KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 9

2025 Mathematics 2026

Unit 12 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

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Unit 12 Booklet – Part 2

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Tasks

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1 Expanding Double Brackets

Worked Example	Your Turn
Expand and simplify: a) $(x+3)(x-4)$ b) $(2x+3)(3x-4)$	Expand and simplify: a) $(x+3)(x-7)$ b) $(2x+3)(3x-7)$

Dr Frost 299a, 299b, 299c, 299d, 299f, 299g, 299h and 299i

Worked Example	Your Turn
Expand and simplify: a) $(x-3)^2$ b) $(2x-3)^2$	Expand and simplify: a) $(x-7)^2$ b) $(3x-7)^2$

Fill in the Gaps

Double Brackets	Split into Two Single Brackets	Unsimplified Expression	Simplified Expression
$(x+5)(x+4)$	$x(x+4)+5(x+4)$	$x^2+4x+5x+20$	$x^2+9x+20$
$(y+9)(y-2)$	$y(y-2)+9(y-2)$		
$(b-3)(b+1)$			
$(n+8)(n-4)$			
	$d(d-6)-2(d-6)$		
	$e(e+8)+7(e+8)$		
		$t^2-3t-3t+9$	t^2-6t+9
		$c^2+5c-4c-20$	
$(h+1)(h-8)$			
	$t(t-3)-9(t-3)$		
		$a^2-3a+2a-6$	
$(k+9)(k+7)$			
		$f^2+4f-f-4$	
			$j^2+13j+36$
			$v^2-3v-40$

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(x+2)(x+3)$	$\times$	x	$+2$	$x^2+2x+3x+6$	
	x	x^2	$+2x$		
	$+3$	$+3x$	$+6$		
$(x+7)(x+4)$	$\times$	x	$+7$		
	x	x^2	$+7x$		
	$+4$	$+4x$	$+28$		
$(x+6)(x+2)$	$\times$	x	$+6$		
	x				
	$+2$				
$(x+5)(x-3)$	$\times$	x	$+5$		
	x				
	-3	$-3x$	-15		
$(x-4)(x+9)$	$\times$	x	-4		
	x				
	$+9$				
$(x+1)(x-3)$	$\times$	x	$+1$		
	x				
	-3				

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(x-5)(x+2)$	$\times$	x	-5		
	x				
	$+2$				
$(x-4)(x-6)$	$\times$	x	-4		
	x		$-4x$		
	-6	$-6x$	$+24$		
$(x-1)(x-7)$	$\times$	x	-1		
	x				
	-7				
$(x-5)(x-8)$	$\times$	x	-5		
	x				
	-8				
	$\times$			$x^2+5x+6x+30$	
	$\times$			$x^2+7x-3x-21$	
	$\times$				$x^2+11x+24$

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(2x+1)(x+5)$	$\times$	$2x$	$+1$	$2x^2+x+10x+5$	
	x	$2x^2$	$+x$		
	$+5$	$+10x$	$+5$		
$(x+3)(3x+2)$	$\times$	x	$+3$		
	$3x$	$3x^2$	$+9x$		
	$+2$	$+2x$	$+6$		
$(2x+7)(x+1)$	$\times$	$2x$	$+7$		
	x				
	$+1$				
$(4x+3)(x+3)$	$\times$				
$(2x+1)(x-3)$	$\times$	$2x$	$+1$		
	x				
	-3	$-6x$	-3		
$(3x+1)(x-5)$	$\times$	$3x$	$+1$		
	x				
	-5				

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(2x-5)(x+3)$	$\times$	$2x$	-5		
	x				
	$+3$				
$(6x-1)(x+5)$	$\times$				
$(2x-3)(x-2)$	$\times$	$2x$	-3		
	x				
	-2		$+6$		
$(3x-1)(x-8)$	$\times$				
$(3x+1)(2x-1)$	$\times$	$3x$	$+1$		
	$2x$	$6x^2$			
	-1				
$(2x+1)^2$	$\times$	$2x$	$+1$		
	$2x$				
	$+1$				
$(3-5x)^2$	$\times$	3	$-5x$		
	3				
	$-5x$				

Fill in the Gaps

Double Brackets Form	Grid			Expanded Form	Simplified Expanded Form
$(a+b)(2a+b)$	$\times$	a	$+b$	$2a^2+2ab+$	
	$2a$	$2a^2$	$+2ab$		
	$+b$				
$(3x-y)(x+2y)$	$\times$	$3x$	$-y$	$3x^2+$	
	x	$3x^2$			
	$+2y$				
$(3-x)(y+5)$	$\times$	3	$-x$		
	y				
	$+5$				
$(a+b)(c-d)$	$\times$				
$(x^2+7)(5-y)$	$\times$				
$(ab+4)(3c-2d)$	$\times$				

Worked Example	Your Turn
Expand and simplify: $(4x^{-5} - 3)(2x^5 - 3)$	Expand and simplify: $(y^4 - 7)(9y^{-4} + 1)$

Worked Example	Your Turn
Expand and simplify: a) $(8x^2 - 7x)(4x^2 + 9x - 5)$ b) $(8x^2 - 9x + 4)(6x^2 + 5x)$	Expand and simplify: a) $(9y^2 - 7y)(5y^2 + 4y + 5)$ b) $(5y^2 - 6y + 7)(5y^2 + 8y)$

Dr Frost 299j and 299k

Fill in the Gaps

Double Brackets Form	Grid				Expanded Form	Simplified Expanded Form
$x(x^2+2x+3)$	$\times$	x^2	$+2x$	$+3$	x^3+2x^2+3x	x^3+2x^2+3x
	x	x^3	$+2x^2$	$+3x$		
$2x(x^2-x+4)$	$\times$	x^2	$-x$	$+4$		
	$2x$	$2x^3$				
$(x+1)(x^2+3x+2)$	$\times$	x^2	$+3x$	$+2$	$x^3+3x^2+2x+x^2+$	
	x	x^3	$+3x^2$	$+2x$		
	$+1$	$+x^2$				
$(x+2)(x^2+4x-3)$	$\times$	x^2	$+4x$	-3		
	x	x^3	$+4x^2$	$-3x$		
	$+2$			-6		
$(x-4)(x^2-x+5)$	$\times$	x^2	$-x$	$+5$		
	x					
	-4	$-4x^2$	$+4x$			

Fill in the Gaps

$(x+5)(2x^2+x-1)$	$\times$	$2x^2$	$+x$	-1		
	x					
	$+5$					
$(x-3)(3x^2+2x-1)$	$\times$	$3x^2$	$+2x$	-1		
	x					
	-3					
$(2x+1)(x^2-5x+2)$	$\times$					
$(3x-2)(6-3x-x^2)$	$\times$					
$(7-2x)(6-x-3x^2)$	$\times$					

Worked Example	Your Turn
Expand and simplify: a) $4x(2x-5)(3x+3)$ b) $5y(3y-2)^2$	Expand and simplify: a) $3y(2y-3)(2y-6)$ b) $3x(2x-6)^2$

Worked Example	Your Turn
Expand and simplify: $(3x - 6)(3x + 1) - 4x(x + 1)$	Expand and simplify: $(2x - 2)(x - 4) - 3(x + 2)$

Dr Frost 299n and 299o

Worked Example	Your Turn
Expand and simplify: $(x+5)(2x-1)-(2x+3)^2$	Expand and simplify: $(2x+5)^2-(3x-4)(3x+6)$

Dr Frost 299p and 299q

Worked Example	Your Turn
Expand and simplify: $(2x + 3)^2 - (2x - 1)^2$	Expand and simplify: $(3x - 1)^2 - (2x - 3)^2$

Worked Example	Your Turn
Expand and simplify: $\left(3-\frac{5}{z}\right)^2$	Expand and simplify: $\left(2-\frac{4}{y}\right)^2$

Extra Notes

2 Factorising by Grouping

Worked Example	Your Turn
Factorise: a) $3x(x+1)-5(x+1)$ b) $3x(x+1)^2-5(x+1)$	Factorise: a) $5x(x+1)-3(x+1)$ b) $5x(x+1)^2-3(x+1)$

Worked Example	Your Turn
Factorise: a) $2(x+8)^2 - 6(x+8)$ b) $2(x+8) - 6(x+8)^2$	Factorise: a) $8(x+2) - 4(x+2)^2$ b) $8(x+2)^2 - 4(x+2)$

Worked Example	Your Turn
Factorise: a) $(a-9)(a-4)-2(a-4)$ b) $(x-7)^4+(x+3)(x-7)^3$	Factorise: a) $(x-8)(x-1)-5(x-1)$ b) $(y-3)^5+(y-8)(y-3)^4$

Dr Frost 365b and 365d

Worked Example	Your Turn
Factorise $35xa - 20x + 21a - 12$	Factorise $10px - 35p + 16x - 56$

Worked Example	Your Turn
Factorise: a) $2x^2 + 2x - 3x - 3$ b) $2x^2 + 2x + 3x + 3$	Factorise: a) $2x^2 - 2x - 3x + 3$ b) $2x^2 - 2x + 3x - 3$

Worked Example	Your Turn
By grouping together pairs of terms, fully factorise: $6a^3 - 36a^2 + 12a - 72$	By grouping together pairs of terms, fully factorise: $6x^3 - 18x^2 + 15x - 45$

Worked Example	Your Turn
By considering groups of terms, factorise fully: $5x^2 - 9x - 2 - 4xy + 8y$	By considering groups of terms, factorise fully: $2a^2 - 9a - 5 + 6ab + 3b$

Extra Notes

3 Factorising Quadratics

Sum and Product

	Sum is Positive	**Sum is Negative**
Product is Positive	$__ \times __ = 14$ $__ + __ = 9$	$__ \times __ = 14$ $__ + __ = -9$
Product is Negative	$__ \times __ = -14$ $__ + __ = 5$	$__ \times __ = -14$ $__ + __ = -5$

	Sum is Positive	**Sum is Negative**
Product is Positive	Positive and Positive	Negative and Negative
Product is Negative	Positive and Negative where the size of the positive is greater than the size of the negative	Positive and Negative where the size of the negative is greater than the size of the positive

Worked Example	Your Turn
$__ \times __ = 140$ $__ + __ = -33$	$__ \times __ = 280$ $__ + __ = -43$

Fill in the Gaps

Q	*Integer 1*	*Integer 2*	*Sum*	*Product*
1	5	7		
2	-5	7		
3			-2	-35
4	-5		-12	
5			8	15
6	-3			-15
7	3			-15
8	-3	-5		
9			10	24
10	-4	6		
11			-2	-24
12		-6	-10	24

Quadratics

Every quadratic expression can be written in the form

$$ax^2 + bx + c$$

- a is the coefficient of x^2
- b is the coefficient of x (or x^1)
- c is the coefficient of x^0 (or the constant term)

Quadratic expressions in x

$x^2 + 4x + 3$

$2x^2 + 7x - \frac{13}{2}$

$4x^2 - 3.5$

$-x^2 + 3x$

$-5x^2 - x$

$7 - x^2$

$7 + \frac{x}{2} + 5x^2$

Not quadratic expressions

$x^3 + 3x^2 + 6x + 7$

$3x^2 + 6x^{-1} + 7$

$x^2 - 6x^{\frac{1}{2}} + 9$

So far, we have factorised monic quadratics, where $a = 1$.
Now we will learn to factorise non-monic quadratics, where $a \neq 1$.

Monic quadratics

$x^2 - x - 1$

$x^2 + 8x$

$x^2 + 9x - 1$

$x^2 + x + 12$

$x^2 - 121$

Non-monic quadratics

$3x^2 + 6x + 7$

$3x^2 - 6x + 7$

$16 - x^2$

$7x^2 - 7x + 1$

Worked Example	Your Turn
Factorise: $3x^2 + 10x + 8$	Factorise: $3x^2 - 10x + 8$

Dr Frost 364a and 364b

Worked Example	Your Turn
Factorise: $3x^2 + 2x - 8$	Factorise: $3x^2 - 2x - 8$

Dr Frost 364c and 364d

Fill in the Gaps

Quadratic	$a \times c$	$\times$ to give ac $+$ to give b	Split the middle term	Group and Factorise	Factorised Quadratic
$2x^2 + 7x + 6$	12	$+4, +3$	$2x^2 + 4x + 3x + 6$	$2x(x + 2) + 3(x + 2)$	$(2x + 3)(x + 2)$
$3x^2 + 19x + 6$	18	$+18, +1$	$3x^2 + 18x + x + 6$	$3x(x + 6) + 1(x + 6)$	
$8x^2 + 6x - 9$	-72	$+12, -6$			
$5x^2 + 12x - 9$					
$9x^2 - 9x - 10$					
$6x^2 + x - 5$					
$8x^2 - 18x + 7$				$2x(4x - 7) - 1(4x - 7)$	
$4x^2 - 12x + 5$					
		$+15, +2$	$6x^2 + 15x + 2x + 5$		
				$4x(3x - 2) + 5(3x - 2)$	
					$(5x - 4)(2x + 1)$

Worked Example	Your Turn
Finish factorising: a) $(x + 2)(10x + 50)$ b) $(4x + 2)(10x + 50)$	Finish factorising: a) $(x + 2)(5x + 15)$ b) $(4x + 2)(5x + 15)$

Worked Example	Your Turn
Factorise: $6x^2 + 20x + 16$	Factorise: $6x^2 - 4x - 16$

Worked Example	Your Turn
Factorise: $x^2 + 20x + 96$	Factorise: $x^2 - 4x - 96$

Dr Frost 362a, 362b, 362c and 362d

Fill in the Gaps

Quadratic Expression	Sum	Product	Pair of Values		Factorised Expression
$x^2 + 8x + 15$	$+8$	$+15$	$+5$	$+3$	$(x + 5)(x + 3)$
$x^2 + 5x + 6$	$+5$	$+6$	$+3$		
$x^2 + 6x + 5$	$+6$	$+5$	$+5$		
$x^2 + 10x + 21$		$+21$			
$x^2 + 14x + 24$	$+14$				
$x^2 - 7x + 10$			-5		
$x^2 - 11x + 18$					
$x^2 + 3x - 10$			$+5$		
$x^2 + 3x - 18$					
$x^2 + 11x + 18$					
$x^2 - 4x - 21$			-7		
$x^2 - 8x - 9$					
$x^2 - 6x + 9$					
$x^2 + x - 20$	$+1$				
$x^2 - x - 6$					
$x^2 - 19x - 42$					
			-8	-3	
	$+4$		-1		

Fill in the Gaps

Quadratic Expression	Expanded Expression	Grid	Factorised Expression
$x^2 + 9x + 20$	$x^2 + 5x + 4x + 20$	$\times$ | x | $+5$ x | x^2 | $+5x$ $+4$ | $+4x$ | $+20$	$(x + 5)(x + 4)$
$x^2 + 12x + 27$	$x^2 + 9x + 3x + 27$	$\times$ | | | x^2 | $+9x$ | $+3x$ | $+27$	
$x^2 + 7x + 12$	$x^2 + 4x + 3x + 12$	$\times$ | | | | | |	
$x^2 - 8x + 15$		$\times$ | | | x^2 | $-3x$ | | $+15$	
$x^2 + 4x - 12$		$\times$ | | | x^2 | | $-2x$ | -12	
$x^2 + 3x - 10$		$\times$ | | | x^2 | $+5x$ | | -10	
$x^2 - 8x - 9$		$\times$ | | | x^2 | | $-9x$ | -9	

Fill in the Gaps

Quadratic Expression	Expanded Expression	Grid	Factorised Expression
$x^2 + 10x + 21$			
$x^2 - 11x + 24$			
$x^2 + 7x - 18$			
$x^2 - x - 12$			
$x^2 - 6x + 8$			
$x^2 + 7x - 30$			
$x^2 - 19x - 42$			

Fill in the Gaps

1st Bracket	2nd Bracket	Expanded Form	Simplified Form
$(x+2)$	$(x+5)$	$x^2+2x+5x+10$	$x^2+7x+10$
$(x-3)$	$(x+6)$		
$(x+4)$	$(x-5)$		
$(x-3)$	$(x-2)$		
$(x+3)$		$x^2+3x+7x+21$	
	$(x+1)$	$x^2-5x+x-5$	
	$(x+2)$		$x^2+11x+18$
$(x+5)$			$x^2+2x-15$
	$(x-2)$		x^2-6x+8
	$(x-4)$		$x^2+5x-36$
		$x^2+4x+7x+28$	$x^2+11x+28$
		$x^2+4x+2x+8$	x^2+6x+8
			$x^2+10x+9$
			$x^2+13x+40$
		$x^2-4x-3x+12$	$x^2-7x+12$
			$x^2+2x-35$
			$x^2+4x-12$
			x^2-7x+6
			$x^2-12x+20$
			$x^2-8x+16$

Worked Example	Your Turn
Show that $(x+5)^2 + (x+8)^2 - (x+11)^2$ can be expressed in the form $(x+a)(x+b)$ where a and b are constants to be found and $a < \mathrm{b}$	Show that $(x+7)^2 + (x+8)^2 - (x+9)^2$ can be expressed in the form $(x+a)(x+b)$ where a and b are constants to be found and $a < \mathrm{b}$

Worked Example	Your Turn
Factorise: $-x^2 - 11x - 28$	Factorise: $-x^2 + 5x - 6$

Worked Example	Your Turn
Factorise: $-6x^2 - 19x + 7$	Factorise: $-12x^2 + 11x - 2$

Worked Example	Your Turn
Factorise: $6x^2 + 41x + 30 - (x + 6)(x + 7)$	Factorise: $6x^2 + 22x + 12 - (x + 3)(x + 5)$

Worked Example	Your Turn
Factorise: $6x^2 - 23xy + 20y^2$	Factorise: $5x^2 - 17xy + 6y^2$

Extra Notes

4 Difference of Two Squares

Worked Example	Your Turn
Factorise: a) $x^2 - 9$ b) $9 - x^2$ c) $x^2 - 9y^6$ d) $16x^2 - 9y^6$	Factorise: a) $x^2 - 25$ b) $25 - x^2$ c) $x^2 - 25y^4$ d) $16x^2 - 25y^4$

Dr Frost 363a, 363b and 363c

Worked Example	Your Turn
Factorise: a) $2x^2 - 8$ b) $2x^2 - 8y^6$	Factorise: a) $2x^2 - 50$ b) $2x^2 - 50y^4$

Worked Example	Your Turn
Factorise: a) $16x^2y - 9yz^2$ b) $2x^2y^3 - 8y^3z^2$	Factorise: a) $16xy^2 - 25xz^2$ b) $2x^4y^2 - 50x^4z^2$

Dr Frost 363e and 363f

Worked Example	Your Turn
Work out $51^2 - 49^2$	Work out $53^2 - 47^2$

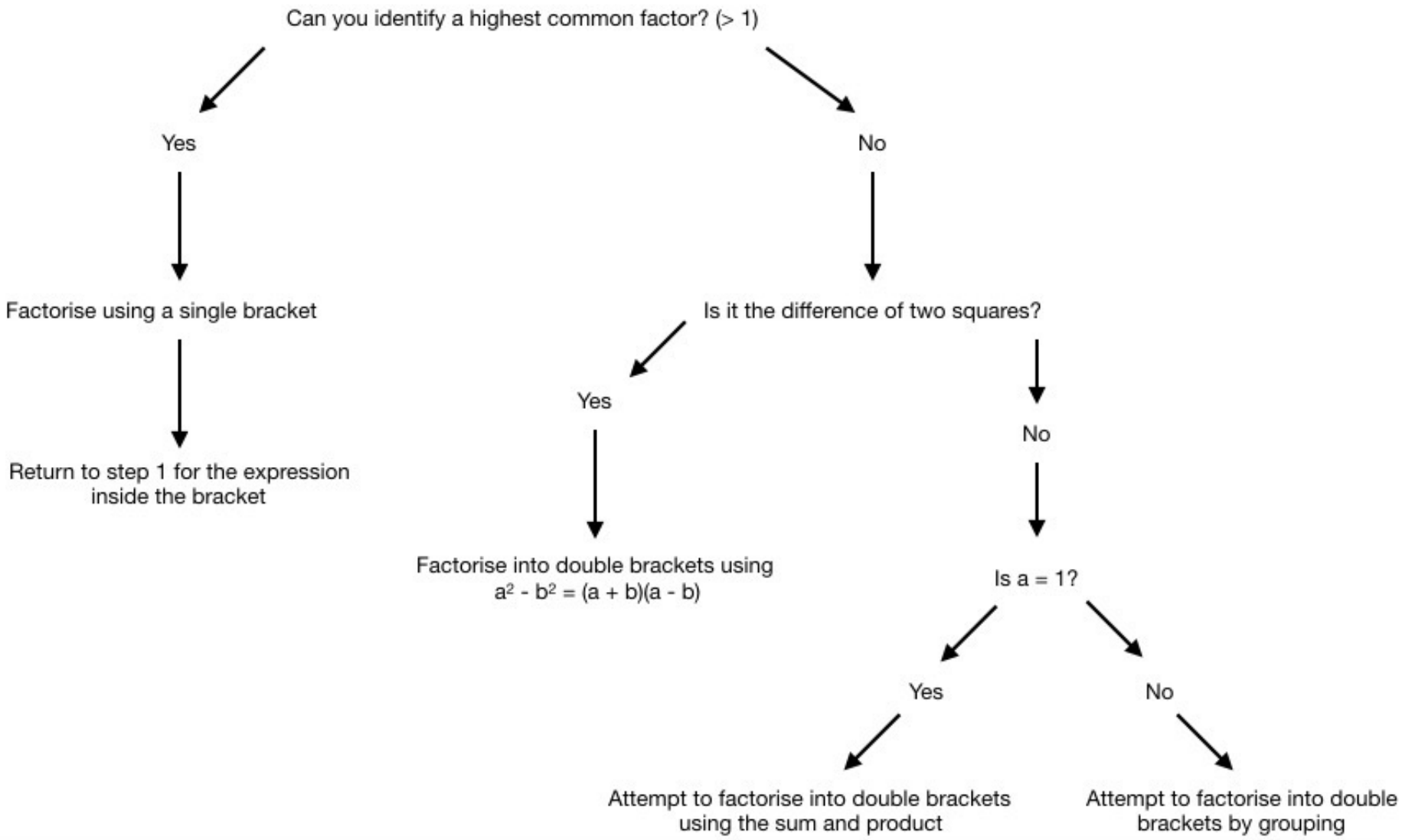
Dr Frost 363g

Worked Example	Your Turn
Factorise: $x^2 - y^2 - 7x + 7y$	Factorise: $x^2 - y^2 - 6x - 6y$

Worked Example	Your Turn
Factorise fully: $81x^4 - 16$	Factorise fully: $16x^4 - 1$

Worked Example	Your Turn
Factorise fully: a) $(11x-8)^2-(4x-7)^2$ b) $(8x-5y)^2-(x+3y)^2$	Factorise fully: a) $(9x-7)^2-(7x+4)^2$ b) $(7x+9y)^2-(4x-y)^2$

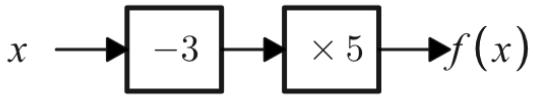
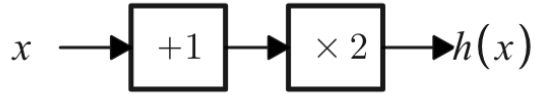
Flowchart


Can you identify a highest common factor? (> 1)
Yes
No
Factorise using a single bracket
Return to step 1 for the expression inside the bracket
Is it the difference of two squares?
Yes
Factorise into double brackets using $a^2 - b^2 = (a + b)(a - b)$
No
Is a = 1?
Yes
Attempt to factorise into double brackets using the sum and product
No
Attempt to factorise into double brackets by grouping

Worked Example	Your Turn
Factorise: a) $2x^3 - 14x^2 + 24x$ b) $54a^3 - 24ab^2$	Factorise: a) $2x^3 + 4x^2 - 70x$ b) $32a^3 - 98ab^2$

Extra Notes

5 Basic Functions

Worked Example	Your Turn
Below is a function machine.  Find an expression for $f(x)$	Below is a function machine.  Find an expression for $h(x)$

Fill in the Gaps

Input	Function Machine		Output	Function
x	$\times 3$	$+8$	$f(x)$	$f(x) = 3x + 8$
x	$\times 5$	-1	$f(x)$	
x	$\times 2$		$g(x)$	$g(x) = 2x - 7$
x	-1		$f(x)$	$f(x) = 4(x - 1)$
x		$+2$	$h(x)$	$h(x) = \frac{x}{3} + 2$
x	$\div 2$	-5	$f(x)$	
x	$+7$	$\div 4$	$f(x)$	
x	*square*	$+3$	$g(x)$	
x	$+2$	*square root*	$f(x)$	
x			$f(x)$	$f(x) = 10x^2$
x				$g(x) = \sqrt{x} + 8$
x				$h(x) = \frac{x^3}{2}$
x	*reciprocal*	$+8$	$f(x)$	

Fill in the Gaps

Input	Function Machine			Output	Function
x →	$\times 3$ →	-1 →	$\div 4$ →	$f(x)$	$f(x) = \dfrac{3x-1}{4}$
x →	$+2$ →	$\div 3$ →	*square root* →	$f(x)$	
x →	$+3$ →	*square* →	-5 →	$h(x)$	
x →	*square root* →	→	$+1$ →	$f(x)$	$f(x) = 4\sqrt{x} + 1$
x →	*reciprocal* →	→	→	$g(x)$	$g(x) = 2\left(\dfrac{1}{x} - 3\right)$
x →	→	→	→	$f(x)$	$f(x) = \dfrac{1}{3x} - 1$
x →	→	→	→		$f(x) = \left(\dfrac{x+2}{3}\right)^2$
x →	→	→	→		$g(x) = \dfrac{1}{4x-3}$

Worked Example	Your Turn
If $f(x) = 3x + 4$, evaluate: a) $f(2)$ b) $f(-4)$	If $g(x) = -3x + 7$, evaluate: a) $g(5)$ b) $g(-2)$

Worked Example	Your Turn
If $f(x) = 2x^2 + 3x$, evaluate: a) $f(4)$ b) $f(-2)$	If $g(x) = 3x^2 - 4x$, evaluate: a) $g(5)$ b) $g(-2)$

Question	Input	Function Machine		Output
$f(x) = 2x - 1$ Find $f(5)$	5 →	$\times 2$ →	-1 →	
$f(x) = 4x + 3$ Find $f(-3)$	→	$\times 4$ →	$+3$ →	
$f(x) = x^2 - 5$ Find $f(4)$	→	*square* →	→	
$g(x) = \frac{x+3}{2}$ Find $g(11)$	→	→	$\div 2$ →	
$f(x) = \sqrt{x+5}$ Find $f(4)$	→	→	→	
$f(x) = \frac{x}{2} + 1$ Find $f(2.5)$	→	→	→	
$f(x) = 3(x+2)$ Find $f(-7)$	→	→	→	
$h(x) = (x+3)^2$ Find $h(-5)$	→	$+3$ →	→	
$f(x) = 4x^2$ Find $f(\sqrt{3})$	→	*square* →	→	
$f(x) = \frac{1}{x} + 9$ Find $f(2)$	→	→	→	
$f(x) =$ Find	10 →	-1 →	$\div 3$ →	
$f(x) =$ Find	-7 →	$+2$ →	*reciprocal* →	

Fill in the Gaps

Question	Input	Function Machine			Output
$f(x) = (2x + 1)^2$ Find $f(4)$				square	
$g(x) = \frac{2x - 5}{3}$ Find $g(6.25)$					
$f(x) = \sqrt{3x - 2}$ Find $f(9)$				square root	
$h(x) = 2(x^3 - 6)$ Find $h(-2)$					
$f(x) = \frac{3}{x} + 7.5$ Find $f(2)$		reciprocal			
$f(x) =$ Find	-2	-1	square	$\times 4$	
$f(x) =$ Find	7	$+2$	reciprocal	square root	

Find the values of:

$f(3), f(-5)$ and $f\left(\frac{1}{2}\right)$ for the following functions:

	$f(3)$	$f(-5)$	$f(\frac{1}{2})$
$f(x) = x + 5$			
$f(x) = 2x + 5$			
$f(x) = \frac{x}{2} + 5$			
$f(x) = \frac{x}{2} + 5x$			
$f(x) = x^2 + 5x$			
$f(x) = 5x^2$			
$f(x) = \frac{5}{x^2}$			
$f(x) = \frac{1}{5x^2}$			
$f(x) =$	16	-24	$\frac{7}{2}$
	7		
		$\frac{25}{2}$	
			-11

For the last three questions, find a possible function for $f(x)$, and therefore, find $f(3), f(-5)$ and $f\left(\frac{1}{2}\right)$.

Worked Example	Your Turn
If $f(x) = 3x + 4$, find x when $f(x) = 19$	If $g(x) = -3x + 7$, find x when $g(x) = 1$

Worked Example	Your Turn
Given $f(x) = 2x^2 + 2x - 2$ $g(x) = 2x^2 + x - 4$ Solve $f(x) = g(x)$	Given $g(x) = 3x^2 + x + 19$ $h(x) = 3x^2 - 3x + 3$ Solve $g(x) = h(x)$

Worked Example	Your Turn
If $f(x) = x^2 - 2$, evaluate: a) $f(x-2)$ b) $f(2x)$	If $g(x) = x^2 + 3$, evaluate: a) $g(x-3)$ b) $g(3x)$

Worked Example	Your Turn
If $f(x) = 3x^2 - 5x - 2$, evaluate $f(x - 2)$	If $g(x) = 5x^2 - 2x + 3$, evaluate $g(x - 3)$

Worked Example	Your Turn
Given $g(x) = 3x^2 + 2x + 1$ Find an expression for: a) $-4g(x)$ b) $g(x) + 2$	Given $h(t) = 2t^2 - t - 3$ Find an expression for: a) $5h(t)$ b) $h(t) - 3$

Worked Example	Your Turn
Given $f(x) = 3x^2 + 4$ Find $\big(h(x+1)\big)^2$, giving your answer in the form $ax^4 + bx^3 + cx^2 + dx + e$	Given $f(t) = 4t^2 + 3t$ Find $\big(f(t-4)\big)^2$, giving your answer in the form $at^4 + bt^3 + ct^2 + dt + e$

Worked Example	Your Turn
Given $f(t) = 4t^2 + 3t$ Find $f(t) - f(t-2)$, giving your answer in the form $at + b$.	Given $f(t) = 3t^2 - 4t$ Find $f(t-4) - f(t)$, giving your answer in the form $at + b$.

Extra Notes

6 Changing the Subject

Is a the subject?

$a = 3x + 1$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a + 1 = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$4a = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$4b + 2 = a$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = 5a - 7b + 3$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a^2 = 3b + 2$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = \frac{1}{2}b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a = \frac{7b + 55c}{2}$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$\sqrt{b} = a$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$\sqrt{a} = b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject
$a + 0 = b$	$\boldsymbol{a}$ is the subject	$\boldsymbol{a}$ is the NOT subject

Intelligent Practice

Formula	Is a the subject?
$a = b + 3$	
$b + 3 = a$	
$a + 3 = b$	
$a + c = b$	
$ac = b$	
$a = bc$	
$a = bc - 6$	
$a = bc - x$	
$a = bc - a$	
$a = bc - a^2$	
$-a = b + 3$	
$\frac{1}{a} = b + 3$	

Formula	Is a the subject?
$a^2 = b + 3$	
$a = b^2 + 3$	
$2a = b^2 + 3$	
$\sqrt{a} = b^2 + 3$	
$a = \sqrt{\frac{b^2 + 3}{2}}$	
$\sqrt{\frac{b^2 + 3}{2}} = a$	
$\sqrt{\frac{b^2 + 3}{2a}} = a$	

Worked Example	Your Turn
Make x the subject of the following formulae: a) $6x + 1 = 7y + 2$ b) $y = \frac{9z+8w}{2x}$	Make a the subject of the following formulae: a) $5a - 3 = 9b + 6$ b) $\mathrm{b} = \frac{2c+5d}{9a}$

Dr Frost 201b and 201d

Worked Example	Your Turn
Make x the subject of the following formula: $y = \frac{7 - 5x}{2}$	Make a the subject of the following formula: $b = \frac{2 - a}{8}$

Worked Example	Your Turn
Make x the subject of the following formula: $3(6z - 5x) = 2z - 4$	Make a the subject of the following formula: $5c - 2 = 3(5c - 2a)$

Worked Example	Your Turn
Make x the subject of the following formula: $2y^2 - 4 = \frac{x}{4u} - 3$	Make a the subject of the following formula: $4b^2 - 1 = \frac{a - 2c}{5}$

Worked Example	Your Turn
Make x the subject of the following formula: $y = \frac{2(3w - 2x)}{5}$	Make a the subject of the following formula: $b = \frac{2(5a - 6)}{9b}$

Worked Example	Your Turn
Make x the subject of the following formula: $7y = \frac{z-4}{3x-2}$	Make a the subject of the following formula: $b = \frac{10c}{7a-2d}$

Worked Example	Your Turn
Make x the subject of the following formula: $5y^3 = 3 - \frac{3}{4x}$	Make a the subject of the following formula: $7b = 9 - \frac{2c}{5a}$

Worked Example	Your Turn
Make x the subject of the following formulae: a) $3y = 6 - 2x^2$ b) $2y = \frac{3x^4+1}{5}$	Make a the subject of the following formulae: a) $4b = 3 - 5a^2$ b) $4b = \frac{3x^4+2}{9}$

Dr Frost 260d and 260e

Worked Example	Your Turn
Make x the subject of the following formulae: a) $5y = 2(y - x^5)$ b) $y = (3 - 2x)^3$	Make a the subject of the following formulae: a) $b = 5(3b - a^5)$ b) $b = (3d - a)^5$

Dr Frost 260f and 260g

Worked Example	Your Turn
Make x the subject of the following formula: $y^3 = \frac{3(x+1)^3}{7}$	Make a the subject of the following formula: $b^3 = \frac{2(3-a^3)}{9}$

Worked Example	Your Turn
Make x the subject of the following formulae: a) $2y^2 = 7 - \sqrt{3x}$ b) $y^3 = \sqrt{7 - 9x}$	Make a the subject of the following formulae: a) $b = 7\sqrt[4]{a} - 1$ b) $3b^3 = \sqrt{5a + 7}$

Fill in the Gaps

Q	$\mathbf{a} =$	$\mathbf{b} =$	$\mathbf{c} =$
1	$a = b + c$		
2	$a = b - c$		
3		$b = ac$	
4			$c = \frac{2b}{a}$
5	$a = 2b + c$		
6		$b = \frac{a + c}{2}$	
7		$b = \frac{a}{2} + c$	
8			$c = b^2 - \frac{a}{2}$
9		$b = \frac{a}{2} + \sqrt{c}$	
10	$a = \frac{2b - 2\sqrt{c}}{3}$		

Worked Example	Your Turn
Make a the subject of the following formula: $ax + ay = 3$	Make a the subject of the following formula: $ak + am = 5$

Worked Example	Your Turn
Make a the subject of the following formula: $2d = 4a^2 - 4a^2c$	Make a the subject of the following formula: $2d = 4a^2 - 6a^2c$

Worked Example	Your Turn
Make a the subject of the following formula: $ax + 2y = 5y + am$	Make a the subject of the following formula: $ab + 3y = 7y + ak$

Worked Example	Your Turn
Make x the subject of the following formula: $ax + ay = cx + by$	Make x the subject of the following formula: $yx + wz = 3xz + 3yz$

Rearrange each formula to make x the subject.

Question	Collect all x terms	Take out x as a common factor	Divide by bracket
$ax = bx + c$	$ax - bx = c$	$x(a - b) = c$	$x = \frac{c}{a - b}$
$ax = c - bx$	$ax + bx = c$		
$ax - b = cx$			
$ax - b = cx - d$	$ax + cx = b - d$		
$ax + b = d - cx$			
$x - a = bx$	$x - bx = a$	$x(1 - b) = a$	
$ax - b = c + x$			
$2 + ax = bx + c$			
$2x - a = b + cx$			
$a^2x - 1 = c - bx$			
$2ax - 1 = b + c^2x$			
$a - b^3x = 2 - cx$			

Worked Example	Your Turn
Make x the subject of the following formula: $w = \frac{x + a}{x - a}$	Make x the subject of the following formula: $w = \frac{x + 2y}{x - y}$

Fill in the Gaps

Rearrange each formula to make x the subject.

Question	Multiply by Denominator	Multiply out bracket	Collect all x terms	Take out x as a common factor	Divide by bracket
$y = \frac{ax}{x-b}$	$y(x-b) = ax$	$xy - by = ax$	$xy - ax = by$	$x(y-a) = by$	$x = \frac{by}{y-a}$
$y = \frac{x}{a-x}$	$y(a-x) = x$				
$y = \frac{2x}{x+b}$					
$2y = \frac{x}{x-1}$					
$y = \frac{x+a}{x+1}$	$y(x+1) = x+a$	$xy + y = x+a$	$xy - x = a - y$		
$y = \frac{x+2}{x-a}$					
$y = \frac{x-a}{x-b}$					

Worked Example	Your Turn
Make a the subject of the formula: $b = \sqrt{\frac{3a+1}{4a-3}}$	Make x the subject of the formula: $y = \sqrt{\frac{5x-6}{2x-1}}$

Worked Example	Your Turn
Given that $a > 0$, make a the subject of the formula: $b = \frac{4 - 3a^2}{5a^2 - 1}$	Given that $x > 0$, make x the subject of the formula: $y = \frac{6x^2 + 4}{3x^2 - 5}$

Worked Example	Your Turn
Make x the subject of the formula: $\frac{4y + 6yw}{3y + 3w} = 2y - 5x$	Make y the subject of the formula: $4 - 3y = \frac{5x^3 - 3y^2}{y + 2}$

Worked Example	Your Turn
Make x the subject of the formula: $\frac{2y}{y-6} = \frac{5x}{x+z}$	Make a the subject of the formula: $\frac{4b}{3b-5} = \frac{a}{a+c}$

Extra Notes

7 Inverse Functions

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2x + 3}{4}$	Find the inverse function: $g(x) = \frac{4x - 3}{2}$

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{3}{2 - 5x}$	Find the inverse function: $g(x) = \frac{4}{5 - 3x}$

Fill in the Gaps

$f(x)$	Write as $y = \cdots$	Swap x and y	Make y the subject	Write as $f^{-1}(x) = \cdots$
$f(x) = 3x - 1$	$y = 3x - 1$	$x = 3y - 1$	$x + 1 = 3y$ $\quad \frac{x+1}{3} = y$	$f^{-1}(x) = \frac{x+1}{3}$
$f(x) = 2x + 5$				
$f(x) = x^2 + 8$				
$f(x) = \sqrt{x-3}$	$y = \sqrt{x-3}$	$x = \sqrt{y-3}$	$x^2 = y - 3$	
$f(x) = \frac{x+2}{7}$				
$f(x) = \frac{x}{3} - 5$				
$f(x) = \frac{9}{x}$				
$f(x) = \frac{4}{x+3}$				

Worked Example	Your Turn
Find the inverse function: $f(x) = \frac{2x - 3}{x + 2}$	Find the inverse function: $g(x) = \frac{4x - 5}{x - 3}$

Fill in the Gaps

$f(x)$	Write as $y = \cdots$	Swap x and y	Make y the subject				Write as $f^{-1}(x) = \cdots$
$f(x) = \frac{x}{x+2}$	$y = \frac{x}{x+2}$	$x = \frac{y}{y+2}$	$xy + 2x = y$	$2x = y - xy$	$2x = y(1-x)$	$y = \frac{2x}{1-x}$	$f^{-1}(x) = \frac{2x}{1-x}$
$f(x) = \frac{x-3}{x}$							
$f(x) = \frac{x}{2x+1}$							
$f(x) = \frac{3x}{x-5}$							
$f(x) = \frac{x+2}{x-1}$							
$f(x) = \frac{x+3}{x+1}$							
$f(x) = \frac{2x+1}{3-x}$							
$f(x) = \frac{5-2x}{x-3}$							

Worked Example	Your Turn
Find the inverse function: $f(x) = \sqrt{\frac{3x-2}{x-4}}$	Find the inverse function: $g(x) = \sqrt{\frac{5x-4}{x+3}}$

Worked Example	Your Turn
Given that $f(x) = \left(\frac{5x-3}{9x-10}\right)^2$, find $f^{-1}(x)$	Given that $g(x) = \left(\frac{2x-9}{10x-1}\right)^2$, find $g^{-1}(x)$

Extra Notes