



KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS



KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM

**Year 9**

**2025 Mathematics 2026**

**Unit 12 Tasks – Part 1**

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**Unit 12 Tasks – Part 2**

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# 1 Expanding Double Brackets

# Fluency Practice

Question 1: Expand and simplify

(a)  $(w + 4)(w + 2)$  (b)  $(y + 1)(y + 2)$  (c)  $(c + 2)(c + 5)$  (d)  $(x + 6)(x + 7)$

(e)  $(a + 5)(a - 3)$  (f)  $(g + 7)(g - 4)$  (g)  $(s - 4)(s + 5)$  (h)  $(x + 1)(x - 3)$

(i)  $(p - 3)(p - 2)$  (j)  $(y - 4)(y - 4)$  (k)  $(k - 5)(k - 6)$  (l)  $(v + 4)(v + 3)$

(m)  $(n + 8)(n - 10)$  (n)  $(b - 3)(b + 7)$  (o)  $(z - 9)(z - 3)$  (p)  $(a - 5)(a + 7)$

(q)  $(w + 2)(w - 8)$  (r)  $(r + 7)(r + 7)$  (s)  $(w - 11)(w + 1)$  (t)  $(t - 8)(t - 7)$

Question 2: Expand and simplify

(a)  $(8 + x)(2 + x)$  (b)  $(9 + y)(4 - y)$  (c)  $(1 + y)(3 + y)$  (d)  $(10 - t)(4 - t)$

(e)  $(4 - w)(w + 2)$  (f)  $(6 - x)(x - 4)$  (g)  $(2 - r)(8 - r)$  (h)  $(x + 2)(8 - x)$

Question 3: Expand and simplify

(a)  $(y + 2)(y - 2)$  (b)  $(w + 7)(w - 7)$  (c)  $(a + 1)(a - 1)$  (d)  $(x - 10)(x + 10)$

(e)  $(g - 8)(g + 8)$  (f)  $(6 - x)(6 + x)$  (g)  $(4 - r)(4 + r)$  (h)  $(11 + y)(11 - y)$

Question 4: Expand and simplify

(a)  $(2c + 1)(2c + 3)$  (b)  $(5x + 1)(2x + 5)$  (c)  $(3w + 2)(w + 1)$

(d)  $(3p + 2)(2p - 1)$  (e)  $(5g - 4)(g + 1)$  (f)  $(2a - 3)(4a + 7)$

(g)  $(4r - 5)(2r - 3)$  (h)  $(2y - 3)(9y - 1)$  (i)  $(5k - 4)(2k - 1)$

(j)  $(2n + 3)(2n + 5)$  (k)  $(3b + 4)(2b + 9)$  (l)  $(2z - 9)(6z - 5)$

(m)  $(4w - 3)(3w - 1)$  (n)  $(4r + 3)(3r + 2)$  (o)  $(5w - 1)(2w - 3)$

(p)  $(3 + 2c)(5 + c)$  (q)  $(9 + 2x)(3 - 2x)$  (r)  $(9 - 4y)(2 + 3y)$

(s)  $(3w + 2)(3w - 2)$  (t)  $(2y + 3)(2y - 3)$  (u)  $(5w - 1)(5w + 1)$

(v)  $(9 - 5a)(9 + 5a)$  (w)  $(1 - 2x)(1 + 2x)$  (x)  $(3 + 2y)(3 - 2y)$

Question 5: Expand and simplify

(a)  $(a + 2)^2$  (b)  $(x + 7)^2$  (c)  $(z - 9)^2$  (d)  $(p + 1)^2$

(e)  $(c - 5)^2$  (f)  $(k + 4)^2$  (g)  $(y - 3)^2$  (h)  $(10 + r)^2$

(i)  $(3g + 2)^2$  (j)  $(2b - 1)^2$  (k)  $(3m - 5)^2$  (l)  $(2v + 9)^2$

(m)  $(7 - a)^2$  (n)  $(4 - 3s)^2$  (o)  $(8 + 5h)^2$  (p)  $(7 - 2p)^2$

Question 6: Expand and simplify

(a)  $(a + 2)(a + 3) + (a + 4)(a + 1)$  (b)  $(2w + 3)(w - 1) + (w - 3)(w - 2)$

(c)  $(x + 9)(3x + 4) - (x + 3)(x - 1)$  (d)  $2(x + 1)(x + 4) - (x + 1)(x + 2)$

(e)  $(x + 4)^2 + (x + 1)^2$  (f)  $(2x + 1)^2 - (x - 5)^2$

Apply

Question 1: Can you spot any mistakes in the questions below.

Expand and simplify  $(5y - 1)(y - 2)$

Expand and simplify  $(y - 7)^2$

$5y^2 - 10y - y - 2$

$y^2 - 49$

$5y^2 - 11y - 2$

## Fluency Practice

Expand and simplify

- (a)  $(x + 7)(x + 3)$
- (b)  $(x + 3)(x + 6)$
- (c)  $(x + 6)(x + 5)$
- (d)  $(x + 6)(x + 1)$
- (e)  $(x + 2)(x + 1)$

Expand and simplify

- (a)  $(x + 4)(x - 2)$
- (b)  $(x - 4)(x + 2)$
- (c)  $(x - 4)(x + 6)$
- (d)  $(x + 6)(x - 3)$
- (e)  $(x - 6)(x + 3)$

Expand and simplify

- (a)  $(x - 4)(x - 2)$
- (b)  $(x - 2)(x - 4)$
- (c)  $(x - 2)(x - 10)$
- (d)  $(x - 3)(x - 10)$
- (e)  $(x - 3)(x - 5)$

Expand and simplify

- (a)  $(x + 4)^2$
- (b)  $(x - 4)^2$
- (c)  $(x + 5)^2$
- (d)  $(x - 5)^2$
- (e)  $(x + 9)^2$
- (f)  $(x - 11)^2$

Expand and simplify

- (a)  $(2x + 1)(x + 5)$
- (b)  $(2x + 3)(3x + 1)$
- (c)  $(2x - 1)(x + 6)$
- (d)  $(3x - 2)(2x + 7)$
- (e)  $(4x - 1)(x - 5)$
- (f)  $(2x - 1)(5x - 3)$
- (g)  $(4x + 1)^2$
- (h)  $(4x - 1)(4x + 1)$

## Fluency Practice

expanding brackets to obtain a quadratic expression

expand and simplify:

1)  $(n + 4)(n + 6)$

10)  $(2a + 7)(2a - 7)$

2)  $(k + 2)(k + 8)$

11)  $(4b - 5)(4b + 5)$

3)  $(x + 5)^2$

12)  $(9c + 1)(9c - 1)$

4)  $(m + 12)(m - 2)$

13)  $(8d + 4)(2d - 1)$

5)  $(k + 13)(k - 3)$

14)  $(9k - 3)(3k + 1)$

6)  $(a - 5)(a + 15)$

15)  $(10p - 2)(5p + 1)$

7)  $(w + 14)(w - 4)$

16)  $(12w + 6)(2w - 1)$

8)  $(2n + 1)(4n + 3)$

17)  $(30n + 5)(6n - 1)$

9)  $(3h + 2)(8h - 2)$

18)  $(25g - 10)(2 + 5g)$

## Purposeful Practice

quadratic expressions with patterns

(a) expand the brackets (showing your steps) and look for patterns

(1)  $(m + 3)(m + 4)$

(7)  $(h + 8)(h + 9)$

(2)  $(m + 6)(m + 1)$

(8)  $(h + 6)(h + 11)$

(3)  $(a + 3)(a + 8)$

(9)  $(z + 15)(z + 10)$

try to find some other  
examples that fit the pattern

(4)  $(a + 5)(a + 6)$

(10)  $(z + 13)(z + 12)$

(5)  $(t + 10)(t + 5)$

(11)  $(y - 7)(y - 6)$

(6)  $(t + 8)(t + 7)$

(12)  $(y - 9)(y - 4)$

(b) expand the brackets (showing your steps) and look for patterns

(1)  $(a + 2)(a + 3)$

(7)  $(d + 18)(d - 3)$

(2)  $(a + 6)(a - 1)$

(8)  $(d + 9)(d + 6)$

(3)  $(b + 4)(b + 6)$

(9)  $(e + 15)(e + 10)$

try to find some other  
examples that fit the pattern

(4)  $(b + 12)(b - 2)$

(10)  $(e + 30)(e - 5)$

(5)  $(c + 12)(c + 5)$

(11)  $(f - 12)(f - 5)$

(6)  $(c + 20)(c - 3)$

(12)  $(f - 20)(f + 3)$

## Purposeful Practice

$(x + 3)$	$(x + 6)$	$(x + 5)$
$(x + 2)$	$(x + 9)$	$(x + 3)$
$(x + 7)$	$(x + 6)$	$(x + 1)$
$(x + 7)$	$(x + 3)$	$(x + 4)$
$(x + 9)$	$(x + 1)$	$(x + 6)$
$(x + 5)$	$(x + 2)$	$(x + 1)$
$(x + 8)$	$(x + 7)$	$(x + 4)$
$(x + 3)$	$(x + 5)$	$(x + 6)$
$(x + 4)$	$(x + 2)$	$(x + 8)$
$(x + 2)$	$(x + 9)$	$(x + 7)$

- (1)  $x^2 + 6x + 8$
- (2)  $x^2 + 8x + 15$
- (3)  $x^2 + 5x + 4$
- (4)  $x^2 + 11x + 18$
- (5)  $x^2 + 13x + 42$
- (6)  $x^2 + 11x + 24$
- (7)  $x^2 + 16x + 63$
- (8)  $x^2 + 7x + 10$
- (9)  $x^2 + 3x + 2$
- (10)  $x^2 + 14x + 48$
- (11)  $x^2 + 13x + 36$
- (12)  $x^2 + 12x + 35$
- (13)  $x^2 + 12x + 36$
- (14)  $x^2 + 8x + 7$
- (15)  $x^2 + 6x + 9$

Pick 2 expressions from the table on the left, multiply them together and try to get the answers on the right-hand side!

## Fluency Practice

1) Expand and simplify fully

a)  $\left(2 + \frac{3}{x}\right)\left(1 + \frac{1}{x}\right)$

b)  $\left(2a + \frac{1}{a}\right)\left(a + \frac{1}{a}\right)$

c)  $\left(2 + \frac{1}{x}\right)\left(1 - \frac{1}{x}\right)$

d)  $\left(n - \frac{1}{n}\right)\left(5n + \frac{1}{n}\right)$

e)  $\left(1 - \frac{1}{x}\right)\left(1 + \frac{1}{x}\right)$

f)  $\left(3c + \frac{1}{c}\right)\left(2c + \frac{1}{c}\right)$

g)  $\left(x + \frac{1}{x}\right)\left(x + \frac{2}{x}\right)$

h)  $\left(x - \frac{1}{x}\right)^2$

Expand and simplify

**a**  $x(x^2 - x^{-1})$

**b**  $2x^3(x^{-1} + 3)$

**c**  $x^{-1}(3x - x^3)$

**d**  $4x^{-2}(3x^5 + 2x^3)$

**e**  $\frac{1}{2}x^2(6x + 4x^{-1})$

**f**  $3x^{\frac{1}{2}}(x^{-\frac{1}{2}} - x^{\frac{3}{2}})$

**g**  $x^{-\frac{3}{2}}(5x^2 + x^{\frac{7}{2}})$

**h**  $x^{\frac{1}{3}}(3x^{\frac{5}{3}} - x^{-\frac{4}{3}})$

**i**  $(x^2 + 1)(x^4 - 3)$

**j**  $(2x^5 + x)(x^4 + 3)$

**k**  $(x^2 - 2x^{-1})(x - x^{-2})$

**l**  $(x^2 - x^{\frac{3}{2}})(x - x^{\frac{1}{2}})$

## Fluency Practice

Expand and simplify, where possible:

- (a)  $6(x + 5) - 4(3 + x)$
- (b)  $(2y + 5)(4 - y)$
- (c)  $a(b + 2a) - 2(ab + 3)$
- (d)  $x(3x - 2) - 2x(1 - x)$
- (e)  $(9 - 2b)(b + a)$

Expand and simplify, where possible:

- (a)  $x + 2 + (x + 7)(x + 4)$
- (b)  $20 - 3y(y + 6)$
- (c)  $a(a + b) + 5a - 7ab$
- (d)  $(2c - 5)(3c - 1) + c^2$
- (e)  $x^2 - 16 + (x + 4)^2$

Expand and simplify, where possible:

- (a)  $4w(5 - 2w) - 3(5 - 2w) - 12$
- (b)  $(5 - 2w)(4w - 3) - 12$
- (c)  $(x - 5)(x - 2) + 5(x - 2)$
- (d)  $(5c - d)(3d - 2c) - 2(c + d)$
- (e)  $6(4 - x) - (x + 1)(x - 1)$

Expand and simplify, where possible:

- (a)  $xy(5 + x) + (y + x)(2y - x) + 3xy$
- (b)  $2a^2 - (a + 2b)(3b - 1) + 4a(b - 2a)$
- (c)  $(5y - 1)(y + 3) - (2y + 3)(y + 7)$

Come up with your own 'expand and simplify' question where the answer is  $5x + 18$  and the question contains:

- (a) Two sets of single brackets
- (b) One set of double brackets
- (c) One set of double brackets and one set of single brackets

## Fluency Practice

Remove the brackets and simplify:

1.  $(x + 4)^2$

2.  $(x + 2)^2$

3.  $(x - 2)^2$

4.  $(2x + 1)^2$

5.  $(y - 5)^2$

6.  $(3y + 1)^2$

7.  $(x + y)^2$

8.  $(2x + y)^2$

9.  $(a - b)^2$

10.  $(2a - 3b)^2$

11.  $3(x + 2)^2$

12.  $(3 - x)^2$

13.  $(3x + 2)^2$

14.  $(a - 2b)^2$

15.  $(x + 1)^2 + (x + 2)^2$

16.  $(x - 2)^2 + (x + 3)^2$

17.  $(x + 2)^2 + (2x + 1)^2$

18.  $(y - 3)^2 + (y - 4)^2$

19.  $(x + 2)^2 - (x - 3)^2$

20.  $(x - 3)^2 - (x + 1)^2$

21.  $(y - 3)^2 - (y + 2)^2$

22.  $(2x + 1)^2 - (x + 3)^2$

23.  $3(x + 2)^2 - (x + 4)^2$

24.  $2(x - 3)^2 - 3(x + 1)^2$

## Fluency Practice

quadratic expression generalisations

find a simpler form for these expressions and possibly factorise the resulting expression

(1)  $(n + 2)(n + 4) - (n + 1)(n + 5)$

(2)  $(2n + 2)(2n + 3) - (2n + 1)(2n + 4)$

(3)  $(n + 1)(n + 2) + (n + 2)^2 - (n + 1)(n + 6)$

(4)  $(2n + 3)(n + 4) - (2n + 1)(n + 5)$

(5)  $(n + 1)(n + 4) + (n + 3)^2 - (n + 5)^2$

(6)  $n(n + 1) + (n + 1)(n + 2) + (n + 2)(n + 3) - (n + 3)(n + 4)$

(7)  $(3n - 7)(n + 2) - (n + 3)(2n - 5)$

(8)  $(3n - 13)(n - 2) - (2n - 5)(n - 7)$

(9)  $(2n - 1)(2n + 7) - (n + 4)(3n - 2)$

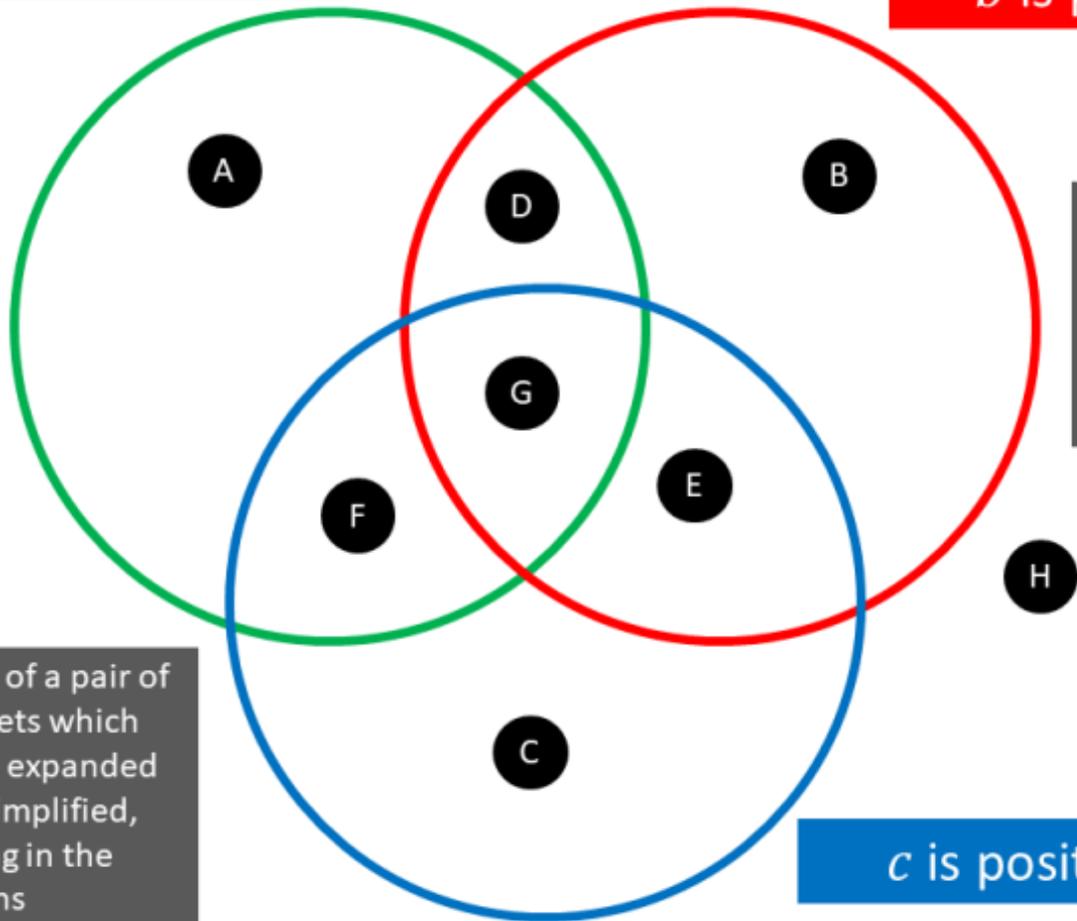
(10)  $(2n - 3)(2n - 1) - (n - 2)(3n - 2)$

# Problem Solving

$$x^2 + bx + c$$

Positive  $x^2$  term

$b$  is positive



If you think a region is impossible to fill, convince me why!

Think of a pair of brackets which when expanded and simplified, belong in the regions

$c$  is positive

## 2 Factorising by Grouping

## Intelligent Practice

Factorise:

1)  $x(x + 1) + 3(x + 1)$

2)  $x(x + 1) + 2(x + 1)$

3)  $x(x + 1) + 1(x + 1)$

4)  $3x(x + 1) + 1(x + 1)$

5)  $3x(x + 1) - 1(x + 1)$

6)  $3x(2x + 1) - 1(2x + 1)$

7)  $3x(1 + 2x) - 1(2x + 1)$

8)  $3x(1 - 2x) - 1(1 - 2x)$

9)  $2x(1 - 2x) - 1(1 - 2x)$

10)  $2x(1 - 2x) - 1(2x - 1)$

## Intelligent Practice

Factorise:

1)  $3x(x + 1) + 2(x + 1)$

2)  $3x(x + 1) - 2(x + 1)$

3)  $3x(x + 1) - 1(x + 1)$

4)  $3x(x + 1) - (x + 1)$

5)  $3x(x + 1) - 4(x + 1)$

6)  $3x(x + 1)^2 - 4(x + 1)$

7)  $3x(x + 1)^2 - 2(x + 1)$

8)  $3x(x + 1)^2 - 2x(x + 1)$

9)  $3x(x + 1)^2 - (2x + 1)(x + 1)$

10)  $3x(x + 1)^3 - (2x + 1)(x + 1)^2$

## Fluency Practice

- 1) Factorise fully  
 $(x + 3)^2 + 4(x + 3)$
- 2) Factorise fully  
 $4(x - 5)^2 + 3(x - 5)$
- 3) Factorise fully  
 $(p + q)^2 + 5(p + q)$
- 4) Factorise fully  
 $(2y + 1)^2 + (x + y)(2y + 1)$
- 5) Factorise fully  
 $(x + 4)^3 + 2(x + 4)^2$
- 6) Factorise fully  
 $2(x + y)^{10} + (x - y)(x + y)^9$

## Fluency Practice

Now factorise the following expressions fully.

1.  $x(x + 4) - 3(x + 4)$ .
2.  $2x(x - 3) + (x - 3)$ .
3.  $4(2x - 1) + 3x(2x - 1)$ .
4.  $x(x + 2) + 2(x + 2)$ .
5.  $3x(y - 1) + 2(y - 1)$ .
6.  $p(x + y) + q(x + y)$ .
7.  $4q(2x - y) - 3p(2x - y)$ .
8.  $4x^2(x + 2)^2 - 2x(x + 2)$ .
9.  $10x^5(2x + 1) + 5x^4(2x + 1)^2$ .
10.  $49y(y - 4)^{10} - 35y^2(y - 4)^9$ .
11.  $(x + 1)^4(x + 2) - 2(x + 1)^3(x + 2)^2$ .
12.  $20(2x + 1)^4(x - 2)^5 - 25(2x + 1)^5(x - 2)^4$ .
13.  $16x^2(x - 1) - 20x(x - 1)^2$ .

## Fluency Practice

Factorise fully the following:

1.  $x(x + 1) + 2(x + 1)$ .
2.  $2x(x - 1) - 3(x - 1)$ .
3.  $x(x + y) + y(x + y)$ .
4.  $p(p - 2q) + q(p - 2q)$ .
5.  $4x(x - 3) + 8(x - 3)$ .
6.  $x^5(x - 5) - x^4(x - 5)$ .
7.  $7x^2(x + 1)^5 + 14x(x + 1)^6$ .
8.  $27x^8(x - 1)^8 - 9x^9(x - 1)^7$ .
9.  $2x^4(x - 1)^4 + 8x^3(x - 1)^3$ .
10.  $(2x - 1)^7(2x + 3)^4 - (2x - 1)^8(2x + 3)^3$ .
11.  $10(t + 2)^6(t + 3)^4 - 5(t + 2)^7(t + 3)^3$ .
12.  $7x^3(x - 1)^8(2x + 1)^3 - 14x(x - 1)^9(2x + 1)^2$ .
13.  $27(2t - 1)^2(t + 1)^2 - 18(2t - 1)(t + 1)^3$ .
14.  $y^4(y + 7)^4(2y - 1)^2 - 2y^2(y + 7)^3(2y - 1)$ .
15.  $6(2z + 5)^7(z - 1)^{10} - 12(2z + 5)^6(z - 1)^{11}$ .

## Fluency Practice

1. Consider the expression  $ax + 2x + 3a + 6$ . What do you notice about the first pair of terms and about the second pair of terms? Rewrite the expression in view of your discovery. You now have two terms. What have they in common? Now factorise the expression. If you are not sure about this result, refer to Exercise 77.
2. Consider the expression  $ax + ay + bx + by$ , and answer the same questions about this expression.
3. Consider the expression  $ax + 6 + 2x + 3a$ . Pick out two pairs of terms with common factors and hence factorise the expression. Compare the working out of this question with that of question 1.
4. Do the same for  $ax + by + ay + bx$ .

Factorise the following expressions where it is possible.

5.  $ab + 2a + 3b + 6$ .      6.  $ac + 2a + 2c + 4$ .      7.  $bd + 3b + 2d + 6$ .
8.  $de + 5d + 3e + 15$ .      9.  $ef + 4e + 6f + 24$ .      10.  $ab + 8 + 2a + 4b$ .
11.  $ac + 12 + 2a + 6c$ .      12.  $de + 15 + 3d + 5e$ .      13.  $xy + 20 + 2x + 10y$ .
14.  $xy + 20 + 4x + 5y$ .      15.  $xy + xz + uy + uz$ .      16.  $pr + qr + ps + qs$ .
17.  $pr + qr + ps + rs$ .      18.  $rs + rt + us + ut$ .      19.  $sx + sy + vx + vy$ .
20.  $hkl + lm + hl + km$ .      21.  $hkl + lm + hm + kl$ .      22.  $x^2 + xy + xz + yz$ .
23.  $a^2 + ab + ac + bc$ .      24.  $e^2 + ef + eg + fg$ .      25.  $h^2 + lh + mh + lm$ .
26.  $h^2 + lh + mh + nh$ .      27.  $n^2 + pn + nq + pq$ .      28.  $ab - 2a + 3b - 6$ .
29.  $ac - 2a + 2c - 4$ .      30.  $bd + 3b - 2d - 6$ .      31.  $bc + 3b + 2d + 6$ .
32.  $de - 5d + 3e - 15$ .      33.  $ef + 4e - 6f - 24$ .      34.  $xy + x + y + 1$ .
35.  $pq + p + q + 1$ .      36.  $pq + 1 + p + q$ .      37.  $1 + cd + c + d$ .
38.  $x^3 + x^2 + x + 1$ .      39.  $r^3 + 1 + r^2 + r$ .      40.  $pq - p + q - 1$ .
41.  $st - 1 + s - t$ .      42.  $su + 1 + s - u$ .      43.  $uv - 1 - u + v$ .
44.  $c^2d - cd + c^2 - c$ .      45.  $k - p - ky + yp$ .      46.  $e - 5 - ex + 5x$ .
47.  $2ht - tz - 2hn + nz$ .      48.  $6cf - 7cp - 6kf + 7kp$ .
49.  $ap + aq + aps - 2aqr$ .      50.  $cy - dy + 5xy$ .
51.  $24cq - 8xq + 15cs - 5sx$ .      52.  $ah + hx - 3ab - 3bx$ .
53.  $bm - km - 6ab + 6ak$ .      54.  $br - kr - 6ab + 6ar$ .
55.  $hx + hy + kx + ky + jx + jy$ .      56.  $ab + ac + ad + eb + ec + ed$ .
57.  $1 + x + y + z + xz + yz$ .      58.  $1 + a - b + x + ax - bx$ .
59.  $3 - r + s - 3u + ru - su$ .      60.  $t - v + st - sv + tx - vx$ .

## Fluency Practice

Factorise:

1)  $ab - 2a + 4b - 8$

2)  $ab + bc - 3a - 3c$

3)  $ac + ad + bc + bd$

4)  $y^2 + cy + dy + cd$

5)  $x^2y - x^2 + 2y - 2$

6)  $x^3 + x^2 + x + 1$

7)  $a^3 - a^2 + a - 1$

8)  $x^3 + x^2 - x - 1$

9)  $x^3 + x^2 + xy^2 + y^2$

10)  $x^3 - 2x^2 + 3x - 6$

## Intelligent Practice

Factorise:

1)  $x^2 + 2x + 3x + 6$

2)  $x^2 + 3x + 2x + 6$

3)  $3x^2 + 3x + 2x + 2$

4)  $3x^2 + 3x + 4x + 4$

5)  $3x^2 - 3x + 4x - 4$

6)  $3x^2 - 3x - 4x + 4$

7)  $3x^2 + 3x - 4x - 4$

8)  $3x^2 - 4x + 3x - 4$

9)  $5x^2 - 4x + 5x - 4$

10)  $-5x^2 + 4x - 5x + 4$

## 3 Factorising Quadratics

## Intelligent Practice

$\begin{aligned} \_ \times \_ &= 18 \\ \_ + \_ &= 11 \end{aligned}$	$\begin{aligned} \_ \times \_ &= 20 \\ \_ + \_ &= 9 \end{aligned}$	$\begin{aligned} \_ \times \_ &= 180 \\ \_ + \_ &= -36 \end{aligned}$
$\begin{aligned} \_ \times \_ &= -18 \\ \_ + \_ &= 7 \end{aligned}$	$\begin{aligned} \_ \times \_ &= -20 \\ \_ + \_ &= -1 \end{aligned}$	$\begin{aligned} \_ \times \_ &= -252 \\ \_ + \_ &= -36 \end{aligned}$
$\begin{aligned} \_ \times \_ &= -18 \\ \_ + \_ &= -7 \end{aligned}$	$\begin{aligned} \_ \times \_ &= -20 \\ \_ + \_ &= 1 \end{aligned}$	$\begin{aligned} \_ \times \_ &= 252 \\ \_ + \_ &= 48 \end{aligned}$
$\begin{aligned} \_ \times \_ &= 18 \\ \_ + \_ &= -11 \end{aligned}$	$\begin{aligned} \_ \times \_ &= 20 \\ \_ + \_ &= -9 \end{aligned}$	$\begin{aligned} \_ \times \_ &= 432 \\ \_ + \_ &= 48 \end{aligned}$
$\begin{aligned} \_ \times \_ &= 18 \\ \_ + \_ &= -9 \end{aligned}$	$\begin{aligned} \_ \times \_ &= -36 \\ \_ + \_ &= -9 \end{aligned}$	$\begin{aligned} \_ \times \_ &= 576 \\ \_ + \_ &= 48 \end{aligned}$
$\begin{aligned} \_ \times \_ &= -18 \\ \_ + \_ &= 3 \end{aligned}$	$\begin{aligned} \_ \times \_ &= -36 \\ \_ + \_ &= 0 \end{aligned}$	$\begin{aligned} \_ \times \_ &= -576 \\ \_ + \_ &= 0 \end{aligned}$

# Fluency Practice

## Number Pairs

What two numbers have a **sum** of 6 and a **product** of 8?

### EXAMPLE

Sum = 6  
Product = 8

&

1) Sum = 7  
Product = 12

&

2) Sum = 11  
Product = 30

&

3) Sum = 4  
Product = 4

&

4) Sum = 8  
Product = 7

&

5) Sum = 10  
Product = 24

&

 How do we get  
a **negative product**? 

7) Sum = 3  
Product = -18

&

8) Sum = 7  
Product = -8

&

9) Sum = 1  
Product = -20

&

10) Sum = -1  
Product = -20

&

11) Sum = -4  
Product = -21

&

12) Sum = 0  
Product = -36

&

 A **negative** sum,  
but a **positive** product? 

14) Sum = -10  
Product = 24

&

15) Sum = -13  
Product = 36

&

16) Sum = -13  
Product = 42

&

17) Sum = -11  
Product = 28

&

18) Sum = -1  
Product = -72

&

19) Sum = -4  
Product = -45

&

# Fluency Practice

① Factorise the following

$2x^2 + 11x + 5$

$5x^2 + 8x + 3$

$3x^2 + 10x + 3$

$2x^2 + 5x + 3$

Identify a, b and c	$a = 2, b = 11, c = 5$			
Multiply a and c	$2 \times 5 = 10$			
What multiples to give ac but adds to give b?	$1 \times 10 = 10$ $1 + 10 = 11$			
Split the middle term	$2x^2 + 10x + x + 5$			
Factorise each pair of terms	$2x(x + 5) + 1(x + 5)$			
Complete the factorisation	$(2x + 1)(x + 5)$			

$3x^2 + 13x - 10$

$3x^2 + 4x - 15$

$2x^2 - x - 1$

$2x^2 - 5x - 12$

Identify a, b and c				
Multiply a and c				
What multiples to give ac but adds to give b?				
Split the middle term				
Factorise each pair of terms				
Complete the factorisation				

## Fluency Practice

Factorise:

- (a)  $2x^2 + 7x + 3$     (b)  $3x^2 + 8x + 5$   
(c)  $2x^2 + 5x + 2$     (d)  $7x^2 + 8x + 1$   
(e)  $6x^2 + 13x + 5$     (f)  $4x^2 + 12x + 9$   
(g)  $3x^2 + 13x + 4$     (h)  $8x^2 + 14x + 3$

Factorise:

- (a)  $16x^2 - 8x + 1$     (b)  $6x^2 - 7x + 2$   
(c)  $3x^2 - 13x + 4$     (d)  $7x^2 - 23x + 6$   
(e)  $4x^2 - 7x + 3$     (f)  $6x^2 - 31x + 5$   
(g)  $12x^2 - 8x + 1$     (h)  $9x^2 - 9x + 2$

Factorise:

- (a)  $2x^2 - x - 6$     (b)  $3x^2 + 11x - 4$   
(c)  $5x^2 - 3x - 2$     (d)  $4x^2 - 4x - 15$   
(e)  $7x^2 + 5x - 2$     (f)  $6x^2 + 7x - 5$   
(g)  $8x^2 - 2x - 3$     (h)  $9x^2 + 6x - 8$

Factorise

- (a)  $3x^2 + 19x + 20$   
(b)  $4x^2 - 23x + 15$   
(c)  $7x^2 - 19x - 6$   
(d)  $3x^2 - 8x - 60$   
(e)  $9x^2 + 48x + 28$   
(f)  $2x^2 - 21x + 52$

# Fluency Practice

Question 1: Factorise each of the following

- (a)  $2x^2 + 7x + 5$       (b)  $2x^2 + 11x + 15$       (c)  $2x^2 + 9x + 10$   
(d)  $3x^2 + 13x + 4$       (e)  $3x^2 + 4x + 1$       (f)  $3x^2 + 8x + 4$   
(g)  $5x^2 + 13x + 6$       (h)  $5x^2 + 26x + 5$       (i)  $7x^2 + 10x + 3$   
(j)  $11x^2 + 47x + 12$       (k)  $2x^2 + 17x + 36$       (l)  $5x^2 + 62x + 24$

Question 2: Factorise each of the following

- (a)  $3x^2 + x - 4$       (b)  $7x^2 + 20x - 3$       (c)  $2x^2 - 13x + 15$   
(d)  $3x^2 - 17x + 10$       (e)  $3x^2 - 16x - 12$       (f)  $3x^2 - x - 4$   
(g)  $5x^2 - 13x - 6$       (h)  $3x^2 + 8x - 3$       (i)  $2x^2 - x - 10$   
(j)  $2x^2 - 3x - 44$       (k)  $7x^2 - 22x + 16$       (l)  $2x^2 + 15x - 38$

Question 3: Factorise each of the following

- (a)  $6x^2 + 13x + 6$       (b)  $9x^2 + 9x + 2$       (c)  $6x^2 + 13x + 2$   
(d)  $8x^2 + 41x + 5$       (e)  $9x^2 + 6x + 1$       (f)  $8x^2 + 26x + 15$   
(g)  $8x^2 + 29x + 15$       (h)  $10x^2 + 9x + 2$       (i)  $9x^2 + 27x + 20$   
(j)  $10x^2 + 17x + 7$       (k)  $12x^2 + 13x + 3$       (l)  $15x^2 + 32x + 16$

Question 4: Factorise each of the following

- (a)  $9x^2 - 12x - 5$       (b)  $4x^2 - 4x - 3$       (c)  $4x^2 - 11x + 6$   
(d)  $6x^2 - 7x + 2$       (e)  $10x^2 - 91x + 9$       (f)  $4x^2 + 25x - 56$   
(g)  $6x^2 - 35x + 49$       (h)  $6x^2 - 7x - 10$       (i)  $8x^2 + 10x - 3$   
(j)  $15x^2 + 31x + 10$       (k)  $12x^2 + 5x - 3$       (l)  $20x^2 - 23x + 6$

Question 1: A quadratic expression,  $3x^2 + ax + 10$ , can be factorised. Find all possible values for a. a can be positive or negative.

Question 2: A quadratic expression,  $4x^2 + bx - 12$ , can be factorised. Find all possible values for b. b can be positive or negative.

Question 3: A quadratic expression,  $2x^2 - x + c$ , can be factorised. Find three possible values for c.

Question 4: Andrew has completed his homework on factorising quadratics. Can you spot any mistakes?

Factorise  $2y^2 + 7y - 15$

$$(2y + 3)(y - 5)$$

Factorise  $2w^2 - 9w + 4$

$$(2w + 1)(w + 4)$$

Factorise  $3y^2 + 10y - 8$

$$(3x - 2)(x + 4)$$

Factorise  $6w^2 - 11w - 10$

$$(3w + 2)(2w - 5)$$

## Fluency Practice

factorise (1)

(1)  $2x^2 + 7x + 6$

(2)  $3x^2 + x - 14$

(3)  $5x^2 + 13x - 6$

(4)  $6x^2 + 5x + 1$

(5)  $9x^2 + 12x + 4$

(6)  $4x^2 + 21x + 5$

(7)  $4x^2 - 5x - 6$

(8)  $6x^2 + 17x + 5$

(9)  $7x^2 - 19x - 6$

(10)  $2x^2 - 35x + 150$

(11)  $3x^2 + 19x + 20$

(12)  $4x^2 - 23x + 15$

(13)  $4x^2 + 5x - 9$

(14)  $2x^2 - 31x + 42$

(15)  $4x^2 + 21x - 18$

(16)  $3x^2 - 34x + 80$

(17)  $10x^2 - 43x - 9$

(18)  $9x^2 - 65x - 56$

## Fluency Practice

factorise (3)

(1)  $4x^2 + 7x + 3$

(10)  $20x^2 - 37x + 8$

(2)  $2x^2 + 13x + 20$

(11)  $4x^2 + 37x - 30$

(3)  $20x^2 + 17x + 3$

(12)  $9x^2 - 30x + 25$

(4)  $6x^2 + 13x + 6$

(13)  $9x^2 + 72x - 25$

(5)  $4x^2 + 11x + 6$

(14)  $12x^2 + 11x - 15$

(6)  $12x^2 + 23x + 10$

(15)  $10x^2 - 19x + 6$

(7)  $2x^2 + 11x - 6$

(16)  $12x^2 - 7x - 12$

(8)  $6x^2 - 25x - 9$

(17)  $30x^2 - 29x + 4$

(9)  $8x^2 - 26x + 15$

(18)  $12x^2 + 8x - 15$

## Intelligent Practice

Factorise:

1)  $3x^2 + 5x + 2$

2)  $3x^2 + 8x + 4$

3)  $3x^2 + 11x + 6$

4)  $3x^2 + 7x - 6$

5)  $3x^2 + 4x - 4$

6)  $3x^2 + 1x - 2$

7)  $3x^2 + x - 2$

8)  $3x^2 - x - 2$

9)  $3x^2 - 4x - 4$

10)  $3x^2 - 7x - 6$

11)  $3x^2 - 11x + 6$

12)  $3x^2 - 8x + 4$

13)  $3x^2 - 5x + 2$

14)  $5x^2 - 3x + 2$

## Intelligent Practice

Where possible, finish factorising:

1)  $(2x + 4)(x + 5)$

2)  $(x + 2)(2x + 10)$

3)  $(x + 2)(2x + 11)$

4)  $3x + 6)(2x + 11)$

5)  $(9x + 6)(2x + 11)$

6)  $(9x + 6)(2x + 10)$

7)  $(9x + 6)(5x + 10)$

8)  $(9x + 6)(15x + 10)$

9)  $2(9x + 6)(15x + 10)$

10)  $2(9x - 6)(15x + 10)$

11)  $2(6 - 9x)(15x + 10)$

12)  $2(6x - 9x^2)(15x + 10)$

13)  $2x^3(6x - 9x^2)(15x + 10)$

14)  $0.2(6 - 9x)(15x + 10)$

## Intelligent Practice

Factorise:

1)  $6x^2 + 10x + 4$

2)  $6x^2 - 10x + 4$

3)  $6x^2 + 2x - 4$

4)  $6x^2 - 2x - 4$

5)  $9x^2 - 3x - 6$

6)  $9x^2 - 30x - 24$

## Fluency Practice

Factorise:

- (a)  $x^2 + 7x + 10$    (b)  $x^2 + 7x + 12$   
(c)  $x^2 + 5x + 4$    (d)  $x^2 + 5x + 6$   
(e)  $x^2 + 10x + 16$    (f)  $x^2 + 10x + 21$   
(g)  $x^2 + 22x + 21$    (h)  $x^2 + 13x + 22$   
(i)  $x^2 + 13x + 42$    (j)  $x^2 + 13x + 12$

Factorise:

- (a)  $x^2 - 7x + 10$    (b)  $x^2 - 7x + 12$   
(c)  $x^2 - 8x + 12$    (d)  $x^2 - 13x + 12$   
(e)  $x^2 - 13x + 30$    (f)  $x^2 - 11x + 30$   
(g)  $x^2 - 11x + 18$    (h)  $x^2 - 9x + 18$   
(i)  $x^2 - 9x + 20$    (j)  $x^2 - 9x + 14$

Factorise:

- (a)  $x^2 + 3x - 10$    (b)  $x^2 - 3x - 10$   
(c)  $x^2 + 3x - 28$    (d)  $x^2 - 3x - 28$   
(e)  $x^2 - x - 30$    (f)  $x^2 - x - 12$   
(g)  $x^2 + 4x - 12$    (h)  $x^2 - 11x - 12$   
(i)  $x^2 + 2x - 15$    (j)  $x^2 - 2x - 35$

Factorise:

- (a)  $x^2 + 18x + 81$    (b)  $x^2 - 2x - 80$   
(c)  $x^2 - x - 42$    (d)  $x^2 - 12x + 27$   
(e)  $x^2 - 2x + 1$    (f)  $x^2 + 11x - 26$   
(g)  $x^2 - 12x + 32$    (h)  $x^2 - 5x - 36$   
(i)  $x^2 + 13x - 30$    (j)  $x^2 - 22x - 75$

# Fluency Practice

Question 1: Factorise each of the following

- (a)  $x^2 + 7x + 12$     (b)  $x^2 + 6x + 8$     (c)  $x^2 + 5x + 6$     (d)  $x^2 + 8x + 7$   
 (e)  $x^2 + 4x + 4$     (f)  $x^2 + 8x + 15$     (g)  $x^2 + 6x + 9$     (h)  $x^2 + 11x + 28$   
 (i)  $x^2 + 10x + 25$     (j)  $x^2 + 12x + 20$     (k)  $x^2 + 25x + 24$     (l)  $x^2 + 11x + 24$   
 (m)  $x^2 + 9x + 14$     (n)  $x^2 + 23x + 60$     (o)  $x^2 + 29x + 100$     (p)  $x^2 + 20x + 51$

Question 2: Factorise each of the following

- (a)  $x^2 + x - 12$     (b)  $x^2 + 5x - 6$     (c)  $x^2 + 3x - 10$     (d)  $x^2 + 3x - 4$   
 (e)  $x^2 + 2x - 48$     (f)  $x^2 + 4x - 32$     (g)  $x^2 + 2x - 35$     (h)  $x^2 + 8x - 33$

Question 3: Factorise each of the following

- (a)  $x^2 - 3x - 10$     (b)  $x^2 - x - 20$     (c)  $x^2 - 6x - 27$     (d)  $x^2 - 2x - 3$   
 (e)  $x^2 - x - 12$     (f)  $x^2 - 4x - 12$     (g)  $x^2 - 4x - 21$     (h)  $x^2 - 6x - 55$

Question 4: Factorise each of the following

- (a)  $x^2 - 6x + 9$     (b)  $x^2 - 9x + 20$     (c)  $x^2 - 9x + 14$     (d)  $x^2 - 13x + 22$   
 (e)  $x^2 - 9x + 8$     (f)  $x^2 - 12x + 32$     (g)  $x^2 - 15x + 36$     (h)  $x^2 - 14x + 48$

Question 5: Factorise each of the following

- (a)  $x^2 - 9x + 8$     (b)  $x^2 + 24x + 23$     (c)  $x^2 - 5x - 14$     (d)  $x^2 - 7x + 12$   
 (e)  $x^2 + 12x + 36$     (f)  $x^2 - 2x - 63$     (g)  $x^2 + 14x + 24$     (h)  $x^2 + 17x + 60$   
 (i)  $x^2 - 11x + 30$     (j)  $x^2 - 4x - 32$     (k)  $x^2 - 2x - 63$     (l)  $x^2 - 16x - 17$   
 (m)  $x^2 - 11x + 18$     (n)  $x^2 - 13x + 22$     (o)  $x^2 + 18x + 56$     (p)  $x^2 - 21x + 110$

- (q)  $x^2 - 16x + 64$     (r)  $x^2 + 22x + 121$     (s)  $x^2 - x - 72$     (t)  $x^2 - 3x - 18$   
 (u)  $x^2 - 4x - 45$     (v)  $x^2 - 16x + 63$

Question 6: Factorise each of the following

- (a)  $x^2 + 8x - 105$     (b)  $x^2 - 18x - 88$     (c)  $x^2 - 75x + 350$     (d)  $x^2 + 22x + 96$   
 (e)  $x^2 + 25x + 154$     (f)  $x^2 - 55x - 300$     (g)  $x^2 - 29x + 180$     (h)  $x^2 - x - 210$

Question 1: A quadratic expression,  $x^2 + ax + 20$ , can be factorised. Find all possible values for a. a can be positive or negative.

Question 2: A quadratic expression,  $x^2 + bx + 16$ , can be factorised. Find all possible values for b. b can be positive or negative.

Question 3: A quadratic expression,  $x^2 - 6x + c$ , can be factorised. Find three possible values for c.

Question 4: Andrew has completed his homework on factorising quadratics. Can you spot any mistakes?

Factorise  $x^2 + x - 6$

$$(x - 3)(x + 2)$$

Factorise  $x^2 + 10x + 9$

$$(x + 3)(x + 3)$$

Factorise  $x^2 - 7x + 12$

$$(x + 5)(x + 2)$$

Factorise  $x^2 + 8x + 16$

$$(x + 4)(x + 4)$$

## Intelligent Practice

Factorise:

1)  $x^2 + 10x + 24$

2)  $x^2 + 10x - 24$

3)  $x^2 - 10x - 24$

4)  $x^2 - 10x + 24$

5)  $x^2 - 11x + 30$

6)  $x^2 - 110x + 3000$

## Fluency Practice

Q	Factorise:	Answers	Q	Factorise:	Answers
1	$x^2 + 5x + 6$		13	$x^2 + 10x + 9$	
2	$x^2 - 5x + 6$		14	$x^2 - 10x + 9$	
3	$x^2 + 5x - 6$		15	$x^2 + 6x + 9$	
4	$x^2 - 5x - 6$		16	$x^2 - 6x + 9$	
5	$x^2 + 6x + 8$		17	$x^2 + 9$	
6	$x^2 - 6x + 8$		18	$x^2 - 9$	
7	$x^2 + 6x - 7$		19	$x^2 + 10x + 16$	
8	$x^2 - 6x - 7$		20	$x^2 - 10x + 16$	
9	$x^2 + 7x + 10$		21	$x^2 + 8x + 16$	
10	$x^2 - 7x + 10$		22	$x^2 - 8x + 16$	
11	$x^2 + 7x - 18$		23	$x^2 + 16$	
12	$x^2 - 7x - 18$		24	$x^2 - 16$	

# Fluency Practice

## Factorising Quadratics

Factorise these expressions into two brackets:

- |  |                      |                     |  |
|--|----------------------|---------------------|--|
| A) $x^2 + 3x + 2$                            | B) $x^2 + 4x + 3$    | C) $x^2 + 5x + 4$   |  |
| D) $x^2 + 8x + 12$                           | E) $x^2 + 11x + 28$  | F) $x^2 + 11x + 24$ |  |
| G) $x^2 + 11x + 18$                          | H) $x^2 + 10x + 25$  | I) $x^2 + 9x + 18$  |  |
| J) $x^2 - 2x - 3$                            | K) $x^2 + 2x - 8$    | L) $x^2 + 4x - 21$  |  |
| M) $x^2 - 4x - 12$                           | N) $x^2 + 4x - 45$   | O) $x^2 - 4x - 5$   |  |
| P) $x^2 - 7x + 10$                           | Q) $x^2 - 11x + 30$  | R) $x^2 - 4x + 4$   |  |
| S) $x^2 - 14x + 49$                          | T) $x^2 - 13x + 42$  | U) $x^2 - 13x + 40$ |  |
| V) $x^2 + 9x - 52$                           | W) $x^2 + x - 30$    | X) $x^2 - x - 72$   |  |
| <hr style="width: 50%; margin: 10px auto;"/> |                      |                     |  |
| 1) $x^2 + 2ax + a^2$                         | 2) $x^2 + mx - 2m^2$ | 3) $x^2 - 9y^2$     |  |

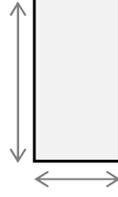
4) The area of this square is  $x^2 + 12x + 36$

What is the length of one side of the square?



5) The area of this rectangle is  $x^2 - 3x - 130$

What is the length & width of the rectangle in terms of  $x$ ?



6) How many ways can you complete this identity?

$$x^2 + 8x \quad \square \quad \equiv \quad (x + \square)(x + \square)$$

## Fluency Practice

Multiply pairs of factors from the first table to make the quadratics in the second table.

Hint: It is much easier to do this by factorizing the quadratics first!

The first pair is done for you.

$(x + 3)$	$(2x - 2)$	$(2x + 1)$	$(2x - 1)$
$(3x + 2)$	$(x + 3)$	$(2x - 1)$	$(x + 4)$
$(2x - 2)$	$(x - 1)$	$(x - 3)$	$(x + 4)$
$(2x + 1)$	$(x + 5)$	$(x - 3)$	$(2x - 3)$
$(x + 8)$	$(x - 7)$	$(x + 4)$	$(2x - 4)$
$(x + 2)$	$(x + 2)$	$2(x + 1)$	$(x - 2)$
$2(x - 2)$	$(3x - 3)$	$(x + 3)$	$(6x - 3)$
$(x - 5)$	$(x - 5)$	$(x + 3)$	$(x - 4)$
$(2x + 4)$	$(x - 1)$	$(x - 5)$	$(x - 2)$
$(2x + 3)$	$(2x - 5)$	$(3x + 1)$	$(x - 9)$
$(x - 3)$	4	$(x + 1)$	$(2x - 8)$

$(x + 3)$	$(x + 4)$	$x^2 + 7x + 12$	$2x^2 + 3x + 1$
		$x^2 - 3x - 10$	$4x^2 - 1$
		$x^2 + 5x + 6$	$2x^2 - 15x - 27$
		$x^2 - 25$	$4x^2 + 2x - 12$
		$x^2 + 4x - 32$	$4x^2 - 12x + 8$
		$x^2 + 2x - 3$	$2x^2 - 4x - 6$
		$x^2 + 8x + 16$	$24x - 12$
		$x^2 - 7x + 10$	$4x^2 - 14x + 10$
		$x^2 - 4x + 3$	$2(3x^2 - 4x - 4)$
		$x^2 - 9x + 14$	$6x^2 - x - 1$
		$x^2 - 9$	$6x^2 - 30x + 24$

# Problem Solving

		$x^2 + 7x + 6$
		$x^2 + 8x + 15$
$x^2 + 4x + 3$	$x^2 + 11x + 30$	<b>Question 1</b>

		$x^2 + 7x + 12$
		$x^2 + 6x + 8$
$x^2 + 8x + 16$	$x^2 + 5x + 6$	<b>Question 2</b>

		$x^2 + 9x + 14$
		$x^2 + 7x + 10$
$x^2 + 7x + 10$	$x^2 + 9x + 14$	<b>Question 3</b>

		$x^2 + 12x + 27$
		$x^2 + 13x + 42$
$x^2 + 16x + 63$	$x^2 + 9x + 18$	<b>Question 4</b>

		$x^2 + 14x + 48$
		$x^2 + 7x + 10$
$x^2 + 8x + 12$	$x^2 + 13x + 40$	<b>Question 5</b>

		$x^2 + 13x + 36$
		$x^2 + 13x + 40$
$x^2 + 17x + 72$	$x^2 + 9x + 20$	<b>Question 6</b>

		$x^2 + 18x + 81$
		$x^2 + 4x + 4$
$x^2 + 11x + 18$	$x^2 + 11x + 18$	<b>Question 7</b>

		$x^2 + 11x + 28$
		$x^2 + 10x + 21$
$x^2 + 7x + 12$	$x^2 + 14x + 49$	<b>Question 8</b>

		$x^2 + 12x + 35$
		$x^2 + 9x + 8$
$x^2 + 15x + 56$	$x^2 + 6x + 5$	<b>Question 9</b>

		$x^2 + 14x + 45$
		$x^2 + 12x + 36$
$x^2 + 15x + 54$	$x^2 + 11x + 30$	<b>Question 10</b>

Fill in each of the gaps in the grid with a bracket so that every row and column multiply to give the quadratic at the end.

There's an example to the right →

$(x + 6)$	$(x + 9)$	$x^2 + 15x + 54$
$(x + 1)$	$(x + 3)$	$x^2 + 4x + 3$
$x^2 + 7x + 6$	$x^2 + 12x + 27$	<b>Example</b>

## Problem Solving

Using the numbers 1-9, at most once each time, fill in the blanks so that each expression factorises.

$$x^2 + \square x + 2$$

$$x^2 - \square x - 32$$

$$x^2 + 5x - \square$$

$$x^2 + 6x + \square$$

$$x^2 - 2x + \square$$

$$x^2 - \square x - 18$$

$$x^2 - \square x + 12$$

$$x^2 + \square x - 15$$

$$x^2 - \square x + 8$$

## Problem Solving

$$\begin{array}{r} \square x^2 + \square x + \square \\ + \square x \\ + \square \\ + \square x + \square x^2 \end{array}$$

Use the digits 1 – 8 exactly once so that all four sides of the square factorise into pairs of linear expressions.

Can you prove that there must be multiple solutions?

## Intelligent Practice

Factorise:

1)  $-x^2 + 11x - 30$

2)  $-x^2 + 13x - 30$

3)  $-2x^2 + 13x - 15$

4)  $-2x^2 + 13x + 15$

5)  $-4x^2 + 26x + 30$

6)  $-6x^2 - 39x + 45$

## Fluency Practice

Factorise:

(a)  $15 + 2x - x^2$

(b)  $15 - 2x - x^2$

(c)  $10 + 3x - x^2$

(d)  $10 + 9x - x^2$

(e)  $36 + 9x - x^2$

(f)  $36 - 16x - x^2$

Factorise:

(a)  $3 + 5x - 2x^2$

(b)  $3 - 5x - 2x^2$

(c)  $5 - 9x - 2x^2$

(d)  $10 - x - 2x^2$

## Fluency Practice

Q	Factorise:	Answers	Q	Factorise:	Answers
1	$x^2 + x - 2$		11	$2x^2 + x - 3$	
2	$x^2 - x - 2$		12	$2x^2 - x - 3$	
3	$x^2 + x - 6$		13	$3x^2 + x - 2$	
4	$x^2 - x - 6$		14	$3x^2 - x - 2$	
5	$x^2 + x - 12$		15	$5x^2 + x - 6$	
6	$x^2 - x - 12$		16	$5x^2 - x - 6$	
7	$x^2 + x - 20$		17	$6x^2 + x - 5$	
8	$x^2 - x - 20$		18	$6x^2 - x - 5$	
9	$x^2 + x - 30$		19	$6x^2 + x - 2$	
10	$x^2 - x - 30$		20	$6x^2 - x - 2$	

## Fluency Practice

Q	Factorise:	Answers	Q	Factorise:	Answers
1	$x^2 + 7x + 12$		13	$x^2 + x - 12$	
2	$2x^2 + 11x + 12$		14	$3x^2 - 5x - 12$	
3	$x^2 + 7x + 6$		15	$x^2 + x - 6$	
4	$2x^2 + 13x + 6$		16	$5x^2 - 7x - 6$	
5	$x^2 + 5x + 6$		17	$x^2 + x - 2$	
6	$3x^2 + 11x + 6$		18	$7x^2 - 5x - 2$	
7	$x^2 - 7x + 12$		19	$x^2 - x - 20$	
8	$2x^2 - 11x + 12$		20	$3x^2 + 7x - 20$	
9	$x^2 - 7x + 6$		21	$x^2 - x - 30$	
10	$2x^2 - 13x + 6$		22	$5x^2 + 19x - 30$	
11	$x^2 - 5x + 6$		23	$x^2 - x - 42$	
12	$3x^2 - 11x + 6$		24	$3x^2 + 11x - 42$	

## Fluency Practice

Q	Factorise:	Answers	Q	Factorise:	Answers
1	$x^2 - 9$		11	$x^2 - 25$	
2	$x^2 - 8x - 9$		12	$x^2 - 24x - 25$	
3	$x^2 - 8x$		13	$x^2 - 24x$	
4	$2x^2 - 8x$		14	$24x^2 - 3x$	
5	$2x^2 + 3x - 9$		15	$3x^2 - 26x - 9$	
6	$x^2 - 16$		16	$x^2 - 36$	
7	$x^2 + 6x - 16$		17	$x^2 - 5x - 36$	
8	$x^2 + 6x$		18	$x^2 - 5x$	
9	$3x^2 + 6x$		19	$10x^2 - 5x$	
10	$3x^2 + 8x - 16$		20	$10x^2 - 9x - 36$	

## Fluency Practice

Q		Answers	Q		Answers
1	Expand $(x + 2)(x + 3)$		13	Expand $(2x + 3)(x + 2)$	
2	Factorise $x^2 + 7x + 6$		14	Factorise $3x^2 + 11x + 6$	
3	Expand $(x + 3)(x + 4)$		15	Expand $(3x + 5)(x + 2)$	
4	Factorise $x^2 + 8x + 12$		16	Factorise $5x^2 + 13x + 6$	
5	Expand $(x - 2)(x + 3)$		17	Expand $(3x - 2)(x + 2)$	
6	Factorise $x^2 - x - 6$		18	Factorise $2x^2 + 3x - 9$	
7	Expand $(x - 6)(x + 1)$		19	Expand $(2x - 3)(x + 5)$	
8	Factorise $x^2 + 5x - 6$		20	Factorise $2x^2 - 7x - 15$	
9	Expand $(x - 3)(x - 4)$		21	Expand $(3x - 2)(x - 5)$	
10	Factorise $x^2 - 7x + 6$		22	Factorise $3x^2 - 11x + 10$	
11	Expand $(x - 2)(x - 5)$		23	Expand $(5x - 3)(2x - 7)$	
12	Factorise $x^2 - 8x + 15$		24	Factorise $21x^2 - 41x + 10$	

## 4 Difference of Two Squares

## Intelligent Practice

Factorise:

1)  $x^2 - 4^2$

2)  $4^2 - x^2$

3)  $5^2 - x^2$

4)  $5^2 - (2x)^2$

5)  $7^2 - (3x)^2$

6)  $8^2 - (3x)^2$

7)  $64 - (3x)^2$

1)  $x^2 - 144$

2)  $144 - x^2$

3)  $144 - (7x)^2$

4)  $144 - 49x^2$

5)  $144 - 25x^2$

6)  $100 - 9x^2$

7)  $100 - 49x^2$

1)  $25x^2 - 121$

2)  $121 - 25x^2$

3)  $(11y)^2 - 25x^2$

4)  $81y^2 - 25x^2$

5)  $81y^4 - 25x^2$

6)  $81y^6 - 25x^2$

7)  $81y^6 - 25x^8$

1)  $81w^2y^6 - 25x^8z^2$

2)  $144w^2y^6 - 25x^8z^2$

3)  $144w^6y^6 - 25x^{10}z^2$

## Intelligent Practice

Factorise:

1)  $2x^2 - 162$

2)  $2x^2 - 98$

3)  $98 - 2x^2$

4)  $98 - 8x^2$

5)  $98 - 18x^2$

6)  $200 - 18x^2$

7)  $200 - 2x^2$

8)  $300 - 3x^2$

1)  $12x^2 - 3y^2$

2)  $3x^2 - 3y^2$

3)  $18x^2 - 32y^2$

4)  $32x^4 - 50y^6$

5)  $48x^4 - 75y^6$

6)  $80x^4 - 125y^6$

7)  $80x^4 - 125y^6z^2$

# Fluency Practice

Question 1: Factorise each of the following

- (a)  $x^2 - 25$       (b)  $y^2 - 49$       (c)  $w^2 - 100$       (d)  $x^2 - 4$   
(e)  $c^2 - 64$       (f)  $x^2 - 1$       (g)  $x^2 - 900$       (h)  $y^2 - 9$   
(i)  $16 - x^2$       (j)  $1 - y^2$       (k)  $81 - x^2$       (l)  $144 - h^2$   
(m)  $x^2 - y^2$       (n)  $a^2 - c^2$       (o)  $9x^2 - 25$       (p)  $4y^2 - 1$   
(q)  $49x^2 - 16$       (r)  $100 - 81x^2$       (s)  $9x^2 - 4y^2$       (t)  $36a^2 - c^2$   
(u)  $121w^2 - 196y^2$       (v)  $225 - 121y^2$

Question 2: Factorise **fully** each of the following

- (a)  $2x^2 - 32$       (b)  $2y^2 - 18$       (c)  $2x^2 - 200$       (d)  $3x^2 - 75$   
(e)  $5c^2 - 20$       (f)  $18x^2 - 2$       (g)  $12x^2 - 147$       (h)  $20y^2 - 320$

Question 3: Factorise each of the following

- (a)  $x^4 - 1$       (b)  $y^4 - 16$       (c)  $a^4 - 25$       (d)  $x^4 - y^4$   
(e)  $h^2 - p^4$       (f)  $16x^4 - 49$       (g)  $y^6 - 36$       (h)  $x^6 - 64$   
(i)  $81p^4 - x^6$       (j)  $144x^8 - 1$

Question 1: Can you spot any mistakes?

Factorise  $x^2 - 16$

$$(x + 8)(x - 8)$$

Factorise fully  $2y^2 - 50$

$$2(y^2 - 25)$$

Factorise  $x^2 - 25$

$$(x - 5)(x - 5)$$

Factorise  $y^2 - 9w^2$

$$(3w - y)(3w + y)$$

## Fluency Practice

Factorise:

- (a)  $x^2 - 4$
- (b)  $b^2 - 49$
- (c)  $y^2 - 9$
- (d)  $x^2 - 1$
- (e)  $x^2 - 64$
- (f)  $x^2 - 100$
- (g)  $a^2 - 144$
- (h)  $x^2 - 225$
- (i)  $100 - x^2$
- (j)  $16 - y^2$

Factorise:

- (a)  $4x^2 - 25$
- (b)  $4y^2 - 49$
- (c)  $16x^2 - 1$
- (d)  $9x^2 - y^2$
- (e)  $25 - 4b^2$
- (f)  $25y^2 - 81$
- (g)  $100x^2 - 9$
- (h)  $16x^2 - 25$
- (i)  $a^2x^2 - 16$
- (j)  $121x^2 - 64y^2$

Factorise:

- (a)  $2x^2 - 50$
- (b)  $2d^2 - 32$
- (c)  $10x^2 - 10$
- (d)  $10x^2 - 250$
- (e)  $5x^2 - 45y^2$
- (f)  $5y^2 - 80$
- (g)  $8x^2 - 32$
- (h)  $8x^2 - 128$
- (i)  $20a^2 - 5b^2$
- (j)  $75x^2 - 27$

Factorise:

- (a)  $x^3 - 25x$
- (b)  $x^4 - 49$
- (c)  $100x^2 - x^6$
- (d)  $4x^5 - 9x^9$
- (e)  $8x^3 - 50xy^2$
- (f)  $72a^2b^3 - 98b^9$

## Fluency Practice

Factorise the following:

1)  $x^2 - 9$

2)  $x^2 - 4$

3)  $x^2 + 16$

4)  $9x^2 - 4$

5)  $4x^2 - 9$

6)  $4 - 9x^2$

7)  $16 - 9y^2$

8)  $100x^2 - 25$

9)  $100x^2 - 25y^2$

10)  $x^2 - y^2$

11)  $100x^{10} - 25y^{10}$

12)  $100x^2y^2 - 25$

13)  $100 - 25$

14)  $99x^2 - 24$

15)  $\frac{1}{4}x^2 - \frac{9}{25}$

## Fluency Practice

$$d^2 - g^2$$

$$w^2 - 1$$

write these as a difference of two squares

$$4t^2 - 1$$

$$81 - k^4$$

$$h^2 - 100$$

$$v^{14} - p^{12}$$

$$25 - m^{20}$$

$$a^8 - 9$$

$$4t^2 - 9p^2$$

$$t^2 - \frac{1}{4}$$

$$16g^6 - 49g^2$$

$$100g^{18} - 49$$

$$t^4 - e^2$$

$$d^6 - 25$$

$$121b^{24} - 0.01$$

$$d^{100} - 2\frac{1}{4}$$

$$0.36r^2 - 0.04$$

$$(n + 1)^2 - (n - 1)^2$$

## Fluency Practice

difference of two squares: factorise fully

1)  $1 - k^2 =$

2)  $4 - 9r^2 =$

3)  $49 - t^4 =$

4)  $\frac{1}{4} - 4b^2 =$

5)  $16g^6 - 4 =$

6)  $a^{2n} - 1 =$

7)  $1 - h^8 =$

8)  $0.64 - 0.25d^2 =$

9)  $64w^{10} - w^4 =$

10)  $(d - 2)^2 - 9 =$

11)  $e^{2f^4} - 36 =$

12)  $4a^2b^4c^8 - 4 =$

13)  $18c^2 - 2 =$

14)  $320 - 20m^2 =$

15)  $12 - 75k^2 =$

16)  $2a^3b - 8ab^3 =$

17)  $(n + 2)^2 - (n + 1)^2 =$

18)  $9(g + 2)^2 - 4(g - 3)^2 =$

19)  $32m^2n - 50n^3 =$

20)  $\frac{1}{4}d^2 - \frac{1}{9} =$

21)  $\left(\frac{k+1}{2}\right)^2 - \left(\frac{k-1}{2}\right)^2$

22)  $\left(\frac{k}{4}+1\right)^2 - \left(\frac{k}{4}-1\right)^2$

23)  $\left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

24)  $2\frac{1}{4}c^2 - 5\frac{4}{9} =$

## Fluency Practice

difference of two squares:  $a^2 - b^2 = (a - b)(a + b)$

use the difference of two squares to work out these calculations, without using a calculator:

1)  $13^2 - 3^2$

2)  $14^2 - 4^2$

3)  $13^2 - 7^2$

4)  $25^2 - 15^2$

5)  $51^2 - 49^2$

6)  $57^2 - 43^2$

7)  $36^2 - 26^2$

8)  $81^2 - 79^2$

9)  $26^2 - 24^2$

10)  $48^2 - 42^2$

11)  $55^2 - 5^2$

12)  $33^2 - 13^2$

13)  $95^2 - 5^2$

14)  $33^2 - 32^2$

15)  $21^2 - 29^2$

16)  $16^2 - 36^2$

17)  $38^2 - 12^2$

18)  $556^2 - 444^2$

19)  $667^2 - 333^2$

20)  $12345^2 - 12344^2$

21)  $1.5^2 - 0.5^2$

22)  $3.6^2 - 1.6^2$

23)  $9.7^2 - 0.3^2$

24)  $4.6^2 - 4.4^2$

25)  $6.6^2 - 3.4^2$

26)  $14.5^2 - 10.5^2$

27)  $27.5^2 - 23.5^2$

28)  $18.5^2 - 28.5^2$

29)  $9.66^2 - 0.34^2$

30)  $9.75^2 - 0.25^2$

try to write 55 as the difference between two square numbers – in two different ways

## Fluency Practice

Factorise:

1)  $3y^2 - 27$

2)  $2x^2 - 32$

3)  $x^3 + 4x^2 - 5x$

4)  $ax^2 - ay^2$

5)  $2x^3 - 8x$

6)  $16x^2y - 36y$

7)  $x^4 - 4x^3$

8)  $x^{10} - 4x^9 + 4x^8$

9)  $x^8 - 1$

## Fluency Practice

### THE FOUR BASIC METHODS OF FACTORING:

- **HIGHEST COMMON FACTOR:** *Always try this first.*
- **DIFFERENCE OF SQUARES:** This involves two terms.
- **QUADRATICS:** This involves three terms.
- **GROUPING:** This involves four or more terms.

Factoring should continue until each factor is *irreducible*, meaning that it cannot be factored further.

Use the techniques of the previous questions to factor each expression.

(a)  $a^2 - 25$

(b)  $b^2 - 25b$

(c)  $c^2 - 25c + 100$

(d)  $2d^2 + 25d + 50$

(e)  $e^3 + 5e^2 + 5e + 25$

(f)  $16 - f^2$

(g)  $16g^2 - g^3$

(h)  $h^2 + 16h + 64$

(i)  $i^2 - 16i - 36$

(j)  $5j^2 + 16j - 16$

(k)  $4k^2 - 16k - 9$

(l)  $2k^3 - 16k^2 - 3k + 24$

(m)  $2a^2 + ab - 4a - 2b$

(n)  $6m^3n^4 + 9m^2n^5$

(o)  $49p^2 - 121q^2$

(p)  $t^2 - 14t + 40$

(q)  $3t^2 + 2t - 40$

(r)  $5t^2 + 54t + 40$

(s)  $5t^2 + 33t + 40$

(t)  $5t^3 + 10t^2 + 15t$

(u)  $u^2 + 15u - 54$

(v)  $3x^3 - 2x^2y - 15x + 10y$

(w)  $1 - 36a^2$

(x)  $4a^2 - 12a + 9$

## Fluency Practice

	Single Bracket	Double Bracket	Difference of two Squares	How do you know?	Factorised Form
$x^2 + 4x - 21$					
$4x^2 + 8x$					
$x^2 - 64$					
$x^2 - 14x + 48$					
$10xy + 25y^2$					
$144x^2 - 25$					
$3x^2 + 6x^3$					
$10x^2 + 5x$					
$x^2 + 6x - 7$					

# Fluency Practice

Mixed Factorising Quadratics					
<b>(a)</b>	<b>(b)</b>	<b>(c)</b>	<b>(d)</b>	<b>(e)</b>	<b>(f)</b>
Factorise $x^2 - 4x$	Factorise $x^2 - 4x + 3$	Factorise $x^2 - 4$	Factorise $x^2 - 4x + 4$	Factorise $2x^2 - 4x$	Factorise $2x^2 - 5x + 3$
<b>(g)</b>	<b>(h)</b>	<b>(i)</b>	<b>(j)</b>	<b>(k)</b>	<b>(l)</b>
Factorise $2x^2 - 8$	Factorise $2x^2 - 15x - 8$	Factorise $x^2 - 15x - 16$	Factorise $x^2 - 16$	Factorise $x^2 + 10x + 16$	Factorise $x^2 + 10x$
<b>(m)</b>	<b>(n)</b>	<b>(o)</b>	<b>(p)</b>	<b>(q)</b>	<b>(r)</b>
Factorise $6x^2 - 10x$	Factorise $6x^2 - 11x + 3$	Factorise $6x^2 + 29x - 5$	Factorise $6x^2 - 24$	Factorise $x^2 + 5x - 24$	Factorise $5x^2 + 26x - 24$
<b>(s)</b>	<b>(t)</b>	<b>(u)</b>	<b>(v)</b>	<b>(w)</b>	<b>(x)</b>
Factorise $x^2 + 26x - 56$	Factorise $2x^2 + 6x - 56$	Factorise $2x^2 + 6x$	Factorise $2x^2 - 50$	Factorise $x^2 - 5x - 50$	Factorise $8x^2 - 9x - 50$

## Fluency Practice

<b>(a)</b>	<b>(b)</b>	<b>(c)</b>	<b>(d)</b>
Factorise $5x - 15$	Factorise $2x^2 + 7x$	Factorise fully $8x - 2x^2$	Factorise fully $15bc + 10abc$
<b>(e)</b>	<b>(f)</b>	<b>(g)</b>	<b>(h)</b>
Factorise fully $9a^3bc^2 + 15ab^2c$	Factorise $x^2 + 7x + 12$	Factorise $y^2 + 5y - 14$	Factorise $x^2 - 11x + 24$
<b>(i)</b>	<b>(j)</b>	<b>(k)</b>	<b>(l)</b>
Factorise $y^2 - 36$	Factorise $3x^2 + 5x + 2$	Factorise fully $2x^2 - 98$	Factorise $4x^2 + x - 14$
<b>(m)</b>	<b>(n)</b>	<b>(o)</b>	<b>(p)</b>
Factorise $25y^2 - 4x^2$	Factorise $6x^2 - 13x + 5$	Factorise $21 + 4x - x^2$	Factorise fully $2x^3 + 9x^2 - 5x$

## Fluency Practice

<b>A1</b> Factorise: $12x + 4$	<b>A2</b> Factorise: $3x - 15$	<b>A3</b> Factorise: $x^2 + 4x$	<b>A4</b> Factorise: $2x^2 - 5x$
<b>B1</b> Fully factorise: $2xy + 4x$	<b>B2</b> Fully factorise: $3xy - 9y$	<b>B3</b> Fully factorise: $4ab - 6abc$	<b>B4</b> Fully factorise: $2a^2b + 8ab^2$
<b>C1</b> Factorise: $x^2 - 36$	<b>C2</b> Factorise: $x^2 - 81$	<b>C3</b> Factorise: $x^2 - 1$	<b>C4</b> Factorise: $x^2 - 9$
<b>D1</b> Factorise: $x^2 + 3x - 54$	<b>D2</b> Factorise: $x^2 + 8x + 7$	<b>D3</b> Factorise: $x^2 - 4x - 32$	<b>D4</b> Factorise: $x^2 - 8x + 15$

## Fluency Practice

**A1** Factorise:  
 $9x + 15$

**A2** Factorise:  
 $4x^2 - 3x$

**A3** Factorise:  
 $3x^2 + 15x$

**A4** Factorise:  
 $9x^2y - 12xy^2$

**B1** Factorise:  
 $x^2 - 81$

**B2** Factorise:  
 $x^2 - y^2$

**B3** Factorise:  
 $4x^2 - 25$

**B4** Factorise:  
 $9x^2 - 16y^2$

**C1** Factorise:  
 $x^2 + 13x + 22$

**C2** Factorise:  
 $x^2 - 11x + 10$

**C3** Factorise:  
 $x^2 + 3x - 18$

**C4** Factorise:  
 $x^2 - x - 20$

**D1** Factorise:  
 $3x^2 + 8x + 4$

**D2** Factorise:  
 $2x^2 - 19x + 42$

**D3** Factorise:  
 $5x^2 + 21x - 20$

**D4** Factorise:  
 $4x^2 - 4x - 15$

## Fluency Practice

$$\textcircled{1} \quad x^2 + 3x + 2$$

$$\textcircled{2} \quad x^2 + 8x + 12$$

$$\textcircled{3} \quad x^2 - 5x + 6$$

$$\textcircled{4} \quad x^2 - 10x + 24$$

$$\textcircled{5} \quad x^2 + 11x - 12$$

$$\textcircled{6} \quad x^2 + x - 6$$

$$\textcircled{7} \quad x^2 - 5x - 14$$

$$\textcircled{8} \quad x^2 - 5x - 36$$

$$\textcircled{9} \quad 2x^2 + 6x + 4$$

$$\textcircled{10} \quad 2x^2 - 20x + 32$$

$$\textcircled{11} \quad 3x^2 - 12x - 36$$

$$\textcircled{12} \quad x^3 - 4x^2 - 12x$$

$$\textcircled{13} \quad 7x^3 - 28x^2 - 84x$$

$$\textcircled{14} \quad 2x^2 + 7x + 5$$

$$\textcircled{15} \quad 3x^2 - 16x + 5$$

$$\textcircled{16} \quad 3x^2 + 8x - 3$$

$$\textcircled{17} \quad 2x^2 - 7x - 15$$

$$\textcircled{18} \quad 8x^2 + 30x + 7$$

$$\textcircled{19} \quad 3x^2 - 17x + 10$$

$$\textcircled{20} \quad 9x^2 - 24x + 16$$

$$\textcircled{21} \quad 8x^2 - 14x - 15$$

$$\textcircled{22} \quad 12x^2 - 2x - 30$$

$$\textcircled{23} \quad 12 - x - x^2$$

$$\textcircled{24} \quad 3 + 8x - 3x^2$$

$$\textcircled{25} \quad cx^2 + 2bcx + b^2$$

$$\textcircled{26} \quad 3t^2 + 5qt + 2q^2$$

$$\textcircled{27} \quad 6r^2 - 5rt - 4t^2$$

$$\textcircled{28} \quad 12a^3 - 26a^2b + 4ab^2$$

## Intelligent Practice

- |                     |                          |                           |                          |
|---------------------|--------------------------|---------------------------|--------------------------|
| 1) $x^2 + 9x$       | 11) $2x^2 - 9x + 4$      | 21) $6x^2 - 19x - 20$     | 31) $40 - 6x - x^2$      |
| 2) $x^2 - 9x$       | 12) $2x^2 - 9x$          | 22) $6x^2 - 150$          | 32) $40 - 3x - x^2$      |
| 3) $x^2 - 9$        | 13) $2x^2y - 9xy^2$      | 23) $3x^2 - 75$           | 33) $ax - 3a - 5x + 15$  |
| 4) $x^2 - 9y^2$     | 14) $x^2 - y^2$          | 24) $3x^2 - 40x - 75$     | 34) $xy - 2x - 4y + 8$   |
| 5) $x^2 + 9x + 8$   | 15) $4x^2 - 4y^2$        | 25) $3x^2 - 40xy - 75y^2$ | 35) $x^2 - 2x - 8$       |
| 6) $x^2 + 9x + 20$  | 16) $6x^2 - 6y^4$        | 26) $3x^6 - 75y^2$        | 36) $x^3 - 2x^2 - 8x$    |
| 7) $x^2 - 9x + 20$  | 17) $6x^2y - 18y^4$      | 27) $(x - 3)^2 - 49$      | 37) $x^3y - 2x^2y - 8xy$ |
| 8) $x^2 - 9x - 22$  | 18) $6x^3y^2 - 18x^2y^3$ | 28) $(2x - 3)^2 - 49$     | 38) $10y^2 - 11y - 8$    |
| 9) $x^2 + 9x - 22$  | 19) $6x^2 + 22x + 20$    | 29) $4x^2 - 6x - 40$      | 39) $10y^4 - 11y^2 - 8$  |
| 10) $2x^2 + 9x + 4$ | 20) $6x^2 + 2x - 20$     | 30) $x^2 - 6x - 40$       | 40) $10y^5 - 11y^3 - 8y$ |

## Fluency Practice

Solve the following equations:

1.  $(x + 1)(x + 2) = x^2 + 5$

2.  $(x - 1)(x + 1) = x^2 + 3x + 17$

3.  $(x + 3)(x + 4) = x^2 + 3x + 17$

4.  $(x + 3)(2x - 3) = 2x^2 - 2x + 3$

5.  $(x + 2)(x + 1) = x^2 + x - 2$

6.  $(x + 3)(x - 4) = x^2 - 8x + 2$

7.  $x^2 - 3x + \frac{1}{4} = (x + 1)(x - 1) - 4x + 2\frac{3}{4}$

8.  $3x - x^2 + 5 = (5 - x)(4 + x) - 9$

9.  $5 + 4x + x^2 = (5 + x)(x + 1)$

10.  $6 + 3x - x^2 = (9 - x)(x - 6)$

11.  $(2x + 3)(x + 2) + x = (x + 2)(x + 3) + x^2 + 9$

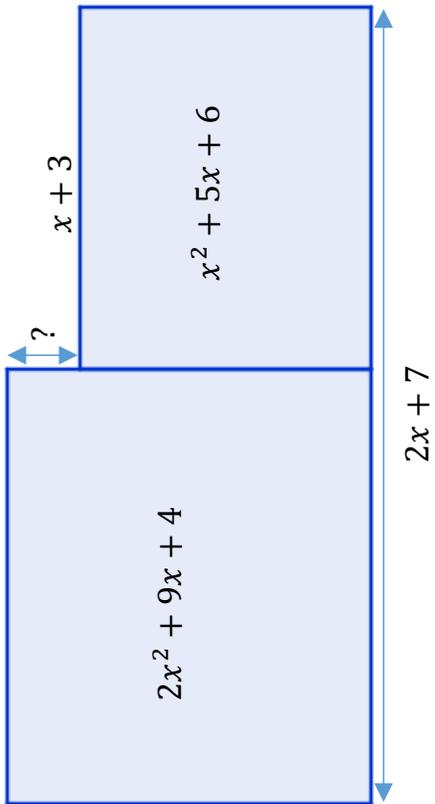
12.  $(2x + 3)(x - 1) + x = (x + 1)(2x - 1) + 2x - 3$

13.  $(x + 3)(2x - 1) + x^2 = (3x + 2)(x - 1) + 3x + 5$

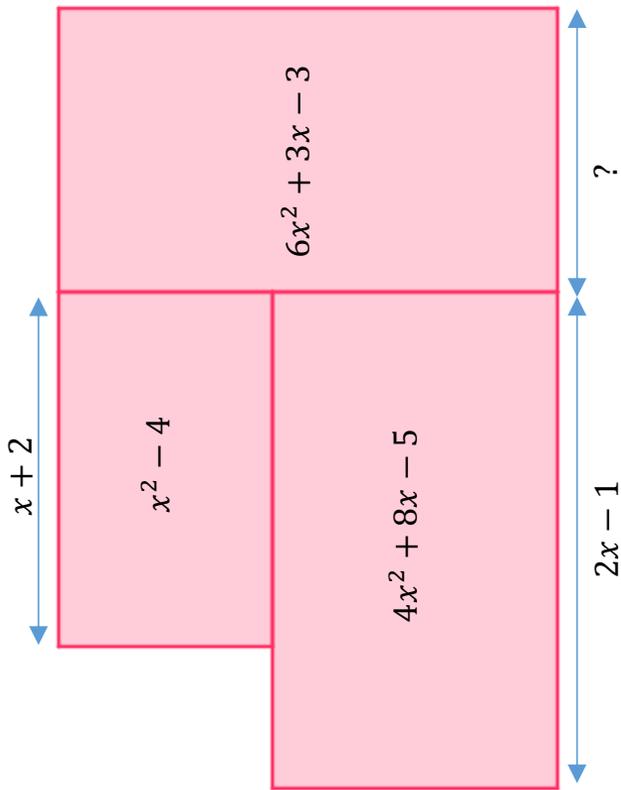
# Problem Solving

In each of these questions, find the marked side or area in terms of  $x$ .

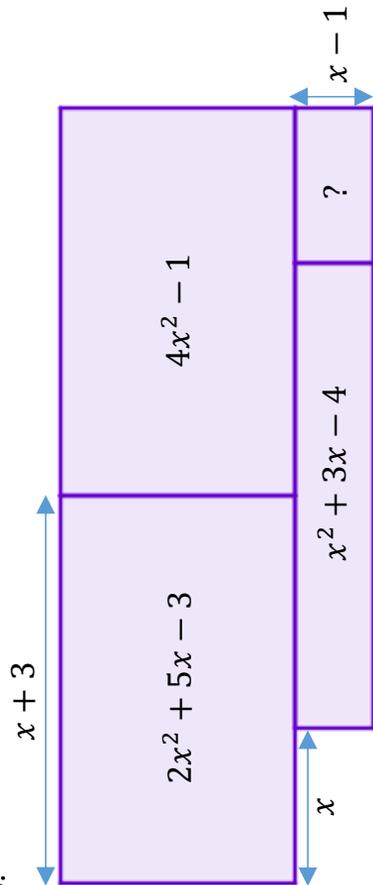
1.



2.

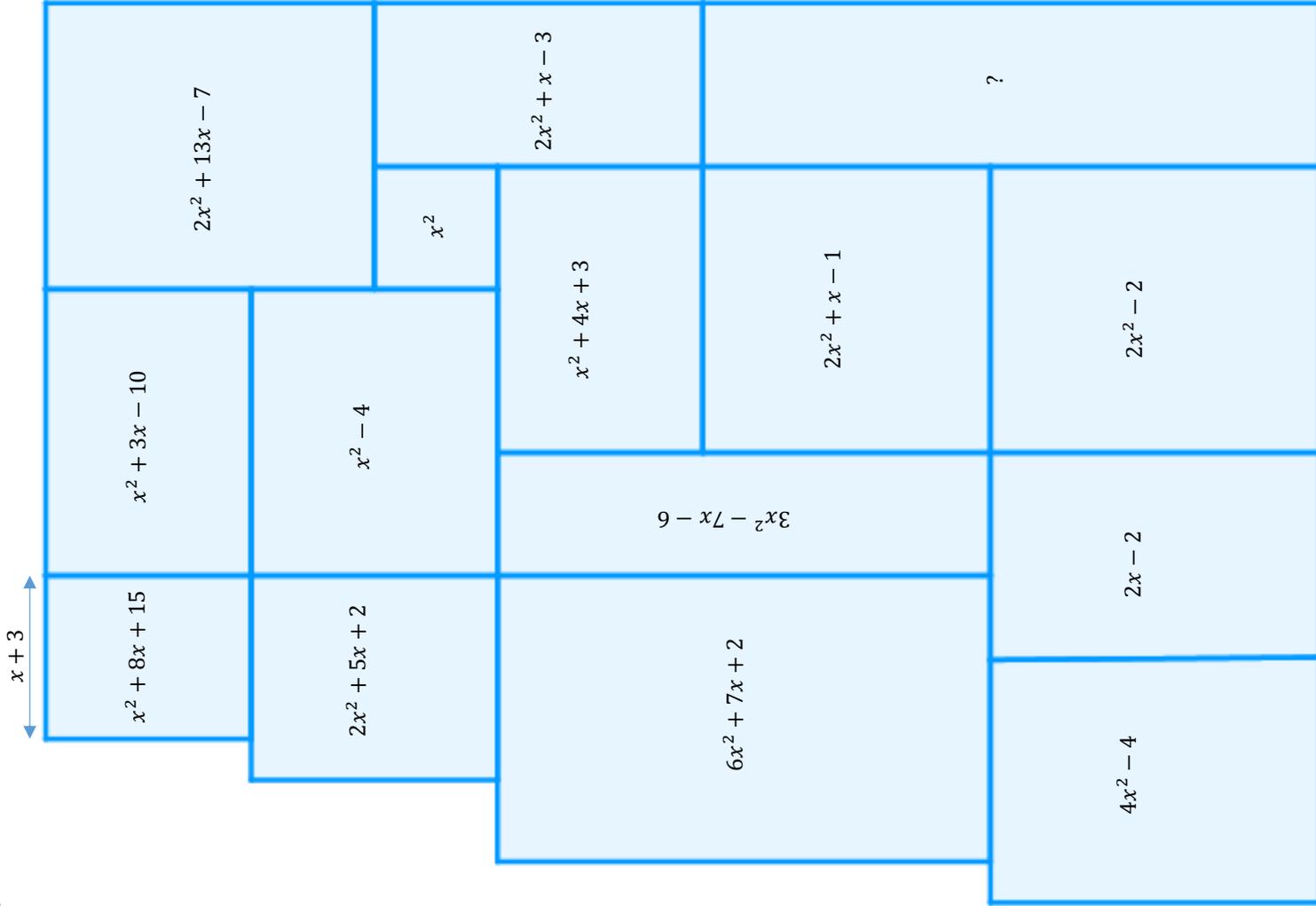


3.



# Problem Solving

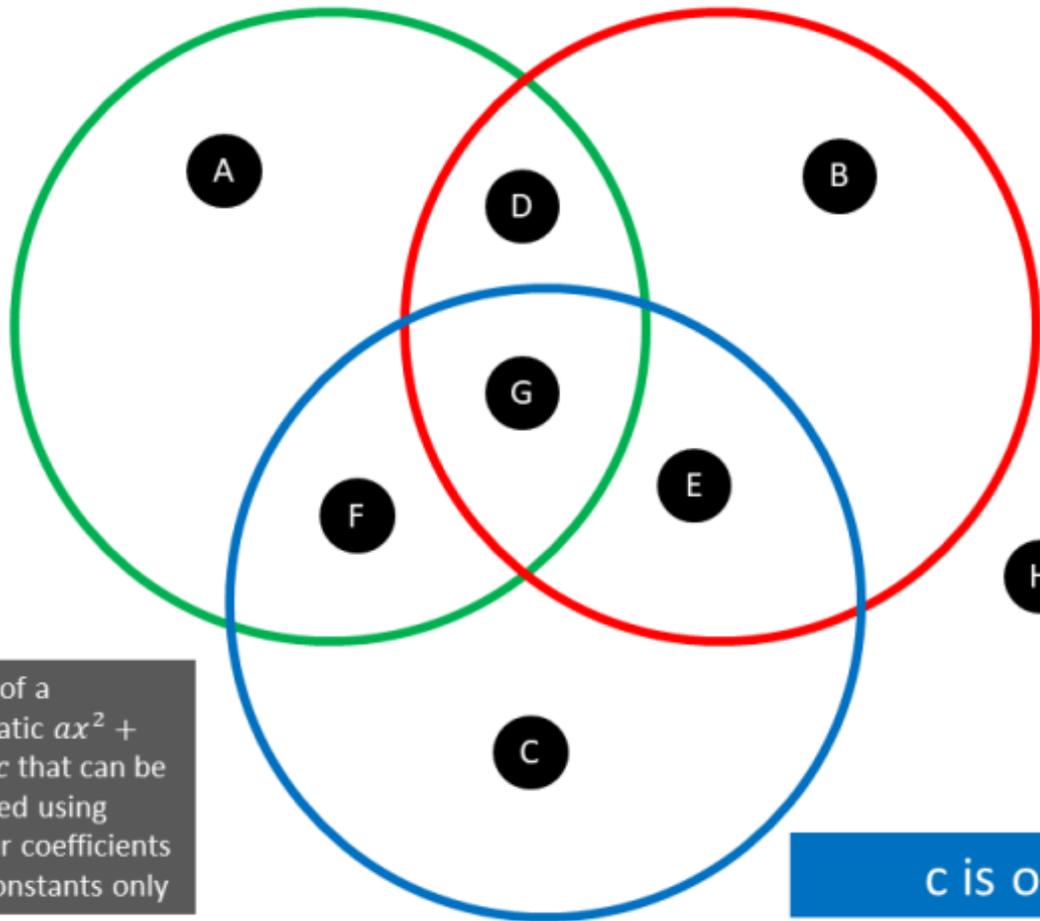
4.



# Problem Solving

$a > 1$

$b$  is even



If you think a region is impossible to fill, convince me why!

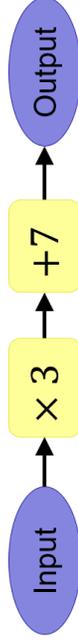
Think of a quadratic  $ax^2 + bx + c$  that can be factored using integer coefficients and constants only

$c$  is odd

## 5 Basic Functions

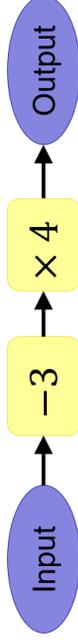
# Fluency Practice

Here is a function machine.



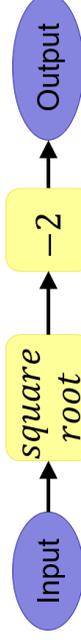
- (a) Find the output when the input is 10
- (b) Find the output when the input is 2.5
- (c) Find the output when the input is -3
- (d) Find the **input** when the **output** is 31

Here is a function machine.



- (a) Find the output when the input is 5
- (b) Find the output when the input is 12
- (c) Find the output when the input is 2
- (d) Find the **input** when the **output** is 28

Here is a function machine.



- (a) Find the output when the input is 25
- (b) Find the output when the input is 49
- (c) Find the output when the input is 121
- (d) Find the **input** when the **output** is 2

Here is a function machine.



- (a) Find the output when the input is 2
- (b) Find the output when the input is 10
- (c) Find the output when the input is -5
- (d) Find the **input** when the **output** is 48

(a) When the input is 3, the output is 10.  
Suggest two possible function machines.

(b) When I input 2, I get an output of 7.  
When I input 4, I get an output of 17.  
Can you work out what my two-step function machine is?

## Fluency Practice

Question 1: Given  $f(x) = 3x + 5$

Work out the values of

(a)  $f(2)$       (b)  $f(8)$       (c)  $f(0)$       (d)  $f(-2)$

Question 2: Given  $g(x) = \frac{2x + 9}{4}$

Work out the values of

(a)  $g(6)$       (b)  $g(-1)$       (c)  $g(0)$       (d)  $g(-10)$

Question 3: Given  $h(x) = x^2 - 5$

Work out the values of

(a)  $h(7)$       (b)  $h(-1)$       (c)  $h(-3)$       (d)  $h(15)$

# Fluency Practice

## functions into functions

Work out the value of each of these functions.  
Match them to the jumbled answers at the bottom.

$$G(x) = \frac{x}{3}$$

$$G(3)$$

$$G(-9)$$

$$G\left(\frac{1}{3}\right)$$

$$G(x+4)$$

$$G(6x + 6)$$

$$G(3x)$$

$$G(9x^3)$$

$$y(x) = 2x - 3$$

$$y(3)$$

$$y(-3)$$

$$y(?) = -3$$

$$y(x+3)$$

$$y(x-3)$$

$$y(4x)$$

$$y\left(\frac{x}{2}\right)$$

$$p(x) = 10 - x$$

$$p(4)$$

$$p\left(\frac{1}{4}\right)$$

$$p(-2)$$

$$p(-9)$$

$$p(x+2)$$

$$p(x-3)$$

$$p(x-7)$$

$$f(x) = \frac{1}{2}x^2$$

$$f(4)$$

$$f(12)$$

$$f(-6)$$

$$f(2x)$$

$$f(4x)$$

$$f\left(\frac{x}{2}\right)$$

$$f(x^2)$$

## jumbled answers

$$0$$

$$2x - 9$$

$$10 - (x+2)$$

$$12$$

$$17 - x$$

$$1$$

$$8x^2$$

$$-3$$

$$6$$

$$\frac{x+4}{3}$$

$$2x + 3$$

$$2x+2$$

$$-9$$

$$18$$

$$13 - x$$

$$72$$

$$x$$

$$9.75$$

$$3x^3$$

$$3$$

$$\frac{1}{9}$$

$$2x^2$$

$$8$$

$$8x - 3$$

$$x - 3$$

$$19$$

$$\frac{x^4}{2}$$

$$\frac{x^2}{8}$$

## Fluency Practice

Question 4: The function  $f$  is such that  $f(x) = 3x - 8$

Solve  $f(x) = 7$

Question 5: The function  $g$  is such that  $g(x) = 19 - 4x$

Solve  $g(x) = 31$

Question 6: The function  $h$  is such that  $h(x) = \frac{5x - 1}{2}$

Solve  $h(x) = 32$

Question 1: Given  $f(x) = 5x + 7$  and  $g(x) = 3x - 18$

Find the value of  $a$  such that  $f(a) = g(a)$

## Fluency Practice

(a)  $f(x) = 5x + 2$

- Find (i)  $f(3)$  (ii)  $f(7)$   
(iii)  $f(-4)$  (iv)  $f(-0.5)$

(b)  $f: \rightarrow x^2 - 4$

- Find (i)  $f(4)$  (ii)  $f(6)$   
(iii)  $f(-2)$  (iv)  $f(0.9)$

(c)  $g(x) = x^3 - 3x^2 - 2x + 1$

- Find (i)  $g(0)$  (ii)  $g(1)$   
(iii)  $g(-1)$  (iv)  $g(1.5)$

(d)  $f(x) = \sqrt{2x + 5}$

- Find (i)  $f(2)$  (ii)  $f(10)$   
(iii)  $f(-2)$  (iv)  $f(-1.78)$

(e) The functions  $f$  and  $g$  are such that  
 $f(x) = 3x - 5$  and  $g(x) = 4x + 1$

- (i) Find  $f(-1)$  and  $g(2)$   
(ii) Find the value of  $x$  for which  
 $f(x) = g(x)$ .

(f) The functions  $f$  and  $g$  are such that  
 $f(x) = 2x^2 - 1$  and  $g(x) = 5x + 2$

- (i) Find  $f(-3)$  and  $g(-5)$   
(ii) Find the two values of  $x$  for which  
 $f(x) = g(x)$ .

## Fluency Practice

Question 4: Given  $f(x) = x^2 + 4x - 1$

Express the following in the form  $ax^2 + bx + c$

(a)  $f(x + 2)$

(b)  $f(x - 1)$

(c)  $f(2x)$

(d)  $f(3x)$

(e)  $f(2x - 1)$

(f)  $f(4x + 3)$

Question 8:  $f(x) = x^2 + 3x + 8$

Show that  $f(x + 1) - f(x) = 2x + 4$

## Fluency Practice

1. The function  $f$  is such that  $f(x) = 5x + 2$   
Find (a)  $f(3)$  (b)  $f(7)$  (c)  $f(-4)$   
(d)  $f(-2)$  (e)  $f(-0.5)$  (f)  $f(0.3)$

2. The function  $f$  is such that  $f(x) = x^2 - 4$   
Find (a)  $f(4)$  (b)  $f(6)$  (c)  $f(-2)$   
(d)  $f(-6)$  (e)  $f(-0.2)$  (f)  $f(0.9)$

3. The function  $g$  is such that  $g(x) = x^3 - 3x^2 - 2x + 1$   
Find (a)  $g(0)$  (b)  $g(1)$  (c)  $g(2)$   
(d)  $g(-1)$  (e)  $g(-0.4)$  (f)  $g(1.5)$

4. The function  $f$  is such that  $f(x) = \sqrt{2x + 5}$   
Find (a)  $f(0)$  (b)  $f(1)$  (c)  $f(2)$   
(d)  $f(-1)$  (e)  $f(-0.7)$  (f)  $f(1.5)$

5.  $f(x) = 3x^2 - 2x - 8$

Express  $f(x + 2)$  in the form  $ax^2 + bx$

6. The functions  $f$  and  $g$  are such that  
 $f(x) = 3x - 5$  and  $g(x) = 4x + 1$   
(a) Find (i)  $f(-1)$  (ii)  $g(2)$   
(b) Find the value of  $x$  for which  $f(x) = g(x)$ .

7. The functions  $f$  and  $g$  are such that  
 $f(x) = 2x^2 - 1$  and  $g(x) = 5x + 2$   
(a) Find  $f(-3)$  and  $g(-5)$

## Fluency Practice

### Functions

1. If  $f(x) = 3x + 5$ , find

a)  $f(5) =$       b)  $f(2) =$       c)  $f(-1) =$

2. If  $g(x) = 4x^2 - 3x$ , find

a)  $g(3) =$       b)  $g(-1) =$       c)  $g\left(\frac{1}{2}\right) =$

3. If  $f(x) = \frac{3x+5}{2x-1}$ , find

a)  $f(5) =$       b)  $f(-1) =$       c)  $f\left(\frac{5}{6}\right) =$

4. Given that  $f(x) = 5x + 3$ , find the value of  $x$  for

a)  $f(x) = 9,$       b)  $f(x) = -4,$

c)  $f(x) = 2x + 7,$

5. Given that  $f(x) = \frac{3x+4}{2x-5}$ , find the value of  $x$  for

a)  $f(x) = 4,$       b)  $f(x) = -3,$

c)  $f(x) = \frac{3}{4},$

7. If,  $f(x) = 10 - 2x$ , determine, by simplifying if possible

a)  $f(3x) =$       b)  $f(x + 3) =$

c)  $f(x - 3) =$

8. If,  $g(x) = x^2 - 2x$ , determine, by simplifying if possible

a)  $g(2x) =$       b)  $g(-3x) =$

c)  $g(x + 1) =$

9. If,  $f(x) = 2x^2 - 5x$ , determine, by simplifying if possible

a)  $f(3x) =$       b)  $f(-x) =$

c)  $f(x - 4) =$

*Extension*

10. If  $f(x) = 2x - 1$

a) Determine  $f(1 + 2f(x - 1)) =$

b) Solve  $f(x + 1) + f(x - 1) = 0,$

## Fluency Practice

1: Work out the value of each function:

a) If  $h(x) = x - 9$ , find  $h(13)$

\_\_\_\_\_

b) If  $g(x) = \frac{-x}{3}$ , find  $g(-30)$

\_\_\_\_\_

2: Work out the value of each function:

a) If  $f(x) = \frac{-x^2 + 8}{8}$ , find  $f(8)$

\_\_\_\_\_

b) If  $h(x) = -7(x - 3)^2$ , find  $h(10)$

\_\_\_\_\_

3: Work out the value of each function:

a) If  $f(x) = \sqrt{\left(\frac{x+8}{5}\right)}$ , find  $f(117)$

\_\_\_\_\_

b) If  $g(x) = \frac{x-3}{10}$ , find  $g(63)$

\_\_\_\_\_

4: Work out the value of each function:

a) If  $f(x) = \frac{x+9}{x+2}$ , find  $f(3)$

\_\_\_\_\_

b) If  $g(x) = \frac{1}{4-3x}$ , find  $g(1)$

\_\_\_\_\_

## Fluency Practice

5: Work out the value of each function:

a) If  $h(x) = \frac{1-4x}{2x}$ , find  $h(-\frac{1}{6})$

\_\_\_\_\_

b) If  $g(x) = \frac{6x-5}{5x-2}$ , find  $g(\frac{21}{46})$

\_\_\_\_\_

6: Solve for x:

a)  $f(x) = x^2$ ;  $f(x) = 81$

\_\_\_\_\_

b)  $h(x) = \sqrt{x}$ ;  $h(x) = 2$

\_\_\_\_\_

7: Solve for x:

a)  $f(x) = (\frac{x+4}{9})^2$ ;  $f(x) = \frac{36}{81}$

\_\_\_\_\_

b)  $g(x) = 7x + 6$ ;  $g(x) = -36$

\_\_\_\_\_

# Fluency Practice

8: Solve for  $x$ :

a)  $h(x) = 6\sqrt{x} - 3$ ;  $h(x) = 1$

b)  $h(x) = \sqrt{-7x + 2}$ ;  $h(x) = 10$

\_\_\_\_\_

\_\_\_\_\_

9: Solve for  $x$ :

a)  $g(x) = \frac{5x - 3}{6x - 4}$ ;  $g(x) = \frac{3}{4}$

b)  $f(x) = \frac{4}{x - 4}$ ;  $f(x) = -\frac{4}{13}$

\_\_\_\_\_

\_\_\_\_\_

10: Solve for  $x$ :

a)  $g(x) = \frac{4}{x} - 10$ ;  $g(x) = 7$

b)  $f(x) = \frac{5x + 10}{x - 1}$ ;  $f(x) = -3$

\_\_\_\_\_

\_\_\_\_\_

## 6 Changing the Subject

## Intelligent Practice

Make  $x$  the subject for each of the following formulae:

1)  $y = ax + b$

2)  $y = b^2 + ax$

3)  $y = ax + bcd$

4)  $y = d^2ef + ax$

5)  $y = ax - b$

6)  $y = -\sqrt{b} + ax$

7)  $y = ax - bcd$

8)  $y = -\sqrt{bcd} + ax$

## Intelligent Practice

Make  $x$  the subject for each of the following formulae:

$$1) \quad y = \frac{x}{a} + c$$

$$2) \quad y = -\frac{x}{a} + c$$

$$3) \quad y = \frac{x}{a^2} - c$$

$$4) \quad y = -\frac{x}{a^2} - c$$

$$5) \quad y = \frac{x}{bc} + \sqrt{qrs}$$

$$6) \quad y = -\frac{x}{bc} - \sqrt{qrs}$$

## Fluency Practice

Make  $x$  the subject of these equations.

There are two ways to rearrange each equation so there are two sets of answer boxes. You only need to fill in your answer once. Use the boxes where your answer fits best.

$$a(x + b) = c \quad x = \boxed{\quad} - \boxed{\quad} \quad \text{or} \quad x = \boxed{\quad}$$

$$2(x + a) = b \quad x = \boxed{\quad} - \boxed{\quad} \quad \text{or} \quad x = \boxed{\quad}$$

$$a(x - 6) = b \quad x = \boxed{\quad} + \boxed{\quad} \quad \text{or} \quad x = \boxed{\quad}$$

$$c(7 + x) = 2 \quad x = \boxed{\quad} - \boxed{\quad} \quad \text{or} \quad x = \boxed{\quad}$$

## Intelligent Practice

Make  $a$  the subject for each of the following formulae:

1)  $2a^2 + b = c$

2)  $2a^2 - b = c$

3)  $6a^2 - b = c$

4)  $2(a - 2b)^2 + b = c$

5)  $2(a + 2b)^2 - b = c$

6)  $6(a + 2b)^2 - b = c$

1)  $2\sqrt{a} + b = c$

2)  $2\sqrt{a} - b = c$

3)  $6\sqrt{a} - b = c$

4)  $2\sqrt{a - 2b} + b = c$

5)  $2\sqrt{a + 2b} - b = c$

6)  $6\sqrt{a + 2b} - b = c$

# Fluency Practice

Question 1: Make  $y$  the subject of each of the following

- |                       |                        |                      |
|-----------------------|------------------------|----------------------|
| (a) $y + w = c$       | (b) $y - p = m$        | (c) $m + y = s$      |
| (d) $y - 2g = n$      | (e) $3y = c$           | (f) $ay = w$         |
| (g) $\frac{y}{c} = w$ | (h) $\frac{y}{a} = 2c$ | (i) $a = y + p$      |
| (j) $c = y - k$       | (k) $y^2 = s$          | (l) $y^3 = x$        |
| (m) $\sqrt{y} = g$    | (n) $\pi y = c$        | (o) $n - y = t$      |
| (p) $ry = c$          | (q) $4\pi y = b$       | (r) $y + 7t = c + r$ |
| (s) $\frac{r}{y} = w$ | (t) $y^2 = k + x$      | (u) $A = xy$         |

Question 2: Make  $x$  the subject of the following formulae

- |                           |                           |                                 |
|---------------------------|---------------------------|---------------------------------|
| (a) $4x + c = w$          | (b) $dx - t = 8$          | (c) $x^2 + 3 = h$               |
| (d) $2x + 2y = P$         | (e) $s = x^2 - 3$         | (f) $y = xz + s$                |
| (g) $\frac{x}{n} + 2 = w$ | (h) $\frac{x}{6} - 5 = w$ | (i) $\frac{x+3}{c} = h$         |
| (j) $3y = 4x + 1$         | (k) $x^2 + a = v$         | (l) $x^3 - 4 = 5y$              |
| (m) $\frac{x+t}{m} = 2c$  | (n) $\frac{w+x}{u} = 3z$  | (o) $A = \pi x^2$               |
| (p) $A = \frac{1}{2}bx$   | (q) $V = abx$             | (r) $v^2 = u^2 + 2ax$           |
| (s) $\frac{a+b}{x} = r$   | (t) $\frac{5cx}{b} = a$   | (u) $\sqrt[3]{\frac{x}{k}} = w$ |

Question 3: Make  $c$  the subject of the following

- |                     |                  |                     |
|---------------------|------------------|---------------------|
| (a) $(a + c)^2 = t$ | (b) $v = u + ac$ | (c) $v = \pi c^2 h$ |
|---------------------|------------------|---------------------|

Question 1: The circumference of a circle is given as  $c = 2\pi r$   
Make the radius,  $r$ , the subject of the formula.

Question 2: The formula to convert degrees Fahrenheit to degree Celsius is  $\frac{5}{9}(F - 32) = C$   
Find the formula to convert from degrees Celsius to degrees Fahrenheit by making  $F$  the subject.

Question 3: Can you spot any mistakes below?

Make  $y$  the subject of the formula:

$$k = y^2 + a$$

$$\sqrt{k} = y + a$$

$$\sqrt{k} - a = y$$

$$y = \sqrt{k} - a$$

Express  $v$  in terms of  $t$

$$t = \frac{v}{4} + 1$$

$$t - 1 = \frac{v}{4}$$

$$\frac{t - 1}{4} = v$$

## Fluency Practice

<b>Rearranging One-Step Formulae.</b> Make the subject the letter given in brackets.			
<b>(a)</b>	<b>(b)</b>	<b>(c)</b>	<b>(d)</b>
$V = I \times R$ ( $R$ )	$A = b \times h$ ( $b$ )	$P = \frac{F}{A}$ ( $F$ )	$D = \frac{M}{V}$ ( $M$ )
<b>(e)</b>	<b>(f)</b>	<b>(g)</b>	<b>(h)</b>
$E = C + D$ ( $C$ )	$a = b + c$ ( $c$ )	$S = U - V$ ( $U$ )	$y = x - 5$ ( $x$ )
<b>(i)</b>	<b>(j)</b>	<b>(k)</b>	<b>(l)</b>
$y = x^2$ ( $x$ )	$u = v^2$ ( $v$ )	$T = \sqrt{S}$ ( $S$ )	$a = \sqrt{b}$ ( $b$ )
<b>(m)</b>	<b>(n)</b>	<b>(o)</b>	<b>(p)</b>
$y = mx + c$ ( $c$ )	$v = u + at$ ( $u$ )	$d = c^2 + b$ ( $b$ )	$V = R - I^2$ ( $R$ )

## Fluency Practice

**Rearranging Two-Step Formulae.** Make the subject the letter given in brackets.

<b>(a)</b>	<b>(b)</b>	<b>(c)</b>	<b>(d)</b>
$v = u + at$ (t)	$y = mx + c$ (m)	$a = 2b - c$ (b)	$s = ut - d$ (u)
<b>(e)</b>	<b>(f)</b>	<b>(g)</b>	<b>(h)</b>
$a = \frac{b+c}{2}$ (b)	$t = \frac{u-v}{4}$ (u)	$d = \frac{a}{b} + c$ (a)	$h = \frac{e}{2} - f$ (e)
<b>(i)</b>	<b>(j)</b>	<b>(k)</b>	<b>(l)</b>
$T = m^2 + d$ (m)	$b = x^2 - a$ (x)	$d = 5c^2$ (c)	$P = RI^2$ (I)
<b>(m)</b>	<b>(n)</b>	<b>(o)</b>	<b>(p)</b>
$s = \frac{uv}{2}$ (v)	$a = b + c - d$ (c)	$v = \sqrt{e+f}$ (e)	$y = \sqrt{\frac{a}{b}}$ (a)

## Fluency Practice

<b>Rearranging Formulae.</b> Make $x$ the subject of these formulae.			
<b>(a)</b>	<b>(b)</b>	<b>(c)</b>	<b>(d)</b>
$y = x + a$	$y = bx$	$y = \frac{x}{c}$	$y = bx + a$
<b>(e)</b>	<b>(f)</b>	<b>(g)</b>	<b>(h)</b>
$y = \frac{x}{a} + b$	$y = \frac{x}{a} - b$	$y = \frac{x + b}{2}$	$y = 4x + a$
<b>(i)</b>	<b>(j)</b>	<b>(k)</b>	<b>(l)</b>
$y = x^2$	$y = x^2 + a$	$2y = x^2 - b$	$y = ax^2$
<b>(m)</b>	<b>(n)</b>	<b>(o)</b>	<b>(p)</b>
$y = ax^2 + b$	$2y = bx^2$	$y = \frac{3x + a}{5}$	$y = \sqrt{x + b}$

## Fluency Practice

<b>Rearranging Multi-Step Formulae.</b> Make $x$ the subject of each of the formulae.			
<b>(a)</b>	<b>(b)</b>	<b>(c)</b>	<b>(d)</b>
$y = \frac{ax}{b} + c$	$y = \frac{ax^3}{b}$	$ax + by = c$	$y = \sqrt{\frac{bx}{10}}$
<b>(e)</b>	<b>(f)</b>	<b>(g)</b>	<b>(h)</b>
$y + a = \frac{x}{2b} - 3a$	$x^2 + y^2 = 9$	$y = ab + \sqrt{cx}$	$y = \frac{2x + a}{b}$
<b>(i)</b>	<b>(j)</b>	<b>(k)</b>	<b>(l)</b>
$y = \sqrt[3]{5x - a}$	$y = \frac{a(x - 2)}{b}$	$y + 6 = \frac{ax}{2} - 4y$	$y^3 = \frac{x^2}{a} - b^2$

## Fluency Practice

Q	Make $x$ the subject of the formula	Answers
<b>1</b>	$a = b + \sqrt{cx}$	
<b>2</b>	$a = \sqrt{b + cx}$	
<b>3</b>	$a = b - \sqrt{cx}$	
<b>4</b>	$a = \sqrt{b - cx}$	
<b>5</b>	$a = \sqrt{b} + cx$	
<b>6</b>	$a = \sqrt{b} - cx$	
<b>7</b>	$a = b + \sqrt[3]{cx}$	
<b>8</b>	$a = b - \sqrt[3]{cx}$	
<b>9</b>	$a = \sqrt[4]{cx - b}$	
<b>10</b>	$a = \sqrt[n]{cx} + b$	

## Fluency Practice

1	weight = mass $\times$ gravitational field strength	$W = m g$	Make m the subject	
2	work done = force $\times$ distance	$W = F s$	Make s the subject	
3	force = spring constant $\times$ extension	$F = k e$	Make e the subject	
4	pressure = $\frac{\text{force normal to a surface}}{\text{area of that surface}}$	$p = \frac{F}{A}$	Make F the subject	
5	distance travelled = speed $\times$ time	$s = v t$	Make v the subject	
6	acceleration = $\frac{\text{change in velocity}}{\text{time taken}}$	$a = \frac{\Delta v}{t}$	Make $\Delta v$ the subject	
7	resultant force = mass $\times$ acceleration	$F = m a$	Make a the subject	
8	momentum = mass $\times$ velocity	$p = m v$	Make m the subject	
9	power = $\frac{\text{work done}}{\text{time taken}}$	$P = \frac{W}{t}$	Make W the subject	
10	density = $\frac{\text{mass}}{\text{volume}}$	$\rho = \frac{m}{V}$	Make m the subject	
11	time period = $\frac{1}{\text{frequency}}$	$T = \frac{1}{f}$	Make f the subject	
12	power = (current) <sup>2</sup> $\times$ resistance	$P = I^2 R$	Make I the subject	

## Fluency Practice

### Changing the Subject of a Formula

If you know all the other information, how do you find...

Volume of a Cuboid

... the height of a cuboid?

$$V = whl$$

Area of a Triangle

... the base length of a triangle?

$$A = \frac{bh}{2}$$

Volume of a Triangle-based Prism

... the length of the prism?

$$V = \frac{1}{2}whl$$

Circumference of a Circle

... the radius?

$$C = 2\pi r$$

Area of a Trapezium

... one of the parallel side lengths ( $b$ )?

$$A = \frac{1}{2}(a + b)h$$

Area of a Circle

... the radius?

$$A = \pi r^2$$

Surface Area of a Sphere

... the radius?

$$A = 4\pi r^2$$

Volume of a Square-based Pyramid

... the base edge length ( $l$ )?

$$V = \frac{1}{3}hl^2$$

Pythagoras' Theorem

... the length of a shorter side ( $a$ )?

$$a^2 + b^2 = c^2$$

Volume of a Sphere

... the radius?

$$V = \frac{4}{3}\pi r^3$$

## Intelligent Practice

Make  $a$  the subject of the following formulae

1.  $ab + ac = 3$

8.  $m(a + y) = x(a + b)$

2.  $3a + ay = x$

9.  $\frac{y-a}{x+a} = b$

3.  $a(3 + y) + 2a = x$

10.  $a - n = \frac{a+2}{n}$

4.  $2a = x + ak$

11.  $\sqrt{\frac{a+x}{a-x}} = 2$

5.  $ay + k = 3a$

12.  $\sqrt{\frac{m(a+n)}{a}} = p$

6.  $ay - x = ab$

7.  $ay + b = 3b + ax$

## Intelligent Practice

Make  $x$  the subject for each of the following formulae:

1)  $5x + b = 2x + a$

2)  $5x + bd = 2x + ac$

3)  $bx + 5 = ax + 2$

4)  $bx + 5d = ax + 2c$

5)  $5(x + b) = 2(x + a)$

6)  $5(x + bd) = 2(x + ac)$

7)  $b(x + 5) = a(x + 2)$

8)  $b(x + 5d) = a(x + 2c)$

## Intelligent Practice

Make  $x$  the subject for each of the following formulae:

$$1) \quad w = \frac{5x+5b}{x+a}$$

$$2) \quad w = \frac{5x+5b}{4x+4a}$$

$$3) \quad w = \frac{5x+5b}{3x-3a}$$

## Fluency Practice

Question 1: Make  $x$  the subject of each of the following

(a)  $A = \frac{1}{2}(x + y)$

(b)  $A = \pi r^2 + 2\pi r x$

(c)  $T = 3x^2 - y$

(d)  $s = \frac{m}{ax}$

(e)  $s = uy + \frac{1}{2}xy^2$

(f)  $\frac{1}{3}w = \frac{1}{4}ax + t$

(g)  $j = \frac{x+3}{d}$

(h)  $g = \frac{t}{x-2}$

(i)  $p = 3(y + 2x)^2$

(j)  $12w = \frac{3}{4}(2x + a)$

Question 2: Make  $m$  the subject of the following formulae

(a)  $5(m + y) = 4(m - 3y)$

(b)  $3(3m + 4) = 7(m + 2a)$

(c)  $15(2m + 3c) = 5(m + 7c)$

(d)  $9m + 4c = 2(a + 3m)$

(e)  $a(c + m) = 2(c + 3m)$

(f)  $w(m + n) = x(m - n)$

(g)  $8 = \frac{m+3c}{m-f}$

(h)  $y = \frac{m+4}{m+5}$

(i)  $y = 3mt - a^2m$

(j)  $r(c + 7) = 3m + 5$

(k)  $x = 4\pi m + am$

(l)  $2 = \frac{m+k}{m-t}$

(m)  $dm = y - em$

(n)  $m(c + d) = m + f$

(o)  $y - mp = np + 2y$

(p)  $m(r + p) = r(h - m)$

(q)  $\pi x = \frac{m+8}{m-1}$

(r)  $\frac{3m+2}{c} = \frac{m+1}{a}$

Question 3: Make  $c$  the subject of the following

(a)  $w = \frac{ac}{a-c}$

(b)  $w = 6 + \frac{a}{c+2}$

Apply

Question 1: The cosine rule is  $a^2 = b^2 + c^2 - 2bc \cos A$ .  
Make  $\cos A$  the subject.

## Fluency Practice

1. Make  $x$  the subject of each of the following formulae.

(a)  $2x + a = x - b$

(b)  $ax - b = cx - d$

(c)  $xa - 4 = bx - 5$

(d)  $3x - 6 = 4a + 2x$

(e)  $b - 2x = c - 5x$

(f)  $a - bx = c - dx$

(g)  $2(x + 1) = a - x$

(h)  $4(x - a) = 3(a - x)$

(i)  $p(x + 1) = q(x - 1)$

(j)  $\frac{x - a}{2} = \frac{x + b}{3}$

(k)  $\frac{2x - a}{5} = x + 1$

(l)  $\frac{x}{a} = \frac{x + b}{4}$

2. Make  $x$  the subject of each of the following formulae.

(a)  $P = \frac{x}{x + 1}$

(b)  $P = \frac{ax + b}{x}$

(c)  $Q = \frac{x + b}{x - a}$

(d)  $q^2 = \frac{x + y}{x - y}$

(e)  $\frac{x - 2}{x + 3} = a$

(f)  $\frac{x - b}{x - c} = 4$

(g)  $p = \sqrt{\frac{x}{x + 1}}$

(h)  $w = \sqrt{\frac{x - 2}{x}}$

(i)  $w = \sqrt{\frac{x - 2}{x + 1}}$

(j)  $p = \frac{x^2 + 2}{x^2}$

(k)  $p = \frac{2 - x^2}{3 - x^2}$

(l)  $g = \frac{x^2 - y}{x^2 + y}$

## Fluency Practice

Make  $x$  the subject of each formula:

(a)  $ax = bx + c$

(b)  $ax = bx - c$

(c)  $ax + c = bx + 2$

(d)  $ax - d = bx + c$

(e)  $ax - 2 = d - bx$

Make  $x$  the subject of each formula:

(a)  $a = \frac{bx}{x-2}$

(b)  $2 = \frac{bx}{a+x}$

(c)  $a = \frac{x+b}{x}$

(d)  $2a = \frac{x}{x-1}$

Make  $x$  the subject of each formula:

(a)  $a(x + 2) = bx - c$

(b)  $a(b - x) = c(x + d)$

(c)  $2b = \frac{x-a}{c+x}$

(d)  $a = \frac{x+b}{x-b}$

Make  $x$  the subject of each formula:

(a)  $d = \sqrt{\frac{x-c}{x}}$

(b)  $bx = c + \frac{x}{a}$

(c)  $x = \sqrt{\frac{a+x^2}{b}}$

(d)  $bx^3 = \frac{a^2-x^3}{c}$

## Intelligent Practice



Make x the subject of the formula

1)  $y = x + a$

2)  $y = ax$

3)  $y = x - a$

4)  $y = a - x$

5)  $y = \frac{x}{a}$

6)  $y = \frac{a}{x}$

7)  $y = ax + b$

8)  $y = abx$



Make x the subject of the formula

1)  $y = \frac{ax}{b}$

2)  $y = x^2 + a$

3)  $y = ax^2$

4)  $y = \sqrt{x} + a$

5)  $y = \sqrt{x - a}$

6)  $y = a\sqrt{x}$

7)  $y = \frac{x^2}{ab}$

8)  $y = \frac{\sqrt{x}}{ab}$



Make x the subject of the formula

1)  $x + a = b - x$

2)  $ax = b - x$

3)  $x - b = ax + c$

4)  $ax = by + x$

5)  $a(x+y) = b - x$

6)  $ax + y = bx + z$

7)  $ax - y = bx + yz$

8)  $a = \frac{bx}{x+c}$

## Intelligent Practice

Make  $a$  the subject of the following formulae:

1)  $2a = b$

2)  $\frac{a}{2} = b$

3)  $a + 2 = b$

4)  $a - 2 = b$

5)  $\frac{a}{c} = b$

6)  $\frac{a}{c} + 2 = b$

7)  $ac = b$

8)  $ac - 2 = b$

9)  $ac - d = b$

10)  $\frac{a+2}{c} = b$

11)  $\frac{2a}{c} = b$

12)  $\frac{d(a+2)}{c} = b$

13)  $2a = b + a$

14)  $2a + c = b + a$

15)  $2a + c = b - a$

16)  $ae + c = b + a$

17)  $ae + c = b + af$

18)  $ae + c = b - af$

## Fluency Practice

Make  $x$  the subject.

(a)  $F = x^2 - 3$

(b)  $p = 5(x - 3)$

(c)  $2x + a = x + b$

(d)  $y^2 = x^2 + a^2$

(e)  $P = \frac{x^2}{4} + c$

(f)  $y - 5 = 2 + \frac{x}{6}$

Make  $x$  the subject.

(a)  $4x - 8 = ax$

(b)  $ax = b - cx$

(c)  $2(x + 3) = bx$

(d)  $a - bx = dx - a$

(e)  $4(x - 2y) = 3(2x - y)$

(f)  $p(x - b) = q(x + b)$

Make  $x$  the subject.

(a)  $A = 2xb^2 + cx$

(b)  $x(c - d) = c(d - x)$

(c)  $y = \frac{x+2}{x-2}$

(d)  $2a = \frac{x-3}{4x}$

(e)  $b = \frac{ax}{a+x}$

(f)  $a = \frac{2+3x}{x-2}$

# Fluency Practice

## Rearranging Harder Formulae

Make  $x$  the subject of each of the formulae.

(a)	(b)	(c)	(d)
$a = b + x^2$	$a^2 = b^2 + x^2$	$b = \frac{x^3}{2a}$	$b = 2a + \sqrt{x}$
(e)	(f)	(g)	(h)
$b = \sqrt{\frac{x}{a}}$	$a + 5 = \sqrt{\frac{x}{b}}$	$ax = x + 5$	$ax + b = a + x$
(i)	(j)	(k)	(l)
$a = \frac{x}{x + 1}$	$a(x - b) = bx$	$b = \frac{x + a}{x - a}$	$a = \sqrt{\frac{x}{x + b}}$

## Fluency Practice

<b>(a)</b>	<b>(b)</b>	<b>(c)</b>	<b>(d)</b>
$y = x^2 + 2x$ Find the value of $y$ when $x = 5$	$b = a^3 - 5a$ Find the value of $b$ when $a = 3$	$w = 2d^2 + 5d$ Find the value of $w$ when $d = -4$	$y = 3x^3 + 5x^2 - 6$ Find the value of $y$ when $x = -2$
<b>(e)</b>	<b>(f)</b>	<b>(g)</b>	<b>(h)</b>
$d = 3a + 5b$ Find $d$ when $a = 7$ and $b = -2$	$t = p^2 + pq$ Find $t$ when $p = -6$ and $q = 2$	$f = \frac{2d + e^2}{de}$ Find $f$ when $d = 5$ and $e = -2$	$y = \frac{3}{4}ab^2$ Find $y$ when $a = 5$ and $b = -0.5$
<b>(i)</b>	<b>(j)</b>	<b>(k)</b>	<b>(l)</b>
Make $b$ the subject of $a = 4b - 7$	Make $x$ the subject of $y = x^2 + 5$	Make $d$ the subject of $e = \frac{c + d}{5}$	Make $a$ the subject of $x = 2a^2 - cd$
<b>(m)</b>		<b>(n)</b>	
Make $x$ the subject of the formula $y = \frac{x}{x-3}$		Make $a$ the subject of the formula $b = \frac{5-2a}{3a+2}$	

## 7 Inverse Functions

## Fluency Practice

Question 13: Find  $f^{-1}(x)$  for each of the following:

(a)  $f(x) = 2x$       (b)  $f(x) = x - 6$       (c)  $f(x) = \frac{x}{3}$

(d)  $f(x) = 5x + 1$       (e)  $f(x) = \frac{2x}{7}$       (f)  $f(x) = \frac{x-2}{6}$

Question 14: Given  $h(x) = \frac{x}{4}$

(a) Find  $h^{-1}(x)$

(b) Calculate the value of  $h^{-1}(1.5)$

Question 15: Given  $f(x) = 2x - 3$



(a) Find  $f^{-1}(x)$

(b) Calculate the value of  $f^{-1}(7)$

Question 16: Given  $g(x) = \frac{3x+1}{2}$

(a) Find  $g^{-1}(x)$

(b) Calculate the value of  $g^{-1}(11)$

Question 17: Given  $f(x) = \frac{4x}{9} - 8$

(a) Find  $f^{-1}(x)$

(b) Calculate the value of  $f^{-1}(-10)$

# Fluency Practice

- Find the inverse function,  $f^{-1}(x)$ , of the following functions:
  - $f(x) = 3x - 1$
  - $f(x) = 2x + 3$
  - $f(x) = 1 - 2x$
  - $f(x) = x^2 + 5$
  - $f(x) = 6(4x - 1)$
  - $f(x) = 4 - x$
  - $f(x) = 3x^2 - 2$
  - $f(x) = 2(1 - x)$
  - $f(x) = \frac{2}{x+1}$
  - $f(x) = \frac{x+1}{x-2}$
- The function  $f$  is such that  $f(x) = 7x - 3$ 
  - Find  $f^{-1}(x)$ .
  - Solve the equation  $f^{-1}(x) = f(x)$ .
- The function  $f$  is such that  $f(x) = \frac{8}{x+2}$ 
  - Find  $f^{-1}(x)$ .
  - Solve the equation  $f^{-1}(x) = f(x)$ .
- The function  $f$  is such that  $f(x) = \frac{1}{x+4}$ ,  $x \neq -4$ .  
Evaluate  $f^{-1}(3)$ .  
[Hint: First find  $f^{-1}(x)$  and then substitute for  $x = -3$ ]
- $f(x) = \frac{x}{x+3}$ ,  $x \in \mathbb{R}$ ,  $x \neq -3$ 
  - If  $f^{-1}(x) = -5$ , find the value of  $x$ .
  - Show that  $ff^{-1}(x) = x$
- Functions  $f$  and  $g$  are such that  
 $f(x) = 3x + 2$                        $g(x) = x^2 + 1$   
Find an expression for  $(fg)^{-1}(x)$                       [Hint: First find  $fg(x)$  ]



## Fluency Practice

Find the inverse function,  $f^{-1}(x)$ , of the following functions:

(a)  $f(x) = 3x - 1$

(b)  $f(x) = 2x + 3$

(c)  $f(x) = 1 - 2x$

(d)  $f(x) = x^2 + 5$

(e)  $f(x) = 6(4x - 1)$

(f)  $f(x) = 4 - x$

(g)  $f(x) = 3x^2 - 2$

(h)  $f(x) = 2(1 - x)$

Find the inverse function,  $f^{-1}(x)$ , of the following functions:

(i)  $f(x) = \frac{2x}{x+1}$

(j)  $f(x) = \frac{x+1}{x-2}$

The function is such that  $f(x) = 7x - 3$

(a) Find  $f^{-1}(x)$

(b) Solve the equation  $f^{-1}(x) = f(x)$

The function is such that  $f(x) = \frac{8}{x+2}$

(a) Find  $f^{-1}(x)$

(b) Solve the equation  $f^{-1}(x) = f(x)$

The function is such that

$$f(x) = \frac{1}{x+4}, \quad x \neq -4$$

Evaluate  $f^{-1}(-3)$

## Fluency Practice

### Inverse Functions

1. Find  $f^{-1}(x)$  for the following functions,

a)  $f(x) = 7x$

b)  $f(x) = x - 12$

c)  $f(x) = 9 - x$

d)  $f(x) = 3x + 5$

e)  $f(x) = \frac{x+10}{4}$

f)  $f(x) = \frac{3x-4}{7}$

g)  $f(x) = \sqrt{x}$

h)  $f(x) = 4\sqrt{x} - 9$

i)  $f(x) = \frac{3}{x+5}$

j)  $f(x) = \frac{1}{2x-5}$

2. If  $f(x) = 3x + 2$

a) Find  $f^{-1}(x)$

b) Hence evaluate  $f^{-1}(5)$

c) Solve  $f(x) = f^{-1}(x)$

3. Find  $f^{-1}(x)$  for the following functions,

a)  $f(x) = \frac{x}{x-5}$

b)  $f(x) = \frac{x+4}{x}$

c)  $f(x) = \frac{x+5}{x-1}$

d)  $f(x) = \frac{3x+2}{x-6}$

e)  $f(x) = \frac{4-x}{2x+5}$

f)  $f(x) = \frac{5x}{2x-7}$

4. If,  $f(x) = \frac{2x}{x-5}$ ,

a) Find  $f^{-1}(x)$

b) Evaluate  $f^{-1}(1)$

c) Solve  $f^{-1}(x) = 10$

## Fluency Practice

1: Work out the following:

a) If  $h(x) = x - 9$ , find  $h^{-1}(4)$

b) If  $g(x) = \frac{-x}{3}$ , find  $g^{-1}(10)$

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2: Work out the following:

a) If  $f(x) = \frac{-x - 8}{8}$ , find  $f^{-1}(8)$

b) If  $h(x) = \sqrt{\left(\frac{-x}{7}\right)} + 3$ , find  $h^{-1}(10)$

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3: Work out the following:

a) If  $f(x) = \frac{5}{x - 8}$ , find  $f^{-1}\left(-1\frac{2}{3}\right)$

b) If  $g(x) = \frac{5x + 10}{6x + 3}$ , find  $g^{-1}\left(1\frac{1}{39}\right)$

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## Fluency Practice

4: Work out the following:

a) If  $h(x) = \frac{-2x + 9}{x - 1}$ , find  $h^{-1}(3)$

b) If  $g(x) = \frac{4}{x} + 3$ , find  $g^{-1}(1)$

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5: Find the inverse of each function:

a) If  $f(x) = x^2$ , find  $f^{-1}(x)$

b) If  $h(x) = \sqrt{x}$ , find  $h^{-1}(x)$

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## Fluency Practice

6: Find the inverse of each function:

a) If  $f(x) = \sqrt{\frac{-x-10}{10}}$ , find  $f^{-1}(x)$

b) If  $g(x) = -8(x-10)^2$ , find  $g^{-1}(x)$

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7: Find the inverse of each function:

a) If  $g(x) = \frac{9}{x+4}$ , find  $g^{-1}(x)$

b) If  $f(x) = \frac{-6x+7}{5x-4}$ , find  $f^{-1}(x)$

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