Pearson Edexcel

# Mark Scheme (Results)

## Summer 2024

Pearson Edexcel GCE

In Mathematics (9MA0)

Paper 01 Pure Mathematics

Summer 2024
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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

**EDEXCEL GCE MATHEMATICS**

**General Instructions for Marking**

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

## General Principles for Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

**<u>Method mark for solving 3 term quadratic:</u>**

**1. Factorisation**

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to $x=\ldots$

**2. Formula**

Attempt to use the correct formula (with values for *a*, *b* and *c*)

**3. Completing the square**

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$, leading to $x=\ldots$

**<u>Method marks for differentiation and integration:</u>**

**1. Differentiation**

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

**2. Integration**

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

**<u>Use of a formula</u>**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

**<u>Exact answers</u>**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1** | $\text{g}(3)=3(3)^3-20(3)^2+3(k+17)+k=0$ | M1 | 3.1a |
| | $4k-48=0 \Rightarrow k=...$ | M1 | 1.1b |
| | $\{k=\}12$ | A1 | 1.1b |
| | | **(3)** | |
| | | | **(3 marks)** |

**Notes**

**Note:** Ignore any use of $\text{f}(x)$ in place of $\text{g}(x)$ throughout.

**M1:** Attempts $\text{g}(3)=0$ to set up a linear equation in *k*. The = 0 may implied by their value of *k*.

Expect to see 3 substituted for *x* at least twice but condone minor slips copying the function.
May be scored for e.g. 81 – 180 + 3(*k* + 17) + *k* = 0
Missing brackets may be recovered.
Attempting $\text{g}(-3)=0$ scores M0 but note that the second M1 is available.

If algebraic division is attempted, they need to achieve a linear remainder in *k* only and set =0 Condone slips in their calculations.
As a minimum, expect to see $3x^2+\lambda x,\ \lambda\neq 0$ as their quotient **leading to a linear remainder in *k* only set = 0** (the = 0 may be implied by their value for *k*).
For reference, the correct division is

$$\begin{array}{r} 3x^2-11x+\ k-16 \\ x-3\overline{)3x^3-20x^2+(k+17)x+k} \\ \underline{3x^3-9x^2}\qquad\qquad\qquad \\ -11x^2+(k+17)x+k \\ \underline{-11x^2+33x}\qquad\quad \\ (k-16)x+k \\ \underline{(k-16)x-3k+48} \\ 4k-48=0 \end{array}$$

You may also see variations on the table below.
Here, the M1 is scored when the sum of both coefficients of *x* are equated to (*k* + 17)

| | $3x^2$ | $-11x$ | $-\frac{k}{3}$ |
|---|---|---|---|
| $x$ | $3x^3$ | $-11x^2$ | $-\frac{k}{3}x$ |
| $-3$ | $-9x^2$ | $33x$ | $k$ |

$33-\frac{k}{3}=k+17$ scores M1

**M1:** Scored for attempting to solve a linear equation in *k* having attempted g (±3) = 0
Do not be concerned about the process, e.g. $-81+180-3(k+17)+k=0\rightarrow k=...$ scores M1.
Via division they must have a linear remainder in *k* set = 0
The = 0 may be implied by their value for *k* in all approaches.

**A1:** Obtains $\{k=\}12$ only. Do not accept e.g. $\frac{48}{4}$ Allow slips in working to be recovered.
Condone e.g. $x=12$ provided it has come from a linear equation in *k*.
Note that e.g. $3(3)^3-20(3)^2+3(k+17)+k\{=0\}\rightarrow k=12$ and
$81-180+3k+51+k\{=0\}\rightarrow k=12$ are sufficient to imply M1M1A1.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)** | $\dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(\pm 9x)^2$ or $\dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-9x)^3$ | M1 | 1.1b |
| | $(1-9x)^{\frac{1}{2}}=1+\frac{1}{2}(-9x)+\dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}(\pm 9x)^2+\dfrac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}(-9x)^3$ | A1 | 1.1b |
| | $(1-9x)^{\frac{1}{2}}=1-\frac{9}{2}x-\frac{81}{8}x^2-\frac{729}{16}x^3$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | Expansion is valid for $\lvert x\rvert<\frac{1}{9}$ and $x=-\frac{2}{9}$ **is outside this range.** | B1 | 2.4 |
| | | **(1)** | |
| | | | **(4 marks)** |

**Notes**

(a)

**M1:** For an attempt at the binomial expansion with $n=\frac{1}{2}$ and obtains the correct structure for term 3 **or** term 4. Award for the correct coefficient with the correct power of $x$.

e.g. $\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(\lambda x)^2$ **or** $\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(\lambda x)^3$ where $\lambda\neq 1$

Condone missing or incorrect brackets around the $x$ terms but the binomial coefficients must be correct. Allow 2! and/or 3! or 2 and/or 6.

Do not allow notation such as $\begin{pmatrix}\frac{1}{2}\\1\end{pmatrix},\begin{pmatrix}\frac{1}{2}\\2\end{pmatrix}$ unless these are interpreted correctly.

**A1:** Correct unsimplified expression as shown but the bracketing must be correct unless any missing brackets are implied by subsequent work.
May be implied by a correct simplified expression.

**OR** allow this mark for at least 2 correct simplified terms from $-\frac{9}{2}x, -\frac{81}{8}x^2$ and $-\frac{729}{16}x^3$

**A1:** $1-\frac{9}{2}x-\frac{81}{8}x^2-\frac{729}{16}x^3$ or simplified equivalent. Correct answer with no working can score full marks. Ignore any extra terms and allow the terms to be listed or in a different order. Apply isw once a correct expansion is seen. Condone $+-$ (equivalent to listing).
Allow recovery if applicable e.g. if an "$x$" is lost then "reappears".
Allow decimal equivalents $1-4.5x-10.125x^2-45.5625x^3$ provided they are exact.
Allow mixed numbers $1-4\frac{1}{2}x-10\frac{1}{8}x^2-45\frac{9}{16}x^3$

**Note:** You may see attempts via direct expansion, but these will be scored using the main scheme, ignoring absence of powers on the 1s. The below attempts both score first M1A1.
If you are unsure, send to review.

$$(1-9x)^{\frac{1}{2}} = 1^{\frac{1}{2}} + \frac{1}{2}\left(1^{-\frac{1}{2}}\right)(-9x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(1^{-\frac{3}{2}}\right)(-9x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(1^{-\frac{5}{2}}\right)(-9x)^3$$

$$9^{\frac{1}{2}}\left(\frac{1}{9}-x\right)^{\frac{1}{2}} = 3\left[\left(\frac{1}{9}\right)^{\frac{1}{2}} + \frac{1}{2}\left(\left(\frac{1}{9}\right)^{-\frac{1}{2}}\right)(-x) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\left(\frac{1}{9}\right)^{-\frac{3}{2}}\right)(-x)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\left(\frac{1}{9}\right)^{-\frac{5}{2}}\right)(-x)^3\right]$$

(b)

**B1:** Expansion is valid for $|x| < \frac{1}{9}$ or $|x| \leqslant \frac{1}{9}$ and $x = -\frac{2}{9}$ **is outside this range.**

Requires:

- an acceptable range of validity given
- an acceptable comparison of $-\frac{2}{9}$ **or** $\frac{2}{9}$ with their range leading to e.g. "not valid".

Examples of acceptable alternatives include:

- (Valid for) $|9x| < 1$ or $|9x| \leqslant 1$ and as $9x = -2$ (the expansion is) not valid.
- (Valid for) $|x| < \frac{1}{9}$ or $|x| \leqslant \frac{1}{9}$ and as $\frac{2}{9} > \frac{1}{9}$ $\left(\text{or } -\frac{2}{9} < -\frac{1}{9}\right)$ the expansion is not valid.
- (Valid for) $-\frac{1}{9} < x < \frac{1}{9}$ and $x = -\frac{2}{9}$ is too small/big (condoned as minimally acceptable).
- (Series converges for) $|-9x| \leqslant 1$ and as $-9x = 2$ the series will diverge.
- (Valid for) $|x| < \frac{1}{9}$ but $\left|-\frac{2}{9}\right| > \frac{1}{9}$ so $-\frac{2}{9}$ cannot be used.

Do not accept vague statements such as "it is too big", "it is outside the range" without any mention of what the range is. $-\frac{2}{9} < -\frac{1}{9}$ alone is insufficient evidence (without any mention of what the range is) and scores B0.
An attempt to evaluate the expansion and compare with $\sqrt{3}$ is not acceptable on its own.

| Question | Scheme | Marks | AOs |
| --- | --- | --- | --- |
| 3(a) | $\{f(3.6)=\}\ 3.6+\tan\left(\frac{1}{2}(3.6)\right) = -0.686\ldots < 0$ **and** $\{f(3.7)=\}\ 3.7+\tan\left(\frac{1}{2}(3.7)\right) = 0.211\ldots > 0$ | M1 | 1.1b |
| | <u>Change of sign</u> and function is <u>continuous</u> in the interval $\Rightarrow$ <u>conclusion</u> e.g. "there is a root in [3.6, 3.7]" * | A1* | 2.4 |
| | | **(2)** | |
| **(b)** | Use of $\tan\left(\frac{1}{2}x\right) \to \ldots\sec^2\left(\frac{1}{2}x\right)$ | M1 | 1.1b |
| | $\{f'(x)=\}\ 1+\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ | A1 | 1.1b |
| | | **(2)** | |
| **(c)** | Attempts $3.7-\dfrac{3.7+\tan\left(\frac{1}{2}(3.7)\right)}{"1+\frac{1}{2}\sec^2\left(\frac{1}{2}(3.7)\right)"}=\ldots$ (N.B. $f(3.7)=0.211\ldots$ and $f'(3.7)=7.58\ldots$) | M1 | 1.1b |
| | $\alpha$ = awrt 3.672 | A1 | 1.1b |
| | | **(2)** | |
| | | **(6 marks)** | |

**Notes**

(a)

**M1:** Attempts both f(3.6) and f(3.7) or a narrower interval that contains the root 3.672… (which may be implied by sight of f(3.6) = … and f(3.7) = … with at least one correct) **and** obtains at least one correct to 1 significant figure (rounded or truncated) for their interval **and** considers their signs. Use of degrees is M0.
Some examples for consideration of sign (which are also sufficient for the change of sign part of reasoning for the A1):

- f(3.6) = –0.6 < 0 and f(3.7) = 0.2 > 0
- $f(3.6)\times f(3.7)<0$
- f(3.6) = –0.7, f(3.7) = 0.2 "change in sign"

For reference f(3.6) = –0.68626… and f(3.7) = 0.21194…

**A1*:** This mark requires:

- both f(3.6) and f(3.7) correct to 1 significant figure (rounded or truncated) (or their values correct to 1 significant figure if using a narrower interval)
- a reference to sign change
- a reference to continuity {of f($x$)}
- a (minimal) conclusion, e.g. "hence root", "proved", ✓, #, QED, $3.6<\alpha<3.7$

Accept as a minimum, "change of sign, continuous, root".
Do not condone "change in sign therefore continuous" or other incorrect statements such as "$x$ is continuous", "the interval is continuous" – these score A0.
Condone "the graph is continuous".
Condone reference to $x$ in place of $\alpha$ in their conclusion, e.g. "hence $x$ lies in the interval".
Condone statements such as "there is at least one root" in place of their conclusion.

| | |
|---|---|
| (b) | **Note:** Their answer to (b) may be seen in part (c) provided that they have not clearly attempted part (b) incorrectly, e.g., an attempt at $f^{-1}(x)$ in (b). |
| **M1:** | For $\tan\left(\frac{1}{2}x\right) \to ...\sec^2\left(\frac{1}{2}x\right)$ o.e. The brackets are not required. You may see attempts at the quotient rule but the method should be correct and they should reach something equivalent to $...\sec^2\left(\frac{1}{2}x\right)$.<br>e.g. $\tan\left(\frac{1}{2}x\right) = \dfrac{\sin\left(\frac{1}{2}x\right)}{\cos\left(\frac{1}{2}x\right)} \to \dfrac{k\cos\left(\frac{1}{2}x\right)\cos\left(\frac{1}{2}x\right) - -k\sin\left(\frac{1}{2}x\right)\sin\left(\frac{1}{2}x\right)}{\cos^2\left(\frac{1}{2}x\right)}$ where $k$ is a **positive** constant scores M1. If the formula is seen it must be correct. |
| **A1:** | $\{f'(x) =\}\ 1+\frac{1}{2}\sec^2\left(\frac{1}{2}x\right)$ o.e. which may be unsimplified and apply isw.<br>The brackets are not required. There is no need for $f'(x) =$ just look for the expression.<br>Note that $\{f'(x) =\}\ \frac{3}{2}+\frac{1}{2}\tan^2\left(\frac{1}{2}x\right)$ is correct and appears occasionally.<br>$\{f'(x) =\}\ 1+\frac{1}{2}\sec\frac{1}{2}x^2$ is condoned for M1A0 only but $1+\frac{1}{2}\left(\sec\frac{1}{2}x\right)^2$ scores M1A1. |
| (c) | |
| **M1:** | Attempts $3.7-\dfrac{f(3.7)}{f'(3.7)}$ and obtains a value following through on their f ′($x$) as long as it is a "changed" function in terms of $x$.<br>Just stating $3.7-\dfrac{f(3.7)}{f'(3.7)} = ...$ without evidence of use of 3.7 in f($x$) (note that this evidence might come from part (a)) **and** in their f ′($x$) is M0 unless implied by a correct value for both f($x$) **and** f ′($x$) **or** by their final answer.<br>Must be a correct N-R formula used – you may need to check their values – accuracy of at least 3s.f. rounded or truncated required.<br>Allow if attempted in degrees. For reference in degrees f (3.7) = 3.73… and f ′(3.7) = 1.50… and gives $\alpha$ = 1.21…<br>Note that the full N-R accuracy is 3.672051617.<br>For reference, the value of $\alpha$ is approximately 3.673194406… and scores M0A0 without other valid work. |
| **A1:** | For awrt 3.672 Ignore any subsequent iterations. |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 4 | $\frac{(x+h)^2-x^2}{h}=\frac{x^2+2xh+h^2-x^2}{h}$ | M1 | 2.1 |
| | $=\frac{2xh+h^2}{h}$ | A1 | 1.1b |
| | $\frac{dy}{dx}=\lim_{h\to 0}\frac{2xh+h^2}{h}=\lim_{h\to 0}(2x+h)=2x$ * | A1* | 2.5 |
| | | **(3)** | |
| | | **(3 marks)** | |

**Notes**

**Note:** Throughout the question allow use of $\delta x$ for $h$ or any other letter e.g. $\alpha$ if used consistently. If $\delta x$ is used then you can condone e.g. $\delta^2 x$ for $\delta x^2$ as well as condoning e.g. poorly formed δ's

**M1:** Begins the process by writing down the gradient of the chord and attempts to expand the correct squared bracket – you can condone "poor" squaring e.g. $(x+h)^2=x^2+h^2$ but the $-x^2$ must be present.

**A1:** Reaches a correct fraction o.e. with the $x^2$ terms cancelled out and with no algebraic errors, e.g. $\frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h}$, $2x+h$ is correct.

**A1*:** Completes the process by applying a limiting argument and deduces that $\frac{dy}{dx}=2x$ with no errors seen. They must have = $2x$ and not just $\lim_{h\to 0}2x$ to complete the proof.

$\frac{dy}{dx}=$ or an equivalent e.g. $f'(x)=$ or "Gradient =" must be evident somewhere in their working or final line. If $f'(x)$ is used then there is no requirement to see f ($x$) defined first. Condone e.g. $\frac{dy}{dx}\to 2x$ or $f'(x)\to 2x$.

Condone missing brackets to allow e.g. $\frac{dy}{dx}=\lim_{h\to 0}\frac{2xh+h^2}{h}=\lim_{h\to 0}2x+h=2x$

Do not allow $h = 0$ if there is never a reference to $h\to 0$.

e.g. $\frac{dy}{dx}=\lim_{h\to 0}\frac{2xh+h^2}{h}=\lim_{h\to 0}2x+0=2x$ is acceptable

but e.g. $\frac{dy}{dx}=\frac{2xh+h^2}{h}=2x+0=2x$ is not unless $h\to 0$ is seen.

The $h\to 0$ does not need to be present throughout the proof e.g. appear on every line but must appear at least once.

They must reach $2x+h$ at the end and not $\frac{2xh+h^2}{h}$ (without the $h$'s cancelled) to complete the limiting argument.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5(a) | $\mathrm{f}(x)=\dfrac{2x-3}{x^2+4}\Rightarrow \mathrm{f}'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2}$ <br> **or** <br> $\mathrm{f}(x)=(2x-3)(x^2+4)^{-1}\Rightarrow \mathrm{f}'(x)=2(x^2+4)^{-1}-2x(2x-3)(x^2+4)^{-2}$ | M1<br>A1 | 1.1b<br>1.1b |
| | $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)** | $-2x^2+6x+8=0\Rightarrow -2(x+1)(x-4)=0\Rightarrow x=-1,\ 4$ | B1ft (M1 on EPEN) | 1.1b |
| | Chooses correct region for their numerator and their critical values <br> $x<-1$ or $x>4$ | M1<br>A1 | 1.1b<br>2.2a |
| | | **(3)** | |
| | | | **(6 marks)** |

**Notes**

(a)

**M1:** Attempts the quotient rule to obtain an expression of the form $\dfrac{P(x^2+4)-Qx(2x-3)}{(x^2+4)^2}$ $P,Q>0$

condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$).

Condone, e.g. $\{\mathrm{f}'(x)=\}\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)}$ provided an incorrect formula is not quoted.

May also see the product rule applied to $(2x-3)(x^2+4)^{-1}$ to obtain an expression of the form $\{\mathrm{f}'(x)=\}P(x^2+4)^{-1}-Qx(2x-3)(x^2+4)^{-2}$ $P,Q>0$ condoning bracketing errors/omissions or minor slips (e.g. $2x+3$ or $x+4$)

**A1:** Fully correct differentiation in any form with correct bracketing which may be implied by subsequent working.

**A1:** $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x^2+4)^2}$ or simplified equivalent, e.g. numerator terms in a different order.

Allow recovery from "invisible" brackets earlier and apply isw once a correct answer is seen. Note that the complete form of the answer is not given so allow candidates to go from e.g.

$\mathrm{f}'(x)=\dfrac{2(x^2+4)-2x(2x-3)}{(x^2+4)^2}\to\dfrac{-2x^2+6x+8}{(x^2+4)^2}$ for the final mark. The denominator $(x^2+4)^2$ may go "missing" on an intermediate line provided it is present in their initial derivative **and** recovered in the final answer. Allow recovery from incorrect expansion of the denominator.

The $\mathrm{f}'(x)=$ must appear at some point but allow e.g. "$\dfrac{\mathrm{d}y}{\mathrm{d}x}=$"

Note that just e.g. $\mathrm{f}'(x)=\dfrac{-2(x^2-3x-4)}{(x^2+4)^2}$ without sight of a correct derivative in the correct form scores A0.

(b) **Note:** it is possible to score B0M1A1 in this question due to the demand to "use algebra".
**Note:** if their numerator from (a) is not a 3 term quadratic then no marks can be scored in (b).

**B1ft:** Uses algebra to solve their $ax^2+bx+c...0$ with $a,b,c\neq 0$ where … is any equality or inequality, finding the correct, real critical values for their 3TQ.
The … 0 may be implied by their method.
They must show their working for this mark, so expect to see factorisation, substitution into the correct quadratic formula or completing the square.
Correct values for their quadratic do **not** imply this mark.
Approaches via factorisation must have completely correct factorisation, e.g.

$-2x^2+6x+8\{=0\}\Rightarrow -2(x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(4-x)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow x^2-3x-4\{=0\}\Rightarrow (x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B1ft

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B0ft

$-2x^2+6x+8\{=0\}\Rightarrow (x+1)(x-4)\{=0\}\Rightarrow x=-1,\ 4$ scores B0ft

**M1:** Selects the "correct" region for their critical values and their *a* from part (a). Must be *x* not f(*x*)
CVs may have been found using a calculator and may be implied if they are correct for their 3TQ. CVs may be incorrect due to errors in their calculations (but not errors in their method).

- For $a<0$ and roots $\alpha<\beta$ they need e.g. $x<\alpha$ , $x>\beta$ (or e.g. $x\leqslant\alpha$ or $x\geqslant\beta$ )
- For $a>0$ and roots $\alpha<\beta$ they need e.g. $\alpha<x<\beta$ (or e.g. $x\geqslant\alpha,\ x\leqslant\beta$)

Do not be overly concerned about their use of = , > , < in reference to their $-2x^2+6x+8...0$ for this mark or for the A1.
Indicating the region on a sketch is not sufficient. Allow , / or / and / $\cup$ / $\cap$ for the M1.
If they have complex roots (or they use the discriminant to find there are no real roots) then they can score this mark for concluding:

- if *a* < 0, "<u>all values for *x*</u> (have f decreasing)" or "f is <u>always decreasing</u>" or <u>$x\in\mathrm{R}$</u>
- if *a* > 0, "<u>no values for *x*</u> (have f decreasing)" or "f is <u>never decreasing</u>"

**A1:** Correct solution $x<-1$ or $x>4$ (allow $x\leqslant -1$ or $x\geqslant 4$) coming from the correct numerator.
Do not isw if they go on to select e.g. $x>4$ or combine incorrectly to $4<x<-1$
Allow full marks to be scored in (b) from an incorrect denominator (but it must be positive for all *x*), e.g. from $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{(x+4)^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{4x^2}$ or $\mathrm{f}'(x)=\dfrac{-2x^2+6x+8}{x^2+16}$

Examples: Just "$x<-1$ or $x>4$" stated scores B0M1A1

$-2x^2+6x+8\{=0\}\Rightarrow -2(x+1)(x-4)=0\Rightarrow x<-1, x>4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\}\Rightarrow x^2-3x-4\{=0\}\Rightarrow (x+1)(x-4)=0\Rightarrow x\leqslant -1,\ x\geqslant 4$ scores B1ftM1A1

$-2x^2+6x+8\{=0\}\Rightarrow (2x+2)(x-4)\Rightarrow x<-1,\ x>4$ scores B0ftM1A1

$-2x^2+6x+8<0\Rightarrow x^2-3x-4<0\Rightarrow (x+1)(x-4)<0\Rightarrow x<-1, x>4$ scores B1ftM1A1 (as this has correct factorisation shown, the region follows from *a* < 0 (M1) and we condone reference to $x^2-3x-4<0$ as part of their working to find critical values (A1).)

Acceptable notation: allow a "," , "or" , "and" or "$\cup$" to link the two regions, which may also be in set notation. e.g. $x<-1$ or $x>4$ ; $x\leqslant -1$ , $x\geqslant 4$ ; $x<-1$ and $x>4$
$x\leqslant -1\cup x\geqslant 4$ ; $\{x: x<-1\cup x>4\}$ ; $\{x\in\mathbb{R}: x\leqslant -1\}\cup\{x\in\mathbb{R}: x\geqslant 4\}$ ;
$x\in(-\infty,-1)\cup(4,\infty)$ ; $(-\infty,-1]\cup[4,\infty)$ etc.

Do not accept $4<x<-1$ or use of the $\cap$ symbol e.g. $(-\infty,-1]\cap[4,\infty)$ for the final mark, but they may be condoned for the M1. Note also that $[-\infty,-1]\cup[4,\infty]$ scores A0.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6(a)** | $x = 2$ **or** $y = 5$ | B1 | 1.1b |
| | $P(2, 5)$ | B1 | 2.2a |
| | | **(2)** | |
| **(b)** | $16 - 4x = 3(x-2) + 5 \Rightarrow x = ...$ | M1 | 1.1b |
| | $x = \frac{17}{7}$ | A1 | 2.1 |
| | | **(2)** | |
| **(c)** | $k_{\max} = 3$ or $k_{\min} = \frac{"5" - 4}{"2"}$ | M1 | 3.1a |
| | $\frac{1}{2} < k < 3$ | A1 | 2.5 |
| | | **(2)** | |
| | | | **(6 marks)** |

**Notes**

(a)

**B1:** One correct coordinate. Either $x = 2$ or $y = 5$ or (2, …) or (…,5) seen.

**B1:** Deduces (2, 5) Accept written separately e.g. $x = 2$, $y = 5$ isw after a correct answer. Condone 2, 5 without the brackets.

(b)

**M1:** Attempts to solve the correct equation without modulus signs $16 - 4x = 3(x-2) + 5 \Rightarrow x = ...$ Must reach a value for $x$. Ignore attempts at e.g. $16 - 4x = 3(2-x) + 5$

**A1:** $x = \frac{17}{7}$ o.e. exact answer and no other values. If other values have been found they must be rejected or the $x = \frac{17}{7}$ clearly selected. Answer only implies both marks.

Note: $x$ = "2.75" coming from $5 = 16 - 4x$ may be found as part of their working to establish which branch of the modulus graph the line $y = 16 - 4x$ intersects. If this is the case it need not be "rejected" provided it is not clearly stated as one of their solutions.

Those that achieve $|x| = \frac{17}{7}$ can score BOD M1A0.

**Alternative by squaring:**

$$16 - 4x = 3|x-2| + 5 \Rightarrow 11 - 4x = 3|x-2|$$

$$\Rightarrow 16x^2 - 88x + 121 = 9\left(x^2 - 4x + 4\right)$$

$$\Rightarrow 7x^2 - 52x + 85 = 0 \Rightarrow x = 5, \frac{17}{7}$$

**M1:** Isolates the $|x-2|$ (or $3|x-2|$), squares both sides and solves the resulting 3TQ using the usual rules and may be by calculator, leading to a value for $x$.

**A1:** Selects the $\frac{17}{7}$ or rejects any other values as in main scheme.

(c)

**M1:** Correct method to find either critical value (following through on their *P*).

Either *k* {=} 3 or *k* {=} $\dfrac{"5"-4}{"2"}$ scores M1 without evidence of an incorrect method.

Note that *k* = 3 occasionally appears from use of the discriminant on $x(k-3)+5=0,$ and scores M0 unless there is an alternative valid reason given.

Allow the use of e.g. *m* = in place of *k* = here but do not allow *x* = or *y* =

**A1:** Correct range in terms of *k* in acceptable notation with no incorrect method seen.

Use of e.g. *x* is A0. Allow "and" or "$\cap$" to join the regions but not "or" or "," or "$\cup$"

Accept e.g. $(0.5,3)$ **;** $k\in\left(\frac{1}{2},3\right)$ **;** $k<3$ and $k>\frac{1}{2}$ **;** $k>\frac{1}{2}\cap k<3$

but not $\frac{1}{2}<x<3$ **;** $\frac{1}{2}\leqslant k\leqslant 3$ **;** $\left[\frac{1}{2},3\right]$ **;** $k>\frac{1}{2}\cup k<3$ **;** $k>\frac{1}{2},k<3$**;** $k>\frac{1}{2}$ or $k<3$

**Alt 1 via solving simultaneous equations:**

$$\text{e.g. } kx+4=3(x-2)+5\Rightarrow kx+4=3x-1\Rightarrow x=-\frac{5}{k-3}$$

$$kx+4=3(2-x)+5\Rightarrow kx+4=11-3x\Rightarrow (k+3)x=7$$

$$\Rightarrow (k+3)\left(\frac{-5}{k-3}\right)=7\Rightarrow k=\frac{1}{2}$$

**M1:** Sets $kx+4=3(x-2)+5$ and $kx+4=3(2-x)+5$, eliminates *x*, and solves for *k*

**A1:** As main scheme.

**Alt 2 via squaring and the discriminant:**

$$kx+4=3|x-2|+5\Rightarrow kx-1=3|x-2|$$

$$\Rightarrow k^2x^2-2kx+1=9\left(x^2-4x+4\right)$$

$$\Rightarrow \left(k^2-9\right)x^2+(36-2k)x-35=0$$

$$\Rightarrow (36-2k)^2-4\left(k^2-9\right)(-35)=0$$

$$\Rightarrow 144k^2-144k+36=0\Rightarrow k=\frac{1}{2}$$

**M1:** Sets $kx+4=3|x-2|+5$, isolates $|x-2|$ (or $3|x-2|$), squares both sides, uses $b^2-4ac\ldots0$ where … is any equality or inequality, and solves the resulting 3TQ using the usual rules and may be by calculator, leading to a value for *k*.

Condone slips in expanding the brackets.

**A1:** As main scheme.

**Alt 3 via Domain for the right hand branch of the modulus graph:**

$$kx+4=3x-1\Rightarrow x=\frac{-5}{k-3}>2\quad \left\{\text{or } x=\frac{5}{3-k}>2\right\}$$

$$\Rightarrow k-3<0\quad \{\text{and}\quad \Rightarrow -5<2(k-3)\}$$

$$\Rightarrow k<3\quad \{\text{and}\quad \Rightarrow 0.5<k\}$$

**M1:** Sets $kx+4=3(x-2)+5$, makes *x* the subject, sets > 2 and deduces a critical value.

**A1:** As main scheme.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 7(a) | $\{H=\}\,0.6\mathrm{e}^{-0.2t}\,\{+c\}$ | M1 | 1.1b |
| | $t=0, H=1.5 \Rightarrow 1.5 = \text{"}0.6\text{"}+c$<br>$\Rightarrow c = 0.9$ | dM1 | 3.4 |
| | $\Rightarrow H = 0.6\mathrm{e}^{-0.2t}+0.9$ | A1 | 2.1 |
| | | **(3)** | |
| (b) | $1.2 = 0.6\mathrm{e}^{-0.2t}+0.9 \Rightarrow 0.6\mathrm{e}^{-0.2t} = 0.3$ | M1 | 3.4 |
| | $\mathrm{e}^{-0.2t} = \frac{1}{2}$<br>$\Rightarrow t = -5\ln\left(\frac{1}{2}\right)$ | dM1 | 1.1b |
| | $\{t=\}\,3\,\text{hours}\ 28\,\text{minutes}$ | A1 | 3.2a |
| | | **(3)** | |
| (c) | {As $t$ gets large $H \rightarrow$} 0.9 | M1 | 3.1b |
| | 0.9 m or 90 cm | A1ft | 2.2b |
| | | **(2)** | |
| | | | **(8 marks)** |

**Notes**

(a)

**M1:** Attempts integration to achieve $\{H=\}\,k\,\mathrm{e}^{-0.2t}\,\{+c\}$ with $k$ a numerical constant $\neq -0.12$
Note that we will condone $k$ = (–0.2)(–0.12) {= 0.024}
If they divide by –0.12 first before integrating they need $aH = b\mathrm{e}^{-0.2t}\,\{+c\}$ with $a$ and $b$ numerical and $b \neq 1$. Condone a spurious integral symbol remaining after integration.

**dM1:** Uses $t=0, H=1.5$ and a model of the form $H = k\,\mathrm{e}^{-0.2t}+c$ (or $aH = b\mathrm{e}^{-0.2t}+c$) to find the value of the constant $c$. They cannot just "make up" a value for $k$.
Do not be concerned with their processing to find $c$ but they cannot just state $B$ (or $c$) is 0.
For reference if they divide by –0.12 first they should reach $-\frac{25}{3}H = -5\mathrm{e}^{-0.2t} - 7.5$ o.e.

**A1:** Correct complete equation in the required form: $H = 0.6\mathrm{e}^{-0.2t}+0.9$ with the $H$ = present.
May be awarded if seen at the start of (b) but not in (c). Condone $-\frac{1}{5}$ in place of –0.2
Finding correct values for $A$ and $B$ is insufficient for this mark.
Allow exact equivalents but they must be in the required form, e.g. $H = \frac{6}{10}\mathrm{e}^{-0.2t}+\frac{9}{10}$
A minimally acceptable answer is $\{H=\}\,0.6\mathrm{e}^{-0.2t}+c \rightarrow H = 0.6\mathrm{e}^{-0.2t}+0.9$ score M1dM1A1.
Note: sight of differentiating the given form to e.g. $\frac{\mathrm{d}H}{\mathrm{d}t} = -0.2A\mathrm{e}^{-0.2t}$ in their working without clear evidence of integration of the original differential equation should be marked using the special case below.

**SC:** For candidates starting with the given answer $H = A\mathrm{e}^{-0.2t}+B$ it is possible to use $\frac{\mathrm{d}H}{\mathrm{d}t} = -0.2A\mathrm{e}^{-0.2t} = -0.12\mathrm{e}^{-0.2t}$ to deduce that $A = 0.6$. This can be awarded SC M1dM0A0
If they go on to find $B$ as in the main scheme then this can be awarded SC M1dM1A0
Answer with no working scores 110.

**Note:** If the special case is applied they may go on to achieve the rest of the marks in (b) and (c).

(b) Note: $A$ and $B$ must be numbers but may be "made up" if they did not have an answer to (a).

**M1:** Uses $H = 1.2$ in a model of the form $H = Ae^{-0.2t} + B,\ B \neq 0$ and rearranges to make $Ae^{\pm 0.2t}$ or $e^{\pm 0.2t}$ the subject. Condone slips in rearranging, e.g. dividing the LHS by 0.9 instead of subtracting 0.9. Rearranging first before substituting is acceptable but they must get to $Ae^{\pm 0.2t}$ or $e^{\pm 0.2t}$ as the subject.

**dM1:** Correct use of ln to make $t$ the subject. Requires $A > 0,\ 0 < B < 1.2$ **and** $e^{\pm 0.2t} = \lambda > 0$
If they had a negative value for $A$ in part (a) they cannot just make it positive at this stage.
Any of $5\ln 2$ or $-5\ln\frac{1}{2}$ or awrt 3.46 or awrt 3.47 following a correct equation will imply M1dM1.
If they do not show their method for an incorrect $H = Ae^{-0.2t} + B$ with $A > 0,\ 0 < B < 1.2$ you may need to check their value for $t > 0$ as it may imply M1dM1.

**A1:** Correct time in hours and minutes $\{t =\}3\text{hours } 28\text{minutes},$ but condone e.g. 3h 28m
Must come from correct values of $A$ and $B$ in (a).

**Note:** If their $B = 0$ then they should end up with $t = -3.46\ldots$ however, they did not score the first M1. They cannot "recover" this by making it positive and finding $t = 3\text{hours } 28\text{minutes}$.

(c) Note that 0.9 or 0.9m must come from a correct value of $B$ in (a) to score any marks.

**M1:** Identifies the requirement to establish the limit as $t$ tends to infinity.
It can be implied by stating that $H = Ae^{-0.2t} + B \to B$ or $\left(\lim_{t\to\infty}\left[0.6e^{-0.2t} + 0.9\right]\right) = 0.9$
Stating "$B$" on its own will score this mark.
Substituting a large value is M0 unless it leads to their value for $B$ at which point the A1 is available as well.

**A1ft:** Correct height including units. Follow through on their value of $B$ where $0 < B < 1.2$
Correct ft height including units implies M1A1, while e.g. 0.9 (no units) would imply M1A0.
Evidence of an incorrect method such as $1.5 - 0.6e^{-0.2(0)} = 0.9\,\text{m}$ scores M0A0.

Misreading as $\frac{dH}{dt} = -0.12e^{0.2t}$ can score a maximum (a) 110 (b) 100 (c) 10.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8(a)** | $\text{fg}(2)=4-3\left(\frac{5}{2(2)-9}\right)^2=...$ | M1 | 1.1b |
| | $\text{fg}(2)=1$ | A1 | 1.1b |
| | | **(2)** | |
| **(b)** | $y=\frac{5}{2x-9}\Rightarrow 2xy-9y=5\Rightarrow 2xy=5+9y$ | M1 | 1.1b |
| | $2xy=5+9y\Rightarrow x=\frac{5+9y}{2y}$ | A1 | 2.1 |
| | $g^{-1}(x)=\frac{5+9x}{2x}x\neq 0\{x\in\mathbb{R}\}$ | A1 | 2.5 |
| | | **(3)** | |
| **(c)(i)** | $\{\text{gf}(x)=\}\ \frac{5}{2(4-3x^2)-9}$ | M1 | 1.1b |
| | $=\frac{5}{-1-6x^2}$ or $\frac{-5}{1+6x^2}$ | A1 | 1.1b |
| **(ii)** | $-5\leqslant \text{gf}(x)<0$ | B1 | 2.2a |
| | | **(3)** | |
| **(d)** | $\text{f}(x)=\text{h}(x)\Rightarrow 4-3x^2=2x^2-6x+k$<br>$\Rightarrow 5x^2-6x+k-4=0$ | M1 | 1.1b |
| | $b^2-4ac<0\Rightarrow 36-4(5)(k-4)<0\Rightarrow k>...$ | dM1 | 3.1a |
| | $k>5.8$ o.e. | A1 | 2.2a |
| | | **(3)** | |
| | | | **(11 marks)** |

**Notes**

(a)

**M1:** Correct method, e.g. attempts to find g(2) $\left(=\frac{5}{4-9}\right)$ and substitutes its value into f to achieve a value.

Alternatively, attempts $\text{fg}(x)=4-3\left(\frac{5}{2x-9}\right)^2$, condoning slips, and substitutes $x=2$ to achieve a value.

**A1:** Correct answer only. If gf(2) is also attempted, then mark the final attempt which is the most complete.

(b)

**M1:** Eliminates the fraction and puts the *xy* term (or *x* term) onto one side of the equation. Alternatively swaps *x*'s and *y*' s, eliminates the fraction and puts the *xy* term (or *y* term) onto one side of the equation. Condone minor slips in rearranging e.g. $-9y$ instead of $+9y$

**A1:** Correct expression for the inverse, *x* in terms of *y* or *y* in terms of *x*. Need not be simplified.

Note that $y=\frac{5}{2x-9}\Rightarrow 2x-9=\frac{5}{y}\Rightarrow 2x=\frac{5}{y}+9$ is M1 and $\Rightarrow x=\frac{\frac{5}{y}+9}{2}$ is A1

**A1:** Fully correct notation for the inverse including its domain and including the e.g. $g^{-1}$ = .
Condone $x \neq 0$ without $x \in \mathbb{R}$ Need not be simplified.
Do not be too worried about $g^{-1}$ looking a bit like $y^{-1}$ due to poor handwriting but if it is clearly $y^{-1}$ then withhold this mark.

Accept e.g. $g^{-1}(x) = \dfrac{5+9x}{2x}$ $x \in \mathbb{R}$, $x \neq 0$ **or** $g^{-1}(x) = \dfrac{5}{2x} + \dfrac{9}{2}$ $x \neq 0$ **or** $g^{-1} = \dfrac{\frac{5}{x}+9}{2}$ $x \neq 0$
Ignore any reference to the range of g.

(c)(i)
**M1:** Correct method. Attempts to substitute f into g, condoning slips, e.g. missing the 3.
**A1:** Correct simplified fraction. Ignore any reference to domains. Do not isw.
There is no need to include the gf($x$) =
If fg($x$) is also attempted, then mark the final attempt which is the most complete.
(ii)
**B1:** Deduces the correct range. May be scored even if gf($x$) is incorrect (but not a follow through).
Allow e.g. $-5 \leqslant y < 0$, $y \in [-5, 0)$, $[-5, 0)$
Do not allow e.g. $-5 \leqslant x < 0$, $y \in (-5, 0)$, $-5 \leqslant f(x) < 0$, $-5 \leqslant g(x) < 0$

(d)
**M1:** Sets f($x$) = h($x$) and attempts to collect terms to obtain a 3TQ = 0
The = 0 may be implied by use of the discriminant. Condone copying slips in f($x$) and h($x$).

**dM1:** Recognises the need to use "$b^2 - 4ac \ldots 0$" on their 3TQ and uses this to establish a value or range of values for $k$. Allow for an attempt to solve $b^2 - 4ac \ldots 0$ or $b^2 \ldots 4ac$, which must be in terms of $k$ only, where ... is an equality or any inequality.

**(Alt 1)** Attempts to complete the square for their 3TQ (usual rules) and uses its minimum value set … 0 to establish a value or range of values for $k$. Their expression for the minimum value must be in terms of $k$ only. Condone any equality or inequality when comparing their minimum value to 0.
e.g. $5x^2 - 6x + k - 4 \to 5\left(x - \frac{3}{5}\right)^2 - \frac{29}{5} + k \to "-\frac{29}{5} + k" > 0 \Rightarrow k > \ldots$ scores dM1

**(Alt 2)** Differentiates their 3TQ with respect to $x$ to achieve a linear expression, sets = 0 (which may be implied), solves for $x$ and substitutes $x$ back into their 3TQ set … 0 to establish a value or range of values for $k$. Here … can be any equality or inequality.
e.g. $5x^2 - 6x + k - 4 \to 10x - 6 \to x = 0.6 \Rightarrow 5(0.6)^2 - 6(0.6) + k - 4 > 0 \Rightarrow k > \ldots$ scores dM1

**A1:** Deduces the correct range for $k$, e.g. $k > \dfrac{29}{5}$ o.e. Must be in terms of $k$ and not e.g. $x$
Accept e.g. $k \in (5.8, \infty)$ or just $\left(\dfrac{29}{5}, \infty\right)$ but **not** e.g. $k \ldots \dfrac{29}{5}$ or $x > \dfrac{29}{5}$ or $[5.8, \infty)$

| Question | Scheme | | Marks | AOs |
|---|---|---|---|---|
| **9(a)** | $\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}} \Rightarrow \frac{3^{2(7-2k)}}{3^{4k-5}}=\frac{3^{2(k-1)}}{3^{2(7-2k)}}$ | | M1 | 3.1a |
| | $\left(3^{2(7-2k)}\right)^2=3^{4k-5}\times 3^{2(k-1)}$ $\Rightarrow 28-8k=6k-7\Rightarrow k=...$ | $3^{2(7-2k)-(4k-5)}=3^{2(k-1)-2(7-2k)}$ $\Rightarrow 19-8k=6k-16\Rightarrow k=...$ | dM1 | 1.1b |
| | $k=\frac{5}{2}$* | | A1* | 2.1 |
| | | | **(3)** | |
| **(b)** | $a=3^{4(2.5)-5}$ and $r=\frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}\Rightarrow$ one of $a=243$ or $r=\frac{1}{3}$ | | M1 | 2.2a |
| | $S_\infty=\frac{a}{1-r}=\frac{"243"}{1-"\frac{1}{3}"}$ | | M1 | 1.1b |
| | $S_\infty=\frac{729}{2}(364.5)$ cao | | A1 | 1.1b |
| | | | **(3)** | |
| | | | **(6 marks)** | |

**Notes**

(a) **Special cases:**

**SC 1:** For those that verify rather than prove a SC 100 is awarded for substituting $k=\frac{5}{2}$ into all three terms to correctly obtain 243, 81 and 27 **with** a statement that this is **geometric** with $r=\frac{1}{3}$ (or equivalent reason). All statements must be correct.

**SC 2:** Be aware that e.g. $\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}\Rightarrow 81^{2(7-2k)}=9^{6k-7}$ is an incorrect process (without some indication that they have intentionally squared both sides) that fortuitously leads to the correct answer and may score maximum SC 010.

**M1:** Uses the 3 terms to set up an equation in $k$ **and**

- **either** reaches a common base by replacing 9 with $3^2$ or by replacing 3 with $9^{0.5}$ and uses the power law of indices correctly
- **or** uses the laws of indices correctly to reach $9^{14-4k}=3^{6k-7}$ condoning slips in e.g. expanding brackets.

Writing down e.g. $2(7-2k)-(4k-5)=2(k-1)-2(7-2k)$ is sufficient to imply the M1.

**dM1:** Correct processing leading to a value for $k$.

**A1*:** Correct value following correct working. Allow $k=2.5$ in place of $k=\frac{5}{2}$

Condone missing/invisible brackets provided they are recovered correctly.

**Alt 1 Using Base 9:**

$$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}\Rightarrow\frac{9^{7-2k}}{9^{2k-2.5}}=\frac{9^{k-1}}{9^{7-2k}} \text{ o.e. scores M1}$$

$$\Rightarrow 9^{9.5-4k}=9^{3k-8}\Rightarrow 9.5-4k=3k-8\Rightarrow 7k=17.5\Rightarrow k=2.5$$

Score dM1 when a value of $k$ is achieved using a correct process and A1* if fully correct.

**Alt 2 Finding $r$ in terms of $k$ and using e.g. $u_3 = ar^2$:**

$$r=\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(7-2k)}}{3^{4k-5}}\left\{=3^{19-8k}\right\} \textbf{ or } r=\frac{3^{2(k-1)}}{9^{7-2k}}=\frac{3^{2k-2}}{3^{2(7-2k)}}\left\{=3^{6k-16}\right\}$$

$$\Rightarrow\ 3^{4k-5}\times\left(3^{19-8k}\right)^2=3^{2k-2} \textbf{ or } \Rightarrow\ 3^{4k-5}\times\left(3^{6k-16}\right)^2=3^{2k-2} \text{ scores M1}$$

$$\Rightarrow\ 3^{4k-5}\times 3^{2(19-8k)}=3^{2k-2}\ \Rightarrow\ 33-12k=2k-2\ \Rightarrow\ 14k=35 \Rightarrow\ k=2.5$$

Score dM1 when a value of $k$ is achieved using a correct process and A1* if fully correct.

**Alt 3 Using Logs Way 1:**

$$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}\ \Rightarrow\ \left(9^{7-2k}\right)^2=3^{6k-7}\ \Rightarrow\ 9^{14-4k}=3^{6k-7} \text{ scores M1}$$

$$\Rightarrow(14-4k)\log_3 9=6k-7$$

$$\Rightarrow\ 2(14-4k)=6k-7$$

$$\Rightarrow\ k=2.5$$

Score dM1 when a value of $k$ is achieved using a correct process and A1* if fully correct.

**Alt 4 Using Logs Way 2:**

$$\frac{9^{7-2k}}{3^{4k-5}}=\frac{3^{2(k-1)}}{9^{7-2k}}$$

$$\Rightarrow(7-2k)\log_3 9-(4k-5)\log_3 3=(2k-2)\log_3 3-(7-2k)\log_3 9 \quad \text{scores M1}$$

$$\Rightarrow\ 2(7-2k)-(4k-5)=2k-2-2(7-2k)$$

$$\Rightarrow\ 19-8k=6k-16$$

$$\Rightarrow\ k=2.5$$

Score dM1 when a value of $k$ is achieved using a correct process and A1* if fully correct.

**Alt 5 Recognising that taking $\log_3$ forms an Arithmetic Sequence:**

$$\{\log_3\}u_1=4k-5,\ \{\log_3\}u_2=2(7-2k),\ \{\log_3\}u_3=2(k-1)$$

$$\Rightarrow 2(7-2k)-(4k-5)=2(k-1)-2(7-2k) \quad \text{scores M1}$$

$$\Rightarrow\ 19-8k=6k-16$$

$$\Rightarrow\ k=2.5$$

Score dM1 when a value of $k$ is achieved using a correct process and A1* if fully correct.
There is no need to see any mention of log in this approach.

(b)

**M1:** Deduces expressions for the first term **and** the common ratio using $k=\frac{5}{2}$ in the <u>correct formulae</u> **and** finds at least one of $a$ = 243 or $r=\frac{1}{3}$. Allow if seen in (a). May be implied by correct values for $a$ and $r$. For reference, $a=3^{4(2.5)-5}$ and $r=\frac{9^{7-2(2.5)}}{3^{4(2.5)-5}}$ $\left\{\text{or } r=\frac{3^{2(2.5-1)}}{9^{7-2(2.5)}}\right\}$

**M1:** Recalls the sum to infinity formula and substitutes their values for $a$ and $r$ provided $|r|<1$
Dependent on a correct attempt to find both $a$ and $r$ using $k$ = 2.5 but allow if neither value is correct or if they are unprocessed e.g. $\dfrac{3^{4(2.5)-5}}{1-\dfrac{9^{7-2(2.5)}}{3^{4(2.5)-5}}}$ scores this mark.

**A1:** cao. Correct sum to infinity. Answer only (with no working) scores full marks. Apply isw.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **10(a)** | $8(4)-4^{\frac{5}{2}}=32-32=0$ | B1 | 1.1b |
| | | **(1)** | |
| **(b)** | $8-\frac{5}{2}x^{\frac{3}{2}}$ | B1 | 1.1b |
| | $x=4\Rightarrow\left\{\frac{dy}{dx}=\right\}8-\frac{5}{2}\times 8=-12$ <br> $\Rightarrow y\{-0\}="-12"(x-4)$ | M1 | 1.1b |
| | $12x+y=48$ * | A1* | 1.1b |
| | | **(3)** | |
| **(c)** | Attempts to find one of the coordinates of the point of intersection <br> $y=8x,\ 12x+y=48\Rightarrow y=19.2$ (or $x=2.4$) | M1 | 1.1b |
| | Triangle area is $\frac{1}{2}\times 4\times "19.2"\left(=38.4 \text{ or } \frac{192}{5}\right)$ <br> or $\int_0^{"2.4"} 8x\ dx+\int_{"2.4"}^{4} "(48-12x)"\ dx$ | dM1 | 3.1a |
| | $\int\left(8x-x^{\frac{5}{2}}\right)dx=4x^2-\frac{2}{7}x^{\frac{7}{2}}$ | B1 | 1.1b |
| | $A=38.4-\left["4x^2-\frac{2}{7}x^{\frac{7}{2}}"\right]_0^4=38.4-64+\frac{256}{7}$ | ddM1 | 3.1a |
| | $=\frac{384}{35}$ | A1 | 1.1b |
| | | **(5)** | |
| | | | **(9 marks)** |

**Notes**

(a)

**B1:** Substitutes $x = 4$ into the equation of the curve and verifies that $y = 0$. Accept "$8(4)-4^{\frac{5}{2}}=0$"

Alternatively, sets $8x-x^{\frac{5}{2}}=0$ and solves with correct processing to achieve $x = 4$.

As a minimum accept e.g. $8x-x^{\frac{5}{2}}=0 \Rightarrow x^{\frac{3}{2}}=8 \Rightarrow \{x=\}\ 4$ which may follow factorisation.

(b)

**B1:** Correct differentiation. The $\frac{dy}{dx}=$ need not be present.

**M1:** Correct method for finding the equation of the tangent at $A(4, 0)$.

Requires substitution of $x = 4$ into their $\frac{dy}{dx}$ and an attempt at the equation of the line using this gradient. If using $y-y_1=m(x-x_1)$ then condone the omission of the – 0.

If $y=mx+c$ is used they must proceed as far as $c=\ldots$

Accept $\frac{dy}{dx}=-12$ or $m = -12$ without explicit substitution of $x = 4$ provided $8-\frac{5}{2}x^{\frac{3}{2}}$ is seen.

**A1*:** Correct work leading to the given equation having scored B1M1.
Condone $y+12x=48$ and apply isw once seen.
Do not condone $12x+y-48=0$ (unless a correct equation = 48 is seen).

(c) **Note:** Condone poor notation such as missing d$x$ or spurious $\int$ symbols throughout.

**M1:** Attempts to find either the $x$ or $y$ coordinate of the intersection of line $l_1$ and line $l_2$
You may need to check the diagram or limits to their integrals.

**dM1:** Correct method for the area of the triangle. e.g. Triangle area is $\frac{1}{2}\times 4\times "19.2"\left(=38.4 \text{ or } \frac{192}{5}\right)$

If integration is attempted then condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x\ \mathrm{d}x+\int_{"2.4"}^{4} "(48-12x)"\ \mathrm{d}x \qquad \left\{=\frac{576}{25}+\frac{384}{25}=23.04+15.36\right\}$

**B1:** Correct integration of **curve** ignoring limits, i.e. $4x^2-\frac{2}{7}x^{\frac{7}{2}}$ but condone e.g. $\frac{8x^{1+1}}{2}-\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$

**ddM1:** Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
Implied by $38.4-\frac{192}{7}$ or a correct final answer $\frac{384}{35}$
Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence of substitution (which need not be evaluated).

**A1:** Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

**Alternative using lines – curve:**

**M1:** Attempts to find either the $x$ or $y$ coordinate of the intersection of line $l_1$ and line $l_2$
You may need to check the diagram or limits to their integrals.

**dM1:** Correct method for at least one part (0 to "2.4" **or** "2.4" to 4) of the area of $R$ including limits.
Condone slips in their rearrangement of $12x+y=48$ to $y=48-12x$ and note that their integrals do need not to be evaluated, so for example

look for $\int_0^{"2.4"} 8x-\left(8x-x^{\frac{5}{2}}\right)\mathrm{d}x$ or $\int_{"2.4"}^{4} "(48-12x)"-\left(8x-x^{\frac{5}{2}}\right)\ \mathrm{d}x$ (or a sum of both)

**B1:** Correct integration of **both** regions ignoring limits. May be completed as a sum or separately.
Condone e.g. $\frac{x^{\frac{5}{2}+1}}{\frac{7}{2}}$ in place of $\frac{2}{7}x^{\frac{7}{2}}$ Note that each integral may have been simplified.

$\int_{...}^{...} x^{\frac{5}{2}}\ \mathrm{d}x$ and $\{+\}$ $\int_{...}^{...} 48-20x+x^{\frac{5}{2}}\ \mathrm{d}x \quad\rightarrow\quad \left[\frac{2}{7}x^{\frac{7}{2}}\right]_{...}^{...}$ and $\{+\}$ $\left[48x-10x^2+\frac{2}{7}x^{\frac{7}{2}}\right]_{...}^{...}$

**ddM1:** Fully correct strategy including substitution which would lead to an **exact** area.
Does not need to reach a value. Dependent on both previous M marks.
This approach requires:

- substitution of 0, their 2.4 and 4 in the correct places
- the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ to be cancelled (may be implied by a correct final answer $\frac{384}{35}$)

Note that the decimal approximation that might be seen is 10.97142857 and implies ddM0 unless there is evidence that the $\frac{2}{7}(2.4)^{\frac{7}{2}}-\frac{2}{7}(2.4)^{\frac{7}{2}}$ has been cancelled e.g. ~~6.118...~~ – ~~6.118...~~

**A1:** Correct exact value. Either $\frac{384}{35}$ or $10\frac{34}{35}$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **11** | Identifies angle $BAO=\frac{\pi}{3}$ or angle $BOC=\frac{2\pi}{3}$ | B1 | 2.2a |
| | Area(segment) $=\frac{1}{2}\times5^2\times\frac{\pi}{3}-\frac{1}{2}\times5^2\sin\frac{\pi}{3}\left\{=\frac{25\pi}{6}-\frac{25\sqrt{3}}{4}\right\}$ **or** Area($AOB$) $=2\times\frac{1}{2}\times5^2\times\frac{\pi}{3}-\frac{1}{2}\times5^2\sin\frac{\pi}{3}\left\{=\frac{25\pi}{3}-\frac{25\sqrt{3}}{4}\right\}$ | M1 | 2.1 |
| | Area of $R=\frac{1}{2}\times5^2\times\frac{2\pi}{3}-\left(\frac{1}{2}\times5^2\times\frac{\pi}{3}-\frac{1}{2}\times5^2\sin\frac{\pi}{3}\right)$ **or** Area of $R=\frac{1}{2}\times\pi\times5^2-\left(\frac{1}{2}\times5^2\times\frac{\pi}{3}-\frac{1}{2}\times5^2\sin\frac{\pi}{3}+\frac{1}{2}\times5^2\times\frac{\pi}{3}\right)$ | dM1 | 3.1a |
| | $=\frac{25}{4}\sqrt{3}+\frac{25}{6}\pi\ (\text{cm}^2)$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(4 marks)** |

**Notes**

**Note: Use of degrees, if used in formulae in degrees, can score full marks.**

**B1:** Deduces angle $BAO$ or angle $BOA$ is $\frac{\pi}{3}$ radians or 60° or deduces that angle $BOC$ is $\frac{2\pi}{3}$ radians or 120°. May be seen on the diagram or their own sketches or embedded in a formula. May be implied if they find e.g. $\frac{1}{6}\times\pi\times5^2$ for the area of the minor sector.

**M1:** Uses a correct process and an angle of $\frac{\pi}{3}$ radians or 60° to find

- the area of the segment bounded by the arc $OB$ and straight line $OB$
- or the area of the segment bounded by the arc $AB$ and straight line $AB$
- or the area of unshaded region $AOB$.

Allow decimal values to imply the method (require 3sf rounded or truncated).
Use of 60° must be in a correct formula in degrees. $0.5\times5^2\times60$ scores M0.
For reference these are the areas of the regions:

- segment $=\frac{1}{2}\times5^2\times\frac{\pi}{3}-\frac{1}{2}\times5^2\times\sin\frac{\pi}{3}\left\{=\frac{25\pi}{6}-\frac{25\sqrt{3}}{4}=2.26\right\}$ (3 s.f.) scores M1
- $AOB=\frac{1}{2}\times5^2\times\frac{\pi}{3}+\frac{1}{2}\times5^2\times\frac{\pi}{3}-\frac{1}{2}\times5^2\times\sin\frac{\pi}{3}\left\{=\frac{25\pi}{3}-\frac{25\sqrt{3}}{4}=15.35\right\}$ (2 d.p.) scores M1
- Minor sector $=\frac{1}{2}\times5^2\times\frac{\pi}{3}\left\{=\frac{25\pi}{6}=13.09\right\}$ (2 d.p.) scores M0 on its own
- Major sector $=\frac{1}{2}\times5^2\times\frac{2\pi}{3}\left\{=\frac{25\pi}{3}=26.18\right\}$ (2 d.p.) scores M0 on its own

- Triangle $AOB = \frac{1}{2}\times 5^2 \times \sin\frac{\pi}{3}\left\{=\frac{25\sqrt{3}}{4}=10.8\right\}$ (3 s.f.) scores M0 on its own.

This mark may be implied if seen as part of a **correct** strategy for the area of $R$.

Note that the area of triangle $AOB$ may also be found using Pythagoras and $\frac{1}{2}bh$ but their method must be correct.

**dM1:** A fully correct strategy for finding the area of $R$.
Follow through on incorrectly simplified areas but the method must be correct.
Allow decimal values to imply the method (require 3sf rounded or truncated).
For reference, the area is approximately 23.92 (2d.p.) and is likely to imply B1M1dM1 but their work should be checked.
Either:

- major sector – segment $=\frac{1}{2}\times 5^2\times\frac{2\pi}{3}-\left(\frac{1}{2}\times 5^2\times\frac{\pi}{3}-\frac{1}{2}\times 5^2\sin\frac{\pi}{3}\right)$

or

- semicircle – $AOB$ $=\frac{1}{2}\times\pi\times 5^2-\left(\frac{1}{2}\times 5^2\times\frac{\pi}{3}-\frac{1}{2}\times 5^2\sin\frac{\pi}{3}+\frac{1}{2}\times 5^2\times\frac{\pi}{3}\right)$

or

- semicircle – 2 × sector $AOB$ + triangle $AOB$ $=\frac{1}{2}\times\pi\times 5^2-2\times\frac{1}{2}\times 5^2\times\frac{\pi}{3}+\frac{1}{2}\times 5^2\sin\frac{\pi}{3}$

or

- sector $AOB$ + triangle $AOB$ $=\frac{1}{2}\times 5^2\times\frac{\pi}{3}+\frac{1}{2}\times 5^2\sin\frac{\pi}{3}$ (this also implies the first M1)

If they go straight to one of these expressions then we would imply the first M1 (and the B1).

**A1:** Correct expression $\frac{25}{4}\sqrt{3}+\frac{25}{6}\pi$ o.e. in the correct form e.g. $\frac{50}{8}\sqrt{3}+\frac{75}{18}\pi$

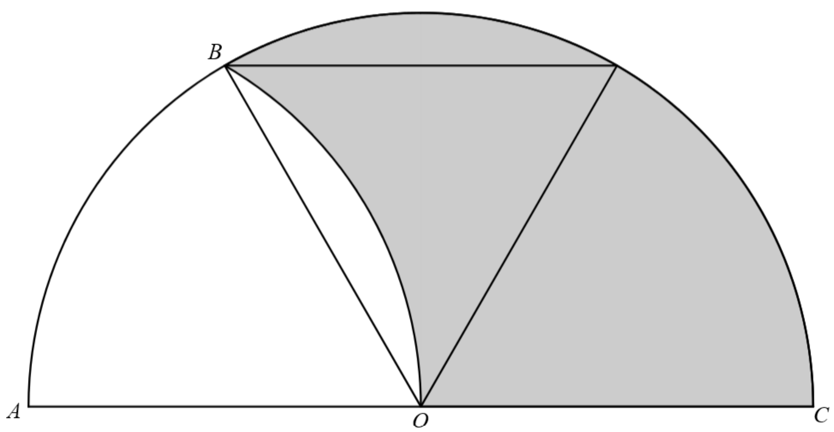
Ignore any reference to (or absence of) units.

Do not apply isw if they go on to add or subtract additional areas.

However, we can condone poor simplification e.g. $\frac{25}{12}\left(\sqrt{3}+\pi\right)$ following a correct answer.

**Note:** Attempts via integration for the area underneath a circle or polar coordinates are unlikely but if seen then use Review. The schemes for these approaches follow the main scheme but the first M1 may also be scored for finding half of the area of $AOB$.

You might find the following diagram helpful:



| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **12(a)** | $K = 500$ | B1 | 1.1b |
| | $\tan\alpha = \dfrac{480}{140} \Rightarrow \alpha = ...$ | M1 | 1.1b |
| | $\alpha =$ awrt $73.74°$ or $500\cos(\theta + 73.74)°$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)(i)** | $R = 1000 + 500\cos(30t + 73.74)°$<br>or<br>$R = 1000 + 140\cos(30t)° - 480\sin(30t)°$ | B1ft | 3.3 |
| **(b)(ii)** | $\{R_{\min} =\}\,500$ | B1ft | 3.4 |
| | | **(2)** | |
| **(c)** | $t = 3.5 \Rightarrow R = "1000" + "500"\cos(30(3.5) + "73.74")° = ...$ | M1 | 3.4 |
| | $R$ = awrt 500.1…<br>so the model is reliable | A1 | 3.5a |
| | | **(2)** | |
| **(d)** | $\sin(30t + 70)° = -1 \Rightarrow 30t + 70 = 270 \Rightarrow 30t = ...$ (or $t$ = ...) | M1 | 3.4 |
| | $30t = 200 \left(\text{or } t = \dfrac{20}{3}\right)$ | A1 | 1.1b |
| | $R = "1000" + "500"\cos\left(30\left("\dfrac{20}{3}"\right) + "73.74"\right)°$<br>or<br>$R = "1000" + 140\cos("200")° - 480\sin("200")°$ | dM1 | 3.4 |
| | $R$ = 1032 (or 1033) | A1 | 1.1b |
| | | **(4)** | |
| | | **(11 marks)** | |

**Notes**

**Note:** Candidates working in radians are able to score all the M and B marks in this question. Condone the absence of the degrees symbol throughout the whole question.

(a)

**B1:** Correct value for $K$. Condone $R$ = 500

**M1:** Award for $\tan\alpha = \pm\dfrac{480}{140} \Rightarrow \alpha = ...$, $\tan\alpha = \pm\dfrac{140}{480} \Rightarrow \alpha = ...$, $\sin\alpha = \pm\dfrac{480}{"500"} \Rightarrow \alpha = ...$ or $\cos\alpha = \pm\dfrac{140}{"500"} \Rightarrow \alpha = ...$

Note $\alpha =$ awrt 1.3 (rad) implies this mark.

**A1:** $\alpha =$ awrt $73.74\{°\}$ or correct expression $500\cos(\theta + 73.74)\{°\}$

(b)(i) Note: mark parts (b)(i) and (b)(ii) together.

**B1ft:** Correct equation of the model in either form including the $R$ = following through on their numerical $K$ ($0 < K \leqslant 750$) and their numerical $\alpha$.

Allow for e.g. $R = 1500 - "500" + "500"\cos(30t + "73.74")\{°\}$ or for

$R = 1500 - "500" + 140\cos(30t)° - 480\sin(30t)°$ but not e.g. $R = 1500 - K + K\cos(30t + \alpha)°$

$R = 1000 + 140\cos 30t - 480\sin 30t$ (without the brackets) is correct.

Allow this mark if they have truncated or rounded an otherwise correct $\alpha$ (to 3s.f.)

(b)(ii)

**B1ft:** 500 or follow through on (their $A$ – their $K$) or (1500 – 2 × their $K$) provided it is non-negative and less than 1500. It must be clear this is their answer to (b)(ii) so expect to see e.g. (b) or $R_{\min} =$ or an indication it is the minimum.

(c) Note: if $\theta$ is used in place of 30$t$ then they must revert back to 30$t$ correctly to access the marks.

**M1:** Substitutes $t$ = 3.5 into their model for the number of rabbits (you may need to check if no method is shown)

**or** substitutes $t$ = 3.5 into their $\cos(30t+\alpha)°$

Condone substitution of a value of $t$ in the range $3 \leqslant t \leqslant 4.5$ for this mark.

**A1:** $R$ = awrt 500.1… **or** 500 (not awrt) following substitution of $t$ = 3.5, suggesting that the model is valid/reliable/appropriate/good.

**or** $\cos(30(3.5)+73.74)\{°\} \approx -1$ suggesting that the model is valid/reliable/appropriate/good.

Allow this mark if they have truncated or rounded an otherwise correct $\alpha$ (to 3s.f.)

**Alt:**

**M1:** Minimum occurs when $A+K\cos(30t+\alpha)° = R_{\min} \Rightarrow \cos(30t+\alpha)° = \lambda$ with $|\lambda| \leqslant 1$ leading to $t = ...$

May just see $\cos(30t+73.74)° = -1 \Rightarrow t = ...$ (or their $\cos(30t+\alpha)° = -1 \Rightarrow t = ...$)

$30t+\alpha = 180 \Rightarrow t = ...$ implies this mark. Condone $30t+\alpha = \pi \Rightarrow t = ...$ for this mark.

**A1:** $t = 3.54...$ (i.e. the middle of April) so the model is valid/reliable/appropriate/good.

Do not condone incorrect statements, e.g., $t = 3.54...$ i.e. the middle of March so close to middle of April. If using $A+K\cos(30t+\alpha)° = R_{\min}$ then $R_{\min}$ must be = their $A -$ their $K$

Allow this mark if they have truncated or rounded an otherwise correct $\alpha$ (to 3s.f.)

**Alt 2 using differentiation (Condoned)**

**M1:** Condone finding the minimum using $...\sin(30t+73.74)° = 0 \Rightarrow t = ...$ (or their $\sin(30t+\alpha)° = 0 \Rightarrow t = ...$)

$30t+\alpha = 180 \Rightarrow t = ...$ implies this mark. Condone $30t+\alpha = \pi \Rightarrow t = ...$ for this mark.

**A1:** $t = 3.54...$ (i.e. the middle of April) suggesting that the model is valid/reliable/appropriate.

Do not condone incorrect statements, e.g., $t = 3.54...$ i.e. the middle of March so close to middle of April.

The complete derivative for $\frac{dR}{dt}$ does not need to be seen.

Allow this mark if they have truncated or rounded an otherwise correct $\alpha$ (to 3s.f.)

(d) Note: if $\theta$ is used in place of 30$t$ then they must revert back to 30$t$ correctly to access the marks.

**M1:** Realises that $\sin(30t+70)° = -1$, reaches $30t+70 = 270$ or –90 and attempts to find $t$ (or 30$t$)

Condone attempts using differentiation. The minimum occurs when $\cos(30t+70)° = 0 \Rightarrow$ $30t+70 = 270 \Rightarrow 30t = ...$ (or $t$ = …). They must use 270 or –90 and **not** 90 to achieve the minimum. Condone $30t+\alpha = \frac{3\pi}{2} \Rightarrow t = ...$ for this mark but not $30t+\alpha = \frac{\pi}{2} \Rightarrow t = ...$.

**A1:** Correct value for 30$t$ (or $t$) Accept rounded or truncated values to at least 3s.f. e.g. 6.66 or 6.67

**dM1:** Substitutes their value of $t$ > 0 (or 30$t$ > 0) coming from $30t+70 = \mathbf{270}$ into their model for $R$

**A1:** Correct number of rabbits. Allow 1032 or 1033 but must be whole numbers and not just 1030. Allow this mark if they have truncated or rounded an otherwise correct $\alpha$ (to 3s.f.)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 13(a) | $x = a\sin^2\theta \Rightarrow \dfrac{dx}{d\theta} = 2a\sin\theta\cos\theta$ | B1 | 1.1b |
| | $\int x^{\frac{1}{2}}\sqrt{a-x}\,dx = \int \sqrt{a}\sin\theta\sqrt{a-a\sin^2\theta} \times 2a\sin\theta\cos\theta\{d\theta\}$ | M1 | 2.1 |
| | $= \int \sqrt{a}\sin\theta\sqrt{a}\cos\theta \times 2a\sin\theta\cos\theta\,d\theta = 2a^2\int \sin^2\theta\cos^2\theta\,d\theta$ <br> $= 2a^2\int\left(\frac{1}{2}\sin 2\theta\right)^2 d\theta$ | dM1 | 3.1a |
| | Replaces or considers limits $\{x=0 \Rightarrow\}\,\theta = 0,\ \{x=a \Rightarrow\}\,\theta = \frac{\pi}{2}$ <br> $= \frac{1}{2}a^2\int_0^{\frac{\pi}{2}} \sin^2 2\theta\,d\theta$ * | A1* | 1.1b |
| | | **(4)** | |
| (b) | $\ldots\int \sin^2 2\theta\,d\theta \to \ldots\int \dfrac{1-\cos 4\theta}{2}\,d\theta$ | M1 | 1.1b |
| | $\to \ldots\left(\frac{\theta}{2} - \frac{1}{8}\sin 4\theta\right)$ | dM1 | 2.1 |
| | $\mu\int \sin^2 2\theta\,d\theta \to \frac{\mu}{2}\left(\theta - \frac{1}{4}\sin 4\theta\right)$ | A1 | 1.1b |
| | $\left\{\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,dx\right\} = \frac{a^2}{4}\left(\frac{\pi}{2} - 0 - 0\right) = \frac{1}{8}\pi a^2$ | A1 | 1.1b |
| | | **(4)** | |
| | | | **(8 marks)** |

**Notes**

(a)

**B1:** $\dfrac{dx}{d\theta} = 2a\sin\theta\cos\theta$ or $dx = 2a\sin\theta\cos\theta\,d\theta$ o.e. seen or implied by their substitution.

Note that writing $x = a\sin^2\theta = \frac{a}{2}(1-\cos 2\theta) \to \frac{dx}{d\theta} = a\sin 2\theta$ is correct. Condone use of $\frac{dx}{da}$

**M1:** Attempts to substitute, fully replacing $x^{\frac{1}{2}}$ and $\sqrt{a-x}$ with $\theta$'s and d$x$ with their d$x$ = …

Look for $x^{\frac{1}{2}}\sqrt{a-x}\,dx \to f(\theta)g(\theta)h(\theta)$ where

- $f(\theta)$ is an attempt at $\sqrt{a\sin^2\theta}$ e.g. allow $a\sin\theta$ but just $a\sin^2\theta$ is not condoned
- $g(\theta)$ is an attempt at $\sqrt{a-a\sin^2\theta}$ but not $\sqrt{a}-\sqrt{a\sin^2\theta}$ unless $\sqrt{a-a\sin^2\theta}$ is attempted first
- $h(\theta)$ = their d$x$ or their $\dfrac{dx}{d\theta}$ or $\dfrac{1}{\text{their } \frac{dx}{d\theta}}$ (in terms of $\theta$ only but condone d$a$ seen)

Condone slips provided the intention is clear, e.g. $x^{\frac{1}{2}} \to \sqrt{a}\sin^2\theta$ but $x$ must be eliminated.
There is no need to replace the limits and condone the absence of $d\theta$ and/or the integral sign for this mark.

**dM1:** Attempts to use $\sin 2\theta = 2\sin\theta\cos\theta$ to convert an integral of the form $\int \sin^2\theta\cos^2\theta \,\mathrm{d}\theta$ or

e.g. the form $\int \sin\theta\cos\theta\sin 2\theta \,\mathrm{d}\theta$ to $\int ...\sin^2 2\theta \,\mathrm{d}\theta$

There is no need to replace the limits and condone the absence of $\mathrm{d}\theta$ and/or the integral sign for this mark.

**A1*:** Replaces or considers limits $\{x = 0 \Rightarrow\}\ \theta = 0,\ \{x = a \Rightarrow\}\ \theta = \frac{\pi}{2}$ at some stage before the given answer and proceeds with no errors to the given answer. The replaced limits may appear with their integral symbol and do not have to be justified and do not have to appear on every line. Condone infrequent slips in notation, e.g. $\sin\theta^2$ in a line as long as it is not consistently poor. You must see the integral sign with the correct limits and the $\mathrm{d}\theta$ together in the given answer.

(b)

**M1:** Adopts an appropriate strategy by using the double angle identity to obtain an integrable form $...\int \sin^2 2\theta \,\mathrm{d}\theta \rightarrow ...\int \frac{\pm 1 \pm \cos 4\theta}{2}\mathrm{d}\theta$ which may be seen as

$\lambda\int \sin^2 2\theta \,\mathrm{d}\theta \rightarrow \frac{\lambda}{2}\int \pm 1 \pm \cos 4\theta \,\mathrm{d}\theta$ with the $\frac{1}{2}$ absorbed into their coefficient of the integral.

**dM1:** Integrates into the form $\pm p\theta \pm q\sin 4\theta$

**A1:** Correct integration of $\mu\int \sin^2 2\theta \,\mathrm{d}\theta$ to $\frac{\mu}{2}\left(\theta - \frac{1}{4}\sin 4\theta\right)$. Here $\mu$ may be 1.

Condone lack of limits here.

**A1:** Applies limits to the correct integral and proceeds to $\frac{1}{8}\pi a^2$ following correct work.

There is no need to see 0 substituted in and condone any omission of integral signs and/or $\mathrm{d}\theta$

Note that $\frac{a^2}{4}\left[\theta + \frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{2}} = \frac{1}{8}\pi a^2$ is incorrect and scores M1dM1A0A0

Use of $\sin^2 2\theta = \frac{\pm 1 \pm \cos k\theta}{2}$ with $k \neq 4$ scores M0dM0A0A0 but may lead to $\frac{1}{8}\pi a^2$

Condone use of $x$ in place of $\theta$ e.g. $\frac{a^2}{4}\left[x - \frac{1}{4}\sin 4x\right]_0^{\frac{\pi}{2}} = \frac{1}{8}\pi a^2$

**See overleaf for some alternative approaches.**

**Alternative 13(a) working backwards:**

$$\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta \qquad \frac{\mathrm{d}x}{\mathrm{d}\theta}=2a\sin\theta\cos\theta \text{ score B1 (as in main scheme)}$$

$$=\frac{1}{2}a^2\int_0^{\frac{\pi}{2}}(2\sin\theta\cos\theta)(2\sin\theta\cos\theta)\,\mathrm{d}\theta$$

$$=a\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta\,\mathrm{d}x$$

Score M1 here for using the double angle identity and replacing $\ldots\sin\theta\cos\theta\ \mathrm{d}\theta$ with $\mathrm{d}x$

$$=a\int_0^{a}\sqrt{\frac{x}{a}}\sqrt{1-\frac{x}{a}}\,\mathrm{d}x$$

Score dM1 here for a full attempt to replace all trig leading to everything in terms of $x$ only

Must come from the form $\int \ldots\sin\theta\cos\theta\,\mathrm{d}x$

A1 fully correct with limits replaced / considered before the final line and the final line fully correct with limits, integral sign and d$x$ as per the main scheme.

**Alternative 13(b) via IBP Way 1:**

$$\int_0^{\frac{\pi}{2}}\sin^2 2\theta\,\mathrm{d}\theta \qquad \left\{\begin{matrix} u=\sin^2 2\theta & v'=1 \\ u'=4\sin 2\theta\cos 2\theta & v=\theta\end{matrix}\right\}$$

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-4\int_0^{\frac{\pi}{2}}\theta\sin 2\theta\cos 2\theta\,\mathrm{d}\theta$$

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\int_0^{\frac{\pi}{2}}\theta\sin 4\theta\,\mathrm{d}\theta \qquad \left\{\begin{matrix} u=\theta & v'=\sin 4\theta \\ u'=1 & v=-\dfrac{\cos 4\theta}{4}\end{matrix}\right\}$$

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\left(\left[-\frac{\theta\cos 4\theta}{4}\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}-\frac{\cos 4\theta}{4}\,\mathrm{d}\theta\right) \qquad \text{Score M1 here.}$$

$$=\left[\theta\sin^2 2\theta\right]_0^{\frac{\pi}{2}}-2\left[-\frac{\theta\cos 4\theta}{4}+\frac{\sin 4\theta}{16}\right]_0^{\frac{\pi}{2}} \quad \text{Score dM1 here, A1 if correct (ignoring limits).}$$

$$=\left[\theta\sin^2 2\theta+\frac{\theta\cos 4\theta}{2}-\frac{\sin 4\theta}{8}\right]_0^{\frac{\pi}{2}}$$

$$=\left(0+\frac{\pi}{4}-0\right)-(0+0-0)=\frac{\pi}{4} \ \rightarrow\ \left\{\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\,\mathrm{d}x=\right\}=\frac{1}{8}\pi a^2 \ \text{ score A1* (with no errors).}$$

**Alternative 13(b) via IBP Way 2:**

$$\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta \qquad \left\{ \begin{array}{ll} u = \sin 2\theta & v' = \sin 2\theta \\ u' = 2\cos 2\theta & v = -\dfrac{\cos 2\theta}{2} \end{array} \right\}$$

$$= \left[ -\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} -\cos^2 2\theta \, d\theta$$

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta + \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \left[ -\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos^2 2\theta \, d\theta + \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta$$

$$\Rightarrow 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \left[ -\frac{\sin 2\theta \cos 2\theta}{2} \right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} 1 \, d\theta$$ Score M1 here.

$$\Rightarrow 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \left[ -\frac{\sin 2\theta \cos 2\theta}{2} + \theta \right]_0^{\frac{\pi}{2}}$$ Score dM1 here, A1 if correct including the 2 (ignoring limits).

$$\Rightarrow \int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta = \frac{1}{2}\left(0 + \frac{\pi}{2}\right) - \frac{1}{2}(0+0) = \frac{\pi}{4}$$

$$\rightarrow \left\{ \int_0^a x^{\frac{1}{2}} \sqrt{a-x} \, dx = \right\} = \frac{1}{8}\pi a^2$$ score A1* (with no errors).

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **14(a)** | $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{k}{\sqrt{r}}$ | B1 | 3.3 |
| | | **(1)** | |
| **(b)** | $0.9=\frac{k}{\sqrt{16}}\Rightarrow k=3.6$ | B1 | 3.4 |
| | $\int\sqrt{r}\ \mathrm{d}r=\int"3.6"\mathrm{d}t\Rightarrow ...$ | M1 | 2.1 |
| | $\frac{2}{3}r^{\frac{3}{2}}="3.6"t\quad\{+c\}$ | A1 | 1.1b |
| | $t=10, r=16\Rightarrow\frac{2}{3}\times16^{\frac{3}{2}}=3.6\times10+c\Rightarrow c=...$ | dM1 | 3.4 |
| | $r^{\frac{3}{2}}=5.4t+10$ * | A1* | 1.1b |
| | | **(5)** | |
| **(c)** | $t=20\Rightarrow r=\left(5.4(20)+10\right)^{\frac{2}{3}}=...$ | M1 | 3.4 |
| | $r$ = 24.1 cm | A1 | 1.1b |
| | | **(2)** | |
| **(d)** | (The model will not hold indefinitely as) the balloon may burst | B1 | 3.5b |
| | | **(1)** | |
| | | | **(9 marks)** |

**Notes**

(a)

**B1:** Correctly sets up the model. $\frac{\mathrm{d}r}{\mathrm{d}t}\propto\frac{1}{\sqrt{r}}$ scores B0 unless e.g. $\frac{\mathrm{d}r}{\mathrm{d}t}=\pm\frac{k}{\sqrt{r}}$ is seen but condone $\frac{\mathrm{d}r}{\mathrm{d}t}=\pm\frac{k}{\sqrt{r}}$ being seen at the start of (b). They may use any letter except $t$ or $r$ in place of $k$.

You may see $\frac{\mathrm{d}r}{\mathrm{d}t}=\pm\frac{1}{k\sqrt{r}}$ which is acceptable provided it is clear that it is not the $k^{\text{th}}$ root.

(b) **Note:** candidates using $\frac{\mathrm{d}r}{\mathrm{d}t}=\pm\frac{1}{\sqrt{r}}$ in (b) can score maximum B0M1A1dM0A0

**B1:** $k$ = 3.6 coming from $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{k}{\sqrt{r}}$ (or $k=\frac{5}{18}$ coming from $\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{1}{k\sqrt{r}}$) and from use of $r$ = 16 and $\frac{\mathrm{d}r}{\mathrm{d}t}=0.9$ but note that this may occur later in their working, which is perfectly fine provided it is from acceptable work. Note e.g. $k$ = –3.6 coming from $\frac{\mathrm{d}r}{\mathrm{d}t}=-\frac{k}{\sqrt{r}}$ is correct.

Note, however, that an attempt to find $k$ from comparing coefficients between $r^{\frac{3}{2}}=1.5kt+c$ and $r^{\frac{3}{2}}=5.4t+10$ is not acceptable (see special case).

They can also find $k$ by differentiating **their** $r$ = f($t$) and substituting $t$ = 10, $r$ = 16 and $\frac{\mathrm{d}r}{\mathrm{d}t}=0.9$ **if** they have also used $t$ = 10, $r$ = 16 in **their** $r$ = f($t$). This sets up simultaneous equations where $c$ can be eliminated. Use Review if you are unsure if their approach is acceptable.

**M1:** Separates the variables for their differential equation **correctly** and attempts to integrate both sides.

Must be a differential equation of the form $\frac{\mathrm{d}r}{\mathrm{d}t} = \mathrm{f}(r)$ for some function f($r$) independent of $t$.

Evidence of $r^n \to r^{n+1}$, or e.g. $\frac{1}{r} \to \ln r$ is sufficient for their attempt to integrate in $r$, but $k$ must be integrated to $kt$ o.e. e.g. $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r^2} \Rightarrow r^2 \frac{\mathrm{d}r}{\mathrm{d}t} = k \Rightarrow \lambda r^3 = kt\{+c\}$ would score this mark.

Note that they may divide by $k$ (or 3.6) prior to integrating. Here, 1 must be integrated to $t$.

**A1:** Correct integration for their $k$. Allow this mark if they have not found $k$, so allow e.g. $\frac{2}{3}r^{\frac{3}{2}} = kt\{+c\}$ with/without the constant of integration but the $\frac{2}{3}$ must be evident in some way.

Condone spurious notation for this intermediate mark e.g. integral signs left in after integrating.

**dM1:** Uses $t$ = 10, $r$ = 16 in their equation to find the constant of integration.

This mark is dependent on the first method mark.

Must have already found a value for $k$ using a valid strategy and the constant of integration must be present.

Those that found $k$ from comparing coefficients between $r^{\frac{3}{2}} = \frac{3}{2}kt + c$ and $r^{\frac{3}{2}} = 5.4t + 10$ may not score this mark.

**A1*:** Correct equation from correct working. May be seen at the start of (c).

Must follow A1 earlier so do check if this has been obtained fortuitously.

**SC:** It is possible to compare coefficients (following integration) between $r^{\frac{3}{2}} = \frac{3}{2}kt + c$ and $r^{\frac{3}{2}} = 5.4t + 10$ to deduce the value of $k$ as 3.6 (or just write their coefficient as 5.4).

The maximum that can be scored this way or by a similar invalid approach is B0M1A1dM0A0

(c)

**M1:** Substitutes $t$ = 20 into the given equation and uses correct processing to find the value of $r$

e.g. substitutes $t = 20$ into $\frac{2}{3}r^{\frac{3}{2}} = 72 + \frac{20}{3} \Rightarrow r = \left(\frac{3}{2}\left(72 + \frac{20}{3}\right)\right)^{\frac{2}{3}} = \ldots$

Their work *should* lead to the correct answer so the index work must be correct e.g. $\sqrt{118^3}$ is M0.

$\sqrt[3]{118^2}$ or $118^{\frac{2}{3}}$ are acceptable as values, i.e., the bracket must be evaluated.

May be implied by awrt 24 (cm) following $r^{\frac{3}{2}} = 118$ or by awrt 24.1 (cm). Ignore units for M1.

**A1:** cao 241mm or 241 or 24.1cm but do not accept 24.1 or e.g. $\sqrt[3]{118^2}$ cm

Correct answer with units implies both marks.

(d)

**B1:** Examples of acceptable answers (which must relate to the **model in context**):

- (The model will not hold indefinitely as) the balloon may burst/pop
- The balloon is unlikely to be (perfectly) spherical (condone circular)
- The model predicts the balloon will increase in size without limit (which is unrealistic)

Note $t \to \infty \Rightarrow r \to \infty$ is unrealistic / impossible scores B0 unless they reference e.g. the radius.

Condone the presence of additional remarks such as "the balloon may not inflate at the same rate" or "the radius of the balloon might not start at 0" that have already been addressed in the model, but these answers alone score B0.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **15(i)** | $k^2-4k+5=(k-2)^2+\ldots$ | M1 | 2.1 |
| | $k^2-4k+5=(k-2)^2+1$ so $k^2-4k+5\ldots1$<br>so $k^2-4k+5$ is always positive* | A1* | 2.4 |
| | | **(2)** | |

**Notes**

(i) Note: Using e.g. *x* throughout is acceptable for both marks.

**M1:** Starts the process of showing the given expression is positive.

Completing the square requires $(k-2)^2\pm\ldots$

Differentiation requires them to differentiate to a linear expression in *k*, set = 0 (which may be implied), solve for *k* and substitute their *k* into $k^2-4k+5$ to reach a value. Ignore e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}$

Discriminant requires a calculation of $b^2-4ac=(\pm4)^2-4\times1\times5\{=-4\}$ and might be seen embedded in the quadratic formula.

Sketches on their own are insufficient without working to find the minimum point algebraically.

Stating the minimum is (2, 1) without any evidence is M0.

**A1*:** Completes the proof with no errors and correct reasoning.

Accept e.g. "hence proved" in place of "so $k^2-4k+5$ is always positive" as long as there is sufficient justification for it being "proved".

As a minimum expect to see e.g.:

- $k^2-4k+5=(k-2)^2+1$ which is always positive as $\underline{(k-2)^2\ldots0}$ (or as squares are always positive **or zero**). Must be a correct statement. Do not condone e.g. $(k-2)^2>0$
- $k^2-4k+5=(k-2)^2+1$ so (2, 1) is the **minimum** (point) so $k^2-4k+5$ is always positive.
- $k^2-4k+5=(k-2)^2+1$ so $\underline{k=2\pm\sqrt{-1}}$ hence $k^2-4k+5$ has **no real roots** and as ***k*² has a positive coefficient** (condone e.g. positive $k^2$ or positive quadratic or $a>0$), hence proved.
- $2k-4=0\Rightarrow k=2\Rightarrow k^2-4k+5=1$ is the **minimum** value so $k^2-4k+5$ is always positive.
- $b^2-4ac=16-20\,\underline{<0}$ so $k^2-4k+5=0$ has **no real roots** and as ***k*² has a positive coefficient** (condone e.g. positive $k^2$ or positive quadratic or $a>0$), hence proved.

Note that stating $k^2-4k+5=(k-2)^2+1>0$ alone is not sufficient for the A1*

Similarly, stating $k^2-4k+5=(k-2)^2+1>0$ because $(k-2)^2>0$ (or $(k-2)^2$ is positive) is incorrect and scores A0.

**SC:** Attempts at completing the square for $k=2n$ and/or $k=2n+1$ (or $k=2n-1$) can score M1 provided at least one is attempted as far as $(\ldots)^2+\ldots$ e.g. to $(2n-2)^2+\ldots$ or e.g. $4(n-1)^2+\ldots$

Candidates are unlikely to complete the argument to score A1 (it is **not** sufficient to just consider odd/even) but if you think they might deserve the A1 then send to Review.

$k=2n\rightarrow k^2-4k+5=4n^2-8n+5$ leading to $(2n-2)^2+1$ or $4(n-1)^2+1$

$k=2n+1\rightarrow k^2-4k+5=4n^2-4n+2$ leading to $(2n-1)^2+1$ or $4\left(n-\frac{1}{2}\right)^2+1$

$k=2n-1\rightarrow k^2-4k+5=4n^2-12n+10$ leading to $(2n-3)^2+1$ or $4\left(n-\frac{3}{2}\right)^2+1$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **15(ii)** | Attempts to solve any 1 pair of the relevant 11 sim. equations.<br>e.g. **one of**<br>$\left.\begin{array}{r} 3x+2y=28 \\ 2x-5y=1 \end{array}\right\} \Rightarrow x=\ldots,(y=\ldots)$<br>**or** … (see notes for all 11) | M1 | 2.1 |
| | Attempts to solve any 2 pairs of the relevant 11 sim. equations with at least one correct and correctly rejected.<br>e.g. **both**<br>$\left.\begin{array}{r} 3x+2y=28 \\ 2x-5y=1 \end{array}\right\} \Rightarrow x=\frac{142}{19},\left(y=\frac{53}{19}\right)$ Not integers<br>**and**<br>$\left.\begin{array}{r} 3x+2y=7 \\ 2x-5y=4 \end{array}\right\} \Rightarrow x=\ldots,\ (y=\ldots)$<br>**or** … (see notes for all 11 options) | dM1<br>(A1 on EPEN) | 2.2a |
| | Attempts to solve all 5 pairs of the relevant sim. equations with positive RHS<br>**or** e.g.<br>$\left.\begin{array}{r} 3x+2y=28 \\ 2x-5y=1 \end{array}\right\} \Rightarrow x=\frac{142}{19},\left(y=\frac{53}{19}\right)$ Not integers<br>**and**<br>$\left.\begin{array}{r} 3x+2y=7 \\ 2x-5y=4 \end{array}\right\} \Rightarrow x=\ldots,\ (y=\ldots)$<br>**and**<br>all other cases are not possible as $3x + 2y$ □ 5 | ddM1 | 2.1 |
| | Requires:<br>• All cases considered, with correct values and rejected<br>• Correct reasons given in each case e.g. "not integers"<br>• Concluding statement e.g. "hence proven" | A1 | 2.4 |
| | | **(4)** | |
| | | **(6 marks)** | |

**Notes**

15(ii)

**General Note:**

Throughout this question we are condoning if candidates do not reference the following two points which are deemed fairly trivial and acceptable to be omitted at A-level:

- As $x$ and $y$ are integers then both $3x + 2y$ and $2x - 5y$ are integers.
- As $x$ and $y$ are positive then $3x + 2y > 0$

As such, we only *require* candidates to prove that there are no positive integer solutions $x$ and $y$ to simultaneous equations that have a positive RHS.

**Note:** Any attempt to solve the given simultaneous equations does not score any marks on its own.
Any attempts that use substitutions such as $x = 2n$ etc. are unlikely to score any marks.
There are other methods to eliminate some pairs of simultaneous equations e.g. by showing that the only solution to $3x + 2y = 7$ is (1, 2) which is not on $2x - 5y = 4$. Use Review in such cases.

**M1:** Attempt to solve any one of the other 11 possible cases (labelled *A*-*K*) below to find a value for *x* or a value for *y*. The attempt may be implied by a value for *x* or *y* which need not be correct.

**dM1:** Attempts to solve any two of the other 11 possible cases below to find a value for *x* or a value for *y* **with** at least one correct **and** correctly rejected. It is not necessary to find both values of *x* and *y* unless the correct value found does not cause a contradiction (see cases *D* and *G*).

**ddM1:** Attempts to solve all 5 pairs (*A*-*E*) of the relevant simultaneous equations with positive RHS **or** attempts to solve cases *A* and *B* and justifies that these are the only cases that need checking using either:

- $3x+2y \geqslant 5$ (because *x* and *y* are positive) **or**
- $3x+2y > 2x-5y$ (because *x* and *y* are positive) [cases *F*, *G* and *H* do not need checking using this approach because of the general note]

Their values for *x* and *y* do not need to be correct for this mark as long as the dM1 is scored.

**A1:** cso Shows that all necessary cases are impossible with correct values, correct reasons, and a minimal conclusion e.g. "hence proved". There is no need to say "contradiction" or restate the objective (you can also ignore any inaccurate attempt to restate the objective).
All 5 pairs of simultaneous may be rejected in one go if the rejection is sufficiently clear.
The mechanics of solving the simultaneous equations does not need to be shown.

$A: \left.\begin{matrix}3x+2y=28\\2x-5y=1\end{matrix}\right\}\Rightarrow x=\frac{142}{19},\left(y=\frac{53}{19}\right)$ Not integers

$B: \left.\begin{matrix}3x+2y=7\\2x-5y=4\end{matrix}\right\}\Rightarrow x=\frac{43}{19},\left(y=\frac{2}{19}\right)$ Not integers

$C: \left.\begin{matrix}3x+2y=4\\2x-5y=7\end{matrix}\right\}\Rightarrow x=\frac{34}{19},\left(y=-\frac{13}{19}\right)$ Not integers/not positive

$D: \left.\begin{matrix}3x+2y=2\\2x-5y=14\end{matrix}\right\}\Rightarrow (x=2),\ y=-2$ Not positive

$E: \left.\begin{matrix}3x+2y=1\\2x-5y=28\end{matrix}\right\}\Rightarrow x=\frac{61}{19},\left(y=-\frac{82}{19}\right)$ Not integers/not positive

$F: \left.\begin{matrix}3x+2y=-1\\2x-5y=-28\end{matrix}\right\}\Rightarrow x=-\frac{61}{19},\left(y=\frac{82}{19}\right)$ Not integers/not positive

$G: \left.\begin{matrix}3x+2y=-2\\2x-5y=-14\end{matrix}\right\}\Rightarrow x=-2,\ (y=2)$ Not positive

$H: \left.\begin{matrix}3x+2y=-4\\2x-5y=-7\end{matrix}\right\}\Rightarrow x=-\frac{34}{19},\left(y=\frac{13}{19}\right)$ Not integers/not positive

$I: \left.\begin{matrix}3x+2y=-7\\2x-5y=-4\end{matrix}\right\}\Rightarrow x=-\frac{43}{19},\left(y=-\frac{2}{19}\right)$ Not integers/not positive

$J: \left.\begin{matrix}3x+2y=-14\\2x-5y=-2\end{matrix}\right\}\Rightarrow x=-\frac{74}{19},\left(y=-\frac{22}{19}\right)$ Not integers/not positive

$K: \left.\begin{matrix}3x+2y=-28\\2x-5y=-1\end{matrix}\right\}\Rightarrow x=-\frac{142}{19},\left(y=-\frac{53}{19}\right)$ Not integers/not positive

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