Year 12
2025　　Mathematics　　2026
Summer Task Booklet

HGS Maths

Dr Frost Course

Name: _____

Class: _____

Contents

Pure Mathematics 1

Pure Mathematics 2

Instructions

Welcome to A Level Mathematics at HGS (https://hgsmaths.com)

This booklet must be completed by **all students** intending to study A Level Mathematics at HGS. When lessons begin, it will be assumed that you are already familiar with the material contained within.

You will sit a **classroom test** at the start of the course to assess your understanding.

Completion of this booklet is mandatory – not optional.

The content is largely crossover material from Higher GCSE Mathematics, although there may be sections that are new to you.

How to Use This Booklet
1. Attempt each example independently before checking the solution.
2. Use the QR code to watch video solutions for all examples in a section (e.g., 2.1).
3. Complete exercises in an organised manner – using an exercise book, notepad, or folder.
4. Check your answers at https://tinyurl.com/yckeua78 or scan the QR code at the bottom of the exercise pages.

Your teacher will check that you have completed this booklet fully at the start of your studies.

1 Algebraic Expressions

[1.1 Index Laws](#)	https://youtu.be/UO19cAFiii4?si=SV6H1P6w2srlze7U	
[1.2 Expanding Brackets](#)	https://youtu.be/w7PDJMErfvM?si=0KNVDC5g0QCiMGGP	
[1.3 Factorising](#)	https://youtu.be/Q1Mm9iF83IA?si=5o2BANfOsMd9mdDi	
[1.4 Negative and Fractional Indices](#)	https://youtu.be/dzPnlZbZRH0?si=5g1PYurpXjkalM8G	
[1.5 Surds](#)	https://youtu.be/k6CZyMEi89s?si=q-9Tb0CAeh3M1fN3	
[1.6 Rationalising Denominators](#)	https://youtu.be/Ps46XZwzyrU?si=Bh32wJAl59y_unOj	

Worked Example

Example 1

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $\dfrac{b^7}{b^4}$ **d** $6x^5 \div 3x^3$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

Worked Example

Example 2

Expand these expressions and simplify if possible:

a $-3x(7x - 4)$

b $y^2(3 - 2y^3)$

c $4x(3x - 2x^2 + 5x^3)$

d $2x(5x + 3) - 5(2x + 3)$

Worked Example

Example 3

Simplify these expressions:

a $\dfrac{x^7 + x^4}{x^3}$

b $\dfrac{3x^2 - 6x^5}{2x}$

c $\dfrac{20x^7 + 15x^3}{5x^2}$

Exercise 1A

1 Simplify these expressions:

a $x^3 \times x^4$

b $2x^3 \times 3x^2$

c $\dfrac{k^3}{k^2}$

d $\dfrac{4p^3}{2p}$

e $\dfrac{3x^3}{3x^2}$

f $(y^2)^5$

g $10x^5 \div 2x^3$

h $(p^3)^2 \div p^4$

i $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$

k $2a^4 \times 3a^5$

l $\dfrac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$

n $3x^3 \times 2x^2 \times 4x^6$

o $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$

q $2a^3 \div 3a^2 \times 6a^5$

r $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$

b $x(x + 9)$

c $-3y(4 - 3y)$

d $x(y + 5)$

e $-x(3x + 5)$

f $-5x(4x + 1)$

g $(4x + 5)x$

h $-3y(5 - 2y^2)$

i $-2x(5x - 4)$

j $(3x - 5)x^2$

k $3(x + 2) + (x - 7)$

l $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$

n $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$

p $7y^2(2 - 5y + 3y^2)$

q $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$

s $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$

u $4x(x + 3) - 2x(3x - 7)$

v $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\dfrac{6x^4 + 10x^6}{2x}$

b $\dfrac{3x^5 - x^7}{x}$

c $\dfrac{2x^4 - 4x^2}{4x}$

d $\dfrac{8x^3 + 5x}{2x}$

e $\dfrac{7x^7 + 5x^2}{5x}$

f $\dfrac{9x^5 - 5x^3}{3x}$

Worked Example

Example 4

Expand these expressions and simplify if possible:

a $(x + 5)(x + 2)$ b $(x - 2y)(x^2 + 1)$ c $(x - y)^2$ d $(x + y)(3x - 2y - 4)$

Worked Example

Example **5**

Expand these expressions and simplify if possible:

a $x(2x + 3)(x - 7)$ **b** $x(5x - 3y)(2x - y + 4)$ **c** $(x - 4)(x + 3)(x + 1)$

Exercise 1B

1 Expand and simplify if possible:

a $(x + 4)(x + 7)$ b $(x - 3)(x + 2)$ c $(x - 2)^2$

d $(x - y)(2x + 3)$ e $(x + 3y)(4x - y)$ f $(2x - 4y)(3x + y)$

g $(2x - 3)(x - 4)$ h $(3x + 2y)^2$ i $(2x + 8y)(2x + 3)$

j $(x + 5)(2x + 3y - 5)$ k $(x - 1)(3x - 4y - 5)$ l $(x - 4y)(2x + y + 5)$

m $(x + 2y - 1)(x + 3)$ n $(2x + 2y + 3)(x + 6)$ o $(4 - y)(4y - x + 3)$

p $(4y + 5)(3x - y + 2)$ q $(5y - 2x + 3)(x - 4)$ r $(4y - x - 2)(5 - y)$

2 Expand and simplify if possible:

a $5(x + 1)(x - 4)$ b $7(x - 2)(2x + 5)$ c $3(x - 3)(x - 3)$

d $x(x - y)(x + y)$ e $x(2x + y)(3x + 4)$ f $y(x - 5)(x + 1)$

g $y(3x - 2y)(4x + 2)$ h $y(7 - x)(2x - 5)$ i $x(2x + y)(5x - 2)$

j $x(x + 2)(x + 3y - 4)$ k $y(2x + y - 1)(x + 5)$ l $y(3x + 2y - 3)(2x + 1)$

m $x(2x + 3)(x + y - 5)$ n $2x(3x - 1)(4x - y - 3)$ o $3x(x - 2y)(2x + 3y + 5)$

p $(x + 3)(x + 2)(x + 1)$ q $(x + 2)(x - 4)(x + 3)$ r $(x + 3)(x - 1)(x - 5)$

s $(x - 5)(x - 4)(x - 3)$ t $(2x + 1)(x - 2)(x + 1)$ u $(2x + 3)(3x - 1)(x + 2)$

v $(3x - 2)(2x + 1)(3x - 2)$ w $(x + y)(x - y)(x - 1)$ x $(2x - 3y)^3$

(P) **3** The diagram shows a rectangle with a square cut out. The rectangle has length $3x - y + 4$ and width $x + 7$. The square has length $x - 2$.

Find an expanded and simplified expression for the shaded area.
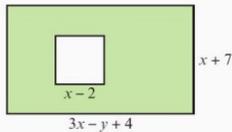
Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:

6cm, 3cm, 10cm

$x + 7$

$x - 2$

$3x - y + 4$
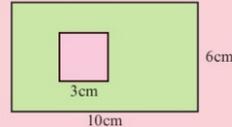
(P) **4** A cuboid has dimensions $x + 2$ cm, $2x - 1$ cm and $2x + 3$ cm. Show that the volume of the cuboid is $4x^3 + 12x^2 + 5x - 6$ cm³.

(E/P) **5** Given that $(2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cxy^2 + dy^3$, where a, b, c and d are constants, find the values of a, b, c and d. **(2 marks)**

Challenge

Expand and simplify $(x + y)^4$.

1.3 Factorising

Worked Example

Factorise these expressions completely:

a $3x + 9$ **b** $x^2 - 5x$ **c** $8x^2 + 20x$ **d** $9x^2y + 15xy^2$ **e** $3x^2 - 9xy$

Worked Example

Example 7

Factorise:

a $x^2 - 5x - 6$ **b** $x^2 + 6x + 8$ **c** $6x^2 - 11x - 10$ **d** $x^2 - 25$ **e** $4x^2 - 9y^2$

Worked Example

Example 8

Factorise completely:

a $x^3 - 2x^2$ b $x^3 - 25x$ c $x^3 + 3x^2 - 10x$

1 Factorise these expressions completely:

a $4x + 8$ b $6x - 24$ c $20x + 15$
d $2x^2 + 4$ e $4x^2 + 20$ f $6x^2 - 18x$
g $x^2 - 7x$ h $2x^2 + 4x$ i $3x^2 - x$
j $6x^2 - 2x$ k $10y^2 - 5y$ l $35x^2 - 28x$
m $x^2 + 2x$ n $3y^2 + 2y$ o $4x^2 + 12x$
p $5y^2 - 20y$ q $9xy^2 + 12x^2y$ r $6ab - 2ab^2$
s $5x^2 - 25xy$ t $12x^2y + 8xy^2$ u $15y - 20yz^2$
v $12x^2 - 30$ w $xy^2 - x^2y$ x $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$ b $2x^2 + 6x$ c $x^2 + 11x + 24$
d $x^2 + 8x + 12$ e $x^2 + 3x - 40$ f $x^2 - 8x + 12$
g $x^2 + 5x + 6$ h $x^2 - 2x - 24$ i $x^2 - 3x - 10$
j $x^2 + x - 20$ k $2x^2 + 5x + 2$ l $3x^2 + 10x - 8$
m $5x^2 - 16x + 3$ n $6x^2 - 8x - 8$
o $2x^2 + 7x - 15$ p $2x^4 + 14x^2 + 24$
q $x^2 - 4$ r $x^2 - 49$
s $4x^2 - 25$ t $9x^2 - 25y^2$ u $36x^2 - 4$
v $2x^2 - 50$ w $6x^2 - 10x + 4$ x $15x^2 + 42x - 9$

Hint For part **n**, take 2 out as a common factor first. For part **p**, let $y = x^2$.

3 Factorise completely:

a $x^3 + 2x$ b $x^3 - x^2 + x$ c $x^3 - 5x$
d $x^3 - 9x$ e $x^3 - x^2 - 12x$ f $x^3 + 11x^2 + 30x$
g $x^3 - 7x^2 + 6x$ h $x^3 - 64x$ i $2x^3 - 5x^2 - 3x$
j $2x^3 + 13x^2 + 15x$ k $x^3 - 4x$ l $3x^3 + 27x^2 + 60x$

(P) 4 Factorise completely $x^4 - y^4$. **(2 marks)**

Problem-solving
Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$

(E) 5 Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

Challenge
Write $4x^4 - 13x^2 + 9$ as the product of four linear factors.

1.4 Negative and Fractional Indices

Worked Example

Simplify:

a $\dfrac{x^3}{x^{-3}}$ b $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ c $(x^3)^{\frac{2}{3}}$ d $2x^{1.5} \div 4x^{-0.25}$ e $\sqrt[3]{125x^6}$ f $\dfrac{2x^2 - x}{x^5}$

Worked Example

Example **10**

Evaluate:

a $9^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $25^{-\frac{3}{2}}$

Worked Example

Example 11

Given that $y = \frac{1}{16}x^2$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{2}}$

b $4y^{-1}$

Exercise 1D

1 Simplify:

a $x^3 \div x^{-2}$

b $x^5 \div x^7$

c $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$

e $(x^3)^{\frac{5}{3}}$

f $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{3}{2}} \div 3x^{\frac{1}{6}}$

h $5x^{\frac{7}{3}} \div x^{\frac{2}{3}}$

i $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$

k $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$

l $\dfrac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$

b $81^{\frac{3}{2}}$

c $27^{\frac{1}{3}}$

d 4^{-2}

e $9^{-\frac{1}{2}}$

f $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$

h $1296^{\frac{1}{4}}$

i $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$

k $\left(\frac{6}{5}\right)^{-1}$

l $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$

b $\dfrac{5x^3 - 2x^2}{x^5}$

c $(125x^{12})^{\frac{1}{3}}$

d $\dfrac{x + 4x^3}{x^3}$

e $\dfrac{2x + x^2}{x^4}$

f $\left(\frac{4}{9}x^4\right)^{\frac{1}{2}}$

g $\dfrac{9x^2 - 15x^5}{3x^3}$

h $\dfrac{5x + 3x^2}{15x^3}$

(E) 4 a Find the value of $81^{\frac{1}{4}}$. **(1 mark)**

b Simplify $x(2x^{-\frac{1}{3}})^4$. **(2 marks)**

(E) 5 Given that $y = \frac{1}{8}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 marks)**

b $\frac{1}{2}y^{-2}$ **(2 marks)**

6 Solve

a $x^{\frac{1}{2}} = 7$

b $y^{\frac{4}{3}} = 81$

c $x^{-\frac{3}{2}} = 8$

d $z^{-\frac{3}{4}} = 1000$

> **Hint** You can undo a fractional index by raising it to the power of its reciprocal:
> $\left(y^{\frac{4}{3}}\right)^{\frac{3}{4}} = y$

(E/P) 7 Solve $27\sqrt{x} = \frac{1}{x}$ **(2 marks)**

> **Problem-solving**
> Multiply both sides by x then simplify the indices.

1.5 Surds

Worked Example

Example 12

Simplify:

a $\sqrt{12}$

b $\dfrac{\sqrt{20}}{2}$

c $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

Worked Example

Example 13

Expand and simplify if possible:

a $\sqrt{2}(5 - \sqrt{3})$

b $(2 - \sqrt{3})(5 + \sqrt{3})$

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\dfrac{\sqrt{12}}{2}$

g $\dfrac{\sqrt{27}}{3}$ **h** $\sqrt{20} + \sqrt{80}$ **i** $\sqrt{200} + \sqrt{18} - \sqrt{72}$

j $\sqrt{175} + \sqrt{63} + 2\sqrt{28}$ **k** $\sqrt{28} - 2\sqrt{63} + \sqrt{7}$ **l** $\sqrt{80} - 2\sqrt{20} + 3\sqrt{45}$

m $3\sqrt{80} - 2\sqrt{20} + 5\sqrt{45}$ **n** $\dfrac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12} + 3\sqrt{48} + \sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2 + \sqrt{3})$ **b** $\sqrt{5}(3 - \sqrt{3})$ **c** $\sqrt{2}(4 - \sqrt{5})$

d $(2 - \sqrt{2})(3 + \sqrt{5})$ **e** $(2 - \sqrt{3})(3 - \sqrt{7})$ **f** $(4 + \sqrt{5})(2 + \sqrt{5})$

g $(5 - \sqrt{3})(1 - \sqrt{3})$ **h** $(4 + \sqrt{3})(2 - \sqrt{3})$ **i** $(7 - \sqrt{11})(2 + \sqrt{11})$

E **3** Simplify $\sqrt{75} - \sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer. **(2 marks)**

Worked Example

Example 14

Rationalise the denominator of:

a $\dfrac{1}{\sqrt{3}}$ **b** $\dfrac{1}{3 + \sqrt{2}}$ **c** $\dfrac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$ **d** $\dfrac{1}{(1 - \sqrt{3})^2}$

1 Simplify:

a $\dfrac{1}{\sqrt{5}}$ **b** $\dfrac{1}{\sqrt{11}}$ **c** $\dfrac{1}{\sqrt{2}}$ **d** $\dfrac{\sqrt{3}}{\sqrt{15}}$

e $\dfrac{\sqrt{12}}{\sqrt{48}}$ **f** $\dfrac{\sqrt{5}}{\sqrt{80}}$ **g** $\dfrac{\sqrt{12}}{\sqrt{156}}$ **h** $\dfrac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\dfrac{1}{1+\sqrt{3}}$ **b** $\dfrac{1}{2+\sqrt{5}}$ **c** $\dfrac{1}{3-\sqrt{7}}$ **d** $\dfrac{4}{3-\sqrt{5}}$ **e** $\dfrac{1}{\sqrt{5}-\sqrt{3}}$

f $\dfrac{3-\sqrt{2}}{4-\sqrt{5}}$ **g** $\dfrac{5}{2+\sqrt{5}}$ **h** $\dfrac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **i** $\dfrac{11}{3+\sqrt{11}}$ **j** $\dfrac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

k $\dfrac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **l** $\dfrac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **m** $\dfrac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\dfrac{1}{(3-\sqrt{2})^2}$ **b** $\dfrac{1}{(2+\sqrt{5})^2}$ **c** $\dfrac{4}{(3-\sqrt{2})^2}$

d $\dfrac{3}{(5+\sqrt{2})^2}$ **e** $\dfrac{1}{(5+\sqrt{2})(3-\sqrt{2})}$ **f** $\dfrac{2}{(5-\sqrt{3})(2+\sqrt{3})}$

(E/P) **4** Simplify $\dfrac{3-2\sqrt{5}}{\sqrt{5}-1}$ giving your answer in the form $p+q\sqrt{5}$, where p and q are rational numbers. **(4 marks)**

Problem-solving

You can check that your answer is in the correct form by writing down the values of p and q and checking that they are rational numbers.

Summary

Summary of key points

1. You can use the laws of indices to simplify powers of the **same base**.
   - $a^m \times a^n = a^{m+n}$
   - $a^m \div a^n = a^{m-n}$
   - $(a^m)^n = a^{mn}$
   - $(ab)^n = a^n b^n$

2. Factorising is the opposite of expanding brackets.

3. A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.

4. $x^2 - y^2 = (x+y)(x-y)$

5. You can use the laws of indices with any rational power.
   - $a^{\frac{1}{m}} = \sqrt[m]{a}$
   - $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
   - $a^{-m} = \dfrac{1}{a^m}$
   - $a^0 = 1$

6. You can manipulate surds using these rules:
   - $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
   - $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

7. The rules to rationalise denominators are:
   - Fractions in the form $\dfrac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.
   - Fractions in the form $\dfrac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.
   - Fractions in the form $\dfrac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.

2 Quadratics

2.1 Solving Quadratic Equations	https://youtu.be/VBweWkrXn1A?si=dB612sGGDvhxUOwE	
2.2 Completing the Square	https://youtu.be/roOgntGk8SA?si=i5cp4zLg4X5cWZ1C	

Worked Example

Example **1**

Solve the following equations:

a $x^2 - 2x - 15 = 0$

b $x^2 = 9x$

c $6x^2 + 13x - 5 = 0$

d $x^2 - 5x + 18 = 2 + 3x$

Worked Example

Example **2**

Solve the following equations

a $(2x - 3)^2 = 25$ **b** $(x - 3)^2 = 7$

Exercise **2A**

1 Solve the following equations using factorisation:

a $x^2 + 3x + 2 = 0$ **b** $x^2 + 5x + 4 = 0$ **c** $x^2 + 7x + 10 = 0$ **d** $x^2 - x - 6 = 0$

e $x^2 - 8x + 15 = 0$ **f** $x^2 - 9x + 20 = 0$ **g** $x^2 - 5x - 6 = 0$ **h** $x^2 - 4x - 12 = 0$

2 Solve the following equations using factorisation:

a $x^2 = 4x$ **b** $x^2 = 25x$ **c** $3x^2 = 6x$ **d** $5x^2 = 30x$

e $2x^2 + 7x + 3 = 0$ **f** $6x^2 - 7x - 3 = 0$ **g** $6x^2 - 5x - 6 = 0$ **h** $4x^2 - 16x + 15 = 0$

3 Solve the following equations:

a $3x^2 + 5x = 2$ **b** $(2x - 3)^2 = 9$ **c** $(x - 7)^2 = 36$ **d** $2x^2 = 8$ **e** $3x^2 = 5$

f $(x - 3)^2 = 13$ **g** $(3x - 1)^2 = 11$ **h** $5x^2 - 10x^2 = -7 + x + x^2$

i $6x^2 - 7 = 11x$ **j** $4x^2 + 17x = 6x - 2x^2$

(P) **4** This shape has an area of $44\,\text{m}^2$. Find the value of x.
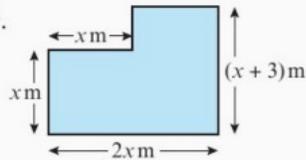
Problem-solving

Divide the shape into two sections:
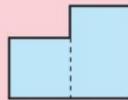
(P) **5** Solve the equation $5x + 3 = \sqrt{3x + 7}$.

Worked Example

Example 3

Solve $3x^2 - 7x - 1 = 0$ by using the formula.

Exercise 2B

1 Solve the following equations using the quadratic formula.
  Give your answers exactly, leaving them in surd form where necessary.
  a $x^2 + 3x + 1 = 0$
  b $x^2 - 3x - 2 = 0$
  c $x^2 + 6x + 6 = 0$
  d $x^2 - 5x - 2 = 0$
  e $3x^2 + 10x - 2 = 0$
  f $4x^2 - 4x - 1 = 0$
  g $4x^2 - 7x = 2$
  h $11x^2 + 2x - 7 = 0$

2 Solve the following equations using the quadratic formula.
  Give your answers to three significant figures.
  a $x^2 + 4x + 2 = 0$
  b $x^2 - 8x + 1 = 0$
  c $x^2 + 11x - 9 = 0$
  d $x^2 - 7x - 17 = 0$
  e $5x^2 + 9x - 1 = 0$
  f $2x^2 - 3x - 18 = 0$
  g $3x^2 + 8 = 16x$
  h $2x^2 + 11x = 5x^2 - 18$

3 For each of the equations below, choose a suitable method and find all of the solutions.
  Where necessary, give your answers to three significant figures.
  a $x^2 + 8x + 12 = 0$
  b $x^2 + 9x - 11 = 0$
  c $x^2 - 9x - 1 = 0$
  d $2x^2 + 5x + 2 = 0$
  e $(2x + 8)^2 = 100$
  f $6x^2 + 6 = 12x$
  g $2x^2 - 11 = 7x$
  h $x = \sqrt{8x - 15}$

Hint You can use any method you are confident with to solve these equations.

(P) 4 This trapezium has an area of $50\,\text{m}^2$.
  Show that the height of the trapezium is equal to $5(\sqrt{5} - 1)\,\text{m}$.

$\leftarrow x\,\text{m} \rightarrow$
$2x\,\text{m}$
$\longleftarrow (x + 10)\,\text{m} \longrightarrow$
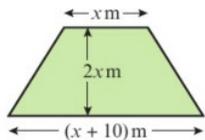
Problem-solving
Height must be positive. You will have to discard the negative solution of your quadratic equation.

Challenge
Given that x is positive, solve the equation
$$\frac{1}{x} + \frac{1}{x + 2} = \frac{28}{195}$$

Hint Write the equation in the form $ax^2 + bx + c = 0$ before using the quadratic formula or factorising.

2.2 Completing the Square

Worked Example

Example 4

Complete the square for the expressions:

a $x^2 + 8x$ b $x^2 - 3x$ c $2x^2 - 12x$

Worked Example

Example 5

Write $3x^2 + 6x + 1$ in the form $p(x + q)^2 + r$, where p, q and r are integers to be found.

1 Complete the square for the expressions:

   a $x^2 + 4x$ **b** $x^2 - 6x$ **c** $x^2 - 16x$ **d** $x^2 + x$ **e** $x^2 - 14x$

2 Complete the square for the expressions:

   a $2x^2 + 16x$ **b** $3x^2 - 24x$ **c** $5x^2 + 20x$ **d** $2x^2 - 5x$ **e** $8x - 2x^2$

> **Hint** In question **3d**, write the expression as $-4x^2 - 16x + 10$ then take a factor of -4 out of the first two terms to get $-4(x^2 + 4x) + 10$.

3 Write each of these expressions in the form $p(x + q)^2 + r$, where p, q and r are constants to be found:

   a $2x^2 + 8x + 1$ **b** $5x^2 - 15x + 3$ **c** $3x^2 + 2x - 1$ **d** $10 - 16x - 4x^2$ **e** $2x - 8x^2 + 10$

(E) 4 Given that $x^2 + 3x + 6 = (x + a)^2 + b$, find the values of the constants a and b. **(2 marks)**

(E) 5 Write $2 + 0.8x - 0.04x^2$ in the form $A - B(x + C)^2$, where A, B and C are constants to be determined. **(3 marks)**

Worked Example

Example 6

Solve the equation $x^2 + 8x + 10 = 0$ by completing the square.
Give your answers in surd form.

Worked Example

Example 7

Solve the equation $2x^2 - 8x + 7 = 0$. Give your answers in surd form.

Exercise 2D

1 Solve these quadratic equations by completing the square. Leave your answers in surd form.

a $x^2 + 6x + 1 = 0$ **b** $x^2 + 12x + 3 = 0$ **c** $x^2 + 4x - 2 = 0$ **d** $x^2 - 10x = 5$

2 Solve these quadratic equations by completing the square. Leave your answers in surd form.

a $2x^2 + 6x - 3 = 0$ **b** $5x^2 + 8x - 2 = 0$ **c** $4x^2 - x - 8 = 0$ **d** $15 - 6x - 2x^2 = 0$

(E) **3** $x^2 - 14x + 1 = (x + p)^2 + q$, where p and q are constants.

    a Find the values of p and q. **(2 marks)**

    b Using your answer to part **a**, or otherwise, show that the solutions to the equation $x^2 - 14x + 1 = 0$ can be written in the form $r \pm s\sqrt{3}$, where r and s are constants to be found. **(2 marks)**

(E/P) **4** By completing the square, show that the solutions to the equation $x^2 + 2bx + c = 0$ are given by the formula $x = -b \pm \sqrt{b^2 - c}$. **(4 marks)**

Problem-solving

Follow the same steps as you would if the coefficients were numbers.

Challenge

a Show that the solutions to the equation

$ax^2 + 2bx + c = 0$ are given by $x = -\dfrac{b}{a} \pm \sqrt{\dfrac{b^2 - ac}{a^2}}$.

b Hence, or otherwise, show that the solutions to the equation $ax^2 + bx + c = 0$ can be written as

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Hint Start by dividing the whole equation by a.

Links You can use this method to prove the quadratic formula. → Section 7.4

Summary

Summary of key points

1. To solve a quadratic equation by factorising:
   - Write the equation in the form $ax^2 + bx + c = 0$
   - Factorise the left-hand side
   - Set each factor equal to zero and solve to find the value(s) of x

2. The solutions of the equation $ax^2 + bx + c = 0$ where $a \neq 0$ are given by the formula:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

4. $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

3 Equations and Inequalities

3.1 Linear Simultaneous Equations	https://youtu.be/5SpbJOzKGF4?si=XB4k1qamxqYym6F7	
3.2 Quadratic Simultaneous Equations	https://youtu.be/luj2ZofSTlw	

3.1 Linear Simultaneous Equations

Worked Example

Example 1

Solve the simultaneous equations:

a $2x + 3y = 8$
  $3x - y = 23$

b $4x - 5y = 4$
  $6x + 2y = 25$

Worked Example

Example 2

Solve the simultaneous equations:

$$2x - y = 1$$
$$4x + 2y = -30$$

1 Solve these simultaneous equations by elimination:

a $2x - y = 6$
$4x + 3y = 22$

b $7x + 3y = 16$
$2x + 9y = 29$

c $5x + 2y = 6$
$3x - 10y = 26$

d $2x - y = 12$
$6x + 2y = 21$

e $3x - 2y = -6$
$6x + 3y = 2$

f $3x + 8y = 33$
$6x = 3 + 5y$

2 Solve these simultaneous equations by substitution:

a $x + 3y = 11$
$4x - 7y = 6$

b $4x - 3y = 40$
$2x + y = 5$

c $3x - y = 7$
$10x + 3y = -2$

d $2y = 2x - 3$
$3y = x - 1$

3 Solve these simultaneous equations:

a $3x - 2y + 5 = 0$
$5(x + y) = 6(x + 1)$

b $\dfrac{x - 2y}{3} = 4$
$2x + 3y + 4 = 0$

c $3y = 5(x - 2)$
$3(x - 1) + y + 4 = 0$

> **Hint** First rearrange both equations into the same form e.g. $ax + by = c$.

E/P **4** $3x + ky = 8$
$x - 2ky = 5$
are simultaneous equations where k is a constant.

a Show that $x = 3$. **(3 marks)**

b Given that $y = \frac{1}{2}$ determine the value of k. **(1 mark)**

> **Problem-solving**
>
> k is a constant, so it has the same value in both equations.

E/P **5** $2x - py = 5$
$4x + 5y + q = 0$
are simultaneous equations where p and q are constants.
The solution to this pair of simultaneous equations is $x = q$, $y = -1$.
Find the value of p and the value of q. **(5 marks)**

Worked Example

Example 3

Solve the simultaneous equations:

$x + 2y = 3$

$x^2 + 3xy = 10$

1 Solve the simultaneous equations:

a $x + y = 11$
$xy = 30$

b $2x + y = 1$
$x^2 + y^2 = 1$

c $y = 3x$
$2y^2 - xy = 15$

d $3a + b = 8$
$3a^2 + b^2 = 28$

e $2u + v = 7$
$uv = 6$

f $3x + 2y = 7$
$x^2 + y = 8$

2 Solve the simultaneous equations:

a $2x + 2y = 7$
$x^2 - 4y^2 = 8$

b $x + y = 9$
$x^2 - 3xy + 2y^2 = 0$

c $5y - 4x = 1$
$x^2 - y^2 + 5x = 41$

3 Solve the simultaneous equations, giving your answers in their simplest surd form:

a $x - y = 6$
$xy = 4$

b $2x + 3y = 13$
$x^2 + y^2 = 78$

> **Watch out** Use brackets when you are substituting an expression into an equation.

(E/P) 4 Solve the simultaneous equations:

$x + y = 3$
$x^2 - 3y = 1$

(6 marks)

(E/P) 5 a By eliminating y from the equations

$y = 2 - 4x$
$3x^2 + xy + 11 = 0$

show that $x^2 - 2x - 11 = 0$. **(2 marks)**

b Hence, or otherwise, solve the simultaneous equations

$y = 2 - 4x$
$3x^2 + xy + 11 = 0$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers. **(5 marks)**

(P) 6 One pair of solutions for the simultaneous equations

$y = kx - 5$
$4x^2 - xy = 6$

is $(1, p)$ where k and p are constants.

> **Problem-solving** If $(1, p)$ is a solution, then $x = 1$, $y = p$ satisfies both equations.

a Find the values of k and p.

b Find the second pair of solutions for the simultaneous equations.

Challenge

$y - x = k$
$x^2 + y^2 = 4$

Given that the simultaneous equations have exactly one pair of solutions, show that

$k = \pm 2\sqrt{2}$

Summary

Summary of key points

1 Linear simultaneous equations can be solved using elimination or substitution.

2 Simultaneous equations with one linear and one quadratic equation can have up to two pairs of solutions. You need to make sure the solutions are paired correctly.

5 Straight Line Graphs

5.1 $y = mx + c$	https://youtu.be/C7meQTODsnA?si=T57mJnp-X34OnlI2	

Worked Example

Example 1

Work out the gradient of the line joining $(-2, 7)$ and $(4, 5)$

Worked Example

Example **2**

The line joining $(2, -5)$ to $(4, a)$ has gradient -1. Work out the value of a.

Exercise **5A**

1 Work out the gradients of the lines joining these pairs of points:

    a $(4, 2), (6, 3)$ **b** $(-1, 3), (5, 4)$ **c** $(-4, 5), (1, 2)$

    d $(2, -3), (6, 5)$ **e** $(-3, 4), (7, -6)$ **f** $(-12, 3), (-2, 8)$

    g $(-2, -4), (10, 2)$ **h** $(\frac{1}{2}, 2), (\frac{3}{4}, 4)$ **i** $(\frac{1}{4}, \frac{1}{2}), (\frac{1}{2}, \frac{2}{3})$

    j $(-2.4, 9.6), (0, 0)$ **k** $(1.3, -2.2), (8.8, -4.7)$ **l** $(0, 5a), (10a, 0)$

    m $(3b, -2b), (7b, 2b)$ **n** $(p, p^2), (q, q^2)$

2 The line joining $(3, -5)$ to $(6, a)$ has a gradient 4. Work out the value of a.

3 The line joining $(5, b)$ to $(8, 3)$ has gradient -3. Work out the value of b.

4 The line joining $(c, 4)$ to $(7, 6)$ has gradient $\frac{3}{4}$. Work out the value of c.

5 The line joining $(-1, 2d)$ to $(1, 4)$ has gradient $-\frac{1}{4}$. Work out the value of d.

6 The line joining $(-3, -2)$ to $(2e, 5)$ has gradient 2. Work out the value of e.

7 The line joining $(7, 2)$ to $(f, 3f)$ has gradient 4. Work out the value of f.

8 The line joining $(3, -4)$ to $(-g, 2g)$ has gradient -3. Work out the value of g.

(P) 9 Show that the points $A(2, 3)$, $B(4, 4)$ and $C(10,7)$ can be joined by a straight line.

> **Problem-solving**
>
> Find the gradient of the line joining the points A and B and the line joining the points B and C.

(E/P) 10 Show that the points $A(-2a, 5a)$, $B(0, 4a)$ and $C(6a, a)$ are collinear. **(3 marks)**

> **Notation** Points are collinear if they all lie on the same straight line.

Worked Example

Example **3**

Write down the gradient and y-intercept of these lines:

a $y = -3x + 2$ **b** $4x - 3y + 5 = 0$

Worked Example

Example **4**

Write these lines in the form $ax + by + c = 0$

a $y = 4x + 3$

b $y = -\frac{1}{2}x + 5$

Worked Example

Example 5

The line $y = 4x - 8$ meets the x-axis at the point P. Work out the coordinates of P.

Exercise 5B

1 Work out the gradients of these lines:

a $y = -2x + 5$
b $y = -x + 7$
c $y = 4 + 3x$
d $y = \frac{1}{3}x - 2$
e $y = -\frac{2}{3}x$
f $y = \frac{5}{4}x + \frac{2}{3}$
g $2x - 4y + 5 = 0$
h $10x - 5y + 1 = 0$
i $-x + 2y - 4 = 0$
j $-3x + 6y + 7 = 0$
k $4x + 2y - 9 = 0$
l $9x + 6y + 2 = 0$

2 These lines cut the y-axis at $(0, c)$. Work out the value of c in each case.

a $y = -x + 4$
b $y = 2x - 5$
c $y = \frac{1}{2}x - \frac{2}{3}$
d $y = -3x$
e $y = \frac{6}{7}x + \frac{7}{5}$
f $y = 2 - 7x$
g $3x - 4y + 8 = 0$
h $4x - 5y - 10 = 0$
i $-2x + y - 9 = 0$
j $7x + 4y + 12 = 0$
k $7x - 2y + 3 = 0$
l $-5x + 4y + 2 = 0$

3 Write these lines in the form $ax + by + c = 0$.

a $y = 4x + 3$
b $y = 3x - 2$
c $y = -6x + 7$
d $y = \frac{4}{5}x - 6$
e $y = \frac{5}{3}x + 2$
f $y = \frac{7}{3}x$
g $y = 2x - \frac{4}{7}$
h $y = -3x + \frac{2}{9}$
i $y = -6x - \frac{2}{3}$
j $y = -\frac{1}{3}x + \frac{1}{2}$
k $y = \frac{2}{3}x + \frac{5}{6}$
l $y = \frac{3}{5}x + \frac{1}{2}$

4 The line $y = 6x - 18$ meets the x-axis at the point P. Work out the coordinates of P.

5 The line $3x + 2y = 0$ meets the x-axis at the point R. Work out the coordinates of R.

6 The line $5x - 4y + 20 = 0$ meets the y-axis at the point A and the x-axis at the point B. Work out the coordinates of A and B.

7 A line l passes through the points with coordinates $(0, 5)$ and $(6, 7)$.
   a Find the gradient of the line.
   b Find an equation of the line in the form $ax + by + c = 0$.

(E) 8 A line l cuts the x-axis at $(5, 0)$ and the y-axis at $(0, 2)$.
   a Find the gradient of the line. **(1 mark)**
   b Find an equation of the line in the form $ax + by + c = 0$. **(2 marks)**

(P) 9 Show that the line with equation $ax + by + c = 0$ has gradient $-\frac{a}{b}$ and cuts the y-axis at $-\frac{c}{b}$

Problem-solving

Try solving a similar problem with numbers first:

Find the gradient and y-intercept of the straight line with equation $3x + 7y + 2 = 0$.

(E/P) 10 The line l with gradient 3 and y-intercept $(0, 5)$ has the equation $ax - 2y + c = 0$. Find the values of a and c. **(2 marks)**

(E/P) 11 The straight line l passes through $(0, 6)$ and has gradient -2. It intersects the line with equation $5x - 8y - 15 = 0$ at point P. Find the coordinates of P. **(4 marks)**

(E/P) 12 The straight line l_1 with equation $y = 3x - 7$ intersects the straight line l_2 with equation $ax + 4y - 17 = 0$ at the point $P(-3, b)$.
   a Find the value of b. **(1 mark)**
   b Find the value of a. **(2 marks)**

Challenge

Show that the equation of a straight line through $(0, a)$ and $(b, 0)$ is $ax + by - ab = 0$.

Summary

Summary of key points

1. The gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
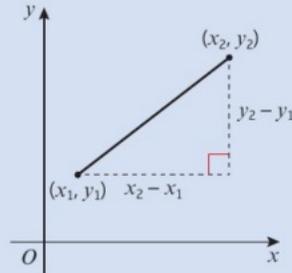
2. • The equation of a straight line can be written in the form

$$y = mx + c,$$

where m is the gradient and $(0, c)$ is the y-intercept.

• The equation of a straight line can also be written in the form

$$ax + by + c = 0,$$

where a, b and c are integers.
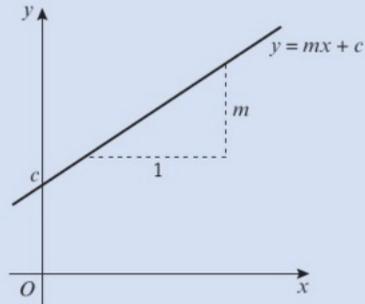
7 Algebraic Methods 1

7.1 Algebraic Fractions	https://youtu.be/zkf1lO5EqXA?si=PgGllqb-zo9aDnp4	

7.1 Algebraic Fractions

Worked Example

Example **1**

Simplify these fractions:

a $\dfrac{7x^4 - 2x^3 + 6x}{x}$ **b** $\dfrac{(x + 7)(2x - 1)}{(2x - 1)}$ **c** $\dfrac{x^2 + 7x + 12}{(x + 3)}$ **d** $\dfrac{x^2 + 6x + 5}{x^2 + 3x - 10}$ **e** $\dfrac{2x^2 + 11x + 12}{(x + 3)(x + 4)}$

1 Simplify these fractions:

a $\dfrac{4x^4 + 5x^2 - 7x}{x}$
b $\dfrac{7x^5 - 5x^5 + 9x^3 + x^2}{x}$
c $\dfrac{-x^4 + 4x^2 + 6}{x}$

d $\dfrac{7x^5 - x^3 - 4}{x}$
e $\dfrac{8x^4 - 4x^3 + 6x}{2x}$
f $\dfrac{9x^2 - 12x^3 - 3x}{3x}$

g $\dfrac{7x^3 - x^4 - 2}{5x}$
h $\dfrac{-4x^2 + 6x^4 - 2x}{-2x}$
i $\dfrac{-x^8 + 9x^4 - 4x^3 + 6}{-2x}$

j $\dfrac{-9x^9 - 6x^6 + 4x^4 - 2}{-3x}$

2 Simplify these fractions as far as possible:

a $\dfrac{(x + 3)(x - 2)}{(x - 2)}$
b $\dfrac{(x + 4)(3x - 1)}{(3x - 1)}$
c $\dfrac{(x + 3)^2}{(x + 3)}$

d $\dfrac{x^2 + 10x + 21}{(x + 3)}$
e $\dfrac{x^2 + 9x + 20}{(x + 4)}$
f $\dfrac{x^2 + x - 12}{(x - 3)}$

g $\dfrac{x^2 + x - 20}{x^2 + 2x - 15}$
h $\dfrac{x^2 + 3x + 2}{x^2 + 5x + 4}$
i $\dfrac{x^2 + x - 12}{x^2 - 9x + 18}$

j $\dfrac{2x^2 + 7x + 6}{(x - 5)(x + 2)}$
k $\dfrac{2x^2 + 9x - 18}{(x + 6)(x + 1)}$
l $\dfrac{3x^2 - 7x + 2}{(3x - 1)(x + 2)}$

m $\dfrac{2x^2 + 3x + 1}{x^2 - x - 2}$
n $\dfrac{x^2 + 6x + 8}{3x^2 + 7x + 2}$
o $\dfrac{2x^2 - 5x - 3}{2x^2 - 9x + 9}$

(E/P) **3** $\dfrac{6x^3 + 3x^2 - 84x}{6x^2 - 33x + 42} = \dfrac{ax(x + b)}{x + c}$, where a, b and c are constants.

Work out the values of a, b and c. **(4 marks)**

Summary

Summary of key points

1. When simplifying an algebraic fraction, factorise the numerator and denominator where possible and then cancel common factors.

9 Trigonometric Ratios

9.1 The Cosine Rule	https://youtu.be/LBvY4APYguo?si=roV11p-XjoQDBZl8	
9.2 The Sine Rule	https://youtu.be/yZqpDMZ1EsU?si=k81MU8bXoQ8MSJJk	
9.3 Areas of Triangles	https://youtu.be/fphNJIonm1I?si=zBvvpW9TKWNf6yJ8	

Worked Example

Example 1

Calculate the length of the side AB of the triangle ABC in which $AC = 6.5\,\text{cm}$, $BC = 8.7\,\text{cm}$ and $\angle ACB = 100°$.

Worked Example

Find the size of the smallest angle in a triangle whose sides have lengths 3 cm, 5 cm and 6 cm.

Worked Example

Example ③

Coastguard station B is 8 km, on a bearing of 060°, from coastguard station A. A ship C is 4.8 km, on a bearing of 018°, away from A. Calculate how far C is from B.

Worked Example

Example **4**

In $\triangle ABC$, $AB = x$ cm, $BC = (x + 2)$ cm, $AC = 5$ cm and $\angle ABC = 60°$.
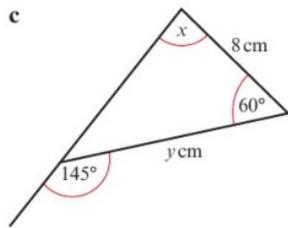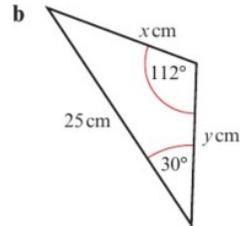Find the value of x.

Exercise 9A

Exercise 9A

Give answers to 3 significant figures, where appropriate.

1 In each of the following triangles calculate the length of the missing side.

a

b

c

d

e

f
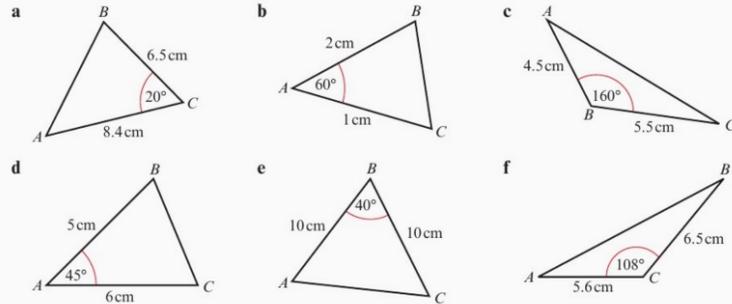
2 In the following triangles calculate the size of the angle marked x:

a

b

c

d

e

f
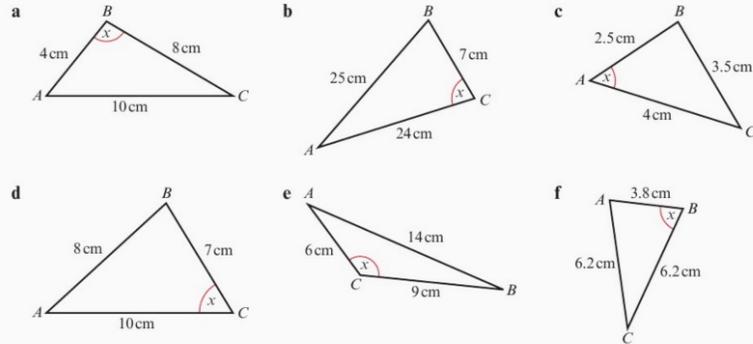
3 A plane flies from airport A on a bearing of 040° for 120 km and then on a bearing of 130° for 150 km. Calculate the distance of the plane from the airport.
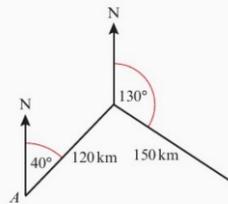
4 From a point A a boat sails due north for 7 km to B. The boat leaves B and moves on a bearing of 100° for 10 km until it reaches C. Calculate the distance of C from A.

5 A helicopter flies on a bearing of 080° from A to B, where $AB = 50$ km. It then flies for 60 km to a point C. Given that C is 80 km from A, calculate the bearing of C from A.

6 The distance from the tee, T, to the flag, F, on a particular hole on a golf course is 494 yards. A golfer's tee shot travels 220 yards and lands at the point S, where $\angle STF = 22°$. Calculate how far the ball is from the flag.

(P) 7 Show that $\cos A = \frac{1}{8}$
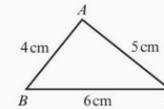
(P) 8 Show that $\cos P = -\frac{1}{4}$
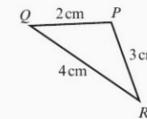
9 In $\triangle ABC$, $AB = 5$ cm, $BC = 6$ cm and $AC = 10$ cm. Calculate the size of the smallest angle.

10 In $\triangle ABC$, $AB = 9.3$ cm, $BC = 6.2$ cm and $AC = 12.7$ cm. Calculate the size of the largest angle.
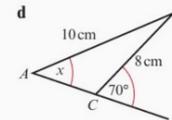
(P) 11 The lengths of the sides of a triangle are in the ratio $2:3:4$. Calculate the size of the largest angle.

12 In $\triangle ABC$, $AB = (x - 3)$ cm, $BC = (x + 3)$ cm, $AC = 8$ cm and $\angle BAC = 60°$. Use the cosine rule to find the value of x.

(P) 13 In $\triangle ABC$, $AB = x$ cm, $BC = (x - 4)$ cm, $AC = 10$ cm and $\angle BAC = 60°$. Calculate the value of x.

(P) 14 In $\triangle ABC$, $AB = (5 - x)$ cm, $BC = (4 + x)$ cm, $\angle ABC = 120°$ and $AC = y$ cm.
  a Show that $y^2 = x^2 - x + 61$.
  b Use the method of completing the square to find the minimum value of y^2, and give the value of x for which this occurs.

P **15** In $\triangle ABC$, $AB = x$ cm, $BC = 5$ cm, $AC = (10 - x)$ cm.

    a Show that $\cos \angle ABC = \dfrac{4x - 15}{2x}$

    b Given that $\cos \angle ABC = -\frac{1}{7}$, work out the value of x.

P **16** A farmer has a field in the shape of a quadrilateral as shown.
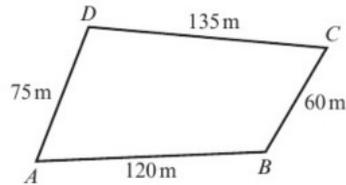
Problem-solving

You will have to use the cosine rule twice. Copy the diagram and write any angles or lengths you work out on your copy.

The angle between fences AB and AD is 74°. Find the angle between fences BC and CD.

E/P **17** The diagram shows three cargo ships, A, B and C, which are in the same horizontal plane. Ship B is 50 km due north of ship A and ship C is 70 km from ship A. The bearing of C from A is 020°.

    a Calculate the distance between ships B and C, in kilometres to 3 s.f.   **(3 marks)**

    b Calculate the bearing of ship C from ship B.   **(4 marks)**
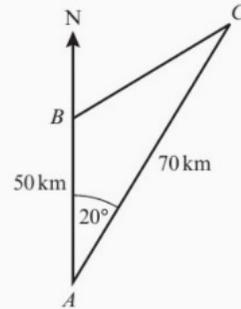
Worked Example

Example 5

In $\triangle ABC$, $AB = 8\,\text{cm}$, $\angle BAC = 30°$ and $\angle BCA = 40°$. Find BC.

Worked Example

In $\triangle ABC$, $AB = 3.8$ cm, $BC = 5.2$ cm and $\angle BAC = 35°$. Find $\angle ABC$.

Exercise 9B

Give answers to 3 significant figures, where appropriate.

1 In each of parts **a** to **d**, the given values refer to the general triangle.

a Given that $a = 8$ cm, $A = 30°$, $B = 72°$, find b.

b Given that $a = 24$ cm, $A = 110°$, $C = 22°$, find c.

c Given that $b = 14.7$ cm, $A = 30°$, $C = 95°$, find a.

d Given that $c = 9.8$ cm, $B = 68.4°$, $C = 83.7°$, find a.
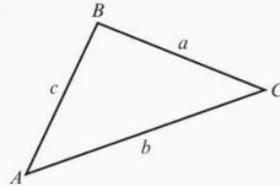
2 In each of the following triangles calculate the values of x and y.

a

b

c

d
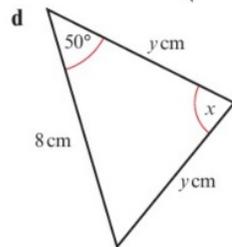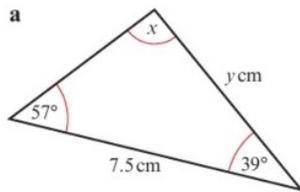
Hint In parts **c** and **d**, start by finding the size of the third angle.

e

f
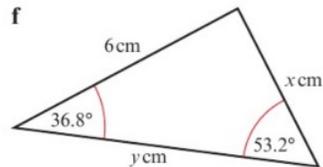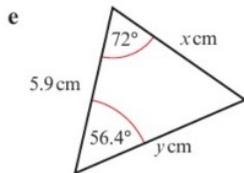
3 In each of the following sets of data for a triangle ABC, find the value of x.

a $AB = 6$ cm, $BC = 9$ cm, $\angle BAC = 117°$, $\angle ACB = x$

b $AC = 11$ cm, $BC = 10$ cm, $\angle ABC = 40°$, $\angle CAB = x$

c $AB = 6$ cm, $BC = 8$ cm, $\angle BAC = 60°$, $\angle ACB = x$

d $AB = 8.7$ cm, $AC = 10.8$ cm, $\angle ABC = 28°$, $\angle BAC = x$
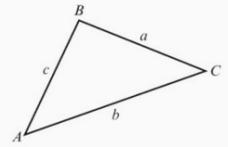
4 In each of the diagrams shown below, work out the size of angle x.

a 7.2 cm, 5.8 cm, 67.5°

b 6.2 cm, 4.5 cm, 80°

c $3\sqrt{2}$ cm, $\sqrt{2}$ cm, 50°

d 10 cm, 8 cm, 70°

e 7.9 cm, 55°, 10.4 cm

f 12.4 cm, 9.7 cm, 60°
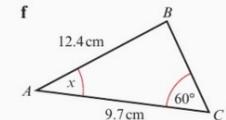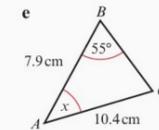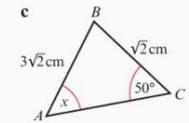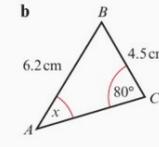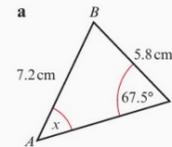
5 In $\triangle PQR$, $QR = \sqrt{3}$ cm, $\angle PQR = 45°$ and $\angle QPR = 60°$. Find **a** PR and **b** PQ.

6 In $\triangle PQR$, $PQ = 15$ cm, $QR = 12$ cm and $\angle PRQ = 75°$. Find the two remaining angles.

7 In each of the following diagrams work out the values of x and y.

a 5.5 cm, 3.9 cm, 75°

b 10.8 cm, 8.5 cm, 45°

c 7 cm, 20°, x cm, 100°, 12.2 cm

d 10°, 6 cm, 78°, 5 cm

e 6.4 cm, 7 cm, 24°, 60°

f 7.5 cm, 80°, 6.2 cm, 8.5 cm
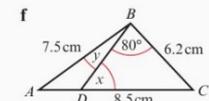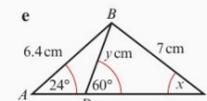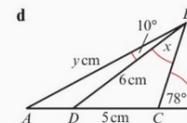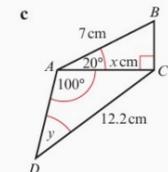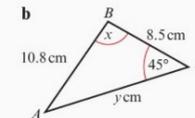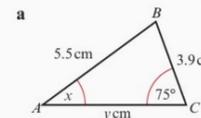
8 Town B is 6 km, on a bearing of 020°, from town A. Town C is located on a bearing of 055° from town A and on a bearing of 120° from town B. Work out the distance of town C from:
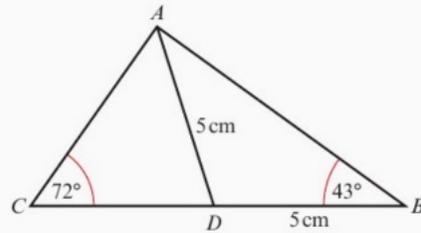
a town A

b town B

Problem-solving

Draw a sketch to show the information.

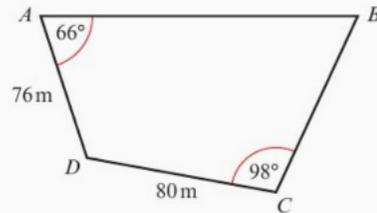9 In the diagram $AD = DB = 5$ cm, $\angle ABC = 43°$ and $\angle ACB = 72°$.
Calculate:
a AB
b CD

10 A zookeeper is building an enclosure for some llamas. The enclosure is in the shape of a quadrilateral as shown.

If the length of the diagonal BD is 136 m

a find the angle between the fences AB and BC

b find the length of fence AB

(E/P) 11 In $\triangle ABC$, $AB = x$ cm, $BC = (4 - x)$ cm, $\angle BAC = y$ and $\angle BCA = 30°$.

Given that $\sin y = \dfrac{1}{\sqrt{2}}$, show that

$x = 4(\sqrt{2} - 1)$ **(5 marks)**

Problem-solving

You can use the value of $\sin y$ directly in your calculation. You don't need to work out the value of y.

(E/P) 12 A surveyor wants to determine the height of a building. She measures the angle of elevation of the top of the building at two points 15 m apart on the ground.

a Use this information to determine the height of the building. **(4 marks)**

b State one assumption made by the surveyor in using this mathematical model. **(1 mark)**
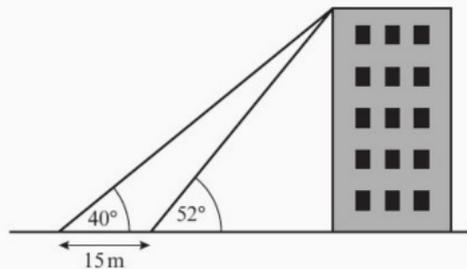
Example 7

In $\triangle ABC$, $AB = 4\,\text{cm}$, $AC = 3\,\text{cm}$ and $\angle ABC = 44°$. Work out the two possible values of $\angle ACB$.

Exercise **9C**

Give answers to 3 significant figures, where appropriate.

1 In $\triangle ABC$, $BC = 6\,\text{cm}$, $AC = 4.5\,\text{cm}$ and $\angle ABC = 45°$.
  a Calculate the two possible values of $\angle BAC$.
  b Draw a diagram to illustrate your answers.

2 In each of the diagrams shown below, calculate the possible values of x and the corresponding values of y.

  a

  b

  c
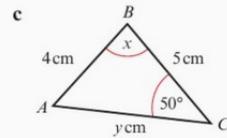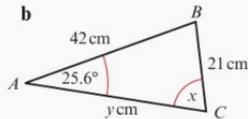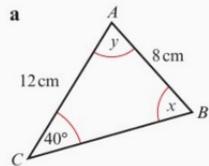
(P) **3** In each of the following cases $\triangle ABC$ has $\angle ABC = 30°$ and $AB = 10\,\text{cm}$.
  a Calculate the least possible length that AC could be.
  b Given that $AC = 12\,\text{cm}$, calculate $\angle ACB$.
  c Given instead that $AC = 7\,\text{cm}$, calculate the two possible values of $\angle ACB$.

(P) **4** Triangle ABC is such that $AB = 4\,\text{cm}$, $BC = 6\,\text{cm}$ and $\angle ACB = 36°$. Show that one of the possible values of $\angle ABC$ is $25.8°$ (to 3 s.f.). Using this value, calculate the length of AC.

(P) **5** Two triangles ABC are such that $AB = 4.5\,\text{cm}$, $BC = 6.8\,\text{cm}$ and $\angle ACB = 30°$. Work out the value of the largest angle in each of the triangles.

(E/P) **6** **a** A crane arm AB of length 80 m is anchored at point B at an angle of 40° to the horizontal. A wrecking ball is suspended on a cable of length 60 m from A. Find the angle x through which the wrecking ball rotates as it passes the two points level with the base of the crane arm at B. **(6 marks)**
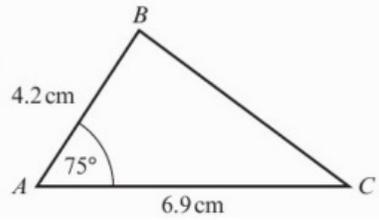 **b** Write down one modelling assumption you have made. **(1 mark)**
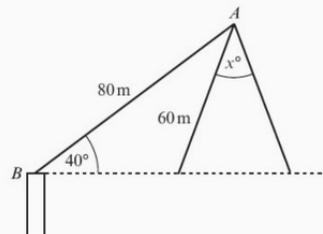
Worked Example

Example **8**

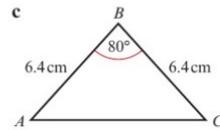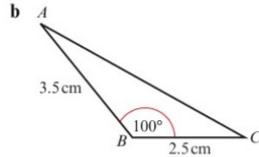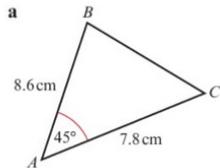Work out the area of the triangle shown below.

Worked Example

Example **9**

In $\triangle ABC$, $AB = 5$ cm, $BC = 6$ cm and $\angle ABC = x$. Given that the area of $\triangle ABC$ is 12 cm^2 and that AC is the longest side, find the value of x.
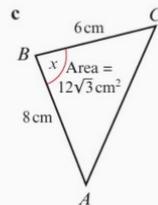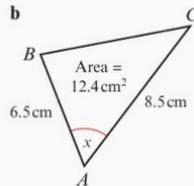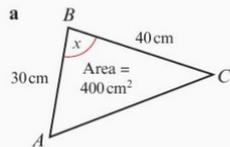
Exercise 9D

Exercise 9D

1 Calculate the area of each triangle.

a

8.6 cm, 7.8 cm, 45°, B, C, A

b

3.5 cm, 2.5 cm, 100°, A, B, C

c

6.4 cm, 6.4 cm, 80°, A, B, C

2 Work out the possible sizes of x in the following triangles.

a

40 cm, 30 cm, x, Area = 400 cm^2, A, B, C

b

6.5 cm, 8.5 cm, x, Area = 12.4 cm^2, A, B, C

c

6 cm, 8 cm, x, Area = 12$\sqrt{3}$ cm^2, A, B, C

3 A fenced triangular plot of ground has area 1200 m^2. The fences along the two smaller sides are 60 m and 80 m respectively and the angle between them is θ. Show that $\theta = 150°$, and work out the total length of fencing.

Ⓟ **4** In triangle ABC, $BC = (x + 2)$ cm, $AC = x$ cm and $\angle BCA = 150°$. Given that the area of the triangle is 5 cm^2, work out the value of x, giving your answer to 3 significant figures.

x cm, 150°, $(x + 2)$ cm, A, B, C
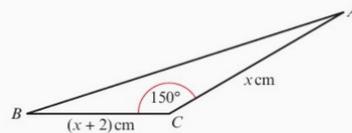
Ⓔ/Ⓟ **5** In $\triangle PQR$, $PQ = (x + 2)$ cm, $PR = (5 - x)$ cm and $\angle QPR = 30°$. The area of the triangle is A cm^2.

   a Show that $A = \frac{1}{4}(10 + 3x - x^2)$. **(3 marks)**

   b Use the method of completing the square, or otherwise, to find the maximum value of A, and give the corresponding value of x. **(4 marks)**

Ⓔ/Ⓟ **6** In $\triangle ABC$, $AB = x$ cm, $AC = (5 + x)$ cm and $\angle BAC = 150°$. Given that the area of the triangle is $3\frac{3}{4}$ cm^2

   Problem-solving

   x represents a length so it must be positive.

   a Show that x satisfies the equation $x^2 + 5x - 15 = 0$. **(3 marks)**

   b Calculate the value of x, giving your answer to 3 significant figures. **(3 marks)**

11 Vectors
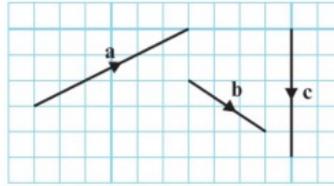
| 11.1 Vectors | https://youtu.be/Oao432GabNQ?si=0VZxY688r1LTz9RC | |

Worked Example

The diagram shows vectors **a**, **b** and **c**.

Draw a diagram to illustrate the vector addition
a + **b** + **c**.

Worked Example

In the diagram, $\overrightarrow{QP} = \mathbf{a}$, $\overrightarrow{QR} = \mathbf{b}$, $\overrightarrow{QS} = \mathbf{c}$ and $\overrightarrow{RT} = \mathbf{d}$.
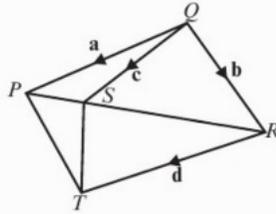
Find in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$:

a $\overrightarrow{PS}$ **b** $\overrightarrow{RP}$

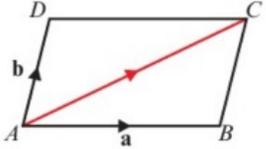**c** $\overrightarrow{PT}$ **d** $\overrightarrow{TS}$

Worked Example

Example 3

$ABCD$ is a parallelogram. $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AD} = \mathbf{b}$. Find $\overrightarrow{AC}$.

Worked Example

Show that the vectors $6\mathbf{a} + 8\mathbf{b}$ and $9\mathbf{a} + 12\mathbf{b}$ are parallel.

Worked Example

Example 5

In triangle ABC, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$.

P is the midpoint of AB.
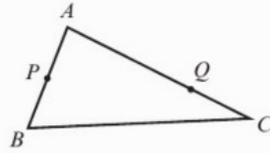
Q divides AC in the ratio $3:2$.

Write in terms of $\mathbf{a}$ and $\mathbf{b}$:

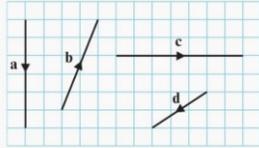a $\overrightarrow{BC}$ b $\overrightarrow{AP}$ c $\overrightarrow{AQ}$ d $\overrightarrow{PQ}$

Exercise 11A

Exercise **11A**

1 The diagram shows the vectors **a**, **b**, **c** and **d**.

Draw a diagram to illustrate these vectors:

a $a + c$

b $-b$

c $c - d$

d $b + c + d$

e $2c + 3d$

f $a - 2b$

g $a + b + c + d$

2 $ACGI$ is a square, B is the midpoint of AC, F is the midpoint of CG, H is the midpoint of GI, D is the midpoint of AI.

$\overrightarrow{AB} = \mathbf{b}$ and $\overrightarrow{AD} = \mathbf{d}$. Find, in terms of **b** and **d**:

a $\overrightarrow{AC}$ b $\overrightarrow{BE}$ c $\overrightarrow{HG}$ d $\overrightarrow{DF}$

e $\overrightarrow{AE}$ f $\overrightarrow{DH}$ g $\overrightarrow{HB}$ h $\overrightarrow{FE}$

i $\overrightarrow{AH}$ j $\overrightarrow{BI}$ k $\overrightarrow{EI}$ l $\overrightarrow{FB}$
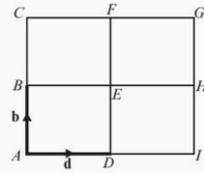
3 $OACB$ is a parallelogram. M, Q, N and P are the midpoints of OA, AC, BC and OB respectively.

Vectors **p** and **m** are equal to $\overrightarrow{OP}$ and $\overrightarrow{OM}$ respectively. Express in terms of **p** and **m**.

a $\overrightarrow{OA}$ b $\overrightarrow{OB}$ c $\overrightarrow{BN}$ d $\overrightarrow{DQ}$

e $\overrightarrow{OD}$ f $\overrightarrow{MQ}$ g $\overrightarrow{OQ}$ h $\overrightarrow{AD}$

i $\overrightarrow{CD}$ j $\overrightarrow{AP}$ k $\overrightarrow{BM}$ l $\overrightarrow{NO}$
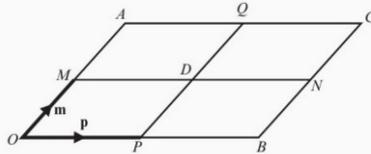
4 In the diagram, $\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{QS} = \mathbf{b}$, $\overrightarrow{SR} = \mathbf{c}$ and $\overrightarrow{PT} = \mathbf{d}$.

Find in terms of **a**, **b**, **c** and **d**:

a $\overrightarrow{QT}$ b $\overrightarrow{PR}$

c $\overrightarrow{TS}$ d $\overrightarrow{TR}$
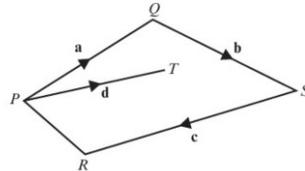
5 In the triangle PQR, $PQ = 2\mathbf{a}$ and $QR = 2\mathbf{b}$.

The midpoint of PR is M. Find, in terms of **a** and **b**:

a $\overrightarrow{PR}$ b $\overrightarrow{PM}$ c $\overrightarrow{QM}$

(P) **6** $ABCD$ is a trapezium with AB parallel to DC and $DC = 3AB$.

M divides DC such that $DM : MC = 2 : 1$. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.

Find, in terms of **a** and **b**:

a $\overrightarrow{AM}$ b $\overrightarrow{BD}$ c $\overrightarrow{MB}$ d $\overrightarrow{DA}$

Problem-solving

Draw a sketch to show the information given in the question.

7 $OABC$ is a parallelogram. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$.

The point P divides OB in the ratio 5:3.

Find, in terms of **a** and **b**:

a $\overrightarrow{OB}$ b $\overrightarrow{OP}$ c $\overrightarrow{AP}$
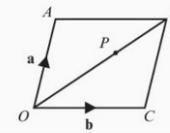
8 State with a reason whether each of these vectors is parallel to the vector $\mathbf{a} - 3\mathbf{b}$:

a $2\mathbf{a} - 6\mathbf{b}$ b $4\mathbf{a} - 12\mathbf{b}$ c $\mathbf{a} + 3\mathbf{b}$ d $3\mathbf{b} - \mathbf{a}$ e $9\mathbf{b} - 3\mathbf{a}$ f $\frac{1}{2}\mathbf{a} - \frac{2}{3}\mathbf{b}$

(P) **9** In triangle ABC, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$.

P is the midpoint of AB and Q is the midpoint of AC.

a Write in terms of **a** and **b**:

i $\overrightarrow{BC}$ ii $\overrightarrow{AP}$ iii $\overrightarrow{AQ}$ iv $\overrightarrow{PQ}$

b Show that PQ is parallel to BC.
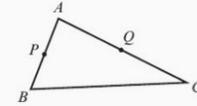
(P) **10** $OABC$ is a quadrilateral. $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OC} = 3\mathbf{b}$ and $\overrightarrow{OB} = \mathbf{a} + 2\mathbf{b}$.

a Find, in terms of **a** and **b**:

i $\overrightarrow{AB}$ ii $\overrightarrow{CB}$

b Show that AB is parallel to OC.
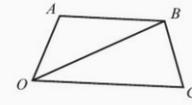
(P) **11** The vectors $2\mathbf{a} + k\mathbf{b}$ and $5\mathbf{a} + 3\mathbf{b}$ are parallel. Find the value of k.

Summary

Summary of key points

1. If $\overrightarrow{PQ} = \overrightarrow{RS}$ then the line segments PQ and RS are equal in length and are parallel.

2. $\overrightarrow{AB} = -\overrightarrow{BA}$ as the line segment AB is equal in length, parallel and in the opposite direction to BA.

3. **Triangle law for vector addition:** $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

   If $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{BC} = \mathbf{b}$ and $\overrightarrow{AC} = \mathbf{c}$, then $\mathbf{a} + \mathbf{b} = \mathbf{c}$

4. Subtracting a vector is equivalent to 'adding a negative vector': $\mathbf{a} - \mathbf{b} = \mathbf{a} + (-\mathbf{b})$

5. Adding the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QP}$ gives the zero vector $\mathbf{0}$: $\overrightarrow{PQ} + \overrightarrow{QP} = \mathbf{0}$.

6. Any vector parallel to the vector $\mathbf{a}$ may be written as $\lambda\mathbf{a}$, where λ is a non-zero scalar.

1 Algebraic Methods 2

1.2 Algebraic Fractions	https://youtu.be/cZHi5UmvRCY	

Worked Example

Example 5

Simplify the following products:

a $\dfrac{3}{5} \times \dfrac{5}{9}$

b $\dfrac{a}{b} \times \dfrac{c}{a}$

c $\dfrac{x+1}{2} \times \dfrac{3}{x^2-1}$

Worked Example

Example 6

Simplify:

a $\dfrac{a}{b} \div \dfrac{a}{c}$

b $\dfrac{x+2}{x+4} \div \dfrac{3x+6}{x^2-16}$

Exercise 1B

1 Simplify:

a $\dfrac{a}{d} \times \dfrac{a}{c}$ **b** $\dfrac{a^2}{c} \times \dfrac{c}{a}$ **c** $\dfrac{2}{x} \times \dfrac{x}{4}$ **d** $\dfrac{3}{x} \div \dfrac{6}{x}$ **e** $\dfrac{4}{xy} \div \dfrac{x}{y}$ **f** $\dfrac{2r^2}{5} \div \dfrac{4}{r^3}$

2 Simplify:

a $(x + 2) \times \dfrac{1}{x^2 - 4}$

b $\dfrac{1}{a^2 + 6a + 9} \times \dfrac{a^2 - 9}{2}$

c $\dfrac{x^2 - 3x}{y^2 + y} \times \dfrac{y + 1}{x}$

d $\dfrac{y}{y + 3} \div \dfrac{y^2}{y^2 + 4y + 3}$

e $\dfrac{x^2}{3} \div \dfrac{2x^3 - 6x^2}{x^2 - 3x}$

f $\dfrac{4x^2 - 25}{4x - 10} \div \dfrac{2x + 5}{8}$

g $\dfrac{x + 3}{x^2 + 10x + 25} \times \dfrac{x^2 + 5x}{x^2 + 3x}$

h $\dfrac{3y^2 + 4y - 4}{10} \div \dfrac{3y + 6}{15}$

i $\dfrac{x^2 + 2xy + y^2}{2} \times \dfrac{4}{(x - y)^2}$

(E/P) 3 Show that $\dfrac{x^2 - 64}{x^2 - 36} \div \dfrac{64 - x^2}{x^2 - 36} = -1$ **(4 marks)**

(E/P) 4 Show that $\dfrac{2x^2 - 11x - 40}{x^2 - 4x - 32} \times \dfrac{x^2 + 8x + 16}{6x^2 - 3x - 45} \div \dfrac{8x^2 + 20x - 48}{10x^2 - 45x + 45} = \dfrac{a}{b}$ and find the values

of the constants a and b, where a and b are integers.   **(4 marks)**

(E/P) 5 a Simplify fully $\dfrac{x^2 + 2x - 24}{2x^2 + 10x} \times \dfrac{x^2 - 3x}{x^2 + 3x - 18}$ **(3 marks)**

(E/P) 6 $f(x) = \dfrac{2x^2 - 3x - 2}{6x - 8} \div \dfrac{x - 2}{3x^2 + 14x - 24}$

     a Show that $f(x) = \dfrac{2x^2 + 13x + 6}{2}$ **(4 marks)**

Worked Example

Example 7

Simplify the following:

a $\dfrac{1}{3} + \dfrac{3}{4}$

b $\dfrac{a}{2x} + \dfrac{b}{3x}$

c $\dfrac{2}{x+3} - \dfrac{1}{x+1}$

d $\dfrac{3}{x+1} - \dfrac{4x}{x^2-1}$

Exercise 1C

1 Write as a single fraction:

a $\dfrac{1}{3}+\dfrac{1}{4}$ **b** $\dfrac{3}{4}-\dfrac{2}{5}$ **c** $\dfrac{1}{p}+\dfrac{1}{q}$ **d** $\dfrac{3}{4x}+\dfrac{1}{8x}$ **e** $\dfrac{3}{x^2}-\dfrac{1}{x}$ **f** $\dfrac{a}{5b}-\dfrac{3}{2b}$

2 Write as a single fraction:

a $\dfrac{3}{x}-\dfrac{2}{x+1}$ **b** $\dfrac{2}{x-1}-\dfrac{3}{x+2}$ **c** $\dfrac{4}{2x+1}+\dfrac{2}{x-1}$

d $\dfrac{1}{3}(x+2)-\dfrac{1}{2}(x+3)$ **e** $\dfrac{3x}{(x+4)^2}-\dfrac{1}{x+4}$ **f** $\dfrac{5}{2(x+3)}+\dfrac{4}{3(x-1)}$

3 Write as a single fraction:

a $\dfrac{2}{x^2+2x+1}+\dfrac{1}{x+1}$ **b** $\dfrac{7}{x^2-4}+\dfrac{3}{x+2}$ **c** $\dfrac{2}{x^2+6x+9}-\dfrac{3}{x^2+4x+3}$

d $\dfrac{2}{y^2-x^2}+\dfrac{3}{y-x}$ **e** $\dfrac{3}{x^2+3x+2}-\dfrac{1}{x^2+4x+4}$ **f** $\dfrac{x+2}{x^2-x-12}-\dfrac{x+1}{x^2+5x+6}$

(E) **4** Express $\dfrac{6x+1}{x^2+2x-15}-\dfrac{4}{x-3}$ as a single fraction in its simplest form. **(4 marks)**

5 Express each of the following as a fraction in its simplest form.

a $\dfrac{3}{x}+\dfrac{2}{x+1}+\dfrac{1}{x+2}$ **b** $\dfrac{4}{3x}-\dfrac{2}{x-2}+\dfrac{1}{2x+1}$ **c** $\dfrac{3}{x-1}+\dfrac{2}{x+1}+\dfrac{4}{x-3}$

(E) **6** Express $\dfrac{4(2x-1)}{36x^2-1}+\dfrac{7}{6x-1}$ as a single fraction in its simplest form. **(4 marks)**

(E/P) **7** $g(x)=x+\dfrac{6}{x+2}+\dfrac{36}{x^2-2x-8},\ x\in\mathbb{R},\ x\neq-2,\ x\neq4$

  a Show that $g(x)=\dfrac{x^3-2x^2-2x+12}{(x+2)(x-4)}$ **(3 marks)**

Summary

3 To multiply fractions, cancel any common factors, then multiply the numerators and multiply the denominators.

4 To divide two fractions, multiply the first fraction by the reciprocal of the second fraction.

5 To add or subtract two fractions, find a common denominator.