



# Name:

**Class:** 

#### Contents

#### 1 <u>Sequences</u>

- 1.1 Finding the Next Term
- 1.2 Constant Differences
- **1.3** <u>Term to Term Rule</u>
- 1.4 **Types of Sequences**
- 1.5 Linear Sequences
- 1.6 **Position to Term Rule**
- 1.7 Generating Linear Sequences
- 1.8 Patterns
- 1.9 Fibonacci-Type Sequences
- 2 Prime Factorisation
- 2.1 Prime Factors
- 2.2 **Product of Prime Factors**
- 2.3 Using Product of Prime Factors
- 2.4 Factors from Prime Factors
- 2.5 <u>Types of Numbers from Prime Factors</u>
- 2.6 Using Prime Factorisation to Simplify Fractions
- 2.7 Using Prime Factorisation to Find Roots
- 2.8 Number of Factors

#### 3 <u>Probability</u>

- 3.1 **Probability Scale**
- 3.2 **Probability of Single Events**
- 3.3 <u>Mutually Exclusive Events</u>
- 3.4 Exhaustive Events
- 3.5 <u>Expectation</u>
- 3.6 <u>Relative Frequency</u>
- 3.7 Listing Outcomes
- 3.8 Sample Space Diagrams

### **1** Sequences

#### Introduction

A **sequence** is simply an ordered list of items (possibly infinitely long), usually with some kind of pattern.

Each item in a sequence is called a term.

## **1.1 Finding the Next Term**

In this section you will look at how to find the next term of a sequence.

#### **1.2 Constant Differences**

In this section you will look at how to find the constant difference given terms in a sequence.

| Worked Example  | Your Turn   |  |  |  |  |  |  |  |  |
|---|---|--|--|--|--|--|--|--|--|
| What is the constant difference in the sequence?                    | What is the constant difference in the sequence?      |  |  |  |  |  |  |  |  |
| The $10^{\text{th}}$ term is 52 and the $18^{\text{th}}$ term is 76 | The $10^{th}$ term is 52 and the $22^{nd}$ term is 76 |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |

| Worked Example  | Your Turn   |  |  |  |  |  |  |  |  |
|---|---|--|--|--|--|--|--|--|--|
| What is the constant difference in the sequence?                    | What is the constant difference in the sequence?      |  |  |  |  |  |  |  |  |
| The $10^{\text{th}}$ term is 76 and the $18^{\text{th}}$ term is 52 | The $10^{th}$ term is 76 and the $22^{nd}$ term is 52 |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |

#### **1.3 Term to Term Rule**

In this section you will look at how to find the term to term rule of a sequence.

Some sequences we can generate by stating a rule to say how to generate the next term given the previous term(s).

#### 3, 7, 11, 15, 19 ...

What is the rule, in words, for this sequence? **We add 4 each time.** 

The problem is that this also describes many other sequences. Can you think of another sequence that adds 4 every time?

We need to both state our **rule** and our **starting term**.

A better rule for this sequence would be: Start with 3, add 4 each time.



## Fill in the Gaps

| Five Terms of Sequence | Term-to-Term Rule |
|------------------------|-------------------|
| 10 14                  |                   |
| 3 1                    |                   |
| 5                      |                   |
| 3 9                    |                   |
| 1.7 2.1                |                   |
| 7 2 -3                 |                   |
| 40 20                  |                   |
| 1 $1\frac{1}{2}$       |                   |
|                        | add 3             |
|                        | add 7             |
| 4                      | subtract 2        |
| 2.5                    | add 0.5           |
| 5                      | subtract 2.5      |
| 2                      | multiply by 2     |
|                        | divide by 10      |
|                        | subtract 3        |

#### **1.4 Types of Sequences**

In this section you will look at the different types of sequences.

**Arithmetic/Linear:** The terms' first difference is constant. e.g., 1, 3, 5, 7, ...

**Geometric:** The terms found by multiplying by the same number each time. e.g., 2, 4, 8, 16, ...

**Quadratic:** The terms' second difference is constant. e.g., 2, 5, 10, 17, ...

**Fibonacci-Type:** The terms found by adding the previous two terms together. e.g., 1, 3, 4, 7, 11, ...

## **Fluency Practice**

| Charlie        | Types of Sequence<br>Tick or Trash<br>Tick one answer and trash<br>the other! | Lola           |
|----------------|---|----------------|
| Arithmetic     | 4, 7, 11, 15, 19, 23,   | Geometric      |
| Quadratic      | 1, 4, 9, 16, 25, 36,  | Square numbers |
| Arithmetic     | 3, 6, 12, 24, 48,   | Geometric      |
| Fibonacci      | 1, 1, 2, 3, 5, 8, 13,   | Quadratic      |
| Quadratic      | 3, 6, 11, 18, 27,   | Arithmetic     |
| Triangular     | -5, -7, -9, -11, -13,   | Arithmetic     |
| Quadratic      | 1, 3, 6, 10, 15, 21,  | Triangular     |
| Square numbers | 1, 8, 27, 64, 125,  | Cube numbers   |
| Fibonacci      | -2, 4, -8, 16, -32,   | Geometric      |
| Quadratic      | 3, 11, 23, 39, 59,  | Arithmetic     |

| Charlie        | Types of Sequence<br>Tick or Trash<br>Tick one answer and trash<br>the other! | Lola           |  |  |  |  |
|----------------|---|----------------|--|--|--|--|
| Arithmetic     | 4, 7, 11, 15, 19, 23,   | Geometric      |  |  |  |  |
| Quadratic      | 1, 4, 9, 16, 25, 36,  | Square numbers |  |  |  |  |
| Arithmetic     | 3, 6, 12, 24, 48,   | Geometric      |  |  |  |  |
| Fibonacci      | 1, 1, 2, 3, 5, 8, 13,   | Quadratic      |  |  |  |  |
| Quadratic      | 3, 6, 11, 18, 27,   | Arithmetic     |  |  |  |  |
| Triangular     | -5, -7, -9, -11, -13,   | Arithmetic     |  |  |  |  |
| Quadratic      | 1, 3, 6, 10, 15, 21,  | Triangular     |  |  |  |  |
| Square numbers | 1, 8, 27, 64, 125,  | Cube numbers   |  |  |  |  |
| Fibonacci      | -2, 4, -8, 16, -32,   | Geometric      |  |  |  |  |
| Quadratic      | 3, 11, 23, 39, 59,  | Arithmetic     |  |  |  |  |
|                |   |                |  |  |  |  |

#### **Special Sequences**

Find the next two terms in each sequence and name the sequence:

- 1, 3, 5, 7, 9, ...
- 2, 4, 6, 8, 10, ...
- 1, 4, 9, 16, 25, ...
- 1, 8, 27, 64, 125, ...
- 2, 4, 8, 16, 32, ...
- 1, 3, 6, 10, 15, ...
- 1, 1, 2, 3, 5, ...
- 2, 3, 5, 7, 11, ...
- 1, 11, 21, 1211, 111221, ...

#### **1.5 Linear Sequences**

In this section you will look at linear sequences which are also known as arithmetic sequences.

| Frayer Model – Linear Sequences |                     |  |  |  |  |  |  |  |  |
|---------------------------------|---------------------|--|--|--|--|--|--|--|--|
| Definition                      | Characteristics     |  |  |  |  |  |  |  |  |
| Examples                        | <u>Non-Examples</u> |  |  |  |  |  |  |  |  |

#### **1.6 Position to Term Rule**

In this section you will look at how to find the position to term rule of a sequence.

It is sometimes more helpful to be able to generate a term of a formula based on its position in the sequence.

We could use it to say find the 300<sup>th</sup> term of a sequence without having to write all the terms out!

We use n to mean the **position in the sequence**. So, if we want the 3<sup>rd</sup> term, n = 3.

The **position to term rule** is also called the  $n^{\text{th}}$  term rule.

This year, we will only look at how to work out the position to term rule for linear sequences. You will learn how to find the position to term rule for geometric and quadratic sequences in year 11.

| Worked Example                      | Your Turn                           |  |  |  |  |  |  |  |  |  |
|-------------------------------------|-------------------------------------|--|--|--|--|--|--|--|--|--|
| Find the $n^{\text{th}}$ term rule: | Find the $n^{\text{th}}$ term rule: |  |  |  |  |  |  |  |  |  |
| 8, 15, 22, 29, 36,                  | 11, 18, 25, 32, 39,                 |  |  |  |  |  |  |  |  |  |
| -6, 1, 8, 15, 22,                   | -3, 4, 11, 18, 25,                  |  |  |  |  |  |  |  |  |  |
| 36, 29, 22, 15, 8,                  | 39, 32, 25, 18, 11,                 |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |
|                                     |                                     |  |  |  |  |  |  |  |  |  |

| Worked Example  | Your Turn   |  |  |  |  |  |  |  |  |
|---|---|--|--|--|--|--|--|--|--|
| Find the $n^{\text{th}}$ term rule:                             | Find the $n^{\text{th}}$ term rule:                           |  |  |  |  |  |  |  |  |
| $\frac{1}{2}, \frac{7}{10}, \frac{9}{10}, 1\frac{1}{10}, \dots$ | $\frac{1}{3}, \frac{7}{9}, 1\frac{2}{9}, 1\frac{2}{3}, \dots$ |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |

| Worked Example   | Your Turn   |  |  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|--|--|
| Find the $n^{\text{th}}$ term rule:                              | Find the $n^{\text{th}}$ term rule:                               |  |  |  |  |  |  |  |  |
| $\frac{5}{12}, \frac{7}{19}, \frac{9}{26}, \frac{11}{33}, \dots$ | $\frac{6}{13}, \frac{8}{20}, \frac{10}{27}, \frac{12}{34}, \dots$ |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |

### **1.7 Generating Linear Sequences**

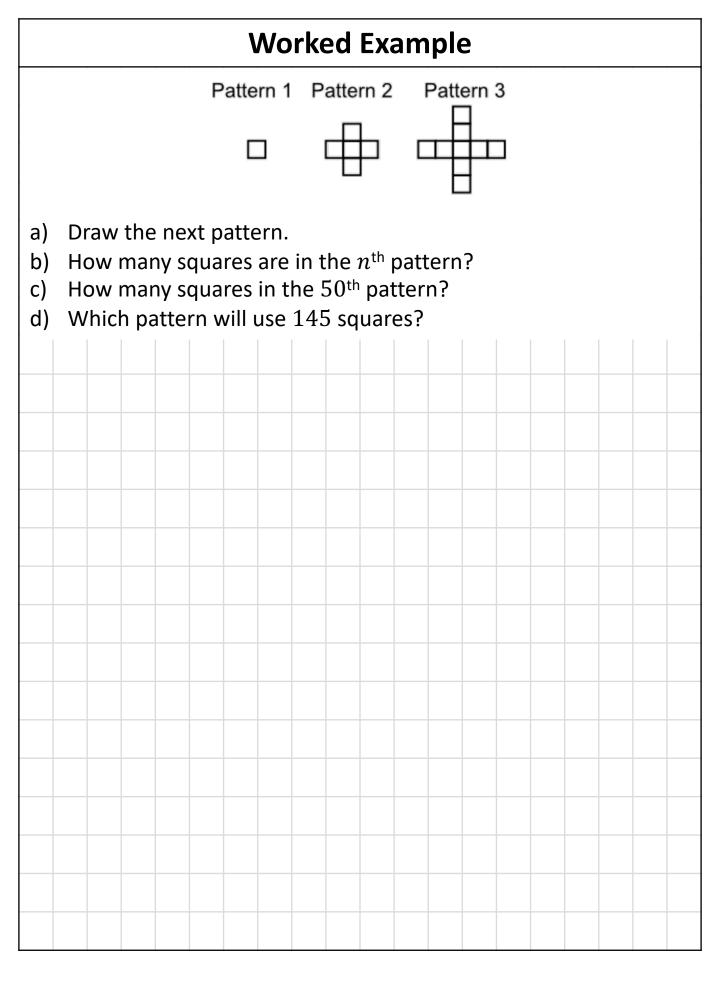
In this section you will look at how to generate terms in a linear sequence.

To generate a term of a linear sequence, substitute n (the position number) into the  $n^{\text{th}}$  term rule.

|    | Worked Example   |  |  |  |  |  |  |  |  |  |  | Your Turn   |  |  |  |  |  |  |  |  |  |
|----|--|--|--|--|--|--|--|--|--|--|--|---|--|--|--|--|--|--|--|--|--|
| a) | Generate the first 5 terms of<br>a) $5n + 3$<br>b) $-3 - 5n$ |  |  |  |  |  |  |  |  |  |  | Generate the first 5 terms of<br>a) $6n - 3$<br>b) $3 - 6n$ |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |
|    |  |  |  |  |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |

#### 1.8 Patterns

In this section you will look at how to apply your sequences knowledge to patterns.



|                      | Your Turn   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|----------------------|---|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|
|                      | Pattern 1 Pattern 2 Pattern 3   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| a)<br>b)<br>c)<br>d) | <ul> <li>b) How many squares are in the n<sup>th</sup> pattern?</li> <li>c) How many squares in the 50<sup>th</sup> pattern?</li> </ul> |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|                      |   |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

### **1.9 Fibonacci-Type Sequences**

In this section you will look at Fibonacci-type sequences.

Recall that the next term of a Fibonacci-type sequence can be found by adding the previous two terms.

| Worked Example   | Your Turn   |
|--|---|
| Find the next three terms in these Fibonacci-type sequences: | Find the next three terms in these Fibonacci-type sequences:    |
| 2, 7, 9, 16,   | 3, 11, 14, 25,  |
| $\frac{2}{3}, \frac{5}{6}, \frac{3}{2}, \frac{7}{3}, \dots$  | $\frac{3}{4}, \frac{5}{6}, \frac{19}{12}, \frac{29}{12}, \dots$ |
| $3a + 4b, a + 7b, 4a + 11b, \dots$                           | 3a - 4b, $2a - 5b$ , $5a - 9b$ ,                                |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |
|  |   |

### **2** Prime Factorisation

#### **2.1 Prime Factors**

In this section you will look at how if a number is a prime factor of another number.

3 is a prime factor of 36 (True / False)

9 is a prime factor of 36 (True / False)

1 is a prime factor of 36 (True / False)

2 is a prime factor of 36 (True / False)

7 is a prime factor of 36 (True / False)

## **Intelligent Practice**

| Intelligent Practice                        |
|---|
| 7 is a prime factor of 12 (True / False)    |
| 6 is a prime factor of 12 (True / False)    |
| 5 is a prime factor of 12 (True / False)    |
| 4 is a prime factor of 12 (True / False)    |
| 3 is a prime factor of 12 (True / False)    |
| 2 is a prime factor of 12 (True / False)    |
| 1 is a prime factor of 12 (True / False)    |
| 1 is a prime factor of 27 (True / False)    |
| 2 is a prime factor of 27 (True / False)    |
| 3 is a prime factor of 27 (True / False)    |
| 7 is a prime factor of 27 (True / False)    |
| 9 is a prime factor of 27 (True / False)    |
| 13 is a prime factor of 27 (True / False)   |
| 13 is a prime factor of 26 (True / False)   |
| 3 is a prime factor of 26 (True / False)    |
| 2 is a prime factor of 26 (True / False)    |
| 2 is a prime factor of 25 (True / False)    |
| 5 is a prime factor of 25 (True / False)    |
| 12.5 is a prime factor of 25 (True / False) |

#### **2.2 Product of Prime Factors**

In this section you will look at if a number is written as a product of prime factors, and how to write a number as a product of prime factors.

| Product of Prime Factors   | Yes / No ? |
|----------------------------|------------|
| $9 \times 11$              |            |
| 19 × 11                    |            |
| $19 \times 11^2$           |            |
| $2 \times 19 \times 11^2$  |            |
| $2 \times 19 \times 101^2$ |            |

# **Intelligent Practice**

| Product of Prime Factors                  | Yes / No ? |
|---|------------|
| 5 + 7                                     |            |
| 5 × 7                                     |            |
| 4 × 7                                     |            |
| 3 × 7                                     |            |
| 2 × 7                                     |            |
| 1 × 7                                     |            |
| $1 \times 7 \times 9$                     |            |
| $2 \times 7 \times 9$                     |            |
| $2 \times 7 \times 11$                    |            |
| $2 \times 7 + 11$                         |            |
| $2 \times 7 \times 11 \times 21$          |            |
| $2 \times 7 \times 11 \times 31$          |            |
| $1 \times 2 \times 7 \times 11 \times 31$ |            |
| $2 \times 7 \times 7 \times 11 \times 31$ |            |
| $2 \times 7^2 \times 11 \times 31$        |            |
| $2^2 \times 7^2 \times 11 \times 31$      |            |
| $2^3 \times 7^2 \times 11 \times 31$      |            |
| $2^3 \times 7^2 \times 11^5 \times 31^4$  |            |
| $1^3 \times 7^2 \times 11^5 \times 31^4$  |            |
| $2^3 \times 7^2 \times 11^5 \times 41^4$  |            |

| Worked Example                           | Your Turn                                |
|--|--|
| Express 24 as a product of prime factors | Express 48 as a product of prime factors |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

| Worked Example                           | Your Turn                                |
|--|--|
| Express 40 as a product of prime factors | Express 80 as a product of prime factors |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

| Worked Example                               | Your Turn                                    |
|--|--|
| Express $2^3 \times 3$ as an ordinary number | Express $3^2 \times 5$ as an ordinary number |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |
|  |  |

## Fill in the Gaps

| Number | Prime Eactor Decomposition                       | Index Form                |
|--------|--|---------------------------|
| Number | Prime Factor Decomposition                       | muex rorm                 |
| 6      |  |                           |
|        | $2 \times 2 \times 3$                            |                           |
| 48     |  |                           |
| 240    |  |                           |
|        |  | $2^4 \times 3^2 \times 5$ |
|        | $2 \times 2 \times 2 \times 3 \times 3$          |                           |
| 216    |  |                           |
|        |  | $2^2 \times 3^2$          |
|        | $2 \times 2 \times 3 \times 3 \times 5 \times 5$ |                           |
|        |  | $2 \times 3 \times 5$     |
| 420    |  |                           |
| 12 600 |  |                           |

### **2.3 Using Product of Prime Factors**

In this section you will look at how to use the prime factorisation of one number to write the prime factorisation of another number.

| Worked Example   | Your Turn $84 = 2^2 \times 3 \times 7$ How is 504 written as its<br>product of prime factors? |  |  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|--|--|
| $84 = 2^2 \times 3 \times 7$<br>How is 840 written as its<br>product of prime factors? |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |

| Worked Example                                   | Your Turn  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|
| $C = 3^a \times 5^b$                             | $D = 3^e \times 7^f$                             |  |  |  |  |  |  |  |  |  |  |
| a) 3 <i>C</i><br>b) 5 <i>C</i><br>c) 25 <i>C</i> | a) 3 <i>D</i><br>b) 7 <i>D</i><br>c) 27 <i>D</i> |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |

In this section you will look at if a number is a factor given the prime factorisation of the number. 10 is a factor of  $2 \times 5 \times 7 \times 11 \times 17$  (True / False) 10 is a factor of  $2 \times 5^3 \times 7 \times 11 \times 17$  (True / False) 15 is a factor of  $2 \times 5^3 \times 7 \times 11 \times 17$  (True / False) 25 is a factor of  $2 \times 5^3 \times 7 \times 11 \times 17$  (True / False) 22 is a factor of  $2 \times 5^3 \times 7 \times 11 \times 17$  (True / False)

| intelligent Flact  |                |
|--|----------------|
| 2 is a factor of $2 \times 3 \times 7 \times 13$                 | (True / False) |
| 3 is a factor of 2 $\times$ 3 $\times$ 7 $\times$ 13             | (True / False) |
| 5 is a factor of $2 \times 3 \times 7 \times 13$                 | (True / False) |
| 7 is a factor of $2 \times 3 \times 7 \times 13$                 | (True / False) |
| 4 is a factor of $2 \times 3 \times 7 \times 13$                 | (True / False) |
| 6 is a factor of $2 \times 3 \times 7 \times 13$                 | (True / False) |
| 14 is a factor of $2 \times 3 \times 7 \times 13$                | (True / False) |
| 21 is a factor of $2 \times 3 \times 7 \times 13$                | (True / False) |
| 15 is a factor of $2 \times 3 \times 7 \times 13$                | (True / False) |
| 15 is a factor of $2 \times 3 \times 5 \times 7 \times 13$       | (True / False) |
| 30 is a factor of 2 $\times$ 3 $\times$ 5 $\times$ 7 $\times$ 13 | (True / False) |
|  |                |

| intelligent i laet   |                |
|--|----------------|
| 9 is a factor of $2 \times 3 \times 5 \times 7 \times 13$      | (True / False) |
| 9 is a factor of $2 \times 3^2 \times 5 \times 7 \times 13$    | (True / False) |
| 9 is a factor of $2 \times 3^2 \times 5 \times 7 \times 23$    | (True / False) |
| 4 is a factor of $2 \times 3^2 \times 5 \times 7 \times 23$    | (True / False) |
| 4 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$  | (True / False) |
| 8 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$  | (True / False) |
| 16 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$ | (True / False) |
| 2 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$  | (True / False) |
| 28 is a factor of $2^3 \times 3^2 \times 5 \times 7 \times 23$ | (True / False) |
| 28 is a factor of $2^2 \times 3^2 \times 5 \times 7 \times 23$ | (True / False) |
| 28 is a factor of $2 \times 3^2 \times 5 \times 7 \times 23$   | (True / False) |
|  |                |

# **Fluency Practice**

| Number | Prime Factor<br>Decomposition      | Factor     | Yes/No |
|--------|------------------------------------|------------|--------|
| 2520   | $2^3 \times 3^2 \times 5 \times 7$ | 15 = 3 × 5 | Yes    |
| 2520   |                                    | 8          |        |
| 2520   |                                    | 25         |        |
| 2520   |                                    | 45         |        |
| 1320   |                                    | 22         |        |
| 1320   |                                    | 45         |        |
| 1320   |                                    | 88         |        |
| 20250  |                                    | 12         |        |
| 20250  |                                    | 27         |        |
| 20250  |                                    | 15         |        |
| 20250  |                                    | 75         |        |
| 15120  |                                    | 16         |        |
| 15120  |                                    | 21         |        |
| 15120  |                                    | 70         |        |
| 15120  |                                    | 18         |        |

### **2.5 Types of Numbers from Prime Factors**

In this section you will look at if a number is a square number or cube number or neither using its prime factorisation.

- Square numbers have even powers in their prime factorisation.
- Cube numbers have powers which are multiples of 3.

| Product of Prime Factors         | Square Number | Cube Number | Neither |
|----------------------------------|---------------|-------------|---------|
| $5^2 \times 11$                  |               |             |         |
| $5^2 \times 11^8$                |               |             |         |
| $5^6 \times 11^8$                |               |             |         |
| 5 <sup>6</sup> × 11 <sup>9</sup> |               |             |         |
| $5^6 \times 11^9 \times 17^{13}$ |               |             |         |

| Product of Prime Factors    | Square Number | Cube Number | Neither |
|-----------------------------|---------------|-------------|---------|
| 2 × 3                       |               |             |         |
| 3 × 3                       |               |             |         |
| 3 <sup>2</sup>              |               |             |         |
| 3 <sup>3</sup>              |               |             |         |
| $3^3 \times 7$              |               |             |         |
| $3^3 \times 7^2$            |               |             |         |
| $3^3 \times 7^3$            |               |             |         |
| $3^2 \times 7^2$            |               |             |         |
| $5^2 \times 7^2$            |               |             |         |
| $2 \times 5^2 \times 7^2$   |               |             |         |
| $2^2 \times 5^2 \times 7^2$ |               |             |         |
| $2^3 \times 5^2 \times 7^2$ |               |             |         |
| $2^3 \times 5^3 \times 7^3$ |               |             |         |

| Product of Prime Factors                      | Square Number | Cube Number | Neither |
|---|---------------|-------------|---------|
| $2^4 \times 5^4 \times 7^4$                   |               |             |         |
| $2^5 \times 5^5 \times 7^5$                   |               |             |         |
| $2^6 \times 5^6 \times 7^6$                   |               |             |         |
| $2^7 \times 5^7 \times 7^7$                   |               |             |         |
| $2^8 \times 5^8 \times 7^8$                   |               |             |         |
| $2^9 \times 5^9 \times 7^9$                   |               |             |         |
| $2^9 \times 5^9 \times 7^6$                   |               |             |         |
| $2^2 \times 5^9 \times 7^6$                   |               |             |         |
| $2^3 \times 5^9 \times 7^6$                   |               |             |         |
| $2^6 \times 5^{18} \times 7^{12}$             |               |             |         |
| $2^6 \times 5^{18} \times 7^{12} \times 11$   |               |             |         |
| $2^6 \times 5^{18} \times 7^{12} \times 11^2$ |               |             |         |
| $2^6 \times 5^{18} \times 7^{12} \times 11^3$ |               |             |         |

### **2.6 Using Prime Factorisation to Simplify Fractions**

In this section you will look at how to use prime factorisation to simplify fractions.

| Worked Example              | Your Turn                   |
|-----------------------------|-----------------------------|
| Simplify $\frac{693}{1925}$ | Simplify $\frac{693}{1155}$ |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |
|                             |                             |

# **2.7 Using Prime Factorisation to Find Roots**

In this section you will look at how to use prime factorisation to find roots.

| Worked Example |   |  |  |  |  |  |  |  | Your Turn                                       |  |  |  |  |  |  |  |  |  |  |
|----------------|---|--|--|--|--|--|--|--|---|--|--|--|--|--|--|--|--|--|--|
| a)<br>b)       | ) Find $\sqrt{784}$<br>) Find $\sqrt[3]{216}$ |  |  |  |  |  |  |  | a) Find $\sqrt{324}$<br>b) Find $\sqrt[3]{512}$ |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |
|                |   |  |  |  |  |  |  |  |   |  |  |  |  |  |  |  |  |  |  |

### **2.8 Number of Factors**

In this section you will look at how to use prime factorisation to find the number of factors of a number.

To get the number of factors of a number in prime factorised form, add one to each power and times the powers together.

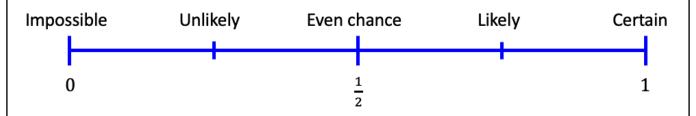
| Worked Example |  |  |  |  |  |  |  |  |                                      |   |  |                                | Yo | ur | Tu | rn |  |  |  |
|----------------|--|--|--|--|--|--|--|--|--------------------------------------|---|--|--------------------------------|----|----|----|----|--|--|--|
| a)             | a) How many factors does 36 have?  |  |  |  |  |  |  |  |                                      |   |  | How many factors does 72 have? |    |    |    |    |  |  |  |
| b)<br>c)       | <ul> <li>b) How many factors does 37<br/>have?</li> <li>c) How many factors does 38</li> </ul> |  |  |  |  |  |  |  |                                      | <ul> <li>b) How many factors does 73</li> <li>have?</li> <li>c) How many factors does 74</li> </ul> |  |                                |    |    |    |    |  |  |  |
|                | ) How many factors does 38 have?   |  |  |  |  |  |  |  | c) How many factors does 74<br>have? |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |
|                |  |  |  |  |  |  |  |  |                                      |   |  |                                |    |    |    |    |  |  |  |

# **3 Probability**

# 3.1 Probability Scale

In this section you will look at the probability scale.

- Probability is a numerical measure of how likely or unlikely an event is to occur.
- Probabilities are usually written as fractions, but can be written in any form equivalent to that fraction, e.g.,  $\frac{3}{4} = 0.75 = 75\%$
- Probabilities can be anywhere between 0 (impossible) and 1 (certain):



|                |            |     | 1              |
|----------------|------------|-----|----------------|
|                | Could it b | e a | a Probability? |
| 0.35674        | Yes        | /   | No             |
| 1.35674        | Yes        | /   | No             |
| 1              | Yes        | /   | No             |
| $\frac{1}{3}$  | Yes        | /   | No             |
| $-\frac{1}{3}$ | Yes        | /   | No             |
|                |            |     |                |
|                |            |     |                |
|                |            |     |                |

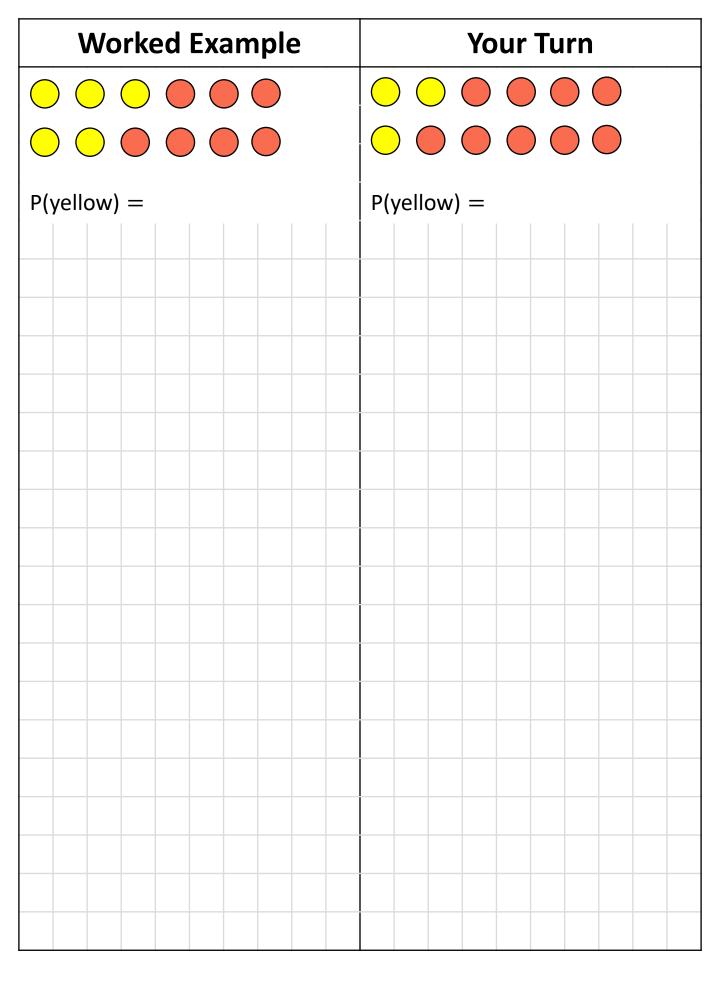
| 0.3         | Yes / No | 1               | Yes / No |
|-------------|----------|-----------------|----------|
| -0.3        | Yes / No | 2               | Yes / No |
| 1.3         | Yes / No | -1              | Yes / No |
| 0.000003    | Yes / No | <u>2</u><br>3   | Yes / No |
| 0.43045783  | Yes / No | $1\frac{2}{3}$  | Yes / No |
| 1.43045783  | Yes / No | $-\frac{2}{3}$  | Yes / No |
| -0.43045783 | Yes / No | <u>3</u><br>2   | Yes / No |
| 0.4         | Yes / No | <u>43</u><br>51 | Yes / No |
| 0           | Yes / No |                 |          |

| Your Turn   |  |  |  |  |  |  |
|---|--|--|--|--|--|--|
| <ul> <li>Describe using impossible,<br/>unlikely, even chance, likely or<br/>certain the probability that:</li> <li>a) You roll an even number on<br/>a fair die.</li> <li>b) The day after Monday is<br/>Wednesday.</li> <li>c) You roll a number between 1</li> </ul> |  |  |  |  |  |  |
| and 6 on a fair die.<br>d) You will go to space in your<br>life.  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |
|   |  |  |  |  |  |  |

# **3.2 Probability of Single Events**

In this section you will look at how to find the probability of single events.

The probability of an event occurring is defined as: Number of desired outcomes  $Probability = \frac{1}{Number of possible outcomes}$ 



### 11: **.**: .\_ . 1

|  | Increase / Decrease / Same? | P(yellow) = |
|--|-----------------------------|-------------|
| 1. O O O O O O O O O O O O O O O O O O O |                             |             |
| 2.                                       |                             |             |
| 3.                                       |                             |             |
| 4.                                       |                             |             |
| 5.                                       |                             |             |
|  | Increase / Decrease / Same? | P(yellow) = |
| 6.                                       |                             |             |
| 7.                                       |                             |             |
| 8.                                       |                             |             |
| 9.                                       |                             |             |
| 10.                                      |                             |             |

| Worke  | d Example   | Your Turn  |  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|--|--|
| red sweets, 2 y<br>4 green sweet<br>a) What is the<br>choosing a<br>b) What is the<br>choosing a<br>sweet? | e probability of<br>red sweet?<br>e probability of<br>red or yellow<br>e probability of | <ul> <li>A bag of sweets contains only 8</li> <li>red sweets, 4 yellow sweets and 8 green sweets.</li> <li>a) What is the probability of choosing a red sweet?</li> <li>b) What is the probability of choosing a red or yellow sweet?</li> <li>c) What is the probability of choosing a mint?</li> </ul> |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |

## **3.3 Mutually Exclusive Events**

In this section you will look at mutually exclusive events.

Mutually exclusive means "cannot happen at the same time".

### **Examples**

- Turning left or turning right (you cannot turn left and right at the same time).
- Going to Liverpool at 9am tomorrow or going to Manchester at 9am tomorrow (you cannot be in two places at once).

### **Non-Examples**

- Turning left and scratching your head can happen at the same time.
- Kings and hearts, because you can have a king of hearts.

## **3.4 Exhaustive Events**

In this section you will look at exhaustive events.

The probabilities of all possible outcomes add up to 1.

| Worked Example   | Your Turn  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|
| Castle FC play football matches every Saturday.                              | Castle FC play football matches every Saturday.                  |  |  |  |  |  |  |  |  |
| The table shows the probability that Castle FC will win or lose.             | The table shows the probability that Castle FC will win or lose. |  |  |  |  |  |  |  |  |
| a) Work out the probability that Castle FC will lose                         | a) Work out the probability that Castle FC will lose             |  |  |  |  |  |  |  |  |
| Win Lose   | Win Lose   |  |  |  |  |  |  |  |  |
| $\frac{3}{4}$  | $\frac{6}{8}$  |  |  |  |  |  |  |  |  |
| <ul> <li>b) Work out the probability<br/>that Castle FC will lose</li> </ul> | b) Work out the probability that Castle FC will win              |  |  |  |  |  |  |  |  |
| Win Lose   | Win Lose   |  |  |  |  |  |  |  |  |
| 0.75   | 0.75   |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

### **3.5 Expectation**

In this section you will look at expectation.

Expectation is the long-run average you would get if a test was repeated many times.

If an event has probability p, the expectation in n trials is  $n \times p$ .

Expectation is used as an estimate for how many times an event will occur.

| Worked Example  | Your Turn  |  |  |  |  |  |  |
|---|--|--|--|--|--|--|--|
| The relative frequency of a<br>teacher throwing a pen in the<br>bin is 0.5. A teacher throws a<br>pen 100 times. How many<br>throws will be successful? | The relative frequency of a<br>teacher throwing a pen in the<br>bin is 0.5. A teacher throws a<br>pen 1000 times. How many<br>throws will be successful? |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |
|   |  |  |  |  |  |  |  |

| Worked Example  | Your Turn   |  |  |  |  |  |  |  |  |
|---|---|--|--|--|--|--|--|--|--|
| If I roll a fair dice 12 times, how many times would you expect it to land on the number 1? | If I roll a fair dice 60 times, how<br>many times would you expect it<br>to land on the number 1? |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |

### **3.6 Relative Frequency**

In this section you will look at relative frequency.

In most events, it is difficult to accurately predict the probability of an event happening.

When there is no theory behind the probability of an event happening, we use **relative frequency** to calculate probabilities.

Because it is often calculated after performing experiments, it is often called **experimental probability**.

The more trials there are, the more accurate that experimental probability becomes.

| Worked Example  | Your Turn   |  |  |  |  |  |  |  |
|---|---|--|--|--|--|--|--|--|
| A coin is flipped 30 times. The results are:  | A coin is flipped 20 times. The results are:  |  |  |  |  |  |  |  |
| нтннннннтттннтт   | тнттттннн   |  |  |  |  |  |  |  |
| тттнннттннтнтнн   | ннтнтннннн  |  |  |  |  |  |  |  |
| <ul> <li>a) What are the relative<br/>frequencies for heads and<br/>tails?</li> </ul>                                       | a) What are the relative<br>frequencies for heads and<br>tails?   |  |  |  |  |  |  |  |
| <ul> <li>b) The coin is flipped 300 more<br/>times. Estimate how many<br/>times the coin will land on<br/>tails.</li> </ul> | <ul> <li>b) The coin is flipped 100 more<br/>times. Estimate how many<br/>times the coin will land on<br/>tails.</li> </ul> |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |

### **Fluency Practice**

| ur<br>e<br>d<br>ww<br>te |   | Frequency<br>5<br>4<br>1<br>7   |  |   |  |  |  |
|--------------------------|---|---|--|---|--|--|--|
| d<br>ww<br>te            |   | 4   |  |   |  |  |  |
| ie                       |   | 1   |  |   |  |  |  |
| te                       |   |   |  |   |  |  |  |
|                          |   | 7   |  |   |  |  |  |
| :k                       |   | 7   |  |   |  |  |  |
|                          |   | 3   |  |   |  |  |  |
|                          | he next (<br>ii)  | car will be<br>red  | iii)   | Not black   |  |  |  |
|                          |   | expect if<br>ii)  | 60 cars  | went past   |  |  |  |
| hrows a drawir           | ng pin 20   | 0 times and rec   | ords how   | v it lands.   |  |  |  |
| ıp                       |   |   |  |   |  |  |  |
| own                      |   |   |  |   |  |  |  |
|                          | he pin w<br>ii)   | ill land<br>pin down  |  |   |  |  |  |
|                          | ld you e><br>ii)  | vas throw<br>iii)   | n<br>400 times   |   |  |  |  |
|                          | Iv Red cars wo<br>00 cars went p<br>hrows a drawir<br>p<br>wm<br>he probability t<br>in up? | he probability the next of<br>ilue ii)<br>y Red cars would you<br>00 cars went past<br>hrows a drawing pin 20<br>up<br>wm<br>he probability the pin w<br>in up? ii)<br>y pin ups would you ex | ik     3       he probability the next car will be<br>liue     ii)       red | ik       3         he probability the next car will be liue       iii)         ne probability the next car will be liii)       iii)         ny Red cars would you expect if 00 cars went past       ii)         ny Red cars would you expect if 00 cars went past       iii)         hrows a drawing pin 200 times and records how         up       160         wn       40         he probability the pin will land in up?       ii)         iii)       pin down |  |  |  |

 A group of children are asked to write for their favourite food, and child is picked at random.

| Favourite Food | Number of people |
|----------------|------------------|
| Chinese        | 20               |
| Pizza          | 16               |
| Mexican        | 18               |

a) What is the probability the person i) liked Chinese? ii)

Didn't like Mexican best.

- b) How many people would you expect to like pizza if i) 100 people were asked
  - ii) 250 people were asked
  - iii) 1000 people were asked?
  - iv) 460 people were asked?

# **3.7 Listing Outcomes**

In this section you will look at listing outcomes.

| Worked Example  |  |  |  |  |  |  |   | Your Turn |  |  |  |  |  |  |  |  |      |
|---|--|--|--|--|--|--|---|-----------|--|--|--|--|--|--|--|--|------|
| List all the ways of arranging the<br>letters in the word:<br>CAT |  |  |  |  |  |  | List all the ways of arranging the<br>letters in the word:<br>DOG |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   | <br>      |  |  |  |  |  |  |  |  | <br> |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   | <br>      |  |  |  |  |  |  |  |  | <br> |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  | <br> |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  | <br> |
|   |  |  |  |  |  |  |   | <br>      |  |  |  |  |  |  |  |  | <br> |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |
|   |  |  |  |  |  |  |   |           |  |  |  |  |  |  |  |  |      |

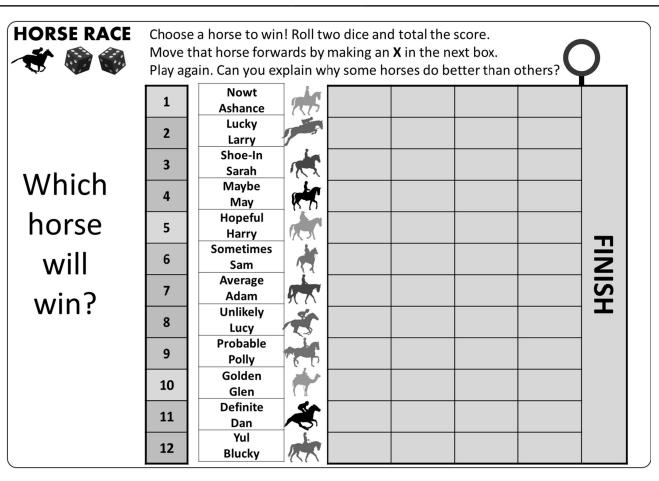
| Worked Example |                       |   |   |  |  |  |  |  |  | Your Turn |  |  |  |  |  |  |  |  |  |
|----------------|-----------------------|---|---|--|--|--|--|--|--|-----------|--|--|--|--|--|--|--|--|--|
| side           | o a c<br>ed di<br>com |   | I flip a coin and then roll a 4-<br>sided die. List the possible<br>outcomes. |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       | _ |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       | _ |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       | _ |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |
|                |                       |   |   |  |  |  |  |  |  |           |  |  |  |  |  |  |  |  |  |

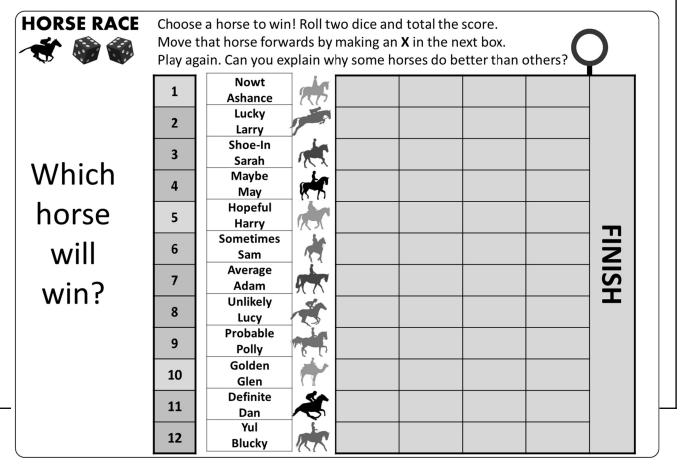
| Worked Example   | Your Turn   |  |  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|--|--|
| The first five positive integers<br>are 1, 2, 3, 4, 5. I choose two<br>numbers from this list. Write<br>down all possible combinations<br>of two numbers I can choose. | The four square numbers are<br>1, 4, 9, 16. I choose two<br>numbers from this list. Write<br>down all possible combinations<br>of two numbers I can choose. |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |

## **3.8 Sample Space Diagrams**

In this section you will look at sample space diagrams.

### **Horse Race**





### **Horse Race**



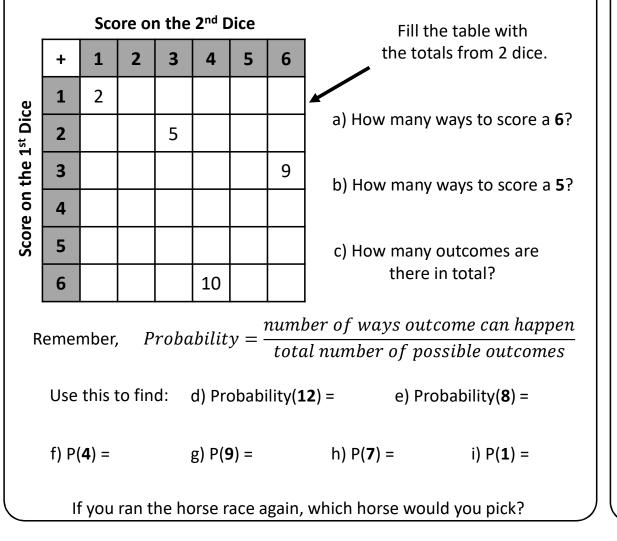
1) Who won the race(s)?

- 2) Who did you expect to win?
- 3) Do some horses have a higher chance of winning? Why?

4) How many ways can you score a 2?

- 5) How many ways can you score a **12**?
- 6) How many ways can you score a 4?
- 7) How many ways can you score a 10?
- 8) How many ways can you score a **7**?

We can make this easier by using a Sample Space Diagram.



| Worked Example  | Your Turn   |  |  |  |  |  |  |  |  |
|---|---|--|--|--|--|--|--|--|--|
| I spin these two spinners then<br>add the numbers together to get<br>a score.<br>Work out the probability that I<br>get a score of 4. | I spin these two spinners then<br>add the numbers together to get<br>a score.<br>Work out the probability that I<br>get a score of 4. |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |
|   |   |  |  |  |  |  |  |  |  |

| Worked Example   |  |  |  |  |  |  |  |  |   |  | Your Turn |  |  |  |  |  |  |   |  |  |
|--|--|--|--|--|--|--|--|--|---|--|-----------|--|--|--|--|--|--|---|--|--|
| Bag A contains four counters,<br>labelled 2, 3, 5 and 7. Bag B<br>contains five counters, labelled<br>1, 4, 9,16 and 25. A counter is<br>taken from each bag at random<br>and the numbers are added<br>together. Draw a sample space<br>to show all possible scores. |  |  |  |  |  |  |  |  | n | Bag A contains four counters,<br>labelled 3, 5, 7 and 9. Bag B<br>contains five counters, labelled<br>1, 8, 27 and 64. A counter is<br>taken from each bag at random<br>and the numbers are added<br>together. Draw a sample space<br>to show all possible scores. |           |  |  |  |  |  |  | n |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |
|  |  |  |  |  |  |  |  |  |   |  |           |  |  |  |  |  |  |   |  |  |

| Worked Example   | Your Turn   |  |  |  |  |  |  |  |  |
|--|---|--|--|--|--|--|--|--|--|
| Two four-sided dice are rolled.<br>The numbers on the two dice<br>are multiplied together. Draw a<br>sample space of the all the<br>possible products. | Two six-sided dice are rolled.<br>The numbers on the two dice<br>are multiplied together. Draw a<br>sample space of the all the<br>possible products. |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |
|  |   |  |  |  |  |  |  |  |  |