. It could for example be the collected heights of people in the class.

x	1	3	4	6
$P(X = x)$	0.1	0.6	0.2	0.1

A **random variable** is simply a variable with **probabilities associated with each outcome**. If the variable is a discrete quantity, then this is known as a **discrete random variable**.

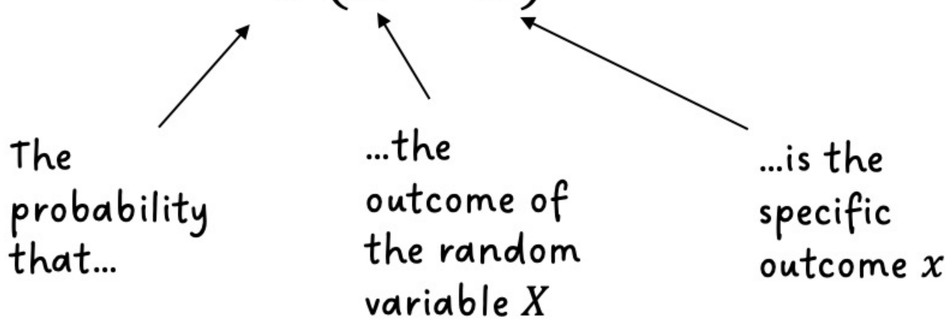
What are Discrete Random Variables?

x	1	3	4	6
$P(X = x)$	0.1	0.6	0.2	0.1

A random variable X represents a single experiment/trial. It consists of outcomes with a probability for each.

Notation:

$$P(X = x)$$



For the "*probability that the outcome was 3*", we could write $\boldsymbol{P(X = 3)}$, or $\boldsymbol{p(3)}$ for shorthand, with a lowercase p. If the latter is used, it should be unambiguous what random variable we are referring to.

Probability Distributions as Tables vs Functions

A probability distribution can be written in two ways:

As a table:

x	1	2	3	4
$P(X = x)$	0.1	0.2	0.3	0.4

Advantage:
Probability of each outcome is explicit.

As a function:

$$P(X = x) = \begin{cases} 0.1x & x = 1,2,3,4 \\ 0 & \text{otherwise} \end{cases}$$

This is known as a **piecewise function** (skill 487) allowing us to have different expressions based on the input (the outcome). Let's explore how to use it...

Advantages:

- More concise representation.
- Can express an **underlying formula** that works out the probability from the outcome (e.g. see Poisson 840 and Binomial distributions 639).
- Can represent distributions with **infinitely many outcomes** (e.g. geometric distribution 843).

Converting from Functional to Table Form

As a function:

$$P(X=x)=\begin{cases}0.1x & x=1,2,3,4\\ 0 & \text{otherwise}\end{cases}$$

The outcomes are on the right of the piecewise function. The 'otherwise' row makes explicit that the probability is 0 for any outcome other than 1 to 4.

x	1	2	3	4
$P(X=x)$	**0.1**	**0.2**	**0.3**	**0.4**

To calculate $P(X=1)$, use the expression on the left associated with this outcome. $0.1\times1=0.1$

Similarly, $P(X=2)=0.1\times2=0.2$

Similarly, $P(X=3)=0.1\times3=0.3$

To check, ensure your probabilities sum to 1.

Notes

Worked Example

A spinner has six equally-sized sections.
Four contain the letter G. 2 contain the letter Y.
The spinner is spun until it lands on Y or has been spun five times in total.
Find the probability distribution of the random variable S, the number of times the spinner is spun.

Discrete Uniform Distributions

A discrete uniform distribution is when the **probability of each outcome is equal**.

For what situations would it be useful to model as a discrete uniform distribution?

For any situation where we want 'fairness' **without bias towards a particular outcome**. e.g. "the throw of a fair/unbiased coin" or "the throw of a fair dice".

Distribution as a table:

x	1	2	...	n
$P(X = x)$	$\frac{1}{n}$	$\frac{1}{n}$		$\frac{1}{n}$

Worked Example

In a game, a player can score 0, 1, 2, 3 or 4 points each time the game is played.

The random variable S, representing the player's score, has the following probability distribution where a, b and c are constants.

s	0	1	2	3	4
$P(S = s)$	a	b	c	0.1	0.15

The probability of scoring less than 2 points is twice the probability of scoring at least 2 points.

Each game played is independent of previous games played.

John plays the game twice and adds the two scores together to get a total.

Calculate the probability that the total is 6 points.

(6)

Your Turn

On a spinner, the numbers 1, 2, 3, 6 or 9 can occur.

The random variable N, representing the number that the spinner points at, has the following probability distribution where a, b and c are constants.

n	1	2	3	6	9
$P(N = n)$	0.18	a	0.12	b	c

The probability of spinning a number which is a multiple of three is three times the probability of spinning a number which is not a multiple of three.

Each spin is independent of the previous number.

The game requires the spinner to be spun twice and the two numbers are multiplied together to get the score.

Calculate the probability that the score is a prime number.

(6)

Worked Example

A biased 4-sided spinner has the numbers 6, 7, 8 and 10 on it.

The discrete random variable X represents the score when the spinner is spun once and has the following probability distribution,

x	6	7	8	10
$P(X = x)$	0.5	0.2	q	q

where q is a probability.

(a) Find the value of q

(1)

Karen spins the spinner repeatedly until she **either** gets a 7 **or** she has taken 4 spins.

(b) Show that the probability that Karen stops after taking her 3rd spin is 0.128

(2)

The random variable S represents the number of spins Karen takes.

(c) Find the probability distribution for S

(4)

The random variable N represents the number of times Karen gets a 7

(d) Find $P(S > N)$

(1)

Your Turn

A biased 5-sided spinner has the numbers 3, 4, 5, 6, and 7 on it.

The discrete random variable X represents the score when the spinner is spun once and has the following probability distribution,

x	3	4	5	6	7
$P(X = x)$	0.15	q	0.25	q	0.4

where q is a probability.

(a) Find the value of q

(1)

Rebecca spins the spinner repeatedly until she **either** gets a 6 **or** she has taken 5 spins.

(b) Show that the probability that Rebecca stops after taking her 4th spin is 0.0729

(2)

The random variable S represents the number of spins Rebecca takes.

(c) Find the probability distribution for S

(4)

The random variable N represents the number of times Rebecca gets a 6

(d) Find $P(S < N)$

(1)

Worked Example

Manon has two biased spinners, one red and one green.

The random variable R represents the score when the red spinner is spun.
The random variable G represents the score when the green spinner is spun.

The probability distributions for R and G are given below.

r	2	3
$P(R = r)$	$\frac{1}{4}$	$\frac{3}{4}$

g	1	4
$P(G = g)$	$\frac{2}{3}$	$\frac{1}{3}$

Manon spins each spinner once and adds the two scores.

(a) Find the probability that

(i) the sum of the two scores is 7

(ii) the sum of the two scores is less than 4

(3)

The random variable $X = mR + nG$ where m and n are integers.

$$P(X = 20) = \frac{1}{6} \quad \text{and} \quad P(X = 50) = \frac{1}{4}$$

(b) Find the value of m and the value of n

(5)

Your Turn

Manon has two biased spinners, one red and one green.

The random variable R represents the score when the red spinner is spun.
The random variable G represents the score when the green spinner is spun.

The probability distributions for R and G are given below.

r	2	3
$P(R = r)$	$\frac{1}{5}$	$\frac{4}{5}$

g	1	4
$P(G = g)$	$\frac{5}{6}$	$\frac{1}{6}$

Manon spins each spinner once and adds the two scores.

(*a*) Find the probability that

(i) the sum of the two scores is 6

(ii) the sum of the two scores is less than 5

(3)

The random variable $X = mR - nG$ where m and n are integers.

$$P(X = 12) = \frac{1}{30} \quad \text{and} \quad P(X = 28) = \frac{2}{3}$$

(*b*) Find the value of m and the value of n

(5)

Worked Example

Julia selects 3 letters at random, one at a time without replacement, from the word

V A R I A N C E

The discrete random variable X represents the number of times she selects a letter A.

(a) Find the complete probability distribution of X.

(5)

Yuki selects 10 letters at random, one at a time **with** replacement, from the word

D E V I A T I O N

(b) Find the probability that he selects the letter E at least 4 times.

(3)

Your Turn

Fatimah selects 3 letters at random, one at a time without replacement, from the word

D E V I A T I O N

The discrete random variable X represents the number of times she selects a letter I.

(a) Find the complete probability distribution of X.

(5)

Kasim selects 12 letters at random, one at a time **with** replacement, from the word

V A R I A T I O N

(b) Find the probability that he selects the letter V at least 3 times.

(3)

Summary

1 A **probability distribution** fully describes the probability of any outcome in the sample space.

2 The sum of the probabilities of all outcomes of an event add up to 1. For a random variable X, you can write $\sum P(X = x) = 1$ for all x.